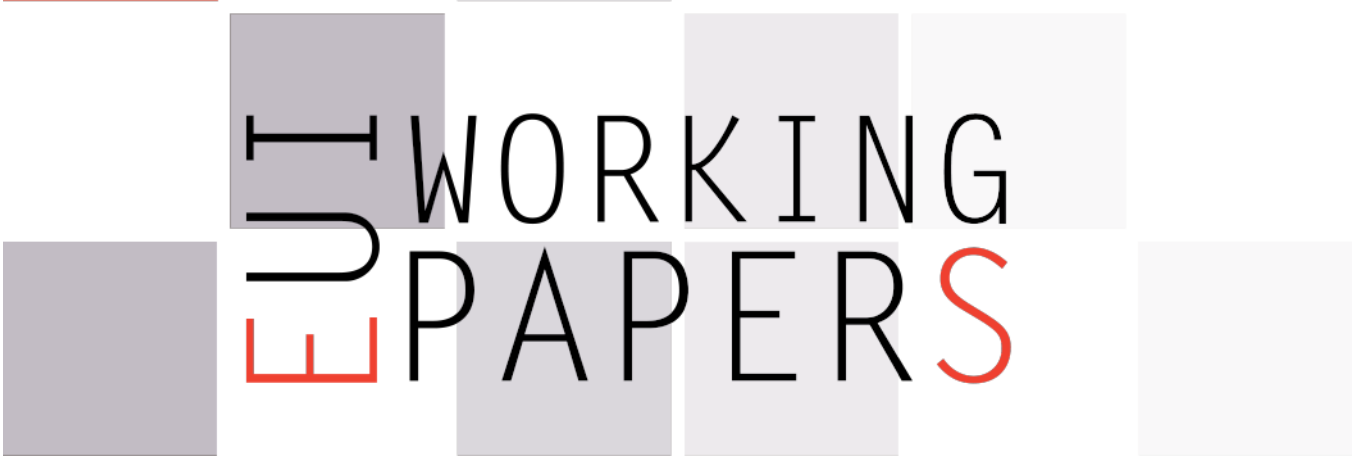




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MORTGAGE DEFAULTS, EXPECTATION-DRIVEN HOUSE PRICES AND MONETARY POLICY

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ABSTRACT

We contribute to the literature on dynamic stochastic general equilibrium models with housing collaterals by including shocks to house price expectations. We incorporate endogenous mortgage defaults which are rarely included in DSGE models with housing collaterals. We show that our theoretical model of mortgage default is consistent with empirical evidence. We use this particular DSGE setup to study the effects of variations in house price expectations on macroeconomic dynamics and their implications for monetary policy. Extensive model simulations show that an increase in expected future house prices leads to a decline in mortgage default rates as well as in interest rates on loans, whereas it leads to an increase in house prices, household debt, bank leverage ratios and economic activity. As opposed to previous studies we find that inflation is low during a house price boom. Finally, we demonstrate that although monetary policy that reacts to household credit growth improves the stability of the real economy and enhances financial stability, yet it jeopardizes price stability.

JEL classification: E32; E44; E52

Keywords: House price expectations; Inflation dynamics; Monetary policy; Mortgage defaults

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1. INTRODUCTION

Fluctuations in house prices can have a great impact on the real economy as it was clearly demonstrated by the burst of the U.S real estate bubble in 2006 as well as the burst of the Japanese housing bubble in the early 1990s. Indeed, Shiller (2007) note right before the burst that the U.S. experienced the biggest house price boom in its history. In Figure 1, which displays the real house prices for the U.S., it is evident that the boom started in 1997 and ended in 2006, when house prices suddenly collapsed and thereafter entered its worst recession since the 1930s.

In their survey of homebuyers Case and Shiller (2003) find that in 2003 a large share of home buyers expected future house prices to rise over the next several years. Shiller (2007) and Case *et al.* (2012) find that expectations of a boost in future house prices played an important role in determining the upcoming U.S. house price boom. Piazzesi and Schneider (2009) demonstrate that a fraction of households whose optimism reflects price appreciations grew during the U.S. boom and generated an after-effect on house price dynamics. Lambertini *et al.* (2013) using data on expectations about future property prices by the Michigan Survey of Consumers, show via a VAR model that the expectations of rising house prices explain a large part of business cycle oscillations during the U.S. price boom. Additionally, Ling *et al.* (2015) find that non-fundamental components of the U.S. housing market sentiment can predict price fluctuations above and beyond movements in fundamentals. Towbin and Weber (2015) after estimating a VAR model for the U.S. economy, illustrated that shocks to house price expectations are the most important drivers of house prices, as they generate a significant persistence component in the GDP. All the aforementioned studies indicate that expectations play an important role as the driving forces of house prices. Interestingly though, shocks to house price expectations are mostly absent in DSGE models including a housing collateral. Such examples can be encountered in the works of Iacoviello (2005), Gerali *et al.* (2010), Iacoviello and Neri (2010), Forlati and Lambertini (2011), Liu *et al.* (2013) and Iacoviello (2015). Hence, unlike the models reported in those studies, our setup incorporates shocks to house price expectations under a DSGE framework with housing collaterals.

The purpose of this paper is to study the role of house price expectations and the implications for macroeconomic dynamics and monetary policy under a DSGE model with housing collaterals. Our model includes many sectors, e.g., household, business, entrepreneur, retail, banking sector and a central bank. In particular, the household sector is comprises two types: financially unconstrained and financially constrained households. Both types consume, work and buy houses. The financially constrained households use their housing stock as collateral to obtain loans from the banking sector to buy houses. The business sector consists of entrepreneurs who

use labor, capital and housing stock to produce intermediate goods. Entrepreneurs are financially constrained agents utilizing their housing stock as collateral to obtain loans from the banking sector. The latter sector is represented by commercial banks which collect deposits from financially unconstrained households and provide funds to financially constrained households and entrepreneurs. The commercial banks face a capital requirement. Furthermore, the retail sector consists of monopolistic retailers who transform intermediate goods into final goods; this sector is the source of price stickiness in the economy. Lastly, the central bank conducts monetary policy. We include two features that are not typically included in a DSGE model with housing collaterals, namely shocks to house price expectations and endogenous mortgage defaults. In our model, a positive shock to price expectations leads to an increase in prices and consequently the boom is caused by optimistic expectations about future property prices. The decision of mortgage default is endogenous in the sense that financially constrained households default when the value of their houses is lower than the stipulated mortgage loan repayment. Furthermore, the commercial bank's lending decisions are influenced by the bank capital position which is affected by house prices. The commercial bank also takes into account expected mortgage defaults when making loans to financially constrained households.

While Kollmann *et al.* (2011) and Iacoviello (2015) also include a banking sector under a similar setup, notably they do not model loan defaults as exogenous shocks. In addition, these works do not include shocks to house price expectations, as opposed to ours. Accordingly, although Forlati and Lambertini (2011) include endogenous mortgage defaults and variations in mortgage defaults with the latter depending on variations in mortgage risk, in our modeling framework mortgage defaults derive from variations in expected future house prices. As such, the models by Forlati and Lambertini (2011) and Iacoviello (2015) imply that variations in mortgage defaults lead movements in house prices, so a decline in mortgage defaults leads to an increase in house prices. In contrast to the aforementioned works, our theoretical model of defaults implies that a rise in optimistic expectations about future house prices leads to an increase in house prices, which in turn leads to a decline in mortgage defaults. Hence, our mortgage default component behaves consistently with econometric evidence which shows that variations in house prices lead movements in mortgage defaults, yet not vice versa. On the whole, we use our proposed DSGE model with endogenous mortgage defaults to examine the effects of shocks to house price expectations on the real economy and financial activities i.e., household debt, business debt, bank leverage ratio, mortgage default rate, interest rates on household and business loans. We also investigate the effects of shocks to house price expectations on inflation dynamics and provide evidence that our DSGE model may explain why inflation is low during a house

price boom. Interestingly, we also demonstrate the mechanism according to which the reaction of monetary policy to household credit growth can reduce the volatility of output and inflation.

We contribute to the relevant literature in the following manner: we embed shocks to house price expectations into a DSGE model with housing collaterals, and we include endogenous mortgage defaults which depend on variations in expected future house prices. Eventually, the DSGE model allows for an interplay between the household sector and the banking sector through the house price and expected mortgage default channels. We learn that optimistic expectations about future property price appreciations can generate a housing market boom and a credit boom, yet at the same time we observe a period of low inflation. Monetary policy that reacts to household credit growth reduces the volatility of output and dampens the credit boom, but these effects lead to higher inflation volatility. Consequently, the central bank when it includes financial stability considerations in its monetary policy decisions, might miss its inflation target in the short-run.

Furthermore, the main findings from the DSGE model simulations are as follows: firstly, we see that a positive house price expectation shock leads to a rise in prices, housing demand, household debt, business debt and bank leverage ratio, whilst it leads to a decline in mortgage default rate and interest rates on household and business loans. The positive shock generates an increase in the real economic indicators and in financial/banking activities. The intuition behind our results is that a rise in expectations about future house prices increases the expected value of housing collaterals and the expected resale value of houses. Hence, these effects eventually induce financially constrained households to increase their demand for houses, which in turn leads to an increase in household debt. The rise in house prices leads to a rise in home equity whereas at the same time it projects a decline in mortgage default rate, which is in accordance with econometric evidence in the relevant literature. The increase in house prices induces commercial banks to expand loan supply, which drives down interest rates on household and business loans, but this effect increases the bank leverage ratio. The increase in loan supply to financially constrained households and entrepreneurs stimulates the real economy. As a side effect also the rise in house prices relaxes entrepreneurs' collateral constraints, which encourages the entrepreneurs to increase capital investment.

Our second finding is that inflation tends to be low during a house price boom and amid a credit boom, whereas previous studies such as by Bernanke and Gertler (2000), Forlati and Lambertini (2011) and Badarau and Popescu (2014) report that inflation tends to be higher during asset price booms in general. This result has important ramifications for monetary policy and financial stability. Specifically, the central bank observes a downward pressure on inflation and

responds to the shock to house price expectations by reducing the policy rate. This further magnifies the rise in real estate demand, household debt and bank leverage ratios, therefore it increases financial instability. Obviously, the real economy becomes more vulnerable to a housing market meltdown. For example, Christiano *et al.* (2010) argued that during the Japanese stock market boom in the 1980s, the Bank of Japan cut the policy rate to stimulate inflation which in turn amplified the stock market boom¹. Ultimately, asset prices collapsed and the Japanese economy underwent a severe recession.

Thirdly, we find that monetary policy which reacts to household credit growth, dampens the response of the real economy to house price expectation shocks, yet it amplifies the response of inflation in the short-run. Monetary policy that takes into account credit growth reduces the volatility of output but increases the volatility of inflation. Moreover, by reacting to household credit growth, the exercised monetary policy reduces housing demand, household debt and bank leverage ratio, hence it enhances real economy and financial stability, albeit to the cost of price stability.

Our proposed setup is related to DSGE modeling with collateral constraints as already mentioned, and in particular is based on the framework of Kiyotaki and Moore (1997) and Iacoviello (2005) wherein the amount economic agents can borrow is tied to the value of their collateral. Similarly in our model, the amount of loans that financially constrained households can obtain are tied to the value of their houses. The collateral constraint framework has been applied also by Monacelli (2009), Iacoviello and Neri (2010), Jermann and Quadrini (2012), Calza *et al.* (2013), Lambertini *et al.* (2013), Liu *et al.* (2013) and Walentin (2014). Nevertheless, as opposed to our DSGE model, their models do not include debt defaults. Unlike Kollmann *et al.* (2011) and Iacoviello (2015) who treat loan defaults as exogenous shocks, we model mortgage defaults as an endogenous process, in a way that households default when the value of their houses is lower than the mortgage loan repayment. Furthermore, in contrast to the model of Kollmann *et al.* (2011) and Iacoviello (2015) our model has sticky prices, so we can directly analyze the effects of variations in house price expectations upon inflation dynamics. Our banking sector accommodates different interest rates under a perfect banking competition framework with a representative bank facing capital requirements (Kollmann *et al.*, 2011).² Overall, all aforementioned studies do not include shocks to house price expectations in their models.

¹ The Japanese economy experienced both a stock market boom and a house price boom in the 1980s.

² Other DSGE studies incorporating a financial intermediary or a banking sector can be found in Gerali *et al.* (2010), Andrés and Arce (2012), Gambacorta and Signoretti (2014), Verona *et al.* (2014) and Bekiros and Paccagnini (2016).

The paper is organized as follows: section 2 presents empirical findings vis-à-vis the relationship between house prices and mortgage default rates. Section 3 introduces a DSGE model with endogenous mortgage defaults, whilst section 4 displays the impact of stochastic processes for shocks to house price expectations. Section 5 presents the calibration of our model parameters and section 6 highlights the quantitative results from extensive model simulations. Section 7 discusses implications for monetary policy and section 8 conducts a sensitivity analysis. Section 9 concludes.

2. EMPIRICAL ANALYSIS OF HOUSE PRICES AND MORTGAGE DEFAULT RATES

In this section, we present our empirical findings regarding the link between house prices and mortgage default rates. We conduct a Granger causality analysis and then we examine the effects of a positive shock to real house prices on mortgage default rate under a bivariate VAR model for the U.S utilizing quarterly data from 1991 Q1 to 2015 Q4.^{3,4} The lag order (four) is determined using the Schwarz information criterion. Table 1 presents the Granger causality test results, which indicate that real house prices Granger cause mortgage defaults. Hence, real house prices carry significant information for mortgage default rates, but not vice versa. This econometric evidence supports our assumption that fluctuations in real house prices lead those in mortgage defaults, as opposed to Forlati and Lambertini (2011) and Iacoviello (2015) who both imply that variations in mortgage defaults lead movements in house prices.⁵

Figure 2 displays the impulse responses in the mortgage default rates and real house prices to positive house price shocks. We use a Cholesky decomposition to identify the structural VAR following an ordering where the mortgage default rate comes first and the house prices are last, implying that i) a house price shock has an immediate impact on house prices and ii) this price shock does not affect the mortgage default rates contemporaneously. These assumptions are realistic taking into account that households do not default immediately as their housing value becomes less than the loan repayment. Furthermore, we want to show that even though a house price shock does not impact mortgage default rates immediately, interestingly the house price shock generates a strong persistence in mortgage rates. In addition, Figure 2 clearly indicates that a positive house price shock has a negative effect on default rates and a positive shock to house prices leads to a strongly persistent mortgage default rate, although their interrelationship is not

³ Mortgage default rate is measured as the delinquency rate on residential mortgages. For more details please see Appendix II.

