European
University
Institute

DEPARTMENT
OF
ECONOMICS

Macroeconomics of Bargain Hunting

Krzysztof Pytka

Thesis submitted for assessment with a view to obtaining the degree of
Doctor of Economics of the European University Institute

Florence, 21 December 2017

European University Institute
Department of Economics

Macroeconomics of Bargain Hunting

Krzysztof Pytka

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Examining Board

Prof. Árpád Ábrahám, EUI, Supervisor
Prof. Piero Gottardi, EUI
Prof. Christian Bayer, University of Bonn
Prof. Guido Menzio, University of Pennsylvania

European University Institute

DEPARTMENT OF ECONOMICS

Researcher declaration to accompany the submission of written work

I Krzysztof Pytka certify that I am the author of the work *Macroeconomics of Bargain Hunting* I have presented for examination for the PhD thesis at the European University Institute. I also certify that this is solely my own original work, other than where I have clearly indicated, in this declaration and in the thesis, that it is the work of others.

I warrant that I have obtained all the permissions required for using any material from other copyrighted publications.

I certify that this work complies with the *Code of Ethics in Academic Research* issued by the European University Institute (IUE 332/2/10 (CA 297).

Signature and Date:

11/13/2017

Pamięci Babci-Mamy, największej twardzielki

Abstract

In most macroeconomic models prices for consumption goods are competitive and consumers are treated as price-takers, which gives rise to the law of one price. However, as the empirical literature documents, prices for the same products are substantially dispersed. The consumers facing the price heterogeneity can affect the effective prices they pay by employing different shopping strategies. In this thesis, I investigate whether price dispersion matters for shaping macroeconomic aggregates.

In chapter 1, I study how income fluctuations are transmitted to consumption decisions in the presence of price dispersion. To this end, I propose a novel and tractable framework to study search for consumption as part of the optimal savings problem. The search protocol can be easily embedded into a standard incomplete-market model. As I show, frictions in the purchasing technology generate important macroeconomic implications for modeling inequality and, in general, household consumption. In economies with those frictions, consumers feature smoother consumption responses to income shocks and the level of wealth inequality is amplified.

In chapter 2, I study equilibrium properties of a standard model of endogenous price distribution by Burdett and Judd (1983). In search economies of this type in most cases there are multiple equilibria. I show that only some allocations can be characterized as stable equilibria. Next, I propose a modification of the original model, which gives rise to one unique symmetric dispersed equilibrium, that can be used for characterizing every feasible allocation.

Finally, in chapter 3, I use the framework from chapter 1 to study the redistributive function of monetary policy. I show that money injection to households might reduce the inefficiency generated by non-competitive behavior of firms thanks to an increase in consumption purchased by bargain hunters. This results in the reduction of the monopolistic power of firms and lower consumption real prices.

Acknowledgements

The dissertation you are about to read is a product of my Ph.D. studies at the European University Institute. The completion of this research endeavor would be impossible without the support of many people.

Firstly, I would like to express my sincere gratitude to my advisors, Árpád Ábrahám and Piero Gottardi, whose complementarity in supervising the thesis was superb. The continuous encouragement and open-mindedness of Árpád and an attention to the finest detail of Piero made me a better researcher. It is very hard to imagine the better combination of advisors.

My deepest appreciation goes to Inga Jańczuk and my parents, Jolanta Mazur-Pytka and Jan Pytka. Their unconditional support and love at all stages of the dissertation were a great motivation for my work.

Besides, I am substantially benefited from many discussions I had with Greg Kaplan and Mark Aguiar during my visit at Princeton. Greg's feedback boosted confidence in my research agenda.

Special thanks to my flatmates, Paweł Kopiec and Radek Michalski. Paweł's sense of humor was so bad that it was very tough to set a lower bar. Who would imagine that Radek will succeed at it. I am the biggest fan of jokes of both of them.

My gratitude also goes to Paweł Doligalski and Luis E. Rojas for their insights and stimulating discussions, Axelle Ferriere and Dominik Sachs for critical hints during my job-market period, and to Anne Banks, Sarah Simonsen, Jessica Spataro, and Lucia Vigna for their outstanding assistance and willingness to help with any issues. A very special mention goes out to Mateusz K. Łącki, who provided a lot of mathematical feedback on my thesis, even though I bet he does not know its topic, and to Adam Kisio, who instilled my interest in mathematics.

The dissertation is dedicated to the memory of Agnieszka Mazur, my bravest grandmother, one of the greatest bargain hunters.

Mannheim, December 7, 2017

Contents

List of Figures

List of Tables

1

Shopping Effort in Self-Insurance Economies

I. Introduction

Household consumption accounts for nearly 70% of the GDP in the US. A good understanding of how household income fluctuations are passed on consumption expenditures is crucial for credible quantitative analysis in a vast class of economic models, when household consumption plays an important role[1]. This paper provides a theory of the pass-through of shocks into consumption when the law of one price does not hold, *i.e.* different retailers charge different prices for exactly the same good. The theory is motivated by recent strong evidence for price dispersion and heterogeneity in household shopping behavior. None of those findings have been integrated into standard consumption models yet and as I show they generate some important implications for modeling household inequalities and, in general, the aggregate consumption.

[1]Just to name but a few: optimal capital income taxation (Aiyagari, 1995), the benefits of insuring unemployed people (Hansen and İmrohoroğlu, 1992), effects of fiscal stimulus payments in a recession (Kaplan and Violante, 2014), the redistributional role of monetary policy (Auclert, 2016; Kaplan, Moll, and Violante, 2016), effects of a credit crunch on consumer spending (Guerrieri and Lorenzoni, 2015), the role of household debt and bankruptcy filing rates (Chatterjee, Corbae, Nakajima, and Ríos-Rull, 2007).

The standard incomplete-markets models with heterogenous households (henceforth, SIM) in the tradition of Bewley (1986), Aiyagari (1994), and Huggett (1993), where consumers insure against future income fluctuations accumulating a risk-free bond, are a workhorse for quantitative analysis of consumption from both macroeconomic and microeconomic standpoints[2]. Nonetheless, they underestimate the level of risk sharing present in the economy and given income process observed in data they have serious problems with generating enough inequality[3]. Especially, the former is of particular importance. As Heathcote, Storesletten, and Violante (2014) point out quantifying existing risk sharing is crucial to evaluate the welfare consequences of counterfactual policy experiments[4]. In addition to this, in their current form the SIM models completely abstract from any kind of price dispersion and assume that the law of one price always holds, namely all households pay the same competitive price.

Substantial and systematic price dispersion is a fact observed in data. A growing literature documents many examples of this phenomenon on both sides of the market, between different households and between different retailers. *(i.a)* Aguiar and Hurst (2007) show that retirees pay approximately 4% less for the same goods than households with heads in their working age. *(i.b)* A similar price differential is observed between non-employed and employed households. Kaplan and Menzio (2015) used the same dataset and found that on average non-employed consumers pay between 1 and 4% less for the same consumption baskets. *(ii)* Moreover, price dispersion is also present from retailers' perspective.

[2]See survey articles by Heathcote, Storesletten, and Violante (2009), Guvenen (2011), and Attanasio and Weber (2010).

[3]Admittedly, there are some models that manage to capture the right level of inequalities, but they either assume the very risky income process as in (Castañeda, Díaz-Giménez, and Ríos-Rull, 2003) or substantial heterogeneity in time preference as in (Krusell and Smith, 1998).

[4]In a similar way, Kaplan and Violante (2010) argue that replicating the level of consumption smoothness, a measure of risk-sharing proposed by Blundell, Pistaferri, and Preston (2008), should be one of the central goals in quantitative macroeconomics.

Kaplan and Menzio (2016) and Kaplan, Menzio, Rudanko, and Trachter (2016) observed the average standard deviation of prices for the same goods amounts to 15.3%. In addition to this, the authors identified that only at most 15% of the price variance is due to variation in the expensiveness of the stores at which a good is sold.

Second, individual shopping effort, which is measured as time spent obtaining goods, varies significantly between households depending on their labor status. Using time diaries Aguiar and Hurst (2007) documented that retirement-age people spend on average 33% more time shopping than households aged 25-29. Similarly, Krueger and Mueller (2012) show that the unemployed people spend between 15 and 30% more time shopping than the employed. Traditionally, higher shopping effort is rationalized by search for lower prices. Thus, both empirical patterns, heterogeneity in shopping intensity and in average prices paid by households, are connected by the price comparison mechanism.

On the empirical side, I contribute to studies on the consumer shopping behavior. Using the American Time-Use Survey I find that unemployed and retired consumers spend on average more time shopping. These observations are consistent with the aforementioned findings made by Aguiar and Hurst (2007) and Krueger and Mueller (2012). What is new in my analysis is that conditioned on being employed, households from top earnings deciles spend about 12% more time shopping than poor employed households. This observation together with the fact that rich households pay higher prices[5] seem to stay at odds with the traditional mechanism relying on the price comparison motive. This suggests that apart from price hunting there must be another motive driving shopping effort in such a fashion that rich people spend more time purchasing goods. High earn-

[5] Aguiar and Hurst (2007) documented households earning more than $70,000 a year pay 2.1 percent more than households earning less than $30,000 a year.

ers are also households that consume more. Therefore, the observed increase in their shopping time can be driven not by the price search intensity, but rather by willingness to increase their consumption. The findings obtained in the empirical part are used for disciplining the behavior of my quantitative model.

In the theoretical part, I integrate search for consumption into a life-cycle version of the SIM model due to İmrohoroğlu, İmrohoroğlu, and Joines (1995); Huggett (1996); Ríos-Rull (1996). The household's income is driven by idiosyncratic productivity shocks. Every household makes the decision about level of savings, that are used to insure against the future income fluctuations and to smooth the future consumption. The remaining disposable resources of the household are spent on consumption. I extend the benchmark SIM economy by adding frictions in the purchasing technology. Households have to exert effort to purchase goods. This effort can be decomposed into two components: 1. *price search intensity* – effort to search for price bargains, 2. *purchase effort* – effort required to purchase consumption of a given size[6]. Both retailers and households' shopping come together at random through a frictional meeting process. Households that search for low price more intensively are able to find lower prices more often. Households exhibiting higher purchase effort are able to obtain more consumption. Retailers set their prices in response to the distribution of household search intensity. Sellers charging relatively high (low) prices sell less (more) often but with higher (lower) markups. In an equilibrium every seller yields the same profits, but for different prices the profit comes from a different combination of appropriation of consumer surplus and stealing customers of other competing retailers. To the best of my knowledge, the proposed model is the first to combine the optimal savings problem and search for consumption in a quantitatively

[6]A size of consumption might be understood in three ways: quantity, variety, and quality. In the model I focus on quantity but it can be extended to variety very easily, if preferences of households are modeled as in Wolinsky (1986).

meaningful way.

Finally, I juxtapose the aggregate consumptions of two versions of calibrated economies, the SIM economy without product-market frictions and the "shopping" economy with frictions in the purchasing technology. I use two alternative approaches to study their properties: 1. consumption responses to idiosyncratic income shocks; and 2. cross-sections of households' decisions (consumption expenditures) and endogenous states (net wealth). Consumption responses reflect the dynamic character of the aggregate demand, while cross-sections of decisions and endogenous states determine the initial state of any counterfactual experiments. Thus, an accurate measure of both is of particular interest for quantitative analyses. Using simulated panels, I show that in the SIM economy without product-market friction households overreact to income changes and 80% of permanent shocks are translated to consumption[7], while in the shopping economy only 60% of permanent shocks are transmitted into consumption expenditures. The responses generated in the shopping model are much closer the empirical counterpart of 64% documented by Blundell, Pistaferri, and Preston (2008). This effect can be explained by the fact that marginal disutility for the shopping effort partially offsets utility of consumption, which makes consumption responses smoother. Even if the shopping generates smoother consumptions responses, it also amplifies wealth and consumption inequality. Consumption expenditures are more dispersed in the shopping economy due to disentangling consumption from consumption expenditures. Poor households with lower consumption exert higher price search intensity. As a result, they pay lower prices, which leads to a lower share in aggregate consumption expenditures accrued to poor groups. On the other hand, net wealth inequality is increased by rich working households,

[7]This finding is consistent with the "excess smoothness" of empirical consumption documented in aggregate data by Campbell and Deaton (1989) and in individual data by Attanasio and Pavoni (2011).

who detain from increasing current consumption due to the high utility cost of additional purchases.

The rest of the paper is structured as follows. Section II reviews connections to the existing literature. In section III, I present empirical patterns of shopping time by American households. In section IV, I present the building blocks of the quantitative model and characterize the equilibrium. Subsequently in section V I carry out the calibration to match moments observed in the US data. In section VI I highlight the nature of price dispersion at play for the calibrated version of the shopping model. In section VII I deconstruct aggregate demands generated by two artificial economies, the standard incomplete-markets economy without frictions in the purchasing technology and the shopping economy with product-market frictions. Section VI concludes.

II. Related Literature

The idea that consumer search might have important macroeconomic implications for modeling aggregates is relatively new. Kaplan and Menzio (2016) propose a theory of amplification of shocks driven by changing shopping behavior of households. To this purpose they build a model that combines consumer search (Burdett and Judd, 1983; Butters, 1977) with labor search (Mortensen and Pissarides, 1994). Despite some similarities in modeling product-market frictions between my framework and theirs, it is important to note some remarkable differences. First, in my setup price search intensities are determined endogenously by household decisions, while the authors calibrate them to exogenous values. Second, Kaplan and Menzio (2016) assume that agents are hand-to-mouth, they can neither save or borrow and are not allowed to smooth their marginal utility of consumption over their lifecycle. Consequently, my model allows addressing important questions related to household consumption, which are beyond interests of the aforementioned article.

Huo and Ríos-Rull (2013) and Bai, Ríos-Rull, and Storesletten (2011) offer an alternative model of search for consumption. In their models the authors employ directed search due to Moen (1997). In directed search retailers are divided into locations that provide goods at different prices and lengths of queues. Household with different earnings and net wealth visit different locations. I argue that random search is a more natural choice for modeling consumption decisions for two reasons. Affluent households are able to obtain goods in locations with short queues and high prices. In this sense they substitute shopping effort with higher prices. This behavior is common for both protocols, but in directed search there is no limit for such a substitution while in random search households can substitute effort with prices up to the limit where they decide to be captive in all transactions where prices are drawn at random. Consequently, without limits of substitution between prices and shopping effort consumption expenditure responses in economies with directed search for consumption can be even higher than in the SIM model without shopping frictions, which problems with generating smoothness of consumption observed in data is well documented (Attanasio and Pavoni, 2011; Kaplan and Violante, 2010). Furthermore, the recent empirical literature due to Kaplan, Menzio, Rudanko, and Trachter (2016) and Kaplan and Menzio (2015) shows that only 15% of the variance of prices is due to variation in the expensiveness of the stores at which a good is sold. This finding suggests to use the random search rather than directed search, where the market is split into locations with different prices.

The search protocol used in my paper also relates to the classical model of random search for consumption due to Burdett and Judd (1983). The household problem is the aspect in which my approach departs from that model substantially. In the economy of Burdett and Judd (1983) all households buy a unit of good and make a decision on their price search. They can either draw one price from the equilibrium dispersion or by paying an additional cost they can draw two prices and choose the lower one. In my model households make a decision about the quantity of consumption and the *probability* of drawing two prices. The former is important for introducing consumption search into the optimal savings problems, while the latter has implications for properties of the equilibrium, stability and multiplicity. I study and compare equilibria of both models in great details in the companion paper (Pytka, 2016).

The potential effect of search for price bargains on aggregates was recently studied by Krueger, Mitman, and Perri (2016). The authors consider an economy, in which output does not depend only on capital and labor inputs but also on consumption. This effect is obtained in a reduced form simply by introducing consumption as another input in the production function. In my shopping economy this aggregate demand externality has intuitive micro foundations. Any transfer targeted to poor households with high search intensity increases the aggregate price search intensity. Retailers respond to this change by charging lower prices and this results[8] in a hike in consumption spendings for all households, not only the recipients of the transfer. As a result, the shopping friction magnifies the effect of stimulus transfers.

The model developed in this paper relates also to concerns raised by Petrosky-Nadeau, Wasmer, and Zeng (2016). The authors used time diaries to document the average decline in shopping time in the Great Recession compared to 2005–2007. This finding was used to call into question whether the shopping effort can be used as an amplifier of shock propagation. They argue that in a contraction households should increase the shopping effort, if the theory of bargain

[8]Under the assumption that consumption is a normal good.

hunting is correct. However, this argument does not have to be true, if the shopping effort is constituted not only by search for price bargains but also by the purchase effort associated with the size of consumption. Therefore, I claim that introducing a new margin of shopping effort proposed in this paper is necessary to reconcile the pattern observed by Petrosky-Nadeau, Wasmer, and Zeng (2016) and the theory of macroeconomic implications of the price dispersion.

III. Empirical Patterns

I start by characterizing the shopping effort observed in the data. I follow the literature (e.g., Aguiar and Hurst, 2007; Aguiar, Hurst, and Karabarbounis, 2013; Kaplan and Menzio, 2016) and I use time spent shopping as a proxy for the shopping effort. For this purpose I study time diaries, which document the allocation of time of the American households. In particular, I am interested in the relationship between shopping and labor market status, i.e. unemployment, retirement, and the level of labor earnings. I show that conditioned on being employed, the level of shopping effort exerted by households is positively correlated with the level of their earnings. Next, the findings from this section are used to discipline the quantitative model outlined in the subsequent section, IV.

Data. In the analysis I use data from the 2003–2015 waves of the American Time-Use Survey. The ATUS is conducted by the U.S. Census Bureau and individuals are randomly selected from a subset of households from the Current Population Survey. Each wave is based on 24-hour time diaries where respondents report the activities from the previous day in specific time intervals. Next the ATUS staff categorizes those activities into one of over 400 types. The 2003 wave includes over 20,000 respondents, while the later waves consist of around 13,000 respondents.

Identification Strategy. To assess how shopping effort is reallocated across different households i with different levels of labor earnings I estimate the following regression:

$$\log shopping_i = \alpha + \sum_j \beta_j earn_i^j + \gamma X_i + \varepsilon_i. \tag{1.1}$$

To abstract from the discussion on the functional specification, I regressed the dependent variable on dummies. In addition to this I logarithmized[9] $shopping_i$ to include possible multiplicative interactions between covariates.

The variable $shopping_i$ measures cumulative daily time (in minutes) spent obtaining goods or services (excluding education, restaurant meals, and medical care) and travels related to these activities. Some examples of activities captured by this variable are: grocery shopping, shopping at warehouse stores (e.g., WalMart or Costco) and malls, doing banking, getting haircut, reading product reviews, researching prices/availability, and online shopping.

There are three types of variables associated with the labor force status: 1. nine categorical variables $earn_i^j$ where each of them represents j-th decile of weekly labor income with the bottom decile as the referential category (Table: A.1), 2. unemployment status (both on lay-off and looking), and 3. retirement. To control other sources of heterogeneity I introduce some demographic variables: the (quadratic) age trend, gender (woman as reference), and race (white as reference).

Aguiar and Hurst (2007) suggest controlling for 'shopping needs,' which stem from differences in the family composition. For this reason, I add dummies indicating: 1. if the respondent has a partner (both spouse and unmarried), 2. whether the partner is unemployed, and 3. the presence of children.

[9] Due to the fact that there are observations with zero values, I use the inverse hyperbolic sine function $\text{arsinh}(x) = \ln(x + \sqrt{1+x^2})$ as an approximation of the logarithmic function.

Table 1.1: Regression results

	Dependent variable		
	$log(shopping)$		
	(I)	(II)	(III)
Earnings dummies (Fig. 1.1)	■	▲	●
Retired	0.147***	0.161***	0.165***
	(0.035)	(0.035)	(0.035)
Unemployed	0.302***	0.314***	0.321***
	(0.032)	(0.032)	(0.032)
Male	−0.484***	−0.466***	−0.470***
	(0.013)	(0.013)	(0.013)
Age	0.007**	−0.002	−0.003
	(0.003)	(0.003)	(0.003)
Age^2	−0.0001*	0.00004	0.00005
	(0.00004)	(0.00004)	(0.00004)
Black	−0.151***	−0.128***	−0.127***
	(0.020)	(0.020)	(0.020)
Single		−0.125***	−0.124***
		(0.016)	(0.016)
Unemployed Partner		−0.170***	−0.170***
		(0.019)	(0.018)
Child		0.041***	0.041***
		(0.015)	(0.015)
Constant	1.979***	2.182***	2.217***
	(0.070)	(0.073)	(0.078)
Shopping needs	No	Yes	Yes
Year and day dummies	No	No	Yes
N	132,131	132,131	132,131
F Statistic	100.574***	90.876***	71.039***

Notes:
***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.

Results. I estimated the model using the pooled regression. All observations were weighted to ensure that each stratification group is correctly represented in the population. I restricted the sample only to households aged 22-74 and excluded top-percentile households with respect to earnings and shopping. Three specification are considered: with labor market variables only (I); with controls for labor market status and shopping needs (II); with controls for market status, shopping needs, and year/day dummies (III). The estimated results of model (1.1) are presented in Table 1.1 and Figures 1.1 and 1.2. The conducted analysis leads to the following observations on the relationship between the shopping behavior and the labor market status:

Pattern 1 (Shopping effort and labor market status) *In the ATUS 2003-2015 the following patterns are observed:*

i *the unemployed people spent on average* $\exp(.321) = 37.85\%$ *more time shopping than the referential earning group;*

ii *the retired people spent on average* $\exp(.165) = 17.94\%$ *more time shopping than the referential earning group;*

iii *top deciles of the labor earnings spent on average more time shopping than the referential earning group.*

Observation 1.i and 1.ii do not differ qualitatively from results present in the literature. Kaplan and Menzio (2016) show that the unemployed people spend between 13% and 20% more time on shopping than the employed. The difference obtained by Krueger and Mueller (2012) is larger and amounts to 28%. My finding is of the same sign but is also quantitatively higher. The reason for this can be attributed to the fact that I used the bottom decile of labor earners as the referential category in my study, whereas in the aforementioned articles the unemployed are compared with the whole population of the employed. Regarding the shopping behavior of retirees, Aguiar and Hurst (2007) compare the cell of retirement-age[10] people with households aged 25-29 and show the older spend on average 32.7 log-points more on shopping. This effect is twice as large as observation 1.ii. However, if instead of regressing on the quadratic age trend I use age bins in the way Aguiar and Hurst (2007) did, then the people who are in the oldest cell *and* are retired spend 30.7 log-points more on shopping. In the further considerations I stick to estimate 1.ii, which disentangles the retirement state from the age effect.

