The Degree of Independence in Goods and Capital Markets: An econometric degustation

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The Degree of Independence in Goods and Capital Markets: An econometric *degustation*

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" Giorgio Calzolari, Università di Firenze
" Søren Johansen, external EUI & University of Copenhagen, Supervisor
" Grayham Mizon, University of Southampton
To my son Øystein, who suffered most from this ego trip.

Writing a thesis indebts one to many. To render justice to all would require a book in itself. I therefore forced to limit myself to few. I hope that my friends at the EUI in particular, will excuse me for the oversight.

My warmest thanks goes of course to those nearest and dearest to me, my wife Monica and her wonderful Sardinian family for having helped and encouraged me in every respect the last couple of years. Without them this thesis would never have seen it's final stage and I would probably also have been fifteen kilos lighter. Mind you Rita!

Then of course, I want to thank Søren Johansen for giving me the opportunity to write a thesis under his supervision. I want particularly to thank him for giving me support and help some years ago in an, to me, extremely difficult period of life. Thanks also to my second supervisor, Mike Artis, for having taken the time to thoroughly read through all my papers, some of them several times, and for offering suggestions, amendments and comments, all of which have contributed significantly to this work. Then of course I wish to extend a particular word of thanks to Katarina Juselius, who through a countless number of encouraging discussions, seminars and workshops gave me the inspiration and motivation I needed to be able to finish this thesis.

I also want to extend a particular word of thanks to Anja Haensch and Matthias Rau for helping me to combat occasional problems with a lack of self-confidence and for encouraging me to go on in times I felt most like giving up. Thanks also to my three “drinking” companions, Hans Van Der Veen, Erik Tangerstad and James Kay, to whom I owe an awful lot, last mentioned also for proofreading parts of Chapter 2 and the introduction of this thesis. Finally, I want to thank those not mentioned in the above who in some way or other has helped and encouraged me during my stay in Italy. I want you all to know that I have not forgotten a single one of you. Thanks.

Oslo, January 20, 2002
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Introduction

The purposes of this thesis are manifold. Nevertheless one may say that one of its governing ideas has been to make use of some of the most advanced and recently developed state of the art time series techniques that exist to analyse actual data in as coherent, scientific and correct environment as possible. The overriding concern for this has primarily been to detach as far as it can be done, discussions of economic matters from non-scientific and manipulative approaches where the focus as suggested by the title, has particularly been on revealing the degree of independence in capital and goods markets. In this respect, as this thesis focuses on the analysis of long-run relationships among the times series of individual data sets, a matter of particular concern has been to avoid resorting to dummies in the process of model design and identification, beyond what has been necessary, of course, to get residuals with sufficiently “nice” properties not to invalidate the statistical analyses. This choice has been made upon a strong and a priori personal belief that long-run common features among time series, like shared common trends, if they at all are to be considered as robust, should not be as heavily affected by outliers that they legitimate a whole battery of dummies. Thus, to let them in should, in any case, not influence the outcome of the analysis of the long-run relationships as opposed to how they might affect a dynamic model specification. The development of fully specified dynamic models has thus been deemed less urgent, though I admit that this might potentially have had the effect of creating certain difficulties with regard to the possibility of developing a congruent dynamic representation of the information contained in data based on the identified long-term structures. Another important purpose of this thesis has been to spark what is meant to be a creative discussion of the different time series techniques involved, hopefully to enhance and elaborate their understanding as well as to pinpoint their implicit limitations and advantages. Finally, in some innocent way I have also tried to contribute to the econometric literature by suggesting new ways to deal with certain kinds of problems and data. Below follows a brief summary of the individual chapters and their aims.

The first chapter deals with the identification of international interest rate linkages between European and international capital markets. Besides identifying the long-term cointegrating relationships among the times series, using the method developed by S. Johansen (1988), the paper also seeks to develop a structural VAR model. The results are rather mixed as the outcome of the cointegration analysis suggests that long-term European interest rates are
driven in the long run by the corresponding international ones at the same time as the dynamic structure implies short-run effects on long-term European rates of changes in short-term European interest rates. Even though the first result might suggest some kind of impotence on the part of Central Banks in the conduct of monetary policy, the second could indeed be taken to indicate quite the contrary. That is that monetary policy is effective through affecting long-term interest rates and expectations with regard to future interest rates in such a way that it neutralizes an eventual effect that short rates might have on long-term interest rates in the long run. However as paper number two, to which we now turn, strongly suggests, this potential explanation might be an illusion as the dynamic structure given to the VAR model of this paper turns out to be highly arbitrary. Furthermore, the choice made in this paper with regard to the number of cointegrating vectors is not a trivial one as the analysis indicates the potential existence of an additional cointegrating vector. The elaboration of this possibility and the discussion of its implications are placed in Chapter 3 of this thesis.

The second paper, Chapter 2, aims at critically discussing the widely adopted perception of central banks having been the crux of the two periods of convergence observed during the eighties and nineties. Based on the results of Chapter one, a paper by Juselius and McDonald (2000) and an independent analysis herein, the paper concludes that there is evidence that the dynamic short-run effect of short rates on long rates identified in the first paper of this thesis, is a spurious one. If so, this could seriously jeopardize the conclusions made with regard to the ECB’s ability to run an independent monetary policy; as long-term interest rates would be totally determined by what is going on in international markets an independent monetary policy can have no bearing whatsoever on long-term European interest rates. Whether this is the case rests on the assumption that monetary policy works mainly through the way that policy rates affect long-term interest rates and in so far as it is correct, this would imply that Central Bank policy cannot have been the only factor influencing the convergence process during the last two decades. Other factors must have had important roles as well and the paper points to fiscal policy and the effect of improved mobility in capital and goods markets as alternative and supplementary explanations. The second part of the paper then tries to bring this discussion one step further as a potential loss of control on part of central banks with regard to the long end of the capital market, could have clear policy implications, particularly with regard to solving the issue of unemployment in Europe. In this respect the paper ends up suggesting the use of regionally directed policies geared towards stimulating investment and boosting demand.
The aim of Chapter three has been to develop a new technique to deal with cointegration when data in addition to varying along a time series dimension, vary along a cross sectional dimension that is not too large. The suggested strategy is a simple one and implies undertaking the analysis in two-steps with the possibility of adding a third step to improve upon the estimates. The first step involves making an ordinary section-wise cointegration analysis. The second step then treats the cointegrating relations of the first step as known and looks for long-run relationships across sectors conditional on these by again using Johansen’s Maximum Likelihood procedure in a straight-forward way, explicit account of course taken of the fact that the distribution of the trace statistics now will deviate from the ordinary asymptotic one. A motivation for the idea of treating some “known” cointegrating vectors as fixed when in fact these have been estimated in a preliminary step, is given in Chapter 4. The recommended third step, after having identified the long-term structure, is to estimate all parameters simultaneously; this is to take into account the potential non-diagonality of the covariance matrix and thus to improve upon the estimates. In the paper the suggested procedure is used to identify cointegrating relationships between and within sectors related to two applied studies, respectively, the international interest rate study of the first two chapters of this thesis and a study of Norwegian exports. In both studies the sector dimension is equal to two, which of course makes them particularly suitable for illustrative purposes, and the suggested procedure turns out to be able to identify cointegrating relationships across sectors as well as between for both of them. With regard to the first of these studies, the procedure is in addition able to pinpoint the existence of a third cointegrating relation, a possibility that to some degree had already been addressed in the first paper of this thesis. Even though the identification of this third long-run relationship does not have a bearing on the conclusion made with regard to the ability of Central Banks to control the long end of the yield curve on the basis of two cointegrating vectors, it nevertheless turns out to have substantial implications with respect to the status of short-term interest rates and how they might be affected by long-term interest rates through their potential capacity of informing policy rules on the part of Central Banks.

Chapter four I have chosen to call: “Bootstrapping or train-spotting: A note on small sample properties of the trace statistics related to specific VARS”, and as the name suggests, not only aims at discussing the small sample properties of the trace test statistics related to the studies of Chapter 1 and 3 herein, but also to critically discuss the value added of undertaking Monte
Carlo, and Bootstrapping in particular, on non-robust coefficients and test statistics when the central premise that one knows the DGP probably is far from satisfied and one in fact is forced to use a substitute that might potentially deviate from it significantly. As the paper is partly based on the preceding chapter, the text also aims at discussing the idea of treating some of the cointegrating vectors as fixed when in fact these have been estimated in a first step.

The final chapter, Chapter 5, is a study of German and Norwegian exports and aims particularly at addressing the issue of I(2)-ness. A central goal has therefore been to unveil potential signs of higher order non-stationarity and in the case this is found to be evident, to identify potentially multi-cointegrating relationships. In this, there has been no intention of forcing I(2)-ness upon the data and as stated in the introduction of the chapter the approach has been more to cling to a null of I(1) than to continue along the dimension of an artificially made supposition. In this respect it may also be added that the I(2) analysis is deemed less urgent as the aim of both studies is to reveal generic properties of the underlying data generating processes. However, when this is said, it must also be stressed that an I(2) analysis may be an interesting exercise to carry out even in the case one might not feel confident about its premises; if nothing else, to compare with and eventually to support the outcome of an I(1) analysis. This more pragmatic view is the preferred one when interpreting the results of the I(2) analysis in Section 4 of this chapter. The paper is also given an economic motivation: to test the claims of foreign trade entrepreneurs that their businesses are extremely vulnerable to vagaries of foreign demand and prices as well as to shocks to supply, like hikes in wages and prices of intermediate products. To be able to discuss this issue in its full generality the theoretical framework has been an encompassing one, meaning that most special cases constitute restrictions on a parameterised version of a general model. My results indicate that there is monopolistic power in the process determining export prices, not only in a big country like Germany but also in a small open one like Norway. Furthermore, exports and export prices seem to be heavily affected by shocks to world quantities like world demand and world prices. This at least is in accordance with the claims of foreign entrepreneurs. Their businesses are heavily influenced by the vicissitudes in international trade.
References


Chapter 1

“Who’s in the driving seat in Europe, International financial markets or the BUBA? ” *

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Abstract

The purpose of this chapter is to reexamine empirically the relationship between long-term interest rates in well integrated financial markets. The analysis focuses on long-term interest rates in the US and Germany and has been carried out within the framework of a five dimensional VAR for the simultaneous determination of short- and long-term interest rates in the US and Germany and the rate of depreciation. The results strongly support the existence of a long-run relationship between the long-term German and the long-term US interest rate and imply a full pass-through of changes in the long-term US rate into the corresponding German rate. The analysis also substantiates that the direction of causality goes from the long-term US to the long-term German interest rate. With regard to the possibility of controlling the long end of the market on the part of the Bundesbank, the paper apparently takes on a rather pessimistic view, as there is nothing to indicate a long-run relationship between domestic short- and long-term German interest rates. However, the strong influence that short-term German interest rates exhibit on German long-term interest rates in the very short run according to the structural model of this paper, might be taken to indicate that the opposite is the case,

*I am grateful for comments by Soren Johansen, Grayham Mizon and participants at the first year student forum at the EUI, Florence. I want also to thank Birger Vikoren with whom I wrote the forerunner of this paper.*
as effects originating from expectations with regard to future short-term interest rates might totally neutralize an unequivocally positive short-run portfolio effect in the long run. If this is the case, there is nothing strange in the fact that one is unable to identify a long run relationship between domestic short- and long-term German interest rates. On the contrary it is exactly what to be expected if the monetary transmission mechanism works appropriately.

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1 Introduction

Recently, there has been some focus on what impact increased capital mobility could have on the determination of long-term interest rates (e.g. Borio and McCauley (1996) and OECD(1996)). These studies have been initiated by the striking co-movements of long-term interest rates in US and Europe so far in the 1990s (Figure 1). A recent study of this relationship substantiates this high degree of correlation and also suggests that long-term European interest rates seem mainly to be determined by US long-term interest rates in the long run, the causality going only one way, from the US to the European economy (Hammersland and Vikøren (1997)). However, the model developed in this paper does not really seem to explain the events of 1997, when the two interest rates start to diverge. Also, problems with interpreting the model's long-run relationship suggest extending the information set to improve on the model. However, before starting the analysis, I will look at two types of explanations, one macroeconomic and one microeconomic, which have been suggested as reasons for the strong co-movements in long-term interest rates.
Figure 1: Long-term interest rates in Germany (R10DME) and the US (R10USE).

in recent years. It is important to realize that these explanations are all based on time series being stationary and that a high degree of correlation may be spurious as a consequence of non stationarity. When analyzing the actual data in the next sections to come, it is therefore extremely important to use a methodology that is capable of identifying the fundamental factors behind the correlation patterns observed between the time series. This is the main reason why when analyzing the data, I pursue a reduced rank VAR analysis in this paper.

A typical macroeconomic explanation for the correlation between nominal long-term interest rates across countries assumes that these rates are roughly equal to the sum of real long-term interest rates and inflation expectations. Disregarding for a moment the problem commented on above with regard to spurious correlation when dealing with non-stationary data, correlation between nominal interest rates must therefore entail that there is a correlation between real interest rates and/or a correlation between inflation expectations. The joint hypothesis of uncovered interest rate parity (UIP) and ex ante Purchasing Power Parity (PPP) leads to real interest rate parity (RIP). Although it is a widely held view that RIP does not hold in the short run, King (1992) argues that RIP is more likely to hold in the long run. In this case, real long-term interest rates will be highly correlated between
countries. There might also be a correlation between inflation expectations in different countries due to either significant changes in commodity prices or to synchronized changes in the assessment of the business cycles in various countries.

A microeconomic explanation looks at the trading strategies of large institutional investors. For instance, the increase in bond rates in the US and Europe during 1994 has been explained by the observation that the fall in bond prices in the US prompted highly leveraged investors to sell US as well as European bonds. This explanation is supported by Borio and McCauley (1996) who examine the rise in long-term interest rates in 1994 and conclude that markets' own dynamics seem to provide a stronger explanation than market participants' apprehensions about economic fundamentals.

So far, I have focused on the relationship between foreign long-term interest rates across countries. However, the expectations theory of the term structure entails that there should also be a relationship between short-term and long-term interest rates in each country. According to this theory, the long-term interest rate is equal to a weighted average of the current and expected future short-term interest rate (see Schiller (1979)). Thus, the impact on the long-term interest rate from a change in the current short-term interest rate depends on how expected future short-term interest rates are affected. A rise in the current short-term interest rate that is regarded as permanent will lead to a full pass-through from short-term to long-term interest rates. On the other hand, if an increase in the current short-term interest rate leads to a significant reduction in inflation expectations, long-term interest rates may even decline.

The discussion above shows that both domestic short-term interest rates and foreign long-term interest rates could have an impact on domestic long-term interest rates. Goodhart (1995) recognizes this and argues that increased capital mobility has led to a greater tension between international pressure (e.g. foreign long-term interest rates) and domestic factors (e.g. the expected time-path of future short rates) in the determination of long-term interest rates. However, uncovered interest parity and relative purchasing power parity, used to explain the correlation between long-term interest rates, also suggests effects from differences in inflation rates or the expected rate of depreciation, and a unified treatment of all these possibilities may be given within the framework of a loanable funds equilibrium approach where interest rates are determined by the demand and supply of funds (Brandson (1977)).
Figure 2: Interest spreads between long-term interest rates in the US and Germany (S10DMUS) and between domestic long-and short-term German interest rates (SD103).

In Figure 2 I plot the spreads between long-term interest rates in Germany and the US and between domestic long- and short-term German interest rates, respectively. Graphical inspection indicates a possible long-run relationship between German and US long-term interest rates, although extended periods are observed in which the long-run relationship does not seem to hold. However, a similar relationship between German short and long-term interest rates does not seem to exist.

Below, I shed further light on these issues by undertaking an empirical analysis of nominal short- and long-term interest rates in Germany, \((i^{GL} \text{ and } i^{GS})\), and the US, \((i^{UL} \text{ and } i^{US})\). The empirical proxies for long-term interest rates have been effective interest rates on Government bonds with ten years to maturity while short-term interest rates are represented by the corresponding three months money market interest rates\(^1\). The information set also consists

\(^1\)The concept effective interest rates refers to the fact that one has taken into account the compound interest rate effect. In the general case with a deposit with a term to maturity less than one year this might be given the following representation:

\[
i = \left( \left( 1 + \frac{r_{\text{nom}}}{100} \right)^n - 1 \right) 100
\]
of the actual rate of depreciation \((Dv)\), where the exchange rate is the log of German marks per US dollar. The rationale for including this variable was alluded to in the above and comes from the arbitrage condition of uncovered interest rate parity, saying that in a steady state the return of investing one unit of domestic currency at home or abroad should be equal. Thus, the domestic interest rate, \(i^D\), should be equal to the foreign interest rate, \(i^F\), plus the expected percentage increase in the value of the foreign currency relative to the domestic currency, that is the expected depreciation of the bilateral domestic exchange rate, over the horizon we are looking at\(^2\). The analysis has been undertaken using monthly data for the period 1990 (1) to 1997 (12) and has been carried out within the framework of a five dimensional VAR model for the simultaneous determination of the four interest rates and the rate of depreciation. To be able to test the Fisher hypothesis and to build a model of inflation, information sets including inflation rates and indicators of domestic activity have been tried out prior to the empirical analysis of this paper. However, these attempts have so far not succeeded and belong to the field to be further explored. Compared to a study undertaken on a data set comprising only the four interest rates, it turns out that the widening of the information set to also include the bilateral exchange rate revises results and makes it possible to identify an interpretable long-run relationship. The model’s forecast ability is also strongly improved compared to a model of interest rates only.

The rest of this chapter is organized as follows. Section 2 examines cointegration and exogeneity. Section 3 then presents the outcome of a structural reinterpretation of the reduced form analysis. Section 4 contains concluding remarks.

\(^2\)In the chapter I have used the monthly change in the logarithm of the bilateral exchange rate, being aware of the fact that it would have been more correct from a theoretical perspective to use the change over three months. However, one may argue that investors operating in the markets are using the monthly change as an indicator because it is a more updated proxy for what it after all seeks to capture, namely the expected rate of depreciation.

where: \(i\) is the effective interest rate, \(r\) the nominal coupon interest rate and \(n\) the number of periods per year. The implicit assumption in the above example is that the principal amount and the accrued interest rate are re-invested at the same nominal rate of interest rate throughout the period. In the case of bonds with fixed coupon dividends the formulas become slightly more elaborate and the interested reader is referred to The Norwegian Society of Financial Analysts (2001).
2 Integration, Cointegration and Weak Exogeneity

This section presents statistics for testing stationarity of the individual time series in the information set. Johansen's maximum likelihood procedure is applied to test for cointegration and the direction of causality among the short- and long-term interest rates in Germany and the US.

Prior to modelling, it is useful to determine the orders of integration of the variables in the information set. Below, I therefore first present the results of using ordinary univariate augmented Dickey-Fuller (ADF) tests for unit roots in individual time series (Dickey and Fuller (1981)). However, I also present the results when using the Johansen method to test for stationarity in a multivariate framework. These two approaches for testing stationarity differ in two important respects. First, when using the Johansen approach the null-hypothesis is that the individual time series is stationary, while Dickey-Fuller tests have non-stationarity as their null-hypothesis. Second, the multivariate test statistics are conditional on the number of cointegrating vectors in the information set.

Table 2.1 lists augmented Dickey-Fuller test statistics for the long- and short-term interest rates in Germany and the US. The last column also gives the tests for the rate of depreciation. The absolute value of the deviation from unity of the estimated largest root appears in parentheses below each Dickey-Fuller statistic: this deviation should be approximately zero if the series has a unit root. Unit root tests are given for the variables in levels and for their first differences. This permits testing whether a given series is I(0), I(1) or I(2), albeit in a pairwise fashion for adjacent orders of integration. According to the unit root tests all variables except for the rate of depreciation appear to be integrated of order one. The rate of depreciation on the other hand seems to be a stationary variable.

Table 2.2 below reports values of a multivariate statistic for testing the times series properties of a given variable. Specifically, these LR-test statis-

---

3 For identification of the cointegration indices using the two-step procedure of Johansen (1995), the reader is referred to the international interest rate analysis in Chapter 3.

4 The diagnostics of the fourth order autoregressive model of the German long-term interest rate reveal problems with heteroscedasticity and autocorrelation as well as non-normality. Strictly speaking therefore, the results of the Dickey Fuller test for this variable is not valid. However, with regard to the other variables all diagnostics are fine.
### Table 1: ADF(4) Statistics for Testing for a unit Root.

Estimates of $|\hat{\rho} - 1|$ in parenthesis$^{1,2}$

<table>
<thead>
<tr>
<th>Variable</th>
<th>$i^{GL}$</th>
<th>$i^{UL}$</th>
<th>$i^{GS}$</th>
<th>$i^{US}$</th>
<th>$D_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I(1)$</td>
<td>-1.1465</td>
<td>-1.5053</td>
<td>-0.9477</td>
<td>-1.9276</td>
<td>-5.072**</td>
</tr>
<tr>
<td></td>
<td>(0.0217)</td>
<td>(0.036)</td>
<td>(0.0074)</td>
<td>(0.0231)</td>
<td>(0.859)</td>
</tr>
<tr>
<td>$I(2)$</td>
<td>-3.5098**</td>
<td>-4.5702**</td>
<td>-3.1376**</td>
<td>-2.7768**</td>
<td>-7.3708</td>
</tr>
<tr>
<td></td>
<td>(0.5419)</td>
<td>(0.747)</td>
<td>(0.469)</td>
<td>(0.3662)</td>
<td>(2.5954)</td>
</tr>
</tbody>
</table>

1 For any variable $x$ and a null hypothesis of $I(1)$, the ADF statistics are testing a null hypothesis of a unit root in $x$ against an alternative of a stationary root. For a null hypothesis of $I(2)$, the statistics are testing a null hypothesis of a unit root in $\Delta x$ against the alternative of a stationary root in $\Delta x$.

2 For a given variable and the null hypotheses of $I(1)$ and $I(2)$, two values are reported. The 4'th-order augmented Dickey-Fuller (1981) statistics, denoted ADF(4) and (in parentheses) the absolute value of the estimated coefficient on the lagged variable, where that coefficient should be equal to zero under the null. A constant-term is included in all regressions. The effective sample is 1990(1)-1997(12).

3 Here and elsewhere in the chapter, asterisks * and ** denote rejection of the null hypotheses at the 5% and 1% significance level, respectively. The critical values for the ADF statistics are -2.892 at a level of 5% and -3.499 at a level of 1% (MacKinnon (1991)).
tics test the hypothesis that one of the cointegrating vectors contains all zeros except for the coefficients corresponding to the variable under consideration and a non-restricted constant term, where the test as alluded to above, is conditional on the number of cointegrating vectors. For instance, the null hypothesis of a stationary long-term German interest rate implies that one of the cointegrating vectors is \((1 \ 0 \ 0 \ 0 \ 0 \ \beta)\), where I have implicitly assumed that long-term German interest rates and the constant are respectively the first and last variable of the variable vector. In Table 2, the statistics quoted are conditional on there being two cointegrating vectors and refer to the same VAR model that is used later to identify the long-run relationships. Empirically, all the stationarity tests, except for the depreciation rate, reject with p-values less than one per cent. These rejections of stationarity are consistent with the inability to reject the null hypothesis of a unit root in all the interest rates when using the Dickey Fuller test statistic. Thus, all four interest rates are treated below as if they are I(1). The rate of depreciation, however, seems to be stationary and will be treated likewise.

The methodology developed by S. Johansen (Johansen (1988), (1992) and Johansen and Juselius (1990)) is used to identify the long-run relationships and to test whether some variables may be considered as exogenous with regard to estimation of the parameters of the long-run relationships. The results of the analysis are given in Table 3. However, the order of the VAR is not known a priori, hence some testing of lag order may be beneficial in order to ensure reasonable power in the Johansen procedure. Beginning with a fifth-order VAR in \(i_{GL}, i_{UL}, i_{GS}, i_{US}\) and \(Dv\) that includes a restricted constant term, we show in Appendix A, Table 8, that it is statistically acceptable to simplify to a second-order VAR. Further reduction to a first-order VAR is rejected. The empirical cointegration analysis is therefore made on a 5 dimensional VAR of order two.

Table 3 shows the results of Johansen’s maximum likelihood procedure. Looking first at Table 4 which gives the diagnostics of the individual equations as well as for the system, it is worth noting that all diagnostics are fine except for a five per cent rejection of normality for the residuals in the equation of the US long-term interest rate and a marginal rejection of the corresponding vector test statistic. Table 3 supports the existence of three cointegrating vectors at a significance level of five per cent, but only two using a test level of one per cent. However, we know that the rate of depreciation is stationary, so if we accept that there are only two cointegrating vectors, we
Table 2:
Multivariate test statistics for testing for stationarity
Two cointegrating vectors and constant in CI-space$^{1,2}$

<table>
<thead>
<tr>
<th>Variables</th>
<th>$t^{GL}$</th>
<th>$t^{UL}$</th>
<th>$t^{GS}$</th>
<th>$t^{US}$</th>
<th>$Dv$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X^2(3)$</td>
<td>17.556**</td>
<td>10.387*</td>
<td>18.814**</td>
<td>12.095**</td>
<td>4.1717</td>
</tr>
<tr>
<td></td>
<td>[0.0005]</td>
<td>[0.0155]</td>
<td>[0.0003]</td>
<td>[0.0071]</td>
<td>[0.2435]</td>
</tr>
</tbody>
</table>

$^1$The test statistics are the LR-tests of restrictions on the cointegration space within the Johansen framework. Specifically, these statistics test the restriction that one of the cointegrating vectors contains all zeros except for a unity corresponding to the coefficient of the variable we are testing whether is stationary and a non-restricted constant coefficient. In Table 2, the statistics quoted are conditional on there being two CI-vectors and refer to the same VAR model that later is used to identify the long-run relationships. The figures in brackets under each test statistics are the tests' significance probabilities and * and ** denote rejection at 5% and 1% critical levels, respectively.
Table 3: Johansens cointegration tests

System: $i^{GL}, i^{UL}, i^{GS}, i^{US}, Dv.$
Deterministic part: Restricted constant¹

<table>
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<tr>
<th>Eigenvalues of $\Pi$:</th>
<th>0.4508</th>
<th>0.2551</th>
<th>0.2064</th>
<th>0.1219</th>
<th>0.0520</th>
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<th>Max Eigenvalue Tests²</th>
<th>Trace Eigenvalue Tests</th>
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</thead>
<tbody>
<tr>
<td>Null Alt. Statistics</td>
<td>95%</td>
</tr>
<tr>
<td>Null Alt. Statistics</td>
<td>95%</td>
</tr>
<tr>
<td>r=0 r≤1 57.53** 34.4</td>
<td>r=0 r≤5 125.6** 76.1</td>
</tr>
<tr>
<td>r≤1 r≤2 28.27* 28.1</td>
<td>r≤1 r≤5 68.08** 53.1</td>
</tr>
<tr>
<td>r≤2 r≤3 22.2 22.0</td>
<td>r≤2 r≤5 39.8* 34.9</td>
</tr>
<tr>
<td>r≤3 r≤4 12.48 15.7</td>
<td>r≤3 r≤5 17.61 20.0</td>
</tr>
<tr>
<td>r≤4 r≤5 5.13 9.2</td>
<td>r≤4 r≤5 5.13 9.2</td>
</tr>
</tbody>
</table>

¹The constant is restricted to lie in the space spanned by the columns of $\alpha$
²The 5 per cent critical values shown in brackets are taken from Osterwald Lenum (1992). An asterisk indicates that a test is significant to a level of five per cent, while two asterisks indicate that the test is significant to a level of one per cent.
only have to identify the second. The unrestricted estimated cointegrating linear combinations and the loading matrix in case of only two cointegrating vectors are given in Table 5 below. The following table, Table 6, quotes tests of different hypotheses with regard to the cointegration space and the space spanned by the α’s. As already noted when tested for stationarity the test of the restrictions identifying the rate of depreciation as the first cointegrating vector is fine. Also, the tests do not reject a homogenous linear combination of the long-term interest rates and the short-term German rate to be the second cointegrating vector. However, the spread between the two long-term interests rates is not rejected either. Anticipating the outcome of the tests for Granger non-causality and exogeneity, this suggests that the German short rate is superfluous and that there is a full pass through of changes in the US rate into the German rate in the long run. This agrees with the former graphical inspection of the spreads made in the introduction. A simple test of weak exogeneity, proposed by Johansen (1992a, 1992b) (see also Urbain (1992)), is simply to test zero restrictions on a subset of the weights in the loading matrix. The results of these tests give support to treating the long-term US interest rate as exogenous with respect to estimation of the long-run parameters of the two restricted cointegrating vectors. With regard to the short-term US interest rate the status is more uncertain as the individual test conditional on the two identified cointegrating relationships and no error correction in the equation of long-term US interest rates, is significant to a level of five per cent (p-value equal to 0.0246). However, the same test when not conditioning on long-term US interest rates as exogenous has a p-value that is only marginally below five per cent which is also the case with regard to the test of considering both US rates as jointly exogenous. This implies that we probably are not making too big a mistake by restricting the two cointegrating vectors and the feedback coefficients to enter only the equations of the long- and short-term German interest rate together with the equation of the rate of depreciation. If so, the two US interest rates can be considered as being exogenous with regard to estimation of the long-run parameters and inference with regard to these would be possible to conduct from a three dimensional model where we condition on US interest rates without a significant loss of information. However to take the additional step of justifying

5 The analysis of this chapter is based on the existence of only two cointegrating long-run relationships. For an elaboration of the alternative of three cointegrating vectors the reader is referred to Chapter 3.
on this basis the simpler modelling strategy implied by a three dimensional conditional system analysis when building a dynamic structural model necessitates further investigation as to whether the two US interest rates might also be considered as weakly exogenous with regard to estimation of the dynamic short-run parameters. A test of strict exogeneity with regard to the two US interest rates related to the structural model developed in the next section, does however not reject. This is indicative of both US interest rates also being weakly exogenous with regard to the dynamic coefficients and together with their status of being exogenous with regard to estimation of the long-run parameters legitimates the sort of conditional analysis pursued in the next section to come. That is a three dimensional structural dynamic analysis of the system consisting of German short- and long-term interest rates and the actual rate of depreciation conditional on the two US interest rates.

The two identified cointegrating relationships together with the restricted loading matrix, are given in Table 7 below. The test of the restrictions is also quoted and does not reject to a level of five per cent. The long-run relationship implies that a 100 basis points change in the long-term US interest rate leads to the same change in the German long-term interest rate in the long run. Thus there is a full pass-through of changes in US long-term interest rates into the corresponding German rates. The recursively estimated eigenvalues of Figure 3 in the appendix show signs of instability. However, taking the scale on the vertical axes into consideration, this instability seems mainly to be a graphical illusion.

The test of strict exogeneity has been undertaken by plugging the residuals of the structural model of Section 3 into the autoregressive marginal processes of order one of the two American interest rates and restricting their coefficients to zero. The joint test of restricting all residual coefficients to zero is \( x^2(6) \) and gave a test statistic equal to 6.25826 [0.3949], where the number in parenthesis is the respective test’s significance probability.

The outcome of an unconditional analysis does not significantly change the outcome of our analysis as the restrictions implied by both US interest rates being univariate autoregressive processes of order one constitute valid restrictions on the full dynamic structure. However, there is some indication of a simultaneous dynamic effect of changes in long-term US interest rates on changes in the corresponding short term US interest rates. For a discussion of this possibility the reader is referred to Chapter 2.
### Table 4:
Individual equation and system diagnostics of the unrestricted VAR\(^1\)

<table>
<thead>
<tr>
<th>Equation/Tests</th>
<th>AR 1-6 F([6,79])</th>
<th>ARCH 6 F([6,73])</th>
<th>Normality (\chi^2) (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta i^{GL})</td>
<td>0.6045[0.7260]</td>
<td>0.5329[0.7815]</td>
<td>4.510[0.1049]</td>
</tr>
<tr>
<td>(\Delta i^{UL})</td>
<td>1.2169[0.3065]</td>
<td>0.8787[0.5149]</td>
<td>9.128[0.0104](^*)</td>
</tr>
<tr>
<td>(\Delta i^{GS})</td>
<td>1.3716[0.2364]</td>
<td>0.6138[0.7185]</td>
<td>1.933[0.3804]</td>
</tr>
<tr>
<td>(\Delta i^{US})</td>
<td>2.0817[0.0646]</td>
<td>1.9673[0.0814]</td>
<td>0.759[0.6842]</td>
</tr>
<tr>
<td>(\Delta Dv)</td>
<td>0.6376[0.6998]</td>
<td>0.2909[0.9394]</td>
<td>1.631[0.4424]</td>
</tr>
</tbody>
</table>

**System tests:** AR 1-5[150,257] VNormality \(\chi^2\) (10) VX\(^2\) F\([300,646]\)

| Statistics: | 1.2128[0.0886] | 14.707[0.1431] | 1.1967[0.0324]\(^*\) |

\(^1\)The Values shown in brackets are the individual test's significance probability. \(^*\) and \(^**\) denote as usual rejection of the corresponding null at levels of 5 and 1 per cent, respectively. VNormality and VX\(^2\) denote the Vector tests of normality and heteroscedasticity. For an explanation of the various test statistics the reader is referred to Chapter 14 of the PcFiml manual (Doornik and Hendry (1999)).
Table 5: The unrestricted cointegrating linear combinations and the loading matrix

\[
\hat{\beta}'(i_{GL}^t i_{UL}^t i_{GS}^t i_{US}^t Dv 1)'
\]

\[
= \hat{\beta}_{11}i_{GL} + \hat{\beta}_{21}i_{UL} + \hat{\beta}_{31}i_{GS} + \hat{\beta}_{41}i_{US} + \hat{\beta}_{51}Dv + \hat{\beta}_{61}
\]

\[
= \hat{\beta}_{12}i_{GL} + \hat{\beta}_{22}i_{UL} + \hat{\beta}_{32}i_{GS} + \hat{\beta}_{42}i_{US} + \hat{\beta}_{52}Dv + \hat{\beta}_{62}
\]

\[
i_{GL} - 0.28i_{UL} - 0.21i_{GS} - 0.23i_{US} + 0.64Dv - 0.025
\]

\[
-0.79i_{GL} + i_{UL} - 0.06i_{GS} - 0.07i_{US} + 0.014Dv - 0.016
\]

<table>
<thead>
<tr>
<th>Equation</th>
<th>Loading matrix$^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta i_{GL}$</td>
<td>$\hat{\alpha}<em>{11}$ $\hat{\alpha}</em>{12}$ = $0.025 [0.0136]$ 0.143 [0.0527] $\hat{\alpha}<em>{41}$ $\hat{\alpha}</em>{42}$ = $0.017 [0.0143]$ 0.022 [0.0552] $\hat{\alpha}<em>{51}$ $\hat{\alpha}</em>{52}$ = $1.340 [0.1777]$ 1.468 [0.6836]</td>
</tr>
<tr>
<td>$\Delta i_{UL}$</td>
<td>$\hat{\alpha}<em>{21}$ $\hat{\alpha}</em>{22}$ = $0.010 [0.0170]$ $-0.101 [0.0656]$</td>
</tr>
<tr>
<td>$\Delta i_{GS}$</td>
<td>$\hat{\alpha}<em>{31}$ $\hat{\alpha}</em>{32}$ = $0.016 [0.0149]$ $0.201 [0.0575]$</td>
</tr>
<tr>
<td>$\Delta i_{US}$</td>
<td>$\hat{\alpha}<em>{41}$ $\hat{\alpha}</em>{42}$ = $0.017 [0.0143]$</td>
</tr>
<tr>
<td>$\Delta Dv$</td>
<td>$\hat{\alpha}<em>{51}$ $\hat{\alpha}</em>{52}$ = $1.340 [0.1777]$ $1.468 [0.6836]$</td>
</tr>
</tbody>
</table>

$^1$The values shown in brackets to the right of the estimated loading coefficients are the respective coefficients' standard error.
Table 6: Test of Hypotheses related to the parameterisation of Table 5

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>LR-test, Rank =2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1): ( \beta_{11} = \beta_{21} = \beta_{31} = \beta_{41} = \beta_{61} = 0, \beta_{51} = 1 )</td>
<td>( \chi^2 (4) = 4.18 \ [0.383] )</td>
</tr>
<tr>
<td>2): ( \beta_{11} = \beta_{21} = \beta_{31} = \beta_{41} = \beta_{61} = 0, \beta_{51} = 1 ) ( \beta_{42} = \beta_{52} = \beta_{62} = 0, \beta_{12} = 1 = - (\beta_{22} + \beta_{32}) )</td>
<td>( \chi^2 (7) = 8.46 \ [0.294] )</td>
</tr>
<tr>
<td>3): ( \beta_{11} = \beta_{21} = \beta_{31} = \beta_{41} = \beta_{61} = 0, \beta_{51} = 1 ) ( \beta_{32} = \beta_{42} = \beta_{52} = \beta_{62} = 0, \beta_{12} = 1 = - \beta_{22} )</td>
<td>( \chi^2 (8) = 13.65 \ [0.091] )</td>
</tr>
<tr>
<td>4): ( \beta_{11} = \beta_{21} = \beta_{31} = \beta_{41} = \beta_{61} = 0, \beta_{51} = 1 ) ( \beta_{32} = \beta_{42} = \beta_{52} = \beta_{62} = 0, \beta_{12} = 1 = - \beta_{22} ) ( \alpha_{21} = \alpha_{22} = 0 )</td>
<td>( \chi^2 (10) = 13.78 \ [0.183] )</td>
</tr>
<tr>
<td>5): ( \beta_{11} = \beta_{21} = \beta_{31} = \beta_{41} = \beta_{61} = 0, \beta_{51} = 1 ) ( \beta_{32} = \beta_{42} = \beta_{52} = \beta_{62} = 0, \beta_{12} = 1 = - \beta_{22} ) ( \alpha_{41} = \alpha_{42} = 0 )</td>
<td>( \chi^2 (10) = 18.75 \ [0.044]^* )</td>
</tr>
<tr>
<td>6): ( \beta_{11} = \beta_{21} = \beta_{31} = \beta_{41} = \beta_{61} = 0, \beta_{51} = 1 ) ( \beta_{32} = \beta_{42} = \beta_{52} = \beta_{62} = 0, \beta_{12} = 1 = - \beta_{22} ) ( \alpha_{21} = \alpha_{22} = 0, \alpha_{41} = \alpha_{42} = 0 )</td>
<td>( \chi^2 (12) = 21.06 \ [0.05]^* )</td>
</tr>
</tbody>
</table>

\(^*\)The value shown in brackets after each individual LR-test is the test’s significance probability. One star, *, behind a test statistic means as before that the test is significant to a level below five per cent.
Table 7: The restricted cointegrating linear combinations and the restricted loading matrix

Restricted cointegrating linear combinations

\[ \beta' \left( \Delta \tilde{G}^L, \Delta \tilde{U}^L, \Delta \tilde{G}^S, \Delta \tilde{U}^S, Dv, 1 \right)' = Dv_t \]

\[ \left( \tilde{G}^L - \tilde{U}^L \right)_t \]

Equation: Restricted estimated loading matrix

<table>
<thead>
<tr>
<th>( i )</th>
<th>( \hat{\alpha}_{i1} )</th>
<th>( \hat{\alpha}_{i2} )</th>
<th>( \hat{\alpha}_{i3} )</th>
<th>( \hat{\alpha}_{i4} )</th>
<th>( \hat{\alpha}_{i5} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \tilde{G}^L )</td>
<td>( -0.112 [0.0078] )</td>
<td>( -0.1067 [0.0324] )</td>
<td>( -0.0094 [0.0096] )</td>
<td>( 0.0000 )</td>
<td>( 0.8302 [0.1097] )</td>
</tr>
<tr>
<td>( \Delta \tilde{U}^L )</td>
<td>( 0.0000 )</td>
<td>( 0.0000 )</td>
<td>( -0.1021 [0.0409] )</td>
<td>( 0.0000 )</td>
<td>( 0.0000 )</td>
</tr>
<tr>
<td>( \Delta \tilde{G}^S )</td>
<td>( 0.0000 )</td>
<td>( 0.0000 )</td>
<td>( -0.1021 [0.0409] )</td>
<td>( 0.0000 )</td>
<td>( 0.0000 )</td>
</tr>
<tr>
<td>( \Delta \tilde{U}^S )</td>
<td>( 0.0000 )</td>
<td>( 0.0000 )</td>
<td>( -0.1021 [0.0409] )</td>
<td>( 0.0000 )</td>
<td>( 0.0000 )</td>
</tr>
<tr>
<td>( \Delta Dv )</td>
<td>( -0.8302 [0.1097] )</td>
<td>( 0.0000 )</td>
<td>( 0.0000 )</td>
<td>( 0.0000 )</td>
<td>( 0.0000 )</td>
</tr>
</tbody>
</table>

\[ \chi^2 (13) = 21.08 [0.0714] \]

1The values in brackets to the right of the estimated loading coefficients are the respective coefficients' standard error.
3 A conditional error correction model for the long- and short-term German interest rate.

