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Evidence from an Estimated DSGE Model

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The Macroeconomic Effects of Quantitative Easing in the Euro Area: Evidence from an Estimated DSGE Model

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14 March 2017

Abstract

This paper analyses the macroeconomic effects of the ECB's quantitative easing programme using an open-economy DSGE model estimated with Bayesian techniques. Using data on government debt stocks and yields across maturities we identify the parameter governing portfolio adjustment in the private sector. Shock decompositions suggest a positive contribution of ECB QE to EA year-on-year output growth and inflation of up to 0.4 and 0.5 pp in the standard linearized version of the model. Allowing for an occasionally binding zero-bound constraint by using piecewise linear solution techniques raises the positive impact up to 1.0 and 0.7 pp, respectively.

JEL classification: E44, E52, E53, F41

Keywords: Quantitative easing, portfolio rebalancing, Bayesian estimation, open-economy DSGE model, real GDP, inflation

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1. Introduction

In early 2015 the European Central Bank (ECB) has joined the group of central banks that have implemented large-scale asset purchase programmes as unconventional policy measures. These asset purchases, also called Quantitative Easing (QE), have led to a strong extension of the central banks' balance sheets. By end-July 2015 the amount of outright purchases on the balance sheet had reached 24% of GDP in the case of the US Federal Reserve, 64% of GDP in the case of the Bank of Japan, 21% of GDP in the case of the Bank of England, and 5% of GDP in the case of the ECB (Constâncio 2015). The ECB's QE programme announced in January 2015 (Public Sector Purchase Programme) foresaw buying €60 billion of assets a month from March 2015 to September 2016, which in sum corresponds to circa 10% of annualised euro area (EA) GDP. In December 2015, the ECB has extended the programme until March 2017, and it has raised the amount of monthly purchases to €80 billion starting from April 2016. In December 2016 the programme has been extended and modified again, lengthening the period of asset purchases until (at least) December 2017, but at a reduced pace of €60 billion of assets a month after March 2017.

Operating close to the zero lower bound (ZLB), the ECB considered its "conventional" monetary accommodation to be insufficient to address weak inflation dynamics, falling inflation expectations and sizeable economic slack in the EA. As a result, the balance sheet interventions were proposed to "achieve the price stability objective, given that interest rates have reached their lower bound" (Draghi 2015). In practice, the ECB purchases public sector financial assets (government debt) of longer maturity and extends liquidity (base money) to the private sector (Claeys et al. 2015, Valiante 2015). Research at the ECB has provided evidence for an impact of the QE programme on long-term bond yields and spillover to other asset prices through portfolio reallocation: Altavilla et al. (2015) in an event study report a 30-50 basis-point decline in 10-year government bond yields with spillover into lower corporate bond, spreads, higher equity prices, and euro depreciation. Andrade et al. (2016) report a decline of EA 10-year government bond yields in the range 27-64 basis points in the context of the ECB's extended asset purchases programme with spillover into higher equity prices and inflation expectations. The evidence in De Santis (2016) suggests that ECB non-standard policy has reduced EA 10-year government bond yields between September 2014 and October 2015 on average by 63 basis points.

This paper analyses the macroeconomic effects of the ECB's QE programme using a two-region DSGE model for the EA and the rest of the world (RoW), estimated with Bayesian techniques. QE is introduced into the model by adding a central bank balance sheet and distinguishing between short-term and long-term government debt. We use a formulation of private-sector portfolio preferences - as, e.g., Andres et al. (2004) - that allows for non-neutral effects of central

bank purchase programmes due to imperfect substitutability between assets of different maturity. More specifically, the central bank alters its balance sheet by purchasing long-term bonds (the latter modelled as in Woodford, 2001) and injecting liquidity to the private sector. Our specification of QE allows us to capture its effects through a large number of the transmission channels put forward by the literature (see Krishnamurthy and Vissing-Jorgensen, 2011) including the saving, financing cost, exchange rate, inflation, and fiscal channels. The exchange rate channel is absent from most model-based studies of QE policies (e.g., Chen et al. 2012, De Graeve and Theodoridis 2016, Gertler and Karadi 2011), which build instead on closed-economy frameworks.

The contribution of this paper is to analyse the macroeconomic impact of ECB QE using a state-of-the-art *estimated* dynamic stochastic general equilibrium (DSGE) model. Combining data on government debt stocks and yields across maturities, the model estimation enables to extract a value for the crucial parameter governing the portfolio adjustment costs of households. The implied strength of these costs dictates the magnitude of the yield spread following QE of a given volume and time path, such as the one announced by the ECB in January 2015. Given that ECB QE has been launched only in 2015, we have little data points for the QE episode itself. Data-driven identification of the degree of substitutability between short-term and long-term bonds in our model therefore has to rely mainly on the pre-QE part of the sample. Lags in the transmission of QE to the real economy furthermore imply that the effects of the ongoing programme have not fully materialised yet.

The analysis starts with a standard linearized version of the model in which the Taylor rule is never hitting the ZLB (we refer to this as "unconstrained model" in the text). We then turn to a model version with occasionally binding constraints in which the ZLB can become binding endogenously when contractionary shocks drive the target ("shadow") interest rate below the lower bound (we refer to this as "constrained model" in the text). According to our shock decompositions from the estimated unconstrained model, QE as captured by the model has increased EA year-on-year output growth and inflation in 2015q1-16q2 by 0.3 pp and 0.3 pp on average, with maximum impact of 0.4 and 0.5 pp in 2016. Including the endogenously and occasionally binding ZLB constraint raises the 2015q1-16q2 average growth and inflation effect of QE to 0.7 and 0.4 pp, respectively, with peaks at 1.0 and 0.7 pp in 2016q2. The stronger QE impact in the constrained model is due to the absence of a countervailing short-term policy rate response when the ZLB binds in 2015-16 in the constrained model.

The remainder of the paper is structured as follows: Section 2 summarizes the closely related literature; section 3 outlines the general structure of the model; section 4 describes the mod-

el solution and estimation methodology; section 5 presents parameter estimates; section 6 discusses the impact of QE in the unconstrained model; section 7 present results from the model with occasionally binding constraint; section 8 summarizes the paper and concludes.

2. Related literature

The empirical relevance of individual channels and the aggregate macroeconomic effects of QE are an empirical matter and likely to vary with structural features of the economy across countries and time. So far there is little (published) research on the effects of the ECB QE, and particularly little model-based analysis. Most existing papers consider unconventional monetary policy in the US and the UK and their spillovers to the world economy. This literature has been summarised, e.g., in Priftis and Vogel (2016). Here, we limit ourselves to the review of model-based general-equilibrium (DSGE) analyses that focus on the impact of QE on portfolio rebalancing and financing costs.

The central contribution of this paper is the incorporation of central bank balance sheet policy (QE) in a large-scale open-economy macroeconomic model and *estimation* of this model on euro area data. The approach is closest to Chen et al. (2012) and De Graeve and Theodoridis (2016) who analyse US QE in estimated closed-economy models of the US economy. Both Chen et al. (2012) and De Graeve and Theodoridis (2016) focus on the portfolio balancing channel of QE transmission.

According to the results in Chen et al. (2012), the US LSAP II (large-scale asset purchase) programme, with a volume of circa 4% of US GDP, combined with a commitment to keep interest rates low for an extended period of time has raised US real GDP growth by around 0.13% and inflation by only 0.03 percentage points (pp). According to the estimates of De Graeve and Theodoridis (2016), "Operation Twist", the purchase of long-term and sale of short-term maturity bonds of circa 2% of US GDP by the Federal Reserve, has increased US real GDP by 0.6% and inflation by up to 0.3 pp.

The models by Gertler and Karadi (2013) and Carlstrom et al. (2017) assume financially constrained financial intermediaries, where QE eases the constraint on financing productive investment in the economy. In these models, productive capital is financed by financial intermediaries based on their net worth and the deposits made by households. In a calibrated version of their model, Gertler and Karadi (2013) quantify the impact of US LSAP with a volume of 2.5% of GDP on output and inflation to 1% and 1.5 pp respectively if policy rates remain unchanged, and to 0.2% and 0.2 pp respectively if the standard monetary policy rule is active and partly offsets expansionary QE effects by an increase in the short-term rate. While the transmission can be

described in terms of the credit channel, recent empirical analyses (see, e.g., Bluwstein and Canova 2016, Tillman and Ludering 2016) do not suggest the credit channel to be a primary transmission mechanism for QE in the EA and spillover to non-EA countries.

Sahuc (2016) borrows the Gertler-Karadi model for an assessment of ECB QE policy that involves asset purchases of circa 9% of EA GDP. Like Gertler and Karadi (2013), the Sahuc (2016) assessment stresses the importance of keeping short-run policy rates low for longer. Keeping the policy rate constant only in 2015 gives a maximum QE effect on output growth and inflation of 0.2 and 0.1 pp in 2015-6, whereas keeping the policy rate unchanged for another year raises the average output growth and inflation effect in 2015-6 to 0.6 and 0.6 pp.

3. Model description

The present analysis uses a modified two-region (EA and ROW) framework of Kollmann et al. (2016) and extends this model to incorporate non-standard monetary policy. The model is estimated using quarterly data for the period 1999q1-2016q2.

The EA region assumes two (representative) households, intermediate and final goods firms and a government. Ricardian households have access to financial markets, whereas liquidity-constrained household consume their disposable income in every period. Preferences of both types of households exhibit habit formation in both consumption and leisure. A monopolistically-competitive sector produces differentiated goods by employing domestic labor and capital. Firms in this sector maximize the present value of their dividends at a discount factor that is strictly larger than the risk-free rate and varies over time, subject to investment and labour adjustment costs and a varying capacity utilization rate. Final goods firms combine a domestic differentiated goods bundle with energy inputs. Nominal differentiated goods prices are sticky as are the wages paid to the workers; the latter being determined by monopolistic trade unions. The fiscal authority imposes distortive taxes and issue debt.

The exposition below describes the QE-relevant extensions. A detailed overview of the general model can be found in Kollmann et al. (2016) and the model appendix.

We extend the model by introducing features of non-standard monetary policy as in Priftis and Vogel (2016), which is similar in spirit to the modelling of QE in Chen et al. (2012) and De Graeve and Theodoridis (2016). Similar to Chen et al. (2012) and De Graeve and Theodoridis (2016), our model emphasises the transmission of QE mainly through portfolio rebalancing. However, unlike these studies, our multi-country model allows for an exchange rate channel and potential trade effects from QE.

In line with the standard notion, QE is introduced as a monetary policy strategy that increases the size of the central bank's balance sheet. In particular, the central bank purchases long-term (government) bonds, with the aim of reducing the interest spread between long and short maturities, i.e. to flatten the yield curve. The central bank finances the bond purchases by providing additional liquidity to the private sector. QE intends to affect private-sector portfolio and saving decisions especially when short-term policy rates are already at or close to the zero lower bound (ZLB).

We introduce short-term and long-term government debt to incorporate investor preferences for a maturity mix and central bank balance sheet operations in the model. Following Woodford (2001) long-term government debt is modelled through bonds for which the nominal coupon *c*, which is a fraction of the principal, depreciates over time at rate δ_b.[1] The price in period t of a long-term bond issued in t (P_t^N) equals the discounted value of future payments $P_t^N = \sum_{n=0}^{T} \frac{\delta_b^n}{(1+i)^{1+n}} c$, where T is the maturity period of the bond. Analogously, the price in period t of a long-term bond issued in *t-1* (P_t^O) equals the discounted sum of outstanding payments, $P_t^O = \sum_{n=0}^{T-1} \frac{\delta_b^{1+n}}{(1+i)^{1+n}} c$. If $\delta_b/(1+i) < 1$ and T is large, the price in t of long-term bonds issued in *t-1* corresponds (approximately) to the price of newly issued long-term bonds times the depreciation rate:

(1) $$P_t^O = \delta_b P_t^N$$

Equation (1) shows that the price of the long-term bond that pays a declining coupon declines over time at the rate δ_b. Total outstanding government debt at face value consists of long-term bonds B_t^L, held by the private sector ($B_t^{L,H}$) and the central bank ($B_t^{L,CB}$), and short-term bonds , B_t^S :

(2) $$B_t = B_t^{L,H} + B_t^{L,CB} + B_t^S$$

The short-term and long-term bonds are imperfect substitutes in the model. In particular, private investors have a preference for holding a mix of short-term and long-term bonds, and deviations from the target value κ for the ratio of long-term over short-term debt induce quadratic adjustment costs (γ_b).[2]

[1] The Woodford (2001) perpetual-bond formulation is also be used by, e.g., Carlstrom et al. (2017).
[2] The same formulation has been used previously by, e.g., Andrés et al. (2004), Falagiarda (2013), Harrison (2012), and Liu et al. (2015).