Patterns 1.i and 1.ii are well known and rationalized by the search for price bargains. Households with low resources pay more attention to expenditures and are more patient to get lower prices. They are able to decrease their prices by increasing the search effort embodied by such activities as visiting more stores for comparison shopping, clipping coupons, or waiting for sales. All of them require some additional amount of time though. Those observations led to a traditional view that equalizes shopping effort with the search for price bargains.

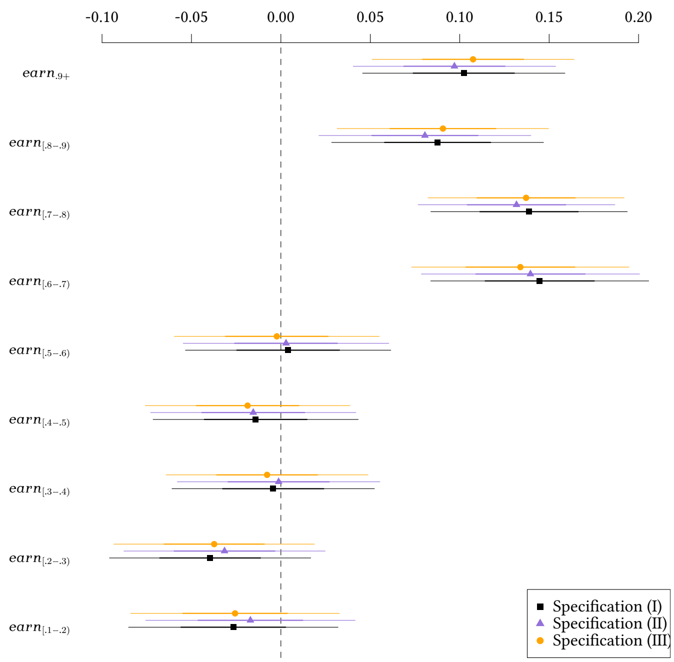
In this sense, observation 1.iii, which to the best of my knowledge is novel, seems to be paradoxical. Conditioned on being employed, affluent households from top deciles spend significantly more time shopping than poor households (see Figure 1.1). Nonetheless, according to the reasoning above we should rather observe the opposite effect[11]. This finding suggests that apart from price hunting there must be another motive driving shopping effort in such a fashion that rich people spend more time shopping. I claim that an effort accompanying the size of consumption is a good candidate for such a motive that rationalizes fact 1.iii.

[10]The authors do not use an explicit dummy for the retirement state.

[11]In this claim I implicitly assume that leisure and consumption are normal goods.

The argument for this is that households with a high level of consumption have to visit more stores. Every shopping trip takes additional amount of time[12].

Figure 1.1: Regression estimates for dummies of earning deciles.

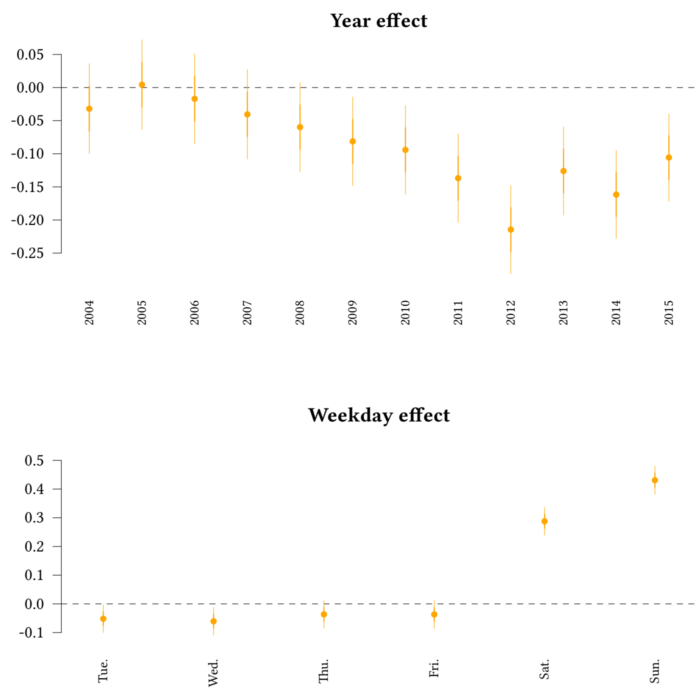


The remaining estimates of variables controlling for shopping needs are consistent with the intuition. Single households spend less time shopping due to lower variety and amount of needed consumption. Respondents with a unemployed partner also spend less time on purchasing goods. This can be explained with delegation of non-market work to unemployed members of a family, who

[12]Admittedly, households can purchase many units of consumption in one store. This concern is discussed thoroughly in section IV. Without going too much into details, such a shopping strategy of households increases the market power of retailers and makes customers captive for a bigger fraction of purchases.

have more time. Having children increases shopping needs too[13].

Figure 1.2: Regression estimates for year and weekdays dummies.



Last but not least, Figure 1.2 shows how the shopping effort varied over years and weekdays. Unsurprisingly, households spend more time shopping in weekends. It is worth noting, there is a visible downward trend from 2003 through 2015. Every year households spent less time shopping, on average 1.28% less every year. One reason for this phenomenon can be the profound improvement of purchasing technology. New technologies developed recently such as online purchases, comparison shopping engine, mobile payments, and many more may caused the decline in the magnitude of product market frictions. Consequently,

[13] In an auxiliary analysis, I verified if a number of children is important. It does not seem so as the increase of shopping time is of about 4% for every number of children (treated as dummies).

it might have given rise to lower time required for obtaining consumption goods.

IV. A Life-Cycle Model of Shopping Effort

My framework integrates random search for consumption into a life-cycle incomplete markets model with heterogenous agents (*e.g.,* İmrohoroğlu, İmrohoroğlu, and Joines, 1995; Huggett, 1996; Ríos-Rull, 1996). Household income is driven by idiosyncratic productivity shocks. Every household makes the decision about level of savings, that are used to insure against future income fluctuations and to smooth the future consumption. The remaining disposable resources of household are spent on consumption. On top of the economy I introduced the frictional transactions technology. Households have to exert effort to purchase goods. This effort has two components: 1. effort to search for price bargains, 2. purchase effort required to purchase consumption of a given size. The former accounts for increasing probability that household during a single purchase samples a lower price, while the latter relates to the assumption that more consumption is possible by increasing a number of purchases[14]. The price search is present and documented in the literature (*e.g.,* Kaplan and Menzio, 2016) and the purchase effort is new and explained in more details later.

I first describe the setup of the economy. Next I characterize the model equilibrium and present some examples to shed some light on the shopping mechanism at work.

[14]In this regard, there is an important difference with the story of long queues with low prices and short queues with high prices offered by the directed search (*e.g.,* Moen, 1997; Bai, Ríos-Rull, and Storesletten, 2011). The recent empirical literature due to Kaplan, Menzio, Rudanko, and Trachter (2016); Kaplan and Menzio (2015) shows that only 15% of the variance of prices is due to variation in the expensiveness of the stores at which a good is sold. This finding suggests to use the random search instead.

Demographics. The model period is one year. The stationary economy is populated by a continuum of households living T periods. Consumers work for T_{work} periods and next go into retirement for $T - T_{work}$ periods.

Preferences. Households exhibit preferences defined over stochastic sequences of consumption and overall shopping effort $\{c_t, f_t\}_{t=1}^{T}$ represented by the instantaneous utility function:

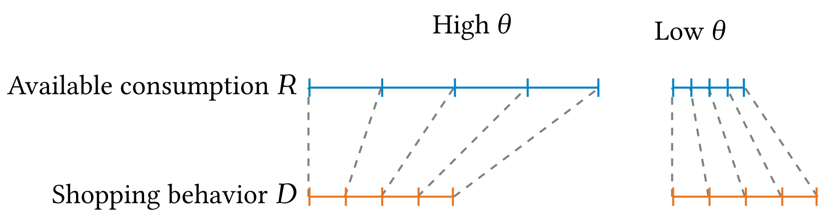
$$u(c_t) - v(f_t), \tag{1.2}$$

and the discount factor β. Households are expected utility maximizers. The utility from consumption, $u(c_t)$ is additively separable from the disutility from shopping effort, $v(f_t)$. Both functions are assumed to be increasing and $u(c_t)$ is concave while $v(f_t)$ is convex. Overall shopping effort f_t is a function of two shopping margins, a number of purchases m_t and search intensity s_t. It increases in both margins, *i.e.* $\frac{\partial f_t}{\partial m_t} > 0$, $\frac{\partial f_t}{\partial s_t} > 0$. Besides, both shopping margins affect each other's impact on f_t as follows $\frac{\partial^2 f_t}{\partial s_t \partial m_t} > 0$. It means that higher shopping effort m_t increases the marginal cost of searching for price bargains, s_t, and vice versa, the higher search intensity leads to a higher marginal cost of shopping effort.

Purchases (m_t). In order to consume goods c_t, households must spend some time for visiting stores. They make many repeated purchases (shopping visits) m_t in a given period. The level of the required effort is strictly increasing with consumption. Consumers make purchases, which are matched with goods offered by the retailers. Let D be the aggregate level of shopping effort (yet to be defined) of all households, R be the total amount of consumption purveyed by the retailers and $\theta = \frac{R}{D}$ be the market tightness of the consumption market. Both sides come together through a constant return to scale Cobb-Douglass function, $M(D,R) = D^{\alpha}R^{1-\alpha}$. A single shopping visit allows a household to purchase $\frac{M(D,R)}{D} = \theta^{1-\alpha}$ units of consumption. Thus, given an equilibrium market "tightness" θ, there is a linear relationship between consumption and the level of required shopping effort, *viz.*

$$c_t = m_t \theta^{1-\alpha}. \tag{1.3}$$

Figure 1.3: Matching shopping effort with retailers



Note, that the efficiency of purchase $\theta^{1-\alpha}$ does not necessarily have to be less than one. This statistics tells us about the level of feasible consumption for a single purchase[15]. Suppose that a household wants to consume a certain amount of goods. In an economy with high θ she has to make fewer shopping trips to be able to purchase it (Figure 1.3).

Price Search (s_t). Apart from the number of purchases (which directly translates to the level of consumption), each household makes a decision on the intensity of search for price bargains, s_t. Suppose prices quoted by retailers are distributed according to a cdf $G(p) = Pr(x \leq p)$ with a lower bound $\underline{p}$, such that $G(\underline{p}) = 0$ and an *exogenously*[16] set upper bound ζ, such that $G(\zeta) = 1$. For

[15] In this regard, the interpretation of the efficiency of shopping effort differs from the probability that an unemployed worker matches with a vacancy used in the labor search literature. It is due to the fact that consumption is intuitively divisible while jobs are not.

[16] The relevance of this assumption is discussed more thoroughly later.

a single purchase the price is sampled independently. Depending on the search intensity s_t, the purchase receives with probability of s_t two independent offers drawn from $G(p)$ and the lower one is paid, or with complementary probability of $1 - s_t$ one price is sampled and the customer is captive for this specific transaction. Thus the distribution of the effective price of a single purchase is a result of the compound lottery:

$$F(p; s_t) = (1 - s_t)G(p) + s_t \left(1 - [1 - G(p)]^2\right). \tag{1.4}$$

The first term, $(1 - s_t)G(p)$ tells us the probability that the purchase is captive *and* the effective price will be lower than p, while the second term is the probability that two prices are drawn and the minimum of those offers are lower than p[17]. A household can decrease the expected value of the price drawn from the lottery by increasing its search intensity s_t, but on the other hand, there is a trade-off since it increases the disutility from shopping visits[18].

The cost of the consumption bundle. The price of every purchase constituting the overall shopping effort (m_t) is sampled independently. It means that the overall cost of the consumption bundle $c_t = m_t\theta^{1-\alpha}$ is the realization of continuum of lotteries, *i.e.*

$$\int_0^{m_t\theta^{1-\alpha}} p(i)di, \tag{1.5}$$

where prices $p(i)$ are drawn from the cdf $F(p; s_t)$. Lemma 1 states that, while the cost of a single purchase is random and ex-ante unknown, the cost of many purchases is certain with probability one.

[17]Clearly, $Pr(x \geq \min\{p', p''\}) = (1 - G(p))^2$, so the cdf of the minimum of two prices is given by $Pr(x \leq \min\{p', p''\}) = 1 - [1 - G(p)]^2$.

[18]It is a consequence of assuming the positive cross partial derivative, $\frac{\partial f_t}{\partial m_t \partial s_t} > 0$.

Lemma 1 (Cost of consumption bundle) *Let the effective price of a purchase be distributed according to the cdf $F(p; s_t)$. Then the cost of consumption c_t given search intensity converges almost surely:*

$$\int_0^{m_t\theta^{1-\alpha}} p(i)di \overset{\text{a.s.}}{\rightarrow} \underbrace{m_t\theta^{1-\alpha}}_{c_t}\mathbb{E}(p|s_t), \tag{1.6}$$

where the effective price of consumption is equal to $\mathbb{E}(p|s_t) = \int pdF(p; s_t)$.

Proof The lemma is an immediate result of applying the weak law of large numbers for random continuum in a version proposed by Uhlig (1996, Theorem 2). ∎

It is convenient to make a decomposition of $\mathbb{E}(p|s_t)$ to disentangle the marginal effect of increasing search intensity on the effective price.

Lemma 2 (Linearity of the effective price function) *For given distribution of the quoted prices $G(p)$ the effective price paid by households is a linear function with respect to search intensity s :*

$$\mathbb{E}(p|s_t) = p^0 - s_t MPB, \tag{1.7}$$

where:

i. $p^0 = \int_{\underline{p}}^{\zeta} xdG(x)$ *is the price for the fully captive consumer;*

ii. $MPB = \mathbb{E}\max\{p', p''\} - p^0 (\geq 0)$ *is the marginal (price) benefit of increasing the search intensity s_t, where $\mathbb{E}\max\{p', p''\}$ is the expected maximum of two independent draws of prices.*

Proof To derive (1.7) I use the fact that the expected value of any non-negative random variable x distributed according to a cdf $H(x)$ can be computed integrating over its survival function (Billingsley, 1995, p. 79), namely:

$$\mathbb{E}(x) = \int_0^\infty (1 - H(x))dx. \tag{1.8}$$

The price of the consumption bundle is then a result of applying this property to equation (3.11):

$$\mathbb{E}(p|s_t) = \int_0^\infty 1 - G(x) - s_t \left(G(x) - [G(x)]^2\right) dx,$$

where $\int_0^\infty 1 - G(x)dx$ is the expected value for the captive offer and, using an analogous reasoning from Lemma 1, is also the price of consumption for the fully captive household that decides not to make any search for prices.

The residual part is equal to:

$$\int_0^\infty \left(G(x) - [G(x)]^2\right) dx =: MPB, \tag{1.9}$$

and which is clearly always positive as $\forall_x G(x) \geq [G(x)]^2$. For better interpretation it is convenient to reformulate equation (1.9):

$$\int_0^\infty \left(G(x) - [G(x)]^2\right) dx = \underbrace{\int_0^\infty 1 - [G(x)]^2 dx}_{\mathbb{E}\max\{p',p''\}} - \underbrace{\int_0^\infty 1 - G(x)dx}_{p^0}.$$

The expected maximum of two independent draws, $\max\{p', p''\}$ is distributed according to $[G(x)]^2$. It can be easily shown by the fact that $Pr(\max\{p', p''\} \leq x) = Pr(p' \leq x, p'' \leq x)$. Assuming independence of p' and p'' we get $Pr(p' \leq x) \cdot Pr(p'' \leq x) = [G(x)]^2$. Therefore, $\mathbb{E}\max\{p', p''\} = \int_0^\infty 1 - [G(x)]^2 dx$.

∎

Lemma 1 shows that thanks to the fact that the cost of consumption is a sum of many repeated purchases the overall cost of the consumption basket can be pinned down deterministically. Each purchase is a result of different lottery price. Lemma 2 goes even further. It says that only two statistics of the price distribution, p^0 and MPB are needed to be known by households for making the optimal decision.

Productivity process. While being active in the labor market ($t \in \overline{1, T_{work}}$) every household faces the idiosyncratic wage risk. Log productivities follow an exogenous stochastic process:

$$\ln y_t = \kappa_t + \eta_t + \varepsilon_t,$$
$$\eta_t = \eta_{t-1} + \nu_t,$$

where $\varepsilon_t \sim_{\text{iid}} \mathcal{N}(0, \sigma_\varepsilon^2)$ and $\nu_t \sim_{\text{iid}} \mathcal{N}(0, \sigma_\nu^2)$. The deterministic part κ_t is a lifecycle component common to all households. The martingale part η_t and the serially uncorrelated part ε_t account for the permanent and transitory components of the productivity, respectively. While being employed all households receive the labor income wy_t.

Retirement. Households older than T_{work} receive a deterministic retirement that is a function of their income in the last working-age period with replacement rate $repl$:

$$\log y_t = log(repl) \cdot \{\kappa_{T_{work}} + \eta_{T_{work}} + \varepsilon_{T_{work}}\}.$$

Budget constraint. Households can hold a single risk-free asset which pays a net return, r. Let a_{t+1} be the amount of asset carried over from t to $t+1$. Every household faces the sequence of intertemporal budget constraints:

$$\mathbb{E}(p|s_t)c_t + a_{t+1} \leq wy_t + (1+r)a_t, \quad \forall_{t\in\overline{1,T}}. \tag{1.10}$$

The effective price of consumption is a function of search intensity and is given by equation (1.7). It is worth noting that the intensity of search for prices s does not affect, at least directly, the level of consumption, but only the price of consumption. The shopping effort m_t affects the cost of consumption bundle only by the level of consumption expressed by the upper limit of the integral in formula (1.5). In addition to this every household faces the exogenous borrowing constraint $a_{t+1} \geq \underline{B}$.

Households' Decision Problem. The dynamic problem of a household of age t whose state is $x = (a, \varepsilon, \nu, \eta)$ is:

$$\mathcal{V}_t(a, \varepsilon, \eta) = \max_{c,f,m,s,p,a'} u(c) - v(f) + \beta \mathbb{E}_{\eta'|\eta} \mathcal{V}_{t+1}(a', \varepsilon', \eta') \tag{1.11}$$

s.t.

$$\begin{aligned}
pc + a' &\leq (1+r)a + wy, \\
c &= m\theta^{1-\alpha}, \\
f &= f(m,s), \\
p &= p^0 - sMPB, \\
a' &\geq \underline{B}, \\
s &\in [0,1], \\
\log y &= \begin{cases} \kappa_t + \eta + \varepsilon, & \text{for } t \leq T_{work}, \\ log(repl) \cdot \{\kappa_{T_{work}} + \eta_{T_{work}} + \varepsilon_{T_{work}}\}, & \text{for } t > T_{work}, \end{cases} \\
\eta' &= \eta + \nu',
\end{aligned}$$

The problem is not convex due to bilinearity in controls s and c in the budget constraint. This may cause that the first order conditions do not suffice and might lead to local solutions. However, this issue is solved by using envelope convexification of the bilinear constraint, which was proposed by McCormick (1976).

Retailers' problem. Sellers buy consumption goods at the cost standardized to one and quotes her price in every period conditioned on being matched with households' purchases. She maximizes the sales revenue:

$$S(p) = \theta^{-\alpha} \sum_{t=1}^{T} \int \frac{\theta^{1-\alpha} m_t(x)(1+s_t(x))}{D} \underbrace{\left(1 - \frac{2s_t(x)}{1+s_t(x)} G(p)\right)}_{\substack{\text{Business} \\ \text{Stealing}}} \underbrace{(p-1)}_{\substack{\text{Surplus} \\ \text{Appropriation}}} d\mu_t(x), \tag{1.12}$$

where $\mu_t(x)$ is the distribution of households of age t over the individual states

$x = (a, \varepsilon, \nu, \eta)$. In the problem of sellers there are two opposite motives. First, the net revenue $(p-1)$ from a single purchase is increasing with the set price. The second motive is generated by lack of information whether the matched buyer has the alternative offer for this purchase. The probability that the household has an alternative that with a better price than p amounts to $\frac{2s_t(x)}{1+s_t(x)}G(p)$. Thus, the probability of acceptance a given price price is the complementary event with probability $\left(1 - \frac{2s_t(x)}{1+s_t(x)}G(p)\right)$. Higher prices decrease the probability that the offer will be accepted by the buyer. Thus, these two motive can generate a price dispersion, in which there are retailers that have higher markups but their prices are rejected more often and retailers that cut their prices to increase the probability of the successful transaction. In an equilibrium the sellers are indifferent[19].

Relevance of exogenous reservation price ζ. A question that arises from the exogenous price ζ is about the commitment of households to pay sampled prices for all purchases. The repeated purchases can be interpreted as consumption in different subperiods of the year. If the subperiods are long enough it is reasonable to say that households agree to pay the lowest offered (but still high) price in order to avoid starving to death due to the lack of consumption. On the other hand, if subperiods are short enough households might prefer setting their own endogenous reservation price $\overline{p}$ and deferring from paying above this price. In this case, the model should be augmented by an additional control, $\overline{p}$. However, this extensions leads to some issues. First, lemma 2 does not hold and the constraint for the effective price is not linear with all controls. This is due to the fact that $\overline{p}$ replaces exogenous ζ. Second, there is no clear distinction between two motives, shopping effort m_t and search intensity s_t anymore. An additional

[19]In this sense, the mechanism is similar to the theory of homogenous hotel rooms with different prices given by Prescott (1975).

increase in shopping effort accompanied by a decrease in $\overline{p}$ plays the same role as an increase in s[20].

Equilibrium. Having outlined the building blocks of the economy, I am in the position to define an equilibrium of the economy.

