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Abstract

This paper presents a structural model of intra-European airline competition. Using a two countries/two airlines (flag-carriers) framework, three different competitive scenarios are analysed. These scenarios reflect the new EC competition rules. The results suggest that, when flag-carriers operate hub-and-spoke networks, the potential welfare gains arising with the abandoning of collusive practices are significant throughout the network. In addition, the model shows that, with increasing returns to density, a cross-border merger between two flag-carriers may increase the net social welfare throughout the network. Consequently, the threat of monopolisation through merger should not be of primary concern to EC antitrust authorities. (JEL L13,L43,L93)

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"...Because carriers produce their "output" by operating over a network, it stands to reason that the only way to examine the effects of regulation change on carrier service provision is by incorporating network detail."

Daughety. 1985. p.476.

I Introduction

Given inherited regulations and European airline industry specificities how should the potential benefits stemming from the new regulatory environment be evaluated ? What airline EC merger policy would be more appropriate ? These are the main questions I aim to answer in this paper. To this end. I suggest the analysis of a structural model of intra-European airline competition[1] that is able to take the main characteristics of this industry into account: **the new EC competition rules**[2] and **the structure of European airline networks**.

Various researchers have stressed the importance of networking and multiproduct aspects in airline economics (see Sarndal & Statton [1975]. Pavaux [1984]. Caves et al. [1984], etc.). Recently, Brueckner & Spiller [1991] provided an analytical framework to study the effect of competition in airline hub-and-spoke networks. My aim is to extend their approach to a two country/two airline model. In fact, while Brueckner & Spiller [1991]. analyse the effect of an exogenous change in the number of firms serving a particular market, this paper is an attempt to analyse the possible effects and the social welfare consequences of the gradual European airline liberalisation. In particular, the model presents various competition scenarios, going from explicit cartel agreement toward more competitive behaviour.

[1] Only flag-carriers operating *scheduled* air passengers services are considered in this paper.

[2] From 1988, as an attempt to promote competition in intra-EC air transport, the Commission gradually introduced three packages of measures, containing regulations concerning competition rules (their enforcement and permissible exemptions). fares, market access, and licensing. The First Package is published in OJ L374. No 3975/87/EEC, 31.12.87. The Second Package, in OJ L217, No 2342,2343/90/EEC. 11.8.90. Finally, the Third Package is published in OJ L240, No 2411/92/EEC, 24.8.92.

Finally, the model offers some insights on the important merger issue. Nowadays, it seems clear that the future of the European airline industry will depend, to a large extent, on a successful EC merger policy.

This paper is organised as follows. In Section II, I specify the assumptions and set up the model. In Section III and Section IV, I present the collusive agreement and the "competitive" solutions, respectively. Section V provides a comparison between both solutions. The merger solution is proposed in Section VI and Section VII provides a comparison between the merger and the "competitive" solutions. Section VIII concludes.

II Assumptions and Model Set-up

The model is based on the following assumptions. The first three are derived from the network characteristics and the regulatory regimes. A tractable model calls for the last assumption.

1. The hub-and-spoke[3] network is *exogenously* given to both airlines (flag-carriers). In a dynamic perspective, it is clear that the route structure is a key endogenous variable[4].

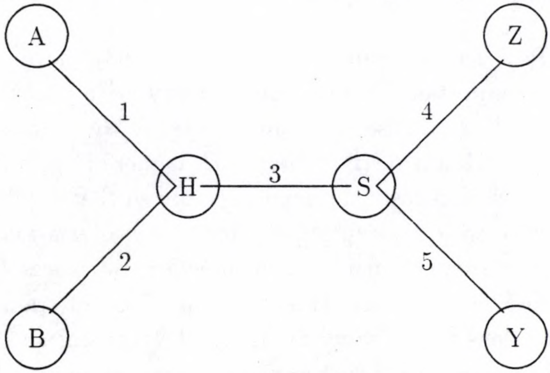
2. Each flag-carrier is a **monopolist** in its *"hinterland"* or *protected market niche.* Consequently, each airline has the monopoly of **two** purely domestic routes. This assumption reflects the fact that previous bilateral agreements did not make provision for the so-called *cabotage rights*, i.e., the ability to carry passengers within a country by an airline of another country with the origin/destination in its home country[5]. (Complete cabotage freedom in Europe will be granted from April 1997).

[3] See Bauer [1987] for a definition of hub-and-spoke routing. For a description of some European hub-and-spoke networks, see Nero [1994].

[4] However, in the short run, given industry and/or regulatory rigidities observed at the European level, this assumption is not too restrictive.

[5] Nero [1994] found, for example, that in 1993 the **domestic** market shares (capacity) of Swissair, KLM and Austrian Airlines were, 92%, 70% and 58%, respectively.

Figure 1: Two simple linked hub-and-spoke networks



3. The two flag-carriers operate **one** intra-European route, on which they are assumed to provide a *homogeneous* service[6]. On this route, I will assume different degrees of **duopolistic**[7] cross-border competition using a quantity setting strategy.

4. For computational convenience, I assume symmetric airlines, i.e., they use the same technology[8], they operate symmetric networks, in particular the **legs**, $l = 1, .., 5$ of Figure 1 all have the same distance. Moreover, airlines face symmetric demand functions.

Figure 1 shows two simple linked hub-and-spoke networks operated by two airlines, Airline 1 in country 1 and Airline 2 in country 2. Cities A, B and H belong to country 1, whereas cities Z, Y and S belong to country 2. Therefore, a total of 6 cities are involved in this structure, implying 15 different city-pairs or markets[9]. Because of their central

[6] European airlines offer very similar ranges of services on intra-European routes.

[7] It should be mentioned that the drawback of this two country/two airline model is that it fails to explicitly take *fifth freedom* competition into account, i.e., the ability of a **third** flag-carrier to serve this intra-European route. However, Nero [1994] found that among the 103 intra-European city-pairs operated by Swissair in 1993, 99 are served by **at most** two airlines, i.e., 96%. Similar figures are found for Austrian Airlines and KLM (85%).

[8] Actually, it should be noticed that most European flag-carriers use similar aircraft.

[9] With n the number of cities, the total city-pairs would be $\frac{n(n-1)}{2}$.

locations, cities H and S serve as the hub for Airline 1 and Airline 2 networks, respectively[10].

In a seminal paper, Caves et al. [1984] have shown that airlines achieve important **returns to density** within a given network. Returns to density arise when an increase of the volume of transportation services[11] **within a given network** is more important than the associated increase in costs. It should be noticed that, in the airline literature, returns to scale are typically defined as the variation in unit costs with respect to proportional changes in *both* network size (for example by increasing the cities served) and the provision of transportation services, holding density constant. Similarly, the economies of scope measure the cost advantage of jointly providing a large number of diversified products (city-pairs) as against specialising in the production of a single product (subadditivity criterion). Since the different scenarios analysed in this paper may affect the density achieved within a given network, only returns to density are considered hereafter[12].

Recent theoretical papers have shown that hub-and-spoke networks are optimal transportation routing[13]. In particular, Hendricks et al. [1992] have shown that if there are economies of density, the optimal network has the hub-and-spoke characteristic. In fact, by consolidating the connecting passengers with the same origin but different destinations (or vice versa) on the same route (spoke), the airline gains, principally, two kinds of advantages[14]:

- It increases the density of traffic along each spoke. Therefore, it can use aircraft more intensively by increasing the load factor (rate of capacity utilisation) and/or using larger, more efficient airplanes.

[10] Cities H and S are, most of the time, the capitals of each country.

[11] Passengers and/or freight.

[12] For a recent survey on the "economies of scale" in the airline industry, see Antoniou [1991]. Levine [1987] provides a general discussion of indivisibilities arising in this industry.

[13] See, for example, Starr & Stinchcombe [1992].

[14] Besides the opportunity of exerting market power in the hub airport, see Borenstein [1989,1991,1992].

In both cases, the unit cost per passenger transported declines[15].

- The potential increase in frequency[16] (for example, two flights per day instead of one) along each spoke may increase demand and therefore density. However, to the extent that this indirect cost advantage is less important, I avoid dealing with it in the present paper.

In summary, in the absence of returns to density, airlines would provide nonstop connections between each pair of cities, for example between A and S. In the presence of such returns, airlines have an incentive to channel passengers through the hub airport, H in the previous example. In other words, it is profit maximising to operate **one stop** services between A and S, for example.

In this paper, I assume that, because of the presence of returns to density, Airline 1 operates aircraft on **three** legs: leg 1, 2 and 3. A similar structure is assumed for Airline 2, which operates aircraft on leg 3, 4 and 5. Leg 3 represents the intra-European leg. It connects the two hub airports, H and S. Given bilateral agreements, it follows that this leg is served by **both** airlines. Airline 1 and Airline 2 operate two domestic legs, leg 1 and 2 and leg 4 and 5, respectively. On these routes, each airline is a **monopolist**. Therefore, in this model:

- Peripheral cities (A and B in country 1 and Z and Y in country 2) are connected through the hub airport, i.e., with a one stop service.
- Similarly, country 1(2) peripheral cities are connected to country 2(1) cities with at least one stop service.

Actually, although the model may seem quite restrictive, there is an empirical evidence that most nonstop intra-European services are provided from hub airports (for example Vienna, Amsterdam, Copenhagen, etc.).

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[15] Oum & Tretheway [1990] and recently Starr & Stinchcombe [1992] suggested that the addition of a new city in this system can stimulate traffic density on the other links of the hub, generating further economies.

[16] An important dimension of the quality of service provided.

I assume that the demand is symmetric across city-pairs. Consequently, the inverse demand function for round-trip travel in any given city-pair market ij is given by $P(Q_{ij})$, with Q_{ij} representing the number of round-trip passengers in the market ij. Accordingly, Q_{ij} represents the number of passengers travelling from city i to city j and back, *plus* the number of passengers travelling from city j to city i and back. Furthermore, I assume a limited demand for international services in the sense that: $D(Q_{AZ}) = D(Q_{AY}) = D(Q_{BZ}) = D(Q_{BY}) = 0$. Put it another way, there is no demand between cross-border peripheral cities. While gaining in simplicity[17], the model captures the following feature: most intra-European traffic flows stop at hub airports. This remark is particularly relevant for central Europe, where capitals attract most leisure and business travellers. In addition, because the change of carrier implies higher risks of missing a connection[18] (often associated with the change of terminal in hubs airports and/or the lack of flight coordination between carriers) or of losing baggage, I assume that a passenger originating its journey in A and willing to fly to city S, for example, will choose the same carrier, i.e., Airline 1. These travellers' preferences ensure each airline the ability to transport their connecting passengers on the HS leg. Airline 1, for example, will carry all the Q_{AS} and Q_{BS} passengers. Similarly, Airline 2 will carry all the Q_{ZS} and Q_{YS} travellers.