Private households with access to financial markets (superscript r for Ricardian) face the following optimisation problem:[3]

$$
\max L^r = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t^r, 1-N_t^r)
$$

(3)
$$
-\mathrm{E}_0 \sum_{t=0}^{\infty} \lambda_t \beta^t \left(\begin{array}{l} \frac{(1+t_t^c)P_t^C}{P_t} C_t^r + \frac{P_t^C (K_t - (1-\delta_k)K_{t-1})}{P_t} + \frac{P_t^N B_t^{L,H}}{P_t} \left(1 + \frac{\gamma_b}{2} (\kappa \frac{B_t^S}{B_t^{L,H}} - 1)^2 \right) \\ + \frac{B_t^S}{(1+i_t)P_t} + \frac{e_t B_t^*}{(1+i_t^*)P_t} + \frac{\gamma_f}{2} \left(\frac{e_t (B_t^* - \overline{B}^*)}{P_t} \right)^2 - \frac{TR_t}{P_t} - \frac{cB_{t-1}^{L,H}}{P_t} - \frac{\delta_b P_t^N B_{t-1}^{L,H}}{P_t} \\ - \frac{B_{t-1}^S}{P_t} - \frac{e_t B_{t-1}^*}{P_t} - \frac{(1-t_t^w) W_t N_t^r}{P_t} - (i_{t-1}^k - (i_{t-1}^k - \delta_k) t_{t-1}^k - \varphi_{t-1}) \frac{P_t^C}{P_t} K_{t-1} - \frac{D_t}{P_t} \end{array} \right)
$$

Ricardian households receive labour income, returns on financial assets, income i_t^k from lending capital to firms net of an (exogenous) risk/insurance premium given revenue uncertainty φ_t, and dividends D_t from firm ownership. $K_t = I_t + (1-\delta_k)K_{t-1}$ is the capital stock as the sum of new investment I_t and the pre-period capital stock depreciated at rate δ_k. The government levies taxes t_t^w on income from labour, t_t^k on corporate income and t_t^c on consumption. The price in period t of a short-term (1-period) bond of nominal value B_t^S is $B_t^S/(1+i_t)$, with i_t being the short-term nominal interest rate. Analogously, $e_t B_t^*/(1+i_t^*)$ is the price in domestic currency of a foreign bond B_t^*, where e_t is the nominal exchange rate as the value in domestic currency of one unit of foreign currency.

The maximisation problem (3) provides the following first-order conditions (FOC):

(4)
$$
\frac{\partial L^r}{\partial B_t^S} \Rightarrow \beta E_t(\frac{\lambda_{t+1}}{\lambda_t}) = E_t(\frac{P_{t+1}}{P_t}) \left(\frac{1}{1+i_t} + \gamma_b \kappa P_t^N (\kappa \frac{B_t^S}{B_t^{L,H}} - 1) \right)
$$

(5)
$$
\frac{\partial L^r}{\partial B_t^{L,H}} \Rightarrow \beta E_t(\frac{\lambda_{t+1}}{\lambda_t}) = E_t(\frac{P_t^N}{\delta_b P_{t+1}^N + c} \frac{P_{t+1}}{P_t}) \left(1 + \frac{\gamma_b}{2} (\kappa \frac{B_t^S}{B_t^{L,H}} - 1)^2 - \gamma_b \kappa (\kappa \frac{B_t^S}{B_t^{L,H}} - 1) \frac{B_t^S}{B_t^{L,H}} \right)
$$

(6)
$$
\frac{\partial L^r}{\partial B_t^*} \Rightarrow \beta E_t(\frac{\lambda_{t+1}}{\lambda_t}) = E_t(\frac{e_t}{e_{t+1}} \frac{P_{t+1}}{P_t}) \left(\frac{1}{1+i_t^*} + \gamma_f \frac{e_t (B_t^* - \overline{B}^*)}{P_t} \right)
$$

(7)
$$
\frac{\partial L^r}{\partial K_t} \Rightarrow \beta E_t(\frac{\lambda_{t+1}}{\lambda_t}) = E_t(\frac{P_{t+1}}{P_t} \frac{P_t^C}{P_{t+1}^C}) \frac{1}{(1+i_t^k - \varphi_t - \delta_k) - t_t^k (i_t^k - \delta_k)}
$$

[3] The description of the budget constraint here omits adjustment costs in the real sector of the economy (price, wage, capital stock, and labour adjustment costs) that do not affect the first-order conditions for portfolio holdings and savings. These adjustment costs (which generate, e.g., nominal price and wage stickiness) are present in the full version of the model that underlies the simulations. Details on the specification of the real-sector adjustment frictions can be found, e.g., in Priftis and Vogel (2016) and in the appendix.

(8) $$\frac{\partial L^r}{\partial C_t^r} \Rightarrow U_t^C = \frac{(1+t_t^c)P_t^c}{P_t}\lambda_t$$

(9) $$\frac{\partial L^r}{\partial N_t^r} \Rightarrow U_t^N = \frac{(1-t_t^w)W_t}{P_t}\lambda_t$$

Combining (4) with (6), (7) and (8) illustrates the transmission channels of QE to the real economy:

(10) $$\frac{1}{1+i_t} + \gamma_b \kappa P_t^N (\kappa \frac{B_t^S}{B_t^{L,H}} - 1) = E_t(\frac{e_t}{e_{t+1}})\left(\frac{1}{1+i_t^*} + \gamma_f \frac{e_t(B_t^* - \bar{B}^*)}{P_t}\right)$$

(11) $$\frac{1}{1+i_t} + \gamma_b \kappa P_t^N (\kappa \frac{B_t^S}{B_t^{L,H}} - 1) = E_t(\frac{P_t^C}{P_{t+1}^C})\frac{1}{(1+i_t^k - \varphi_t - \delta_k) - t_t^k(i_t^k - \delta_k)}$$

(12) $$\frac{1}{1+i_t} + \gamma_b \kappa P_t^N (\kappa \frac{B_t^S}{B_t^{L,H}} - 1) = \beta \frac{(1+t_t^c)P_t^c U_{t+1}^C}{(1+t_{t+1}^c)P_{t+1}^c U_t^C}$$

The impact on asset prices of the central bank's purchase of long-term bonds derives from the private investors' portfolio adjustment costs ($\gamma_b > 0$), i.e. imperfect substitutability between different financial assets. If $\gamma_b > 0$, the effects of reducing $B_t^{L,H}$ relative to B_t^S in the household portfolio are similar to the impact of a reduction of the short-term interest rate i_t in equation (4), and unconventional monetary policy can, hence, mimic the effect of reductions in the short-term interest rate.

In particular, when the central bank intervenes by purchasing long-term bonds, private investors that aim at re-establishing the portfolio mix of short-term and long-term assets can respond by holding more corporate equity and foreign bonds, and by reducing savings. The first response means a portfolio reallocation towards equity and foreign-currency assets that increases the price of corporate equity (rising stock market) and the price of foreign currency (exchange rate devaluation). Equation (10) shows that QE leads to higher demand for foreign assets and depreciation of the domestic currency (increase in e_t) for given levels of i_t (restricted, e.g., at the ZLB) and i_t^*. Equation (11) illustrates the portfolio reallocation from government bonds towards corporate equity. Equation (12) shows that QE reduces private saving similar to a reduction in the short-term interest rate.

Concerning the transmission to the real economy, i) rising stock markets reduce the financing costs of corporations and, dampening the required return to capital, translate -under decreasing returns to capital- into stronger investment and capital accumulation, ii) exchange rate depreciation strengthens net exports provided that export and import demand are sufficiently

price elastic, and iii) reduced savings to restore the preferred portfolio mix strengthen contemporaneous consumption demand.

The stock of government debt, which is, composed of short-term bonds and long-term bonds follows:

$$(13)\qquad \frac{B_t^S}{(1+i_t)P_t}+\frac{P_t^N}{P_t}B_t^L=\frac{B_{t-1}^S}{P_t}+\frac{(\delta_b P_t^N+c)B_{t-1}^L}{P_t}+\frac{PGE_t}{P_t}-\frac{TAX_t}{P_t}-\frac{PR_t^{CB}}{P_t}$$

where PGE_t, TAX_t and PR_t^{CB} are, respectively, the primary government expenditure (public consumption, public investment, transfers), total tax revenue (labour, consumption, and corporate taxes), and the operating profit of the central bank as additional source of government revenue.

The operating profit of the central bank equals the sum of base money issuance and interest income minus the current expenditure on buying long-term bonds, where the latter equals the change of the value of long-term bonds on the central bank's balance sheet:

$$(14)\qquad PR_t^{CB}=\Delta M_t+cB_{t-1}^{L,CB}-(P_t^N B_t^{L,CB}-\delta_b P_t^N B_{t-1}^{L,CB})$$

Under the central bank's budget constraint (14), purchases of long-term government bonds can be financed either by increasing liquidity (money issuance), or by reducing the central bank's operating profit.[4] In line with the standard definition of QE and the ECB announcement, we consider only the case of enhanced liquidity provision.

Purchases of long-term bonds by the central bank can be modelled as endogenous response to the economic environment (e.g., the economy's position in the business cycle, or the slope of the yield curve) similar to a Taylor rule, or as exogenous path. The set-up in this paper models QE as exogenous path that replicates the announced ECB programme in timing and size.

4. Model solution and econometric approach

We compute an approximate model solution by linearizing the model around its deterministic steady state. Following the recent literature that estimates DSGE models, we calibrate a subset of parameters to match long-run data properties, and we estimate the remaining parameters using Bayesian methods. The observables employed in estimation are listed in the Data Appendix.[5] The estimation uses quarterly data for the period 1999q1-2016q2. We also perform estimation on the subsample 1999q1-2014q4 to test the stability of parameter estimates, especially the adjustment

---

[4] A third option, in general, is for the central bank to sell other assets from its portfolio to sterilise the impact of its intervention on the central bank balance sheet.

[5] The observables are not demeaned or de-trended prior to estimation. The model is estimated on first differences of real GDP, real demand components and price indices, and on nominal ratios of aggregate demand components to GDP.

cost/portfolio preference parameter γ_b, with respect to the implementation of QE. The model has been estimated using the slice sampler algorithm proposed by Neal (2003).[6]

We calibrate the model steady state so that steady-state ratios of main economic aggregates to GDP match average historical ratios over the sample period. The EA steady state ratios of private consumption and investment to GDP are set to 56% and 19%, respectively. The steady state shares of EA GDP in world GDP is 17%. The steady state trade share (0.5*(exports+imports)/GDP) is set at 18% in the EA (excluding intra-EA trade), and the quarterly depreciation rate of capital is 1.4%. We set the steady state government debt/annual GDP ratio at 80% of GDP in the EA. The steady state real GDP growth and inflation rates are set to 0.35% and 0.4% per quarter, respectively, and the effective rate of time preferences to 0.25% per quarter.

With respect to the model extension to imperfect asset substitutability and non-standard monetary policy, we set the steady-state portfolio share of long-term to short-term government debt to 0.916 in line with the average of outstanding EA government debt over the sample period. We use data on swap rates of EA government bonds to determine the yield spread between short-term and long-term bonds. In particular, we use the current and 3-month-ahead swap rates on 10-year bonds to calculate the implied expected period-on-period return on long-term bonds. This approach is consistent with our modelling assumption that agents are not obliged to hold long-term bonds to maturity, but rather can trade these bonds in the secondary market at each period in time.

The results displayed in the following sections treat ECB QE as an AR(1) shock for which the estimated persistence is very high, as in Chen et al. (2012) and De Graeve and Theodoridis (2016). The specification as AR(1) implies that bond purchases by the central bank are not anticipated by the private sector (e.g. the agents react only to action, even though further steps have already been announced by the ECB), which mutes the impact of announcement effects with respect to further bond purchases. The high persistence – innovations have a half-life of 12 years – also implies that agents expect only very gradual exit from QE, however. Alternatively, we have tested a QE shock with random walk assumption. The random walk assumption implies that private investors expect the balance sheet expansion to be permanent, i.e. no exit from QE in the sense of a return of the central bank balance sheet to pre-QE levels. Results for the highly persistent AR(1) and the random walk shock specification are very similar.[7]

[6] See also Planas et al. (2015) for a detailed description on the theory and practice of slice sampling.

[7] We have also tested an AR(2) specification of the QE shock. The AR coefficients are both close to 0.5 in this case and estimation results very similar to the AR(1) case.

5. Posterior parameter estimates[8]

The posterior estimates of key model parameters for the EA are reported in Table 1. These estimates are based on the unconstrained linearized version of the model and also used for the solution with occasionally binding constraint.

Table 1: Prior and posterior distributions of key estimated EA model parameters.

Description	Prior Distribution		Posterior Distribution
	Dist.	Mean (Std.)	Mode (Std.)
Preferences			
Consumption habit persistence	B	0.5 (0.20)	0.89 (0.02)
Risk aversion	G	1.5 (0.20)	1.42 (0.15)
Inverse Frisch elasticity of labor supply	G	2.5 (0.50)	2.29 (0.43)
Import price elasticity	G	2 (1)	1.98 (0.34)
Steady state consumption share of Ric. HH	B	0.65 (0.10)	0.85 (0.02)
Nominal and real frictions			
Portfolio adjustment costs	G	0.0015 (0.0006)	0.0009 (0.0003)
Price adjustment cost	G	60 (40)	26.6 (7.76)
Nominal wage adj. cost	G	5 (2)	5.86 (1.37)
Real wage rigidity	B	0.5 (0.20)	0.97 (0.01)
Monetary Policy			
Interest rate persistence	B	0.7 (0.12)	0.78 (0.03)
Response to inflation	B	2 (0.4)	1.68 (0.29)
Response to GDP	B	0.5 (0.2)	0.06 (0.03)
Response to cumulative deflation gap	G	0.005 (0.002)	0.006 (0.002)
Autocorrelations of shocks			
QE (purchases of long-term bonds)	B	0.5 (0.2)	0.987 (0.01)
Bond risk premium	B	0.5 (0.20)	0.77 (0.09)
Domestic price mark-up	B	0.5 (0.20)	0.19 (0.09)
Standard deviations (%) of innovations			
Monetary Policy	B	1 (0.40)	0.10 (0.00)
QE (purchases of long-term bonds)	G	1 (0.40)	2.1 (0.15)
Investment risk premium	G	0.1 (0.40)	0.19 (0.02)
Bond risk premium	G	1 (0.40)	0.54 (0.15)
Domestic price mark-up	G	2 (0.80)	3.05 (0.99)

Notes: Cols. (1) lists model parameters and shocks. Cols. (2)-(3) indicates the prior distribution function (B: Beta distribution; G: Gamma distribution). Cols. (4)-(5) show the mode and the standard deviation (Std) of the posterior distributions of EA parameters.