Definition 3 (Rational Stationary Equilibrium) *A stationary equilibrium is a sequence of consumption and shopping plans $\{c_t(x), m_t(x), s_t(x)\}_{t=1}^{T}$, and the distribution of quoted prices $G(p)$ and paid prices $F(p; s_t(x))$, distribution of households $\mu_t(x)$ and interest rate r such that:*

1. *$c_t(x), m_t(x), s_t(x)$ are optimal given r, w, $G(p)$, $\underline{B}$, and θ;*

2. *individual and aggregate behavior are consistent:*

$$D = \sum_{t=1}^{T} \int (1 + s_t(x)) m_t(x) d\mu_t(x); \tag{1.13}$$

3. *retailers post prices p to maximize the sales revenues taking as given households' behavior;*

4. *the private savings sum up to an exogenous aggregate level $\overline{K}$:*

$$\sum_{t=1}^{T} \int a_t(x) d\mu_t(x) = \overline{K}; \tag{1.14}$$

5. *$G(p)$ and $F(p; s_t(x))$ are consistent given the household distribution $\mu_t(x)$;*

6. *$\mu_t(x)$ is consistent with the consumption and shopping policies.*

[20]However, search intensity s_t is still necessary for generating price dispersion.

The dispersed distribution of posted prices is consistent with the solution to the maximization of the retailers' net sales revenue, (1.12). Lemma 4 presents properties of an equilibrium of this kind. The proof of the lemma is similar to ones used in Burdett and Judd (1983) and Kaplan and Menzio (2016).

Lemma 4 (Characterization of the Equilibrium Price Dispersion) *The c.d.f. $G(p)$ exhibits following properties:*

i. $G(p)$ *is continuous.*

ii. $\text{supp } G(p)$ *is a connected set.*

iii. the highest price charged by retailers is equal to ζ*,*

iv. all retailers yield the same profit, $\forall_{p \in \text{supp } G(p)} S(p) = S^*$*,*

where $\text{supp } G(p)$ *is the smallest closed set whose complement has probability zero.*

Proof The two first properties are an immediate result of Lemma 1 from (Burdett and Judd, 1983). Suppose that $G(p)$ has a discontinuity at some $p' \in \text{supp } G(p)$. The retailer posting an infinitesimally smaller price $p' - \epsilon$ would increase its profit as the probability of making a sale would change by a discrete amount. Furthermore, $\text{supp } G(p)$ is a connected set. Suppose there is a gap of zero probability between p' and p''. The seller's gain would be strictly higher at p'' as $p'' > p'$, and $G(p') = G(p'')$. This cannot occur in an equilibrium.

Next, suppose that (iii) is not true. Then $\max \text{supp } G(p) =: \overline{p} \leq \zeta$.[21] Moreover, we know that $G(\overline{p}) = G(\zeta) = 1$. If we substitute values of the c.d.f. for both prices into (1.12) all firms will have incentives to set higher price for higher

[21] Recall that there is the exogenous upper-bound for prices ζ, so $\overline{p} \geq \zeta$ is not considered.

demand, which leads us to contradiction. As a result, max supp $G(p) = \zeta$. Fact (iv) is an equilibrium condition. If there would be such a price p that would yield higher profit, each individual retailer would have incentives to set this price. ∎

It is convenient to decompose the aggregate shopping effort D defined in (1.13) into two components:

$$\Psi_{(-)} := \sum_{t=1}^{T} \int m_t(x)(1 - s_t(x)) d\mu_t(x), \tag{1.15}$$

$$\Psi_{(+)} := \sum_{t=1}^{T} \int m_t(x) 2 s_t(x) d\mu_t(x), \tag{1.16}$$

$$D = \sum_{t=1}^{T} \int m_t(x)(1 + s_t(x)) d\mu_t(x) = \Psi_{(-)} + \Psi_{(+)}. \tag{1.17}$$

Notice that $\Psi_{(-)}$ in (1.15) is an aggregate measure of visits where customers are captive and $\Psi_{(+)}$ in (1.16) where households draw two prices and choose the lower one. D from (1.17) is the measure of the aggregate shopping defined in (1.13) and is a sum of $\Psi_{(-)}$ and $\Psi_{(+)}$. Consequently, $\frac{\Psi_{(-)}}{D}$ and $\frac{\Psi_{(+)}}{D}$ are probabilities that a single draw is captive or matched with an alternative offer, respectively. By construction all offers of $\Psi_{(-)}$ are effective for the reason that buyers are captive during these purchases. On the other hand, only half of $\frac{\Psi_{(+)}}{D}$ is accepted by buyers and the remaining part is rejected. It is so because for this measure of offers consumers get two price offers and choose the lower one.

Properties from Lemma 4 can be used to derive the formula for an equilibrium price dispersion.

Theorem 5 (Equilibrium Price Dispersion) *Given aggregate statistics of households' shopping decisions* $\{\Psi_{(-)}, \Psi_{(+)}, D\}$, *(where* $\Psi_{(-)}, \Psi_{(+)} > 0$*), the equilib-*

rium price dispersion can be expressed in a closed form:

$$G(p) = \begin{cases} 0, & \textit{for } p < \underline{p}, \\ \frac{D}{\Psi_{(+)}} - \frac{\Psi_{(-)}}{\Psi_{(+)}} \cdot \frac{\zeta - 1}{p-1}, & \textit{for } p \in [\underline{p}, \zeta], \\ 1, & \textit{for } p > \zeta, \end{cases} \tag{1.18}$$

where the lower bound of $\operatorname{supp} G(p)$ *is:*

$$\underline{p} = \frac{\Psi_{(+)}}{D} + \frac{\Psi_{(-)}}{D}\zeta. \tag{1.19}$$

Proof The proof is relegated to Appendix II. ∎

Discussion of Theorem 5. Given p, the equilibrium price dispersion $G(p)$ is a linear function decreasing in: 1. the inverse odds ratio[22] of being matched with a non-captive customer, $\left(\frac{\Psi_{(+)}}{\Psi_{(-)}}\right)^{-1}$ and 2. the probability that a visiting buyer draws an alternative offer, $\frac{\Psi_{(+)}}{D}$. Suppose that there are two economies with the same aggregate shopping effort D and different level of search intensity, $\Psi'_{(+)} > \Psi''_{(+)}$. Due to the fact that $\frac{\partial G(p)}{\partial \frac{\Psi_{(+)}}{D}} > 0$ for every p from the the interior of supp $G(p)$, the price lottery of the economy with higher search intensity $\Psi'_{(+)}$ first-order stochastically dominates the price lottery of the economy with lower search intensity $\Psi''_{(+)}$. This observation leads to an immediate remark that economies with higher search intensity exhibit the lower expected value of the price lottery. The result is consistent with economic intuition. The higher fraction of buyers with alternative offers is, the stronger competition between retailers is observed. For a better understanding how the price equilibrium changes in $\Psi_{(+)}$ consider three cases:

[22]Notice that $\frac{\Psi_{(+)}}{\Psi_{(-)}} = \frac{\frac{\Psi_{(+)}}{D}}{1 - \frac{\Psi_{(+)}}{D}}$.

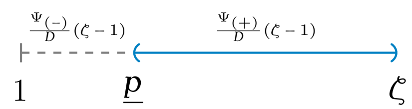
1. $\Psi_{(+)} = 0$ – the business stealing motive from (1.12) embodied by $\left(1 - \frac{2s_t(x)}{1+s_t(x)} G(p)\right)$ disappears and only the surplus appropriation motive occurs. Every customer is captive and this leads to a degenerate Diamond (1971)-type equilibrium, where all retailers charge the monopolistic price, ζ;

2. $\Psi_{(+)} = D$ *(hypothetical)* – every consumer draws two prices and chooses the lower one. Consequently, all retailers start playing a Bertrand game and the only price equilibrium is a degenerate competitive one, $p = 1$. Nonetheless, it is a purely hypothetical case since an equilibrium from Definition (3) with $\Psi_{(+)} = D$ never exists. If all prices are set competitively, then none of households have incentives to make any search. To them it pays off to be captive all the time but then $\Psi_{(-)} = D$ and $\Psi_{(+)} \neq D$, which contradicts the constituting assumption of the case that $\Psi_{(+)} = D$;

3. $\Psi_{(+)} \in (0, D)$ – there occurs a tug of war between two motives, 1. the appropriation of consumers' surplus and 2. business stealing. In every point of the support of the equilibrium price dispersion supp $G(p) = [\underline{p}, \zeta]$ retailers yield the same profit S^*. However, for each price there is a different composition of sources of this profit. The business stealing motive is the only motive for retailers charging $\underline{p}$, while the surplus appropriation only rationalizes the behavior of sellers that set ζ. Prices from the interior of supp $G(p)$ are supported by a combination of both. As the aggregate search $\frac{\Psi_{(+)}}{D}$ increases, retailers set lower prices and the lowest quoted price, $\underline{p}$ gets closer to the competitive pricing.

The lower bound $\underline{p}$ of supp $G(p)$ also depends on the aggregate search intensity in the economy. Interestingly, it is a convex combination of the competitive price (normalized to 1) and the monopolistic price ζ, where $\frac{\Psi_{(+)}}{D}$ and $\frac{\Psi_{(-)}}{D}$ are weights. The higher $\frac{\Psi_{(+)}}{D}$ is, the further $\underline{p}$ is from the monopolistic price and closer to the

Figure 1.4: The equilibrium support of $G(p)$.



competitive price (see Figure 1.4).

Equilibrium price moments. Finally, p^0 and $\mathbb{E}\max\{p', p''\}$ from Lemma 2 can be pinned down using the closed form solution from Theorem 5.

Proposition 6 *Given households' aggregate shopping efforts $\Psi_{(-)}$ and $\Psi_{(+)}$, the price for captive customers (p^0) and the expected maximum of two independent draws ($\mathbb{E}\max\{p', p''\}$) can be expressed in a closed form:*

i. price of the captive customer:

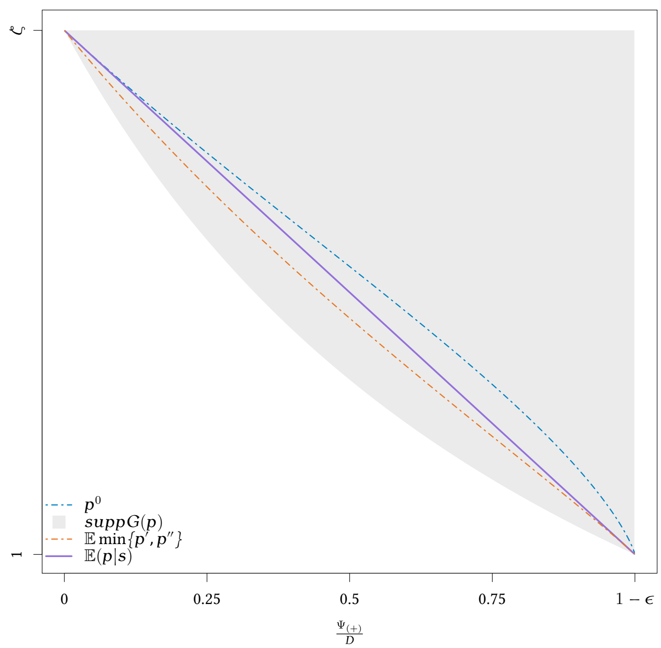
$$p^0 = \underline{p} + \frac{\Psi_{(-)}}{\Psi_{(+)}}(\zeta - 1)\log\left(\frac{\zeta - 1}{\underline{p} - 1}\right) + \left(1 - \frac{D}{\Psi_{(+)}}\right)(\zeta - \underline{p})\,; \tag{1.20}$$

ii. the expected maximum of two independent draws:

$$\mathbb{E}\max\{p', p''\} = \zeta - \left(\frac{D}{\Psi_{(+)}}\right)^2(\zeta - \underline{p}) + 2\frac{D\Psi_{(-)}}{\Psi_{(+)}^2}(\zeta - 1)\log\left(\frac{\zeta - 1}{\underline{p} - 1}\right) - \\ - \left(\frac{\Psi_{(-)}}{\Psi_{(+)}}\right)^2(\zeta - \underline{p})\frac{\zeta - 1}{\underline{p} - 1}.$$

Proof *The proof is relegated to Appendix II.* ■

Figure 1.5: Moments of equilibrium price distribution and the aggregate search intensity.



Note: The figure depicts summary shopping moments for $\frac{\Psi_{(+)}}{D} \in [0, 1)$.

The moments from Proposition 6 are "sufficient" price statistics[23] that are required in the household's problem (1.11). For gaining a better insight into the mechanics of the equilibrium it is helpful to conduct the comparative statics with respect to the search intensity. Without loss of generality, in this exercise I focus on the representative consumer framework. For this case there is a one-to-one mapping between the individual search intensity of the consumer and the aggregate search intensity, *i.e.* $\frac{\Psi_{(+)}}{D} = \frac{2s}{1+s}$[24]. Figure 1.5 shows how

[23]Recall that $MPB = \mathbb{E}\max\{p', p''\} - p^0$.

[24]Besides, notice that in this case the equilibrium cdf is distributed according to $G(p) = \frac{1+s}{2s} -$

the key price characteristics change in the probability of being matched with a non-captive customer, $\frac{\Psi_{(+)}}{D}$. First, the average effective price $\mathbb{E}(p|s)$ varies between the price of the fully captive customer p^0 and the expected minimum of two draws $\mathbb{E}\min\{p', p''\}$. Even though prices are sampled from a whole interval supp $G(p) = [\underline{p}, \zeta]$, the (unit) cost of the consumption bundle is the average price $\mathbb{E}(p|s)$ drawn from $F(p)$ and given by (1.7). As mentioned before, for $\Psi_{(+)} = 0$ there exists only the degenerate Diamond (1971)-type equilibrium, where $\mathbb{E}(p|s) = p^0 = \zeta$. An increase in $\frac{\Psi_{(+)}}{D}$ makes $\mathbb{E}(p|s)$ further from the captive price p^0 and closer to the expected minimum $\mathbb{E}\min\{p', p''\}$. In the limit case you can observe[25]:

$$\lim_{s \to 1^-} \mathbb{E}(p|s) = 2p^0 - \mathbb{E}\max\{p', p''\} = \mathbb{E}\min\{p', p''\}. \tag{1.21}$$

Higher search intensity in the economy fosters higher competition between retailers. As a result, all price statistics (p^0, $\mathbb{E}\min\{p', p''\}$, $\mathbb{E}\max\{p', p''\}$, $\mathbb{E}(p|s)$) tend towards the competitive solution, which in the model is normalized to unity.

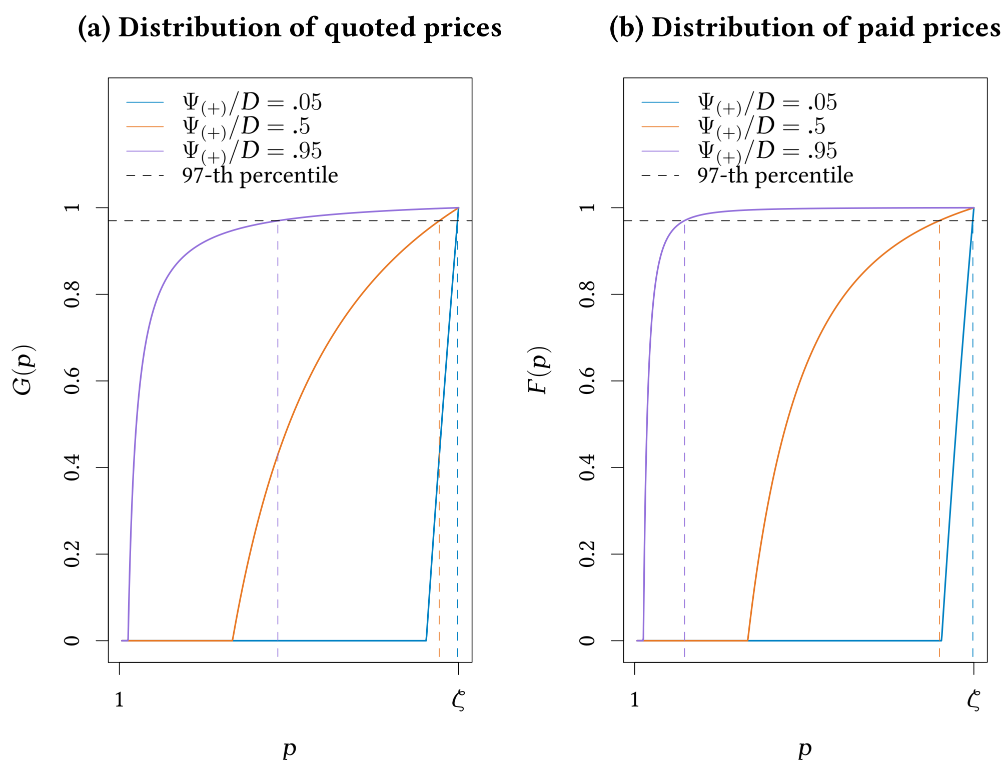
A natural concern that arises here is the assumption on the exogeneity of the upper bound ζ of supp$G(p)$. The minimum price quoted by retailers responds to the level of search intensity, while the maximum price is constant all the time. However, it is not a problem. As Figure 1.6.a shows top percentiles decrease in search intensity. Effective price ($\mathbb{E}(p|s)$ in Figure 1.6.b) decreases even faster. For instance, 97^{th} percentile in a low search economy is close to the upper bound, ζ. In fact, the whole support is concentrated in this neighborhood. On the contrary, the same percentile is much closer to the competitive price in a high search

$\frac{1-s}{2s}\frac{\zeta-1}{p-1}$. Only the search intensity s matters, while the number of purchases m cancels out. In the heterogenous-agent framework it is analogous. The latter margin plays only a weighting role for purchases made by various households.

[25]Note $\max\{p', p''\} = \frac{p'+p''}{2} + |p' - p''|$ and $\min\{p', p''\} = \frac{p'+p''}{2} - |p' - p''|$. Then $\mathbb{E}\min\{p', p''\} + \mathbb{E}\max\{p', p''\} = \mathbb{E}p' + \mathbb{E}p'' = 2p^0$, which gives the latter equality in (1.21).

economy. This observation is true especially for the paid prices (Figure 1.6.b). In fact, in spite of the exogeneity of ζ, prices paid by consumers can be successfully reduced by increasing search intensity, s.

Figure 1.6: The equilibrium price dispersion for the different aggregate search intensities, $\frac{\Psi_{(+)}}{D}$.



Solution to the household's problem. Finally, I am in the position to write the first order conditions that constitute the solution to the households' problem (1.11). The intertemporal decision is determined by:

$$\frac{u'(c)\theta^{(1-\alpha)} - v'(f)\frac{\partial f}{\partial m}}{p^0 - sMPB} \geq \beta(1+r)\mathbb{E}_{x'|x}\frac{u'(c')\theta^{(1-\alpha)} - v'(f')\frac{\partial f'}{\partial m'}}{p^0 - s'MPB}, \tag{1.22}$$

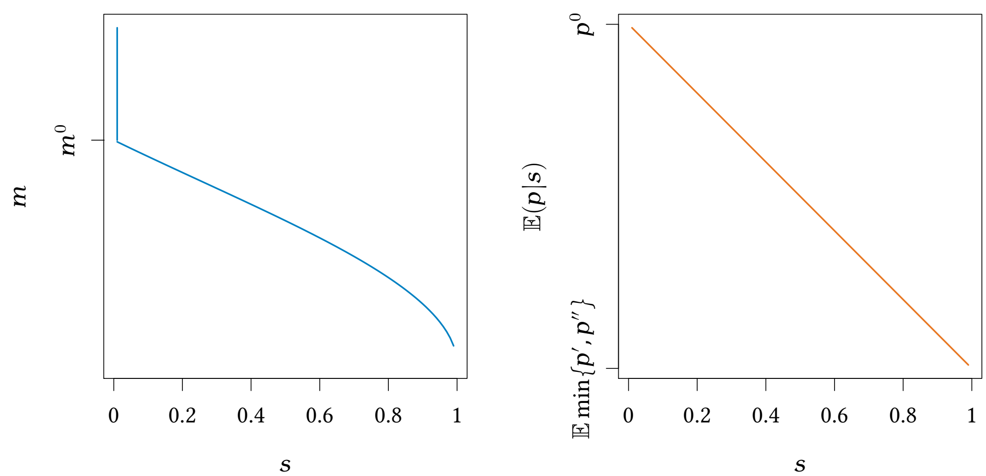
and $a' \geq \underline{B}$, with complementary slackness. The main departure from the textbook Euler equation is the additional convex cost, $v(f_t)$ and varying price, $p = p^0 - sMPB$, which is a function of the control, s in the considered case. For the CRRA specification the household makes also an intratemporal decision on its shopping behavior:

$$m \geq \left\{ \frac{\theta^{(1-\sigma)(1-\alpha)}}{(\frac{1+s}{1-s})^{\phi}\left(1 + \frac{2p^0}{(1-s)MPB}\right)} \right\}^{\frac{1}{\sigma+\phi}} \tag{1.23}$$

and $s \geq 0$, with complementary slackness[26]. First, as in the standard model, consumption goes along with the level of wealth. By construction, it affects m_t in the same way due to the linear relationship, $c_t = m_t\theta^{1-\alpha}$. Second, both shopping margins are Frisch complements to each other in the disutility function. Consequently, households with higher consumption exert lower search for prices, s_t. There is also a certain number of purchases m^0 (which translates directly into $c^0 = m^0\theta^{1-\alpha}$), above which households decide to be captive in every purchase, $\mathbb{E}(p|s = 0) = p^0$ (see Figure 1.7). However, as mentioned before it does not make them to pay ζ all the time because there is a positive externality generated by households with high search. This is embodied by $p^0 < \zeta$.

[26]Condition (1.23) is not defined in $s = 1$. However, the assumed functional specification meets an Inada-like condition, $\lim_{s \to 1^-} v(m_t, s_t) = \infty$, which guarantees that such a search intensity is never chosen.

Figure 1.7: Optimal relationship between the number of purchases (m_t), price search (s_t) and the effective consumption price $\mathbb{E}(p|s)$.



V. Taking the Model to Data

In this section I present my strategy for parametrization of the model and computation of the equilibrium. Model parameters are divided into two groups. Values of the first group (Table 1.2) are preset exogenously to standard values drawn from the literature. Values of crucial parameters which account for the shopping technology (Table 1.4) are determined internally using the method of simulated moments.

Demographics. The model is annual. Households enter the labor market when they are 25, they retire at age of 60 and die at age 90. This implies $T_{work} = 35$ and $T = 65$.

