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TESTING FOR UNIT ROOTS WITH THE k-th AUTOCORRELATION COEFFICIENT

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Abstract

Some applied research has recently used the regression of a variable on its k-th lag, rather than on its first lag, as a test for unit roots. By doing so, series which otherwise would be accepted to be I(1), are instead treated as I(0). This paper shows that when one uses a lag other than the first one, the standard Dickey-Fuller distribution changes. The paper also provides an easy correction for the Dickey-Fuller critical values, so that the standard tables can be adapted for the inference.

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I Introduction

Autoregressive time series with a unit root are the subject of much attention in the econometrics literature, not only with data from financial and commodity markets where it has a long history but also with aggregate time series. However, in the applied literature it is found that series which appear to be integrated of order 1, write I(1), are treated as stationary series, write I(0). For instance Svensson (1991, 1993), tries to estimate the expected future exchange rate of a set of currencies in the European Monetary System (EMS) and to do that, he basically estimates by linear regression the equation:

$$y(t+k) = \rho_k \; y(t) + u(t+k) \qquad \textbf{(1)}$$

with the inclusion of dummies in Svensson (1991) and two interest rates in Sevensson (1993), where y(t) is the exchange rate (in logs).

Equation (1) is based on the Bertola-Svensson (1990) model, where the single determinant of the expected future rate of depreciation, of a currency in a target zone model, k periods ahead with information up to and including Y(t)={y(t), y(t-1).....}, is the current exchange rate y(t).

Once the equation is estimated, and with the scope of studying whether there is mean reversion in the exchange rates (that is, whether the exchanges rates are weakly stationary), inference is performed on ρ_k, noting that if the true ρ_k is unity, there is a downward bias in the estimated coefficient and the usual t-statistics do not apply.

Thus, they compare the t-statistics on the estimated ρ_k's, which I shall denote r_k, corrected to account for serial correlation because of overlapping data, with the standard Dickey-Fuller (DF) tables. Notice that this is equivalent to testing for a unit root with the k-th autocorrelation coefficient which, in the random walk model, continues to be 1.

However, in this paper it is shown that special care should be taken when one is testing for a unit root, when the test is based on an autocorrelation coefficient other than the first one. In particular, the paper shows that if one uses Newey-West standard errors which are consistent under both serial correlation and heteroscedasticity, while ensuring that the covariance matrix is nonnegative, the asymptotic distribution of the t-statistic changes and depends on the lag of the autocorrelation coefficient k. The rest of the paper is organized as follows: Section II, contains the asymptotic distribution of the autocorrelation coefficient at lag k-th of the random walk model. Section III, presents a Montecarlo experiment where the size of the tests based on different k's is explored. Section IV, applies the results of Section II to six currencies of the EMS. Finally, Section V closes the paper with some conclusions.

II Limit theory

Let $\{y(t)\}_{t=1}^{\infty}$ be a stochastic process generated in discrete time according to:

$$y(t+1)=\rho y(t)+e(t+1)$$
$$\rho=1 \quad E(e(t+1))=0; \quad E(e(t+1)^2)=\sigma^2 \tag{2}$$
$$t=1,2...; \quad y(0)=0$$

Under (2) it is possible to write y(t) in terms of the partial sum $S(t)=\Sigma_1^t u(j)$, and the initial condition y(0), that is y(t)=S(t)+y(0).

Now, we are interested in the limiting distributions of standardized sums such as:

$$a)\ X_T(s)=(T\sigma^2)^{1/2}S[Ts]=(T\sigma^2)^{1/2}S(t-1) \quad \frac{(t-1)}{T}\le s<\frac{t}{T} \quad t=1,..,T$$
$$b)\ X_T(1)=(T\sigma^2)^{-1/2}S(T) \tag{3}$$

where [.] denotes the integer part of its argument. Note that the sample paths $X_T(s)$ belong to the space of all real valued functions on [0,1], D=D[0,1], that are right continuous at each point of [0,1] and have finite left limits. D will also be endowed with the uniform metric defined by $||f-g||=\sup_s|f(s)-g(s)|$, for any f, g in D.

