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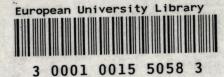## Testing the Joint Hypothesis of Rationality and Neutrality under Seasonal Cointegration: The Case of Korea

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TESTING THE JOINT HYPOTHESIS OF RATIONALITY AND NEUTRALITY UNDER SEASONAL COINTEGRATION: THE CASE OF KOREA

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Abstract

With seasonally adjusted data, and using a procedure based on non-seasonal cointegration, the macro rational expectations hypothesis of rationality and money neutrality is rejected at the 10% level. However, with seasonally unadjusted data, and using a procedure based on seasonal cointegration, the same hypothesis is not rejected. The paper thus provides an example of how the application of deseasonalization procedures variable by variable can distort empirical inference, in a manner which parallels the practice of differencing variable by variable without taking into account the presence of cointegrating relations.

**Testing the Joint Hypothesis of Rationality and Neutrality
under Seasonal Cointegration: the Case of Korea**

1. Introduction

The purpose of this paper is twofold: firstly, to illustrate with seasonally *unadjusted* Korean data a procedure based on seasonal cointegration (SC) for testing the macro rational expectations hypothesis of rationality and money neutrality (RMN); secondly, to provide, by comparing the result of this test with that obtained with seasonally *adjusted* data, an example of how the application of deseasonalization procedures variable by variable (such as the X-11 procedure) can distort empirical inference. As explained below, this distortion parallels the one associated with the detrending practice of differencing variable by variable without taking into account the presence of cointegrating relations.

The RMN hypothesis was initially tested by Barro [1981] and Mishkin [1982a, 1982b] for the US economy with seasonally adjusted data. With different choices of regressors and dynamics, but with essentially the same framework, they rejected neutrality but not rationality. Since then, much research has been devoted - mostly with US data - to modify or expand Barro's and Mishkin's models (for recent summaries, see Spencer [1989], Sephton [1990], and Bohara [1991], among others): trend-stationarity has been replaced with difference-stationarity of the relevant variables; the implied unidirectional causality of money to output has been replaced with bi-directional causality in a multi-variate approach (specifically, VAR); the bivariate (money, output) analysis has been extended to a trivariate (money, output, interest rate) analysis. Although most of this research has confirmed Barro's and Mishkin's results, recently Ermini and Chang [1993] could not reject money neutrality, in addition to rationality. Using US seasonally adjusted data and applying Johansen's [1988] multi-variate cointegration analysis to a tri-variate system of money, output and interest rate, they attribute the non-rejection of money neutrality to the significance of an error-correction term that takes into account the presence of a cointegrating relation within the system. It is now well accepted that analyzing a vector

autoregressive system (VAR) in first differences without taking into account this possibility may lead to incorrect inference due to mis-specification (as in Bohara [1991]). To distinguish from seasonal cointegration, we refer to Johansen's [1988] approach as non-seasonal cointegration (NSC).

Under the rational expectations hypothesis, only the unexpected component of money may have real effects; money neutrality thus refers only to the ineffectiveness of its anticipated or predictable component. In Barro's and Mishkin's framework, money is neutral if the explanatory power of predictable money for real output growth is negligible compared to the explanatory power of unpredictable money. It follows that money neutrality may also be the consequence of the predictable component of money being quite small in variance compared to the unpredictable component. In this sense, the non-rejection of money neutrality for the US case, obtained in Ermini and Chang [1993], is consistent with the consensus view that monetary policy in the US has been typically conducted with a high degree of unpredictability. A similar consensus does not exist for the case of South Korea: its monetary policy is believed to have been effective to promote development in the past three decades, but it is also believed to have been conducted in a systematic and predictable way, and this would imply ineffectiveness under the rational expectations neutrality hypothesis (see *inter alia* Cole and Park [1983]). It is thus quite interesting to test the RMN hypothesis for Korea.

Unlike US data, Korean data is published seasonally unadjusted. Until recently, that is until the development of SC (see Hylleberg, Engle, Granger and Yoo [1990], henceforth HEGY, and Lee [1992]), an investigator conducting empirical work on Korea's economy would have deseasonalized the data series by series, using for example the X-11 procedure. The recent development of SC instead provides a framework to analyze seasonally unadjusted data directly; moreover, it does so in a system context rather than variable by variable. A growing consensus among economists recognizes this approach as more appropriate, as seasonal adjustment procedures may eliminate from the data valuable information and may distort unit root tests, cointegration tests, and the like (see, for example, Jaeger and Kunst [1990], and Ghysels and Perron [1993]; however, see also Lee and Siklos [1991a] for a different conclusion). By testing the RMN

hypothesis for South Korea with both approaches, the paper provides an interesting comparison. Indeed, whereas with X-11 adjusted data and the application of NSC analysis the RMN hypothesis is rejected at the 10% significance level, RMN is no longer rejected with seasonally unadjusted data and the application of SC analysis.