Preferences. The preferences over consumption are represented by a CRRA specification, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. The elasticity of relative risk aversion parameter $\frac{1}{\sigma}$

was set to $.5$. The disutility from overall shopping effort is modeled by an isoelastic function:

$$v(f) = \begin{cases} \frac{f^{1+\phi}}{1+\phi}, & \text{for } t \in 1, \dots, T_{work}, \\ \chi^{ret} \frac{f^{1+\phi}}{1+\phi}, & \text{for } t \in T_{work}+1, \dots, T. \end{cases} \quad (1.24)$$

Factor $\chi^{ret}(< 1)$ is supposed to capture a lower opportunity cost of shopping time for the retired consumers. The function of overall shopping effort f is chosen to meet assumptions on increasing in both margins and mutual complementarity. It is represented by a functional specification $f = \frac{1+s}{1-s}m$. Besides this form is convenient in the computational procedure, which is described and explained more carefully in the end of the section. Finally the discount factor β was chosen to replicate an aggregate wealth-income ratio of 2.5.

Interest rate and assets. I calibrate the discount factor β to generate an aggregate wealth-income ratio of 2.5. Following the RBC literature (Cooley and Prescott, 1995), the interest rate r was set to $.04$. Household debt contributes very little to wealth distribution. In aggregate it poses less than 1% of the total wealth and a median quarterly credit limit reported by households from the SCF amounts to merely 74% of quarterly labor income, which is not much in comparison to the mean net worth equal to over 900% of the labor income. For this reason, I assume households can save but cannot borrow, $\underline{B} = 0$ as modeled in Carroll (1997) or more recently in Krueger, Mitman, and Perri (2016). The sales tax, τ_c was set to the average US sales tax.

Income process. The income process is a combination of two components, transitory $\{\varepsilon_t\}$ and permanent $\{\eta_t\}$. Following the literature, the log variances of those shocks were set to $\sigma_\varepsilon^2 = .05$, $\sigma_\eta^2 = .01$. The age-dependent deterministic component, κ_t is approximated by a quadratic regression using the PSID data as in Kaplan and Violante (2010). In retirement, households receive a social security income payment that is a function of their income in the last working-age period with replacement rate $repl$ (Guvenen and Smith, 2014; Berger, Guerrieri, Lorenzoni, and Vavra, 2015).

Table 1.2: External choices

Parameter	Interpretation	Value	Source
T_{work}	Age of retirement	35	–
T	Length of life	65	–
σ	Risk aversion	2.0	–
$repl$	Retirement replacement rate	.45	Guvenen and Smith (2014)
σ_ε^2	Variance of the transitory shock	.05	Kaplan and Violante (2010)
σ_η^2	Variance of the permanent shock	.01	Kaplan and Violante (2010)
r	Interest rate	.04	Cooley and Prescott (1995)
τ_{cons}	Consumption tax	.08454	Reuters
κ_t	Deterministic life-cycle income profile	–	Kaplan and Violante (2010)
$\underline{B}$	Borrowing constraint	0	–

Shopping parameters. The key shopping parameters are determined internally using a simulated method of moments. There are six parameters to be pinned down: the discount factor (β), curvature of the disutility from shopping (ϕ), matching efficiency of a single purchase[27] ($\theta^{1-\alpha}$), wage (w), the maximum price quoted by retailers (ζ), and lower disutility from shopping of retired households (χ^{ret}). The quantitative behavior of the model is disciplined by seven internal targets, which can divided into three categories: shopping effort, price dispersion and aggregate state. The calibration consists in simulating artificial

[27] Admittedly, θ is an equilibrium object. However, it is easy to show that every $\theta^{1-\alpha}$ can be rationalized either by fixing the measure of consumption available in aggregate or by setting a fixed entry cost for retailers. In partial equilibrium both approaches are tantamount. Rationalization with a fixed entry cost is used in a work-in-progress paper that studies shopping in general equilibrium (Kaplan and Pytka, 2016).

panels of data for shopping economies and the final parametrization is chosen to set values of simulated moments as close as possible to their empirical counterparts.

Shopping effort. The shopping effort targets are matched using indirect inference (Gourieroux, Monfort, and Renault, 1993). First, I estimate an auxiliary regression model using the ATUS data, which captures the empirical findings 1.i-iii presented in Section III:

$$\log shopping_i = \alpha + \beta earn_i^{\frac{2}{3}+} + \delta_u unemp_i + \delta_r retir_i + \delta_a Age_i + \gamma X_i + \varepsilon_i, \tag{1.25}$$

where $earn_i^{\frac{2}{3}+}$ is a dummy accounting for the top labor income tertile. In this regard, one single dummy variable replaces nine dummies ($\sum_j \beta_j earn_i^j$) of the baseline specification (1.1). This change makes calibration more straightforward and at the same time not much information is lost[28]. Table 1.3 shows the results of the estimation. The regression of the reduced specification exhibits estimated values that are similar to the baseline version. Finally, estimates of variables associated with retirement, age, and being in the top earning tertile were used for disciplining the structural model.

Price dispersion. In the parametrized model I want to capture certain cross-sectional price characteristics of the US economy. For this reason, I targeted two price differentials: 1. between high earners and low earners, 2. between employed and retired. Both moments were observed by Aguiar and Hurst (2007). The authors using scanner data showed that retirement-age households pay on average 3.9 percent less than young households and that households earning more than $70,000 a year pay 2.1 percent more than households earning less

[28]Recall that only top deciles of the earnings spend more time shopping.

Table 1.3: Coefficients of interest for the auxiliary model

	$log(shopping)$
Retired	0.245^{***}
	(0.035)
Unemployed	0.368^{***}
	(0.031)
Age	0.0005
	(0.003)
$earn_{\frac{2}{3}+}$	0.110^{***}
	(0.017)
Shopping needs	Yes
Year and day dummies	Yes
Demographic controls	Yes

Notes: ***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.

than \$30,000 a year. These moments are differences in average prices paid by households with different characteristics. In the model this is embodied by heterogeneity in price search intensity, s amongst households. Consequently, this gives rise to variety in average prices, $\mathbb{E}(p|s)$. Apart from heterogeneity in first moments I use also range statistics of prices paid by households. Kaplan and Menzio (2016) used the same dataset as Aguiar and Hurst (2007) to measure price ranges of *transactions* for various goods and markets. They observed that the average 90-to-10 percentile ratio of paid prices varies between 1.7 and 2.6. The heterogeneity in marginal cost, which is not present in the model, is very likely to contribute to a higher price dispersion[29]. Thus, similarly to the aforementioned paper I decide to target the price ratio of 1.7. It is noteworthy that the authors use the consumer panel dataset which collects data on prices of *effective* transactions. This observation has an important implication as it means the percentile ratio should be computed not from the distribution of prices set by retailers, $G(p)$, but rather the distribution of prices *accepted* by households[30].

Computation. An equilibrium allocation and equilibrium prices are determined by solutions to the household's dynamic problem (1.11) given prices set by retailers and solutions to the retailer's problem (1.12) given households' consumption and shopping decisions. The allocation is computed iteratively. The household's problem is solved given an initial guess on pricing strategy of retailers. Then, the retailer's problem is solved given the solution from the previous step. Next, the pricing strategy of retailers is updated and used for solving the household's problem. The process is repeated until convergence to a fixed point, in which the household's decisions generate the retailers' pricing and vice

[29]This conjecture relies on theoretical results about the impact of firm productivity differentials on the wage dispersion (*e.g.*, Burdett and Mortensen, 1998; Mortensen, 2003).

[30]In aggregate they are distributed according to $(1 - \frac{\Psi_{(+)}}{D})G(p) + \frac{\Psi_{(+)}}{D}[1 - (1 - G(p))^2]$, where $\frac{\Psi_{(+)}}{D}$ is the probability that a transaction is non-captive.

Table 1.4: Calibration targets and model values

Target	Data Value	Source	Model Value
Shopping effort:			
Shopping time of retired relative to the referential group	1.245	This paper	1.251
Shopping time of the top earn. tercile relative to the referential group	1.11	This paper	1.112
Age trend for shopping time	0	This paper	.010
Price dispersion:			
$^{95^{th}\text{decile}}/_{5^{th}\text{decile}}$ of paid prices	1.7	Kaplan and Menzio (2016)	1.369
Price differential between high earners and low earners	.021	Aguiar and Hurst (2007)	.011
Price differential between retirees and working-age households	-.039	Aguiar and Hurst (2007)	-.051
Aggregate state:			
Aggregate wealth-income ratio	2.5	Kaplan and Violante (2010)	2.498

versa. The algorithm is intuitive and relatively fast (it takes 10-15 iterations to converge). A drawback of this approach is a risk of computing a degenerate Diamond (1971)-type equilibrium[31]. Given a "wrong" initial guess the alghorithm might converge to an allocation where all households want to exert the maximum search intensity, $s = 1$. In response to this all retailers set monopolistic prices, $p = \zeta$, and in the next iteration all households become captive all the time, $s = 0$. The decision to make the minimum price search effort supports degenerate pricing, $p = \zeta$, in subsequent iterations. Nonetheless, this problem can be tackled by imposing an Inada-like assumption, $\lim_{s \to 1^-} v(m_t, s_t) = \infty$, which guarantees that a search intensity ($s = 1$) is never chosen by any house-

[31] In a companion paper I show that this type of equilibrium is unstable for this model setup (Pytka, 2016). This property stays in contrast with the classical model of search for consumption due to Burdett and Judd (1983).

Table 1.5: Calibrated parameters

Parameter	Value	Description
ϕ	.104	curvature of disutility from shopping
$\theta^{1-\alpha}$	.113	matching efficiency
w	14.041	wage
ζ	84.605	maximum price
χ^{ret}	.588	lower disutility from shopping for retirees
β	.951	discount factor

hold. The functional specification ($f = \frac{1+s}{1-s}m$ together with (1.24)) used in the calibration meets this condition.

Calibration results. The parameter values are presented in Table 3.4 and the targeted data moments with their model counterparts are summarized in Table 1.4. The model matches the shopping effort statistics and the aggregate wealth-income ratio very well. On the other hand, the price range and the price differential between high earners and low earners are too low while the price differential between retirees and working-age households is slightly overestimated. However, in the calibration there are seven targets and six parameters, so the system of targeted moments is overdetermined. It makes exact identification impossible. Overall, the simulated moments are pretty close to the data targets.

VI. Price Dispersion(s) at Play

In the equilibrium allocation the price dispersion can be characterized in four dimensions, that is:

i. the distribution of prices *quoted* by retailers,

ii. the distribution of prices *accepted* by households,

iii. the distribution of prices of *individual purchases* for a household exerting a certain search intensity, $s_t(x)$,

iv. the distribution of *average* prices paid by households with different search intensities $s_t(x)$ distributed according to the distribution of types given by $\mu_t(x)$.

Table 1.6 presents moments of the equilibrium price dispersion for the calibrated version of the model. Aggregate search intensity is equal to $\frac{\Psi_{(+)}}{D} = .271$. From retailers' perspective this number can be interpreted as the probability that a visiting customer received an alternative offer from another retailer. This probability constitutes the equilibrium price dispersion given by (3.2). Consequently, the offered prices are a connected set $[\underline{p}, \zeta]$, where the equilibrium lower bound $\underline{p}$ amounts to about 70% of the monopolistic price. If a seller is matched with a consumer who is also matched with a lower alternative price, then the offer with the higher price is rejected. Only offers of measure $\frac{\Psi_{(+)}}{2D} = .1355$ drawn with an alternative competitive offer come into force. As a result, 86.45% offers[32] quoted by retailers are accepted, while the complementary 13.55% is rejected. The accepted offers are distributed according to:

$$\left(1 - \frac{\Psi_{(+)}}{D}\right) G(p) + \frac{\Psi_{(+)}}{D} \left\{1 - [1 - G(p)]^2\right\}. \tag{1.26}$$

Both distributions, quoted prices and accepted prices, are depicted in Figure 1.8. Intuitively, lower prices have higher probability for being accepted by customers. This property is embodied by the fact that $G(p)$ is first-order stochastically dominated by formula (1.26).

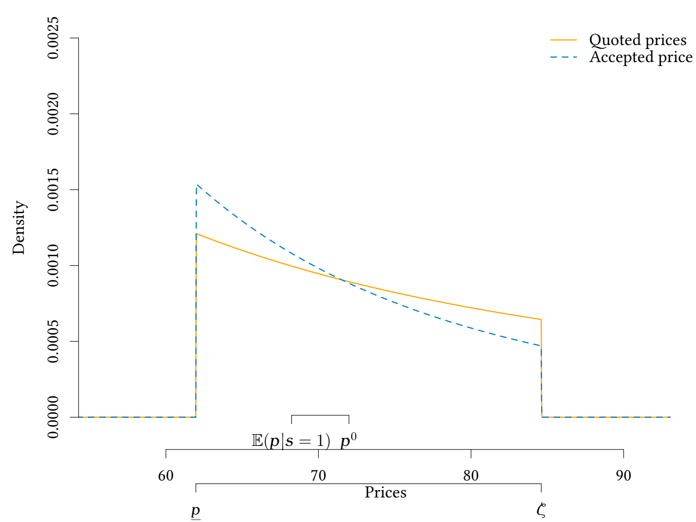
Given the distribution of prices quoted by retailers, households decide on their individual search intensity, s. Consequently, every consumer draws prices

[32] This number is a sum of 13.55% offers matched with competitors with higher prices and 72.9% transactions with captive customers.

Table 1.6: Moments of the Equilibrium Price Dispersion

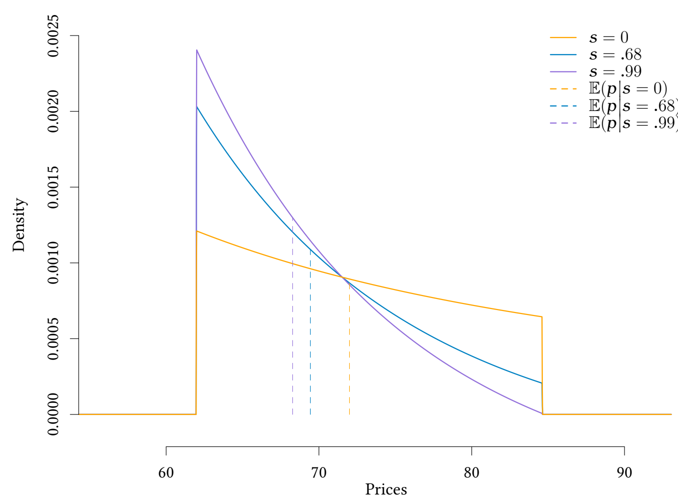
Moment	Value	Description
Price moments:		
$\underline{p}/\zeta$	.703	Min-max ratio of quoted prices
$\Psi_{(+)}/D$	.271	Aggregate search
p^0/ζ	.851	Captive price-max ratio
MPB/p^0	.052	Marginal price benefits
Shopping moments:		
$\mathbb{E}s$	.201	Average search intensity
$\mathbb{E}(s\|s>0)$	.683	Average price search conditioned on being non-captive

Figure 1.8: Distribution of quoted and accepted prices.



from her individual price lottery generated by equation (3.11), which is illustrated in Figure 1.9. Households with low search intensity sample prices from a price lottery with a higher expected value. Recall that a consumption bundle, c consists of continuum of shopping lotteries. Thanks to Lemma 1 the overall cost of consumption is pinned down deterministically. Therefore, the cost of a unit of consumption belongs to an interval, $(p^0 - MPB, p^0]$. Households that decide to be captive in every purchase ($s = 0$) pay p^0, while consumers with positive search intensity ($s > 0$) spend $p^0 - sMPB$ on every unit of consumption. In the limit case, for $s = 1$ they would pay the minimum possible price $p^0 - MPB$, which is equal to the expected minimum from two draws[33], $\mathbb{E}\min\{p', p''\}$. The fact that retailers cannot distinguish between captive and non-captive transactions, the expected value of a single draw (p^0) is 15.5% lower than the monopolistic pricing. The marginal price benefit from increasing search intensity allows to reduce prices up to 5.2% compared with prices paid by captive customers, p^0.

Figure 1.9: Individual price lotteries.



The equilibrium price dispersion characterizing the economy is supported

[33]However this shopping strategy is never chosen for the assumed utility function. In the calibrated economy maximum effort is set to $s = .979$ for the employed households and $s = .998$ for the retirees.

Table 1.7: Captive and non-captive households

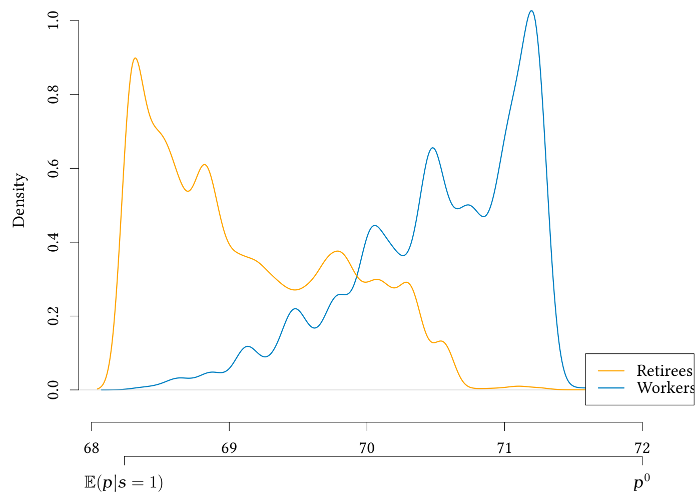
Type of households	Non-captive	Captive	$\mathbb{E}(s \vert s > 0)$
Working-age	.129	.871	.417
Retired	.485	.515	.765
Overall	.293	.707	.683

by non-captive households that exert positive price search intensity ($s > 0$). As shown in Table 1.7, less than 30% households decide to make some search effort to draw (with some probability) two prices. Households exerting a positive price search effort draw two prices and choose the lower one with the probability of 68.3%. Interestingly, if households are broke down into two groups, those in the working-age and retirees, then it turns out that more than half of retirees and merely 12.9% of workers are non-captive. In intensive margins there is also a substantial difference. The average search intensity of non-captive retirees equals to 76.5%, whilst the average search intensity of non-captive amounts to 41.7%. This result stems directly from lower opportunity cost modeled by χ^{ret}. Finally, the last type of price dispersion, heterogeonous average prices are generated by different search intensities, presented in Figure 1.10.

VII. Deconstructing the Aggregate Consumption

The aggregate demand generated by artificial economies is characterized using two alternative approaches, consumption responses to idiosyncratic income shocks and cross-sectional distributions of households' decisions (consumption expenditures) and endogenous states (net wealth positions). All results stem from simulating the invariant distribution of two versions of the economy, the standard incomplete-markets model *without* frictions in the purchasing technology (SIM) and the incomplete-markets economy augmented with the search

Figure 1.10: Distribution of average prices for non-captive households.



friction described in Section IV. Both economies are calibrated to replicate the same level of aggregate savings (wealth-income ratio of 2.5). The remaining non-shopping parameters are set at the same level for both models (see Table 1.2). Next, simulated statistics are compared with their empirical counterparts. It is worth pointing out that none of statistics presented in this section were used in the calibration procedure. In this sense, results of this section provide natural yardsticks to measure which model is better for the quantitative analysis.

A. Consumption Responses to Shocks

To study consumption responses to income shocks I employ an identification strategy proposed by Blundell, Pistaferri, and Preston (2008). The authors, using data on non-durable consumption from the Panel Study of Income Dynamics and making imputations basing on food demand estimates from the Consumer Expenditure Survey, assessed the transmission of income shocks into consumption. To this end they used the log-linear approximation of the Euler equation and run the regression:

$$\Delta \ln c_{it} = \alpha + MPC^{\varepsilon}\varepsilon_{it} + MPC^{\eta}\eta_{it} + \xi_{it}, \tag{1.27}$$

where MPC^{ε} and MPC^{η} are the pass-through coefficients of income shocks into consumption. Intuitively, they can be interpreted as marginal propensity to consume out of different types of shocks, permament (η) and transitory (ε).

In the data the distinction between different types of shocks is difficult. The authors offer an estimator of MPC that is consistent under two assumptions: short history dependences and no advanced information. This estimator is as follows:

$$\widehat{MPC^{x}} = \frac{cov(\Delta(p_{it}c_{it}), g(x_{it}))}{var(g(x_{it}))}, \tag{1.28}$$

where:

$$g(\varepsilon_{it}) = \Delta y_{i,t+1}, \quad g(\eta_{it}) = \Delta y_{i,t-1} + \Delta y_{i,t} + \Delta y_{i,t+1}.$$

It is worth presenting values of the coefficients for some notable examples:

1. complete markets (with separable labor supply): $MPC^{\varepsilon} = MPC^{\eta} = 0$ – households are able to smooth the marginal utility of consumption fully and all shocks are insured away,

2. autarky with no storage technology: $MPC^{\varepsilon} = MPC^{\eta} = 1$,

3. the classical version of the permanent income-life cycle model: $MPC^{\eta} = 1$ and the response to transitory shocks MPC^{ε} depends on the time hori-

zon. For a long horizon it should be very small and close to zero, while for a short horizon it tends to one.

In their empirical study Blundell, Pistaferri, and Preston (2008) documented that on average 64% of permanent shocks are translated directly into consumption. Kaplan and Violante (2010) applied an analogous procedure to an artificial panel generated from a calibrated version of the life-cycle SIM model. In their simulation they reported that in the artificial economy between 78 and 93% of permanent shocks are passed on consumption, depending on the borrowing limit.

