Exchange Rate Targets: Models and Design

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It happened that a grower gave [the blind man] a bunch of grapes as charity. As they were very ripe, the bunch of grapes broke up in his hands. He decided to have a feast. We sat on a fence and he said:

-Now I would like to be liberal with you. I would like for both of us to eat this bunch of grapes, and that you have as much from it as I. We will divide it in this way: we will take turns picking from it, but you must promise not to take more than one grape each time. I will do the same until we finish it and in this way there will be no deception.

With this pac*, we began; but then in his second turn, the traitor changed his mind and began to take them two by two, supposing that I should do the same. I was not content with catching him up, and went further still: two by two, and three by three. And however I could eat them. The grapes finished, he sat there a bit with the naked stalk in his hand and, shaking his head, he spoke:

-Lazarino, you have deceived me. I would swear to God that you have eaten the grapes three by three.

-I did not— I replied; but why do you suspect me?

The sagacious blind man replied:

-You know how I see that you ate them three by three? Because I ate them two by two and you kept quiet.

-Lazaro, me has engañado. Asegurarte que tú has comido las uvas de a tres.

-No es cierto— respondí yo; más ¿por qué sospechas eso?

Contesto el sagacísimo viejo.

-¿Sabes en que veo que las comiste de tres a tres? En que yo comía de dos a dos y callabas.

Lazarillo de Tormes, Anónimo
A mis padres

A mi abuelo,
admírador aquí y en el cielo
This dissertation has been written between 1992 and 1995. These have been critical years for the process of European Unification, in which a wave of pessimism has swept the rosy perspectives at the beginning of the decade. Indeed, the Maastricht Treaty (December 1991, ratified in 1992) set the programme to reach a Monetary Union and closer political ties among the European countries. However, the structure on which Maastricht had to be developed immediately crumbled: the Treaty was hardly endorsed by the citizens who were given the chance to vote on it; the war in ex-Yugoslavia has revealed the lack of cohesion and resolve among the European countries and, finally, German unification and economic slowdown pushed the European Monetary System to the verge of collapse in 1992 and the year after.

Although we are exclusively concerned with the latter, monetary issue, we are aware that the whole idea of a united Europe rests on the existence of institutional and economic mechanisms which are suited to an heterogeneous group of countries. Therefore I believe that the exchange rate arrangement sustaining the transition to a Common Currency should be carefully devised in order to reinforce rather than weaken the cohesion and cooperation among the members of the European Union.

The shift from Europhoria to Euroskepticism has also influenced my mood deeply, and this is observable in the critical position that I adopt throughout the dissertation regarding the European Monetary System. The reader will already realise in chapter one my reservations towards the main result of target zone models, namely, that fixing the exchange rate around a band exerts a stabilizing effect on the exchange rate through agents’ expectations.

This conclusion crucially depends on the credibility of the arrangement. As a matter of fact, credibility is the central issue of this dissertation. We take up the task of showing how and why the lack of credibility makes the exchange rate arrangement unsustainable. Much as in the story of the **Lazarillo** above, the authority cannot expect to be believed when its ‘wicked nature’ prevents her from respecting the commitment implied by the arrangement.
In the second part of the thesis, I propose some mechanisms to bypass this credibility problem; retaking the idea advanced above, the design of the system should be flexible and transparent enough to be acceptable for every country. I recognise that the ideas I put forward here are based on very simple models and therefore they have no immediate applicability. I hope though that they serve as a reflection on the economic foundations to build a Monetary Union.

I have enjoyed four years at the European University Institute and I have glimpsed the appeal of a real ‘European community’ of cultural exchange while respecting diversity. I recognise that here all the means work in favour of a positive atmosphere, but it should be our compromise to translate this climate of enriching co-existence to all the levels of the European Union.

Therefore, this work has a special value for me, because it deals with the idea of an integrated Europe. This is not at odds with the critical stand I have adopted. On the contrary, I think that a constructive critique is necessary to push this project ahead and that it is worthwhile to try it.

Acknowledgements

First I want to remember the people who encouraged me to come here, Paco Pérez, Javier Quesada and, in particular, Vicente Orts, who has been a continuous source of support. I am also grateful to Fundación Ramón Areces and in particular to Carmen Aguí who have made me forget about the real monetary issue for three years. Finally, the help and support of Jacqueline, Jessica and Marcia in the Department are simply invaluable.

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Many times during these years, I have thought that writing a thesis is not much more than a by-product of an experience of several years in a marvellous place among marvellous people. This feeling mainly came to my mind in moments of discouragement; in some of these occasions I sometimes got relief just by going out to the loggia to watch
the twilight behind the Duomo, up to the Bar Fiasco or away to the gorgeous Tuscany landscape. When these remedies proved insufficient I could always rely on the people around me, who are the ones who make the Institute such an unique place.

I am indebted to all these friends, but in particular to those who have had, by choice or contract, to put up with me more intensively: my flatmates, Daniela and Melanie. I cannot forget the Carlucci family - Alice, Carlo & Chiara- whose support, generosity and joy rescued me from desperation this last term. Special mention deserves Rut who has not only suffered my moody behaviour but has also succeeded in diluting it with her magic.

Finally, I want to remember my grandad Angel, from whom I have inherited the ‘economist gen’ and who has died shortly before seeing his grandson becoming a Doctor; and my parents who have endured the costs of this thesis in terms of distance and separation. I want to dedicate this work to them.
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Methodological Note

Equations and charts are labelled chapter by chapter. When there is a reference to an equation or figure of other chapters, the number of the chapter will be specified before the figures (i.e. if I refer to figure or equation 3 of the first chapter in the second chapter I will write figure 1.3, equation [1.3], respectively).

The bibliography is common to all chapters and it appears at the end of the dissertation.
In the next few years, the European Union will take decisive steps towards shaping its future as economic and political entity. One of the cornerstones of this process is the prospect of a common currency, for which an institutional schedule exists (Title VI of the Maastricht Treaty). The abolition of national currencies, if fulfilled, will culminate a story of around thirty years of exchange rate arrangements, which have sustained the configuration of the European Union.

Indeed, it is hard to imagine how the European Union could have been kept together without the existence of an exchange rate system. The building of the European Union has been based on multiple compromises and channels to achieve cooperation. On the monetary side, the successive exchange rate arrangements have provided the framework for policy coordination among the member countries.

While the contribution of exchange rate arrangements to the deepening of the Single European Market is beyond any doubt, its desirability on pure economic terms is open to discussion. Any exchange rate arrangement limits the scope for autonomous monetary and—to a lesser extent—fiscal policy in the member countries. This constraint on economic policy is the basis for the endless debate on flexible as opposed to fixed exchange rates and on optimal currency areas; although an analysis of these issues is beyond the scope of this dissertation, they underlie most of it.

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1-See Friedman (1953) for the classical argument in favour of floating exchange rates and Johnson (1973) for a formalisation of these arguments. The contributions of Poole (1970), Kindleberger (1970) and Boyer (1978) challenge these conclusions. For an updated account of the debate, see Genberg (1989).

Starting with the seminal contribution by Mundell (1961), the Optimum Currency Areas literature deals with the conditions which favour the establishment of a common currency. MacKinnon (1963) and Kenen (1969) provided additional considerations. This area has recently been revived due, precisely, to the horizon of a Common Currency in Europe (See Masson & Taylor (1992), de Grauwe (1992), Corden (1993), Tavlas (1993)).
Introduction

The Exchange Rate Mechanism (ERM) in the European Monetary System is the current exchange rate arrangement among the European economies. The ERM establishes a target zone for the nominal exchange rate by setting a central parity around which the exchange rate can fluctuate within a band. The central parity can be modified by agreement among the members (realignments), giving rise to a regime of semifixed but adjustable exchange rates.

Although the ERM has not changed in essence since its inception in 1979, its nature has evolved according to the changing political and economic environment. When it was established, its main aim was to provide a stable mechanism for the deepening of the European Common Market and, in particular, for the implementation of the Common Agricultural Policy. However, the occurrence of the oil price shocks in the seventies changed the nature of the system completely. The relatively successful performance of Germany in a period of severe inflation induced the development of the 'EMS-inflation' hypothesis, whereby keeping the exchange rate fixed with respect to the Deutsche Mark would allow other countries to borrow credibility for antinflationary policy from Germany, reducing as a result the rate of domestic inflation. Therefore, the ERM started to be considered as a disciplinary device to rein in domestic inflationary tensions and keeping the parity fixed became a key component of the policy strategy for the ERM's high inflation countries.

The practical implication has been that the ERM has changed from a loose arrangement with multiple realignments to a harder system with few realignments. Indeed, between the Basel-Nyborg Agreements in 1987 to the 1992 crises of the EMS, the system behaved as a 'de facto' fixed exchange rate system. This period of the EMS has been known as the 'new EMS' (Giavazzi & Spaventa (1991)). This period was characterised by the strength of the currencies in high inflation countries. This striking feature came about because progress in convergence was being achieved and short-run strains in the policy management were not apparent in this period; This 'convergence play' (Portes (1993)) contributed to increase the credibility of the system and the resulting parities were sustained by the financial markets.

After this period of relative stability, the system has suffered in the last few years its worst crises since its establishment. Essentially, these difficulties arise from the conflict between the common objective of maintaining the established parities alongside the

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domestic goals of the member countries, in a time of recession and uncertainty. 

Currently, uncertainty also reigns over the future of the Common Currency. Let us recall that the Maastricht Treaty assigns a central role to the ERM in the transition process. Now that the stability of the system has vanished, it seems that there is a void of ideas as how to manage the transition to a Common Currency, although the original schedule of the Treaty continues to be officially maintained.

In any case, it should be recognised that the official view correctly emphasises that the situation is better than before the crises: first, the large devaluations have roughly restored ‘equilibrium’ exchange rates; second, the crisis may have been helpful in indicating those countries which may not converge, as Dornbusch (1991) suggested and, finally, recovery seems to be well underway. All these factors might clear the horizon of some of the obstacles and make the eventual EMU target feasible.

However, inflation could again become a central concern when recovery gets stronger, due to the large deficits accumulated in the last years. Consequently, there may be attempts to return to the old rigid EMS in order to rebuild the disciplinary mechanism which was thought to work so well in the eighties. If this were the case, some difficulties could appear. First, it is not clear how the credibility of the system can be restored in the short term. The reaction of the markets to the restoration of the old bands is feared because of the concern that the markets will begin by testing the commitment of the authorities when it is achieved. Second, even if the new system is stable, why should it promote more discipline than before?

What then are the alternatives? An adequate answer should consider in turn the following questions: is the ERM an adequate mechanism for the transition process to a Common Currency or, more generally, is a target zone desirable?. If so, is there a better arrangement than the current ERM?


4- After the crises, numerous answers have been given to these questions. A first group of reactions has dismissed the need of a target zone, and offered a wide variety of alternatives. Some have advocated for the continuity of the pseudo-floating regime which followed the August ‘93 crisis (a 30% fluctuation band); This alternative is open to competitive devaluations and it will not be valid when inflationary pressures reappear.

Other possibility would be a sudden move to a Common Currency. It assumes that the only source of divergence is the existence of different monetary policies and that a Common Currency would consequently eliminate the divergences.

This and similar proposals dismiss some important facts: first, countries are still vulnerable to asymmetric shocks as the evidence gathered by Bayoumi & Eichengreen (1992) show; second, the deepening of the integration process induced by the Single Market may increase this possibility, because economic integration work in favour of higher degrees of specialisation, as the Economic Geography literature has highlighted (See Krugman (1991a). Finally, the absence of a safety net of fiscal transfers (see Sachs & Sala-i-Martin (1991)) to offset
Note that, in fact, the ERM was not originally designed to serve as a disciplinary device. Indeed, the empirical evidence does not clearly show that it has succeeded in this objective. This suggests that the design of an exchange rate arrangement should be consistent with the goals it is intended to achieve and this conclusion applies to the design of an exchange rate system for the transition to a Common Currency. In my view an adequate exchange rate system should:

a/ induce discipline and at the same time be politically feasible;
b/ avoid irresponsible behaviour through transparent rules;
c/ be capable of dealing with unexpected events in an orderly manner and
d/ facilitate policy coordination.

An adequate combination of these ingredients would deliver a suitable degree of commitment and would render the system credible from its inception. Then, a proper management of the regime could allow for a rapid accumulation of reputation, and a smooth transition to a common currency could be achieved.

In the light of these comments, it is clear that the management of exchange rates has played a fundamental role in the evolution of the European Union and will continue to do so until a Common Currency is adopted. In this dissertation, the focus is placed on the 'new EMS' both because it has inspired the analysis of an important set of models for exchange rate determination (target-zone models) and because it is the accepted framework to manage the transition to a Common Currency.

These two reasons suggest the division of this thesis into two well-defined parts. In the first part, we extensively study target zone models; the second part is primarily normative and it focuses on the design of exchange rate targets. I adopt a quite critical stand in both parts. On the one hand, target zone models are of limited interest if the issue of credibility is not properly addressed; indeed, existing target zone models have failed to do so. On the other hand, it seems clear to me that the ERM is not appropriate for the transition to a Common Currency.

Our main objective is to shed some new light upon the underlying conflict between an international commitment to an exchange rate arrangement and the domestic policy goals and constraints. The two parts are thus closely related. While the first part highlights such a conflict, the second attempts to provide the means to overcome it. Now we present in more detail the plan of the thesis.

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these shocks makes the adoption of a Common Currency unadvisable, at least for the periphery countries.

5-See Giavazzi & Giovannini (1989) and Collins (1988)
Introduction

Plan of the thesis

The first part is composed of three chapters. In the first chapter, theoretical target zones models are considered. The second chapter surveys the empirical evidence on target zones, which serves to introduce the analysis of the Spanish experience in a target zone. Finally, in the third chapter, a model of heterogenous beliefs is presented which permits the consideration of dynamics in the design of a target zone.

Despite the multiple extensions and improvements that have been made since the seminal target-zone model of Krugman, some important aspects are still excluded from the modelling. Our theoretical research has been consequently directed towards fulfilling some of the gaps.

To begin with, the assumption of perfect capital mobility has not been relaxed yet, despite the well-documented recurrence of capital controls preceding realignments in the ERM and the current academic debate on the need to reconsider them again. In section III of the first chapter, we relax this assumption.

The explicit consideration dynamics in target zone models is even more relevant, since only then can questions of sustainability be properly addressed. The hypothesis of rational expectations allows us to obtain a state-dependent solution between fundamentals and exchange rates but, in my view, it severely limits the degree of realism of these models.

By relaxing such strong hypothesis and allowing for heterogenous expectations, dynamic trajectories can be generated for the exchange rates. The model of chapter three allows us to give an interpretation to the actual behaviour of the exchange rate in target zones, by considering the dynamics of credibility. It will be shown that a target zone with divergences in the fundamentals is unsustainable in the long-run and that credibility, which evolves endogenously in this model, determines the collapse of the target zone.

Empirical issues are also addressed in this thesis, by contemplating the Spanish experience in the ERM in chapter two. In this case our approach has slightly differed from the standard methods: instead of imposing an 'a priori' econometric model on the data, we have opted for a very flexible method through the application of the Kalman filter.

The second part approaches exchange rate targets from a policy perspective, although the framework of analysis differs in the two chapters contained in this part.

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6 This model is the result of joint work with J. Humberto Lopez y Vicente Orts. The econometric implementation of the model is due to the first author and the economic interpretation belongs to Prof. Orts and myself.
However, both share the idea that flexibility is key to the design of an adequate exchange rate strategy and that the exchange rate should receive the consideration of intermediate target for policymakers.

In chapter four we use the dynamic target zone model to explore the convergence gains that a target zone may provide. The deflationary effect of an overvalued currency contributes to reduce inflation differentials. The first section analyses the effects of convergence in inflation differentials on the sustainability of the band and on the chances of success of a target zone. It will be shown that the EMS target zone may fall even in this case. This raises the question which we address in this part: How should appropriate exchange rate targets be designed?

Our first answer is what we have labelled the 'convergence bands' proposal which implies a reform of the EMS. According to this scheme, the target zone requires a continuous adjustment of the central parity, according to the evolution of the inflation differentials plus, an eventual a progressive tightening of the bands. The system avoids large exchange rate misalignments and monitors the rate of exchange rate variation as long as convergence is achieved. An 'entry clause' could be added to the rules, such that countries which attain convergence can enter the monetary union automatically. The choice of the correction parameter and the width of the band is central for the success of the zone, for it determines the degree of discipline required to achieve convergence.

Therefore, not only can countries enter the monetary union at different moments, but this alternative also has the advantage of containing 'à la carte' components, which should be agreed upon beforehand. This makes the gains and losses of the system transparent from the outset. In this way I believe that the convergence bands proposal delivers a stronger commitment to discipline and, consequently, a higher degree of credibility.

The second alternative addresses the question of exchange rate targeting from a general optimizing approach, when governments have an explicit loss function to minimise. We consider a static model in which two countries that care not only about inflation, but also about employment, have to decide whether or not to target the exchange rate and, if they decide to do so, what is the best targeting strategy. This is in the spirit of the policy coordination literature in which targeting only pays if the outcome is better than the non-cooperative solution, so that the exchange rate acts as a surrogate for policy coordination. The results show that the choice of exchange rate target depends on the type of shocks affecting the economy although policymakers may keep a certain discretion regarding the choice of exchange rate targets.
This dissertation appears at a time when exchange rate targeting may not be a popular issue. The recent experience of the EMS is similar to the end of the Bretton Woods system. Yet the free float period which immediately proceeded the collapse of Bretton Woods eventually proved to be also unsatisfactory. New attempts to design exchange rate arrangements soon followed (McKinnon (1984,1988) and Williamson (1985)), and the European Union set up the 'snake' before Bretton Woods was officially abandoned because the stability of exchange rate was held to be essential for the Common Market.

In any case, a more open-minded approach to the design of exchange rate targets is called for. The flexible approach adopted in part two shows that optimal strategies for targeting the exchange rate can be found. We have to recognize that these are just the first building blocks of a more ambitious project. This dissertation points out the difficulties of the issue. In my opinion, the effort is nonetheless worthwhile because there is much more at stake beyond the economic goal of a common currency for Europe.
Chapter 1
Target-Zones Models

Target-zones models deal with the behaviour of exchange rates in a regime of fixed parities with fluctuation bands. These models have mushroomed in the literature since the seminal work by Krugman (1991, circulated in 1987) and have reached their zenith in the beginning of the nineties, when they appeared regularly in the economic journals. This chapter describes target zones models from a theoretical point of view but it should not be considered a technical survey. Rather, we intend to evaluate the virtues and flaws of these models and to provide a framework to introduce the modifications which we propose.

The next chapter will address the empirical implications and evidence on target zones. We have adopted this division for convenience, but both are complementary, because the empirical findings stimulated the extensions of the target zone basic model.

The research on target zones has evolved in three distinct, albeit overlapping phases. In a first stage the basic target zone model was set up, which already contained the main insights. However, as we will see in the second chapter, when the empirical implications were tested against evidence, the results were so unfavourable as to call for a second phase, in which a series of extensions were developed in order to approach the models to reality. The most significant step was the relaxation of the perfect credibility assumption. Once the possibility of exchange rate realignments was contemplated, some of the empirical problems were overcome. Finally, the third phase has less to do with the functioning of exchange rate target zones and more with the rationale for target zones.

The basic model is presented in section I. In section II, some of the strong assumptions of the previous model are relaxed to consider the issues of credibility and

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1. Pesenti (1991), Bertola (1993) and Svensson (1992a) also provide accounts of the literature. The volume edited by Krugman & Miller (1992) is wholly devoted to target zone models.
capital controls. The role of interventions and reserves deserves special mention, so that section III is devoted to it. We will see that alternative assumptions on interventions affect not only the results of the basic model but also influence the credibility and raises the possibility of sterilized interventions in the case of imperfect capital mobility. Finally, section IV explores the attempts to introduce target-zones models in an optimizing framework, in order to approach the issue from a normative perspective.
of fundamentals, that is, of the current fundamentals and the present discounted value
of future fundamentals—where the inverse of the interest rate semielasticity \((1/\delta)\) turns out
to be the discount factor:

\[
s_t = \frac{1}{\alpha} E[f_t \int_{-\infty}^{\infty} e^{\xi \xi} d\xi] Z_t
\]  

(4)

This expression implicitly rules out bubbles because the stability condition of
standard forward looking models applies:

\[
\lim_{\tau \to -\infty} e^{-\frac{1}{\delta} \tau} s_t = 0
\]

Free float solution

Let us consider first that no target zone is imposed on the exchange rate, so that
it floats freely. Therefore, we take money supply as given \((\Delta m = 0)\). The process for the
velocity shocks and hence the fundamentals are assumed to follow a Brownian motion
which is the analogue to a random walk in continuous time:

\[
df = \mu dt + \sigma dz
\]  

(5)

where \(dz\) is a standard Wiener process, such that \(E(dz) = 0\), \(E(dz^2) = dt\), so \(\sigma\) is the variance
in the growth of \(v\) and \(f\) in this case—by unit of time and \(\mu\) is the a drift term. It also follows
from this specification that:

\[
(f_t | f_0 = f_0) \sim N(f_0, \mu t \sigma^2)
\]

So

\[
E[f_t | f_0 = f_0] = f_0 + \mu (t - T)
\]

\(\forall t \geq T\)

Integrating then the saddlepath solution (4), we obtain the free float solution for the
exchange rate:

\[
s_t = f_t + \alpha \mu
\]  

(6)

that is, the exchange rate expectation is simply equal to the drift of fundamentals and the
exchange rate is a linear function of the level of fundamentals, yielding a 45° degree line
in Figure 1.

Target zone

Let us assume now the effects of imposing a target zone on this model. In this case,
the monetary authorities are expected to defend the band through non-sterilized
Interventions in the foreign exchange markets in order to prevent the exchange rate from surpassing the band. Interventions are assumed to take place at the edges of the band (marginal interventions) in order to control the process for the fundamentals. This monetary rule is perfectly credible and as such it appears in the information set of agents.

It is convenient first to give an intuitive explanation for the result we derive below. Let us imagine that we are close to the upper edge of the band. For simplicity, we assume that the process for the fundamentals has no drift \( \mu = 0 \). What is the conditional expectation for the change in the exchange rate near the edges of the band?

Let us consider the upper limit of depreciation. In a free float regime, the exchange rate expectation is zero, but note that now, if the fundamental decreases, the exchange rate will fall, but if the fundamental increases, the exchange rate will only increase until the edge of the band, because the monetary authority will intervene with certainty. Therefore, the exchange rate will be expected to appreciate, \( E_{\text{rt}ds/\text{dt}} < 0 \), due to the expected intervention at the edges of the band. It also follows from this that, the closer the exchange rate gets to its upper limit \( s_u \), the greater the appreciation expectation, since the room for the fundamentals to increase becomes smaller and the room to decrease larger. As a result, the linear relation of the free float case between the exchange rates and the fundamentals no longer holds. We now show this result more formally.

The intervention of the authorities at the edge of the band is represented by two regulators \( dL, dU \) which take the form of purchases and sale of reserves and are only activated at the lower and upper limits of fluctuation, respectively. This modification affects to the process for the money supply, which is now \( dm = dL + dU \) and consequently the fundamentals now follow a regulated Brownian motion:

\[
df = \mu \text{dt} + \sigma \text{dz} + dL - dU
\]

\[
dL > 0, \forall s \leq s_u
\]

\[
dU > 0, \forall s = s_u
\]

\[
dL = dU = 0, \forall s \in (s_u, s_u)
\]

Note that in this case, the original process is modified at the edges of the band and the rational expectation solution has to account for this modification. By Ito's lemma, the exchange rate expectations can then be expressed as:

\[
E_{r \frac{ds}{dt}} = \mu s(t) \text{dt} + \frac{1}{2} \sigma^2 s(t) \text{dt}, \forall s < s_u;
\]

\[
E_{r \frac{ds}{dt}} = \mu s(t) \text{dt} + \frac{1}{2} \sigma^2 s(t) \text{dt} - s(t) \sigma \text{df}, \text{if } s = s_u;
\]

\[
E_{r \frac{ds}{dt}} = \mu s(t) \text{dt} + \frac{1}{2} \sigma^2 s(t) \text{dt} + s(t) \sigma \text{df}, \text{if } s = s_u.
\]
If we compare the free float solution with the expectation in the interior of the band, we can observe that to achieve the previous free float result the derivative $s_{t}$ should be one and the second derivative $s_{tt}$ zero, such that fundamentals and exchange rates are linearly related. Substituting (8) into (3), the model above implies a second order differential equation, whose solution has the following form:

$$s_{t} = e^{-\alpha_{t}} + A_{1} e^{-\lambda_{1} t} + A_{2} e^{-\lambda_{2} t}$$

where $\lambda_{1}, \lambda_{2}$ are the roots of the characteristic equation and $A_{1}, A_{2}$ are constants of integration, which take zero values in a free float. On the contrary, they are different from zero when a target zone is in place.

The characteristic equation of this differential equation is

$$\frac{\alpha \sigma^{2}}{2} \lambda^{2} + \alpha \mu \lambda - 1 = 0$$

and the roots are given by:

$$\lambda_{1} = -\frac{\mu}{\sigma^{2}} \pm \frac{B}{\sigma^{2}}; \quad \lambda_{2} = -\frac{\mu}{\sigma^{2}} - \frac{B}{\sigma^{2}}$$

$$B = \sqrt{(\sigma \mu)^{2} + 2\alpha \sigma^{2}}$$

The values of the roots just depend on the interest rate semielasticity ($\alpha$) and the parameters of the brownian motion (the drift $\mu$ and the standard deviation $\sigma$). Now the constants $A_{1}, A_{2}$ have to be solved for the boundary conditions. These easily follow from observing (8). The continuity of the expectation at the edges of the band and, consequently, of the exchange rate-fundamental relationship implies that $s_{t} f_{t} = s_{f} f_{t} = 0$. These boundary conditions are also called ‘smooth pasting’ or value matching conditions in the literature, and they are solved as follows$^{4}$.

Taking the derivative of the differential equation, we get

$$0 = 1 + A_{1} \lambda_{1} e^{\lambda_{1} t} + A_{2} \lambda_{2} e^{\lambda_{2} t},$$

$$0 = 1 + A_{1} \lambda_{1} e^{\lambda_{1} t} + A_{2} \lambda_{2} e^{\lambda_{2} t},$$

from which $A_{1}, A_{2}$ is solved as a function of the parameters of the model. Then the saddlepath value of the target zone is defined by the following expression:

$^{4}$Here I have followed Froot & Obstfeld’s (1989) explanation, but there are other kind of derivations like those of Obstfeld (1989), Krugman (1989) or, relating it to the reserve position and attacks, Flood & Garber (1989) and Krugman & Rotemberg (1990)).
The non-linear component of the solution (Ω) is the reason for the curvature of the new S-curve in Figure 1\textsuperscript{5}. The relation between fundamentals and exchange rate bends down, implying that the effect of the fundamentals on the exchange rates decreases as the limits of fluctuations are approached. The slope is lower than in the free float case within the fundamentals range and at the boundaries. Variations in the fundamentals have no effect whatsoever on the exchange rates, as revealed by the smooth pasting conditions. It can also be shown that the smaller the roots, the wider is the wedge between the free float and the target zone solution. The effects of the parameters on the characteristics of the curve are derived from the expressions of the roots.

The existence of a drift disturbs the symmetry of the model because the roots are different. From (10), we can observe that with a positive drift the exchange rate is at the upper part of the band when the fundamentals are zero; since the smooth pasting conditions continue to hold, it is clear that the slope of the curve is lower (steeper) at the upper (lower) part of the band than in the driftless case. Higher values of α reduce the values of the roots and increase the slope of the curve. This result implies that the target zone increases the stabilization effect with higher levels of volatility in the fundamentals. An opposite result holds for the parameter κ.

Finally, the slope of the curve can be shown to decrease when the band narrows. This suggests that the stabilizing effect of the target zone becomes more important as the bands are reduced, matching the findings of Delgado & Dumas (1992).

These features deliver the so-called ‘honeymoon effect’ mentioned by the literature; the imposition of a target zone has a stabilizing effect on the exchange rate, reflected in the higher divergence in the fundamentals with respect to the free float solution. In the case shown in the figure the band has a 12% width, the process is assumed to be driftless, the instantaneous standard deviation α is equal to 10% and κ equal to 1.

The stabilizing effect of the target zone can be more formally shown by computing the (instantaneous) exchange rate variability from the standard deviation of the exchange rate process σ\textsuperscript{t} (Svensson (1991a)).

\textsuperscript{5}Ω highly simplifies if the process is driftless (μ=0). Firstly, from (8) λ\textsubscript{1}=λ\textsubscript{2}=1. Secondly, since the bands are supposed to be symmetric f\textsubscript{1}=f\textsubscript{2}. Then, it is straightforward to show that A\textsubscript{t}=A\textsubscript{t}=A\textsubscript{t}=1(\exp(μτ)−e^{−μτ}); finally, the exponential equations are substituted by hyperbolic trigonometric functions and the solution is σ\textsuperscript{t}=\sqrt{\text{sinh}(κ\,T)/\text{cosh}(κ\,T)}. Now, it is straightforward to check that the derivative at the boundaries is zero.
Figure 1 - Exchange rate and fundamentals in a target zone

Figure 2 - The stabilizing effect of a target zone. Interest rates and position of the exchange rate in the band.
Since the exchange rate is a function of the current and expected fundamentals, it will also follow a brownian motion of the type: \( ds = \mu dt + \sigma dw \). Applying the Ito’s lemma to this expression, it turns out that \( \sigma \) is just the product of the standard deviation of the process for the fundamentals multiplied by the slope of the S-curve\(^6\): \( \sigma^2 = s\sigma \). Since the slope is less than one, the instantaneous standard deviation is less in a target zone than in a free float regime (\( \sigma \)).

**Interest rates**

The difficulties in the definition and measurement of the fundamentals imply that interest rate differentials (\( \delta = \mu' \)), being directly observable are the most convenient variable for an empirical assessment of the model, as we will see in chapter 2. The new relationship— which we will denote curve (\( \delta, s' \))— is implicit in the structure of the model. The UIP condition implies the equivalence between interest rate differentials and exchange rate expectations in \( \delta = E(ds/dt) \). Substituting this expression in (3) we get:

\[
\delta(\eta) = \frac{\delta(\eta) - \theta}{\alpha} \tag{11}
\]

which we define as the magnitude of the ‘honeymoon effect’\(^7\). From figure 2 we observe that at the lower (upper) part of the band, the interest differential is positive (negative) and equivalent smooth pasting conditions hold. The intuition behind this figure corresponds to the expectation mechanism we described at the beginning: a positive interest rate differential reveals an expected depreciation of the currency in the lower part of the bands.

As above, the standard deviation of the interest rate process (\( \sigma^2 \)) can be obtained. From (11), this turns out to be \( \sigma^2 = (\sigma^2 - \alpha)/\alpha \), while in the free float case, it is equal to zero. Therefore, we observe that stability in the exchange rates in a target zone is attained at the cost of a higher volatility in interest rates.

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\(^6\) The drift of the process is simply the interest rate differential, from the UIP condition: \( \mu = \delta \).

\(^7\) Observe that in the free float case of expression (6), the interest differential is constant and equal to the drift in the fundamentals \( \mu \).
II-Extensions to the model

The structural characteristics of the economy (conveyed in our model by the parameters of the monetary model and the process which drives the fundamentals) and the institutional setup of the band (fluctuation bands, intervention rules) determine the features of the basic target zone model. The empirical tests which followed the first wave of target zone models and which are described in chapter two revealed the weaknesses of the basic model and encouraged further research.

The basic model is solved under quite strong assumptions which, once relaxed, affect both the results of the model and the ability to defend the band, by broadening the set of instruments in the hands of the authorities. Figure 3 displays a scheme with the determinants of a sustainable target zone. We can observe that the sustainability of the

![Diagram](Figure 3-Determinants of the target-zone sustainability)
The first factor which appears in the chart is discipline in the conduct of monetary policy to keep the fundamentals within the band. This is conveyed in the commitment of the authorities to keep the range of variation of the fundamentals within the bounds of the target zone \( f_l < f < f_U \). This is the implicit argument which underlies the basic model and it is the key feature in the long-run success of a target zone. The long period of exchange stability in the ERM would suggest that it has been a strong disciplinary mechanism.

However, the existence of wide divergences between countries and the ultimate collapse of the system defy this view. One of the main purposes of this dissertation is to present arguments against such belief.

**Credibility**

Given the tangency points at the edges of the band, the assumption of perfect credibility implies that the band cannot collapse whatever the level of fundamentals. The reason is that the authorities are always expected to restrain the behaviour of the fundamentals. The basic model implies not only that the band is completely credible but also that this full credibility is exogenously given.

The scheme reveals that there are determinant elements in the formation of credibility. The role of reserves will be explored in the next section; the second element is reputation. In a more realistic model the system's past performance should be included in the information set, such that agents would take it into account when forming their expectations. Hence, a system with frequent realignments may not enjoy reputation and consequently the bands may not be credible. On the contrary, a band which has been successfully defended even in objectively difficult circumstances may render the system more credible. From this perspective, the interaction between reputation and credibility can be seen as a self-feeding mechanism: the defence of the band builds up a stock of reputation which delivers a flow of credibility, facilitating the defence of the band.

The dynamic nature of this interaction cannot be grasped by the state dependent solution of the model, we are considering in this chapter. The reader will have to wait until the chapter three, where a time dimension is introduced to observe the evolution of this...
feedback. Here the degree of credibility is taken as given or, at the very most, state-dependent.

Let us start by writing the exchange rate expectation in terms of conditional expectations. We assume that when the bands cannot be defended there is a realignment and that in this case, the exchange rate jumps to its free float solution:

\[ E_s(t) = \omega_s E_s(t) | \text{realignment} \times (1 - \omega_s) E_s(t) | \text{no realignment} \]

where \( \omega_s \) is the probability of realignment at time \( t \). Up to now, the assumption of perfect credibility has kept \( \omega_s \) constant and equal to zero.

What are the effects of a positive probability of realignment? The answer depends on the characterization of the realignment size. To introduce the issue, we will follow Krugman (1991b) and Pesenti (1991).

Let us dispense for a moment with the assumption of infinitesimal interventions and consider point \( s^c(f) \) in figure 4. When the exchange rate reaches its upper limit the authorities face the following decision: either defend the regime and push the exchange

![S-curve relating exchange rate & fundamentals](image)

**Figure 4** A target zone with imperfect credibility. Credibility degree, \( \omega = 0.3 \).
rate into the band \((s^{c}(f) - s(f))\), in which case the exchange rate jumps downwards; or it, letting the exchange rate jump to its free float level \((s^{c}(f) - f)\). These terms represent the conditional expectations above. In particular, the latter term is in this framework the size of the realignment\(^8\).

The ‘a priori’ exchange rate expectation is determined by the no-arbitrage condition, which implies that the expected jump in the exchange rate is zero:

\[
E(\frac{ds}{dt} | Z) = \omega_{s}(f^{c} - s^{c}(f^{c})) + (1 - \omega_{s})(s(f^{c}) - s^{c}(f^{c})) = 0
\]  

Rearranging,

\[
s^{c}(f^{c}) = \omega_{s}f^{c} + (1 - \omega_{s})s(f^{c})
\]

where \(s(f^{c})\) is given by the complete credibility solution in (10).

The existence of a positive realignment probability affects the properties of the model in the following way. Let us express for convenience, the new, unknown solution as

\[
s^{c}(f) = f^{c} + \Omega^{
}
\]

where \(\Omega^{
}\) has yet to be determined.

Substituting the perfect credibility and this new solution into (14), we get

\[
f^{c} + \Omega^{
} = \omega_{s}f^{c} + (1 - \omega_{s})(f^{c} + \Omega)
\]

Solving out \(\Omega^{
}\) it turns out that it is equal to:

\[
\Omega^{
} = (1 - \omega_{s})\Omega
\]

which renders the solution for the imperfect credibility model extremely simple; after substituting this expression into (10), we get the basic equation of the model with imperfect credibility:

\[
s = f^{c} + (1 - \omega_{s})\Omega
\]

Note that now the slope in the curve within the band is steeper than in the perfect credibility model. The stabilizing effect of the target zone is therefore reduced. With no credibility (18) collapses into the free float solution (10) and there exists no honeymoon effect at all.

Another interesting feature of this approach to introduce imperfect credibility into the model is the implied breakdown of the smooth pasting conditions, as it is apparent in figure 4. Now the S-curve is not tangent at the edges of the band; the derivative is positive and therefore, the exchange rate is not insensitive anymore to changes in the fundamentals at the edge of the bands.

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\(^8\): Without loss of generality, we assume that the process is driftless.
Bertola & Svensson (1993), Bertola & Caballero (1993) and Tristani (1994) propose alternative approaches to modelling imperfect credibility, which emphasize other points of interest.

Bertola and Svensson define the realignment risk ($c_r$) as the product of the probability of realignment and the size of the realignment. In terms of the above model, this is simply given by

$$c_r = \alpha (E(ds/dt | \text{realignment}) - E(ds/dt | \text{no realignment})) = \alpha (f - s^2(f))$$

The novelty in their approach is that they assume an additional continuous time process for $c_r$, which now appears as a second state variable of the model.

If $c_r$ is assumed to be positive and uncorrelated with the fundamental process, the new solution shifts the S-curve to the left in figure 1 and shifts up the (S,f) curve in figure 2. The simulations they carry out show that these modifications describe better the observed behaviour of the interest rate differentials.

The advantage of the realignment risk approach will become apparent in chapter two. With a further refinement in the derivation of the conditional expectations, it allows for a method to extract an estimate of the expected realignment from the data. On the contrary, this specification does not dispense with the smooth pasting conditions and this implies that even with imperfect credibility the model does not effectively bind the range for the fundamentals.

Also note from the above expression that, according to the Pesenti-Krugman specification, the value of $c_r$ positively depends on the position of the exchange rate in the band, because the wedge $f - s(f)$ widens as the exchange rate approaches the bounds of fluctuation. However, in the specification of Bertola and Svensson the realignment risk is given exogenously.

The model of Bertola and Caballero is closer in spirit to the specification of figure 4. They take into account the effect of the fundamentals on the credibility of the target zone. Their result is noteworthy because in this case not only the smooth pasting conditions do not hold but the slope of the curve is greater than in the free float and it reaches a maximum, instead of a minimum, at the edges of the band. This implies that an imperfectly credible target zone may have destabilizing effects on the exchange rate or, as they put it, a 'divorce effect' instead of a honeymoon effect may arise.

Finally, Tristani (1994) proposes a more flexible way of modelling of the realignment process which conveys the previous alternatives. Although the slope of the target zone

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[^9]: Actually, Bertola & Svensson indirectly allow that the realignment risk depends on the position of the fundamentals, by making possible a positive correlation between the processes which drive fundamentals and the realignment risk.
may be larger than one, the smooth pasting conditions are preserved.

We will see in the next chapters that credibility issues have been of a crucial importance in restating the significance of target zone models in practice.

Financial integration. Capital controls

The Uncovered Parity assumption implies complete financial integration among markets. Capital markets are perfectly integrated when the possibility to impose explicit capital controls is ruled out and there is perfect substitutability among assets. The literature has in general embraced the assumption of complete financial integration.

Nevertheless, the effective imposition of capital controls or the threat of their potential imposition have constituted an important barrier to the integration of financial markets. Empirical evidence in Rogoff (1985b) and Artis & Taylor (1988) also reveals that capital controls have played an outstanding role in maintaining the exchange rate stability in the ERM, especially in turbulent periods.

Directives on perfect capital mobility have only been implemented at the end of the eighties. In spite of this, the recent turmoil in the ERM has resuscitated the possibility to impose some sort of capital controls to achieve a smooth transition to EMU.