⁴ All variables are deviated from linear trend. The starting date for our econometric model is reflected by data availability with respect to the mortgage default rate.

⁵ The comprehensive version of our theoretical model - which incorporates mortgage defaults - is included in section 3.2.

contemporaneous. Our specific ordering of house prices before the mortgage default rate does not change the qualitative response to house price shocks, however when it is reversed the effects of house price shocks on mortgage default rate become slightly stronger.

In general, the aforementioned econometric results support our assumption that an increase in house prices causes a decline in mortgage defaults. In the next section, we elaborate on the concept of endogenous mortgage defaults and thereby we incorporate endogenous mortgage defaults and house price expectation shocks into a novel DSGE model.

3. DSGE MODEL

We incorporate a mortgage default channel, a banking sector and house price expectation shocks in our DSGE model. The economy is composed of six types of agents, i.e., financially unconstrained and constrained households, entrepreneurs, commercial banks, monopolistic retailers and the central bank. Both types of households supply their labor supply to entrepreneurs, consume and then buy houses. Financially constrained households take loans from a commercial bank and borrow in accordance with the value of their house. The financially constrained households default when the value of their house is lower than the stipulated mortgage loan repayment. Entrepreneurs produce intermediate goods and perceive the price of intermediate goods as given. Also, entrepreneurs are credit constrained and pay back all loans which means that they do not default on their loans. The maturity of loans to the financially constrained households and entrepreneurs is one-period. Commercial banks obtain deposits from financially unconstrained households and provide loans to financially constrained households as well as to entrepreneurs. Commercial bank assets comprise household and business loans. Moreover, commercial banks face capital requirements while their balance sheets are affected by mortgage defaults and house prices. Monopolistic retailers transform intermediate goods to final goods and the retail sector is the source of price rigidity in the economy. Lastly, the central bank conducts monetary policy. Below we describe each agent in detail.

3.1 Financially Unconstrained Household

The expected utility of a representative financially unconstrained household is

$$E_0 \sum_{t=0}^{\infty} \beta_U^t \left(\ln C_{U,t} + v_h \ln H_{U,t} - \frac{(N_{U,t})^{\eta+1}}{\eta+1} \right) \quad (1)$$

where β_U represents the discount factor, $C_{U,t}$ the current consumption, $H_{U,t}$ denotes the holding of housing stock, $N_{U,t}$ the labor hours, v_h the weight on housing and η the inverse Frisch labor supply elasticity.

The financially unconstrained household receives the gross interest income upon last period deposits i.e., $R_{t-1}D_{t-1}/\pi_t$, where R_{t-1} is the gross nominal interest rate on deposits, D_{t-1} the last period deposits and π_t the inflation. The household earns the real wage rate $W_{U,t}$ for supplying $N_{U,t}$ hours to an entrepreneur. The real house prices are denoted by q_t . The financially unconstrained household uses income for consumption $C_{U,t}$ buys houses at $q_t(H_{U,t} - H_{U,t-1})$ and makes deposits D_t at a commercial bank. As the owner of a retail firm, the financially unconstrained household receives the lump-sum profit F_t . The budget constraint is expressed as follows

$$C_{U,t} + D_t + q_t(H_{U,t} - H_{U,t-1}) = \frac{R_{t-1}}{\pi_t} D_{t-1} + W_{U,t}N_{U,t} + F_t \quad (2)$$

The financially unconstrained household chooses $C_{U,t}$, D_t , $H_{U,t}$, and $N_{U,t}$ to maximize equation (1) subject to (2); the first-order conditions are

$$\frac{1}{C_{U,t}} = \beta_U E_t \left(\frac{1}{C_{U,t+1}} \frac{R_t}{\pi_{t+1}} \right) \quad (3)$$

$$\frac{q_t}{C_{U,t}} = \frac{v_h}{H_{U,t}} + \beta_U E_t \left(\frac{q_{t+1}}{C_{U,t+1}} \right) \quad (4)$$

and

$$\frac{W_{U,t}}{C_{U,t}} = N_{U,t}^\eta \quad (5)$$

Equation (3) is a standard consumption Euler equation that captures the household's intertemporal choice between current and future consumption. Current consumption depends negatively on the real deposit interest rate $E_t(R_t/\pi_{t+1})$. Equation (4) is the optimal holding of housing stock. The left-hand side captures the marginal cost of acquiring an additional unit of house $q_t/C_{U,t}$, whereas the right-hand side depicts the marginal benefit of purchasing an extra unit of house, which comprises the marginal utility of having a house $v_h/H_{U,t}$ and the expected resale value of the house $\beta_U E_t(q_{t+1}/C_{U,t+1})$. The optimal hours worked are determined by equation (5).⁶

3.2 Financially Constrained Household

The expected utility of a representative financially constrained household is given by

$$E_0 \sum_{t=0}^{\infty} \beta_F^t \left(\ln C_{F,t} + v_h \ln H_{F,t} - \frac{(N_{F,t})^{\eta+1}}{\eta+1} \right) \quad (6)$$

The discount factor of the financially constrained household is symbolized as β_F . $C_{F,t}$ is the consumption while $H_{F,t}$ denotes the holding of housing stock, $N_{F,t}$ the labor hours, v_h the weight on housing and η similarly to equation (1), the inverse Frisch labor supply elasticity.

⁶ The exact derivation of first order conditions can be found in Appendix I.

This type of household receives the real wage rate $W_{F,t}$ for supplying $N_{F,t}$ hours to the entrepreneur, and obtains new loans $L_{F,t}$ from a commercial bank. The financially constrained household buys houses $q_t(H_{F,t} - H_{F,t-1})$. Furthermore, the amount of debt that the financially constrained household has agreed to pay back is $R_{F,t-1}L_{F,t-1}/\pi_t$, where $R_{F,t-1}$ is the gross nominal interest rate on one-period mortgage loans (including repayment) and $R_{F,t-1}/\pi_t$ represents the gross real interest on mortgage loans. However, the financially constrained household can default on its loans by paying back less than the contractual obligations, hence $Z_{F,t}$ is the amount of mortgage defaults which will be determined below. The budget constraint is expressed as

$$C_{F,t} + q_t(H_{F,t} - H_{F,t-1}) + \frac{R_{F,t-1}}{\pi_t}L_{F,t-1} - Z_{F,t} = L_{F,t} + W_{F,t}N_{F,t} \quad (7)$$

The financially constrained household cannot borrow more than a fraction m_F of the expected value of the house. The collateral constraint is written similarly as in Iacoviello (2005), namely

$$E_t \left(\frac{R_{F,t}}{\pi_{t+1}} \right) L_{F,t} \leq m_F E_t (q_{t+1} H_{F,t}) \quad (8)$$

Thereafter we explain the mechanism of endogenous mortgage default; the financially constrained household consists of many members i who all purchase houses $H_{F,t}^i$. The total housing stock of the household is $H_{F,t} = \int_{i=0}^1 H_{F,t}^i di$. In each period, each member's house value is subject to an idiosyncratic *iid* shock ω^i such that the value of the house becomes $q_t H_{F,t}^i (1 + \omega^i)$. The household member defaults when the value of the house is lower than the mortgage loan repayment i.e., $(R_{F,t-1}L_{F,t-1}/\pi_t) > q_t H_{F,t}^i (1 + \omega^i)$. Let $\hat{\omega}$ be the threshold value of the shock for which the member will pay back the mortgages, thus $\hat{\omega} = \frac{R_{F,t-1}L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - 1$. If the household member draws an ω lower than $\hat{\omega}$, the member will default. Let $-\bar{\omega}$ be the lower bound of the distribution where $\bar{\omega} > 0$ and $f(\omega)$ the probability density function of ω . The amount of mortgage defaults $Z_{F,t}$ is determined by the following integral

$$Z_{F,t} = \int_{-\bar{\omega}}^{\left(\frac{R_{F,t-1}L_{F,t-1}}{\pi_t q_t H_{F,t-1}}\right)-1} \left(\frac{R_{F,t-1}L_{F,t-1}}{\pi_t} - q_t H_{F,t-1} (1 + \omega) \right) f(\omega) d\omega \quad (9)$$

The intuition behind equation (9) is the following: the household defaults on the loans if the *iid* shock ranges between its lowest possible value and the critical value of the shock at which the default occurs. In the case of default, the amount of mortgage defaults is the promised repayment of the loan minus the value of the house that is taken over by the commercial bank. $Z_{F,t}$ is the realized mortgage defaults. To simplify, we assume that $f(\omega)$ is constant in the relevant interval such that $f(\omega) = f$. By evaluating the integral, we can obtain the following amount of mortgage defaults

$$Z_{F,t} = f \frac{q_t H_{F,t-1}}{2} \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - (1 - \bar{\omega}) \right)^2 \quad (10)$$

We now take the derivative of $Z_{F,t}$ with respect to $L_{F,t-1}$ and $H_{F,t-1}$

$$\frac{\partial Z_{F,t}}{\partial L_{F,t-1}} = f \frac{R_{F,t-1}}{\pi_t} \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - (1 - \bar{\omega}) \right) > 0 \quad (11)$$

and

$$\begin{aligned} \frac{\partial Z_{F,t}}{\partial H_{F,t-1}} &= -\frac{f}{2H_{F,t-1}} \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - (1 - \bar{\omega}) \right) \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t} \right) \\ &\quad - \frac{f}{2H_{F,t-1}} \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - (1 - \bar{\omega}) \right) (1 - \bar{\omega}) (q_t H_{F,t-1}) < 0 \end{aligned} \quad (12)$$

Equation (11) implies that one more unit of loans will lead to more mortgage defaults given that the holding of the housing stock is constant. Equation (12) implies that an additional unit of houses leads to less mortgage defaults given that the unit of loans is constant.⁷ The financially constrained household chooses $C_{F,t}$, $L_{F,t}$, $H_{F,t}$ and $N_{F,t}$ to maximize (6) subject to (7), (8) and (10). The first-order conditions are as follows

$$\frac{1}{C_{F,t}} = \beta_F E_t \left(\frac{1}{C_{F,t+1}} \frac{R_{F,t}}{\pi_{t+1}} \right) + \lambda_{F,t} E_t \left(\frac{R_{F,t}}{\pi_{t+1}} \right) - \beta_F E_t \left(\frac{1}{C_{F,t+1}} \frac{\partial Z_{F,t+1}}{\partial L_{F,t}} \right) \quad (13)$$

$$\frac{q_t}{C_{F,t}} = \frac{v_h}{H_{F,t}} + \beta_F E_t \left(\frac{q_{t+1}}{C_{F,t+1}} \right) + \lambda_{F,t} m_F E_t (q_{t+1}) + \beta_F E_t \left(\frac{1}{C_{F,t+1}} \frac{\partial Z_{F,t+1}}{\partial H_{F,t}} \right) \quad (14)$$

and

$$\frac{W_{F,t}}{C_{F,t}} = N_{F,t}^\eta \quad (15)$$

Equation (13) is the Euler condition for consumption with collateral constraint and endogenous loan default. $\lambda_{F,t}$ is the multiplier on the collateral constraint (8). Evidently, equation (13) implies that when the household maximizes utility, the marginal utility of consuming one more unit of goods today ($1/C_{F,t}$) is equal to the marginal utility of consuming one more unit of goods in the next period, taking into account interest rate $\beta_F E_t \left(\frac{1}{C_{F,t+1}} \frac{R_{F,t}}{\pi_{t+1}} \right)$ plus two terms: the first of these terms is $\lambda_{F,t} E_t \left(\frac{R_{F,t}}{\pi_{t+1}} \right)$, which reflects the fact that the more financially constrained the household is, the less it will consume today. The last term $\beta_F E_t \left(\frac{1}{C_{F,t+1}} \frac{\partial Z_{F,t+1}}{\partial L_{F,t}} \right)$ arises because for a given investment in houses, one more unit of loans increases the expected default probability. This effect reduces the repayment of the loan, which in turn reduces the cost of consumption and obviously increases consumption.

⁷ The derivation of the mortgage default conditions is analyzed in Appendix I.

Moreover, equation (14) determines the optimal holding of houses. The left-hand side is the marginal cost of purchasing an extra unit of house. As the household buys one more unit of houses, the household reduces consumption. The right-hand side contains four components: the direct utility gain of having a house $v_h/H_{F,t}$, the expected resale value of the house $\beta_F E_t(q_{t+1}/C_{F,t+1})$, the expected marginal benefit of using house as collateral $\lambda_{F,t} m_F E_t(q_{t+1})$, and the expected mortgage defaults $\beta_F E_t\left(\frac{1}{C_{F,t+1}} \frac{\partial Z_{F,t+1}}{\partial H_{F,t}}\right)$. The housing collateral channel term $\lambda_{F,t} m_F E_t(q_{t+1})$ implies that a more relaxed collateral constraint signifies lower $\lambda_{F,t}$, which sequentially induces the household to buy more houses. The expected mortgage default term $\beta_F E_t\left(\frac{1}{C_{F,t+1}} \frac{\partial Z_{F,t+1}}{\partial H_{F,t}}\right)$ implies that by holding more houses the financially constrained household can borrow more, which in turn provides the household with an incentive to buy more houses. However, holding more houses for given loans reduces the expected defaults, which makes it less attractive to buy more houses. Finally, the labor supply decision for the financially constrained household is determined by equation (15).