Incidentally, as the test of the RMN hypothesis is conducted by estimating stable cointegrating relations within a four-variable system comprising money, interest rate, price and output, the paper also provides estimates of money demand functions at various frequencies. To limit the paper's focus to the RMN hypothesis and its implication for Korea, a study of these cointegrating relations as money market equilibrium conditions, and the tests of appropriate parameters restrictions, will be the object of a separate work (for cointegration studies of money demands, see Johansen and Juselius [1990] for Finland-Denmark, and Hafer and Jansen [1991] for the U.S.).

The paper is organized as follows. Section 2 describes the RMN test with seasonally *adjusted* data, based on non-seasonal cointegration. The procedure adopted is identical to the procedure used in Ermini and Chang [1993] with seasonally adjusted US data, except for the addition of the price level. Section 3 summarizes seasonal cointegration analysis. Section 4 describes the RMN test with seasonally *unadjusted* data; this test procedure, based on seasonal cointegration, is new in the literature. Finally, Section 5 provides some concluding remarks.

2. The RMN Test with Seasonally Adjusted Data

The variables of interest are money M_t (the broad measure $M2$), interest rate r_t (a government bond rate), price P_t (the GNP deflator, 1985 = 100), and output Y_t (real GNP). The series are the natural log of the seasonally unadjusted, quarterly, 1970.I-1991.IV, series published in the Monthly Statistical Bulletin of the Bank of Korea. For the RMN test with seasonally adjusted data, the three series of money, price and GNP were filtered through the X-11 ARIMA procedure of the SAS [1990] program. The X-11 procedure is known for successfully eliminating spectral power peaks at the seasonal frequencies, i.e. $\pi/2$ and π corresponding to one cycle and two cycles per year respectively; it is also known for distorting the series spectra

at other frequencies thus eliminating useful information from the data (for a discussion, see Granger and Newbold [1986], Hylleberg [1992]).

The test procedure follows Ermini and Chang [1993], which extends the original Barro's [1981] and Mishkin's [1982a] framework consisting of two equations in growth rates: a forecasting equation for money, whose role is to decompose money growth into a predictable and an unpredictable component; an equation for real output to assess the relative explanatory power of these two components on output growth. In this framework, the money forecasting equation is given the general form:

$$A(B)\,\Delta M_t = \mu + C(B)\,Z_{t-1} + u_t \tag{1}$$

where ΔM_t is the growth rate of $M2$ (approximated by the log-difference); μ is a constant; Z_t is a vector of variables that have predictive power for ΔM_t (for example , the interest rate and the inflation rate); and u_t is the one-step ahead forecasting error. $A(B)$ is a polynomial in the lag operator B (i.e., such that $M_{t-k} = B^k\,M_t$), and $C(B)$ is a row-vector of polynomials in B. If the forecast of money is optimal (under mean square error) and unbiased, u_t is a zero-mean white noise process; this is a necessary condition under which the forecast is rational in the rational expectations sense. The output equation is given the general form:

$$F(B)\,\Delta Y_t = G(B)\,(\Delta M_t - P_{t-1}[\Delta M_t]) + H(B)\,P_{t-1}[\Delta M_t] + e_t\,, \tag{2}$$

where $P_{t-1}[.]$ stands for forecast, not necessarily rational. The autoregressive-distributed-lag model (2) explains output growth both with the predicted and unpredicted component of money growth. Money neutrality in the rational expectations sense implies $H(B) \equiv 0$; in this case, output growth depends only on the unpredicted component of money. In addition to testing neutrality, we can also jointly test forecast rationality by imposing the additional restriction that $\Delta M_t - P_{t-1}[\Delta M_t]$ is a zero-mean white noise process (i.e., the innovation u_t of (1)).

This two-equation framework can be generalized by noting that equation (1) entails only unidirectional causality from the vector Z_t to money. As we cannot exclude a priori multidirectional causality among these variables, the money equation can be thought of being part of a larger vector autoregressive (VAR) system:

$$Q(B) X_t = \mu + \nu_t , \qquad (3)$$

where X_t is the vector $[M_t, r_t, P_t, Y_t]'$; μ is a vector of constants; the matrix polynomial $Q(B)$ of degree k is equal to $I - \sum_{j=1}^{k} Q_j B^j$; and ν_t is a vector of innovations[1]. As we are interested in growth rates, this four-variable system in levels can be transformed into a system in first differences, by making use of Johansen's [1988] isomorphic representation:

$$D(B) \Delta X_t = \mu + \Pi X_{t-k} + \nu_t , \qquad (4)$$

where the matrix polynomial $D(B)$ of degree $k - 1$ is equal to $I - \sum_{j=1}^{k-1} D_j B^j$, with $D_j = -(I - \sum_{i=1}^{i=j} Q_i)$, and $\Pi = - Q(1)$. The money equation now corresponds to the first row of the system (4), in which money forecasts depend on past values of the interest rate, the price level, output and money.

