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Abstract

The paper deals with unobserved components in economic time series within a general model–based approach. The component, its final estimator, and the preliminary one (which also includes the forecast) are seen to follow different ARIMA models, which can be expressed in terms of the series innovations. Analytical expressions are derived for the different types of associated errors.

Two applications are presented. The first one shows how the use of unobserved components can increase substantially forecasting precision, and how the model–based approach can rigorously answer questions of applied concern. The second application illustrates the dangers of using unobserved components in some macroeconomic models. It is first shown how unobserved component estimators, such as for example a series seasonally adjusted with X11 or with a model–based procedure, will most likely be noninvertible, and hence invertible models (for example, a VAR model) are not appropriate for them. Second, the recent Stock and Watson model aimed at forecasting recessions is used to illustrate how probabilities computed over the distribution of the component, of its final estimator, and of its preliminary one will be poor estimators of each other. As a consequence, the recession forecasts will be systematically biased.

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Introduction

The evolution of economic time series is subject to a variety of short-term movements that may provide a distorted view of the underlying growth of a variable of interest. Typical examples of such movements are the seasonal fluctuations, and the transitory and erratic movements that tend to cancel out over relatively short periods (i.e., the short–term noise). Since a seasonal or a noise component are never directly observed, the relevant underlying growth of the series is, as a consequence, an unobserved component (i.e., a "signal"). Typical signals are, for instance, the Seasonally Adjusted (SA) series or the trend component.

Therefore, interest in obtaining a less distorted view of how the economy is evolving leads to interest in unobserved components and, naturally, their forecasts. In this paper I address the issue of unobserved components and their forecasts within a model–based approach: the components, and hence the observed series, are outcomes of linear stochastic processes which shall be parametrized as Autoregressive Integrated Moving Average (ARIMA) models.

In section 1 the basic unobserved components model is presented, as well as the main assumptions, and section 2 discusses optimal estimators and forecasts of the signal of interest. The properties of the estimators and forecasts, and in particular the structure of their Mean Squared Error (MSE) are analysed in section 3. Section 4 contains a straightforward application, where forecasts of different signals are compared, and it is seen how it is possible to obtain a substantial improvement in forecasting precision. It is further seen how the results in sections 2 and 3 can provide solutions to several problems of applied interest.

The distinction between the theoretical unobserved component, its final or historical estimator, and the preliminary estimator (and forecast), is often a source of confusion in applied work. Section 5 discusses some of the dangers associated with not modeling the distinction properly. The discussion is illustrated with a model similar to that recently developed by Stock and Watson to analyse and forecast the business cycle; it is also seen how the results are easily extended to other important types of models. Finally, section 6 contains a summary of the results.

1 The Model and Assumptions

Let x_t be a time series which is the sum of a signal m_t, and a nonsignal component n_t, as in

$$x_t = m_t + n_t, \tag{1.1}$$

where the two components are outcomes of ARIMA models, which we write in short as

$$\phi_m(B)m_t = \theta_m(B)b_t, \tag{1.2}$$

$$\phi_n(B) n_t = \theta_n(B) c_t; \tag{1.3}$$

B denotes the lag operator, $\phi(B)$ and $\theta(B)$ are finite polynomials in B that may contain unit roots, and b_t and c_t are orthogonal white-noise variables, with variances V_b and V_c. (Throughout the paper, a white–noise variable denotes a variable that is normally, identically, and independently distributed.) It is assumed that the roots of the autoregressive (AR) polynomials $\phi_m(B)$ and $\phi_n(B)$ are different; since AR roots for the same frequency should belong to the same component, this is not a restrictive assumption. The two polynomials $\phi_m(B)$ and $\theta_m(B)$ share no root in common, and the same is true of $\phi_n(B)$ and $\theta_n(B)$. The paper is mostly aimed at quarterly or monthly data, so that, in terms of (1, 1), the signal of interest can be the seasonally adjusted series (in which case the nonsignal is the seasonal component), or the trend (in which case the nonsignal is the sum of the seasonal plus the irregular component).

Combining (1.1), (1.2), and (1.3), it is obtained that

$$\phi_m(B)\,\phi_n(B)\,x_t = \phi_n(B)\,\theta_m(B)\,b_t + \phi_m(B)\,\theta_n(B)\,c_t,$$

and hence x_t also follows an ARIMA model of the type

$$\phi(B)\,x_t = \theta(B)\,a_t, \tag{1.4}$$

where $\phi(B) = \phi_m(B)\,\phi_n(B)$, and $\theta(B)\,a_t$ is the moving average (MA) process such that

$$\theta(B)\,a_t = \phi_n(B)\,\theta_m(B)\,b_t + \phi_m(B)\,\theta_n(B)\,c_t, \tag{1.5}$$

with a_t a white–noise variable with variance V_a (see Anderson, 1971, p. 224). Without loss of generality, V_a is set equal to 1, so that the variances of the component innovations will be implicitly expressed as fractions of V_a, the variance of the one–period–ahead forecast error for the observed series. Since the sum of two uncorrelated MA processes [as in (1.5)] can only be noninvertible when the same unit root is shared by both MA polynomials, if we further assume that $\theta_m(B)$ and $\theta_n(B)$ have no common unit root, it follows that model (1.4) will be invertible. On the other hand, given that the concept of a trend or a seasonal component is intimately linked to nonstationary behavior, models (1.2) and (1.3) will typically be nonstationary (see Hillmer, Bell, and Tiao, 1983). We shall still use the representation

$$\psi(B) = \theta(B)/\phi(B) \tag{1.6}$$

when the series is nonstationary, and similarly for $\psi_m(B)$. Further, letting ω denote frequency (in radians), the Fourier transform of $\psi(B)\,\psi(F)\,V_a$, where $F = B^{-1}$ is the forward operator, will be referred to as the spectrum of x_t, $g_x(\omega)$ (for nonstationary series, it is often called the "pseudospectrum"; see Hillmer and Tiao, 1982, or Harvey, 1990). In a similar way, $g_m(\omega)$ will denote the spectrum of the signal.

Identification of the models (1.2), (1.3), and (1.4) can be reached in several ways. Broadly, two basic approaches can be distinguished: the so–called ARIMA–Model–Based approach (important references are Burman, 1980, and Hillmer and Tiao, 1982), and the Structural Time Series approach (important references are Engle, 1978, and Harvey and Todd, 1983). Since observations are only available on x_t, the first approach starts with model (1.4), which can be identified and estimated directly from the data using Box–Jenkins techniques; then, derives the models for the signal that are compatible with (1.4). From the set of all admissible models, some additional requirements permit the selection of a unique one. The second approach proceeds in an inverse manner, by identifying a–priori models (1.2) and (1.3) for the components. Ultimately, since (1.2) and (1.3) imply a model of the type (1.4), both approaches are closely linked; in fact, the results in this paper are valid for both approaches, and do not depend on the particular identification restrictions used in order to specify models (1.2) and (1.3).

2 Optimal Estimation and Forecasting of an Unobserved Component

Given that the signal is never observed, one is forced to use an estimator. For known models (an assumption that will be made throughout the paper), and having available a finite realization of the series $X_T = [x_1, \ldots, x_T]$, the signal estimator is given by the conditional expectation

$$\hat{m}_{t|T} = E(m_t|X_T), \tag{2.1}$$

which, under our assumptions, yields the Minimum Mean Squared Error (MMSE) estimator. When $t < T$, (2.1) provides an estimator of a past signal; when $t = T$, (2.1) is the concurrent estimator of the signal, and when $t > T$, $\hat{m}_{t|T}$ is the $(t-T)$–periods–ahead forecast. The analytical treatment of these different types of estimators is the same; I shall nevertheless focuss attention on the signal forecast.