[8] The presentation of the results focuses on the role of QE and the related parameter estimates, impulse responses and historical decompositions. Broader discussion of results can be found in Kollmann et al. (2016) and the related not-for-publication appendix, including predicted business cycle statistics (standard deviations and cross-correlations of key macro variables); these statistics are broadly consistent with empirical statistics.

The steady state consumption share of the Ricardian household is estimated at 0.85. Estimated habit persistence in consumption is high (0.89), which indicates a sluggish adjustment of consumption to income shocks. The risk aversion coefficient is in the range of 1.4 and the inverse of the elasticity of labor supply elasticity is estimated to be 2.3. The estimated price elasticity of aggregate imports is 2.0. The model estimates also suggest substantial nominal price and wage stickiness, and strong real wage rigidity. The estimated interest rate rule indicates a strong response of the EA policy rate to domestic inflation, and a weak response to domestic output. Important in our context, the posterior estimate of the adjustment cost parameter attached to the maturity structure of the private sector portfolio of government debt is 0.0009.[9] The fiscal feedback rules for government transfers (not shown in Table 1) exhibit very weak responses to public debt and deficit levels. The estimates also suggest that most exogenous variables are highly serially correlated. The model properties discussed in what follows are evaluated at the posterior mode of the model parameters.

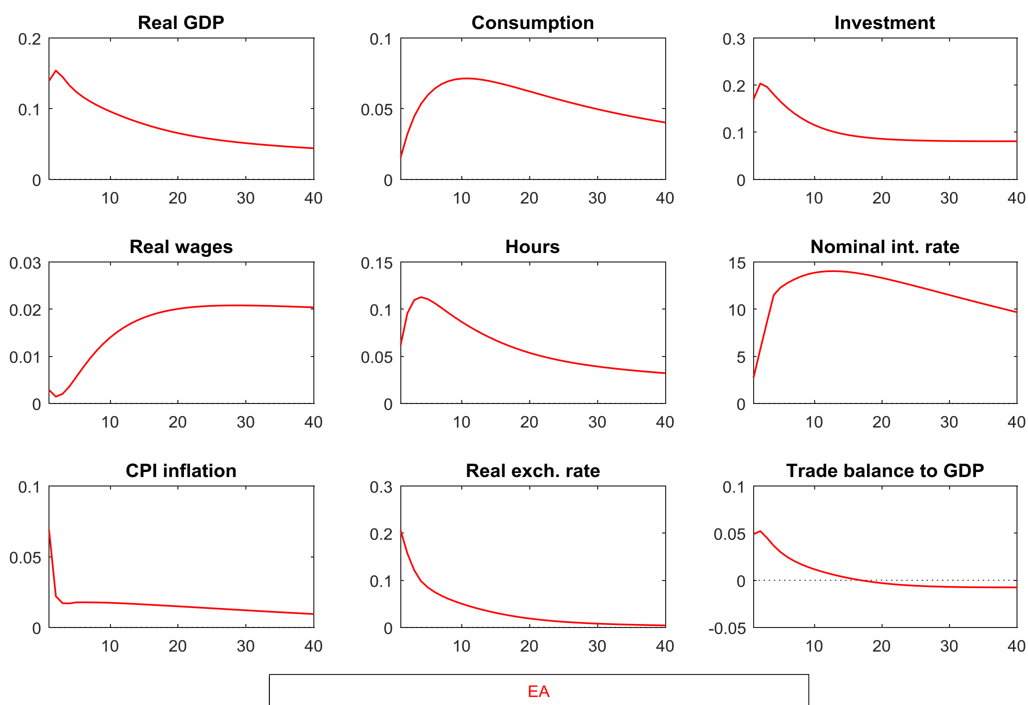
6. QE in the unconstrained model

This section provides results on the impact of QE for the unconstrained linearized. The discussion focuses on impulse responses and shock decompositions for real GDP growth and inflation. Using the unconstrained linearized model implies that the standard monetary policy (Taylor) rule is operational at any period of time. An operational Taylor rule offsets part of effective QE as it reacts to rising output growth and inflation by tightening interest rates.

6.1. Dynamic effects of QE shocks

Figure 1 provides impulse responses of EA endogenous variables for a positive QE shock that illustrate the transmission of QE in the model. More precisely, the shock is a purchase of long-term government bonds by the central bank (CB) that is financed by additional liquidity. The shock is a one-time increase in CB holdings of long-term bonds and calibrated to match the quarterly amount of ECB purchases of around 200 billion euro, which corresponds to circa 8% of baseline quarterly GDP. Given the specification of QE as highly persistent AR(1), the one-time purchase implies a long-lasting extension of the CB's balance sheet by the given amount; only half of the initial balance sheet extension will be undone after 12 years.

[9] We have tested the robustness of the parameter estimate by starting from different (lower and higher) prior values. The estimation has converged to the same value in both cases.



Note: Time intervals on the x-axis are quarters; units on the y-axis are %, except for inflation and the trade balance (both pp) and the short-term nominal interest rate (bp).

Figure 1: Impulse responses for positive EA QE shock (long-term bond purchase of 8% of quarterly EA GDP)

The purchase of long-term government debt by the CB reduces the amount of long-term debt available to private investors. Given the preference of private investors for a maturity mix, i.e. imperfect substitutability between short-term and long-term bonds, the price of long-term bonds rises; their expected yield, consequently, declines. The decline in the yield of long-term bonds leads to portfolio rebalancing towards equity and foreign assets. Higher demand for equity lowers the equity premium and causes an increase in private investment as shown in Figure 1. Higher demand for foreign assets causes depreciation of the domestic currency (an increase in the real exchange rate in Figure 1 corresponds to real effective depreciation), which leads to an improvement in the EA trade balance. The declining yield on long-term bonds also reduces private savings as private investors are less inclined to invest into short-term bonds away from the preferred maturity mix in the portfolio; the decline in savings raises private consumption. The joint increase in consumption, investment and net exports implies an increase in real GDP and (demand-driven) inflation. Note that the assumption of a very persistent expansion of the CB's balance sheet implies a very persistent decline in the equity premium and the savings rate, which is behind the very persistent increase in investment and consumption. Higher investment in productive capital raises the capital stock and the productivity of workers, so that labour demand and employment increase and real wages rise to some extent. Note that the efficiency of QE in Figure 1

is dampened by an offsetting response of the short-term policy rate. Given the positive impact of QE on activity and inflation, standard monetary policy as captured by the Taylor rule becomes less expansionary than in the non-QE baseline. Section 7 below will present corresponding impulse responses with binding zero lower bound (ZLB) constraint, i.e. without tightening of short-term policy rates.

6.2. Decomposing EA output growth and inflation

To quantify the role of different shocks as drivers of output and inflation we plot the estimated contribution of the different shocks to year-on-year real GDP growth (Figure 2) and to the year-on-year growth of the GDP deflator (Figure 3). To focus the discussion on the contribution of QE we group the remaining shocks coarsely in three groups: domestic demand shocks (which include financial shocks affecting consumption, investment and net export demand, as well as fiscal shocks); domestic supply shocks (productivity and price and wage mark-up shocks); and trade and foreign shocks (containing shocks to trade demand and mark-ups and to foreign demand and supply, including oil price shocks). The black solid line presents the data, the blue surface the contribution of the respective group of shocks.

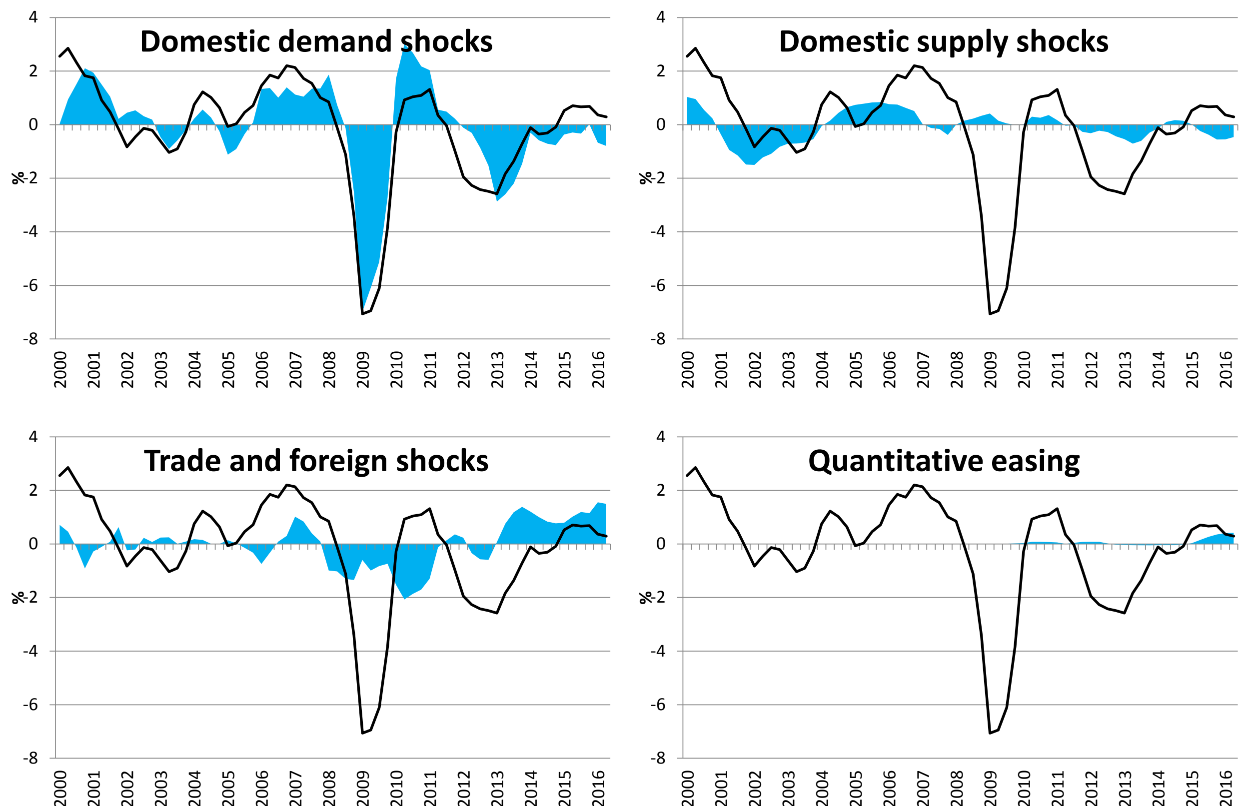


Figure 2: Decomposition of year-on-year growth of EA real GDP

The decomposition in Figure 2 mirrors the finding by Kollmann et al. (2016) that domestic demand shocks (in particular those driving investment demand) account for a large share of the

fluctuation in GDP growth during 1999-2016 and, in particular, the EA double-dip recession. The role of domestic supply shocks appears comparatively weak. Trade and foreign shocks have contributed to the 2009 recession (low external demand and decline in trade), but have supported GDP growth in more recent years (global recovery). Finally, and most interestingly in our context, the decomposition points to a positive impact of EA QE as captured in our model, i.e. taking into account changes in the ECB balance sheet up to 2016Q2, to EA growth. The impact of QE is increasing gradually given the gradual path of asset purchases and our assumption that future CB asset purchases are not anticipated by private investors. Even under this limiting assumption, Figure 2 suggests a contribution of EA QE to EA year-on-year real GDP growth of 0.3 pp on average in 2015q1-16q2, with the maximum of 0.4 pp in 2016q1-2.[10]

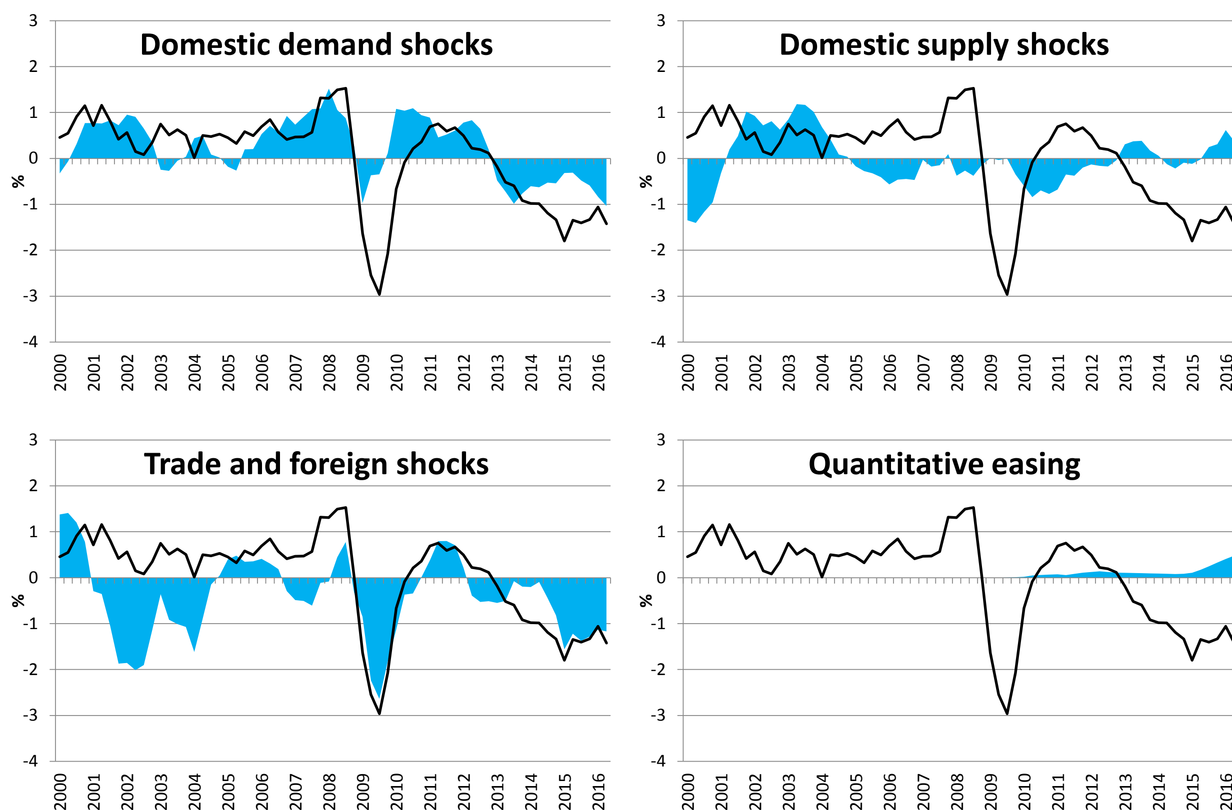


Figure 3: Decomposition of year-on-year EA CPI inflation

The decomposition of CPI inflation in Figure 3 suggests a more balanced role for domestic demand and supply shocks compared to Figure 2. The pre-crisis demand boom has added to inflation pressure, whereas the following contraction of domestic demand has pushed inflation down. Domestic supply shocks, notably negative productivity trends and sluggish wage and price ad-