Johansen shows that the rank r of the matrix Π determines the nature of integratedness of the system: for an N-dimension vector X_t, $N - r$ gives the number of unit roots in the system, that is the number of first-order integrated, or I(1), components; and r gives the number of cointegrating relations, that is the number of stationary, or I(0), components. Here for system components we mean linear combinations of the elements of the vector X_t (which, in fact, could all be I(1), regardless of the value of r). If $r = 0$, that is if $\Pi \equiv 0$, then all the N components of X_t are I(1), and no cointegrating relation exist among them. If $r = N$, then all the components of X_t are I(0). Finally, if $0 < r < N$, only $N - r$ components of X_t are I(1). It follows that, if the system is estimated as a VAR in first differences without the additional term ΠX_{t-k} as in (4), r components would be overdifferenced. This would amount to a loss of information about their long-run behavior (the spectral power at zero frequency of the overdifferenced component

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1. In fact, due to symmetry of the X-11 filter, the filtered series will contain a non-invertible moving average component. Thus, a pure VAR representation of the seasonally adjusted series may not exist (Maravall [1993]). This adds a further argument against the use of seasonally adjusted data for empirical inference.

would be forced to zero). The term ΠX_{t-k} is thus added precisely to restore this loss. In this case, the matrix Π can be decomposed into the product $\alpha \beta'$ of two Nxr matrices, and the r elements of $\beta' X_{t-k}$ are called error-correction terms. System (4) is correspondingly called an error-correction model (ECM).

Johansen develops a test to determine the rank of Π, based on maximum likelihood estimation; Johansen and Juselius [1990] extend this test to the case with constant terms and seasonal dummies. Although not widely recognized in the literature, this test is simultaneously a unit root test, in that it does not require a pretest of each single component of X_t for unit root[2]. We first estimated (4) for several values of k, and obtained $k = 4$ as the optimal lag length based on likelihood ratio tests (the same value was also obtained optimally using the AIC criterion in system version)[3]. Then, we applied Johansen and Juselius [1990] procedure to the case $k = 4$, and found one significant cointegrating relation (table 1). The corresponding error-correction term, obtained by multiplying the first row of the estimated β' with the vector X_t, is:

$$W_t = M_t + 1.50\, r_t - 0.88\, P_t - 1.2\, Y_t. \qquad (5)$$

The interpretation of this stationary cointegrating relation is based on the notion of equilibrium in the money market: real money supply (in log, $M_t - P_t$) equals money demand plus a stationary error W_t; money demand is a linear function of output and interest rate. Notice that the parameters in (5) have all the right signs: the income elasticity of money demand is positive; the interest rate elasticity is negative [4].

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2. Nonetheless, unit root tests on each single component of X_t were performed, based both on Dickey-Fuller's (Dickey and Fuller [1981]) and Perron's (Perron [1989]) procedure. Based on these tests, we cannot reject the hypothesis of a unit root in each of the elements of X_t. These results are available on request from the authors.

3. Starting with $k = 5$, the sequence of LR test statistics was 29.15 for 5 vs. 4, 40.48 for 4 vs. 3, 30.35 for 3 vs 2, against a critical value of $\chi^2(16)$ of 26 at 5% and 32 at 10%.

4. Johansen and Juselius [1990] also provide means to test restrictions on the parameters of (5), with test statistic distributed as χ^2 with $r(N - s)$ degrees of freedom, where s is the number of restrictions. For consistency with the money demand interpretation of (5) (see also Hafer and Jansen [1991] with U.S. data), we tested whether the interest rate elasticity of money demand is zero, when the coefficients of P_t and Y_t are both restricted to one: a test statistic of 3.21 against a 5% critical value of $\chi^2(1)$ of