Based on the results of the vector autoregressive analysis above, I started by specifying a three dimensional conditional structural error correction model incorporating only one lag of differences and the two error correction mechanisms given by the rate of depreciation and the long-term interest rate spread lagged one period. The structure identified was informed by theory and the desire to explain the correlation pattern of the reduced form residuals as the result of a solved data generating structure. From preliminary data analysis we know that the regression model is balanced, i.e. that the model includes only variables with consistent temporal properties. The error correction specification makes it easy to distinguish between short- and long-run effects. The short-run effects are represented by the differenced variables, while the long-run effects are associated with the level variables. In order to find a parsimonious simplification, we then imposed restrictions on the short-term coefficients of the model. The restrictions- like the identification scheme were informed by theory and the desire to explain the correlation pattern of the reduced form.

The structural model below shows the regression result when using Full Information Maximum Likelihood (FIML) on monthly data for the period January 1990 to December 1997. Looking at the diagnostics quoted below the identified structural model, the LR test for over-identifying restrictions implies that the structure imposed constitutes a valid reduction of a just-identified structure. Also, we cannot reject a joint test of imposing linear homogeneity in the equation for long-term German interest rates, together with a linear restriction identifying the first difference of the long-term interest spread as an explanatory variable in the equation determining the bilateral exchange rate. The negative impact from the first difference of the spread on the first difference of the depreciation rate is consistent with an overshooting effect in case of changes to long-term interest rates. That is, to generate increased depreciation expectations in the wake of long-term interest hikes the depreciation rate will have to decrease. From the identified

---

8Note that one lag of a difference includes the second lag of the level, matching the order of the VAR in Section 2.
structure we note that the two variables $i^{GS}$ and $i^{UL}$ both seem to explain the German long-term interest rate in the short run, but only the latter in the long run. The coefficient on $\Delta i^{UL}$ shows the impact (after one month) on the German long-term interest rate of a change in the US long-term rate. The estimate of this coefficient is 0.40, implying that a 100 basis point change in US long-term interest rates leads to a 40 basis points change in the German long-term interest rate after one month. Moreover, we note that this effect is considerably weaker than the short-run impact from a 100 basis points change in the short-term German interest rate which changes the German long-term interest rate by as much as 60 basis points. As the long-run effect of a change in short rates on long-term interest rates is neutral this suggests that the strong short-run effect is neutralized in the long run through affecting expectations of future short-term interest rates. The long run relationship is derived by setting all the differenced variables in the reduced form of the structure equal to zero and implies as commented on before, that there is a complete pass-through into German long-term interest rates of a change in the US long-term interest rate. Thus, a 100 basis points change in the long-term US interest rate leads in the long run to an equal change in the German long-term rate. Hence, US long-term interest rates have a considerably stronger impact on German long-term interest rates in the long run than in the short run.

The Identified Structural model

\[
\begin{align*}
\Delta i_t^{GL} &= 0.156 \Delta i_{t-1}^{GL} + 0.398 \Delta i_t^{UL} + 0.602 \Delta i_t^{GS} + \tilde{\epsilon}_t \\
\tilde{\sigma}_1 &= 0.001756 \\
\Delta i_t^{GS} &= 0.234 \Delta i_{t-1}^{GS} + 0.0151 \Delta Dv_t - 0.114 \{i_t^{GL} - i_t^{UL}\}_{t-1} + \tilde{\epsilon}_2 \\
\tilde{\sigma}_2 &= 0.00201395 \\
\Delta Dv_t &= 0.173 \Delta Dv_{t-1} - 3.819 \Delta \{i_t^{GL} - i_t^{UL}\}_{t-1} - 0.834 Dv_{t-1} \\
&\quad + 2.517 \Delta i_t^{US} - 3.742 \Delta i_{t-1}^{US} + \tilde{\epsilon}_3 \\
\tilde{\sigma}_3 &= 0.022361
\end{align*}
\]
Some Diagnostics of The Structural model

\[
\begin{align*}
T &= 96 \ (1990(1)-1997(12)) \\
LR: \chi^2 (2) &= 0.4298 \ [0.807] \\
VAR 1-6 \ F (54,218) &= 0.8356 \ [0.7809] \\
\chi^2 (36) &= 21.346 \ [0.9749] \\
\chi^2 (36) &= 20.947 \ [0.9786] \\
VNorm \chi^2 (6) &= 12.12 \ [0.0594] \\
F (36,80) &= 0.59294 \ [0.9583] \\
F (36,80) &= 0.58186 \ [0.9634]
\end{align*}
\]

In Section 2 we found that we could estimate the long-run parameters conditionally on both US interest rates, without having to pay attention to their marginal distributions. This suggests that the direction of causality goes from the US to the German economy. To further substantiate this result, however, we have to test against lagged effects of German long- and short-term interest rates as well as of the rate of depreciation on both US interest rates. However, a test for Granger non-causality (Granger (1969)) does not reject the null of no lagged effects on US interest rates of these variables. Thus, there is evidence of a one-way causality between US and German interest rates, the direction of causality going from the US economy to the German economy.

The system diagnostics for serial correlation, non-normality and parameter constancy are all fine. However, both tests for vector heteroscedasticity reject to a level of one per cent. Also, by formulating a structural model we were unable to get rid of the residual correlations across equations in the unrestricted reduced form of the system, the correlation between the residuals of the two German interest rates in fact increasing instead of decreasing. These facts both indicate some sort of misspecification; the two obvious candidates are wrongly imposed structural restrictions and a too small information set.

9The test of Granger non-causality is made on an error correction model for US long- and short-term interest rates where we together with the lagged error correction terms and lagged changes in US long- and short-term interest rates, only have regressed on lagged changes of German long- and short-term interest rates in addition to lagged changes in the rate of depreciation. When incorporating only two lags of differences, the joint reduction of all lagged effects from these model endogenous variables and error correction terms in the marginal models of the two US interest rates gives a test statistic with a significance probability of 0.09.

10The vector \( \chi^2 \) and vector \( X_i^* X_j \) tests are respectively \( F(108,402)=1.63[0.0004]** and \( F(324, 206)=1.58[0.0002]**.
However, as mentioned in the introduction, I have so far not been able to find an adequate understanding of the structure underlying alternative information sets and will therefore leave the ground open for further research. With regard to whether the structural representation might represent wrongly imposed identifying as well as over identifying restrictions the reader is again referred to Chapter 2. The forecast statistics together with Figures 6 to 7, which show static (one step ahead) and dynamic ex post forecasts for the rate of depreciation and the German long- and short-term interest rate for 1997, indicate that our model seems to make fairly good ex post forecasts within the sample period even though the error bands are wide. Figure 5 shows dynamic forecasts of the differenced series. All these forecasts have been undertaken by a model estimated on data only for the period 1990 (1) to 1996 (12) and thus are ex post forecasts in the sense that they are made for the period after the estimation period. Figure 4 shows some graphical test statistics for parameter stability. These graphs do not indicate a serious problem with unstable parameters during the sample period and are in line with the formal tests given under the structural model above.

4 Summary and conclusions

The purpose of this chapter has been to reexamine empirically the relationship between long-term interest rates in well integrated financial markets. The analysis has been carried out within the framework of a five dimensional VAR for the simultaneous determination of short- and long-term interest rates in the US and Germany, and the rate of depreciation. An important motivation for using this framework has been to carefully examine cointegration and exogeneity. Interestingly, our results indicate that both US interest rates are exogenous with regard to estimation of the long-run coefficients in a three dimensional regression model for German long- and short-term interest rates and the rate of depreciation. Also, German long-term interest rates do not seem to Granger cause US interest rates. Thus, the direction of causality seems to be unidirectional, namely from the US to the European economy. This could have important macroeconomic consequences in Germany since much of the debt to households and firms is linked to long-term rates. Moreover, we find that short-term German and long-term US interest rates both have a significant impact on long-term German rates in the short run. However, domestic interest rates do not seem to enter the long-
run relationship. In addition to implying that there is a full pass-through of long-term US interest rates into the corresponding German rate in the long run, this suggests that monetary policy could be effective through affecting expectations with regard to future short-term interest rates in a way that neutralizes the short-run effect that short-term domestic interest rates have on long-term German interest rates in the long run. The forecastability of the model improves significantly compared to a model where one excludes the bilateral exchange rate in the information set, and does give decent forecasts even for 1997. This suggests that the increase in the long-term interest spread in 1997 is partly due to increased depreciation expectations as a consequence of different growth patterns in the US and Germany and probable overvaluation of the dollar in this period.

References


## A Tables and graphs

Table 8: F and related Statistics for the Sequential Reduction from the fifth-order VAR to the First-order VAR.

<table>
<thead>
<tr>
<th>System</th>
<th>k</th>
<th>SC</th>
<th>Maintained Hypothesis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>VAR (5)</td>
</tr>
<tr>
<td>VAR (5)</td>
<td>130</td>
<td>-57.05</td>
<td>0.884</td>
</tr>
<tr>
<td>↓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VAR (4)</td>
<td>105</td>
<td>-54.89</td>
<td>0.974</td>
</tr>
<tr>
<td>↓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VAR (3)</td>
<td>80</td>
<td>-55.72</td>
<td>1.048</td>
</tr>
<tr>
<td>↓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VAR (2)</td>
<td>55</td>
<td>-56.54</td>
<td>1.388*</td>
</tr>
<tr>
<td>↓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VAR (1)</td>
<td>20</td>
<td>-57.05</td>
<td>1.814**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[0.001]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(100,33)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. The first three columns report the vector autoregression with its order, and for that model: the number of unrestricted parameters k and the Schwartz criterion SC.
2. The three entries within a given block of numbers in the last four columns are: the approximate F-statistic for testing the null hypothesis (indicated by the model to the left of the entry) against the maintained hypothesis (indicated by the model above the entry), the tail probability associated with that value of the F-statistic (in square brackets), and the degrees of freedom for the F-statistic (in parentheses). See Doornik and Hendry (1994) for details on the algebra underlying these calculations. * and ** denote as usual rejection of the corresponding null at levels of 5 and 1 per cent, respectively.
Figure 3: Recursively estimated eigenvalues
Figure 4: Chow test statistics for parameter stability of the Structural model
Figure 5: Dynamic forecasts of differenced variables. Estimation period: 1990 (1) to 1996 (12).
Figure 6: Static forecasts for long- and short-term interest rates in Germany and the rate of depreciation.
Figure 7: Dynamic forecasts of the rate of depreciation and long- and short-term interest rates in Germany.
Chapter 2

“We are arrogant because we are good”*, revisited†.
A critical appraisal of Central Banking versus fiscal policy in accomplishing the community wide convergence of the eighties and the nineties‡.

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Florence

Abstract
This chapter addresses the relative importance of monetary versus fiscal authorities in the construction of convergent and stabilizing economic policies within the European community. The perspective is retrospective as well as forward looking in that the chapter analyses

*Statement made by Bundesbank representative (ref. David Marsh (1992), Chapter 1, page 16).
†This chapter is a revised version of an earlier paper entitled, “We are arrogant because we are good”. A critical appraisal of Central Banking in accomplishing the community wide convergence of the eighties and the nineties”, published in Series “Work & Society”: No 28, PIE Lang.
‡I am grateful for comments by Barbara MacLennan, Katarina Juselius, Michael Ehrmann, Andreas Beyer, Mike Artis and participants in the working group for the cooperation project between the Robert Schuman Centre, Florence and The Working Life Institute, Stockholm, called From the Werner plan to the EMU. I would also like to thank Henrik Hansen and Gerdie Everaert. The first for an inspiring discussion with regard to interpretation of structural dynamic coefficients in structural VARs, the second for providing me with data on Primary Government balances for Europe.
historical structures governing the last two decades at the same time as it makes extrapolations with regard to the feasibility of alternative future policy stances. Based on econometric evidence, the chapter particularly aims at discussing the common conviction that Central Banking has been the crux of the observed convergencies within the community during the last two decades. The results indicate that the role of Central Banks has been severely overrated in this respect and suggest looking also at other plausible reasons like fiscal policy and the effect of increased capital mobility. With regard to the making of a future policy stance where one of the most imperative tasks has to do with how to resolve the problem of unemployment in Europe, the chapter strongly argues against the pursuit of “blind structuralism” in the sense of legitimating policies of *laissez faire* or policies geared towards a general diminution of social security protection, most probably both leading to social distress and an ever widening gap of social and economic differences in standards of living within the community. This leaves us of course with an important question to ask: if it is true that monetary policy is not as effective as we want to believe, and an extended use of structural measures is out of question due to its socially unacceptable consequences, is there anything at all we can do? The chapter seeks to answer this question by pointing to the need for regionally directed fiscal policies and policies geared towards regional stimulation of investment and growth, policies that most certainly will bring back the social dimension to the European policy agenda.

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1 Introduction

During the last two decades the European community has witnessed two periods of convergence. The first as a result of the disciplinary effect implicitly imposed on member states of the community by adherence to the EMS. The second period being a consequence of community countries pursuing a policy of commitment to meet the terms of the Treaty on European Union for creating EMU\(^1\). However, in both instances one gets the impression that monetary policy has been given the full credit for being the policy instrument through which this convergence was possible, without even taking as much as a glance at other possible explanations such as changed economic environments or the role played by fiscal authorities. For instance, as Lars E.O. Svensson (1999), the leading Swedish economist, firmly puts it:

There is little doubt that the decline of inflation has largely been due to the growing commitment on the part of monetary policy makers in the euro area to achieve and maintain low inflation. The gradual decline in inflation can therefore be interpreted as corresponding to a fall in the average (implicit) inflation objective of the central banks in the euro area.

Two other examples are Cecchetti (2000) and Ball (1996). The latter study is implicitly based upon the premise that inflation is a valid indicator of monetary policy without even discussing the potential falsity of such a proposition. This means that Ball, in his study, takes as granted that inflation and monetary policy are two sides of the same coin, a presumption that at least must be considered as controversial knowing that there is more to inflation than interest rates and monetary policy. Checchetti on his part is more implicit when stating that, referring to the new perception of how to run monetary policy under the heading these issues:

As consensus has grown on these issues, many countries have redesigned their central banks and for the most part, achieved, remarkable reductions in inflation.

\(^0\)For an interesting account of credit policy to reduce unemployment by stimulating social investment see MacLennan (2001).

\(^1\)Both periods were characterized by a narrowing of interest differentials and a significant drop in the number of realignments as well as inflation. However, as there are strong interdependencies between these three quantities, I will by “convergence” in the following mainly refer to convergence in inflation.
These points of view are particularly strange as the two periods in addition to being characterized as one of tight monetary policy, precisely were periods of both structural change and fiscal consolidation, therefore potentially being both reasons for converging inflation rates within the community as much as strict monetary policy. For the last point I refer to Figure 1 and Figure 2, which give the graphs of the primary government balance for Germany and France, respectively. In the case of Germany the figures clearly identify the periods from about 1983 to 1988 and from 1994 and onwards as periods of considerable improvements in government balances, the periods coinciding almost fully with the two periods of convergence mentioned earlier on. The figures for France are less clear with regard to time intervals, but show the same pattern. Why focus on these matters has almost solely been on monetary policy, and whether the conviction that Central Banking has been the crux of convergence is well founded, or could be regarded as a result of Central Bankers desire to gain influence and power in the process governing the Community, are questions that need to be analyzed.

A huge bulk of empirical literature (e.g. Cukierman (1992) and Alesina and Summers (1993)) has "established" that, in the case of industrialized countries, a higher degree of central bank independence goes hand in hand with lower inflation. A typical interpretation of this has been that monetary

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2I have deliberately chosen to focus on primary and not overall government balances, as these are free from the disturbing movements of interest rate payments and thus should better reflect the degree of expansiveness in government budgets.
Figure 2: Primary Government Balance for France, Cyclically Adjusted, % Potential GDP

Policy through independent central banks, seems to have provided the appropriate response to help promote a culture of monetary stability. Further, in the case of the US, two independent studies (Cohen and Wenninger(1994), Lee and Prasad(1994)) have shown that the correlation between policy rates and long-term interest rates in the US has increased. Typically, they interpret this as indicating control gained by central banks in the conduct of monetary policy. However, making statements of this kind based on bi-variate correlations and non stationary data, is to run the very real risk of attempting inference from spurious or nonsense correlations. The alleged ability of central bank policy to effectively control inflation is further reinforced by neoliberal theory, where a central message is precisely the necessity of getting control over the most serious threat to stable prices, namely the supply of money. This of course leaves Central Banks with an important role to play in both the formulation and the execution of a general policy stance at national levels and notwithstanding the empirical facts, may have contributed to the conviction of Central Banking being the main policy option through which the convergencies during the last two decades were made possible.

The recent contribution of New Keynesian ideas has shown that monetary policy by no means should be considered as the only effective measure of control. Also, within this framework the effectiveness of policies aimed at demand management heavily depends on whether we have a fixed exchange rate system and on the degree of mobility in capital and goods markets, the simultaneous appearance of which imply a total loss of money as a policy
instrument. A more fundamental question is therefore whether monetary policy in a world close to perfect capital mobility, is at all effective. This suggests that even the common appreciation of monetary policy as a legitimate policy instrument through which real demand may be affected, may be related to extrapolations based on historical structures that radically differ from the structure which prevails in the period we are looking at. That is, it may have been the case that the excellent post-war record of Germany in fighting inflation based on tight money under a regime that radically differed from one of perfect capital mobility, has been used to argue for the pursuit of a similar policy at the European level, but now in times where the structure of the economy is changed towards one of high capital mobility across borders. As already mentioned, a well known result in economic theory is that near perfect arbitrage in the capital market may totally undermine any possibility of control on behalf of monetary authorities, not only in real terms but also with regard to controlling a nominal variable like inflation. This result hinges on the economy being one of fixed exchange rates. Whether one can consider EMS to have implied some kind of “fixedness” is therefore crucial for the argument. However, it is all too apparent that the EMS did not render realignments a thing of the past. In the period between 1979 and 1990 there were no less than twelve realignments within the members of the system. However, as pointed out by Swann (1996), the tendency was a declining one and compared with currencies such as the Japanese yen and US dollar the scheme had the effect of reducing day to day fluctuations between Member State currencies considerably. After the currency crisis of 1992 the currencies stabilized even more in the run up to the locking of national exchange rates. At best, we may therefore say that the two periods were two of a kind of fixed exchange rates, the more correct description perhaps being a very dirty float among the Member States. Even though national exchange rates of the Member States were floating against currencies outside the ERM area, the mere possibility of EMS and increased capital mobility having had a neutralizing effect on monetary policy deserves attention. Particularly as the ERM-area as a whole constituted a fairly closed economic entity.

New research based on time series analyses that takes into account both potential non stationarities and the typical multivariate simultaneity in macroeconomic data, persistently reveals exogeneity of long-term interest rates in the period after 1983 (e.g. Juselius (1996), (1998a) and (1998b)). In particular, long-term interest rates neither seem to have been much affected by domestic monetary policy nor inflation to have been as much affected by
changes in the money stock or the short-term interest rate as is usually believed. These results therefore constitute strong evidence of impotence with regard to monetary policy, and taken at face value could be taken to precisely demonstrate the aforementioned mutual inconsistency of an independent monetary policy with fixed exchange rates and near to perfect capital mobility. Admitting that there is more to monetary transmission than the effect coming directly through the interest rate channel, the fact that long-term interest rates in some way are determined by a process that is beyond the control of monetary authorities could nevertheless constitute a major impediment to the effectiveness of monetary policy. This is particularly so as bank lending in most Continental European countries is overwhelmingly linked to long-term interest rates and Europe as a whole constitutes a fairly closed economy, the last point making it less susceptible to effects coming through the exchange rate channel. This would in so far as it is correct, constitute a serious threat to the boldly expressed goal of accomplishing price stability by exerting influence on price inflation directly or indirectly through economic activity by the conduct of monetary policy alone. It would also imply that monetary policy most probably cannot have been the (only) central cause of convergence in the last two decades, suggesting other reasons to have played a role as well, such as i.e. fiscal policy and the abolition of controls on capital and the free movement of goods and services. The aim of this chapter however, is more limited and is in the spirit of substantiating the first point more than making a serious attempt to look at alternative explanations for the convergencies during the two last decades. In doing so, I will heavily draw on the results of Juselius and MacDonald (2000) and confront their findings with the results of my own ongoing research.

3 In fact, as Taylor (1995) points out, if it is the long-term interest rate that is important for consumption and investment demand, the monetary transmission mechanism depends on how monetary policy affects the long-term interest rates. A lack of a relationship between the policy rate and the long end of the yield curve will thus imply that the channel through which monetary policy affects the economy is severely impaired.

4 As illustrated by Borio(1995), the share of outstanding debt bearing interest rates which were either predominantly fixed or indexed to long-term interest rates for six of the seven largest European economies, amounted in 1993 to more than 55%. The only country among the seven with a significantly lower share at that time was Italy. Recent evidence shows however that things have changed dramatically in Italy since then and that the share of mortgages at fixed long-term interest rates has increased from 25 per cent in 1993 to more than 50 per cent in 1997 (European Mortgage Federation (1998)).
The chapter is organized as follows. To motivate and to be able to interpret the results of the econometric analysis, Section two discusses briefly the mechanism through which policy rates may affect the economy. In most Continental European countries bank lending is overwhelmingly linked to long-term interest rates (ref. footnote 4), and the impact of monetary policy on the economy therefore depends crucially on how changes in policy rates are transmitted to the long end of the yield curve. Section two will therefore mainly deal with this part of the transmission mechanism, that is the mechanism through which policy rates affect long rates. Section three then presents and interprets the results of Juselius and MacDonald and compares their findings both with the results in Chapter 1 and the outcome of a separate structural analysis undertaken in this section. Based on the results of the preceding section, Section four seeks to discuss the relative role played by monetary vs. fiscal policy in the future conduct of policy geared towards resolving the problem of unemployment in Europe. Section five contains concluding remarks.

2 The monetary transmission mechanism and stylized facts

The link from monetary policy actions to the economy is far from trivial and can be identified to work through a number of channels. As pointed out by among others, Mishkin (1995, 2001), these transmission mechanism channels include an interest rate channel, an exchange rate channel, an asset price channel and in addition to a channel through which expectations about the future course of the economy and the confidence with which these are held might affect the economy as pointed out by MPC (2001), a so-called credit channel. However, what is common for all these channels is that they largely work through the influence they have on aggregate demand in the economy and to a less extent through affecting the trend path of supply. The channel referred to above as the interest rate channel represents the traditional Keynesian view of how a monetary stimulus is transmitted to the real economy and works through the effect a hike in policy rates has on costs of capital and thus investments and aggregate demand. Because bank lending in Continental Europe mainly is of a long-run character, however, the impact of monetary policy through this channel will depend crucially on how changes in policy
rates are transmitted to the long end of the yield curve. However, it is not only via this channel that long-term interest rates play an important role as the potential effects propagating through the two channels of respectively asset prices and credit mainly originate from the effect that monetary policy might have on equity prices and the net worth of firms. These two quantities are naturally linked to the present value of expected future net income streams which means that monetary policy does not affect the market value of firms only through its effect on contemporaneous short-term interest rate but in fact to a much greater extent through its effect on expectations with regard to future short-term interest rates and thus long-term interest rates.

Without going into details, the two propagating mechanisms are spelled out through respectively, changes in equity prices affecting investment through a change in the ratio of market value to replacement cost of capital (Tobin's q) and changes in net worth of firms affecting the propensity to lend via changed risk of adverse selection and moral hazard on part of banks. Thus, based on the fact that the exchange rate channel in Europe probably is of a minor importance due to its relatively closed nature (ECB (2001)) and disregarding for a moment the effects originating through expectations and confidence, there seems to be a lot to the argument that the propagation mechanism of monetary policy works predominantly through the effect that monetary policy might have on long-term interest rates. This is not to say...

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5 Beyond what is said in footnote 4 above, this fact is given further momentum by the Executive Board of the ECB (2001) which states that: "Longer term interest rates may be especially important since they have a maturity which matches the horizon of many corporate investment and households' saving decisions, such as saving for retirement".

6 The policy rate refers to the policy instrument of the central bank and thus represents the discount rate. The short rates refer to interest rates in the money market for short term assets. As Central Banks have a fairly tight control over the short end of the money market, I will in the following deliberately use these two concepts interchangeably, being aware that they are far from being equivalent. With long rates I am in the following going to mean interest rates on Government bonds with up to ten years to maturity. In the econometric analyses of Section 3 short- and long-term interest rates are represented by three month money market interest rates and interest rates on government bonds with ten years to maturity, respectively.

7 Without denying that the credit channel may be of significant importance, there seems to be a high degree of persistence among economic model builders in incorporating credit channel effects in their models. Suffice it to mention that out of the 18 models analyzed in Taylor (2000), only one is based on a propagation mechanism with a financial accelerator.

8 Apparently, this view is relatively widespread among econometric model builders as is reflected by the fact that among the 18 models analyzed by Taylor (2000), only four...
that short rates are of no importance. No doubt, an increase in the Central Bank’s official rate will have a direct effect on the profitability of firms that rely on bank borrowing linked to short-term money market interest rates which will certainly increase the return that these firms will require from new investment projects and thus lead to a decline in demand and investment activity. However, that this effect could be of a decisive importance for how policy rates affect the economy is rather doubtful, given the dominant role played by debt bearing interest rates which are either predominantly fixed or indexed to long-term interest rates in continental Europe. In the following I will therefore concentrate on the part of the transmission mechanism that deals with how policy rates affect the long end of the yield curve, keeping in mind of course that there are effects coming via other channels as well. As pointed out in Buti et al. (1998) this transmission depends heavily on how a hike in policy rates through interactions of future expectations with regard to inflation, exchange rates, the development of the real economy, and, as a function of these, future monetary policy, affects the long end of the marked. Hence, a monetary contraction will in general not lead to an unambiguous effect on long rates, but will depend on how it is perceived to affect the expectations with regard to the future development of certain key economic variables. Depending on the political and economic situation a monetary contraction may therefore convey a different kind of information and thus potentially both lead to a fall and to a rise in long-term interest rates. However, that this link is not missing is of crucial importance for the argument that monetary policy had a central bearing on the convergence during the last two decades and for monetary policy to be effective in pursuing its goal of price stability, the sign of this relationship being of minor importance. To clarify matters further, I will bear on the classification made in Buti et al. (1998) and classify the effects on long rates into two categories, the effects from portfolio considerations and expectations, respectively.

The portfolio effect describes the effect of reallocation between assets imply a transmission mechanism that is propagated through short-term interest rates, all the rest implying a transmission mechanism that goes through long-term interest rates.

9 In this respect it is important to stress that the credit channel as described in Bernanke and Gertler (1995) is not though of, as they express it: “a distinct, free-standing alternative to the traditional monetary transmission mechanism, but rather as a set of factors that amplify and propagate conventional interest rate effects”. This means that though there may be effects coming through other channels than interest rates, these must be considered to be of a secondary importance.
of imperfect substitutability in the case of changes in their relative yields. This effect is unequivocally positive in the sense that a hike in the yield of one type of asset will lead to an increase also in the yield of the imperfect substitute. This effect goes through increased demand for the asset which has experienced a yield increase and reduced demand for the assets which have experienced a relative yield decline, the last effect leading to reduced prices and through a fixed coupon dividend, an increased percentage return on the new value of the asset. Treating assets of different maturities as imperfect substitutes, then simply means that a hike in the short money market instrument, through encouraging investors to redirect their funds from assets with a long maturity to the instrumental asset, will force yields on assets with a long maturity to increase as well.

The effects of expectations is based on two arbitrage conditions, the uncovered interest parity (UIP) and the expectations theory of the term structure (Schiller (1979)), respectively. The first of these is a relationship between foreign and domestic interest rates on assets of the same maturity and says that in a steady state the return from investing one unit of domestic currency must be the same whether one invests domestically or abroad. The long rates should therefore be equal to the corresponding foreign long rates plus the expected rate of depreciation of the home currency against the foreign currency. The expectation theory of the term structure on the other hand is a relationship between interest rates of different degrees of maturity and says that long rates should be equal to a weighted average of current and expected future short-term interest rates. Thus, the impact on long-term interest rates from a change in current short-term interest rates depends on how expected future short-term interest rates are affected. A rise in current short-term interest rates that is regarded as permanent will lead to a full pass through from short-term to long-term interest rates. On the other hand, if an increase in the current short-term interest rate leads to a significant reduction in inflation expectations, long-term interest rates may even decline.

A change in policy rates may affect expectations with regard to future short rates and exchange rates in different ways. For instance as Buti et al. (1998) point out, in the case of a central bank with a good anti inflationary reputation and high credibility, a hike in the policy rate can be seen as signalling the determination of the central bank to fight inflation. Thus the hike could lead to expectations of an appreciating trend and a downward movement in future interest rates, both potentially leading to a decline in long-term interest rates. On the other hand, in the case of a central bank
with a less good reputation, a hike in interest rates may be taken to signal the build up of an inflationary pressure, and thus most probably leading to increased expectations with regard to the necessity of undertaking upward adjustments in policy rates also in the future. In this case the hike will lead to an increase in long-term interest rates. In the case of Germany the anti inflationary reputation may have been so good that a hike may not have had an effect on either exchange rate expectations or expectations of future inflation. If so the only observable effect should be the portfolio effect implying an unambiguous positive effect on long rates.

Looking at the stylized facts of Figure 3, the spread between German short- and long-term interest rates seems to be far from stationary in the sense of exhibiting a stable mean reverting process\(^\text{10}\). This implies that there does not seem to have been any long-run relationship between short and long rates in the sample interval we are looking at. On the other hand, looking at the spread between US and German long rates, the data clearly reveal a stable long-run international interest rate relationship. This observation has implications far beyond the vague recognition made by Goodhart (1995) that increased capital mobility lately has led to a greater tension between international pressure (e.g. foreign long-term interest rates) and domestic factors (e.g. the expected time path of future short rates) in the determination of long-term interest rates. Taken at face value it implies that the long end of

\(^{10}\) An important caveat with regard to time series being characterized as stationary or not is that these are sample sensitive and do not represent generic properties of the data.
the yield curve has been almost totally dependent on what has been going on in international capital markets in the long run, the influence of domestic monetary policy in fact having been of minor importance. Figure 4, showing long German and US interest rates, further indicates that the direction of causality has been unidirectional in the sense that German long rates seem to be determined by US long rates.

Below, I will shed further light on these issues by referring to two independent papers analysing the determination of long-term interest rates in a simultaneous framework, the paper by Juselius and MacDonald (2000) and my own paper, included as Chapter 1 in this thesis, respectively. Both papers use the cointegrating VAR methodology developed by Johansen (1988), but cover different sample periods and are based on different information sets. While my paper undertakes an analysis covering the period 1990 to 1998 based on a VAR of dimension 5 for long- and short-term interest rates in Germany and the US together with the bilateral exchange rate, Juselius and MacDonald also include nominal prices and money in an extended analysis which covers both periods of convergence.
3 What data tells us: Two independent analyses of the monetary transmission mechanism based on the cointegrating VAR.

In their analysis Juselius and MacDonald use monthly data for the period 1975 to 1998. Their sample thus comprises both periods of convergence within the Community until the realisation of the EMU at the beginning of January 1999. In an information set comprising seven variables\(^{11}\), they are able to identify no less than three long-run relationships\(^{12}\), out of which only one includes short rates. However, as neither of the two long-term interest rates show evidence of adjusting to any of the long-run relations and the structural model developed does not support any short-run effects of short rates on long rates, there is no evidence of a causal relationship implying that short rates feed long rates. Further, the lack of significant inflationary effects is evident in all four interest rate equations. This rules out the possibility that monetary policy might have had an important bearing on long rates through affecting expectations with regard to future inflation. These results are strong evidence against the expectation theory of the term-structure and indicate that an important channel through which monetary policy affects the economy does not seem to exist, either directly or through the channel of inflation expectations. One might expect that these results in some way are fortuitous. However, as Juselius and MacDonald put it: "these are very strong results and have also been found in Danish, Spanish, and Italian data. The references are Juselius (1992), Juselius and Toro (1999) and Juselius and Gennari (2000). Another aspect of their analysis is the lack of short rate effects on inflation in the case of Germany. Taken literally this finding undermines even the possibility of running a policy geared towards controlling inflation, the boldly expressed goal of the ECB.

As I have commented, the results of Juselius and MacDonald are extremely strong and suggest not only that long-term interest rates in Ger-

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11 The seven variables are respectively, long- and short-term interest rates in both Germany and the US, the bilateral real exchange rate and finally the two country specific inflation rates.

12 The three long-run relationships are respectively; a relationship between the real exchange rate and the real long-term interest rate spread, a relationship between German inflation, US inflation and domestic long-term interest rates and a relationship between real short-term interest rates and the long-term interest rate spread.
many are exogenous, but also that US long rates seem to be determined by factors outside the information set considered. This last point is especially surprising as there has been a common appraisal among economists that the Fed in the conduct of monetary policy, has exerted a significant impact on the real economy through affecting long rates. However, in contrast to the results for Germany, Juselius and MacDonald do find that short rates play a significant role in the determination of inflation. Thus, the Fed does at least seem to have a handle through which to manage inflation. The stylised facts of Section two clearly suggest a long-run relationship between long US and German interest rates. This calls into question the exogeneity status assigned to German long rates in the study by Juselius and MacDonald. As this result is controversial I will elaborate further on this by referring to research that diverges on this point.

In contrast to the analysis by Juselius and MacDonald, the analysis undertaken in Chapter 1 of this thesis confirms that there is a long-run relationship between long rates in Germany and the US. The results also indicate that the direction of causality is unidirectional in the sense that the latter ones seem to lead the first ones and not vice versa. Thus, the results in Chapter 1 seem to be totally in line with the stylised facts of Section two. In the paper by Juselius and MacDonald this relationship is implicitly identified and categorised as spurious as they do not find support for an adjustment mechanism where the long spread functions as an attractor towards which the processes of long-term interest rates seek to move. However, even though the relationship identified in Chapter 1 may be identified as spurious in an extended information set, the fact that the model in Chapter 1 does not rely on dummies together with the fact that the analysis confirms the striking evidence set out by Figure 4, should legitimate that the analysis deserves attention, if not from a statistical at least from a theoretical and practical point of view. Also, the argument of spuriousness is one that always can be

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13 The same data have been analyzed in Chapter 3 using an alternative procedure to deal with time series data with a small cross sectional dimension. As for the existence of a strong long-run relationship between German and US long-term interest rates the results herein are totally in line with the findings referred to in the text. However, in contrast to the analysis in Chapter 1 this analysis suggests an additional long-run relationship between US and German domestic interest rate spreads. As discussed herein this relationship is indicative of long rates affecting short rates through their capacity of informing policy rules on part of central banks and seriously call into question the exogeneity status assigned US short-term interest rates in Chapter 1 of this thesis.
addressed to partial economic analyses, the analysis by Juselius and MacDonald notwithstanding, and in general must be sought to be substantiated within the framework of prior beliefs and the reliability of results in conjunction with thoroughly testing for exogeneity in information sets extended in directions proposed by priors and theory. The analysis undertaken in Chapter 1 differs from the one by Juselius and MacDonald also by focusing only on the period after the reunification of Eastern and Western Germany, the argument being one of structural change and lack of credible long time series for unified Germany. However, like Juselius and MacDonald, in Chapter 1, I do not find any long-run relationship between short and long rates and long US rates are found to be exogenous with regard to the information set. In large measures therefore, the two analyses seem to be in line with each other, the main difference being that the long end of the German yield curve in Chapter 1, is directly linked to the US bond market and thus is driven by a common underlying trend originating from international capital markets.