It is well known that the conditional expectation (2.1) can be efficiently computed with the Kalman filter (see, for example, Harvey, 1989). For our purposes, however, it will prove more useful to work with an alternative representation of the conditional expectation, namely, the Wiener–Kolmogorov (WK) filter, particularly suited for analytical discussion. In order to derive the WK filter consider, first, the case of an infinite realization of the series x_t, to be denoted X. The optimal estimator of the signal

$$\hat{m}_t = \hat{m}_{t|\infty} = E(m_t|X) \tag{2.2}$$

can then be expressed, using the notation (1.6), as

$$\hat{m}_t = V_b \, \frac{\psi_m(B)\,\psi_m(F)}{\psi(B)\,\psi(F)}\, x_t. \tag{2.3}$$

Replacing the ψ–polynomials by their rational expressions, after cancelling common factor, (2.3) becomes

$$\hat{m}_t = \nu(B, F)\, x_t, \tag{2.4}$$

where $\nu(B,F)$ is the WK filter, given by

$$\nu(B,F) = V_b \frac{\theta_m(B)\,\phi_n(B)}{\theta(B)} \frac{\theta_m(F)\,\phi_n(F)}{\theta(F)}. \tag{2.5}$$

(see, for example, Whittle, 1963, and Cleveland and Tiao, 1976). The filter is, thus, centered at t, symmetric, and convergent in B and in F due to the invertibility of $\theta(B)$. In fact, (2.5) shows that the WK filter is equal to the autocovariance–generating function of the process

$$\theta(B)\, z_t = \theta_m(B)\,\phi_n(B)\, b_t.$$

(Notice that this process is stationary even when model (1.6) is nonstationary.) Convergence of the filter weights implies that, in practice, the filter can be truncated after a certain point at both ends. To simplify the discussion, we shall assume that the available series is long enough so that, when considering recent estimates, the filter in B has converged. (In practice, this is not a restrictive assumption.)

To project m_t on a finite realization X_T, since $X_T \subset X$, by a well–known property of conditional expectations,

$$\begin{aligned} \hat{m}_{t|T} &= E(m_t|X_T) = E\left(E(m_t|X)|X_T\right) = \\ &= E(\hat{m}_t|X_T), \end{aligned} \tag{2.6}$$

which implies that $\hat{m}_{t|T}$ can be expressed as

$$\hat{m}_{t|T} = \nu(B,F)\, x_{t|T}, \tag{2.7}$$

where $\nu(B,F)$ is the WK filter given by (2.5), and

$$x_{t|T} = E(x_t|X_T).$$

Since $x_{t|T}$ is the forecast of x_t done at time T (equal to x_t if $T \geq t$), the estimator (2.7) can be seen as the WK filter applied to the available series extended at both ends with forecasts and backcasts (i.e., applied to the "extended series"). For a large enough (positive) $T-t$, (2.7) provides in practice the final or historical estimator of m_t, equivalent to (2.4). As t approaches T, (2.7) provides preliminary estimators of recent signals; for $t > T$, (2.7) yields the $(t-T)$–periods–ahead forecast of the signal.

Given an overall ARIMA model (1.4) for the observed series, the polynomials $\phi_m(B)$ and $\phi_n(B)$ are immediately obtained by factorizing $\phi(B)$, and assigning the roots to m_t or n_t according to the type of behavior they induce in the series (i.e., the frequency with which they are associated). In general, however, the polynomials $\theta_m(B)$ and $\theta_n(B)$, as well as the variances V_b and V_c, are not uniquely determined. This is easily seen from the following consideration.

Let models (1.2) and (1.3) represent an admissible decomposition of x_t, with at least one of the components invertible. Let this invertible component be, for example, m_t, and denote by g_m the positive number:

$$g_m = \min\, g_m(\omega), \qquad 0 \leq \omega \leq \pi.$$

It follows that m_t in (1.2) can be further decomposed into orthogonal signal and noise components, as in

$$m_t = \tilde{m}_t + u_t, \tag{2.8}$$

where u_t is a white–noise variable with variance equal to any number in the interval $[0, g_m]$. If u_t is removed from m_t and, consequently, added to n_t, so that

$$\tilde{n}_t = n_t + u_t,$$

it is straightforward to find that $\tilde{m}_t$ and $\tilde{n}_t$ have also expressions of the type (1.2) and (1.3), and, since their spectra will be nonnegative, they provide another admissible decomposition of x_t. Different admissible decompositions can be obtained by setting V_u equal to the different points in the interval $[0, g_m]$ or, in other words, by deciding how much white–noise (within the admissibility bounds) should be assigned to the signal/nonsignal components. (If n_t is invertible, an analogous reasoning would apply.) For our purposes, this lack of identification causes no problem. It is true that different admissible decompositions will imply different estimators of the past and concurrent signals. But, since the forecast of independent future white–noise is zero, (2.8) implies that all admissible decompositions will provide the same forecast of the signal.

3 Mean Squared Estimation and Forecasting Error

Let the error in the forecast of the signal be

$$e_{t|T} = m_t - \hat{m}_{t|T}.$$

It can be rewritten as

$$e_{t|T} = (m_t - \hat{m}_t) + (\hat{m}_t - \hat{m}_{t|T}), \tag{3.1}$$

where the first parenthesis in the right–hand–side (r.h.s.) of the equation represents the error in the final estimator

$$d_t = m_t - \hat{m}_t,$$

and the second parenthesis represents the difference between the final and preliminary estimators. This is the revision error in the preliminary estimator

$$d_{t|T} = \hat{m}_t - \hat{m}_{t|T};$$

of course, when $t > T$, this preliminary estimator is the forecast. The structure of the two types of errors (in particular, their variances and covariances) can be easily derived simply from the models for the components in section 1. First, as shown in Pierce (1979), the error in the final estimator, d_t, and the revision error, $d_{t|T}$, are independent, so that

$$V(e_{t|T}) = V(d_t) + V(d_{t|T}). \tag{3.2}$$

Concerning the final estimation error, Pierce further shows that d_t can be seen as the output of the stationary model

$$\theta(B)\, d_t = \theta_m(B)\, \theta_n(B)\, g_t, \tag{3.3}$$

where g_t is white noise with variance $V_g = V_b\, V_c/V_a$. Therefore, the variance of d_t is finite, and its Autocorrelation Function (ACF) will converge; both, variance and ACF, are immediately obtained from models (1.2), (1.3), and (1.4).

As for the revision error $d_{t|T}$, a simple way to derive its properties is the following. Replacing x_t in (2.4) with (1.4), the final estimator of the signal can be expressed as a linear filter on the innovations a_t of the observed series:

$$\hat{m}_t = \eta(B,F)\, a_t, \tag{3.4}$$

where, considering (2.5), the filter $\eta(B,F)$ is given by

$$\eta(B,F) = V_b\, \frac{\theta_m(B)}{\phi_m(B)}\, \frac{\theta_m(F)\, \phi_n(F)}{\theta(F)}. \tag{3.5}$$

The filter will not be convergent in B when m_t is nonstationary; however, it will always be convergent in F. Assuming some suitable starting conditions (see Bell, 1984), the estimator (3.4) can then be expressed as (for $t > T$)

$$\hat{m}_t = \eta^{(1)}_{t|T}(B)\, a_T + \eta^{(2)}_{t|T}(F)\, a_{T+1}, \tag{3.6}$$

where

$$\begin{aligned} \eta^{(1)}_{t|T}(B) &= \eta_{T-t} + \eta_{T-t-1}\, B + \ldots + \eta_{-t}\, B^T, \\ \eta^{(2)}_{t|T}(F) &= \eta_{T-t+1} + \eta_{T-t+2}\, F + \ldots + \eta_0\, F^{t-T-1} + \ldots \end{aligned}$$

The term $\eta^{(1)}_{t|T}\, a_T$ represents the effect of the innovations up to (and including) a_T on the estimator $\hat{m}_t$. From (3.5), the polynomial $\eta^{(2)}_{t|T}(F)$ is convergent; further, the η–coefficients are straightforward to compute, as shown in the Appendix. Since $E_T\, a_{T+k} = 0$ for $k > 0$, applying (2.6) to (3.6), the forecast of the signal is given by

$$\hat{m}_{t|T} = \eta^{(1)}_{t|T}(B)\, a_T,$$

and substracting this expression from (3.6), the revision error is found to be

$$d_{t|T} = \eta^{(2)}_{t|T}(F)\, a_{T+1}. \tag{3.7}$$

The moving average representation (3.7) can be used to derive the variance and ACF of $d_{t|T}$. Invertibility of (1.4) guarantees that the variance will be finite and the ACF will converge.