Regarding the output equation, consider the general (or unrestricted) case that the money forecast is not rational. In this case, $P_{t-1}[\Delta M_t]$ would be determined linearly through a structure similar to the first row of (4), but with different parameters; that is, rewriting $\sum_{j=1}^{k-1} D_j B^j$ as a 4x4 matrix $[\sum_{j=1}^{k-1} d_{ik,j}B^j] \equiv [d_{ik}(B)]$, and letting X_j be the j-th element of the vector X, one would have:

$$P_{t-1}[\Delta M_t] = \sum_{j=1}^{4} d_{1j}^*(B)\, \Delta X_{j,t} + \mu_1 + \alpha_1^* W_{t-4}, \tag{6}$$

where * indicates parameters in general different from those corresponding to a rational forecast. Substituting (6) into the output equation (2), and replacing the fourth row of system (4) with the output equation so derived, we obtain the following four-variable "unrestricted" [5] error-correction model:

$$\begin{bmatrix} \overline{D}(B) \\ N_1^*(B)\ N_2^*(B)\ N_3^*(B)\ F(B) \end{bmatrix} \begin{bmatrix} \Delta M_t \\ \Delta r_t \\ \Delta P_t \\ \Delta Y_t \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha^*(B) \end{bmatrix} W_{t-4} + \mu + \nu_t, \tag{7}$$

where $\overline{D}(B)$ corresponds to the first three rows of $D(B)$ in (4) (with the restriction $d_{14}^*(B) = 0$), and where

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3.84 seems to corroborate the classical money demand for Korea.

5. "Unrestricted" with respect to the rationality and neutrality hypotheses; the model still exhibits some zero restrictions, imposed after parsimonializing (4) by eliminating from further estimation insignificant coefficients. Particularly, it was found that output has no predictive power for money, except for its presence in the error-correction term. Correspondingly, we set $d_{14}^*(B) \equiv 0$ in (7).

$$N_1^*(B) = d_{11}^*(B) [G(B) - H(B)] - G(B) \qquad (8)$$
$$N_2^*(B) = d_{12}^*(B) [G(B) - H(B)]$$
$$N_3^*(B) = d_{13}^*(B) [G(B) - H(B)]$$
$$\alpha^*(B) = \alpha_1^* [H(B) - G(B)].$$

In (8) $N_j^*(B)$, $j = 1, 2, 3$, are polynomials of degree 4, and $\alpha^*(B)$ is a polynomial of degree 1, so that the error-correction term in (7) appears as W_{t-4} and W_{t-5}[6]. Also, note that the fourth element of μ includes a linear combination of μ_1.

To test the RMN hypothesis, we impose on (7) the neutrality condition $H(B) \equiv 0$ and the rationality condition $d_{1j}^*(B) = d_{1j}(B)$ for $j = 1, 2, 3$, obtaining the following set of eight non-linear cross-equation restrictions:

$$\phi_{11}^1 \phi_{41}^1 + \phi_{41}^2 = 0 \qquad (9)$$
$$\phi_{12}^1 \phi_{41}^1 + \phi_{42}^2 = 0$$
$$\phi_{13}^1 \phi_{41}^1 + \phi_{43}^2 = 0$$
$$\phi_{11}^2 \phi_{41}^1 + \phi_{41}^3 = 0$$
$$\phi_{13}^2 \phi_{41}^1 + \phi_{43}^3 = 0$$
$$\phi_{11}^3 \phi_{41}^1 + \phi_{41}^4 = 0$$
$$\phi_{13}^3 \phi_{41}^1 + \phi_{43}^4 = 0$$
$$\psi_1^0 \phi_{41}^1 + \psi_4^1 = 0$$

where ϕ_{ij}^k is the (i,j)-th element of the 4x4 matrix of coefficients at lag k of the "unrestricted" model (7), and ψ_j^s is the j-th element of the vector of coefficients associated with the error-correction term W_{t-4} for $s = 0$, and W_{t-5} for $s = 1$.

Using the Wald test, these restrictions can be tested without re-estimating the restricted model (for a detailed description of the procedure, see, for example, Granger and Newbold [1986]). We obtained a test statistic of $W = 13.20$, which has a P-value of about 10% (the χ^2 critical value for

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6. Through sequential likelihood ratio tests, the orders for the polynomials $F(B)$, $G(B)$ and $H(B)$ to derive (7)-(8) were parsimoniously set at 2,1 and 1 respectively, although these values are marginally rejected against 2,2,2.

eigth degrees of freedom is 13.36 at 10% confidence level, and 15.51 at 5%). So, at the 10% confidence level, we may reject, although not unambiguously, the RMN hypothesis.

3. Seasonal Cointegration Analysis

In recent years, much research has been devoted to the controversy on the deterministic vs. stochastic nature of the trend component in macroeconomic series. Although the issue of seasonality parallels the issue of the trend component, a similar debate has not occurred for the treatment of seasonality, perhaps as a result of most empirical work being done with US data which is mostly available only seasonally adjusted. The usual treatment of seasonality is to model it deterministically through seasonal dummies, although Box and Jenkins [1976] had already advocated an alternative model based on the seasonal differencing operator, $1 - B^s$, with s the number of periods per year (for example, 4 for quarterly data, 12 for monthly data). It is easily seen that the relation between seasonal dummies and seasonal differencing is similar to the relation between detrending and first-differencing (for a discussion, see also Kunst [1993]).