As Juselius and MacDonald, in Chapter 1 I develop a structural dynamic model based on the observed covariance structure of the residuals in the system of marginal processes in the information set. In contrast to the analyses of Juselius and MacDonald, this structural model however, is based upon the assumption that US long- and short-term interest rates are explained by processes not included in the information set and thus both being exogenous. The short-run structure of this model shows that there are strong effects of short-term interest rates in the very short run. In fact the chapter suggests that a one percent increase in the short-term interest rate will lead to an about 0.6 percentage increase in the long-term German rate after the first month. Based on the fact that I in Chapter 1 am not able to identify any

\footnote{Stochastic structural model builders are often confronted with the critique from statisticians that dynamic coefficients of structural models in general cannot be given the interpretation of \textit{ceteris paribus} elasticities. This critique is based upon the recognition of marginal vector processes being the Data Generating Process (DGP) and that structural models represent deduced representations of these. In that case, unless the covariance matrix of the marginal vector process is diagonal, dynamic structural coefficients cannot be given a \textit{ceteris paribus} interpretation as a shock to one of the processes in general will feed simultaneously into other variables through a non-diagonal reduced form covariance matrix. However, and this is a question appropriate to ask statisticians, what if the structure has the status of a Data Generating Process and the marginal process itself thus being interpreted as a reduced form representation of the structure? In that case the marginal process will have a covariance matrix that only reflects the structure and thus is explained by it. If so, should not the interpretation of dynamic coefficients in a dynamic structural}
long-run relationship between long- and short-term interest rates, either in Germany or in the US, this effect is only transitory. What is important, however, is to recognize that this could be indicative of monetary policy being effective through lowering expectations with regard to future interest rates and thus neutralizing the effect of a hike in policy rates on long-term interest rates in the long run. However, the results of Juselius and MacDonald clearly indicate that inflation rates do not explain interest rates. This should indicate that expectations of future short rates are not very sensitive to inflation and is evidence against a channel through which changes in expectations of future short rates might neutralize the effect on long-term interest rates of a hike in policy rates in the long run. In light of what is said in the above footnote, another circumstance that calls into question the plausibility of expectational changes having a neutralizing effect of hikes in policy rates on long-term interest rates, is the fact that the structural model of Chapter 1 is not able to fully explain the observed correlation pattern of the unrestricted reduced form (URF). Particularly worrying in this respect is the remaining high correlation of 0.48 between the residuals of the equations of the short- and long-term interest rates of Germany in the structural model (conditional on short-and long-term US interest rates). This indicates that the structural model does not serve its purpose of explaining the correlation pattern of the reduced form. In fact with regard to the correlation between the residuals of the two domestic German interest rates it seems to do the opposite, as the correlation between the residuals of the corresponding equations in the model as \textit{ceteris paribus} structural elasticities then hinge on the assumption of a diagonal structural covariance matrix and a deduced non-diagonal reduced form covariance matrix have no bearing on the interpretation of these, only reflect the inherent simultaneity of the structure itself? Shocks to one of the variables will in this case not be interpreted as a shock to its marginal process, but as a structural shock to the behavioral equation of the variable in the structural model. One may say that the issue is related to whether we choose the perspective of a marginal or a structural DGP as the underlying process governing the system of variables in an information set. In the first case a structural model is just another way to present the information contained in the system of marginal processes, while the latter gives the structural model the status of explaining these marginal processes. Anyhow, in the paper by Juselius and MacDonald the marginal perspective is used, and their structural model is therefore only another way to represent the results that better illustrates the economic content of their findings. The low degree of observed correlation between the residuals of the marginal model further allows them to interpret the structural coefficients as elasticities. The structural model in Hammersland, however, is based on a marginal model where the covariance matrix is far from triangular, and the elasticity interpretation hinges on the interpretation of the structure being the DGP.
Table 1: Correlation of residuals of the structural model of Chapter 1 augmented with the marginal processes of US short-and long-term interest rates

<table>
<thead>
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<th>$\Delta i_t^{GL}$</th>
<th>$\Delta i_t^{GS}$</th>
<th>$\Delta Dv_t$</th>
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<td>$\Delta Dv_t$</td>
<td>-0.22214</td>
<td>-0.20561</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta i_t^{UL}$</td>
<td>-0.08969</td>
<td>-0.10798</td>
<td>0.24077</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\Delta i_t^{US}$</td>
<td>-0.20133</td>
<td>0.045917</td>
<td>0.24463</td>
<td>0.46664</td>
<td>1</td>
</tr>
</tbody>
</table>

structural model increases compared with the corresponding residuals of the unrestricted reduced form. Furthermore, the structural model in Chapter 1 is conditional on the processes governing US interest rates, which in addition to hide a huge unexplained correlation between the residuals of the marginal models of short- and long-term US interest rates of 0.47, could severely bias our estimates in case the conditioning argument turns out to be invalid. To improve upon these insufficiencies I will turn to the task of finding an alternative structural representation of the information contained in the data.

The main feature to be modelled is the unexplained correlation of 0.44 between the residuals of the short and long-term German interest rate equations of the structural model in Chapter 1 augmented with the autoregressive marginal processes of both US interest rates\(^{15}\). However, looking at the correlation matrix of the residuals in Table 1, there seems to be another correlation that needs to be explained as well, namely the one between the residuals of the equations of the two domestic US interest rates. These correlations could derive from any of several factors in practice. First, the implied causal structure of the model in Chapter 1, saying that long-term German interest rates are explained by and therefore caused by German short-term interest rates.

\(^{15}\)To be able to augment the structural model of Chapter 1 with the autoregressive marginal processes of both US interest rates I had to make some small identifying modifications.
rates might be wrong. In fact, as proposed in Chapter 3 and as alluded to in an above footnote, the direction of causality might go in the other direction, implying that German long-term interest rates cause the corresponding short-term interest rates through their capacity of informing a policy rule on part of the Bundesbank. With regard to the high correlation observed between the residuals of the corresponding short- and long-term US interest rate equations, the same issue arises. That is, whether long-term US interest rates cause short-term US interest rates through their capacity of informing a policy rule on part of the Fed or short-term US interest rates cause the corresponding long-term interest rates through a term-structure relationship between short- and long-term interest rates, is an empirical issue to be settled. The estimated structural model given in (1) below, is an alternative structural representation of the information contained in data. As is seen when comparing the two correlation matrices of Table 1 and Table 2, the high empirical correlation between the residuals of the two German interest rate equations drops in absolute value from 0.44 to 0.04 while the corresponding drop when considering the correlation between the residuals of the corresponding US interest rate equations is from 0.47 to 0.1475. Thus, the alternative “struct” in (1) seems at least to be able to get rid of the worst instances of high unexplained correlation in the structural model of Chapter 1. Looking at the identified structure in (1) this is seen to be accomplished by reversing the direction of causality between short-and long-term German interest rates as well as endogenizing US short-term interest rates such that these become explained by domestic long-term interest rates. This means that short-term interest rates of both countries in the short run are caused by domestic long-term interest rates, probably through their capacity of informing policy rules on part of the corresponding central banks, and is in striking contrast to a term-structure relationship between domestic short-and long-term German interest rates and exogenously determined short-term US interest rates as in Chapter 1.

Even though the analysis undertaken in Chapter 1 and the alternative structural model presented in this section do not fully confirm the analysis of

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16 The structural model of (1) shows slightly worse diagnostics than the model presented in Chapter 1. Particularly worrying in this respect is the test of normality which rejects to a level below five per cent. However, as indicated by the LR-test for overidentifying restrictions which is chi square with twenty degrees of freedom we are not able to reject the over identifying restrictions imposed on an identified structure that encompasses the structure in Chapter 1.
Juselius and MacDonald who find that long-term German interest rates are exogenous, the results clearly indicate that monetary policy through changes in short-term interest rates does not affect long-term interest rates in the long run. Furthermore, the results of the separate analysis undertaken in this section indicate that an eventual short-run term-structure relationship as found in Chapter 1, might be characterized as the outcome of wrongly imposed identifying restrictions on the structure. This result is of the greatest importance as in most Continental European countries (particularly in Germany) bank lending is overwhelmingly linked to long interest rates, and the impact of monetary policy on the real economy depends crucially on how changes in policy rates are transmitted to the long end of the yield curve.

\[ \Delta i_{GL}^{t} = 0.18828 \Delta i_{t-1}^{GL} + 0.29538 \Delta i_{t}^{UL} - 0.096285 \left( i_{GL}^{t} - i_{UL}^{t} \right) + \varepsilon_{1t} \]

\[ \Delta i_{GS}^{t} = 0.32076 \Delta i_{t-1}^{GS} + 0.31271 \Delta i_{t}^{GL} + \varepsilon_{2t} \]

\[ \Delta Dv_{t} = -3.4328 \Delta \left( i_{GL}^{t} - i_{UL}^{t} \right)_{t-1} - 0.75883 Dv_{t-1} + \varepsilon_{3t} \]  \hspace{1cm} (1)

\[ \Delta i_{UL}^{t} = 0.43840 \Delta i_{t-1}^{UL} + \varepsilon_{4t} \]

\[ \Delta i_{US}^{t} = 0.49509 \Delta i_{t-1}^{US} + 0.28921 \Delta i_{t}^{UL} + \varepsilon_{5t} \]

\[ T = 96 \text{ Effective sample 1990(1)-1997(12)} \quad \text{LR } x^{2}(20) = 0.0764 \]

4 Some remarks about the future role of monetary vs. fiscal policy

EMU will have the effect of creating a huge currency area with an economic weight similar to that of the United States and with a large single and deep
Table 2: Correlations between the residuals of the equations in "struct" (1)

<table>
<thead>
<tr>
<th></th>
<th>$\Delta t^G$</th>
<th>$\Delta t^G$</th>
<th>$\Delta Dv$</th>
<th>$\Delta t^U$</th>
<th>$\Delta t^U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta t^G$</td>
<td>1</td>
<td>-0.03727</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta t^G$</td>
<td>-0.21130</td>
<td>-0.02404</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta Dv$</td>
<td>0.10695</td>
<td>-0.20319</td>
<td>0.24490</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$\Delta t^U$</td>
<td>0.14074</td>
<td>0.20168</td>
<td>0.18273</td>
<td>0.1475</td>
<td>1</td>
</tr>
</tbody>
</table>

financial market. These characteristics should promote the development of the Euro as an international currency and lead to an increased influence in international capital markets, with the possible effect of changing a possible unidirectional link between US and European long-term interest rates\textsuperscript{17}. The

\textsuperscript{17}In contrast to the positive exchange rate effect expected to originate from the promotion of Euro as an international currency, the Euro exchange rate has experienced an almost continuous deterioration in its external value after the locking of national exchange rates on 1st of January 1999. As late as in December 2001, several articles in Financial Times rejected this to be the result of speculation and adduced the role of the US dollar as an invoicing currency for international settlements as an alternative explanation. However, it is all too apparent that this cannot have been the whole story behind the decline in the euro exchange rate. The role of the US dollar as a safe haven currency coupled with a widespread scepticism for the European project and the strong development of the US economy must have played a significant role as well. A more controversial issue is the impact of reallocations of private exchange rate portfolios. The prevailing view among economists has until today been that exchange rate effects of such portfolio movements cannot have played an important role in this respect as they have been a part of a diversification trend initiated already in the eighties. However, the simple conversion of assets held in European currencies into euros meant that more than a third of the world portfolio was denominated in euros from 1st of January 1999 and onwards. This percentage is notably close to the proportion of what today is held in US dollars. Whether private investors wishing to maintain the degree of diversification of their portfolios, were willing to absorb such an amount of euros without a fall in the price of the currency is therefore not too obvious. Also, as pointed out by Notermans (2001) the relatively lax stance of policy taken by the ECB in the wake of the continuous weakening of the Euro exchange rate, may have been due to trade off considerations between output and exchange rate
transmission of changes in policy interest rates to market interest rates may also be affected, and whether the ECB will gain or lose control with regard to the long end of the financial market, is an open question. However, taking the US as an example, Juselius and MacDonald’s analysis suggests that EMU will not lead to increased control over the long end of the yield curve, only improving upon the possibility of inflationary control. Monetary policy might therefore attain importance as a measure of controlling inflation, but probably have a more limited effect in the conduct of policies geared towards managing real demand\textsuperscript{18}. Notwithstanding, the mere possibility of the European Central Bank having to run a single-minded mandatory policy of price stability to establish its counter inflationary reputation, may severely undermine the possibility of the ECB to even run an offsetting policy in wake of bad shocks. If it had not been for the Stability and Growth Pact, this would leave national fiscal policies with an important role to play, especially since EMU implies the loss of national exchange rates as a means of correcting real national imbalances. The criteria and procedures enshrined in the Stability and Growth Pact however, may turn out to be too limited to bring about social cohesion and to cushion impacts of possible future shocks within the community through nationally conducted fiscal policies only. Also, as most unemployment in Europe recently has been characterized as natural, there has been an increasing acceptance that the scope for real demand management is small and that what is called for is policies geared towards the improved functioning of labor markets. However, to baptize the phenomenon of unemployment as structural or “natural” based on the concept of the NAIRU may represent a huge disservice to the goal of understanding unemployment. Either one uses Elmeskov’s procedure (Elmeskov (1993)) or alternative measures based on a univariate smoothing of the unemployment series, the different ways to measure the so called NAIRU in my view are almost all severely biased towards giving unemployment a structural interpretation thus implying certainly the risk of throwing out the baby with the bath water. Notwithstanding the paradox of characterising unemployment rates in the range above 10 percent as natural, the fact that unemployment rates generally show a high degree of persistence, implies that some of its natural part is heavily influenced by shocks to the economy. Not only shocks

\textsuperscript{18}For an extended list of reasons why the value of a stabilising monetary policy may be lesser than it is usual to assume, see Røste (2001).
originating from the labor market and thus characterized as structural, but also shocks that are brought about from the demand side. This process is spelled out through shocks leaving a trace to the natural rate, the trace being bigger the higher is the degree of persistence in unemployment, and is commonly denoted as hysteresis in the economic literature. Thus, even in the worst case of unemployment being completely characterized as structural, there is a channel through which real demand management not only works, but will be of most importance the higher is the degree of persistence. However, notwithstanding all that is said so far, the effects of structural policies are slow in coming and may turn out to be extremely costly both in terms of transitory output losses and increased unemployment as sociological and distributional changes needed to recover the economy when being confronted with huge real imbalances or bad shocks. Forgotten are perhaps the times when employers in their desire to maximize profits, unscrupulously exploited employees and more than half of the European population lived close to or under the subsistence level. However, structural policies geared towards reducing minimum wages and social protection in general could easily have the effect of reintroducing this status quo ante to some of the poorest members of the community. Especially since the lack of harmonization in labor standards across countries easily may lead to, as Artis (1999) puts it, “a race to the bottom” in the level of social protection. However, not only the mere insufficiency of structural policy to solve the unemployment problem alone, but also the fact that unemployment in Europe is far from being exclusively structural, calls for instruments outside the structural sphere. That European unemployment is characterized by having a strong regional19 dimension, further points to the need of instruments which affect regions without feeding into other parts of the economy. A pronounced regional policy is also imperative from the perspective of bringing about internal balance within the community. Without regional measures to bring about social cohesion by reducing disparities between regions and backwardness of less favored regions, EMU could easily end in a battle of national interests. A prerequisite for undertaking these kind of policies however, is that there is a considerable budgetary slack. Due to unsustainable dept to GDP ratios, a revision of the criteria and procedures enshrined in the Stability and Growth Pact is probably not on the agenda. This leaves out the possibility of running additionally directed regional policies at national levels and in addition to an

19 In this context regional is also taken to mean national.
intensification of policies geared towards stimulating investment and growth through community banks like EIB and EBRD, calls either for a considerable strengthening of the Structural Funds of the community, notably through a significant increase of the community budget, or to undertake constitutional changes within the community giving rise to something like a federal common policy unit.

If not, the unaccommodating macro environment offered by the ECB in combination with the Stability and Growth Pact would probably merely promise a continuation of the unemployment problem and in the longer run also severely undermine solidarity within the community.

5 Summary and Conclusions

In this chapter I have discussed the role of monetary authorities in accomplishing the two periods of convergence observed during the mid-1980's and the seven years before the realisation of the EMU in January 1999. The chapter also addresses the issue of the future role of monetary vs. fiscal policy in the conduct of policies geared towards solving the problem of unemployment in Europe.

My findings indicate that during the 1990's, European Central Banks only to a certain extent can be said to have had control over the long end of the market and that this control eventually must have been of a very short-run character. As far as we perceive the long-term interest rate to be the central interest rate through which monetary policy affects the economy, this result stands in glaring contrast to the claimed position of central banking having been the main reason for the observed convergence and suggests that other sources must have had an important saying as well. In this respect the solemn commitment by national governments to pursue a policy of convergence enshrined in the Growth and Stability Pact, may have been one of the factors that played a significant role by tying the hands of national treasuries vis à vis the domestic public opinion. Based on the works of Juselius, the fact that

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20 The structural funds of the community normally constitute respectively, the European Regional Development Fund (ERDF), the European Social Fund (ESF) and the guidance part of the European Agricultural Guidance and Guarantee Fund (EAGGF). In this context the concept is also taken to include the Cohesion Fund of the Maastricht Treaty. EIB and EBRD stand for the European Investment Bank and European Bank for Reconstruction and Development, respectively.
also the mid-1980's was a period of fiscal contraction supports a similar point of view for this period. A point worth mentioning and which has not been discussed in the text, is whether the sole process of increased capital mobility independently in some way may have contributed to the convergence. This process is spelled out through strict inflationary control imposed by a regime of fixed exchange rates and the meeting of wage increases by laying off the least productive part of the labor force to uphold a constant profit share in the trading sector. This last point is given some support by the observed high degree of correlation between average productivity and unemployment rates in Europe and signals that the European wage bill to a large extent has been paid by higher unemployment.

The results of Juselius and MacDonald indicate that monetary policy may gain control in the conduct of future anti-inflationary policy. However, as a device to deal with real imbalances there is nothing to indicate a similar prosperity. US long-term interest rates are as exogenous as the German ones. Also, the fact that the ECB will have to pursue a policy geared towards establishing an anti-inflationary reputation will further impair the possibility of undertaking real demand management through the conduct of monetary policy. Structural policies on the other hand may turn out to have unwanted social consequences. Different ways of measuring the "natural" rate may in addition be severely biased in favor of assigning unemployment a structural interpretation. Together with the mere possibility of hysteresis this substantiates the imperative of a future fiscal policy stance, possibly regionally directed, to deal with the problem of unemployment in Europe. The criteria and procedures enshrined in the Stability and Growth Pact further imply that this probably will have to take place at a European level.

In all, this paper strongly suggests that monetary policy by no means can have been the whole story behind the two periods of convergence. Fiscal policy and the inherent mechanism of high capital mobility are both factors that probably have had an important saying as well. Furthermore, the reduced possibility of monetary policy to deal with future real imbalances combined with huge regional unemployment problems within Europe, leaves regional fiscal policy and policies geared towards stimulation of investment and growth as imperative policy measures through which dealing with real imbalances and unemployment in the future.
References


Chapter 3

Large T and small N:
A two-step approach to the identification of cointegrating relationships in time series models with a small cross-sectional dimension.*†

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Florence

Abstract

This chapter addresses cointegration in panel data models with small cross-sectional dimensions. In addition to dealing with cointegrating relationships within sectors, the paper explicitly addresses the issue of cointegration between sectors. The approach is based upon a well-known distributional result with regard to identification of cointegrating relationships when some CI vectors are known a priori, and advocates a two-step procedure to identify cointegrating relations which first identifies the CI-relations within each sector separately and uses these as known CI-vectors in the next step when analyzing the sectors jointly.

Identification of the long-run structures of Norwegian exports and international interest rate relationships are used as examples. The Norwegian mainland exports are divided into two sectors, the traditional goods sector and the service sector. While in the study of international interest rate

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*I want particularly to thank Henrik Hansen in helping me out with the implementation of known cointegrating restrictions in Cats in Rats. In addition I am grateful for comments by Soren Johansen and Andreas Beyer and participants at the conference on the monetary transmission mechanism at Schaeffergaard in Copenhagen.

†The analyses have been undertaken by using a combination of CATS in RATS (Hansen and Juselius (1995)) and PcFiml 9.20 (Doornik and Hendry (1999)). The I(2) tests have been undertaken by using Clara Jorgensen’s I(2) procedure in Cats in Rats.
relationships the two sectors investigated are Germany and the US. The examples are used to address the more general issues of the degree of independence in capital markets and in goods markets of small open economies.

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1. Introduction

Early in the 1990’s, several studies like Breitung and Meyer (1991), Quah (1994), and not least Levin and Lin (1992, 1993), developed the asymptotic properties and studied finite sample properties of unit-root tests on panel data as both time series and cross-section dimensions grow arbitrarily large. Their results showed that by using data varying not only along one dimension, but along two dimensions, the power of the unit root test in most cases increases dramatically against stationary alternatives. In the spirit of Engle and Granger (1987), these tests have recently been further extended to various tests for cointegration in a panel data framework by, e.g., Pedroni (1996) and McCoskey and Kao (1998). The implied
increased possibility of identifying cointegrating relationships in this setting has also initiated renewed interest in solving parity puzzles, like that of purchasing power parity (e.g. MacDonald (1996), Frankel and Rose (1996), Pedroni (1997)). However, as a means of identifying cointegrating relationships in the multivariate case, with the possible existence of several cointegrating relationships, this method is far from sufficient; there is a need to develop a multivariate system approach along the lines of Johansen (1988). Even though there is a lot of ongoing research aimed at meeting this requirement, a fully general system framework to deal with cointegration in the case of multivariate panel data has still not been developed, the one coming closest to meeting this end perhaps being the paper by Larsson and Lyhagen (1999). Also, even though the problem may have a general solution this most probably will turn out to be totally inadequate as a practical device for undertaking panel data cointegration analysis, as the level of complexity in the general case is already almost unwieldy with even a very small number of sectors.

The primary aim of this chapter is to offer an easily accessible strategy for dealing with time-series models with a small cross sectional dimension and is therefore written in the spirit of developing a kind of ad hoc solution to a case that is less general than the general problem of panel data cointegration. It is based on the result in Horvath & Watson (1995) which gives the asymptotic distribution of the Wald test in vector autoregressive models when some cointegrating vectors are known, and advocates a two-step approach which first identifies the cointegrating relations in each sector separately and then uses these as known CI-relations in the next step when analyzing the sectors jointly. The first step can be done in the usual way by analyzing the sector specific VARS. The second step implies interpreting the estimated cointegrating relations in the first step as representing known cointegrating relations, and then to use the distributional results of Horvath and Watson as tabulated for the likelihood-ratio cointegration rank test in Paruolo (1999), to determine the cointegrating rank of the full system, given these. The contribution lies in the use of a result developed originally for a pure time-series model to help with the identification of cointegrating relations in the case where we also have to deal with a cross-sectional dimension. In addition to allowing for heterogeneous long-run cointegration relationships within each group or sector and cross-sectional dependencies through error-correction terms and short-run effects, this approach explicitly takes into account the possibility of cointegration between sectors. Larsson and Lyhagen (1999) develop a framework where cointegrating relationships are only allowed for within each sector and as such therefore disregard the possibility of long-run cointegrating relationships
between sectors. However, in contrast to an earlier paper, Larsson et al (1998), they explicitly allow for cross-sectional long-run effects through the potential inclusion of all sector-specific cointegrating relationships as equilibrium-correction terms in the equations of the panel data model. Larsson and Lyhagen also allow simultaneous modelling of the long run relationships within sectors, taking into account possible cross-sectional dependencies in the error structure of the model. This last point suggests a third and last step in our procedure to identify cointegrating relations in time series with a small cross-sectional dimension, namely after having gone through the two steps of my suggested procedure, to re-estimate all free parameters of the identified cointegrating relationships in a VAR where the known rank together with the known structure of the cointegrating space are imposed.

The external part of most large-scale econometric models of small open economies has traditionally been modelled as one of monopolistic competition, opening up the possibility of a certain degree of independence in the determination of prices. Within this framework it is therefore appropriate to ask whether the increased degree of internationalization, abolition of barriers to trade and deregulation of capital markets during the eighties and nineties, have had a significant impact on the possibility to deviate from long-run relative purchasing power parity (RPP) in the process governing external trade prices. Another important issue in the wake of deregulation of capital markets and increased internationalization, is whether the possibility of running independent monetary policies in Europe has been considerably weakened during the last decade. And if so, whether this has been accomplished through a stronger dependence on what is going on in international capital markets.

To provide examples of the suggested procedure and to analyze the political issues addressed above, this paper undertakes two independent analyses. The first seeks to identify the long-run structure of Norwegian exports between the first quarter of 1980 and the last quarter of 1998. To provide a cross-sectional time series data set, Norwegian mainland exports are divided into two sectors, the traditional goods sector and the service sector, respectively. The implications with regard to the identification of a RPP relationship are then compared with the results in Hammersland (1996), which based on an aggregate model of Norwegian exports, is not capable of identifying a RPP relationship and reveals significant signs of monopolistic power in the determination of prices over the period 1966 (4) to 1992 (4). The other study is a study of US and German interest rates and seeks to reveal the degree of European autonomy through identification of short-
and long-term interest rate relationships over the period 1990 (1) to 1997 (12). The two sectors are naturally given by the two countries and the results of the analysis are compared with the results in Hammersland and Vikøren (1997) and Chapter 1.

The chapter is organized as follows. Based on Horvath and Watson (1995), Section 2 introduces the model used to analyze the two-dimensional data set and gives a brief motivation for the choice of tables to be used when dealing with identification of cointegrating rank in the case of known cointegrating relationships. Section 3 then contains the two examples of using my suggested two step procedure on two actual “panel” data sets. Before presenting the results, however, subsection 3.1.1 deduces theoretical hypotheses on long-term relations based upon the theory of monopolistic competition and Armington demand theory, extensively reviewed in an appendix. Based on the theories of uncovered interest rate parity, UIP, and the expectation theory of the term structure, Section 3.2.1 does the same for the interest rate study. The results of the econometric analyses where we first identify the cointegrating relationships in the two sectors separately for finally ending up with a joint analysis of the full model, are then given for the two examples in 3.1.3 and 3.2.3, respectively. Section 4 concludes.

2. The Model

The general autoregressive I(1) model is given by:

\[ \Delta X_t = \alpha \beta' X_{t-1} + (\gamma, \mu) \left( \begin{array}{c} Z_t \\ d_t \end{array} \right) + \epsilon_t, \]

where \( X_t \) and \( \epsilon_t \) are both \( p \times 1 \) vectors, \( Z_t = (\Delta X'_{t-1}, ..., \Delta X'_{t-k+1})' \) is \( p(k-1) \times 1 \), \( \epsilon_t \) is assumed to be i.i.d. \( N(0, \Omega) \) and \( d_t \) is a \( q \times 1 \) vector of deterministic terms like a constant term, trend and seasonal dummies. \( \gamma = (\Gamma_1, ..., \Gamma_{k-1}) \) is a \( p \times p(k-1) \) matrix, \( \mu (p \times q) \) and \( \alpha \) and \( \beta \) are both \( p \times r \) matrices assumed to be of full rank, \( r \), such that the \( I(1) \) condition of \( \alpha'(I_p - \sum_{i=1}^{k-1} \Gamma_i)\beta \) having full rank, \( p - r \), is fulfilled when assuming that all the roots of the characteristic polynomial of \( X_t \) lie at one or outside the unit circle. For our purpose \( X_t \) consists of sector specific variables as well as variables that do not vary across the sectors. Thus, in the case of two sectors \( X_t = (X'_{1,t}, X'_{2,t}, X'_{3,t})' \), where \( X_{i,t} = (X_{i1,t}, ..., X_{iN_{i,t}}) \) represents the \( N_i \) numbers of sector specific variables in sector \( i \), \( i = 1,2, \) and \( X_{3,t} = (X_{31,t}, ..., X_{3N_{3,t}}) \) represents the \( N_3 \) numbers of common variables. We are going to look at the case where \( \beta \) can be partitioned into two submatrices, \( \beta_1 = b \).
and $\beta_2$, of dimensions $p \times s$ and $p \times m$ respectively. The first set of cointegrating vectors, $b$, represents the $s$ a priori "known" cointegrating relationships that we are getting to "know" from a preliminary cointegration analysis undertaken at the sectoral level, while $\beta_2$ represents the $m = r - s$ remaining cointegrating vectors to be identified using the whole information set given the "known" relationships identified in the first step. $r$ represents the total number of cointegrating vectors in the two-dimensional data set. Representing the estimated cointegrating vectors at the sectoral level by $\hat{\beta}_1 = \left(\left(\hat{\beta}_{11}', \alpha', \hat{\beta}_{13}'\right)', \left(\alpha', \hat{\beta}_{22}', \hat{\beta}_{23}'\right)'\right)$, this means that $\hat{\beta}_1 = b$ and the level term in 2.1 can be given the equivalent representation of

$$
\alpha \beta' X_{t-1} = (\alpha_1, \alpha_2) \begin{pmatrix} \beta_1' \\ \beta_2' \end{pmatrix} X_{t-1} = (\alpha_1, \alpha_2) \begin{pmatrix} \hat{\beta}_1' \\ \hat{\beta}_2' \end{pmatrix} X_{t-1}
$$

(2.2)

$$
= (\alpha_1^*, \alpha_2^*, \alpha_2) \begin{pmatrix} \beta_{11}^* & 0^* & \hat{\beta}_{13}' \\ 0^* & \beta_{22}' & \hat{\beta}_{23}' \\ \beta_{31}^* & \beta_{32}^* & \beta_{33}^* \end{pmatrix} \begin{bmatrix} X_{1,t-1} \\ X_{2,t-1} \\ X_{3,t-1} \end{bmatrix},
$$

where we have partitioned the remaining cointegrating vectors to be estimated in the second step, $\beta_2$, in conformity with the partitioning of the variable vector, $X_t$. The argument for treating the sector specific cointegrating vectors as known when they in fact have been estimated in a preliminary step, hinges on the super consistency property of the cointegrating vectors. For an elaboration on this point the reader is referred to Chapter 4. In the following I will resort to the simpler notation of denoting the estimated cointegrating vectors as $b$ and the remaining unknown ones as $\beta_2$, keeping in mind the partitioning in 2.2 when interpreting the significance of the vectors.

Our analyses in the next section will be confined to the case where the deterministic term $\mu d_t$ is subdivided in $\mu_1 d_{1t} + \mu_2 d_{2t}$ and $\mu_1$ is restricted to lie in the $\alpha$ space such that $\mu_1 = \alpha \kappa$, where $\kappa = \begin{pmatrix} k \\ \kappa_2 \end{pmatrix}$ is a $r = (s + m) \times q$ matrix. To accommodate these changes 2.1 must be transformed according to:

$$
\Delta X_t = (\alpha_1, \alpha_2) \begin{pmatrix} b_1' \\ b_2' \end{pmatrix} X_{t-1} + (\Upsilon, (\alpha_1, \alpha_2) \begin{pmatrix} k \\ \kappa_2 \end{pmatrix}, \mu_2) \begin{pmatrix} Z_t \\ d_{1t} \\ d_{2t} \end{pmatrix} + \epsilon_t
$$

64
where $\alpha$ and $\kappa$ has been decomposed conformably with the partitioning of $\beta = (b, \beta_2)$. This expression may equivalently be expressed as:

$$\Delta X_t = \alpha_2(\beta_2', \kappa_2) \left( \begin{array}{c} X_{t-1} \\ d_{1t} \end{array} \right) + (\alpha_1, \gamma, \mu_2) \left( \begin{array}{c} X_{t-1} + kd_{1t} \\ Z_t \\ d_{2t} \end{array} \right) + \epsilon_t \quad (2.3)$$

It is this set up of the model we are going to use in the determination of $m = r - s$, the number of cointegrating vectors beyond the known number of relationships, $b^* = (b', k)'$, following from the sectoral analysis, $\beta_2'' = (\beta_2', \kappa_2)'$.

In Section 3, when identifying the cointegrating relationships in the first example, model 2.3 in addition to including unrestricted centralized seasonal dummies, is specified with a trend restricted to lie in the cointegration space and a non-restricted constant term, implying that $d_{1t} = t$ and $d_{2t} = (1, S_1, S_2, S_3)'$. Therefore, the most appropriate critical values to use in identifying the cointegration rank are given by Table 5 in Paruolo (1999). In the second example where I study interest rate relationships, I have deliberately neglected a trend term and the constant term has been restricted to lie in the space spanned by the loading matrix $\alpha_2$. In model 2.3 this is equivalent to $d_{1t} = 1$ and $d_{2t} = 0$. This implies that the correct critical values to use are given by Table 3 in the same paper by Paruolo. It is worth noting that in neither case has it been necessary to fall back on measures to improve diagnostics.$^2$

3. Identification of cointegrating relations using times series data with a small cross-sectional dimension: two examples

This section provides two examples with regard to using the advocated two-step procedure in analyzing real data. In both cases I analyze data with a two dimensional structure where the cross sectional dimension is equal to two.$^3$ The first seeks to identify the structure of exports in small open economies by looking at Norwegian data. To be able to apply the suggested procedure, Norwegian

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$^1$The $S_i$'s are the three centred seasonal dummies.
$^2$The only exception is the use of seasonal dummies in the case of the export study.
$^3$I have deliberately avoided using the term “panel data” as this concept usually is confined to the case of a large cross sectional dimension.
mainland exports have been divided into two sectors, the traditional goods sector and the service sector, respectively. The second example concerns identification of international and domestic interest rate relationships and looks particularly at linkages between European and US long- and short-term interest rates as well as the degree of domestic control over the long end of the yield curve as reflected by a term structure relationship between short- and long-term domestic interest rates.

3.1. Example 1: Modelling of export volumes and export prices in a small open Economy: The Norwegian case.

As alluded to in the abstract, the analyses are partly motivated from the perspective of identifying the degree of independence in capital markets and in goods markets of small open economies. In the case of exports, a model of monopolistic competition is well suited for this purpose as it takes into account the possibility of monopolistic power in the process governing the determination of quantity and prices. Before presenting the results, therefore, a brief review of theory and its implications with regard to cointegration will be given.

3.1.1. Monopolistic competition and its implications with regard to cointegration

Most models for the determination of export volumes are pure demand relationships based on Armington’s theory of demand distinguished by place of production (Armington (1968)). They are often explained by models of monopolistic competition (Bruno (1979)) in which export prices are determined ex ante⁴ and export volumes for fixed prices ex post. In doing so, it is common practice to assume a constant price elasticity in demand and constant returns to scale. However, when looking at the export price and export volume determination simultaneously, these assumptions may be mutually inconsistent with data and the possibility of developing a stable representation in the shape of an econometric model of the information contained in these. For example, under monopolistic competition these assumptions imply that there is no channel through which demand may affect prices. As these effects turn out to be significantly estimated in most econometric works, export price relationships are often implicitly based on either

⁴The ex ante decision refers to a plan made before having complete knowledge of all variables affecting the decision-making, implying that the decision must be based on their expectations. The ex post decision, however, is made on the basis of complete knowledge of all variables.
an assumption of decreasing returns to scale or that a non-constant price elasticity of demand creates cyclical movements in the mark-up. A non constant elasticity of demand will on the other hand necessarily imply an unstable Armington based model of the export volume which makes it inappropriate to assume production processes with constant returns to scale when combining Armington's theory with the theory of monopolistic competition. In this chapter I have chosen to assume decreasing returns to scale, thus enabling us to model both price and volume determination under a consistent set of assumptions. For a mathematical exposition of both theories the reader is referred to the Appendix.

An ex ante, ex post approach requires export prices to be completely fixed according to the ex ante plan while the export volume is allowed to depart from the same plan ex post. The existence of long-term contracts, advertisements, price lists etc. may motivate such a commitment of export prices to the plan. Thus, a representative producer will exercise price-taking behavior ex post and one may be faced with one of two possible situations. In the first case, the consumer will be rationed on the product market and the production will be determined by the price-taking level of production. In the second case, it is the producer that will be rationed on the product market and the export volume will be determined exclusively by real demand. Thus, we are in a situation in which prices are determined ex ante by the behavior of a monopolist facing decreasing returns to scale while at the same time the level of exports is determined exclusively by ex post demand.

The export price equation will thus be defined by the first order condition, price equal to marginal costs multiplied by a mark-up factor greater than one. In practice export prices, PA, may therefore be modelled as a log linear function of unit labor costs, WC/Y,\(^5\) world market prices, PW and foreign real income R.

\[
\ln(PA_t) = c + \phi (\ln(WC_t) - \ln(Y_t)) + (1 - \phi) \ln(PW_t) + \rho \ln(R_t) + \epsilon_{it}\quad (3.1)
\]

The parameter \(\phi\) is the partial elasticity of export prices to unit labor costs. From (3.1) it appears that the export prices are homogenous of degree one in unit labor costs and world market prices. \(\epsilon_{it}\) is a stochastic disturbance term for the export price equation.

We follow Armington(1968) and assume that demand is specific to the producer. Thus, the demand for exports, denoted A, may be specified as a log linear

\(^5\)WC and Y represents wage costs per man-hour and output per man-hour, respectively.
function of the foreign real income, \( R \), and the relative price given by the ratio of export prices to world market prices.

\[
\ln(A_t) = \mu + \beta \ln(R_t) - \sigma(\ln(PA_t) - \ln(PW_t)) + \epsilon_{2t} \tag{3.2}
\]

The producers of small open economies generally have a very small market share, implying that the parameter of relative prices can be interpreted both as a relative price elasticity with regard to export demand and as the elasticity of substitution. This can be shown mathematically (again look at the attached appendix), but it also has some intuitive appeal since the income effect of an increase in the export prices of a small open economy on foreign demand will be virtually negligible. The price elasticity expresses thus the percentage change in the ratio of the goods produced in the small open economy to foreign goods and an elasticity less than zero will therefore imply a decreasing market share in real terms with regard to relative price changes. \( \beta \) bigger than or less than one will imply whether the economy's market share is increasing or not when facing a growing world market. \( \epsilon_{2t} \) is a stochastic disturbance term in the export volume equation. In both equations all prices are given in the currency of the small open economy.

Economic theory contributes in an important way to our empirical analysis by providing suggestions to possible explanatory variables and also to what kind of basic relationships we may expect to find between them. The interpretation of such relationships will however typically be as long-run relationships. Given the non-stationary properties of many of the relevant macro economic time series, such long-run relationships will be associated with the statistical concept of cointegration, which has the implication that an empirical long-run relation exists between the variables. To empirically substantiate economic theory, we will therefore have to require that the results of the cointegration analysis are consistent with theory. The cointegration analysis in this section is therefore based on the export volume and price equations in (3.1) and (3.2), respectively. Theory consistency requires that there are at least two cointegrating relationships and that both disturbance terms in (3.1) and (3.2) are I(0). If we find support for two and only two cointegrating relationships, this will especially require that export prices, unit labor costs and world market prices form a cointegrating linear combination, possibly with an additional demand effect from abroad. On the other hand we would also expect the export volume to be cointegrated with a linear combination of foreign real income and the relative price of export prices to world market prices.

To further develop the implications theory consistency may have with regard
to cointegration, (3.1) may be reformulated as

$$\ln(PA_t) - \ln(PW_t) = c + \rho \ln(R_t) + \phi (\ln(WC_t) - \ln(Y_t) - \ln(PW_t)) + \epsilon_{1t}$$

First, let us assume that the logarithm of the ratio of unit labor costs to world market prices cointegrates. As theory consistency necessarily implies that $$\epsilon_{1t} \sim I(0)$$, this will then either imply RPP or for $$\rho$$ different from 0 and $R \sim I(1)$, that the real exchange rate cointegrates with foreign real income. For $$\phi$$ different from 0, we see that the implication may also go in the other direction, as RPP in the case of $$\rho = 0$$ or $R \sim I(0)$, then would imply constant wage or profit share in the external sector. Further, looking at (3.2), we have that this, under the assumption of $$\beta$$ different from 0 and $R \sim I(1)$, implies that real foreign income must cointegrate with the volume of exports.

Evidently, the imposition of theoretical restrictions still leaves us with lots of degrees of freedom to identify theoretically consistent long-run structures. A more heuristic interpretation with regard to what is consistent with regard to theory may in addition even further increase the possibility set, examples in this respect being removal of homogeneity restrictions, exclusion of variables etc. In the next section these issues are further investigated.