3.3 Entrepreneur

An entrepreneur has the following objective function

$$E_0 \sum_{t=0}^{\infty} \beta_E^t (\ln C_{E,t}) \quad (16)$$

where β_E is her discount factor and $C_{E,t}$ the entrepreneur's consumption. She produces an intermediate good $Y_{E,t}$ and sells the intermediate good to a retailer at a price P_t^W . The retailer transforms the intermediate good without costs into a final good and then the retailer sells the final good to other agents, namely to the entrepreneur, to the financially unconstrained and constrained households and to the banker at the retail price of P_t . The markup of the final good over the intermediate good is defined as $X_t = P_t/P_t^W$. The entrepreneur uses capital, houses and labor to produce an intermediate good. The production function is given by

$$Y_{E,t} = K_{t-1}^\mu H_{E,t-1}^\nu N_{U,t}^{\alpha(1-\mu-\nu)} N_{F,t}^{(1-\alpha)(1-\mu-\nu)} \quad (17)$$

where K_{t-1} is the capital that depreciates at rate δ , $H_{E,t-1}$ the housing input, and $N_{U,t}$ and $N_{F,t}$ respectively are the financially unconstrained and constrained household labor inputs. The parameter α determines the labor income share of the financially unconstrained household and the parameters μ and ν represent the capital and housing shares in the production function. The constraint assumption for the flow of funds is given by

$$C_{E,t} + q_t(H_{E,t} - H_{E,t-1}) + \frac{R_{E,t-1}}{\pi_t} L_{E,t-1} + W_{U,t} N_{U,t} + W_{F,t} N_{F,t} + I_t + \xi_{K,t} = \frac{Y_{E,t}}{X_t} + L_{E,t} \quad (18)$$

Wage payments to the households are estimated as $W_{U,t}N_{U,t} + W_{F,t}N_{F,t}$, and capital investment is denoted I_t . The entrepreneur faces capital adjustment costs $\xi_{K,t} = (\psi_k/2\delta) \left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1}$, where ψ_k is a capital adjustment cost parameter. The capital accumulation is $K_t = (1 - \delta)K_{t-1} + I_t$. The entrepreneur invests in the housing stock $q_t(H_{E,t} - H_{E,t-1})$ and obtains loans $L_{E,t}$ from a commercial bank. The entrepreneur pays back the loans $R_{E,t-1}L_{E,t-1}/\pi_t$ to the commercial bank, where $R_{E,t-1}$ is the gross nominal interest rate on business loans.

Next, the entrepreneur faces a collateral constraint and she cannot borrow more than a fraction m_E of the expected value of housing stock $E_t(q_{t+1}H_{E,t})$. The collateral constraint is written as

$$E_t \left(\frac{R_{E,t}}{\pi_{t+1}} \right) L_{E,t} \leq m_E E_t(q_{t+1}H_{E,t}) \quad (19)$$

The entrepreneur chooses $C_{E,t}$, $L_{E,t}$, K_t , I_t , $N_{U,t}$, $N_{F,t}$, $H_{E,t}$ to maximize the objective function (16) subject to equations (17), (18) and (19). The first-order conditions are calculated as follows

$$\frac{1}{C_{E,t}} = \beta_E E_t \left(\frac{1}{C_{E,t+1}} \frac{R_{E,t}}{\pi_{t+1}} \right) + \lambda_{E,t} E_t \left(\frac{R_{E,t}}{\pi_{t+1}} \right) \quad (20)$$

$$\frac{q_t}{C_{E,t}} = E_t \left[\frac{\beta_E}{C_{E,t+1}} \left(v \frac{Y_{E,t+1}}{X_{t+1}H_{E,t}} + q_{t+1} \right) \right] + \lambda_{E,t} m_E E_t(q_{t+1}) \quad (21)$$

$$\mu_{E,t} = \frac{1}{C_{E,t}} \left[1 + \frac{\psi_k}{\delta} \left(\frac{I_t}{K_{t-1}} - \delta \right) \right] \quad (22)$$

$$\begin{aligned} \mu_{E,t} = \beta_E E_t \left[\frac{1}{C_{E,t+1}} \left(\frac{\psi_k}{\delta} \left(\frac{I_{t+1}}{K_t} - \delta \right) \frac{I_{t+1}}{K_t} - \frac{\psi_k}{2\delta} \left(\frac{I_{t+1}}{K_t} - \delta \right)^2 \right) \right] \\ + \beta_E E_t \left[\frac{\mu_{E,t+1}}{C_{E,t+1} X_{t+1} K_t} + \mu_{E,t+1} (1 - \delta) \right] \quad (23) \end{aligned}$$

$$\alpha(1 - \mu - \nu) \frac{Y_{E,t}}{X_t} = W_{U,t} N_{U,t} \quad (24)$$

$$(1 - \alpha)(1 - \mu - \nu) \frac{Y_{E,t}}{X_t} = W_{F,t} N_{F,t} \quad (25)$$

The entrepreneur's Euler consumption condition is given by equation (20), where $\lambda_{E,t}$ is the multiplier on her borrowing constraint (19). The optimal investment in the housing market is determined by equation (21) and the marginal cost of holding an extra unit of the house is $q_t/C_{E,t}$. The marginal benefit of an additional investment in the housing market comprises the expected future marginal product of house $E_t \left(\frac{\beta_E}{C_{E,t+1}} \frac{vY_{E,t+1}}{X_{t+1}H_{E,t}} \right)$, the expected resale value of house $\beta_E E_t(q_{t+1}/C_{E,t+1})$ and the marginal benefit of using house as collateral $m_E E_t(\lambda_{E,t} q_{t+1})$. A more relaxed collateral constraint implies that $\lambda_{E,t}$ is lower; this increases the marginal benefit of having houses as collateral, which in turn increases the incentive to buy more houses. The decision on capital investment is given by equations (22) and (23). Equations (24) and (25) are labor demand equations.

3.4 Commercial Bank

For the banker the preference is defined as

$$E_0 \sum_{t=0}^{\infty} \beta_B^t (\ln C_{B,t}) \quad (26)$$

where her discount factor is β_B and $C_{B,t}$ the banker's consumption. The commercial bank's assets comprises loans to the financially constrained household $L_{F,t}$ as well as loans to the entrepreneur $L_{E,t}$. The commercial bank obtains deposits D_t from the financially unconstrained household to finance its loan operations. The commercial bank capital is expressed as

$$K_{B,t} = L_{F,t} + L_{E,t} - D_t \quad (27)$$

We follow Kollmann *et al.* (2011) in assuming that a commercial bank faces a capital to asset ratio requirement γ . When a commercial bank deviates from a required or desired capital ratio the bank faces a ϕ_t cost, which is a function of bank's excess capital $X_{B,t}$. The bank excess capital $X_{B,t}$ is defined as the difference between the bank capital $K_{B,t}$ and a fraction γ of the bank's assets ($L_{F,t} + L_{E,t}$) as

$$X_{B,t} = K_{B,t} - \gamma(L_{F,t} + L_{E,t}) \quad (28)$$

where $\phi_t = \phi(X_{B,t})$ is a convex function with the following properties: the first derivative is $\phi' < 0$ and the second derivative is positive $\phi'' > 0$. The first derivative implies that a higher excess capital reduces the cost of deviating from the required capital ratio, while the second derivative implies that a higher excess capital reduces the cost but at a decreasing rate. Using equations (27) and (28), the commercial bank's excess capital can be rewritten as

$$X_{B,t} = (1 - \gamma)(L_{F,t} + L_{E,t}) - D_t \quad (29)$$

We now consider the commercial bank's flow of funds. The expenditure side of the banker includes current consumption, the interest payment $\left(\frac{R_{t-1}}{\pi_t}\right) D_{t-1}$ to the financially unconstrained household, new loans to the financially constrained household $L_{F,t}$ (household loans) and to the entrepreneur $L_{E,t}$ (business loans), as well as the cost of deviating from the required capital ratio $\phi(X_{B,t})$. The flow of income includes the household deposits, the repayment of loans by the entrepreneur $\left(\frac{R_{E,t-1}}{\pi_t}\right) L_{E,t-1}$ and the repayment of loans by the financially constrained household $\left(\frac{R_{F,t-1}}{\pi_t}\right) L_{F,t-1}$. However, the financially constrained household can default on the mortgage loans and the amount of mortgage defaults is $Z_{F,t}$. An increase in $Z_{F,t}$ has a negative impact on the banker's flow of income. The constraint of her flow of funds is expressed as

$$C_{B,t} + \frac{R_{t-1}}{\pi_t} D_{t-1} + L_{E,t} + L_{F,t} + \phi(X_{B,t}) = D_t + \frac{R_{E,t-1}}{\pi_t} L_{E,t-1} + \frac{R_{F,t-1}}{\pi_t} L_{F,t-1} - Z_{F,t} \quad (30)$$

The bank chooses $C_{B,t}$, D_t , $L_{E,t}$, and $L_{F,t}$ to maximize (26) subject to (10), (29) and (30). The first-order conditions are

$$\beta_B E_t \left(\frac{R_t}{\pi_{t+1}} \right) = E_t \left(\frac{C_{B,t+1}}{C_{B,t}} \right) [1 + \phi'(X_{B,t})] \quad (31)$$

$$\beta_B E_t \left(\frac{R_{E,t}}{\pi_{t+1}} \right) = E_t \left(\frac{C_{B,t+1}}{C_{B,t}} \right) [1 + (1 - \gamma)\phi'(X_{B,t})] \quad (32)$$

and

$$\beta_B E_t \left(\frac{R_{F,t}}{\pi_{t+1}} \right) = E_t \left(\frac{C_{B,t+1}}{C_{B,t}} \right) [1 + (1 - \gamma)\phi'(X_{B,t})] + \beta_B E_t \left(\frac{\partial Z_{F,t+1}}{\partial L_{F,t}} \right) \quad (33)$$

Equation (31) defines the optimal holding of deposits, whilst equation (32) determines the optimal loan supply to the entrepreneur. Equation (33) determines the optimal loan supply to the financially constrained household. Therefore, the main difference between equation (32) and (33) is that the expected mortgage default channel $E_t(\partial Z_{F,t+1}/\partial L_{F,t})$ is incorporated into the optimal loan supply for the financially constrained household.

Furthermore, equation (32) implies that a rise in the bank's excess capital reduces the cost of deviating from the capital ratio requirement, which in turn induces the bank to increase the loan supply. This effect leads to a decrease in the interest rate on business loans. In the absence of the capital requirement γ the interest rates on deposits and loans would be the same, and variations in the bank capital would have a small impact on the borrowing cost for the entrepreneur. Thus, a positive shock to house price expectations leads to an increase in the bank's excess capital which in turn leads to a decline in interest rate on business loans. Then, equation (33) implies that an increase in the excess capital reduces the cost of deviating from the capital ratio requirement; thereby this reduction induces the banker to increase loans to the financially constrained household which leads to a decline in the interest rate on household loans. Consequently, a positive shock to house price expectations leads to an increase in bank capital and excess capital, that will lead to a decline in household loan rates given the expected mortgage defaults. On the other hand, for a given unit of houses an increase in the unit of loans to the financially constrained household leads to a rise in the expected mortgage defaults, and in turn dampens an expansion of the supply of loans to the financially constrained household and slows down the decline in the interest rate on household loans.

In summary, the commercial bank's lending decisions are influenced by the bank capital position which is affected by house prices. The commercial bank also takes into account the expected mortgage defaults when making loans to the financially constrained household.

3.5 Retailer

We now turn to the retail sector which introduces price stickiness into the model (Iacoviello, 2005). The model features a continuum of retailers indexed by $z \in [0,1]$ where they transform an intermediate good into a final good. Each retailer z purchases intermediate goods from entrepreneurs at a price P_t^W and each retailer transforms them without cost into the differentiated goods $Y_t(z)$, sold at a price $P_t(z)$. The differentiated goods are then aggregated into total final goods Y_t as such

$$Y_t = \left[\int_0^1 Y_t(z)^{(\varepsilon-1)/\varepsilon} dz \right]^{\varepsilon/(\varepsilon-1)} \quad (34)$$

where $\varepsilon > 1$ is the price elasticity of demand for goods z . The aggregate price index P_t is defined as

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon} dz \right]^{1/(1-\varepsilon)} \quad (35)$$

Each retailer faces the following demand curve

$$Y_t(z) = (P_t(z)/P_t)^{-\varepsilon} Y_t \quad (36)$$

We assume that there is a Calvo price-setting mechanism and a retailer can set her price with probability $(1 - \theta)$, and with θ must keep the price unchanged. $P_t^*(z)$ is the price that the retailer is able to change. Each retailer maximizes the following expected profit

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \left[\Lambda_{t,k} \left(\frac{P_t^*(z)}{P_{t+k}} - \frac{X}{X_{t+k}} \right) \right] Y_{t+k}^*(z) \right\} \quad (37)$$

where $Y_{t+k}^*(z) = (P_t^*(z)/P_{t+k})^{-\varepsilon} Y_{t+k}$. Recall that, the financially unconstrained household is the owner of the retail sector, therefore $\Lambda_{t,k} = \beta_U (C_{U,t}/C_{U,t+k})$ represents the financially unconstrained household's stochastic discount factor. X_t is the markup of final over intermediate goods and her steady state is $X = \varepsilon/(\varepsilon - 1)$. The Calvo price evolves as

$$P_t = [\theta P_{t-1}^\varepsilon + (1 - \theta)(P_t^*)^{1-\varepsilon}]^{\varepsilon/(1-\varepsilon)} \quad (38)$$

Combining (37) and (38) and then log-linearizing, we obtain the following Phillips curve

$$\hat{\pi}_t = \beta_U E_t \hat{\pi}_{t+1} - \kappa \hat{X}_t \quad (39)$$

where $\kappa = (1 - \theta)(1 - \beta_U \theta)/\theta$. Equation (39) indicates that inflation depends negatively on the markup \hat{X}_t and positively on expected inflation.⁸

3.6 Central Bank

The central bank follows a Taylor rule that reacts to inflation, output and household credit growth

$$\hat{R}_t = \rho_i \hat{R}_{t-1} + (1 - \rho_i) \left(\rho_\pi \hat{\pi}_t + \rho_y \hat{Y}_t + \rho_F (\hat{L}_{F,t} - \hat{L}_{F,t-1}) \right) \quad (40)$$

⁸ The (^) "hat" variables denote percentage changes from the steady state.

where $0 \leq \rho_i \leq 1$ is the parameter associated with interest rate inertia and $\rho_\pi \geq 0$, $\rho_y \geq 0$ and $\rho_F \geq 0$ measure the response of the policy rate to current inflation, output and household credit growth respectively. Equation (40) is referred to as an augmented Taylor rule, in which the central bank reacts to the deviations of inflation from its steady state, the deviations of output from its steady state and household credit growth. With $\rho_F = 0$, we have a standard Taylor rule where the central bank reacts to inflation and output.