In summary, the error in forecasting, at period T, the signal m_t is equal to

$$e_{t|T} = m_t - \hat{m}_{t|T} = d_t + d_{t|T},$$

where d_t and $d_{t|T}$ are independent. From the autocovariance function of the two, the variance and ACF of $e_{t|T}$ is easily computed. Notice that the fact that the variances of $e_{t|T}$, d_t, and $d_{t|T}$ are finite, implies that when the signal is nonstationary, the signal, its final estimator, and its forecast will be pairwise cointegrated.

4 An Application

As an example, consider the Italian monetary aggregate targeted in monetary policy, $M2$ (kindly provided to me by the Bank of Italy). It is a monthly series with 84 observations, starting in 1985. The model

$$\nabla \nabla_{12} x_t = (1 - .634B^{12})\, a_t, \tag{4.1}$$

where $\nabla = 1 - B$, $\nabla_{12} = 1 - B^{12}$, and $x = \log M2$ fits very well the series, and the standard deviation of the innovation is $\sigma_a = .00723$.

Model (4.1) accepts a decomposition as in (1.1)–(1.3), with $\phi(B) = \nabla \nabla_{12}$, $\phi_m(B) = \nabla^2$, and $\phi_n(B) = 1 + B + \ldots + B^{11}$. The last two polynomials represent the trend–type autoregressive unit roots, and the seasonal autoregressive unit roots, respectively. It is found that the identification problem mentioned in section 2 is, in this case, due to the fact that a white–noise component, say u_t, with variance in the range $(0, .179V_a)$ can be freely interchanged between the two components m_t and n_t without violating the admissibility of the decomposition. Consider two particular cases of interest:

(a) The maximum noise variance is assigned to the component n_t. In this case the signal follows the model

$$\nabla^2 m_t = (1 + .04B - .96B^2)\, b_t; \qquad V_b = .168V_a \tag{4.2}$$

and is equal to the trend component in the ARIMA–model–based approach referred to earlier. (The nonsignal component is the sum of the seasonal and irregular components.)

(b) The maximum noise variance is assigned to the signal m_t. The model for the signal becomes then

$$\nabla^2 m_t = (1 - .97B + .01B^2)\, b_t; \qquad V_b = .682V_a \tag{4.3}$$

and the signal is the seasonally adjusted series in the ARIMA–model–based approach. Notice that the seasonally adjusted series follows a model very close to the "random walk plus drift" specification. The nonsignal component is in this case simply the seasonal component. Within the set of admissible specifications for the signal, cases (a) and (b) represent two extreme cases: the trend (model (4.2)) and the seasonally adjusted series (model (4.3)) provide the smoothest and the noisiest signal, respectively.

Monetary policy provides an important application of unobserved component forecasting. Overwhelmingly, short–term policy formally uses as signal the seasonally adjusted series, and seasonal adjustment is indeed an issue of serious concern. Typically, at the end of the year, the monetary authority forecasts the seasonal factors or components for the next year. These factors will be used in control and monitoring of the monetary aggregate, in order to accomodate the supply of money to the seasonal fluctuation in money demand, avoiding thus seasonal fluctuations in interest rates. The standard procedure to forecast the seasonal factors is to apply an "ad hoc" formula (as in X11 or X11 ARIMA), with no underlying standard error associated with the forecast.

The actual practice of seasonal adjustment in the conduct of monetary policy has been questioned on several occasions. In fact it has been suggested that the trend should play a more important role (Box et al., 1987, Kenny and Durbin, 1982, Maravall and Pierce, 1986, Moore et al., 1981). One reason, among several, that may make the trend an attractive signal is that, due to its more stable behavior, it could be forecasted with more precision.

Applying the results of the previous section to model (4.1) and the two specifications ((4.2) and (4.3)) for the signal, it is straightforward to obtain the precision of the different forecasts. The standard errors of the 1– and 6–month–ahead forecasts of the $M2$ series (in logs), of its trend, and of its seasonally adjusted component are compared in the following table:

Standard Error of Forecast

(Monetary Aggregate Series)

	1–period–ahead	6–periods–ahead
Series	.0072	.0177
Seas. Adj. Series	.0071	.0166
Trend	.0066	.0163

Clearly, for the short–term horizons we consider, the trend outperforms the seasonally adjusted series and the original series in terms of its forecasting accuracy. This relative performance of forecasts is, in fact, found in other important macroeconomic series. The following table gives the same standard errors of the previous table for the Spanish export and import series (in logs) discussed in Maravall (1986):

Standard Error of Forecast

(Foreign Trade Series)

		1–period–ahead	6–periods–ahead
Series	Exports	.126	.143
	Imports	.117	.133
Seas. Adj. Series	Exports	.110	.129
	Imports	.101	.127
Trend	Exports	.056	.088
	Imports	.054	.083

Since the "true" trend component is never observed, we will never be able to measure the exact forecast error. But if the overall ARIMA models shows no sign of misspecification, we can reasonably assume that the (unobservable) forecast error of the trend displays the precision derived from the model. As seen in figure 1 for the export series, a 95% confidence interval around the trend forecast would be substantially narrower than the corresponding one around the original series forecast (the interval around the SA series would lie in between the two previous ones). Since, in the previous examples, the difference between the trend and the observed series is simply the seasonal component plus random noise (which tends to cancel out over relatively short periods), the relative precision of the trend makes it an attractive candidate for a short–term signal.

Expressions (3.4) and (3.7) — completely determined from the structure of models (1.2), (1.3), and hence (1.4) — allow us to compute, for a given model, the exact standard error of the different signal forecasts. The models derived for d_t and $d_{t|T}$ provide further answers to a variety of questions of applied interest. For example, rates of growth are typically easier to interpret than levels, and are thus heavily used. From the ACF of d_t, $d_{t|T}$, and $e_{t|T}$, standard errors for the (linear approximations to the) different rates of growth and their forecasts are readily obtained. A closely watched rate, for example, is the annual rate at which money is growing at the present moment, measured as the growth of the signal over the last six months for which there are observations, plus the forecasted growth over the next six months. For the Italian monetary aggregate example, the standard error of the above rate (expressed in percent points) is 1.91 for the original series, 1.88 for the seasonally adjusted ones, and 1.85 for the trend component. Over the larger span implicit in the annual rate, the forecasting improvement from using unobserved components naturally decreases.

The final illustration we mention concerns another problem of applied concern. As mentioned before, the standard operating procedure for the monetary authority is to seasonally adjust once a year, and compute then the seasonal factors to be used during the following year. It is well known that there is a loss in precision with respect to a procedure whereby seasonal adjustment is done concurrently every month, and it is important to know how much precision is lost with the suboptimal procedure. Within the framework of section 3, this loss of precision can be quantified as follows.