3.1.2. Data and time series properties

Before presenting the results of the cointegration analysis, I will first draw attention to a brief description of the empirical data set, herein undertaking a preliminary analysis with regard to time series properties of the individual data. Together with graphs of levels and first differences of all variables in the information set, all empirical results of these tests for stationarity, except for the I(2) analysis undertaken below, are placed in the appendix section of this chapter.

The econometric analysis is based on quarterly seasonally unadjusted data over the period 1979 (2) to 1998 (2). The data set consists of observations on the following empirical proxies of the theoretical quantities:\(^6\)

\(^6\)From now on, I will stick to the convention of using small letters for variable names when in fact the variables are logarithmic transformations of the original series, the only exception being the foreign real demand indicator where capital R indicates the logarithm of foreign real demand.
Export volume index of traditional goods

Export volume index of services

Export price deflator on traditional goods

Export price deflator on services

World market price index

Foreign demand indicator

Unit labor cost indicator

Before examining the long-run relationships between the variables, it is useful to first determine the orders of integration of the individual time series in the information set. In the appendix, I therefore first present the results of testing for stationarity within the multivariate framework based on the methodology developed by Johansen for estimation and identification of cointegrating relationships (Johansen (1988), (1995)). This test is conditional on the number of cointegrating relationships and differs in a very important respect from univariate Dickey-Fuller tests by testing the null of stationarity against a non-stationary alternative. These system-tests are superior to univariate testing for stationarity of individual time series. However, due to a generic bias towards these tests among time series econometricians, I have chosen also to present the results of Augmented Dickey Fuller tests. To avoid the problem of nuisance parameters in the DGP all these tests are made similar, implying the joint appearance of a trend and a constant term in the specification of the autoregressive equation. To get rid of as many anomalies as possible, I have also included seasonal dummies. Testing the null of $I(2)$ vs. the alternative of $I(1)$, however, has been done by only including a constant term in the equation to avoid the problem of having to deal with a possible quadratic trend under the alternative. A common problem with all these tests is the rather asymmetric treatment of the null and alternative concerning the status of nuisance parameters. However, this problem can easily be dealt with by undertaking a joint test of both the lagged level variable and the trend, and using Table 4.5 in Banerjee, Dolado, Galbraith and Hendry (1993), which gives the simulated critical values in finite samples for these F-type tests. However, to be able fully to address the issue of higher order integration, I have instead chosen
to undertake a full analysis of the cointegrating indices based on the two-step I(2) procedure developed by Johansen (1995b).

The multivariate test statistics strongly suggest the rejection of the null of stationarity for most variables. However, it is worth noting that with regard to world market prices and export prices in the traditional goods sector we cannot reject the null of stationarity at conventional levels of significance. However, the overwhelmingly strong support for treating all variables in the information set as non-stationary I(1) variables based on univariate testing together with the fact that the significance probabilities of the multiple test statistics for these two variables are close to a nominal level of five per cent, indicates that we probably are not going to make too a serious mistake by treating prices as I(1). This is also indicated by the I(2) tests in Table 3.1 below, even though strictly speaking there is some evidence of an I(2) trend in the sample. These tests of I(2)-ness have been carried out by specifying a seven dimensional VAR of order three, where a drift term has been restricted to the cointegrating space and the constant restricted not to induce quadratic trends in the processes. The test procedure starts from top left testing the null of seven common I(2) trends versus less than or equal to full rank and continues to the right until one reaches the last column which is the ordinary test of seven I(1) trends versus more than or equal to nil common trends. In the case where one rejects all nulls in the first row of seven common trends, one continues this stepwise testing from left towards right by moving down to the next row of six common trends. The number of cointegrating vectors, I(1) and I(2) trends are given by the first null that one cannot reject. In Table 3.1 below, this process of rejection does not end until the number of common trends are equal to three and the number of I(1) trends are identified to two which implies that the number of common I(2) trends are equal to \( p - r - s = 7 - 4 - 2 = 1 \). However as the critical ten per cent level is equal to 49.69, the statistic is hardly significant to a level of ten percent. This could indicate that the cointegration indices are given by \( r = 4, s = 3, p - r - s = 0 \). If so, there are no common I(2) trends and the analysis can be undertaken by ordinary reduced rank analysis for times series integrated of order one. This would be in accordance with the conclusions made on behalf of multivariate testing and the univariate Dickey Fuller tests referred to above. Looking carefully at Table 3.1 it is also worth noting that to a level of slightly above ten percent we are in fact able to reject all combined nulls of more than one common trend and that some of these are I(2). However, the test of more

\[ \text{To be able to fit the table in the text, the numbers have been rounded off to their nearest one decimal representation.} \]
Table 3.1: The trace test of cointegrating indices

<table>
<thead>
<tr>
<th>p-r</th>
<th>r</th>
<th>S_{r,s}</th>
<th>Q_r</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0</td>
<td>553.4</td>
<td>465.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>351.6</td>
<td>311.2</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>445.0</td>
<td>359.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>269.2</td>
<td>233.8</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>340.0</td>
<td>259.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>198.2</td>
<td>167.9</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>237.2</td>
<td>165.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>137.0</td>
<td>113.0</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>138.2</td>
<td>72.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>58.2</td>
<td>36.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>47.6</td>
<td>34.4</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>21.3</td>
<td>6.00*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19.9</td>
<td>12.5</td>
</tr>
</tbody>
</table>

\[p-r-s\]

Table 3.1 is based upon a seven dimensional VAR of order three for the variables \(a_1, a_2, p_{a1}, p_{a2}, pw, ulc\) and \(R\). A drift term has been restricted to lie in the cointegrating space and a constant is included such that it does not induce a quadratic trend in the process. The figure in italics under each test statistic is the 95 per cent fractile as tabulated by Paruolo (1996). The three preferred outcomes discussed in the text are marked with stars.

than or equal to one common I(1) trends versus less than or equal to full rank does not reject. This indicates that there could be as many as six cointegrating vectors and no I(2) trends in the information set\(^8\).

\(^8\)This last hypothesis is also given some support by sectorial identification of the cointegration indices, as both sectors separately indicate the existence of three cointegrating vectors and no I(2) trends.
System: a1, pα1, pw, R, ulc.
Deterministic part: Unrestricted constant, centered seasonals and restricted Trend.

<table>
<thead>
<tr>
<th>Max Eigenvalue Tests</th>
<th>Trace Eigenvalue Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null</td>
<td>Alternative</td>
</tr>
<tr>
<td>( r = 0 )</td>
<td>( r \leq 1 )</td>
</tr>
<tr>
<td>( r \leq 1 )</td>
<td>( r \leq 2 )</td>
</tr>
<tr>
<td>( r \leq 2 )</td>
<td>( r \leq 3 )</td>
</tr>
<tr>
<td>( r \leq 3 )</td>
<td>( r \leq 4 )</td>
</tr>
<tr>
<td>( r \leq 4 )</td>
<td>( r \leq 5 )</td>
</tr>
</tbody>
</table>

Table 3.2: Rank tests for the trading sector

3.1.3. Cointegration Analysis

In estimating the two sectors the effective sample used for estimation has been from 1980 (1) to 1998 (2). In both sectors we have started out with a five dimensional VAR of order three. Both econometric models include a restricted trend term to avoid problems with regard to nuisance parameters when testing for the cointegration rank. Furthermore, constant terms and seasonal dummies have not been restricted to lie within the \( \alpha \)-space.

The trading sector First, I want to draw attention to the diagnostics of the VAR for the traded sector given in Table A.3 of the appendix. Except for some hardly significant signs of autocorrelation and conditional heteroscedasticity in the processes governing export prices and the export volume, all single equation and system diagnostics are fine. Also looking at the recursively estimated Chow tests in Figure B.8 of appendix B, does not reveal any signs of parameter instability whatsoever. Thus, our VAR should be a good starting point for identification of cointegrating relationships.

Table 3.2 shows that both the trace- and maximum-eigenvalue tests strongly support the existence of two cointegrating vectors, implying three common trends
in the system. As complex eigenvalues come in pairs and the first eigenvalue of the companion form is real and the next one is complex (see Table A.6), this should further substantiate the existence of three common trends. To rephrase the message, as long as we believe that the second root really is complex this implies that there will either be two or four cointegrating relationships. However, the trace statistics of a rank less than or equal to two versus less than or equal to full rank has got a p-value slightly above 10 per cent, the asymptotic upper ten per cent fractile being approximately equal to 39.08. Also, complex roots may be realizations of stochastic processes with expectation values lying on the real line. If this is the case, three cointegrating vectors and two common trends will not represent an inconsistency problem but an interesting hypothesis that one should be able to at least investigate and later test formally given the distribution governing the roots. With regard to this last possibility, one may come a step further by undertaking a graphical inspection of how the roots within the unit circle are affected by the imposition of unit roots as to erroneously impose a complex root to lie at one should show up through its complex conjugate assuming a real value lying significantly far away from the unit circle. Looking at the four graphs in Figure B.10 of the eigenvalues of the companion matrix of the trading sector, we see that the imposition of the first common trend seems to reduce the second complex root to two real roots. Whether this change constitutes a significant change or not has to be formally tested, but based on the results of the trace-test statistics and the fact that the two new real roots seem to lie fairly close to each other, a rejection of the null of correct imposition of one unit root, would be very surprising. The third unit root, however, is slightly more controversial and one may discuss whether the imposition of the third unit root is accepted or not by looking at what happens to the complex conjugates of the complex root when imposing one of them to lie at one. I have chosen to decide on two common trends, even though the real transformation of the not restricted complex conjugate does not lie that much further away from the other one, restricted to lie at one, than in the preceding case. The decision of three cointegrating vectors, has been made more to be able to identify relationships interpretable in light of the theory outlined in Section 3.1.1 than to be consistent with the outcome of statistical tests. However, this is not to say that this has been a decision without controversy.

Table 3.3 gives the result of the identification of three cointegrating vectors. Three economically meaningful relationships are here identified to a level of 0.1182 for the LR-test with three degrees of freedom. The first CI-vector is a pure demand
relationship for Norwegian exports with the implication of a quasi elasticity of relative prices in demand of about -0.6 and a foreign income quasi elasticity of 2.7.\(^9\) Even though the second CI-vector is not homogenous in prices and does include a trend, it may be interpreted heuristically as a monopolistic price setting rule. The third CI vector says that the ratio of exports to foreign real income cointegrates with a deterministic trend, implying a yearly growth rate of about 3.8. Figure B.0 in the appendix, showing the graphs of the three concentrated restricted cointegrating relationships, does not reveal any threatening signs of non-stationarity, though the non-concentrated series (not shown here) do indicate a potential problem with the period before 1985\(^10\). Furthermore, the recursively estimated eigenvalues of Figure B.01, show no ominous signs of instability.

<table>
<thead>
<tr>
<th>a1</th>
<th>const. - 0.605 (p\text{a}-p\text{w}) + 2.7R</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-0.0808) (0.1614)</td>
</tr>
<tr>
<td>pa1</td>
<td>const. + 1.775 p\text{w} + 0.854ulc - 0.0085Trend</td>
</tr>
<tr>
<td></td>
<td>(0.2933) (0.2098) (0.001695)</td>
</tr>
<tr>
<td>a1</td>
<td>const + R +0.0096Trend</td>
</tr>
<tr>
<td></td>
<td>(0.00078)</td>
</tr>
</tbody>
</table>

Table 3.3: Restricted cointegrating relationships for the trading sector

The service sector The appendix, Table A.4, also gives us the diagnostics of the five dimensional VAR of the service sector. All individual equation diagnostics are fine, except perhaps for some marginal indications of autocorrelation in the

\(^9\)Strictly speaking, these coefficients cannot be interpreted as elasticities as changes in the residuals of the marginal processes will work through the whole simultaneous system. A one per cent increase in the marginal process governing i.e. foreign income, may therefore even lead to a percentage decline in the process governing exports in the long-run. However, one may modify the meaning of elasticity such that it fulfills the outcome of a feasible hypothetical experiment, an experiment where we change the initial values such that the outcome of a one per cent change in a marginal process gives the percentage change of another variable in the long-run as implied by the traditional interpretation of elasticity.

\(^10\)The difference between the concentrated and non-concentrated cointegrating relationships could be taken to indicate a potential problem with higher order common trends. For a further investigation of this possibility the reader is referred to Chapter 5.
Deterministic part: Unrestricted constant, centered seasonals and restricted.
Trend

<table>
<thead>
<tr>
<th>Eigenvalues of II:</th>
<th>0.9356</th>
<th>0.8134</th>
<th>0.7389</th>
<th>0.6521</th>
<th>0.5104</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max Eigenvalue Tests</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Null</td>
<td>Alternative</td>
<td>Statistics</td>
<td>95%</td>
<td>Null</td>
<td>Alternative</td>
</tr>
<tr>
<td>r=0</td>
<td>r≤1</td>
<td>49.77**</td>
<td>37.5</td>
<td>r=0</td>
<td>r≤5</td>
</tr>
<tr>
<td>r≤1</td>
<td>r≤2</td>
<td>31.64*</td>
<td>31.5</td>
<td>r≤1</td>
<td>r≤5</td>
</tr>
<tr>
<td>r≤2</td>
<td>r≤3</td>
<td>22.39</td>
<td>25.5</td>
<td>r≤2</td>
<td>r≤5</td>
</tr>
<tr>
<td>r≤3</td>
<td>r≤4</td>
<td>15.28</td>
<td>19.0</td>
<td>r≤3</td>
<td>r≤5</td>
</tr>
<tr>
<td>r≤4</td>
<td>r≤5</td>
<td>4.925</td>
<td>12.3</td>
<td>r≤4</td>
<td>r≤5</td>
</tr>
</tbody>
</table>

Table 3.4: Rank tests for the service sector

Equations for pw and ulc. However, this nearly significant problem of autocorrelation at the individual equation level contributes to distorting the system test statistic, which in fact is significant to 1 per cent. However, to be able to precisely identify the long-run structures, I have given priority to the task of keeping the systems to as low order as possible. In this case, it is my belief that what is gained by giving priority to a parsimonious dynamic specification goes far beyond what is lost in terms of efficiency due to autocorrelation. The graphs showing the stability tests, Figure B.9, all indicate that the system seems to be fine.

The trace test statistic in Table 3.4 indicates two common trends. However, the third cointegrating vector is only marginally significant to a level of five per cent, so the additional information given by the fact that the first root is real and the second is complex may lead us to conclude that there are as many as three common trends in the system (see Table A.6). However, taking a closer look at what happens when imposing the third unit root, Figure B.11, we clearly see that the second remaining complex root which is the one affected, cannot have been the outcome of a stochastic processes with expectation values lying on the real line. When imposing one of the complex conjugates to lie at one, the one that is not restated does not reduce to a real value that is anyway close to the unit circle. This leads me to conclude that there seems to be evidence of three cointegrating
vectors in the service sector. The recursive eigenvalues shown in Figure B.9, do all seem to be fairly stable.

The identified system of cointegrating relationships presented in Table 3.5, gives us again three economically meaningful relationships. First, we have that the ratio of exports to foreign income is constant in the long run. Contrary to the results of the traditional goods sector this implies that the relative market share of products in the service sector does not show any tendency to grow in the long run. The two last relationships constitute again two different types of pricing behavior, the first resembling the price setting behavior of a competitive firm which sets its prices as a markup over unit labor costs, the only difference being a trend term which may catch up a non constant markup over time. The other reflects the behavior of a monopolistic competitor.

So far, we have separately identified the long-run properties of the two sectors. Contrary to prior beliefs with regard to the effects of increased internationalisation, my results strongly indicate that small open economies like the Norwegian, still seem to have a considerable degree of monopolistic power when setting their prices. This finding is in accordance with the results in Bowitz and Cappelen(1994) who in fact find unit labor costs to be the single most important explanatory variable in all of their preferred equations for different subsectors of the Norwegian economy and is a expression of the fact that small countries can be "big" in what they produce11. Furthermore, to a large extent both sectors’ export volumes seem to be driven by demand, which is the case when agents accommodate demand ex post to fixed prices ex ante. However, instead of elaborating further on these results, I will now look at the possibility of identifying long-run cross-sectional linkages when considering the long run structure of each sector as known and given by the identified relationships of this section.

The pooled sector  The VAR of the pooled data is of dimension 7 and order 3. As before, the trend is restricted to lie in the cointegration space and the constant and seasonal dummies enter unrestrictedly. Table A.5 of the appendix

11The findings of Bowitz and Cappelen however contrast with the finding in a more recent study, Naug (2001), who claims that Norwegian exporters of raw materials have limited power to set their own prices. A study that confirms the evidence of Bowitz and Cappelen at an aggregate level, is Hammersland (1996), who finds significant signs of monopolistic power in the process governing prices of exports in a study undertaken on aggregate data for the Norwegian mainland economy.
\[ a_2 = \text{const.} + R \]
\[ pa_2 = \text{const.} + 0.506 \text{ ulc} + 0.0025 \text{ Trend} \]
\[ \quad (0.0588) \quad (0.00046) \]
\[ pa_2 = \text{const.} + 0.653 R + 0.362 \text{ ulc} \]
\[ \quad (0.0966) \quad (0.0660) \]

Table 3.5: Restricted Cointegrating Relationships for the service sector

System: \( a_1, a_2, p_{a1}, p_{a2}, pw, R, \text{ ulc}. \)
Deterministic part: Restricted Trend, Unrestricted Constant and centered seasonals
VAR order: 3. Effective Sample period: 1980 (1)-1998 (2)
Number of known cointegrating vectors \( s = 6 \)

<table>
<thead>
<tr>
<th>Null</th>
<th>Alternative</th>
<th>Test Statistics</th>
<th>HW 95% Critical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>m=0</td>
<td>m\leq1</td>
<td>14.1784</td>
<td>&gt;26.4</td>
</tr>
</tbody>
</table>

Table 3.6: Rank tests for the pooled sector conditional on 6 known cointegrating vectors.
contains the single equation and system diagnostics and all test statistics are fine. The imposition of the six known cointegrating vectors from the first step of our analysis has been implemented by using the option restrictions of subsets in CATS, and the LR test for overidentifying restrictions is fine\textsuperscript{12}. The result of the cointegration analysis using the critical values of the trace test in Table 5 of Paruolo (1999), are given in Table 3.6\textsuperscript{13} and clearly indicates that we cannot reject the null of one stochastic trend. This indicates that we may have identified all cointegrating relationships in the information set already in the first step of the identification scheme and at first sight could seem to imply that there is no cointegration across sectors. However, taking a closer look at the cointegrating linear combinations of Table 3.3 and Table 3.5 above, reveals that in the long run $a_1t = a_2t + 0.009Trend$. This implies that we by undertaking a sector specific analysis have been lucky enough to also identify cointegration across sectors. To study this phenomenon closer, I have therefore undertaken an analysis where I have only treated the two first cointegrating vectors of the trading sector together with the three identified in the service sector, as known in the second step\textsuperscript{14}.

The trace test statistics in the case when $s$ is equal to five are given in Table 3.7 and strongly support the existence of another cointegrating vector\textsuperscript{15}. The LR test of whether this additional cointegrating relationship is equal to the one identified from the two sector-specific analyses, gives a $\chi^2$ equal to 0.18, implying that the significance probability of the test statistic is close to 0.67. Thus, by imposing five known cointegrating vectors we have been able to identify a cointegrating relationship across sectors which is consistent with the outcome of the sector specific analysis. It is imperative to point out that this relationship would not have been identified if we in the analysis of the first sector had accepted the outcome of the trace test without looking at the eigenvalues of the companion form. The

\textsuperscript{12}The LR test for the imposition of seven restrictions in each of the six known cointegrating equations is $\chi^2$ with 12 degrees of freedom and is equal to 15.6, implying that the p-value is approximately equal to 0.21.

\textsuperscript{13}Table five in Paruolo does not calculate critical values for more than $s=5$ known cointegrating vectors. This is the reason for the bigger than sign in front of the five percent critical value which is taken from the $s = 5$ column in Paruolo (1999).

\textsuperscript{14}To a certain degree this was also suggested by the rank test of the trading sector as neither the Trace nor the Max eigenvalue tests gave support to a rank beyond two.

\textsuperscript{15}Based on small sample simulations of just about all test statistics of this paper this result is seriously called into question in Chapter 4.
System: $a_1$, $a_2$, $p_{a1}$, $p_{a2}$, $p_w$, $R$, $u_l$.
Deterministic part: Restricted Trend, Unrestricted Constant and centered seasonals
VAR order: 3. Effective Sample period: 1980 (1)-1998 (2)
Number of known cointegrating vectors $s = 5$

| Trace Eigenvalue Tests: $-2\ln(Q_m) = -T(\log(\det(\Omega(p)))-\log(\det(\Omega(r - s)))$ |
|-----------------|-----------------|-----------------|
| Null            | Alternative     | Test Statistics | HW 95% Critical values |
| $m=0$           | $m \leq 2$      | 52.116          | 44.5                       |
| $m \leq 1$      | $m \leq 2$      | 8.816           | 26.5                       |

Table 3.7: Rank tests for the pooled sector conditional on five known cointegrating relationships

\[
a_1 = \text{const.} + a_2 + 0.009 \text{Trend}
\]

LR-test: $\chi^2(1) = 0.18[0.67]$

Table 3.8: Restricted long-run relationship in the pooled analysis

relationship together with the LR test statistics are given in Table 3.8 and implies that exports in the Norwegian trading sector are growing approximately 3.6% faster than exports of the service sector, but that there is a strong long-run link between them. This coincides well with the perceived view of external sector being the main origin for innovative productivity improvements of the economy.

The third step In the introduction I alluded to a possible third step in my procedure of reestimating all free parameters in the identified structure to account for a possible non-diagonal covariance matrix. The results of this reestimation are given in Table 3.9 and reveal a very interesting change in the equation for the export prices of the trading sector. With regard to the other equations all coefficients are pretty much the same as before. The coefficient for the world market price in the export price equation of the trading sector, however, changed from be-
ing significantly positive to being insignificantly negative. The null restriction on this coefficient did not represent a binding restriction on the cointegrating space so the restriction was not testable. However, we see that the restriction of the Trend in the same equation is a testable hypothesis and the test statistic clearly indicates that the restriction is valid in the sense of not being rejected. The important point however, is that by imposing these two restrictions we seem to have identified another across sector cointegrating relationship. Equations number 2 and 4 of Table 3.9 implies namely that we in the long run have the following relationship:

\[ p_{a1} = 1.37p_{a2} - 0.004trend \]

The test statistic for the imposition of a unit quasi elasticity is \( \chi^2 \) with one degree of freedom and is extremely close to zero. This implies that we cannot reject a null of a unit quasi elasticity and the final relationship becomes:

\[ p_{a1} = p_{a2} - 0.002trend \]

Thus, in addition to the identified strong link between exports of the two sectors, there seem to be a strong long-run relationship between export prices of the two sectors. The relationship implies that the yearly inflation rate is about 0.8 per cent higher in the service sector than in the trading sector and could be explained by a more competitive environment in the trading sector.

3.2. Example 2: Identification of international and domestic interest rate relationships: The case of Germany and the US.

As clearly indicated by Figure 3.1 below, the spread between long-term US and German interest rates reveals an extraordinarily high degree of correlation between the two countries' long-term interest rates. The figure also indicates that there seem to have been a lack of a similar relationship between domestic short and long rates in Germany. Taken together with the empirical evidence of a one way causality going from the US economy to the German (ref. Chapter 1), these observations suggest that long-term interest rates in Germany during the nineties have been influenced more by what is going on in international capital markets than
Eq.: Cointegrating relationships:

1: \( a_1 = \text{const.} -0.535(p_{a1-pw}) + 2.380R \)
2: \( p_{a1} = \text{const.} + 0.694ulc \)
3: \( a_2 = \text{const.} + R \)
4: \( p_{a2} = \text{const.} + 0.506ulc + 0.003\text{Trend} \)
5: \( p_{a2} = \text{const.} + 0.707R + 0.329ulc \)
6: \( a_1 = \text{const.} + a_2 + 0.008\text{Trend} \)

LR-tests:

All overidentifying restrictions: \( \chi^2(4) = 2.99[0.56] \)

Restriction on the trend term in eq. 2: \( \chi^2(1) = 0.73[0.39] \)

Table 3.9: Restricted long-run relationships in the pooled analysis when all free parameters have been estimated freely.
by policy run by an independent German central bank. This is a circumstance that could be taken to indicate a lack of independence in the conduct of monetary policy and that who really seems to have been in the driving seat of Europe is not so much perhaps the Bundesbank as international capital markets. The aim of this analysis, however, is not so much to go into a detailed discussion about this as undertaking an alternative empirical analysis based on the approach suggested in this chapter. I will therefore leave most of this discussion for other papers and refer the interested reader to the previous two chapters. Notwithstanding, when discussing the possibility of a third cointegrating vector in the last part of this section, the issue will be forced upon us as this hypothesis radically affects the implications of the analysis. Thus whether there are two or three cointegrating vectors in the information set is not going to be a trivial decision which might be left to the stochastic outcome of test statistics alone. As the decision will have a central bearing on the outcome of the analysis it should therefore be substantiated within the framework of prior beliefs, reliability of results and not least theory in conjunction with the results of the statistical analysis.

To study the degree of independence in European capital markets it is natural to base the analysis on different theories of arbitrage and especially to look at the long end of the market. To clarify matters further, I will therefore in the next subsection give a brief review of two dominating theories concerning the determination of long-term interest rates, the theory of uncovered interest parity (UIP) and the expectation theory of the term structure, respectively.
3.2.1. Some theories of interest rate determination and their implications with regard to cointegration

The theory of uncovered interest parity is a relationship between foreign and domestic interest rates on assets of the same maturity and says that in a steady state the expected return of investing one unit of domestic currency must be the same whether one invests domestically or abroad. The long rates should therefore be equal to the corresponding foreign long rates plus the expected rate of depreciation of the home currency against the foreign currency\(^{16}\). Given a stochastic representation, this may be expressed as:

\[
\begin{align*}
    i_t &= i^*_t + Dv + \epsilon_t
\end{align*}
\]  

(3.3)

I have here assumed rational expectations such that \(Dv = Dv^e + \epsilon\)\(^{17}\). Furthermore \(\epsilon_t\) in (3.3) is assumed to be stationary, I(0), such that the spread between domestic and foreign long term interest rates, \(i_t - i^*_t\), cointegrates with the depreciation rate. It is worth noting that in the case of a stationary rate of depreciation the interest rate spread will be stationary as well.

The expectation theory of the term structure on the other hand is a relationship between interest rates of different degree of maturity and says that long rates should be equal to a weighted average of current and expected future short-term interest rates. Thus, the impact on long-term interest rates from a change in current short-term interest rates depends on how expected future short-term interest rates are affected. A rise in current short-term interest rates that is regarded as permanent will lead to a full pass-through from short-term to long-term interest rates. On the other hand, if an increase in the current short-term interest

\(^{16}\)An important caveat in the following, is that the treatment below deliberately disregards the potential existence of disturbing risk and term premiums. As these probably are two of the most important reasons why econometricians have problems identifying long-run cointegrating arbitrage relationships between yields of different maturities as well as between yields of different countries of origin, as i.e. the UIP hypothesis, it is important to realize that they might apply in this study as well.

\(^{17}\)Even in the case this highly disputed assumption is fulfilled to perfection, the so called peso problem might pose problems in small samples. This happens because rationality does not guarantee that the empirical mean of actual realignments coincides with the realignment expectations, particularly when the probability of observing small changes in the exchange rate within a band is high whereas the opposite is the case with regard to observing a realignment. Besides a non-zero risk premium, this is the most frequent explanation met in the literature to explain why the hypothesis of uncovered interest rate parity often is rejected in actual data sets. So also in this study as our sample not exactly is a big one.
rate leads to a significant reduction in inflation expectations, long-term interest rates may even decline. In the case of a full pass through from the short to the long end of the market, the relationship can be given the following stochastic representation:

\[ i_t^L = i_t^S + \epsilon_t \]  \hspace{1cm} (3.4)

As above the noise term is assumed to be stationary, \( I(0) \), such that the spread between the long rate, \( i_t^L \), and the short rate, \( i_t^S \) is stationary as well.

Taken at face value this implies that one should expect there to be at least two long-run relations. One that concerns the arbitrage across borders and another one representing a domestic arbitrage condition of bonds with different degree of maturity.

3.2.2. Data and time series properties

The econometric analysis is based on monthly observations of short and long term interest rates in Germany and the US together with the bilateral exchange rate between the two countries. The period I am looking at is from 1990 (1) to 1997 (12). More explicitly the data set consists of monthly observations on the following variables:

- \( i^{GL} \); Effective interest rate on German Government bonds with ten years to maturity
- \( i^{GS} \); Three months money market interest rate for Germany
- \( i^{UL} \); Effective interest rate on US Government bonds with ten years to maturity
- \( i^{US} \); Three months money market interest rate for the US
- \( D_v \); The change in the bilateral exchange rate, German marks per US dollar\(^{18}\).

\(^{18}\) I have chosen to use the first difference of the logarithm of the bilateral exchange rate knowing that it would have been more correct theoretically to use either the three months or the twelve months difference. In this respect the one month difference must be viewed as a compromise and as an indicator of what it after all is meant to proxy, the expected rate of change in the exchange rate.
With regard to the time series properties of the data I refer to Chapter 1. The results of multivariate tests and Dickey Fuller tests of stationarity herein, all indicate that interest rates are $I(1)$, while the change in the bilateral exchange rate is stationary, $I(0)$. However, to further substantiate the claim of no higher order of non-stationarity than of order one, I have run the data through Johansen's two-step procedure for estimation and identification of the cointegration indices (Johansen (1995b)). The $I(2)$ test, fully described in Paruolo (1996) and Jorgensen, Kongsted and Rahbek (1999), has been based on the specification of a second order VAR of dimension five where the constant term is restricted to lie in the cointegration space and a possible drift term has been restricted such that it does not generate quadratic trends. In Table 3.10 below the joint test of the number of cointegrating vectors, $r$, and the number of $I(1)$ trends, $s$, is denoted $S_{r,s}$. The test statistics are given in bold letters while the 95% fractiles simulated in Paruolo (1996), are given in italics below. As explained before the test procedure starts from top left testing the null of five common $I(2)$ trends versus less than or equal to full rank and continues to the right until one reaches the last column which is the ordinary test of five $I(1)$ trends versus more than or equal to nil common trends. In the case where one rejects all nulls in the first row of five common trends, one continues this stepwise testing from left towards right by moving down to the next row of four common trends. The number of cointegrating vectors, $I(1)$ and $I(2)$ trends are given by the first null that one cannot reject. In our analysis this happens in the row of three common trends and in the column where all trends are $I(1)$, clearly indicating that there are no $I(2)$ trends in the data and that the number of cointegrating vectors is equal to two. The results are thus fully in line with the tests referred to above and confirms the finding of an order of non-stationarity not higher than one. Also, already at this stage the results give support to the finding in Chapter 1 that there are no more than two cointegrating vectors among the variables in the information set. This finding however, will be further scrutinized in the last section in view of the outcome of the second step of the analysis.

3.2.3. Cointegration analysis

The two sectors we are looking at are Germany and the US. In analyzing the separate sectors I have in both cases started out with a three dimensional VAR of order two where in addition to the country specific interest rates, I have also
Table 3.10: The trace test of cointegrating indices

<table>
<thead>
<tr>
<th>p-r</th>
<th>r</th>
<th>$S_{r,s}$</th>
<th>Q(R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>380.46</td>
<td>285.33</td>
</tr>
<tr>
<td></td>
<td>198.2</td>
<td>167.9</td>
<td>142.2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>222.77</td>
<td>165.43</td>
</tr>
<tr>
<td></td>
<td>137.0</td>
<td>113.0</td>
<td>92.2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>135.10</td>
<td>81.31</td>
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<td></td>
<td>86.7</td>
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<td>53.2</td>
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<td>2</td>
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<td>58.34</td>
<td>36.07</td>
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<td></td>
<td>47.6</td>
<td>34.4</td>
<td>25.4</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>15.47</td>
<td>5.13</td>
</tr>
<tr>
<td></td>
<td>19.9</td>
<td>12.5</td>
<td></td>
</tr>
</tbody>
</table>

1) Table 3.10 is based upon a five dimensional VAR of order two for the variables $\bar{G}L$, $\bar{G}S$, $\bar{U}L$, $\bar{U}S$ and $DV$. A constant is restricted to lie in the cointegration space and a possible drift term has been restricted not to generate quadratic trends.

2) The figure in italics under each test statistic is the 95 per cent fractile as tabulated by Paruolo(1996). The preferred outcome is marked with a star.
System: $i^{GL}, i^{GS}, Dv.$

Deterministic part: Restricted constant and no trend


<table>
<thead>
<tr>
<th>Max Eigenvalue Tests</th>
<th>Trace Eigenvalue Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null</td>
<td>Alternative</td>
</tr>
<tr>
<td>$r=0$</td>
<td>$r \leq 1$</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$r \leq 2$</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>$r \leq 3$</td>
</tr>
</tbody>
</table>

Table 3.11: Rank tests for the German sector

$$
\beta'(i^{GL}, i^{GS}, Dv, 1) = \beta_{11}i^{GL} + \beta_{12}i^{GS} + \beta_{13}Dv + \beta_{14} = 0.591i^{GL} - 0.096i^{GS} + Dv - 0.036
$$

Hypotheses:

LR-test, Rank=1.

$\beta_{13} = 0$

$\chi^2(1) = 28.74[0.00]$

$\beta_{11} = -\beta_{12}, \beta_{13} = \beta_{14} = 0.$

$\chi^2(3) = 41.25[0.00]$

$\beta_{11} = \beta_{12} = \beta_{14} = 0, \beta_{13} = 1.$

$\chi^2(3) = 01.75[0.63]$

Table 3.12: The unrestricted cointegrating linear combinations and tests of restrictions on the cointegrating space for the German sector

included the bilateral exchange rates. The econometric models do not include a trend and the constant terms are restricted to lie in the cointegrating space\textsuperscript{19}. As these data have been extensively examined in Chapter 1, I will here only comment on the diagnostics of the sector specific VARS to the extent that these deviate significantly from the full VAR diagnostics in this chapter.

\textbf{Germany} Table 3.11 gives strong support to the existence of only one cointegrating vector, and as both tests are way off the ninety five percent critical values of 15.7 and 20, respectively, I have chosen to accept this outcome without resort-

\textsuperscript{19}The inclusion of a restricted trend term to assure that the tests are similar, has been avoided on behalf of the a priori very unlikely finding of a trend stationary interest rate relationship.
System: $i^{UL}, i^{US}, Dv.$
Deterministic part: Restricted constant and no trend

<table>
<thead>
<tr>
<th></th>
<th>Max Eigenvalue Tests</th>
<th>Trace Eigenvalue Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null Alternative Statistics</td>
<td>90%</td>
<td>Null Alternative Statistics</td>
</tr>
<tr>
<td>$r=0$ $r \leq 1$</td>
<td>45.92**</td>
<td>$r=0$ $r \leq 3$</td>
</tr>
<tr>
<td>$r \leq 1$ $r \leq 2$</td>
<td>6.99</td>
<td>$r \leq 1$ $r \leq 3$</td>
</tr>
<tr>
<td>$r \leq 2$ $r \leq 3$</td>
<td>2.79</td>
<td>$r \leq 2$ $r \leq 3$</td>
</tr>
</tbody>
</table>

Table 3.13: Rank tests for the US sector

According to a discussion of the eigenvalues of the companion matrix\footnote{The two first eigenvalues of the companion matrix are real and equal to 0.9971 and 0.9093, respectively. The trace test clearly shows that the second one is significantly close to the unit circle. The other eigenvalues are complex but their norms are all less than 0.43 so disregarding these should not represent any problem.}. The unrestricted cointegrating linear combinations together with some LR-tests for overidentifying restrictions are given in Table 3.12. The first thing to note is the rejection of excluding the rate of depreciation, $Dv$. This indicates that either this must be a stationary linear combination in itself, or that it in some way cointegrates with the two interest rates. However, the significance probability for the test of a stationary depreciation rate is equal to 0.63, implying that we are way off rejecting the null of stationarity and in the following this is therefore going to be treated as one of the known cointegrating vectors in the pooled analysis to come\footnote{The diagnostics of the VAR rejects normality and there are signs of ARCH effects too in the model. However, simulation studies have shown that the trace test is fairly robust against certain form of deviations from normality and ARCH effects do not in general seem to invalidate the analyses.}.

The US As for Germany the trace test statistics of Table 3.13 clearly indicate that there is only one cointegrating vector among the variables in the information set\footnote{The first eigenvalue of the companion form is complex with a norm equal to 0.9349. Disregarding the possibility that this root could be the outcome of a stochastic process with expectation value lying on the real line, this gives immediate support to an hypothesis of two common trends and thus only one cointegrating vector.}. Table 3.14 shows furthermore that the picture is very much the same as for the German sector. The stationarity of the depreciation rate cannot be rejected
\[
\beta'(i^{UL}, i^{US}, Dv, 1) = \beta_{11}i^{UL} + \beta_{12}i^{US} + \beta_{13}Dv + \beta_{14} \\
= 1.056i^{GL} - 0.197i^{GS} + Dv - 0.064
\]

Hypotheses: LR-test, Rank=1.

\[
\begin{align*}
\beta_{13} &= 0 \\
\beta_{11} &= -\beta_{12}, \beta_{13} = \beta_{14} = 0. \\
\beta_{11} &= \beta_{12} = \beta_{14} = 0, \beta_{13} = 1.
\end{align*}
\]

\[
\chi^2(1) = 38.52[0.00] \\
\chi^2(3) = 44.62[0.00] \\
\chi^2(3) = 06.33[0.10]
\]

Table 3.14: The unrestricted cointegrating linear combinations and tests of restrictions on the cointegrating space for the US sector

and there does not seem to be any support for a stationary interest spread or other linear combinations which includes the depreciation rate among the variables.

Pooled sector Based on the result that both country specific analyses gave the same outcome with regard to the identified cointegrating vector, the pooled analysis in this section will be contingent on one of the cointegrating relationships being the rate of depreciation. In the terminology of Section two this means that the unit vector with zeros for all coefficients except for the one for the depreciation rate will be treated as the known cointegrating vector.