In the shopping economy I depart from the law of one price so the price component is not constant and does not cancel out. Hence, I have to modify the baseline specification to the following form:

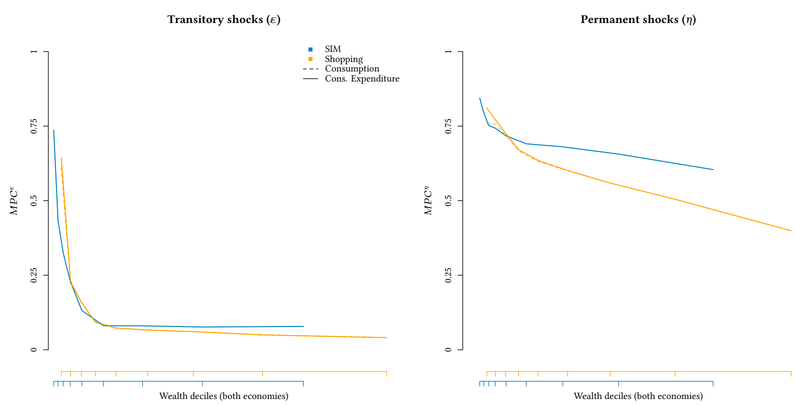
$$\Delta \ln(p_{it}c_{it}) = \alpha + MPC^{\varepsilon}\varepsilon_{it} + MPC^{\eta}\eta_{it} + \xi_{it}. \tag{1.29}$$

I estimate equation (1.29) using artificial panels of two versions of the economies and I compare the results with the empirical counterparts. Table 1.8 presents the obtained estimates. In the shopping economy average consumption responses are smoother and are closer to the empirical counterparts than in the SIM economy without product market-frictions. This phenomenon can be explained by the fact that marginal disutility for the shopping effort partially offsets utility of consumption, which makes consumption responses smoother. This effect increases in the level of consumption, which is conformed by the values of the pass-through coefficients for different wealth groups depicted in Figure 1.11. The interesting implication of the model is also higher heterogeneity in consumption responses. Households with low wealth exhibit similar willingness to consume in both economies In the presence of the shopping friction they are even slightly higher since households decide to search less intensively for lower prices in response to permanent shocks. The discrepancy between two economies is larger for consumers from the wealthiest groups. For those households the cost of obtaining additional units of consumption is so high that they detain from consuming more.

Table 1.8: BPP MPC

Economy	$\widehat{MPC^{\eta}}$	$\widehat{MPC^{\varepsilon}}$
USA (BPP 2008)	.64	.05
Shopping	.602	.152
SIM	.8	.280

Figure 1.11: Distribution of MPCs.



B. Cross-sectional Distribution

The cross-sectional distributions of consumption expenditures and net wealth are another dimension describing the aggregate demand. For this exercise I generate simulated moments from both artificial economies and compare with data.

Following the macroeconomic literature of inequalities (*e.g.*, Castañeda, Díaz-Giménez, and Ríos-Rull, 2003; Krueger, Mitman, and Perri, 2016), distributions are compared with the use of Gini indices and share of the total value held by chosen groups of households. The data counterparts were calculated using the 2006 wave of the Panel Study of Income Dynamics. I focus on households aged between 25 and 90 to make computed statistics compatible with the calibration of the models. Following Blundell, Pistaferri, and Saporta-Eksten (2016) I dropped observations with extremely high net wealth (>$ 20 millions). In the theoretical framework I do not model the household decision to purchase durables, so I focus on non-durables and services.

Table 1.9: Consumption (expenditure) distribution

		Quintile					Top Percentiles		
Economy	Gini	First	Second	Third	Fourth	Fifth	90-95	95-99	99-100
USA (PSID 2006)	.353	.051	.113	.165	.224	.440	.087	.088	.121
Shopping	.402	.053	.112	.163	.208	.457	.073	.061	.200
SIM	.234	.100	.150	.190	.235	.330	.083	.078	.025

Table 1.9 presents the distributions of consumption expenditures in both economies and observed in the data. The shopping economy mirrors inequalities remarkably better than the SIM economy without product-market frictions. The Gini indices for consumption in the baseline SIM and in the shopping economy amount to .234 and .401 , respectively. The empirical counterpart computed from the PSID is equal to .353. This effect is generated mainly by groups exerting high search for price bargains, households with low consumption and retirees. First, households with low consumption search more intensively, which leads to lower effective prices paid by them. Consequently, the fraction of aggregate

consumption expenditures is smaller than in the benchmark SIM model. Second, a drop in consumption expenditures after retirement is higher in the shopping economy as can be seen in Table 1.10. This result is generated by the lower opportunity cost of time for retirees and is consistent with findings made by Aguiar and Hurst (2005, 2007).

Table 1.10: Gini consumption: working-age households vs retirees

Economy	Gini working-age	Gini retirees	overall Gini	$\frac{\mathbb{E}(pc \mid retired)}{\mathbb{E}(pc \mid working)}$
USA (PSID, 2006)	.330	.383	.353	.701
Shopping	.380	.381	.402	.742
SIM	.214	.243	.235	.809

The distributions of net wealth are presented in Table 1.11. As can be seen the shopping friction amplifies the wealth inequalities as well. The Gini indices in the baseline SIM and in the shopping economy amount to .569 and .667 , respectively. The empirical counterpart computed from the PSID is equal to .771. If we look at the fine print, the higher Gini index comes from the higher share of the total wealth held by the top quintile. In the shopping economy households from the top quintile own nearly 70% of total wealth, while in the data 82.6% is observed. In the SIM model only 55% is owned by households from the top quintile. The improvement in this moments is generated in the analogous way to consumption responses from the previous subsection. For this group of households increasing the current consumption is too costly. Instead it is beneficial to them to save more and increase consumption during retirement when the opportunity cost of time is lower. There is still discrepancy between inequalities generated by the shopping economy and observed in data. Admittedly, there are models outperform the shopping economy in this regard. Nonetheless, recall that those statistics were not used in the calibration process, while for instance in Castañeda, Díaz-Giménez, and Ríos-Rull (2003) moments describing wealth inequalities were targets. Moreover, the shopping economy presented in this paper abstracts from important motives for wealth accumulation, such as bequests.

Table 1.11: Wealth distribution

		Quintile					Top Percentiles		
Economy	Gini	First	Second	Third	Fourth	Fifth	90-95	95-99	99-100
USA (PSID 2006)	.771	-.015	.005	.042	.142	.826	.157	.256	.244
Shopping	.667	.011	.031	.065	.198	.696	.1636	.211	.114
SIM	.569	.014	.052	.128	.258	.549	.138	.149	.056

VIII. Concluding Comments

The article advances a novel theory of the search for consumption as part of the optimal savings model. Motivated by recent empirical findings on price dispersion and systematic heterogeneity in shopping time I use the model to address the question how income fluctuations are passed on consumption expenditures when the law of one price does not hold. I show that frictions in the purchasing technology generate important implications for the aggregate consumption. The shopping effort increases the level of risk sharing and brings predictions of the model much closer to the empirical counterparts than the standard incomplemarkets model without frictions in purchasing technology. Moreover, the level of consumption and wealth inequalities are amplified as well and in this sense the theory contributes to the literature of inequality as well.

More broadly, the model is a first step to understand macroeconomic implications of search for consumption. It can provide interesting structural insight into some recent empirical findings on household consumption. For instance, Stroebel and Vavra (2016) documented reactions of retail prices to changes in local house prices, with elasticities of 15–20%. Using my model it can be rationalized by an increase in share of consumption of homeowners, who are most likely captive customers. Consequently, the aggregate price search decreases and retailers adjust their pricing strategy to the new distribution of consumption by charging on average higher prices.

The model can be extended to a more general setup. In another paper (Kaplan and Pytka, 2016), which is being developed, we use the proposed search protocol in an economy with both idiosyncratic and aggregate risk. The model is tailored to quantitatively address policy-related questions, in which household consumption plays an important role. The framework allows studying the relationship between the retail market and the production sector from a macroeconomic standpoint. In addition to this, the shopping effort generates a real effect on output. Two possible applications are suggested. First, our model is likely to give some insight into the source of transition of unskilled workers from the production sector to the retail market observed in the data. Second, the model can be used for studying monetary policy in a framework where price stickiness is a result, not an assumption. We claim that product market frictions in the transactions technology can replace the price-setting rigidities. Moreover, unlike existing Keynesian models, our economy does not require firms or households to be off their optimality conditions.

2

Bargain Hunting in Equilibrium Price Dispersion

I. Introduction

A renewed interest in the explicit role of the demand in shaping economic aggregates resulted in increased popularity of models where the market power of firms depends on individual consumption decisions of buyers. One of prominent examples of such economies is a model of endogenous price distribution due to Burdett and Judd (1983). This simple and elegant model allows to generate price dispersion, which arises as a result of a game between the sellers and the buyers. A good understanding of the equilibrium properties of this model is very useful if not essential for any further applications. Especially, the stability with respect to the buyers' behavior is of particular interest as their changing consumption decisions lie at the heart of the propagation mechanism proposed by Kaplan and Menzio (2016).

In this paper I contribute towards existing literature in two ways: *(i)* I study equilibrium properties of the model of Burdett and Judd (1983) and I show that only some allocations can be supported by stable equilibria, *(ii)* I present the modification of the original model that allows me to recover equilibrium stability of all allocations. The former contribution could be used to criticize the quantitative results of models calibrated to unstable equilibria. However, the latter contribution provides the modified framework that allows to support existing results without any recalibrations.

In the course of the analysis I proceed in the following way. First, I show how in the original model the multiplicity of dispersed equilibria arises. The fact of the multiplicity is present in the original paper, but the actual discussion on its source is omitted. Next, I study stability properties of both equilibria using a tâtonnement process in which the consumers change their price search behavior in the direction that minimize their cost of search. It turns out that according to this notion of stability the low-search equilibrium is unstable while the high-search one and the degenerate Diamond (1971)-type are stable. Any perturbation to the low-search equilibrium gives rise to convergence towards one of stable equilibria. Then I show that a feasible allocation can be supported either as the low-search equilibrium or the high search one and it is never the case that it can be characterized in two ways. This observation is important because it shows that allocations characterized by the low-search equilibria cannot be stable.

To recover the equilibrium stability of all allocations I propose the modification of the original model on the side of buyers. In the original model every buyer is sampled with one price quotation costlessly and they can receive one additional offer by paying a fixed search cost. In equilibrium the consumers are indifferent between both actions and mix both strategies. The search cost is indivisible and stochastic (whether it is paid depends on a pure strategy sampled by the agent). Here I consider an alternative specification of the buyer's problem. I assume that the search effort is divisible, deterministic, and features decreasing marginal returns. The consumers decide upon the probability of two draws and its marginal cost is increasing. The number of price offers received by the consumer is drawn and the probability of two price quotations depends on the search effort. As I show this economy features the unique equilibrium in the class of the symmetric and dispersed equilibria and the degenerate Diamond (1971)-type. The former is stable while the latter is unstable.

Relations to the existing literature. Stability properties of equilibria in the model of Burdett and Judd (1983) were analyzed before in papers due to Hopkins and Seymour (1996, 2002) and Lahkar (2011). The authors use an evolutionary approach to study the stability of the sellers' behavior in the version of the model where the price space is discretized. My article differs from those papers in three dimensions. First, the notion of stability I use is different because I consider tâtonnement stability. This concept was proposed in the formal way by Samuelson (1947) for competitive markets and was adapted to the case of coordination games by Matsuyama (1999, 2002). Second, I focus on possible distortions originated from the buyers' behavior, while the sellers always charge prices that are the best responses to the buyers' actions. Finally, the approach I employ does not require the discretization of the price space and in this sense I work on the original version of the model. While the evolutionary stability and the sellers' behavior might be interesting for the literature of industrial organization, my approach is more suitable for macroeconomic applications of the model for the reason mentioned below.

In the macroeconomic literature for some time a renewed interest in the explicit role of demand in determining aggregates has been observed. Krueger, Mitman, and Perri (2016) embed a reduced-form demand externality into the business cycle model with household heterogeneity and they show that this extension adds an endogenous persistence and amplification of responses of the model to technology shocks. The microfoundations for this effect can be provided either using the competitive search due to Moen (1997) as in Bai, Ríos-Rull, and Storesletten (2017) or using the model of Burdett and Judd (1983) as in Kaplan and Menzio (2016). The behavior of the buyers lies at the heart of this mechanism. Consequently, a good understading of the possible unstability originated from the buyers' side is a prerequisite for using the Burdett and Judd (1983) model in this context.

Finally, the result of the paper which shows that every allocation can be generated by an appropriate choice of the price search cost function is in the similar spirit to the one obtained by Mortensen (2005) for monetary search models due to Head and Kumar (2005). The author presented how to simplify the model by assuming an exogenous Poisson distribution of the number of received price offers per period and how this distribution can be supported as the solution to the buyer's problem. In this sense my result is analogous but for the class of Burdett and Judd (1983) models. Every price search intensity can be supported as the solution to the buyer's problem.

The rest of the paper is structured as follows. In section II, I set out the problem of retailers, which is common to both specifications. In section III, I present the problem of buyers in the original version proposed by Burdett and Judd (1983) and characterize equilibrium allocations. In section II, I propose the alternation. Section V concludes.

II. Firms

The problem of firms is common to both models and for this reason it is presented in one section.

Let $\overline{p}$ be the reservation price common to all buyers. There is a continuum of firms of fixed measure. All firms face the same marginal cost standardized to one. Every retailer is matched with a single buyer only. The buyer is captive and committed to purchasing at any price below $\overline{p}$ with probability $1-q$ and with probability q is matched with two different retailers and chooses the lower price. Given a price distribution of prices set by all firms denoted $F^{q^*}(p)$, the probability that the price p quoted by the retailer will be accepted is:

$$(1-q) + 2q\left(1 - F^{q^*}(p)\right), \tag{2.1}$$

where $2q\left(1 - F^{q^*}(p)\right)$ is the probability that the firm will be visited by a buyer with an alternative offer that is higher than the quoted price p. Then the profit function for this price is equal to:

$$\pi(p) = \left\{(1-q) + 2q\left(1 - F^{q^*}(p)\right)\right\}(p-1). \tag{2.2}$$

Burdett and Judd (1983) show that for a *given* probability $q > 0$ of two draws, hereinafter also referred to as an average price search intensity, there exists the unique best-response[1] price dispersion:

$$F^q(p) = \begin{cases} 0 & \text{for } p < \underline{p}, \\ 1 - \frac{1-q}{2q}\left[\frac{\overline{p}-p}{p-1}\right] & \text{for } p \in [\underline{p}, \overline{p}], \\ 1 & \text{for } p > \overline{p}, \end{cases} \tag{2.3}$$

where $\underline{p} = 1 + (\overline{p} - 1)\frac{1-q}{1+q}$.

The function that defines the price distribution is an element of an equilibrium. Even though different retailers charge different prices, the profit must be the same for all equilibrium prices.

[1] I use the term *best-response* to stress that an *equilibrium* requires the endogenization of the behavior of buyers q. The subsequent sections are dedicated to this matter.

III. Indivisibility of Price Search

In this section I outline the problem of the buyers in the original version due to Burdett and Judd (1983). Next I characterize the equilibrium allocations. There always exists one degenerate equilibrium and for sufficiently low search cost there are additional two dispersed equilibria (or for a knife-edge case there is the unique dispersed equilibrium). The equilibrium with the lower average price search is shown to be unstable. Finally, I present a result saying that no allocation with the average price search lower than $q \approx .634815$ can be supported as the stable equilibrium. This result is illustrated by examples of calibrations of unstable allocations used in the literature.

A. The Problem of the Buyers

There is a continuum of consumers of measure one. Every buyer is sampled with one price quotation costlessly. The consumer can receive an additional offer, but she must decide on it before learning the first drawn price. If she decides to sample the second price, she must pay a search cost equal to c. The problem of the consumer minimizing the expected cost of buying one unit of consumption is as follows:

$$\min_{q\in[0,1]}(1-q)\left(\int_{\underline{p}}^{\overline{p}} p dF^{q^*}(p)\right)+q\left(c+\iint_{(p_1,p_2)\in[\underline{p},\overline{p}]^2}\min\{p1,p2\}dF^{q^*}(p_1)dF^{q^*}(p_2)\right) \tag{2.4}$$

where:

- $\int_{\underline{p}}^{\overline{p}} p dF^{q^*}(p)$ is the expected price from one draw,

- $\iint_{(p_1,p_2)\in[\underline{p},\overline{p}]^2} \min\{p1,p2\} dF^{q^*}(p_1) dF^{q^*}(p_2)$ is the expected minimum from two independent draws,

- q^* is the average choice of all consumers and is taken as given by the consumers.

The consumer chooses the probability q that she samples two prices and accepts the lower one.

Definition 7 (Equilibrium price dispersion) *A dispersed equilibrium is constituted by a cdf $F^{q^*}(p)$, probability of two draws q^*, and profits of the retailers π such that:*

i. every price from the support of $F^{q^*}(p)$ maximizes the problem of the retailers (3.1) *and yields the same profit, π;*

ii. price search q^* minimizes the expected cost of purchasing a unit of consumption according to (2.4)*.*

In a dispersed equilibrium consumers must be indifferent between sampling one price with the higher expected value and two prices with the lower expected value but with the additional cost of search c. This equilibrium condition can be derived from the first-order problem of (2.4):

$$c = \underbrace{\int_{\underline{p}}^{\overline{p}} p dF^{q^*}(p) - \iint_{(p_1,p_2)\in[\underline{p},\overline{p}]^2} \min\{p1,p2\} dF^{q^*}(p_1) dF^{q^*}(p_2)}_{:=\mathcal{W}(q^*)}, \tag{2.5}$$

where $\mathcal{W}(q^*)$ is the expected price difference between a lottery with one price offer and a lottery with two price offers. In the equilibrium allocation the solution to (2.5) coincides with q^* observed by the firms.

There are two possible interpretations of the solution q. First, it can be considered as a mixed strategy randomizing over two pure strategies, two draws with additional cost or one draw. Alternatively, it can be considered as a fraction of households that *ex-ante* decide upon drawing two prices, in the analogous way to the problem of indivisible labor supply due to Rogerson (1988, Sections 2 and 3). Then q is such a fraction that exactly equalizes the cost of search with the price gain from two draws. Both interpretations are actually equivalent. The only difference is that the former is the characterization that is stochastic and constitutes a symmetric equilibrium, everyone chooses the same probability q, while the latter is deterministic and the equilibrium is not symmetric, buyers of measure q sample two prices and the complementary measure of $1-q$ draw one price only.

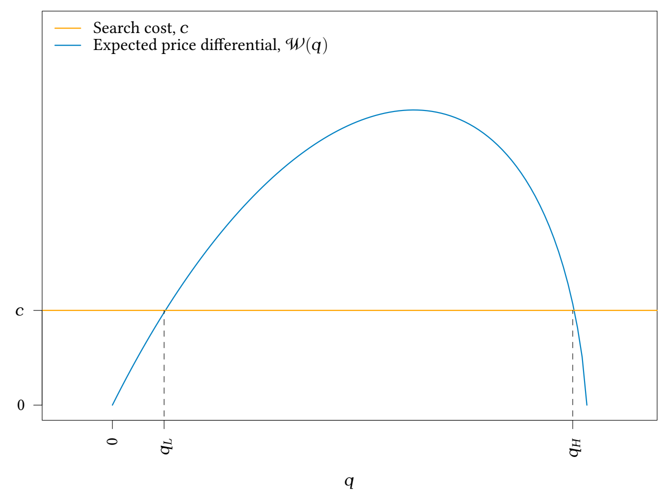
B. Non-Uniqueness

Burdett and Judd (1983) show there can be zero, one, or two dispersed equilibria and one degenerate Diamond (1971)-type equilibrium with $q = 0$ and the monopoly price $p = \overline{p}$ charged by all retailers. The expected price difference between lotteries $\mathcal{W}(q^*)$ is not monotonous in q^*. Given the form of the equilibrium dispersion, the expected values of both price lotteries, $\int p dF^{q^*}(p)$ and $\iint \min\{p1, p2\} dF^{q^*}(p_1) dF^{q^*}(p_2)$ decrease in q. However, for values below a level of the price search intensity maximizing the expected price difference, $\overline{q} := \arg\max_q \mathcal{W}(q)$, the expected price of the lottery with two offers decreases faster than the former one. This constrasts the relationship for $q > \overline{q}$, where the effect of an increase in q is higher for the lottery with one price. Consequently, $\mathcal{W}(q^*)$ increases in q for $q < \overline{q}$ and decreases for for $q > \overline{q}$.

The economy features two equilibria if the cost c is lower than $\mathcal{W}(\overline{q})$, one equilibrium if $c = \mathcal{W}(\overline{q})$, and no dispersed equilibria if $c > \mathcal{W}(\overline{q})$. Figure 2.1

depicts the non-uniqueness with two dispersed equilibria, low-search q_L and and high search q_H, and the degenerate Diamond (1971)-type one with $q = 0$.

Figure 2.1: Non-uniqueness of equilibria



C. Non-Stability

Having defined and characterized the equilibrium allocation(s) I am in the position to study their stability properties. To this end I employ the methodology adapted from the literature of strategic complementarities and coordination failure. Assume that households are split into two fractions, with measure of μ and $1 - \mu$, which are also used to denote the types of consumers. The problem of the buyers is as the one presented in the previous section and is identical for both types. Denote their probability of sampling two draws by q_μ and $q_{1-\mu}$, respectively. Then the average probability of two draws of a visiting buyer is $q^* = \mu q_\mu + (1 - \mu) q_{1-\mu}$. Let $\overline{\mathcal{W}}(q_\mu, q_{1-\mu}) := \mathcal{W}(\mu q_\mu + (1 - \mu) q_{1-\mu})$ be the expected difference between two lotteries for consumers playing q_μ and $q_{1-\mu}$.

To study the stability properties of equilibria I use the following process:

$$\dot{q}_t = \overline{\mathcal{W}}_t(q_\mu, q_{1-\mu}) - c. \tag{2.6}$$

The tâtonnement dynamics of this kind is inspired by Matsuyama (1999, 2002). The logic is that, for fixed actions of the consumers of type $1-\mu$, the buyers of type μ change their price search behavior in the direction that decreases their expected cost of purchasing a good. An equilibrium is considered to be stable if a small change in actions played by one type is not propagated further and the system (2.6) does not evolve to a different equilibrium. The stability properties of all equilibria are discussed below.

For further analysis it is convenient to introduce following definitions in the spirit of Cooper and John (1988):

Definition 8 (Price Differential Spillovers) *Actions made by households of one type generate* spillover effects *on the expected difference between the price lotteries of the other type in the following way:*

i. *if* $\partial_{q_{1-\mu}}\overline{\mathcal{W}}(q_\mu, q_{1-\mu}) > 0$, *then the game features* positive price differential spillovers *on the players playing* q_μ;

ii. *if* $\partial_{q_{1-\mu}}\overline{\mathcal{W}}(q_\mu, q_{1-\mu}) < 0$, *then the game features* negative price differential spillovers *on the players playing* q_μ;

iii. *if* $\partial_{q_\mu}\overline{\mathcal{W}}(q_\mu, q_{1-\mu}) > 0$, *then the game features* positive price differential spillovers *on the players playing* $q_{1-\mu}$;

iv. *if* $\partial_{q_\mu}\overline{\mathcal{W}}(q_\mu, q_{1-\mu}) < 0$, *then the game features* negative price differential spillovers *on the players playing* $q_{1-\mu}$.