[10] Note that the panels for QE in Figures 2 and 3 show smaller positive growth and inflation contributions already before 2015, which are due to (smaller) variations in the CB holding of long-term debt prior to 2015 that are captured by the same shock but not part of the programme initiated in 2015.

justment, have upheld inflation more recently according to the decomposition. Trade and foreign shocks have negatively contributed to inflation through lower export demand during the global recession and falling import prices (including oil) more recently. Finally, Figure 3 points to a sizable contribution of QE to year-on-year EA CPI inflation, with 0.3 pp on average in 2015q1-16q2 and a peak at 0.5 pp in 2016Q2. The sizable positive contribution of QE to real GDP growth (up to 0.4 pp) and inflation (up to 0.5 pp) in the EA occurs despite the fact that effective QE triggers a tightening of standard monetary policy in the unconstrained linearized ("normal times") model. QE effects are stronger when accounting for the ZLB constraint as will be discuss in the following section.

7. QE in the model with occasionally binding constraint

In this section we assess the contribution of QE when we allow the zero-bound on monetary policy to be occasionally binding. A binding ZLB implies that the target ("shadow") policy rate is below the lower bound. By implication, an increase in output and inflation through QE or other factors does not lead to tightening of the short-term rate while the constraint is binding, i.e. while the shadow rate remains below the lower bound.

7.1. Implementation of the occasionally binding constraint

We use the OccBin method developed by Guerrieri and Iacoviello (2015) to treat the occasionally binding constraint via a piecewise linear solution. Moreover, we use an algorithm as in Giovannini and Ratto (2017) to obtain smoothed estimates of latent variables as well as the sequence of regimes along the historical sample.[11] We set the lower bound for quarterly short-term interest rates at 0.0.[12] The unconstrained nominal interest rate i_{kt}^{NC} follows the usual Taylor rule without monetary shock:

$$i_{kt}^{NC} - \bar{\imath} = \rho^{i}(i_{kt-1} - i) + (1 - \rho^{i})\left(\eta^{i\pi}\left(0.25\left(\sum_{r=0}^{3}\pi_{kt-r}^{c+g}\right) - \bar{\pi}^{c+g}\right) + \eta^{iy}(\tilde{y}_{kt}) + \eta^{ip}(c\pi_{kt}^{c+g})\right)$$

As long as the actual policy rate is above the lower bound, the nominal interest rate is:

$$i_{kt} = i_{kt}^{NC} + u_{kt}^{inom}$$

If $i_{kt}^{NC} \leq i^{LB}$ the policy rate is constrained:

$$i_{kt} = i^{LB} + u_{kt}^{inom}$$

[11] See Giovannini and Ratto (2017) for a detailed description of the algorithm and the methodology.

[12] Previous studies on the ZLB use a slightly higher lower bound for the interest rate (around 0.2-0.4% annualized). In our context, this assumption would imply a constrained regime starting from 2013q4 onwards, which would imply a stronger impact of the asset purchase shock on real activity particularly in the years 2013 and 2014, where variations in the CB holding of long-term debt are unrelated to the 2015 QE programme.

The variable i_{kt}^{NC} acts as an "shadow" interest rate under a constrained regime. Under the Occbin algorithm, it allows to be determined endogenously when the constraint is no longer binding.[13]

The algorithm used for estimating latent variables yields initial conditions and a sequence of smoothed shocks that are consistent with the observables and take into account the occasionally binding constraint. The sequence of regimes is reported in Table 2.

Table 2: Estimation of the historical sequence of occasionally binding regimes from 2013q1 – 2016q2.

Time	Regime sequence[1]	Starting period of regime[2]
2013q1	0	1
2013q2	0	1
2013q3	0	1
2013q4	0	1
2014q1	0	1
2014q2	0 1 0	1 2 4
2014q3	1 0	1 6
2014q4	1 0	1 7
2015q1	1 0	1 9
2015q2	1 0	1 8
2015q3	1 0	1 8
2015q4	1 0	1 7
2016q1	1 0	1 6
2016q2	1 0	1 6

Notes: (1 column) 0 = unconstrained, 1 = constrained. [1 0] indicates a constrained regime. [0 1 0] indicates a regime that anticipates future constraints. (2 column) Periods for which the regime starts: [1 6] indicates a constrained regime for 5 periods. [1 2 4] indicates a regime that anticipates future constraints starting in period 2 until period 4.

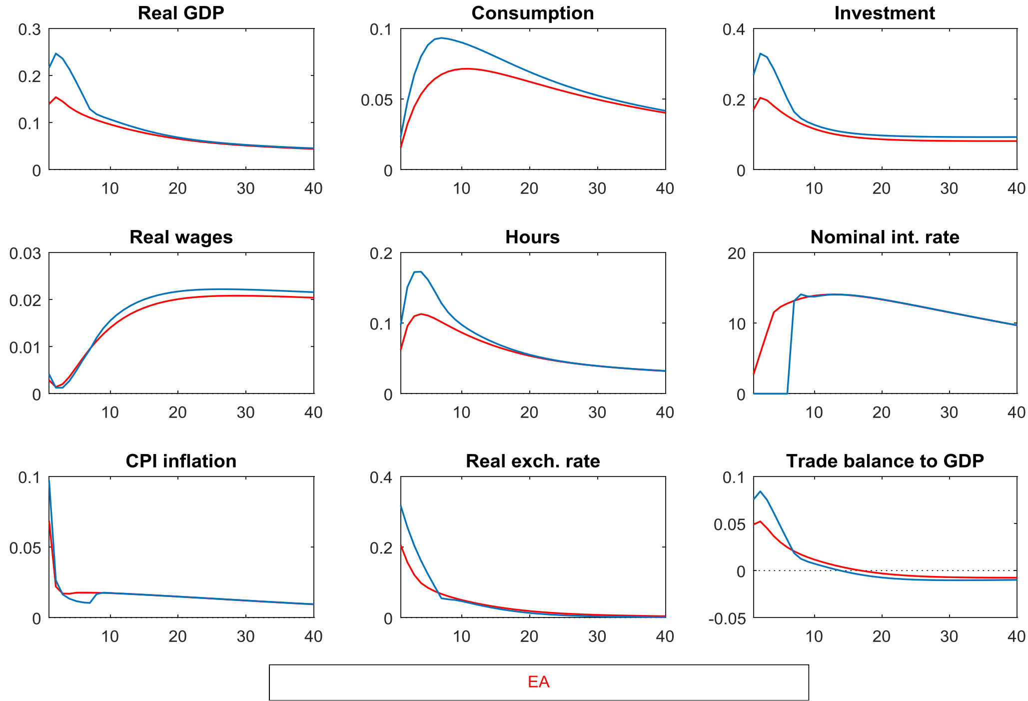
Based on our definition of the ZLB as zero short-term nominal interest rate, EA monetary policy is constrained from 2014q3 onwards, whereby agents have anticipated the ZLB since 2014q2. Our simulation also indicates a relatively long-lasting constrained regime. More precisely, in 2016q2 it anticipates a constrained regime for additional 5 quarters.

7.2. Dynamic effects of QE shocks

Based on the sequence of regimes, we perform IRFs with ZLB that are consistent with the estimated timing and duration of the constrained regime. In particular, we perform the following exercise: We use as a starting point the smoothed variables in 2015q1, which is a period of constrained monetary policy according to Table 2 and the official start of QE. We shut off all QE shocks and simulate the model with all other shocks. Then we perform another simulation adding a positive ECB QE shock of the same size as in Figure 1, i.e. long-term bond purchases of 8% of

[13] We still use an exogenous monetary shock under the constrained regime in order to keep observing the actual policy rate in the data. The shock does not affect the behavior of the piecewise linear solution in terms of transmission mechanisms under the ZLB constraint.

steady-state quarterly EA GDP. The difference between the two simulations provides the IRF of the ECB QE shock under the constrained regime. Figure 4 shows the comparison of the IRFs between the linear (unconstrained) and piecewise linear (constrained) solution, where the former is represented by red and the latter by blue solid lines.



Note: Red is the linear (unconstrained) model, blue the piecewise linear model (ZLB). The linear solution is equivalent to the IRFs in Figure 1.

Figure 4: IRF for positive EA QE shock under unconstrained and constrained monetary policy

Since we use 2015q1 as starting point of our simulations, monetary policy is constrained for 6 quarters until the end of our sample in 2016q2. It implies that monetary policy is going back to 'normal' times from 2016q3 onwards, which is reflected by an immediate jump to the unconstrained impulse response. The change back to the "normal regime" is the consequence of softening of contractionary forces, i.e. abating of contractionary shocks and of the effectiveness of QE itself, not the result of an exogenous regime switch. The IRFs suggest that the impact of the QE shock on real GDP (annualized 0.9 instead of 0.5 pp on impact) and inflation (annualized 0.4 instead of 0.3 pp on impact for CPI inflation) becomes significantly larger in a zero-bound environment.

7.3. Decomposing EA output growth and inflation

Finally, we investigate the shock contributions to the observed data that are consistent with the piecewise linear solution, i.e. the extension of the standard historical shock decompositions to the

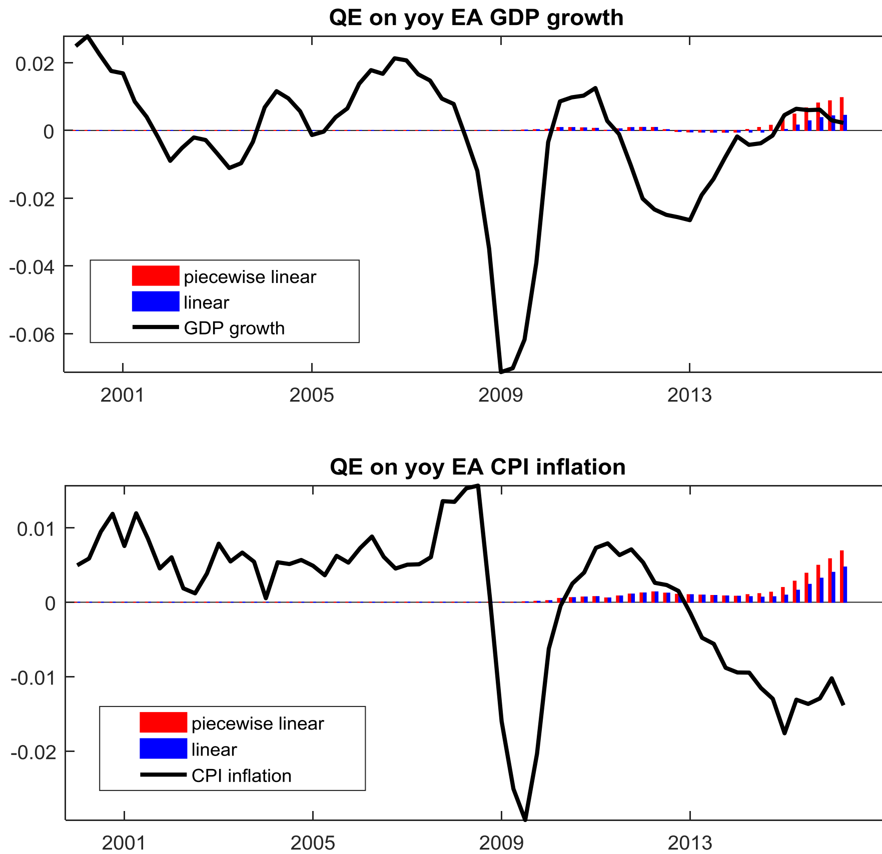
case of occasionally binding regimes. The contribution of individual smoothed shocks is not the mere additive superposition of each shock in this case. Instead, it is a *non-linear function* of the whole set of shocks *simultaneously* affecting the economy. Hence, the contribution of one shock it is *conditional* on the sequence and combination of shocks simultaneously hitting the economy.[14]

We use the *complement/residual contribution* to account for the impact of the ZLB on the contribution of shocks to observed variables. In practice, we compute the contribution of QE by setting the QE shock to zero and perform simulations using initial conditions and the sequence of all the other shocks. The contribution of QE will be the complement/residual of this simulation to the smoothed variable.[15]

Figure 5 compares the contribution of QE on year-on-year real GDP growth and CPI inflation in the EA under the linear (unconstrained) solution and the piecewise linear (ZLB constraint) solution, respectively. Accounting for the temporarily binding ZLB strengthens the *positive contribution* of QE to real GDP growth and inflation in 2015-16. The average impact of QE on year-on-year real GDP growth in 2015q1-16q2 more than doubles from 0.3 to 0.7 pp, and the maximum impact increases from 0.4 to 1.0 pp in 2016q2, which suggests that QE is much more effective under the constrained monetary policy regime. The impact on year-on-year CPI inflation rises from 0.3 to 0.4 pp for the 2015q1-16q2 average, and the maximum impact increases from 0.5 to 0.7 pp in 2016q2. The more positive contribution under the ZLB owes particularly to the absence in this context of the countervailing monetary policy tightening that would occur in "normal" times.