Table 3.15 shows the trace eigenvalue tests when conditioning on the rate of depreciation as a known cointegrating relationship. The critical values are based on the distributional results in Horwath and Watson and the 95 per cent fractiles, as tabulated by Paruolo (1999), are given in the last column. The first thing to notice is that the test statistic strongly rejects the null of 4 common trends versus more than or equal to nil. This means that there is at least one cointegrating vector in the pooled information set beyond the one we found from the country specific analyses. Noteworthy, the statistics give also relatively strong evidence for the existence of a third cointegrating vector as the critical value to a level of one per cent of the test of the hypothesis of more than or equal to three common I(1) trends versus less than or equal to full rank is approximately equal to 46.2, implying that the test’s significance probability lies somewhere between one and five per cent\(^{23}\). However, based on the results of former analyses and the outcome

\(^{23}\)Based on the small sample Monte Carlo experiments in Chapter 4 and the outcome of the
System: $i^G_L$, $i^G_S$, $i^U_L$, $i^U_S$, $Dv$
Deterministic part: Restricted constant and no trend
VAR order: 2. Effective Sample period: 1990 (1)-1997 (12)
Number of known cointegrating vectors $s = 1$

| Trace Eigenvalue Tests: $-2\ln(Q_m) = -T(\log(\det(\Omega(p))) - \log(\det(\Omega(r - s))))$ |
|---|---|---|---|
| Null | Alternative | Test Statistics | HW 95% Critical values |
| $m = 0$ | $m \leq 4$ | 72.18 | 59.0 |
| $m \leq 1$ | $m \leq 4$ | 43.43 | 40.0 |
| $m \leq 2$ | $m \leq 4$ | 21.06 | 24.1 |
| $m \leq 3$ | $m \leq 4$ | 6.67 | 12.1 |

Table 3.15: Conditional rank tests for pooled sector

of the Johansen two-step procedure for identification of cointegrating indices in Section 3.2.2. which all give support to the hypothesis of only two cointegrating vectors in the full information set, I will first comment on the case with only one additional cointegrating vector to the one “known” from before. In this respect it will be of particular interest to find out whether this relationship can be given the interpretation of being the spread between German and US long-term interest rates. Then I will turn to the possibility of an additional third cointegrating vector and the consequences such an hypothesis might have with regard to the central conclusions of this paper concerning the ability of the Federal Reserve and the Bundesbank to control the long end of their respective capital markets.

The joint restriction of a stationary depreciation rate and interest rate spread gives a LR test statistic of 13.68. As this statistic is $\chi^2$ square with 8 degrees of freedom, this implies a significance probability of about 0.09. The test of a stationary spread conditional on the depreciation rate being stationary, is distributed $\chi^2(4)$ and the p-value of the test statistics is approximately equal to 0.05. In my view this should give sufficient support to the hypothesis that long rates in Germany and the US cointegrate and we have been able to identify a cointegrating relationship across the two countries by using the suggested two-step procedure. Before turning to the alternative analysis involving three cointegrating vectors, I will have a look at the finding in Hammersland and Vikoren (1997) that long-term unconditional cointegration analysis in Chapter 1 of the pooled information set this seems to be a relatively robust finding.
German interest rates seem to cointegrate with a homogenous linear combination of domestic short and US long-term interest rates. The test statistic for this hypothesis is $\chi^2$ with three degrees of freedom and gave a test statistic of 3.55. This implies that the null cannot be rejected even to a level as high as 30 per cent. However, we have already seen that the restriction implying a stationary spread is not rejected either. Thus German short interest rates do not seem to significantly enter the long-run relationship which could be taken to indicate reduced domestic control on part of the German central bank, the Bundesbank, with regard to the long end of the market. Also the tests for exogeneity undertaken in Chapter 1 imply that both US rates might be considered driven by two independent processes outside our information set. In particular this could be taken to mean that long-term interest rates in the US are beyond control of monetary authorities, a possible explanation being that these are driven by what is going on in international capital markets. However, such a finding also rejects the possibility of a causal relationship going in the opposite direction, that is that long rates would affect short rates through its capacity of being a variable entering the information set of a policy rule on part of the Fed. As these issues are of most importance to get scrutinized I will indulge in a more comprehensive discussion of these matters when analyzing the possibility of three cointegrating relationships in the next section to come.

Table 3.16 gives the outcome of the analysis when accepting the existence of three cointegrating vectors. The cointegration space as implied by the identified structure in addition to the two relationships identified assuming three common trends, is spanned by a negative relationship between German and US domestic interest rate spreads. To reject the hypothesis of correctly imposed identifying restrictions for the system as a whole is not possible to a level below ten per cent. Furthermore, the p-value of the test of the identifying restrictions concerning the third vector conditional on a stationary rate of depreciation and a stationary spread between US and German long-term interest rates is close to 0.74. Thus the long-run relationship implied by the identifying restrictions on the third relationship constitutes a valid restriction on the cointegrating space. That this relationship between national spreads is a negative one might however seem rather puzzling as one intuitively would think of a hike in the US spread either through a hike in long term interest rates or a fall in short term interest rates, to cause a similar increase in German spreads through a redirection of funds on the part of investors. However, as already alluded to in the text, central banks seem to have a fairly tight control over the short end of the capital market. Based
The cointegration parameters

\[
\begin{bmatrix}
\iGL \\
\iUL \\
\iGS \\
\iUS \\
Dv \\
1
\end{bmatrix}
\]

\[
\beta' = \begin{bmatrix}
\beta_{11}iGL + \beta_{12}iUL + \beta_{13}iGS + \beta_{14}iUS + \beta_{15}Dv + \beta_{16} \\
\beta_{21}iGL + \beta_{22}iUL + \beta_{23}iGS + \beta_{24}iUS + \beta_{25}Dv + \beta_{26} \\
\beta_{31}iGL + \beta_{32}iUL + \beta_{33}iGS + \beta_{34}iUS + \beta_{35}Dv + \beta_{36}
\end{bmatrix}
\]

Hypotheses:

- LR-test, Rank=3.

\[
\begin{align*}
\beta_{11} &= \beta_{12} = \beta_{13} = \beta_{14} = \beta_{16} = 0, & \chi^2(3) &= 0.04[0.26] \\
\beta_{15} &= 1. \\
\beta_{23} &= \beta_{24} = \beta_{25} = \beta_{26} = 0, & \chi^2(6) &= 12.64[0.05] \\
\beta_{21} &= -\beta_{22}. \\
\beta_{31} &= -\beta_{33}, \beta_{32} = -\beta_{34}, \beta_{35} = 0. & \chi^2(8) &= 13.24[0.10]
\end{align*}
\]

The identified system of cointegrating relationships

\[
\hat{\beta} (iGL, iUL, iGS, iUS, Dv, 1) = Dv
\]

- \(iGL - iUL\)
- \((iGL - iGS) + (iUL - iUS) - 0.023\)

Table 3.16: The cointegration parameters, tests of restrictions on the cointegrating space and the identified system in case of three cointegrating vectors.
on the anti-inflationary reputation of the Bundesbank it is therefore not at all surprising that a fall in short-term US interest rates was not allowed to affect the corresponding German interest rates in a way that could seriously jeopardize its anti-inflationary reputation. Thus, the predominant effect of a fall in US short-term interest rates on the German spread would come via the effect it might have on domestic long-term interest rates, either directly via arbitrage or indirectly through changes in the bilateral exchange rate. Disregarding the possibility that the effect of a fall in short rates on expectations with regard to future domestic short-term interest rates might totally offset or even dominate the unequivocally positive portfolio effect, a fall in short rates will lead to a fall also in long-term interest rates. This will make investments in domestic assets with a long-term to maturity less favorable compared to foreign alternatives and lead to increased demand for foreign assets with a long term to maturity. As monetary authorities do not have the same possibility of controlling the long end of the market as the short, the outcome of this arbitrage activity would predominantly be determined by capital markets alone implying that long rates would fall. When a national central bank chooses to cut its policy rate, this will in addition have consequences for the bilateral exchange rates between the country that undertakes the cut and its trading partners, whose bilateral exchange rates naturally would have to appreciate. The appreciation would surely contribute to reduce contemporary inflation as well as expectations with regard to future rates of inflation such that long-term interest rates would fall even more. Going back to our example of a fall in short-term US interest rates and its potential effect on the German economy, this would imply that prices on German assets with long term to maturity will rise and thus corresponding interest rates to fall. All in all therefore, there seem to be good reasons why we should see a negative relationship between domestic US and foreign German interest rate spreads when a widening of the US spread is due to a cut in the Federal funds rate. In the case the widening is due to rising international long-term interest rates, probably originating from shocks to international capital markets reflecting increased expectation of a future inflationary pressure, and short rates are informed by long rates through their capacity of informing policy rules on part of central banks, it is still possible to argue for a negative correlation between the domestic interest rate spreads of Germany and the US. This is due to

24 The cut implies that the expected return of investing one unit of the domestic currency abroad will be higher than investing it domestically for a given expected rate of depreciation. For a given foreign interest rate and assuming uncovered interest parity, this implies that the bilateral exchange rate must appreciate to generate a higher expected rate of depreciation.
the strong potentially offsetting effect that a hike in German policy rates might have had on expectations with regard to future inflation and thus future short term interest rates and long term interest rates just because of high credibility in its pursuit of an anti-inflationary policy stance. The anti-inflationary reputation of Germany further suggests that long-rates might have played a more significant role in forming a policy rule on part of the Bundesbank than on part of the Fed. An increase in international long-term interest rates might therefore in addition to affecting policy rates, have had the effect of increasing the spread between German and US short-term interest rates. As alluded to above this would bring about an immediate appreciation of the bilateral exchange rate and thus contribute to reduce inflationary expectations, future short-term interest rates and thus the long-term German interest rate. Whether the German spread will shrink will depend on whether the combined effect of the induced hike in the German policy rate and the appreciation that follows a potential widening of the spread between short-term international interest rates is sufficiently strong to offset the initial hike in long rates. To be able to discuss the implication of the third cointegrating vector with regard to the central theme of this paper, that is the degree of independence in the making of a monetary policy, one may give the identified long-run structure the equivalent representation of Table 3.17 below. From this formulation we clearly see that both US as German short-run interest rates enter into the cointegrating relationships. Thus there certainly seems to be a kind of link between domestic short and long rates even though at this stage it is too early to say anything about the direction of causality without making further inquiries into which processes might be characterized as exogenous and not. With regard to a potential relationship between short rates we have not been able to identify a relationship giving support to the hypothesis of uncovered interest parity (UIP). Assuming therefore that Central banks are able to control the short end of the yield curve we may at least eliminate a direct causal long-run relationship between short rates. The important question to answer is therefore whether the link between domestic short- and long-term interest rates comes predominantly through the role played by domestic long-term interest rates as indicators of build ups in inflationary pressures and thus as explanatory variables informing a policy rule on part of Central banks, or through a pure term structure relationship between interest rates with different times to maturity. Tests on the loadings strongly

25This result is in accordance with a great bunch of economic literature. Two references are MacDonald and Taylor (1992) and Froot and Thaler (1990). For a more recent account see MacDonald and Juselius (2002a,2002b).
indicate that US long-term interest rates seem to be determined by a process outside control of the monetary authorities, the Fed. However, the corresponding test of whether also long-term German interest rates are exogenous with regard to estimation of the long-run coefficients, rejects to a level below one per cent. As the corresponding tests of the short-term interest rates of both countries also reject, this could be turned to account of domestic short-term interest rates being set in accordance with policy-rules informed by movements in domestic long-term interest rates of both countries. Note that this eliminates the sort of argument used to argue for a negative relationship between spreads in the case of changes to domestic short-run interest rates, as these changes are motivated from shocks to processes that inform the policy rule and not from shocks to exogenous processes governing short-term interest rates. With regard to the central theme of the chapter both the Fed and the Bundesbank seem to be able to set short-term interest rates at their discretion following policy rules informed by long-term interest rates. However, with regard to controlling the long-end of the market there does not seem to be a similar propensity. First, US long-term interest rates seem to be totally determined by processes outside the domain of US monetary authorities. The endogeneity status of the corresponding German interest rates and the fact that there is a long-run relationship between these and the long-term US interest rates suggest that also the German long-term interest rates mainly seem to be driven by the same forces that govern US long term interest rates.

4. Conclusion

In this chapter I have used the concept of known cointegrating vectors to come up with a suggested two-step procedure for how to deal with cointegration in the case of times series with a small cross sectional dimension. The first step of this procedure implies getting to know the “known” cointegrating vectors by preliminary identification of long-run relationships along the sector dimension. Given these, the next step then implies identification of further long-run relationships across sectors by exploiting both dimensions jointly. The first step of this procedure

---

26 The test statistic of the joint null restriction on all loadings in the equation of long-term US interest rates conditional on the identified long-run structure of Table 3.16, is \( \chi^2(3) = 0.208 \) with a p-value of about 0.98. The statistics of the corresponding hypothesis test of German long and short rates and US short rates are respectively, 17.6 (0.0005), 9.56 (0.0188) and 11.88 (0.0078), all statistics being \( \chi^2 \) with three degrees of freedom. The values in brackets are the corresponding statistics' p-values.
Table 3.17: Alternative representation of the identified structure of cointegrating relationships in case of three cointegrating vectors

\[ \beta' (i^{GL}, i^{UL}, i^{GS}, i^{US}, Dv, 1) = i^{GL} - \frac{1}{2}(i^{US} + i^{GS} + 0.023) \]

\[ i^{UL} - \frac{1}{2}(i^{US} + i^{GS} + 0.023) \]

uses ordinary reduced rank methodology to estimate the rank and to identify the CI-relationships. To identify the rank in the second step, however, we have to exploit the fact that these statistics will have the asymptotic distributions given in Horvath and Watson (1995) and simulated in Paruolo (1999).

To illustrate the procedure, I have undertaken two separate analyses. One where I estimate a two-sector model of exports for a small open economy on Norwegian data and another where I look at interest rate relationships between Germany and the US as well as relationships within each individual country. In the first study we find no less than six theoretically consistent cointegrating relationships of which four represent sector specific long-run relationships. The other two are relationships between sectors and implies that there are strong ties between the export sectors of traditional goods and services in Norway. In particular, they imply that exports grow approximately at an annual rate of 3.6 per cent faster in the trading sector than in the service sector and that inflation seems to be approximately 0.8 percentage higher in the service sector than in the trading sector. This could be explained by a more competitive environment in the trading sector. Furthermore, the analysis does not give support to a long-run PPP relationship. On the contrary, my empirical results indicate that small open economies like the Norwegian, still have considerable market power in the export market. With regard to the interest rate study, the second step of the analysis clearly identified a relationship between German and US long-term interest rates. The first step of the analysis was not able, however, to reveal a similar relationship between short and long rates within each country. Taken at face value these findings could be taken to indicate that Europe in its conduct of monetary policy may have lost control to international capital markets. Although the picture changes somewhat with regard to how short-term interest rates are determined, the alternative anal-
ysis based on the existence of three cointegrating vectors does not change the central message that central banks do not seem to be able to control the long end of the capital market. However, the results clearly indicate that the exogeneity status given to US short-term interest rates based on two cointegrating vectors is incorrect. Short term US interest rates seem to be determined in accordance with a policy rule informed by long-term interest rates. Whether the Fed on basis of this can be said to have been successful with regard to controlling inflation is difficult to say and needs further investigation with an extended information set. However, the fact that the Fed seems to have lost control over the long end of the market, as implied by exogenous long-term interest rates, indicates that an important channel through which monetary policy could affect the real economy has been literally blocked. A plausible interpretation of this might be the increasing importance and dependence on international capital markets. With regard to Germany, there seems to be a similar causal relationship between short and long rates. However, in contrast to US long-term interest rates the corresponding German long-term interest rates are endogenous in the sense of being caused by movements to the US long-term interest rate. Again this finding substantiates the claim that the German central bank, the Bundesbank, during the nineties was not able to control the long end of the market, a conclusion that might have far-reaching implications given that bank lending in Continental Europe mainly is of a long-run character. However, as in the US the German Bundesbank seems to have been able to set their short-term interest rates independently in accordance with a policy rule. As for the US, to substantiate whether Bundesbank has succeeded in its endeavor of controlling inflation one will have to undertake further analysis on extended information sets.

All in all, in this chapter I have proposed a procedure to deal with cointegration of data that in addition to vary along a time series dimension varies along a relatively short cross sectional dimension. Even though the dimensions of the information sets in the two examples used to demonstrate the procedure of this paper have been too small to really demonstrate its full potential, the examples at least served to show that the suggested two step procedure of this paper may be an useful device in helping out with the identification of cointegrating relationships across sectors when dealing with times series with a small cross sectional dimension.
References


A. Tables

Table A.1:
Multivariate statistics for testing stationarity

<table>
<thead>
<tr>
<th>Variables</th>
<th>a1</th>
<th>a2</th>
<th>pa1</th>
<th>pa2</th>
<th>pw</th>
<th>R</th>
<th>ulc</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2 (3)$</td>
<td>15.87**</td>
<td>16.19**</td>
<td>7.11</td>
<td>12.84**</td>
<td>6.87</td>
<td>11.6**</td>
<td>9.59*</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.07)</td>
<td>(0.005)</td>
<td>(0.076)</td>
<td>(0.009)</td>
<td>(0.022)</td>
</tr>
</tbody>
</table>

1) The test statistics are the LR-tests of restrictions on the cointegration space within the Johansen framework. Specifically, these statistics test the restriction that one of the cointegrating vectors contains all zeros except for a unity corresponding to the coefficient of the variable we are testing for stationarity. The test is conditional on the number of cointegrating vectors. In Table A.1, the statistics quoted are conditional on there being three CI-vectors and refer to the same VAR model that later is used to identify the long-run relationships. The figures in brackets under each Statistics are the tests' significance probabilities and * and ** denote rejection at 5% and 1% critical levels, respectively.
Table A.2: ADF(N) Statistics for testing for a unit root.
Estimates of |p - 1| in$^{1,2}$

<table>
<thead>
<tr>
<th>Variables</th>
<th>H₀</th>
<th>a1</th>
<th>a2</th>
<th>p₁</th>
<th>p₂</th>
<th>p₂</th>
<th>R</th>
<th>u₁c</th>
</tr>
</thead>
<tbody>
<tr>
<td>I (1)</td>
<td></td>
<td>-2.11</td>
<td>-2.13</td>
<td>-1.884</td>
<td>-2.37</td>
<td>-0.68</td>
<td>-3.48</td>
<td>-2.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.22)</td>
<td>(0.35)</td>
<td>(0.080)</td>
<td>(0.167)</td>
<td>(0.027)</td>
<td>(0.127)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>I (2)</td>
<td></td>
<td>-11.10**</td>
<td>-12.72**</td>
<td>-4.26**</td>
<td>-5.10**</td>
<td>-5.99**</td>
<td>-4.52**</td>
<td>-4.60**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.708)</td>
<td>(2.992)</td>
<td>(0.929)</td>
<td>(1.272)</td>
<td>(1.41)</td>
<td>(0.755)</td>
<td>(1.624)</td>
</tr>
</tbody>
</table>

$^{1}$For any variable x and a null hypothesis of I(1), the ADF statistics are testing a null hypothesis of a unit root in x against an alternative of a stationary root. For a null hypothesis of I(2), the statistics are testing a null hypothesis of an unit root in $\Delta x$ against the alternative of a stationary root in $\Delta x$.

$^{2}$For a given variable and the null hypothesis of I(1) and I(2), two values are reported. The N'th-order augmented Dickey-Fuller (1981) statistics, denoted ADF(N) and (in parentheses) the absolute value of the estimated coefficient on the lagged variable, where that coefficient should be equal to zero under the null hypothesis. Both a constant- and a trend-term together with seasonal dummies are included in the corresponding regressions when testing the null of I(1), whereas only a constant is specified when testing for I(2). N varies across the variables for both tests and is equal to three for $a_1$, $a_2$, $p_1$ and $R$, two for $p_2$ and $u_1$, and four for $p_2$ and $u_1$, and three for $p_1$, $p_2$, $p_2$ and $u_1$. The effective sample-periods have been 1980(1) - 1998(2).

$^{3}$Here and elsewhere in the paper, asterisks * and ** denote rejection of the null hypotheses at the 5% and 1% significance level, respectively. The critical values for the ADF statistics for testing I(1) are -3.47 at a level of 5% and -4.084 at a level of 1% (MacKinnon (1991)).
Table A.3:
Individual equation and system diagnostics of the unrestricted VAR of the Trading sector

<table>
<thead>
<tr>
<th>Equation/Tests</th>
<th>AR 1-5 F[5,49]</th>
<th>ARCH 4 F[4,46]</th>
<th>Normality χ² (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δa1</td>
<td>0.9517[0.4565]</td>
<td>2.9843[0.0285]*</td>
<td>2.9351[0.2305]</td>
</tr>
<tr>
<td>Δpa1</td>
<td>2.9069[0.0224]*</td>
<td>1.5551[0.2023]</td>
<td>0.2446[0.8849]</td>
</tr>
<tr>
<td>Δpw</td>
<td>1.8161[0.1271]</td>
<td>0.4855[0.7463]</td>
<td>0.1556[0.9252]</td>
</tr>
<tr>
<td>ΔR</td>
<td>0.9739[0.4431]</td>
<td>0.6538[0.6272]</td>
<td>0.3227[0.8510]</td>
</tr>
<tr>
<td>Δulc</td>
<td>1.7365[0.1439]</td>
<td>0.6161[0.6532]</td>
<td>0.6316[0.7292]</td>
</tr>
</tbody>
</table>

System tests: AR 1-5[125,127] VNormality χ²(10) VX² F[480,155]

| Statistics    | 1.2571[0.1001] | 6.4595[0.7753] | 0.31062[1.000] |

1 The Values shown in brackets are the individual test's significance probability. * and ** denote as usual rejection of the corresponding null at levels of 5 and 1 per cent, respectively. VNormality and VX² denotes the Vector tests of normality and heteroscedasticity. For an explanation of the various test statistics the reader is referred to Chapter 14 of the PcFiml manual (Doornik and Hendry (1999)).
Table A.4:
Individual equation and system diagnostics of the unrestricted VAR of the Service sector

<table>
<thead>
<tr>
<th>Equation/Tests</th>
<th>AR 1-5 F[5,49]</th>
<th>ARCH 4 F[4,46]</th>
<th>Normality $\chi^2(2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta a_1$</td>
<td>1.5588[0.1810]</td>
<td>0.1525[0.9609]</td>
<td>1.1181[0.5717]</td>
</tr>
<tr>
<td>$\Delta p_{a1}$</td>
<td>0.3199[0.8986]</td>
<td>0.5639[0.6900]</td>
<td>0.8459[0.6551]</td>
</tr>
<tr>
<td>$\Delta p_{w}$</td>
<td>2.3861[0.0515]</td>
<td>0.4787[0.7511]</td>
<td>0.1229[0.9404]</td>
</tr>
<tr>
<td>$\Delta R$</td>
<td>1.5309[0.1976]</td>
<td>0.1656[0.9548]</td>
<td>3.1257[0.2095]</td>
</tr>
<tr>
<td>$\Delta u_{lc}$</td>
<td>2.3399[0.0554]</td>
<td>0.1408[0.9662]</td>
<td>1.2142[0.5449]</td>
</tr>
</tbody>
</table>

System tests: AR 1-5[125,127] VNormality $\chi^2(10)$ VX$^2$ F[480,155]

Statistics: 1.7455[0.0010]* 8.5836[0.5720] 0.359[1.000]

*The Values shown in brackets are the individual test's significance probability. * and ** denote as usual rejection of the corresponding null at levels of 5 and 1 per cent, respectively. VNormality and VX$^2$ denotes the Vector tests of normality and heteroscedasticity. For an explanation of the various test statistics the reader is referred to Chapter 14 of the Pcfiml manual (Doornik and Hendry (1999)).
Table A.5:
Individual equation and system diagnostics of the unrestricted VAR of the Pooled data

<table>
<thead>
<tr>
<th>Equation/Tests</th>
<th>AR 1-5 F[5,43]</th>
<th>ARCH 4 F[4,40]</th>
<th>Normality χ² (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δa1</td>
<td>1.3025[0.2808]</td>
<td>1.9082[0.1279]</td>
<td>3.4755[0.1759]</td>
</tr>
<tr>
<td>Δa2</td>
<td>2.2296[0.0685]</td>
<td>0.1409[0.9660]</td>
<td>0.8086[0.6674]</td>
</tr>
<tr>
<td>Δpa1</td>
<td>1.2499[0.3030]</td>
<td>0.2586[0.9027]</td>
<td>0.1110[0.9460]</td>
</tr>
<tr>
<td>Δpa2</td>
<td>0.5975[0.7020]</td>
<td>0.4717[0.7562]</td>
<td>1.0866[0.5808]</td>
</tr>
<tr>
<td>Δpw</td>
<td>1.0591[0.3962]</td>
<td>0.2464[0.9102]</td>
<td>0.1202[0.9417]</td>
</tr>
<tr>
<td>ΔR</td>
<td>2.1560[0.0768]</td>
<td>0.6230[0.6488]</td>
<td>0.7565[0.6851]</td>
</tr>
<tr>
<td>Δulc</td>
<td>1.9239[0.1101]</td>
<td>0.0361[0.9974]</td>
<td>1.6451[0.4393]</td>
</tr>
</tbody>
</table>

System tests: VAR 1-5[245,60] VNormality χ²(14) VX² χ²[1232]

Statistics: 1.8569[0.0026]** 6.5985[0.9491] 1172.9[0.8844]

¹The Values shown in brackets are the individual test's significance probability. * and ** denote as usual rejection of the corresponding null at levels of 5 and 1 per cent, respectively. VNormality and VX² denotes the Vector tests of normality and heteroscedasticity. For an explanation of the various test statistics the reader is referred to Chapter 14 of the PcFiml manual (Doornik and Hendry (1999)).
<table>
<thead>
<tr>
<th>Eigenvalue number/Part of eigenvalue</th>
<th>real</th>
<th>complex</th>
<th>Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9482</td>
<td>0.0000</td>
<td>0.9482</td>
</tr>
<tr>
<td>2</td>
<td>0.8024</td>
<td>±0.202</td>
<td>0.8275</td>
</tr>
<tr>
<td>3</td>
<td>−0.737</td>
<td>0.0000</td>
<td>0.7365</td>
</tr>
<tr>
<td>4</td>
<td>0.7272</td>
<td>±0.057</td>
<td>0.7995</td>
</tr>
<tr>
<td>5</td>
<td>−0.159</td>
<td>±0.679</td>
<td>0.6974</td>
</tr>
<tr>
<td>6</td>
<td>0.3674</td>
<td>±0.587</td>
<td>0.6921</td>
</tr>
<tr>
<td>7</td>
<td>0.1904</td>
<td>±0.452</td>
<td>0.4900</td>
</tr>
<tr>
<td>8</td>
<td>−0.3867</td>
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Service sector

Trading sector

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Chapter 4

"Bootstrapping or train-spotting: A note on small sample properties of the trace statistics related to specific VARs"*†

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Abstract

This chapter is a study of the small sample properties of trace test statistics related to specific vector autoregressive data generating processes (DGP's) governing respectively, a system of international interest rate relationships and Norwegian exports. The chapter aims particularly at discussing the properties of the statistics in the case when the analysis is conditional on the fact that some of the cointegrating vectors are known, in which case the asymptotic distributions of the LR test statistics are given in Paruolo (1999). In this respect the chapter also aims at discussing the idea of treating some of the cointegrating vectors as fixed when in fact these have been estimated in a preliminary step, to help with the identification of cointegrating vectors in the case of times series with a cross sectional dimension. The simulation results clearly indicate a problem of size even when dealing with a relatively large number of observations. Furthermore, the discrepancy between the two simulated critical values does seem

*I want particularly to thank Chiara Osbat for introducing me to OX as a programming language. I am also grateful to my supervisor Søren Johansen for lots of useful comments and suggestions that surely have contributed to improving the chapter considerably.

†All results in this chapter have been generated by using the programming language Ox version 2.00 (see Doornik, 1998). The DGP's have been estimated by using PcFiml 9.20 (see Doornik and Hendry (1999)). All macros and data referred to in this chapter are available on request from the author.
to depend more on the dimension of the VARs than on the fact that one conditions on some a priori known cointegrating vectors. However, to substantiate this claim further simulation experiments have to be undertaken.

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1 Introduction

Applied works in the field of macro econometrics are almost without exception based on information sets with a relatively small number of observations. Notwithstanding, inference with regard to the number of cointegrating vectors continues to be conducted using asymptotic critical values based on an infinite number of observations. As small sample properties of test statistics most probably deviate significantly from their asymptotic equivalents, the possibility of making valid inference may be severely compromised. A typical example in this respect could be two recent studies made on international interest rate relationships and Norwegian exports (Chapters 1 and 3). In these studies all inference made is contingent on the validity of the asymptotic approximation to the small sample case. In addition, the last analysis uses the concept of known cointegrating vectors to come up with a procedure to help with the identification of CI-vectors in the case of times series that vary along a cross sectional dimension which involves a conditioning argument that significantly affects the asymptotic as well as the small sample distributions. The central finding of this chapter of several incidences
of cointegration across sectors based on a two-step analysis where one first estimates the cointegrating relations within sectors and then treating these as fixed in a second step when trying to identify possible cointegrating relationships between sectors, is therefore a finding that deserves to be scrutinized along the dimension of a possible small sample bias. In addition, the idea of using the concept of known cointegrating vectors when in fact these are estimated needs further justification.

To study small sample properties of the trace test statistics within the context of an economic problem worth scrutiny, the last study referred to above is in this chapter used as a DGP generating source. To be more specific this means that estimated versions of the vector autoregressive systems related respectively to the international interest rate study and the study of Norwegian exports, are used to generate data in a repeated experiment of estimation and testing to study small sample properties of the test statistics. Whether the estimated VARs of the systems represent good approximations of the true underlying DGPs is not taken into consideration beyond the fact that they all seem to satisfy most requirements of a congruent representation thereof. This means that the results of the experiments are all going to be contingent on the eventuality that our estimated VARs constitute the true underlying processes, an assumption that is anything but trivial. Empirical models, like any model of remote processes of immense complexity can never be more than at best good approximations of their immensely complex equivalents and in most cases certainly are far from meeting such a requirement. It is therefore already at the outset imperative to call into question the expected utility of such an experiment. In fact, as the literature on robust inference suggests, a naive implementation of different simulation techniques to investigate small sample properties of specific test statistics and estimators may represent a huge disservice to the goal of improving upon inference. At least as long as we do not have perfect information about the true underlying process governing the specific data under study. E.g. as Huber(1981), Hampel et al. (1986) and more recently, a paper by Ronchetti and Ventura (2001) demonstrate, even relatively small deviations from the assumed parametric model might wipe out the accuracy of asymptotic distributions derived from the presumption that the model holds. This is a caveat that most applied work in the mushrooming field of studies that include Monte Carlo or Bootstrapping to investigate small sample properties of different statistics and estimators, seems to have conveniently swept under the carpet. However, this is not to say that Monte Carlo and Bootstrap as techniques have no
rationale. On the contrary, in the strongly hypothetical situation that one knows the DGP the techniques certainly are able to throw light on what happens to distributions of test statistics when the assumption of an infinite number of observations is not fulfilled. However, to take the additional step of saying something about small sample properties of the same test statistics in the case of an immensely complex and unknown DGP, as is certainly the case with real world, is a totally different exercise that should only be undertaken with the highest degree of caution. Notwithstanding the caveat given above, this chapter uses the technique of parametric Bootstrapping to illuminate small sample properties of the trace statistics related to the specific DGPs alluded to earlier in this introduction. The question that is sought to be answered is therefore something like: Under the assumption that the DGPs are given by the estimated vector autoregressive systems what would the small sample distributions of the trace statistics look like?

The rest of the chapter is organized as follows. Section two presents the models and gives the data generating processes used to study the small sample properties of the trace statistics, both in the ordinary case where we do not know anything about the cointegrating vectors and in the case where some of the cointegrating vectors are known a priori. Then, to be able to discuss economic matters related to the process governing interest relationships between Europe and international capital markets on the one hand and Norwegian exports on the other, the main results in Chapter 3 are briefly reviewed in Section three. However, before presenting these, the vector autoregressive model in the case where some of the cointegrating vectors are known a priori is recast in an interpretational framework of two sectors with individually known cointegrating vectors. In this context the use of fixed cointegrating vectors when in fact they have been estimated in a preliminary step to help with the identification of CI vectors in the pooled analyses will be given a justification. The results of the Monte Carlo exercises together with a brief discussion of their implications, are given in Section four. Based on the central findings of the chapter, Section five seeks to conclude.
2 The estimated models, the test statistics and the DGPs

The models

The general version of our models is given by equation (1) below.

\[ \Delta X_t = \alpha \beta' X_{t-1} + (\Upsilon, \mu) \begin{bmatrix} Z_t \\ d_t \end{bmatrix} + \varepsilon_t \]  

(1)

In equation (1), \( X_t \) and \( \varepsilon_t \) are both \( p \times 1 \) vectors, \( Z_t = (\Delta X'_{t-1}, \ldots, \Delta X'_{t-k+1})' \) is \( p(k-1) \times 1 \), \( \varepsilon_t \) is assumed to be i.i.d. \( \mathcal{N}(0, \Omega) \) and \( d_t \) is a vector of deterministic terms like a constant, trend and seasonal dummies. \( \Upsilon = (\Gamma_1, \ldots, \Gamma_{k-1}) \) is a \( p \times p(k-1) \) matrix, \( \mu (p \times q) \) and \( \alpha \) and \( \beta \) are both \( p \times r \) matrices assumed to be of full rank \( r \) such that the I(1) condition of \( \alpha'_1 (I_p - \sum_{i=1}^{k-1} \Gamma_i) \beta_{\perp} \) having full rank \( p - r \), is fulfilled when assuming that all the roots of the characteristic polynomial of \( X_t \) lie at one or outside the unit circle. We are in addition going to look at the case where \( \beta \) can be partitioned into two sub matrices, \( \beta_1 = b \) and \( \beta_2 \), of dimensions \( p \times s \) and \( p \times m \) respectively, where the first set of cointegrating vectors, \( b \), represents the \( s \) a priori known cointegrating relationships, while \( \beta_2 \) represents the \( m = r - s \) remaining unknown ones. \( r \) represents the full number of cointegrating vectors in the two dimensional data set. Using the identity of orthogonal projections such that \( \mu = \alpha \kappa + \alpha'_1 \bar{\kappa} \), where \( \kappa = (\alpha' \alpha)^{-1} \alpha' \mu \) and \( \bar{\kappa} = (\alpha'_1 \alpha_1)^{-1} \alpha'_1 \mu \), equation (1) can be given the equivalent representation of:

\[ \Delta X_t = \alpha_2 (\beta'_2 X_{t-1} + \kappa_2 d_t) + (\Upsilon, \alpha_1, \alpha'_{\perp} \bar{\kappa}) \begin{bmatrix} Z_t \\ d_t \end{bmatrix} + \varepsilon_t \]  

(2)

\( \kappa \) is here partitioned conformly with the partitioning of the \( \alpha = (\alpha_1, \alpha_2) \) matrix such that \( \kappa = (\kappa'_1, \kappa'_2)' \), where \( \kappa_1 = k \) represents the coefficients of the deterministic term entering the \( s \) known cointegrating vectors and \( \kappa_2 \) the corresponding coefficients in the remaining \( r - s \) unknown cointegrating vectors.

All models of exports are VARs of order three. This implies in particular that \( Z_t = (\Delta X'_{t-1}, \Delta X'_{t-2})' \) and \( \Upsilon = (\Gamma_1, \Gamma_2) \). The deterministic
term, $d_t$, consists of a constant term, a trend and centered seasonal dummies, $S_i$ ($i = 1, 2, 3$). While the constant term and the centered seasonal dummies enter unrestrictedly, the trend coefficient has been restricted to lie in the alpha space not to generate a quadratic trend. By partitioning the coefficient vector of the deterministic term accordingly into $\mu = (\mu_1, \mu_2, \mu_3')$ and taking into account that the trend coefficient is restricted such that $\mu_2 = \alpha \kappa = (\alpha_1, \alpha_2)(k', \kappa')'$, the general expression for the export models reduces to equation (3) below. Note particularly that contrary to the general expression given above, $\mu_1$ and $\mu_3$, have here not been split into their orthogonal projections and the parts orthogonal to the space spanned by these.

$$\Delta X_t = \alpha_2 (\beta_2', \kappa_2) \left[ \begin{array}{c} X_{t-1} \\ X_{t-2} \\ \vdots \\ X_{t-\ell} \\ \varepsilon_t \end{array} \right] + (\Gamma_1, \Gamma_2, \alpha_1, \mu_1, \mu_2') \left[ \begin{array}{c} \Delta X_{t-1} \\ \Delta X_{t-2} \\ \varepsilon_{t-1} \end{array} \right] + \varepsilon_t \quad (3)$$

The model of the international interest rate study is slightly less elaborate as in addition to being VARs of order two, they neither include seasonal dummies nor a trend term. The constant term is further restricted to lie in the the alpha space such that $\mu = \alpha (\alpha')^{-1} \alpha' \mu = \alpha \kappa$. Taking into account the partitioning of the beta and alpha vectors and that $\kappa_1 = k$ implies thus that $\kappa = (k', \kappa')'$ and the models will all be special versions of equation (4) below.

$$\Delta X_t = \alpha_2 (\beta_2', \kappa_2) \left[ \begin{array}{c} X_{t-1} \\ X_{t-2} \\ \vdots \\ X_{t-\ell} \\ \varepsilon_t \end{array} \right] + (\Gamma_1, \alpha_1) \left[ \begin{array}{c} \Delta X_{t-1} \\ \Delta X_{t-2} \\ \varepsilon_{t-1} \end{array} \right] + \varepsilon_t \quad (4)$$

The case with no a priori information as to known cointegrating vectors implies in both (3) and (4) that $b = \beta_1, k = \kappa_1$. The $\Pi$ matrix is therefore given by $\Pi = (\alpha_1, \alpha_2) \left( \begin{array}{cc} \beta_1' & \kappa_1 \\ \beta_2' & \kappa_2 \end{array} \right) = \alpha \beta'$ and there will be no level parts in the exogenous part of the VARs.

The statistics The aim of this analysis is to study the small sample properties of the LR test for the hypothesis of cointegrating rank, the trace test,
associated with the estimated models in Chapter 3. In the case where there is no a priori knowledge with regard to cointegrating vectors and restrictions on the deterministic terms in accordance with (3) and (4), this test is given by the expression

\[-2 \log Q (H^* (r) \mid H^* (p)) = -T \sum_{i=r+1}^{p} \log (1 - \lambda_i^*) , \tag{5}\]

where \(\lambda_i^*\) solves the eigenvalue problem \(|\lambda^* S^*_{11} - S^*_{10} S^*_{00}^{-1} S^*_{01}| = 0\) for eigenvalues \(1 > \lambda_1^* > \ldots > \lambda_2^* > \ldots \lambda_p^* = 0\) and \(S^*_{ij}, i, j = 0, 1\), are the product moments of the residuals we would obtain by regressing respectively, \(\Delta X_t\) and \((X_{t-1}, t)'\) on \((\Delta X'_{t-1}, \Delta X'_{t-2}, 1, S_i)'\) in the case of exports and by regressing \(\Delta X_t\) and \((X_{t-1}, 1)'\) on the lagged first difference, \(\Delta X_{t-1}\), in the case of the international interest rate study. In the case of known cointegrating vectors where the remaining cointegrating vectors are forced to lie in the orthogonal space of the space spanned by these, that is \(\beta_1 = b\) is \(p \times s\) and \(\beta_2 = b_\perp\varphi\) where \(\varphi\) is \(p - s \times m\), the expression for the trace test statistics is given by

\[-2 \log Q (H^* (m) \mid H^* (p - s)) = -T \sum_{i=m+1}^{p-s} \log (1 - \lambda_i^*) . \tag{6}\]

\(\lambda_1 > \ldots > \lambda_{p-s} > 0\) are the eigenvalues of

\[|\lambda^* b_\perp^S S^*_{11} b_\perp - b_\perp^S S^*_{10} S^*_{00}^{-1} S^*_{01} b_\perp| = 0 , \tag{7}\]

where \(S^*_{ij} = S^*_{ij} - S^*_{ii} b (b'S^*_{11} b)^{-1} b'S^*_{ij}\) and \(S^*_{ij}, i, j = 0, 1,\) are the product moments defined in each case as above\(^1\).