Definition 1: Brownian motion process

A stochastic process {W(t); t=1,....,T} is called a Brownian motion process, defined on (H, F, P(.)) where H, is the sample space, F is a σ-field and P(.) is defined as:

P(.):F->[0,1]

if:

i) W(t)=0 for t=0 (a convention)

ii) W(t) is a process with stationary independent increments

iii) these increments are normally distributed

Definition 2: Weak Convergence

Let $\{z(t)\}$ be a sequence of random variables with joint distribution function $\{F_t\}$. If $F_t(z)$->$F(z)$ as t->∞ for every continuity point z, where F is the distribution function of a random variable Z, then z(t) converges in distribution to the random variable Z, denoted z(t)~d~Z. We also say that z(t) converges in law to Z denoted z(t) L Z, z(t) is asymptotically distributed as F, denoted z(t)~a~F, or z(t) converges weakly to Z, denoted z(t)=>Z.

Some important results due to Phillips (1987) are:

If $\{e(t)\}_1^\infty$ in (2) is a sequence that satisfies,

(a) E(e(t))=0 for all t

(b) $\sup_t E|e(t)|^\beta<\infty$ for some $\beta>2$,

(c) $\sigma^2=\lim_{T\to\infty}E(T^{-1}S_T^{\,2})$ exists and $\sigma^2>0$

(d) $\{e(t)\}_1^\infty$ is strong mixing with mixing coefficients α_m, such that

$$\sum_1^\infty \alpha_m^{1-2/\beta}<\infty$$

then as T->∞:

i) $X_T(s)$=>W(s) a standard Brownian motion process

if also:

d) $\sup_t E|e(t)|^{\beta+\eta}<\infty$ for some $\beta>2$, and $\eta>0$, then as T->∞

$$ii)\quad T^{-2}\sum_{1}^{T} y(t-1)^2 \Rightarrow \sigma^2 \int_0^1 W(s)^2 ds$$

$$iii)\quad T^{-1}\sum_{1}^{T} y(t\ \ 1)(y(t)\ \ y(t\ \ 1)) \Rightarrow (\sigma^2/2)(W(1)^2\ \ 1)$$

$$iv)\quad T(r-1) \rightarrow (1/2)(W(1)^2-1)/\int_0^1 W(s)^2 ds$$

$$v)\quad plim(r)=1$$

$$vi)\quad t_\alpha \Rightarrow (1/2)(W(1)^2-1)/(\int_0^1 W(s)^2 ds)^{1/2}$$

Proposition 1: If $\{e(t)\}_1^\infty$ is a sequence of iid $(0,\sigma^2)$ with any finite moment of order larger than 3, then as T->∞:

$$vii)\quad T^{\ 1}\sum_{t=k}^{T} y(t-k)(y(t)-y(t-1)) \rightarrow (\sigma^2/2)(W(1)^2-1)$$

$$viii)\quad T(r_k-1) \Rightarrow (k/2)(W(1)^2-1)/\int_0^1 W(s)^2 ds$$

$$xix)\quad plim(r_k)=1$$

(For Proof, see Appendix)

Notice that the existence of serially correlated errors (as in (1)) does not invalidate the Dickey-Fuller test, based on either the first autocorrelation coefficient or the k-th, as long as the standard errors are estimated consistently. This is so, given that unlike a stable Autoregressive (AR) process with $|\rho<1$, Ordinary Least Squares in both (1) and (2) retains the property of consistency when there is a unit root even in the presence of substantial serial correlation. This can be understood given that the sample variation of y(t) dominates the noise by a factor O(T), so that as T->∞ the effects of any regressor-error correlation are anhililated. So the problem reduces to computing consistent standard errors for r_k in (1), or equivalently, to computing a consistent variance estimator of the process $(y(t+k)-r_k y(t))$. The simplest estimator for the variance of these residuals which has been proposed takes the form of:

$$(a) \quad s_L^2 = s_0 + 2\sum_{i=1}^{l} s_i$$

where **(4)**

$$(b) \quad s_i = T^{-1}\sum_{t=i+1}^{T} v(t)v(t-i) \qquad i=0,1,....l$$

which under suitable conditions guarantees a consistent covariance matrix estimator, and as a result consistent standard errors. However s_L^2 is not constrained to be nonnegative, since large negative sample serial covariances can result in s_L^2 taking negative values. To avoid this problem, Newey and West (1987) suggest an estimator for the variance s_N^2 which ensures that they are nonnegative and is as simple to compute as s_L^2. The modification they propose is:

$$s_N^2 = s_0 + 2\sum_{i=1}^{n} w(n,i)s_i$$

where **(5)**

$w(n,i) = 1 - i/(n+1)$, $\quad s_i \ (i=0,1,...,n)$ *as above*

Proposition 2: If $\{e(t)\}_1^{\infty}$ is a sequence of iid $(0,\sigma^2)$ and n>(k-1), then the Newey-West estimator of the variance of the process $(y(t+k)-r_k y(t))$ as T->∞:

$$xx) \quad plim(s_n^2) = \sigma^2(2k^2+1)/3$$

(For Proof, see Appendix)

Proposition 3:If $\{e(t)\}_1^\infty$ is a sequence of iid $(0,\sigma^2)$ with any finite moment of order larger than 3, and the variance of the process $(y(t+k)-r_k y(t))$ is computed using the Newey-West estimator, then as $T\to\infty$:

$$xxi) \quad t_{\alpha k} \to \frac{k}{((2k^2+1)/3)^{\frac{1}{2}}}(1/2)(W(1)^2-1)/(\int_0^1 W(s)^2 ds)^{1/2}$$

also as $k \to \infty$

$$xxii) \quad t_{\alpha k} \Rightarrow 1.225 t_\alpha$$

where $t_{\alpha k}$ is the Newey-West consistent t-statistic for r_k in (1).

(For Proof, see Appendix)

Result (xi) generalizes the asymptotic distribution of the t-statistic in the random walk model, of the autocorrelation coefficient at lag 1 to those of the autocorrelation coefficients at lag k>1 and obviously reduces to it when k=1.

In practice, (xi) implies that the inference on the k-th autocorrelation coefficient (k>1) should not be based on the standard DF tables, but on a correction of these tables which depends on k. The values of this multiplicative correction for k=2,3,4,5 are 1.15, 1.19, 1.20 and 1.21 respectively, and as a rule of thumb a good approximation for lags larger than 5, can be given by $(3/2)^{1/2}$=1.225.

It is also important to note that this result holds even in the case of including a constant in the regression, by operating as above. And so, for example, consider k=22, the 10% DF critical value t_μ (the t-statistic if a constant is included in the regression) for a sample size larger than 500 would pass from -2.57 to -3.14, while the 5% from -2.86 to -3.50 and, in consequence, comparison of the obtained t-statistics with the standard DF tables will tend to reject too frequently the unit root hypothesis.

III Monte Carlo Evidence

In order to investigate the behavior of this correction in finite samples, a small Monte Carlo experiment has been performed. The idea is to examine the empirical changes at the 1, 5 and 10 percent values of the DF tables. 5000 replications on three sample sizes, T= 100, 500 and 1000, have been simulated with DGP

$$\begin{gathered} y(t) = y(t-1) + u(t) \\ u(t) \sim Nid(0,1) \end{gathered} \qquad \textbf{(6)}$$

to estimate the model

$$y(t) = a + by(t-k) + u(t) \quad k=1,2,5,10,15,20 \qquad \textbf{(7)}$$

with NW standard errors using k-1 lags.

The random numbers have been generated using MATLAB generator, and the seed has been set equal to 100 in all cases.

The results are reported in Table I. At first glance, we see that for T=1000, k=1, we obtain the DF critical values for t_{μ}. For a larger k, and as was expected for T=1000, we find the closest results to the theoretical asymptotic distribution, even if the behavior in T=500 can be considered reasonable.

In the sample sizes T=500, and T=1000, the lags 2, 5 and 10 at the 10 percent, would change in the proportion given by the theoretical correction factor, while for lags 15 and 20, they increase slightly. For T=100, the 1, 5 and 10 % empirical critical values are always higher than the theoretical ones, increasing as k increases.

Examining the asymptotic bias, we see that for the sample sizes T=500 and 1000, the asymptotic bias at lag k equals k times the bias at lag 1, with the only difference that the biases for T=500, are twice those at T=1000. For T=100, up to lag 10 the biases are k times the bias at lag 1, decreasing for lags 15 and 20.