Given the common distance of the legs of Figure 1, I assume a common cost function, $C_l(Q_l)$, **applying to each of the legs**, $l = 1,..,5$. Therefore, this cost function gives the round-trip cost of carrying Q_l travellers on one leg. From the previous assumptions, it follows that Q_l represent both local as well as connecting passengers. On leg 1, for example, aircraft carry both local, i.e., A to H passengers, as well as connecting, i.e., with the same origin but with different destinations, passengers[19]. In this case, $Q_{l=1}$ would correspond to $Q_{AH} + Q_{AB} + Q_{AS}$, i.e., all traffic routing through leg 1. The cost function allows for increasing returns to density stemming from hubbing operations. Put differently, the cost function

[17] The model can be reduced to 11 different city-pairs.

[18] See Carlton et al. [1980] for example.

[19] Of course, traffic also includes passengers returning from A to different destinations.

reflects the cost complementarity arising from producing a bunch of air transportation services (products) in a hub-and-spoke network. Consequently, $C_l(Q_l)$ satisfies the following properties: $C_l(Q_l) > 0$, $C_l'(Q_l) > 0$ and $C_l''(Q_l) \leq 0$.

Following Brueckner & Spiller [1991], I adopt the following inverse demand and cost specifications:

$$P(Q_{ij}) = \alpha - \frac{Q_{ij}}{2} \begin{cases} \text{with } i, j = A, B, H, S,\ i \neq j & \text{for Airline 1} \\ \text{with } i, j = Y, Z, H, S,\ i \neq j & \text{for Airline 2} \end{cases} \tag{1}$$

and with $\alpha > 0$;

$$C = \sum_l C_l(Q_l) = \sum_l (Q_l - \frac{\theta(Q_l)^2}{2}) \begin{cases} \text{with } l = 1, 2, 3 & \text{for Airline 1} \\ \text{with } l = 3, 4, 5 & \text{for Airline 2} \end{cases} \tag{2}$$

where Q_l is the traffic volume of the relevant city-pairs markets routing through leg l and $\theta \geq 0$[20].

Consequently, the intercept of the demand function in (1), α, is identical for all city-pairs markets. This is equivalent to assuming that the cities are similar in size. By eliminating differences in size between cities, this assumption allows us to highlight the effects of the network structure and of the returns to density on the equilibria in the different competition scenarios. It should be noticed that the demand for travelling in the ij market does not depend upon prices in any of the other markets[21]. For simplicity, fixed costs are assumed to be zero under this cost specification. Moreover, (2) reflects both ground and flight operating costs of transporting a given amount of passengers on a given leg[22]. The extent of increasing returns to density is measured by θ in (2). Notice that constant returns to density would imply $\theta = 0$. From (2), it should be noticed that, as long that $\theta \neq 0$, the marginal cost of the leg is inferior to its (declining) average cost[23].

[20] But not too large, see below.

[21] The idea is that customers who wish to travel from city i to j have no desire to travel any where else in the network.

[22] This assumption implies ground and flight costs to be proportional, which is realistic if the fuel price is stable.

[23] Alternatively, it can be verified that the cost elasticity, $\frac{\partial C_l(Q_l)}{\partial Q_l}\frac{Q_l}{C_l(Q_l)}$, is less than one.

III The Collusive Agreement Solution

In order to analyse the effects of liberalisation, I first develop the cartel solution as a bench mark case. This case corresponds closely to the pre-liberalisation case. Under the assumption of (explicit) collusive agreement, Airline 1 and Airline 2 form a **cartel** on the HS market. Therefore, on this city-pair, the cartel provides a quantity so as to maximize joint profit[24]. On the other markets, each airline behaves as a monopolist. Given these assumptions, the Airline 1 profit function, Π_1, is

$$\begin{aligned}
\Pi_1 &= P(Q_{AH})Q_{AH} + P(Q_{AB})Q_{AB} + P(Q_{BH})Q_{BH} + P(Q_{AS})Q_{AS} \\
&\quad + P(Q_{BS})Q_{BS} + P(Q_{HS})Q^1_{HS} - C_1(Q_{AH} + Q_{AB} + Q_{AS}) \\
&\quad - C_2(Q_{BH} + Q_{AB} + Q_{BS}) - C_3(Q^1_{HS} + Q_{AS} + Q_{BS}),
\end{aligned}$$

or expressing it explicitly,

$$\begin{aligned}
\Pi_1 &= (\alpha - \frac{Q_{AH}}{2})Q_{AH} + (\alpha - \frac{Q_{AB}}{2})Q_{AB} + (\alpha - \frac{Q_{BH}}{2})Q_{BH} \\
&\quad + (\alpha - \frac{Q_{AS}}{2})Q_{AS} + (\alpha - \frac{Q_{BS}}{2})Q_{BS} + (\alpha - \frac{Q^1_{HS} + Q^2_{HS}}{2})Q^1_{HS} \\
&\quad - (Q_{AH} + Q_{AB} + Q_{AS} - \frac{\theta(Q_{AH} + Q_{AB} + Q_{AS})^2}{2}) \\
&\quad - (Q_{BH} + Q_{AB} + Q_{BS} - \frac{\theta(Q_{BH} + Q_{AB} + Q_{BS})^2}{2}) \\
&\quad - (Q_{BS} + Q_{AS} + Q^1_{HS} - \frac{\theta(Q_{BS} + Q_{AS} + Q^1_{HS})^2}{2}). \qquad (3)
\end{aligned}$$

From (3), we observe that Airline 1 revenues are generated from its 6 markets, whereas its costs correspond to aircraft flown on three legs. Notice that in the HS market, the demand function is given by $P(Q_{HS}) = \alpha - \left(\frac{Q^1_{HS}+Q^2_{HS}}{2}\right)$, with $Q_{HS} = Q^1_{HS} + Q^2_{HS}$, i.e., the total quantity is the sum of the individual quota offered by Airline 1 and Airline 2, respectively. Similarly, the Airline 2 profit function, Π_2, is

$$\begin{aligned}
\Pi_2 &= P(Q_{ZS})Q_{ZS} + P(Q_{ZY})Q_{ZY} + P(Q_{YS})Q_{YS} + P(Q_{ZH})Q_{ZH} \\
&\quad + P(Q_{YH})Q_{YH} + P(Q_{HS})Q^2_{HS} - C_4(Q_{ZS} + Q_{ZY} + Q_{ZH}) \\
&\quad - C_5(Q_{YS} + Q_{YZ} + Q_{YH}) - C_3(Q^2_{HS} + Q_{ZH} + Q_{YH}). \qquad (4)
\end{aligned}$$

[24] See Doganis [1985] for an example of inter-airline pooling agreements.

Joint profit maximisation boils down to maximising $\Pi^{car} = \Pi_1 + \Pi_2$. Assuming interior solutions, the solution of the cartel problem implies that the following 12 first order conditions be satisfied:

$$\frac{\partial \Pi^{car}}{\partial Q_{AH}} = 0 \implies \alpha - Q_{AH} = 1 - \theta(Q_{AH} + Q_{AB} + Q_{AS}) \qquad (5)$$
$$\frac{\partial \Pi^{car}}{\partial Q_{AB}} = 0 \implies \alpha - Q_{AB} = 2 - 2\theta Q_{AB} - \theta(Q_{AH} + Q_{AS} + Q_{BH} + Q_{BS}) \qquad (6)$$
$$\frac{\partial \Pi^{car}}{\partial Q_{BH}} = 0 \implies \alpha - Q_{BH} = 1 - \theta(Q_{BH} + Q_{AB} + Q_{BS}) \qquad (7)$$
$$\frac{\partial \Pi^{car}}{\partial Q_{AS}} = 0 \implies \alpha - Q_{AS} = 2 - 2\theta Q_{AS} - \theta(Q_{AH} + Q_{AB} + Q^1_{HS} + Q_{BS}) \qquad (8)$$
$$\frac{\partial \Pi^{car}}{\partial Q_{BS}} = 0 \implies \alpha - Q_{BS} = 2 - 2\theta Q_{BS} - \theta(Q_{BH} + Q_{AB} + Q^1_{HS} + Q_{AS}) \qquad (9)$$
$$\frac{\partial \Pi^{car}}{\partial Q^1_{HS}} = 0 \implies \alpha - Q^1_{HS} - Q^2_{HS} = 1 - \theta(Q^1_{HS} + Q_{BS} + Q_{AS}) \qquad (10)$$
$$\frac{\partial \Pi^{car}}{\partial Q_{ZS}} = 0 \implies \alpha - Q_{ZS} = 1 - \theta(Q_{ZS} + Q_{ZY} + Q_{ZH}) \qquad (11)$$
$$\frac{\partial \Pi^{car}}{\partial Q_{ZY}} = 0 \implies \alpha - Q_{ZY} = 2 - 2\theta Q_{ZY} - \theta(Q_{ZS} + Q_{ZH} + Q_{YS} + Q_{YH}) \qquad (12)$$
$$\frac{\partial \Pi^{car}}{\partial Q_{YS}} = 0 \implies \alpha - Q_{YS} = 1 - \theta(Q_{YS} + Q_{YZ} + Q_{YH}) \qquad (13)$$
$$\frac{\partial \Pi^{car}}{\partial Q_{ZH}} = 0 \implies \alpha - Q_{ZH} = 2 - 2\theta Q_{ZH} - \theta(Q_{ZS} + Q_{ZY} + Q^2_{HS} + Q_{YH}) \qquad (14)$$
$$\frac{\partial \Pi^{car}}{\partial Q_{YH}} = 0 \implies \alpha - Q_{YH} = 2 - 2\theta Q_{YH} - \theta(Q_{YS} + Q_{YZ} + Q^2_{HS} + Q_{ZH}) \qquad (15)$$
$$\frac{\partial \Pi^{car}}{\partial Q^2_{HS}} = 0 \implies \alpha - Q^1_{HS} - Q^2_{HS} = 1 - \theta(Q^2_{HS} + Q_{ZH} + Q_{YH}). \qquad (16)$$

The economic interpretation of (5)-(16) is simple. Optimality requires to equalise the marginal revenue (LHS) in city-pair market ij with its associated marginal cost (RHS). Solving the system (5)-(16) yields the optimal quantities. It should be pointed out that, given the symmetry, in equilibrium, it must be the case that $Q^1_{HS} = Q^2_{HS}$, i.e., the traffic on the intra-European market is equally divided among both airlines. The optimal quantities[25] are

$$Q^1_{HS} = Q^2_{HS} \equiv Q^{car}_0 \;=\; \frac{(1-2\theta)(\alpha(1-2\theta)-1)}{16\theta^2 - 13\theta + 2} \qquad (17)$$

[25] For the sake of simplicity these quantities are indexed but they should not be confused with the total volume of the leg.

$$Q_{AH} = Q_{BH} = Q_{ZS} = Q_{YS} \equiv Q_1^{car} = \frac{(2-3\theta)(\alpha(1-2\theta)-1)}{16\theta^2-13\theta+2} \quad (18)$$

$$Q_{AS} = Q_{BS} = Q_{ZH} = Q_{YH} \equiv Q_2^{car} = \frac{\alpha(2-2\theta-2\theta^2)+7\theta-4}{16\theta^2-13\theta+2} \quad (19)$$

$$Q_{AB} = Q_{ZY} \equiv Q_3^{car} = \frac{\alpha(2-\theta-4\theta^2)+6\theta-4}{16\theta^2-13\theta+2}. \quad (20)$$

Therefore, the symmetric structure reduces the joint profit maximisation problem to a four variables problem. This symmetric structure provides a simple way to check the necessary and sufficient second order conditions. In the Appendix (Section IX.1), it is shown that interior solutions exist for $\theta \leq \frac{13-\sqrt{41}}{32} \cong 0.2062$ (see (53)). Similarly, it can be shown that in equilibrium, in order to have both positive quantities and marginal revenues(costs), the following inequalities hold

$$\frac{4-7\theta}{2-2\theta-2\theta^2} < \alpha < \frac{2-3\theta}{\theta(6-10\theta)}. \quad (21)$$

Hereafter, I assume that, in equilibrium, both (21) and (53) are satisfied. Under these conditions, it is straightforward to verify that (17)-(20) are increasing in α.