4. STOCHASTIC PROCESSES

In this section we discuss the stochastic processes associated with a shock to house price expectations. A house price expectation shock can be defined as a shift in expectations about future house prices. In our paper, house price expectation shocks only affect financially constrained households and the main reasons are as follows: firstly, as Piazzesi and Schneider (2009) report, there is a heterogeneity in households' view about the housing market so there is a cluster of households that believe it is a good time to purchase a house because they believe house prices will rise in the future. Households' optimism reflects changes in their house price expectations and some of the households are more optimistic than others. Motivated by Piazzesi and Schneider (2009), we assume that households have different perspectives about future price increases. Hence, we model financially constrained households to be optimistic about price appreciations by incorporating expectation shocks to the financially constrained households' housing demand equation. Note that unconstrained households are not directly affected by house price expectation shocks. Secondly, the fact that house price expectation shocks only affect the financially constrained households, creates a housing market boom and a credit boom and at the same time a period of low inflation in our model simulations; this is better demonstrated in the relevant section where we also outline how this outcome has important ramifications for macroeconomic and financial stability.

In particular, the stochastic process of a house price expectation shock ε_t follows an AR(1) process: $\ln \varepsilon_t = \rho_q \ln \varepsilon_{t-1} + u_{q,t}$, where $0 < \rho_q < 1$ and $u_{q,t}$ follows a white noise process with mean zero and variance σ_q^2 . The shock is incorporated into the financially constrained household housing demand equation. In fact, the housing demand equation (14) now is rewritten as

$$\frac{q_t}{C_{F,t}} = \frac{v_h}{H_{F,t}} + \beta_F E_t \left(\frac{\varepsilon_t q_{t+1}}{C_{F,t+1}} \right) + \lambda_{F,t} m_F E_t (\varepsilon_t q_{t+1}) + \beta_F E_t \left(\frac{1}{C_{F,t+1}} \frac{\partial Z_{F,t+1}}{\partial H_{F,t}} \right) \quad (41)$$

and the expected mortgage default becomes

$$\frac{\partial Z_{F,t+1}}{\partial H_{F,t}} = -\frac{f}{2H_{F,t}} \left(\frac{R_{F,t} L_{F,t}}{\pi_{t+1} \varepsilon_t q_{t+1} H_{F,t}} - (1 - \bar{\omega}) \right) \left(\frac{R_{F,t} L_{F,t}}{\pi_{t+1}} \right)$$

$$-\frac{f}{2H_{F,t}} \left(\frac{R_{F,t}L_{F,t}}{\pi_{t+1}\varepsilon_t q_{t+1}H_{F,t}} - (1 - \bar{\omega}) \right) (1 - \bar{\omega})(\varepsilon_t q_{t+1}H_{F,t}) < 0$$

Equation (41) implies that a shock to expected future house prices will have a direct impact on the demand for housing through the expected resale value of the house $\beta_F E_t \left(\frac{\varepsilon_t q_{t+1}}{C_{F,t+1}} \right)$, the expected collateral value $\lambda_{F,t} m_F E_t(\varepsilon_t q_{t+1})$ and the expected mortgage default channel $E_t(\partial Z_{F,t+1} / \partial H_{F,t})$.

5. CALIBRATION AND PARAMETERIZATION

We calibrate the baseline model to match U.S. data, thus most of the values of model parameters are derived by Iacoviello (2005) and Finocchiaro and Heideken (2013). The model economy is log-linearized around its deterministic steady state. Table 2 summarizes the calibration.

The financially unconstrained household's discount factor β_U is set to 0.9925 to obtain a steady-state interest rate on deposits R approximately 3 % on annual basis. We set the financially constrained household's discount factor β_F and the entrepreneur's discount factor β_E to 0.95, which is smaller than the financially unconstrained household's discount factor, because i) we want the financially constrained household and the entrepreneur to borrow in equilibrium and ii) the collateral constraints to be binding.

The value of the weight on housing in the households' utility function ν_h is set to 0.0905, which implies that the ratio of residential housing to annual output is approximately 145 % in line with the U.S. utilized dataset. The housing share ν in the production function is set to 0.104 i.e., so that the ratio of commercial real estate to annual output of 70 % remains in line with the U.S. data as well. The household loan-to-value (LTV) ratio m_F is 0.83, and the business LTV ratio m_E is set to 0.64. These values indicate that the household sector is more leveraged than the business sector, as documented in Cecchetti *et al.* (2011). The values of the banking parameters are based on Kollmann *et al.* (2011). The discount factor of banker β_B is set to 0.97, the cost parameter ϕ'' to 0.25, while the bank capital ratio γ is set to 0.05 in accordance with the empirical findings of D'Hulster (2009), that the capital ratios of the major European and U.S. investment banks range between 3 % and 5 % over the period 1995 - 2010. Furthermore, following D'Hulster (2009) the U.S. regulator recommends a minimum leverage ratio of 5 % in order for a bank to be considered well-capitalized prior to the financial crisis.

The parameter on the magnitude of losses $\bar{\omega}$ is set to 0.61 to obtain a steady-state interest rate on household loans R_F of approximately 5 % on annual basis. The higher $\bar{\omega}$ is, it will increase the magnitude of mortgage defaults in the steady state, thus increase the steady-state interest rate on

household loans. f measures the sensitivity of the mortgage default to housing prices. A higher f will make mortgage defaults more sensitive to house prices. We set f to 0.83, which implies that the annual mortgage default rate is approximately 2.5 %, in line with U.S. evidence (Forlati and Lambertini, 2011). Recall that the deposit rate R is approximately 3 % annually. The lending rate on household loans R_F is higher than the deposit rate R because the parameters that capture the possibility of default ($\bar{\omega}$ and f) are included in the interest rate on household loans. The annualized steady-state interest rate on business loans (i.e., the interest rate for lending to entrepreneurs) R_E is approximately 3.12 %, which is slightly higher than the deposit rate because this lending rate on business loans does not include the possibility of default.

We already mentioned that monetary policy reacts to quarterly inflation and output. So, for the monetary policy parameters we set $\rho_i = 0.7$, $\rho_y = 0.125$, and $\rho_\pi = 1.8$, based on the U.S. estimated values indicated by Finocchiaro and Heideken (2013). Under the standard Taylor rule, we set $\rho_F = 0$. In the next section, we vary the credit growth coefficient in the augmented Taylor rule. According to our data, the autocorrelations of the linearly detrended U.S. real house prices are approximately 0.95. Thereby, we set the persistence of the house price expectation shock ρ_q to 0.95.

6. MODEL SIMULATIONS

We simulate the response of the economy to a positive shock to house price expectations. Figure 3 shows the impulse response functions to a positive house price expectation shock. All values are expressed in percentage deviations from steady state values, and one period represents one quarter in this model. The central bank follows a standard Taylor rule in which the policy rate responds to inflation and output. Total consumption comprises consumption of financially unconstrained households, financially constrained households, entrepreneurs and the banker. For illustrative purpose, we set an one-standard deviation shock to house price expectations. Figure 3 shows that a positive house price expectation shock yields an increase in total consumption, investment, employment, housing demand, household loans, household debt, business loans and bank leverage ratio, whereas it yields a reduction in inflation, mortgage default rate, interest rates on household and business loans.

A rise in expectations about future house prices induces financially constrained households to increase their demand for housing, also triggering a rise in household loans and household debt. The rise in expected future house prices relaxes the collateral constraint, which further encourages the financially constrained households to borrow more, leading to even higher household debt.

The increase in expected future house prices leads to a rise in current house prices and a boom. The rise in current house prices increases the value of houses beyond the household debt, and as a result the mortgage default rate declines. This is consistent with the empirical evidence shown in section 2. The increase in current house prices expands the commercial bank's balance sheet by increasing the value of bank assets (household and business loans) relative to the liabilities (household deposits). The positive house price expectation shock generates a rise in bank assets and leverage ratio (i.e., bank assets to capital ratio). The rise in house prices leads to a rise in bank capital and the bank's excess capital, which in turn both reduce the cost of deviating from the capital to asset ratio requirement. This effect induces the bank to increase loans to the business sector, hence accordingly the interest rate on business loans decline. Likewise, the rise in bank capital and the excess capital leads the commercial bank to increase loans to financially constrained households, which in turn causes a decline in the interest rate on household loans. On the other hand, a rise in the amount of loans to financially constrained households augments the expected mortgage defaults given the housing stock. The bank takes into account the expected mortgage defaults when making a decision on lending to financially constrained households. Consequently, a grow in the defaults tends to dampen down the expansion of loans to financially constrained households and weakens the decline of interest rates on household loans.

The interplay between the household and the banking sector occurs through the house price and expected mortgage default channels. The increase in house prices causes the interest rate on household loans to decrease, while the rise in expected mortgage defaults boosts the interest rates on loans to financially constrained households. Interestingly, the house price channel is stronger than the expected mortgage default one, and as a result the interest rate on household loans declines. Moreover, the rise in expected future house prices induces financially constrained households to cut consumption goods and leisure to invest more in housing. Nonetheless, financially unconstrained households consume more, increase leisure and buy less houses. As house prices increase, the collateral constraints of entrepreneurs are relaxed, which in turn induces the entrepreneurs to increase investment and consumption. Also, business loans and business debt both increase, and total consumption rises during a house price boom as well.

Now we discuss the effect of a shock to house price expectations on inflation dynamics. The response of inflation rate is negative and persistent. Notably, it is the desire to invest in housing that causes financially constrained households to cut consumption of goods and leisure so as to invest more. The increase in the labor supply by financially constrained households leads to a rise in total employment, that creates downward pressure on the wage rate and the marginal cost of production. This outcome leads to an increase in the markup of final goods over intermediate

goods and places downward pressure on inflation. An increase in the markup tends to put downward pressure on inflation indicated by equation (39), which describes the Phillips curve. Thus, a positive shock creates a negative response in inflation. The central bank responds to a decline in inflation by lowering the policy rate, which leads to lower loan rates and in turn encourages financially constrained households to take on more debt. Hence, the lower policy rate may increase the risk of financial crisis and instability, in that the economy becomes more sensitive to shocks to house price expectations and a collapse of house prices.

In conclusion, our results complement the study of Christiano *et al.* (2010), who show that it is possible for a DSGE model to generate a fall in inflation while the economy experiences a stock market boom. In their model, firms receive a signal about a new cost-saving technology that will be available in the future. The anticipation of the new technology leads to an increase in investment and a boom in stock prices because firms believe this new technology will improve production and lower the marginal cost of production. The fall in the marginal cost of production creates downward pressure on inflation. In our setup, we showed that a positive shock to house price expectations yields an increase in house prices (a housing market boom) and a decline in inflation at the same time.

7. HOUSE PRICE EXPECTATIONS AND IMPLICATIONS FOR MONETARY POLICY

This section discusses house price expectation shocks and their implications for monetary policy. In particular, we investigate whether monetary policy reacting to household credit growth can dampen the responses of the economy to house price expectation shocks and reduce the volatility of output and inflation. We conduct two exercises: in the first one, we compare the responses of the economy under the standard Taylor rule, and under the augmented Taylor rule wherein the central bank reacts to household credit growth. Under the standard Taylor rule regime, the central bank reacts to the deviations of inflation and output from their steady state. Under the augmented Taylor rule, the central bank reacts to the deviations of inflation and output from their steady state, yet also from household credit growth. For illustrative purposes, under the augmented Taylor rule, the household credit growth coefficient ρ_F in the generalized rule equation (40) is set to be greater than zero, i.e., 0.15. In the second exercise, we vary the household credit growth coefficient in the Taylor rule between 0 and 2. We apply a Monte Carlo simulation method where our model is simulated 20000 times, and compute the standard deviations for output and inflation.

7.1 First Exercise

Figure 4 shows the responses of the economy under the two different Taylor rule cases. Specifically, the solid line demonstrates the responses to a positive house price expectation shock

under the standard monetary policy, whilst the dashed one illustrates the responses under the augmented Taylor rule with credit growth. Under the standard rule, the central bank observes a downward pressure on inflation and responds to the shock to house price expectations by reducing the policy rate. However, under the augmented Taylor rule (the central bank reacts to household credit growth), the policy rate path is higher than the one of the standard rule. By taking into account household credit growth, monetary policy weakens the response of economic activity to the real estate expectation shock. Further, the reaction of the central bank to the credit growth leads to an even lower inflation rate because of a fall in aggregate demand in the short-run. Taking household credit growth into account, moderates bank lending activities, a fact which curbs the housing demand and dampens the rise in household debt and bank leverage ratio.

7.2 Second Exercise

We now compare the standard deviations of output and inflation under the two Taylor rule cases. Figure 5 displays the standard deviations of output for different values of the household credit growth coefficient, and Figure 6 shows the standard deviations of inflation. Both figures show that as the response of monetary policy to household credit growth increases, the output volatility decreases whereas inflation volatility increases. An advantage of monetary policy reacting to household credit growth, is that it reduces the output volatility. Moreover, monetary policy that incorporates household credit growth, slows down the rise in household debt and the bank leverage ratio, i.e., improves financial stability. Nevertheless, the disadvantage is that inflation volatility rises and the central bank is likely to miss the inflation target in the short run. In summary, taking household credit growth into account reduces the volatility of output and improves financial stability, but worsens price stability.

8. SENSITIVITY ANALYSIS

We conduct a sensitivity analysis with respect to house price expectation shocks and infer upon their implications for monetary policy. In the previous section, under the standard and augmented Taylor rule, the central bank reacts to the deviations of output from its steady state. Now we examine whether the output volatility decreases and inflation volatility increases when the central bank reacts to output growth, instead of deviations of output from its steady state. Under the standard Taylor rule regime, the central bank reacts to the deviations of inflation from its steady state and output growth, whereas in case of the augmented Taylor rule, the central bank reacts to the deviations of inflation from steady state, output growth and household credit growth. As previously, we vary the household credit growth coefficient in the Taylor rule between 0 and 2.

Similarly, the model is simulated for 20000 times and then we estimate the standard deviations for output and inflation.

Figure 7 depicts the standard deviations of output for different values of the household credit growth coefficient and Figure 8 presents the standard deviations of inflation. Those figures indicate that as the response of monetary policy to household credit growth rises, the output volatility declines and inflation volatility increases. Based on both previous and present sections, we may safely conclude that monetary policy incorporating household credit growth improves macroeconomic stability, dampens the credit boom, yet it jeopardizes price stability.