\(^1\)Note that by redefining \(S^*_{ij} , i,j = 0,1\), such that they constitute the residuals we would obtain by regressing respectively, \(\Delta X_t\) and \((X_{t-1}, t)'\) on \((\Delta X'_{t-1}, \Delta X'_{t-2}, (b'X_{t-1} + kt)', 1, S_i)'\) and \(\Delta X_t\) and \((X_{t-1}, 1)'\) on \((\Delta X'_{t-1}, (b_\perp X_{t-1} + k)'\), (7) will coincide with the notation used in Paruolo(1999), the only difference being the star notation to indicate the restrictions made with regard to the deterministic terms. That is, the equivalent eigenvalue problem will be given by:

\[|\lambda^* b_\perp^S S^*_{11} b_\perp - b_\perp^S S^*_{10} S^*_{00}^{-1} S^*_{01} b_\perp| = 0 .\]
The DGPs The models represented by (3) and (4) have been chosen to be in accordance with the corresponding DGPs used to generate artificial data in our Monte Carlo simulation. As alluded to in the introduction this means that I have made the common but not uncontroversial assumption that the models hold. To be more specific the DGPs are quantified parametric representations of the models in the sense that all parameters are the outcome of ML estimation on the original data sets. The DGPs are therefore all special cases, given by (3) and (4), of the general formulation:

\[ \Delta X_t = \hat{\alpha} \beta' X_{t-1} + (\hat{T}, \hat{\mu}) \begin{bmatrix} Z_t \\ d_t \end{bmatrix} + \hat{\varepsilon}_t \]

where a hat indicates the corresponding ML estimate of the parameter. With regard to exports the effective estimation period has been 1980 (1) to 1998 (2) while the corresponding period in the case of the international interest rate study is 1990 (1)-1997(12). These periods constitute effective samples sizes of respectively 74 and 96 observations. As the aim of this study is to reveal the actual distributions of the LR test statistics related to the two studies in Chapter 3, these sample sizes are also the ones used in our Monte Carlo simulations. With regard to the generation of artificial data the residuals, \( \varepsilon_t \), have been drawn from the empirical distributions under the assumption that these represent identically and independently distributed observations from the normal distribution, implying that \( \varepsilon_t \sim IN(0, \hat{\Omega}) \), where \( \hat{\Omega} \) represents the ML estimated covariance matrices. Also, our Bootstrap data have all been generated from the model under the null. This means that when testing the null of less than or equal to \( r \) cointegrating vectors versus less than or equal to full rank, the data generating processes have been estimated VARs with \( r \) unrestrictedly estimated cointegrating relationships. To start the recursive generation of data related to each Monte Carlo experiment the initial values in the case of exports and a VAR of order three has been set to the realized observations of 1978 3, 1978 4 and 1979 1, while in the case of the international interest study and a VAR of order two, the initial values are the corresponding observations of 1989 9 and 1989 10.
3 Norwegian exports and international interest rate relationships: A Review of Results and the two-step procedure.

In a recent study of Norwegian exports (Chapter 3) I suggest a two-step approach to the identification of cointegrating relationships when dealing with data that in addition to vary along a times series dimension vary along a relatively short cross-sectional dimension. The approach implies first identifying the cointegrating relationships within sectors by undertaking an ordinary cointegration analysis of each sector separately, then treating these as known when trying to identify potentially cointegrating relationships across sectors in a second step when looking at all sectors simultaneously. In the context of the general framework of Section 2, this would imply redefining the known cointegrating vectors, $b$, to consist of all estimated sector-individual $C_l$ vectors and $\beta_2$ of all remaining $C_l$ vectors to be estimated in the second step involving more than one sector simultaneously. Assuming two sectors and no common variables across sectors this would thus imply the following level term in (1):

$$\alpha \beta' X_{t-1} = (\alpha_1, \alpha_2) \begin{pmatrix} \beta'_1 \\ \beta'_2 \end{pmatrix} X_{t-1}$$

(9)

where $X_{i,t-1} = (X_{i1,t-1}, ..., X_{iN_i,t-1})'$, $i = 1, 2$, and $N_i$ the number of sector specific variables in sector $i$. In (9) $b = (b_1, b_2) = \left( (b'_{11}, 0')', (0', b'_{22})' \right)$, where $b_{1i} = \hat{\beta}_{1i}$ for $i = 1, 2$, to illustrate that in this context the known cointegrating vectors have the interpretation of being estimated. The argument for treating some cointegrating vectors as known when estimating the level matrix $\Pi$ of the VAR, even though they strictly speaking have been estimated in a preliminary step, hinges on the super consistency property of the cointegrating vectors. This point may be clarified by looking at the asymptotic distribution.
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<td>( a_2 = \text{const} + R )</td>
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<td>4</td>
<td>( pa_2 = \text{const} + 0.707R + 0.329ulc )</td>
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<td>5</td>
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<tr>
<td>6</td>
<td>( a_1 = \text{const} + a_2 - 0.008Trend )</td>
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Table 1: Restricted Long-run relationships in a two-sector model of Norwegian exports

\[
T^{\frac{1}{2}} \left( \tilde{\Pi} - \Pi \right) = T^{\frac{1}{2}} \left( \tilde{\alpha} \beta' - \alpha \beta' \right) = \left( T^{\frac{1}{2}} \left( \tilde{\alpha} - \alpha \right) \right) \beta' + \alpha \left( T^{\frac{1}{2}} \left( \tilde{\beta} - \beta \right) \right) .
\]

While \( \tilde{\beta} \) is superconsistent in the sense that \( \tilde{\beta} - \beta \in o_p \left( T^{-\frac{1}{2}} \right) \), \( \tilde{\alpha} \) converges to \( \alpha \) at rate \( T^{\frac{1}{2}} \), implying that \( \tilde{\alpha} - \alpha \in O_p \left( T^{-\frac{1}{2}} \right) \). Thus the term \( T^{\frac{1}{2}} \left( \tilde{\beta} - \beta \right) \) converges to zero while the first one, \( \left( T^{\frac{1}{2}} \left( \tilde{\alpha} - \alpha \right) \right) \beta' \), converges to \( N \left( 0, \Omega \otimes \beta \Sigma_{\beta}^{-1} \beta' \right) \), where \( \Omega \otimes \Sigma_{\beta}^{-1} \) is the variance of \( T^{1/2}(\tilde{\alpha} - \alpha) \), and the scaled distribution of \( \tilde{\Pi} \) is asymptotically independent of the estimated cointegrating vectors. However this argument does not explain why we do not estimate all cointegrating vectors simultaneously in a pooled analysis at the outset. This is more an argument of feasibility as the problem of identifying all cointegrating vectors simultaneously becomes intractable when the possibility set increases. To reduce the dimension of the estimated CI space will therefore serve to enhance the interpretability as well as the identifiability of the system of cointegrating vectors.

The fully identified system in the case of Norwegian exports as given in Table 1, implies the existence of no less than six cointegrating vectors of which the last two imply cointegration across sectors. In the table \( a_j \) and \( pa_j (j = 1, 2) \) stand respectively for the logarithms of exports and export
prices in the trading sector \((j = 1)\) and the service sector \((j = 2)\) of the Norwegian economy. \(\bar{R}\) represents the logarithm of foreign real income, while \(\text{pw}\) and \(\text{ulc}\) represent the logarithms of world market prices and unit labor costs, respectively. The system reveals strong evidence of monopolistic competition in both sectors. The lack of supply side effects in the volume equation, however, indicates that exports are determined ex post for prices determined ex ante. More interesting in this context perhaps are the two last relationships of Table 1, which imply the existence of strong long-run links also between the sectors. Relationship 5 for instance, is a cointegrating relationship between the two sector’s export prices which implies that prices grow approximately at a yearly rate of 0.8 per cent faster in the service sector than in the trading sector. This finding is consistent with a more competitive environment in the trading sector. The last equation of a long-run relationship between the two sector’s export volumes says that exports grow approximately at a yearly rate of 3.6 per cent faster in the trading sector than exports in the service sector. This coincides well with the perceived view of the external sector being the main origin for innovative productivity improvements. The identification scheme leading to the system in Table 1, however, depends heavily on the critical values of ordinary trace statistics as well as critical values governing the case when we condition on a priori known cointegrating relationships. Especially critical is perhaps the inference conditional on five known cointegrating vectors which gives rise to the long-run relationship between exports in the two sectors. However, the inference regarding the decision between whether there are two or three cointegrating vectors in the trading and service sectors is not trivial. To be able to discuss these matters related to the outcome of the small sample simulations I will therefore briefly review the identification scheme of the Norwegian export study and relate each single step to one of the tables in the appendix section of the chapter.

As alluded to above, the first step of the analysis implies identification of the cointegrating vectors in each sector individually. To be more explicit this means that in the study referred to above, initially three cointegrating vectors are identified in both the service sector and the trading sector based on five dimensional vector autoregressive systems where the information sets in addition to including sector specific volumes and prices consist of three common variables, respectively, world market prices, unit labor costs and foreign real income. The Tables giving the statistics and the outcome of the simulations related to this step are represented by Tables 2 and 3 in the appendix. In Chapter 3, when looking at all six cointegrating relations identified at the
sectorial level simultaneously, two relationships are perceived to reveal cointegration between the two sectors export volumes. Before embarking on the second step, one of these relationships was therefore taken out to see if this was confirmed when conditioning on the five other identified relationships in a seven dimensional pooled analysis where all sector specific variables together with the three common variables constitute the information set. The statistics and simulation results of this seven dimensional conditional analysis are found in Table 4.

Chapter 3 also takes a closer look at interest relationships between European and international capital markets and in addition to a stationary rate of depreciation, finds strong evidence of a long-run relationship across sectors as represented by a stationary spread between long-term interest rates in Germany and the US. However, the additional finding in this chapter of a possible long-run relationship between domestic German and US spreads deserves particular attention as such a relationship, if correct, would change dramatically conclusions made on the basis of only two cointegrating vectors with regard to the ability of Central Banks to control the long end of the capital market. The small sample simulated critical values of the pooled model where in addition to the country specific short and long rates, the rate of depreciation is included in the information set, are given by Tables 5 in the case where we do not condition on any preliminary identified relationships and Table 6 when we have conditioned on a stationary depreciation rate.

In the section to come all these issues will be further discussed in relation to the results of small sample simulations based on estimated representations of the general models given by (3) and (4).

4 Results

The results of $M=10000$ Monte Carlo simulations of different models and under different assumptions with regard to a priori knowledge about known cointegrating vectors, are given in the tables below. The first thing to notice is that the asymptotic critical values seem to be closer to the simulated the more cointegrating vectors there are under the null. Also, the discrepancy between the two simulated critical values does seem to depend more on the dimension of the VARs than the fact that one conditions on some a priori known cointegrating vectors. However, to substantiate this claim further simulation experiments are necessary.
With regard to the results in Chapter 3, the trace statistics of Table 1 and Table 2 clearly suggest that the inference of three cointegrating vectors of both sectors is at stake. In fact, the small sample simulations imply that we must operate with critical levels of beyond twenty per cent to be able to reject a hypothesis of two cointegrating vectors. Perhaps more surprising is the lack of support for the existence of a sixth cointegrating vector in the pooled analysis conditional on five known cointegrating vectors. However, as commented on in the introduction, the estimated DGPs of this analysis are probably far from representing the underlying "true" DGPs. Thus, even though the outcome of these small sample simulations may represent a significant improvement with regard to inference compared with using asymptotic critical values when the models hold, it might represent a huge disservice to the same objective when the opposite is the case.

Besides giving strong support to the existence of at least two long-run relationships, the simulation results of the international model of interest rates leave the door open for a third cointegrating vector. This is further confirmed in the case where we condition on the bilateral exchange rate as a known cointegrating vector and therefore gives support to the results of the alternative analysis made in Chapter 3, where a third cointegrating relationship between the domestic spreads of Germany and the US was identified.

5 Summary

In this chapter we have discussed small sample properties of trace test statistics related to the specific VARs in Chapter 3. The general impression is that simulated small sample critical values seem to deviate significantly from the corresponding asymptotic ones, the deviation being bigger the fewer cointegrating vectors there are under the null and the higher is the dimension of the VAR. Whether the results indicate a problem when dealing with some a priori known cointegrating vectors is hard to say based on the experiments of this chapter and necessitates further investigation along the dimensions of both different numbers of a priori known cointegrating vectors and VARs with different dimensions. With regard to the results in Chapter 3, the simulated critical values do not change inference related to the international interest study and leave the door open for a third cointegrating vector. With regard to the Norwegian export study however, the situation is quite the opposite; taken literally this indicates an overspecification of cointegrating
within and between sectors and legitimates thus further investigation with regard to the possibility of only two cointegrating vectors in both sectors. Finally, it is important to point out that the validity of the simulated small sample distributions in this chapter might all be severely affected by the fact that the assumed parametric DGPs might deviate significantly from the true underlying ones. As pointed out in the text, the true underlying DGPs are probably of immense complexity. To believe that our simple models have the capacity to duplicate the central mechanism based upon different demands on empirical congruency may therefore be if not naive, overly optimistic. In fact, the eventuality that the models hold in the sense of being close substitutes to DGPs, is probably more of the exception than the rule. In view of the first-order non-robustness of most test statistics, including the LR tests for the hypothesis of cointegrating rank, this fact suggests that one should in some way or another, take into account the effects of different kinds of deviations from the assumed model when undertaking Monte Carlo simulations to study small sample properties of different test statistics and estimators. One way to do this would be to resort to robust inference.

References


A Tables

Table 2: The trace statistics in the trading sector. VAR-dimension:5.

<table>
<thead>
<tr>
<th>Null</th>
<th>Alt.</th>
<th>Statistics 1</th>
<th>95% A 2</th>
<th>90%MC 3</th>
<th>95%MC 3</th>
<th>P-val 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0$</td>
<td>$r \leq 5$</td>
<td>142.70</td>
<td>87.3</td>
<td>101.29</td>
<td>107.07</td>
<td>0.3687</td>
</tr>
<tr>
<td>$r \leq 1$</td>
<td>$r \leq 5$</td>
<td>83.55</td>
<td>63.0</td>
<td>69.558</td>
<td>73.535</td>
<td>0.2311</td>
</tr>
<tr>
<td>$r \leq 2$</td>
<td>$r \leq 5$</td>
<td>39.89</td>
<td>42.4</td>
<td>46.449</td>
<td>49.518</td>
<td>0.1961</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>$r \leq 5$</td>
<td>17.4</td>
<td>25.3</td>
<td>29.295</td>
<td>31.990</td>
<td>0.2159</td>
</tr>
<tr>
<td>$r \leq 4$</td>
<td>$r \leq 5$</td>
<td>3.107</td>
<td>12.3</td>
<td>13.327</td>
<td>15.262</td>
<td>0.1379</td>
</tr>
</tbody>
</table>

1 The figures quoted in the third column, Statistics, are the outcome of the tests undertaken on the original data set in Chapter 3.
2 The 95 per cent asymptotic fractiles in the fourth column with the heading 95% A are taken from Osterwald-Lenum (1992)
3 The figures in the column 90%MC and 95%MC are respectively the 90 and the 95 per cent fractiles of a Monte Carlo simulation with 10 000 replications
4 The figures in the column P-val are the probability values of the 95 per cent simulated asymptotic fractiles of Osterwald-Lenum in the simulated MC distribution
Table 3: The trace statistics in the service sector. VAR-dimension: 5.

<table>
<thead>
<tr>
<th>Null</th>
<th>Alt.</th>
<th>Statistics</th>
<th>95%A</th>
<th>90%MC</th>
<th>95%MC</th>
<th>P-val</th>
</tr>
</thead>
<tbody>
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<td>124</td>
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<td>100.63</td>
<td>105.85</td>
<td>0.3445</td>
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<td>r ≤ 1</td>
<td>r ≤ 5</td>
<td>74.24</td>
<td>63.0</td>
<td>72.740</td>
<td>77.101</td>
<td>0.3125</td>
</tr>
<tr>
<td>r ≤ 2</td>
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<td>42.6</td>
<td>42.4</td>
<td>47.921</td>
<td>51.537</td>
<td>0.2356</td>
</tr>
<tr>
<td>r ≤ 3</td>
<td>r ≤ 5</td>
<td>20.21</td>
<td>25.3</td>
<td>27.717</td>
<td>30.373</td>
<td>0.1614</td>
</tr>
<tr>
<td>r ≤ 4</td>
<td>r ≤ 5</td>
<td>4.925</td>
<td>12.3</td>
<td>13.075</td>
<td>14.856</td>
<td>0.1256</td>
</tr>
</tbody>
</table>

1The figures quoted in the third column, Statistics, are the outcome of the tests undertaken on the original data set in Chapter 3.
2The 95 per cent asymptotic fractiles in the fourth column with the heading 95%A are taken from Osterwald-Lenum (1992)
3The figures in the column 90%MC and 95%MC are respectively the 90 and the 95 per cent fractiles of a Monte Carlo simulation with 10 000 replications
4The figures in the column P-val are the probability values of the 95 per cent simulated asymptotic fractiles of Osterwald-Lenum in the simulated MC distribution

Table 4: The trace statistics in the pooled model of exports conditional on a priori knowledge of five cointegrating vectors. VAR-dimension: 7

<table>
<thead>
<tr>
<th>Null</th>
<th>Alt.</th>
<th>Statistics</th>
<th>95%P</th>
<th>90%MC</th>
<th>95%MC</th>
<th>P-val</th>
</tr>
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<tbody>
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<td>m = 0</td>
<td>m ≤ 2</td>
<td>52.116</td>
<td>44.5</td>
<td>60.881</td>
<td>66.031</td>
<td>0.5157</td>
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<tr>
<td>m ≤ 1</td>
<td>m ≤ 2</td>
<td>8.816</td>
<td>26.5</td>
<td>32.679</td>
<td>36.213</td>
<td>0.2844</td>
</tr>
</tbody>
</table>

1The figures quoted in the third column, Statistics, are the outcome of the tests undertaken on the original data set in Chapter 3.
2The 95 per cent asymptotic fractiles in the fourth column with the heading 95%P are taken from Table 5 in Paruolo (1999)
3The figures in the column 90%MC and 95%MC are respectively the 90 and the 95 per cent fractiles of a Monte Carlo simulation with 10 000 replications
4The figures in the column P-val are the probability values of the 95 per cent simulated asymptotic fractiles of Paruolo in the simulated MC distribution
Table 5: The trace statistics of the pooled model in the international interest rate study. VAR-dimension: 5

<table>
<thead>
<tr>
<th>Null</th>
<th>Alt.</th>
<th>Statistics 1</th>
<th>95%A 2</th>
<th>90%MC 3</th>
<th>95%MC 3</th>
<th>P-val 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>r = 0</td>
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<td>125.60</td>
<td>76.1</td>
<td>82.909</td>
<td>87.695</td>
<td>0.2115</td>
</tr>
<tr>
<td>r ≤ 1</td>
<td>r ≤ 5</td>
<td>68.08</td>
<td>53.1</td>
<td>56.717</td>
<td>60.552</td>
<td>0.1563</td>
</tr>
<tr>
<td>r ≤ 2</td>
<td>r ≤ 5</td>
<td>39.80</td>
<td>34.9</td>
<td>36.245</td>
<td>39.331</td>
<td>0.1240</td>
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<tr>
<td>r ≤ 3</td>
<td>r ≤ 5</td>
<td>17.61</td>
<td>20.0</td>
<td>20.447</td>
<td>22.614</td>
<td>0.1099</td>
</tr>
<tr>
<td>r ≤ 4</td>
<td>r ≤ 5</td>
<td>5.137</td>
<td>9.2</td>
<td>8.9132</td>
<td>10.248</td>
<td>0.0889</td>
</tr>
</tbody>
</table>

1 The figures quoted in the third column, Statistics, are the outcome of the tests undertaken on the original data set in Chapter 3.
2 The 95 per cent asymptotic fractiles in the fourth column with the heading 95%A are taken from Osterwald-Lenum (1992)
3 The figures in the columns 90%MC and 95%MC are respectively the 90 and the 95 per cent fractiles of a Monte Carlo simulation with 10,000 replications
4 The figures in the column P-val are the probability values of the 95 per cent simulated asymptotic fractiles of Osterwald-Lenum in the simulated MC distribution.

Table 6: The trace statistics in the pooled model of international interest rates conditional on a priori knowledge of a stationary exchange rate. VAR-dimension: 5

<table>
<thead>
<tr>
<th>Null</th>
<th>Alt.</th>
<th>Statistics 1</th>
<th>95%P 2</th>
<th>90%MC 3</th>
<th>95%MC 3</th>
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</tr>
</thead>
<tbody>
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<td>m = 0</td>
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<td>72.18</td>
<td>59.0</td>
<td>60.408</td>
<td>64.448</td>
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<tr>
<td>m ≤ 1</td>
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<td>43.43</td>
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<td>0.1368</td>
</tr>
<tr>
<td>m ≤ 2</td>
<td>m ≤ 4</td>
<td>21.06</td>
<td>24.1</td>
<td>24.087</td>
<td>26.628</td>
<td>0.0996</td>
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<tr>
<td>m ≤ 3</td>
<td>m ≤ 4</td>
<td>6.67</td>
<td>12.1</td>
<td>11.029</td>
<td>12.560</td>
<td>0.0613</td>
</tr>
</tbody>
</table>

1 The figures quoted in the third column, Statistics, are the outcome of the tests undertaken on the original data set in Chapter 3.
2 The 95 per cent asymptotic fractiles in the fourth column with the heading 95%P are taken from Table 3 in Paruolo (1999)
3 The figures in the columns 90%MC and 95%MC are respectively the 90 and the 95 per cent fractiles of a Monte Carlo simulation with 10,000 replications
4 The figures in the column P-val are the probability values of the 95 per cent simulated asymptotic fractiles of Paruolo in the simulated MC distribution.
The degree of independence in European goods markets*

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Florence

Abstract

It is common knowledge that foreign trade in Europe is characterized by an acceptance of prices set by the world market. Coupled with a constant profit share in domestic sectors this makes European exports vulnerable to vagaries of international demand and prices as well as to crowding out in the wake of shocks to supply. These circumstances have been used to legitimate special measures geared towards shielding the sector from adverse shocks and general preferential treatment in the past.

In fact econometric evidence is not totally at odds with this view. However, neither exports in a large European economy like Germany nor in a small open one, like Norway, are characterized by price taking behavior. On the contrary, both nations show strong evidence of monopolistic power in the process governing external prices, implying that supply shocks to a large extent can be passed on to prices. On the other hand exports seem to be heavily subject to the vicissitudes of international trade. As opposed to cost aid through tax breaks, subsidies etc., this advocates, if society deems support of the foreign sector to be important, a continuation of arrangements geared towards shielding the sector from temporary fluctuations in international demand and prices.

*The analyses were undertaken using a combination of CATS in RATS (Hansen and Juselius(1995)) and PcFiml 9.20 (Doornik and Hendry(1999)). The I(2) analyses and tests were undertaken by using Clara Jørgensen’s I(2) procedure in Cats in Rats.
1 Introduction

The notorious woes of private entrepreneurs in foreign trade is perhaps one of the most characteristic features of quotidian media in our contemporary society. Almost daily we are reminded of how vulnerable foreign trade businesses are to the vagaries of international demand and prices and how important it is to avoid excessive domestic wage claims and to promote a culture of productivity growth to avoid a general crowding out of the external sector. Now that the project for monetary union is under way, clearly founded on neo-liberal ideas of promoting so called “level playing fields” in just about all kinds of economic areas, this is perhaps even more so. An indication of this and the influence that this kind of movements might have on decision takers were demonstrated quite recently when the more than two centuries old Smithian threat of removing all special measures for privileging exports slipped off the agenda from a recent EU meeting. This is a threat that if realized, certainly could have profound negative effects on the segment of foreign trade that has traditionally received preferential treatment such as export guarantees, price subsidies etc. That such a threat therefore gave rise
to a lot of opposition is no wonder. However, after having had to listen to all kinds of mercantilistic propaganda for centuries, it is legitimate to ask how much of it might represent the truth and what might only have had the effect of bewildering the wider populace. Even though theoretical models, like the Scandinavian (Aukrust 1970, 1977), give strong support for their hypothesis, in that price taking behavior in the trading sector coupled with constant profit shares and a domestic wage leading sector lead to crowding out in wake of excessive wage claims as well as an external sector that is vulnerable to the vicissitudes of the industrialized world, it is imperative to scrutinize these points of view by looking at what a more objective source can tell us. To confront prejudiced attitudes and theories used to support them with reality, this chapter undertakes an in-depth analysis of European exports based on data and an interpretational framework that theoretically encompasses the predictions of the Scandinavian model. A main motivation in this context has also been to reveal the degree of monopolistic power in the process determining European exports and export prices and thus either to confirm or to reject the hypothesis of “the law of one price”. In this context it will be of particular interest to find out whether the size of the economy might also play a significant role and thus whether small open economies in Europe are more susceptible to international influence than one of the so-called “Big Five”. To address this issue the chapter looks at two European economies, the Norwegian as a representative of a typical small economy in an European context, and the German as a representative of one of the “Big Five”. As opposed to the German analysis which is based on an aggregate analysis of data for the whole economy, the Norwegian study has been undertaken based upon data for two subsectors. This has mainly been done to compare with, and further elaborate on, the results in Chapter 3 and in this context to particularly scrutinize the indication in this paper of a possible common (2) trend. However, it may also be given a rationale from the perspective of looking at the status with regard to international independence of still smaller entities.

As alluded to in the above, an important aspect of this study has also been to reveal a potential occurrence of higher order non stationarity. However, the attitude has been rather relaxed in this respect insofar as there has been no intention of forcing (2)-ness upon the data. Rather, in the lack of sufficient support the approach has been more to cling to a null of (1) than to continue along the dimension of an artificially made supposition. Additionally it may be added that as the chapter intends to reveal generic properties of the
underlying data generating processes the legitimacy of undertaking an I(2) analysis is deemed less urgent. However, when this is said, it must also be stressed that an I(2) analysis may be an interesting exercise to carry out even in the case one might not feel confident about its premises. If nothing else, to compare with and eventually to support the outcome of an I(1) analysis. This more pragmatic view is the preferred when interpreting the results of the I(2) analysis for Germany in Section 4.

The chapter is organized as follows. Section two gives a brief review of theory used to help with the interpretation and identification of long-run relationships. The choice of monopolistic competition as an encompassing framework has not been made only because its predictions encompass the ones of the Scandinavian model and thus is convenient from the perspective of explicitly testing the claims of private entrepreneurs, but also because the theory of monopolistic competition is particularly suited to unveil eventual power in the process governing prices. Dependent on whether there is evidence of a second order trend or not, the section also goes one step further and presents alternative hypothetical scenarios based on theory. Section three describes the data and their properties. A particularly important feature of this section is to reveal the existence of potential common trends of a second order. The analysis of the data then follows in the next section, Section four. The last section, Section five, seeks to conclude.

2 An encompassing theory

As alluded to in the introduction the theory of monopolistic competition has been used as an encompassing framework to help with identification and interpretation of long-run cointegrating relations. To make the approach as general as possible theory has been recast in an ex ante ex post framework making the outcome dependent on whether exporters have complete knowledge of all variables or have to make their decisions relying on plans formulated on the basis of expected quantities. In a highly stylized case the situation of the monopolist may be depicted as in Figure 1 below\(^1\). Ex ante the producer does not know the exact position of the demand curve and has to base his or her plans with regard to prices as well as volume on an expected demand curve, denoted \(A^E\) in the figure. Assuming that our representative

\(^1\)For a comprehensive treatment the reader is referred to the mathematical exposition of the appendix.
exporter is a profit maximizer and has perfect knowledge with regard to costs, he will therefore plan to produce the volume where his expected marginal income equals marginal costs, $A^P$, and set a price which is expected to clear the market, $PA^R$.

In case the ex post realized quantities perfectly match the expected ones, the export volume and price relationships may be given the following stochastic log-linear representations:

$$a_t = c + \alpha(pw_t - ulc_t) + \beta R_t + \varepsilon_t$$  \hspace{0.5cm} (1)

$$pa_t = c + \varphi ulc_t + (1 - \varphi)pw_t + \rho R_t + \varepsilon_t$$  \hspace{0.5cm} (2)

where $a_t$ and $pa_t$ represent export volume and export prices while $pw_t$, $ulc_t$ and $R_t$ represent world market prices, unit labor costs and an indicator for "world" demand, respectively\(^2\).

However, a more likely scenario is that demand deviates significantly from the expected, ex post. The existence of long-term contracts, advertisements, price lists etc. may make it costly for the producer to deviate ex post from

\(^2\)All variables are logarithmic transformations of the original series.
the ex ante decided price level. Thus, assuming that our representative monopolist is bound by its ex ante quoted price we will have to distinguish between two cases. In the first case demand is not sufficiently high to meet the volume that exporters want to produce for the fixed price ex ante. Our monopolist will therefore be rationed on the export market and the level of exports fully determined by ex post demand. In the second case ex post demand will exceed supply for the given price and exports will be given by supply. In the figure the first case is depicted by the intersection of the $A^{RI}(\cdot)$ demand curve with the horizontal curve representing the ex ante fixed export price, $PA^R$, while the second case is represented by the price taking level of production, $A^{R2}$, for which the marginal cost curve intersects with the fixed ex ante price line\(^3\). Following Armington (1968) assuming that demand is specific to the producer, the demand for exports may be specified as a log linear function of the world demand indicator and the relative price ratio of export prices to world market prices. This gives us the following relationship:

$$a_t = c - \sigma(pa_t - pw_t) + \beta R_t + \epsilon_t$$  

In the case of a small open economy $\sigma$ can be interpreted both as a relative price elasticity with regard to export demand and as the elasticity of substitution. This can be shown mathematically (again look at the appendix), but it has also some intuitive appeal since the income effect of an increase in the export price of a small economy will be virtually negligible. Thus, the price elasticity will express the percentage change in the ratio of goods produced for export in the small open economy to foreign goods and an elasticity less than zero will imply a decreasing market share in real terms with regard to relative price changes. It is important to note that this interpretation hinges on the fact that the economy is relatively small and that the income effect of an export price increase in a relatively big economy like i.e. Germany cannot be neglected.

### 2.1 Some I(1) scenarios

Economic theory contributes in an important way to our empirical analysis by providing suggestions for possible explanatory variables and also what kind

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\(^3\)Provided that the profit is positive the first case could equivalently be presented by a shift of the demand curve to the left. For ease of exposition this possibility has deliberately been left out in the figure.
of basic relationships we may expect to find between them. The interpretation of such relationships will however typically be as long-run relationships. Given the non stationary nature of many of the relevant macro economic time series, such long-run relationships will be associated with the statistical concept of cointegration, which has the implication that an empirical long-run relation exists between the variables. To empirically substantiate economic theory, we will therefore have to require that the results of the cointegration analysis are consistent with theory. The cointegration analysis in this chapter is therefore based on the export volume and price equations referred to above and consistency requires, in the I(1) case, that there are at least two cointegrating relationships such that all disturbances in (1), (2) and (3) are I(0), i.e. stationary variables. If we find support for two and only two cointegrating relationships, this will especially require that export prices, unit labor costs and world market prices form a cointegrating linear combination, possibly with an additional demand effect from abroad. On the other hand we would also expect the export volume to be cointegrated with a linear combination of foreign real income and the relative price of world market prices to either export prices or unit labor costs. In the case of I(2) variables the analysis complicates somewhat as we in addition to have directly cointegrating vectors also will have relationships that cointegrate polynomially. This will be further dealt with in subsection 2.2 below.

To further elaborate on the implications theory consistency may have for cointegration in the case where we are dealing with I(1) variables, (2) may be reformulated as

\[ p_{at} - pw_t = c + \rho R_t + \varphi (ulc_t - pw_t) + \epsilon_{at} \]

First, let us assume that the logarithm of the ratio of unit labor costs to world market prices cointegrates. As theory consistency necessarily implies that \( \epsilon_{at} \sim I(0) \), this will then either imply relative purchasing power parity (RPP) or for \( \rho \) different from 0 and \( R \sim I(1) \), that the real exchange rate cointegrates with the world demand indicator. For \( \varphi \) different from 0, we see that the implication may also go in the other direction, as RPP in the case of \( \rho = 0 \) or \( R \sim I(0) \), then would imply constant wage or profit share in the external sector\(^4\). Looking at the two alternative volume equations, we have

---

\(^4\)The last assertion follows from the fact that the wage share is given by \( \frac{W}{PA} \), where \( W \) denotes the nominal wage level, \( N \) the number of wage takers and \( Y \) the level of production. \( PA \) denotes as before the export price. As unit labor costs, \( ULC \), are given...
that this, under the assumption of $\beta$ differs from 0 and $R \sim I(1)$, implies that real foreign income must cointegrate with the volume of exports.

Evidently, the imposition of theoretical restrictions leaves us with lots of degrees of freedom to identify theoretically consistent long-run structures in the $I(1)$ case. A more heuristic interpretation with regard to what is consistent and not together with the possibility of multicointegrating relationships in the case of $I(2)$ trends in the data, may in addition increase the possibility set further, examples in this respect being removal of homogeneity restrictions, exclusion of variables etc. To particularly look at the implications multicointegration may have for the identification scheme the next subsection presents an alternative scenario based on the assumption that export prices and unit labor costs are $I(2)$ and cointegrate to $I(1)$, that is that they are cointegrated $CI(2, 1)$.

2.2 Some $I(2)$ scenarios

The moving average representation of the VAR when dealing with $I(2)$ variables is in the general case given by$^5$:

$$X_t = C_2 \sum_{s=1}^{t} \sum_{i=1}^{s} (\varepsilon_i + \phi D_i) + C_1 \sum_{i=1}^{t} (\varepsilon_i + \phi D_i) + C(L)(\varepsilon_t + \phi D_t) + A + Bt, \quad (4)$$

where $C_2 = \beta_{12}(\alpha_{12} + \Theta \beta_{12})^{-1} \alpha_{12}'$, $C_1 = \beta \alpha' \Gamma C_2 + \beta_{11} \alpha_{11}'(I - \Theta C_2) + \beta_{12}$ $C_{12} \Delta X_t(1)$ and $\alpha_{11}, \alpha_{12}, \beta_{11}, \beta_{12}$ and $\Theta$ given respectively by $\alpha_{11} = \alpha_{11}', \beta_{11} = \beta_{11}'$, and $\Gamma = \beta \alpha' \Gamma + \sum_{i=1}^{k} i \Gamma_i$.

In the expressions for the different $C$ matrices the shorthand notation $\kappa = \kappa(k'k)^{-1}$ is used for $k$ being equal to respectively $\alpha$, $\alpha_{11}$, $\alpha_{11}'$, $\beta$, $\beta_{11}$, $\beta_{11}'$

$^5$The usual assumptions apply. That is that all unit roots of the characteristic polynomial, $|(1 - z)I - \Pi z - \sum_{i=1}^{k} \Gamma_i (1 - z)^2|$ lie at one or outside the unit circle, that the matrices $\Pi$ and $\alpha_{11}' \Gamma \beta_{11}$ have reduced rank such that $\Pi = \alpha' \beta$ and $\alpha_{11}' \Gamma \beta_{11} = \xi \eta'$ for matrices $\alpha$ and $\beta$ of dimension $p \times r$ and $\xi$ and $\eta$ of dimension $p \times s$, respectively, all of full rank, and finally that the matrix $\xi_{11}' \alpha_{11}' (\Gamma \beta \alpha' \Gamma + \sum_{i=1}^{k} i \Gamma_i) \beta_{11} \eta_1$ is of full rank, where $\beta = \beta(\beta' \beta)^{-1}$ and $\alpha = \alpha(\alpha' \alpha)^{-1}$.
and $\beta_{12}$ and the matrices $\alpha$, $\beta$, $\xi$ and $\eta$ are all defined in Footnote 5. $C_{\beta_{12}} \Delta x_i(z) = C_{\beta_{12}} \Delta x_i(1) + C_{\beta_{12}} \Delta x_i(z)(1 - z)$ is a convergent power series for $|z| < 1 + \delta$ for some $\delta > 0$ and constitutes the stationary part in the moving average representation of $\beta_{12} \Delta x_i$. $D_t$ is a deterministic term and may constitute a constant term, trend and dummies of various kinds. The coefficients $A$ and $B$ depend on the initial conditions and satisfy $(\beta, \beta_{11})' B = 0$ and $\beta' A - \alpha' \Gamma \beta_{12} \beta_{12}' B = 0$. For a proof the reader is referred to Johansen (1995a). Assuming that $\beta_{12} C_1 \alpha = 0$ such that $C_{\beta_{12}} \Delta x_i(1) = C_{\beta_{12}} \Delta x_i(2) \alpha'_{11} + C_{\beta_{12}} \Delta x_i(3) \alpha'_{12}$, and defining the common $I(2)$ trends as $\sum_{t=1}^T \sum_{i=1}^s u_{it}^2 = \sum_{t=1}^T \sum_{i=1}^s (u_{ii}, \ldots, u_{is})' = \sum_{t=1}^T \sum_{i=1}^s \alpha_{12} \varepsilon_i$ and $I(1)$ trends as $\sum_{t=1}^T u_i = \sum_{t=1}^T (u_{it}^{(2)}, u_{it}^{(1)})' = \sum_{t=1}^T ((\alpha_{12} \varepsilon_i)', (\alpha_{11} \varepsilon_i)')'$, where $u_{it}^{(2)} = (u_{ii}, \ldots, u_{is})'$ are the $s_2$ linear combinations of the errors that cumulate to an $I(2)$ trend and $u_{it}^{(1)} = (u_{(s_2+1)i}, \ldots, u_{(p-r)i})$ are the corresponding $p - r - s_2 = s_1$ linear combinations that cumulate to an $I(1)$ trend, (4) may alternatively be written as

$$X_t = \tilde{\beta}_{12} \sum_{s=1}^T \sum_{i=1}^s u_{it}^2 + [C_{11}, C_{12}] \sum_{t=1}^T (u_{it}^{(2)}, u_{it}^{(1)})' + C(L)(\varepsilon_t) + A + Bt \quad \text{(5)}$$

where we have deliberately suppressed the deterministic term, $D_t$, to make the representation more appropriate for a discussion of the cointegrating properties. In (5) $\tilde{\beta}_{12} = \beta_{12}(\alpha_{12} \theta \beta_{12})^{-1}$ and $C_{11}$ and $C_{12}$ respectively equal to $\overline{\beta} \alpha' \Gamma \beta_{12} = \beta_{12} \alpha'_{11} \Theta \beta_{12} + \overline{\beta}_{12} C_{\beta_{12}} \Delta x_i(2) \alpha_1' \alpha_{11}$ and $\beta_{11} \alpha_{11}^{-1} + \overline{\beta}_{12} C_{\beta_{12}} \Delta x_i(1)$.