Limiting the discussion to quarterly data, and thus to the seasonal operator $\Delta_4 = 1 - B^4$, Box and Jenkins [1976] proposed the general SARIMA model

$$A(B) \Delta_4 X_t = C(B) \varepsilon_t, \tag{10}$$

which would capture, in addition to autoregressive and moving-average stationary components, the long-run peak at zero frequency and seasonal peaks at the seasonal frequencies $\pi/2$ and π. The fourth-differencing operator has four unit roots, at ± 1 and at $\pm i$. The unit root at $+1$ produces a (theoretically infinite) peak at zero frequency; the root -1 produces a (theoretically infinite) peak at frequency π, corresponding to 2 cycles per year with quarterly data; the root $+i$ produces a peak at $\pi/2$, corresponding to one cycle per year; finally, the root $-i$ produces a peak at $-\pi/2$, again corresponding to one cycle per year. Thus, although the fourth-differencing operator exhibits four unit roots, in practice we can concentrate the analysis on three unit roots, at 0, $\pi/2$ and π. Correspondingly, we use the notation $I_k(1)$, with $k = 0, \pi/2, \pi$, to indicate the presence of unit root at each of the

three frequencies respectively.

Considering that $1 - B^4 = (1 - B)(1 + B)(1 + B^2)$, and following HEGY, define:

$$
\begin{aligned}
Y_{1,t} &= (1 + B)(1 + B^2)\, X_t = (1 + B + B^2 + B^3)\, X_t \qquad (11) \\
Y_{2,t} &= (1 - B)(1 + B^2)\, X_t = (1 - B + B^2 - B^3)\, X_t \\
Y_{3,t} &= (1 - B^2)\, X_t \,.
\end{aligned}
$$

Thus, $Y_{1,t}$ is a $I_o(1)$ process, as it is fully deseasonalized and retains only the unit root at zero frequency (in fact, from (10), $A(B)(1-B)\,Y_{1,t} = C(B)\,\varepsilon_t$); $Y_{2,t}$ is a $I_\pi(1)$ process, as it retains only the unit root at -1; and $Y_{3,t}$ is a $I_{\pi/2}(1)$ process, as it retains only the unit roots at $\pm\, i$.

In a multi-variable framework, similarly to the concept of cointegration for I(1) series, we can define seasonal cointegration for seasonal integrated series: there can exist linear combinations of them which no longer exhibit unit roots at the zero frequency, or at $\pi/2$, or at π; moreover, these linear combinations need not be the same for all the frequencies. More specifically, we have cointegration at zero frequency when a linear combination of X_t retains only the unit roots at $\pi/2$ and π; a double cointegration at zero and $\pi/2$ when a linear combination of X_t retains only the unit root at π; and so forth for all the possible cases.

Consider an N-dimensional VAR system like (3), where X_t is now seasonally unadjusted. Under the hypothesis that seasonality occurs only at $\pi/2$ and π (or equivalently, under the hypothesis that the system belongs to the class of fourth-differencing), Lee [1992] extends to seasonal cointegration Johansen's [1988] approach, by analyzing the isomorphic seasonal error-correction model (SECM):

$$
D(B)\,\Delta_4 X_t = \mu + \sum_{j=1}^{4} \Pi_j\, Y_{j,t\text{-}1} + \varepsilon_t \,, \qquad (12)
$$

where $Y_{j,t}$ for $j = 1, 2, 3$ are the series defined in (11), and $Y_{4,t\text{-}1} = Y_{3,t}$; the matrix polynomial $D(B) = I - \sum_{j=1}^{k} D_j B^j$ is appropriately redefined from $Q(B)$ of (3). (Note that if the order of $Q(B)$ is p, the order of $D(B)$ is $p - 4$.)

Similarly to Johansen's approach to non-seasonal cointegration, Lee's approach to seasonal cointegration establishes that the nature of

integratedness of the seasonal system (12) is determined by the rank r_j of the matrices Π_j. Specifically, $N - r_j$ gives the number of unit roots in the system at frequency zero ($j = 1$), π ($j = 2$), and $\pi/2$ ($j = 3$) respectively, and r_j the number of cointegrating relations, or $I_j(0)$ components, in the system at the same frequencies[7]. If $0 < r_j < N$, Π_j can be decomposed into the product $\alpha_j \beta_j{'}$ of two $N x r_j$ matrices, and the r_j stationary elements of $\beta_j{'} \, Y_{j,t}$ are the cointegrating relations or error-correction terms associated with that frequency. Procedures have been developed to test whether these relations are the same or not across frequencies (again, HEGY and Lee [1992]). Given the paper's focus on the RMN hypothesis, these tests are not performed here.