The existence of extremely sophisticated and sensitive foreign exchange rate markets may introduce excessive instability in the exchange rates. It is widely recognized that short term movements in the exchange rates are driven by fads, rumours or other types of seemingly irrational behaviour rather than by economic fundamentals. Reports of the Bank for International Settlements (1986) and empirical evidence gathered by Allen & Taylor (1989) for instance, stress the importance of these phenomena in actual exchange rate expectations. Therefore, imposing capital controls would be directed to inhibit the destabilizing influence of the markets.

The implementation of the controls build on the seminal proposal of Tobin (1978) to ‘throw sand in the wheels’ of capital flows (1978). The sand to which Tobin referred was a small tax on short-term foreign exchange market transactions. This idea was retaken by Dombusch (1986) who coupled it with the possibility of a dual exchange rate market and, more recently, it has been the subject of a Policy Forum whose contributions discuss the need to resort to them to preserve the possibility of EMU10.

Economic efficiency and technical arguments advise against these controls. These measures prevent an optimal allocation of resources and worsen consequently welfare;

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moreover, bearing in mind the sophistication of the markets, capital controls may be rather difficult to implement technically and prone to be circumvented (see Garber & Taylor (1995) and Kenen (1995)). Nonetheless, this has to be balanced against the huge losses that the collapse of the ERM, provoked by a panicking market, may bring about. In any case, the debate remains open.

Now, we explore how these aspects modify the model through the Interest Parity Condition and provide the authorities with new instruments to defend the band. In this section, we will deal with the imposition of capital controls and in the next section the consequences of imperfect substitutability on the role of interventions will be examined. The relevance of these implicit barriers are evident from the extended Mundell-Fleming model: the lower the degree of capital market integration, the more room there exists for an autonomous management of the monetary policy.

The domestic demand for money depends on the money market or on-shore interest rates, denoted as \( r \); consequently, the interest rate differential \( \delta \) refers to the on-shore rates. However, as Giavazzi & Giovannini (1989) stress, there will exist a wedge (denoted by \( d \)) between on-shore rates and off-shore rates \( (l_{off}) \), when capital controls are present:

\[
d = l - l_{off}
\]  

(19)

This differential arises because transactions in the Euromarkets (determining off-shore rates) are not subject to the imposition of capital controls while the domestic rates are. Domestic rates higher than Eurorates reveal then the will to insulate the domestic market from capital inflows: this positive differential would indicate the existence of controls for capital inflows, and vice versa negative differential reveal the existence of controls on capital outflows. This provides the authorities with an additional tool to support the band.

Since the demand for foreign currency depends on the off-shore rates, the interest parity condition in (1) should include this differential, which represents the existence of capital controls:\(^{11}\):

\[
\delta_t = (l_t - l_{off}) = E_t \frac{ds_t}{dt} + d_t
\]  

(20)

Capital controls appear as a simple additive term in the new interest parity condition and the authorities have the possibility of imposing them when they wish.

However, from the previous discussion of credibility we can observe that a perfectly

\(^{11}\) Here we have assumed that the foreign country does not impose controls, such that \( l_{off} = 0 \)
Figure 5.a - Capital controls in a target zone. \( \gamma = 0.5 \)

Figure 5.b - The gap derived from capital controls on the interest differential.
credible target zone would not require the imposition of controls to defend it; controls should be required then only in the cases of imperfect credibility.

In particular, when the band is under strain. If the position of fundamentals determines the credibility of the band, as in most of the models considered above, it follows that the imposition of the controls will also depend on the position of the fundamentals: control on inflows \((d>0)\) would be imposed when fundamentals approach the lower edge \(f_\ell\), control on outflows \((d<0)\) when they approach the upper edge \(f_u\).

To introduce this intuition into our specification, Alberola (1992) defines the parameter \(\gamma\) to express (20) as:

\[
\gamma \delta = E \frac{ds}{dt}
\]

(21)

where \(d < 1 - \gamma \delta\), \(0 \leq \gamma \leq 1\)

where \(\gamma = 1\) represents a situation with no capital controls. Substituting this new IP into the monetary model the basic equation now is

\[
s = f + (\alpha / \gamma) E(ds/dt)
\]

In this modified model, the characteristic roots are smaller in absolute value, so that the S-curve stretches out as \(\gamma \rightarrow 0\); figure 5a displays the effects of capital controls on the S-curve. When there is no capital mobility at all \(\gamma = 0\), fundamentals are unbounded. In figure 5b we can observe the behaviour of the interest rates in the face of capital controls: note that when the currency is strong (lower band) and controls are imposed, larger interest rate differentials are possible than those implied by a situation without controls. The dashed line represents then the off-shore interest rate and the wedge between both lines is the magnitude of \(d\).

Comparing 4 with 5a, we can observe that capital controls exert an opposite effect than the loss of credibility. Therefore, capital controls in a target zone model are designed to offset losses in credibility and eventually safeguard the zone from collapse. In Alberola (1992), this trade-off between credibility and capital controls is explored in greater detail.

As we have mentioned, the current role of capital controls is open to debate. In any case, the removal of legal barriers to capital mobility precludes the use of this instrument at the moment. The loss of this facility consequently should weaken the target zone, unless it is interpreted as a stronger commitment to discipline the domestic policies.
The Intervention mechanism conveyed in the basic model implies that when the exchange rate hits the band (i.e. \( s=s_j \)), the Central Bank intervenes through a sale of reserves \( (dR<0) \), reducing the level of fundamentals and driving up the interest rate, in order to push the exchange rate back into the band. There are however different ways to model interventions and their effects.

These modifications affect the implications of the model in many respects and this is explored in this section. First, we change the assumption of Intervention at the edges to allow for intramarginal interventions. Secondly, the costs of intervention in terms of reserves are considered to show that there is a link between the sustainability of the band, its credibility and the stock of reserves. Finally, we highlight the possibility of sterilized interventions when there capital markets are imperfectly integrated.

**Intramarginal Interventions**

The assumption of marginal and infinitesimal interventions does not fit the actual behaviour of Central Banks in the ERM. On the one hand, discrete interventions should be considered; on the other hand, it is recognized that, especially after the Basel-Nyborg agreements, interventions are carried out throughout the band (see Mastropasqua et al. (1988), Dominguez & Kenen (1991), Lindberg & Söderlín (1992), Lewis (1990)).

The consideration of discrete intervention schemes does not present difficulties and the basic insights of the model are maintained (see Avesani (1991)). However, the introduction of more realistic intervention schemes in the model comes up against the technical difficulties of stochastic calculus. Therefore, the assumption of intramarginal interventions is introduced through very simple mechanisms in Froot & Obstfeld (1991a), Delgado & Dumas (1992), Lindberg & Söderlín (1992).

It is then assumed that the Central Bank leans against the wind and sells reserves as fundamentals deviate from the central parity. This sale is assumed to be a constant proportion \( (h) \) of the fundamentals deviation; this feature is conveyed in the parameter \( h \) which indicates the degree of mean-reversion in the proces. At the edges marginal
Figure 6.a - Intramarginal interventions within a target zone. Degree of reversion parameter $\gamma = 0.7$

Figure 6.b - Interest rate differentials with intramarginal interventions. Note that the relation is almost linear.
Interventions are assumed as before. Intramarginal interventions imply that the exchange rate is controlled by the authorities throughout the bands, not only at the edges. The driving process in (7) is consequently transformed in the following mean-reverting Ornstein-Uhlenbeck process:

$$df = (\mu - hf)dt + \sigma dz + dL - dU$$

(22)

The resolution of the model proceeds as in the marginal intervention case, but now the general solution to the differential equation is given by:

$$s = \frac{f + \alpha \mu}{1 + \alpha h} + A_1 M(\frac{1}{2a\sigma^2} \frac{1}{\frac{2a-h}{\sigma^2}}) + A_2 M(\frac{1}{2a\sigma^2} \frac{1}{\frac{2a-h}{\sigma^2}}) \frac{\mu - hf}{\sigma^2}$$

(23)

where $M(\ldots)$ is the confluent hypergeometric function. The solution is derived in the mentioned papers, applying the smooth pasting conditions to derive $A_1, A_2$.

For our purposes it is enough to observe figure 6 which compares the cases of marginal and intramarginal interventions. Note that the non-linearities are of smaller order, such that the curve is only significantly bent close to the edges in order to fulfill the smooth pasting conditions.

The slope of the curve is now lower than in the marginal intervention case, implying that the stabilizing properties are enhanced. However, the intramarginal intervention, not the interventions at the edge, explain this result. From above, the managed float solution is $s = (f + \alpha \mu) / (1 + \alpha h)$, which is also displayed in figure 6. With a lower slope than the free float solution. Therefore, Intramarginal Interventions have additional stabilizing effects on exchange rate through expectations even when no target zone is in force, but fundamentals are only bounded when a target zone is imposed.

**Reserves and credibility**

We could conclude from above that intramarginal interventions facilitate the defence of the band. However, the benefits of interventions should be compared with the costs in terms of reserves that defending the target zone implies, both in the case of intramarginal and marginal interventions.

Target zone and speculative attacks models have strong affinities because both of them emphasize the interaction between fixed and floating regimes through expectations. The basic target zone model dispenses with the problem of reserve drain when intervention takes place. Krugman & Rothenberg (1990), Flood & Garber (1991) and Dumas & Svensson (1991) for unilateral bands and Delgado & Dumas (1992) in the case of bilateral bands focus on this shortcoming and attempt to lay a bridge between
both strands of the literature.

Although the focus is on fixed regimes, the seminal paper by Krugman on speculative attacks (1979) tackles the insertion of dynamic constraint on the interventions in a target zone. His model of balance of payment crises asserts that a currency which tends to depreciate makes the peg unsustainable in the long run, since the continuous leaking of reserves exhaust them at a certain point.

Agents, however, can anticipate the future depletion and know that after the reserve stock reaches a minimum level (to be determined), no more intervention will be possible. At that point, an speculative attack against the currency is carried out, the exchange rate returns to its free float level and the reserve stock jumps to zero. However, the depth of capital markets precludes arbitrage, so that the exchange rate is not expected to jump. This no-arbitrage condition is useful to compute the minimum level of reserves.

This setup is adapted to target zones by Krugman and Rothemberg (1990). Assuming a one-sided band and substituting the expression for the fundamentals into (3), we can rewrite (8) as

\[ s = m + v + \alpha u + A e^{\lambda \cdot v} \]

Just before the speculative attack is triggered at \( v = v' \), the (modified) value matching condition holds for the last time\(^{12}\):

\[ s_v = 1 + \lambda_1 A e^{\lambda \cdot v'} = 0 \]

The constant \( A \) can then be solved to obtain the expression for the exchange rate just before the speculative attack is triggered

\[ s_* = m + v' + \frac{1}{\lambda_1} \alpha u - \frac{1}{\lambda_1} \]

which has to be equal to the exchange rate after the attack, given by the free float expression:

\[ s_+ = m' + v' + \alpha u \]

where \( m' \) is the money supply after the attack. The loss of reserves or size of the attack is therefore given by \( 1/\lambda_1 \) and from (2) \( m' = \ln(D) \) and the reduction in the money supply is:

\[^{12}\text{Note that there exists an inconsistency between this claim and the treatment of credibility above, since imperfect credibility precludes the tangency point solution. Indeed, Krugman & Rothemberg stress that the smooth pasting condition only should be expected to hold when there is a large enough stock of reserves. Therefore, they imply that the credibility process just takes two values: } \omega = 0 \text{ before the speculative attack and } \omega = 1 \text{ at the moment of the attack. This clarification makes both analysis compatible.}\]
Equating this equation with the size of attack the minimum ratio of reserves to deposits that makes the band sustainable is computed:

$$\frac{R}{D} > e^{\frac{\sigma}{\epsilon} - 1}$$

The message of this extension is that target zones are unsustainable in the long run if one-sided interventions are needed to defend the band. In a similar framework Dumas & Svensson (1992) give an answer to the related question of how long does an unilateral target zone last. They show that the drift in the fundamentals and the initial stock of reserves are the determinants of the duration of the target zone.

The approach of Krugman and Rothenberg provides interesting insights but it hardly considers the link between reserves and credibility (see footnote 11). Bertola & Caballero (1991) attempt to fill this gap by making the probability of realignment ($\omega$) depend on the number of interventions. If the number of net interventions (the sum of positive and negative interventions) is denoted by $J$, this implies that $\omega = \omega(J)$. $\omega > 0$. The implications of these modifications is the existence of a realignment risk which varies with time (or interventions) and that the long-run properties of the exchange rate are similar to the case of a free float or, more precisely, to a crawling peg.

Reserves and financial Integration. Sterilized Interventions

Up to now, we have assumed that there is no scope for sterilized interventions. However, Obstfeld (1988) and Dominguez & Frankel (1990) show that sterilized interventions can be effective in the short run through two different channels, expectations and asset equilibrium, leading to two different effects: the signalling effect and the portfolio effect.

This question has been addressed by Alberola, López and Orts (1994a), where we take as starting point a simplified Central Bank balance sheet:

$$R + B_c = H$$

We can observe that the reserve stock plus the stock of domestic assets held by the Central Bank ($B_c$) equals the high-powered money supply ($H$). This magnitude, multiplied by the monetary multiplier ($x$) constitutes the money supply. Substituting and taking logarithms we arrive at the following expression for the fundamentals which is equivalent to (2):
A sterilized intervention compensates the variation in reserves with an equal variation of the opposite sign in the stock of domestic assets held by the Central Bank \((dR=-dB_c)\) such that the money supply remains unaltered.

The signalling effect determines the structure of the basic model. Recall that the model is solved assuming expected interventions at the edges of the band; therefore, the stabilizing effect of the target zone depends on the signal derived from future interventions. Moreover, when the band is not completely credible, the situation of the stock of reserves may determine the sustainability of the zone, as we have just seen. However, it should also be mentioned that a continued and decisive defence of the band may increase the credibility of the band, despite the simultaneous reduction in the reserve stock. In figure 4, we can observe that a gain in credibility pushes the exchange rate into the band with no variation in the fundamentals. This implies that the intervention can be sterilized as far as it provides gains in credibility.

As a matter of fact in our model of chapter 3 this mechanism plays a role. More precisely, we consider the two opposite effect of interventions: the positive effect which is linked to this signaling effect; the negative effect has to do with the loss of reserves attached to the intervention.

In order to introduce the portfolio effect, the assumption of perfect capital integration has to be removed again. In this case, assets are not assumed to be perfect substitutes.

Let us consider with the help of a simple portfolio model how the degree of asset substitutability allows the sterilization of the interventions. There are two countries and the non-monetary financial wealth \((W)\) is given and consist of the domestic \((B)\) and foreign \((B')\) bond supplies which are also fixed: \(W=B+B'\).

The supply of domestic bonds \((B)\) is demanded by the private sector \((B_p)\) and the Central Bank \((B_c)\). Changes in the private sector demand for domestic bonds \((dB_p)\) depends on the uncovered interest differential and is weighted by a parameter \((q)\) which stands for the degree of asset substitutability as we will see below:

\[
dB_p = q((l_i - l_f) - E_r \frac{ds_t}{dt})
\]  

A negative uncovered differential reduces the demand for domestic bonds. This negative excess in demand is offset by the Central Bank sterilized intervention consisting in the purchase of domestic bonds \((dB_c > 0)\) and a simultaneous sale of reserves, such that
the money supply remains unaltered:

\[ dB_a = dB_c = dR < 0 \]

Substituting the above equality into (28) and solving for the interest differential we arrive to the new expression for the interest parity:

\[ \delta = E_r \frac{ds}{dt} + r, \]

where

\[ r = \frac{dR}{q} \]

can be seen as a premium which arises from the imperfect substitutability. The parameter \( q \) is positive and increasing with the degree of substitutability and it tends to infinity when the substitutability is perfect. Perfect substitutability implies then a political risk equals zero.

Contemplating (29) as an equilibrium condition the portfolio effect can be easily explained. For instance, if \( \delta \) is smaller than the exchange rate expectation, the currency will depreciate and in order to defend the band a sale of reserves takes place.

In the basic model, the adjustment is born by the interest differential (increase in \( \delta \)), through the reduction of the money supply and fundamentals derived from the unsterilized intervention.

However, now we can count on an additional way to establish the equilibrium, which operates through the right-hand side term: for positive values of \( q \), if the sale of reserves is sterilized, \( r \) is negative, fulfilling equivalently the equilibrium condition.

Also note that for higher degrees of asset substitutability, the size of the sterilized intervention must be larger; this means that the higher the financial integration between markets, the more difficult is to carry out effective interventions.

Imperfect substitutability has the same implications that the imposition of capital controls. More precisely, considering the transformation \( q = \gamma / (1 - \gamma) \), we observe that as \( \gamma \to 1 \), \( q \to \infty \) and \( d_r \to 0 \). Thus, we can generalize the meaning of the parameter \( \gamma \) as representing the degree of financial integration.
IV-Why target zones?

The basic target zone model puts forward a basic argument in favour of fixing the exchange rate around a band of fluctuation: when there is credibility, market expectations help to stabilize the exchange rate around the central parity. It is no wonder then that the papers which address the usefulness of target zones have focused on stability issues. Stability is considered not only with respect to the exchange rates, but also with respect to interest rates, output and prices.

The second point emphasized by the results of target zone models is the scope for discretion in the management of economic policy. However, this limited degree of monetary autonomy may vanish when credibility problems arise. Some of the contributions we consider now have considered this issue; this in our view represents an important step in extending the implications and in examining more realistically the rationale for exchange rate target zones models.

Exchange rates, interventions and interest rates

Krugman & Miller (1992) provide an original argument in favour of target zones. They recognize that rational expectations is a strong assumption to explain exchange rate behaviour. In particular, they consider the existence of stop-loss traders which do not care about fundamental but invest (disinvest) in a currency when its price increases (falls) above (below) a given threshold. They show that this leads to an Inverted S-shaped curve for the exchange rate. However, when a target zone is set and the market recognizes the commitment of the authorities to defend the announced band inside the threshold, the behaviour of these agents will be stabilizing rather than destabilizing.

A more relevant contribution is contained in the papers by Svensson (1992) and Miller & Zhang (1994) where the consideration of the problem is taken from an optimal control perspective.

The results of the basic model are determined by the expected evolution of exchange rate and fundamentals, which in turn depend on the behaviour of the
authorities regarding the defence of the target zone. Therefore, a time inconsistency problem arises here: authorities may find convenient to renege on the arrangement after having benefitted from the expectations of the agents on the initial announcement. These papers address this problem in the specification of the target zone. Thus, in both of them, the target zone is seen as a commitment technology to tie down agents expectations.

In the paper by Svensson, a quadratic loss function for the Central Bank is considered and exchange rate and interest volatility appear as arguments. The non-linearities implied by the target zone impose serious obstacles to the analytical solution of the problem. This is overcome by considering a linear approximation which is quite accurate in the case of target zones with intramarginal interventions, as we observed above.

The basic model shows that the choice of regime implies a trade-off between exchange rate and interest rate volatility. Svensson uses this optimizing framework to find the most satisfactory trade-off. This is obtained either by a strong commitment to a target zone or by setting short-run intermediate targets. As a consequence, the degree of monetary autonomy that a target zone seems to grant is limited by the need to commit or follow short-term targets.

These ideas are more elaborated by Miller & Zhang (1994). They build on previous work on optimal interventions on target zones by Avesani (1991). Costs of intervention are assumed proportional to the size of the intervention and they are included in the loss function. On the contrary, interest rates considerations are dropped.

The problem is set up as a game between the Central Bank and the private sector. The outcome of this game is that announced bands are optimal for both participants and time consistent. While Avesani concludes that marginal interventions are optimal, the results derived by Miller and Zhang stress that the optimal rule implies continuous interventions throughout the band.

The problem that both contributions share is that the optimal solution is given at the onset, in the form of an open-loop solution. No feedback on the solution is considered and therefore the target zone design is not affected by the evolving circumstances.

This is a serious limitation in a system like the ERM where credibility evolves over time. Avesani et al. (1994a) attempt to overcome this drawback by setting up a target zone in an optimizing approach which allows for the feedback of the evolving environment on the design of the target zone. Apart from the goal of stability, it is also acknowledged that the target zones imposes undesired restrictions on the authorities and consequently, flexibility is an additional objective. Flexibility is attained through the
possibility to surprise markets. Of course, this has costs in terms of credibility which are conveyed in their specification, too.

This framework allows the authors to design flexible target zones with soft bands. There is again an explicit game between the authorities and the private agents. The announcement of the central parity is done then on a strategic basis and exchange rate targets are corrected when the costs reach a predetermined threshold. More interestingly, the evolution of credibility is explicitly considered and it is shown that the target zone can be hardened when authorities accumulate reputation. We return to the idea of credibility dynamics and reputation in the second part of the dissertation.

**Final targets**

The formalization of exchange rates in an optimization approach considered up to now considers the choice of the band from a Central Bank perspective, narrowly focused on exchange rates or interest rates. However, these are after all intermediate targets for policymakers.

Following the approach of the policy coordination literature, the loss function should really contain some consideration of employment or output and inflation. Some attempts have been made in this direction: first, we consider the contributions of Sutherland (1992), Gros (1990) and Beetsma & Van der Ploeg (1992) and then that of Cuklerman et al. (1994). In chapter five we will consider the issue in a static monetary model of policy coordination.

The three first papers consider the issue from the point of view of stability of output and prices. The loss functions set penalties for the deviations of output and prices from the equilibrium levels. Sutherland compares a completely fixed exchange rate, a free float and a target zone with marginal interventions and finds that the target zones is superior to the other two regimes. Gros (who does not explicitly consider a target zone model) and Beetsma and van der Ploeg (who assume sticky prices) reach the same conclusion: Intramarginal interventions are optimal, contrary to the results of Avesani (1991). The second set of papers also demonstrate the superiority of a managed float over a target zone. Therefore, given these broader considerations, a target zone seems to be a second best alternative to a managed float.

Evidently, the incorporation of final targets improves the framework of analysis. Nevertheless, I am of the opinion that the objective of a target zone in practice goes beyond simple considerations of stability. In the introduction we have seen than the ERM has become a disciplinary device in the minds of policymakers and that this role has been
enhanced in the Maastricht Treaty. Other target zones seem to have been set up for similar reasons, as the Scandinavian experience documented in Lehmussarl et al (1993), Lindberg et al (1992), etc., and the more recent unilateral pegs of developing countries to the dollar, studied by Helpman et al. (1994) illustrate\textsuperscript{13}.

Thus, the approaches which have been considered in this section ignore at least some of the reasons why target zones are adopted. The contribution of Cuklerman, Klugei and Leiderman is a notable attempt to remedy this flaw.

They start by allowing the costs of pegging the exchange rate to depend on the variations in competitiveness which, in turn, affect output. Therefore, if the policymaker is not satisfied with the current level of the real exchange rate, the temptation to manipulate the real exchange rate will arise.

This is formalized in a loss function of the following type:

\[ W(s) = Q(s^e - s) + s^2 \]

where the second term is the cost in terms of exchange rate instability and the first term refers to the desired real exchange rate.

If some degree of stickiness is assumed, a real depreciation (appreciation) can be achieved by making the nominal exchange rate, \( s \), larger (smaller) than \( s^e \); thus, as in Avesani et al. (1994a), surprises in the nominal exchange rate is the channel through which policymakers can satisfy its objectives.

The value of \( Q \) is revealed to the market at each period and it determines the weight of the 'real' objective in the loss function with respect to the stability objective. A value for \( Q \) different from zero reveals that the policymaker views the current exchange rate unfavourably.

The sign of \( Q \) gives information about the real exchange rate objective of the policymaker. \( Q < 0 \) reflects a desire to alleviate internal recession through a nominal depreciation (this would be, in general, the case for periphery countries) and a positive \( Q \) reveals a wish to appreciate the currency. It is evident that a higher absolute value of \( Q \) increases the incentive to cheat on the agreement and therefore reduces the perceived commitment of the authorities. The credibility of the arrangement is in this way linked, although in an implicit way, to real variables.

The model is not exempt of problems though. In particular, the solution is not forward looking as the rest of models, but it makes use of autoregressive expectations, such that the stabilizing behaviour of target zones is not built around the future expected path of the fundamentals, but on the history of the process.

\textsuperscript{13} The references for the ERM can be found in the introduction.
Despite this caveat, the results of this model show that there exists a trade-off between credibility and flexibility: the higher is the reputation of the policymaker commitment, the narrower can be the width of the band. The model also shows that the realignment probability increases at the upper limit of the band. As a result the realignment risk once again turns out to be positively related to the divergence in the fundamentals (defined in terms of the real exchange rates).

These insights will be used to explore the relations between fundamental divergences and sustainability of the band in chapter three.
Part I: Models

1-Target-zones models

V-Final remarks

This survey has presented the elegant models of target zones. The intricacies of stochastic calculus make difficult to extend the modelization and explains why the extensions have regarded the basic model with flexible prices instead of sticky prices which are more realistic.

However, the major limitation of target-zones models is, in my view, the deterministic and state-dependent nature of the solution. The lack of empirical evidence has encouraged extensions of the model, mainly on the issue of credibility, but the introduction of dynamic elements has been hindered by the restrictive assumption of rational expectations. The static nature of the model also limits the insights of the model. The emphasis on the stability properties of target zones which are derived from the models has biased the profession towards justifying target zones as stabilizing devices, except for the notable exceptions which we have reviewed.

The shift of emphasis from stability to discipline in the perception of the ERM took place in the middle of the eighties, that is, it was prior to the development of target zone models. This fact is striking and unfortunate alike and it may have contributed to the discredit of these models after the fall of the ERM.

Notwithstanding this, the efforts made by the optimization strand of the target zone literature to convey the issues of credibility and commitment explicitly in the model should be acknowledged. This step is significant because it leads to the consideration of the actual reasons for the adoption of a target zone and on its sustainability in an optimization framework.

These remarks suggest that there still exists a wide field for target zone models to explore. Our contribution is presented in the third and fourth chapter of this thesis. There, we attempt to introduce explicit dynamics into the model, at the cost of dispensing with the derivation of a closed form solution. This opens up the possibility of considering several crucial issues (credibility, reputation) and more realistic objectives (sustainability, convergence) in a dynamic framework.

The second promising area is to develop further the optimization perspective
reviewed in the last section. More precisely, a global optimizing framework could fill most of the gaps we have pointed out in this chapter. By global we refer both to the consideration of bilateral target zones and more comprehensive economic models. We will return to this point in the conclusions.
This chapter complements the previous survey on theoretical target zone models. We roughly follow the same structure of chapter one: the empirical evidence on the basic model is first examined and then we consider the empirical implications of the extended models.

The empirical analysis of target zones models has focused on the experience of the ERM and the Scandinavian peg to the Deutsche Mark. The feature that differentiates these regimes is that the first is a bilateral target zone and the second is an unilateral peg. Nonetheless, the results do not differ substantially.

The empirical evidence regarding the basic model has determined to some extent the development of the extended models surveyed in chapter one. The speed at which empirical research has evolved is remarkable. This has helped us to learn more about the real functioning of the target zone and, as we will see in the conclusions, it has moderated the initial enthusiasm for target zone models. Particularly disturbing for the literature has been the collapse of the ERM.

The empirical search has been mainly directed to characterize the imperfect credibility of the band because it has been considered the main reason for the empirical failure of the basic model.

I will review several approaches followed by the literature, and, among them Alberola, López and Orts (1994a). The latter consists of an unobserved component decomposition of the elements determining the behaviour of the interest rate differentials in Spain. Our conclusions are similar to the majority of the literature, but we believe that we have overcome some statistical problems within the literature.

The existence of realignment risks has led to the empirical exploration of its causes: reputation, the position of the exchange rate in the band and economic fundamentals. The reputation issue is closely related to the credibility of the band and consequently to
the design of sustainable target zones: the evidence on the band position has endorsed a negative view of the ERM as destabilizing mechanism. The role of fundamentals merits a special mention in the conclusions, because their obscure influence has inspired our dynamic model in the next chapter.

The remainder of this chapter is organized as follows. Section I studies the evidence on the basic model; section II analyses the elements which disturb the basic relationship: interventions, controls and especially realignment risks. Section III provides an account of the trajectory of the Spanish peseta in the ERM until its first realignment, using an unobserved component analysis. Section IV presents an overall interpretation of the results of the empirical evidence and discusses the extensions which are partially developed in the rest of the thesis.
I-TESTS ON THE BASIC MODEL

The basic target zone model presented in chapter one highlights the relationship between three economic variables: exchange rates, fundamentals and interest rates. The behaviour of these variables is explored in this section. The most extensive empirical analysis of the basic model has been carried out by Flood, Mathieson and Rose (1991) and consequently it constitutes our main reference. However, their analysis is supplemented with other contributions. We use the Spanish data from the first regime of the peseta in the ERM, from its entrance (19/06/89) until the first realignment (15/09/92) to illustrate the empirical evidence.

Fundamentals and exchange rates

The central implication of the model is the non-linear relationship between fundamentals and exchange rates represented by the S-curve in figure 1.1. Therefore, the most direct implication to test the relationship implied by (1.3).

The main problem is how to define the fundamentals. In the simple theoretical model, they consist of the money supplies and outputs or, alternatively, of the price level differentials between the two countries whose exchange rate is considered. However, as we will see below, the concept of fundamentals is much wider, depending on which structural model of exchange rate determination is being used: moreover, structural models have failed to explain the evolution of the exchange rate (Meese & Rogoff (1983), Frankel & Rose (1994), McDonald & Taylor (1994)).

Flood et al. (1991) overcome this problem in a rather naïve way by letting the fundamentals be determined by the data. Note that from (1.3), fundamentals can be

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1. The data are daily (788 observations) Sources for the data are Servicio de Estadística (Banco de España) and Datastream. Interest rates are Euromarkets one-month deposits. The central parities with respect to the German Mark and French Franc were set at 65 pts/DM and 19.38 pts/FF, respectively, and a 12% fluctuation band with respect to the central parity was allowed.
solved out as $f = s - a/(1 + a)$. Flood & Rose (1993) carry out the same exercise and label the resulting estimation as virtual fundamentals. The right-hand side terms are observable and consequently a measure for fundamentals can be obtained once the value of the interest rate semielasticity ($a$) has been determined.

Flood et al. (1991) consider two measures for $a$. The first consists of estimating the parameter directly. Taking the theoretical process for the fundamentals to be in discrete time (i.e. assuming they follow a random walk) and taking differences from the expression (1.3), $a$ can be estimated. The existence of many currencies and regimes in which $a$ turns out to be negative does not deter the authors, which simply exclude these cases from the sample. The second method simply uses the value for $a$ from the literature.

The results from these rough methods are hardly surprising: few linearities appear and even in these cases, they do not take the form foreseen by the theory. Moreover, they do not tend to appear in the countries more committed to the ERM, as it should be expected too.

The lack of evidence on the theoretical non-linearities is not exclusively derived from the unreliability of the method. Neither Smith & Spencer (1991) reach conclusive results, applying the Method of Simulated Moments$^2$.

On the whole, testing a hypothesis with a variable which is not observable and has to be constructed creates difficulties, and few conclusions can indeed be derived. The exercise has still less value when confronted with the clear results obtained by using more direct and observable measures to test the impact of a target zone on exchange rates.

**Statistical properties of the exchange rates**

Exchange rates and interest rates are directly observable, so that the empirical implications of the model are immediately apparent. Figures 1.a,b display the position of the peseta in the ERM with respect to the DM and French Franc, respectively and the interest rate differentials in such period.

Regarding the behaviour of the exchange rates, three statistical properties are derived from the basic target zone model:
- The unconditional distribution is U-shaped within the band.
- The conditional distribution of the standard deviation is $\circ$-shaped.
- The exchange rate displays mean-reversion.

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2. The Method of Simulated Moments consists of choosing the model parameters to generate the series of fundamentals and exchange rate which best replicate the statistical moments of the observed data. The results show that the historical moments are quite different from the simulated ones.
Figure 1.a - Interest rate differentials and deviations from the central parity (1989-93)

Figure 1.b - Interest rate differentials and deviations from central parity (1989-93)
Figure 2a-Distributions derived from the data. Germany

Figure 2b-Distributions derived from the data. France
Applying the results of Harrison (1985) to the target zones, Svensson (1991a) shows that since, in the driftless case, the fundamental distribution is uniform, the S-curve relation implies that the mass of probability is higher at the edges of the band. Empirical evidence in Bertola & Caballero (1993), Flood et al. (1991) and Lindberg & Söderlind (1994) contradicts this result. Rather, the probability tends to accumulate at the center of the band.

Figures 2.a,b show a particular feature in the Spanish case: the exchange rate distribution exhibits a peak (for France) close to the lower limit of the band, reflecting the strength of the peseta in most of the period, but a U-shaped distribution is not observable. Chen & Giovannini (1993) derive a technique to directly estimate the distribution functions for the exchange rate in a target zone. Their results also show that the estimated distributions do not match the theoretical shapes.

The second property derives from the standard deviation expression in chapter 1: \( \sigma^* = s_\phi \). Since the slope of the S-curve is decreasing until it reaches zero at the edges of the band, the distribution reduces its probability mass as it approaches the edges, where it reaches a null value. In figures 2.a,b we observe a quite uniform distribution and similar patterns are also found in the literature.

Testing the hypothesis of mean reversion is more interesting because, it is related to some of the credibility tests which are discussed in the following section. The hypothesis is conveyed by the negative relation between interest rate differentials (in this case expected exchange rate depreciation) which appears in figure 1.2. The assumption of perfect credibility implies that the exchange rate will return to the center of the band. However, Svensson (1993) argues that this feature is robust to the specification of the model and that it persists in the case of imperfect credibility because the mean reversion is defined with respect to the expected long-run value, which can well not be the central parity.

The statistical implication is that, contrary to floating regimes (see Meese & Rogoff (1983)) the exchange rate is expected to be stationary-I(0). Therefore, unit root tests can be applied to reveal the mean-reversion property of the series. Notwithstanding this, we have to be aware in what follows that the experiment with simulated data carried out by Froot & Obstfeld (1991b), reveals that this mean-reversion hypothesis only holds for narrow bands and with large samples.

The most popular unit root test is the (Augmented) Dickey-Fuller test, whose general form is:

\[
\Delta_s_t = \beta_0 + \beta_1 s_{t-1} + \sum_{j=1}^{\nu} \beta_j \Delta u_{t-j} \tag{1}
\]

where \( \nu = 1 \) and the null hypothesis states that the series contain a unit root, i.e. \( \beta_1 = 0 \).
Svensson (1993) finds implicit evidence that the null hypothesis is rejected, so that the series are stationary\(^3\). His results are confirmed by Weber (1991), Caramazza (1993) and Thomas (1994). However, these results are biased by the way the unit root tests are performed. Daily data are applied on a monthly basis and Hansen & Hodrick (1980) stress that the overlapping observations can affect the correlation of the residuals and consequently the estimation of the model. However, as López (1995) and also Svensson point out, if standard errors are computed through the application of the Newey-West procedure in order to account for autocorrelated residuals (Newey & West (1987)) this problem is bypassed and the estimation of \( \beta \), in the test is superconsistent under the null hypothesis.

The main problem arises on the validity of the Dickey-Fuller critical values of the distribution. López (1995) shows that testing for unit roots with overlapping data requires a correction of the critical values, otherwise the test is biased towards the rejection of the null hypothesis\(^4\). After the appropriate corrections are applied, it turns out that mean reversion only appears for the strongest currencies of the system: Guilder, Belgian Franc, French Franc and Danish Krone, that is, those currencies which \textit{a fortiori} did not realign. Hence, on this basis we could argue that the basic model is only supported for these currencies.

### Interest rates

Interest rate differentials equal exchange rate expectations when the UIP is fulfilled. Svensson (1992b) argues that the risk premium in a target zone should be negligible and the empirical evidence in Rose & Svensson (1991) confirms this hypothesis for the DM/FF parity in the ERM\(^5\). Therefore, interest rate differentials (\( \delta \)) should be negatively correlated with the position of the exchange rate in the band\(^6\). Although the relationship displayed

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\(^3\) López (1995) computes the implicit values for the unit roots test reported in Svensson’s paper.

\(^4\) The analysis of Svensson includes the interest rates in the regression and this may also bias the test towards the stationarity hypothesis, as he himself recognizes. The correction factor as calculated by López is 1.22, shifting the critical value of the Dickey-Fuller test at 5\% from -2.89 to -3.50 for one month maturity assets (20-22 observations).

\(^5\) An alternative to interest rates is to work with forward premia because they have the advantage of requiring the fulfillment of the covered parity, on this alternative Bartolini & Bodnar (1992) find no support for the basic model in forward premia.

\(^6\) Regarding the statistical properties of the interest rate distribution is U-shaped and the standard deviation is also \( \rho \)-shaped. These properties directly derive from the exchange rate distribution results. Figures 2.a.b and 4.c.d do not display such features.
In figure 1.2 is not linear, Svensson (1991c) shows that it becomes steeper and more linear as the maturity term for the considered asset increases (the result resembles figure 1.6 for Intramarginal Interventions). Therefore, a linear relationship of the following type can be postulated for estimation, which is the observable counterpart of (1.11):

\[ \delta_t = c + b s_t^* + \epsilon_t \]

where \( s^* \) is the deviation of the exchange rate with respect to the central parity, \( b \) is expected to be negative and \( \epsilon \) is an error term.

Apart from being a simple, linear and directly observable specification, the attraction of this expression is that it is general because the variable \( c \) may be defined in so as to capture the degree credibility of the band, as we will observe below.

The basic model requires however that the value of \( c \) is constant and equals zero, because perfect credibility is assumed. In practice, this negative relationship is difficult to find. Graphical methods (Svensson (1991a), Weber (1991), Flood et al. (1992) and Lindberg & Söderlin (1994)) display quite diffuse scatterplots of the relation, similar to the plots in figures 3.a,b. Direct estimation of the parameter yields no better results.

The overall picture on the evidence of the basic target zone models is apparently appalling. No single prediction of the model is fulfilled in practice and, in several cases, completely opposite results are found. Nevertheless, I think that the econometric analysis to test the basic model has been poor and that, therefore, inferring the failure of the basic model or, more generally, of target zones from these result is inadequate. We will return to this point at the end of the chapter.

In any case, the immediate consequence of these findings has been to 'stimulate researchers to get back to the drawing board' as Svensson (1992c,p.129) puts it. New models of target zones which relax the most unrealistic assumptions have been developed, as we have seen In chapter one. On the empirical side, efforts have been concentrated in devising refined methods to study the rest of factors which play an empirical role in target zones.

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7 Note that this specification resembles the unit root test in (1) where interest rate differentials are substituted by changes in the exchange rate. When UIP holds, expression (2) is a particular case of (1). The parameter \( c \) is in this case equal to the long-run expected value of the exchange rate, so that the point which Svensson makes is that in the unit root test the exchange rate will be stationary around a constant if the expected long-run mean value is different from the central parity.
Fig. 3.a - Interest differentials and deviations from the central parity Scatterplot.

Fig. 3.b - Interest rate differentials and deviations from the central parity Scatterplot.
II-TESTING THE EXTENSIONS OF THE MODEL

Relaxing the assumptions of marginal interventions and perfect credibility affects the critical features of the model. We briefly review in the first place the effects of and evidence on intramarginal interventions and capital controls so as to concentrate later on the realignment risk and its determinants.

Intramarginal Interventions and capital controls

The existence of intramarginal intervention is a documented fact in the case of the ERM and in other unilateral target zones (see references in chapter one). The hypothesis of intramarginal intervention changes the S-curve relationship (see figure 1.6) and, consequently the testable implications of the model. The implied non-linearities are now reduced, which would explain the results of Flood et al. (1991). Lindberg and Söderlin (1992) show that, when the fundamental process is driftless, the distribution of the exchange rate is approximately a truncated normal, which is closer to the observed behaviour of the exchange rate data; such evidence of intramarginal interventions allows them to explain much better the empirical features of Swedish data.