9. CONCLUSIONS

We contribute to the literature by embedding two features rarely encountered in DSGE models with housing collaterals, namely shocks to house price expectations, and endogenous mortgage defaults. We find that a rise in expected future house prices leads to an increase in house prices, housing demand, household debt, business debt, bank leverage ratio, aggregate consumption, investment, employment and output, whereas it leads to a decline in mortgage default rates and interest rates on loans and inflation.

As opposed to Iacoviello (2015) who models mortgage defaults as exogenous shocks, we address endogenous defaults on mortgages. Specifically, in our model financially constrained households default when the value of their houses is lower than the mortgage loan repayment. Moreover, contrary to Forlati and Lambertini (2011) who consider that variations in mortgage defaults depend on variations in mortgage risk, we show that mortgage defaults depend on variations in expected future house prices. The results from our DSGE model simulations illustrate that an increase in expected future house prices leads to a rise in house prices, which causes a decline in mortgage default rates. Our results are consistent with econometric evidence. Additionally, Forlati and Lambertini (2011) demonstrate that variations in mortgage risk generate a weak persistence in house prices as well as in aggregate consumption and output. They also indicate that house prices, aggregate consumption and output deviate from the steady state and then rapidly rebound. Instead, in our model, a shock to house price expectations produces strong persistence in house prices, aggregate consumption and output, in accordance with the empirical VAR study by Towbin and Weber (2015).

Furthermore, we show that a rise in expected future house prices leads to a fall in inflation and a rise in current house prices, simultaneously. Hence, our finding complements the work of Christiano *et al.* (2010), in that optimistic expectations about future technology lead to a fall in inflation and a rise in stock prices. Bernanke and Gertler (2000) report that stock prices tend to be

high during an inflationary period. Also, Forlati and Lambertini (2011) confirm that a drop in mortgage risk entails a rise in house prices and inflation. The findings reported by Bernanke and Gertler (2000) and Forlati and Lambertini (2011) have different ramifications for monetary policy and financial stability compared to our results. In their works, the central bank reacts to upward pressure on inflation by increasing the policy rate. This reaction may reduce financial instability by dampening an increase in leverage in the economy, so that the economy becomes less sensitive to disturbances. Based on our setting, the central bank reacts to downward pressure on inflation by cutting the policy rate. This reaction amplifies the rise in housing demand, household debt and bank leverage ratio, therefore increases financial instability. Thus, the highly-leveraged economy is more vulnerable to a collapse in house prices.

Finally, we explore whether monetary policy that takes into account household credit growth can improve macroeconomic stability and contribute to financial stability. We find that monetary policy responding to household credit growth reduces the volatility of output, but increases the volatility of inflation. By reacting to household credit growth, monetary policy dampens the rise in housing demand, household debt and bank leverage ratio. Consequently, this reaction may reduce the sensitivity of the economy to shocks to the housing market. Overall, monetary policy that reacts to household credit growth improves the stability of the real economy and enhances financial stability, albeit it jeopardizes price stability in the short-run as inflation deviates further from its target.

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TABLE 1: GRANGER CAUSALITY TEST BETWEEN HOUSE PRICES AND MORTGAGE DEFAULTS

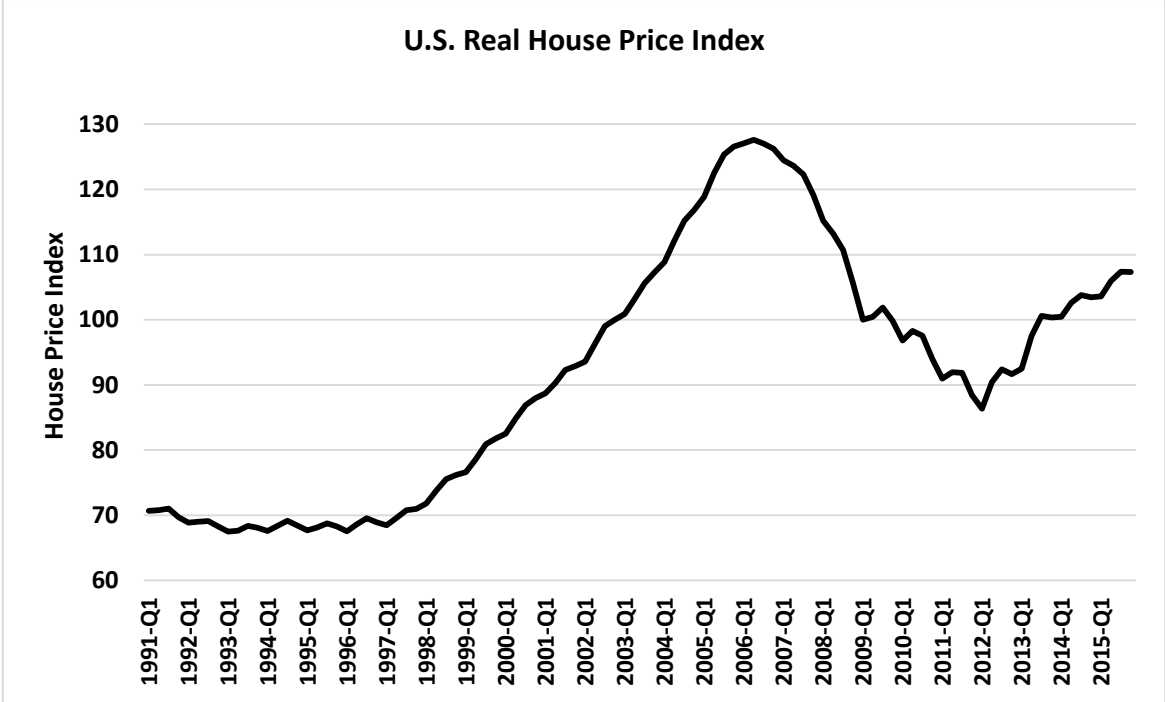
Null Hypothesis	F-Statistic	Prob
<i>Real house prices do not Granger cause mortgage defaults</i>	5.99786	0.0003
<i>Mortgage defaults do not Granger cause real house prices</i>	1.91080	0.1158

TABLE 2: CALIBRATION

Parameter	Description	Value
β_U	<i>Unconstrained household's discount factor</i>	0.9925
β_F	<i>Constrained household's discount factor</i>	0.95
β_E	<i>Entrepreneur's discount factor</i>	0.95
β_B	<i>Banker's discount factor</i>	0.97
γ	<i>Required bank capital ratio</i>	0.05
η	<i>Labor supply aversion</i>	1.01
α	<i>Share of unconstrained household's labor income</i>	0.59
v_h	<i>Weight of housing in household's utility function</i>	0.0905
v	<i>Housing share in the production function</i>	0.104
μ	<i>Capital share in the production function</i>	0.3
δ	<i>Depreciation rate of physical capital</i>	0.03
ψ_k	<i>Capital adjustment cost</i>	0.81
θ	<i>Probability fixed prices</i>	0.75
m_F	<i>Loan-to-value household</i>	0.83
m_E	<i>Loan-to-value entrepreneur</i>	0.64
$\bar{\omega}$	<i>Magnitude of losses</i>	0.61
ϕ''	<i>Cost of deviation from the required capital ratio</i>	0.25

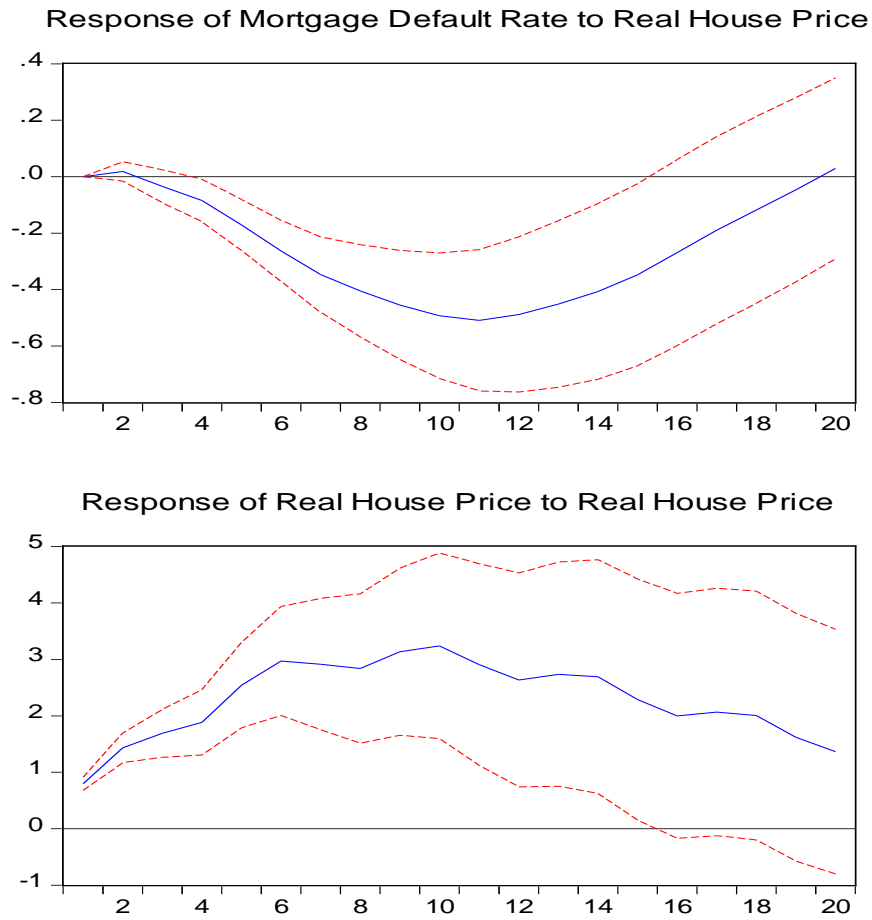
f	<i>Sensitivity of the mortgage default to house prices</i>	0.83
ρ_y	<i>Taylor rule output parameter</i>	0.125
ρ_π	<i>Taylor rule inflation parameter</i>	1.8
ρ_i	<i>Taylor rule interest rate parameter</i>	0.7
ρ_q	<i>Persistence of house price expectation shock</i>	0.95

FIGURE 1: U.S. REAL HOUSE PRICE INDEX



Notes: The data source is Federal Reserve Bank of St. Louis.

FIGURE 2: IMPULSE RESPONSES TO A HOUSE PRICE SHOCK



Notes: The impulse responses are estimated via the Cholesky decomposition method, and are displayed with one S.D Innovations ± 2 SE.