As for the case of non-seasonal cointegration, these seasonal cointegrating relations can be interpreted as equilibrium conditions based on economic theory. However, economic theory is typically concerned with long-run equilibria (zero-frequency), more than with stable relations describing seasonal behavior. It is thus an open and interesting area of research the formulation of theoretical model of agents' behavior that may support cointegrating relations at the seasonal frequencies, if these are found to be significantly different from the long-run relations.

4. The RMN Test with Seasonally Unadjusted Data

Similarly to the previous section, the first step is to estimate the SECM (12) with $X_t = [M_t, r_t, P_t, Y_t]{'}$, from which the forecast equation for money can be derived. Following Lee's [1993] suggestion of adding deterministic seasonal dummies to the system (12), the RMN hypothesis was tested in both cases, with and without deterministic seasonal dummies, obtaining identical results as far as the RMN hypothesis is concerned, but different

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7. Cointegration at $\pi/2$ can be studied for $\Pi_3 \neq 0$ and $\Pi_4 = 0$ (corresponding to synchronous seasonal cycles), or for $\Pi_3 = 0$ and $\Pi_4 \neq 0$ (corresponding to cycles shifted in phase)(see HEGY and Lee [1992] for details). Similarly to other works on seasonal cointegration, this paper also assumes synchronous cycles, and investigates cointegration at $\pi/2$ under the assumption $\Pi_4 = 0$ (see also Lee [1992], Kunst [1993] and Engle, Granger, Hylleberg and Lee [1993]).

cointegrating relations at the seasonal frequencies. For the sake of exposition, we report only the results obtained with deterministic seasonal dummies added to $(12)^8$. Through sequential likelihood ratio tests, the degree of the autoregressive component $D(B)$ was set at two (with statistics of 26.65 for 3 vs. 2, and of 40.5 for 2 vs. 1, against critical values of $\chi^2(16)$ of 26.3 at 5% and 32 at 10%). Table 2 reports the results of seasonal cointegration analysis. At the 5% level we found one cointegrating relation at the zero frequency, one relation at the biannual frequency π, but no cointegrating relation at the annual frequency $\pi/2^9$. An important comment on the sources of the critical values for these tests is reported in the footnote attached to the table. The interpretation of these results is as follows. At the zero frequency, the system exhibits three unit roots and one cointegrating relation, obtained by multiplying the first row of $\beta_1{}'$ by $Y_{1,t}$, e.g:

$$W_{1,t} = \Delta_1 M_t + 0.38\,\Delta_1 r_t - 0.88\,\Delta_1 P_t - 1.27\,\Delta_1 Y_t , \qquad (13)$$

where $\Delta_1 = (1 + B)(1 + B^2)$. This cointegrating relation can be given the same money market equilibrium interpretation of the previous section, once the relevant variables are filtered through the filter Δ_1. Interestingly, with the exception of the coefficient for the interest rate, (13) has the same coefficients of (5) for price and output.

At the biannual frequency π, the system also exhibits three unit roots and one cointegrating relation, obtained by multiplying the first row of $\beta_2{}'$ by $Y_{2,t}$, e.g.:

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8. Other authors also have tested stochastic seasonality with and without deterministic seasonal dummies (for example, Lee and Siklos [1991a] for unit root tests of selected Canadian macrovariables; Kunst [1993] for a comparison of seasonal cointegration across a set of European countries). No final answer is yet available as to whether a SECM should be estimated with or without seasonal dummies. It is worth noting, however, that adding deterministic seasonal dummies overparametrizes seasonality, thus granting it too much freedom; it may also reduce the power to discriminate between deterministic and stochastic seasonality.

9. The absence of cointegration at the annual frequency is somewhat questionable - see the footnote to table 2. However, by adding one cointegrating relation at this frequency did not change the result of the RMN test.

$$W_{2,t} = \Delta_2 M_t - 0.40 \, \Delta_2 r_t + 0.01 \, \Delta_2 P_t - 0.05 \, \Delta_2 Y_t \,, \qquad (14)$$

where $\Delta_2 = (1 - B)(1 + B^2)$. For lack of a pertinent economic theory describing the seasonal behavior of money demand at the frequency of two cycles per year, this cointegrating relation cannot be given at this stage a specific interpretation in terms of money market equilibrium. Finally, for the case of the annual frequency $\pi/2$, as $r_3 = 0$ is not rejected, we can conclude that the system exhibits four unit roots and no cointegrating relation. Thus, the forecast equation of money contains two error-correction terms, associated with the frequencies 0 and π. Compare with the case of seasonally adjusted data.