The possibility of capital controls has been completely neglected by the literature, as pointed out in chapter one. However, figure 4 displays the differential between the onshore (one month interbank rates) and off-shore (Euromarkets) interest rates in Spain. We observe that there is a positive, yet decreasing wedge, indicating the existence of controls on inflows. Only in the last few periods, prior to the realignment did the differential turn negative, as a result of the devaluation expectations. This suggests that the role of capital controls may have been disregarded too quickly.

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*The Economist also stressed the existence of this wedge in France during the EMS crises, despite the theoretical assumption of perfect capital mobility, after the implementation of the Directive on free flows of capitals. The reason pointed out by the magazine is the plea of the Banque of France to the commercial banks to sustain the French franc during the crisis.
Capital controls measured as the difference between domestic and Euromarket interest rates

dt - \Delta t^*.

Realignment risk

Imperfect credibility in a target zone has been the most studied empirical issue of the target-zones literature. Any empirical implication of the target zone model, even the mean reversion property may be reversed when a (variable) realignment risk is present in the data. We observed in the discussion on credibility in the previous chapter that any relation is then possible between the fundamentals and the exchange rate or between the interest rate differentials and the position of the exchange rate in the band. Two types of analysis can be distinguished in this area. The first type regards the methods to extract the realignment risk from the data; the second has attempted to analyse the determinants of the realignment risks in the target zone. We will review each of them in turn.

The first approach to test the credibility of the model was devised by Svensson (1991b) and is extremely simple, as he explicitly recognises in the title of the paper. The simplest test compares the expected returns of the domestic and foreign asset of the same maturity (\( \sigma \)), taking into account the feasible variation of the currency within the band. This imposes implicit upper (\( \sigma_{u,\sigma} \)) and lower (\( \sigma_{l,\sigma} \)) limits on the rate of return of the domestic assets if the band is credible, represented by the dashed lines in figure 7:
When the domestic rate of return surpasses one of the implied limits, this is evidence of imperfect credibility of the target zone. Empirical evidence in Svensson (1991b) and Flood et al. (1991) show that the hypothesis of perfect credibility is in general rejected before realignments for most currencies and regimes.

Nevertheless, this test implicitly assumes that the exchange rate can only be realigned when it reaches the edges of the band. Thus, it provides a sufficient but not necessary condition for the existence of a realignment. In practice, a realignment can also occur even though the test suggests perfect credibility and consequently this method overestimates the credibility of the band.

Figure 5 displays results for the Spanish case for assets of a one-year maturity. We can observe that the band turns out to be credible for the French Franc and for most of the period against the Deutsche Mark. Before the ERM crises there is no problem of credibility, justifying the above remarks.

For the rest of currencies the zone was deemed to be credible according to this method right until just before the first crises: only just before the September turmoil and then for most of the period until the widening of the bands (period September '92-August '93) is the band seen to be non-credible for the French Franc (see Avesani et al. (1994b)).

The 'drift adjustment' method has been proposed by Bertola & Svensson (1993) and has been implemented in most of the successive empirical analysis of target zones, starting with Rose & Svensson (1991), Lindberg et al. (1994) and again Svensson (1993). It consists of extracting the realignment risk from the data in order to give a explicit measure of the credibility of the band.

The expected rate of depreciation of the exchange rate in a target zone can be decomposed into two elements: the expected rate of depreciation within the band (or respect to the central parity (s')) and the expected rate of change in the central parity (C):

\[ E\Delta s_{t+1} = E\Delta s'_{t+1} + E\Delta C_{t+1} \]  

Expectations of realignments inside the band can introduce peso problems in the estimation, as we will discuss later on. Therefore, it is convenient to express the expectation terms in (4) as conditional expectations, as we did in expression (1.12):

\[ E\Delta s'_{t+1} = \omega_r (E(\Delta s'_{t+1} | \text{realignmen})) + (1 - \omega_r) (E(\Delta s'_{t+1} \ | \text{no realignment})) \]

\[ E\Delta C_{t+1} = \omega_r (E(\Delta C_{t+1} \ | \text{realignmen})) \]
Figure 5.a - Svensson's credibility test for Germany.

Figure 5.b - Svensson's credibility test for France.
where we recall that $\omega_i$ is the probability of realignment from time $t$ up to time $t$ and the expectation of change in the central parity conditional on no realignment is of course zero.

Assuming that the UCP is fulfilled the expected rate of change in the central parity, is simply equal to, from (4):

$$EAC_t^C = \Delta s_t^C + \omega_t(E(\Delta s_t^C | realignment) - E(\Delta s_t^C | no realignment))$$

The expected rate of devaluation ($c_t^i$) captures the variation of the exchange rate exclusively due to the realignment, and is then defined as the sum of the expected jump on the central parity plus the expected net jump of the exchange rate within the band, when a realignment occurs:

$$c_t^i = EAC_t^C + \omega_t(E(\Delta s_t^C | realignment) - E(\Delta s_t^C | no realignment))$$

From the above expressions, the expected rate of devaluation can be simply expressed as the difference between the interest rate differential and the expected rate of depreciation within the band, conditional upon no realignment taking place:

$$c_t^i = \Delta s_t^C + \omega_t$$

where the conditional expectation has to be estimated.

The estimation is straightforward. Since the no realignment condition applies to the perfect credibility case, the basic target zone relation applies; the consideration of assets of a relatively long maturity and the existence of intramarginal interventions, allow us to assume in the rest of the chapter that the relation is linear.

It is then reasonable to regress the effective depreciation within the band (i.e. conditional upon no realignment) on the position of the exchange rate in the band:

$$\Delta s_t^C = k_0 + k_1 s_t^C - \omega_t$$

where $k_0$ is a dummy variable for each period between realignments of the currency and the sign of $k_1$ is expected to be negative. Taking expectations, we obtain an estimate of the expected depreciation conditional on no realignment.

The results of these tests are reported in the literature for different asset maturities $t=1,3,6,12$. In general they show that for most of the currencies the expected rate of devaluation with respect to the DM is small but not negligible between realignments and it shows a peak in the periods immediately prior to the realignment. There is a tendency for $c_t^i$ to decrease, indicating that the ERM has consolidated its credibility. In any case, the magnitude of the crises of 1992 does not seem to have been anticipated by the agents in their realignment expectations (Rose and Svensson (1994)). The expected rates of devaluation move closely together between different maturities because the short-term interest rates prompt shifts in the yield curve. As a matter of fact, the realignment risk series are very similar to the interest rate differential series in most of the sample, which
is to be expected given the estimation method.

One apparent feature of this method is that the regression only makes sense if the exchange rate series are stationarity; otherwise, the parameter \( k \) is not significantly different from zero. This does not mean however that the test is invalidated by the existence of a unit root in the exchange rates, because in this case the expected rate of devaluation would be simply given by the interest rate differentials.

The papers which apply the drift adjustment method find significant negative values for these parameters, although not all of them carry out unit root tests explicitly. However, we must interpret these results with caution. In the discussion above we stressed that in equation such as (9) it is necessary to correct the Dickey-Fuller critical values; none of these papers takes such a consideration into account.

In any case, comparing (2) with (8,9), we can observe that, the Intercept term in the former coincides with the expected realignment risk and that \( k = \beta \), so that the drift adjustment method removes from the data the noise due to imperfect credibility; if the parameter is significantly negative the implication of the (extended) target zone model is endorsed by the data: once the disturbance term due to imperfect credibility is accounted for, the data display a negative relationship between the interest rate differentials and the position of the exchange rate in the band, as implied by the basic theoretical target zone model.

Weber (1991) uses an alternative method to study the credibility of the target zone. He allows for a time-varying devaluation risk to be estimated through a Bayesian multi-process Kalman filter. The idea is to convey in a State Space representation the relationship between interest rate differentials and the position of the exchange rate in the band, appearing in (2).

The stimulus to Weber's approach is precisely his observation that the exchange rate series are not stationary in some regimes of the ERM. This makes him think that the realignment risk shifts between a stationary behaviour such as in Svensson (1991a) and a non-stationary process, as in Bertola & Svensson (1993).

Consequently, two different specifications are proposed for the devaluation risk \( (c) \) as sub-models of the general State Space form. The Bayesian component of the filter determines the stochastic shifts using the information provided by the data. The results obtained by Weber coincide with the drift-adjustment method: a reduction of the devaluation risk throughout the FRM and significant negative values for the parameter \( b \) which supports the target zone's extended model.
The contribution of Alberola, López and Orts (1994b) is developed in the next section. This methodology is based on the specification of Weber (1991), although the implementation is quite different. We also make use of the Kalman Filter because we consider that it is an adequate method to extract information from the data. However, we dispense with the multi-process setting and the specification of the devaluation risk is set up beforehand by observing in detail the statistical behaviour of the series under study. This approach is probably less sophisticated than Weber’s but it is much more robust on a statistical basis.

Other contributions (Frankel & Phillips (1991)) use a survey of forecasts to calibrate the credibility of the system which is seen to increase with time. Finally, it is noteworthy to mention the contribution of Avesani et al. (1994b) which combines a study of the credibility of the system with some economic policies guidelines. They use their optimal targeting strategy (commented in the previous chapter) to study the credibility of the FF/DM parity. Their method simply involves to generate exchange rate series in their model until finding the one which most closely follows the behaviour of the FF/DM until the realignment. Then, they use the credibility measures conveyed in their model and compare them with other measures as the simplest credibility test in Svensson and a detailed series of statements in the Economist. The comparison reveals a close relationship between the theoretical measure and that actually perceived in the markets. Finally, the information accumulated in the flexible target zone model allows to determine endogenously the optimal band width of the new announcement. This is used to suggest the reaction of the ERM members to avoid the collapse of the system: widening the band.

What’s in a risk

The existence of a well-defined measure for the realignment risk (with the statistical caveats we indicated above) has encouraged research into the factors which determine its evolution. The methodology is similar in all the cases, and consists on a more or less sophisticated linear regression of $c_t$ on the explanatory variables. Several types of factors have been considered: the position of the exchange rate in the band, reputation of the authorities and economic fundamentals. The next table summarises the components of the realignment risk and the results of the empirical evidence.

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5. We will see that the parameters are chosen by maximum likelihood and that the priors of the Kalman Filter are obtained with accurate computational algorithms. On the contrary, Weber seems to have set these parameters ad hoc and this shed doubts on the reliability of his results.
### GENERAL REGRESSION

\[ c = \delta s' + [\delta_s \alpha^* + \delta_{\alpha^*}] + [\delta_y (m - m^*) + \delta_{m^*} (y - y^*) + \delta_{y^*} (\pi - \pi^*)] + d_4 XN + \delta_4 FEER + \delta_6 (\text{def} / \text{GDP}) + [\delta_5 \mu + \delta_5 \alpha + \delta_5 \pi] + \delta_6 R. \]

<table>
<thead>
<tr>
<th>DETERMINANTS OF ( c_t )</th>
<th>REFERENCES</th>
<th>EFFECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1- Deviations of the exchange rate ((s')), ( \delta_s &gt; 0 )</td>
<td>Chen &amp; Giovannini, Caramazza, Thomas</td>
<td>YES</td>
</tr>
<tr>
<td>2- Measures of reputation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- SD of future exchange rate ((\sigma^-)), ( \delta_{\sigma^-} &gt; 0 )</td>
<td>Rose &amp; Svensson</td>
<td>YES</td>
</tr>
<tr>
<td>- Time span since the last realignment, ((T)), ( \delta_T &lt; 0 )</td>
<td>Chen &amp; Giovannini</td>
<td>YES</td>
</tr>
<tr>
<td>3- Model fundamentals</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Money supplies ((m, m')), ( \delta_m &gt; 0 )</td>
<td>Rose &amp; Svensson</td>
<td>NO</td>
</tr>
<tr>
<td>- Outputs ((y, y')), ( \delta_y &lt; 0 )</td>
<td>Rose &amp; Svensson</td>
<td>NO</td>
</tr>
<tr>
<td>- Inflation differentials ((\pi, \pi')), ( \delta_{\pi} &gt; 0 )</td>
<td>Rose &amp; Svensson, Ayuso et al.</td>
<td>Mixed</td>
</tr>
<tr>
<td>4- Economic imbalances</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Trade balance ((XN)), ( \delta_X &lt; 0 )</td>
<td>Ayuso et al.</td>
<td>NO</td>
</tr>
<tr>
<td>- Fund Eq. Exchange Rate ((FEER)), ( \delta_{FEER} &lt; 0 )</td>
<td>Thomas</td>
<td>NO</td>
</tr>
<tr>
<td>- Fiscal imbalance as ( % ) GDP ((\text{def} / \text{GDP})), ( \delta_{\text{def}/\text{GDP}} &gt; 0 )</td>
<td>Rose &amp; Svensson, Ayuso et al.</td>
<td>NO</td>
</tr>
<tr>
<td>5- Deflationary bias</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Unemployment rate ((\mu)), ( \delta_{\mu} &gt; 0 )</td>
<td>Thomas, Caramazza</td>
<td>YES</td>
</tr>
<tr>
<td>- Real exchange rate ((\alpha)), ( \delta_\alpha &gt; 0 )</td>
<td>Klein &amp; Marion</td>
<td>NO</td>
</tr>
<tr>
<td>- Interest rates ((\pi)), ( \delta_\pi &gt; 0 )</td>
<td>Klein &amp; Marion</td>
<td>NO</td>
</tr>
<tr>
<td>6- Reserves ((R)), ( \delta_R &lt; 0 )</td>
<td>Ayuso et al.</td>
<td>NO</td>
</tr>
</tbody>
</table>

NB. The regression above is an simplification. The literature uses sometimes this variables in levels, others in differences, sometimes lags or leads, etc.

The sign of the parameters refers to the expected sign.

**Table 1** - Decomposition of the realignment risk according to the empirical research
The inclusion of the position of the exchange rate in the band (s') has special relevance. We observed in figure 3 the blurred relation between it and the interest rate differential (a good proxy for realignment risk), contrary to the standard assumptions. The empirical evidence suggests that the position of the exchange rate in the band is positively related to the realignment risk (Chen & Giovannini (1993b), Thomas (1994), Caramazza (1993)). This implies that the actual relation between interest rates and the position of the exchange rate in the band may be positive as the imperfect credibility models of Bertola & Caballero (1992) and Tristani (1994) suggest. This contradicts again the central prediction of target zone models because in this case the target zone may have a destabilizing effect on the exchange rate.

Rose & Svensson (1994) and Chen & Giovannini (1993b) include two different measures of reputation in the target zone. Rose & Svensson (1994) build on the idea that the ERM leaves some scope for monetary autonomy and not using it increases the credibility of the authorities (Svensson (1992a)). They use the standard deviation of the future expected exchange rate as a proxy of such autonomy and find a significantly positive relationship, as the theory suggests. The measure used by Chen and Giovannini is simpler. They use the time span from the previous realignment as a measure of reputation. Using this method a negative sign is found in the regression.

Both empirical results confirm the importance of the commitment of the authorities to enhance the stabilizing effect of the target zones.

The factor which is common to all the papers is the explicit consideration of some economic variable representing the fundamentals. The choice of the variables is based on economic considerations derived from different areas of the literature.

The most obvious candidates are money and output which appear in the underlying model of target zones (Rose & Svensson); prices, in the form of inflation differentials is another obvious candidate if price stickiness is considered and some sort of PPP is assumed (Rose & Svensson (1994), Ayuso et al. (1993)).

External and fiscal imbalances should also be taken into account; its rationale can be found both in the currency crisis and the Fundamental Equilibrium Exchange Rate (FEER) literature. External Imbalances can be introduced simply by taking the current account deficit (Ayuso et al. (1993)) or more elaborate measures based on FEERs which also convey the effect of inflation differentials, too (Thomas (1994). Fiscal imbalances
affect long-run expectations of inflation and consequently the credibility of the zone\textsuperscript{10};
and can be measured by the deficit/GDP ratio as in Rose & Svensson and Ayuso et al.

Some measure of unemployment (such as NAIRU in Thomas (1994) and the unemployment rate in Caramazza (1993)) can capture some of the real costs derived from pegging the currency to a low inflation country. The overvaluation of the real exchange rate and the higher interest rates that a devaluation risk imposes on the country imply a deflationary bias which may affect the credibility of the target zone (Klein & Marlon (1994)).

Finally, reserves is other obvious candidate because it is a signal on the sustainability of the band. However, Ayuso et al. (1993) find no significative correlation.

The conclusions from these analysis are mixed. An important restriction lies in the difficulties to find high frequency data for many of the assumed fundamentals; a second is that the joint consideration of a great number of variables, many of them highly correlated, makes it difficult to pick up reliable response values. Finally, the explanatory power of these variables, taken separately is usually quite low. Despite this caveats, one interesting implication found in Ayuso et al. (1993) is that when longer maturities are considered, fundamentals seem to be able to explain better the behaviour of realignment risk. Apart from this, each paper identifies different measures of fundamental as being significant. Some consensus only seems to appear around the unemployment variable and to a lesser extent inflation differentials (Rose and Svensson in certain cases, Ayuso et al. for long maturities, but not in the rest).

On the whole, there does seem to be an effect fundamentals on the realignment risk, but it is difficult to grasp it statistically. The reason, which we develop at length in the next chapter is that some kind of fundamental imbalance justifies the final collapse of the parities, but they only occasionally influence the behaviour of exchange rates and realignment risk in the short run. This is consistent with the results of Allen & Taylor (1989) who underline that foreign exchange markets are mostly driven by short-run considerations.

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\textsuperscript{10} The problem of debt crises, stressed in the currency crisis literature is not likely to have much relevance in a European context.
III-An approximation to the Spanish experience in the ERM

Spain entered the EC on 1 January 1986 whereas the Incorporation to the ERM took place on 19 June 1989. The strength of the peseta until its first realignment is remarkable, especially given the magnitude of inflation differentials with Germany and that it was one of the first currencies to realign in the monetary crises of the ERM. This strength can be explained by the economic environment in Spain in these years. The economic consequences of Spanish integration into the EC and the ERM are explored for instance in Viñals et al. (1990), Viñals (1990) and Dolado and Viñals (1991).

Spain being a relative large country, the prospect of joining the Common Market boosted growth from 1985 onwards, but this extremely expansive cycle was constrained by structural inefficiencies which kept inflation rates high. Economic policy was therefore aimed at easing these constraints through an expansive fiscal policy while restraining at the same time monetary growth to control domestic inflation.

Time was ripe to consolidate the reduction of the inflation differential achieved up to that moment: Spain joined the ERM to narrow further the inflation gap by benefitting from the German reputation, despite the attached costs in terms of the external balance. In spite of the discipline imposed by the system, interest rate differentials have remained relatively high at a time when an intense process of financial integration has been implemented. Finally, macroeconomic indicators started to converge to ‘European levels’ (as Spaniards use to put it), but too slowly to close the gaps. However, this seemed to satisfy the markets which accepted the rules of the convergence play (see Portes (1993)) in a EMS which was living its golden days.

High interest rates and economic expansion kept then the peseta strong in the system, and caused a sharp overvaluation of the real exchange rate (see figure 8 below), providing a classic example of the ‘new EMS’. The process of financial integration which accompanied the strength of the peseta was such as to call for an active defence of the (lower!) band through interventions which boosted the Spanish stock of reserves.

After four years of currency stability and economic expansion, growth slowed down and the mirage vanished: inflation had not converged, the public deficit boomed, capital poured out of the country and the accumulated fundamentals imbalances placed the
peseta in the eye of the EMS storm.

The model

In this section we undertake an empirical analysis of the Spanish experience in the ERM target zone. We intend to measure the stabilizing effect of the existence of the ERM on exchange rate expectations by exploring the Spanish data with respect to two other currencies of the ERM: the French Franc and the Deutsche Mark.

Let us recall that the target zone will be considered to stabilize expectations if a negative relation is found between the interest rate differentials and the position of the exchange rate in the band, that is:

$$\delta_t = c_r + b\delta_x + \epsilon_t$$

where $b < 0$ under the target zone hypothesis.

We have already pointed out that the implications of the basic model do not hold for the peseta. Exploring the reasons for the rejection of the basic model requires a statistical analysis of the exchange rate and interest rate differential series. The results of this analysis will also help us to reformulate the structure of the empirical model. In this approach, we depart from previous empirical models, in that no process for the variables of the model (especially $c_r$) is assumed 'a priori'. Therefore, we formulate a flexible model for $c_r$ of the form:

$$c_r = \mu + p c_{r-1} + \theta(B) \eta_t$$

where $p \in (0,1)$; $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots$; $E(\eta_t) = 0$; $E(\eta_t^2) = \sigma^2$

with $B$ is the backshift operator, such that $B^r x_t = x_{t-r}$, and all the roots of $\tau(B) = 0$ lie on or outside the unit circle. Note that the only restriction imposed on the empirical model is that $\epsilon_t$ in (2') follows a white noise process.

Given this specification, we are able to explore the actual process for $c_r$, given our data set. The behaviour of the data specifies completely the structure of the model and then we can estimate it applying the Kalman Filter.

The summary statistics of the differenced series contained in Table 2 show several noteworthy points. Firstly, the series present excess kurtosis—mainly in the interest rate differentials—indicating that they are highly nonnormal; further, the column labeled $S(0)$ provides estimates of the spectral densities at zero frequency. As noted by several authors, (e.g. Cochrane (1988)), this provides a useful diagnostic on the permanent (that is, for the Spanish case we are considering see Serrat (1992), Rodriguez (1992) and Ayuso et al. (1993). They show that the band is credible in the studied period ('89-beginning 1992).
components with a unit root) and transitory components in the series. Judging from point estimates, there seems to be far more persistence of shocks in $s^*$-the deviations with respect to the central parity. On the contrary, the interest rate differentials exhibit some degree of mean reversion towards a downwards trend, and this could lead to the rejection of a random walk model. Consequently, we test for the existence of a unit root in the autoregressive representation of each series against two alternatives: one consistent with fluctuations around a constant mean $\tau^*$, the other with stationary fluctuations around a deterministic linear trend $\tau^*$. Table 3 contains the corresponding Augmented Dickey-Fuller tests; in both cases eight lags are included to account for serial correlation, although after the first lag the results appear to be insensitive to the number of lags.

We can see that there is very little evidence against the unit root hypothesis if the considered alternative is stationarity around a constant mean. However, as noted above, a more relevant alternative would be to consider whether the interest differentials could present stationary fluctuations around a deterministic linear trend. As the results show, the French differentials ($\delta_f$) are actually stationary ($I(0)$)-around a deterministic trend, whereas the German differentials ($\delta_G$) contain a unit root ($I(1)$). Finally, both exchange rate

**Table 2: Statistics of the daily changes in the series**

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>$S(0)_{x0}$</th>
<th>Skew</th>
<th>Ex. Kurt</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_g$</td>
<td>-.006</td>
<td>.202</td>
<td>.01</td>
<td>-1.28</td>
<td>17.31</td>
</tr>
<tr>
<td></td>
<td>(.004)</td>
<td></td>
<td>(.093)</td>
<td></td>
<td>(.186)</td>
</tr>
<tr>
<td>$s_g$</td>
<td>-.001</td>
<td>.273</td>
<td>.05</td>
<td>.3142</td>
<td>1.513</td>
</tr>
<tr>
<td></td>
<td>(.008)</td>
<td></td>
<td>(.092)</td>
<td></td>
<td>(.186)</td>
</tr>
<tr>
<td>$\delta_f$</td>
<td>-.003</td>
<td>.197</td>
<td>.01</td>
<td>-1.09</td>
<td>19.66</td>
</tr>
<tr>
<td></td>
<td>(.005)</td>
<td></td>
<td>(.093)</td>
<td></td>
<td>(.186)</td>
</tr>
<tr>
<td>$s_f$</td>
<td>-.000</td>
<td>.253</td>
<td>.04</td>
<td>.3509</td>
<td>1.661</td>
</tr>
<tr>
<td></td>
<td>(.007)</td>
<td></td>
<td>(.092)</td>
<td></td>
<td>(.186)</td>
</tr>
</tbody>
</table>

*S(0)_{x0} is a Bartlett estimate of the spectral density at zero frequency using 40 lags. Newey-West Standard errors in brackets.*
Table 3 - Augmented Dickey Fuller tests

<table>
<thead>
<tr>
<th></th>
<th>8_0</th>
<th>8_1</th>
<th>8_2</th>
<th>8_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>8(8)</td>
<td>-76</td>
<td>-2.09</td>
<td>-2.03</td>
<td>-1.99</td>
</tr>
<tr>
<td>8(8)</td>
<td>-3.94</td>
<td>-2.07</td>
<td>-3.82</td>
<td>-2.00</td>
</tr>
</tbody>
</table>

Numbers for 8_0 are t statistics on 8_0 in the regression \( \delta = \alpha + \beta x_0 + \gamma x_1 + \epsilon \). Numbers for 8_1 are t statistics on 8_1 in the regression \( \delta = \alpha + \beta x_0 + \gamma x_1 + \epsilon \). Critical values at 5% are -2.86 for 8_0 and -3.41 for 8_1.

deviations (s_0^2, s_1^2) are I(1).^{12}

From these results, we can observe that for France a constant parameter \( c \), would imply that the residuals \( e \), in (2') should contain a unit root, given that-8_0 being I(0), around a deterministic trend, and 8_1 is I(1)- these series cannot be cointegrated. For Germany a cointegration relationship could be possible, as both series are I(1), hence we test for it. EG(5) in Table 3 shows that both series are not cointegrated, and as a consequence maintaining a constant parameter for \( c \) would also imply a unit root in \( e_1 \). This explains to some extent the blurred relationship between \( \delta \) and \( s_0 \) when a constant \( c \) is assumed as displayed in Figures 3. This is the reason to formulate the flexible model for \( c \), in (10).

The lack of cointegration between the series \( \delta \) and \( s_0 \) implies that the hypothesis of \( p=1 \) is not rejected, and \( c_1 \) is I(1) for both countries. Note that the unit root previously contained in the residuals (when \( c_1 \) was constant) is now shifted to the variable parameter \( c_1 \). The second element to determine is the order of \( \tau(B) \); this is done with the help of the autocorrelations of the differenced series, displayed in Table 4. There, we can see that for \( s_0 \) an IMA(1,1) process is an acceptable representation, whereas the series \( \delta \) seem to be well represented by a pure random walk.\(^{13}\) Therefore, the MA polynomial for

---

12. Here we do not have the problem of overlapping observations in the exchange rate because \( \theta = 1 \) in our case. As a matter of fact, it was the contradiction between the outcome in the unit root test here and using the drift-adjustment test that motivated López (1995) to study the correction mechanism.

13. Note that this would in some sense contradict the previous integrability analysis, in which \( \delta_0 \) was considered to be I(0) around a deterministic trend. In order to reconcile these results we have regressed \( \delta_0 \) on a constant, a linear trend and the lagged variable with these results:

\[
\delta_0 = 0.051 - 0.0004 t + 0.89 \delta_0 + \epsilon
\]

(0.06) (0.0001) (0.01) R² = 99
c, is the sum of a MA(1) and a white noise, and a model for c, consistent with the data would be given by an IMA(1,1) plus a drift (μ). Finally, the value of the drift will arise from the estimation below.

Table 4 - Statistics of the daily changes in the series

<table>
<thead>
<tr>
<th>lag</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ_ε</td>
<td>.03</td>
<td>.02</td>
<td>-.05</td>
<td>-.07</td>
<td>-.01</td>
</tr>
<tr>
<td>s_ε</td>
<td>-.14</td>
<td>.05</td>
<td>-.09</td>
<td>.01</td>
<td>-.04</td>
</tr>
<tr>
<td>δ_f</td>
<td>.07</td>
<td>.01</td>
<td>-.00</td>
<td>-.06</td>
<td>.05</td>
</tr>
<tr>
<td>s_f</td>
<td>-.19</td>
<td>.07</td>
<td>-.09</td>
<td>.00</td>
<td>-.05</td>
</tr>
</tbody>
</table>

Standard Errors ±2/10 = .076

Estimation procedure

The estimation of the model conveyed in equations (2). is designed to obtain the relationship between Interest rates and position of the exchange rate in the band and check whether the theoretical hypothesis is fulfilled. This implies to extract the intercept term (c,) from the Interest rate differential series δ in order to remove the 'noise' that imperfect credibility brings about in the basic relation. Previous to applying the Kalman Filter, the model has to be expressed in State Space form.

where the residuals show no autocorrelation. This result would indicate that δ is follows an ARMA process with AR parameter p = .89 with a drift equal to -0.0004, and a MA polynomial with a unit root. This ARMA representation would imply the following MA representation for δ: \(8(B) = 1 - .11B - .096B^2\). However, there is something we should note now: the presence of strong outliers is known to cause additional problems in the identification of ARMA models, due to their effects on the autocorrelation function. In fact, simply by dropping out the first five observations of the series, and smoothing observations 183-92, the correlograms for δ, present an IMA(1,1) structure (θ = .10), which is closer to what we would expect according to the process we have identified for δ.
Rewriting the equations in State Space form we obtain the following expression

\[ \Psi_t = Z \alpha_t + \varepsilon_t, \]
\[ \alpha_t = \theta \alpha_{t-1} + Ru_t, \]

\[ E(\varepsilon_t) = 0; E(\varepsilon_t^2) = \alpha^2; E(u_t) = 0; E(u_t u_{t-i}) = I; E(c_t) = 0 \]

where \( \Psi_t = \delta_t - bs^t, \)

with

\[ Z = (1 0 0 0); \alpha_t = (c_t, \mu_t, \eta_t, \eta_{t-1}) \]

and

\[ u_t = (\eta_t, v_{1t}, v_{2t}, v_{3t}) \]

\[ T = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad R = \begin{bmatrix} \sigma_n & 0 & 0 & 0 \\ 0 & \sigma_\mu & 0 & 0 \\ 0 & 0 & \sigma_\eta & 0 \\ 0 & 0 & 0 & \sigma_\eta \end{bmatrix} \]

\[ E(u_t u_{t-i}) = I, \text{ the identity matrix of order 4} \]

where the unknown parameters \( b, \mu, \sigma_n^2, \sigma_\mu^2, \) and \( \tau \) are chosen so that they satisfy the restrictions implied for the model and they maximize the likelihood function, or equivalently, the log likelihood:

\[ \log L = - \frac{T}{2} \log 2\pi - \frac{1}{2} \sum_{t=1}^{T} \log |F_t| - \frac{1}{2} \sum_{t=1}^{T} e^t F_t^{-1} e_t \]

where

\[ e_t = \delta_t - bs^t, -Z \alpha_{t-1} \]

The first equation is known as the measurement equation, and the second is the transition equation. Notice that the unobserved component \( c_t \) and the drift for its process \( (\mu_t) \) are included in the state vector \( \alpha_t \).

Some remarks are in order at this point about some of the assumptions made above: first, given that when we deal with integrated variables, our estimation problem could be understood as the estimation of a cointegrating vector for \( \delta_t, c_t \), and \( s^t; \) of the form \( (1 -1 -b)^t \); in this case, Stock (1987) shows that the estimator of \( b \), is superconsistent, even when we work with daily data on monthly maturity assets and the problem of overlapping observations arises as we discussed above. Secondly, we have assumed homoskedasticity \( E(\varepsilon_t^2) = \alpha^2 \) for all \( t \), which, for financial data, can be considered a strong assumption. Nevertheless, as Florentini and Maravall (1993) show, the changes in the estimates of the parameters and of the unobserved component between models with and without Autoregressive Conditional Heteroskedastic (ARCH) residuals, are negligible.
Having cast the model in State-Space form, the Kalman Filter is a common method to compute the Gaussian likelihood function for a trial set of parameters (for a discussion see, for instance, Harvey (1989)). The filter recursively produces minimum mean square error (MMSE) estimates of the unobserved state vector given observations on $\Psi_t$. The filter consists of two sets of equations, the prediction and the updating or correction equations. Denoting by $a_{t,1}$ the optimal estimator of $a_{t,1}$ based on information up to and including $\Psi_{t-1}$, and letting $P_{t-1}$ denote the $4 \times 4$ covariance matrix of the estimation error, that is:

$$P_{t-1} = E((a_{t,1} - a_{t,1})(a_{t,1} - a_{t,1})')$$

the Kalman Filter consists on the recursive application of the following set of equations to extract a value for the state $a_t$:

**Prediction Equations**

$$a_{t,1} = T a_{t-1}$$

$$P_{t,1} = T P_{t-1} T' + R Q R'$$

**Parameter correction equations**

$$\theta_t = \Psi_t - Z P_{t,1} Z' a_{t,1}$$

$$F_t = Z P_{t,1} Z' a_{t,1}^2$$

$$K_t = P_{t,1} Z' F_t^{-1}$$

$$a_t = a_{t,1} + K_t \theta_t$$

$$P_t = P_{t,1} - K_t Z P_{t,1}$$

where the subscript $t/t-1$ indicates that the optimal estimators for $P$ and $a$ are at time $t$ are computed with the information available at time $t-1$. Given initial values $a_0$, $P_0$ the filter computes the estimate $a_t$ for $a_t$ at time $t$ ($t=1, 2, ..., T$). However, given that the state vector is not stationary, those initial conditions turn out to play a very important role. In fact, with incorrect initial conditions in the nonstationary case, the results of the estimation procedure may be inaccurate (see, Gomez & Maravall (1993)). In order to compute these initial values several alternatives are offered in the literature, such as de Jong (1988), which is explained in the appendix and it is the method used here.

**Results and Interpretation**

Maximization of the log likelihood of the above state space model gives the results summarized in Table 5. Figure 6 shows the intercept terms $c_t$ for Germany and France and figures 7.a,b display the adjusted interest rate differentials ($S^{aj}$) obtained by subtracting $c_t$ from $\delta_t$, that is, they display differentials net of the intercept term. Some results are worth mentioning:

- First, the sign of the response parameter $b$ is negative, but the theory states,
though its value is very small—see the slope of the \((\delta^2, s')\) scatterplot, and note the different scale with respect to the \((\delta, s')\) scatterplot in figure 3—, highlighting the small influence of the band on the interest rate differentials (for France the band would explain 6.15% of the differentials; for Germany only 1.42%). Moreover, in the case of Germany the sign is in the limit of significance.

- Secondly, and as a consequence of the previous result, the term \(c\), accounts for most of the differentials (see figure 6).

- Finally, the residuals \((e_t)\) are white noise and of small magnitude as figure 8 displays.

<table>
<thead>
<tr>
<th>Table 5—Results of the analysis</th>
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<tr>
<td>GERMANY</td>
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<td>FRANCE</td>
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Standard Errors computed numerically.

The disappointing results of the target zone model are explained by the behaviour of the series: the effect of the band on \(\delta\) relies on the expectation that the exchange rate will appreciate when it approaches the upper band, and vice versa; this suggests that the exchange rate is expected to revert to the interior of the band. However, the non-stationarity of \(s'\) indicates that the exchange rate does not display such mean reversion; thus, the position of the currency in the band cannot significantly influence the interest rate differentials.\(^\text{14}\)

The parameter \(c\), picks up the unit root implied in the regression and this is precisely the realignment risk underlying the agents' expectations. The trajectory of \(c\), very close to \(\delta\), shows that the interest rate differentials have reflected this implicit realignment risk, rather than the expectations derived from the position of the currency in the band.

The obvious conclusion to draw from this analysis is that the band has not been completely credible during the sample under study. However, the decreasing profile of

\(^\text{14}\)-We should be cautious at this stage, since the lack of mean reversion can also be influenced by the wide band in which the peseta is allowed to fluctuate. For instance, Svensson (1993) rejects that \(s'\) are I(1) for all the ERM original currencies, but the Italian Lira, that is, the currency generally acknowledged to have the worst reputation currency in the ERM, but also the only one fluctuating within a wide band in most of the period under study.
Fig. 6.a - Adjusted interest differentials and deviations from the central parity. Scatterplot.

Fig. 6.b - Adjusted interest rate differentials and deviations from the central parity. Scatterplot.

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Fig. 7 - Intercept term (c)

Fig. 8 - Residuals of the model.
Part I: Models

2-Empirical Evidence. The Spanish case

c, reveals that the zone has gained credibility throughout the period, except for the last months, as if the realignment which subsequently took place were anticipated.

Note that our results roughly match the empirical evidence found using the alternative methods reviewed above. However, it is clear from the data that the drift-adjustment method cannot be applied because the exchange rate series contain a unit root. Therefore, the expected realignment risk would equal the interest rate differential which is in practical terms, the same conclusion we derive. This result can be reconciled with Svensson’s assertion that mean reversion is a general feature of the model, even with imperfect credibility. Actually, this feature depends on the degree of credibility. In our case, given the large existing realignmeht risk, the target zone seems to have enjoyed a low degree of credibility, low enough to reverse the expected behaviour of the series and to obscure the influence of the target zone under a thick veil of uncertainty and risk.

Despite the apparent robustness of this result we have to be extremely cautious in interpreting the entire interest rate differential as a realignment risk. As Ayuso et al. (1993) point out, the expected realignment rate -or in our case c- picks up expected exchange rate regime shifts, and this change in the implicit regime can actually occur with the same central parity (for instance, a jump from the lower to the upper part of the same band). This implies that the interest rate differentials would overestimate the realignment risk. Note that this possibility is even more likely in the case of a wide band.

As regards the determinants of the estimated the realignment risk, the variability showed by c, and, especially, its progressive reduction until mid-1992 suggests that its evolution has been determined by several elements. On the one hand, nominal convergence is reflected in the reduction of the interest differentials; on the other hand, the process of financial integration implemented during this period may have allowed meaningful gains in the credibility of the exchange rate commitment; these gains have been reflected in the lower probability of the current band being abandoned—at least in the short run—and, consequently, in a lower realignment risk.

Finally, the factors which allow for the reduction in c, vanish around the end of the period (Summer ’92): the deviations of the fundamentals from the policy targets, the general instability in the ERM and the uncertainties regarding eventual Monetary Union swiftly erode the previous credibility gains and the term c displays a change of trend, reflecting that the probability of a realignment starts to increase. This increase in the realignment risk is not stopped either by the increase in the interest rate differentials nor by the depreciation of the currency within the band nor by the massive sale of reserves, and results in the strong speculative pressures which ended up causing the first realignment of the Spanish peseta on the 16th of September 1993.
IV-Evaluation of the evidence. Research prospects

The empirical evidence on target zone models is at least weak. The different methods reviewed in this chapter have shown that target zones do not seem to have greatly affected the behaviour exchange rates: the regime has lacked in general perfect credibility and even mean-reversion in the exchange rate is a feature which is only verified beyond doubt for a few currencies at the core of the system.

However, this is a too strict interpretation of the ERM. We have to distinguish between the empirical findings of target zones models and target zones regimes. It is true that the type of stabilizing non-linearities predicted by the models are not found in the data, but it is also true that the ERM has reduced the volatility of exchange rates as all the studies on the issue have shown (Artis and Taylor (1988,1993), Ungerer et al. (1990), etc.). However, the foreign exchange market is subject to so much noise (rumours, fads) that any empirical evidence on any structural model is illusive. In my view, attempting to test in practice the bold implications of such stylized models is a rather vain exercise.

The role that fundamentals have played in the empirical analysis is quite ambiguous. At the beginning of the chapter they have been used in the attempt to validate the basic model; they have reappeared in the end to explain the existence of realignment risk, that is, to 'validate the invalidation' of the basic model. In none of the cases, the search has been very fruitful.

As pointed out above, economic fundamentals do not influence the exchange rate (or the realignment risk) on a day-to-day basis, but they are presumably crucial in the long-run behaviour of the exchange rates. Moreover, the evolution of current fundamentals inform the markets about the future sustainability of the system and consequently they exert a reputational effect on the system and determine the probability of a realignment. The problem is that fundamentals are unobservable and it is difficult to define a reliable measure for them.

We saw in the introduction and in the overview of the Spanish case, the large misalignments that the ERM has allowed; figure 9 displays the evolution of the nominal
exchange rate and the real exchange rate in terms of consumer prices (a reasonable measure for the fundamentals) for the Spanish peseta. The sample period has been extended to include the three realignments of the peseta between 1992 and 1993. They can be interpreted in the light of this graph as natural episodes which return the exchange rate to its equilibrium.