Basically, this small Montecarlo experiment confirms the previous theoretical findings for sample sizes larger than 500, highlighting however that in small samples, say smaller than 500, the empirical critical values can be considerably higher, in absolute value, than the corrected theoretical values. Consequently, in small samples there could be two reasons to account for this: firstly the theoretical correction, and secondly the behavior of the tests.

IV Empirical Results

In this section we compare the results obtained by Svensson (1991) and (1993), to the new critical values. Svensson (1991) rejects a unit root in six European currencies, the Belgian Franc (BF), the Danish Krone (DK), the French Franc (FF), the Italian Lira (IL), the Irish Pound (IP) and the Ducth Guilder (NG) with respect to the Deutsche Mark (DM). His Table 2 (pg.20) reports the slope and NW standard errors (22) lags for the regression model

$$y(t+22) = \sum \alpha_j d_j + \beta y(t) + e(t-22) \tag{8}$$

where d_j are dummy variables for the realignments.

The implied t-statistics are reported in Table II. If we now compare these t's with the corrected DF critical values for k=22 (1% -4.19, 5% -3.50, 10% -3.14), they lead to accepting the unit root hypothesis at the 5% for the IL and the IP. Nevertheless, the inclusion of several dummies will have the effect of increasing the critical values, and in consequence a test at the 1% is considered more reliable. In this case we also accept a unit root for the NG. Notice also that since the FF is not very far from the critical value at the 1%, accounting for the dummies a unit root could also be accepted.

In Svensson (1993) the estimated model is

$$y(t+65) = \sum \alpha_j d_j + \beta_1 y(t) + \beta_2 i(t)^{65} + \beta_3 i(t)^{*65} + e(t+65) \tag{9}$$

where d_j are again dummy variables for the realignments, and $i(t)^{65}$ and $i(t)^{*65}$ are the home currency and the DM 3-month interest rates respectively.

In this case, his Table 2 (pg. 777) reports the results; Table III contains the implied t-statistics, from where at the 5% the unit root hypothesis should be rejected in the six currencies (note that for k=65, the corrected DF critical values are the same as for k=22). Nevertheless at the 1%, which can be considered more accurate due to the inclusion of the dummies, it is accepted for the IL and the IP as above, although for the NG is rejected in this case. This can be explained in part, by noting that the inclusion of the variables $i(t)^{65}$ and $i(t)^{*65}$, motivates variants of the Dickey-Fuller test that are likely to have critical values somewhat larger than the standard test, and so in some sense, the test for the rest of the currencies can be considered inconclusive.

V Conclusions

In this paper I have tried to unify the classical approach to test for a unit root, with that used in some applied work where a variable is regressed on its lag k-th. It has been shown that the standard DF tables are no longer valid when k is different from 1, but that the DF critical values can be easily corrected. For k larger than 5, as a good rule of thumb a correction of 1.225 times the original DF values works well. Finally I have compared the results of using these new values to those of other empirical work with six currencies of the EMS. While Svensson rejects the unit root hypothesis in all the cases he studies, with the new critical values the unit root hypothesis is accepted in three of them from Sevensson (1991), named the Italian Lira, the Dutch Guilder and the Irish Pound, against the Deutsche Mark, and in two of them from Svensson (1993), named the Italian Lira and the Irish Pound against the Deutsche Mark.

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Table I

T	K	1%	5%	10%	A.B.
DF t_μ		-3.43	-2.86	-2.57	
100	1	-3.63	-2.91	-2.58	-.05
	2	-4.27	-3.41	-3.03	-.10
	5	-4.95	-3.80	-3.35	-.24
	10	-6.29	-4.48	-3.81	-.43
	15	-7.83	-5.49	-4.44	-.60
	20	-10.8	-6.69	-5.41	-.73
500	1	-3.40	-2.80	-2.55	-.01
	2	-3.96	-3.24	-2.94	-.02
	5	-4.22	-3.43	-3.11	-.05
	10	-4.40	-3.54	-3.18	-.10
	15	-4.53	-3.66	-3.23	-.14
	20	-4.67	-3.76	-3.29	-.19
1000	1	-3.43	-2.86	-2.56	-.005
	2	-3.95	-3.31	-2.95	-.01
	5	-4.18	-3.50	-3.11	-.02
	10	-4.21	-3.54	-3.16	-.05
	15	-4.28	-3.56	-3.20	-.07
	20	-4.37	-3.59	-3.22	-.10

1%, 5%, 10% empirical critical values and Asymptotic Bias (AB); theoretical correction factors are 1.15 for k=2, 1.21 for k=5 and 1.22 for k>5.