In the once–a–year adjustment, the seasonal factors used are $\hat{s}_{t|t}$, $\hat{s}_{t+1|t}$, $\ldots$, $\hat{s}_{t+11|t}$; their estimation error can be expressed as

$$e_{t+j|t} = d_t + d_{t+j|t}, \qquad j = 0, 1, \ldots, 11.$$

The variance of d_t is, as before, that of model (3.3), and, letting η_{-j} denote the coefficient of B^j in $\eta(B, F)$ of (3.4), the variance of $d_{t+j|t}$ becomes, for $j = 0$,

$$V(d_{t|t}) = \sum_{i=1}^{\infty} \eta_i^2,$$

a convergent sum, and, for $j = 1, 2, \ldots, 11$,

$$V(d_{t+j|t}) = \eta_{-j+1}^2 + \ldots + \eta_0^2 + V(d_{t|t}).$$

For the monetary aggregate series (model (4.1)), operating in this way, and averaging the results for the 12 estimation errors $d_{t|t}, \ldots, d_{t+11|t}$, it is found that concurrent adjustment would reduce the root mean-squared error of the seasonal factor estimators by 15.2%, on average. This seems to be a somewhat boundary value: the improvement is certainly not negligible, but neither are the practical requirements of a concurrent seasonal adjustment.

5 Some Extensions; The Danger of Using Unobserved Components

5.1 The Basic Model

In the previous section I have illustrated some straightforward applications of unobserved components forecasting. However, when used in econometric models, the distinction between the theoretical unobserved component, its final estimator, and the preliminary one is often a source of confusion in applied work. Since one is always forced to work with estimators, lack of a proper consideration of the different stochastic structures may have serious effects. I shall address two types of effects, one related to the specification of the models; the second one related to probability statements concerning the distribution of the unobserved components.

The two types of complications appear in the important work that Stock and Watson have recently completed, aimed at analysing the business cycle and forecasting recessions (see Stock and Watson, 1989, 1991, 1993). Thus, in what follows, I shall maintain their framework and use for illustration a particularly simple case of their model. Leaving aside constants and leading indicators, which are irrelevant to our discussion, the model can be expressed as

$$\nabla x_t = \gamma(B) \nabla c_t + u_t, \tag{5.1.a}$$

$$\phi(B) \nabla c_t = b_t, \tag{5.1.b}$$

$$D(B)\, u_t = \omega_t, \tag{5.1.c}$$

where x_t is a vector of k observed economic variables, c_t is a scalar unobserved component that follows model (5.1.b), and u_t is a vector of k residuals, assumed to follow the VAR model (5.1.c). The polynomial $\phi(B)$ is stationary, and so is the polynomial matrix $D(B)$. The residuals b_t and ω_t are mutually independent white-noise variables, Normally distributed, with zero mean, and variances V_b and V_ω, respectively. (While c_t is a scalar I(1) variable, u_t contains k I(0) variables.) The elements in the vector x_t share the common nonstationary component c_t, and hence x_t is the sum of an effect due to the common factor, plus a VAR model. Notice that the specification rules out the possibility of cointegration among the x-variables, since, in such a case, it is easily seen that u_t could not follow a finite VAR model as in (5.1.c).

5.2 Invertible Models and Seasonally Adjusted Series

In the Stock and Watson (SW) model, some of the variables in the x_t vector have been seasonally adjusted with X11. Properly speaking, thus, some of the variables in x_t are not observed variables, but estimators of an unobserved component (the SA series). Although the distinction is often ignored in applied econometric work, it is in no way trivial, and may have strong implications in terms of the model specification, as I proceed to show.

Leaving aside the preliminary seasonally adjusted series at both ends of the series, for the historical estimator, the SA series obtained with X11 (ignoring outlier corrections) can be expressed as the linear filter

$$x_t^a = \nu_{X11}(B,\, F)\, x_t, \tag{5.2}$$

where $\nu_{X11}(B,\, F)$ is centered and symmetric. Ghysels and Perron (1993) present the weights (up to 68 leads and lags) of the linear X11 monthly filter. The Fourier Transform of the filter is displayed in figure 2. The zeroes for the seasonal frequencies correspond to seasonal unit roots and, in fact, factorizing the filter, it is found that the seasonal unit roots appear in duplicate, and the X11 filter can be written as

$$\nu_{X11}(B,\, F) = \alpha(B,\, F)\, S(B)\, S(F), \tag{5.3}$$

where $\alpha(B,\, F)$ is a finite linear filter (centered and symmetric, with decaying weights, as shown in figure 3) and

$$S(B) = 1 + B + \ldots + B^{11}$$

contains the seasonal unit roots for the 1, 2, ..., 6 times–a–year frequency. Expression (5.3) also applies to the filters implied by the model–based approximation to X11 of Cleveland and Tiao (1976), and Burridge and Wallis (1984). Since, in both modelizations, the seasonal component contains the autoregressive operator $S(B)$, in both cases $\phi_n(B) = S(B)$, and hence the seasonal adjustment filter, given by (2.5), has a factorization as in (5.3).

The series for which seasonal adjustment by X11 is appropriate are typically series with nonstationary seasonality. In general, thus, the roots of $S(B)$ will be part of their autoregressive polynomial, so that the model for the original series can be, quite generally, written as

$$\delta(B)\, S(B)\, x_t = \lambda(B)\, a_t, \tag{5.4}$$

where $\delta(B)$ is the nonseasonal autoregressive nonstationary polynomial (typically ∇ or ∇^2), and $\lambda(B)\, a_t$ is a stationary process. From (5.2), (5.3), and (5.4), it is obtained that

$$\delta(B)\, x_t^a = \alpha(B,\, F)\, S(F)\, \lambda(B)\, a_t. \tag{5.5}$$

This is the model that generates the SA series from the set of innovations $[a_t]$. Since $\alpha(B,\, F)$ and $\lambda(B)$ are convergent polynomials, they cannot contain the inverse of any of the roots of $S(F)$, which are all of unit modulus. As a consequence, x_t^a will be a noninvertible series, with (at least) 11 unit roots in its moving average expression. (If the seasonality of x_t is assumed stationary, then the polynomial $S(B)$ in the r.h.s. of (5.3) does not cancel out, and there will be at least 22 unit roots present!)

When the SA series is used, SW model (5.1.a) is

$$\nabla x_t^a = \gamma(B) \nabla c_t + u_t, \tag{5.6}$$

where $D(B)\, u_t = \omega_t$, so that u_t is an invertible process. Since the two components in the r.h.s. of (5.6) are independent and at least one is invertible, it follows that the r.h.s. of (5.6) is an invertible process. But this cannot be true of the left–hand–side, since x_t^a is noninvertible. As a consequence, SW model cannot be applied to seasonally adjusted series with X11.

At a more basic level, since the SA series are noninvertible, no finite autoregressive representation will capture their structure, nor will it be admissible, of course, to fit a vector autoregression to a set of series some of them seasonally adjusted. The use of autoregressive models on series adjusted with X11 is, however, a common practice.

If instead of seasonally adjusting with X11, the model–based approach of sections 2 and 3 is used, a similar result is obtained. Letting m_t in (1.1) denote the SA series and n_t the seasonal component, expressions (3.4) and (3.5) imply that the MMSE estimator of the SA series can be seen as the output of the model:

$$[\theta(F)\, \phi_m(B)]\, \hat{m}_t = [V_b\, \theta_m(B)\, \theta_m(F)\, \phi_n(F)]\, a_t. \tag{5.7}$$

Therefore $\hat{m}_t$ will be noninvertible if the seasonal component has unit AR roots (also, if $\theta_m(B)$ has unit roots). Since practically all model–based approaches specify a seasonal component with $S(B)$ included in $\phi_n(B)$, the SA series obtained with a model–based procedure will likely be noninvertible.