The expression for conditional expectations can be used to illustrate these points. Let us recall from chapter one that the overall exchange rate expectation is made up of two components:

\[ E_\tau \frac{ds}{dt} = \omega \left(E_\tau \frac{ds}{dt} | \text{realignment} \right) + \left(1-\omega \right) \left(E_\tau \frac{ds}{dt} | \text{no realignment} \right) \]

\[ 0 \leq \omega \leq 1 \]

which can be redefined in discrete terms and subdivided between expectations within the band and expected jumps of the central parity as in (5) above.

The important point to stress here is the evolution of the probability of realignment (\( \omega \)) in the expression, which appears multiplying the expected magnitude of the realignment. From what has been said above, one can accept that the size of the realignment depends on the expectations that agents assign to the evolution of the fundamentals, on their actual evolution and on the difference between these two; this component should behave smoothly in absence of news. On the contrary, the probability
attached by the agents to a realignment can be highly volatile and very sensitive to news; a jump in $\omega$ can trigger a speculative attack against the existing parities$^{15}$.

The peculiar behaviour of the fundamentals in determining devaluation expectations and the exchange rates can consequently be explained by a probability term which is close to zero most of the time between realignments, but switches to high values when some negative news hit the system. These shocks can be related or not to the evolution of fundamentals. If the fundamentals imbalance is large this can lead to the collapse of the band.

This argument is similar to the peso problem literature. If the probability of a realignment is positive ($\omega>0$) and the subjective distribution of the exchange rate incorporates this possible event; 'a fortiori', if the devaluation does not occur, the observed data will tend to underestimate the actual risk which agents perceive in the foreign market. Indeed, the existence of peso problems in exchange rates create problems of non-stationarity in the exchange rates (Krasker (1980), Obstfeld (1987)) and prevents convergence in interest rates. This is a common problem to exchange rates pegs as the parallelism between the ERM turmoil and the recent new 'peso problem' in Mexico illustrate.

The consequences which can be drawn from this analysis will be developed at length in the remainder of the thesis: the first consequence regards the need to improve target zone models and the second is related to policy issues.

From a theoretical point of view, it would be desirable if the target zone models conveyed the variable influence of fundamentals. The next chapter presents a dynamic model in which the evolution of expectations is only occasionally affected by the behaviour of the fundamentals.

The economic policy implications of the empirical analysis are developed in chapters four and five. The idea is linked to the question we put at the end of last chapter, that is, the reasons for imposing a target zone on the exchange rate.

The evidence shows that in the long run, the target zone almost behaves like a crawling peg, as Bertola & Caballero (1993) have anticipated for a target zone with imperfect credibility. Why then, is it necessary to go through those stressing periods of turmoil, where the reputation of governments is put at stake, in the end the markets go their way? This is of course the pessimistic interpretation of the ERM experience. In fact, the idea of managing exchange rates is not very popular at the moment, but we are of

$^{15}$Ayuso et al. (1994) use a similar approach. However, they assume that the evolution of fundamentals is linear (instead of attempting to measure it) and then compute the probability of realignment.
the opinion that exchange rate management is not impossible and it may be beneficial. Finally, it is a fact that, sooner or later, the issue reappears because the exchange rate is a key variable to manage economic interdependence and, hence, policy coordination.

Therefore, a more constructive interpretation would be to stress the benefits of a exchange rate peg and to identify the flaws of the system, so that it can be redesigned. The results given in the last part of chapter one take this perspective. Emphasis is placed on the importance of reputation and commitment for the sustainability of the target zone. These ideas are taken up again in the second part of the thesis.
Appendix. Computation of initial conditions

In principle the starting values for the Kalman Filter $a_0$ and $P_0$ are given by the mean and the covariance matrix of the unconditional distribution of the state vector. However, when the state equation is nonstationary the unconditional distribution of the state vector is not defined. Unless genuine prior information is available, the initial distribution of $a_0$ must be specified in terms of a diffuse prior. If we write $P_0 = \kappa I$, where $\kappa$ is a positive scalar, the diffuse prior is obtained as $\kappa^{-1}I$. This corresponds with $P_0^{-1} = 0$. The distribution is an improper one since it does not integrate to one. However, as Gomez & Maravall (1993) note, the use of the so-called big $\kappa$ method is not only numerically dangerous but also inaccurate. Alternative initialization procedures can be found in de Jong (1988) (the one used), Carraro & Sartore (1987) and Gomez & Maravall (1993). In de Jong’s algorithm it is assumed that the initial state vector can be partitioned into D nonstationary and N-D stationary elements, and expressed as:

$$a_0 = C_1 y_1 + C_2 y_2$$

where $y_1$ is a D×1 vector containing the D nonstationary elements, with a diffuse prior, that is $\text{Var}(y_1) = \infty$ or $(\text{Var}(y_1))^{-1} = 0$, while $y_2$, an N-D×1 vector has a proper prior, that is,

$$E(y_2) = m \quad \text{Var}(y_2) = V$$

In our model with stationary errors $m$ is a 3×1 where the first element is set equal to a consistent estimate of $\mu$ and the two other elements are zero, while $V$ is diagonal with the first element equal to zero, as given in Appendix 1; $y_1$ would be set equal to $c$, while $y_2 = (\mu \eta_1, \eta_2)'; C_1 = (1, 0, 0)'$ and $C_2 = (0, 1)'$. The use of the big $\kappa$ approximation for $y_1$ is avoided by extending the Kalman Filter as follows. Define

$$X_0 = (Y, 0)$$

where 0 in (.) is a null matrix with D columns. (so in the model D=1). Then the standard Kalman Filter recursions would be initiated with

$$P_{0|0} = C_2 V C_2'$$

The recursion for the state vector is augmented so as to become a recursion for the matrices $A_t$ and $A_{t|t}$. Defined as:

$$A_t = A_{t|t-1} + P_{t|t-1} Z'_t F_t' N_t, \quad t = 1, \ldots, T$$
where

\[ N_t = X_t Z A_{1t} \]
\[ A_{1t} = T A_{1t-1} \]

with \( A_{10} = (C_1 m C_1) \)

and

\[ S_t = S_{t-1} + N_t F_t^{-1} N_t \]
\[ t = 1, \ldots, T \]

with \( S_0 = 0 \)

The output from the above recursions is used to construct the required statistics from the diffuse prior model as follows. First, partition \( N_t, A_t, \) and \( S_t \) as follows:

\[ N_t = (v_t, N_t') \quad A_t = (a_t, A_t') \quad \text{and} \quad S_t = (S_t, S_t') \]

with \( S_t = (S_t', S_t) \)

Then, as shown by de Jong (1988), the estimator of the vector for the diffuse prior starting values is:

\[ \hat{a}_0 = A_0 \left( I - S_t' S_t^{-1} \right) \]

and

\[ \hat{P}_0 = P_0 A_0^* S_t^{-1} A_0^* \]

where \( S_t^{-1} \) is the generalized inverse of \( S_t \).

The log likelihood function, ignoring the constant term, is given by

\[ \ell_i = -\frac{1}{2} \Sigma \ln | F_t | - \frac{1}{2} S_t^{-1} - \frac{1}{2} \Sigma e'_t F_t^{-1} e_t \]

The expression for the log likelihood assumes that the recursions are carried on until the end of the sample. However once \( S_t \) becomes nonsingular the diffuse filter can be collapsed to the usual filter. If \( S_t \) is nonsingular at some point \( t=\tau \), then the usual Kalman Filter can be employed starting from values obtained for \( a_\tau \) and \( P_\tau \) for \( t=\tau \).

It can be shown that \( S_t \) is non singular if

\[ r(C_t, Z^t, C_t' Z^t, C_t' T' Z^t, \ldots, C_t' T' Z^t') = 1 \]

Since in our particular case \( r(C_t, Z^t) = 1 \), we can truncate at \( t=1 \), and the log likelihood function for the \( T \) observations becomes

\[ \ell = \ell_i - \frac{1}{2} \Sigma \ln | F_t | - \frac{1}{2} \Sigma e'_t F_t^{-1} e_t \]

with \( \ell_i \) defined as above and the sums evaluated only at \( t=1 \).
Chapter 3

A Dynamic Model of Target Zones

After the Basel-Nyborg Agreements, the ERM survived for more than five years without realignments. However, the diverging evolution of the fundamentals generated large implicit misalignments in the exchange rate. The apparent currency stability in this period went hand in hand with the accumulation of deeper tensions and when these tensions eventually came to light, a series of exchange rate crises and multiple realignments followed which roughly reestablished exchange rate equilibrium, as we have seen in figure 2.9.

The main objective of this chapter is to develop a dynamic model of target zones in order to give a formal explanation for the complex relation between fundamentals and exchange rates in a target zone which we highlighted in the previous chapter. We claim that fixing the nominal exchange rate within a band while divergences in the fundamentals persist may allow deviations from the equilibrium exchange rate in the short-run (the so-called honeymoon effect), but it will eventually become unsustainable in the long-run. More precisely, we intend to show that a target zone with diverging fundamentals conveys a perverse mechanism which sooner or later leads to its collapse. The experience of the ‘new EMS’ and its ultimate collapse are consistent with this interpretation.

We have seen in chapter one that when the target zone regime is credible, economic fundamentals between two countries are able to diverge more for a given exchange rate than in a floating regime. We have outlined the attempts to build target zone models which explain why a target zone loses credibility and eventually collapses in the face of speculative attacks, but these models are unable to endogenously generate exchange rate collapses. One reason is that the surveyed target-zones models do not treat adequately all the dynamic aspects of the model, in particular the evolution of the credibility. Svensson & Dumas (1992) compute precisely the expected duration of
a target zone regime, given a finite stock of reserves; however, they also miss dynamic credibility considerations, which is in our view the factor which determines the collapse of the zone. This limitation reduces the interest of their results. Therefore, developing an endogenous dynamic process for the credibility in a target zone model seems necessary to us.

We observed in the models of the previous chapter that in the different models the honeymoon effect was fixed over time. We also noted that imperfect credibility reduced the honeymoon effect. The rational expectations hypothesis (REH) which underlies the solution, while being essential to obtaining a closed form solution in target-zones models, strongly constrains the relationship between the exchange rate and the fundamentals in the form of a fixed honeymoon effect. Thus, if we are able to dispense with the limiting REH hypothesis, while preserving the rationality of agents, we may be able to introduce explicit dynamics in the target zone, such that the the credibility and the honeymoon effect evolve with time.

This is done in section I by setting up a model with heterogeneous beliefs; each belief is based on a different model of exchange rate determination. This implies departing from the expectation formation mechanism implicit in the standard models. Nevertheless, there is still a representative agent who solves an optimization problem, so that rational behaviour is preserved; he is simply faced with two different beliefs from which to make up his expectation, rather than a single model or belief as in the standard REH. This specification stresses the existence of an implicit conflict between beliefs in a target zone system with diverging fundamentals.

After obtaining some insights from a mathematical analysis of the model in section II, we simulate the model to examine different exchange rate trajectories (section III). We examine the characteristics of these trajectories, which depend on the degree of divergence between the fundamentals of the two economies. The observed stability of the 'new EMS' while the fundamentals between member countries continued to diverge can be explained by our model; we refer to this feature as the dynamic honeymoon effect since it changes over time. Although the target zone may endure periods of strain (attacks), the zone necessarily collapses, given the divergence of fundamentals. In the next section (IV), we show that this outcome can be interpreted in terms of reputation and credibility; this perspective also highlights the reasons for the empirical failure of target zone models. Section V sums up the conclusions.
I-A TARGET ZONE MODEL WITH HETEROGENEOUS BELIEFS

It has long been recognized that structural models incorporating rational expectations have failed to explain the evolution of exchange rates after the collapse of the Bretton-Woods system (Meese & Rogoff (1983)). This failure has generated a series of studies on how expectations are actually formed in the market and it has opened a vast field of theoretical research in which the strongest assumptions of the REH have been abandoned.\(^1\)

The departure from REH is always hazardous since, quoting Goldberg & Frydman (1993), "the great appeal of the Rational Expectations paradigm is that ... the implied expectations of the agents have superior theoretical properties in terms of unbiasedness and consistency...and any attempt at moving away from the RE framework immediately implies the presence of some degree of arbitrariness and inconsistency". Thus, we have to be careful to maintain the rational behaviour by agents when specifying of the model but allow the market to hold heterogeneous beliefs.

Agents in financial markets form their expectations under uncertainty and presumably do their best in a rational way to produce forecasts. The uncertain environment is crucial for the generation of heterogeneous expectations which is invariably found among individual agents. Actually, it is difficult to reconcile the omniscience implied by the REH with the behaviour of the markets. Kurz (1991) stresses that agents may exhibit drastic differences in beliefs even when they share the same information; this does not imply at all that they are inefficient, but that after efficiently processing the information, different agents draw different conclusions, according to their different but rational beliefs. Kurz concludes that "scientific knowledge is limited and many different theories are compatible with the behaviour of the data. The existence of wide gaps in the knowledge of the agents about the economy results in a speculative search, which forms conjectures, hypotheses or theories and these are the main source of heterogeneity of beliefs".

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This heterogeneity of beliefs can be captured by a model in which the market expectation is the result of aggregating the private beliefs of individuals agents. Individual agents form their expectations conditionally on their information set and according to different structural models, not just the one model or view of the world implied by the REH. The agent’s information set mainly consists of the observed past data and of their knowledge about the evolving environment.

Note that none of the models which agents have in mind necessarily corresponds with the ‘true’ model, from which the exchange rate process actually derives. As a matter of fact, since the effective exchange rate is influenced by market expectations, it will be driven by the dominant model in which the market believes, regardless of whether it makes economic sense or not.

**Fundamental and institutional beliefs**

The existence of heterogeneous beliefs is an inherent feature of target zones regimes since the co-existence of two different well-established models for the determination of the exchange rate are implicit:

- On the one hand, there is the assumption that the exchange rate has to return to its equilibrium level determined by macroeconomic fundamentals. This level would correspond to the free float solution and it determines in our model the fundamental beliefs.

- On the other hand, when an institutional exchange rate arrangement is established there exists a commitment adopted by the authorities to keep to the arrangement and this commitment to defend the parities influences the beliefs of the agents. We refer to these beliefs as institutional beliefs.

Then, each model generates a distinct belief structure. The market expectation will result in an aggregate of the expectations generated by both beliefs. Hence, one of the crucial questions to settle is how this aggregation is determined. We start by describing the expectation which each type of belief generates.

**Fundamental beliefs** (*F*). The core hypothesis of any structural model is the assumption that the exchange rate must ultimately be consistent with the level of fundamentals so that the equilibrium exchange rate (*₅*) fulfills some specific fundamental equilibrium relationship, such as Purchasing Power Parity (PPP). Fundamental beliefs evolve in the form of adaptive expectations so as to capture this idea: we assume that the nominal exchange rate, *s*, is expected to return to the equilibrium ₅, at an adjustment
speed determined by the parameter $p$:

$$F_t: \Delta s_{t+1}^F = p(E(s_{t+1}) - s_t) \quad (1)$$

The superscript (in this case $F$) denotes the expected exchange rate change on the basis of the corresponding type of belief; the corresponding subscript refers to expectation at time $t$ regarding $t+1$. When the current exchange rate is lower than the equilibrium exchange rate, it is expected to depreciate and vice versa.

Institutional beliefs (!) on the other hand, are determined by the existence of a fully credible target zone, corresponding to the basic model in chapter one. The commitment of the authorities to intervene at the margins of fluctuation to defend the band influences exchange rate expectations throughout their whole range of fluctuation.

This result has been formally obtained in chapter one. Fundamentals ($F$) follow a Brownian motion process which is regulated at the margin when a target zone is set up. The solution to this model is of the following form, which is equivalent to the expression (1.10) in chapter one:

$$s = s^* + \Omega(s, s_t) \quad (2)$$

where $s^*$ is the upper and lower limits of fluctuation and $\Omega$ is a non linear function of the fundamentals, in our case $F$. The non-linearity implies a wedge between the evolution of the fundamentals and the exchange rate, which delivers the so-called honeymoon effect.

Exchange rate expectations conditional on the existence of the target zone are determined by the mapping between the position of the exchange rate in the band and exchange rate expectations in this model, such as it appears in figure 1.2. Writing the exchange rate in terms of deviations from the central parity ($s' = s - c$), the institutional beliefs are defined as:

$$I_t = \Delta s_{t+1}^I = \Omega'(s') \quad (3)$$

$$\forall s \leq s' \leq s^* \Rightarrow \Omega'_c < 0$$

where $\Omega'$ is a non-linear function derived from $\Omega$, and corresponding to expression (1.11). The closer the exchange rate is to the upper limit-$s^*$ (lower limit-$s$), the larger the appreciation (depreciation) that is expected.

It is important to note that the magnitude of the honeymoon effect is largely determined by the width of the band and the drift in the fundamentals, with no explicit consideration of time and, in this sense, the honeymoon effect is static. We will see below that on the contrary our model allows the projection of the honeymoon effect in time.
Market expectations

Given the two models which potentially enter in any individual agents' beliefs, the market aggregates private beliefs to form the aggregate expectation. We assume that the market is composed of an arbitrarily large number of agents \((i=1,2,...,n)\), whose individual expectations \((\Delta s_{i,t+1})\) are a weighted average of the expectations derived from the fundamental \((F_i)\) and the institutional \((I_i)\) beliefs they hold, with weights equal to \(w'_i, 1-w'_i\), respectively. It is further assumed that the expectations derived from the two types of beliefs are homogeneous across agents:

\[ F_i = F'_i; I_i = I'_i \forall i \]

However, each agent is assumed to place a different weight to each belief, according to their individual perceptions of reality; individual beliefs are consequently heterogeneous.

The market aggregate expectation is then simply the weighted average of the fundamental and institutional beliefs:

\[
\Delta s_{M,t+1} = w'_i F_i + (1-w'_i) I'_i; \quad w'_i = \frac{1}{n} \sum_{i=1}^{n} w'_i 
\]

\[ 0 < w'_i < 1 \]

Aggregating the agents, the overall market expectation can be obtained:

\[
\Delta s_{M,t+1} = \frac{1}{n} \sum_{i=1}^{n} \Delta s_{i,t+1} = w'_i F_i + (1-w'_i) I'_i; \quad w'_i = \frac{1}{n} \sum_{i=1}^{n} w'_i 
\]

\[ 0 < w'_i < 1 \]

This specification is similar to that in Frankel & Froot (1988) in which each type of belief represents a type of agent in the market. We adopt however a slightly different interpretation. The market is thought to be a representative agent which forms its forecast as a linear combination of the forecasts generated by two different models or beliefs. Zellner (1986) gives a formal Bayesian justification to this approach and shows its optimality under uncertainty. Now we turn to explore the issue of optimality in our model.

Optimal forecasts

The market aims at maximizing the yield of its investment in foreign exchange and

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Also note that (4) is a restatement of the exchange rate expectations in conditional form which appeared in the previous chapters in which \( F \) represents the free float solution. Therefore, our model provides an alternative interpretation of that expression.
In doing so, it needs to formulate the most accurate forecast, conditional to its knowledge of the economy (conveyed by the two models) and the information contained in the data.

Since the error variance of any linear combination of forecasts is lower than the variance of any individual forecast, the combined forecast of the models improves upon any single forecast. Consequently, the market will find it optimal to use a linear combination of forecasts (see Granger & Newbold (1986)), such as in (4).

The values for F and I are now measured at each period from the data. Therefore the question to solve is how to select the optimal weight \( w \), which minimizes the one-step ahead forecast errors \( \xi^M \). Subtracting (4) from the actual exchange rate change, the respective one-step-ahead forecast errors of the period are:

\[
\xi^M_t = \omega_t (\xi^F_t) + (1-\omega_t) (\xi^I_t)
\]

where

\[
\xi^M = \Delta s_{t+1} - \Delta s^{M}_{t+1}; \quad \xi^F = \Delta s_{t+1} - F_t; \quad \xi^I = \Delta s_{t+1} - I_t;
\]

It is immediate to show that the market one-step-ahead forecast errors are minimized by obtaining the Least Square estimator of \( w \) from the regression:

\[
Y_t = w_t X_t + \epsilon_t
\]

where

\[
Y_t = \Delta s_t - I_t; \quad X_t = F_t - I_t; \quad k=1, ..., t-1
\]

which, rearranging terms is equal to (4), but with actual exchange rate changes replacing the market forecast.

Allen & Taylor (1989) show that in the foreign exchange market the recent past influences the current forecast more than distant periods. Thus, we will consider that the memory of the market is short and most recent observations carry more weight in the determination of the current forecast. This idea can be grasped with a Discounted Least Square (DLS) estimator, which updates the weight of the estimator according to the following expression:

\[
\begin{align*}
\hat{w}_{t+1} &= \hat{w}_t + G_t (Y_t - w_t X_t) \\
G_t &= \frac{X_t}{\lambda \left( \sum_{k=1}^{t-1} \lambda^{t-1-k} X_{k+1}^2 + X_t^2 \right)}
\end{align*}
\]

The estimator has been expressed in recursive form for latter convenience, where the term \( G_t \) is the recursive gain (see Harvey (1981), ch.7). The parameter \( \lambda \) is the discount

---

3. This type of estimation is a particular case of Generalized Least Square, and the estimator is unbiased and efficient (see Harvey (1981)).
factor; the lower $\lambda$, the more weight is carried by the most recent past and the faster expectations are updated ($\lambda=1$, in the ordinary least squares case).

Thus, the optimal rule is simply a DLS updating rule of the parameter $w$, derived from minimizing the past forecast errors. The model which has performed better in the recent past will gain weight in the current period forecast and vice versa.

Finally, as we indicated above, structural knowledge of the economy is incomplete and the market ignores the 'true' process governing the exchange rate, which we approximate in discrete time by:

$$
\begin{align*}
S_t &= S_{t-1} + \theta \Delta S_{t-1} + \nu_t; \\
\nu_t &\sim \mathcal{N}(0,\sigma^2)
\end{align*}
$$

(7)

This can be interpreted as a random walk with a drift which depends linearly on market expectations (we go into greater detail on the interpretation of the parameter $\theta$ below). This disparity between the 'true' process and the interpretation of the realizations of the process by the market provides the driving force for the model dynamics.

**Divergence and conflicting expectations**

We assume that the equilibrium exchange rate ($S$) follows a random walk (that is the discrete time version of a Brownian motion) with drift $\mu$, such that in each period it diverges from the central parity at an expected rate equal to $\mu$:

$$
\bar{S}_{t+1} = \bar{S}_t + \mu + \epsilon_t; \quad \epsilon_t \sim \mathcal{N}(0,\sigma^2)
$$

(8)

Recalling that the equilibrium exchange rate is given by PPP and denoting the log of domestic and foreign prices by $p$ and $p^*$, respectively, where 'foreign' can be interpreted as an average of the member countries of the zone, it follows immediately that the drift in the equilibrium exchange rate is equal to the expected inflation differential $(\pi - \pi^*)^*$ between the countries:

$$
\bar{S} = p - p^* \Rightarrow \Delta \bar{S} = \Delta p - \Delta p^* = \pi - \pi^*; \quad E(\Delta \bar{S}) = \mu = (\pi - \pi^*)^*
$$

(9)

Therefore, we can treat the drift of the process for the equilibrium exchange rate, $\mu$, as a measure of divergence between the economies.

Figure 1 compares the expectations generated by both models in a target zone with a 12% fluctuation band. The unit period adopted is one month, so that the plot displays monthly expectations; the exchange rate is expressed in (log) deviations from the central parity, which is normalised to one (hence, its log is equal to zero). Assuming that the equilibrium exchange rate is set equal to the central parity ($S=c=0$), both types of belief predict that the currency depreciates when it is in the lower part of the band and
Figure 1- Exchange rate expectations: Fundamentalists and Institutionals.
Fundamentalists have linear expectations but, if there exists divergence in the fundamentals, their expectations are not constant with respect to the position of the exchange rate in the band. Institutionals have non-linear expectations which are constant with respect to the exchange rate position within the band.

What happens as fundamentals drift away from the central parity? If we assume a positive drift, the expectation derived from the fundamental beliefs shifts upwards by a magnitude equal to the drift each period, as we can see in figure 1.

On the contrary, for the case of institutional beliefs, the outcome is in principle ambiguous because they can be seen to be based either on the evolution of fundamentals from (2) or on the position of the exchange rate in the band from (3); these two mappings are both consistent in the static setting represented in figure 1.
However, if the fundamental belief contributes to the exchange rate movements \((w>0)\), the mappings become inconsistent, because the position of the exchange rate in the band will not correspond with the current level of fundamentals\(^4\).

We assume then that, when forming their institutional beliefs, agents take into account the position of the exchange rate in the band, which is immediately observable and the level of fundamentals is only used to form fundamental beliefs: if the exchange rate qualitatively behaves within the band as the institutional belief implies, this assumption is consistent with what agents are observing. Thus, institutional beliefs are static with respect to the position of the exchange rate within the band, as figure 1 displays.

Under these circumstances, while the institutional belief forecasts an appreciation of the currency, the fundamental belief will eventually forecast a further depreciation even when it reaches the limit of fluctuation. Consequently, the expectations deriving from the different beliefs are not only heterogeneous, but also eventually conflicting; hence, one of the models must predict incorrectly. The potential for conflict cannot be ultimately avoided given the establishment of a target zone among economies with diverging fundamentals, as the ERM.

Even if the ERM parities approximately corresponded to their equilibrium exchange rate at the time they were originally determined, the exchange rate equilibrium has subsequently deviated from the central parity over time, provoking misalignments in the nominal exchange rate. Furthermore, the ERM has displayed recurrent periods of calm which have been invariably followed by exchange rate crisis and realignments.

The obscure relation between fundamentals and the exchange rate documented in chapter two can be interpreted in the light of our model as the evolution of an underlying conflict between the two types of beliefs, where one or another belief has dominated the market at any particular moment. The endurance of ERM parities for years while fundamentals were still diverging can be basically thought of as periods when the institutional belief dominated the market, whereas the turbulent periods preceding realignments of intrinsically weak currencies represent episodes when the market followed the fundamental belief. Our analysis will allow us to explore these issues after the effects of interventions are taken into account.

\(^4\)The parameters which determine the solution to the institutional belief underlying model could be updated in order to reconcile both mappings. However, the dynamics of the model may be such that long deviations of the equilibrium exchange rate correspond to exchange rate equal to the central parity, and, in this case, reasonable parameters cannot be found.
The signalling effect of interventions

When a target zone is established for an exchange rate, there is an assumed commitment to defend the band. The effect of interventions on market expectations raises the issue of the sustainability of the exchange rate commitment. We assume that the authorities only intervene at the edges of the band and that these interventions are infinitesimal. In chapter one we discussed the effects of interventions and reserves on the credibility of the target zone; the approach here tries to incorporate those considerations in our specification.

Let \( s' \) be the exchange rate for the next period conditional on no intervention, and \( t \) the period when an intervention is necessary. Then, when the exchange rate is driven outside the (upper) band \( (s_t < s_{t+1} + \theta \Delta s_{t+1} + \nu) \), a sale of reserves \( (R) \) will take place to keep the exchange rate at the edge of the band \( (s_t = s_j) \). For simplicity, we will assume that \( \Delta R = -(s' - s_j) < 0 \). The effects of this intervention can be considered from three perspectives: fundamental beliefs, institutional beliefs and the overall market expectation.

**Fundamental beliefs:** expressions (1) and (9) imply that unsterilized interventions affect the fundamental belief: the monetary contraction derived from a loss of reserves could affect the price level and would consequently push down the equilibrium exchange rate, reducing the depreciation expectation arising from the fundamental belief and hence easing the pressure on the currency. Despite the appeal of introducing a more formal intervention-sterilization framework into the model, we will assume in this paper that interventions are sterilized to emphasize the role of expectations. This being the case, the authorities only intervene when it is necessary (marginal interventions) and rely completely on the signalling effect of interventions to manage the exchange rate within the band, so that fundamental beliefs are not affected.

**Institutional beliefs** already account for the expectation that the authorities will intervene at the edge of the band when producing forecasts. Therefore, the signalling effect of interventions is automatically incorporated into the target zone model and it generates the characteristic non-linear relationship displayed in Figure 1. Since by definition, institutional beliefs always attach perfect credibility to the band, effective interventions do not affect their expectations, either.

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5. This is the assumption underlying the basic target-zone model which determines institutional beliefs. It is straightforward to introduce intramarginal interventions. We already know that the result would be a higher slope of the institutional beliefs curve and therefore a higher stabilizing effect on the target zones. However, it would also imply a larger drain of reserves.
Market expectations are on the contrary conceivably affected by interventions and we have to devise a method so as to incorporate them into the model.

We saw in chapter one that interventions may send two conflicting signals to the market: on the one hand, when the band is put under strain and it is successfully defended, it is reasonable to think that the authorities will gain credibility (a positive signal); on the other hand, interventions erode the stock of reserves of the Central Bank and the ability of the authorities to defend the band in successive periods of pressure. Furthermore, the recurrence of interventions may alert the market to the future viability of the zone and consequently about its eventual sustainability; both factors send negative signals to the market.

Thus, the question to settle is which type of belief will gain dominance when an intervention takes place. In other words, how interventions affect the dynamics of \( \omega \), given that a net negative signal will presumably be reflected in a larger weight of the fundamental belief in the market, and vice versa. We introduce the following mechanism to account for this intuition.

According to the updating mechanism of the model in (6), it can be observed that the effective exchange rate change \( \Delta s_t = s_t - s_{t-1} \) should be used in the next period’s estimation of the weights. This would imply taking fully into account the positive signal derived from intervention. At the other extreme, if the intervention had been dismissed by the market, the updating would be made as if no intervention had been carried out: \( \Delta s_t = s_t - s_{t-1} \). The signalling effect of interventions is then easily incorporated into the model taking these two options as extreme cases, and modifying the exchange rate change which effectively enters into the updating process (\( \Delta s^* \)) in the following way:

\[
\Delta s^* = \Delta s_t - \gamma AR_t = (s_t - s_{t-1}) - \gamma (s^* - s_t)
\]

where \( 0 < \gamma < 1 \) is an increasing function of the number of interventions:

\[
\gamma = (\#AR) / (\#AR + 1)
\]

where \( \#AR \) is the number of interventions. This specification captures the intuition that highest positive signal is obtained with the first intervention, and it decreases thereafter:

\[
\#AR = 1 \Rightarrow \gamma = 0 \Rightarrow \Delta s^* = s_t - s_{t-1}
\]

\[
\#AR \rightarrow \infty \Rightarrow \gamma \rightarrow 1 \Rightarrow \Delta s^* \rightarrow s_t - s_{t-1}
\]

At the first intervention, the market takes fully into account the intervention; whilst, at the limit the market will update as if there would have been no intervention at all. This mechanism conveys the intuition that the negative influence given by the drain of reserves eventually offsets the positive signal derived from a successful defence of the band.
II-MATHEMATICAL ANALYSIS

The aim of the model outlined above is to explore the dynamics of the exchange rate in a target zone system, where the interaction between different and eventually conflicting beliefs in the market determines the evolution of the exchange rate. This section explores, as far as possible, the characteristics and restrictions of our model from a mathematical point of view and the next section relies on a simulation analysis to overcome the limitations of the mathematical approach.

Some restrictions are necessary to obtain explicit results from the mathematical analysis. In particular, we constrain the range of exchange rate fluctuation to explore to the upper part of the band (s>c) and to overvalued exchange rates (s > s)\(^6\). These are not strong assumptions: on the one hand, for positive drifts, below the central parity fundamental expectations drag the exchange rate towards the upper band, whereas institutional expectations pull it towards the central parity, so that both forces are upwards and the exchange rate is expected to remain in the upper part of the band; on the other hand, if the exchange rate depreciates above its equilibrium level, both beliefs anticipate an appreciation. With these conditions, it follows that

\[ \forall t, F>0, l<0 \rightarrow x>0, g>0. \]

The model can be expressed as a system of four difference equations, governing the market expectations s\(^m\), the weight attributed to fundamental beliefs, w, the equilibrium exchange rate s and the effective exchange rate, s. The latter equation is redundant and the dynamics of s are exogenous; therefore after solving for l in equation (4) and substituting terms in (6), this can be seen as a system of two difference equations with a forced motion conveyed in (8). Only in the cases in which interventions are necessary, the mechanism which appears in equation (10) substitutes the effective exchange rate

---

\(^6\) We also assume in what follows that \( \epsilon > 0 \), i.e. the process for the fundamentals is deterministic and, unless otherwise stated, that \( \nu = 0 \). Finally, \( l(0) = 0 < \mu \) which is only the case when the process assumed for the fundamentalist is driftless. These assumptions do not change the qualitative results of the analysis, but saves us from complicated notation of minor order terms.
change. We can then write:

\[ \Delta s_{t+1} = w_t F_t + (1 - w_t) I_t \]  \hfill (4')

\[ G_t((\theta - 1)(\Delta s_{t+1} + \nu_p)) \quad \forall s < s_t \]  \hfill (6')

\[ \Delta w_{t+1} = G_t((s_t - s_{t-1}) + \nu(s^* - s_{t-1}) - \frac{1}{\theta}((s^* - s_{t-1}) + \nu)) \quad \forall s_t = s \]  \hfill (6'')

\[ \Delta s_{t+1} = \mu + \varepsilon \]  \hfill (8')

From (6') we can see that, for the system to have endogenous dynamics, i.e: dynamics which do not depend on the stochastic terms, it is necessary that \( \theta \) is different from one, otherwise \( w_t \) would be constant. The parameter \( \theta \) can be interpreted as follows: there exists a group of agents who do not contemplate any particular model, but who follow the mood of the market, represented by the market expectation (and consequently does not participate in forming such expectation).

Similar kinds of behaviour have recently received much attention in the literature, under different names (stop-loss traders, chartists, noise traders, etc). The important point to make is that its implies that the structural models’ forecast overshoot. We will then assume that \( \theta \) is larger than one but the magnitude of such noise trading is unknown by the market when forming expectations and this gap of knowledge is crucial in the generation of exchange rate dynamics.

We can then think of the model as a system of recursive difference equations in which changes in the exchange rate imply changes in the same direction in the weights (intuitively, when the exchange rate depreciates, the fundamental belief gains weight, because the institutional belief is predicting less well). However, the opposite does not necessarily hold, see equation (14) below. This ‘asymmetry’, along with the forced motion component in the equilibrium exchange rate process, generates the complex dynamics of the model.

Nevertheless, we can still characterise central aspects of the dynamic behaviour of the model. First, we study the stability of the solutions, then we obtain a sufficient condition for the collapse of the zone and finally some indicative results on the dynamics are worked out.

Solutions and stability

The recursive structure of the model implies that stationary points for \( s (S) \) are also stationary points for \( w (W) \). We can concentrate on the solution for the weights. From (4')
The value of this solution largely depends on two factors: non-linearly on the position of the exchange rate in the band and on the equilibrium exchange value, which is in turn a function of time. The complexity of the system makes it unfeasible to find a general closed form solution. Nonetheless, since the values of the weights are bounded between zero and one, an important question to investigate is whether the process will converge to those points and, in this case, whether these are stable solutions. It follows immediately that \( W=0 \) \( \rightarrow S=c=0 \) and \( W=1 \) \( \rightarrow S=s \) do constitute solutions to the system. In fact, they correspond to the phase planes displayed in figure 1.

The critical question is whether these solutions are stable. Taking the total differential to the expression above, where \( dS/dt=\mu \); \( dt/dt=1 \); \( ds/dt=v \), (note that disturbances are now being considered), we obtain:

\[
\frac{dW}{ds} = - \frac{A^{-1} l(S)(p(\mu t-v_\tau)-l'(\mu t-S))}{A=(p(\mu t-S)-l(S))^2}
\]

Let us now evaluate the sign of these derivatives at the solutions.

\[
dW|_{W=0} = -A\frac{l'(\mu t-S)}{l(S)} \forall \mu \geq 0 \Rightarrow dW|_{W=0} > 0.
\]

\[
dW|_{W=1} = A\frac{p}{l(S)} (\mu - v_\tau ) \forall \mu > 0 \Rightarrow dW|_{W=1} < 0.
\]

Therefore, when there is a positive disturbance at \( W=0 \) the system will follow an unstable trajectory, since the increase in \( w \) will provoke an exchange rate depreciation (if \( S=s \)) and this will translate in a further increase in \( w \), and so on. The inverse process occurs for \( W=1 \) when the disturbance is larger than the drift in the fundamentals.

Relaxing the assumptions used in this analysis (i.e. no disturbances in the equilibrium exchange rate, no drift implied in institutional belief) adds complexity to the above expressions but does not change the conclusion, which is that the system is potentially unstable at the extreme market beliefs.

**Sustainability of the target zone and collapse condition**

The second issue to explore is the potential for collapse of the target zone or, in other words, the sustainability of the band. The eventual collapse of the band is provoked by the persistence of market expectations which drive the exchange rate outside the band. This depends on the behavior of the system at the edges of the band. Let us retake the previous discussion on intervention and assume again that the exchange rate
increases and eventually the edge of the target zone band is breached at some time \( t \), so that a Central Bank intervention is necessary to defend the band. From (4), next period's change in the market expectation is:

\[
\Delta s_{x,t+1}^M = \Delta s_{x,t} + \Delta w_{t+1} \left( F_{x,t} - k(s) \right) + w_{t+1} \Delta F_{x,t+1} \cdot \left( 1 - w \right) \Delta l_{x,t+1}.
\]

This expression has no definite sign. When it is negative the exchange rate returns to the band with no further intervention. However, when this expression is positive, a second consecutive intervention will be necessary to keep the exchange rate at the edge of the band. Since \( \Delta F_{x,t}=0 \Delta l_{x,t}=0 \) and \( l_{x,t}=l(s) \), updating (13) for the period of the second intervention simplifies to:

\[
\Delta s_{x,t+2}^M = \Delta s_{x,t+1} + \Delta w_{t+1} \left( F_{x,t} - k(s) \right)
\]

The sign of this expression only depends on the change in the weight (the rest of the terms are positive). Consequently, a sufficient condition for (14) to be positive is that \( \Delta w_{t+1} \geq 0 \). The sign is determined from \( (6') \): while the gain \( G \) is positive, the term in brackets may be negative. Ignoring the stochastic term, and taking into account that \( s_{x,t}=s_x \), this term simplifies to \((y-1/\theta)(s'-s)\) so that a necessary and sufficient condition for a positive increase in the weight in the period is that \( 1/\theta < y < 1 \).

When that is the case the market will expect further depreciations. More importantly, since \( y \) is an increasing function of the number of interventions, the above condition for increasing weights verifies thereafter \( 1/\theta < y_{x,t}, y_{x,t+1}, \ldots < 1 \), so that additional interventions are required for the band to be defended. Given the dynamics of the system, these new interventions drain the stock of reserves but are unable to reverse the expectations of the market. At this point, the band will be perceived as not sustainable and institutional beliefs will swiftly be abandoned, precipitating the realignment.

This sufficient condition for collapse may seem quite complicated, but actually we can restate it in a simple practical rule:

**COLLAPSE CONDITION**

For \( 1/\theta < y_x < 1 \) and positive misalignments \( (s - s) > 0 \) the target zone will collapse when two consecutive interventions are necessary, i.e. when the exchange rate does not return to the interior of the band after one intervention.
Fig 2.a.b: The dynamic evolution of \( s, w \) for given weights and exchange rates, respectively. In the right plot, we assume an initial state \( w_0 = 0 \) and a perturbation at time 1 \( \nu^c \) in the exchange rate, such that \( s_i = 2 \). Following the corresponding mapping, \( w \) increases, and we assume that in the next period \( s_i = 4 \). With the corresponding switch in the mapping and increase in \( w \), finally, the exchange rate reaches the band and an intervention takes place, but now the weight decreases. The inverse process follows and reductions in the exchange rate are accompanied by reductions in the weight. The envelope trajectory \( (T^w) \) bends back to the origin, suggesting a cyclical movement. The trajectory of collapse \( (T_c) \) displays the case in which the exchange rate does not decrease when the upper band is reached.
Fixed points

Up to this point we have shown that the system is potentially unstable and under certain circumstances, it may collapse. In this sub-section we aim at getting some understanding on the complex dynamics of the model by exploring the characteristic of the fixed points and dynamic mappings of the exchange rate ($s$) and weight ($w$).