Proposition 1 *As long as $\theta \in [0, 0.2062]$ and inequalities (21) hold, we have that $Q_1^{car} \geq 2Q_0^{car}$. PROOF see Appendix.*

Therefore, the model shows that, for a given **nonstop** market, i.e., a market implying only one leg, the quantity provided by each airline on its monopoly domestic city-pair market (i.e., Q_1^{car}) is larger than the quantity provided by the cartel, i.e., $Q_{HS}^1 + Q_{HS}^2 = 2Q_0^{car}$, on the intra-European HS market. This result shows that, for a given nonstop market, output or traffic is more restricted under the collusive arrangement than it is under monopoly. As a consequence, ceteris paribus, the price is higher on the intra-European (HS) market.

Proposition 2 *As long as $\theta \in [0, 0.2062]$ and inequalities (21) hold, we have that $Q_3^{car} \geq Q_2^{car}$. PROOF see Appendix.*

Consequently, for a given **one stop** market, i.e., a market implying two legs, the quantity provided on markets connecting two domestic peripheral cities (markets AB, ZY) is higher than the quantity provided on markets connecting a peripheral city and the foreign hub (markets AS, BS, ZH, YH). Therefore, although markets AB and ZY are served by monopoly airlines, consumers are better off on these markets in comparison with markets affected by the collusive agreement. This result is due to the presence of increasing returns to density. Since each airline has to share the HS market, under increasing returns to density, lower traffic per airline on that leg raises the marginal cost on the leg and generates a negative externality on markets using that leg (AS, BS, ZH, YH). This explains why, for a given one stop market, the quantity provided by the monopoly airline (Q_3^{car}) is higher than the quantity in a market routing through the collusive leg (Q_2^{car}).

Corollary: Under the (limit) case of constant returns to density ($\theta = 0$), (21) implies that $2 < \alpha < \infty$[26], we have that $2Q_O^{car} = Q_1^{car} = \alpha - 1$, and $Q_2^{car} = Q_3^{car} = \alpha - 2$.

Therefore, total traffic in the HS market, $2Q_0^{car}$, is equal to the traffic of the nonstop domestic markets, Q_1^{car}. This means that, for a given nonstop market, the cartel and the monopoly outcomes are the same. What is the intuitive explanation of this result ? On its domestic route, when $\theta > 0$, the monopolist airline fully recognises it, setting a larger quantity than the quantity provided on the collusive route. Notice also that, as expected, in longer journey markets, i.e., markets using two legs (Q_2^{car} and Q_3^{car}), the traffic is less (higher costs imply, ceteris paribus, higher price and lower demand). Again, the previous argument may explain why the traffic is the same ($Q_2^{car} = Q_3^{car}$), whereas under $\theta \neq 0$ these quantities are different (see Proposition 2).

In addition, in equilibrium, the model prevents arbitrage opportunities from arising, which is a useful requirement in a transportation network model. In effect, in order to prevent arbitrage opportunities, fares must be set such that the sum of the individual fare for the two legs

[26] In fact, the right hand side of (21) tends to a vertical asymptote as $\theta \to 0$.

of the trip (e.g., AH plus HS) is larger than the fare for a given city-pair market involving one stop (e.g., AS). If this would not be the case, it would be profitable for the traveller to purchase the tickets separately. Given the inverse demand function (1) and (17)-(20), it follows that the arbitrage conditions are reduced to the following inequalities

$$P(Q_1^{car}) + P(2Q_0^{car}) > P(Q_2^{car}) \quad \text{and} \quad 2P(Q_1^{car}) > P(Q_3^{car}). \qquad (22)$$

It can be shown that these inequalities are verified when the first order conditions (5)-(16) and the second order conditions (53) (see Appendix) are satisfied.

IV The "Competitive" Solution

The introduction of the Third Package of regulations (January 1993) will not promote airline competition on all EC routes in the same way. On the one hand, access to domestic routes is restricted since cabotage traffic rights will still be severely regulated until 1997. As a consequence, flag-carriers' hinterlands are unlikely to disappear within the Third Package. On the other hand, intra-European routes are the subject of more competitive rules. Firstly, any EC certificated airline can provide capacity between two countries (fifth freedom competition). Secondly, the Commission is going to seriously prevent airlines from making bilateral collusive agreements on capacity and fares. In the short run, while the former decision is likely to only affect the most profitable intra-European routes[27], the latter decision is likely to affect **many** intra-European routes[28]. Consequently, this section focuses on this second effect.

I assume that both flag-carriers, Airline 1 and Airline 2, compete in the intra-European market HS, while continuing to exercise monopoly power in their domestic markets. Therefore, although **tacit** collusion could not, a priori, be excluded under the new regulatory environment,

[27] The sunk costs on new routes are likely to be high. According to Betts & Gardner [1992], European airlines estimate the introduction of a new route to cost £10-12 million. See also Levine [1987] for a discussion on sunk cost in this industry.

[28] Therefore, the drawback of this two country/two airline model (it fails to explicitly take fifth freedom competition into account) is less important.

I assume that both airlines act individually (i.e., noncooperatively) on the HS market. Airlines are assumed to play a Cournot static ("one shot") game. Two lines of argument are in favour of a quantity setting model. Firstly, Cournot behaviour in the airline industry has found empirical support in the literature (Reiss & Spiller [1989], Brander & Zhang [1990,1993]). Brander & Zhang's [1990] paper is particularly relevant for our analysis since they estimate conjectural variation parameters for *duopoly* airline routes[29]. They found that, in general, the Cournot assumption is consistent with the data[30]. Secondly, to the extent that the two flag-carriers have been keeping, until recently, stable bilateral agreements, it is unlikely that they would compete more vigorously (e.g., Bertrand behaviour) than Cournot competition would imply. More importantly, perhaps, is the general perception among airline managers that capacity (and therefore frequency) is the key variable in this industry. It is not surprising that American Airlines Chairman, Robert Crandall, recently reported that

"Capacity is how we compete in this business."[31]

In order to provide a simple comparison with the cartel solution, I assume the same symmetric networks (Figure 1) and same demand assumptions on travellers' preferences for intra-European air services. Consequently, the Airline 1 problem reduces to maximising (3). Similarly, the Airline 2 problem is to maximise its profit function (4). However, under the "competitive" assumption each individual airline has to select a quantity of output to maximise its own profit. The Cournot behaviourial assumption implies that when Airline 1(2) maximises its own profit, it takes Airline's 2(1) quantity as given. The symmetry of the model allows the analysis to concentrate on the symmetric Cournot-Nash equilibrium

[29] Their data set consists of 33 duopoly routes served by American Airlines and United Airlines in Chicago in 1985.

[30] Their related paper of 1993 examines the dynamic pattern of firm conduct. Although they are able to reject the hypothesis that the dynamic path of quantities and prices was characterised by mere repetition of the Cournot one shot solution, they suggest that data are more consistent with a quantity setting regime-switching model.

[31] Fortune Magazine, 14 July 1993, p.53.

where $Q^1_{HS} = Q^2_{HS}$. For simplicity, I work out the solution in terms of Airline 1. Given (1) and (2), it follows that the maximisation of (3) implies that first order conditions (5)-(9) be satisfied and that, in addition,

$$\frac{\partial \Pi_1}{\partial Q^1_{HS}} = 0 \implies \alpha - Q^1_{HS} - \frac{1}{2}Q^2_{HS} = 1 - \theta(Q^1_{HS} + Q_{BS} + Q_{AS}). \tag{23}$$

Notice that the marginal revenue in (23) is now different from the marginal revenue in (10), reflecting the Cournot assumption. Solving the system (5)-(9) and (23) yields the following Cournot-Nash equilibrium quantities

$$Q^1_{HS} \equiv Q_0^{comp} = \frac{(2-4\theta)(\alpha(1-2\theta)-1)}{26\theta^2 - 20\theta + 3} \tag{24}$$

$$Q_{AH} = Q_{BH} \equiv Q_1^{comp} = \frac{(3-5\theta)(\alpha(1-2\theta)-1)}{26\theta^2 - 20\theta + 3} \tag{25}$$

$$Q_{AS} = Q_{BS} \equiv Q_2^{comp} = \frac{\alpha(3-3\theta-4\theta^2)+11\theta-6}{26\theta^2 - 20\theta + 3} \tag{26}$$

$$Q_{AB} \equiv Q_3^{comp} = \frac{\alpha(3-2\theta-6\theta^2)+10\theta-6}{26\theta^2 - 20\theta + 3}. \tag{27}$$

In equilibrium, it can be shown that $Q^2_{HS} = Q_0^{comp}, Q_{ZS} = Q_{YS} = Q_1^{comp}, Q_{ZH} = Q_{YH} = Q_2^{comp}, Q_{ZY} = Q_3^{comp}$, i.e., Airline's 2 optimal quantities are similar.

As previously, this symmetric structure provides a simple way to check the second order conditions. In the Appendix (Section IX.4), it is shown that interior solutions exist for $\theta < \frac{20-\sqrt{88}}{52} \cong 0.2042$ (see Appendix (56)). Furthermore, in equilibrium, in order to have both positive quantities and marginal revenues(costs), the following inequalities must hold

$$\frac{6-11\theta}{3-3\theta-4\theta^2} < \alpha < \frac{3-5\theta}{\theta(9-16\theta)}. \tag{28}$$

Hereafter, I assume that, in equilibrium, both (28) and (56) are satisfied. Under these conditions, it is straightforward to verify that (24)-(27) are increasing in α.