[14] In general, we can consider two definitions that generalize the concept of shock contributions in the non-linear case, which degenerate to the standard shock decomposition for the linear case: The *conditional contribution* and the *complement contribution*. See Giovannini and Ratto (2017) for a detailed description of both methods.

[15] Note that each of these simulations provides a different sequence of regimes, which in general will be different from the historical one. The *complement/residual contribution* triggers key non-linear features associated to the interaction between shock realization and the occasionally binding constraints.



Note: Real GDP growth and CPI inflation are both shown as deviations from steady state, which is calibrated as the mean over the sample period from 1999q1-2016q2. In both sub-plots 0.01 on the y-axis corresponds to 1 pp.

Figure 5: QE contribution to year-on-year EA real GDP growth and CPI inflation in linear and piecewise linear solution

The effects of QE on EA GDP and inflation in our estimated model are comparable to existing literature for EA QE and comparable in order of magnitude to results from similar exercises for the US QE: Sahuc (2016) find effects of ECB QE (9% of EA GDP) on EA real GDP growth (inflation) of 0.2 (0.1 pp) in 2015-6 for short-term rates constant in 2015, whereas keeping the policy rate unchanged for another year raises the average growth (inflation) effect in 2015-6 to 0.6 pp (0.6 pp). For the US, Chen et al. (2016) report GDP growth (inflation) effects of 0.1 pp (0.3 pp) of US QE measures of a volume of 4 pp of GDP. De Graeve and Theodoridis (2016) report GDP growth (inflation) effects of 0.6 pp (0.3 pp) of the Federal Reserve's "Operation Twist" (2%-of-GDP). Both studies work with scenarios in which short-term policy rates do not respond (immediately) to higher growth and inflation. Gertler and Karadi (2013) quantify the impact of US LSAP with a volume of 2.5% of GDP on output growth (inflation) to 1 pp (1.5 pp) if policy rates remain unchanged, and 0.2 pp (0.2 pp) if the standard monetary policy rule is active and partly offsets expansionary QE effects.

8. Conclusion

We have introduced imperfect substitutability between bonds of different maturities and central bank balance sheet operations in a New Keynesian open-economy DSGE model. We have estimated a two-region (EA and rest of the world) version of the model to assess the impact of the ECB's large-scale asset purchase programme (QE) on economic activity and inflation in the EA. The detailed modelling of QE and portfolio adjustment enables us to capture a large number of the transmission channels put forward in the literature, including the saving, financing cost, exchange rate, inflation, and fiscal channels. We use data on government debt stocks and yields across maturities to identify the parameter that governs portfolio adjustment in the private sector and yield, exchange rate and savings effects of central bank asset purchases. The shock decompositions for year-on-year real GDP growth and CPI inflation in the EA suggest a positive average contribution of ECB QE to EA output growth and inflation of 0.3 and 0.3 pp during 2015q1-16q2 in the standard unconstrained ("normal times") model and 0.7 and 0.4 pp respectively in the model that accounts for the temporarily binding zero-bound constraint.

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APPENDIX: MODEL DESCRIPTION

We introduce elements of quantitative easing into a two-region world consisting of the Euro Area (EA) and the rest of the world (RoW). The EA region is rather detailed, while the RoW block, which also includes US, is more stylized.[16]

The EA region assumes two (representative) households, a number of layers of firms and a government. EA households provide labor services to firms. One of the two households (savers, or 'Ricardians') in each country has access to financial markets, and she owns her country's firms. The other (liquidity-constrained, or 'non-Ricardian') household has no access to financial markets, does not own financial or physical capital, and in each period only consumes the disposable wage and transfer income. The preferences of both types of household exhibit habit formation in both consumption and leisure, a feature which allows for better capturing persistence of the data.

There is a monopolistically-competitive sector producing differentiated goods, using domestic labor and capital. The firms in the sector maximize the present value of dividends at a discount factor that is strictly larger than the risk-free rate and varies over time. This is a short-cut for capturing financial frictions facing firms; it can, e.g., be interpreted as a 'principal agent friction' between the owner and the management of the firm. Optimization is subject to investment and labor adjustment costs and a varying capacity utilization rate, which lets the model better capture the dynamics of the current account and other macro variables.

Total output is produced by combining the domestic differentiated goods bundle with energy input. EA wages are set by monopolistic trade unions. Nominal differentiated goods prices are sticky as are the wages paid to the workers. Fiscal authorities in the EA impose distortive taxes and issue debt.

The RoW block is simplified compared to the EA block. Specifically, the RoW consists of a budget constraint for the representative household, demand functions for domestic and imported goods (derived from CES consumption good aggregators), a production technology that uses la-

[16]The EA block builds on, but is considerably different than the QUEST model of the EU economy (Ratto et al., 2009).

bor as the sole factor input, and a New Keynesian Phillips curve. The RoW block abstracts from capital accumulation.

The behavioral relationships and technology are subject to autocorrelated shocks denoted by ε_t^x , where *x* stands for the type of shock. ε_t^x will generally follow an AR(1) process with autocorrelation coefficient $\rho^x < 1$ and innovation u_t^x:

$$(\varepsilon_t^x) = \rho^x(\varepsilon_t^x) + u_t^x$$

There is also a separate category of shocks, denoted A_t^x, whose logs are integrated of order 1.[17] With the exception of the TFP shocks, these shocks are modelled as ARIMA(1,1,0) shocks.[18]

We next present a detailed description of the EA block, followed by an overview of the RoW model block. Throughout the derivation the following indexing convention will be preserved. Indices i and j index firms and households, respectively. These indices will usually be dropped when the equilibrium conditions are derived due to the representative household/firm assumption. Index l indicates sovereign states or economic regions. Finally, index *k* will always indicate the 'domestic' economy. This index will be generally dropped for parameters (even if they are country-specific), but will be usually preserved for variables.

A.1. EA households

The household sector consists of a continuum of households $j \in [0;1]$. There are two types of households, savers ("Ricardians", superscript *s*) who own firms and hold government and foreign bonds and liquidity-constrained households (subscript *c*) whose only income is labor income and who do not save. The share of savers in the population is ω^s.

Both households enjoy utility from consumption C_{jkt}^r and incur disutility from labor N_{jkt}^r ($r = s, c$). On top of this, Ricardian's utility depends also on the financial assets held.

Date *t* expected life-time utility of household *r*, is defined as:

$$U_{jkt}^r = \sum_{s=t}^{\infty} \varepsilon_{kt}^c \beta^{s-t} u_{jkt}^r(\cdot)$$

where β is the (non-stochastic) discount factor (common for both types of households) and ε_{kt}^c is the saving shock.

A.1.1. Ricardian households

[17] These, in particular, include the TFP shock and the final demand productivity shocks.
[18] TFP is driven by 3 shocks, see below.

The Ricardian households work, consume, own firms and receive nominal transfers T_{jkt}^{S} from the government. Ricardians have full access to financial markets and are the only households who own financial assets $\frac{A_{jkt}}{P_{kt}^{c,vat}}$ where $P_{kt}^{c,vat}$ is consumption price, including VAT.[19] Financial wealth of household j consists of bonds $\frac{B_{jkt}}{P_{kt}^{c,vat}}$ and shares $\frac{P_{kt}^{S}S_{jkt}}{P_{kt}^{c,vat}}$, where P_{kt}^{S} is the nominal price of shares in t and S_{jkt} the number of shares held by the household. It is assumed that households invest only in domestic shares.

$$\frac{A_{jkt}}{P_{kt}^{c,vat}} = \frac{B_{jkt}}{P_{kt}^{c,vat}} + \frac{P_{kt}^{S}S_{jkt}}{P_{kt}^{c,vat}}$$

Bonds consist of domestic government bonds $\frac{B_{jkkt}^{g}}{P_{kt}^{c,vat}}$, foreign bonds $\frac{e_{lkt}\ \ B_{jlkt}^{g}}{P_{kt}^{c,vat}}$ and private risk-free bonds $\frac{B_{jkt}^{rf}}{P_{kt}^{c,vat}}$ (in zero supply):

$$\frac{B_{jkt}}{P_{kt}^{c,vat}} = \frac{B_{jkt}^{rf}}{P_{kt}^{c,vat}} + \sum\nolimits_{l} e_{lkt}B_{jlkt}^{g}$$

with e_{lkt} the bilateral exchange rate and $e_{kkt} \equiv 1$.[20]

Total government bonds consist of long-term bonds $B_{jkkt}^{g,L}$ and short-term bonds $B_{jkkt}^{g,S}$:

$$B_{jkkt}^{g} = B_{jkkt}^{g,L} + B_{jkkt}^{g,S}$$

From the outstanding long-term bonds $B_{jkkt}^{g,L,H}$ are held by the private sector and $B_{jkkt}^{g,L,CB}$ by the central bank:

$$B_{jkkt}^{g,L} = B_{jkkt}^{g,L,H} + B_{jkkt}^{g,L,CB}$$

Short-term and long-term bonds are imperfect substitutes in the model. In particular, households have a preference for holding a mix of short-term and long-term bonds, and deviations from the target value κ for the ratio of long-term over short-term debt induce quadratic adjustment costs (

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[19] Note that $P_{kt}^{c,vat}$ is related to P_{kt}^{C}, the private consumption deflator in terms of input factors, by the formula: $P_{kt}^{c,vat} = (1+\tau_{k}^{C})P_{kt}^{C}$ where τ^{c} is the tax on consumption.

[20] For simplicity, at this moment the model assumes only one type of foreign bonds, B_{RoWkt}^{g}, issued by RoW and denominated in RoW currency.

γ_b). The same formulation of portfolio preferences or adjustment costs has been used previously by, e.g., Andrés et al. (2004), Falagiarda (2013), Harrison (2012), and Liu et al. (2015).

The budget constraint of a Ricardian household j is given by:

$$P_{kt}^{c,vat}C_{jkt}^{s}+\frac{P_{kt}^{N}B_{kkt}^{g,L,H}}{P_{kt}^{Y}}\left(1+\frac{\gamma_b}{2}(\kappa\frac{B_{kkt}^{g,S}}{B_{kkt}^{g,L,H}}-1)^2\right)+\frac{B_{kk,t-1}^{g,S}}{P_{kt}^{Y}}+\frac{e_{lkt}B_{lkt-1}^{g}}{P_{kt}^{Y}}+\frac{B_{t}^{S}}{(1+i_{t}^{g})P_{kt}^{Y}}=(1-\tau^{N})W_{kt}N_{jkt}^{s}$$
$$+\left(1+i_{t-1}^{rf}\right)B_{jkt-1}^{rf}+\left(P_{kt}^{S}+P_{kt}^{Y}d_{kt}\right)S_{jkt-1}+div_{kt}+T_{jkt}^{s}-tax^{s}{}_{jkt}+\frac{cB_{kkt-1}^{g,L,H}}{P_{kt}^{Y}}+\frac{\delta_b P_{kt}^{N}B_{kkt-1}^{g,L,H}}{P_{kt}^{Y}}$$

where W_{kt} is the nominal wage rate, P_{kt}^{Y}, is GDP price deflator, i_{lt-1}^{g} are interest rates on government bonds of region l, i_{t-1}^{rf} is interest rate on risk-free bond, T_{jkt}^{s} are government transfers to savers and $tax^{s}{}_{jkt}$ are lump-sum taxes paid by savers. Note that savers own all the firms in the economy. div_{kt} represent the profits of all firms other than differentiated goods producers (the latter producers transfer profits to savers by paying dividends d_{kt}).

The price in period t of a short-term (1-period) bond of nominal value $B_{kkt}^{g,S}$ is $B_{kkt}^{g,S}/(1+i_{t}^{g})$, with i_{t}^{g} being the short-term nominal interest rate. Following Woodford (2001) long-term government debt is modelled through bonds for which the nominal coupon c, which is a fraction of the principal, depreciates over time at rate δ_b. The price in period t of a long-term bond issued in t (P_{kt}^{N}) equals the discounted value of future payments:

$$P_{kt}^{N}=\sum_{n=0}^{T}\frac{\delta_b^{n}}{(1+i^{g})^{1+n}}c$$

where T is the maturity period of the bond.