To facilitate the econometric analysis and the economic interpretation of the subsequent empirical results of the $I(2)$ analysis for Germany, equation (6) below presents a moving average representation similar to (5) of the five dimensional system for exports implicitly defined by equations (1) to (3) above.
\[
\begin{bmatrix}
\alpha_t \\
p\alpha_t \\
w t \\
u l c_t \\
R_t \\
\Delta p\alpha_t \\
\Delta u l c_t 
\end{bmatrix} = \begin{bmatrix} 0 \\ \gamma_{21} \\ \gamma_{41} \end{bmatrix} + \sum_{s=1}^{t} \sum_{i=1}^{s} u_{1i} + \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \\ c_{51} & c_{52} & c_{53} \\ \gamma_{21} & 0 & 0 \\ \gamma_{41} & 0 & 0 
\end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t} u_{1i} \\ \sum_{i=1}^{t} u_{2i} \\ \sum_{i=1}^{t} u_{3i} \end{bmatrix} + X_0 \tag{6}
\]

In (6), \(X_0 = \tilde{C}(L) + A + Bt\) in (5), and the presence of three common trends out of which one is of a second order, is made upon the anticipation of subsequent empirical results. (6) also implies that only export prices and unit labor costs are \(I(2)\) and that a linear combination of the two reduces the order of non-stationarity from two to one. An implication of (6) is that neither export prices nor unit labor costs can separately enter into a cointegrating relationship. This implies particularly that (6) rules out the possibility of a purchasing power parity (PPP) relationship or perhaps more correctly denoted, the hypothesis of "one price," already at the outset. Assuming that exports ex post do not deviate from the ex ante planned level so that exports are given by a variant of (1), this seems either to imply that the concept of unit labor costs must be in real terms or that unit labor costs do not enter into the cointegrating relationship at all. The latter case could perhaps more likely be taken to mean that exports are determined exclusively by demand ex post without changes in own prices affecting the output. Assuming that the long-run export volume relationship constitutes a directly cointegrating relationship, this would therefore imply that the linear relation \(\alpha_t - \omega_1 w t - \omega_2 R_t\) is stationary. For this to be the case we must have that the vector \((1, 0, -\omega_1, 0, -\omega_2, 0, 0)\) is a cointegrating vector and hence that it is orthogonal to all the vectors in the matrices in (6). Thus we must have that

\[
c_{1j} - \omega_1 c_{3j} - \omega_2 c_{5j} = 0, \ j = 1, 2, 3,\]

for weights \(\omega_1\) and \(\omega_2\). The first situation, however is slightly more elaborate and implies that the linear combination \(\alpha_t - \omega_1 w t - \omega_2 R_t + \omega_3 (\gamma_{21} u l c_t - \gamma_{41} p\alpha_t)\) is stationary. This implies that the vector

\[
(1, -\omega_3 \gamma_{41}, -\omega_1, \omega_3 \gamma_{21}, -\omega_2, 0, 0)
\]
must be a cointegrating one and hence that also this is orthogonal to the matrices in (6) so that

\[ c_{1j} - \omega_3 \gamma_{41} c_{2j} - \omega_1 c_{3j} + \omega_3 \gamma_{21} c_{4j} - \omega_2 c_{5j} = 0, \quad j = 1, 2, 3, \]

where \( \omega_3 \) is the weight given to the cointegrating CI(2, 1) linear combination of unit labor costs and export prices. Assuming a polynomially cointegrating export price relationship the most plausible candidate would be that the CI(2, 1) linear combination of export prices and unit labor costs cointegrates with world market prices, the indicator of foreign demand and growth in either export prices or unit labor costs. This implies that the linear combination

\[ \omega_3 (\gamma_{41} p a_t - \gamma_{21} u lct) - \omega_1 p w_t - \omega_2 R_t - \bar{\alpha}' \Gamma \bar{\beta}_{12} \bar{\beta}'_{12} x \]

is stationary, \( x \) representing either relative growth in export prices, \( \Delta p a_t \), or in unit labor costs, \( \Delta u lct \). Thus the cointegrating vector is given by

\[ (0, \omega_3 \gamma_{41}, -\omega_1, -\omega_3 \gamma_{21}, -\omega_2, -\bar{\alpha}' \Gamma \bar{\beta}_{12} \bar{\beta}'_{12}, 0) \]

or

\[ (0, \omega_3 \gamma_{41}, -\omega_1, -\omega_3 \gamma_{21}, -\omega_2, 0, -\bar{\alpha}' \Gamma \bar{\beta}_{12} \bar{\beta}'_{12}) \]

depending on which of the growth rates enters in the cointegrating relationship, and its orthogonality property implies that

\[ \omega_3 \gamma_{41} c_{2j} - \omega_3 \gamma_{21} c_{4j} - \omega_1 c_{3j} - \omega_2 c_{5j} = 0, \quad j = 2, 3 \]

and

\[ \omega_3 \gamma_{41} c_{2j} - \omega_3 \gamma_{21} c_{41} - \omega_1 c_{31} - \omega_2 c_{51} - \bar{\alpha}' \Gamma \bar{\beta}_{12} \bar{\beta}'_{12} \gamma_{j1} = 0, \quad j = 2 \text{ or } 4 \]

Normalizing on export prices such that \( \omega_3 = \frac{1}{\gamma_{41}} \), these conditions might equivalently be expressed as \( c_{22} = \frac{2a_1}{\gamma_{41}} c_{42} + \omega_1 c_{32} + \omega_2 c_{52} \), \( c_{23} = \frac{2a_1}{\gamma_{41}} c_{43} + \omega_1 c_{33} + \omega_2 c_{53} \) and that \( c_{21} = \frac{2a_1}{\gamma_{41}} c_{41} + \omega_1 c_{31} + \omega_2 c_{51} - \bar{\alpha}' \Gamma \bar{\beta}_{12} \bar{\beta}'_{12} \gamma_{j1} = 0 \), for \( j = 2 \) or \( j = 4 \). Anticipating the result of the subsequent analysis it is particularly interesting to look at the case where \( \gamma_{41} = \gamma_{21} \) and \( \omega_1 = \omega_2 = 0 \). That is, neither world market prices nor world demand enter into the multicointegrating relationship and the spread between unit labor costs and exports prices is cointegrated CI(2, 1). If so, the implied restrictions must be that \( c_{22} = c_{42}, c_{23} = c_{43} \) and finally that \( c_{21} - \omega_3 c_{41} - \bar{\alpha}' \Gamma \bar{\beta}_{22} \bar{\beta}'_{22} \gamma_{j1} = 0 \).
3 Data and time series properties

Before presenting the results of the cointegration analysis, I will in this section first draw attention to a brief description of the empirical data set, herein undertaking a preliminary analysis with regard to time series properties of the individual data. I will concentrate on commenting on the German data as data for Norwegian exports have been extensively commented on in Chapter 3.

The econometric analyses are based on quarterly seasonally unadjusted data for the period 1960 (1) to 1998 (4) in the case of Germany and for 1979 (2) to 1998 (2) for Norway\(^6\). The data set consists of observations on the following empirical proxies of the theoretical counterparts\(^7\):

\[ a_j \] Aggregate export volume for Germany \((a)\), the trading sector \((j = 1)\), and the service sector \((j = 2)\) of Norway.

\[ p_{aj} \] Aggregate export prices for Germany \((pa)\), the trading sector \((j = 1)\), and the service sector \((j = 2)\) of Norway.

World market prices in domestic prices. A weighted average of GDP-deflators for the nine, respectively ten, most significant trading partners of Norway and Germany.

\[ pw \] Unit labor costs

\[ R \] World demand indicator. A weighted average of GDP, respectively imports of the same trading partners as for world market prices of Norway and Germany.

The weights used to create the world demand, world price and an effective exchange rate, the last one used to convert world market prices into their do-

\(^6\)As unadjusted data were not available for unit labor costs of Germany these have been included seasonally adjusted. However, this is deemed less serious as unit labor costs should not show a strong seasonal pattern.

\(^7\)In the whole chapter I will stick to the convention of using small letters for variable names when in fact the variables are logarithmic transformations of the original series, the only exception being the indicator of world demand where \(R\) also indicates a logarithmic transformation.
mestic currency equivalents, have been the average share of total exports exported to individual trading partners. Plots of all German series, both levels and first differences, are shown in the graphical part of the appendix. Based on graphical inspection there is scarce evidence of I(2)-ness in the data. The series closest in agreement with such a description perhaps are export prices and unit labor costs. However, to further investigate the issue of whether the data are I(1) or I(2) one will have to formally determine the orders of integration by thorough testing. In the appendix I therefore anticipate events somewhat by first presenting the results of testing for stationarity of order one within a multivariate framework based on the methodology developed by Johansen for estimation and identification of cointegrating relationships (Johansen (1988), (1995a)). The model used is the same as in Section 4 when identifying the cointegrating relationships when data are supposed to lie in the I(1) space. The tests are therefore conditional on two cointegrating relationships and differ in a very important respect from univariate Dickey-Fuller tests by testing the null of stationarity against non-stationary alternatives. The test statistics are the LR-tests of restrictions on the cointegrating space and imply particularly testing the hypotheses that one of the cointegrating vectors consists of zero coefficients for all variables except for the coefficient of the variable we want to test for stationarity. In Table 9 the coefficient of the restricted trend has also been left unrestricted implying in fact that we are testing the null of trend-stationarity versus non-stationary alternatives. These system-tests are superior to univariate testing for stationarity of individual time series. However, due to a generic bias towards these tests among time series econometricians, I have in the same appendix chosen also to present the results of univariate Augmented Dickey-Fuller (ADF) tests. To avoid the problem of nuisance parameters in the DGP these univariate ADF-tests are made similar, implying the joint appearance of a trend and a constant term in the specification of the autoregressiv equation. To get rid of as many anomalies as possible, I have in addition included seasonal dummies. To avoid the problem of having to deal with a possible quadratic trend under the alternative, testing the null of I(2) vs. the alternative of I(1), however, has been done by only including a constant term in the equation. To be able to fully address the issue of higher order integration, however, I have finally

\footnote{In the German data the weights used are based on data on exports to individual countries for the period 1960-1980. It is a weakness that these weights are old and that for some countries they turn out to be highly unstable. Notably for the US where the weights show a significantly increasing tendency.}
chosen to present a full analysis of the cointegrating indices based on the multivariate two step I(2) procedure developed by Johansen (1995b).

The multivariate test statistics for stationarity in Table 9 of the appendix strongly suggest rejection of the null of stationarity for all the variables. This is further confirmed by looking at the Augmented Dickey-Fuller tests of the subsequent table, Table 10, which are not able to reject the null of a non-stationary I(1) process for any of the variables. With regard to a possible second order trend the results of univariate testing are far from promising. All tests reject the null of a second order trend to a level far below one percent. On the other hand looking at the multivariate tests of the cointegration indices as reported in Table 1 below, the tests clearly indicate that there is a second order trend in the information set, however. These multivariate tests have been carried out by specifying a five dimensional VAR of order five, where a drift term has been restricted to lie in the cointegrating space and a constant restricted not to induce quadratic trends in the process. In addition to centered seasonal dummies the specification involves two unrestricted dummies out of which one implies a shift in the constant term and the other a transitory shift in levels\(^9\). Thus, our model does not contain intervention dummies that cumulate to trends in the DGP and therefore potentially might invalidate inference based on standard asymptotic tables (Johansen and Nielsen (1994)). We should therefore be able to proceed by using the asymptotic tables for the I(2) model of Paruolo (1996). The procedure starts testing from the upper left corner of Table 1 at a null of five common I(2) trends against the alternative of less than or equal to full rank. If this first test statistics is bigger than the 95 percent fractile given in italics under each statistics, the procedure continues towards the right reducing the number of common I(2) trends under the null by one. This process goes on to the end of the first row and proceeds similarly row-wise from left to right until the test statistics is insignificant to a level of five percent, in which case the cointegration indices are jointly identified by a rank equal to \(r\), the number of I(1) trends under the null, \(s_1\), and the number of I(2) trends given by \(p - r - s_1 = s_2\). In Table 1, this process of rejection does not end until the number of common trends are equal to three and the number of I(1) trends are identified to two, implying that the number of common I(2) trends are equal to \(p - r - s_1 = 5 - 2 - 2 = 1\). This finding is in accordance with

\(^9\)The reader is referred to Section 4 and footnote 11 for a more complete presentation of the model.
Table 1: The trace test of cointegrating indices for German exports

<table>
<thead>
<tr>
<th>p-r</th>
<th>r</th>
<th>S_{r,s}</th>
<th>Q(R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0</td>
<td>309.20</td>
<td>227.97</td>
</tr>
<tr>
<td></td>
<td>198.2</td>
<td>167.9</td>
<td>142.2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>187.73</td>
<td>126.90</td>
</tr>
<tr>
<td></td>
<td>137.0</td>
<td>113.0</td>
<td>92.2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>102.75</td>
<td>70.60</td>
</tr>
<tr>
<td></td>
<td>86.7</td>
<td>68.2</td>
<td>53.2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>49.72</td>
<td>23.68</td>
</tr>
<tr>
<td></td>
<td>47.6</td>
<td>34.4</td>
<td>25.4</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>28.66</td>
<td>5.32</td>
</tr>
</tbody>
</table>

\[ \text{The figure in italics under each test statistic is the 95 per cent fractile as tabulated by Paruolo(1996). The preferred outcome of the sequential testing is marked by a star.} \]

\[ \text{The multivariate tests have been carried out by specifying a five dimensional VAR for the variables } a, pa, pw, ulc \text{ and } R \text{ of order five. A drift term has been restricted to lie in the cointegrating space and a constant included in such a way that it does not induce quadratic trends in the process. In addition to centered seasonal dummies the specification also involves two unrestricted dummies of which one implies a shift in the constant term and the other a transitory shift in levels.} \]

the suggested scenario of Section 2 and will form the basis of the analysis to come.

4 The analyses

In Section two I presented the moving average representation of the I(2) model in the general case when no restrictions are imposed on the deterministic terms. In general, if \( X_t \) is integrated of order two and a linear regressor as well as a constant is included unrestrictedly in the model, this would allow for cubic as well as quadratic trends in the process governing the data.
while in the case \( X_t \) is integrated of order one an unrestricted trend term would generate a quadratic trend. As these peculiarities are not very likely to prevail in practice we will have to place restrictions on the deterministic components of the model. In the \( I(1) \) model this implies that the linear regressor has been restricted to lie in the cointegrating space while the constant, seasonals and dummies have been left unrestricted. This restriction has also the advantage of making inference similar with respect to the level and linear trend parameters of the DGP. As for the \( I(2) \) models the constant regressor has in addition been restricted not to generate a quadratic trend and in such a way that it allows for linear trends in all linear combinations of \( X_t \). Keeping this in mind and suppressing the deterministic terms, the general error correction form of the \( I(2) \) model

\[
\Delta^2 X_t = \alpha \beta' X_{t-1} - \Gamma \Delta X_{t-1} + \sum_{i=1}^{k-2} \psi_i \Delta^2 X_{t-1} + \varepsilon_t
\]

may be given the equivalent representation to be used as reference in the following when interpreting the results of the \( I(2) \) analyses\(^{10}\):

\[
\Delta^2 X_t = \tilde{\alpha}_1 \tilde{\beta}'_1 X_{t-1} + \tilde{\alpha}_2 (\tilde{\beta}'_2 X_{t-1} + \tilde{\delta} \beta'_{12} \Delta X_{t-1}) + \tilde{\kappa} \beta' \Delta X_{t-1} + \tilde{\kappa} \beta'_{11} \Delta X_{t-1} + \sum_{i=1}^{k-2} \psi_i \Delta^2 X_{t-1} + \varepsilon_t
\]  

(7)

In (7) \( \tilde{\beta}'_1 X_{t-1} \) denotes the directly cointegrating \( CI(2,2) \) relationships while \( \tilde{\beta}'_2 X_{t-1} + \tilde{\delta} \beta'_{12} \Delta X_{t-1} \) are the multicointegrating relationships. Their respective loading matrices are denoted by \( \tilde{\alpha}_1 \) and \( \tilde{\alpha}_2 \). More details with regard to the specified VARS will be given in connection with the country specific analysis below.

\(^{10}\) In 7 the different parameters as functions of the original parameters are given by:

\[
\tilde{\kappa} = \left[ \alpha \psi \tau + (\alpha' \Omega^{-1} \alpha)^{-1} \alpha' \Omega^{-1} \tilde{\alpha}_\perp + \Omega \tilde{\alpha}_\perp (\alpha'_\perp \Omega \alpha_\perp)^{-1} \right] \kappa', \quad \tilde{\beta}'_1 = \delta' \rho \tau', \quad \tilde{\beta}'_2 = \delta' \rho \tau', \quad \tilde{\alpha}_1 = \alpha \delta \tilde{\alpha}_\perp, \quad \tilde{\alpha}_2 = \alpha \delta \text{ and } \tilde{\delta} = \delta' \delta, \text{ where } \rho' = (I_{r \times r}, 0_{r \times s}), \quad \rho = (\beta, \beta_\perp), \quad \psi' = -\alpha' \Gamma \text{, } \delta = \psi \tau \text{ and } \kappa' = -\alpha'_\perp \Gamma. \text{ The result follows by straightforward use of the identity } a(\alpha' \Omega^{-1} \alpha)^{-1} a' \Omega^{-1} \alpha_\perp = I \text{ in combination with the orthogonal projection identities } P_\Theta + P_{\Theta_\perp} = \Theta (\Theta' \Theta)^{-1} \Theta' + \Theta_\perp (\Theta'_\perp \Theta_\perp)^{-1} \Theta'_\perp = I \text{ for } \Theta = \tau, \delta, \alpha.
4.1 Germany

The analysis of German exports is based on a five dimensional VAR of order five. In addition to the restrictions alluded to above with regard to deterministic regressors, the model has been specified with two dummies, respectively, D7334 and D741. While the first one effectuates a transitory shift in the levels of the series in the third quarter of 1973, the second imposes a permanent shift in the constant term in the first quarter of 1974. The single equation diagnostics of the system are given in the upper part of Table 11 of the appendix and except for some hardly significant signs of conditional heteroscedasticity in the process governing German exports, they seem to fulfil most requirements for a congruent representation of a DGP. The system tests indicate though a marginal problem with regard to normality. This seems however mainly to originate from excess kurtosis and based on the results of Gonzalo(1994) is therefore deemed less serious. Also, even though there seem to be some problems in the last part of the seventies/early eighties, the recursively estimated Chow tests of the appendix do not reveal any ominous signs of parameter instability. Thus our VAR should be a good starting point for identification and estimation of cointegrating relationships.

The ordinary trace test statistics of Table 2 clearly identify the presence of at least two cointegrating vectors. We also note that two of the estimated eigenvalues of the $\Pi - I$ matrix are quite big consistent with three cointegrating vectors. However, the eigenvalues of the companion matrix indicate that imposing two unit roots leaves an unrestricted root with a large modulus, 0.965, in the model. Three unit roots are consistent with either two cointegrating vectors or in the case of three cointegrating vectors, that one of the three common trends is a trend of second order, $I(2)$. As seen from the Table, imposing a third unit root still leaves us with an unrestricted eigenvalue with a relatively large modulus, 0.942. As a unit root in the characteristic polynomial that belongs to an $I(2)$ trend cannot be removed by lowering the number of cointegrating vectors, this is certainly an indication of $I(2)$-ness. However, as mentioned in the introduction the aim of this analysis is to un-

---

11 To be more specific this means that the two dummies are model specific. In the $I(1)$ case this means that the dummy, $D7334$, is given by $(...,0,1,-2,1,0,....)$ such that when it is cumulated ones it assumes the values 1 in 1973 (3), -1 for 1973 (4) and nil otherwise. The dummy $D741$ is a blip dummy equaling 1 in 1974 (1) and nil otherwise and cumulates therefore to a level shift in 1974 (1). The corresponding dummies in the $I(2)$ models are given respectively by $(...,0,1,-3,3,-1,0,....)$ and $(...,0,1,-1,0,....)$ such that when they are cumulated twice they assume the same values as for the cumulated $I(1)$ case.
veil generic properties of the DGP and not to design a model that may better explain certain local phenomena. Before embarking on the more complicated $I(2)$ analysis I have therefore chosen first to undertake an analysis based on the more plausible presumption of $I(1)$. The outcome of this analysis will then function as a basis for comparison with other analyses and particularly with the one made on the assumption of a second order trend in the data.

4.1.1 The $I(1)$ case

The results of the $I(1)$ analysis based upon the presumption of two cointegrating vectors are given in Table 3 below. The first thing to notice is the lack of unit labor costs and the relatively strong effects of changes in international demand and prices in the volume equation. Furthermore, the export price equation is a pure markup relationship over unit labor costs and implies that effects from international conditions play a negligible role in the long run. If this is correct it means that German exporters when setting their prices, almost act as though the rest of the world does not exist and feel free to pass increases in unit labor costs on to prices without even considering international demand and price conditions. On the other hand, exports seem to depend heavily on international factors and even though the relationship is not of an Armington type, it suggests, as in the Norwegian case, an ex post interpretation of its origin. Why it is the nominal and not the real world market price denoted in units of the export price that enters in the equation is a conundrum. However, it may indicate that German exporters have some kind of money illusion.

With regard to the loadings the most puzzling artifact is perhaps the strongly significant positive error correction of the discrepancy of the aggregate export price from its long-run solution in the equation for unit labor costs. Even though such an effect may seem a little bit far fetched, it may be explained if expected (as opposed to unexpected) hikes in export prices are perceived as the result of excessive wage claims made by trade unions in their tripartite negotiations with the employers associations. Another puzzling effect though not significant, is the corresponding negative weight in the equation for foreign demand. Otherwise, we see that there is significant error correction in the export price equation from deviations of export prices from their long-run solution and a close to significant error correction in the volume equation from the deviation of exports from its equilibrium level$^{12}$.

$^{12}$In this context it is worth mentioning that a simultaneous reduction, incorporating
System: a, pa, pw, R, ulc.
Deterministic part: Restricted Trend, Unrestricted Constant, Centered Seasonals and the dummies D741 and D7334.
VAR order: 5. Effective Sample period: 1961 (2)-1998 (4)

<table>
<thead>
<tr>
<th>Common trends imposed</th>
<th>Modulus of the eigenvalues of the companion form and the estimated eigenvalues of the II matrix:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.986 0.986 0.901 0.901 0.889</td>
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<tr>
<td>1</td>
<td>1 0.988 0.988 0.895 0.895</td>
</tr>
<tr>
<td>2</td>
<td>1 1 0.965 0.965 0.885</td>
</tr>
<tr>
<td>3</td>
<td>1 1 1 0.942 0.942</td>
</tr>
<tr>
<td>4</td>
<td>1 1 1 1 0.914</td>
</tr>
</tbody>
</table>

Eigenvalues of the II matrix: 0.941 0.918 0.847 0.781 0.726

Trace Eigenvalue Tests: $-2\ln(Q) = T (\log(\det(\Omega(p))) - \log(\det(\Omega(r))))$

<table>
<thead>
<tr>
<th>Null $r=0$</th>
<th>Alternative $r \leq 5$</th>
<th>Test Statistics</th>
<th>95% Critical values</th>
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</thead>
<tbody>
<tr>
<td>$r \leq 1$</td>
<td>$r \leq 5$</td>
<td>133 **</td>
<td>87.3</td>
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<tr>
<td>$r \leq 2$</td>
<td>$r \leq 5$</td>
<td>84.6**</td>
<td>63.0</td>
</tr>
<tr>
<td>$r \leq 3$</td>
<td>$r \leq 5$</td>
<td>47.3*</td>
<td>42.4</td>
</tr>
<tr>
<td>$r \leq 4$</td>
<td>$r \leq 5$</td>
<td>22.16</td>
<td>25.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.213</td>
<td>12.3</td>
</tr>
</tbody>
</table>

Table 2: Rank tests and modulus of eigenvalues of the companion form for the German system of exports
### Restricted cointegrated vectors in the $I(1)$ model

<table>
<thead>
<tr>
<th>$a$</th>
<th>$pa$</th>
<th>$pw$</th>
<th>$ulc$</th>
<th>$R$</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1'$</td>
<td>1</td>
<td>0</td>
<td>-2.9425</td>
<td>0</td>
<td>-3.5469</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.412)</td>
<td>(0.341)</td>
<td>(0.0065)</td>
</tr>
<tr>
<td>$\beta_2'$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Loading matrix

<table>
<thead>
<tr>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0382 (0.0255)</td>
<td>-0.0037 (0.0037)</td>
<td>0.0285 (0.0118)</td>
<td>-0.0456 (0.0104)</td>
<td>0.0182 (0.0192)</td>
</tr>
<tr>
<td>-0.0645 (0.0638)</td>
<td>-0.0349 (0.0093)</td>
<td>0.0142 (0.0295)</td>
<td>0.0834 (0.0261)</td>
<td>-0.0828 (0.0481)</td>
</tr>
</tbody>
</table>

LR-test, rank=2: $\chi^2(4) = 1.1979$ [0.8784]

Table 3: The restricted cointegrated vectors, loadings and test for the overidentifying restrictions in the German model of exports
As alluded to above a typical sign of higher order common trends is that lowering the number of cointegrating vectors does not remove an additional unit root associated with $I(2)$-ness. Another sign is that the graphs of the concentrated and non-concentrated cointegration relations exhibit significantly different behavior, in particular if the former looks more stationary than the latter. Before proceeding to the $I(2)$ analysis, it is therefore worth taking a closer look at Figures 2 and 3 which depict the graphs of the concentrated and not-concentrated restricted cointegrating relations where the upper panels contain the uncorrected relations, $\beta'X_t$, and the lower panels the cointegration relations corrected for short-run dynamics, $\beta'R_{tt}$. With regard to the first cointegrating relationship the two graphs do not seem to differ significantly. The second however reveals significant differences where the concentrated cointegrating relation looks considerably more stationary than the un-concentrated one. Coupled with the fact that we were not able to get rid of an extra unit root by lowering the number of cointegrating vectors, this clearly legitimates further investigation along the dimension of a potential second order common trend.

4.1.2 The $I(2)$ case

Table 4 gives the unrestricted outcome of the $I(2)$ analysis when imposing three common trends of which one is of a second order. The first thing to notice is that the common $I(2)$ trend mainly seem to feed into export prices and unit labor costs. The $I(2)$ trend itself originates from a linear combination of residuals in export prices, world market prices and unit labor costs and can thus be characterized as a purely nominal trend. In light of theory, the directly cointegrating relationship denoted in the table as a stationary linear combination of levels, is close to the outcome of monopolistic competition, the main difference being that there seems to be some measure of real unit labor costs that enters into the relationship. If so, this means that an increase in unit labor costs will only affect output insofar the increase also implies an increase in “real” terms. The multicointegrating relationship is interpretable as a monopolistic price setting rule. Getting rid of the export volume coefficient by multiplying the first directly cointegrating vector $\tilde{\beta}_1$ by 0.339 and

the identifying restrictions of Table 3 together with zero restrictions on all loadings except for $\alpha_{31}$, $\alpha_{21}$, $\alpha_{22}$, and $\alpha_{41}$, is marginally significant to a level of five per-cent with a $p$-value equal to 0.0465.
Figure 2: Concentrated, $\beta_1'Rk(t)$, and not-concentrated, $\beta_1'Zk(t)$, restricted cointegrating relation number 1

Figure 3: Concentrated, $\beta_2'Rk(t)$, and not-concentrated, $\beta_2'Zk(t)$, restricted cointegrating relation number 2
adding it to the second concerning the level part of the multicointegrating
relationship, $\hat{\beta}_2$, gives namely the relationship:

$$\hat{p}a_t = 1.48pw_t + 0.4396ulc_t + 1.156R_t - 0.023Trend$$

The polynomial part seems to be dominated by the coefficients of growth
in prices and/or unit labor costs. However, following the suggested proce­
dure in Rahbek et al (1999), these coefficient estimates can be made more
interpretable by adding stationary relations to $\beta'_2X_t + \delta'\beta'_2\Delta X_t$. That is, to
combine $\beta'_2X_t$ with linear combinations, $\nu'\Delta X_t$, where $\nu$ is a $p \times (p - r - s)$
matrix such that $\nu'\beta'_2$ has full rank, see Johansen (1992). This may be
 clearer if we consider the alternative relation, $\beta'_2X_t + \delta'\nu'\Delta X_t$, and rewrite
it as in Rahbek et al, as

$$\beta'_2X_t + \delta'\nu'\beta'_2\Delta X_t + \delta'\nu'(\beta, \beta_1')(\beta, \beta_1')\Delta X_t$$

The last term is stationary as $(\beta, \beta_1)'X_t$ is $I(1)$, and the first terms define a
polynomially cointegrating relation if $\delta'\nu'\beta'_2 = \delta$.

Focusing on the role of export prices, I will assume that $\nu = (0,1,0,0,0)'$
is a valid choice. Based on the estimated coefficients, this means that the
loading to $\nu$ is $\delta = \delta'\beta'_2\beta'_2(\nu'\beta'_2)^{-1} = 16.175$ and the polynomially cointe­
grating relation is therefore given by:

$$\hat{p}a_t - 1.48pw_t - 0.4396ulc_t - 1.156R_t + 0.023Trend + 16.175\Delta pa_t.$$  (8)

(8) is a dynamic export price relation and could readily constitute an error-
correction model in an $I(1)$ framework. However, in this context (8) is a
polynomially cointegrating relation and functions as an error correction term
in the $I(2)$ model.

In the $I(1)$ analysis we were able to identify a system where unit labor
costs did not enter into the volume equation at the same time as export
prices are pure markup relationships over unit labor costs. To investigate
whether this also could be the case when having to deal with a second order
trend I present below the results when these restrictions are imposed on the
level part of the cointegrating vectors. Looking at Table 5, the first thing
to notice is the LR-test for overidentifying restrictions which is not able to
reject the null of correctly imposed restrictions as the p-value is equal to
0.24. Otherwise, the results are strikingly similar to the results of the $I(1)$
Table 4: Unrestricted estimates in the I(2) model for Germany

\[ pa_t - ulc_t + 10.926\Delta pa_t + 0.0013Trend. \quad (9) \]

Figure 4 compares the second cointegrating relation of the I(1) case with the multico-integrating relationship (9). Even though different scales contribute to give a somewhat biased impression, it is readily seen that the new long-run relationship appears much more stationary than the old static
Table 5: Restricted estimates in the I(2) model for Germany

Figure 4: Static cointegrating relationship vs. multicointegrating relationship for the German economy.
one. The residual in 1974 first quarter has been properly dealt with by the inclusion of the transitory intervention dummy $D741$ in the VAR.

### 4.2 Norway

In Section 3 I deliberately avoided commenting on the time series properties of the Norwegian data as these have been extensively commented on elsewhere. However, to briefly summarize the results, neither system tests nor univariate Dickey Fuller tests are able to question the inherent non-stationarity of the data. The only matter for concern thus seems to be whether this non-stationarity might be of a higher order or not. Table 6 below reproduces the results with regard to determination of cointegration indices of the pooled seven-dimensional analysis in Chapter 3, where the data in addition to including exports and export prices of both the service and trading sectors, consist of unit labor costs and indicators of world market prices and world demand. As can be seen from the table, there is slight evidence of three common trends of which one is of a second order. However, the rejection of a common $I(2)$ trend is only marginally insignificant and if one is willing to reject at a level of close to ten per cent, the outcome could easily be accepting as many as six cointegrating vectors among the variables in the information set. If so, this would be totally in line with the outcome of the two-step analysis in Chapter 3 designed particularly to deal with identification of cointegrating vectors in the case of times series with a small cross sectional dimension.

Looking at Table 7 below, there does not seem to be a problem of getting rid of a potential high additional unit root. Already, after imposing the first, the second largest has namely a modulus significantly below 0.9. Also, looking at the graphs of the concentrated and un-concentrated restricted cointegrating relations in the appendix does not reveal that the un-concentrated series exhibit a significantly different behavior from the concentrated ones. In total therefore, there seem to be little evidence of higher order non-stationarity and I have thus chosen to present the outcome of an analysis based on the existence of no less than six cointegrating vectors and a com-

---

*13 To be able to fit the Table in the text, the numbers have been rounded off to their nearest integer representation.
Table 6: The trace test of cointegrating indices for the Norwegian pooled data

<table>
<thead>
<tr>
<th>p-r</th>
<th>r</th>
<th>S_{r,s}</th>
<th>Q_r</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0</td>
<td>553 466</td>
<td>392 288</td>
</tr>
<tr>
<td></td>
<td></td>
<td>352 311</td>
<td>274 241</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>445 359</td>
<td>287 225</td>
</tr>
<tr>
<td></td>
<td></td>
<td>269 234</td>
<td>203 175</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>340 260</td>
<td>188 152</td>
</tr>
<tr>
<td></td>
<td></td>
<td>198 168</td>
<td>142 120</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>237 165</td>
<td>104 78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>137 113</td>
<td>92 75</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>138 72</td>
<td>50* 38*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>87 68</td>
<td>53 42</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>58 37</td>
<td>21*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>47 34</td>
<td>25</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>21 6*</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>20 13</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 is based upon a seven dimensional VAR of order three for the variables a1, a2, pa1, pa2, pw, ulc and R. A drift term has been restricted to lie in the cointegrating space and a constant is included such that it does not induce a quadratic trend in the process.

The figure in italics under each test statistic is the 95 per cent fractile as tabulated by Paruolo(1996). The non-significant test statistics are marked with stars.
Table 7: Moduli of the eigenvalues of the companion matrix under the imposition of common trends

<table>
<thead>
<tr>
<th>Common trends</th>
<th>Moduli</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9005 0.9005 0.8464 0.8464 0.8218 0.8218</td>
</tr>
<tr>
<td>1</td>
<td>1 0.8618 0.8618 0.8440 0.8440 0.7371</td>
</tr>
<tr>
<td>2</td>
<td>1 1 0.8761 0.8761 0.7312 0.7312</td>
</tr>
<tr>
<td>3</td>
<td>1 1 1 0.8014 0.7435 0.7435</td>
</tr>
<tr>
<td>4</td>
<td>1 1 1 1 0.7392 0.7392</td>
</tr>
</tbody>
</table>

mon trend of order one. For an account of the identification scheme the interested reader is referred to Chapter 3.

The first point to notice is that the hypothesis of one price does not appear to be supported by the data. On the contrary, the empirical results of Table 8 indicate that small open economies like the Norwegian, still have a considerable degree of monopolistic power in the export market when setting their prices. Also, both sectors' export volumes seem to be totally driven by demand, which is the case when agents accommodate demand ex post for prices fixed ex ante. Finally, and perhaps even more interesting is the finding of strong long-run links across sectors both with regard to the determination of prices and the determination of volume. Equation (5) for instance is a cointegrating relationship between the two sectors' export prices, saying that export prices in the service sector grow at approximately a yearly rate of 0.8 per cent faster than in the trading sector. This finding is completely in accordance with the perceived view of a more competitive environment in the trading sector. Likewise, equation (6) which is a cointegrating relationship between the two sectors' export volumes, implies that exports grow at a yearly rate of approximately 3.6 per cent faster in the trading sector than in the service sector. This also coincides well with another perceived view: namely that the trading sector is the main origin for innovative productivity improvements.
Eq.: Cointegrating relationships:

1: \( a_1 = \text{const.} -0.533(pa_1-pw) +2.366R \)  
\( (0.040) \)  
\( (0.082) \)

2: \( pa_1 = \text{const.} + 0.657ulc \)  
\( (0.033) \)

3: \( a_2 = \text{const.} + R \)

4: \( pa_2 = \text{const.} + 0.474R +0.542ulc \)  
\( (0.041) \)  
\( 0.033 \)

5: \( pa_1 = \text{const.} + pa_2 - 0.002\text{Trend} \)  
\( (0.0005) \)

6: \( a_1 = \text{const.} + a_2 +0.008 \text{Trend} \)  
\( (0.0007) \)

**LR-tests:**

All overidentifying restrictions: \( \chi^2(5) = 3.98[0.55] \)

Table 8: Restricted long-run relationships in the pooled analysis when all parameters have been estimated freely.
5 Summary and Conclusions

In this chapter I have tried to unveil the degree of independence in European goods markets. The results are rather divergent. On the one hand there is strong evidence of monopolistic power in the process governing external prices among European exporters. The perceived view that shocks to supply may crowd out the foreign sector is therefore seriously called into question as the effects of wage hikes, intermediate price shocks etc. readily can be passed on to prices. Further, as this makes goods arbitrage ineffective, hypotheses like PPP and "the law of one price" are of course far from being confirmed. It is important to point out that this lack of arbitrage may have the effect of increasing the legitimacy of policies geared towards the management of domestic demand in case of severe recessions. On the other hand, exports seem to be extremely vulnerable to changes in international demand and world market prices which helps explain entrepreneurs' cry for arrangements geared towards shielding the sector from the vicissitudes of international trade conditions.

References


A  Data and tests

Figure 5: German export prices in levels and differences
Figure 6: World market prices in levels and differences

Figure 7: Unit labor costs in levels and differences
Figure 8: World demand in levels and differences

Figure 9: Recursively estimated Chow tests for German exports
Figure 10: Recursively estimated Chow tests for German export prices.

Figure 11: Recursively estimated Chow tests for the World market price equation.
Figure 12: Recursively estimated Chow tests for parameter stability of the German unit labor costs equation

Figure 13: recursively estimated Chow-tests for parameter stability of the world demand equation for Germany
Figure 14: Concentrated, $\beta_1/R(t)$, and unconcentrated, $\beta_1/Z(t)$, restricted cointegrating relation number 1 in the Norwegian study.

Figure 15: Concentrated, $\beta_2/R(t)$, and unconcentrated, $\beta_2/Z(t)$, restricted cointegrating relation number 2 in the Norwegian study.
Figure 16: Concentrated, $\beta_3 Rk(t)$, and unconcentrated, $\beta_3 Zk(t)$, restricted cointegrating relation number 3 in the Norwegian study.

Figure 17: Concentrated, $\beta_4 Rk(t)$, and unconcentrated, $\beta_4 Zk(t)$, restricted cointegrating relation number 4 in the Norwegian study.
Figure 18: Concentrated, \( \text{beta}_5 R_k(t) \), and unconcentrated, \( \text{beta}_5 Z_k(t) \), restricted cointegrating relation number 5 in the Norwegian study.