Proposition 3 *As long as $\theta \in [0, 0.2042]$ and inequalities (28) hold, we have that $2Q_0^{comp} > Q_1^{comp}$. PROOF see Appendix.*

Therefore, for a given **nonstop** market, i.e., a market implying only one leg, the model shows that the total quantity provided on the competitive intra-European market ($2Q_0^{comp}$) is larger than the quantity provided on the monopoly domestic market (Q_1^{comp}). In other words, this result shows that, for a given nonstop market, output or traffic is more restricted under monopoly than it is under Cournot competition. As a consequence, ceteris paribus, the price is higher on the domestic market. It should be noticed that, as expected, Proposition 3 is the reverse of Proposition 1.

Proposition 4 *As long as $\theta \in [0, 0.2042]$ and inequalities (28) hold, we have that $Q_3^{comp} \geq Q_2^{comp}$. PROOF see Appendix.*

Consequently, although market AB is served by Airline 1 as a monopolist, the traffic between these two domestic peripheral cities is higher than the traffic between a peripheral city and the foreign hub (markets AS and BS). This counter intuitive result is due to the presence of increasing returns to density. On the HS market, Airline 1 has to divide the market with its competitor. In the presence of increasing returns to density, lower traffic per airline on the HS leg raises the marginal cost on the leg and generates a negative externality on markets using that leg (AS and BS). This explains why, for a given **one stop** market, the quantity provided by the monopoly airline (Q_3^{car}) is higher than the quantity in a market routing through the competitive segment (Q_2^{car}).

Corollary: Under the (limit) case of constant returns to density ($\theta = 0$), (28) implies that $2 < \alpha < \infty$, we have that $2Q_O^{comp} = \frac{4(\alpha-1)}{3}$, $Q_1^{comp} = \alpha - 1$, and $Q_2^{comp} = Q_3^{comp} = \alpha - 2$.

It follows that, in equilibrium, total traffic in the HS market ($2Q_0^{comp}$), is *always* greater than the traffic of the nonstop domestic markets (Q_1^{comp}), independent of the degree of returns to density. This result is an important feature of the model and departs from the previous result under collusive agreements (Section III), where we found that $2Q_0^{car} = Q_1^{car}$ when $\theta = 0$. It can be noticed that, as expected, in longer journey markets, i.e., markets using two legs (Q_2^{comp} and Q_3^{comp}), the optimal quantity is less than in shorter journey markets. Moreover, under constant returns to density, we have that $Q_2^{comp} = Q_3^{comp}$. This latter result is similar to the result derived in the collusive solution (Section III).

As previously, it can be shown that arbitrage conditions hold when (28) and (56) are satisfied. Therefore, in equilibrium,

$$P(Q_1^{comp}) + P(2Q_0^{comp}) > P(Q_2^{comp}) \quad \text{and} \quad 2P(Q_1^{comp}) > P(Q_3^{comp}). \qquad (29)$$

V The Collusive versus the "Competitive" Solution

Given the results obtained under the cartel solution (Section III) and the "competitive" solution (Section IV), it would be interesting to compare both scenarios, in order to assess which solution is socially preferable. In fact, until now I have compared quantities **within** a given solution. In this section, I provide a comparison of quantities between the two solutions and I measure the change in welfare arising from the more competitive environment[32]. Therefore, these results could provide an assessment of the new regulatory rules introduced in the EC airline industry.

A proper comparison implies the restriction of α in order to satisfy both solutions. In fact, in order to satisfy (21) and (28), the following inequalities must hold

$$\frac{4-7\theta}{2-2\theta^2-2\theta} < \alpha < \frac{3-5\theta}{\theta(9-16\theta)}. \qquad (30)$$

Hereafter, I assume that both (30) and (56) are satisfied in equilibrium.

Proposition 5 *As long as* $\theta \in [0, 0.2042]$ *and inequalities (30) hold, we have that* $2Q_0^{comp} > 2Q_0^{car}, Q_1^{comp} \geq Q_1^{car}, Q_2^{comp} \geq Q_2^{car}$ *and* $Q_3^{comp} \geq Q_3^{car}$. *Therefore, given the specifications of the model, we have that the "competitive" solution provides a strictly greater quantity in the HS market and greater or equal quantities in all other markets. PROOF see Appendix.*

[32] It should be remembered that our bench mark case is the collusive solution.

Corollary: Under the (limit) case of constant returns to density ($\theta = 0$), (30) implies that $2 < \alpha < \infty$, we have that $2Q_0^{comp} > 2Q_0^{car}, Q_1^{comp} = Q_1^{car}, Q_2^{comp} = Q_2^{car}$, and $Q_3^{comp} = Q_3^{car}$.

This is an important theoretical result. Competition on the intra-European leg not only increases the quantity provided on that market. but also increases the quantity on **all** other markets as soon as returns to density are increasing. This outcome occurs whenever the demand is weak or strong, so long as α satisfies (30). Therefore, all consumers of the network benefit from a greater competition on the intra-European route. This positive externality arises since the greater quantity on the HS market (as a whole) lowers the marginal cost on the leg, which in turn implies a lower price for the markets routing through the competitive segment. It should be stressed that this positive externality occurs also on markets not directly affected by the intra-European route, i.e., the purely domestic markets. This result tends to show that competition on one important market[33] has widespread *positive* effects throughout the network. A misrepresentation of such effects would provide an important bias in the analysis of the EC airline liberalisation.

The next step is to compute the net social welfare (NSW) arising from both solutions. Net social welfare is defined as the sum of consumers' surplus (CS) on each market ij plus the economic profit of the industry[34]. In the case of the linear inverse demand (1), the CS is represented in Figure 2. Therefore, in a given market ij, the CS is equal to

$$CS = \frac{[\alpha - (\alpha - \frac{Q_{ij}^*}{2})]Q_{ij}^*}{2} = \left(\frac{Q_{ij}^*}{2}\right)^2. \tag{31}$$

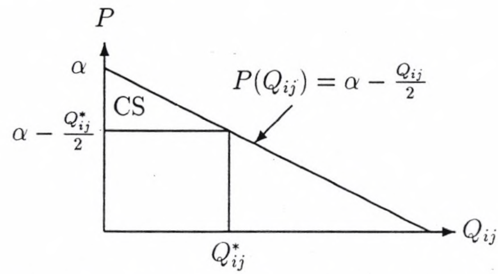
Given (31) and (17)-(20), we can show that the consumer surplus throughout the network under the collusive solution, CS^{car}, is given by the following expression

$$CS^{car}(\theta, \alpha) = (Q_0^{car})^2 + (Q_1^{car})^2 + (Q_2^{car})^2 + \frac{(Q_3^{car})^2}{2} \tag{32}$$

[33] It is important because it connects two hub airports.

[34] For the sake of comparison with the merger solution (Section VI), I consider NSW on the entire network to be composed of country 1 and country 2.

Figure 2: Linear Demand and Consumer Surplus



$$= \frac{\alpha^2(22 - 92\theta + 171\theta^2 - 208\theta^3 + 128\theta^4) - \alpha(68 - 224\theta + 184\theta^2) + 58 - 192\theta + 160\theta^2}{2(16\theta^2 - 13\theta + 2)^2}.$$

Similarly, given (31) and (24)-(27), the consumer surplus throughout the network under the "competitive" solution, CS^{comp}, is

$$\begin{aligned} CS^{comp}(\theta,\alpha) &= (Q_0^{comp})^2 + (Q_1^{comp})^2 + (Q_2^{comp})^2 + \frac{(Q_3^{comp})^2}{2} \qquad (33) \\ &= \frac{\alpha^2(53 - 244\theta + 492\theta^2 - 624\theta^3 + 396\theta^4)}{2(26\theta^2 - 20\theta + 3)^2} + \\ &\quad \frac{-\alpha(160 - 576\theta + 536\theta^2 - 32\theta^3) + 134 - 476\theta + 424\theta^2}{2(26\theta^2 - 20\theta + 3)^2}. \end{aligned}$$

Given (1), (2), ((51) see Appendix) and the optimal quantities (17)-(20), the economic profit of the industry under the collusive arrangement is

$$\Pi^{car}(\theta,\alpha) = \frac{\alpha^2(11 - 23\theta + 8\theta^2) - \alpha(34 - 56\theta) + 29 - 48\theta}{16\theta^2 - 13\theta + 2}. \qquad (34)$$

Similarly, the profit under the "competitive" solution is twice Airline's 1 profit ((54) see Appendix). Given the optimal quantities (24)-(27), this expression is equal to

$$\begin{aligned} \Pi^{comp}(\theta,\alpha) = & \frac{\alpha^2(49 - 438\theta + 1196\theta^2 - 1204\theta^3 + 332\theta^4)}{(26\theta^2 - 20\theta + 3)^2} + \\ & \frac{-\alpha(152 - 1292\theta + 3144\theta^2 - 2360\theta^3) + 130 - 1106\theta + 2696\theta^2 - 2028\theta^3}{(26\theta^2 - 20\theta + 3)^2}. \qquad (35) \end{aligned}$$

Since we have demonstrated that quantities under the competitive solution are greater than under the collusive solution, it must be the case that $CS^{comp} > CS^{car}$. Of course, the profit of the industry is larger under the collusive agreement. Therefore, the comparison of the NSW reflects these two contrasting effects. Given (32), (33), (34) and (35), we can show that $NSW^{comp} - NSW^{car} > 0$, for any $\theta \in [0, 0.2042]$ and α satisfying (30). Figure 3 (see Appendix) illustrates this difference in a three dimension space. The positive surface indicates that the NSW under the "competitive" solution is greater than the NSW under the collusive solution. This difference is increasing in the return to density and the appropriate demand parameters.

By setting $\theta = 0$, i.e., constant returns to density, the NSW is reduced to a single variable problem and it becomes straightforward to show that, in equilibrium, $NSW^{comp} - NSW^{car} > 0$. In effect, we have that

$$\begin{aligned}
NSW^{comp} - NSW^{car} > 0 &\iff CS^{comp} + \Pi^{comp} > CS^{car} + \Pi^{car} \\
&\implies \frac{151\alpha^2 - 464\alpha + 394}{18} > \frac{66\alpha^2 - 204\alpha + 174}{8} \\
&\implies \alpha^2 - 2\alpha + 1 > 0 \\
&\implies (\alpha - 1)^2 > 0 \quad \text{which is always true.}
\end{aligned}$$

The results obtained in this section indicate that, given the assumptions of the model, it would be socially desirable to set a more competitive environment on intra-European routes when airlines operate hub-and-spoke networks. Therefore, from a theoretical point of view, the establishment of more competition on the intra-European routes[35], **even without new entrants**, should be encouraged.