Figure 2.a,b maps the current value on the future value of $s$ and $w$, respectively, obtained from expressions (4,6). Each line represents the corresponding mapping for given values of $w$ and $s$ respectively. This analysis is merely indicative and we have made several simplifications, such as keeping the gain and the equilibrium exchange rate fixed ($s = 10\%$ larger than the upper limit of fluctuation) and dismissing the effect of interventions on the mapping. The intersection of the lines with the 45° line represents a fixed or equilibrium point of the mapping. Observe that the different equilibrium points are stable for the mapping $(s, s_+)$, although for high values of $w$ the equilibria are beyond the target zone. On the contrary, the equilibrium points are unstable for the mapping $(w, w_+)$.

This is a partial picture of the system dynamics, since it lacks the link between the dynamics for the exchange rate and the dynamics for the weights. As a matter of fact, the evolution of the system makes the mapping switch between both curves in figure 2, so that the actual mapping is obtained by joining points between different lines. We use this intuition to draw a non-collapse (NC) and a collapse (C) trajectory in the $(w, w_+)$ mapping which highlights the sort of dynamics expected in the model and sums up the results obtained in this section. In principle, the non collapse trajectory differs from the collapse path only in the magnitude of the perturbation, $v^C > v^{NC}$, but the difference in the outcome is dramatic. In the latter case, the exchange rate reaches the upper limit above the 45° line and this generates an intervention spiral which causes the band to collapse.

The analysis carried out in this section has provided useful insights on the workings of the model, but it is constrained by the complexity of this non-linear dynamic system. The chaotic feature above suggests that the outcomes are dependent on the magnitude and distribution of the shocks. We turn next to a simulation analysis, in which exchange rate trajectories are generated in order to fully investigate the characteristics of the model.
III-THE DYNAMIC HONEYMOON EFFECT

The formal evaluation of the model does not show how the honeymoon effect is projected in time. By simulating exchange rate trajectories, we intend to characterize the evolving divergence between the exchange rate and the fundamentals. We will also be able to show that the conflict derived from the co-existence of a target zone and an increasing exchange rate misalignment (overvaluation) culminates in the eventual collapse of the target zone. Since this conflict arises from the existence of divergences in the process driving the equilibrium exchange rate, the drift parameter \( y \) becomes essential. Thus, we will analyse the outcome for different drifts in the equilibrium exchange rate process.

*Exchange rate trajectories*

We intend to generate exchange rate trajectories that have similar characteristics to the recent ERM experience: a pattern of quiet and turbulent periods, periodic interventions by the authorities and realignments. These features depend on the values assigned to the parameters, so they will be chosen accordingly. In the previous section, some restrictions on the parameters were suggested, and to these we have added other principles in the choice of parameters. First, the white noise terms \((v,e)\) are chosen to have a low variance, such that they do not dominate the drift in the fundamentals and they introduce low *a priori* volatility in the exchange rate; secondly, the weights and the exchange rate should be allowed to vary widely in time, so as to generate the required features mentioned at the beginning of this paragraph.

The initial setting is the following: a 12% fluctuation band with respect to the exchange rate central parity \( \epsilon \) is established between two countries. The central parity is

\[ \epsilon = 0.05; \sigma = 0.005; \rho = 0.5; \lambda = 10; \lambda = 0.8 \]

Also note that, given the function for \( y \), the sufficient condition for collapse holds from the outset.
set equal to the equilibrium exchange rate at the starting period \((t=0)\). We also assume that the exchange rate fulfills its initial equilibrium condition \((\bar{s}_x=c=\bar{s}_y=1)\). Finally, the market is assumed to initially maintain fundamental beliefs \((w_x=1)\). A positive drift is assumed, so inflation rates differentials are positive, inducing an upward trajectory in the equilibrium exchange rate process. We take one month as unit of time so that all plots refer to monthly observations.

With these initial conditions and for the chosen parameters, exchange rate trajectories can be generated from the model. We will first consider one simulation to explain how the model works and then we will replicate the process one thousand times in order to infer more robust conclusions.

Figure 3.a displays one realization of the exchange rate trajectory, where the equilibrium exchange rate \((\bar{s})\) is driven by a drift equal to 1% per year. Although the equilibrium exchange rate eventually exceeds the upper band, the target zone lasts much longer before it collapses (162 instead of 75 periods). We can see that periods of calm and turbulence alternate in the exchange rate, while the misalignment \((\delta - \bar{s})\) tends to widen. This trajectory can be explained by Fig 3.b. The solid line represents the relative weight given to fundamental beliefs by the market \((w; \text{ the institutional weight is } 1-w)\); the dashed line and the asterisks display the evolution of fundamental and institutional beliefs, respectively. The relative weight assigned to each type of belief by the market evolves in time. When the market "goes institutional" the exchange rate remains close to the central parity, but it can be seen that there are periods in which fundamental beliefs gain strength in the market and the exchange rate depreciates.

The magnitude of such a depreciation is determined by equation (4). Nevertheless, figure 3 illustrates a remarkable feature: the relative weight of fundamental beliefs tends to decrease, but in spite of this, the exchange rate jumps to its fluctuation limit with small increases in \(w\).

The resolution to this apparent paradox can be found considering the size of the misalignment \((\delta - \bar{s})\). On the one hand, as the misalignment increases, market expectations have decreasing effects on the weight attached to fundamental beliefs; or, more intuitively, as fundamental beliefs depart from the actual exchange rate evolution, they tend to lose appeal in the market. On the other hand, large misalignments imply that small increases in the weight given to the fundamental beliefs can cause large exchange rate depreciations. Notice that this is precisely the interpretation of the realignment risk we gave in the last section of chapter two. We will return to this point when we relate the results of the models to the credibility of the target zone.

The conflict and interaction between both types of expectations is the crucial Fig
Fig 3.a,b-Exchange rate trajectories simulated by the model. The upper graphs display the trajectories of the equilibrium (s) and effective (e) exchange rates. The lower graph presents the evolution of fundamental (F) and institutional (I) beliefs in that period with the trajectories of the weights.
Fig 3.c,d: Exchange rate expectation with respect to the position of the exchange rate in the band
feature of the model. Further insight on how the conflict of beliefs determines the outcome can be obtained from Figures 3.c and 3.d, which display the expectations for both types of beliefs derived from the model with respect to the position of the exchange rate in the band. As pointed out above, while the institutional beliefs lead to an expectation which is constant relative to the position of the exchange rate in the band (Fig. 3.d), the phase diagram of the fundamental beliefs show the divergence in time (the line in 3.c follows an upwards spiral trajectory). Once the equilibrium exchange rate has exceeded the upper limit of fluctuation, the exchange rate may be driven above the band and an intervention is necessary. The target zone finally collapses (when our collapse condition is fulfilled), and the exchange rate equilibrium is reestablished in the form of a realignment ($S=c$).

The main implication of the model can now be advanced: despite the underlying potential for collapse, the realignment only takes place long time after the equilibrium exchange rate surpasses the upper band. Hence, the zone allows a wide divergence between the level of fundamentals (embedded in $S$) and the effective exchange rate which is reflected in the exchange rate misalignment. This divergence can be seen as a dynamic version of the honeymoon effect stressed in the target zone literature. It has two different dimensions, since it can be measured both in magnitude by the size of the misalignment and duration by the time interval between the equilibrium exchange rate surpassing the band and the realignment.

**Degree of divergence and collapse of the band**

The timing of the collapse, given the rest of parameters, essentially depends on the magnitude of the drift. Furthermore, we have intuitively shown above that slightly different disturbances may lead to completely different outcomes. The working of the model has been explained with the help of one particular realization of the model, whose results cannot be readily generalized.

In order to draw more robust conclusions, one thousand ($D=1000$) simulations of the model have been made for a wide range of drifts ($0.2\% \leq \mu \leq 5\%$). Figure 4 and Table 1 summarize the results of these simulations. Plots in Figure 4 display the period of collapse histograms, for selected drifts. In the first place, it is remarkable that the distributions are not normal and become increasingly skewed to the left, suggesting again the chaotic nature of the model. Looking at the table we can see that both for the first intervention

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8. We define attacks as episodes of strain for the band in which interventions are necessary; each attack may last several periods. In the particular draw shown in the graphs, no attack can be sustained by the target zone.
Fig. 5.- Histograms of the replications for the collapse period
<table>
<thead>
<tr>
<th>µ %</th>
<th>APCT</th>
<th>ATTACKS</th>
<th>1st INTERVENTION</th>
<th>COLLAPSE TIME</th>
<th>DHE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>s,µ</td>
<td>Mean</td>
<td>Range</td>
<td>Median</td>
<td>Mean</td>
</tr>
<tr>
<td>0.20</td>
<td>360</td>
<td>0.89</td>
<td>0-5</td>
<td>547</td>
<td>553.4</td>
</tr>
<tr>
<td>0.50</td>
<td>144</td>
<td>0.68</td>
<td>0-5</td>
<td>235</td>
<td>239.4</td>
</tr>
<tr>
<td>1.00</td>
<td>72</td>
<td>0.43</td>
<td>0-4</td>
<td>127</td>
<td>132.3</td>
</tr>
<tr>
<td>1.50</td>
<td>48</td>
<td>0.30</td>
<td>0-3</td>
<td>77</td>
<td>80.4</td>
</tr>
<tr>
<td>2.00</td>
<td>36</td>
<td>0.20</td>
<td>0-3</td>
<td>56</td>
<td>58.7</td>
</tr>
<tr>
<td>2.50</td>
<td>29</td>
<td>0.15</td>
<td>0-3</td>
<td>43</td>
<td>46.2</td>
</tr>
<tr>
<td>3.00</td>
<td>24</td>
<td>0.13</td>
<td>0-3</td>
<td>35</td>
<td>37.3</td>
</tr>
<tr>
<td>4.00</td>
<td>18</td>
<td>0.12</td>
<td>0-2</td>
<td>26</td>
<td>27.6</td>
</tr>
<tr>
<td>5.00</td>
<td>14</td>
<td>0.10</td>
<td>0-2</td>
<td>18</td>
<td>20.1</td>
</tr>
</tbody>
</table>

Test of skewness: The statistic \( sk = \frac{s^3}{\mu^3} \) is distributed as a \( AN(0,6/T) \). Critical value at 95% is 0.1518.

Normality: The statistic \( no = T/6 sk^2 + T/24 (s^4 \mu^4/T-3) \) is distributed as a \( \chi^2_1 \). Critical value at 95% is 5.99.
distribution (i.e., when the exchange rate reaches for the first time the upper limit of the band) and for the collapse distribution, the statistical tests reject the hypothesis of symmetry and normality for the distributions. We can also identify the existence of several peaks in the distributions, and some of them are bimodal. In these latter cases, the reason is the existence of one or more sustained attacks, which spread out the distribution and generate the possibility of large outliers on the right of the distributions; these outliers imply that, when the target zone resists more than one or two attacks, the collapse time is postponed considerably in time (see range for the collapse time in the table). Looking at the table, we can also observe that the mean of number of attacks sustained is lower than one and the mode is zero for all the drifts, although its range is wide.

These are general results derived from the simulations. Further inference regarding the effect of different drifts have to be made within the appropriate framework for hypothesis testing. The resulting non-normal distributions prevents us from using the central moments for inference. Actually, although the mean of the distributions is very close to the median (see table), the standard deviation can be misleading. Having the empirical distributions gives us the chance to make inference on them directly. Instead,

Figure 5.a shows the mean, median and fifth quantile of the collapse time distribution. Figure 5.b gives us an equivalent picture, but relative to the 'a priori' collapse time (denoted by APCT) defined as the period in which the equilibrium exchange rate is expected to break through the band, that is, when it reaches the upper limit of fluctuation (s). Since the expected value of the equilibrium exchange rate per period is given by the drift $\mu$, this is simply given by:

$$APCT = \frac{s}{\mu}$$

This value appears in the first entry of the table. In the former figure, we can see that the collapse time decreases with the size of the drift, as expected. From the table, we can also see that the first intervention and the mean for the number of attacks follow the same pattern.

Figure 5.b tells us about the length of the honeymoon effect, defined as the ratio between the corresponding values of the simulations (median, fifth and tenth quantiles) and the respective APCT. It suggests that the length of the honeymoon effect tends to decrease with higher drifts. The hypothesis of a significative spell in the honeymoon is also shown in the graphs (see footnote 8): the spell turns out to be significantly different from zero for all the drifts, but for $\mu = 4.7\%, 5\%$, at a 90% significance level, and for drifts smaller

---

*For any null hypothesis to test, it will be rejected at, for instance, 95% significance level, if the 95% of the distribution does not comply with the hypothesis. Hence, the null hypothesis in Figure 5, $H_0:'a priori' collapse time = effective collapse time$, is tested by obtaining the fifth quantile of the collapse distribution. If that value is higher than the 'a priori' collapse time, the null hypothesis is rejected.*
Fig 5.0 - Collapse time for different drifts. The fifth quantile represents the 95% significance level.

Fig 5.1 - The figure shows the ratio between the respective statistics of the collapse distribution and the 'a priori' collapse time, i.e., the expected collapse when institutional beliefs play no role.
than 4.5% for a significance level of 95%.

What about the magnitude of the dynamic honeymoon effect? Given the process for the equilibrium exchange rate and the stochastic terms, we can derive a direct link between spell and magnitude, such that the previous inference is also valid for the magnitude. The difference between this equilibrium exchange rate at the time of collapse and the upper part of the band measures the magnitude of the effect. The median values of the magnitude of the honeymoon effect appear in the last column of the table, where we can see that it decreases with the size of the drift.

---

10. Substituting in the above expression the effective collapse time for APCT, and solving for the exchange rate (s instead of $s$), the expected equilibrium exchange rate at the collapse time is obtained.
IV-PESO PROBLEMS, CREDIBILITY AND REPUTATION

Up to this point, we have just described the numerical results of the model. The outcome could shed some additional light on the reasons for the empirical failure of target zone models. We can observe in figure 6 how the simulated behaviour of our heterogeneous beliefs target-zone model differs from the behaviour of the basic model.

Figure 6 shows the scatterplot of the effective exchange rate changes with respect to the position of the exchange rate in the band for the example we have considered throughout the chapter. In chapter two we have used this relationship (using interest rate differentials to approximate the exchange rate changes) to test (and reject) the basic target-zone hypothesis. Looking at the graph and comparing it with the institutional beliefs, it would be difficult not to reject the target zone hypothesis. The co-existence of two models interacting in the foreign exchange market disturbs the basic target-zone relationship.

Let us imagine, for instance, an analyst attempting to test the target zone model on a data sample generated by our model, where a realignment has not yet occurred. Let us recall that institutional beliefs rest upon the assumption of a perfectly credible target zone. Since the curve representing institutional expectations has a negative slope, we have observed in chapter two that the simplest way to test the hypothesis of a credible target zone formally would be to estimate the regression
\[ \Delta s_{t+1} = a + bs_t \epsilon_t \]
and to test for \( b \) having a negative sign.

This test, performed on our simulated data rejected the hypothesis in 61% of the replications\(^\text{11}\). The analyst would then explore the reasons for this rejection; following the literature reviewed in chapter two, he would probably point out that there exists a realignment risk implied in the expectations.

This realignment risk is explicitly derived in our model from the existence of two different (and conflicting) perceptions regarding the exchange rate behaviour, which break the theoretical relation derived from either one of the models. Let us recall again the conditional expectation expression in (1.12), from where the expression for a

\[^{11}\text{More precisely the } F\text{-test on the significance of parameter } b \text{ has been applied, correcting the residuals with the Newey-West method to have into account error autocorrelation (see section two of chapter two).}\]
Fig 6 - Position in the band and exchange rate variations for a particular realization of the process.

Fig 7 - The figure shows the ratio between the respective statistics of the collapse distribution and the 'a priori' collapse time, i.e. the expected collapse when institutional beliefs play no role.
realigniment risk was subsequently derived in the second chapter. In our model two outcomes are also possible;

- that the system prevails or
- that a realignment comes about.

The market assigns a probability \((1-w,w)\), respectively) to each of these events. Therefore, expression \((4)\) is equivalent to \((1.12)\) and \((2.5)\) as we have already underlined:

\[
\Delta s_{t+1} = \alpha(\Delta s_{t+1} | \text{no realignment}) + (1-\alpha)(\Delta s_{t+1} | \text{realignment}) = wF_t + (1-w)F_t
\]  

\((15)\)

From here, the realignment risk \((c_r)\) is given by the divergences between market expectations and institutional beliefs:

\[
c_r = w(F_t - F_t')
\]

\((16)\)

and \(w\) can be interpreted not only as the weight of the fundamental beliefs but also as the probability of a realignment (when the equilibrium exchange rate surpasses the band).

Expression \((15)\) reflects the existence of a **peso problem** in the data (Krasker (1980), Obstfeld (1987), Lizondo (1983)). We already mentioned this possibility when explaining the failure of the empirical evidence in chapter two. Now the same interpretation can be applied to our model. Recall that if during the period under study there has been no realignment (that is, the event has not occurred) but that possibility affects expectations (and forward exchange rates) in the form of a realignment risk, the data will show anomalies, with respect to the target zone hypothesis. These anomalies as we have seen in the test on our simulated data consist of the presence of a unit root which lead us to reject the target zone hypothesis.

Underlying the peso problem then is the problem of the imperfect credibility of the target zone. We are now going to see how credibility evolves endogenously in the model and is affected by the evolution of the fundamentals, as Bertola & Caballero (1993) suggest.

**Credibility and reputation**

The above expression has also been used by Cuklerman & Metzler (1986) in a different context to define the average credibility \((AC)\) of a monetary announcements. \(AC\) can be defined as the difference between the expectation derived from the announcement and the expectation which the market effectively holds. In our setup, the former is evidently given by the institutional beliefs, which represent the case of a perfectly credible target zone. Therefore, it is straightforward to show that the average credibility is just equal to the realignment risk defined in \((16)\):
The smaller $AC$, the higher the credibility of the announcement. The monetary announcement in our context is the target zone. The announcement is only perfectly credible in the periods when institutional beliefs dominate the market ($w=0$) and the average credibility is zero. Figure 7 shows $w$ and $AC$. The plot suggests that the model endogenously generates dramatic changes in the credibility of the target zone.

At this point, it is important to consider the distinction between credibility and reputation which Weber (1990) makes. Reputation can be seen as a stock variable which is determined by the evolution of the system, and it interacts with the credibility of the system which evolves as a flow. We are now in the position to show the feedback between both elements.

In our model, the parameter driving the credibility of the regime is the weight $w$, whose dynamics are determined by equation (6'). The gain $G$, in that expression contains information about the past performance of the model, in particular on the expectations generated by both beliefs in the past (recall that $X_k = F_k - F_{k-1}$, $k=1,...,t-1$). Consequently, there may exist a link between the gain and the reputation of the system. Indeed, on the one hand, the gain is a positive term which determines the loss of credibility induced by exchange rate shocks; the larger the gain, the larger loss of credibility in the face of a shock, since from (6'):

$$dAw/dv = G(0-1) > 0 \Rightarrow AC < 0$$

On the other hand, if we take the derivative of the gain with respect to the exchange rate misalignment, we can show that the gain decreases with the size of the misalignment:

$$\frac{\partial G}{\partial (s-s_0)} < 0, \text{ with } s_0 = c/(\Sigma X^2)$$

$$\Sigma \sum_{s=1}^{s-1} \lambda^{s-1} - \lambda^2 X^2 < X^2$$

From the first result, it follows that the marginal credibility loss increases with the size of the gain. The second result implies that the gain is reduced with the size of the misalignment. Hence, considering both results together, the larger the misalignment the smaller the marginal loss of credibility. This outcome is consistent with the trajectories of our simulations, where it is observed that the model generates widening realignments; the fact that the target zone survives with widening realignments should strengthen the credibility of the system and this is reflected in a downward trend of $w$ in figure 7.

The two previous expressions provide us with the elements to define a measure of
reputation. The most suitable candidate is the inverse of the gain $G^{-1}$, since a small gain derived from a large misalignment is the observable measure of a high reputation\(^{12}\).

The feedback between credibility and reputation is now apparent from figure 7, where the increases in $W$ are inversely related to the degree of reputation ($G^{-1}$). The opposite feedback can also be identified in the plot: losses in credibility erode the reputation of the system, because larger values of $W$ are associated with narrowing misalignments, which, in turn, increase the value of the gain.

Several points are important to stress. Firstly, the concepts of credibility and reputation just presented are defined with respect to the relative performance of both models, such that the credibility and reputation of the target zone (with respect to the fundamental beliefs) will be high as long as the expectations are conflicting. Secondly, the results above show that the magnitude and spell of the dynamic honeymoon effect tend to decrease with larger drifts; the reason is that large drifts make the equilibrium exchange rate diverge more rapidly, inhibiting the cumulation of enough reputation to postpone further the time of collapse.

Finally, we can rephrase the mechanics of collapse in the model in the light of this definition of reputation. Since, from (6) perfect reputation ($G=0$) is not attainable; this implies that $W$ may suddenly jump and, more importantly, that the danger of collapse is always present even in systems with high reputation.

The smooth divergence in the fundamentals implies then that the expected size of realignment increases with time. Therefore, when $W$ jumps and the misalignment is large the exchange rate may be driven outside the band, even though the jumps tend to be smaller due to the accumulated reputation.

\(^{12}\text{This inverse relation between reputation and the value of the gain is equivalent to the interpretation given in Basar & Salmon (1969), where our concepts correspond to information-credibility and Kalman gain, respectively. Furthermore, deriving the gain with respect to the discounting parameter }\frac{dG}{d\lambda}<0, \text{ we obtain a positive relationship between sluggishness in the adjustment of expectations (high }\lambda\text{) and reputation, which also confirms their results.}\)
This chapter has offered a rationalization of the evolution and collapse of a target zone regime with divergences in the fundamentals. Note that the intuitive explanation of chapter two regarding the sustainability of the target zone follows almost word for word the interpretation of the model given in the previous paragraphs. Therefore, the schizophrenic role of the fundamentals, to which we referred in the conclusions of the previous chapter has been explained by a formal model of target zones: on the one hand, the fundamentals determine the collapse of the target zone, due to the large expected size of realignment which is accumulated; on the other hand, they allow for the accumulation of reputation and credibility. These two opposing forces generate the rich dynamics of the model. All in all, the exchange rate misalignment is the decisive factor for the collapse of the zone even if the system enjoys high reputation.

The main insight is that a target zone generates a dynamic honeymoon effect, that is, the target zone may last for a long time despite divergences in the fundamentals and even endure attacks against the parities, thanks to the accumulated reputation. However, this short run success is eventually challenged by the existence of sufficiently large exchange rate misalignments. The overall result is a honeymoon effect which abruptly vanishes.

Thus, the model highlights the existence of a perverse self-defeating mechanism built into a target zone between divergent economies and underlines the internal inconsistency of exchange rate pegs: the exchange rate misalignment which the target zone itself generates becomes the ultimate cause for the collapse of the system. This comes about due to the existence of a continuous clash between backward looking expectations, which sustain the credibility of the zone, and forward looking expectations which foresee the return of the exchange rate to its equilibrium level.

The simultaneous accumulation of both reputation and future instability seems to have been neglected by policy makers, who instead enjoyed the vision of a secure path to Monetary Union. The "new EMS" developed a high reputation prior to 1992; in the face
of the large misalignments of the high-inflation currencies. The existing parities were credible because the market tended to hold an institutional view which was self-sustaining and avoided tensions. When diverse shocks hit the system, its credibility and the reputational gains were eroded and the market judged that the parities were unsustainable, provoking the speculative attacks that followed.

This paper could be contemplated as a contribution to the current debate on the future of the European Monetary System. It subscribes to the view that a rigid target zone regime in a world of integrated financial markets faces important problems in the presence of divergences in the economies; although the system can deliver stability for a long time and even contribute to the convergence of the economies (as we will see in next chapter), exchange rate disequilibria are simultaneously built up and this leads to the eventual collapse of the target zone.

In our opinion, the direct implication of this Intrinsic Instability is the need for a reform of the system, which renders it more in line with the behaviour of the fundamentals and, as a consequence, more stable. Notwithstanding this, the current specification of the model does not allow us to consider the issues of design adequately, because the fate of the target zone is predetermined. Only if the target zone becomes an instrument of convergence might collapse be avoided.

The consideration of convergence requires then a further step: relaxing the assumption of a constant drift in the fundamentals. When this is done, we can analyse the possibility of convergence in a target zone, that is, the case for a successful exchange rate peg. It also allows us to consider alternative and more flexible designs along the lines mentioned above. This is the object of the next chapter.
When there are divergences in the fundamentals, the stabilisation argument in favour of target zones only holds in the short-run and the model in the previous chapter has shown that the cost of maintaining a nominal exchange rate parity today is a sudden collapse of the target zone in the future. Given this conclusion regarding the fate of target zone systems when persistent divergences exist in the economies, one could reasonably wonder what are the advantages of joining a target zone regime.

The ERM has evolved from a stabilizing device to a disciplinary device, in order to force convergence in inflation. In this chapter, we assume that inflation convergence is indeed the main objective of the authorities who peg their currency to an anchor country with no inflation. The underlying claim of this and the next chapter is that the exchange rate should not be regarded as an objective in itself, but as an intermediate target to attain other objectives.

In chapter three the drift in the fundamentals, which was equal to the inflation differentials has been taken as exogenous and fixed. Relaxing this assumption allows us to consider the possibility of convergence in inflation differentials. The deflationary effects of fixed exchange rates are attained through two channels: discipline in monetary policy and deflation through the external position.

If pegging the exchange rate to a strong currency is credible, monetary authorities will be able to exploit the reputation of the foreign Central Bank (the Bundesbank in the ERM case). This argument has been exhaustively studied in the literature on international policy coordination (see also next chapter). The main problem is how reputation is attained to make this option credible. The framework of our model does not allow us to explore this question in a straightforward way, although in the final section we will indicate extensions to make this possible and then the chapter five we will retake the issue.
Thus, we will consider the second channel: deflation through external imbalances. The overvaluation of the currency exerts a downward pressure on inflation due to the reduction of the price of imported goods. This deflationary effect is stronger, the more open is the economy. It only requires a minor modification into the model which is considered in the next two sections.

Once the model allows for convergence, we are able to explore in section III the possibility of designing a target zone towards such an objective, extending the analysis outlined in Alberola (1995). The recent crisis of the EMS provides the motivation for this exercise. We present a proposal to reform the ERM in the transition to the European Monetary Union. The proposal supports the adoption of convergence bands instead of the fixed bands of the ERM in order to avoid large misalignments. Convergence bands are implemented through a correction of the central parity and/or the band width to adjust for inflation differentials. These corrections should endow the system with sufficient flexibility to avoid exchange rate crises derived from the support of unrealistic central parities.

After setting up the framework to consider this proposal, it is evaluated through two different simulation experiments, depending on whether the band is allowed to narrow or not. The outcomes of this simulation establish a ranking between the different alternatives. It turns out that there is an important trade off between flexibility and convergence and that the current system of fixed central parity and rigid bands is not the best alternative in general.

The final section of the chapter elaborates further the convergence bands proposal and takes into account other considerations which may change the conclusions. In particular, other positive implications that flexible convergence bands may have are examined, before being more formally developed in the next chapter.
I-ENDOGENOUS DRIFT

The first step to consider the possibility of an endogenous drift is to provide a rationale for it. Let us assume without loss of generality that the inflation of the foreign country is zero \((p_t' = 0, \forall t)\) so that the equilibrium exchange rate is simply equal to the price of the domestic goods: \(s_t = p_t\). Therefore, if the domestic country has an interest in reducing inflation differentials, one beneficial strategy would be to adopt a target zone regime\(^1\).

We can demonstrate this claim more formally by expressing the consumer price index \((p_c)\) as a weighted average of the domestic and foreign goods prices:

\[
p_c^t = (1-h)p_t + h(s_t + p_t')
\]

where the parameter \(h\) is the share of imports in the consumer basket and also can be seen as a deflator for the economy or the degree of integration between the economies. It is convenient to rewrite the above expression as follows:

\[
p_c^t = s_t - h(s_t - s_{t-1})
\]  

Let us assume on the other hand that wages are fully indexed to the consumer prices\(^2\), and this indexation translates into increases in the price of the domestic good: \(\Delta p_r^t = \Delta p_r^t, t\). Since the expected price is equal to the expected equilibrium exchange rate, taking differences in (1) and substituting, we can obtain an expression for the changes in the drift for the fundamentals:

\[
\Delta p_r^t = \Delta^2 s_t^e = h\Delta(s_t - s_{t-1})
\]

This expression is appended now to the original model of heterogeneous beliefs to study the effects of a target zone on convergence. The rest of the model equations and

---

\(^1\) We implicitly assume the case of a small country which does not create inflation problems to the foreign country by pegging the currency. As a matter of fact, this is the view adopted by the literature which has not provided yet a plausible reason for the anchor country (Germany) to accept the arrangement. The next chapter treats both countries on equal basis.

\(^2\) This seemingly innocuous assumption discards the announcement effects of the target zone on inflation expectations. In any case, there is little empirical evidence that the ERM has greatly affected inflation expectations (see Giovazzi & Giovannini (1989), Collins (1988)). Such announcement effect and the credibility attached to it is the issue of the other disciplinary channel of exchange rate pegging. Thus, here the game played between the authorities and the private sector is dismissed and the time consistency problem does not arise.
assumptions are carried through to this section. We will see that the existence of a
dynamic honeymoon effect, reduces the drift in the fundamentals and promotes
convergence.

Expression (2) could also be expressed in terms of real exchange rate
\[ z = s - (p - p') = s - 5 \]

Then, we can see that an overvaluation of the real exchange rate \( z < 0 \) exerts a
deflationary effect through the reduction in the drift. This effect is more important the
higher the share of imports. The introduction of a variable drift feedbacks into the model
and affects its dynamics, as we will explore in detail below.

Entry clause and convergence gains

It is useful to define more precisely the concept of convergence. First of all, the
simplicity of the setup precludes any reference to real convergence. We are dealing
exclusively with nominal convergence on inflation rates, although the deflation imposed
by the system is presumably costly in terms of output. Nevertheless, this approach is in line
with the philosophy of the Maastricht Treaty, which ignores the real effects of nominal
convergence altogether. Therefore, we bypass the conflict of objectives between real
and nominal targets, i.e. the trade-off between inflation and unemployment, which may
arise in the target zone. We will take up this point in the final remarks.

The question to settle is then whether convergence is understood in terms of the
level of prices or in terms of differences, that is, in terms of inflation. Each interpretation
conveys different implications for the behaviour of the exchange rate.

The Maastricht Treaty demands convergence in inflation rates and the
maintenance of the nominal exchange rate in the ERM during the two years prior to the
Monetary Union. This suggests an interpretation in terms of differences. However, this has
important consequences for the exchange rates and the real economy. In chapter three,
we observed that if no realignment is made and inflation divergences remain, the real
exchange rate becomes overvalued.

Let us suppose that the countries adopt a common currency when inflation
converges but that the equilibrium exchange rate is outside the band and that the
established conversion is equal to the central parity or any exchange rate within the band
(in order to avoid one-way bets). This would mean that the domestic country enters the
union with an overvalued currency, with no possibility to be subsequently corrected by
‘nominal’ policy actions.

This unsatisfactory situation implied by the Maastricht conditions gives rise to the
problem of a 'last realignment clause': countries with an overvalued currency will go for 
a final realignment that reestablishes their competitive position and this, when recognised 
by the markets, will leave scope for one-way bets, introducing instability in the last stages 
of the ERM (Giovannini (1993), Begg et al. (1991)).

The alternative would be an approach based on convergence in the level of 
fundamentals, whereby not only inflation rates but also prices levels must have converged 
to join the Common Currency. This approach implies a much stronger concept of 
convergence. A period of negative inflation differentials is required in this case, in order 
to return the real exchange rate to the central parity or, at least, within the band.3

Here we will follow the Maastricht approach, so that convergence is defined in 
terms of inflation differentials. We will assume that the domestic country starts with a two 
percent drift (µ=2%) and converges (in inflation) at a speed determined by the share of 
imports in the consumer basket. When convergence is achieved, the target zone is 
substituted by a common currency.

An entry clause has to be defined for the high inflation country. Since the inflation 
rate depends on the misalignment and this is variable, it is sensible to consider the 
evolution of inflation over several recent periods, let us say one year, instead of just the 
last period. Therefore, a country will be allowed to enter the currency union, when the 
average annual rate of inflation is below a certain threshold:

\[
\text{ENTRY CLAUSE} \quad \frac{1}{12} \sum_{t=1}^{12} \mu_t \leq \mu^* 
\]

The entry clause plus the variable drift assumption introduce the possibility of an 
additional outcome in the model. Now the target zone will either collapse or be finally 
abandoned through the adoption of a common currency.4 We can then talk either of 
the failure or success of the target zone strategy. However, this interpretation might not 
be accurate. In particular, it may be possible that after the collapse not all the 
convergence gains are wiped out, despite the jump in inflation that the collapse and the 
consequent return to the equilibrium exchange rate level may imply.

In order to make this point more clear, let us take the expression for the evolution 
of the drift (2) for all the periods. We can then write:

where the sum of error terms is expected to be zero, so that we can dismiss it. If the target 
zone collapses at time \( t \), the last line in (4) is:

\[3 \text{-This is the criterion assumed in Alberola (1995).} \]

\[4 \text{-In the simulation exercise we will see that an intermediate outcome is possible: the drift varies around an} \]
\[\text{stationary value close to zero and the band does not collapse.} \]
where the last term represents the jump in the drift in the collapse time. Some of the convergence gains will be consolidated if the drift, after the collapse, is lower than the initial drift, that is, if \( \mu_i - \mu_i < 0 \). From (5) this condition holds if:

\[
\mu_i - \mu_j = ((1-h)^{I-1} - 1)\mu_1 + \sum_{n=2}^{\infty} (1-h)^{I-n}\Delta\bar{s}_{s-n+1} < 0
\]  

The first term is negative and the second term is positive, but the sum of both terms is also negative, highlighting that the endurance of the target zone reduces the inflation differentials. However, at the time of collapse the jump in the drift may reverse this gain. In the simulation exercise below we will compute this expression to determine the sign. We can say in advance that the overall outcome is the existence of small gains, although there are particular realisations in which the result is positive (\( \mu_i > \mu_j \)).
II-SIMULATING CONVERGENCE PATHS

The analysis carried out in this section is analogous to that in the previous chapter, but with the modified model. The focus of the analysis shifts now from the magnitude of the drift to the degree of integration, represented by $h$, and from the duration of the band to the possibility of convergence.

For simplicity, we will assume in what follows that full convergence in inflation is required ($\mu = 0$) instead of a divergence of 1.5% which is contemplated by the Maastricht Treaty. The initial drift is assumed to be equal to 2% annual ($\mu_e = 0.02$), so that a sizable degree of convergence is required. We let the process run until the band collapses or the entry clause (zero annual inflation) is satisfied. Therefore, two different outcomes of the model are now possible, since we now recognise the possibility of convergence within the target zone regime. Now we present several examples to illustrate the possible outcomes.

The first example assumes a closed economy, such that $h = 0$. In order to compare these results with the following examples, when the drift is assumed to be variable. With these assumptions, the only change in the dynamics of the model is given by the drift correction equation (3), which is in turn driven by the degree of interdependence and the size of the misalignment.

Figures 1.a, b are equivalent to figures 3.a and 8 in chapter three. In figure 1.a the evolution of the equilibrium and effective exchange rates is shown and in the second the evolution of credibility, reputation and the weight of fundamental beliefs are displayed. The plot labelled 'b' will be considered in the interpretation of the results, after the replication exercise.

Let us assume now that that the share of imports $h$ is equal to 10%. Figures 2.a, b display the result of the simulation; they do not greatly differ from the fixed drift case. However, an additional line appears in the figure 2.a, which displays the equilibrium exchange rate in the case that no convergence was allowed in inflation. This has to be compared with the dashed line of the effective trajectory of the equilibrium exchange.

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5. A limit of periods ($T = 500$) has been imposed in the simulation. This implies that there may be cases in which the system neither collapses nor converges.
Figure 1.a - Exchange rates trajectories: Equilibrium and actual exchange rate

Figure 1.b - Probability of realignment, average credibility and reputation
**Figure 2.a** - Exchange rates trajectories. Equilibrium and actual exchange rate

**Figure 2.b** - Probability of realignment, average credibility and reputation.
**Figure 3.a** - Exchange rates trajectories: Equilibrium and actual exchange rate

**Figure 3.b** - Probability of realignment, average credibility and reputation
Figure 4.a - Exchange rates trajectories. Equilibrium and actual exchange rate

Figure 4.b - Probability of realignment, average credibility and reputation.
Part II-Design

4. Inflation convergence and convergence bands

**Figure 5.a** - Exchange rates trajectories. Equilibrium and actual exchange rate.

**Figure 5.b** - Probability of realignment, average credibility and reputation.

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Convergence in the (annual average) inflation rate ($\mu$) can be observed in the line labelled (1) of figure 6. There, we observe that the inflation differential jumps to values slightly lower than the initial drift of 2% at the collapse time. We observe that the target zone collapses anyway; the convergence in inflation only contributes to slightly postpone the collapse. This delay can be explained by the more sluggish behaviour of the equilibrium exchange rate; it passes through the band around 8 periods later than when no convergence is assumed (see the horizontal distance between the two equilibrium exchange rates on the upper bound line). This behaviour has a direct effect on the dynamics of the fundamental beliefs which also evolve slower, postponing the strains in the expectation formation process.

The next two examples assume an equal deflator ($h=30\%$), but the outcome varies
dramatically\(^7\). In the first case (figure 3.a,b) the target zone collapse as before, although the collapse time is deferred; on the contrary, in the second case (figures 4.a,b), full convergence is achieved. In both cases, inflation is largely reduced (see lines (2) & (3) in figure 6) but in the first case this convergence is not fast enough to avoid the collapse.

Note that these two examples are replications of the same models, so that the different result is exclusively due to the different shocks hitting the model, highlighting again the importance of the stochastic components in the outcome of the model.

The final example in figure 5 assumes a very high level of integration (\(h=50\%\)) so that full convergence is achieved before the equilibrium exchange rate surpasses the upper bound of fluctuation, due to the sharp reduction in inflation (see line (4) in figure 6). This is a case in which the strong concept of convergence (in levels) can be applied because in the case of a final realignment, the exchange rate would not need to jump.

Intuition is consistent with the message derived from these examples: convergence can be attained given a determined level of integration. As in the previous chapter, this intuition has to be confirmed by a replication exercise. Here, the crucial variable to consider is the degree of integration (\(h\)). The rest of the parameters are kept at the same values.

Table 1 and figure 7 sum up the results from 1000 draws and different values of \(h\). The first row recalls the case in which the drift is assumed fixed (\(h=0\)). The table consists of two blocks of data. We have divided the table into two parts: the first block refers to the replications in which a collapse occurs and the second to the draws in which the band succeeds in providing convergence. The first row reproduces the values for the constant drift case, which appear in Table 1 of chapter 3.

The first column in each block computes the percentage of draws in which the band collapses-column (1)—and in which convergence is achieved-column (7), respectively; the plot (a) in figure 7 displays graphically the same information.

The next two columns -(2,3) and (8,9)- in the first (second) block and graph (b) in figure 7 list the median period and the range of periods respectively, for which the band collapses (inflation converges).