Figure 19: Concentrated, \( \text{beta}_6 R_k(t) \), and unconcentrated, \( \text{beta}_6 Z_k(t) \), restricted cointegrating relation number 6 in the Norwegian study.
Table 9:
Multivariate statistics for testing stationarity of the German data
Two cointegrating vectors and trend in CI space

<table>
<thead>
<tr>
<th>Variables</th>
<th>a</th>
<th>pa</th>
<th>pw</th>
<th>ulc</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2(3)$</td>
<td>20.045</td>
<td>17.23</td>
<td>20.668</td>
<td>14.345</td>
<td>16.751</td>
</tr>
<tr>
<td>[0.0002]**</td>
<td>[0.0006]**</td>
<td>[0.0001]**</td>
<td>[0.0025]**</td>
<td>[0.0008]**</td>
<td></td>
</tr>
</tbody>
</table>

The test statistics are the LR-tests of restrictions on the cointegration space within the Johansen framework. Specifically, these statistics test the restriction that one of the cointegrating vectors contains all zeros except for a unity corresponding to the coefficient of the variable we are testing for stationary. The test is conditional on the number of cointegrating vectors. In Table 9, the statistics quoted are conditional on there being three CI-vectors and refer to the same VAR model that later is used to identify the long-run relationships in Section 4. The figures in brackets under each Statistic are the tests’ significance probabilities and * and ** denote rejection at 5% and 1% critical levels, respectively.
Table 10:  
ADF(N) Statistics for Testing for a unit Root in German data.  
Estimates of $|\hat{\rho} - 1|$ in parenthesis

<table>
<thead>
<tr>
<th>Variable</th>
<th>H0</th>
<th>a</th>
<th>pa</th>
<th>pw</th>
<th>ulc</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>I(1)</td>
<td>-1.7928</td>
<td>-0.45905</td>
<td>-2.8923</td>
<td>-0.15304</td>
<td>-2.3491</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.04152)</td>
<td>(0.00337)</td>
<td>(0.079)</td>
<td>(0.00191)</td>
<td>(0.0635)</td>
<td></td>
</tr>
<tr>
<td>I(2)</td>
<td>-5.6386**</td>
<td>-4.2226**</td>
<td>-5.8016**</td>
<td>-3.9442**</td>
<td>-6.0913**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.63109)</td>
<td>(0.39685)</td>
<td>(0.71758)</td>
<td>(0.56312)</td>
<td>(0.8318)</td>
<td></td>
</tr>
</tbody>
</table>

1For any variable x and a null hypothesis of I(1), the ADF statistics are testing a null hypothesis of a unit root in x against an alternative of a stationary root. For a null hypothesis of I(2), the statistics are testing a null hypothesis of an unit root in $\Delta x$ against the alternative of a stationary root in $\Delta x$.

2For a given variable and the null hypotheses of I(1) and I(2), two values are reported. The N'th-order augmented Dickey-Fuller (1981) statistics, denoted ADF(N) and (in parentheses) the absolute value of the estimated coefficient on the lagged variable, where that coefficient should be equal to zero under the null. Both a constant- and a trend-term together with seasonal dummies are included in the corresponding regressions when testing the null of I(1), whereas only a constant is specified when testing for I(2). N varies across the variables for both tests and is equal to one for a and pa, three for pw, four for ulc and five for R when testing I(1) versus I(0), while two for pw, three for a, four for pa and R, and five for ulc when testing I(2)-ness.

3Here and elsewhere in the paper, asterisks * and ** denote rejection of the null hypotheses at the 5% and 1% significance level, respectively. The critical values for the ADF statistics for testing I(1) are -3.44 at a level of 5% and -4.022 at a level of 1% (MacKinnon (1991)) while the corresponding values are -2.881 and -3.475 when testing I(2).
Table 11:  
Individual equation and system-diagnostics for the unrestricted VAR-model of German exports$^1$

<table>
<thead>
<tr>
<th>Equation/Tests</th>
<th>AR 1-5 F[5,114]</th>
<th>ARCH 4 F[4,46]</th>
<th>Normality $X^2_n(2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta a$</td>
<td>1.5477 [0.1807]</td>
<td>2.6367 [0.0377]*</td>
<td>0.3664 [0.8326]</td>
</tr>
<tr>
<td>$\Delta pa$</td>
<td>0.4368 [0.8220]</td>
<td>1.6502 [0.1667]</td>
<td>3.3688 [0.1856]</td>
</tr>
<tr>
<td>$\Delta pw$</td>
<td>0.9679 [0.4405]</td>
<td>0.8903 [0.4723]</td>
<td>4.3124 [0.1158]</td>
</tr>
<tr>
<td>$\Delta R$</td>
<td>1.206 [0.3108]</td>
<td>0.2837 [0.8880]</td>
<td>1.8333 [0.3999]</td>
</tr>
<tr>
<td>$\Delta ulc$</td>
<td>0.5425 [0.7437]</td>
<td>0.1414 [0.9664]</td>
<td>3.1946 [0.2024]</td>
</tr>
</tbody>
</table>

System tests:  
VAR 1-5[125,447]  VNorm $X^2(10)$  $VX^2 F(780,822)$
1.0905 [0.2622]  20.854 [0.022]*  0.61706 [1.000]

$^1$The values shown in brackets are the individual test's significance probability. * and ** denote as usual rejection of the corresponding null at levels of 5 and 1 per cent, respectively. VNormality and VX^2 denotes the Vector tests of normality and heteroscedasticity. For an explanation of the various test statistics the reader is referred to Chapter 14 of the PcFiml manual (Doornik and Hendry (1999)).
Appendix
1 Determination of export volumes and export prices within a theory of monopolistic competition \(^1\).

In this appendix I study the formation of export volumes and export prices within a theory of monopolistic competition with price discrimination (Bruno (1979)). As alluded to in two of the chapters of this thesis, Armington's theory can be integrated with the theory of monopolistic competition with price discrimination and a country-specific demand curve without having to assume production processes with constant returns to scale and a constant price elasticity. Section 1 of this appendix reviews the theory of monopolistic competition in the case where a representative producer sells on the domestic and foreign market. Within this framework, I discuss possible ex post export volume realizations when the export price is determined ex ante. If the producer is rationed on the export market, the export volume will be determined by demand. In section 2, therefore, the Armington model with two countries only, "an individual economy" and "the rest of the world", is given a formal treatment. It turns out that when the individual country we are looking at can be considered as a small open economy, this approach gives rise to an interesting interpretation of the elasticity of exports with regard to export prices. The use of symbols in this appendix does not coincide with the one given in the two thesis chapters dealing with exports. However, it is my hope that the symbol list given below at least partly compensates for this lack of consistency.

1.1 Monopolistic competition with price competition.

This section reviews the theory of monopolistic competition with only one representative producer selling on two markets, respectively the domestic and the foreign market.

Symbol list:\(^2\)

\[
\begin{align*}
P_D & \quad \text{Domestic price} \\
P_F & \quad \text{Export price} \\
A_D & \quad \text{Indicator of domestic demand (expected quantity)} \\
A_F & \quad \text{Indicator of foreign demand (expected quantity)} \\
X_D & \quad \text{The quantity demanded, supplied or realized on the home market}
\end{align*}
\]

\(^1\)I want particularly to thank Bergljot Barkbu for helping me with proofreading the manuscript.

\(^2\)All nominal quantities are denoted in domestic currency. Moreover, in the text I have chosen to use the same symbol for the quantity demanded, supplied and realized hoping that the context brings out the meaning.
The quantity demanded, supplied or realized on the export market

$X_F$

Import price

$P_{DC}$

World market price

$P_{FC}$

Indicator for technological progress

$Y$

Total gross production

$X$

Labor costs per man-hour

$w$

Price of intermediate products

$q$

Number of man-hours

$L$

Quantity of intermediate products

$M$

Variable costs

$b$

Cost function

$b^*$

Profit

We assume the following demand functions for the domestic and foreign market respectively:

\[
P_D = A_f(X_D, P_{DC}), \quad \text{El}_{X_D} P_D = \sigma_{D1} \leq 0 \quad \text{and} \quad \text{El}_{P_{DC}} P_D = \sigma_{D2} \geq 0 \quad (1)
\]

\[
P_F = A_f(X_F, P_{FC}), \quad \text{El}_{X_F} P_F = \sigma_{F1} \leq 0 \quad \text{and} \quad \text{El}_{P_{FC}} P_F = \sigma_{F2} \geq 0 \quad (2)
\]

The restrictions $A_f = 1$, $\sigma_{F1} = 0$ and $\sigma_{F2} = 1$ imply that $P_F = P_{FC}$. The representative producer will thus be a price-taker on the export market. However, rather than incorporating these restrictions at the outset of the theoretical exposition, we are going to deal with the general case of (1) and (2). The implications of various restrictions are then going to be commented on within this framework.

The variables $P_{DC}, P_{FC}, A_D, A_F, w, q, Y$ are all assumed to be exogenous. Splitting up total production, $X$, into what is produced respectively for the domestic market, $X_D$, and the foreign market, $X_F$, the maximizing problem of the representative is given by:

\[
\max_{X_D, X_F} A_f(X_D, P_{DC})X_D + A_f(X_F, P_{FC})X_F - b(X_D + X_F, Y, w, q), \quad (3)
\]

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The aggregate cost function \( b(X_D + X_F, Y, w, q) = b^* = \min_{L,M} (wL + qM) \)
given \( YF(L, M) = X \), and \( P_D \) and \( P_F \) are defined by (1) and (2)\(^3\). Assuming an inner solution and a strictly concave production function, the first order conditions:

\[
A_D f(X_D, P_{DC}) [E]_{X_D P_D + 1} = b'_{X_D} (X_D + X_F, Y, w, q)
\]

\[
A_F f(X_F, P_{FC}) [E]_{X_F P_F + 1} = b'_{X_F} (X_D + X_F, Y, w, q)
\]
together with the definitional relations

\[
P_D = A_D f(X_D, P_{DC})
\]

\[
P_F = A_F f(X_F, P_{FC}),
\]
expressed equivalently as

\[
P_D(\sigma_{D1} + 1) = b'_{X_D} (X_D + X_F, Y, w, q)
\]

\[
P_F(\sigma_{F1} + 1) = b'_{X_F} (X_D + X_F, Y, w, q)
\]

\[
P_D = A_D f(X_D, P_{DC})
\]

\[
P_F = A_F f(X_F, P_{FC}),
\]
implicitly define \( X_D, X_F, P_D \) and \( P_F \) as functions of the exogenous variables.

Thus we have that

\[
X_F = G(w, q, Y, A_D, P_{DC}, \sigma_{D1}, A_F, P_{FC}, \sigma_{F1})
\]

\[
P_F = H(w, q, Y, A_D, P_{DC}, \sigma_{D1}, A_F, P_{FC}, \sigma_{F1}),
\]

which implies that the relative time derivatives are given by

\(^3\) \( YF(\cdot) \) is the production function with Hicks-neutral technical progress, in the case of only two factors of production being characterized by an unchanged cost minimizing factor ratio when holding constant the ratio between factor prices (see e.g. Varian (1992)).
and
\[
\frac{\dot{X}_F}{X_F} = \alpha_1 \frac{\dot{w}}{w} + \alpha_2 \frac{\dot{q}}{q} + \alpha_3 \frac{\dot{Y}}{Y} + \alpha_4 \frac{\dot{A}_D}{A_D} + \alpha_5 \frac{\dot{P}_{DC}}{P_{DC}}
\]
\[
+ \alpha_6 \frac{\dot{\sigma}_{D1}}{\sigma_{D1}} + \alpha_7 \frac{\dot{A}_F}{A_F} + \alpha_8 \frac{\dot{P}_{FC}}{P_{FC}} + \alpha_9 \sigma_{F1} \frac{\dot{F}}{\sigma_{F1}}
\]

and
\[
\frac{\dot{P}_F}{P_F} = \beta_1 \frac{\dot{w}}{w} + \beta_2 \frac{\dot{q}}{q} + \beta_3 \frac{\dot{Y}}{Y} + \beta_4 \frac{\dot{A}_D}{A_D} + \beta_5 \frac{\dot{P}_{DC}}{P_{DC}}
\]
\[
+ \beta_6 \frac{\dot{\sigma}_{D1}}{\sigma_{D1}} + \beta_7 \frac{\dot{A}_F}{A_F} + \beta_8 \frac{\dot{P}_{FC}}{P_{FC}} + \beta_9 \sigma_{F1} \frac{\dot{F}}{\sigma_{F1}}
\]

In (6) and (7) the $\alpha_i's$ and $\beta_i's$ have the interpretation of being the partial elasticities of respectively the functions $G(\cdot)$ and $H(\cdot)$ with regard to argument number $i$. In the following I assume decreasing or constant returns to scale, i.e. $\varepsilon \leq 1$, where $\varepsilon$ is the elasticity to scale. I will show that this enables us to uniquely determine the signs of the coefficients in (6) and (7).

Logarithmic differentiation of the first order conditions gives:
\[
\frac{\dot{P}_D}{P_D} + \frac{\dot{\sigma}_{D1}}{1 + \sigma_{D1}} = \frac{b'_{X_D}(X,Y,w,q)}{b_{X_D}(X,Y,w,q)}
\]
\[
\frac{\dot{P}_F}{P_F} + \frac{\dot{\sigma}_{F1}}{1 + \sigma_{F1}} = \frac{b'_{X_F}(X,Y,w,q)}{b_{X_F}(X,Y,w,q)}
\]
\[
\frac{\dot{P}_D}{P_D} = \frac{\dot{A}_D}{A_D} + \sigma_{D1} \frac{\dot{X}_D}{X_D} + \sigma_{D2} \frac{\dot{P}_{DC}}{P_{DC}}
\]
\[
\frac{\dot{P}_F}{P_F} = \frac{\dot{A}_F}{A_F} + \sigma_{F1} \frac{\dot{X}_F}{X_F} + \sigma_{F2} \frac{\dot{P}_{FC}}{P_{FC}}
\]

To solve this system for the relative time derivatives, it is necessary to deduce an explicit expression for the relative time derivative of the cost function. The
first order condition of the problem \( \min_{L,M} \{wL + qM\} \) given \( YF(L,M) = X \) are:

\[
\begin{align*}
wL &= \lambda X \varepsilon_L \\
qM &= \lambda X \varepsilon_M \\
YF(L,M) &= X
\end{align*}
\]

, where \( \varepsilon_L \) and \( \varepsilon_M \) are the partial elasticities of production to labor and intermediate goods, respectively. The first order conditions in (12) define \( L, M, \) and \( \lambda \) implicitly as functions of \( w, q, Y \) and \( X \), implying that the cost function, \( b^* \), is given by \( wL(X,Y,w,q) + qM(X,Y,w,q) = b(X,Y,w,q) \). The envelope theorem\(^4\) implies that \( b'_{X}(X,Y,w,q) = \lambda \). Since

\[
\frac{\bar{b}(X,Y,w,q)}{b'_{X}(X,Y,w,q)} = \varepsilon, \text{ where } \bar{b} = \frac{b}{X}, \text{ will } \lambda X = \frac{b(X,Y,w,q)}{\varepsilon}.
\]

Substituting for \( \lambda X \) in the first order conditions, (12) gives

\[
wL = \frac{\varepsilon_L}{\varepsilon} b(X,Y,w,q) \quad \text{and} \quad qM = \frac{\varepsilon_M}{\varepsilon} b(X,Y,w,q)
\]

By taking the time derivative of the variable costs \( b = wL + qM \), replacing \( wL \) and \( qM \) gives

\[
\frac{\dot{b}}{b} = \frac{1}{\varepsilon} (\varepsilon_L \frac{\dot{w}}{w} + \varepsilon_M \frac{\dot{q}}{q} + \frac{\dot{X}}{X} - \frac{\dot{Y}}{Y}), \text{ where } \frac{\dot{L}}{L} + \varepsilon_M \frac{\dot{M}}{M} = \frac{\dot{X}}{X} - \frac{\dot{Y}}{Y}
\]

follows directly from the assumption of a production function with Hicks neutral technical progress. From \( b(X,Y,w,q) = \varepsilon X b'(X,Y,w,q) \) it follows that

\[
\frac{\dot{b}X_{X}}{b'_{X}(X,Y,w,q)} = \frac{\dot{b}(X,Y,w,q)}{b(X,Y,w,q)} \frac{\dot{X}}{X} - \frac{\dot{e}}{\varepsilon}
\]

\[
= \frac{\varepsilon_L}{\varepsilon} \frac{\dot{w}}{w} + \frac{\varepsilon_M}{\varepsilon} \frac{\dot{q}}{q} + \left( \frac{1}{\varepsilon} - 1 \right) \frac{\dot{X}}{X} - \frac{1}{\varepsilon} \frac{\dot{Y}}{Y} - \frac{\dot{e}}{\varepsilon}
\]

By substituting (10) and (11) into (8) and (9), respectively, and inserting (13) for the relative time derivative of the marginal cost function we get:

\(^4\)The envelope theorem says that the partial derivative of an indirect function can be obtained by partially differentiating the function being optimized. In a constrained problem this last function will be the Lagrange function (c.f. H. Varian (1992)).
\[
\begin{align*}
\frac{\dot{A}_D + \sigma_{D1} \frac{\dot{X}_D}{X_D} + \sigma_{D2} P_{DC}}{A_D} + \sigma_{D1} &= \frac{\epsilon_L \dot{w} + \epsilon_m \dot{q} + (\frac{1}{\epsilon} - 1) \frac{1}{X} \frac{1}{Y} \frac{\dot{e}}{e}}{\epsilon} \\
\frac{\dot{F}_D + \sigma_{F1} \frac{\dot{X}_F}{X_F} + \sigma_{F2} P_{FC}}{A_F} + \sigma_{F1} &= \frac{\epsilon_L \dot{w} + \epsilon_m \dot{q} + (\frac{1}{\epsilon} - 1) \frac{1}{X} \frac{1}{Y} \frac{\dot{e}}{e}}{\epsilon}
\end{align*}
\]  

(14)

Since \( X = X_D + X_F \), the relative time derivative of total production is

\[
\frac{\dot{X}}{X} = V_F \frac{\dot{X}_F}{X_F} + V_D \frac{\dot{X}_D}{X_D}, \text{ where } V_j = \frac{X_j}{X} (j = F, D)
\]

After some algebra (14) gives the following price and quantity equations:

\[
\begin{align*}
\frac{\dot{X}_F}{X_F} &= \frac{\sigma_{D1} \epsilon_L \dot{w} + \sigma_{D1} \epsilon_m \dot{q} - \frac{\sigma_{D1} \frac{1}{X} \frac{1}{Y} \frac{\dot{e}}{e}}{D} \left( \frac{1}{\epsilon} - 1 \right)}{D} V_D \frac{\dot{A}_D}{A_D} \\
&- \frac{\sigma_{D2}}{D} \left( \frac{1}{\epsilon} - 1 \right) V_D \frac{P_{DC}}{P_{DC}} \left( \frac{1}{\epsilon} - 1 \right) V_D \frac{\dot{A}_D}{A_D} \\
\frac{\dot{F}_F}{P_F} &= \frac{\sigma_{F1} \epsilon_L \dot{w} + \sigma_{F1} \epsilon_m \dot{q} - \frac{\sigma_{F1} \frac{1}{X} \frac{1}{Y} \frac{\dot{e}}{e}}{D} \left( \frac{1}{\epsilon} - 1 \right)}{D} V_D \frac{\dot{A}_D}{A_D} \\
&- \frac{\sigma_{F1} \frac{1}{X} \frac{1}{Y} \frac{\dot{e}}{e}}{D} \left( \frac{1}{\epsilon} - 1 \right) V_D \frac{P_{DC}}{P_{DC}} \left( \frac{1}{\epsilon} - 1 \right) V_D \frac{\dot{A}_D}{A_D}
\end{align*}
\]  

(15)

\[
\begin{align*}
\frac{\dot{A}_F}{A_F} &= \frac{\sigma_{D1} \epsilon_L \dot{w} + \sigma_{D1} \epsilon_m \dot{q} - \frac{\sigma_{D1} \frac{1}{X} \frac{1}{Y} \frac{\dot{e}}{e}}{D} \left( \frac{1}{\epsilon} - 1 \right)}{D} V_D \frac{\dot{A}_D}{A_D} \\
&- \frac{\sigma_{D1} \frac{1}{X} \frac{1}{Y} \frac{\dot{e}}{e}}{D} \left( \frac{1}{\epsilon} - 1 \right) V_D \frac{P_{DC}}{P_{DC}} \left( \frac{1}{\epsilon} - 1 \right) V_D \frac{\dot{A}_D}{A_D}
\end{align*}
\]  

(16)
Under each of the coefficients in equations (15) and (16) we have indicated the coefficient sign under the assumptions we have made as to the partial elasticities, $\sigma_{D1}$ and $\sigma_{F1}$, and the elasticity of scale $\varepsilon$. In particular, these assumptions imply that

$$D = \sigma_{F1}\sigma_{D1} - \left(\frac{1}{\varepsilon} - 1\right)(V_F\sigma_{D1} + V_D\sigma_{F1}) > 0$$

Comparing (15) and (16) with (6) and (7), respectively, we see that we have uniquely determined the sign on all the coefficients. In addition, we have introduced the elasticity to scale, $\varepsilon$, as a new explanatory variable.

Price-taking behavior on the export market implies that $A_F = 1$, $\sigma_{F1} = 0$ and $\sigma_{F2} = 1$, eliminating both $A_F$ and $\sigma_{F1}$ as explanatory variables in the export volume equation. Beyond that, these restrictions have only quantitative and not qualitative implications with respect to the coefficients relating to the other variables. Moreover, equation (16) confirms that there will be no demand side effects in the export price relation when having constant returns to scale. In case of a falling demand curve, the export volume is determined by the ex ante production decision. This yields something between the pure Keynesian and Classical case, in that both demand and supply components related to the export market affect the determination of the export quantity. In the case of price-taking behavior on the export market, the foreign demand geared to our products will have an infinite price elasticity and the export quantity will be exclusively determined from the supply side.

**An example with decreasing returns to scale.** To provide an illustration and to construct a simple relationship which can serve as a starting point for estimation of ex ante export price and volume relations, we will have a further look at the behavior of a monopolistic competitor with a one factor Cobb-Douglas production function. We will assume decreasing returns to scale, implying that the elasticity to scale, $\phi$, is strictly less than one. The elasticity of demand is assumed strictly greater than one. A representative agent is assumed to face the following product and demand functions:

$$X_j = (Y L_j)^{\phi_j} \quad 0 < \phi_j < 1 \quad j = D, F$$

$$X_j = A_j \left(\frac{P_j C}{I_j}\right)^{\theta_j} \quad \theta_j > 1 \quad j = D, F$$

---

5 Economic agents who distribute their products between several distinct markets and where the prices are differentiated according to the degree of market power are denoted as monopolistic competitors.
Our assumptions imply that:

\[ \frac{1}{1-\phi_j} \phi_j > 0 \quad \text{and} \quad \frac{1}{1-\left(\phi_j + \frac{1}{\phi_j}\right)} < 0 \]  

(18)

The second order conditions for a maximum are satisfied. Together with (17) the first order conditions then define the following export volume and price relations:

\[
\ln X_F = \frac{1}{1 - \left(\frac{1}{\theta_F} + \frac{1}{\phi_F}\right)} \ln \left\{ \left(\frac{1}{\theta_F} - 1\right) \phi_F \right\} \\
+ \frac{1}{1 - \left(\frac{1}{\theta_F} + \frac{1}{\phi_F}\right)} (\ln w - \ln Y - \ln P_{FC}) \\
- \frac{1}{\theta_F} \frac{1}{1 - \left(\frac{1}{\theta_F} + \frac{1}{\phi_F}\right)} \ln A_F \\
= \kappa + \gamma (\ln w - \ln Y - \ln P_{FC}) - \delta \ln A_F ,
\]  

(19)

\[
\ln P_F = \frac{1}{\theta_F} \kappa + \frac{1}{\theta_F} \left(1 + \frac{1}{\theta_F} \gamma\right) \ln A_F - \frac{1}{\theta_F} \gamma (\ln w - \ln Y) \\
+ \left(1 + \frac{1}{\theta_F} \gamma\right) \ln P_{FC} \\
= c + \zeta \ln A_F + \varphi (\ln w - \ln Y) + (1 - \varphi) \ln P_{FC}
\]

The formulation in (19) gives a rationale for the simultaneous presence of real wages and aggregate real demand as arguments in the export volume equation. Aggregate analyses often assume constant returns to scale implying that price equations are pure mark-up relationships over nominal wage costs. Under the assumption that demand is homogenous of degree 0 in nominal prices, unchanged demand in the wake of a hike in one of the commodity prices requires an equivalent increase also in all the others. This makes it impossible to change the real wage given the level of real demand (Carlin and Soskice (1985)). Layard and Nickell (1985, 1986) circumvent this problem by assuming that agents base their decisions on an irrational expectation as to the aggregate price level. This lack of rationality can be explained by the existence of many small agents. In such a case a rise in the price level caused by a hike in the nominal wage level could be perceptible as specific to the individual agent and therefore having no

\[6\]From the assumption \( \phi_j < 1 \) it follows that \( \phi_j < 1/(1 - \theta_j) \). The rightmost inequality in (18) follows from the equivalence of \( \phi_j < 1/(1 - \theta_j) \) and \( 1/\theta_j + 1/\phi_j > 1 \).
expectational consequences with regard to the aggregate price level. However, virtually all prices increase proportionally which keeps real demand unchanged. Thus, it will be possible to change the expected real wage for a given level of real demand. An alternative approach is to make a more realistic classification of the economy. Bhaskar (1992) splits the economy into one monopolistic competing sector producing for the home market and two sectors producing for the foreign market. The two sectors producing for the foreign market are assumed to show oligopolistic and price-taking behavior, respectively. By introducing a sector facing prices as given from the world market, the homogeneity between wage costs and the price index is removed. This makes a change in the real wage possible.

So far we have assumed full correspondence between expected and realized quantities. This is a very unrealistic description of reality. The producers may want to change their ex ante decision when the realized quantities appear thereby creating a gap between the decision made ex ante and ex post. For the endogenous price variables it may be realistic to assume producers committed to their prices decided ex ante through price lists, advertisements etc. We assume that the producers have perfect expectations with regard to future wage levels. They will therefore be price takers ex post on both the market for intermediate goods and for end products. This gives us two idealized situations. In one case the consumer will be rationed on the product market and exports given by the price taking level of production. In the other case the producer will be rationed on the product market and exports exclusively determined by real demand. In the latter case we will therefore have the following export volume equation:

\[ X_F = B_F g_F (P_F, P_{FC}) \]  

implying that

\[ \frac{\dot{X}_F}{X_F} = \frac{B_F}{B_F} + E_l P_F \frac{\dot{P}_F}{P_F} + E_l P_{FC} X_F \frac{\dot{P}_{FC}}{P_{FC}}. \]  

Solving (21) for export prices and inserting the coefficients from (11) gives

\[ \frac{\dot{X}_F}{X_F} = \frac{1}{\sigma_{F2} F_{1F} A_F} + \frac{1}{\sigma_{F1} F_{1F} P_F} \frac{\dot{P}_F}{P_F} + \frac{\sigma_{F2}}{\sigma_{F1} F_{1F} P_{FC}} \frac{\dot{P}_{FC}}{P_{FC}}. \]  

The equations (15) and (16) are relations between relative time derivatives. Therefore without reservation we cannot use these relations to say anything qualitative about the long run theoretical implications. However, assuming a Cobb-Douglas function for both the export volume and price equation leads to correspondence between the coefficients in (15) and (16) and the coefficients in a log linear relation for the same variables. Thus, in the following we will ensure that this assumption is fulfilled for both the export volume and price relation.

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1.2 Armington demand functions

Traditionally, the basis for econometric analysis of exports has been Armington's theory of demand for products distinguished by place of production. Within the theory of monopolistic competition the ex post decision rule described above can motivate these models. In this section I will therefore try to clarify the relation between (20)-(22) and the demand equations implied by Armington's theory.

Consider a world consisting of m countries producing n goods. Therefore, m product varieties exist for each good. One variety for each country producing the good. For the consumers of the economy these varieties appear to be different in the sense of not constituting perfect substitutes. Each representative consumer of the individual country is assumed to have the utility function

\[ U = U \left[ u_1 (X_1), \ldots, u_i (X_i), \ldots, u_n (X_n) \right] , \]  

where

\[ X_i = (X_{i1}, \ldots, X_{ij}, \ldots, X_{im}) \quad i = 1, \ldots, n \]

and

\[ u_i = \left( \sum_{j=1}^{m} \alpha_{ij} X_{ij}^{\beta_i} \right) \frac{d_{ij}}{d_{ik}} \]

One should particularly note the interpretation of \( X_{ij} \) as the demand of a country's representative producer for good number \( i \) directed towards country \( j \). The specification implies that the utility function is assumed separable as to the various goods, meaning that the marginal rate of substitution between two product varieties is independent of all product varieties of other goods. The inner utility function, \( u_i \), is homogenous of degree one and shows a constant elasticity of substitution, \( \sigma_i \), for every pair of product varieties within good \( i \).\(^7\)

\(^7\)Later I will simplify the analysis by merely looking at an economy consisting of two countries, an individual open economy and "the rest of the world". Exports from the individual economy will then result from the decision of a representative consumer for "the rest of the world". To make the exposition easier to follow, I have omitted subscripts for quantities that are specific for the consumers as utility, utility functions, demand for products and income.

\[ u_i = \left( \sum_{j=1}^{m} \alpha_{ij} X_{ij}^{\beta_i} \right) \frac{d_{ij}}{d_{ik}} \Rightarrow MRS_{jk} = \frac{\partial u_i / \partial X_{ik}}{\partial u_i / \partial X_{ij}} = \alpha_{ij} \left( \frac{X_{ij}}{X_{ik}} \right)^{1-\beta_i} \]

\[ \Rightarrow X_{ij} = X_{ik} \left( \frac{\alpha_{ij}}{\alpha_{ik}} \right)^{1-\beta_i} \Rightarrow MRS_{jk}^{\frac{1}{1-\beta_i}} = \sigma_i = E(MRS_{jk}) \left( \frac{X_{ij}}{X_{ik}} \right) = \frac{1}{1-\beta_i} \Rightarrow \beta_i = 1 - \frac{1}{\sigma_i} \]
The consumer is assumed to have a given income, \( Y \), for consumption of the various country specific products of the economy. Out of this income the amount \( Y_i \) is used on products of good \( i \). The consumer is assumed to maximize his utility. Subject to the marginal utility of the outer utility function on the inner utility function being positive, the budget constraint

\[
\sum_{i=1}^{n} \sum_{j=1}^{m} P_{ij} X_{ij} \leq Y,
\]

will be satisfied with equality. The consumer will thus face the problem

\[
\max_{\substack{X_{ij} \in \mathbb{R}_+ \forall i \in \{1, \ldots, n\} \forall j \in \{1, \ldots, m\}, \sum_{j=1}^{m} P_{ij} X_{ij} = Y}} U\left[ u_1 (X_{11}, \ldots, X_{1m}), \ldots, u_n (X_{n1}, \ldots, X_{nm}) \right]
\]

given \( \sum_{i=1}^{n} \sum_{j=1}^{m} P_{ij} X_{ij} = Y \)

This problem may be solved in two steps, the first step being:

\[
\max_{\substack{X_{ij} \in \mathbb{R}_+ \forall i \in \{1, \ldots, n\} \forall j \in \{1, \ldots, m\}, \sum_{j=1}^{m} P_{ij} X_{ij} = Y_i \forall i \}} \left( \sum_{j=1}^{m} \alpha_{ij} X_{ij}^\alpha \right)^{\frac{1}{\alpha}}
\]

given \( \sum_{j=1}^{m} P_{ij} X_{ij} = Y_i \forall i \in \{1, \ldots, n\} \)

Assuming an inner solution and a strictly concave utility function, the first order conditions of (25) will define the demand functions \( X_{ij}(P_i, Y_i) \), where \( P_i \) is the product price vector of good \( i \). The second step of the problem is:

\[
\max_{\substack{X_{ij} \in \mathbb{R}_+ \forall i \in \{1, \ldots, n\} \forall j \in \{1, \ldots, m\}, \sum_{j=1}^{m} P_{ij} X_{ij} = Y_i \forall i \}} U\left[ u_1 (P_1, Y_1), \ldots, u_n (P_n, Y_n), \ldots, u_n (X_{n1}, \ldots, X_{nm}) \right]
\]

given \( \sum_{i=1}^{n} Y_i = Y \)

Again assuming an inner solution and a strictly concave utility function, the first order conditions of this problem will define the expenditure functions \( Y_i(P, Y) \), where \( P \) is the product price vector of all the goods. Inserting these in the demand functions of the first step, \( X_{ij}(P, Y_i) \), defined by (25), we will arrive at the demand function of the consumer, \( X_{ij}(P, Y) \). The first order conditions of the problem in the first step, (25), are given by

\[
X_{ij} = \lambda^{-\sigma_i} P^{-\sigma_i} \alpha^{\sigma_i} u_i
\]

\[
\sum_{j=1}^{m} P_{ij} X_{ij} = Y_i
\]
Replacing $X_{ij}$ in (28) by (27), we have

$$ u_i \lambda^{-\sigma_i} = \frac{Y_i}{\sum_{j=1}^m P_{ij}^{1-\sigma_i} \alpha_{ij}^{\sigma_i}}. $$

Inserting this expression in (27) gives the demand functions:

$$ X_{ij} = \frac{P_{ij}^{-\sigma_i} Y_i}{\sum_{j=1}^m P_{ij}^{1-\sigma_i} \alpha_{ij}^{\sigma_i}} = \frac{P_{ij}^{-\sigma_i} Y_i}{\sum_{j=1}^m P_{ij}^{1-\sigma_i} \alpha_{ij}^{\sigma_i}} $$

(29)

The denominator of the two parentheses can be interpreted as a utility weighted price index. This can be demonstrated by deriving the compensated demand functions from the minimization problem

$$ \min_{\lambda} \sum_{j=1}^m P_{ij} X_{ij} \quad \text{given} \quad \left( \sum_{i=1}^n \alpha_{ij} X_{ij} \right)^{\frac{1}{\sigma_i}} = u_i $$(30)

and define "the cost per unity", $c_i(P_i)$, by

$$ c_i(P_i) = \frac{\sum_{j=1}^m P_{ij} X_{ij}^*}{u_i} $$

where $X_{ij}^*$ is the compensated demand function following from (31) and (32) below. Since $\beta_i = 1 - 1/\sigma_i$, the first order conditions are given by

$$ X_{ij}^* = P_{ij}^{-\sigma_i} \lambda^{\sigma_i} u_i \alpha_{ij}^{\sigma_i} $$

(31)

$$ \left\{ \sum_{j=1}^m \alpha_{ij} X_{ij}^{\sigma_i-1} \right\}^{\frac{1}{\sigma_i-1}} = u_i $$

(32)

Substituting (31) for $X_{ij}$ in (32) gives

$$ \lambda^{\sigma_i} u_i = \frac{u_i}{\left\{ \sum_{j=1}^m \alpha_{ij} P_{ij}^{1-\sigma_i} \right\}^{\frac{1}{\sigma_i-1}}} $$

(33)

Inserting (33) in (31) gives then the compensated demand functions:
Substituting this equation, (34), in the expression for "the cost per unity" gives:

\[ c_i \left( P_i \right) = \left( \sum_{j=1}^{m} \alpha_{ij}^{s_i} \frac{P_j^{1-s_i}}{c_i \left( P_i \right)} \right)^{-\frac{1}{s_i}}. \]  

This last expression is nothing but the denominator of the two parentheses in (29). From (35) it is seen that \( c_i \left( P_i \right) \) is homogenous of degree one, implying that the interpretation as a utility weighted price index is plausible. We can now write (29) as:

\[ X_{ij} = \alpha_{ij}^{s_i} \left( \frac{P_j}{c_i \left( P_i \right)} \right)^{-\sigma_i} \left( \frac{Y_i}{c_i \left( P_i \right)} \right). \]  

Because \( c_i \left( P_i \right) \) can be interpreted as a price index, the two parentheses can be given the interpretations of the real export price of good \( i \) in country \( j \) and the real value of the importer's total expenditures, respectively. From equation (36) it is seen that the demand for products of country \( j \) is homogenous of degree null in the prices and that the real expenditure elasticity equals one. Taking the elasticity of (29) to \( P_{ik} \) we have

\[ El_{P_{ik}} \left( X_{ij} \right) = El_{P_{ik}} \left( P_{ij} \right)^{-\sigma_i} - \frac{\alpha_{ik}^{s_i} P_{ik}^{1-s_i} \sum_{j=1}^{m} \alpha_{ij}^{s_i} P_j^{1-s_i}}{\sum_{j=1}^{m} \alpha_{ij}^{s_i} P_j^{1-s_i}} \]

\[ = -\sigma_i \delta_{jk} - (1 - \sigma_i) \frac{\alpha_{ik}^{s_i} P_{ik}^{1-s_i}}{\sum_{j=1}^{m} \alpha_{ij}^{s_i} P_j^{1-s_i}}, \]  

where \( \delta_{jk} = \begin{cases} 1 & \text{for } k = j \\ 0 & \text{for } k \neq j \end{cases} \)

The purpose of the analysis is to model aggregate export price and export volume equations. Therefore, it will be appropriate to simplify the theoretical analysis by only considering the case with one good and two countries. The two countries may be given the interpretations of respectively "an individual economy" and "the rest of the world". Thus, the demand for products of the individual economy will be determined by the decision of a representative consumer for "the rest of the world". Moreover, the notion of income and expenditure will coincide, implying that \( Y_i \) can be interpreted as the income of "the rest of the world". The budget share of country \( k \) is thus given by
For a small open economy this budget share will be very small. The direct Cournot elasticity ($j = k$) will therefore be numerically close to the elasticity of substitution, $\sigma$. Moreover, the cross price elasticity will differ significantly from zero and the export price of that economy will enter into the expression in (35) with a relatively small weight. Thus the price index will approximately equal a weighted world market price index. In addition the demand indicator, $B_F$ in (20), will be a function of foreign real income. Thus, the Armington function, (36), could be said to be a special case of the demand function in (20). Omitting the subscript $i$, letting $j = F$ and substituting the world market price, $P_{FC}$, for the price index $c(P)$, logarithmic derivation gives

$$
\frac{X_F}{X_F} = \frac{\frac{\dot{X}_F}{X_F}}{\frac{\dot{R}}{R}} - \sigma \left( \frac{\dot{P}_F}{P_F} - \frac{\dot{P}_{FC}}{P_{FC}} \right),
$$

where $R$ symbolizes foreign real income. Compared with equation (22) this imposes the restriction $\sigma_{F2} = 1$. The Armington function, (36), shows a constant market share by changes in the real income. What kind of restrictions this will lay on $\sigma_{F1}$ in (22) completely depends on how real income is represented in $A_F$. In the case $A_F = (R)^{\sigma_{F1}}$, $\sigma_{F1}$ can vary freely.