The second step is to replace the forecast for money into the output equation, which is now written in fourth-differencing as

$$F(B) \, \Delta_4 Y_t = G(B)(\Delta_4 M_t - P_{t-1}[\Delta_4 M_t]) + H(B) \, P_{t-1}[\Delta_4 M_t] + e_t \,. \quad (15)$$

As in the seasonally adjusted data case, the unrestricted model (unrestricted with regard to the rationality hypothesis) contains a forecast of money exhibiting a structure similar to the first row of (12), but with different parameters; that is, with notation identical to (6) (intercept and seasonal dummies not reported):

$$P_{t-1}[\Delta_4 M_t] = \sum_{j=1}^{4} d^*_{1j}(B) \, \Delta_4 X_{j,t} + \alpha^*_{1,1} W_{1,t-1} + \alpha^*_{1,2} W_{2,t-1} \,, \qquad (16)$$

where the two error-correction terms are defined in (13) and (14). Substituting (16) in (15), and replacing the fourth row of (12) with the output equation so derived, we obtain the following "unrestricted" seasonal error-correction model (intercepts and seasonal dummies not reported):

$$\begin{bmatrix} \overline{D}(B) \\ N^*_1(B) \ N^*_2(B) \ N^*_3(B) \ N^*_4(B) \end{bmatrix} \begin{bmatrix} \Delta_4 M_t \\ \Delta_4 r_t \\ \Delta_4 P_t \\ \Delta_4 Y_t \end{bmatrix} = \begin{bmatrix} \alpha_{1,1} \\ \alpha_{2,1} \\ \alpha_{3,1} \\ \alpha^*_1(B) \end{bmatrix} W_{1,t-1} + \qquad (17)$$

$$\begin{bmatrix} \alpha_{1,2} \\ \alpha_{2,2} \\ \alpha_{3,2} \\ \alpha^*_2(B) \end{bmatrix} W_{2,t-1} + \varepsilon_t \,,$$

where $\overline{D}(B)$ corresponds to the first three rows of $D(B)$ in (12); where

$N_j^*(B)$, $j = 1, 2, 3$, are defined as in (8); and where

$$N_4^*(B) = F(B) - d_{14}^*(B)[H(B) - G(B)] \tag{18}$$
$$\alpha_k^*(B) = \alpha_{1,k}[H(B) - G(B)] \ , \ k = 1, 2.$$

Through sequential likelihood ratio tests, the order of $F(B)$, $G(B)$ and $H(B)$ was chosen, as in the previous case, as 2, 1, and 1 respectively. Correspondingly, the order of $N_j^*(B)$ is 2, and of $\alpha_k(B)$ is one; the two error-correction terms thus appear as $W_{k,t-1}$ and $W_{k,t-2}$, $k = 1,2$. No zero restrictions were imposed on $\overline{D}(B)$.

To impose the RMN hypothesis, we impose on (17) the neutrality condition $H(B) \equiv 0$ and the rationality condition $d_{1j}^*(B) = d_{1j}(B)$, for all j, obtaining the following set of six non-linear cross-equation restrictions:

$$\begin{aligned}
\phi_{11}^1 \phi_{41}^1 + \phi_{41}^2 &= 0 \\
\phi_{12}^1 \phi_{41}^1 + \phi_{42}^2 &= 0 \\
\phi_{13}^1 \phi_{41}^1 + \phi_{43}^2 &= 0 \\
\phi_{11}^2 \phi_{41}^1 + \phi_{41}^3 &= 0 \\
\psi_{11}^0 \phi_{41}^1 + \psi_{14}^1 &= 0 \\
\psi_{21}^0 \phi_{41}^1 + \psi_{24}^1 &= 0,
\end{aligned} \tag{19}$$

where ϕ_{ij}^k is the (i, j)-th element of the 4x4 matrix of coefficients at lag k of the "unrestricted" model (17), and ψ_{ij}^s is the j-th element of the column vector i ($= 1, 2$) of coefficients associated with the error-correction terms $W_{i,t-1}$ for $s = 0$, and $W_{i,t-2}$ for $s = 1$.