FIGURE 3: RESPONSES OF THE ECONOMY TO A HOUSE PRICE EXPECTATION SHOCK

FIGURE 3A

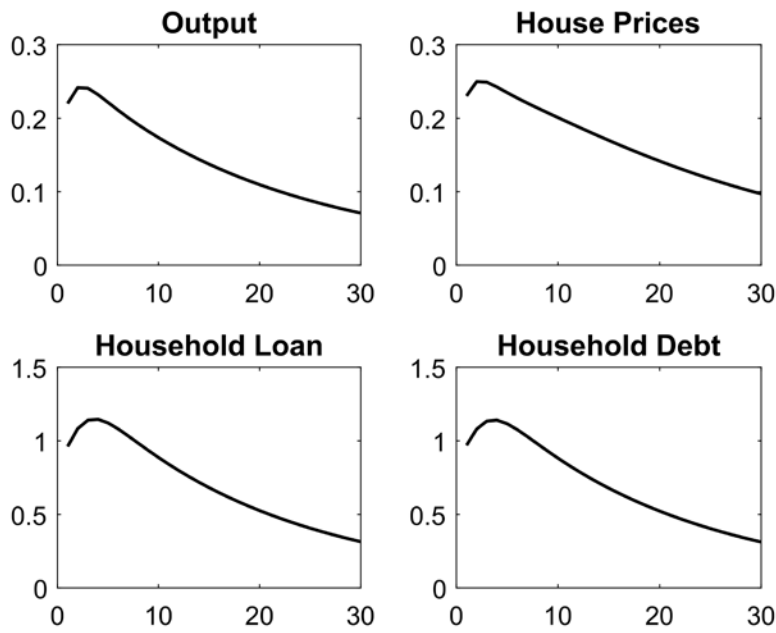


FIGURE 3B

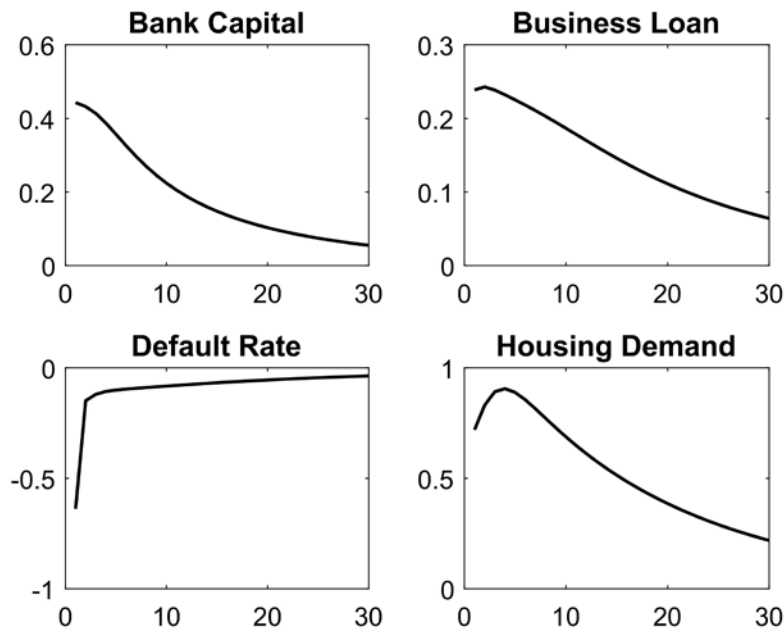


FIGURE 3C

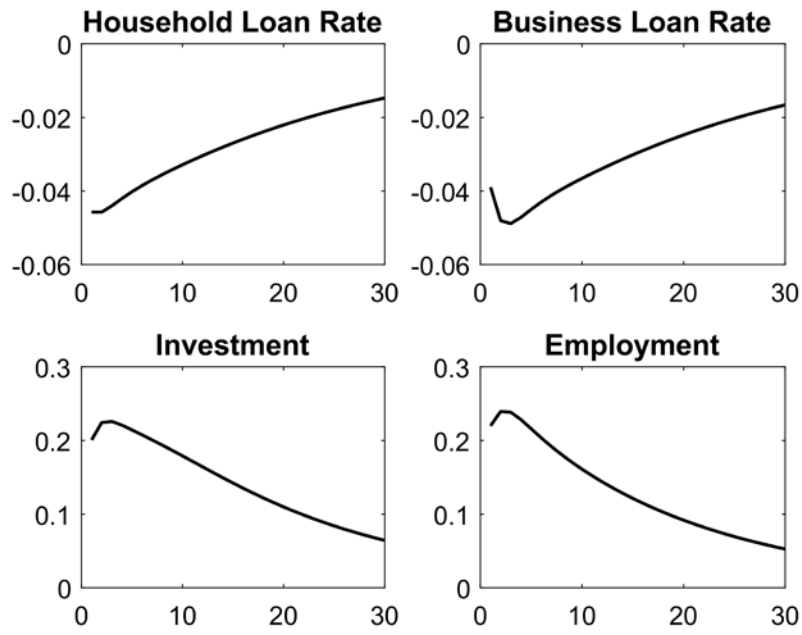


FIGURE 3D

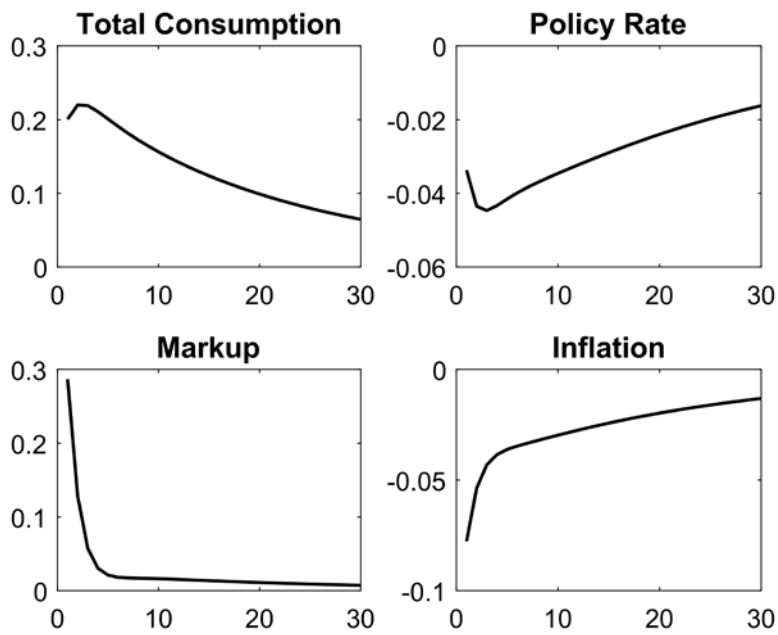
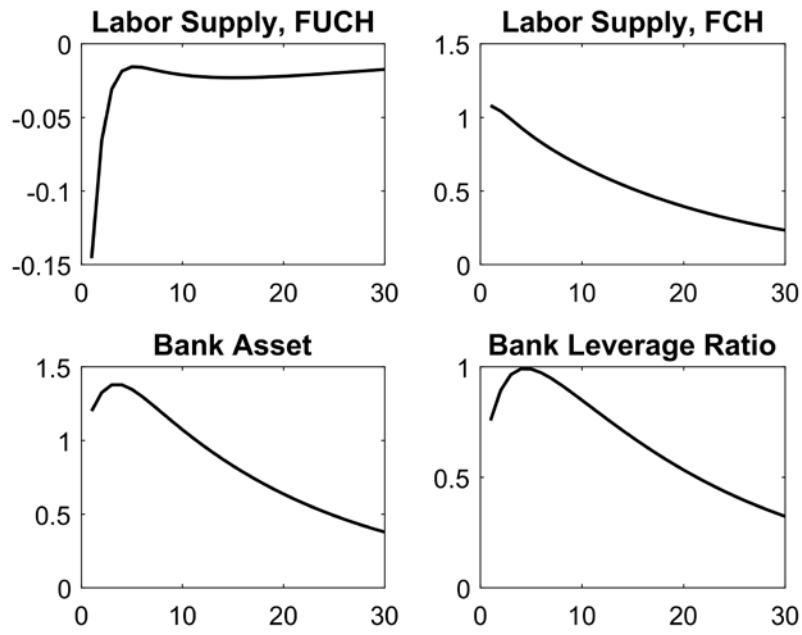


FIGURE 3E



Notes: The IRFs are depicted in case of a house price expectation shock. FUCH denotes financially unconstrained household, while FCH represents a financially constrained household.

FIGURE 4: HOUSE PRICE EXPECTATIONS AND MONETARY POLICY

FIGURE 4A

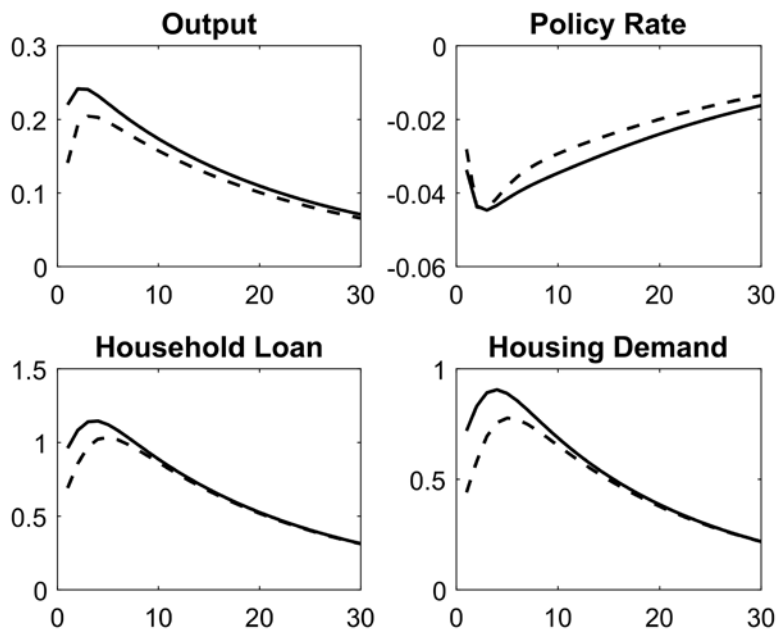


FIGURE 4B

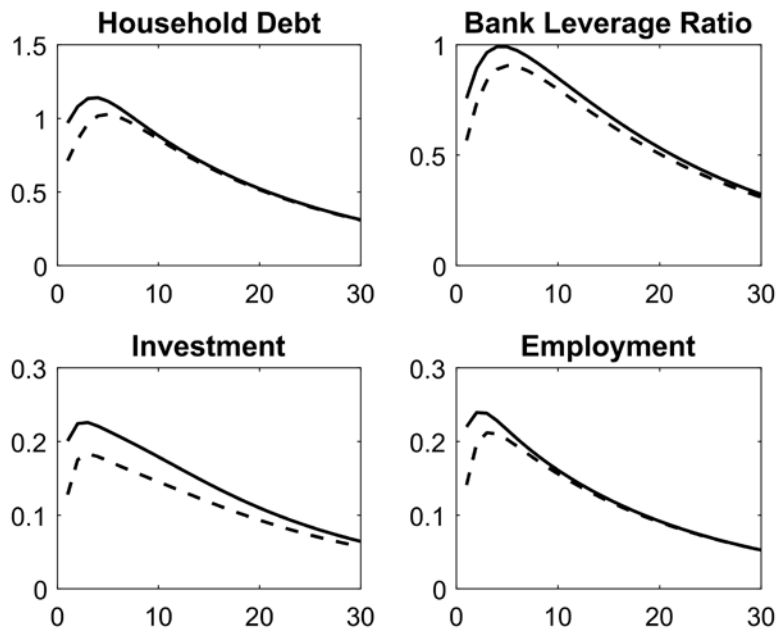
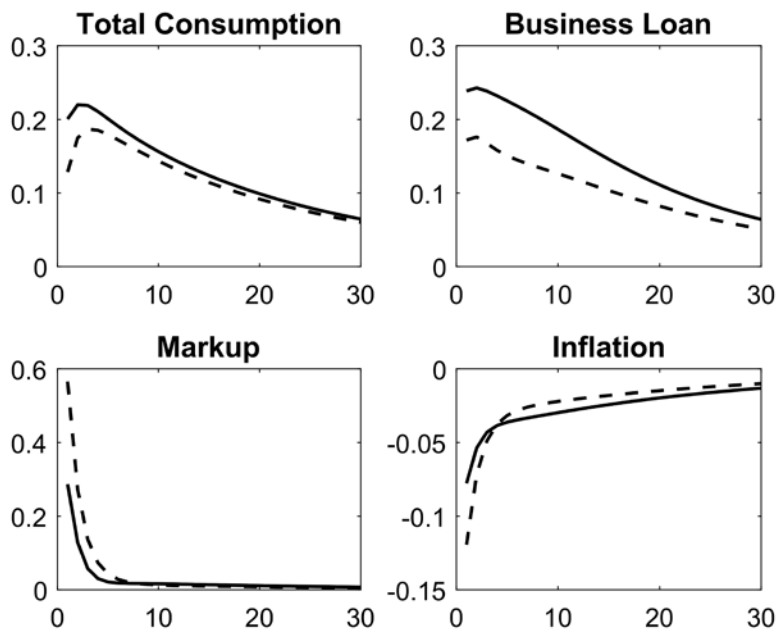
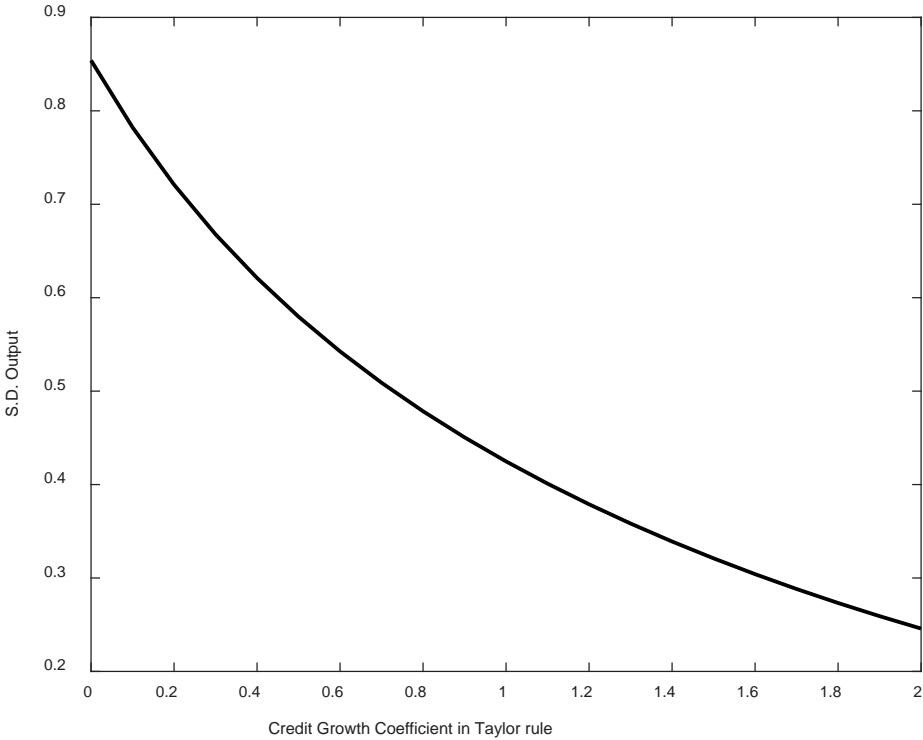


FIGURE 4C



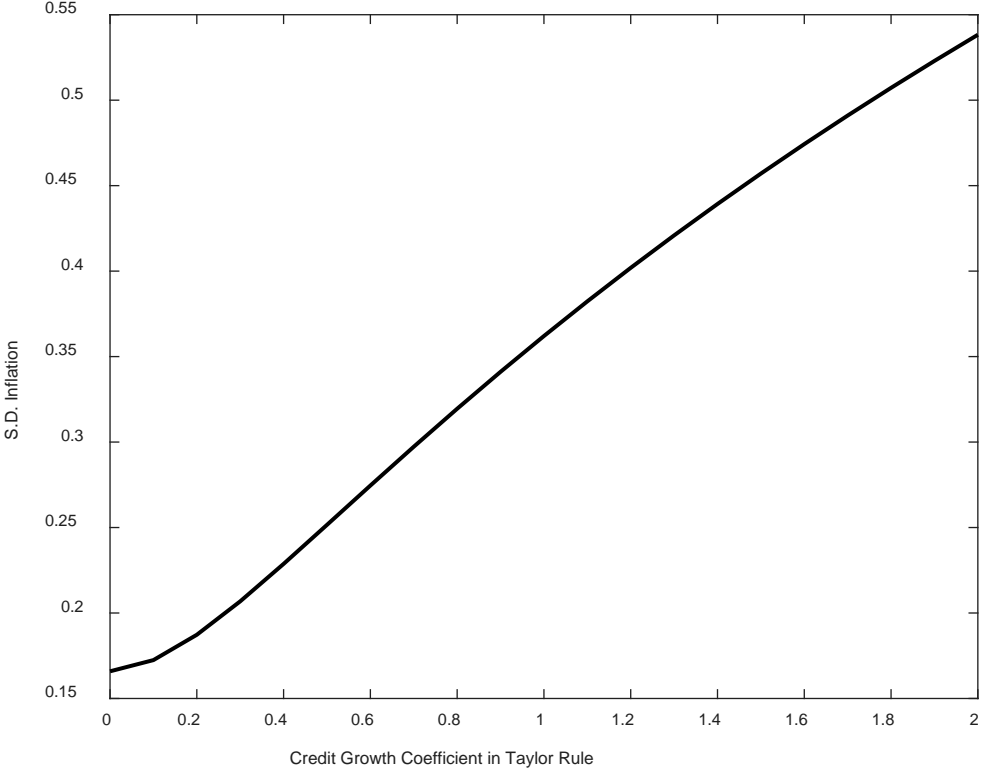
Notes: The solid line shows the responses of the economy to a positive house price expectation shock under the standard monetary policy. The dashed line shows the reaction of the economy under the augmented Taylor rule with credit growth.