We define the gross nominal return on domestic shares as:

$$1+i_{kt}^{s}=\frac{P_{kt}^{S}+P_{kt}^{Y}d_{kt}}{P_{kt-1}^{S}}$$

The instantaneous utility functions of savers, $u^{s}(\cdot)$, is defined as:

$$u^s\left(C_{jkt}^s, N_{jkt}^s, \frac{U_{jkt-1}^A}{P_{kt}^{c,vat}}\right)$$

$$= \frac{1}{1-\theta}\left(C_{jkt}^s - hC_{kt-1}^s\right)^{1-\theta} - \frac{\omega^N \varepsilon_{kt}^U}{1+\theta^N}(C_{kt}^s)^{1-\theta}\left(N_{jkt}^s - h_N N_{kt-1}^s\right)^{1+\theta^N}$$

$$- \ (C_{kt}^s - hC_{kt-1}^s)^{-\theta} \ \frac{U_{jkt-1}^A}{P_{kt}^{c,vat}}$$

where $C_{kt}^s = \int C_{jkt}^s$, $C_{kt} = \omega^s C_{kt}^s + (1-\omega^s)C_{kt}^c$; $h, h_N \in (0;1)$ measure the strength of the external habits in consumption and labor and $\varepsilon_{kt}^U$is the labor supply (or wage mark-up) shock . The disutility of holding financial assets, U_{jkt-1}^A, is defined as:

$$U_{jkt-1}^A = \sum_l \left(\left(\alpha_{lk}^{bB0} + \varepsilon_{lkt-1}^B\right) e_{lkt-1} B^g{}_{jlkt-1}\right) +$$

$$\left(\left(\alpha_k^{sS0} + \varepsilon_{kt-1}^S\right) P_{st-1}^s \, S_{jkt-1}\right)$$

The Ricardian household problem leads to the following first order conditions (FOC).[21]

The FOC w.r.t. savers' consumption produces:

$$\varepsilon_{kt}^C(C_{kt}^s - hC_{kt-1}^s)^{-\theta} = \lambda_{kt}^s$$

where λ_{kt}^s is the Lagrange multiplier on the budget constraint.

FOC w.r.t. domestic risk-free bond:

$$\beta E_t\left[\frac{\lambda_{kt+1}^s}{\lambda_{kt}^s}\frac{1+i_{kt}^{rf}}{1+\pi_{kt+1}^{C,vat}}\right] = 1$$

FOC w.r.t. domestic short-term government bonds:

$$\beta E_t\left(\frac{\lambda_{kt+1}^s}{\lambda_{kt}^s}\right) = E_t\left(\frac{P_{kt+1}^Y}{P_{kt}^Y}\right)\left(\frac{1}{1+i_t^g} + \gamma_b \kappa P_{kt}^N \left(\kappa \frac{B_{kkt}^{g,S}}{B_{kkt}^{g,L,H}} - 1\right)\right)$$

FOC w.r.t. domestic long-term government bonds:

$$\beta E_t\left(\frac{\lambda_{kt+1}^s}{\lambda_{kt}^s}\right) = E_t\left(\frac{P_{kt}^N}{\delta_b P_{kt+1}^N + c}\frac{P_{kt+1}^Y}{P_{kt}^Y}\right)\left(1 + \frac{\gamma_b}{2}\left(\kappa \frac{B_{kkt}^{g,S}}{B_{kkt}^{g,L,H}} - 1\right)^2 - \gamma_b \kappa\left(\kappa \frac{B_{kkt}^{g,S}}{B_{kkt}^{g,L,H}} - 1\right)\frac{B_{kkt}^{g,S}}{B_{kkt}^{g,L,H}}\right)$$

FOC w.r.t. RoW government bonds:

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[21] See subsection A.1.3 for the labor supply condition.

$$\beta E_t \left[\frac{\lambda_{kt+1}^s}{\lambda_{kt}^s} \frac{\left(1 + i_{RoWkt}^g\right) \frac{e_{RoWkt+1}}{e_{RoWkt}} - \varepsilon_{RoWkt}^B - \left(\alpha_{RoWk}^{b0} + \alpha_{RoWk}^{b1} \frac{e_{RoWkt} B_{RoWkt}^g}{P_k^Y Y_k} \right)}{1 + \pi_{kt+1}^{C,vat}} \right] = 1$$

where ε_{RoWkt}^B the risk-premium on RoW bonds.

FOC w.r.t. domestic stocks:

$$\beta E_t \left[\frac{\lambda_{kt+1}^s}{\lambda_{kt}^s} \frac{(1 + i_{kt+1}^s) - \varepsilon_{kt}^S - \alpha_{kk}^{s0}}{1 + \pi_{t+1}^{C,vat}} \right] = 1$$

where ε_{kt}^S the risk-premium on stocks. The above optimality conditions are similar to a textbook Euler equation, but incorporate asset-specific risk premia, which depend on an exogenous shock ε_{kt}^A as well as the size of the asset holdings as a share of GDP. Taking into account the Euler equation for the risk-free bond and approximating, they simplify to the familiar expressions:

$$i_{kt}^g = i_{kt}^{rf} + rprem_{kt}^g$$

$$E_t \left[\frac{e_{RoWkt+1}}{e_{RoWkt}} \right] i_{RoWkt}^g = i_{kt}^{rf} + rprem_{RoWkt}^g$$

$$i_{kt}^s = i_{kt}^{rf} + rprem_{kt}^s$$

In the equations above, $rprem_{kt}^g$ is the risk premium on domestic government bonds. Similarly, $rprem_{RoWkt}^g$ is the risk premium on domestic government bonds sold abroad (to RoW). This feature of the model, hence, helps capture international spillovers that occur via the financial market channel, see Vitek (2013, 2014). Finally, $rprem_{kt}^s$ is a crucial risk premium on domestic shares. It is introduced to capture in a stylized manner financial frictions that are commonly believed to have contributed to the first phase of the financial crisis and may have contributed to its second phase, see also subsection A.2.2, below.[22]

[22] Observationally, this approach is equivalent to exogenous risk premia as well as risk premia derived in the spirit of Bernanke, Gertler & Gilchrist.

A.1.2. Liquidity-constrained household

The liquidity-constrained household consumes her disposable after-tax wage and transfer income in each period of time ('hand-to-mouth'). The period t budget constraint of the liquidity-constrained household is:

$$(1+\tau_k^C)P_{kt}^C C_{jkt}^c = (1-\tau_k{}^N)W_{kt}N_{kt}^c + T_{kt}^c - tax^c{}_{jkt}.$$

The instantaneous utility functions for liquidity-constrained households. $u^c(\cdot)$, is defined as:

$$u^c\left(C_{jkt}^c, N_{jkt}^c\right) = \frac{1}{1-\theta}\left(C_{jkt}^c - hC_{kt-1}^c\right)^{1-\theta} - \left(C_{kt}^c\right)^{1-\theta}\frac{\omega^N exp(u_{kt}^U)}{1+\theta^N}\left(N_{jkt}^c - h_N N_{kt-1}^c\right)^{1+\theta^N}$$

with $C_{kt}^c = \int C_{jkt}^c$.

A.1.3. Labor supply

Trade unions are maximizing a joint utility function for each type of labor. It is assumed that types of labor are distributed equally over Ricardian and liquidity-constrained households with their respective population weights. The wage rule is obtained by equating a weighted average of the marginal utility of leisure to a weighted average of the marginal utility of consumption times the real wage adjusted for a wage mark-up. Nominal rigidity in wage setting is introduced in the form of adjustment costs for changing wages. The wage adjustment costs are borne by the household. Real wage rigidity is also allowed, given the following optimality condition:

$$\left((1+\mu_t^W)\frac{\omega^s V_{1-l,jkt}^s + (1-\omega^s)V_{1-l,jkt}^c}{\omega^s U_{c,jkt}^s + (1-\omega^s)U_{c,jkt}^c}(1+\tau_k^C)p_{kt}^C\right)^{1-\gamma^{wr}}\left((1-\tau_k{}^N)\frac{W_{kt-1}}{P_{kt-1}^Y}\right)^{\gamma^{wr}} = (1-\tau_k{}^N)\frac{W_{kt}}{P_{kt}^Y} +$$

$$\gamma^w(\pi_t^w - (1-sf^w)\pi_{t-1}^w)(1+\pi_t^w) - \gamma^w\frac{L_{t+1}}{L_t}\frac{1+\pi_{t+1}^y}{1+i_{t+1}^{sd}}(\pi_{t+1}^w - (1-sf^w)\pi_t^w)(1+\pi_{t+1}^w)$$

where μ_t^w is the wage mark-up, γ^{wr} is the degree of real wage rigidity, γ^w is the degree of nominal wage rigidity and sf^w is the degree of forward-lookingness in the labor supply equation. $V_{N,jkt}^x$, for x=s,c, is the marginal disutility of labor, defined as:

$$V_{N,jkt}^x = \omega^N exp(u_{kt}^U)C_{kt}{}^{1-\theta}\left(N_{jkt}^x - h_N N_{kt-1}^x\right)^{\theta^N}$$

A.2. EA production sector

A.2.1. Total output demand

Total output O_{kt} is produced by perfectly competitive firms by combining value added, Y_{kt}, with energy input, Oil_{kt}, using the following CES production function:

$$O_{kt} = \left[\left(1 - s^{Oil}\right)^{\frac{1}{\sigma^O}} (Y_{kt})^{\frac{\sigma^O - 1}{\sigma^O}} + \left(s^{Oil}\right)^{\frac{1}{\sigma^O}} (OIL_{kt})^{\frac{\sigma^O - 1}{\sigma^O}} \right]^{\frac{\sigma^O}{\sigma^O - 1}}$$

where s^{Oil} is the energy input share in total output and elasticity σ^O is inversely related to the steady state output price gross mark-up. It follows that the demand for Y_{kt} and OIL_{kt} by total output producers is, respectively:

$$Y_{kt} = \left(1 - s^{Oil}\right) \left(\frac{P_{kt}^Y}{P_{kt}^O}\right)^{-\sigma^O} O_{kt}$$

$$OIL_{kt} = s^{Oil} \left(\frac{P_{kt}^{Oil}}{P_{kt}^O}\right)^{-\sigma^O} O_{kt}$$

where P_{kt}^Y and $P_{kt}^{Oil}$are price deflators associated with Y_{kt} and Oil_{kt}, respectively, and the total output deflator P_{kt}^O is such that:

$$P_{kt}^O = \left[\left(1 - s^{Oil}\right) (P_{kt}^Y)^{1 - \sigma^O} + s^{Oil} \left(P_{kt}^{Oil}\right)^{1 - \sigma^O} \right]^{\frac{1}{1 - \sigma^O}}$$

A.2.2. Differentiated goods supply

Each firm $i \in [0; 1]$ produces a variety of the domestic good which is an imperfect substitute for varieties produced by other firms. Because of imperfect substitutability, firms are monopolistically competitive in the goods market and face a downward-sloping demand function for goods. Domestic final good producers then combine the different varieties into a homogenous good and sell them to domestic final demand goods producers and exporters.

Differentiated goods are produced using total capital K_{ikt-1}^{tot} and labour N_{ikt} which are combined in a Cobb-Douglas production function:

$$Y_{ikt} = (A_{kt}^Y N_{ikt})^\alpha (cu_{ikt} K_{ikt-1}^{tot})^{1-\alpha}$$

where A_{kt}^Y is labour-augmenting productivity shock common to all firms in the differentiated goods sector and cu_{ikt} is firm-specific level of capital utilization. Total Factor Productivity, TFP_{kt}, can therefore be defined as:

$TFP_{kt} = (A_{kt}^Y)^\alpha$.

We allow for three types of shocks related to the technology: a temporary shock ε_{kt}^{AY} which accounts for temporary deviations of A_{kt}^{Y} from its trend, $\overline{A}_{kt}^{Y}$, and two shocks related to the trend components itself:

$$\log(A_{kt}^{Y}) - log(\overline{A}_{kt}^{Y}) = \varepsilon_{kt}^{AY}$$

$$\log(\overline{A}_{kt}^{Y}) - log(\overline{A}_{kt-1}^{Y}) = g_{kt}^{\overline{AY}} + \varepsilon_{kt}^{L\overline{AY}}$$

$$g_{kt}^{\overline{AY}} = \rho^{\overline{AY}} g_{kt-1}^{\overline{AY}} + \varepsilon_{kt}^{G\overline{AY}} + \left(1 - \rho^{\overline{AY}}\right) g^{\overline{AY}}$$

with $g^{\overline{AY}}$ being the long-run technology growth.