Finally, the first block displays three additional columns: (4) lists the median of the drift after the collapse and (5) computes the median of minimum inflation achieved in the cases of collapse; they are also displayed in graph (c); column (6) and graph (d) compute the median magnitude of the honeymoon effect, computed as the difference...

---

\(^7\) This choice of the deflator is intentional. In table 1 below we can observe that for this value the zone converges in almost half of the cases.
<table>
<thead>
<tr>
<th>ERM</th>
<th>COLLAPSE</th>
<th>CONVERGENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cases% (1)</td>
<td>Time (2)</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
<td>60</td>
</tr>
<tr>
<td>5</td>
<td>99</td>
<td>64</td>
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<td>99.5</td>
<td>63</td>
</tr>
<tr>
<td>15</td>
<td>98</td>
<td>68</td>
</tr>
<tr>
<td>20</td>
<td>86</td>
<td>73</td>
</tr>
<tr>
<td>25</td>
<td>78.5</td>
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<td>27</td>
<td>72</td>
</tr>
<tr>
<td>50</td>
<td>21</td>
<td>64</td>
</tr>
</tbody>
</table>

Table 1: Results from simulation. Fixed central party and bands
Figure 7 - Results of the simulations with endogenous drifts. All the results are expressed in medians.
between the equilibrium exchange rate and the upper edge of the band (3-s). Several conclusions can be drawn from these results:

i) From columns (1,7), it is observed that the higher the level of integration, the higher is the probability of convergence and the smaller the probability of collapse. Moreover, column (5) shows that even in the cases of collapse the degree of convergence is larger for higher values of $h$.

ii) For low degrees of integration $h<10\%$, convergence is not feasible. There are however some cases in which there is no collapse, either; this is revealed in the sum of columns (2) and (7) which do not reach 100\% for low degrees of convergence, because the time period taken is too short.

iii) More remarkable is the fact that convergence is not guaranteed even for very high degrees on integration. For $h=50\%$, for instance, the band collapses in more than 20\% of the cases.

iv) The collapse and convergence times are consistent with the first conclusion. Higher degrees of integration imply that the median of the convergence time comes earlier and the collapse time is postponed. However in the column corresponding to the ranges, we observe that for large values of $h$ there are realizations in which the band collapses earlier than in the constant drift case; as a consequence, the resulting distribution is more scattered and skewed than in the constant drift case.

v) The magnitude of the honeymoon effect is smaller the larger the degree of integration. For $h>40\%$ it is less than 1\%. Finally, when there is a collapse the drift jumps but it does not reach its initial level in most of the cases.

Interpretation of the results

Most of these conclusions confirm intuition. As seems reasonable, the target zone is more likely to succeed when convergence is faster, that is, when the economy is more integrated. Nevertheless, there are some results which are rather striking and counterintuitive. Therefore, the interpretation will be done along the lines of the last section in chapter three.

The first is that the band has a sizable probability of collapsing even for the maximum level of integration allowed. The second is that for some realizations the

\(^8\) Since the constant drift distribution is not normal, these distributions are non-normal too (we have carried out the same normality tests than in the previous chapter and the normality hypothesis is rejected in all cases). This explains why we have also the median as reference in this chapter.
collapse may arrive earlier with higher degrees of integration. These patterns of behaviour can be explained by the mechanisms which underlie the model, in particular on the credibility process. The reduction of the inflation differentials exerts two opposing effects on the determination of the collapse time:

- On the one hand, as we saw in the previous examples, lower inflation differentials imply that it takes longer for the exchange rate to pass through the band limit; this implies than the strains in the system are postponed because the fundamental beliefs also start to expect a collapse of the target zone later; this factor works in favour of a delay in the collapse of the band.

- On the other hand, the slower evolution of the misalignment also slows down the accumulation of reputation; as we saw in the constant drift case, this affects credibility because credibility is defined in relative terms. We can observe in figures 1-5.b that the average credibility is closer to zero when the degree of integration is low. This implies that reputation gains are obtained at a faster rate than when inflation converges rapidly. When this happens, the wedge between the actual and equilibrium exchange rate does not widen as fast as in the constant drift case; fundamental beliefs are more credible and the updating mechanism is less biased towards the institutional beliefs. Therefore, the evolution of \( w \) tends to give more weight to the fundamental beliefs, as we can see in figures 3-5.b.

Thus, for larger degrees of integration, the slowdown in the accumulation of reputation and the longer time span to build it has to be confronted with the faster rate of convergence. The dominance of one or other factor will determine the success or failure of the target zone.

The unfavourable factor tends to be more than compensated for by the longer time span to build up reputation that the inflation convergence provides. However, when reputation is built up too slowly, the system collapses. As a matter of fact, this may happen when the equilibrium exchange rate has just surpassed the upper bound; this explains the narrowing of the honeymoon effect and the earlier collapse in certain cases for large levels of integration.

The positive probability of collapse, regardless of the value of the deflator \( h \), stresses the risk component derived from the stochastic environment. Nevertheless, we can see that the lower the initial drift and the more open the economy, the probability of convergence is higher, while for very low degrees of integration and/or large initial drifts the collapse is guaranteed.
III-CONVERGENCE BANDS

We have seen that when the assumption of a constant drift is relaxed, the resulting misalignment reduces inflation differentials. However, if the band collapses, the gains derived from convergence virtually vanish.

These results have been obtained with the target zone taken as given. The failure or collapse of the target zones just depend on the initial conditions (initial drift, parameters of the model) and on the continuum of shocks hitting the process. In this part of the chapter, we explore the possibility that the policymaker plays an active role in designing the target zone. The main goal of the policymaker is to achieve convergence and the secondary objective is the sustainability of the target zone.

The design of a target zone can be seen to depend on the choice of two parameters in particular: the width of the band and the central parity. Therefore, it would be desirable to contemplate a flexible and simple mechanism to design and analyse the target zone. We will refer to this mechanism as convergence bands. As above, the complexity of the model again precludes a mathematical analysis of optimality; thus, we will carry out another simulation analysis in order to choose the most favourable design.

The rationale for the convergence bands mechanism can be found both in the ERM experience and in the results of our dynamic target zone model. Large misalignments without perfect reputation are in both cases behind the collapse of the target zone. It is then evident that if the target zone would have been built around a real exchange rate target instead of a nominal target, large misalignments would not be possible, and the target zone would be sustainable.

This alternative reminds the Williamson proposal (1985, 87) of real exchange rate parities. However, given our Inflation modelization, no convergence would be possible, because no deflationary effect would be attained. An equivalent argument can be

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9: Alternative management of reserves could be another possible instrument considered by governments. Here we maintain the assumption of marginal interventions, although relaxing it would be an interesting extension.
applied to the band width. If the bands are too wide there would be no problem of surpassing them, but it would also imply that they are not binding, and little or no honeymoon effect would arise.

Chapter three formally showed that excessive rigidity in the ERM has led to the collapse of the system. Therefore, it may be suitable a target zone design in which the conflict of objectives is not so apparent. The alternatives proposed in the aftermath of the crisis ranged, from the immediate locking of parities to a return to floating. The Ecofin finally decided to widen the bands dramatically (15%). This implies that the potential of exploiting a honeymoon effect or ensuring convergence derived from the existence of the target zone is doubtful.

Another question of interest is the degree of monetary autonomy that a target zone should allow. One of the main reasons for the doubts of some countries, in particular the United Kingdom to enter and then stay in the system was the loss of control on monetary policy.

Which type of target zone is required then? The mechanism we propose intends to be flexible enough to allow for a wide choice of target zones and to be simple enough to be testable in our general framework. Furthermore, as it will be immediately apparent, the sacrifice of monetary autonomy in this proposal is chosen by each country, albeit at the cost of the disciplinary effects that the system can deliver.

The results which will follow the proposal are intended to give us an indication about the optimal target zone arrangement.

Elements

The convergence bands link the evolution of the fundamentals to the adjustment of the central and/or the band width by a functional form of the following type:

CONVERGENCE BANDS

<table>
<thead>
<tr>
<th>Central parity</th>
<th>$C_i(1-p)(\bar{s}_i+\mu_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band width</td>
<td>$(s_i^*-C_j)=\gamma_0+\gamma_i\mu_i$</td>
</tr>
</tbody>
</table>

$0 \leq p \leq 1; \gamma_0, \gamma_i > 0$
Figure 8 - Exchange rate trajectories when the central parity partially accommodates inflation differentials.

Figure 9 - Exchange rate trajectories when the bands narrow as inflation differentials decrease.
Let us consider first the expression for the central parity. The model is defined in terms of the nominal exchange rates and the ERM displays a fixed nominal parity between realignments. Consequently, an overvaluation of the exchange rate is expected when price differentials are positive, since the nominal exchange rate is bounded. Therefore, a value for \( p \) equal to 1 portrays the ERM case.

On the contrary, the Williamson proposal implies a fixed real parity; hence, the exchange rate target equals the expected equilibrium rate \( S+\mu \), which is defined in terms of PPP. Thus, the central parity adjusts to the evolution of the equilibrium exchange rate, such that \( p=0 \).

These proposals represent in our specification the extreme cases of flexible and rigid target zones. The virtues and disadvantages of each option have already been mentioned. Note that now given the choice of the convergence bands parameters, countries can keep more autonomy than in a fixed central parity regime. While in this latter case the cost of non-discipline is a collapse of the band, now the costs are measured in terms not only of the future collapse, but also on the continuous nominal exchange rate depreciation of the currency, which difficulties the attainment of convergence.

The essence of the convergence band proposal is to consider the intermediate options, too. The range of intermediate values would partially offset the inflation differentials in the exchange rate target. This possibility is considered because there may exist a certain range of values for \( p \) which reconciles the advantages of discipline with the sustainability and monetary autonomy of a more flexible regime. In order to obtain a flavour of how the central parity correction mechanism works, figure 8 shows the case of a target zone with adjusting central parity \( (p=-.5) \) and a low share of Imports \( (h=10\%) \) which finally collapses.

The choice of the band width also has important implications. In chapter one we observed that the degree of exchange rate stabilization in the static model negatively depends on the width of the band. This conclusion is obtained when perfect credibility is assumed. When this assumption is relaxed and credibility is endogenous as in our model, a too narrow exchange rate band may be counterproductive, due to the lack of reputation. In any case, it should be noted that the concept of 'too narrow' or 'too wide' is related to the magnitude of the drift. If the band is narrow but the drift is also small the reputation problem may not arise. As a matter of fact, the ERM considered two

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10. The second difference is that Williamson favours the existence of approximate band widths (soft bands) while in the ERM the band width is well determined. We will return to this issue in the next chapter.
different band widths before 1992: a narrow band (2.25%) for the core countries and a wide band (6%) for the periphery countries.

These remarks have been taken into account in the expression for the width of the target zone band above. The width of the band depends negatively on the current drift in the fundamentals, so that as convergence progresses the bands of fluctuation narrow up to a lower bound, given by the parameter $\gamma_0$. Figure 9 illustrates this case.

Intuition suggests that the convergence band proposal faces the same sort of trade-off between reputation and convergence as before. On the one hand, a flexible central parity will be more easily maintained and the time to accumulate reputation is longer; however, it also implies smaller misalignments and therefore a slower accumulation of reputation. On the other hand, the width of the band is also linked to the issue of reputation: too tight a target zone may leave too little horizon for the accumulation of reputation, even if the exchange rate misalignment is larger; for a wide band the opposite reasoning applies.

**Testing the convergence bands**

The optimal solution to this trade-off can only be found by performing simulations experiments. A complete analysis of the question would require an extensive consideration of initial and final values of the band widths for every value of $p$ and every degree of integration; a simulation run should be applied on each combination. This is a cumbersome task, so we have simplified the problem by running just two types of experiments.

Fixed band widths will be assumed in the first experiment, so that $\gamma_0=6\%$, $\gamma_1=0$; in the second experiment, we will allow a narrowing of the band by setting $\gamma_0=2.25\%$, $\gamma_1=1.875$. The choice of these values imply that for the initial drift we consider $\mu=2\%$, the initial band width is $6\%$, coincident with the wide ERM band and with the earlier analysis and the floor is given by the ERM narrow band. In both experiments, the two options will be considered for different choices of the central parity, that is from $p=0.1$ to $p=1$, with .1 steps. Given the different values for the share of imports (.05 $\leq h \leq .5$, with 5% steps), we then build a 'grid' of 100 different subcases (ten cases for each value of $p$).

The cases considered in the evaluation of the convergence bands are now

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11. The complete offsetting of inflation differentials $p=0$, i.e. the Williamson proposal, is not considered because it precludes convergence and is always sustainable.
summed up, where the experiments correspond to the rows of the table:

<table>
<thead>
<tr>
<th>Band width correction</th>
<th>Central parity correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st exp. NO</td>
<td>ERM CC</td>
</tr>
<tr>
<td>2nd exp. YES</td>
<td>CW CWC</td>
</tr>
</tbody>
</table>

The cases can be classified in terms of rigidity. The most flexible case is the possibility of central parity correction (CC) and the most rigid case is represented by the correction of the band widths (CW). The two intermediate cases are the current situation of fixed central parity and band width, namely the ERM, and the simultaneous correction of central parity and band width (CWC).

The ERM is a particular case of the first experiment with \( p = 1 \) and has been analysed in Table I. The second case (CW) is composed of 10 subcases, equivalent to the ERM with the only difference being that now the bands narrow with declining inflation. Table 2 and figure 10 present the results of this modification.

The rest of the cases contain 90 subcases each. Tables 3-5 present the cases CW (tables labelled 'a') and CWC (tables labelled 'b'), for several values of \( p \). The structure of these tables is identical to the first table to allow for comparison. Figures 11-14 give an overall picture of the results of the two experiments.

**Evaluation of the proposal**

The objective of this section is to rank the four cases according to their respective merits. The analysis of these simulated data can be done in two steps. First, we will compare column-wise the cases of central parity correction and then we will consider the band width. Apart from the results summarised in the tables and graphs we have constructed a test on the difference of median values in order to make inference on the simulated data. The hypothesis to test is \( H_0: M_1 = M_2 \) vs. \( H_1: M_1 > M_2 \) or \( M_1 < M_2 \), where \( M_1, M_2 \) are the medians of the samples to compare \( S_1, S_2 \).

The test is quite simple. The combined \( N_1 + N_2 = N \) values from \( S_1, S_2 \) are ordered by increasing size, the median \( M \) is determined and the values in each sample are arranged according to whether they are larger or smaller than the common median in
the following scheme \((a,b,c,d)\) are frequencies:

<table>
<thead>
<tr>
<th>Number of occurrences of the value</th>
<th>(&lt;M)</th>
<th>(&gt;M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_1):</td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>(S_2):</td>
<td>(c)</td>
<td>(d)</td>
</tr>
</tbody>
</table>

The statistic \(\chi^2 = \frac{(N(a+c)^2)}{(a+b)(c+d)}\) which is distributed as a \(\chi^2\) with one degree of freedom, is then computed. If the value of the statistic is larger than the critical value of the distribution the null hypothesis is rejected (from Sachs (1984, pp.302,347-348). The results of these tests can be found in tables 6-8.
Table 2 - Results from simulation. Fixed central parity and correction of the bands.
Figure 10 - Results of the simulations with endogenous drifts and narrowing of the bands (CW). All the results are expressed in medians.
### Table 3.a: Central parity correction with fixed bands (1-p)=.2

<table>
<thead>
<tr>
<th>h %</th>
<th>Cases% (1)</th>
<th>Time (2)</th>
<th>Range (3)</th>
<th>Posterior drift (4)</th>
<th>Minimum drift (5)</th>
<th>DHE (1-s) (6)</th>
<th>Cases% (7)</th>
<th>Median (8)</th>
<th>Range (9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
<td>60</td>
<td>31-160</td>
<td>2</td>
<td>2</td>
<td>4.00</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>99.5</td>
<td>69</td>
<td>38-174</td>
<td>1.96</td>
<td>1.74</td>
<td>2.13</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>98</td>
<td>74</td>
<td>42-262</td>
<td>1.94</td>
<td>1.50</td>
<td>2.38</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>98</td>
<td>78</td>
<td>44-266</td>
<td>1.91</td>
<td>1.32</td>
<td>2.60</td>
<td>2</td>
<td>370</td>
<td>253 469</td>
</tr>
<tr>
<td>20</td>
<td>92</td>
<td>81</td>
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<td>1.87</td>
<td>0.91</td>
<td>2.44</td>
<td>8</td>
<td>197</td>
<td>164 340</td>
</tr>
<tr>
<td>25</td>
<td>79</td>
<td>82.5</td>
<td>48-172</td>
<td>1.85</td>
<td>0.70</td>
<td>2.63</td>
<td>21</td>
<td>153.5</td>
<td>129 183</td>
</tr>
<tr>
<td>30</td>
<td>65</td>
<td>82.5</td>
<td>48-164</td>
<td>1.82</td>
<td>0.53</td>
<td>1.98</td>
<td>35</td>
<td>130</td>
<td>92 176</td>
</tr>
<tr>
<td>35</td>
<td>61.5</td>
<td>80</td>
<td>50-154</td>
<td>1.82</td>
<td>0.51</td>
<td>1.32</td>
<td>36.5</td>
<td>110</td>
<td>80 189</td>
</tr>
<tr>
<td>40</td>
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<td>82</td>
<td>51-129</td>
<td>1.74</td>
<td>0.50</td>
<td>0.86</td>
<td>52</td>
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<td>0.48</td>
<td>60</td>
<td>95</td>
<td>63 141</td>
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<td>50</td>
<td>34</td>
<td>84.5</td>
<td>50-131</td>
<td>1.64</td>
<td>0.29</td>
<td>0.19</td>
<td>66</td>
<td>87</td>
<td>57 140</td>
</tr>
</tbody>
</table>

### Table 3.b: Central parity correction with narrowing bands (1-p)= 2

<table>
<thead>
<tr>
<th>h %</th>
<th>Cases% (1)</th>
<th>Time (2)</th>
<th>Range (3)</th>
<th>Posterior drift (4)</th>
<th>Minimum drift (5)</th>
<th>DHE (1-s,) (6)</th>
<th>Cases% (7)</th>
<th>Median (8)</th>
<th>Range (9)</th>
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</thead>
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<tr>
<td>0</td>
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<td>2</td>
<td>4.00</td>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>99</td>
<td>68</td>
<td>43-217</td>
<td>1.96</td>
<td>1.70</td>
<td>2.56</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>98.5</td>
<td>70</td>
<td>40-217</td>
<td>1.94</td>
<td>1.43</td>
<td>2.13</td>
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<td>-</td>
<td>-</td>
</tr>
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<td>15</td>
<td>98.5</td>
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<td>35-240</td>
<td>1.90</td>
<td>1.16</td>
<td>2.17</td>
<td>1.5</td>
<td>447</td>
<td>237 478</td>
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<tr>
<td>20</td>
<td>87.5</td>
<td>79</td>
<td>46-173</td>
<td>1.89</td>
<td>0.98</td>
<td>2.56</td>
<td>12.5</td>
<td>178</td>
<td>154 392</td>
</tr>
<tr>
<td>25</td>
<td>79</td>
<td>83</td>
<td>45-154</td>
<td>1.84</td>
<td>0.80</td>
<td>2.81</td>
<td>21</td>
<td>137.5</td>
<td>108 276</td>
</tr>
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### Table 4.a: Central parity correction with fixed bands (1-p)=.5

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### Table 4.b: Central parity correction with narrowing bands (1-p)=5

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### Table 5.a: Central parity correction with fixed bands \((1-p)=0.8\)

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### Table 5.b: Central parity correction with narrowing bands \((1-p)=0.8\)

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Figure 11 - Results of the simulations. Cases of convergence and collapse for the two experiments. All the results are expressed in medians.
Figure 12 - Results of the simulations. Times of convergence and collapse for the two experiments. All the results are expressed in medians.
Figure 13 - Results of the simulations. Minimum and final drifts for the two experiments. All the results are expressed in medians.
Figure 14 - Results of the simulations. Magnitude of the dynamic honeymoon effect for the two experiments. All the results are expressed in medians.
Table 6-Test on the difference of convergence time. The first results compare the fixed band/variable band for the different degree of correction in the band, given by p. The second set of results compare fixed bands with variable bands. Finally, an overall comparison between both cases is carried out.

Table 7-Test on the difference of convergence time. The first results compare the fixed band/variable band for the different degree of correction in the band, given by p. The second set of results compare fixed bands with variable bands. Finally, an overall comparison between both cases is carried out.
# Table 8: Test on the difference of convergence time.

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| ... | ERM | ERM | ERM | ERM | ERM | ERM | ERM | ERM | ERM | ERM |
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| ... | ERM-CC | ERM-CC | ERM-CC | ERM-CC | ERM-CC | ERM-CC | ERM-CC | ERM-CC | ERM-CC | ERM-CC |
| ... | -   | -   | -   | -   | -   | -   | -   | -   | -   | -   |
| ... | CWC | CWC | CWC | CWC | CWC | CWC | CWC | CWC | CWC | CWC |

**Note for the tables 6-8**

The null hypothesis \( (H_0) \) is that the median \( (M) \) in the two samples is the same. The alternative hypothesis \( H_1 \) is that the median of one of the samples is larger than the other. Thus:

\[
H_0: M_1 = M_2 \\
H_1: M_1 > M_2 \text{ or } M_2 > M_1
\]

The statistic to be considered is \( \chi^2 \) which is distributed as a \( \chi^2 \) (see text); the critical values are 2.706 for a 95% significance level and 1.642 for a 90% significance level.

- 'o' implies rejection at 90% significance level.
- 'oo' implies rejection at 90% and 95% significance level.
- 'x' implies rejection at 90%, 95% and 99% significance level.
- 'xx' implies rejection at 90%, 95% and 99% significance level.
Considering first the question of adjusting the central parity to partially offset inflation differentials (cases ERM.CW vs. CC.CWO), a careful inspection of the tables and graphs allows to reach the following conclusions.

I) From tables 3-5, we observe that keeping the central parity fixed improves the probability of convergence (7% more cases) but it also increases the probability of collapse (4% more cases); there are however subcases in which the result is reversed (33%), that is, with fixed parities the band collapses and with correction of the central parity there exists convergence. This happens in general when both the degree of integration and the parameter of adjustment are high.

Notwithstanding this, the results of the test in table 6 shows that the hypothesis of an equal probability of convergence cannot be rejected in this case, except when the comparison is done between the ERM case and CC with $p=0.2$. Therefore, allowing for a correction of the bands in general does not improve the probability of convergence.

II) The correction of the central parity allows to postpone the collapse time (in median) between 9 and 273 periods (for $p=0.9$, 1.1, respectively). As a matter of fact, when $p$ is low, there is an important number of subcases in which the maximum number of iterations is exceeded without collapse. Table 6 confirms that the collapse time is significantly postponed in time, even for low degrees of integration.

III) In a similar way, correction of the central parity also postpones the time of convergence, provided that the value of $p$ is not too high, as it can be inferred from the test in table 8.

IV) For high degrees of integration and fixed bands, the degree of convergence is higher when the central parity is corrected; for low $h$ and for narrowing bands, this result is reversed. Also, the dynamic honeymoon effect improves 1.46% (in median) when the central parity is corrected. These results are robust to statistical inference (not reported).

The overall conclusion is that correcting the central parity does improve the endurance of the target zone, because the misalignment is smaller; however, the performance in terms of convergence is worse: the probability of convergence does not improve and convergence arrives later. The interpretation for this unsatisfactory outcome is straightforward: the gains in terms of time that correcting the central parity provides are not enough to compensate the costs in terms of credibility that a smaller misalignment brings about. As a consequence, correcting the central parity is in general an adequate strategy only if the emphasis is placed on the sustainability of the target zone, not on convergence.
The second set of options to consider are either the bands fixed or allow for a hardening of the band as convergence progresses, that is, the ERM-CC and CW-CWC cases. We can mainly focus on the comparison between ERM and CW contained in tables 1 and 2, respectively, because the results do not differ greatly when correcting the central parity is considered. Close inspection of the tables reveals that:

i) with narrowing bands (CW) there are more cases in which convergence is attained and less cases of collapse for low and intermediate degrees of integration (h=.05-.3) but ERM performs better in both aspects when h is large. However, there is no significant difference in the probability of collapse (see table 6).

ii) From the tables of results and table 7, we observe that keeping the band fixed allows to postpone the collapse time for large values of h. On the contrary, for intermediate degrees of integration (h=.25-.30) narrowing the bands is shown to significantly postpone the collapse time (see two last rows in table 7).

iii) For all cases, convergence is achieved earlier when the bands narrow. This result is confirmed by the hypothesis testing in table 8 (see two last rows).

iv) Fixed bands imply higher degrees of convergence in the collapse subcases. The final drift, on the other hand, is smaller, too. Finally, the dynamic honeymoon effect is larger in the CW-CWC case.

Thus, narrowing the bands tends to dominate the choice of fixed bands. Only for large values of h, the ERM case seems to perform better in terms of sustainability, although convergence is achieved latter and the dynamic honeymoon effect is smaller.

Ranking the alternatives

The overall consideration of these results allows to rank the alternatives on the basis of two main criteria: convergence and sustainability. However, the main conclusion of our analysis is that, except for one case, the hypothesis that one choice increases the probability of convergence (or sustainability) is rejected. This implies that neither of the analysed choices improves the ability to converge and that the ranking has to be exclusively made in terms of speed of convergence or collapse, in which we can observe some differences among the options. According to the sustainability criterion (postponement in the collapse time) and the convergence criterion (anticipation of the convergence time), the options can be ranked:

<table>
<thead>
<tr>
<th>SUSTAINABILITY</th>
<th>CWC &gt; CC &gt; CW &gt; ERM (for intermediate h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CC &gt; CWC &gt; ERM &gt; CW (for large h)</td>
</tr>
</tbody>
</table>

| CONVERGENCE          | CW > ERM > CWC > CC                      |

150
Flexible management of the central parity improves the sustainability of the band. However, it is somehow striking that the most flexible alternative (CC) does not always rank first under the sustainability criterion. On the contrary, the most rigid option (CW) is the best choice under the convergence criterion and the most flexible (CC) ranks worst when the convergence time is considered.

The rationale for these results is again related to reputation considerations. In the case of a flexible central parity, reputation is only slowly accumulated. This slowness is decisive, because in this case, convergence is also slowly achieved. Under these circumstances, the first strains on the system, despite appearing later, may not be counteracted by high credibility of the target zone and collapse follows.

These rankings do not reveal an unique best alternative. Notwithstanding this, the emphasis of this chapter has been placed on the question of how a target zone can deliver convergence. It turns out that the misalignment and reputation necessary in this model to achieve convergence is best administered by a rigid target zone.

From this point of view, with all the cautions and caveats that will be considered in the final remarks, hardening the target zone bands appears as the best alternative while correcting the central parities seems to be the counterproductive, because reputation is acquired too slowly. However, the design of more flexible target zones should not be dismissed before taking into account other considerations, which cannot be properly analysed in the current framework.
IV-FINAL REMARKS & POLICY IMPLICATIONS

The validity of the previous analysis and its conclusions should be considered in the light of our approach, taking into account the limitations inherent in the model. Therefore, the results should be seen rather as an illustration of the effects of flexibility on a target zone rather than a recipe for policymaking.

In the introduction we listed the required features for a successful targeting strategy. Our specification conveys only some of these characteristics, while others, in particular, the policy coordination aspects cannot be included in our model. On the one hand, we only consider one country which pegs the currency to a foreign country, which acts as an anchor; this second country is not affected by the evolution of the domestic exchange rate, so that the latter is implicitly assumed to be a small economy. On the other hand, the policymaker is assumed to contemplate one policy objective: inflation convergence; there is no reference to real variables. Therefore, some crucial considerations are left aside. Some of them work in support of more flexibility and some others would advise more rigidity. Let us start with the latter.

The option of a rigid peg is supported on the basis of the reputation of exchange rate announcements, which is precisely the second source of inflation convergence that a peg may provide. We are already familiar with this argument: full convergence is achieved by pegging the exchange rate to the anchor currency if such an announcement is credible. In this case, the best option is the most rigid target zone.

However, as we know, acquiring credibility is not a free lunch. Giavazzi & Pagano (1988) show that the costs in terms of misalignment of non-disciplining in the face of a peg would provide the necessary credibility, but this commitment would only be slowly recognised by the markets and misalignment would appear anyway. This view emphasizes the real costs of misalignment. Viñals (1994), Eichengreen & Wyplosz (1993) etc., have recognized that this is an unsatisfactory feature of the system because inflation differentials are persistent. The first author suggests that more flexibility would increase the chances for further convergence.

There remains an open question then: which option is better, a rigid system which is not credible given the expected evolution of fundamentals, or a less tight regime which can enjoy credibility? The policy coordination literature (see references in next chapter) show that a
coordination equilibrium (attainable through an exchange rate peg) would be the worst option if reputation is absent, suggesting that the second alternative might be better. All these considerations have guided our quest for flexibility.

Giavazzi & Pagano rightly underline the importance of real variables in the determination of the credibility. The account of the ERM and the peseta crises above has highlighted the importance of real variables.

As a matter of fact, considering the existence of real targets in the implicit objective function of policymakers provides some additional arguments for the flexibility case. Correcting the central parity has been observed to reduce the deflationary effect of the target zones; this observation which conflicts with the achievement of credibility and convergence in our model is, on the contrary, a bonus if output is an additional target. In this case, losses in terms of output are reduced and this can improve the credibility of the regime, since it is perceived to be less costly to maintain.

The second caveat-the small country assumption-does not seem realistic in the case of the ERM and its relaxation further favours the flexibility option. If the anchor country is not immune to exchange rate movements of the domestic country, it will import inflation in the same way that the domestic country imports deflation. This raises doubts about the willingness of the anchor country to respect the parity. In this situation a correction of the central parity would relieve the inflationary pressure on the anchor country and the credibility of this bilateral band would increase.

There is a final consideration which further strengthens the case of flexible parities. The way in which our specification builds up reputation is at odds with the convergence play, whereby progresses in convergence are enough to strengthen the credibility of the system. In our model, the size of the misalignment is positively related to the reputation of the zone, which is defined in relative terms (see chapter three, section V); this implies that progress in convergence is, ceteris paribus, counterproductive for the reputation of the model. If the credibility mechanism was modified somehow to take into account this progress, the conclusions might be reversed.

All in all, taking into account some factors which are difficult to introduce in our framework would have positive implications for the robustness of the regime, beyond the formal analysis of the model.

In the first place, flexible central parities endorse more sustainable parities and therefore they are more easily justifiable and consequently, easier to defend. Secondly, the system is transparent because rules are automatically enforced; countries immediately perceive the costs
of irresponsible behaviour through a depreciation of their currency; this mechanism induces discipline and should increase the attraction of commitment for the domestic country, but also for the anchor country, which will not suffer from the indiscipline of the others. Finally, these elements should contribute to improving the chances of policy coordination, because the exchange rate regime around which they are implemented has more chance of being incentive compatible. In the next chapter, we explicitly analyse this case, albeit in a static model, and incentive compatibility will become a necessary condition to design the exchange rate targets.

Secondly, while ruling out the main source of instability, convergence bands do not guarantee the absence of speculative attacks, provoked by expected policy shifts or other types of shocks not foreseen when setting the bands. Nevertheless, the cooperation and coordination climate which the new system is expected to provide could help to overcome these crises. Furthermore, we should not forget that the ERM turmoil affected all currencies with a domino effect. First, the currencies with larger misalignments were attacked and after their success the speculators turned to other currencies in the system, including the healthy ones. Our system, in principle, would avoid this sort of contagion, because there is no scope for excessive misalignments to develop.

A final consideration concerns the feasibility of the system. We should be aware that technical difficulties could arise: how often to adjust, how to correct the deviations from expected inflation differentials, which body should determine the parities, how much scope to allow for discretion and how much for rules, how to channel the parities and so on. Besides there remains the question of the political feasibility of this option. While core countries might prefer it to the current ERM, the periphery might be reluctant to accept a system which imposes a visible cost on divergences. But they should also recognize the reputation gains that a good performance in this system would grant and the higher commitment from the core countries to defend the system. From this perspective, the goal to join the currency union would be perceived as costly but also as challenging, and this sense of challenge and the dangers of not facing it could stimulate the will to push ahead the necessary reforms.

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12 Obstfeld (1986,94) shows how expected shifts in policy can induce self-fulfilling attacks. In practice, the strains to which the French Franc was exposed in the recurrent crises of the ERM did not essentially reflect fundamental disequilibria, but rather the expectation that the French Government would eventually abandon its franc fort policy for the sake of a much needed reduction in the interest rates.
In this chapter we turn to consider explicitly real variables and how exchange rate targeting may be justified in a wider optimal policy coordination framework. We will also see that the design of the exchange rate arrangement will be seen to be equivalent to the convergence band proposal.

This chapter deals with the design of exchange rate targets and their use as stabilizing device in the face of economic shocks. The policymakers have to choose among a set of targeting strategies the subset which provides the best outcome according to their policy objectives. However, as we will see targeting the exchange rate may be counterproductive under certain circumstances.

In the last section of the chapter one, we reviewed the efforts to embed target zone models in an optimizing framework and judged positively the attempts to include real targets in the loss function, albeit in a unilateral target zone. In this chapter, we dispense with the target zone models we have been using up to now and adopt a static game-theoretic perspective which has been standard in the early policy coordination literature.

The assumptions are more realistic. The actions of policymakers spillover to other countries in interdependent economies; the existence of an arrangement between two symmetric countries is also more accurate for the European environment than the implicit small country assumption of target zone models\(^1\). Welfare considerations then determine the gains from coordination and, in our case, the virtues of a exchange rate arrangement.

\(^{1}\) Of course, the approach does not lack problems. A multicountry setting would complicate the results of the analysis, the underlying behaviour of the economy is stylized and dispensing with the restrictive assumption of a quadratic loss function would have generated many technical problems (see Currie & Levine (1991)). Even the use of different empirical models can provide contradictory results (Frankel & Rockett (1988)).
Players in the policy game only agree to follow a cooperative strategy if both gain with respect to the non-cooperative (Nash) outcome; in other words, the outcome from cooperation has to be incentive compatible.

Countries are considered on an equal strategic basis, in contrast with the targeting strategies traditionally contemplated in the literature, where leader-follower schemes have implicitly been used. Therefore, it will also be interesting to compare how the respective results differ.

The policy coordination literature has in general been more pessimistic than target zone models regarding the possibility of gaining from exchange rate pegs. The outcome can even be worse than the non-cooperative solution if reputation is not previously acquired. This result arises both in a static setting (Rogoff (1985a), Canzoneri & Henderson (1988), Currie et. al (1992)) and in dynamic games (Miller & Salmon (1985,1990), Levine & Currie (1987)).

A form of cooperation is implicit in our model through an exchange rate arrangement on which countries commit beforehand. The idea is similar to the optimal institutional contracts considered in the literature, albeit here both agents (countries) enjoy the same strategic position.

Incentive compatibility is the condition used to reach an agreement. Otherwise, private agents in the market would realize the incentive to renge that a return to a non-cooperative equilibrium offers. The system would not be credible and, as a consequence, it would be prone to instability and speculative attacks.

The optimal contract should also ideally deliver an efficient outcome. Since Pareto optimal solutions correspond to the cooperative solution in our static model, the exchange rate target would then act as a perfect surrogate for cooperation.

The feasibility of the arrangement is thus defined by two conditions: incentive compatibility and Pareto optimality. Note that although the optimality condition just improves the virtue of the arrangement, it is not necessarily required. Therefore, an incentive compatible target zone may turn out to be just a second best to explicit monetary policy coordination.

This instrumental role for the exchange rate makes it qualitatively different from the rest of targets in the economy. On the one hand, it implies a common target for both countries which is appended to the original optimization function of both policymakers; on the other hand, the exchange rate is an intermediate target to attain the actual policy.

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objectives but it has no value on its own right.

The search for an optimal targeting strategy is developed in two stages. In the first stage, we consider all the types of shocks which may hit the economy, making it diverge from its equilibrium level; then we explore the possibility of targeting the exchange rate for the different types of shocks. This is formally done by delimiting the type of shocks for which the exchange rate arrangement is incentive compatible and Pareto optimal. It turns out that the feasibility of targeting the exchange rate depends on the type of shock.

The second stage directly tackles the question of designing optimal exchange rate targets within the relevant shock subset. We will observe that there is some scope for the discretion of the policymakers who can choose between a wide range of exchange rate targets, provided that they also choose the optimal commitment to the selected target.

All these questions will be explored in more detail in this chapter. Next section presents a flexible framework to generate the set of targeting strategies. This allows to consider both nominal and real exchange rate targets. Section II describes the model used in this chapter, originally developed by Canzoneri & Henderson (1991). The optimal targeting strategy is derived from the concepts of incentive compatibility and Pareto optimality defined in the third section. Section IV and V develop the two stages to targeting. The conclusions summarise our results and compare them with alternative proposals of exchange rate management and to the recent experience and future prospects of the EMS.
We start by considering two identical and interdependent economies (subscripts \(1,2\)) and their loss functions \((L_1, L_2)\) defined so as to penalize the deviations of employment \(n\) and consumer prices \(q\) from their desired levels (superscript \(d\)). Deviations from the desired values imply welfare losses:

\[
L_1 = \frac{1}{2} \sigma (n_1 - n_d)^2 + (q_1 - q_d)^2 \\
L_2 = \frac{1}{2} \sigma (n_2 - n_d)^2 + (q_2 - q_d)^2
\]

The quadratic form and the arguments of these loss functions are standard in the policy coordination literature, pioneered by Hamada (1976). Penalty values are normalized with respect to the weight of inflation in the loss function.

Hamada adopts a game-theoretic approach to measure the gains from cooperative macroeconomic policy with respect to the non-cooperative solution. The outcome also determines implicitly the type of exchange rate management required.

Each country is a player in this policy game. The policy instruments are given by the money supplies. When countries do not cooperate, they use their respective money supplies to minimize welfare losses, taking the actions of the other player as given. On the contrary, when countries cooperate, a common welfare function is jointly minimised.

The implications for the exchange rates differ, depending on the features of the model employed. In Hamada’s deterministic environment, fixing the exchange rate mimics the efficient cooperative outcome; however, in a stochastic setting with rational expectations the conclusion is not so straightforward. In the Henderson model (1984), shifts in demand require the exchange rate to float while shifts in asset preferences advise the use of a peg; in the case of supply shocks, there is a conflict between the preferences of the players. The Canzoneri-Henderson (1991) model which we use here, highlights in the case of symmetric supply shocks- the need for a leader to determine the money supply and a follower who pegs the exchange rate, yielding an asymmetric arrangement. The
The diversity of results suggests that it is advisable to adopt a flexible approach to exchange rate targeting, in which the different alternatives are considered.

**Exchange rate targets as optimal contracts**

The ERM and Bretton Woods regimes targeted the nominal exchange rate around a narrow band while the Williamson (1985) proposal advocated maintaining the real exchange rate around a predetermined level; in this case, the band was loosely defined. The ‘Program for Monetary coordination’ proposed by McKinnon (1984) suggested an agreement on the global money supply to maintain exchange rates fixed. This global money supply had then to be allocated between countries, on the basis of a price stability target.

The central role of the exchange rate in the practice of policy coordination emphasises the value of exchange rate targets as surrogates for explicit policy coordination. One of the main reasons is observability. In the words of Kenen ‘...governments are prone to cheat and will not engage in optimal coordination because they cannot trust each other. A government cannot cheat on a firm commitment to exchange rate pegging without being caught. Therefore exchange rate pegging is viewed as a viable alternative to full-fledged coordination’ (Kenen (1989), p. 54).