From the previous discussion it is seen that noninvertibility of the SA series is the result of requiring that the sum of the seasonal component over a year span should not be too far from zero (or, in other words, that $S(B)\, s_t$ be a stationary process). Given that this requirement seems a minimal requirement for any seasonal adjustment method (of a moving–average type), noninvertibility of the SA series seems a fairly general property. (For a similar result concerning the trend component, see Maravall, 1993.)

From a more general perspective, what expression (5.7) indicates is that the estimator of a signal (be that an SA series, a trend, or a cycle) will be noninvertible whenever the nonsignal component contains nonstationarity. When this happens, invertible models (such as the SW model or a VAR model) fitted to the signal estimator will be misspecified.

5.3 Sequences of Unobserved Components and Forecasting Recessions

Leaving aside intercepts and leading indicators, Stock and Watson (1993) use model (5.1) to forecast recessions. In brief, with model (5.1) as the data-generating process, they define first a recessionary pattern as a sequence of ∇c_t (the monthly growth of the common unobserved component) that are below a certain threshold; an expansionary pattern is a sequence of ∇c_t above some threshold values. The economy is in a recession in month t if (and only if) that month belongs to a recessionary pattern. In that case, the variable R_t takes the value 1; otherwise, $R_t = 0$. Similarly, the variable E_t takes the value 1 when month t falls in an expansionary pattern, and $E_t = 0$ otherwise. To forecast a recession, SW estimate the probability that, for some future t, $R_t = 1$, conditional on the present information. This probability is estimated by Monte Carlo simulation in the following way:

Suppose information is available up to period $T(t > T)$. SW consider the joint distribution of a sequence $M_t = [\nabla c_{t-k_1}, \ldots, \nabla c_t, \ldots, \nabla c_{t+k_2}]$ conditional on information through month T. Thus, they consider the joint distribution of the preliminary estimator $\hat{M}_{t|T}$. Then, preudo-random realizations are drawn from that distribution, and R_t and E_t are computed for each realization. The probability of a recession is estimated as the number of times $R_t = 1$ divided by the number of times R_t and E_t are 1 over all realizations.

The probability that a future Δc_t falls into a recessionary pattern depends on the joint distribution of the sequence M_t. This is obvious from the following consideration: When the recessions tend to be very long and the expansions very short, the probability of a future Δc_t falling into a recessionary pattern will be larger than when recessions are very short and expansions very long. This probability, as already mentioned, is measured by SW over the distribution of the preliminary estimator $\hat{M}_{t|T}$.

The authors match the in-sample (and a few out-of-sample) forecasts of their model with the official dating of recessions by the NBER Business Cycle Dating Committee (BCDC). If their model has nothing to do with the way in which the BCDC proceeds, then good performance of the SW forecast would be a product of luck. This is unlikely; as the authors state, their model "attempts to capture, in a simple way, the institutional process in which recessions are categorized" by the BCDC, and they certainly develop an attractive framework for analysing recessions. Yet the use of the unobserved component c_t brings a point that casts some doubts as to the proper relationship between their definition of a recession, the way the recession probabilities are estimated, and the actual dating of recessions by the BCDC.

Assume the most favorable case, in which the SW model exactly duplicates the BCDC behavior. In that case, the definition of a recession would be based on the "true" unobserved component, the recession forecast on the preliminary estimators of the component, while the BCDC measurements would be based on the (optimal) final estimator. Concerning this last assumption it is worth mentioning that indeed the BCDC identifies recessions with a two–sided filter, characteristic of final estimators. Thus, for example, the BCDC reported on December 22, 1992, the dating of the last official recession: it had started in July 1990, and had ended in March 1991. It took, thus, the BCDC 21 periods after the recession ended to reach the final "official" estimator. (This behavior is clearly rational: after all, what the BCDC does is to look at observations and extract some type of stochastic signal. It would not make sense to stick to an old preliminary estimator when new evidence points to a revision.)

While it is true that the final estimator is the best estimator of the unobserved component when all relevant information has become available, and that the preliminary estimator is the best estimator of the component (and of its final estimator) when part of the information is still not available, the distribution functions of the unobserved component, of the final estimator, and of the preliminary estimator will be (structurally) different. As a consequence, measures of probability computed over those distributions can be poor estimators of each other.

To illustrate the point it will be enough to consider the simplest (nontrivial) case of model (5.1), namely the one given by

$$\begin{aligned} \nabla x_t &= \nabla c_t + u_t \\ (1 - \phi B) \nabla c_t &= b_t, \end{aligned}$$

where u_t and b_t are independent white–noise variables with variances V_u and V_b, respectively. By defining $z_t = \nabla x_t$, and $m_t = \nabla c_t$, the previous model can be rewritten

$$z_t = m_t + u_t \tag{5.8.a}$$

$$(1 - \phi B)\, m_t = b_t. \tag{5.8.b}$$

Model (5.8) is a simple "signal–plus–noise" decomposition, with the signal following a stationary AR(1) process. It is easily seen that (5.8) implies that the observed series follows an ARMA(1, 1) model of the type

$$(1 - \phi B)\, z_t = (1 - \theta B)\, a_t, \tag{5.8.c}$$

where $(1 - \theta B)\, a_t$ is the invertible MA(1) process such that its autocovariance function is that of $[b_t + (1 - \phi B)\, u_t]$. The parameters θ and V_a are easily obtained from V_u, V_b, and ϕ.

For this simplified model, define a recession as two consecutive negative values of the signal. Consider an infinite realization of the process $\nabla c_t = m_t$, as well as the corresponding series of final estimators $\hat{m}_t$, and of preliminary estimators $\hat{m}_{t+k|t}$, for some positive value of k. Let us ask ourselves the question: what is the probability that m_t and m_{t-1} are both negative? This probability is easily obtained from model (5.8). According to it

$$\begin{pmatrix} m_t \\ m_{t-1} \end{pmatrix} \sim N \left[\underline{0},\, V_m \begin{pmatrix} 1 & \phi \\ \phi & 1 \end{pmatrix} \right], \tag{5.9}$$

where $V_m = V_b/(1-\phi^2)$. From this joint distribution, $P(m_t < 0,\, m_{t-1} < 0)$ is readily obtained.

The distribution (5.9) of the unobserved component concerns the definition of a recession. The BCDC however never observes the component and is forced to use its optimal estimator when all relevant data has been observed, i.e. $\hat{m}_t$. So, let us ask now the question: what is the probability that $\hat{m}_t$ and $\hat{m}_{t-1}$ be both negative? To compute it, notice that equation (5.7) becomes, for the particular case of model (5.8),

$$(1-\phi B)\,(1-\theta F)\,\hat{m}_t = V_b\, a_t, \tag{5.10}$$

from which the autocovariances of $\hat{m}_t$ are easily obtained. The joint distribution of the final estimators $\hat{m}_t$ and $\hat{m}_{t-1}$ is

$$\begin{pmatrix} \hat{m}_t \\ \hat{m}_{t-1} \end{pmatrix} \sim N\left[\underline{0},\, V_f \begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}\right], \tag{5.11}$$

where $r = (\phi+\theta)/(1+\phi\,\theta)$, and $V_f = (1+\phi\,\theta)\, V_b^2\, V_a/[(1-\phi\,\theta)\,(1-\phi^2)\,(1-\theta^2)]$. From (5.11), one can obtain $P(\hat{m}_t < 0,\, \hat{m}_{t-1} < 0)$.