Total capital is a sum of private installed capital, K_{ikt}, and public capital, K_{ikt}^{g}:

$$K_{ikt}^{tot} = K_{ikt} + K_{ikt}^{g}$$

The producers maximize the value of the firm, V_{kt}, equal to a discounted stream of future dividends, $V_{kt} = d_{kt} + E_t[sdf_{kt+1} V_{kt+1}]$, with the stochastic discount factor

$$sdf_{kt} = \left(1 + i_{kt}^{sd}\right) / \left(1 + \pi_{kt}^{C,vat}\right) \approx \left(1 + i_{kt-1}^{rf} + rprem_{kt-1}^{s}\right) / \left(1 + \pi_{kt}^{C,vat}\right)$$

which depends directly on the investment risk premium, $rprem_{kt-1}^{s}$. The dividends are defined as:

$$d_{ikt} = (1 - \tau_k^K) \left(\frac{P_{ikt}^{Y}}{P_{kt}^{Y}} Y_{ikt} - \frac{W_{kt}}{P_{kt}^{Y}} N_{ikt} \right) + \tau_k^K \delta \frac{P_{kt}^{I}}{P_{kt}^{Y}} K_{ikt-1} - \frac{P_{kt}^{I}}{P_{kt}^{Y}} I_{ikt} - adj_{ikt}$$

where I_{ikt} is physical investment, P_{kt}^{I} is investment price, τ_k^K is the profit tax, δ is capital depreciation rate and adj_{ikt} are adjustment costs associated with price P_{ikt}^{Y} and labour input N_{ikt} adjustment or moving capacity utilization cu_{ikt} and investment I_{ikt} away from their optimal level:

$$adj_{ikt} = adj(P_{ikt}^{Y}) + adj(N_{ikt}) + adj(cu_{ikt}) + adj(I_{ikt})$$

where

$$adj(P_{ikt}^{Y}) = \frac{\gamma^{p}}{2} Y_{kt} \left(\frac{P_{ikt}^{Y}}{P_{ikt-1}^{Y}} - 1 \right)^2$$

$$adj(N_{ikt}) = \frac{\gamma^{n}}{2} Y_{kt} \left(\frac{N_{ikt}}{N_{ikt-1}} - 1 \right)^2$$

$$adj(cu_{ikt}) = \frac{P_{kt}^{I}}{P_{kt}^{Y}} K_{ikt-1} \left(\gamma^{u,1} (cu_{ikt} - 1) + \frac{\gamma^{u,2}}{2} (cu_{ikt} - 1)^2 \right)$$

$$adj(I_{ikt}) = \frac{P^I_{kt}}{P^Y_{kt}}\left(\frac{\gamma^{I,1}}{2}K_{kt-1}\left(\frac{I_{ikt}}{K_{kt-1}} - \delta\right)^2 + \frac{\gamma^{I,2}}{2}\frac{(I_{ikt} - I_{ikt-1})^2}{K_{kt-1}}\right)$$

The maximization is subject to production function, standard capital accumulation equation:

$$K_{ikt} = (1-\delta)K_{ikt-1} + I_{ikt}$$

and the usual demand condition which inversely links demand for variety i goods and the price of the variety:

$$Y_{ikt} = \left(\frac{P^Y_{ikt}}{P^Y_{kt}}\right)^{-\sigma^y} Y_{kt}$$

Let $adj_{X,ikt}$ for $X = P^Y, N, cu, I$ denote additional dynamic terms due to the existence of adjustment costs. Let also define $g^X_{kt} := \frac{X_{kt} - X_{kt-1}}{X_{kt-1}}$ the net growth rate of variable $X = N, Y, I, C, \ldots$ and $\pi^X_{kt} := \frac{\Delta P^X_{kt}}{P^X_{kt-1}}$ the inflation rate of a price deflator associated with variable $X = N, Y, I, C, \ldots$ The main optimality conditions of the differentiated goods producers are as follows.

The usual equality between the marginal product of labor and labor cost holds, with a wedge driven by the labor adjustment costs:

$$\mu^y_{kt}\alpha\frac{Y_{kt}}{N_{kt}} - adj_{N,ikt} = (1-\tau^k)\frac{W_{kt}}{P^Y_{kt}}$$

with μ^y_{kt} being inversely related to the price mark-up. The capital optimality condition reflects the usual dynamic trade-off faced by the firm:

$$\frac{1+\pi^y_{kt+1}}{1+i^{sd}_{kt+1}}\frac{P^I_{kt+1}/P^Y_{kt+1}}{P^I_{kt}/P^Y_{kt}}\left(\mu^y_{kt+1}(1-\alpha)\frac{P^Y_{kt+1}Y_{ikt+1}}{P^I_{kt+1}K^{tot}_{ikt}} + \tau^k\delta - adj^{cu}_{kt}/K_{ikt} + (1-\delta)Q_{kt+1}\right) = Q_{kt}$$

where Q_{kt} has the usual Tobin's interpretation.

FOC w.r.t. investment implies that Tobin's Q varies due to the existence of investment adjustment costs:

$$Q_{kt} = 1 + adj_{I,ikt}$$

Firms adjust their capacity utilization depending on the conditions on the market via the optimality condition:

$$\frac{\mu_{kt}^{y}}{P_{kt}^{I}/P_{kt}^{Y}}(1-\alpha)\frac{Y_{kt}}{cu_{kt}} = adj_{cu,ikt}$$

Finally, the FOC w.r.t. differentiated output price pins down the price mark-up:

$$\frac{\sigma^{y}}{(\sigma^{y}-1)}\mu_{kt}^{y} = (1-\tau^{k}) + \frac{adj_{P^{Y},ikt}}{(\sigma^{y}-1)} + \varepsilon_{kt}^{\mu}$$

with ε_{kt}^{μ} being the markup shock. The latter equation, combined with the FOC w.r.t. labor implies the Phillips curve of the familiar form.

A.3. Trade

A.3.1. Import sector

Aggregate demand components

The final aggregate demand component goods C_{kt} (private consumption good), I_{kt}, (private investment good) G_{kt} (government consumption good) and I_{kt}^{G} (government investment good) are produced by perfectly competitive firms by combining domestic output, O_{kt}^{Z} with imported goods M_{kt}^{Z}, $Z = C, I, G, I^{G}$, using the following CES production function:

$$Z_{kt} = A_{kt}^{p^{z}}\left[(1-\varepsilon_{kt}^{M}s^{M,Z})^{\frac{1}{\sigma^{z}}}(O_{kt}^{Z})^{\frac{\sigma^{z}-1}{\sigma^{z}}} + (\varepsilon_{kt}^{M}s^{M,Z})^{\frac{1}{\sigma^{z}}}(M_{kt}^{Z})^{\frac{\sigma^{z}-1}{\sigma^{z}}}\right]^{\frac{\sigma^{z}}{\sigma^{z}-1}}$$

with $A_{kt}^{p^{z}}$ a shock to productivity in the sector producing goods Z and ε_{kt}^{M} is a shock to the share $s^{M,Z}$ of imports in domestic demand components. We assume that the log difference of the specific productivities, $A_{kt}^{p^{z}}$ is an AR(1), $\varepsilon_{kt}^{p^{z}}$ with mean $g^{p^{z}}$. It follows that the demand for the domestic and foreign part of demand aggregates is:

$$O_{kt}^{Z} = \left(A_{kt}^{p^{z}}\right)^{\sigma^{z}-1}(1-\varepsilon_{kt}^{M}s^{M,Z})\left(\frac{P_{kt}^{O}}{P_{kt}^{Z}}\right)^{-\sigma^{z}} Z_{kt}$$

$$M_{kt}^{Z} = \left(A_{kt}^{p^{z}}\right)^{\sigma^{z}-1}\varepsilon_{kt}^{M}s^{M,Z}\left(\frac{P_{kt}^{M}}{P_{kt}^{Z}}\right)^{-\sigma^{z}} Z_{kt}$$

where P_{kt}^{Z} are price deflators associated with Z_{kt}; they satisfy:

$$P_{kt}^{Z} = \left(A_{kt}^{p^{z}}\right)^{-1}\left[(1-\varepsilon_{kt}^{M}s^{M,Z})(P_{kt}^{O})^{1-\sigma^{z}} + \varepsilon_{kt}^{M}s^{M,Z}(P_{kt}^{M})^{1-\sigma^{z}}\right]^{\frac{1}{1-\sigma^{z}}}$$

Economy-specific final imports demand

Final imported goods are produced by perfectly competitive firms combining economy-specific homogenous imports goods, M_{lkt}, using CES production function:

$$M_{kt} = \left(\sum_{l} (s_{lkt}^{M})^{\frac{1}{\sigma^{FM}}} (M_{lkt})^{\frac{\sigma^{FM}-1}{\sigma^{FM}}} \right)^{\frac{\sigma^{FM}}{\sigma^{FM}-1}}$$

where σ^{FM} is the price elasticity of demand for country l's goods and $\sum_{l} s_{lkt}^{M} = 1$ are import shares. The demand for goods from country l is then:

$$M_{lkt} = s_{lkt}^{M} \left(\frac{P_{lkt}^{M}}{P_{kt}^{M}} \right)^{-\sigma^{FM}} M_{kt}$$

while the imports price:

$$P_{kt}^{M} = \left(\sum_{l} s_{lkt}^{M} (P_{lkt}^{M})^{1-\sigma^{FM}} \right)^{\frac{1}{1-\sigma^{FM}}}$$

with P_{lkt}^{M} being the country-specific imports good prices.

Supply of economy- and sector-specific imports

The homogenous goods from country l are assembled by monopolistically competitive firms from economy- and sector- specific goods using a linear production function and subject to adjustment costs. All products from country l are initially purchased at export price P_{lt}^{X} of this country. Firms then maximize a discounted stream of profits, div_{kt}^{IM}, such that :

$$div_{ilkt}^{IM} = \frac{P_{ilkt}^{M}}{P_{kt}^{Y}} M_{ilkt} - e_{lkt} \frac{P_{lt}^{X}}{P_{kt}^{Y}} M_{ilkt} - adj_{ilkt}^{PM}$$

where adj_{ilkt}^{PM} are the adjustment costs that producers face when choosing the bilateral import price.[23] The maximization is subject to the usual inversely-sloping demand equation. These assumptions result in a simple expression for price P_{lkt}^{M} of homogenous goods from country l:

$$P_{lkt}^{M} = e_{lkt} P_{lt}^{X} - adj_{M,ilkt}^{PM}$$

where $adj_{M,ilkt}^{PM}$ are additional dynamic terms due to costs of adjustment.

A.3.2. Export sector

The exporting firms are supposed to be competitive and set their prices equal to the output price, up to a shock, ε_{kt}^{X}:

[23] $adj_{ilkt}^{PM} = \frac{\gamma^{pM}}{2} \frac{P_{lkt}^{M}}{P_{kt}^{Y}} M_{lkt-1} \left(\frac{P_{ilkt}^{M}}{P_{ilkt-1}^{M}} - 1 \right)^{2}$

$$P^X_{kt} = \varepsilon^X_{kt} P^O_{kt}$$

A.4.1. Monetary policy

The operating profit of the central bank equals the sum of base money issuance and interest income minus the current expenditure on buying long-term bonds, where the latter equals the change of the value of long-term bonds on the central bank's balance sheet:

$$PR^{CB}_{kt} = \Delta M_{kt} + cB^{g,L,CB}_{jkk,t-1} - (P^N_{kt} B^{g,L,CB}_{jkkt} - \delta_b P^N_{kt} B^{g,L,CB}_{jkk,t-1})$$

Under the central bank's budget constraint, purchases of long-term government bonds can be financed either by increasing liquidity (money issuance), or by reducing the central bank's operating profit.[24] Purchases of long-term bonds by the central bank are modelled as an exogenous path that replicates the announced ECB programme in timing and size.

Monetary policy in "normal times" is modelled by a Taylor rule where the ECB sets the policy rate i_{kt} in response to area-wide inflation and real GDP growth. The policy rate adjusts sluggishly to deviations of inflation and GDP growth from their respective target levels; it is also subject to random shocks:

$$i_{kt} - \bar{\imath} = \rho^i (i_{kt-1} - i) + (1 - \rho^i)\left(\eta^{i\pi}\left(0.25\left(\sum_{r=0}^{3} \pi^{c+g}_{kt-r}\right) - \bar{\pi}^{c+g}\right) + \eta^{iy}(\tilde{y}_{kt}) + \eta^{ip}(c\pi^c_{kt})\right) + u^{inom}_{kt}$$

where $i = r + \pi^{c+g}$ is the steady state nominal interest rate, equal to the sum of the steady state real interest rate and CPI inflation, $\tilde{y}_{kt} = log(Y_{kt}) - \bar{y}_{kt}$ is the output gap with $\bar{y}_t$ as (log) potential output, $c\pi^{c+g}_{kt} = c\pi^{c+g}_{kt-1} + 0.25(\pi^{c+g}_{kt} - \bar{\pi}^{c+g})$ is the cumulated inflation gap as a proxy for a form of forward-guidance or price level target. The Taylor rule may be extended to deal with economies with managed exchange rates and other exchange rate regimes. It is assumed that the risk-free rate is equal to the policy rate: $i^{sd}_{kt} \equiv i_{kt}$.

A.4.2. Fiscal policy

Government expenditure and receipts can deviate temporarily from their long-run levels in systematic response to budgetary or business-cycle conditions and in response to idiosyncratic shocks. Concerning government consumption and government investment, we specify the following autoregressive equations:

[24] A third option, in general, is for the central bank to sell other assets from its portfolio to sterilise the impact of its intervention on the central bank balance sheet. We discard this possibility in our model by limiting central bank assets to long-term government bonds.

$$\frac{G_{kt}}{\bar{Y}_{kt}A_{kt}^{PG}}-\bar{G}=\rho^{G}\left(\frac{G_{kt-1}}{\bar{Y}_{kt}A_{kt}^{PG}}-\bar{G}\right)+u_{kt}^{G}$$

$$\frac{I_{kt}^{G}}{\bar{Y}_{kt}A_{kt}^{PI}}-\bar{I}^{G}=\rho^{IG}\left(\frac{I_{kt-1}^{G}}{\bar{Y}_{kt}A_{kt}^{PI}}-\bar{I}^{G}\right)+u_{kt}^{IG}$$

$$\frac{T_{kt}}{\overline{P_{kt}^{Y}Y_{kt}}}-\bar{T}=\rho^{T}\left(\frac{T_{kt-1}}{\overline{P_{kt}^{Y}Y_{kt}}}-\bar{T}\right)+\eta^{DEF,T}\left(\frac{\Delta B_{kt}^{gtot}}{P_{kt}^{Y}Y_{kt}}-def^{T}\right)+\eta^{B,T}\left(\frac{B_{kt}^{gtot}}{P_{kt}^{Y}Y_{kt}}-\bar{B}_{k}^{G}\right)+u_{kt}^{T}$$

with B_{kt}^{gtot} total nominal government debt. Government transfers react to the level of government debt and the government deficit relative to the associated debt and deficit targets $\bar{B}_{k}^{G}$ and def^{T}.