Table II

	BF/DM	DK/DM	FF/DM	IL/DM	IP/DM	NG/DM
t	-6.00	-5.00	-4.25	-2.50	-3.50	-3.85

DF critical values (cv), (t_μ) at the 1%, 5% and 10% are -3.43, -2.86 and -2.57. Corrected DF cv (k=22) at the 1%, 5% and 10% are -4.19, -3.50 and -3.14.

Table III

	BF/DM	DK/DM	FF/DM	IL/DM	IP/DM	NG/DM
t	-4.80	-7.57	-5.27	-3.64	-4.16	-4.98

DF critical values (cv), (t_μ) at the 1%, 5% and 10% are -3.43, -2.86 and -2.57. Corrected DF cv (k=65) at the 1%, 5% and 10% are -4.19, -3.50 and -3.14.

Mathematical Appendix

PROOF OF PROPOSITION 1. To prove (vii) we write each statistic as a functional of $X_T(s)$, and compute recursively for k=2,3...;

thus

$$y(t)^2=(y(t-2)+e(t)+e(t-1))^2=$$

$$=y(t-2)^2+e(t)^2+e(t-1)^2-2e(t)e(t-1)+2y(t-2)e(t)+2y(t-2)e(t-1)$$

so that

$$y(t-2)e(t)=(1/2)(y(t)^2-y(t-2)^2-e(t)^2-e(t-1)^2-2y(t-2)e(t-1)-2e(t)e(t-1))$$

then

$$T^{-1}\sum_{t=2}^{T} y(t-2)e(t)=$$

$$=(1/2)(T^{-1}\sum_{t=2}^{T}(y(t)^2-y(t-2)^2)-T^{-1}\sum_{t=2}^{T}e(t)^2-T^{-1}\sum_{t=2}^{T}e(t-1)^2+$$
$$-2T^{-1}\sum_{t=2}^{T}y(t-2)e(t-1)-2T^{-1}\sum_{t=2}^{T}e(t)e(t-1))$$

Notice now, that

$$T^{-1}\sum_{t=2}^{T}e(t)^2=T^{-1}\sum_{t=1}^{T}e(t)^2-T^{-1}e(1)^2$$

$$\text{as } T\to\infty \quad \text{plim}\,T^{-1}\sum_{t=2}^{T}e(t)^2=\text{plim}\,T^{-1}\sum_{t=1}^{T}e(t)^2=\sigma^2$$

similarly

$$\text{plim}\,T^{-1}\sum_{t=2}^{T}e(t-1)^2=\sigma^2$$

$$\text{plim}\,T^{-1}\sum_{t=2}^{T}e(t)e(t-1)=0$$

$$T^{-1}\sum_{t=2}^{T}(y(t)^2-y(t-2)^2)=T^{-1}(y(T)^2+y(T-1)^2+\sum_{t=2}^{T-2}y(t)^2-\sum_{t=2}^{T}y(t-2)^2)=$$

$$T^{-1}(y(T)^2+y(T-1)^2+\sum_{t=4}^{T}y(t-2)^2-\sum_{t=2}^{T}y(t-2)^2)=T^{-1}(y(T)^2+y(T-1)^2-y(0)^2-y(1)^2)$$

and using results (i), (ii) and (iii)

$$T^{-1}\sum_{t=2}^{T}y(t-2)e(t)\Rightarrow(1/2)(2\sigma^2W(1)^2-2\sigma^2)-(\sigma^2/2)(W(1)^2-1);$$

$$T^{-1}\sum_{t=2}^{T}y(t-2)e(t)\Rightarrow T^{-1}\sum_{t=1}^{T}y(t-1)e(t)\Rightarrow(\sigma^2/2)(W(1)^2-1)$$