Notwithstanding this, in the policy coordination literature the choice of the exchange rate usually appears as a by-product of the coordination solution and it is not explicitly considered. Nevertheless, a strand of this literature (see Hughes Hallet et. al (1989) and, in particular, Hughes Hallet (1993)) has explicitly included exchange rate targets in the policymakers optimization problem. We can justify this inclusion more formally using an analogy with the optimal contract literature, adapted to the context of policy coordination.

This literature has its origin in the credibility problem of monetary policy pointed out by Kydland & Prescott (1977) and Barro & Gordon (1983). Their contribution revealed the utility of setting up rules for the management of policy to which the monetary authority (principal) has to commit to cancel the inflationary bias in agents expectations. Barro and Gordon underlined the incentive to cheat on these rules, due to time inconsistency.
This incentive problem was addressed by Rogoff (1985b) which highlighted that it could be overcome if monetary policy was entrusted to the Central Bank through some kind of institutional provision regarding some intermediate target (optimal contract). If the commitment to the rule could be achieved, the incentive to renege would be eliminated.

In our case the contract is specified in terms of exchange rate targets, for the reasons of observability and controllability we have mentioned above. Two sovereign countries decide to commit to an exchange rate arrangement if they are expected to gain from it; that is, the incentive problem is redefined here in terms of incentive compatibility. Note that principal-agent considerations are no central here. The behaviour of the agents is guided from their rational expectations and both players are on an equal strategic footing. Reneging on the arrangement opens the possibility of retaliation by the second country and the suspension of an agreement which is beneficial for both countries. Therefore the incentive to renege is eliminated.

The contract is appended to the optimizing problem of the countries. Since the countries are equal from a strategic point of view, the contract is the same for both of them. The function which each country considers is then modified to become:

\[ W_1 = L_1 + V \]
\[ W_2 = L_2 + V \]

The contract is defined in terms of the real exchange rate (\(z\)) and enters in the optimizing problem with a weight \(\beta\). More precisely, we propose a contract of the following form:

\[ V = \frac{1}{2} \beta (z - z')^2 \]

At this point, it should be clear that the exchange rate is an intermediate target in the modified loss functions (\(W_1, W_2\)). It serves as a vehicle to attain the final goals represented by employment and inflation, but it is not a final goal in itself. Consequently, the gains from targeting are not directly derived from fulfilling the exchange rate goal but from the benefits on the rest of variables that respecting the agreed parity provides; therefore, in the evaluation of the gains, the deviations from \(V\) are not taken into account.

3 Later works refined these ideas and orientated the research to the design of institutional regulations. See Baron (1989), Walsh (1992), Lohman (1992) and Persson & Tabellini (1993)
The design of the targeting strategy

The contract is signed by the countries at the beginning of the period in order to minimise the welfare loss derived from unanticipated shocks. The question is then how to design a contract which is incentive compatible when a shock occurs. Note that the design of the contract is determined by the choice of a desired level for the real exchange rate and by its weight ($\beta$) in the modified loss function. Thus, the design of the optimal contract is equivalent to the choice of the optimal targeting strategy and we will use both terms interchangeably.

Following Rogoff (1985b) we can define the parameter or weight $\beta$ as the optimal degree of commitment to the intermediate target, in this case the exchange rate. The parameter $\beta$ is constrained to be positive, otherwise what is being targeted is the exchange rate to avoid. If $\beta$ were equal to zero, no constraint is imposed on the exchange rate and the result corresponds to the non-cooperative free-float solution.

Since positive values of $\beta$ penalize deviations from the desired values, it represents a soft band of fluctuation for the desired exchange rate target; the larger the value of $\beta$, the narrower will be the implied band. This specification allows us to think of the targeting strategy as a target zone with soft bands, where $z^\circ$ represents the central parity. As a matter of fact, the specification for the exchange rate target is also familiar to us, from the convergence bands proposal above.

How is the central parity chosen? In other words, how do players agree on the desired level for the real exchange rate? The exchange rate target in our model is inspired by the exchange rate arrangements mentioned above, especially on the nominal ERM regime and the Williamson proposal. The singularity of our approach is the consideration of a continuum of exchange rate targets, which spans between both alternatives. This setup allows for flexibility in the design of the contract; a feature which adds new insights to the question of exchange rate targeting.

The ERM and Williamson’s proposal have in common the choice of a target exchange rate which is allowed to fluctuate within a band. If rigidity is defined with respect to the equilibrium exchange rate in terms of Purchasing Power Parity, they represent the polar choices of a rigid system with ‘hard’ bands and a flexible regime with ‘soft’ bands, respectively. This suggests the use of an approach which incorporates both of these proposals as special cases, as well as the range of intermediate options, as we did in the convergence bands proposal. This idea can be formalised as follows.
Let us take the real exchange rate \( (z) \) identity, in terms of purchasing power parity:

\[ z = e^{-\left(p_1 - p_2\right)} \]

where \( p \) is the price level, the subscripts denote countries one and two respectively and the nominal exchange rate \( e \) is defined as the price of country 2 currency in terms of country 1 currency.

The ERM regime aims at maintaining a fixed nominal exchange rate parity, i.e. \( e^d = 0 \). Substituting above, we observe that this is equivalent to a real exchange rate target equal to the negative of price differentials:

\[ z^d = -(p_1 - p_2). \]

The Williamson target zone on the contrary implies a desired value for the real exchange rate equal to zero \( z^d = 0 \) or, equivalently, a depreciation of the nominal exchange rate equal to the price differentials:

\[ e^d = (p_1 - p_2). \]

Let us now define the parameter \( p \), such that \( e^d = (1-p)(p_1 - p_2) \). Note that \( (1-p) \) represents then the degree which the nominal exchange rate target offsets price differentials. Thus, we can write the exchange rate target in general form as a function of the price differentials

\[ z^d = e^d - (p_1 - p_2) = -p(p_1 - p_2) \quad (2) \]

It immediately follows that \( p = 1 \) corresponds to a nominal exchange rate target, as in the ERM regime and \( p = 0 \) corresponds to the Williamson target zone, that is, a real exchange rate target. As \( p \) moves away from zero the desired exchange rate partially accommodates inflation differentials. These intermediate values present special interest because they provide flexibility in the choice of exchange rate target, according to the preferences of policymakers.

We can observe that the design of the targeting strategy is determined by the choice of just two parameters: \( p \) and \( \beta \). The first specifies the extent to which the real exchange rate is the target and the second determines the degree of commitment to such a target. We will refer to them as target parameters. The range of parameters is constrained to positive values of \( \beta \) and to values of \( p \) between zero and one, i.e. between real and nominal exchange rate targets.

Countries can choose among a wide combination of exchange rate targets and values for \( \beta \), which represent the set of targeting strategies. This set, denoted as \( \lambda \) can be formally defined as:

\[ \lambda = (\beta; X_p), \forall \beta > 0, 0 \leq p \leq 1 \]

In this way we have a very simple and general method to explore whether
targeting the exchange rate pays when countries are placed on an equal strategic footing. When the answer is in the affirmative, our specification will be able to determine which is the optimal targeting strategy within the set. Prior to this, we have still to develop the economic model of reference and to define the concepts which will be used in the formal analysis. This is done in the two next sections.
In this section we present the Canzoneri & Henderson (1988,1991) two-country model of symmetric monetary economies. For a more detailed explanation of the model we refer the reader to their paper.

This economy is subject to shocks on the demand \((u_1,u_2)\) and the supply side \((x_1,x_2)\). Rational expectations are assumed, so that only unanticipated shocks can affect equilibrium. The disaggregation of shocks and the treatment of the exchange rates introduce some minor modifications into the Canzoneri-Henderson model.

Two countries are considered, each producing a different good with the corresponding subscripts. All the variables except the interest rates are expressed in logs and represent deviations of actual values from equilibrium. Symmetry holds in the strong sense that all the parameters are the same in both countries (for the exchanges rates they have opposite signs).

The output of each country \((y_i, i=1,2\) where the subscript \(i\) refers to the variable in each country) is obtained through a Cobb-Douglas production function. It is an increasing function of domestic employment \(n\), and it decreases when some adverse supply shock \(x'\) hits the economy:

\[
y_1 = (1-\alpha)n_1-x'_1, \quad y_2 = (1-\alpha)n_2-x'_2.
\]

For convenience we present in the table in next page the values and definitions of the parameters appearing in the model and those derived from the transformations which follow.

Firms hire labour up to the point in which real wages equal the marginal product of labour:

\[
w_1-p_1 = \alpha n_1-x'_1, \quad w_2-p_2 = \alpha n_2-x'_2.
\]
where $w_t$ and $p_t$ are nominal wages and prices, respectively. We can assume that the supply shocks take the form of adverse labour productivity shocks. Contracts are signed at the beginning of each period, so that shocks are unanticipated. These contracts specify nominal wages and employment rules and workers agree to supply whatever quantity of labour firms want at the nominal wage specified in the contracts.

Consumer price indexes $q_t$ are weighted averages of domestic and foreign goods prices:

$$q_1 = (1-\xi)p_1 + \xi(\theta + p_2) = p_1 + \xi \theta$$

$$q_2 = (1-\xi)p_2 - \xi(\theta + p_2) = p_2 - \xi \theta$$

The market equilibrium conditions for the demands of goods are:

$$y_1 = 2 \xi (1-\xi) \epsilon \rho + \xi \epsilon \rho_2 - (1-\xi) \theta \rho_1 - 1 + \beta \rho_1$$

$$y_2 = -2 \xi \epsilon \rho + \xi (1-\xi) \epsilon \rho_2 - (1-\xi) \theta \rho_1 - 1 + \beta \rho_1$$

where $r_1$ are the real interest rates and $\theta$ stand for positive demand shocks. Uncovered interest parity holds, so that $r_1 - r_2 = \pi^* - \pi$. The superscript stands for expected value.

Finally, the equilibrium in the money market is given by the Cambridge equations:

$$m_1 = p_1 + y_1$$

$$m_2 = p_2 + y_2$$

Nominal wages are set as follows. From the output and the wages equations

<table>
<thead>
<tr>
<th>Par.</th>
<th>Definition</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1-\alpha$</td>
<td>Labour coefficient in the production function</td>
<td>$0 &lt; 1-\alpha &lt; 1$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Marginal propensity to spend</td>
<td>$0 &lt; \epsilon &lt; 1$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Effect of exchange rate in demand</td>
<td>$0 &lt; \delta &lt; 1$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Interest rate elasticity</td>
<td>$0 &lt; \nu &lt; 1$</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Share of import goods in domestic basket</td>
<td>$0 &lt; \zeta &lt; \frac{1}{2}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$\gamma = 2 \delta + (1-2 \zeta) \epsilon \nu_{1}'$</td>
<td>$0 &lt; \gamma &lt; 1$</td>
</tr>
<tr>
<td>$\zeta'$</td>
<td>$\zeta' = (1-2 \zeta) \epsilon$</td>
<td>$0 &lt; \zeta' &lt; 1$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$\phi = \zeta' \gamma (1-\alpha)$</td>
<td>$0 &lt; \phi &lt; \zeta$</td>
</tr>
<tr>
<td>$v_\eta$</td>
<td>$v_\eta = (\alpha + \gamma) \nu_{1}'$</td>
<td>$0 &lt; v_\eta &lt; 1$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$\theta = \frac{1}{2} v_\eta \phi$</td>
<td>$0 &lt; \theta &lt; \frac{1}{2}$</td>
</tr>
</tbody>
</table>
above, and using the money market equilibrium equations, employment can be expressed as a function of the money supplies and nominal wages.

\[ n_1 = m_1 - w_1 \]
\[ n_2 = m_2 - w_2 \]

Firms and workers choose the nominal wage that minimises the expected square deviations of employments from the full-employment value, set equal to zero. Optimizing the square of the expression above, we observe that the respective nominal wages are set equal to the expected money supplies:

\[ \frac{\partial(n_1^2)^*}{\partial w_1} = w_1 - m_1^* = 0 ; \quad \frac{\partial(n_2^2)^*}{\partial w_2} = w_2 - m_2^* = 0 \]

The reduced forms are obtained by expressing all the variables of interest in terms of the instruments and the shocks. The expressions for the employment are straightforward:

\[ n_1 = m_1 - m_1^* \]
\[ n_2 = m_2 - m_2^* \]

The expressions for the consumer price indexes require several steps. First, we solve for the output prices and then for the exchange rates. The output prices expression is straightforward. Substituting the equilibrium values for nominal wages and the expression for employment in the product prices, we can express them in reduced form as:

\[ p_1 = m_1^* (\alpha - 1)(m_1 - m_1^*) + x_1^* \]
\[ p_2 = m_2^* (\alpha - 1)(m_2 - m_2^*) + x_2^* \]

To obtain the reduced form for the real exchange rate, we note first that the sum of excess demands must be zero. Substracting the expressions for the demand for goods, we obtain

\[ -\zeta (y_1 - y_2) + 2\delta - (1 - 2\zeta )\nu (r_1 - r_2) + u_1' - u_2' = 0 \]

Using the equations for the real exchange rate and the uncovered interest parity, the real interest differential from this expression is given by:

\[ r_1 - r_2 = (1 - 2\zeta ) (z^2 - z) \]

Substituting this value above and eliminating \( y_1 - y_2 \) with the help of the output expressions, we obtain the reduced form for the real exchange rate:

\[ z = \zeta (1 - \alpha) ((m_1 - m_1^*) - (m_2 - m_2^*)) \cdot (1 - 2\zeta )\nu z^* \cdot (u_1' - u_2') \]

In this expression, we can reasonably assume that the expected real exchange rate is equal to zero \((z^2 = 0)\). On the one hand, the expected values of the disturbances is zero; on the other hand, since nominal wages and prices are perfectly flexible, the expected real exchange rate is independent of today’s money supply. Finally, the existence of an exchange rate target does not influence ‘a priori’ exchange rate
expectations, because they are only binding in the case of shocks.

The nominal exchange rate reduced form is obtained from the real exchange rate expression. Substituting in the reduced forms for product prices and the real exchange rate, we obtain

$$e = (m_1 - m_2) \times (y - 1)(1 - \alpha)((m_1 - m_1) - (m_2 - m_2)) - \chi(u_1 - u_2)$$

Finally, the expression for the consumer price indexes is obtained by direct substitution:

$$q_1 = m_1 + (\phi + \alpha - 1)(m_1 - m_1) - (m_2 - m_1) + x_1 \cdot - \chi(u_1 - u_2)$$
$$q_2 = m_2 - \phi(m_1 - m_1) + (\phi + \alpha - 1)(m_2 - m_2) + x_2 \cdot - \chi(u_1 - u_2)$$

Finally, in the appendix is shown that the expected money supplies are zero. The shocks are redefined as follows

$$x_i = \sqrt{n}x_i, \quad u_i = \sqrt{n}u_i, \quad i = 1, 2$$

and the reduced forms for the variables of interest take the following form:

$$n_1 = m_1, \quad n_2 = m_2$$
$$q_1 = (\sqrt{n})^{-1}(m_1 - 2\theta m_2) \cdot x_i - (u_1 - u_2)$$
$$q_2 = (\sqrt{n})^{-1}(m_2 - 2\theta m_1) \cdot x_j - (u_1 - u_2)$$

The implications of the model are standard. A domestic monetary expansion positively affects domestic employment and prices and negatively affects foreign prices; adverse supply shocks ($x_i > 0$) reduce domestic output and increase domestic consumer prices through the increase in the price of the domestic good. Positive demand shocks in country one ($u_i > 0$) push domestic consumer prices down because of the appreciation of the nominal exchange rate induced by the excess demand for the domestic good. More precisely, the reduced forms for the exchange rates are:

$$e = (\alpha + \chi(1 - \alpha))(m_1 - m_2) \cdot \sqrt{\eta}^{-1}(x_1 - x_2) - (\sqrt{\eta}^{-1})^1(u_1 - u_2)$$
$$z = \chi(1 - \alpha)(m_1 - m_2) - (\sqrt{\eta}^{-1})^1(u_1 - u_2)$$

In this model, we can see that the nominal exchange rate is affected by both demand and supply shocks, while the real exchange rate is only affected by demand shocks.

The excess demand created by demand shocks require a correction of relative prices and consequently of the real exchange rate. Since the price of the good is not
affected, the burden of adjustment falls upon the nominal exchange rate. On the contrary, a domestic adverse supply shock does not affect the real exchange rate. The reason is that the resulting nominal exchange rate depreciation in this particular model offsets the price differential in the real exchange rate expression.

It is also convenient to write the real exchange rate target derived from the correction mechanism derived in the previous sections. Substituting terms we obtain:

$$z^d = p(\sigma(m_1 - m_2)x(x_1 - x_2))$$

From (4), it is evident that supply shocks would not affect the desired exchange rate and in this case $p=0$, others things being equal.

However, shocks also affect the policy goals and countries react to them using their policy instruments to stabilize the economy around the desired levels. For instance, in the face of the asymmetric demand shock considered above, the domestic country has an incentive to inflate and the foreign country an incentive to disinflate. The resulting exchange rate is the result of both influences and consequently the desired exchange rate value (determined by the choice of $p$) has to be chosen so as to induce the optimal response of both countries, to stabilise the economy for any given shocks. This is the issue to be explored in the remaining part of this chapter.

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Incentive compatibility and sustainability

In this section we define the conditions to derive optimal targeting strategies. They are encompassed by the requirements of sustainability and optimality. However, before reaching a formal definition we need to make use of other related concepts which are now presented.

The derivation of the model allows us to substitute the reduced forms (4.5) in the modified loss functions. Assuming that the desired inflation and unemployment levels are set equal to zero ($n^d=0$, $q^d=0, i=1,2$), the expression to minimize turns out to be just a function of the instruments ($m_1, m_2$) and the different shocks.

\[
W_1 = \frac{1}{2} \sigma (m_1)^2 + (m_1 - 20m_2 + x_1 - (u_1 - u_2))^2 + \\
+ \beta (\sqrt{p} (m_1 - m_2) + \frac{p}{\eta} (x_1 - x_2) - \frac{1}{\xi \eta} (u_1 - u_2))^2)
\]

\[
W_2 = \frac{1}{2} \sigma (m_2)^2 + (m_2 - 20m_1 + x_2 + (u_1 - u_2))^2 + \\
+ \beta (\sqrt{p'} (m_1 - m_2) + \frac{p'}{\eta} (x_1 - x_2) - \frac{1}{\xi \eta} (u_1 - u_2))^2)
\]

where $\sqrt{p'} = \rho \alpha + \sigma \gamma (1 - \alpha)

The introduction of a exchange rate target in the optimization problem implies that the exchange rate target acts as an indirect cooperation device. It is indirect because the optimization problem facing each country is equivalent to the non-cooperative case: each country maximizes its welfare with respect to its instrument, taking as given the actions of the foreign country, but also taking into account the common exchange rate:

\[
\frac{\partial W_1}{\partial m_1} = 0 \Rightarrow R_1: m_1 = \psi_1 (\psi_2 m_2 - (1 + p) \psi_3 x_1 + p \psi_4 x_2 + \psi_5 (u_1 - u_2))
\]

\[
\frac{\partial W_2}{\partial m_2} = 0 \Rightarrow R_2: m_2 = \psi_1 (\psi_2 m_1 - (1 + p) \psi_3 x_2 + p \psi_4 x_1 - \psi_5 (u_1 - u_2))
\]
where

\[ \psi_1 = 1 - \sigma - \beta \rho' \quad \psi_2 = 2 \theta + \beta \rho' \quad \psi_3 = \frac{\beta \sqrt{\rho'}}{\sqrt{\eta}} \quad \psi_4 = 1 - \frac{\psi_3}{2} \]

The points where the reaction functions \( R_1, R_2 \) intersect represent the solutions. The reaction functions and the rest of concepts defined in this section are represented in figures 1 and 2, below. We operate in the \((m_1, m_2)\) space, where the ellipses are isoclines of the loss functions and the reaction functions are represented by straight lines.

The solutions depend on the parameters of the model, the shocks and the values of the target parameters; while the two former are taken as given, the latter are chosen by the authorities in some optimal way to be defined.

The choice of the target parameters in the strategy set induce a set of targeting equilibria which can be specified as a function of \( \lambda \), \( T(\lambda) = (m_1^T, m_2^T) \), where for convenience, here we redefine \( \lambda \) as the product \( p \rho \).

\[ m_1^T = \frac{-\psi_1 x_1^2 + \psi_2 x_2^2 + \psi_3 (x_1 - x_2) + \psi_4 (x_1-x_2)(u_1-u_2)}{\psi_1^2 - \psi_2^2} \]

\[ m_2^T = \frac{-\psi_3 x_1 x_2 - \psi_4 (x_1-x_2)(u_1-u_2)}{\psi_1^2 - \psi_2^2} \]

It is important to explore the characteristics of the targeting solutions because they will be crucial to derive the next propositions. We show in the appendix that the set of targeting solutions is a straight line (target line, hereafter); More formally, it is claimed that:

**Proposition 1**: The set of targeting equilibria, \( T(\lambda) \), is contained in a straight line with slope equal to -1 in the \((m_1, m_2)\) space, where:

\[ T(\lambda) = \{ (m_1^T, m_2^T) \mid m_2^T = -\frac{x_1 x_2}{1 - \sigma - 2 \theta} - m_1^T \} \]

One particular point of this line refers to the case in which the exchange rate is not targeted, i.e. \( \beta = 0, \forall \rho \). This is of course the Nash non-cooperative solution \( N = T(0) = (m_1^*, m_2^*) \) of our model\(^4\). In this case, where \( \psi_1 = 1 + \alpha, \psi_2 = 2 \theta, \psi_3 = 0, \psi_4 = 1, \) it is straightforward to see by direct substitution into (9) that:

---

\(^4\) See Canzoneri & Henderson (1991, pgs.21 and 37). They only consider symmetric supply shocks \( x_1 = x_2 \) and opposite demand shocks \( u_1/2 = u_2/2 \). Substituting in (11) we obtain the same expressions as theirs.
Part II-Design

5-Optimal exchange rate targets

\[ m_i \mid_{\theta=0} = \frac{(1+\sigma)x_i + 2\theta x_i - (1-\sigma - 2\theta)(u_i - u_j)}{(1+\sigma)^2 - (2\theta)^2} \]

\[ m_j \mid_{\theta=0} = \frac{(1+\sigma)x_j - 2\theta x_j + (1-\sigma - 2\theta)(u_i - u_j)}{(1+\sigma)^2 - (2\theta)^2} \]

The set of incentive compatible solutions or bargaining area (A) is defined as the set of points which are Pareto superior to the Nash solution:

\[ A \{ \forall m_1, m_2 \mid W_1(m_1, m_2) \leq W_1(m_1^*, m_2^*) , W_2(m_1, m_2) \leq W_2(m_1^*, m_2^*) \} \]

The explicit cooperative solution is obtained by the minimization of the weighted joint loss function \( W^C \), where \( \delta, 1-\delta \) are the weights assigned to each country:

\[ W^C = \delta W_1 + (1-\delta) W_2 \]

The contract curve \( C(\delta) \) is obtained by minimizing this loss function with respect to each instrument, and setting the rate of marginal substitution equal to minus one to each instrument, and setting the rate of marginal substitution equal to minus one.

\[ \frac{\partial W^C}{\partial m_i} - \frac{\partial W^C}{\partial m_j} \Rightarrow \]

\[ \delta \Sigma m_1 - 2\theta m_2 = (1-\delta)2\theta x_j - (\delta + (1-\delta)2\theta)(u_i - u_j) = 0 \]

\[ = (1-\delta)(1+\sigma) + (1-\delta)(1+\sigma) \]

where \( \Sigma = \delta(1+\sigma) + (1-\delta)(1+\sigma) + (1-\delta)(1+\sigma) \)

Setting \( \delta = 1, (\delta = 0) \), the instruments of both countries are chosen to maximize the welfare of the first (second) country. These solutions are known as bliss points \( (B_1, B_2) \)

\[ C(1) = B_1, I_{\delta=1} = (m_1^*, m_2^*) = (0, x/20, (u_1 - u_2)) \]

\[ C(0) = B_2, I_{\delta=0} = (m_1^*, m_2^*) = (x/20, (u_1 - u_2)) \]

Since the bliss points correspond to zero welfare losses, \( B_1, B_2 \) are placed at the center of the respective ellipses.

Finally, it is convenient to define the segment which joins the bliss points as the bliss line \( (R) \). Taking the expressions for the bliss points, it is straightforward to derive the following segment:

\[ B : (m_1, m_2) \in (R) \mid m_2 = \frac{x_1 + u_i - u_j}{2} + bm_1 \]

where \( b = \frac{x_1}{x_2}, \forall x \neq 0; b = 1, \forall u \neq 0, i = 1, 2 \)
Incentive compatible and optimal targets

The only novel concept among the above definitions is the set of targeting equilibria derived from the exchange rate arrangement. Since we have shown that this set is embedded in the target line, we can conveniently make use of the latter to specify the sustainability and optimality of the targeting strategy.

Recall that, in general, incentive compatible strategies must be Pareto superior to the Nash solution, that is, they have to belong to the bargaining area. Therefore a target strategy will be incentive compatible if it lies on the target line that passes through the bargaining area:

\[
\text{SUSTAINABILITY CONDITION} \quad (14) \\
\exists \lambda \mid T(\lambda) \cap A \neq \emptyset
\]

Similarly Pareto optimal strategies are placed on the contract curve of cooperative solutions \( C(\delta) \). Therefore the targeting strategy will be Pareto optimal when the target line intersects the contract curve:

\[
\text{OPTIMALITY CONDITION} \quad (15) \\
\exists \lambda, \delta \mid T(\lambda) = C(\delta)
\]

Countries aim at designing contracts that are incentive compatible and Pareto optimal at the same time. However, it could be the case that there is no possible means to design a Pareto optimal strategy which is sustainable or, vice versa, an incentive compatible strategy which is Pareto optimal.

Note that the first situation would preclude the arrangement; on the contrary, an incentive compatible strategy which is not Pareto optimal could be accepted because incentive compatibility is a sufficient condition. Nevertheless in this latter case, it should be convenient to contemplate some complementary criterion (suboptimality criterion) to achieve the most efficient targeting strategy which is sustainable; for instance, the sustainable targeting equilibrium which minimizes the distance to the contract curve point.

Considering both conditions jointly and taking into account the suboptimality criterion, the optimal targeting strategy (or optimal contract) can be defined as follows:

\[
\lambda^* \mid T(\lambda^*) = (m_1^*, m_2^*) \\
(m_1^*, m_2^*) = T(\lambda^*) \cap C(\delta^*), \quad \text{if} \quad T(\lambda) \cap C(\delta) \in A
\]

or, in the case that the Pareto optimal criterion cannot be fulfilled:

\[
(m_1^*, m_2^*) = (m_1^*, m_2^*) \in A \mid \min(d_{\text{dist}}(A, C(\delta))) \quad \text{if} \quad T(\lambda) \cap C(\delta) \notin A
\]
IV-Economic shocks and optimal targets

Supply and demand shocks have different effects on welfare as an inspection of (6.7) reveals. Consequently, the scope for targeting may crucially depend on the type and magnitude of the shock hitting the economy. In this section we formally analyse the possibility of designing an optimal targeting strategy for the two different types of shocks we are considering. The results take the form of propositions whose proofs appear in the appendix.

We will apply the above conditions when the shocks arise on the demand side and on the supply side: first, we will explore incentive compatibility, then optimality and finally the joint hypothesis.

It is important to keep in mind that in this section we focus on the whole target line, without any constraint on the target parameters. However, in the next section we will restrict the relevant subset of target strategies, which corresponds to positive values of $\beta$ and $\rho$ between zero and one\textsuperscript{5}.

Figures 1 and 2 give a flavour of the results in this section. They represent the concepts defined in the previous section in the $(m_1,m_2)$ space when different types of shocks hit the economy. We consider the case of symmetric ($x_1=x_2$), idiosyncratic ($x_1>x_2>0,u_1>u_2>0$) and opposite ($x_1=-x_2,u_1=-u_2$) shocks. From the observation of the reduced forms (4.5), we can see that symmetric demand shocks ($u_1-u_2$) have no effect whatsoever on the economy.

---

\textsuperscript{5} Therefore, strictly speaking the propositions below are necessary but not sufficient conditions for targeting the exchange rate appropriately. Only when the value of $\beta$ of the targeting parameters are specified, the propositions will be completed. In the next section we will focus in more detail on the conditions for which $\beta$ is positive when $\rho$ is between zero and one. Nevertheless, we can anticipate that for demand shocks positive $\beta$ can always be found, while for supply shocks the sufficient condition does not apply in all the cases.
Effect of demand shocks. Asymmetric and idiosyncratic shocks

Figure 1 - Strategic behaviour and demand shocks

Effect of supply shocks. Opposite shocks

Figure 2a - Strategic behaviour and opposite supply shocks
Effect of supply shocks. Idiosyncratic shock

Figure 2.b-Strategic behaviour and idiosyncratic supply shocks

Effect of supply shocks. Symmetric shock

Figure 2.c-Strategic behaviour and symmetric supply shocks
Demand shocks

Figure 1 suggests the conclusions of the formal analysis below. An asymmetric demand shock can be interpreted as a shift in demand from one to another country (from country one to country two if \( u_1 \) is positive) and an idiosyncratic shock may be the result of an expansionary fiscal policy in any of the countries.

The plot shows that the target line crosses the bargaining area (A) and the contract curve (C(\( \delta \))) for both asymmetric and idiosyncratic shocks. This suggests that an incentive compatible and Pareto optimal targeting strategy can be specified. This intuition is confirmed by the formal analysis of the model, which is carried out in the appendix. The results can be summed up in the following three propositions:

Proposition 2: For demand shocks of any type or magnitude, there exists at least one exchange rate arrangement which is incentive compatible.

Proposition 3: For demand shocks of any type or magnitude, there exists a Pareto optimal exchange rate arrangement.

Proposition 4: For demand shocks of any type or magnitude, the Pareto optimal exchange rate arrangement is incentive compatible and corresponds to the point \( C(\frac{1}{2}) \), where

\[
C(\frac{1}{2}) = (m_1^*, m_2^*) \in \mathcal{T}(\lambda)
\]

\[
m_1^* = \frac{(1+2\delta)(u_1-u_2)}{\sigma(1+2\delta)^2} = -m_2^*
\]

The two first results are linked by proposition four which shows that whatever the shock, the targeting equilibrium which intersects the contract curve belongs to the bargaining area, so that the Pareto optimal equilibrium is also incentive compatible. Therefore demand shocks present no problem for the design of optimal targeting strategies.

Supply shocks

The question is not as straightforward as in the case of supply shocks. The different plots of figure 2.a-c display a quite different picture: only in the case of asymmetric supply shocks, does the target line cross A and C(\( \delta \)).

We can interpret an asymmetric supply shock as a shift in productivity from one
country to another, due, for instance to the transfer of an industry from one country to
another. More standard are the other two cases. A symmetric supply disturbance may be
due to price shocks on raw materials used to produce both goods (typically an oil shock);
if the shock affects only the production of one good, the case of an idiosyncratic supply
shock would arise.

The mathematical analysis of the appendix shows that the case for targeting is not
general in the case of supply shocks. As before, we can convey the result in three
propositions:

**Proposition 5:** For supply shocks at least one incentive compatible exchange rate
target will exist if \(-1+\alpha)x/2\theta \leq x \leq -2\theta x/1+\alpha\), \(\forall x > 0\), \(i,j=1,2\) is.

**Proposition 6:** For supply shocks there exists an optimal exchange rate target only
if \(-1+\alpha)x/2\theta \leq x \leq -2\theta x/1+\alpha\), \(\forall x > 0\), \(i,j=1,2\) is.

**Proposition 7:** For opposite supply shocks \((x_j=-x_j)\), the Pareto optimal exchange rate
arrangement is incentive compatible and corresponds to the point \(C(1/2)\):

\[
\forall x_1=-x_2 \Rightarrow C(1/2) = (m_1^*, m_2^*) \in T(\xi)
\]

\[
m_1^* = \frac{(1+2\theta)(2x_1)}{(\alpha + (1+2\theta)^2)} = -m_2^*
\]

From propositions five and six, it follows that the existence of sustainable or Pareto
optimal targeting strategies is restricted to a range of shocks determined by the values of
the rest of the parameters of the model. The parameter \(\theta\) is positive and less than \(\frac{1}{2}\) (see
table) and \(\alpha\) is positive. Therefore,

\[-(1+\alpha)/2\theta < -1, -2\theta /1+\alpha > 1\]

and consequently \(x_1=-x_2\) always belongs to this range. The implications of this result are the
following:

- Idiosyncratic and symmetric supply shocks (figures 2.b,c) preclude the design of
  incentive compatible or Pareto optimal targeting strategies.

- A necessary condition for the exchange rate arrangement to be incentive
  compatible is that the shocks are of opposite sign.

- In particular, opposite shocks (figure 2.a) allow for incentive compatible targeting
  strategies, for a range of parameters to be determined in the next section.

Furthermore, proposition seven states that only in the case of opposite supply
shocks is the existence of a incentive compatible strategy which is Pareto optimal
attainable.
For the rest of the cases in the relevant range we have had to proceed by numerical simulation. The outcome depends on the values of $\sigma$ and $\theta$. In particular, for values close to the extremes of the range the Pareto optimal point does not fall within the bargaining area. In any case, for these latter situations, the suboptimality criterion indicated above (see expression (17)) can be applied and the incentive compatible targeting equilibrium which minimizes the distance to the contract curve point would be chosen.

Interpretation of the results

Note that in our model the cases for which a targeting strategy is feasible have some features in common. Comparison of figure 1 (demand shocks) with figure 2.c (opposite supply shocks) actually reveals an equivalent outcome in graphical terms. In particular, note that in both cases the equilibria are found on the quadrants where monetary supplies have different signs; of course, if the sign of the shocks is reversed, the solution would appear in the same quadrant. Note that in the quadrants where money supplies have different signs, that is, when one country reacts by expanding and the other by contracting the money supply in the face of a shock, there may be scope for an optimal targeting strategy. This suggests the following claim:

Proposition 8: The necessary condition for the existence of an incentive compatible or Pareto optimal exchange rate arrangement is that, for any type of shocks

$$\text{sign}(m^1_n) \neq \text{sign}(m^2_n)$$

This conjecture is proved in the appendix. The economic significance of this result is quite clear: when the reaction of one country exerts a positive externality on the other country there is scope for an optimal exchange rate strategy. Regarding the exchange rate, we can observe that both countries agree on the direction of change for the exchange rate; the wish of country one to inflate in order to depreciate the exchange rate can be alternatively satisfied with a deflation of the second country.

The magnitude of the desired exchange rate change is what differs between countries; for instance, in the case of a positive global demand shock ($u_1, u_2 > 0$), the individual effort to depreciate the exchange rate is too cautious when countries act non-cooperatively and it will result in an insufficient depreciation, yielding an inefficient outcome. However, the joint effort to depreciate derived from the targeting strategy will induce an optimal outcome.
Note that this implies a more activist role for monetary policy derived from the targeting strategy. Comparing the expressions for the Nash solution in (11) and the optimal targeting strategy for demand and opposite supply shocks (expressions (17) and (18) respectively), we can express the latter as a function of the Nash solution; it turns out that:

\[
1 \frac{m_i^N}{1 + \sigma + 2\theta} < 1 \frac{m_i^*}{1 + \sigma + 2\theta} < 2 x_i^* + u_i - u_j \]

This result shows that the optimal strategy will always imply a larger change in the money supply, both for the expansionary and the deflationary country. This result is confirmed by the graphical analysis where we can observe that, in the relevant figures, the optimal solution is more distant from the origin than the Nash solution.

Finally, it is interesting to point out that the slope of the target line implies that the global money supply does not change when the solution shifts from the Nash to the targeting equilibrium. More formally, the expression for the target line in (10) shows that at the targeting equilibrium the global money supply remains constant and equal to the Nash solution: for supply shocks

\[
m_1 + m_2 = (x_1 + x_2)/(1 + \sigma - 2\theta)
\]

and for demand shocks the global money supply is simply zero.

Thus, the effect of the targeting strategy is to allocate more efficiently a given global money supply than in a non-cooperative situation, which is a conclusion similar to McKinnon’s (1984,1988) proposal for Monetary Stabilization.
V-The design of optimal targeting strategies

Up to now, we have shown the feasibility of designing optimal targeting strategies. Once the set of possible shocks has been determined, attention can be shifted to the more concrete issue of designing optimal contracts.

As a matter of fact, the analysis of the previous section is incomplete unless the characteristics of the target line in the bargaining area are determined. The position on the target line depends on the target parameters $\beta, \rho$. In principle, they are independent parameters which are chosen at the discretion of policymakers, but the range of values that each of them can take is constrained. The parameter $\rho$, which determines the desired exchange rate target, is bounded between real and nominal exchange rate targets and $\beta$, the optimal commitment to the desired exchange rate target, must be positive. Thus, the feasible range for the parameters are:

$$0 < \rho \leq 1; \quad \beta > 0$$

We will first explore the conditions to obtain a positive degree of commitment, for the considered range of exchange rate targets and then we will specify the relationship between them.

Let us recall first that the quadratic optimization solution minimizes deviations from the desired targets; hence, when $\beta$ is positive, it is obvious that the new solution must reduce the deviation from the now binding exchange rate target relative to the Nash solution, where the exchange rate is not targeted.

Therefore, comparing the effects of a particular exchange rate target at the optimal solution with the Nash solution determines the existence of a positive value for $\beta$. More precisely if

$$|z - z^*| > |z - z^*| \Rightarrow \beta > 0$$

and vice versa. The appendix shows that

---

6-From (5), the exchange rates and the targets are defined with respect to one country. Countries one and two have then to be labelled, depending on the type of shocks, such that a nominal exchange rate target correspond to $\rho = 1$, not to $\rho = 1$ (the nominal exchange rate target for country two).
Proposition 9: In the case of demand shocks, the degree of commitment is positive for both nominal and real exchange rate targets, that is,
\[ \forall 0 \leq p \leq 1 \Rightarrow |(z - z^0)^n| > |(z - z^0)| \Rightarrow \beta > 0 \]  \hfill (22)

Proposition 10: In the case of opposite supply shocks, targeting the real exchange rate is counterproductive \((p = 0 \text{ implies } \beta < 0)\), while a nominal exchange rate target may be appropriate. More precisely
\[ \forall p \geq p^* = \frac{2 \theta}{1 + 2 \theta + \frac{\sigma}{1 - 2 \theta} - \zeta \sqrt{n}} > 0 \Rightarrow |(z - z^0)^n| > |(z - z^0)| \Rightarrow \beta > 0 \]  \hfill (23)

Thus, for demand shocks the existence of a feasible targeting strategy is confirmed, while existence is not clear for opposite supply shocks because \(p^*\) may indeed be larger than one. Note that expression (23) imposes an additional condition on the existence of an optimal targeting strategy (with positive \(\beta\)) in addition to the conditions derived in the previous section. Targeting the real exchange rate \((p = 0)\) is therefore discarded for supply shocks, because \(p^*\) is strictly positive.

Inspection of the exchange rate expressions in (5) may help to explain this result. Since the optimal target implies a more activist monetary policy and supply shocks do not affect the real exchange rate, deviations from \(z\) are actually maximized, rather than minimized if the real exchange rate is targeted, implying a negative \(\beta\). This is not the case for the nominal exchange rate, which depreciates in the case of an adverse productivity shock, other things being equal. Since the optimal policy response may offset this effect more than the Nash solution, targeting the nominal exchange rate may or may not turn out to be appropriate, depending on the parameter values. On the contrary, demand shocks do affect both nominal and real exchange rates, so that targeting either of them will be adequate.

The degree of commitment to a specified target

The optimal degree of commitment \((\beta')\) to the exchange rate target is the value of \(\beta\) which attains an optimal solution for a given value of \(p\). The previous expressions have shown that the sign of \(\beta\) depends on the value \(p\). This suggests that the optimal degree of commitment \((\beta')\) can be expressed as a function of the exchange rate target and the

\[ ^{7} \text{In particular, we will see below that low levels of goods market integration (low } \sigma \text{) would make even nominal exchange rates inappropriate. For low values of } \sigma \text{, a similar result holds. See figures 3.b and 4.b.} \]
structural parameters of the economy: $\beta^*=f(p,\alpha)$. Two of these parameters are particularly important: the weight given to the employment objective (\(\alpha\)) and the degree of openness of the economy (\(\zeta\)).