Finally, the recession forecasts are based on the joint distribution of the preliminary estimator $\hat{m}_{t+k|t}$, for $k > 0$. Again, let us ask the question: what is the probability that the preliminary estimators of two consecutive monthly growths ($\hat{m}_{t+k|t}$ and $\hat{m}_{t+k-1|t}$) be both negative? In order to derive the appropriate distribution, notice first that, from (5.10), $\hat{m}_{t+k}$ can be expressed as

$$\hat{m}_{t+k} = \eta(B,\, F)\, a_{t+k},$$

where

$$\eta(B,\, F) = \ldots + \eta_{-1}\, B + \eta_0 + \eta_1\, F + \ldots = (1-\phi B)^{-1}\,(1-\theta F)^{-1}.$$

Hence

$$\hat{m}_{t+k|t} = E_t\, \hat{m}_{t+k} = \eta_{-k}\, a_t + \eta_{-k-1}\, a_{t-1} + \ldots, \tag{5.12}$$

where use has been made of the fact that $E_t\, a_{t+j} = 0$ for $j > 0$. It is straightforward to find that, for $k > 0$,

$$\eta_{-k} = \frac{\phi^k}{1-\theta\,\phi} V_b,$$

so that $\eta_{-k-1} = \phi\,\eta_{-k}$. Therefore, from (5.12),

$$\begin{aligned} \hat{m}_{t+k|t} &= \eta_{-k}\,[a_t + \phi\, a_{t-1} + \phi^2\, a_{t-2} + \ldots] = \\ &= \frac{\phi^k\, V_b}{1-\theta\,\phi}\, \frac{1}{1-\phi B}\, a_t, \end{aligned}$$

and hence the preliminary estimator follows the model

$$(1-\phi B)\,\hat{m}_{t+k|t} = c_o\, a_t, \tag{5.13}$$

where $c_0 = \phi^k V_b/(1-\theta\,\phi)$ and B operates on t. (Notice that the ACF of the component forecast will be closer to that of the true component than the ACF of the final estimator.) Replacing k by $(k-1)$, it is finally obtained that

$$\hat{m}_{t+k|t} = \phi\,\hat{m}_{t+k-1|t}. \tag{5.14}$$

As a consequence, the joint distribution of $\hat{m}_{t+k|t}$ and $\hat{m}_{t+k-1|t}$ is degenerate; the correlation between the two preliminary estimators is 1, and hence

$$P(\hat{m}_{t+k|t} < 0,\ \hat{m}_{t+k-1|t} < 0) = P(\hat{m}_{t+k|t} < 0) = .5.$$

This probability is constant, independent of the model parameters.

In summary, the probability of a recession, as measured by the unobserved component (according to the definition), by the final estimator (as the official dating committee supposedly does), or by the preliminary estimator (used for forecasting purposes), are structurally different. This is so because they are measured over different distributions, as shown in figure 4, where the same scale is used for both distributions. For example, for the model with parameter values $V_u = 1$, $V_b = .1$, $\phi = -.6$, the three probabilities are

$$\begin{aligned} P(m_t < 0,\ m_{t-1} < 0) &= .15 \\ P(\hat{m}_t < 0,\ \hat{m}_{t-1} < 0) &= .08 \\ P(\hat{m}_{t|t-k},\ \hat{m}_{t-1|t-k}) &= .50, \end{aligned}$$

for $k > 0$. If the sign of ϕ is reversed, the above probabilities become .35, .42, and .50, respectively. For any given model, thus, probabilities computed over the joint distribution of the preliminary estimator would provide a biased forecast of the underlying probabilities for the true unobserved component or for its final estimator. Figure 5 compares the three types of probability for $V_b/V_u = .1$ and different values of ϕ. Even in this very simple case of a stationary AR(1) signal with added noise, the three types of probability can be quite distant.

In particular, in model (5.8), since (5.14) holds for any k, the probability of a recession computed over the joint distribution of the forecasts (equal to .5 in all cases) is always larger than that probability computed over the joint distribution of the final estimators. In other words, for any forecast horizon, the probabilty given by the forecasts will overestimate the probability in the official dating. It is worth noticing that SW find precisely the same bias: the forecasted probability tends to be larger than the one obtained from the BCDC classification, for all forecast horizons.

The example we have discussed has focussed on the joint distribution of the signal and of its estimator. A similar result can be derived for the conditional distribution of the unobserved component. To illustrate it, consider a slightly different version of the previous model, often used to analyse business cycles in macroeconomic (see, for example, Stock and Watson, 1988, Clark, 1987, and Watson, 1986). The observed series is assumed equal to the sum of a trend component, which follows a random–walk process, and an orthogonal stationary process, which represents the business cycle. As before, it will be enough to consider the simplest case, given by

$$x_t = m_t + u_t \tag{5.15.a}$$

$$\nabla\, m_t = b_t. \tag{5.15.b}$$

where m_t is the nonstationary trend, and u_t is a white–noise variable, orthogonal to b_t. Model (5.15) implies that x_t follows an IMA(1, 1) model

$$\nabla\, x_t = (1 - \theta B)\, a_t, \qquad (\theta > 0), \tag{5.16}$$

where $(1 - \theta B)\, a_t = b_t + (1 - B)\, u_t$.

Since u_t is white–noise, $P(u_t < 0 | u_{t-1} < 0) = .5$. As for the MMSE estimator $\hat{u}_t$, for a full realization of the series, from (5.7) it is seen that $\hat{u}_t$ follows the model

$$(1 - \theta F)\, \hat{u}_t = V_u (1 - F)\, a_t, \tag{5.17}$$

so that its ACF is that of the inverse or dual model of (5.16). Expression (5.17) implies that $(\hat{u}_t,\, \hat{u}_{t-1})$ have a joint distribution as in (5.11), with $r = (\theta - 1)/2$, and $V_f = 2\, V_u^2\, V_a / (1 + \theta)$; from that, the probability of interest can be easily computed. For the case $V_u = .1$, $V_b = 1$, it is obtained that

$$P(\hat{u}_t < 0 | \hat{u}_{t-1} < 0) = .35,$$

certainly different from the value of .5 obtained for the unobserved component. As before, the use of the distribution of the MMSE estimator induces a bias in the computed probability. In this case, the probability associated with the definition of a recession (i.e., with the unobserved component) would be underestimated. Figure 6 compares the conditional distribution of the component and of its estimator.

6 Summary

The paper analyses unobserved component (or signal) forecasting within a model–based approach, whereby the unobserved components (and hence the observed series) follow ARIMA processes. The model approach, described in section 1, is quite general and, in particular, is valid for the so–called ARIMA–Model–Based and Structural Time Series metodologies.

In section 2 the optimal estimator and forecasts of the unobserved component are derived. The forecast can be seen as a preliminary estimator, which shall be revised until the final or historical estimator is obtained. The preliminary and final estimators are expressed in terms of the Wiener–Kolmogorov filter, and as filters applied to the innovations of the observed series. It is seen that the distribution function of the theoretical component will be different from that of the final estimator, which in turn will differ from that of the preliminary one. In section 3, analytical expressions are derived for the error in the final estimator (i.e., the difference between the unobserved component and its final estimator), and for the revision error (i.e., the difference between the preliminary estimator and the final one). Both types of errors are seen to be stationary processes, determined from the overall model, and computation of their variances and autocovariances is straightforward.

Section 4 contains an application to the monthly series of the Italian money supply. Models for alternative short–term signals (in particular, the seasonally adjusted series and the trend) are derived. It is found that the use of the trend may produce a substantial improvement in forecasting precision, and that this is also true for other types of macroeconomic variables. Knowledge of the models for the errors permits us to answer a variety of questions of applied interest. For example, the precision of the forecast of alternative rates of growth of interest, as measured with different signals, can be readily assessed. As another example, it is possible to quantify the loss in precision implied by a once–a–year seasonal adjustment procedure instead of a concurrent one.