The government budget constraint is

$$\frac{B_{jkkt}^{g,S}}{(1+i_{kt})P_{kt}^{Y}}+\frac{P_{kt}^{N}}{P_{kt}^{Y}}B_{jkkt}^{g,L}=\frac{B_{kt-1}^{g,S}}{P_{kt}^{Y}}+\frac{(\delta_{b}P_{kt}^{N}+c)B_{jkk,t-1}^{g,L}}{P_{kt}^{Y}}-\frac{PR_{kt}^{CB}}{P_{kt}^{Y}}-R_{kt}^{G}+P_{kt}^{G}G_{kt}+P_{kt}^{IG}I_{kt}^{G}+T_{kt}$$

where government (nominal) revenue:

$$R_{kt}^{G}=\tau_{k}^{K}(P_{kt}^{Y}Y_{kt}-W_{kt}N_{kt}-P_{kt}^{I}\delta_{k}K_{kt-1})+\tau^{N}W_{kt}N_{kt}+\tau^{C}P_{kt}^{C}C_{kt}+tax_{kt}$$

consists of taxes on consumption, labor and corporate income as well as lump-sum tax.

Finally, the accumulation equation for government capital is:

$$K_{kt}^{G}=\left(1-\delta^{G}\right)K_{kt-1}^{G}+I_{kt}^{G}$$

A.5. The RoW block

The model of the RoW economy (subscript k=RoW) is a simplified structure with fewer shocks. Specifically, the RoW consists of a budget constraint for the representative household, demand functions for domestic and imported goods (derived from CES consumption good aggregators), a production technology that uses labor as the sole factor input, and a New Keynesian Phillips curve. The RoW block abstracts from capital accumulation. There are shocks to labor productivity, price mark-ups, the subjective discount rate, the relative preference for domestic vs. imported goods, as well as monetary policy shocks in the RoW.

More specifically the budget constraint for the RoW representative household is:

$$P_{RoWt}^{Y}Y_{RoWt}+P_{RoWt}^{Oil}OIL_{RoWt}=P_{RoWt}^{C}C_{RoWt}+P_{RoWt}^{X}X_{RoWt}-\sum_{l}\frac{size_{l}}{size_{RoW}}e_{lRoWt}P_{lt}^{X}M_{lRoWt}$$

where X_{RoWt} are non-oil exports by the RoW, and the intertemporal equation for aggregate demand derived from the FOC for consumption:

$$\beta_{t}\frac{\lambda_{RoWt+1}}{\lambda_{RoWt}}\frac{1+i_{RoWt}}{1+\pi_{RoWt+1}^{C}}=1$$

with $\beta_t = \beta \exp(\varepsilon^C_{RoWt})$, $\left(C_{RoWt} - hC_{RoWt-1}\right)^{-\theta} = \lambda_{RoWt}$ and ε^C_{RoWt} as the RoW demand shock. Note that

$$i_{RoWt} \equiv i^g_{RoWkt}$$

As for the EA, final aggregate demand C_{RoWt} (in the absence of investment and government spending in the RoW block) is a combination of domestic output, Y_{RoWt} and imported goods, M_{RoWt}, using the following CES function:

$$C_{RoWt} = A^p_{RoWt}\left[(1-\varepsilon^M_{RoWt}s^M)^{\frac{1}{\sigma}}(Y^C_{RoWt})^{\frac{\sigma-1}{\sigma}} + (\varepsilon^M_{RoWt}s^M)^{\frac{1}{\sigma}}(M^C_{RoWt})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

which gives the demand for the domestic and foreign goods in RoW demand:

$$Y^C_{RoWt} = \left(A^p_{RoWt}\right)^{\sigma-1}(1-\varepsilon^M_{RoWt}s^M)\left(\frac{P^Y_{RoWt}}{P^C_{RoWt}}\right)^{-\sigma} C_{RoWt}$$

$$M^C_{RoWt} = \left(A^p_{RoWt}\right)^{\sigma-1}\varepsilon^M_{RoWt}s^M\left(\frac{P^M_{RoWt}}{P^C_{RoWt}}\right)^{-\sigma} C_{RoWt}$$

where the consumer price deflator P^C_{RoWt} satisfies:

$$P^C_{RoWt} = \left(A^p_{RoWt}\right)^{-1}[(1-\varepsilon^M_{RoWt}s^M)(P^Y_{RoWt})^{1-\sigma} + \varepsilon^M_{RoWt}s^M(P^M_{RoWt})^{1-\sigma}]^{\frac{1}{1-\sigma}}$$

The RoW non-oil output is produced with the technology:

$$Y_{RoWt} = A^Y_{RoWt}N_{RoWt}$$

Price setting for RoW non-oil output follows a New Keynesian Phillips curve:

$$\pi^Y_{RoWt} - \bar{\pi}^Y_{RoW} = \beta\frac{\lambda_{RoWt+1}}{\lambda_{RoWt}}(sfp(E_t\pi^Y_{RoWt+1} - \bar{\pi}^Y_{RoW}) + (1-sfp)(\pi^Y_{RoWt-1} - \bar{\pi}^Y_{RoW}))$$
$$+ \varphi^Y_{RoW}\ln(Y_{RoWt} - \bar{Y}_{RoW}) + \varepsilon^Y_{RoWt}$$

Monetary policy in the RoW follows the Taylor rule:

$$i_{RoWt} - \bar{\iota} = \rho^i(i_{RoWt-1} - \bar{\iota}) + \left(1-\rho^i\right)\left(\eta^{i\pi}(\pi^Y_{RoWt} - \bar{\pi}^Y_{RoW}) + \eta^{iy}\tilde{y}_{RoWt}\right) + \varepsilon^{inom}_{RoWt}$$

where $\tilde{y}_{RoWt}$ is the deviation of actual output from trend output.

The RoW net foreign asset (NFA) position equals minus the EA NFA position.

Finally, oil is assumed to be fully imported from the RoW and the oil price is assumed as follows:

$$P^{Oil}_{RoWt} = \frac{\bar{P}^Y}{A^{p^{oil}}_{ROWt}e_{RoW,US}}$$

where $A^{p^{oil}}_{ROWt}$ is oil-specific productivity and oil is priced in USD.

Total nominal exports are defined as:

$$P^X_{RoWt}X_{RoWt} = \sum_l P^X_{lRoWt}M_{RoWlt}$$

with the bilateral export price being defined as the domestic price subject to a bilateral price shock:

$$P^{X}_{lRoWt} = \exp(\varepsilon^{PX}_{lRoWt})\, P^{Y}_{RoWt}$$

A.6 Closing the economy

Market clearing requires that:

$$Y_{kt}P^{Y}_{kt} + div^{M}_{kt}P^{Y}_{kt} = P^{C}_{kt}C_{kt} + \; P^{I}_{kt}I_{kt} + P^{IG}_{kt}IG_{kt} + TB_{kt}$$

Export is a sum of imports from the domestic economy by other countries:

$$X_{kt} = \sum_{l} M_{klt}$$

where M_{klt} stands for imports from the domestic economy to economy l. The total imports are defined as:

$$P^{Mtot}_{kt}M^{tot}_{kt} = P^{M}_{kt}M_{kt} + P^{oil}_{kt}OIL_{kt}$$

where non-oil imports

$$P^{M}_{kt}M_{kt} = P^{M}_{kt}(M^{C}_{kt} + M^{I}_{kt} + M^{G}_{kt} + M^{IG}_{kt})$$

$$e_{RoWk,t}B^{w}_{k,t} = +\left(1 + i^{bw}_{t-1}\right)e_{RoWk,t}B^{w}_{k,t-1} + P^{X}_{kt}X_{kt} - \sum_{l}\frac{size_l}{size_k}e_{lkt}P^{X}_{lt}M_{lkt} - P^{Oil}_{kt}OIL_{kt} + ITR_k\overline{P^{Y}_{kt}Y_{kt}}$$

where $P^{X}_{kt}X_{kt} - \sum_{l}\frac{size_l}{size_k}e_{lkt}P^{X}_{lt}M_{lkt} - P^{Oil}_{kt}OIL_{kt} = TB_{kt}$ defines the trade balance, with domestic importers buying the imported good at the price $\mathrm{P}^{\mathrm{X}}_{\mathrm{lt}}$ .We allow non-zero trade balance and include an international transfer, ITR_k, calibrated in order to satisfy zero NFA in equilibrium.

Finally, net foreign assets of each country sum to zero:

$$\sum_{l} NFA_{lt}size_l = 0.$$

$size_l$ is the relative size of economy l.

APPENDIX: DATA

1. Data sources

Data for the EA (quarterly national accounts, fiscal aggregates, quarterly interest and exchange rates) are taken from Eurostat. ROW series are constructed on the basis of the IMF International Financial Statistics (IFS) and World Economic Outlook (WEO) databases.

2. Constructing of data series for ROW variables

Series for GDP and prices in the ROW starting in 1999 are constructed on the basis of data for the following 59 countries: Albania, Algeria, Argentina, Armenia, Australia, Azerbaijan, Belarus, Brazil, Bulgaria, Canada, Chile, China, Colombia, Croatia, Czech Republic, Denmark, Egypt, Georgia, Hong Kong, Hungary, Iceland, India, Indonesia, Iran, Israel, Japan, Jordan, Korea, Lebanon, Libya, FYR Macedonia, Malaysia, Mexico, Moldova, Montenegro, Morocco, New Zealand, Nigeria, Norway, Philippines, Poland, Romania, Russia, Saudi Arabia, Serbia, Singapore, South Africa, Sweden, Switzerland, Syria, Taiwan, Thailand, Tunisia, Turkey, Ukraine, United Arab Emirates, United Kingdom, United States, and Venezuela. The ROW data are annual data from the IMF International Financial Statistics (IFS) and World Economic Outlook (WEO) databases.

3. List of observables

The estimation uses the following time series for the EA: GDP, GDP deflator, population, total employment, employment rate, employment in hours, participation rates, relative prices with respect to GDP deflator (VAT-consumption, government consumption, private investment, export, and import), government investment price relative to private investment, nominal policy rate, and nominal shares of GDP (consumption, government consumption, investment, government investment, government interest payment, transfers, public debt, wage bill and exports). The list of observables also includes the oil price and the effective exchange rate of the EA. For the ROW we use data on population, GDP, GDP deflator and the nominal policy rate. The EA specific QE observables are securities held for monetary policy purposes as proxy for long-term bond holdings by the ECB and the share of long-term debt in total government debt. Furthermore, we use current and 3-month-ahead swap rates on 10-year government bonds to calculate the implied expected period-on-period return on long-term bonds.

4. Model fit

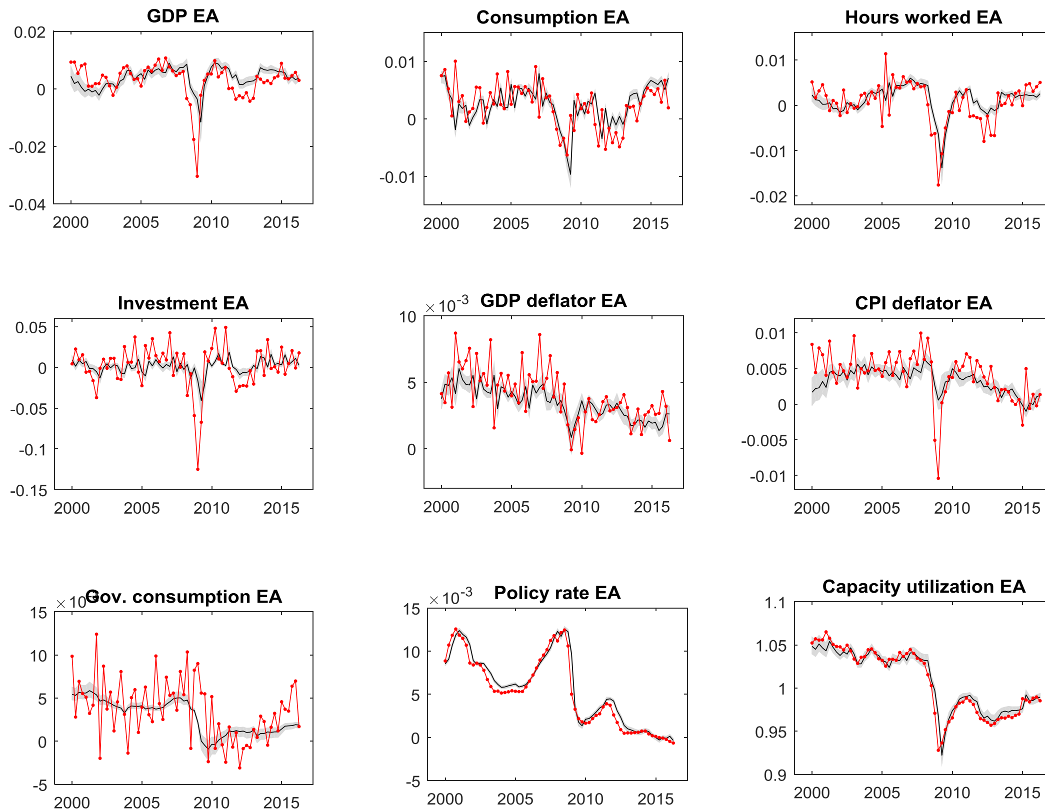

Figure A.1: Bayesian 1-step-ahead prediction

Figure A.1 shows the Bayesian 1-step-ahead prediction for various observable variables. The red line indicates the observed series from 1999q1-2016q2. The black line depicts the posterior mean estimate of the 1-step-ahead forecast of the endogenous variable calculated by the Kalman filter. The grey confidence bounds represent the posterior parameter uncertainty.