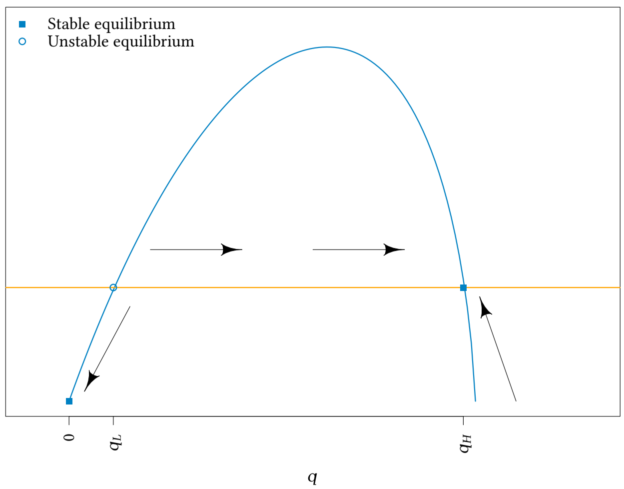
According to this definition price search of households of one type has a direct impact on the price difference between lotteries of the other type. In the Burdett and Judd (1983) economy the positive price differential spillovers for both types of agents exist for search allocations $q \in [0, \overline{q}]$ The higher average price search exerted by one type of households amplifies the difference between the price lotteries and creates incentives for making a higher price search. By contrast, for price search higher than $\overline{q}$ the negative price differential spillovers are observed. The higher average price search exerted reduces the difference between the price lotteries and creates incentives for making a lower price search.

A small *positive* perturbation $\varepsilon > 0$ in $q = q_H + \varepsilon$ make retailers decrease their prices. The change in the expected price of one draw will be higher than for the lottery with two draws, thus decreasing the expected difference between two lotteries. The benefit of sampling a second offer $\mathcal{W}(q)$ will be lower than the cost of search c. Consequently, consumers would not have more incentives to increase probability of two draws even more. The same logic applies to a positive disturbance in the degenerate equilibrium $q = 0$.

In the similar manner, a small *negative* perturbation $\varepsilon < 0$ in $q = q_H + \varepsilon$ make retailers increase their prices. The change in the expected price of one draw will be higher than for the lottery with two draws, thus increasing the expected difference between two lotteries. The benefit of sampling a second offer $\mathcal{W}(q)$ will be higher than the cost of search c. Consequently, consumers would not have more incentives to decrease probability of two draws even more for the positive disturbance.

Those two observations show that in the neighborhood of both equilibria price search exhibits strategic substitutability making the allocations self-sustainable. The higher (lower) price search is not amplified by similar actions of other buyers. On the contrary, it fosters the opposite action. This leads to the conclusion that both the degenerate equilibrium and the high-search equilibrium are stable.

Figure 2.2: Stability of equilibria.



On the other hand, a small *positive* perturbation $\varepsilon > 0$ in $q = q_L + \varepsilon$ make retailers decrease their prices. The change in the expected price of one draw will be lower than for the lottery with two draws, thus increasing the expected difference between two lotteries. In the neighborhood of this equilibrium price search exhibits strategic complementarity. Any disturbances in the allocation entail similar actions by households of the other type. As a result, the jittered allocation gives rise to convergence towards one of the stable equilibria, the degenerate in case of negative shocks or the high-search equilibrium in case of positive ones.

The discussion is depicted in Figure 2.2 and is concluded with the following theorem:

Theorem 9 *Suppose* $c < \mathcal{W}(\overline{q})$. *Then there exist two dispersed equilibria, high-search with* $q = q_L$ *and low-search with* $q = q_H$, *and one degenerate* $q = 0$. *The degenerate and high-search equilibria are stable. The low-search equilibrium is unstable.*

The next lemma is very useful for characterizing allocations q that can be rationalized as stable equilibria:

Lemma 10 *The maximizer* $\overline{q}$ *of the expected difference between lotteries* $\mathcal{W}(q)$ *does not depend on the reservation price* $\overline{p}$.

Proof Here I present the sketch of the proof and full derivations are relegated to Appendix IV. The idea of the proof is to show that the maximizer $\overline{q}$ does not change in $\overline{p}$, *i.e.*

$$\forall_{\overline{p}>1} \frac{\partial \overline{q}}{\partial \overline{p}} = 0.$$

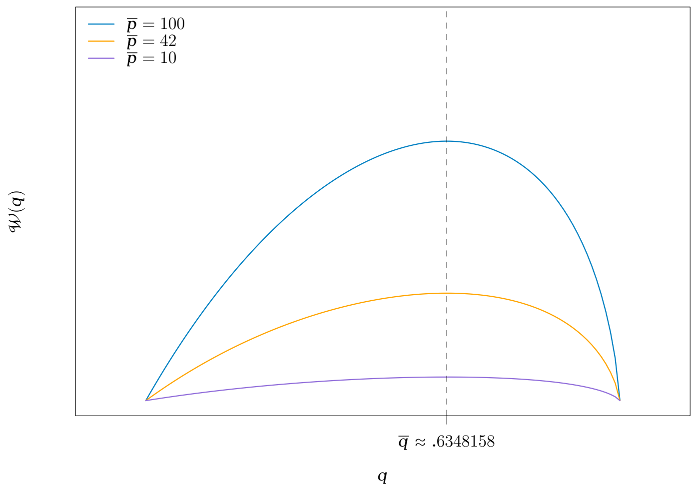
First I start by deriving the first-order condition consituting $\overline{q}$:

$$\partial_q \mathcal{W} = 0. \tag{2.7}$$

The equation does not have the closed-form solution so I use the implicit function theorem to determine $\frac{\partial q}{\partial \overline{p}}$ provided that $\partial_q \mathcal{W} = 0$. Next I make a guess, basing on a numerical solution to (2.7), that $\overline{q} \approx .634815$. If the lemma is true then for every value $\overline{p}$, the marginal effect of $\overline{p}$ on $\overline{q}$ should be zero (or very close to zero due to the numerical error). It turns out that the effect $\left.\frac{\partial q}{\partial \overline{p}}\right|_{q=.634815}$ is of the order of magnitude -7, which can be equalized with zero. $\blacksquare$

Figure 2.3 illustrates Lemma 10 at work. The reservation price $\overline{p}$ influences the value of $W(q)$ for every $q \in (0,1)$ and plays also a role in the value of

Figure 2.3: The maximum of $\mathcal{W}(q)$ does not depend on $\overline{p}$.



$\partial_q W(q)$ for $q \in (0,1) \setminus \{\overline{q}\}$. However, the value of the maximizer $\overline{q}$ is constant and does not change in $\overline{p}$.

Both Theorem 9 and Lemma 10 set out the stability condition for allocation q, which has an important quantitative implication. All allocations with price search intensity $q < q \approx .634815$ are unstable in the version of the economy proposed by Burdett and Judd (1983)[2]. The quantitative results of models calibrated to unstable equilibria can be criticized for describing allocations that can be transitory. However, in the subsequent section I propose a method that can be used to recover the stability of all allocations.

[2]There are examples of such parameterizations existing in the literature (*e.g.*, Kaplan and Menzio, 2016; Albrecht, Postel-Vinay, and Vroman, 2015).

IV. Price Search with Bargain Hunting

In this section I propose a refinement of the original model that overcomes the drawbacks presented in the previous section. It allows to generate the unique dispersed symmetric equilibrium, which is shown to be stable regardless of allocation q. As a result, the destructive implication of Theorem 9 and Lemma 10 can be fixed, if one uses the proposed framework as the rationalization of calibrated allocations in place of the original version. The alternation is introduced on the consumer side, in which the cost of search is exerted in the deterministic way and is strictly convex. The problem of the firms is the same as in the original framework.

A. The problem of the Buyers

There is a continuum of consumers of measure one. Every buyer is sampled with one price with probability $1-q$ or two price quotations with some probability q. The probability depends on the effort exerted by the consumer and is convex in q. In particular, assume that effort is equal to $\frac{q^2}{2}$. The problem of the consumer minimizing the expected cost of buying one unit of consumption is as follows:

$$\min_{q\in[0,1]}(1-q)\int_{\underline{p}}^{\overline{p}} pdF^{q^*}(p)+q\iint_{(p_1,p_2)\in[\underline{p},\overline{p}]^2}\min\{p1,p2\}dF^{q^*}(p_1)dF^{q^*}(p_2)+\frac{q^2}{2}. \tag{2.8}$$

In the original framework, the consumer mixes two pure strategies, $q=0$ and $q=1$. The cost of search c is made only conditioned on sampling the action that chooses to draw to two price offers $(q=1)$. However, in the described problem, it is not the case. All households make costly effort and having exerted this effort, it is sampled whether the consumer receives one or two offers. This structure of the problem resembles *bargain hunting.* The consumers that look for price bargains pay on average lower price but also they make search effort with *certainty.*

The solution to the problem of consumer is:

$$q = \mathcal{W}(q^*), \tag{2.9}$$

where q^* is taken as given.

An equilibrium for this setup is analogous to the original one:

Definition 11 (Equilibrium price dispersion with bargain hunting) *A dispersed equilibrium is constituted by a cdf $F^{q^*}(p)$, probability of two draws q^*, and profits of the retailers π such that:*

i. every price from the support of $F^{q^*}(p)$ maximizes the problem of the retailers (3.1) *and yields the same profit, π;*

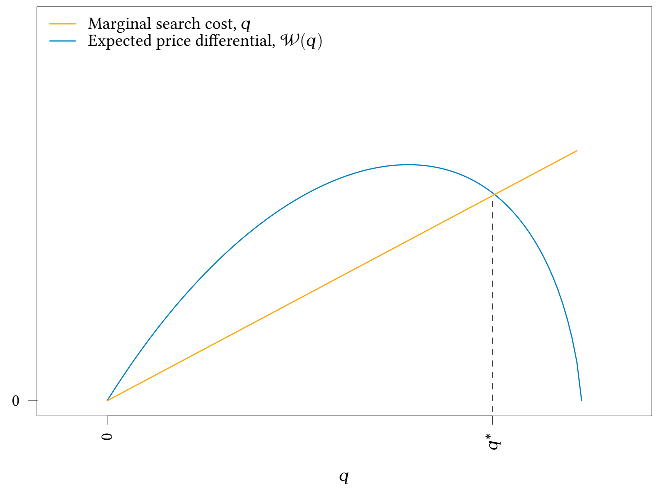
ii. price search q^* minimizes the expected cost of purchasing a unit of consumption according to (2.8).

B. Uniqueness and Stability

The economy features one degenerate equilibrium $q = 0$ and zero or one dispersed equilibrium q^*. For $q > 0$, the expected price difference $\mathcal{W}(q)$ and the marginal cost of search q cross each other at most once. This fact is depicted in Figure 2.4.

To study the stability properties of equilibria in this economy I employ the same methodology and an analogous tâtonnement process (2.6) to the one from the previous section:

Figure 2.4: Uniqueness of dispersed equilibrium



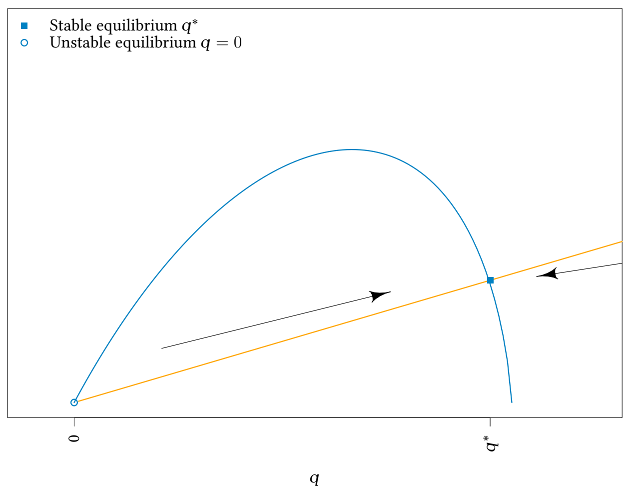
$$\dot{q}_t = \overline{\mathcal{W}}_t(q_\mu, q_{1-\mu}) - q. \tag{2.10}$$

The subtracted term is the only difference between this process and (2.6). Here the marginal search cost is increasing. Using analogous logic to the previous economy, a small *positive* perturbation $\varepsilon > 0$ in $q = q^* + \varepsilon$ leads to reducing the expected difference between two lotteries. At the same time, due to strong convexity of the search cost function, the cost of higher q will increase. Similarly, a small *negative* perturbation $\varepsilon < 0$ in $q = q^* + \varepsilon$ will leads to an increase in the expected difference between two lotteries and to a decrease in the marginal cost. Therefore, in $q = q^*$ consumers would not have incentives to change their actions in response to small perturbations. In addition to this the degenerate equilibrium is unstable[3]. A small positive perturbation results in convergence to

[3] I am grateful to Guido Menzio for suggesting this point.

the stable equilibrium with $q = q^*$.

Figure 2.5: Only the dispersed equilibrium is stable



The discussion is depicted by Figure 2.5 and is concluded with the following theorem:

Theorem 12 *There exist one dispersed equilibrium with price search* $q = q^*$ *and one degenerate* $q = 0$*. The dispersed equilibrium is stable. The degenerate equilibrium is unstable.*

Finally, I am in the position to present a result that enables to rationalize any allocation $q \in (0, 1)$ as the stable equilibrium from Theorem 12:

Lemma 13 (Rationalizability) *Given the reservation price* $\overline{p}$*, every allocation* $q^* \in (0, 1)$ *can be rationalized by the search cost function:*

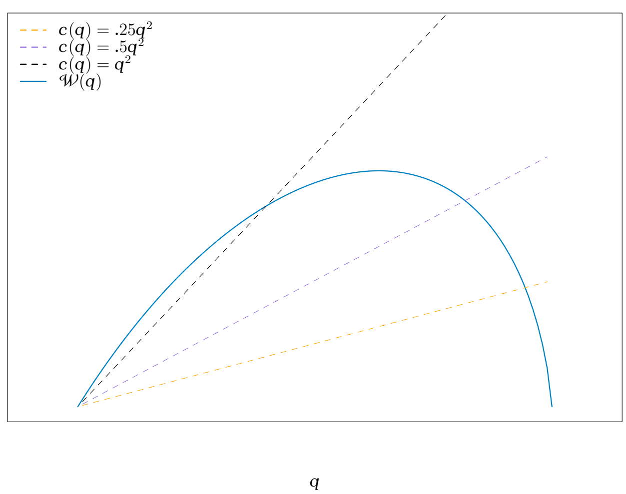
$$c(q) = \frac{\mathcal{W}(q^*)}{q^*} \cdot \frac{q^2}{2}, \tag{2.11}$$

where q^* is taken exogenously by the buyer.

Proof Let the cost function be of the following form $c(q) = \alpha \frac{q^2}{2}$. Then the solution to the problem of buyers satisfies: $\alpha q = W(q^*)$. In equilibrium this solution coincides with q^*. Consequently, $\alpha = \frac{\mathcal{W}(q^*)}{q^*}$, which gives (2.11).

Lemma 13 is illustrated in Figure 2.6. This result can be used to recover the stability of all allocations, especially those that cannot be supported by the stable equilibrium in the original economy of Burdett and Judd (1983). What is interesting this lemma does not imply that recalibrations of the existing quantitative results are required. The only change that should have be done is applying the argument basing on bargain hunting with convex search cost in place of the original one with the indivisible search margin.

Figure 2.6: Every allocation is rationalizable



V. Concluding Comments

In this article I studied equilibrium properties of a standard model of endogenous price distribution due to Burdett and Judd (1983). In search economies of this class typically there are two dispersed equilibria, low-search and high-search one. I show that the low-search equilibrium is unstable while the high-search equilibrium is stable. What is important every allocation can be characterized only as one of those types. This finding substantially narrows the range of allocations, in which the price dispersion is stable and its form is not a temporary phenomenon. To recover the stability of every allocation I proposed an extension of the original model, which gives rise to one unique symmetric dispersed equilibrium. This equilibrium is shown to be stable and it can be used to rationalize *every* allocation. In addition to this, in contrast to the original model the degenerate Diamond (1971)-type equilibrium is unstable.

3

Monetary Policy and the Price Search Channel

I. Introduction

In this paper, I study interactions between a bargain hunting behavior of households and a redistributive channel of monetary policy. Traditionally, monetary policy is analyzed using the New Keynesian perspective, in which the main driving force is the intertemporal substitution of household consumption in the presence of nominal rigidities. Most models of this class assume the existence of the representative consumer, which results in abstracting from the role of household heterogeneity. However, as recent studies by Auclert (2017) and Kaplan, Moll, and Violante (2017) show the redistribution channel is also important part of the transmission mechanism. The effect of expansionary monetary policy is amplified by the redistributive channel because households that benefit from the expansion exhibit substantially higher marginal propensity to consume than households who lose.

This paper contributes to the existing literature of the redistributive role of monetary policy by introducing *bargain hunting*, an additional effect that may be important to account for the redistributive channel. This effect consists in higher search effort exerted by low-income households than high-income consumers, which has a direct impact on prices quoted by retailers. I show that money injection to households reduces the inefficiency generated by non-competitive behavior of firms. This result stems from the fact that bargain hunters also feature higher marginal propensity to consume. An increase in the fraction of the aggregate consumption purchased by buyers with higher price search intensity causes a reduction of the monopolistic power of the firms. As a result, a monetary expansion is amplified by lower consumption prices for all households. As I show the quantitative model suggests that thanks to this effect the tension between equity and efficiency is mitigated.

In the course of the analysis, I proceed in the following way. First, I start by formulating an illustrative example of an economy where the redistributive fiscal policy can foster efficiency of the equilibrium allocation. The tension between equity and efficiency motives is completely reduced and all households benefit from higher redistribution and the output increases. This seemingly counterintuitive result is possible thanks to the price channel. The redistribution reduces market power of firms due to an increase in the number of transactions with bargain hunters, thus causing equilibrium prices to fall. The effect of this policy is that all households immediately benefit from paying lower prices. This mechanism is similar to the one outlined by Bai, Ríos-Rull, and Storesletten (2011), where preference shocks in disutility from search has the real effect on shaping aggregates.

Next, I embed consumption search into a pure-currency economy with idiosyncratic productivity risk, where money is the only storable good that can be used as the mean of payment. As mentioned before the monetary policy is usually studied with the use of the channel relying on nominal rigidities. Here my approach is different. The starting point of my analysis is a class of incomplete markets models which allows to focus on the redistributive effects generated by monetary policy in the tradition of Bewley (1986), Scheinkman and Weiss (1986), İmrohoroğlu (1992) or more recently Rocheteau, Weill, and Wong (2015) and Lippi, Ragni, and Trachter (2015). On top of the model I introduce the search for consumption in the spirit of Burdett and Judd (1983). The currency injection leads to anticipated inflation. Households decide on their price search intensity and firms set their prices depending on the aggregate shopping effort. In the considered economy money transfers plays a double role. First, it provides an additional instrument of insurance for smoothing consumption. This property is typical for monetary models with incomplete markets as in İmrohoroğlu (1992) or Scheinkman and Weiss (1986). Second, what is new in my analysis, lump-sum transfers increase a fraction of consumption purchased by bargain hunters. In response to this policy the firms decrease their real prices and their margins. As a result, redistributive policy decreases inefficiency generated by the non-competitive pricing behavior of the firms.

Burdett and Menzio (2017a,b) also use the search protocol in the tradition of Burdett and Judd (1983) to study the transmission of monetary policy to the real side of the economy. Nonetheless, there are two substantive differences between my framework and the one proposed by the authors. Namely, they assume that: *(i)* there exists the representative buyer, whose *(ii)* price search intensity is set exogenously. Consequently, money injection does not provide insurance (due to lack of idiosyncratic risk) and the market power of firms does not depend on inflation (due to both exogenous price search and lack of idiosyncratic risk). In their framework higher inflation unambiguously causes welfare to fail. As I show this is not the case in my framework.

On the empirical side, the paper is motivated with two strands of the literature. First, Aguiar and Hurst (2007) and Kaplan and Menzio (2016) used price scanner data to document evidence that retired and unemployed households pay on average lower prices than employed individuals for exactly the same kinds of goods. Second, Auclert (2017) and Kaplan, Moll, and Violante (2017) show that the inflation can generate sizable wealth redistribution effect. Those two observations combined can be a premise for studying what is the response of the distribution of prices to redistributive monetary policy.

The rest of the paper is structured as follows. In section II, I present an exemplary economy which shows that redistributive policy can increase output and overall welfare. In section III, I introduce the price search friction into a pure-currency monetary economy. Section IV is dedicated to the calibration and numerical solution of the model. In section V I use the monetary model to quantitatively evaluate the cost of inflation in the presence of price search. Section VI concludes.

II. Efficiency and Bargain Hunting

I start by presenting an illustrating example how policies exploiting bargain hunting can improve the welfare of *all* agents. To this end I use linear taxation of labor income imposed on low-search (and high-income) households, which is transferred to the bargain hunters. I show that thanks to this redistribution policy in this example all agents may be better off. The source of this result is that the bargain hunting behavior decreases the non-competitive behavior of the firms. The retailers facing a higher mass of transactions with bargain hunters respond by setting prices closer to the competitive prices, which support the efficient allocation. Consequently, this redistribution generates positive price externalities on all consumers, who thanks to this policy pay lower consumption prices and their real labor income increases.

A. Model Environment

Setup. Time is discrete and goes on forever. There are two types of households, savers of measure λ that have access to the market of shares of producers and hand-to-mouth households of measure $1-\lambda$ that consume their labor income every period. Households obtain their consumption through a matching process with producers of measure one. In one match only one unit of consumption can be purchased. Thus consumption bundle of size c is a realization of a continuum of matches. There is no uncertainty at the aggregate and individual level.