The distinction between the theoretical unobserved component, its final estimator, and the preliminary one, is often a source of confusion in applied work. Section 5 discusses some of the dangers associated with this confusion. The first danger concerns the fitting of models to series seasonally adjusted with X11, instead of using the original unadjusted series. It is shown that the final seasonally adjusted series will be noninvertible, and hence invertible models cannot be used on them. Examples of invertible models are finite autoregressive or VAR models, as well as the model recently developed by Stock and Watson to analyse and forecast the business cycle. Use of invertible models on seasonally adjusted series is, however, a common practice. Using the model–based approach, the same result is obtained: the final seasonally adjusted series will typically be noninvertible. More generally, the final estimator of an unobserved component will be noninvertible whenever some of the other components present in the series is nonstationary.

The second danger discussed concerns bias in inferences drawn from the joint distribution of the unobserved component. The Stock and Watson model referred to above is used as illustration. In this model the probability of a recession depends on the joint distribution of sequences of an unobserved component. While the definition of a recession is based on the theoretical component, the forecasted probability of a recession depends on the conditional distribution of the preliminary estimator. Moreover, the final dating of recessions is based on the final estimator. While it is true that the final and preliminary estimators are the best estimators of the component for a complete and a partial realization of the series, respectively, the distribution function of the component, of the final estimator, and of the preliminary estimator will be structurally different. As a consequence, measures of probability computed over those distributions will be poor estimators of each other and, as shown in the paper, will display systematic bias. This bias also affects the computation of conditional probabilities, as illustrated with a slightly different model (also used by macroeconomists to analyse the business cycle).

Appendix

Decomposition of a Two–Sided Asymmetric Filter

The filter (3.5), that expresses the MMSE estimator of the signal in terms of the series innovations, is a two–sided asymmetric filter. Its knowledge, as repeatedly shown in the paper, permits to answer many questions of applied interest. I proceed to sketch a simple procedure to obtain the η–coefficients and express the filter as the sum of a filter in B and a filter in F. Thus, consider, in general, the filter

$$\eta(B,F) = \frac{N_1(B)\,N_2(F)}{D_1(B)\,D_2(F)} = \eta^{(1)}(B) + \eta^{(2)}(F). \tag{A.1}$$

From the N– and D–polynomials, we wish to obtain the η–coefficients.

Let n_i and d_i denote the orders of the polynomials $N_i(z)$ and $D_i(z)$, respectively $(i = 1, 2)$. Let $n = n_1 + n_2$, and $d = d_1 + d_2$. The filter (A.1) can be rewritten:

$$\eta(B,F) = \frac{F^{n_2}}{F^{d_2}}\,\frac{N_1(B)\,N_2'(B)}{D_1(B)\,D_2'(B)}, \tag{A.2}$$

where, if $P(z) = p_0 + p_1 z + \ldots + p_{r-1} z^{r-1} + p_r z^r$, $P'(z)$ denotes the polynomial $P'(z) = p_r + p_{r-1} z + \ldots + p_1 z^{r-1} + p_0 z^r$. A partial fraction decomposition of the filter in B in the r.h.s. of (A.2) yields

$$\frac{N_1(z)\,N_2'(z)}{D_1(z)\,D_2'(z)} = q(z) + \frac{E(z)}{D_1(z)} + \frac{G'(z)}{D_2'(z)},$$

where the order of $E(z)$ and $G'(z)$ are $(d_1 - 1)$ and $(d_2 - 1)$, respectively, and the order of $q(z)$ is $(n - d)$ when $n \geq d$ and 0 otherwise. It is easily seen that this decomposition is unique. Compute

$$q(z) + \frac{G'(z)}{D_2'(z)} = \frac{H'(z)}{D_2'(z)};$$

then

$$\frac{N_1(z)\,N_2'(z)}{D_1(z)\,D_2'(z)} = \frac{E(z)}{D_1(z)} + \frac{H'(z)}{D_2'(z)},$$

where the order of $H'(z)$ is $(n - d_1)$ when $n \geq d$, and $(d_2 - 1)$ when $n < d$. The coefficients of $E(z)$ and $H'(z)$ are easily obtained from the linear system of equations implied by the identity

$$E(z)\,D_2'(z) + H'(z)\,D_1(z) = N_1(z)\,N_2'(z).$$

As a consequence,

$$\begin{aligned}
\eta(B,F) &= \frac{F^{n_2}}{F^{d_2}}\left[\frac{E(B)}{D_1(B)} + \frac{H'(B)}{D_2'(B)}\right] = \\
&= F^{n_2-d_2}\,\frac{E(B)}{D_1(B)} + F^{n_2}\,\frac{H'(B)}{D_2(F)} = \\
&= F^{n_2-d_2}\,\frac{E(B)}{D_1(B)} + F^{r}\,\frac{H(F)}{D_2(F)},
\end{aligned} \tag{A.3}$$

where $r = d_1 - n_1$ when $n \geq d$ and $r = n_2 - d_2 + 1$ when $n < d$. Once the coefficients of the two one–sided filters $E(B)/D_1(B)$ and $H(F)/D_2(F)$ have been obtained (see, for example, Box, Hillmer and Tiao, 1978, p. 334), each array of coefficients is multiplied by the appropriate power of F, as indicated by (A.3), so as to be properly centered. In this way, the desired decomposition (A.1) is obtained.

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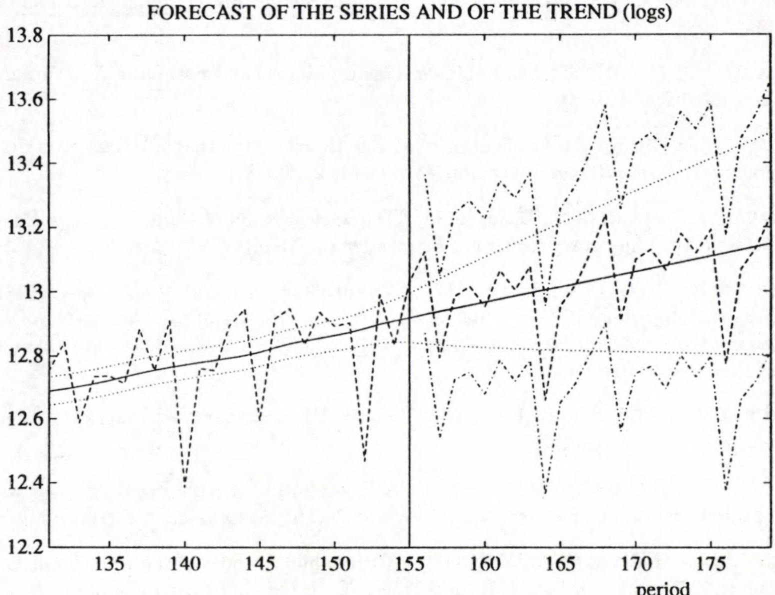


Fig. 1

- - - Original Series (last observation for T = 155)