By testing the restrictions (19) through the Wald test, we obtained a statistic $W = 0.21$, compared with a χ^2 critical value for six degrees of freedom of 12.59 at 5% level, and 10.64 at 10%. It is thus seen that applying seasonal cointegration analysis to seasonally unadjusted data, we strongly cannot reject the RMN hypothesis at the 10% level, as opposed to the case with seasonally adjusted data in which at the same confidence level we obtained a border-line statistic. This reversal of test result can be attributed to the significance of the error-correction term associated with frequency π. Heuristically, the application of the X-11 filter is essentially equivalent to the application of the filter $(1 + B)(1 + B^2)$ variable by variable, which removes 4 unit roots at each seasonal frequency. However, while at frequency $\pi/2$ this removal is legitimate in that the absence of cointegrating relations at

this frequency entails the existence of precisely 4 unit roots in the system, at frequency π this removal is not legitimate: the existence of one significant cointegrating relation at this frequency implies the existence of only three unit roots in the system at π. Therefore, the application of the filter overdifferences one system component at this frequency, resulting in mis-specification. The presence of the error-correction $W_{2,t-1}$ in (17) eliminates this mis-specification.

5. Conclusions

This paper illustrates with seasonally *unadjusted* Korean data a procedure based on seasonal cointegration for testing the macro rational expectations hypothesis of rationality and money neutrality. Based on a Wald test, we cannot reject the hypothesis at the 10% confidence level. The paper also tests the hypothesis with the same data seasonally *adjusted* through application of the X-11 filter; interestingly, the Wald statistic for this case has a *P*-value of about 10%, thus making the test result somewhat ambiguous at the 10% level.

Indirectly, this comparison provides an example of how the application of deseasonalization procedures variable by variable can distort empirical inference, in a manner which parallels the practice of detrending by differencing variable by variable, without taking into account the presence of cointegrating relations. More specifically, the paper finds that at the seasonal frequency of π the system composed of money, interest rate, price and real output exhibits three unit roots and one cointegrating relation. As the X-11 procedure is essentially equivalent to differencing at that frequency (it eliminates the spectral power peak at that frequency), the application of the X-11 filter to each of the four variables entails overdifferencing - and thus loss of information - of the system component corresponding to the cointegrating relation. Seasonal cointegration analysis, on the other hand, permits to deseasonalize the system without losing this information, as it restores this information by identifying and adding appropriate error-correction terms. The message then is to use seasonally unadjusted data whenever possible, and to deseasonalize the series by directly modelling stochastic seasonality, instead of filtering the data through procedures of seasonal adjustment like the X-11. An additional problem arising from

seasonal adjustment filters is their induction of non-invertible moving average components in the filtered series. These components invalidate the existence of the pure autoregressive representations typically adopted in the literature with seasonally adjusted data.

As a side product, the paper identifies stable cointegrating relations among the four variables both at frequency zero and π. The interpretation of these relations as money market equilibrium conditions, together with a complete analysis of these relations in terms of parameter restrictions, will be the object of a separate work.

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TABLE 1	Cointegration Test at Zero Frequency with Seasonally Adjusted Data [11]			
$r\,(\Pi)$	λ_{max}	$\lambda_{max}(.05)$	Trace	Trace (.05)
$r \leq 3$	0.226	8.083	0.226	8.083
$r \leq 2$	9.747	14.595	9.974	17.844
$r \leq 1$	17.039	21.279	27.013	31.256
$r = 0$	35.626	27.341	62.639	48.419

11. Critical values from table A2 of Johansen and Juselius [1990].

TABLE 2 *Cointegration Tests with Seasonally Unadjusted Data -*
Case with Intercept and Seasonal Dummies [13]

r (Π)	λ_{max}		Trace	Trace (.05)
		$\omega = 0$		
$r \leq 3$	0.005		0.005	8.6
$r \leq 2$	15.910		15.911	19.3
$r \leq 1$	18.564		34.480	34.5
$r = 0$	43.604		78.084	48.42
		$\omega = \pi$		
$r \leq 3$	1.673		1.673	8.6
$r \leq 2$	7.344		9.017	19.3
$r \leq 1$	15.101		24.119	34.4
$r = 0$	27.303		51.422	48.42
		$\omega = \pi/2$		
$r \leq 3$	0.162		0.162	11.9
$r \leq 2$	4.144		4.306	24.3
$r \leq 1$	12.265		16.571	40.6
$r = 0$	26.072		42.643	..

13. Lee and Siklos [1991b] report the finite-sample trace critical values for the case with intercept and seasonal dummies, but only for $N = 3$ (also, they do not report the finite-sample critical values for the λ_{max} statistic). So, the first three critical values reported here for each frequency are taken from Lee and Siklos, for a sample size of 100. As Lee [1992] shows that the asymptotic distribution of the rank test for both Π_1 and Π_2 is the same as the asymptotic distribution of non-seasonal cointegration, regardless of whether it is known *a priori* that some of the Π's are zero, the fourth trace critical values for frequency zero and frequency π are taken from the asymptotic distributions reported in Johansen and Juselius's [1990] table A2. For the case of frequency $\pi/2$, the critical value is not available, but a simple extrapolation shows that the hypothesis $r = 0$ is likely not to be rejected.

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