Recursive computation for k=3,4...., gives an expression

$$T^{-1}\sum_{t=k+1}^{T}y(t-k)e(t)\Rightarrow(k\sigma^2/2)W(1)^2-(k/2)\sigma^2-((k-1)\sigma^2/2)(W(1)^2-1)$$

so

$$T^{-1}\sum_{t=k}^{T}y(t-k)e(t)\Rightarrow T^{-1}\sum_{t=1}^{T}y(t-1)e(t)\Rightarrow\sigma^2/2(W(1)^2-1)$$

To prove (viii) and (xix), write

$$y(t)=\rho_k y(t-k)+u(t)\quad \rho_k=1$$

notice

$$u(t)=e(t)+e(t-1)+...+e(t-k+1)$$

so,

$$r_k = \sum_{t=k}^{T} y(t-k)y(t) / \sum_{t=k}^{T} y(t-k)^2 = \rho_k + \sum_{t=k}^{T} y(t-k)u(t) / \sum_{t=k}^{T} y(t-k)^2$$

$$T(r_k - 1) = T^{-1} \sum_{t=k}^{T} y(t-k)(e(t) + e(t-1) + \ldots + e(t-k+1)) / T^{-2} \sum_{t=k}^{T} y(t-k)^2$$

asymptotically for a fixed k as $T \to \infty$

$$T(r_k - 1) \Rightarrow kT^{-1} \sum_{t=1}^{T} y(t-1)e(t) / T^{-2} \sum_{t=1}^{T} y(t-1)^2 \Rightarrow k(1/2)(W(1)^2 - 1) / \int_0^1 W(s)^2 ds$$

also

$$r_k \sim a - 1 \text{ equivalently } plim(r_k) = 1$$

PROOF OF PROPOSITION 2

Notice that

$$plim(s_i) = (k-i)\sigma^2 \qquad \text{for } i = 0, 1, \ldots, k-1$$
$$= 0 \qquad \text{for } i \geq k$$

where s_i as defined (4b)

and write the Newey-West estimator as

$$s_N^2 = s_0 + 2 \sum_{i=1}^{n} (1 - i/(n+1)) s_i$$

then

$$plim(s_N^2) = plim(s_0) + 2 \sum_{i=1}^{n} (1 - i/(n+1)) plim(s_i) =$$

using $s_i = 0$ for $i \geq k$

$$=k\sigma^2+2\sum_{i=1}^{k\ 1}(1-i/k)(k-i)\sigma^2=$$

$$\sigma^2(k+2\sum_{i=1}^{k-1}(k\ \ i)\ \ 2\sum_{i=1}^{k-1}i(k\ \ i)/k=$$

using $\sum_{i=1}^{m} i-m(m-1)/2;\ \sum_{i=1}^{m} i^2-m(m+1)(2m+1)/6$

we get

$plim(s_N^2)=\sigma^2(k+2(k\ \ 1)k\ \ (k\ \ 1)k\ \ k(k\ \ 1)+(k\ \ 1)(2k\ \ 1)/3);$

finally

$$plim(s_N^2)=\sigma^2(2k^2+1)/3$$

PROOF OF PROPOSITION 3

Write

$$t_{\alpha k}=(r_k-1)/std(r_k)=(r_k-1)/(s_N^2/\sum_{t=k}^{T}y(t-k)^2)^{\frac{1}{2}}=$$

$$=T(r_k-1)(T^{-2}\sum_{t=k}^{T}y(t-k)^2)^{\frac{1}{2}}/s_N$$

using results (viii), (x) above

$$t_{\alpha k}\Rightarrow\frac{k}{((2k^2-1)/3)^{\frac{1}{2}}}(1/2)(W(1)^2-1)/(\int_0^1 W(s)^2ds)^{\frac{1}{2}}$$

so

$$t_{\alpha k}\Rightarrow\frac{k}{((2k^2-1)/3)^{\frac{1}{2}}}t_\alpha$$

finally

$$\lim_{k\to\infty}\frac{k}{((2k^2-1)/3)^{\frac{1}{2}}}=(3/2)^{\frac{1}{2}}=1.2247$$

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