—— Trend Component

-.-.- Confidence interval for series forecast function

..... Confidence interval for preliminary estimator of trend

FOURIER TRANSFORM OF X11 FILTER

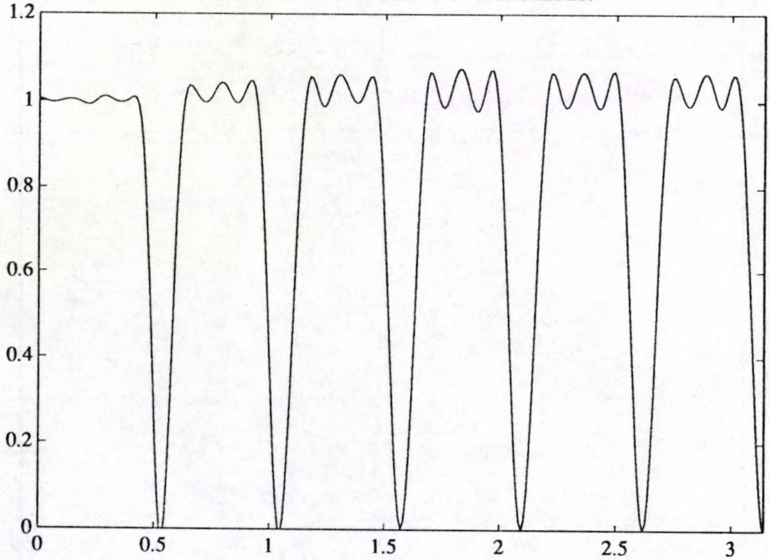


Fig. 2

X11 FILTER WEIGHTS WITH AND WITHOUT SEASONAL UNIT ROOTS

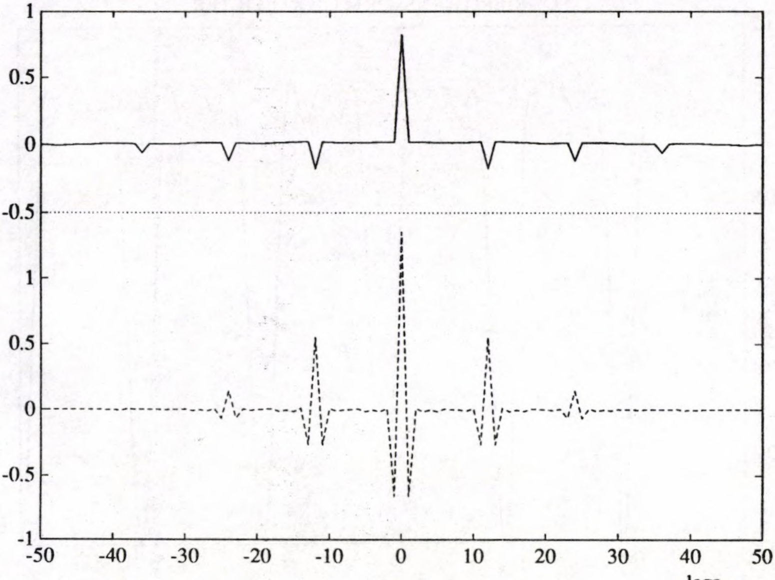


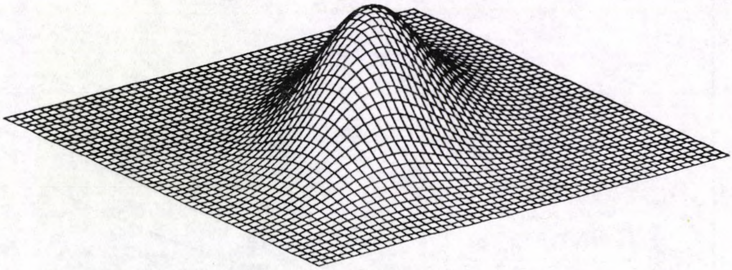
Fig. 3

_____ Weights for full filter

----- Weights for filter without seasonal unit roots

JOINT DISTRIBUTION: COMPONENT AND ESTIMATOR

a) Theoretical Component



b) Final Estimator

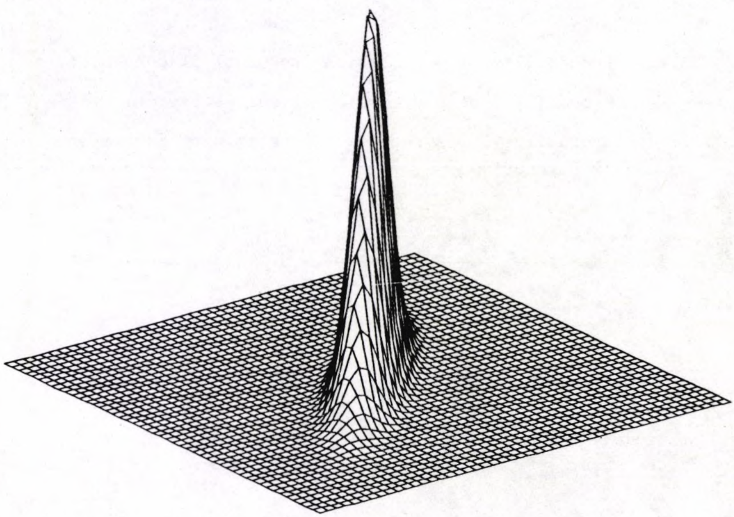


Fig. 4

COMPONENT AND ESTIMATORS: THEORETICAL PROBABILITIES

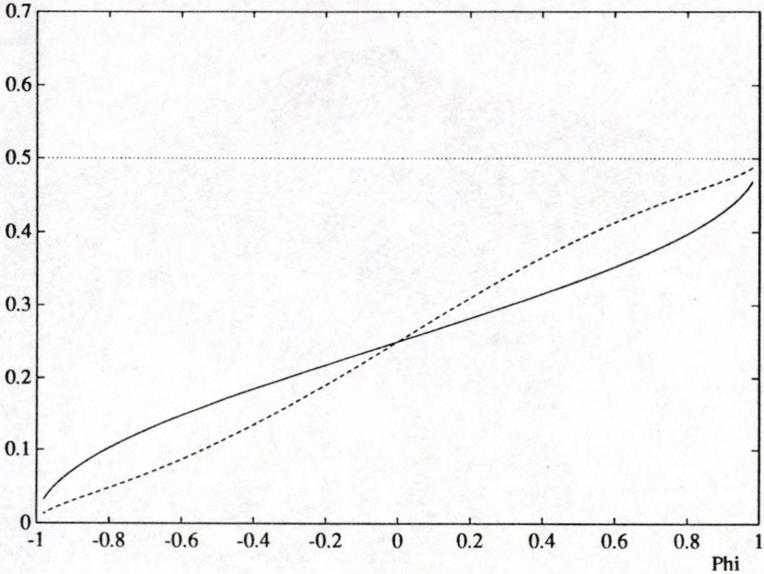


Fig. 5

———	Probability of a recession (theoretical component)
-----	Probability of a recession (final estimator)
.....	Probability of a recession (preliminary estimator or forecast)

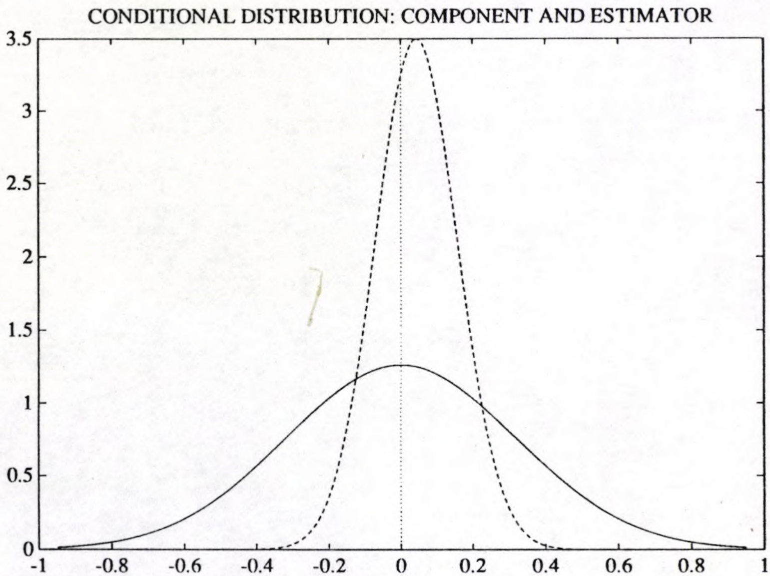


Fig. 6

——— Theoretical Component

----- Final Estimator

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