FIGURE 5: STANDARD DEVIATION OF OUTPUT



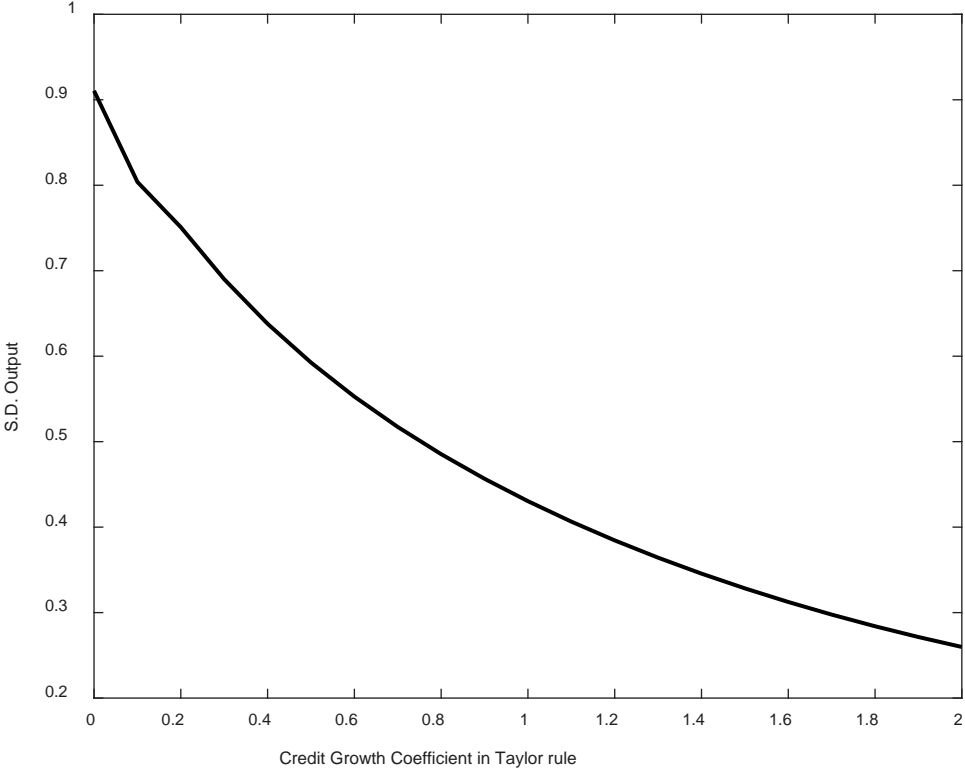
Notes: The central bank reacts to the deviations of inflation from its steady state, the deviations of output from steady state and to household credit growth.

FIGURE 6: STANDARD DEVIATION OF INFLATION



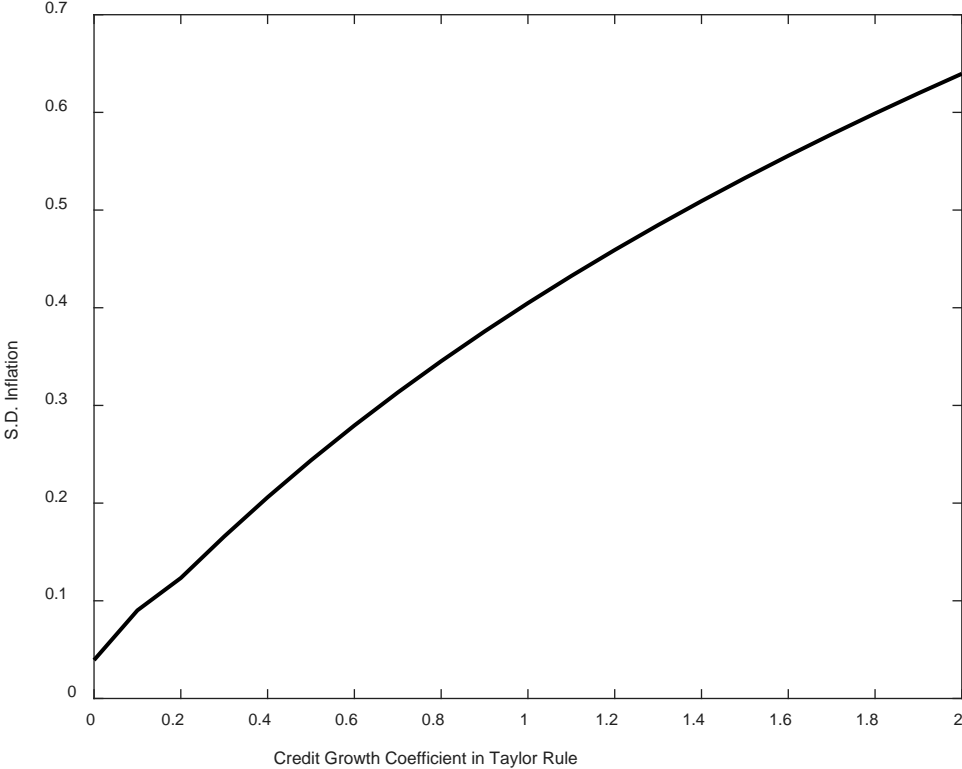
Notes: The central bank reacts to the deviations of inflation from its steady state, the deviations of output from its steady state, and household credit growth.

FIGURE 7: SENSITIVITY ANALYSIS AND STANDARD DEVIATION OF OUTPUT



Notes: The central bank reacts to the deviations of inflation from its steady state, output growth and from household credit growth.

FIGURE 8: SENSITIVITY ANALYSIS AND STANDARD DEVIATIONS OF INFLATION



Notes: The central bank reacts to the deviations of inflation from its steady state, output growth and household credit growth.

APPENDIX I: TECHNICAL DESCRIPTION

I.1 DEBT DEFAULTS

This section presents the full derivations of the mortgage default. A financially constrained household can default on mortgages. The household consists of many members i who all purchase houses $H_{F,t}^i$. The total stock of the household is $H_{F,t} = \int_{i=0}^1 H_{F,t}^i di$. All members buy houses of the same size $H_{F,t}^i = H_{F,t}$. In every period, each member's house value is subject to an idiosyncratic *iid* shock ω^i such that the house value becomes $q_t H_{F,t}^i (1 + \omega^i)$. The household member i defaults when the value of the house is lower than the mortgage loan repayment $\frac{R_{F,t-1}}{\pi_t} L_{F,t-1} > q_t H_{F,t-1} (1 + \omega^i)$. Let $\hat{\omega}$ be the threshold value of the shock for which the member will pay back the mortgages $\hat{\omega} = \frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - 1$. If the household member i draws ω lower than $\hat{\omega}$, the member will default. Let $-\bar{\omega}$ be the lower bound of the distribution where $\bar{\omega} > 0$. We assume that $\hat{\omega}$ is always larger than $-\bar{\omega}$, thus there are always some defaults. The amount of mortgage defaults $Z_{F,t}$ is determined by the following integral

$$Z_{F,t} = \int_{-\bar{\omega}}^{\left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}}\right) - 1} \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t} - q_t H_{F,t-1} (1 + \omega) \right) f(\omega) d\omega$$

To simplify, we assume that $f(\omega)$ is constant in the interval $-\bar{\omega} < \omega < \hat{\omega}$ such that $f(\omega) = f$. Therefore, the following integration becomes

$$\begin{aligned} & \int_{-\bar{\omega}}^{\left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}}\right) - 1} \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t} - q_t H_{F,t-1} (1 + \omega) \right) f(\omega) d\omega \\ &= \left[\left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t} - q_t H_{F,t-1} \right) \omega - q_t H_{F,t-1} \frac{\omega^2}{2} \right]_{-\bar{\omega}}^{\left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}}\right) - 1} f d\omega \\ &= f \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t} - q_t H_{F,t-1} \right) \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - 1 \right) \\ &\quad - \frac{q_t H_{F,t-1} f}{2} \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - 1 \right)^2 + \left[\left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t} - q_t H_{F,t-1} \right) \bar{\omega} - q_t H_{F,t-1} \frac{\bar{\omega}^2}{2} \right] f \\ &= f q_t H_{F,t-1} \left[\left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - 1 \right)^2 - \frac{1}{2} \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - 1 \right)^2 \right] \\ &\quad + f q_t H_{F,t-1} \left[\left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - 1 \right) \bar{\omega} + \frac{\bar{\omega}^2}{2} \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{f q_t H_{F,t-1}}{2} \left[\left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - 1 \right)^2 + 2 \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - 1 \right) \bar{\omega} + \bar{\omega}^2 \right] \\
&= \frac{f q_t H_{F,t-1}}{2} \left[\left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - 1 \right)^2 - 2(1 - \bar{\omega}) \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} \right) + (1 - \bar{\omega})^2 \right]
\end{aligned}$$

This calculation provides the following amount of mortgage defaults

$$Z_{F,t} = f \frac{q_t H_{F,t-1}}{2} \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - (1 - \bar{\omega}) \right)^2$$

I.2 FINANCIALLY UNCONSTRAINED HOUSEHOLD

The utility function of a financially unconstrained household is

$$E_0 \sum_{t=0}^{\infty} \beta_U^t \left(\ln C_{U,t} + v_h \ln H_{U,t} - \frac{(N_{U,t})^{\eta+1}}{\eta+1} \right) \quad (\text{I.1})$$

The financially unconstrained household maximizes its utility subject to the following budget constraint

$$C_{U,t} + D_t + q_t (H_{U,t} - H_{U,t-1}) = \frac{R_{t-1}}{\pi_t} D_{t-1} + W_{U,t} N_{U,t} + F_t \quad (\text{I.2})$$

The Lagrangian for the financially unconstrained household is

$$\begin{aligned}
\Gamma &= E_0 \sum_{t=0}^{\infty} \beta_U^t \left(\ln C_{U,t} + v_h \ln H_{U,t} - \frac{(N_{U,t})^{\eta+1}}{\eta+1} \right) \\
&+ \mu_{U,t} \left(\frac{R_{t-1}}{\pi_t} D_{t-1} + W_{U,t} N_{U,t} + F_t - C_{U,t} - D_t - q_t (H_{U,t} - H_{U,t-1}) \right)
\end{aligned}$$

The first-order conditions with respect to $C_{U,t}$, D_t , $H_{U,t}$, and $N_{U,t}$ are as follows respectively

$$\frac{\partial \Gamma}{\partial C_{U,t}} = \frac{1}{C_{U,t}} - \mu_{U,t} = 0 \quad (\text{I.3})$$

$$\frac{\partial \Gamma}{\partial D_t} = \beta_U E_t \left(\mu_{U,t+1} \frac{R_t}{\pi_{t+1}} \right) - \mu_{U,t} = 0 \quad (\text{I.4})$$

$$\frac{\partial \Gamma}{\partial H_{U,t}} = \frac{v_h}{H_{U,t}} - \mu_{U,t} q_t + \beta_U E_t (\mu_{U,t+1} q_{t+1}) = 0 \quad (\text{I.5})$$

and

$$\frac{\partial \Gamma}{\partial N_{U,t}} = -(N_{U,t})^\eta + \mu_{U,t} W_{U,t} = 0 \quad (\text{I.6})$$

Combining equations (I.3) and (I.4), we obtain the Euler consumption equation, which can be written as

$$\frac{1}{C_{U,t}} = \beta_U E_t \left(\frac{1}{C_{U,t+1}} \frac{R_t}{\pi_{t+1}} \right) \quad (\text{I.7})$$

Jointly from equations (I.3) and (I.5), we obtain the following housing demand equation

$$\frac{q_t}{C_{U,t}} = \frac{v_h}{H_{U,t}} + \beta_U E_t \left(\frac{q_{t+1}}{C_{U,t+1}} \right) \quad (\text{I.8})$$

From equations (I.3) and (I.6), we obtain the following labor supply equation

$$\frac{W_{U,t}}{C_{U,t}} = N_{U,t}^\eta \quad (\text{I.9})$$

I.3 FINANCIALLY CONSTRAINED HOUSEHOLD

The utility function of a financially constrained household is

$$E_0 \sum_{t=0}^{\infty} \beta_F^t \left(\ln C_{F,t} + v_h \ln H_{F,t} - \frac{(N_{F,t})^{\eta+1}}{\eta+1} \right) \quad (\text{I.10})$$

The financially constrained household maximizes its utility subject to the constraints

$$C_{F,t} + q_t(H_{F,t} - H_{F,t-1}) + \frac{R_{F,t-1}}{\pi_t} L_{F,t-1} - Z_{F,t} = L_{F,t} + W_{F,t} N_{F,t} \quad (\text{I.11})$$

$$E_t \left(\frac{R_{F,t}}{\pi_{t+1}} \right) L_{F,t} \leq m_F E_t(q_{t+1} H_{F,t}) \quad (\text{I.12})$$

and

$$Z_{F,t} = f \frac{q_t H_{F,t-1}}{2} \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - (1 - \bar{\omega}) \right)^2 \quad (\text{I.13})$$

The financially constrained household maximizes its utility (I.10) subject to the budget constraint (I.11), the collateral constraint (I.12) and the mortgage default condition (I.13). The Lagrangian for the financially unconstrained household is

$$\begin{aligned} \Gamma = & E_0 \sum_{t=0}^{\infty} \beta_U^t \left(\ln C_{F,t} + v_h \ln H_{F,t} - \frac{(N_{F,t})^{\eta+1}}{\eta+1} \right) \\ & + \mu_{F,t} \left(L_{F,t} + W_{F,t} N_{F,t} + Z_{F,t} - C_{F,t} - q_t(H_{F,t} - H_{F,t-1}) - \frac{R_{F,t-1}}{\pi_t} L_{F,t-1} \right) \\ & + \lambda_{F,t} \left(m_F E_t(q_{t+1} H_{F,t}) - E_t \left(\frac{R_{F,t}}{\pi_{t+1}} \right) L_{F,t} \right) \end{aligned}$$