VI The Merger Solution

Until the late 80s., the strategic response from the EC airlines to an increasingly liberalised, competitive worldwide market, was essentially that

[35] As the introduction of the Third Package is supposed to bring about.

of cooperation. This cooperation took mainly the form of (technical) collaboration and partnership among established European flag-carriers together with major airlines from the U.S. and other continents. Recently, the current trend of consolidation in the European airline industry suggests that the flag-carriers' strategies are 1) to absorb the small, principally domestic, regional airlines and 2) to form cross-border flag-carriers mergers[36]. Whereas the main goal for taking over regional airlines is to "feed" central hub airports by regional traffic, incentives for cross-border mergers are more directly related to achieving higher levels of efficiency. This is particularly true for mergers involving medium-sized flag-carriers, where specialists and airline managers recognize that the prospects for cost savings are impressive[37]. Since the second type of merger is likely to become an important issue in the EC airline industry, this section will deal with the cross-border merger problem. Given the framework developed in Section IV, let us suppose that, in response to a more liberal environment, the two hub-and-spoke airlines (Airline 1 and Airline 2) decide to merge and form a (cross-border) common entity. Should the EC regulatory authority approve the merger of these two flag-carriers and therefore authorize the formation of a monopoly over the entire network? What are the effects of the elimination of competition on the intra-European leg? What are the spillovers on the other markets? The purpose of the following analysis is to explore these important questions.

[36] Air France Chairman, B. Attali, recently reported that: Cannibalism has become a strategic model in this industry. (Fortune Magazine, 2 Nov. 1992, p.26.)

[37] For an airline specialist view see, for example, the Economist, 13 Nov. 1993, p.70. and H. Carnegy & I. Rodger in Financial Times, 24 Nov. 1993. According to an internal Swissair's document, the cost savings from a merger with Austrian Airlines, SAS and KLM (the project is known as Alcazar) could account for 1100 million Ecus in 1997. According to KLM Annual Report (1992/93), this hypothetical merger should principally

- achieve a greater efficiency and lower cost levels at the four airlines,
- strengthen the joint market position of the partners based on various European hubs,
- form a customer-driven global route network.

It should be stressed that the negotiations for this merger failed last winter, but opportunities for a merger involving fewer partners are still open at the moment.

Farell & Shapiro [1990] demonstrated that, in general, horizontal mergers in Cournot oligopoly raise prices if they generate no synergies between the merging firms. The important theoretical analysis of airline mergers is due to Brueckner & Spiller [1991]. Under some conditions, they show that 1) the merger of a hub airline and a nonhub competitor may raise the social welfare, and 2) in a network, the merger may lead to welfare gains outside the markets which are of primary concern with the increase in market power. Borenstein [1990] has studied the effect on airfares of two U.S. airlines mergers: the TWA-Ozark and Northwest-Republic mergers. He found a significant increase in relative airfares on routes affected by the Northwest-Republic merger, but no evidence of fare increases associated with the TWA-Ozark merger. Kim & Singal's [1993] paper provides insights into how market power and efficiency gains interact in U.S. airline mergers. They find that, in general, airline mergers during the 1985-1988 sample period led to higher fares on routes affected by the merger, creating wealth transfers from consumers. However, according to these authors, most of the effect of increased market power takes place during the merger discussion. Once the merger is completed, they find, in fact, that the efficiency gains offset much of the impact of increased market power, at least when mergers do not involve financially distressed airlines. In effect, they report that (p.567)

> *"Efficiency gains start to kick in after merger completion, mainly for routes with potential sources of direct operating synergies, such as routes on which the merging firms have common hubs or provide overlapping service. For these routes, efficiency gains offset much of the impact of increased market power."*

In the present model, the potential efficiency gains stemming from the merger are simply captured by the increase in traffic densities on the **overlapping** intra-European leg. Following Kim & Singal's [1993] terminology, because prior to the merger the airlines operated an overlapping leg without a common hub, it is likely that *"in the air"* synergies[38] arise from the use of fewer aircraft and/or a better load factor, i.e., capacity

[38] In contrast, *"on the ground"* synergies arise from better use of gates/slots and ground crews.

utilisation[39]. Therefore, the interesting question is: how much economies of density (and corresponding demand level) are needed for a merger to increase output and reduce price with respect to the "competitive" solution ? Putting it another way, how much efficiency gains (in terms of economies of density) should the merger generate in order to offset the effects of exercising additional market power by virtue of reducing the number of competitors by one?

Let us assume that because of the network complementarity (see Figure 1), the merged airline, Airline M, operates on the same network structure. In particular, it maintains the two separate and specialised hubs[40]. Again, in order to provide a simple comparison with the previous solutions, I assume the same demand and cost specifications as in (1) and (2) except that now $i = A, B, H, S, Z, Y$, $j = A, B, H, S, Z, Y$, (with $i \neq j$) and $l = 1, 2, 3, 4, 5$, in order to properly take the new merger structure into account. I maintain also the same assumptions on travellers' preferences for intra-European air services (see Section II). In particular, it is assumed that $D(Q_{AZ}) = D(Q_{AY}) = D(Q_{BZ}) = D(Q_{BY}) = 0$. Consequently, the Airline M problem reduces to maximizing

$$\begin{aligned}
\Pi^M &= P(Q_{AH})Q_{AH} + P(Q_{AB})Q_{AB} + P(Q_{BH})Q_{BH} + P(Q_{AS})Q_{AS} \\
&\quad + P(Q_{BS})Q_{BS} + P(Q_{HS})Q_{HS} + P(Q_{SZ})Q_{SZ} + P(Q_{ZH})Q_{ZH} \\
&\quad + P(Q_{ZY})Q_{ZY} + P(Q_{YH})Q_{YH} + P(Q_{YS})Q_{YS} \\
&\quad - C_1(Q_{AH} + Q_{AB} + Q_{AS}) - C_2(Q_{BH} + Q_{AB} + Q_{BS})
\end{aligned}$$

[39] In this model, since both airlines are equally efficient, a merger does not offer an opportunity to rationalise production in the traditional sense, i.e., without changing the total level of output, to shift output to the more efficient airline. Nor is it the case that by combining their aircraft (capital) they would produce more efficiently, since as pointed out by Brueckner & Spiller [1991] the existence of an active rental market for aircraft gives no particular advantage associated with acquiring another airline's capital. Of course, it is clear that the existence of complementary resources (capital) could enhance the efficiency gains stemming from the merger. Increases in efficiency can arise from economies of scale or scope related to aircraft maintenance, marketing and sales services, management, extended network, airport gates acquisition, etc.. To the extent that the model only captures the economies of density, it is likely that the efficiency gains provided in this section are underestimated.

[40] For instance, city H could serve as the hub for the southern markets, whereas city S could serve as the hub for the northern markets. See Oum & Tretheway [1990] for a discussion on the various types of hub-and-spoke structures.

$$-C_3(Q_{HS}+Q_{AS}+Q_{BS}+Q_{ZH}+Q_{YH})$$
$$-C_4(Q_{ZS}+Q_{ZY}+Q_{ZH})-C_5(Q_{YS}+Q_{YH}+Q_{YZ}). \tag{36}$$

where Π^M is the profit function of the merged airline. It should be noticed from (36) that Airline M generates its revenue from 11 city-pair markets whereas its costs correspond to aircraft flown on five different legs. It should be stressed also that on the intra-European leg, Airline M carries now **all** the travellers of the HS market (Q_{HS}) as well as all the connecting travellers ($Q_{AS}+Q_{BS}+Q_{ZH}+Q_{YH}$) using the intra-European leg. Assuming interior solutions, the solution of Airline M implies that the first order conditions (5), (6), (7), (11), (12), (13) be satisfied and that in addition,

$$\frac{\partial \Pi^M}{\partial Q_{AS}}=0 \implies \alpha - Q_{AS} = 2-2\theta Q_{AS}$$
$$-\theta(Q_{AH}+Q_{AB}+Q_{HS}+Q_{BS}+Q_{ZH}+Q_{YH}) \tag{37}$$
$$\frac{\partial \Pi^M}{\partial Q_{BS}}=0 \implies \alpha - Q_{BS} = 2-2\theta Q_{BS}$$
$$-\theta(Q_{BH}+Q_{AB}+Q_{HS}+Q_{AS}+Q_{ZH}+Q_{YH}) \tag{38}$$
$$\frac{\partial \Pi^M}{\partial Q_{HS}}=0 \implies \alpha - Q_{HS} = 1-\theta(Q_{HS}+Q_{AS}+Q_{BS}+Q_{ZH}+Q_{YH}) \tag{39}$$
$$\frac{\partial \Pi^M}{\partial Q_{ZH}}=0 \implies \alpha - Q_{ZH} = 2-2\theta Q_{ZH}$$
$$-\theta(Q_{ZS}+Q_{ZY}+Q_{YH}+Q_{HS}+Q_{AS}+Q_{BS}) \tag{40}$$
$$\frac{\partial \Pi^M}{\partial Q_{YH}}=0 \implies \alpha - Q_{YH} = 2-2\theta Q_{YH}$$
$$-\theta(Q_{YS}+Q_{ZY}+Q_{ZH}+Q_{HS}+Q_{AS}+Q_{BS}). \tag{41}$$

It should be pointed out that the new first order conditions (37)-(41) are related to markets routing through the HS leg. Solving the system constituted by (5)-(7),(11)-(13) and (37)-(41) yields the optimal quantities offered by the merged airline. These optimal quantities are

$$Q_{HS} \equiv Q_0^M = \frac{\alpha(1-4\theta+8\theta^2)-1}{16\theta^2-9\theta+1} \tag{42}$$

$$Q_{AH}=Q_{BH}=Q_{ZS}=Q_{YS} \equiv Q_1^M = \frac{\alpha(1-6\theta+6\theta^2)+4\theta-1}{16\theta^2-9\theta+1} \tag{43}$$

$$Q_{AS}=Q_{BS}=Q_{ZH}=Q_{YH} \equiv Q_2^M = \frac{(1-2\theta)(\alpha(1+\theta)-2)}{16\theta^2-9\theta+1} \tag{44}$$

$$Q_{AB} = Q_{ZY} \equiv Q_3^M = \frac{(1-4\theta)(\alpha(1+\theta)-2)}{16\theta^2 - 9\theta + 1}. \tag{45}$$

Consequently, the Airline M problem is reduced to a four variables problem. It should be mentioned that, in contrast to the previous solutions, (42) corresponds to the total traffic in the HS city-pair market. In the Appendix (Section IX.8), it is shown that the second order conditions are satisfied as long as $\theta \leq \frac{9-\sqrt{17}}{32} \cong 0.1524$ (see Appendix (59)). Moreover, in order to have both positive quantities and marginal revenues, we must ensure that in equilibrium the following inequalities hold

$$\frac{2}{1+\theta} < \alpha < \frac{1}{\theta(5-8\theta)}. \tag{46}$$

Hereafter, I assume that both (46) and (59) are satisfied in equilibrium. Under these conditions, it is straightforward to verify that (42)-(45) are increasing in the demand parameter α.