The function $f(p,\alpha)$ can be obtained as follows. The optimal money supplies in the cases of demand and opposite supply shocks must belong to the target solution which appears in \(10\). Noting that $x_i=-x_2$ and

$$\frac{(\psi_1,\psi_2)\cdot(\psi_1^2,\psi_2^2)}{1+\alpha+2\theta+2\beta\rho'}=1$$

It follows that:

$$m_1'=\frac{(1+2\theta)}{(\sigma+(1+2\theta)^2)}((u_1-u_2)-2x_1)=\frac{(1+2\theta)\sqrt{\eta}(u_1-u_2)+(1+2\theta)\rho^2 x_1}{1+\alpha+2\theta+2\beta\rho'}$$

Considering each shock separately, we can equate both terms to solve for $\beta$:

$$\forall u_1-u_2, \beta=\frac{\alpha 2\theta \sqrt{\eta}}{\sqrt{\rho^2 ((1+2\theta)^2-2+\zeta/\eta (1+2\theta)\sqrt{1+2\theta})}}$$

$$\forall x_1=-x_2, \beta=\frac{\alpha 2\theta \sqrt{\eta}}{\sqrt{\rho^2 (\rho (1+2\theta)^2+\alpha)-\zeta/\eta (1+2\theta)\sqrt{1+2\theta})}}$$

This is the general expression for the optimal degree of commitment to a given exchange rate target. In other words, this result gives a formula to define the optimal targeting strategy, taking into account the parameter constraints given above.

Given the complexity of the expression, it is helpful to plot $f(p,\alpha)$ for the most relevant parameters: $f(p,\zeta,\alpha)$, $f(p,\zeta,\alpha)$. Figures 3.4.a and 3.4.b. display the weight assigned to the exchange rate target relative to the weight attached to the other final targets for different values of $\alpha$ and $\zeta$, respectively. The value of $\beta$ corresponds to the optimal targeting strategy; recalling that the Nash solution corresponds to $\beta=0$, all the values between zero and $\beta$ are also incentive compatible, albeit suboptimal and correspond to the segment of the target line between the Nash point and the contract curve in figures 1 and 2.

Note that $f(p,\alpha)$ is continuous for demand shocks, but for opposite supply shocks it presents a discontinuity at $p=p^*$; for values of $p$ lower than $p^*$ would imply a negative $\beta^*$, precluding the possibility of targeting the exchange rate.

---

8. More precisely, the weight is normalized relative to the unemployment penalty. In figure 3 and 4 (a) $\alpha=1$; however, but figures (b) the optimal degree of commitment is expressed as $\beta/\alpha$ and the weight attached to the to the price objective is $1/\alpha$ otherwise.

9. Actually, there is also a range of values above $p^*$ for which the target zone is also sustainable. This range corresponds to the segment of the target line between the contract curve and the outer limit of the bargaining area in figures one and two.
Figure 3.a.-Demand shocks. Optimal commitment for a given exchange rate target when the degree of integration varies.

Figure 3.b.-Demand shocks. Optimal commitment for a given exchange rate target when the weight of employment in the loss function varies.
Figure 4.a.- Opposite supply shocks. Optimal commitment for a given exchange rate target when the degree of integration varies.

Figure 4.b.- Opposite supply shocks. Optimal commitment for a given exchange rate target when the weight of employment in the loss function varies.
It may also be interesting to comment briefly the consequences of changes in the relevant parameters for the targeting strategy. The derivatives of $\beta$ with respect to these parameters are:

\[
\text{Demand shocks} \\
\frac{\partial \beta}{\partial \zeta} > 0; \frac{\partial \beta}{\partial \alpha} < 0;
\]

\[
\text{Supply shocks (\forall \beta > 0)} \]

\[
\frac{\partial \beta}{\partial \zeta} < 0; \frac{\partial \beta}{\partial \alpha} < 0
\]

The effects of the level of market integration ($\zeta$) on the targeting strategy are different, depending on the type of shocks. We already mentioned that low and medium values of $\zeta$ preclude exchange rate targets in the case of opposite supply shocks. For demand shocks, we observe that higher levels of integration which increase the spillover effects call for tighter exchange rate targets (larger $\beta$), as intuition suggests.

The parameter $\alpha$ - the weight assigned to the employment goal - has equivalent effects for both cases. Let us note from [6,7] that the unemployment goal penalizes any deviation of the money supplies from zero. Since the optimal policy is more activist than the Nash solution, an increase in $\alpha$ will reduce the need for targeting.

A first apparent result is that in the case of demand shocks (Fig 3,a,b) the emphasis on targeting is less than that on the final goals in $L_1, L_2$; this seems reasonable, given that the exchange rate is an intermediate objective, For opposite supply shocks this is also the general result, although for values of $\zeta$ close to the discontinuity term, we find $\beta > 1$.

**Flexible targets**

Another interesting feature of the outcome is that it depends on the type of shock affecting the economy, but not on the magnitude nor the sign of the shocks. This is a result which is also found in the existing policy coordination literature, and it greatly facilitates the design of the optimal contract. As a matter of fact, for any demand shock the formula above can be applied; for supply shocks, we already know that only in certain cases will it be possible to find an optimal targeting strategy. In this case, the corresponding expression can be applied. Otherwise, an alternative type of targeting strategy should be employed, as we observed above.

The third and most relevant conclusion of this analysis is that there is not an unique optimal design for the exchange rate target. Although the optimal degree of commitment is determined by [24], it ultimately depends on the definition of the optimality embraced by the policymakers who can choose between different combinations of commitment
and exchange rate targets given the function \( r(\rho, \ldots) \). This trade-off is conveyed in the derivative of \( \beta^* \) with respect to \( \rho \) in (24) and it gives an idea of the type of targeting strategy to adopt:

\[
\begin{align*}
\text{Demand shocks: } & \quad \frac{\partial \beta}{\partial \rho} > 0; \\
\text{Supply shocks: } & \quad \frac{\partial \beta}{\partial \rho} < 0, \forall \beta > 0
\end{align*}
\]

If the policymaker's goal is to design a target zone which minimizes exchange rate volatility relative to the desired target, the higher \( \beta^* \), the better. On the contrary, if the aim is to provide exchange rate flexibility reaping the full benefits of coordination, the value of \( \rho \) which allows for the highest exchange rate flexibility will be chosen. According to this second criterion, a real exchange rate target would be the optimal choice in the case of demand shocks while for the case opposite supply shocks, a nominal peg would be the solution.
and exchange rate targets given the function \(f(p, \cdot)\). This trade-off is conveyed in the derivative of \(\beta^*\) with respect to \(p\) in (24) and it gives an idea of the type of targeting strategy to adopt:

Demands shocks: \(\frac{\partial \beta}{\partial p} > 0\);

Supply shocks: \(\frac{\partial \beta}{\partial p} < 0\), \(\forall \beta > 0\)

If the policymaker’s goal is to design a target zone which minimizes exchange rate volatility relative to the desired target, the higher \(\beta^*\), the better. On the contrary, if the aim is to provide exchange rate flexibility reaping the full benefits of coordination, the value of \(p\) which allows for the highest exchange rate flexibility will be chosen. According to this second criterion, a real exchange rate target would be the optimal choice in the case of demand shocks while for the case opposite supply shocks, a nominal peg would be the solution.
VI-Conclusions. The need for flexibility

Welfare considerations should be the basis for any exchange rate arrangement among countries. Consequently, exchange rate targets which are not incentive compatible cannot be sustained. Given this idea we have set up a framework for analysing exchange rate targeting in the form of an optimal contract between countries which are on an equal strategic basis.

The exchange rate target is viewed as an intermediate objective. The policymakers in order to attain their final targets, agree on a targeting strategy consisting of a desired exchange rate target and their optimal commitment to it. The approach allows us to design simple rules which do not depend on the nature of the shock, but on its magnitude and this appears to be general enough to encompass all reasonable exchange rate arrangements.

Despite this generality, targeting the exchange rate in such a way turns out not to be always appropriate. While for demand shocks of any type or magnitude an optimal contract can be devised, for supply shocks the answer depends on their differential impact on each country. The reason is that the optimal contract can only be designed when both countries are interested in moving the exchange rate in the same direction; this is the case of demand shocks and opposite supply shocks. Even in this latter case, the feasibility of the targeting strategy may be constrained by the structural parameters of the model.

Flexible targeting strategies

The assumption of an identical strategic role for each country in the targeting strategy turns out to be a crucial feature. The resulting optimal contract allocates a fixed global money supply more efficiently than in a non-cooperative Nash situation but it does not pay when the reactions of one country exert a negative externality on the other country.

The following table compare our results with those of Canzoneri and Henderson
Part II-Design

(1991). It is interesting to make this comparison, not only because the model is the same but also because they devise a different type of targeting strategy.

### TARGETING STRATEGY

<table>
<thead>
<tr>
<th>DOES TARGETING PAY?</th>
<th>OPTIMAL CONTRACT (Our model)</th>
<th>LEADER-FOLLOWER (Canz&amp;Hen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S H O C K S</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DEMAND SHocks $(u)$</td>
<td>General</td>
<td>General</td>
</tr>
<tr>
<td></td>
<td>YES</td>
<td>YES, but suboptimal</td>
</tr>
<tr>
<td>SUPPLY SHocks $(x)$</td>
<td>Opposite shocks</td>
<td>Symmetric shocks</td>
</tr>
<tr>
<td></td>
<td>YES</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(but parameter dependent and may be suboptimal)</td>
<td></td>
</tr>
<tr>
<td>Rest of cases</td>
<td>NO</td>
<td>YES</td>
</tr>
</tbody>
</table>

Canzoneri and Henderson consider an exchange rate target from strategically asymmetric perspective, where one country (the follower) pegs the exchange rate to the leader. The leader sets the value of its money supply to minimize its own welfare function and the follower only cares about maintaining the parity. Note that in this situation, there is no exchange rate arrangement but simply an unilateral exchange rate peg by the follower.

In the table we observe that this asymmetric strategy pays in the case of symmetric supply shocks. In this case, as it is apparent in figure 2.a, both countries respond by changing their money supplies in the same direction, provoking an overshooting of the exchange rate with respect to the efficient solution. Therefore, the result of a leader-follower strategy is to offset this negative externality and place the economy on the point $H$, which is optimal.

Note that in this case, the existence of a leader exerts a disciplinary effect on the actions of the follower because changes in the money supplies are smaller. Thus, when the optimal response to a shock requires a restraint or discipline in the management of the money supply a leader-follower strategy is desirable because the leader provides an anchor to the monetary policy. From the table we can observe that this is the only
situation in which the Canzoneri-Henderson strategy is superior.

However, when countries act on an equal strategic basis, they can only agree on an optimal contract which has the same effect on the behaviour of both countries; in this case, no disciplinary effect can be attained. Therefore, the optimal contract strategy may only be beneficial when a more activist response is required. This implies that our alternative dominates for demand and opposite supply shocks\(^{10}\).

These results emphasize the importance of the environment to design exchange rate regimes. The risks of devising a targeting strategy in inappropriate circumstances becomes apparent. The rules and goals with which the arrangements were conceived may not apply in practice and the likely result is a collapse of the system.

The principle which has guided this research is that a more flexible approach to exchange rate management is advisable. In the light of the subsequent analysis, the advantages of flexibility are manifest.

On the one hand, there is a flexibility of response. Our outcome suggests that the adequate targeting strategy depends on the type of shock hitting the economy: in certain situations it will be convenient to adopt a leader-follower strategy; in others, both countries should play an equal strategic role; in this latter cases, sometimes it will be convenient to target the real exchange rate, in others it may be possible to target both, etc. Thus, a flexible approach to exchange rate targeting may cover most of the possible range of shocks.

On the other hand, there is a flexibility of design. We have shown that the trade-off between the exchange rate target and the optimal commitment to it leaves some room for discretion in the choice of the target to policymakers, as long as the optimal degree of commitment to the selected target is chosen.

Practical implications

We have not shown a general dominance of a nominal exchange rate target on a real target or vice-versa. Indeed, we have proved that in certain cases (demand shocks) both are valid and in others neither of them is (a wide range of supply shocks). Only for the case of opposite supply shocks the nominal exchange rate target-if any- is superior to a real exchange rate target. As we have mentioned, our results encompass

\(^{10}\) As we have mentioned, Canzoneri and Henderson only consider shifts in demand (opposite demand shocks) and common (symmetric) supply shocks. Notwithstanding this, the results for the demand shocks can be generalised, as it suggested by the table.
Mckinnon's proposal because the global money supply target can be agreed beforehand and then allocated according to the optimal strategy. However, the choice of final targets also includes here unemployment explicitly, not only prices, as in MacKinnon's formulation.

The message from this paper is then twofold:

- First, it is the economic and not the political environment which should determine the targeting strategy. Therefore, when the outcome of the bargaining process does not adjust to the economic demands the system will be inadequate and it is no wonder that it cannot satisfy the goals for which it was originally devised. This remark recognises the objective difficulties to designing optimal exchange rate targets.

- Second, the nature of the regime should be revised when the economic environment or the goals of the system change.

These conclusions provide ammunition against the excessive rigidity envisaged in the new EMS. On the one hand, major demand shocks as German reunification are at odds with a nominal target plus narrow bands; The optimal response would have been a move to a real exchange rate target or a widening of the band, as the markets ultimately achieved through the crisis of 1992-93. On the other hand, when the emphasis shifts from exchange rate stability to inflation convergence to attain a Common Currency, a reflection on the adequacy of the old rules is in order.

It goes without saying that the convergence process needed in Europe to attain a common currency includes dynamic considerations which fall beyond the scope of this analysis. In the conclusions to the dissertation we will examine possible extensions to improve on this analysis and reconcile it with the previous work. We have also omitted the issue of policy delegation as a means by which conducting policy. The concept of the optimal contract is assumed in our approach to concern sovereign countries, tied by a commitment to respect the contract. This contract could also have been viewed as the constitution of a supranational Central Bank, which in a first stage has the responsibility to monitor the behaviour of the exchange rate between the countries, much in the way that the European Monetary Institute is supposed to operate at present.

All in all, we wish to point out that in the transition to a Common Currency, where sound stabilization strategies are required, some sort of exchange rate management is clearly needed. From the analysis above, we suggest that the approach be flexible and adapted to the circumstances.
This appendix shows that expectation on money supplies equal zero and develops the proofs to propositions one to ten.

**Money supply expectations**

Substituting the reduced forms in which expectations explicitly appear in the functions to optimize and taking the derivative with respect to the instruments, we obtain:

\[
\frac{\partial W_1}{\partial m_1} = 0 = \alpha(m_1 - m_1^*) + \eta(\phi + \alpha)(m_1 - m_1^*) - \phi(m_2 - m_2^*) + x_1' - \zeta(y_1' - y_2') + \\
\beta\sqrt{p}((m_1 - m_2) - (m_1^* - m_2^*)) + (1 - p)(x_1' - x_2') - \gamma(y_1' - y_2') = 0
\]

\[
\frac{\partial W_2}{\partial m_2} = 0 = \alpha(m_2 - m_2^*) + \eta(\phi + \alpha)(m_2 - m_2^*) - \phi(m_1 - m_2^*) + x_2' + \zeta(y_2' - y_1') - \\
\beta\sqrt{p}((m_1 - m_2) - (m_1^* - m_2^*)) + (1 - p)(x_1' - x_2') - \gamma(y_1' - y_2') = 0
\]

Taking expectations from these expressions, and noting that xle = u_i = 0, i=1,2, because they represent unanticipated shocks, it is straightforward to conclude that the expected money supplies equal zero. Setting m_1^* = m_2^* = 0 the reduced forms net of expectation which appear in (4.5) are obtained.

**Proof to proposition 1 (Target line)**

The slope of T(\lambda) in the (m_1, m_2) space is obtained by the cocent of the derivatives of the target solutions appearing in (9) with respect to \lambda:

\[
\frac{dm_1^*/dm_2^*} = \frac{\partial m_1^*/\partial \lambda)}{\partial m_2^*/\partial \lambda).}
\]

Let us consider the different terms in equation (9). We observe that the last two terms in the expressions and consequently their partial derivatives, are equal and of opposite sign.

On the contrary, the first term is different in both expressions. Let us denote these term as m_1^*, m_2^*, respectively and let us express x_2 in terms of x_1, x_2 = Kx_1, K \in R. Taking the
partial derivative with respect to $\beta$ of $m_1^*(t1), m_2^*(t1)$, we get:

$$\frac{\partial m_1^*}{\partial \lambda} = \frac{(1-K)(1-\alpha-2\theta)^2}{((1-\alpha+\beta)^2-(28+\beta\beta)^2)^2} = \frac{\partial m_2^*}{\partial \lambda}$$

Therefore

$$dm_1^*/dm_1^* = (\partial m_1^*(\partial \lambda))/(\partial m_1^*(\partial \lambda)) = -1$$

Finally, from (11) the Nash solution is known to represent one point in this line, so that we can derive the equation of the straight line:

$$\tau(\lambda) = \{ (m_1^*, m_2^*) \mid m_1^* = \frac{x_1 + x_2}{1+\alpha-2\theta} - m_1^* \}$$

Proofs to propositions 2 and 5. (Sustainability)

Taking as reference the $m_1$-axis, the slope of the target line is equal to -1, so that $t g(\omega) = -1$, where $\omega = 135^\circ, 315^\circ$ and $\omega$ represents the angle formed by the target line and the $m_1$-axis at the Nash point.

Let us note first that the bargaining area, $A$, is formed by the area within the ellipses crossing at the Nash solution (N). Thus $A$ is placed between the tangent lines to the two ellipses at $N$. Secondly, from (10,11), N is known to be a point on the target line. Therefore, as the figures suggest, if the target line lies between the angle formed by those two tangents: $\omega_m < \omega < \omega_M$, $|\omega| = 12$, the target line will cross the bargaining area. The general expression for the ellipses slope at Nash point are given by:

$$\frac{dm_2}{dm_1} \bigg|_{\omega,\alpha} = \frac{dW_1/dm_1}{dW_2/dm_2} = \frac{(1-\alpha)m_1^N-20m_2^N+x_1-u_2)}{20(m_1^N-20m_2^N+x_1-u_2)}$$

Let us consider first demand shocks ($x_1=0, x_2=0$). Substituting the Nash solution (11) into the previous expression, we obtain

$$\frac{dm_2}{dm_1} \bigg|_{\omega,\alpha} = \frac{0}{\alpha} (u_1-u_2) = 0 \quad \frac{dm_2}{dm_1} \bigg|_{\omega,\alpha} = \frac{\alpha}{0} (u_1-u_2) = \infty$$

The angles formed by these tangent lines depend on the sign of $(u_1-u_2)$. In particular, for the ellipse corresponding to $W_1$: 192
\[ \forall u_1, u_2, \lim_{m \to m^*} \frac{dm_2}{dm_1} \big|_{\omega_w^*, \omega_w^*} = 0 \Rightarrow \tan(\omega_w) = 0 \]

and \( \exists \epsilon > 0 \mid \)

\[ \forall u_1, u_2 < 0, m_1 \epsilon + (m_1^N - \epsilon, m_1^N), \Rightarrow \frac{dm_2}{dm_1} \big|_{\omega_w^*, \omega_w^*} > 0; \]

\[ \forall u_1, u_2 > 0, m_1 \epsilon + (m_1^N - \epsilon, m_1^N), \Rightarrow \frac{dm_2}{dm_1} \big|_{\omega_w^*, \omega_w^*} < 0. \]

Thus \( \forall u_1, u_2 < 0, \omega_w^* = 360^\circ; \forall u_1, u_2 > 0, \omega_w^* = 180^\circ \)

and for the second ellipse \( W_2 \):

\[ \forall u_1, u_2, \lim_{m \to m^*} \frac{dm_2}{dm_1} \big|_{\omega_w^*, \omega_w^*} = \infty \Rightarrow \tan(\omega_w) = \infty \]

and \( \exists \epsilon > 0 \mid \)

\[ \forall u_1, u_2 < 0, m_2 \epsilon + (m_2^N - \epsilon, m_2^N), \Rightarrow \frac{dm_2}{dm_1} \big|_{\omega_w^*, \omega_w^*} > 0; \]

\[ \forall u_1, u_2 > 0, m_2 \epsilon + (m_2^N - \epsilon, m_2^N), \Rightarrow \frac{dm_2}{dm_1} \big|_{\omega_w^*, \omega_w^*} < 0. \]

Thus \( \forall u_1, u_2 < 0, \omega_w^* = 90^\circ; \forall u_1, u_2 > 0, \omega_w^* = 270^\circ \)

It follows then that

\[ \forall u_1, u_2 < 0 \Rightarrow \omega_w^* < \omega_t < \omega_w^*; \]

\[ \forall u_1, u_2 > 0 \Rightarrow \omega_w^* < \omega_t < \omega_w^*; \]

and the target line is precisely the bisectrix of the angle formed by the tangents to both ellipses. Thus, the target zone will always cross the bargaining area.

Proceeding as before we obtain for supply shocks \( u_1 = 0, u_2 = 0 \):

\[ \frac{dm_2}{dm_1} \big|_{\omega_w^*, \omega_w^*} = \frac{0}{0} \Rightarrow \tan(\omega_w) = \frac{\sigma 2^{\left( (1+\sigma)x_1 + 2\sigma x_z \right)}}{0} \]

Applying the same reasoning, we have for \( W_1 \) and \( W_2 \), respectively:

\[ x_2 \leq -\frac{1+\sigma}{2} \Rightarrow \omega_w^* = 360^\circ; \; x_2 \geq \frac{1+\sigma}{2} \Rightarrow \omega_w^* = 180^\circ \]

\[ x_2 \leq \frac{1+\sigma}{2} \Rightarrow \omega_w^* = 90^\circ; \; x_2 \geq -\frac{2}{1+\sigma} \Rightarrow \omega_w^* = 270^\circ \]

Noting that \( 2 \sigma < 1 < 1+\sigma \) (see table) the following cases arise:

\[ \forall x_1 < 0: \]

(a) \( -\frac{2}{1+\sigma} \leq x_1 \leq -\frac{1+\sigma}{2} \Rightarrow \tan(\omega_w) = 360^\circ; \tan(\omega_w) = 270^\circ \)

(b) \( x_2 \geq -\frac{1+\sigma}{2} \Rightarrow \tan(\omega_w) = 180^\circ; \tan(\omega_w) = 270^\circ \)

(c) \( x_2 \geq -\frac{2}{1+\sigma} \Rightarrow \tan(\omega_w) = 360^\circ; \tan(\omega_w) = 90^\circ \)
Thus, the target zone constitutes the bisectrix of the angle formed by the tangents to both ellipses in cases labelled (a); for the rest of cases, the target line is perpendicular to the bisectrix and consequently are ruled out. The range of shocks for which, the target zone crosses the bargaining area can then be established:

\[ \forall x > 0, I = 1.2 I, \text{if } -(1 + \sigma)x / 2 \leq x_2 \leq 20 \sigma x / 1 + \sigma \Rightarrow \exists \lambda, \text{IT}(\lambda) \cap A \]

Proof of propositions 3 and 6

Let us consider the bliss line B, instead of the contract curve, since the latter is too complex to work with. As a previous step, it is claimed that if the target line intersects the bliss line it also implies intersects the contract curve. We prove this claim as follows.

Since the contract curve, the target line and the bliss line are continuous and differentiable, we can express:

- \( m_2^C = f(b) \) as a function of \( m_1^C = g(C) \):
  \( m_2^C = f(g^{-1}(m_1^C)) = C(m_1^C) \)
- \( m_2^T = f'(\lambda) \) as a function of \( m_1^T = g'(\lambda) \):
  \( m_2^T = f'(g^{-1}(m_1^T)) = I(m_1^T) \)
- \( m_2^B \) as a function of \( m_1^B \):
  \( m_2^B = B(m_1^B) \)

Then the lemma below, based on the Bolzano-Weierstrass theorem can be directly applied and our claim is proved.

The coordinates for which \( T = B \) are now found for the two different types of shocks.

The intersection between \( T \) and \( B \) is obtained by equating expressions (10) and (13).

For demand shocks \((x_1 = 0, x_2 = 0)\), we get

\[ m_1^T = m_1^B = \frac{1}{2} \frac{U_1 - U_2}{20}, m_2^T = m_2^B = -\frac{1}{2} \frac{U_1 - U_2}{20} \]

which corresponds precisely to the middle point of the bliss line. Thus, in the case of demand shocks the target line will always cross the contract curve.

The resolution is more complex when supply shocks hit the economies \((u_1 = 0, u_2 = 0)\).

Equating the target and bliss lines, the solution is given by
where it is not straightforward to ascertain whether this point falls within the relevant segment of the bliss line. Thus, all the possible combinations of supply shocks are examined to obtain the range of shocks which make the target line to intersect the bliss segment. The solution turns out to be given by the following range:

\[ \forall x > 0, \forall j = 1, 2 \text{ if } -(1 + \sigma)x_1 \leq x \leq -2\theta x_1 \Rightarrow 3 \lambda, \delta \text{ if } (\lambda, \delta) = C(\delta) \]

the same than above.

**Lemma (Bolzano-Weierstrass)**

The lines C(\delta) and B have at least two common points at \( \delta = 0, \delta = 1 \). Then, if \( \exists m_1, I(T(\lambda)) = B \Rightarrow \exists m_2, I(T(\lambda)) = C(\delta) \).

**Proof:** The claim is that \( \exists m_1, I(T-C)m_1 = 0 \).

Adding and subtracting B, we get \( (T-B+C)m_1 \).

Let us assume that \( \exists m_1, I(B(m_1)) = T(m_1) = m_2 \).

Recall that \( C(0) = B_1, C(1) = B_2 \), so that \( (C-B)m_1 = (C-B)m_2 = 0 \).

Since \( B_1, B_2 \) are the extremes of the bliss line, this implies that if

\[ B(m_1 = C'(m_1), B(m_1) = C'(m_2) < T'(m_1), \forall j = 1, 2 \text{ and } \exists m_1, I(C'(m_1) = T'(m_1)) \]

by Bolzano-Weierstrass, and the lemma is proved.

**Proof of proposition 4 (and 7). (Optimal and incentive compatible points)**

From (10), we observe that the \( m_1 = m_2 \) on the target line. Substituting \( m_1, \) for \( -m_2 \)

in the first order conditions of the cooperative solution (which correspond to the left-side term and right-side term of the expression for \( C(\delta) \) in (12)) and substituting \( \Sigma_i \) for the respective values, the following equalities must hold:

\[ ((1 + \delta) - (1 - \delta)2\theta) + 2\theta \delta, m_1 - (\delta + (1 - \delta)2\theta)(u_1 - u_2) = 0 \]

Finally, the demand shocks. this is a non-linear system in \( \delta \) and \( m_1 \). However, we showed that \( T \) intersects the bliss line at the middle point; this suggest that a reasonable value for solving the system is \( \delta = 1/2 \). Substituting this guess above, it is immediate to see that \( \delta = 1/2 \) satisfy both equations. The Pareto optimal point on the target line is then given...
by:

\[ C(\lambda) = \{ m^*_1, m^*_2 \} \in \mathcal{T}(\lambda) \]

\[ m^*_1 = \frac{(1+\Theta)(u_1 - u_2)}{\alpha + (1+\Theta)^2} = -m^*_2 \]

Substituting \( N \) and \( C(1/2) \) in the loss functions, the welfare loss is obtained:

\[ W^*_1 = W^*_2 = \frac{\sigma}{2(1+\Theta)^2}(u_1 - u_2); \]

\[ W^*_1 = W^*_2 = \frac{(1+\Theta+\Theta^2)}{(1+\Theta)^2}(1+\Theta)^2 \sigma \] \( \Rightarrow W^*_1 < W^*_N \), \( i = 1, 2 \)

and consequently the Pareto optimal point is incentive compatible. 

Proof of proposition 8 (Positive externality)

Substituting the value of the shocks in the Nash solution (11) we get

\[ \forall x > 0, x \in \left(-\frac{1+\sigma}{2\Theta} x, -\frac{2\Theta}{1+\sigma} x\right), i = 1, 2, k, j \Rightarrow \text{sign}(m^*_1) = \text{sign}(m^*_N) \]

\[ \forall x > 0, x \in \left(-\frac{1+\sigma}{2\Theta} x, -\frac{2\Theta}{1+\sigma} x\right), i = 1, 2, k, j \Rightarrow \text{sign}(m^*_1) = \text{sign}(m^*_N) \]

\[ \forall u_1, u_2 \Rightarrow m^*_1 = -m^*_2 \]

Note that the first case covers just the range of values for which neither optimal nor incentive compatible exchange targeting strategies can be devised and other two cases conveyed the range of shocks for which propositions 2-7 apply. Thus, the proposition is proved.

Proof of propositions 9 and 10

Exchange rate deviations appear in (5). Substituting the Nash (11) and optimal solutions (18, 19) for the values of the instruments, the expressions to compare are expressed as function of the shocks. Rearranging terms, we can write, in the case of demand shocks:

\[ (z-z^9)^n = 2 \left(\frac{\sqrt{p'}}{1+\Theta} - \frac{1}{2\sqrt{\eta}}\right)(u_1 - u_2) \]

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5-Optimal exchange rate targets

\[
(z-z\,^*) = 2\sqrt{\frac{\rho'}{\sigma(1-\theta)^2}} \cdot \frac{1}{2\zeta\sqrt{\eta}} (u_i - u_j)
\]

and, when the shock arise in the supply side (opposite shocks):

\[
(z-z\,^*) = -2\left(\frac{2\sqrt{\rho'} - \frac{\rho}{\sqrt{\eta}}}{1-\sigma+2\theta}\right) x_i
\]

From these expressions, where we note that only source of divergence in these expressions is the term in brackets, we can deduct the sign of \(\beta\) subtracting the optimal strategy from the Nash strategy. If the result is positive this is indication that \(\beta\) is positive.

Let us start by considering the demand shocks. After consulting the parameter table, we can see that this term is negative in both expressions. However, the first term in the brackets is less than one for the considered range of \(\rho\), and the second larger than one and it is straightforward to show that the Nash solution in absolute value is larger than the optimal targeting solution. Thus, for demand shocks:

\[
\forall \, 0 < \sigma < 1 \Rightarrow |(z-z\,^*)| > |(z-z\,^*)| \Rightarrow \beta > 0
\]

The outcome is not so straightforward for opposite supply shocks because \(\rho\) appears both in the numerator of the first and second term. However, operating and simplifying, the following condition for a positive \(\beta\) can be derived:

\[
\forall \rho > \rho^* = \frac{2\theta}{1+2\theta + \frac{\sigma}{1-\theta} - \zeta\sqrt{\eta}} > 0 \Rightarrow |(z-z\,^*)| > |(z-z\,^*)| \Rightarrow \beta > 0
\]

where we have made use of \(2\theta = (V_1)\). Given the values of the parameters, it follows that real exchange targets produce negative \(\beta\) and that, for reasonable values of the structural parameters, nominal targets are valid.

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The debate on exchange rate targeting can be summed up in the conflict of objectives which such a strategy generates. On the one hand, it implies the subordination monetary autonomy to the exchange rate objective, preventing in consequence the unhindered reaction to adverse domestic or external circumstances. On the other hand, targeting the exchange rate tends to provide exchange rate stability which is a goal in itself and contributes to the coordination of macroeconomic policies among interdependent countries.

In the ERM experience, which has been the main reference in this dissertation, these considerations have to be complemented with other circumstances which render the inherent policy conflict even more apparent. The use of the ERM as a disciplinary device to reduce inflation expectations in the periphery countries has been considered as the main bonus of the system in the last few years but, at the same time, maintaining the exchange rate parity weakens the competitiveness of these countries, and imposes significant real costs in terms of output and employment.

The sluggish progress in inflation convergence, due to the slow gains in the credibility of the policymakers commitment, explains this situation. The longer the target is maintained, the more apparent are its costs, the greater is the temptation to recover monetary autonomy to stabilize the economy and consequently, the more difficult it is to achieve credibility gains.

This inconsistency of objectives has always been present under different forms. In our research, it is reflected in the credibility of the commitment to the announced target. Note that the basis for successful exchange rate targeting is the credibility of the target and therefore the evolution of the conflict is reflected in the evolution of credibility.
Conclusions

In the first part of the dissertation we have attempted to uncover the nature of the conflict using target-zone models. As a matter of fact, the initial claim of these models is that a certain degree of monetary autonomy can be kept in a target zone. However, this result depends on the credibility of the exchange rate target and the empirical evidence reveals the difficulties in achieving credibility in a target zone. In the final chapter of this first part, we propose a model which displays the nature of the conflict; the persistence of inflation differentials imposes a series of real costs (in the form of real exchange rate misalignments) and itself creates credibility problems which make the band unsustainable in the long-run.

With these insights, the second part of the thesis considers the possibility of designing exchange rate targets to overcome this inherent policy conflict. The recipe we put forward is for a more flexible approach to the design of exchange rate targets.

These ideas constitute the basis which have guided our research. However, there are still many loose ends which will need to be tied up in future work. For the moment, we sum up the main conclusions which have been reached in the dissertation and list the directions of further research.

Theoretical models

Target zones models have stressed the stabilizing behaviour of exchange rate expectations when the nominal exchange rate is pegged within a narrow band of fluctuation. In chapter one we have pointed out the limitations of these models. Some of these limitations are derived from the techniques used and from the economic models and perspective adopted.

The tools of stochastic calculus allow to present a closed-form non-linear solution for the relation between exchange rate and fundamentals in a rational expectation setting, but their use is limited to very simple economic models. In spite of this, target-zone models highlight interesting insights which could only be informally considered before. In particular the stabilizing role of expectations in a target zone. This is the great innovation of these models.

However, one main problem limits interest in target-zone models. We have often stressed that stabilization has not been the primary objective of target zones in practice. They have become in the ERM disciplinary devices to place fundamentals in line with the anchor country. This conflict in objectives has received relatively little attention in the literature and has created an inaccurate view of the rationale for target zones. In this sense, we can say that the research has been misguided. A closer approach to the policy
coordination literature, where the exchange rate has been effectively considered as a disciplinary mechanism is therefore in order.

As hinted above, this alternative perspective allows us to place the issue of credibility at the center of the discussion. All in all, the lack of perfect credibility which has characterized the target zone reveals a system which behaves more as a crawling peg in the long run, albeit in the short run the exchange rate can remain stable. The reason is the subtle role that fundamentals play in determining the exchange rate. ‘Fundamentals’, whatever may be its definition, do not seem to determine the exchange rate in the short run, but they are decisive in explain realignments of exchange rates. The Spanish case analysed in chapter two provides a good example of this.

These features have motivated a different approach to modelling target zones in chapter three. By relaxing the rational expectation hypothesis and substituting for it the notion of heterogenous beliefs, the new model uncovers the interaction between the evolution of the fundamentals and the credibility and reputation of the model, which now become explicitly dynamic processes. It is formally shown that the sustainability of the band in the face of larger divergences increases the reputation of the regime but this is a self-defeating mechanism because, in the absence of convergence in the fundamentals, misalignments finally bring about the collapse of the regime. In other words, the conflict of objectives becomes unsustainable in the end.

In my view, the model developed in chapter 3 captures some of the important characteristics of actual behaviour of the exchange rate in a target zone and offers an interpretation based on the evolution of credibility, which determines both the endurance of the regime and Its sudden collapse.

The model also allows a shift in the emphasis from stability (sustainability in a dynamic setting) to convergence, when divergences in the fundamentals are endogenously determined. The choice of the target zone must be seen under this prism as a long-run strategy for convergence which is not exempt of risks: pegging the exchange rate in a target zone can only be successful if the depth of the conflict (in terms of real costs) is kept under control and this can only be achieved if gains in convergence are attained with enough speed; otherwise, the peg will not enjoy credibility and the regime will finally collapse.
Exchange rate targeting strategies

This interpretation raises the issue of designing the optimal target zone, which brings together the appropriate mix of convergence, credibility and flexibility to minimise the magnitude of the policy conflict. Thus, the second part of the dissertation is guided by a quest for designing flexible exchange rate targets.

This flexible approach has been motivated by the observation that the ERM became excessively rigid before the crises and allowed to appear misalignments that were too large and clearly unsustainable in the long run. On the methodological side, the flexible targeting strategies enabled the analysis of a wide range of possibilities for the management of exchange rates.

In both chapters four and five, the exchange rate acts as an intermediate target to attain convergence and to stabilize the economy with respect to desired levels, respectively. Notwithstanding these similarities, the approach is completely different in each model. Chapter four extends the dynamic target zone model presented earlier, while chapter five uses a small static model to explore the strategic interactions of symmetric exchange rate regimes.

Judging from the simulations of the extended dynamic target zone model, more flexible target zones -defined with respect to the behaviour of the fundamentals- do not appear as superior alternatives to more rigid schemes, such as the new EMS. The reason is that, although flexible schemes postpone the conflict of objectives, the achievement of convergence and, consequently, of credibility is also postponed.

This conclusion should be weighed up with other considerations which could bias the balance towards more flexible targets: transparency, real variables, the consideration of second country objectives, etc. The cost of taking these considerations into account implied that we had to dispense with the rich dynamics of the previous models.

In chapter five, the targeting strategy is considered from an optimization framework in which the nature of the policy conflict is explicitly considered. The concept of a target zone is substituted by that of an optimal contract between countries taken on an equal strategic footing. The optimal design of this contract determines the optimal commitment to a desired exchange rate target. It turns out that exchange rate targets are a sustainable and efficient alternative to explicit coordination for certain shocks, while for others it would be better too pursue a leader-follower strategy.
Conclusions

The results of this part have not generated a general answer, regarding the optimal targeting strategy, because it depends on the nature of the shocks. What is clear however is that these results suggest a more flexible approach to targeting the exchange rate in order to minimise the policy conflict.

Directions for future research

This dissertation has considered possible extensions in exchange rate targeting which should be considered in future research.

In my view, any further extension of the target zone models is difficult for the reasons mentioned above. The dynamic target zone model in chapter three could be refined in several aspects. The first concerns the expectations mechanism. More sophisticated updating methods could be considered and, more interestingly, noise traders and chartist could play an explicit role in the model, which in this case would be defined in terms of three and not two different beliefs. The second aspect has to do with the way to model interventions. In particular, the role of intramarginal interventions.

The study of the empirical evidence has left open a promising issue. We have explained the existence of realignment risk in terms of conditional expectations, but no confirmation of such hypothesis has been attempted. Therefore, it would be challenging to devise an estimation method to extract conditional expectations from the data, along the lines of switching process estimation or Bayesian multi-process Kalman filter. Both methods would first require the estimation of fundamentals, which is not an easy task. At this point, the expression to estimate would be similar to (3.15), where the data would reveal the evolution of the weights.

Regarding the second part of the thesis, the convergence band proposal and the optimal contract approaches should be put in a unifying framework. The idea would be to pick up the dynamic characteristics of the target zone model within the optimizing setting of chapter five. The natural framework to implement this project is the macroeconomic dynamic game literature.

The first step would be to choose an economic model of reference and an objective function for both countries. The exchange rate would play a analogous role as in chapter five. The difference would be that, in this dynamic problem, the emphasis could be shifted from stabilization to convergence.

This extension would overcome the limitations of the two models used in the second part of the thesis. Furthermore, with an adequate specification of the objective function
Conclusions

(for instance, considering the Maastricht criteria), the optimal transition to a common currency could be explored by considering alternative exchange rate regimes. Thus, it is hoped that the policy implications of the model, which at this stage seem quite loose, could be strengthened and they could contribute to provide insights to attain a smooth transition into a Common Currency.


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