The first order conditions with respect to $C_{F,t}$, D_t , $H_{F,t}$ and $N_{F,t}$ are respectively

$$\frac{\partial \Gamma}{\partial C_{F,t}} = \frac{1}{C_{F,t}} - \mu_{F,t} = 0 \quad (\text{I.14})$$

$$\frac{\partial \Gamma}{\partial L_{F,t}} = \mu_{F,t} - \beta_F E_t \left(\mu_{F,t+1} \frac{R_{F,t}}{\pi_{t+1}} \right) - \lambda_{F,t} E_t \left(\frac{R_{F,t}}{\pi_{t+1}} \right) + \beta_F E_t \left(\mu_{F,t+1} \frac{\partial Z_{F,t+1}}{\partial L_{F,t}} \right) = 0 \quad (\text{I.15})$$

$$\frac{\partial \Gamma}{\partial H_{F,t}} = \frac{v_h}{H_{F,t}} - \mu_{F,t} q_t + \beta_F E_t (\mu_{F,t+1} q_{t+1})$$

$$+ \lambda_{F,t} m_F E_t(q_{t+1} H_{F,t}) + \beta_F E_t \left(\frac{\partial Z_{F,t+1}}{\partial H_{F,t}} \right) = 0 \quad (\text{I.16})$$

and

$$\frac{\partial \Gamma}{\partial N_{F,t}} = -(N_{F,t})^\eta + \mu_{F,t} W_{U,t} = 0 \quad (\text{I.17})$$

From equations (I.14) and (I.15), we attain the following Euler consumption equation with collateral and mortgage default channels

$$\frac{1}{C_{F,t}} = \beta_F E_t \left(\frac{1}{C_{F,t+1}} \frac{R_{F,t}}{\pi_{t+1}} \right) + \lambda_{F,t} E_t \left(\frac{R_{F,t}}{\pi_{t+1}} \right) - \beta_F E_t \left(\frac{1}{C_{F,t+1}} \frac{\partial Z_{F,t+1}}{\partial L_{F,t}} \right) \quad (\text{I.18})$$

Furthermore, combining equations (I.14) and (I.16) we obtain the housing demand equation with collateral channel

$$\frac{q_t}{C_{F,t}} = \frac{v_h}{H_{F,t}} + \beta_F E_t \left(\frac{q_{t+1}}{C_{F,t+1}} \right) + m_F E_t (\lambda_{F,t} q_{t+1}) + \beta_F E_t \left(\frac{1}{C_{F,t+1}} \frac{\partial Z_{F,t+1}}{\partial H_{F,t}} \right) \quad (\text{I.19})$$

Additionally, from equations (I.14) and (I.17), we set the labor supply equation, written as

$$\frac{W_{F,t}}{C_{F,t}} = N_{F,t}^\eta \quad (\text{I.20})$$

I.4 ENTREPRENEUR

An entrepreneur has the following objective function

$$E_0 \sum_{t=0}^{\infty} \beta_E^t (\ln C_{E,t}) \quad (\text{I.21})$$

She maximizes its objective function (I.21) subject to the following constraints

$$Y_{E,t} = K_{t-1}^\mu H_{E,t-1}^\nu N_{U,t}^{\alpha(1-\mu-\nu)} N_{F,t}^{(1-\alpha)(1-\mu-\nu)} \quad (\text{I.22})$$

$$\begin{aligned} C_{E,t} + q_t (H_{E,t} - H_{E,t-1}) + \frac{R_{E,t-1}}{\pi_t} L_{E,t-1} + W_{U,t} N_{U,t} + W_{F,t} N_{F,t} + I_t + \xi_{K,t} \\ = \frac{Y_{E,t}}{X_t} + L_{E,t} \end{aligned} \quad (\text{I.23})$$

where $\xi_{K,t} = (\psi_k/2\delta) \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1}$

$$K_t = (1 - \delta)K_{t-1} + I_t \quad (\text{I.24})$$

and

$$E_t \left(\frac{R_{E,t}}{\pi_{t+1}} \right) L_{E,t} \leq m_E E_t (q_{t+1} H_{E,t}) \quad (\text{I.25})$$

The Lagrangian is

$$\begin{aligned} \Gamma = E_0 \sum_{t=0}^{\infty} \beta_F^t (\ln C_{E,t}) \\ + \mu_{E,t} \left(\frac{Y_{E,t}}{X_t} + L_{E,t} - C_{E,t} - q_t (H_{E,t} - H_{E,t-1}) - \frac{R_{E,t-1}}{\pi_t} L_{E,t-1} - W_{U,t} N_{U,t} - W_{F,t} N_{F,t} - I_t - \right. \\ \left. \xi_{K,t} \right) + \lambda_{E,t} \left(m_E E_t (q_{t+1} H_{E,t}) - E_t \left(\frac{R_{E,t}}{\pi_{t+1}} \right) L_{E,t} \right) \end{aligned}$$

The entrepreneur chooses $C_{E,t}$, $L_{E,t}$, K_t , I_t , $N_{U,t}$, $N_{F,t}$, and $H_{E,t}$. The first-order conditions are as follows

$$\frac{\partial \Gamma}{\partial C_{E,t}} = \frac{1}{C_{F,t}} - \mu_{E,t} = 0 \quad (\text{I.26})$$

$$\frac{\partial \Gamma}{\partial L_{E,t}} = \mu_{E,t} - \beta_E E_t \left(\mu_{E,t+1} \frac{R_{E,t}}{\pi_{t+1}} \right) - \lambda_{E,t} E_t \left(\frac{R_{E,t}}{\pi_{t+1}} \right) = 0 \quad (\text{I.27})$$

$$\frac{\partial \Gamma}{\partial H_{E,t}} = \beta_E E_t \left[\mu_{E,t+1} \left(v \frac{Y_{E,t+1}}{X_{t+1} H_{E,t}} + q_{t+1} \right) \right] + m_E \lambda_{E,t} E_t (q_{t+1}) = 0 \quad (\text{I.28})$$

$$\begin{aligned} \frac{\partial \Gamma}{\partial K_t} &= \beta_E E_t \left[\mu_{E,t+1} \left(\frac{\psi_k}{\delta} \left(\frac{I_{t+1}}{K_t} - \delta \right) \frac{I_{t+1}}{K_t} - \frac{\psi_k}{2\delta} \left(\frac{I_{t+1}}{K_t} - \delta \right)^2 \right) \right] \\ &+ \beta_E E_t \left[\mu_{E,t+1} \frac{\mu Y_{E,t+1}}{X_{t+1} K_t} + \mu_{E,t+1} (1 - \delta) \right] - \mu_{E,t} = 0 \end{aligned} \quad (\text{I.29})$$

$$\frac{\partial \Gamma}{\partial I_t} = \mu_{E,t} \left[1 + \frac{\psi_k}{\delta} \left(\frac{I_t}{K_{t-1}} - \delta \right) \right] - \mu_{E,t} = 0 \quad (\text{I.30})$$

$$\frac{\partial \Gamma}{\partial N_{U,t}} = \mu_{E,t} \alpha (1 - \mu - \nu) \frac{Y_{E,t}}{X_t N_{U,t}} - \mu_{E,t} W_{U,t} = 0, \quad (\text{I.31})$$

and

$$\frac{\partial \Gamma}{\partial N_{F,t}} = \mu_{E,t} (1 - \alpha) (1 - \mu - \nu) \frac{Y_{E,t}}{X_t N_{F,t}} - \mu_{E,t} W_{F,t} = 0 \quad (\text{I.32})$$

Equations (I.26) and (I.27) provide the Euler consumption with a collateral constraint

$$\frac{1}{C_{E,t}} = \beta_E E_t \left(\frac{1}{C_{E,t+1}} \frac{R_{E,t}}{\pi_{t+1}} \right) + \lambda_{E,t} E_t \left(\frac{R_{E,t}}{\pi_{t+1}} \right) \quad (\text{I.33})$$

Combining equations (I.26) and (I.28), we set the real estate demand equation with collateral constraint as follows

$$\frac{q_t}{C_{E,t}} = E_t \left[\frac{\beta_E}{C_{E,t+1}} \left(v \frac{Y_{E,t+1}}{X_{t+1} H_{E,t}} + q_t \right) \right] + m_E E_t (\lambda_{E,t} q_{t+1}) \quad (\text{I.34})$$

Moreover, equations (I.26), (I.29) and (I.30) produce the investment equation

$$\begin{aligned} \frac{1}{C_{E,t}} \left[1 + \frac{\psi_k}{\delta} \left(\frac{I_t}{K_{t-1}} - \delta \right) \right] &= \beta_E E_t \left[\frac{1}{C_{E,t+1}} \left(\frac{\psi_k}{\delta} \left(\frac{I_{t+1}}{K_t} - \delta \right) \frac{I_{t+1}}{K_t} - \frac{\psi_k}{2\delta} \left(\frac{I_{t+1}}{K_t} - \delta \right)^2 \right) \right] \\ &+ \beta_E E_t \left[\frac{1}{C_{E,t+1}} \left(\frac{\mu Y_{E,t+1}}{X_{t+1} K_t} + (1 - \delta) \right) \right] \end{aligned} \quad (\text{I.35})$$

Finally, equations (I.31) and (I.32) give us the following labor demand conditions

$$\alpha (1 - \mu - \nu) \frac{Y_{E,t}}{X_t} = W_{U,t} N_{U,t} \quad (\text{I.36})$$

and

$$(1 - \alpha) (1 - \mu - \nu) \frac{Y_{E,t}}{X_t} = W_{F,t} N_{F,t} \quad (\text{I.37})$$

I.5 COMMERCIAL BANK

A banker has a preference defined as

$$E_0 \sum_{t=0}^{\infty} \beta_B^t (\ln C_{B,t}) \quad (\text{I.38})$$

The banker maximizes her utility (I.38) subject to the following constraints

$$X_{B,t} = (1 - \gamma) (L_{F,t} + L_{E,t}) - D_t \quad (\text{I.39})$$

$$C_{B,t} + \frac{R_{t-1}}{\pi_t} D_{t-1} + L_{E,t} + L_{F,t} + \phi(X_{B,t}) = D_t + \frac{R_{E,t-1}}{\pi_t} L_{E,t-1} + \frac{R_{F,t-1}}{\pi_t} L_{F,t-1} - Z_{F,t} \quad (\text{I.40})$$

and

$$Z_{F,t} = f \frac{q_t H_{F,t-1}}{2} \left(\frac{R_{F,t-1} L_{F,t-1}}{\pi_t q_t H_{F,t-1}} - (1 - \bar{\omega}) \right)^2 \quad (\text{I.41})$$

The Lagrangian for the banker is the one below

$$\Gamma = E_0 \sum_{t=0}^{\infty} \beta_B^t (\ln C_{B,t}) + \mu_{B,t} \left(\begin{array}{l} D_t + \frac{R_{E,t-1}}{\pi_t} L_{E,t-1} + \frac{R_{F,t-1}}{\pi_t} L_{F,t-1} - Z_{F,t} \\ -C_{B,t} - \frac{R_{t-1}}{\pi_t} D_{t-1} - L_{E,t} - L_{F,t} - \phi(X_{B,t}) \end{array} \right)$$

In that, the banker chooses $C_{B,t}$, D_t , $L_{E,t}$, and $L_{F,t}$. The first-order conditions are

$$\frac{\partial \Gamma}{\partial C_{B,t}} = \frac{1}{C_{B,t}} - \mu_{B,t} = 0 \quad (\text{I.42})$$

$$\frac{\partial \Gamma}{\partial D_t} = \mu_{B,t} - \beta_B E_t \left(\mu_{B,t+1} \frac{R_t}{\pi_{t+1}} \right) + \mu_{B,t} \phi'(X_{B,t}) = 0 \quad (\text{I.43})$$

$$\frac{\partial \Gamma}{\partial L_{E,t}} = \beta_B E_t \left(\mu_{B,t+1} \frac{R_{E,t}}{\pi_{t+1}} \right) - \mu_{B,t} - \mu_{B,t} (1 - \gamma) \phi'(X_{B,t}) = 0 \quad (\text{I.44})$$

and

$$\begin{aligned} \frac{\partial \Gamma}{\partial L_{F,t}} &= \beta_B E_t \left(\mu_{B,t+1} \frac{R_{F,t}}{\pi_{t+1}} \right) - \mu_{B,t} \\ &- \mu_{B,t} (1 - \gamma) \phi'(X_{B,t}) - \beta_B E_t \left(\mu_{B,t+1} \frac{\partial Z_{F,t+1}}{\partial L_{F,t}} \right) = 0 \end{aligned} \quad (\text{I.45})$$

The condition for the demand of deposits is given by equations (I.42) and (I.43)

$$\beta_B E_t \left(\frac{R_t}{\pi_{t+1}} \right) = E_t \left(\frac{C_{B,t+1}}{C_{B,t}} \right) [1 + \phi'(X_{B,t})] \quad (\text{I.46})$$

From equations (I.42) and (I.44), we model the condition for loan supply to the entrepreneur

$$\beta_B E_t \left(\frac{R_{E,t}}{\pi_{t+1}} \right) = E_t \left(\frac{C_{B,t+1}}{C_{B,t}} \right) [1 + (1 - \gamma) \phi'(X_{B,t})] \quad (\text{I.47})$$

Lastly, the condition for loan supply to the financially constrained household is given by incorporating the effects from equations (I.42) and (I.45), as such

$$\beta_B E_t \left(\frac{R_{F,t}}{\pi_{t+1}} \right) = E_t \left(\frac{C_{B,t+1}}{C_{B,t}} \right) [1 + (1 - \gamma) \phi'(X_{B,t})] + \beta_B E_t \left(\frac{\partial Z_{F,t+1}}{\partial L_{F,t}} \right) \quad (\text{I.48})$$

APPENDIX II: DATA DESCRIPTION

- *Delinquency Rate on Single-Family Residential Mortgages*: Booked in Domestic Offices, All Commercial Banks, Percent, Quarterly, Seasonally Adjusted, Federal Reserve Bank of St. Louis.
- *S&P/Case-Shiller U.S. National Home Price Index*: Index Q1 2009=100, Quarterly, Not Seasonally Adjusted, Federal Reserve Bank of St. Louis.
- *Gross Domestic Product Implicit Price Deflator*: Index 2009=100, Quarterly, Seasonally Adjusted, Federal Reserve Bank of St. Louis.