Proposition 6 *As long as* $\theta \in [0, 0.1524]$ *and inequalities (46) hold, we have that* $Q_0^M \geq Q_1^M$. *PROOF see Appendix.*

This result suggests that, for a given **nonstop** market, the traffic transported on the HS city-pair market is higher than the traffic in any other city-pair market. This intuitive result can be explained by the position of the cities H and S in the network. Airline M, as unique operator of the central leg of the network (the leg 3 connects the two hubs), is able to achieve higher economies of density, reducing, ceteris paribus, the cost of this leg. Actually, it is interesting to note that the "competitive" solution offered the same qualitative result (see Proposition 3).

Proposition 7 *As long as* $\theta \in [0, 0.1524]$ *and inequalities (46) hold, we have that* $Q_2^M \geq Q_3^M$. *PROOF see Appendix.*

Consequently, for a given **one stop** market, the quantity provided on markets connecting two peripheral cities (markets AB, ZY) is lower than the quantity provided on markets connecting a peripheral city and the foreign hub (markets AS, BS, ZH, YH). Therefore, consumers are, ceteris paribus, better off on these latter markets. This intuitive result is due to the presence of increasing returns to traffic density. Due to its monopoly position in the HS market, Airline M is able to transport a higher traffic level on this market, which generates a positive externality on markets using that leg. It is interesting to note that Proposition 7 is the reverse of Proposition 2 and Proposition 4, where we found that, under the collusive and the "competitive" solutions, consumers were better off in the former markets, i.e., AB and ZY.

Corollary: Under the (limit) case of constant returns to density ($\theta = 0$), (46) implies that $2 < \alpha < \infty$, we have that $Q_O^M = Q_1^M = \alpha - 1$, and $Q_2^M = Q_3^M = \alpha - 2$.

Therefore, in equilibrium, when $\theta = 0$, Airline M provides the same quantity on the nonstop city-pairs markets ($Q_0^M = Q_1^M$). As expected, a lower quantity is provided in longer journey markets, i.e., on markets routing through two legs. Moreover, in these latter markets, the quantity provided is identical ($Q_2^M = Q_3^M$).

Finally, it can easily be shown that, in equilibrium,

$$P(Q_1^M) + P(Q_0^M) > P(Q_2^M) \quad \text{and} \quad 2P(Q_1^M) > P(Q_3^M), \qquad (47)$$

i.e., the arbitrage conditions hold if the first order conditions (5)-(7),(11)-(13) and (37)-(41) and the second order conditions (59) are satisfied.

VII The Merger versus the "Competitive" Solution

In order to give an answer to the questions arising from the preceding section, I propose to compare the "competitive" solution (Section IV) with the merger solution (Section VI). To this end, I follow the same methodology developed in Section V. A proper comparison implies the restriction of α in order to satisfy both solutions. In order to satisfy (28) and (46), the following inequalities must hold

$$\frac{6-11\theta}{3-3\theta-4\theta^2} < \alpha < \frac{1}{\theta(5-8\theta)}. \tag{48}$$

Notice that (48) is satisfied for $\theta \in [0, 0.1479]$. Consequently, restricting θ to $[0, 0.1479]$, ensures that all the appropriate conditions of the model, i.e., (48), (56) and (59), are satisfied in equilibrium.

Proposition 8 *As long as $\theta \in [0, 0.1479]$ and inequalities (48) hold, we have that $Q_1^M \geq Q_1^{comp}, Q_2^M \geq Q_2^{comp}, Q_3^M \geq Q_3^{comp}$ and $Q_0^M > (<) 2Q_0^{comp}$. PROOF see Appendix.*

Therefore, given the specification of the model, we have an ambiguous result in the HS market which depends on a complex relation between the returns to density and the demand parameters. It can be shown that when the returns to density are sufficiently strong, i.e., $\theta \in [0.093, 0.1479]$ **and** the demand satisfies[41] $\alpha^* < \alpha < \frac{1}{\theta(5-8\theta)}$, the merger solution provides a **greater** quantity on the HS market. In all the other markets, the merger solution provides greater or equal quantities with respect to the "competitive" solution, for all values of α and θ. This result highlights the importance of the network structure in the analysis of the airline industry. In effect, it suggests that the merger may lead to greater(lower) quantities(prices) on the markets which are not directly affected by an increase in market power, i.e., (Q_1^M, Q_2^M and Q_3^M). Brueckner & Spiller [1991] found a similar result using another hub-and-spoke structure.

Corollary: Under the (limit) case of constant returns to density ($\theta = 0$), (48) implies that $2 < \alpha < \infty$, we have that $Q_0^M < 2Q_0^{comp}, Q_1^M = Q_1^{comp}, Q_2^M = Q_2^{comp}$, and $Q_3^M = Q_3^{comp}$.

Therefore, the "competitive" solution provides a greater quantity on the market where competition is effective (HS). On the other markets, both solutions provide the same quantities. It should be noticed that under the absence of increasing returns to density, the merger solution

[41] See the Proof.

mimics the cartel outcome. The comparison of the merger and the "competitive" solutions is shown in Table I (see Appendix) for some selected values of θ up to 0.147.

This second theoretical result deserves some comments. In effect, the model suggests that, for some variety of parameters, the merger solution provides a greater quantity on *all* city-pairs markets. To understand the intuition behind this result, it is important to remember that in the "competitive" solution, competition is really effective on the intra-European market HS[42]. This means that on all the other markets, only one airline actually provides air services. Therefore, in these latter markets, the merger and the "competitive" solutions are, basically, similar in terms of market power. The merged airline provides a larger quantity in these markets, because it is able to exploit the increasing returns to density on the intra-European leg HS. In fact, the decrease of the marginal cost on that leg has positive effects on the other markets throughout the network (even in those markets which are not routing through the HS leg). Matters are quite different in the HS city-pair market, where the market power changes in function of the solution analysed. Not surprisingly, the model suggests that when the efficiency gains (through the returns to density) are sufficiently important ($0.0925 < \theta < 0.1479$) **and** the demand is relatively strong ($\alpha > \alpha^*$), consumers on the HS market are better off under the merger solution. Conversely, when the returns to density are relatively weak ($\theta < 0.0925$), the "competitive" outcome is preferred from the HS consumers' point of view since the quantity(price) is larger(lower). It should be noticed that, when $\theta < 0.0925$ ($\theta > 0.139$), the "competitive" (merger) outcome is preferred **whatever** the demand is weak or strong[43]. Hence, these results emphasize the key role played by the returns to density.

The final desirability of the merger should be assessed after comparing the NSW under the merger and the "competitive" solutions. Given (31) and (42)-(45), the consumer surplus throughout the network under

[42] By assumption, the model excludes cabotage competition.

[43] These figures are approximated by computer simulation. See Table I.

the merger solution, CS^M, equals

$$\begin{aligned} CS^M(\theta,\alpha) &= \frac{(Q_0^M)^2}{4} + (Q_1^M)^2 + (Q_2^M)^2 + \frac{(Q_3^M)^2}{2} \\ &= \frac{\alpha^2(11 - 76\theta + 214\theta^2 - 288\theta^3 + 256\theta^4)}{4(16\theta^2 - 9\theta + 1)^2} + \\ & \quad \frac{-\alpha(34 - 192\theta + 320\theta^2) + 29 - 160\theta + 256\theta^2}{4(16\theta^2 - 9\theta + 1)^2}. \end{aligned} \tag{49}$$

Given (36) and the optimal quantities (42)-(45), the economic profit of Airline M, hence of the industry, is

$$\Pi^M(\theta,\alpha) = \frac{\alpha^2(11 - 38\theta + 16\theta^2) - \alpha(34 - 96\theta) + 29 - 80\theta}{2(16\theta^2 - 9\theta + 1)}. \tag{50}$$

The NSW under the merger solution, NSW^M, is obtained by adding (49) and (50). Unfortunately, the difference between NSW^M and NSW^{comp} has not a closed form solution. This difference depends in a complex relation between the parameters of the model (θ and α). The comparison of NSW^{comp} and NSW^M is shown in Table II (see Appendix) for some selected values of θ up to 0.14. It is interesting to note that NSW^M is superior to NSW^{comp} when the **low** returns to density are balanced with a relatively **high** demand. When the returns to density are relatively important ($\theta > 0.1$), Table II shows that the merger solution is always preferable from the welfare point of view.

Figure **??** (see Appendix) illustrates this difference in the three dimension space. As expected, the "competitive" solution dominates the merger solution if the returns to density **and** the demand are relatively weak. An increase in θ sustained by a relatively high demand reverses the previous result. When $\theta > 0.1$ there is no ambiguity, the merger outcome is socially preferable.

By setting $\theta = 0$, the NSW is reduced to a single variable problem. As expected, in this case the "competitive" solution dominates the merger solution. In effect, we have that

$$\begin{aligned} NSW^{comp} - NSW^M > 0 &\iff CS^{comp} + \Pi^{comp} > CS^M + \Pi^M \\ &\Longrightarrow \frac{151\alpha^2 - 464\alpha + 394}{18} > \frac{33\alpha^2 - 102\alpha + 87}{4} \end{aligned}$$

$$\Longrightarrow \quad \alpha^2 - 2\alpha + 1 > 0$$
$$\Longrightarrow \quad (\alpha - 1)^2 > 0 \quad \text{which is always true.}$$

The results obtained in this section give some insights into the opportunity of a socially desirable cross-border merger. In particular, this model shows that a merger between two airlines organised in hub-and-spoke networks may increase the social welfare when the efficiency gains (obtained through the returns to density) are relatively important[44]. Consequently, under increasing returns to density, the threat of monopolisation through the merger should not be of primary concern to EC antitrust authorities. In addition, given that the markets which benefit from the merger (Q_1^M, Q_2^M and Q_3^M) are outside the market of direct concern with an increase in market power (Q_0^M), exclusive focus on gains and losses in this latter market may have the effect of blocking a socially desirable cross-border merger. This model also suggests that purely domestic consumers should not, a priori, be harmed by such a cross-border merger. Finally, it should be noticed that, although it is not a matter of concern to this section, the merger solution dominates the collusive solution for all the values of θ and α allowed by the model.

VIII Conclusion

This paper provides an analysis of the intra-European airline competition within an explicit (hub-and-spoke) network. Using the quantity setting paradigm, optimal solutions are derived for various competition scenarios. The model highlights that under sufficient increasing returns to density, the threat of monopolisation through the merger should not be of primary concern to EC antitrust authorities. The model rather suggests that EC authorities should ban bilateral collusive agreements between flag-carriers.

This analysis could be extended in the following different directions.