Firms. Every firm has access to common technology that produces perishable output using only labor input, according to the linear production function $y = N$. The produced output is then sold on the decentralized market, where households and firms meet each other. During one shopping visit the firm is able to sell only a unit of consumption. The bargaining power is different for both types of households. Savers meet two retailers with probability ψ_{sa} and choose the lower sampled price and only one search with $1-\psi_{sa}$, while hand-to-mouth households make two searches with probability ψ_{HtM}, where $\psi_{sa} < \psi_{HtM}$. The producer does not know whether a visiting household has drawn an alternative offer from another firm. Shopping visits that constitute the aggregate consumption is distributed equally amongst all firms. For further analysis it is convenient to define some aggregate shopping statistics:

$$
\begin{aligned}
\Psi_{(-)} =& (1-\psi_{HtM})(1-\lambda)c_{HtM} + (1-\psi_{PI})\lambda c_{PI}, \\
\Psi_{(+)} =& 2\psi_{HtM}(1-\lambda)c_{HtM} + 2\psi_{PI}\lambda c_{PI}, \\
D =& \Psi_{(+)} + \Psi_{(-)}.
\end{aligned}
$$

With probability $\frac{\Psi_{(-)}}{D}$ a household that is matched with the firm is captive and accepts any sampled price set by the firm below the upper bound, R. With the complementary probability $\frac{\Psi_{(+)}}{D}$, the visiting household samples an alternative offer from another producer and chooses the minimum of the drawn prices. Given the realized demand for pricing strategy p denoted $y(p)$ the firms maximize their expected profit by choosing the real price (in terms of labor):

$$\max_{p}(p-1)\cdot y(p) \tag{3.1}$$

$s.t.$

$$y(p)=\begin{cases}\Psi_{(-)}+\Psi_{(+)}\left(1-G(p)\right) & \text{for } p\leq R,\\ 0 & \text{for } p>R.\end{cases}$$

I assume that the retailers cannot differentiate themselves to increase the number of matches and they are indistinguishable for households before making a visit. [1] Therefore, every retailer is visited by the same number of times, D. Depending on the pricing strategy of the retailer a certain number of visits become effective. Retailers setting the maximum price $p = R$ target only at captive customers, which leads to $\Psi_{(-)} = \frac{\Psi_{(-)}}{D}\cdot D$ realized transactions.[2] Offers made by retailers setting such a price $\underline{p}$ that $G(\underline{p}) = 0$ will be accepted by all visiting customers,

[1]For example, you can think of advertising as one way of differentiation. However, as Butters (1977) shows it does not change much in the behavior of the model. From the modeling perspective the only difference is that the buyers are matched with advertisements rather than with firms. All theoretical results stay unchanged.

[2]The maximum price is set here exogenously but consider a model where households have access to home production technology generating a perfect substitute for market consumption at the rate of transformation leisure to consumption equal to $\frac{1}{R}$. No price above R would be accepted as households would substitute working and spending their labor income on the market consumption with producing home production goods.

$D = (\frac{\Psi_{(-)}}{D} + \frac{\Psi_{(+)}}{D}) \cdot D$. In equilibrium all retailers receive the same profit denoted $\overline{\Pi}^*$.

Equilibrium price distribution. The distribution of prices quoted by producers is consistent with the aggregate shopping behavior of households in the economy. Using the logic presented in the original paper by Burdett and Judd (1983) it is given by[3]:

$$G(p) = \begin{cases} 0, & \text{for } p < \underline{p}, \\ \frac{D}{\Psi_{(+)}} - \frac{\Psi_{(-)}}{\Psi_{(+)}} \cdot \frac{R-1}{p-1}, & \text{for } p \in [\underline{p}, R], \\ 1, & \text{for } p > R, \end{cases} \tag{3.2}$$

where $\underline{p} = \frac{\Psi_{(+)}}{D} + \frac{\Psi_{(-)}}{D} R$.

$$p^{HtM} = (1 - \psi_{HtM}) \int p dG(p) + \psi_{HtM} \mathbb{E} \min\{p_1, p_2\}, \tag{3.3}$$

$$p^{sa} = (1 - \psi_{sa}) \int p dG(p) + \psi_{sa} \mathbb{E} \min\{p_1, p_2\}, \tag{3.4}$$

where $\int p dG(p)$ is the expected price of a shopping trip with one visit only and $\mathbb{E} \min\{p_1, p_2\} = \int\int \min\{p_1, p_2\} dG(p_1) dG(p_2)$ is the expected minimum price from a lottery with two sampled prices from different producers. Intuitively, the price of consumption basket of savers is higher than the price paid by hand-to-mouth consumers. This result comes immediately from the fact that the expected value of one price offer is higher than the expected minimum of two draws and from the fact that the savers draw one price more often.

Savers. Savers of measure λ decide upon consumption c_t^{sa}, labor supply l_t^{sa} and the size of purchased shares of the production firms denoted a_t at price p_t^f.

[3]In the companion paper (Pytka, 2017) I explain in detail how to derive (3.2).

One share pays dividend $\overline{\Pi}_t^*$ every period. Then the problem of the savers reads as follows:

$$\max_{\{c_t, l_t\}} \sum_{t=0}^{\infty} \beta^t \{u(c_t) - g(l_t)\}, \tag{3.5}$$

s.t.

$$p_t^{sa} c_t + p_t^f a_{t+1} = l_t + a_t(p_t^f + \overline{\Pi}_t^*) \quad \forall_t,$$

where p_t^{sa} is defined by (3.4).

HtM households. Hand-to-mouth households of measure $1-\lambda$ decide on labor supply l_t^{sp} which is fully consumed in the same period, c_t^{sp}. The problem of HtM households reads then:

$$\max_{\{c_t, l_t\}} u(c_t) - g(l_t) \quad \text{s.t.} \tag{3.6}$$

$$p_t^{HtM} c_t = l_t,$$

where p_t^{HtM} is defined by (3.3).[4]

B. Equilibrium

Definition 14 (Equilibrium) *An equilibrium for the economy is constituted by a distribution of prices of consumption goods $G(p)$, a price of firm stocks p^f, consumption and labor decisions of both types of households c_t^{sa}, l_t^{sa}, c_t^{HtM}, l_t^{HtM} such*

[4]Note that the labor is the numéraire in both problems. The same convention is used in Section III.

that:

i. *households maximize their utility.*

ii. *producers maximize* (3.1) *and all equilibrium prices yield the same profit,* $\overline{\Pi}^*$.

iii. *the market of shares:*

$$\lambda a_t = 1,$$

iv. *the labor markets clears:*

$$(1-\lambda)l_{HtM} + \lambda l_{PI} = \int y(p)dG(p),$$

v. *the goods market clears:*

$$(1-\lambda)c_{HtM} + \lambda c_{PI} = \int y(p)dG(p),$$

vi. *the aggregate consumption is consistent with the aggregate shopping trips:*

$$(1-\lambda)c_{HtM} + \lambda c_{PI} = \Psi_{(-)} + \frac{\Psi_{(+)}}{2}.$$

The equilibrium and its efficiency can be characterized in the following way:

Proposition 15 *i. Consumption and labor supply* $\forall_t c_t^{sa} = c_{sa}^*, \forall_t c_t^{HtM} = c_{HtM}^*, l_t^{sa} = l_{sa}^*, l_t^{HtM} = l_{HtM}^*$ *are constant over time and are summarized by first-order conditions:*

$$\frac{u'(c_{HtM}^*)}{p^{HtM}} = g'(l_{HtM}^*), \tag{3.7}$$

$$\frac{u'(c_{sa}^*)}{p^{sa}} = g'(l_{sa}^*). \tag{3.8}$$

ii. *The price of stocks is equal to* $p^f = \frac{\beta}{1-\beta}\overline{\Pi}^*$.

iii. *The equilibrium price dispersion is given by* (3.2).

iv. *The equilibrium allocation is inefficient.*

Proof Property (i) can be derived using first order condition of the household problems. Property (ii) comes from the Euler condition of the saver problem. The proof of property (iii) can be found in (Pytka, 2017) and is analogous to the proof due to Burdett and Judd (1983). In the efficient allocation chosen by the benevolent social planner the marginal rate of transformation equalizes the marginal rate of substitution. This implies $u'(c^*_{HtM}) = g'(l^*_{HtM})$ and $u'(c^*_{sa}) = g'(l^*_{sa})$. Nonetheless, the equilibrium prices p^{HtM} and p^{sa} in this economy are always greater than one, whenever at least $\psi_{HtM} > 0$. Therefore solutions to the problems of households (3.7) and (3.8) cannot be compatible with the allocation chosen by the social planner. This observation concludes the proof. ∎

The price search friction creates a wedge, which is the source of inefficiency. The scale of this wedge could be mitigated by reducing prices paid by households. One way to achieve this could be a hypothetical increase in the fraction of purchases with two draws holding other things constant:

Remark 16 *An increase in the fraction of shopping visits with two draws* $\frac{\Psi_{(+)}}{D}$ *in the aggregate shopping behavior leads to a decrease in prices paid by* both *types of households.*

Proof The implied prices paid by households p^{HtM} and p^{sa} can be written in the following way:

$$p_i = \underline{p} + \int_{\underline{p}}^{\infty} 1 - G(x) - \psi_i G(x) + \psi_i [G(x)]^2 dx,$$

where $i \in \{sa, HtM\}$.

The negative impact of prices can be shown by applying Leibniz's integration rule:

$$\frac{\partial p_i}{\partial \frac{\Psi_{(+)}}{D}} = (1 - R) + \int_{\underline{p}}^{R} (2\psi_i G(x) - 1 - \psi_i) \left(\frac{R - x}{x - 1} \right) \left(\frac{D}{\Psi_{(+)}} \right)^2 dx.$$

The term $(1 - R)$ is always negative and $2\psi_i G(x) - 1 - \psi_i$ is never positive.[5] The factor $\frac{R-x}{x-1}$ is positive for the interior of the price support. As a result, the implied price p_i decreases in $\frac{\Psi_{(+)}}{D}$ for both types of households. $\blacksquare$

C. Numerical example

In this subsection I present a possible implication of Remark (16) for redistributive policies in economies characterized by the price search friction. Let the utility of both types of households be of the GHH form, $u(c_t, l_t) = \log(c - \frac{l^{1+\phi}}{1+\phi})$ and parameters be equal to values presented in Table A.5. The values of the parameters were set to generate high heterogeneity in price search intensities ($\psi_{HtM} \gg \psi_{sa}$) and high home production cost R.

Table 3.1: Equilibrium allocation.

Parameter	R	ϕ	λ	ψ_{HtM}	ψ_{sa}
Value	5	.5	.5	.7	0

Now suppose that a linear labor income tax is imposed on savers $\tau = .05$, which finances the lump-sum transfer to hand-to-mouth households. In a frictionless economy the savers are always worse off after such a reform. However,

[5] Just note that $\psi_i G(x) - \psi_i \leq 0$ and $\psi_i G(x) - 1 \leq 0$.

in the considered economy transfers plays an additional role in decreasing inefficiency by reducing the bargaining power of producers. The fraction of consumption purchased by hand-to-mouth households increases and this increases $\frac{\Psi_{(+)}}{D}$. In response to that the producers set lower prices distributed according to formula (3.2). In the parameterized example the benefits of lower prices offset the cost of lower profits even for the savers. The tax reform has also a positive effect on the labor supply as it increases the real wages for both types of households, $\frac{1}{p_{HtM}}$ and $\frac{1}{p_{sa}}$.

Table 3.2: Numerical example.

	Laissez-faire		Tax intervention
Spenders:			
c_{HtM}	.010	<	.015
l_{HtM}	.0466	<	.0495
p_{HtM}	4.634	>	4.495
Period utility	-4.601	<	-4.209
Savers:			
c_{sa}	.0817	<	.0822
l_{sa}	.0452	<	0.0476
p_{sa}	4.701	>	4.585
Period utility	-2.516	<	-2.512
Aggregate:			
$\frac{\Psi_{(+)}}{D}$	.142	<	.194
$\int p dG(p)$	4.701	>	4.585
$\mathbb{E} \min\{p_1, p_2\}$	4.606	>	4.456
$\overline{\Pi}^*$	3.339	>	3.142
Total output	.0918	<	.0971

III. Monetary Economy under Price Search

In this section I set out a pure cash-currency economy that features the frictions in the purchasing economy in a similar way to the one described in Section II. The quantitative behavior of the economy is disciplined in the subsequent section. Then the model is used to evaluate the welfare cost of inflation in the presence of the price search friction.

A. Building Blocks of the Economy

Time is continuous and lasts forever. The economy is inhabited with a unit measure of households and a fixed measure of producers. There is a single perishable consumption good produced with the use of labor input. The economy is pure-currency in this sense that fiat money is the only storable good that can be used as the mean of payment. The money supply grows at a constant rate and the new money augments individual balances of households as lump-sum transfers. The currency injection leads to anticipated inflation, which is embodied by a constant increase in nominal prices of consumption and a constant decrease in the value of money. Households face idiosyncratic risk in income, which gives rise to a precautionary demand for liquidity. The goods produced by the firms are traded on a decentralized market. Each household exerts some price search effort. Given a price distribution, higher effort translates into lower expected prices paid by a household. The production firms set their prices that maximize their expected profit and which are the best response to the aggregate shopping behavior of households.

Preferences. The households maximize their expected lifetime utility, which is defined over flows of discounted consumption c_t and price search intensity s_t with the time discount rate ρ :

$$\mathbb{E}_0 \int_0^\infty e^{-\rho t} \left\{ u(c_t) - h(s_t) \right\} \mathrm{d}t, \tag{3.9}$$

where $u(\cdot)$ is increasing and strictly concave and $h(\cdot)$ is increasing and strictly convex.

Income risk. Households can be workers ($i = w$) or entrepreneurs ($i = e$). Transition between occupations of a single household evolves exogenously and stochastically over time and is modeled with a two-state Poisson process $i \in \{w, e\}$ with intensities λ^w and λ^e, respectively. The entrepreneurs receive an equal share of profits generated by producers, $\overline{\Pi}^*$. Each worker supplies labor, which productivity is subject to idiosyncratic risk. In particular, I assume that log labor earnings follow the Ornstein–Uhlenbeck process:

$$\dot{z} = \theta(\overline{z} - z)\mathrm{d}t + \sigma \mathrm{d}W, \tag{3.10}$$

where the drift exhibits mean reversion to $\overline{z}$ with the speed equal to θ and the diffusion component is generated with Wiener increments, $\mathrm{d}W$.[6]

Price search of households. Each household makes a decision on the intensity of search, s. The consumption purchased by households is the result of continuum of shopping lotteries. In a single shopping lottery a households can be matched with one or two sellers and only a unit of goods can be purchased. The probability of meeting two retailers depends on the price search intensity. With probability $1 - s$, the household is captive during a lottery and accepts any sampled price below the upper bound, R.With the complementary probability s the buyer samples offers from two firms and chooses the minimum of the drawn prices. Let $G(p)$ be a cdf of relative prices (in terms of labor) observed in the economy. Then the distribution of the effective price of a single purchase is a result of the following compound lottery:

$$F(p, s) = (1 - s)G(p) + s\left(1 - [1 - G(p)]^2\right). \tag{3.11}$$

[6]This process is a continuous-time analogue of AR(1) with autocorrelation $e^{-\theta} \approx 1 - \theta$. The process is characterized by stationary distribution $\mathcal{N}(\overline{z}, \frac{\sigma^2}{2\theta})$.

Using the law of large numbers as in Pytka (2017) I am in the position to reduce the dimensionality of the household problem and replace the distribution $G(p)$ with two sufficient statistics:

- the expected price of a single lottery with one drawn price:

$$p^0 := \int p dG(p)$$

- the difference between expected prices from a lottery with one offer and two offers:

$$MPB := p^0 - \iint \min\{p_1, p_2\} dG(p_1) dG(p_2).$$

Consequently, the price of a unit of consumption is as follows:

$$p(s) = p^0 - s \cdot MPB. \tag{3.12}$$

Money holdings. In the economy asset markets are incomplete and households can store value using only fiat money, which is used to self-insure against future fluctuations in the individual income. The money is issued by the government in the supply of $\tilde{M}_t$ and grows at a constant rate π. The nominal money balances of individual households $\tilde{m}$ are augmented with a nominal lump-sum transfer equal to $\tilde{\tau}_t$. Then the drift of nominal money holding of an individual household is given by:

$$\dot{\tilde{m}}_t = \begin{cases} \frac{z_t - p(s_t) \cdot c_t}{q_t} + \tilde{\tau}_t, & \text{for } i = w, \\ \frac{\overline{\Pi}_t^* - p(s_t) \cdot c_t}{q_t} + \tilde{\tau}_t, & \text{for } i = e, \end{cases} \tag{3.13}$$

where the real price of money (in terms of labor) is denoted q_t. There is no credit market and borrowing is not allowed:

$$\tilde{m}_t \geq 0. \tag{3.14}$$

The real money balance in terms of labor is equal to $m_t := q_t \tilde{m}_t$.[7] Its dynamics evolves according to:

$$\dot{m}_t = \dot{\tilde{m}}_t q_t + \tilde{m}_t \dot{q}_t. \tag{3.15}$$

In a stationary environment the rate of return of money holdings coincides with the rate of growth of the money supply, $\frac{\dot{q}_t}{q_t} = -\pi$. If we combine (3.13) and (3.15) we obtain the evolution of the real money holdings of workers:

$$\dot{m}_t = z_t - p(s_t) \cdot c_t + \frac{\tilde{\tau}_t}{q_t} + m_t \frac{\dot{q}_t}{q_t} = z_t - p(s_t) \cdot c_t - \pi_t m_t + \tau_t, \tag{3.16}$$

and for entrepreneurs:

$$\dot{m}_t = \overline{\Pi}_t^* - p(s_t) \cdot c_t - \pi_t m_t + \tau_t. \tag{3.17}$$

The value of real lump-sum transfers $\tau_t := q_t \tilde{\tau}_t$ is equal to a decrease in the real value of the existing individual money balances:

$$\tau_t = \pi \cdot \left\{ \frac{\lambda^w}{\lambda^e + \lambda^w} \iint m g^w(m, z) \mathrm{d}m \mathrm{d}z + \frac{\lambda^e}{\lambda^e + \lambda^w} \int m g^e(m) \mathrm{d}m \right\}, \tag{3.18}$$

where $\frac{\lambda^w}{\lambda^e + \lambda^w}$ and $\frac{\lambda^e}{\lambda^e + \lambda^w}$ are the fractions of workers and entrepreneurs in a sta-

[7] The real value of money balance in terms of consumption is equal to $\frac{q_t}{p(s_t)} \tilde{m}_t$ and depends on the price search intensity of the household.

tionary equilibrium and $g^w(m,z)$ and $g^e(m)$ (such that $\iint g^w(m,z)\mathrm{d}m\mathrm{d}z = 1$ and $\int g^e(m)\mathrm{d}m = 1$) are density functions of workers and entrepreneurs.

Firms. The problem of the firms is analogous to the one from Section II. The only difference is that the aggregate shopping statistics are given by:

$$\begin{aligned}
\Psi_{(-)} &= \int c^w(m,z)\left(1 - s^w(m,z)\right) g^w(m,z)\mathrm{d}m\mathrm{d}z + \int c^e(m)\left(1 - s^e(m)\right) g^e(m)\mathrm{d}m, \\
\Psi_{(+)} &= \int c^w(m,z) 2s(m,z) g^w(m,z)\mathrm{d}m\mathrm{d}z + \int c^e(m) 2s^e(m) g^e(m)\mathrm{d}m, \\
D &= \Psi_{(+)} + \Psi_{(-)}.
\end{aligned}$$

B. Equilibrium

Definition 17 (Stationary Equilibrium) *A stationary equilibrium of the economy is constituted by a sequence of prices of money in terms of labor $\{q_t\}$, a distribution of prices of consumption goods in terms of labor $G(p)$, a set of decision rules of workers $c^w(m,z)$, $s^w(m,z)$ and entrepreuners $c^e(m)$, $s^e(m)$, and invariant distributions of workers $g^w(a,z)$ and enterpreneurs $g^e(m)$ such that:*

i. households maximize the expected utility (3.9) subject to the evolution of real money holdings (3.16) and (3.17), income dynamics driven by idiosyncractic labor productivity and occupational risk, and the borrowing limit (3.14);

ii. producers maximize (3.1) and all equilibrium prices yield the same profit, $\overline{\Pi}^*$.

iii. the money market clears:

$$M = \frac{\lambda^w}{\lambda^e + \lambda^w}\int m g^w(m,z)dmdz + \frac{\lambda^e}{\lambda^e + \lambda^w}\int m g^e(m)dm.$$

iv. the labor markets clears:

$$\iint zg^w(m,z)\mathrm{d}m\mathrm{d}z = \int y(p)dG(p),$$

v. the goods market clears:

$$\frac{\lambda^w}{\lambda^e+\lambda^w}\iint c^w(m,z)g^w(m,z)\mathrm{d}m\mathrm{d}z + \frac{\lambda^e}{\lambda^e+\lambda^w}\int c^e(m)g^e(m)\mathrm{d}m = \int y(p)dG(p)$$

vi. the aggregate consumption is consistent with the aggregate shopping trips

$$\frac{\lambda^w}{\lambda^e+\lambda^w}\iint c^w(m,z)g^w(m,z)\mathrm{d}m\mathrm{d}z + \frac{\lambda^e}{\lambda^e+\lambda^w}\int c^e(m)g^e(m)\mathrm{d}m = \Psi_{(-)} + \frac{\Psi_{(+)}}{2}.$$

C. Equilibrium Characterization

A stationary equilibrium with individual decisions on consumption and price search intensities of households, a distribution of prices set by retailers, and joint distribution of money holdings and income can be summarized with: a cdf of prices $G(p)$, and two systems of recursive differential equations: a Hamilton-Jacobi-Bellman equation (henceforth, HJB) and Kolmogorow forward equation (henceforth, KF).

The HJB equation for entrepreneurs with the real money holding m is as follows:

$$\begin{aligned}\rho v^e(m) = \max_{c,s\in[0,1]} u(c) - h(s) + \partial_m v^e(m)\left[\overline{\Pi}^* - p(s)\cdot c - \pi m\right] + \\ + \lambda^w\left(v^w(m,\overline{z}) - v^e(m)\right).\end{aligned} \tag{3.19}$$

Thanks to the fact that the problem is studied in continuous time the income risk of entrepreneurs for becoming a worker with the average labor income $\overline{z}$ is captured in an additive way by the term $\lambda^w \left(v^w(m,\overline{z}) - v^e(m) \right)$. Notice that this element does not affect the first-order condition for consumption and price search. The entrepreneurs make decision on the flows of the utility from the current consumption c and the disutility from the current price search intensity s, which for the optimal solution of the problem are equalized with benefits of increasing the real money holdings, $\partial_m v^e(m)\dot{m}$.