[44] Recently, Röller & Sickles [1993] argued that competition policy in Europe should allow mergers or strategic alliances to be formed if they do translate in costs savings (especially from the elimination of cost inefficiencies) and increased international competitiveness.

Firstly, it would be interesting to see if these results are confirmed under a price setting strategy with product (air service) differentiation. Secondly, a market specific demand parameter (α_{ij}) could be introduced. Thirdly, this model could analyse the potential effects of cabotage competition. In that case, it is assumed that access to domestic routes is open to foreign flag-carriers. Consequently, flag-carriers loose their "hinterland" and duopolistic competition arises throughout the network. Given that airlines face each other in several markets, it would be interesting to see how they could compete less vigorously in one market due to the fear of retaliation in another (multimarket contact argument).

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IX Appendix

IX.1 Second Order Conditions for the Collusive Solution

In fact, given (17)-(20), the cartel profit, $\Pi^{car} = \Pi_1 + \Pi_2$ can be simplified to the following expression

$$\begin{aligned}\Pi^{car} &= 2(\alpha - Q_0)Q_0 + 4(\alpha - \frac{Q_1}{2})Q_1 + 4(\alpha - \frac{Q_2}{2})Q_2 + 2(\alpha - \frac{Q_3}{2})Q_3 \\ &\quad -4(Q_1 + Q_2 + Q_3 - \frac{\theta(Q_1 + Q_2 + Q_3)^2}{2}) \\ &\quad -2(Q_0 + 2Q_2 - \frac{\theta(Q_0 + 2Q_2)^2}{2}). \end{aligned} \tag{51}$$

From (51), it turns out that second order conditions reduce to the following symmetric Hessian matrix

$$\begin{vmatrix} \theta - 2 & 0 & 2\theta & 0 \\ 0 & 2(\theta - 1) & 2\theta & 2\theta \\ 2\theta & 2\theta & 2(3\theta - 1) & 2\theta \\ 0 & 2\theta & 2\theta & 2\theta - 1 \end{vmatrix} \tag{52}$$

The maximisation of (51) requires (52) to be negative semidefinite. This condition is verified if and only if the principal minor determinants of order q have sign $(-1)^q$ for $q = 1, 2, 3, 4$ [45]. It can be easily verified that the sign of the principal minor determinants of (52) properly alternates if

$$16\theta^2 - 13\theta + 2 \geq 0. \tag{53}$$

[45] See Varian [1992].

This holds for any $\theta \leq \frac{13-\sqrt{41}}{32} \cong 0.2062$ [46]. It should be noticed that the quadratic function (53) corresponds to the denominator of the optimal quantities (17)-(20).

IX.2 Proof of Proposition 1

$$
\begin{aligned}
Q_1^{car} \geq 2Q_0^{car} \quad &\Longleftrightarrow \quad \frac{(2-3\theta)(\alpha(1-2\theta)-1)}{16\theta^2-13\theta+2} \geq \frac{(2-4\theta)(\alpha(1-2\theta)-1)}{16\theta^2-13\theta+2} \\
&\Longrightarrow \quad \theta \geq 0 \quad \text{since} \quad \alpha > \frac{1}{1-2\theta} \quad \text{under (21).}
\end{aligned}
$$

IX.3 Proof of Proposition 2

$$
\begin{aligned}
Q_3^{car} \geq Q_2^{car} \quad &\Longleftrightarrow \quad \frac{\alpha(2-4\theta^2-\theta)+6\theta-4}{16\theta^2-13\theta+2} \geq \frac{\alpha(2-2\theta^2-2\theta)+7\theta-4}{16\theta^2-13\theta+2} \\
&\Longrightarrow \quad \theta(\alpha(1-2\theta)-1) \geq 0 \\
&\Longrightarrow \quad \theta \geq 0 \quad \text{since} \quad \alpha > \frac{1}{1-2\theta} \quad \text{under (21).}
\end{aligned}
$$

IX.4 Second Order Conditions for the "Competitive" Solution

As previously, using (1) and (2) and (24)-(27), the Airline 1 problem (3) reduces to maximising the following expression

$$
\begin{aligned}
\Pi_1 \;=\; & (\alpha - Q_0)Q_0 + 2(\alpha - \frac{Q_1}{2})Q_1 + 2(\alpha - \frac{Q_2}{2})Q_2 + (\alpha - \frac{Q_3}{2})Q_3 \\
& -2(Q_1 + Q_2 + Q_3 - \frac{\theta(Q_1+Q_2+Q_3)^2}{2}) \\
& -(Q_O + 2Q_2 - \frac{\theta(Q_0+2Q_2)^2}{2}). \qquad (54)
\end{aligned}
$$

[46] Given that this quadratic function admits two roots, I assume that $\theta \in [0, 0.2062]$. Actually, this restriction is consistent with the empirical findings provided by Caves et al. [1984].

Consequently, the second order conditions from (54) reduce to the following symmetric Hessian matrix

$$\begin{vmatrix} \theta-2 & 0 & 2\theta & 0 \\ 0 & 2(\theta-1) & 2\theta & 2\theta \\ 2\theta & 2\theta & 2(3\theta-1) & 2\theta \\ 0 & 2\theta & 2\theta & 2\theta-1 \end{vmatrix} \tag{55}$$

It should be notice that this matrix is similar to (52). As a consequence, (55) is negative semidefinite if (53) is verified, i.e., if $16\theta^2 - 13\theta + 2 \geq 0$. For computational convenience, I restrict θ in order to have positive quantities. This is, in part, satisfied if the denominator in (24)-(27) is positive, i.e., if

$$26\theta^2 - 20\theta + 3 > 0. \tag{56}$$

In turn, this implies $\theta < \frac{20-\sqrt{88}}{52} \cong 0.2042$ [47].

IX.5 Proof of Proposition 3

$$\begin{aligned} 2Q_0^{comp} > Q_1^{comp} \quad &\Longleftrightarrow \quad \frac{(4-8\theta)(\alpha(1-2\theta)-1)}{26\theta^2-20\theta+3} > \frac{(3-5\theta)(\alpha(1-2\theta)-1)}{26\theta^2-20\theta+3} \\ &\Longrightarrow \quad (4-8\theta) > (3-5\theta) \quad \text{since} \quad \alpha > \frac{1}{1-2\theta} \quad \text{under (28)}, \\ &\Longrightarrow \quad \theta < \frac{1}{3}. \end{aligned}$$

IX.6 Proof of Proposition 4

$$\begin{aligned} Q_3^{comp} \geq Q_2^{comp} \quad &\Longleftrightarrow \quad \frac{\alpha(3-2\theta-6\theta^2)+10\theta-6}{26\theta^2-20\theta+3} \geq \frac{\alpha(3-3\theta-4\theta^2)+11\theta-6}{26\theta^2-20\theta+3} \\ &\Longrightarrow \quad \theta(\alpha(1-2\theta)-1) \geq 0 \\ &\Longrightarrow \quad \theta \geq 0 \quad \text{since} \quad \alpha \geq \frac{1}{1-2\theta} \quad \text{under (28)}. \end{aligned}$$

[47] Actually, it can be noticed that this figure is very close to the figure of condition (53), i.e., $\theta < 0.2062$.

IX.7 Proof of Proposition 5

$$
\begin{aligned}
2Q_0^{comp} > 2Q_0^{car} &\iff \frac{(4-8\theta)(\alpha(1-2\theta)-1)}{26\theta^2-20\theta+3} > \frac{(2-4\theta)(\alpha(1-2\theta)-1)}{16\theta^2-13\theta+2} \\
&\Longrightarrow 2(16\theta^2-13\theta+2) > 26\theta^2-20\theta+3 \\
&\Longrightarrow 6\theta^2-6\theta+1>0 \quad \text{which is satisfied for} \quad \theta \in [0, 0.2113].
\end{aligned}
$$

$$
\begin{aligned}
Q_1^{comp} \geq Q_1^{car} &\iff \frac{(3-5\theta)(\alpha(1-2\theta)-1)}{26\theta^2-20\theta+3} \geq \frac{(2-3\theta)(\alpha(1-2\theta)-1)}{16\theta^2-13\theta+2} \\
&\Longrightarrow (3-5\theta)(16\theta^2-13\theta+2) \geq (2-3\theta)(26\theta^2-20\theta+3) \\
&\Longrightarrow \theta^2(1-2\theta) \geq 0 \\
&\Longrightarrow \theta \leq \frac{1}{2}.
\end{aligned}
$$

$$
\begin{aligned}
Q_2^{comp} \geq Q_2^{car} &\iff \frac{\alpha(3-3\theta-4\theta^2)+11\theta-6}{26\theta^2-20\theta+3} \geq \frac{\alpha(2-2\theta^2-2\theta)+7\theta-4}{16\theta^2-13\theta+2} \\
&\Longrightarrow \theta(1-2\theta)(1-3\theta)[\alpha(1-2\theta)-1] \geq 0 \\
&\Longrightarrow \alpha \geq \frac{1}{1-2\theta} \quad \text{which is true under (30).}
\end{aligned}
$$

$$
\begin{aligned}
Q_3^{comp} \geq Q_3^{car} &\iff \frac{\alpha(3-2\theta-6\theta^2)+10\theta-6}{26\theta^2-20\theta+3} \geq \frac{\alpha(2-4\theta^2-\theta)+6\theta-4}{16\theta^2-13\theta+2} \\
&\Longrightarrow 2\theta^2(1-2\theta)[\alpha(1-2\theta)-1] \geq 0 \\
&\Longrightarrow \alpha \geq \frac{1}{1-2\theta} \quad \text{which is true under (30).}
\end{aligned}
$$

IX.8 Second Order Conditions for the Merger Solution

Using (42)-(45), the Airline M problem (36) can be simplified to the following expression

$$
\begin{aligned}
\Pi^M &= (\alpha - \frac{Q_0}{2})Q_0 + 4(\alpha - \frac{Q_1}{2})Q_1 + 4(\alpha - \frac{Q_2}{2})Q_2 + 2(\alpha - \frac{Q_3}{2})Q_3 \\
&\quad -4(Q_1+Q_2+Q_3 - \frac{\theta(Q_1+Q_2+Q_3)^2}{2}) \\
&\quad -(Q_O + 4Q_2 - \frac{\theta(Q_0+4Q_2)^2}{2}). \qquad (57)
\end{aligned}
$$

Although similar to the joint profit maximisation problem (51), it is important to stress that (57) corresponds to a different expression. It turns out that the second order conditions from (57) reduce to the following symmetric Hessian matrix

$$\begin{vmatrix} \theta-1 & 0 & 4\theta & 0 \\ 0 & 4(\theta-1) & 4\theta & 4\theta \\ 4\theta & 4\theta & 4(5\theta-1) & 4\theta \\ 0 & 4\theta & 4\theta & 2(2\theta-1) \end{vmatrix} \quad (58)$$

It can be shown that (58) is negative semidefinite if

$$16\theta^2 - 9\theta + 1 > 0. \quad (59)$$

This holds for any $\theta \leq \frac{9-\sqrt{17}}{32} \cong 0.1524$ [48]. It should be pointed out that (59) corresponds to the denominator of the optimal quantities (42)-(45).

IX.9 Proof of Proposition 6

$$Q_0^M \geq Q_1^M \iff \frac{\alpha(1-4\theta+8\theta^2)-1)}{16\theta^2-9\theta+1} \geq \frac{(\alpha(1-6\theta+6\theta^2)+4\theta-1)}{16\theta^2-9\theta+1}$$
$$\implies \theta \geq 0 \quad \text{since} \quad \alpha > \frac{2}{1+\theta} \quad \text{under (46).}$$

IX.10 Proof of Proposition 7

$$Q_2^M \geq Q_3^M \iff \frac{(1-2\theta)(\alpha(1+\theta)-2)}{16\theta^2-9\theta+1} \geq \frac{(1-4\theta)(\alpha(1+\theta)-2)}{16\theta^2-9\theta+1}$$
$$\implies \theta \geq 0 \quad \text{since} \quad \alpha > \frac{2}{1+\theta} \quad \text{under (46).}$$

[48] Given that this quadratic function admits two roots, I assume that $\theta \in [0, 0.1524]$.

IX.11 Proof of Proposition 8

$$
\begin{aligned}
Q_1^M \geq Q_1^{comp} &\iff \frac{\alpha(1-6\theta+6\theta^2}{16\theta^2-9\theta+1} \geq \frac{(3-5\theta)(\alpha(1-2\theta)-1)}{26\theta^2-20\theta+3} \\
&\implies \alpha(7-10\theta-4\theta^2)-13+24\theta \geq 0 \\
&\implies \alpha \geq \frac{13-24\theta}{7-10\theta-4\theta^2} \quad \text{which is always true under (48).}
\end{aligned}
$$

$$
\begin{aligned}
Q_2^M \geq Q_2^{comp} &\iff \frac{(1-2\theta)(\alpha(1+\theta)-2)}{16\theta^2-9\theta+1} \geq \frac{\alpha(3-3\theta-4\theta^2)+11\theta-6}{26\theta^2-20\theta+3} \\
&\implies \alpha(7-10\theta-4\theta^2)-13+24\theta \geq 0 \\
&\implies \alpha \geq \frac{13-24\theta}{7-10\theta-4\theta^2} \quad \text{which is always true under (48).}
\end{aligned}
$$

$$
\begin{aligned}
Q_3^M \geq Q_3^{comp} &\iff \frac{(1-4\theta)(\alpha(1+\theta)-2)}{16\theta^2-9\theta+1} \geq \frac{\alpha(3-2\theta-6\theta^2)+10\theta-6}{26\theta^2-20\theta+3} \\
&\implies \alpha(7-10\theta-4\theta^2)-13+24\theta \geq 0 \\
&\implies \alpha \geq \frac{13-24\theta}{7-10\theta-4\theta^2} \quad \text{which is always true under (48).}
\end{aligned}
$$

$$
\begin{aligned}
Q_0^M > 2Q_0^{comp} &\iff \frac{\alpha(1-4\theta+8\theta^2)-1}{16\theta^2-9\theta+1} > \frac{(4-8\theta)(\alpha(1-2\theta)-1)}{26\theta^2-20\theta+3} \\
&\implies \alpha > \frac{1-24+110\theta^2-128\theta^3}{1-20\theta+94\theta^2-136\theta^3+48\theta^4} = \alpha^* \text{ for } \theta > 0.0717.
\end{aligned}
$$

Table I: **Comparison of Merger and "Competitive" equilibria**

	Proper solutions require	*Outcome*	
$\theta = 0$	$2 < \alpha < \infty$	$Q_0^M < 2Q_0^{comp}$	$Q_1^M = Q_1^{comp}, Q_2^M = Q_2^{comp}, Q_3^M = Q_3^{comp}$
$\theta = 0.02$	$1.967 < \alpha < 10.33$	$Q_0^M < 2Q_0^{comp}$	$Q_1^M > Q_1^{comp}, Q_2^M > Q_2^{comp}, Q_3^M > Q_3^{comp}$
$\theta = 0.04$	$1.935 < \alpha < 5.342$	$Q_0^M < 2Q_0^{comp}$	$Q_1^M > Q_1^{comp}, Q_2^M > Q_2^{comp}, Q_3^M > Q_3^{comp}$
$\theta = 0.06$	$1.903 < \alpha < 3.687$	$Q_0^M < 2Q_0^{comp}$	$Q_1^M > Q_1^{comp}, Q_2^M > Q_2^{comp}, Q_3^M > Q_3^{comp}$
$\theta = 0.08$	$1.872 < \alpha < 2.867$	$Q_0^M < 2Q_0^{comp}$	$Q_1^M > Q_1^{comp}, Q_2^M > Q_2^{comp}, Q_3^M > Q_3^{comp}$
$\theta = 0.09$	$1.857 < \alpha < 2.596$	$Q_0^M < 2Q_0^{comp}$	$Q_1^M > Q_1^{comp}, Q_2^M > Q_2^{comp}, Q_3^M > Q_3^{comp}$
$\theta = 0.10$	$1.842 < \alpha < 2.380$	$Q_0^M < 2Q_0^{comp}$ if $\alpha < 2.234$ $Q_0^M > 2Q_0^{comp}$ if $\alpha > 2.234$	$Q_1^M > Q_1^{comp}, Q_2^M > Q_2^{comp}, Q_3^M > Q_3^{comp}$
$\theta = 0.11$	$1.827 < \alpha < 2.207$	$Q_0^M < 2Q_0^{comp}$ if $\alpha < 2.026$ $Q_0^M > 2Q_0^{comp}$ if $\alpha > 2.026$	$Q_1^M > Q_1^{comp}, Q_2^M > Q_2^{comp}, Q_3^M > Q_3^{comp}$
$\theta = 0.12$	$1.812 < \alpha < 2.063$	$Q_0^M < 2Q_0^{comp}$ if $\alpha < 1.905$ $Q_0^M > 2Q_0^{comp}$ if $\alpha > 1.905$	$Q_1^M > Q_1^{comp}, Q_2^M > Q_2^{comp}, Q_3^M > Q_3^{comp}$
$\theta = 0.13$	$1.798 < \alpha < 1.943$	$Q_0^M < 2Q_0^{comp}$ if $\alpha < 1.829$ $Q_0^M > 2Q_0^{comp}$ if $\alpha > 1.829$	$Q_1^M > Q_1^{comp}, Q_2^M > Q_2^{comp}, Q_3^M > Q_3^{comp}$
$\theta = 0.14$	$1.783 < \alpha < 1.841$	$Q_0^M > 2Q_0^{comp}$	$Q_1^M > Q_1^{comp}, Q_2^M > Q_2^{comp}, Q_3^M > Q_3^{comp}$
$\theta = 0.147$	$1.773 < \alpha < 1.779$	$Q_0^M > 2Q_0^{comp}$	$Q_1^M > Q_1^{comp}, Q_2^M > Q_2^{comp}, Q_3^M > Q_3^{comp}$

Table II: NSW^{comp} **versus** NSW^M

	Proper solutions require	*Outcome*
$\theta = 0$	$2 < \alpha < \infty$	$NSW^{comp} > NSW^M$
$\theta = 0.02$	$1.967 < \alpha < 10.33$	$NSW^{comp} > NSW^M$ if $\alpha < 3.418$ $NSW^{comp} < NSW^M$ if $\alpha > 3.418$
$\theta = 0.04$	$1.935 < \alpha < 5.342$	$NSW^{comp} > NSW^M$ if $\alpha < 2.314$ $NSW^{comp} < NSW^M$ if $\alpha > 2.314$
$\theta = 0.06$	$1.903 < \alpha < 3.687$	$NSW^{comp} > NSW^M$ if $\alpha < 2.058$ $NSW^{comp} < NSW^M$ if $\alpha > 2.058$
$\theta = 0.08$	$1.872 < \alpha < 2.867$	$NSW^{comp} > NSW^M$ if $\alpha < 1.934$ $NSW^{comp} < NSW^M$ if $\alpha > 1.934$
$\theta = 0.10$	$1.842 < \alpha < 2.380$	$NSW^{comp} > NSW^M$ if $\alpha < 1.853$ $NSW^{comp} < NSW^M$ if $\alpha > 1.853$
$\theta = 0.11$	$1.827 < \alpha < 2.207$	$NSW^{comp} < NSW^M$
$\theta = 0.12$	$1.812 < \alpha < 2.063$	$NSW^{comp} < NSW^M$
$\theta = 0.13$	$1.798 < \alpha < 1.943$	$NSW^{comp} < NSW^M$
$\theta = 0.14$	$1.783 < \alpha < 1.841$	$NSW^{comp} < NSW^M$

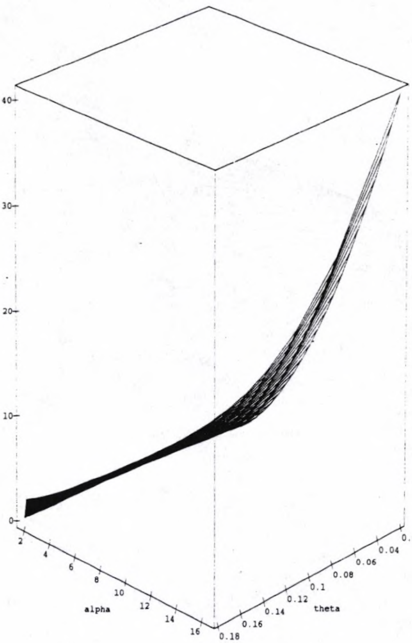

Figure 3: $NSW^{comp} - NSW^{car}$

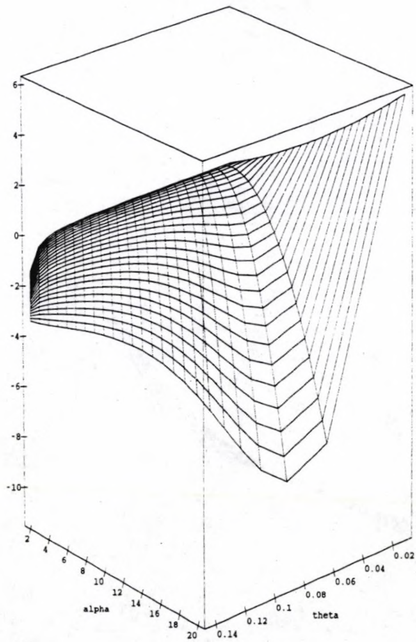

Figure 4: $NSW^{comp} - NSW^{M}$

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