The HJB equation for workers with the real money holding m and the income z is represented in the following way:

$$\begin{aligned}\rho v^w(m,z) = \max_{c,s\in[0,1]} u(c) - h(s) + \partial_m v^w(m,z)\left[z - p(s)\cdot c - \pi m\right] + \\ + \lambda^e \left(v^e(m) - v^w(m,z)\right) + \\ + \partial_z v^w(m,z)\theta(\overline{z} - z) + \frac{1}{2}\partial^2_{zz} v^w(m,z)\sigma^2.\end{aligned} \tag{3.20}$$

The main difference from the HJB equation of entrepreneurs comes from the fact that the workers are subject to additional source of risk in labor earnings modeled as a diffusion process. Those terms accounting for this kind of labor risk are obtained directly from Itô's lemma.

The stationary distribution of money holdings of the entrepreneurs satisfies the KF equation:

$$0 = -\frac{\mathrm{d}}{\mathrm{d}m}\left\{(\overline{\Pi}^* - p(s^e(m))\cdot c^e(m) - \pi m)\cdot g^e(m)\right\} - \lambda^w g^e(m) + \lambda^e \int g^w(m,z)dz. \tag{3.21}$$

The term $-\lambda^w g^e(m)$ is the outflow of entrepreneurs who become workers and the term $\lambda^e \int g^w(m,z)dz$ is the inflow of workers with real money holding m and different productivities z distributed according to $g^w(m,z)$.

The stationary distribution of real money holdings of workers satisfies the KF equation:

$$0 = -\frac{\mathrm{d}}{\mathrm{d}m}\left\{(z - p(s)\cdot c - \pi m)\cdot g^w(a,z)\right\} - \lambda^e g^w(m,z) + \mathbb{I}_{\{z=\overline{z}\}}\lambda^w g^e(m)$$
$$- \partial_z(\theta(\overline{z} - z)g^w(m,z)) + \frac{1}{2}\partial_{zz}\sigma^2 g^w(m,z). \qquad (3.22)$$

The term $-\lambda^e g^w(m)$ is the outflow of workers who become entrepreneurs and the term $\mathbb{I}_{\{z=\overline{z}\}}\lambda^w g^e(m)$ (where $\mathbb{I}_{\{z=\overline{z}\}}$ is an indicator function) is the inflow of entrepreneurs with real money holding m to the state with mean labor productivity $\overline{z}$. For other labor income states we do not observe any inflows of entrepreneurs. The last term $-\partial_z(\theta(\overline{z}-z)g^w(m,z))+\frac{1}{2}\partial_{zz}\sigma^2 g^w(m,z)$ is obtained by applying Itô's lemma.

The stationary equilibrium allocation is the solution to the system of equations (3.19), (3.20), (3.21), (3.22), and (3.2).

IV. Mapping the Model to Data

Having outlined the structure of the economy I calibrate the model to replicate empirical counterparts. The environment is quite complex and it is very hard to find the analytical solution. To this end I employ numerical methods for solving heterogeneous agent models in continuous time recently proposed by Achdou, Han, Lasry, and Lions (2017).

The solution to the HJB equations (3.19) and (3.20) is constituted by first-order conditions:

$$\begin{aligned} u'(c) &= \partial_m v^w(m,z)p, & u'(c) &= \partial_m v^e(m)p, \\ h'(s) &= \partial_m v^w(m,z)MPBc, & h'(s) &= \partial_m v^e(m)MPBc, \\ p &= p^0 - sMPB. \end{aligned} \tag{3.23}$$

These conditions always hold with equality thanks to the fact that the borrowing condition (3.14) is replaced by state constraint boundary conditions:

$$u'\left(\frac{z}{p(s)}\right) \leq \partial_m v^w(0,z)p(s), \quad u'\left(\frac{\overline{\Pi}^*}{p(s)}\right) \leq \partial_m v^e(0)p(s). \tag{3.24}$$

Those constraints guarantee that marginal utility from consumption violating the budget constraint is lower than the marginal value of wealth on the borrowing constraint. Consequently, households never choose to consume more than that.[8]

The solution algorithm I implement relies on a finite difference method. Barles and Souganidis (1991) presented conditions for which a computational scheme using this method converges to the unique solution of HJB equations. In particular, I use an upwind scheme which in my setup consists in using a forward difference approximation of $\partial_m v^e(m)$ and $\partial_m v^w(m,z)$ for the positive drift of real money holdings and a backward difference approximation when the drift is negative. Achdou, Han, Lasry, and Lions (2017) have shown that a so called

[8]Notice that in the continuous-time version of the household problem the drift is smooth and the borrowing constraint is never binding in the interior of the state space. The only possible state in which the constraint can be violated is $m = 0$.

implicit version of the scheme always converges.[9]

The prices set by producers depend on decisions on consumption and search intensity of households and vice versa. In order to compute the numerical solution to the equilibrium allocation I use the following algorithm:

1. Start with an initial guess on value functions v^e, v^w, price statistics p^0, MPB and real lump-sum transfers τ.

2. Solve HJB equations (3.19), (3.20) using first-order conditions (3.23), where $\partial_m v^w(m,z)$ and $\partial_m v^e(m)$ are approximated using the upwind scheme and implicit method. Compute KF equations, (3.21) and (3.22), and equilibrium distribution of real money holdings.

3. Given new distribution of real money holdings and policy function compute $G(p)$ from (3.2) and sufficient statistics p^0, MPB and real lump-sum transfers τ from (3.18). Stop if changes in p^0, MPB, τ are small enough. Otherwise, go to step (2) with updated statistics p^0, MPB and real lump-sum transfers τ, but with initial guesses v^e, v^w.

B. Calibration

Having outlined the setup and the numerical implementation I am in the position to discipline the quantitative behavior of the model. In the procedure I chose four internal targets featuring the U.S. economy to calibrate values of five parameters.[10]

Functional specification. All working households exhibit the common utility function of the following form:

[9] The logic behind the implicit scheme is explained in great detail by Tourin (2013).

[10] One of the parameters, λ^e accounting for the probability of becoming an entrepreneur is standardized to generate mass of 1% of the richest households.

$$u(c) - h(s) = \log(c) - \eta_1 z^{\eta_2} \frac{1}{1-s}. \tag{3.25}$$

Then the flows of consumption and price search intensity that solves the problem of the household are as follows:

$$\frac{1}{c} = \partial_m v^e(m,z)p, \tag{3.26}$$

$$\eta_1 z^{\eta_2} \left(\frac{1}{1-s}\right)^2 = p^0 - sMPB. \tag{3.27}$$

Given the value function $v^e(m,z)$ (or more precisely the finite-difference approximation of its derivative) and the effective price p, the consumption can be pinned down analytically. For the chosen specification the price search intenstiy s, which one-to-one maps to p by (3.12), can be solved numerically for every level of the income and it does not depend on the money holdings. The optimal intensity decreases in z, which is consistent with empirical observations made by Aguiar and Hurst (2007). At the same time, (3.27) does not depend on the real money holdings. This choice is motivated by two things. First, to the best of my knowledge, there are no studies on the relationship between effective consumption prices and money holdings. Second, solving (3.27) requires implementing root-finding routines. Focusing only on the relationship between prices and income reduces the numerical complexity of the problem significantly.

The time discount rate $\rho = .0838$ was calibrated to match the relative ratio of real money holdings to GDP of .32. The model was parameterized to capture two targets corresponding to the product market, markups and the level of price dispersion. The former one was set to 30%. Admittedly, it is higher than the level of 10% from Basu and Fernald (1997), which is traditionally used in the New

Table 3.3: Internal targets

Target	Data	Model
Money supply	.32	.305
Markup	.30	.281
Corporate profits	.10	.112
Price dispersion	1.7	1.655

Keynesian literature. However, it is consistent with new microeconomic studies on this matter such as Faig and Jerez (2005), Stroebel and Vavra (2016), or very recent Anderson, Rebelo, and Wong (2017) and Loecker and Eeckhout (2017). Kaplan and Menzio (2016) using price scanner data documented that the ratio of the maximum to minimum prices varies between 1.7 and 2.6. Due to the fact that the model does not feature heterogeneity in marginal cost, which may amplify the price dispersion, I set this target to 1.7. The ratio of profits to the output was set to be equal to $.1$, the average ratio of corporate profits after tax to GDP in the US in the last ten years. The parameter λ^w was taken from Guvenen, Kaplan, and Song (2014), which is the probability of leaving top 1% in one year. Given λ^w, the parameter λ^e was set to generate the richest group of mass of 1%.[11] The labor income process is parameterized to match the annual autocorrelation $\rho_\varepsilon = .967$ and variance of and $\sigma_\varepsilon^2 = .017$ documented by Flodén and Lindé (2001).[12] Finally I set the inflation rate to be equal to $\pi = .02$.

V. Cost of Inflation: A Quantitative Analysis

In this section I use the calibrated version of the model to quantitatively evaluate the cost of inflation in the presence of the price search friction. In the

[11]In the stationary environment this can be pinned down from $\frac{\lambda^e}{\lambda^w+\lambda^e} = .01$.

[12]Using the Euler-Maryuama approximation those statistics can be mapped into the parameters characterizing the Ornstein-Uhlenbeck process, $\theta = e^{-\rho}$ and $\sigma^2 = \frac{\sigma_\varepsilon^2}{\theta}$.

Table 3.4: Calibrated parameters

Parameter	Value	Description
ρ	.0838	discount rate
R	1.5524	maximum price
η_1	.000475	relative utility
η_2	5.4594	curvature
λ^e	.00252	prob. of becoming entrepreneur
λ^w	.25	prob. of becoming worker

traditional literature of incomplete markets there is a tradeoff between equity and efficiency. The inflation leads to eroding the real value of assets and makes consumption smoothing through self-insurance more costly. On the other hand, the money injection, which generates inflation, finances lump-sum transfers and this provides an additional insurance, especially for cash-poor households. In the economy featuring frictions in the purchasing technology there exists an additional channel. Namely, thanks to money transfers the fraction of cash-poor households with higher price search increases. Consequently, in the line of the example from Section II the bargain hunters reduce the inefficiency generated by non-competitive pricing.

In the quantitative exercise I employ the methodology developed by Flodén (2001) and inspired by Bénabou (2002). This approach relying on certainty-equivalent consumption measures allows to decompose the welfare cost of inflation into two components, the cost of uncertainty and the cost of inequality. Let the utility of the benevolent social planner be over utility of households in the economy under regime k be as follows:

$$\mathcal{W}^k\left(c_t^k(\cdot), s_t^k(\cdot), g_k^e, g_k^w\right) := \frac{\lambda^e}{\lambda^e+\lambda^w}\int\left(\mathbb{E}\int_0^\infty e^{\rho t}\{u(c_t^k)-h(s_t^k)\}dt\right)g_k^e(m)dm+ \\ +\frac{\lambda^w}{\lambda^e+\lambda^w}\iint\left(\mathbb{E}\int_0^\infty e^{\rho t}\{u(c_t^k)-h(s_t^k)\}dt\right)g_k^w(m,z)dmdz. \tag{3.28}$$

The social welfare increases if consumption increases and price search decreases. Convex preferences (concave $u(c)$ and convex $h(s)$) implies that equity and certainty is preferred.

The overall cost of inflation can be calculated by solving for ω :

$$\mathcal{W}^{\pi=.02}\left((1+\omega)c_t^{\pi=.02}(\cdot), s_t^{\pi=.02}(\cdot), g_{\pi=.02}^e, g_{\pi=.02}^w\right) = \mathcal{W}^{\pi=0}\left(c_t^{\pi=0}(\cdot), s_t^{\pi=0}(\cdot), g_{\pi=.02}^e, g_{\pi=0}^w\right) \tag{3.29}$$

where ω is the cost in terms of consumption and is interpreted as the percentage amount of consumption in every state that the social planner is willing to give to sacrifice to fully eliminate the inflation. Given social welfare in both regimes, with the 2% inflation and without inflation the cost can be computed analytically:

$$\omega := \exp\left(\frac{\mathcal{W}^{\pi=0}}{\mathcal{W}^{\pi=.02}}\right)-1. \tag{3.30}$$

This cost can be further decomposed into two components stemming from different motives of the social planner, uncertainty aversion and inequality aversion. For further analysis it is convenient to define individual certainty-equivalent consumptions, $\overline{c}^w(m,z)$ and $\overline{c}^e(m)$, which are solutions to following equations:

$$\frac{\ln \overline{c}^w(m,z) - h(\mathcal{S})}{\rho} = \mathbb{E}\int_0^\infty e^{\rho t}\left(\ln c_t^w - h(s_t)\right), \tag{3.31}$$

$$\frac{\ln \overline{c}^e(m) - h(\mathcal{S})}{\rho} = \mathbb{E}\int_0^\infty e^{\rho t}\left(\ln c_t^e - h(s_t)\right), \tag{3.32}$$

where $\mathcal{S}$ is the average price search in the economy. These terms can be interpreted as consumption bundles that households in given states are willing to accept for eliminating future uncertainty.[13] Let $\mathcal{C}$ and $\overline{\mathcal{C}}$ be the average consumption and certainty-equivalent consumption in the economy:

$$\mathcal{C} := \frac{\lambda^w}{\lambda^e + \lambda^w}\iint c^w(m,z)g^w(m,z)dmdz \quad + \frac{\lambda^e}{\lambda^e + \lambda^w}\iint c^e(m)g^e(m)dm,$$
$$\overline{\mathcal{C}} := \frac{\lambda^w}{\lambda^e + \lambda^w}\iint \overline{c}^w(m,z)g^w(m,z)dmdz \quad + \frac{\lambda^e}{\lambda^e + \lambda^w}\iint \overline{c}^e(m)g^e(m)dm.$$

Then the social cost associated with fluctuations in the individual income of the households, p_{unc} can be expressed as the solution to the following equation:

$$\frac{\ln \overline{\mathcal{C}} - h(\mathcal{S})}{\rho} = \frac{\ln\left([1 - p_{unc}]\mathcal{C}\right) - h(\mathcal{S})}{\rho}. \tag{3.33}$$

This statistics measures how much the social planner is willing to sacrifice the *average* consumption to reduce the income uncertainty completely. Notice that this statistics is defined only over the average characteristics, thus it does not depend on the distribution amongst households.

The complementary statistics, p_{ineq} evaluates the social cost associated with

[13] There are many possible ways of determining price search in the deterministic allocation. In particular, Flodén (2001) suggests either fixing the value chosen in the uncertain economy for given states or setting the average level in the economy. I use the latter method.

inequality and solves the following equation:

$$\frac{\ln\left([1-p_{ineq}]\overline{\mathcal{C}}\right)-h(\mathcal{S})}{\rho}=\mathcal{W}^k\left(\overline{c}(\cdot),\mathcal{S},g^e,g^w\right). \tag{3.34}$$

This measure tells us how much the social planner is willing to reduce the average certainty-equivalent consumption to equalize the distribution of consumption completely. To compare allocations in both regimes, with inflation $\pi=.02$ and without inflation the following statistics are useful:

$$\omega_{unc}:=\frac{1-p_{unc}^{\pi=0}}{1-p_{unc}^{\pi=.02}}-1,$$
$$\omega_{ineq}:=\frac{1-p_{ineq}^{\pi=0}}{1-p_{ineq}^{\pi=.02}}-1,$$

where ω_{unc} is the welfare gain of reducing uncertainty and ω_{ineq} is the welfare gain of reducing inequality caused by eliminating inflation. Using the logic similar to Flodén (2001), the overall cost of inflation can be expressed in the following way:

$$\omega=(1+\omega_{unc})(1+\omega_{ineq})-1.$$

Having outlined the methodology I am in the position to present the results of the quantitative experiment. In this exercise I compare two allocations, with inflation $\pi=.02$ and without inflation. In addition to this to better understand the role of the purchasing friction I compare allocations for fixed price decisions of the households or the firms:

1. *full price adjustments* - households and firms play the best responses to actions of others. It is the baseline comparison between the stationary

Table 3.5: Welfare cost of inflation

Type of adjustment	ω_{unc}	ω_{ineq}	ω
Full adjustment	.0486	-.0051	.0434
Quoted prices only	.0450	-.0047	.0401
Price search only	.0502	-.0036	.0462
No adjustments	.0502	-.0036	.0462

equilibria with and without inlfation,

2. *no price adjustments* - the distribution of prices and price search policies are set to the optimal ones in the no-inflation equilibrium,

3. *adjustments in price search only* - households in both regimes play their best responses but the distribution of quoted prices is set to the no-inflation equilibrium,

4. *adjustments in quoted prices only*- firms play their best responses and set prices optimally while the price search intensity of households depend is fixed to policy functions constituting the no-inflation equilibrium.

Table 3.5 presents the results of the experiment. In the baseline scenario when I compare both equilibrium allocations the welfare gain of reducing inflation is equal to $.0434$. The welfare gains from inflation is lowest if households keep their price search intensities and only retailers change their pricing. This effect is generated by the fact that the composition of households with lower consumption and higher search intensity increases and this reduces inefficiency generated by retailers. Households that are able to adjust their price search intensities decrease their search which increases the welfare cost of inflation. The cost of the inflation is highest if retailers stick to their pricing strategy. For such an economy the redistributive price search channel is shut.

VI. Concluding Comments

In this paper I studied the redistributive function of monetary policy in the presence of frictions in the purchasing technology. Using the calibrated model I show that the welfare cost of inflation can be overstated when this channel is not accounted for. The bargain hunting reduces inefficiency generated by the non-competitive behavior of the firms. Thus, the redistributive policy targetting at households with high marginal propensity to consume and low reservation prices may be welfare improving.

The further work can be carried out in a few different directions. First, the friction can be incorporated into a richer environment with a more realistic structure of assets with different returns and liquidity as in Kaplan and Violante (2014) or Kaplan, Moll, and Violante (2017). Second, the model can be used to provide microfoundations for cyclicality of markups, consistent with very recent empirical findings due to Hall (2014), Nekarda and Ramey (2013), Stroebel and Vavra (2016), and Anderson, Rebelo, and Wong (2017). The cyclicality of markups in this framework is quite intuitive and stems from the fact that an increase in average individual earnings reduces the number of bargain hunters, which gives rise to higher market power of the firms and higher consumption prices. To study this phenomenon, aggregate uncertainty should be introduced to the model. To this end the methodology developed by Ahn, Kaplan, Moll, Winberry, and Wolf (2017) can be extremely useful.

Bibliography

Achdou, Y., J. Han, J.-M. Lasry, and P.-L. Lions (2017): "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach," .

Aguiar, M., and E. Hurst (2005): "Consumption versus Expenditure," *Journal of Political Economy*, 113(5), 919–948.

——— (2007): "Life-Cycle Prices and Production," *American Economic Review*, 97(5), 1533–1559.

Aguiar, M., E. Hurst, and L. Karabarbounis (2013): "Time Use during the Great Recession," *American Economic Review*, 103(5), 1664–96.

Ahn, S., G. Kaplan, B. Moll, T. Winberry, and C. Wolf (2017): "When Inequality Matters for Macro and Macro Matters for Inequality," in *NBER Macroeconomics Annual 2017, volume 32*. University of Chicago Press.

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, 109(3), 659–84.

——— (1995): "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting," *Journal of Political Economy*, 103(6), 1158–75.

Albrecht, J., F. Postel-Vinay, and S. Vroman (2015): "Burdett-Judd Redux," Discussion paper.

Anderson, E., S. Rebelo, and A. Wong (2017): "The Cyclicality of Gross Margins," Discussion paper.

Attanasio, O. P., and N. Pavoni (2011): "Risk Sharing in Private Information Models With Asset Accumulation: Explaining the Excess Smoothness of Consumption," *Econometrica*, 79(4), 1027–1068.

Attanasio, O. P., and G. Weber (2010): "Consumption and Saving: Models of Intertemporal Allocation and Their Implications for Public Policy," *Journal of Economic Literature*, 48(3), 693–751.

Auclert, A. (2016): "Monetary Policy and the Redistribution Channel," Discussion paper.

——— (2017): "Monetary Policy and the Redistribution Channel," Discussion paper.

Bai, Y., J.-V. Ríos-Rull, and K. Storesletten (2011): "Demand Shocks that Look Like Productivity Shocks," Discussion paper.

Bai, Y., J.-V. Ríos-Rull, and K. Storesletten (2017): "Demand Shocks that Look Like Productivity Shocks," Discussion paper.

Barles, G., and P. Souganidis (1991): "Convergence of approximation schemes for fully nonlinear second order equations.," *Asymptotic Anal.*, 4(3), 271–283.

Basu, S., and J. G. Fernald (1997): "Returns to Scale in U.S. Production: Estimates and Implications," *Journal of Political Economy*, 105(2), 249–283.

Berger, D., V. Guerrieri, G. Lorenzoni, and J. Vavra (2015): "House Prices and Consumer Spending," Working Paper 21667, National Bureau of Economic Research.

Bewley, T. (1986): "Stationary Monetary Equilibrium with a Continuum of Independently Fluctuating Consumers," in *Contributions to Mathematical Economics in Honor of Gerard Debreu*, ed. by W. Hildenbrand, and A. Mas-Collel. North-Holland, Amsterdam.

Billingsley, P. (1995): *Probability and Measure*. Wiley, 3 edn.

Blundell, R., L. Pistaferri, and I. Preston (2008): "Consumption Inequality and Partial Insurance," *American Economic Review*, 98(5), 1887–1921.

Blundell, R., L. Pistaferri, and I. Saporta-Eksten (2016): "Consumption Inequality and Family Labor Supply," *American Economic Review*, 106(2), 387–435.

Burdett, K., and K. L. Judd (1983): "Equilibrium Price Dispersion," *Econometrica*, 51(4), 955–69.

Burdett, K., and G. Menzio (2017a): "The (Q,S,s) Pricing Rule," *The Review of Economic Studies*, forthcoming.

——— (2017b): "The (Q,S,s) Pricing Rule: A Quantitative Analysis," *Research in Economics*, forthcoming.

Burdett, K., and D. T. Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39(2), 257–73.

Butters, G. R. (1977): "Equilibrium Distributions of Sales and Advertising Prices," *The Review of Economic Studies*, 44(3), 465–491.

Bénabou, R. (2002): "Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?," *Econometrica*, 70(2), 481–517.

Campbell, J. Y., and A. Deaton (1989): "Why Is Consumption So Smooth?," *Review of Economic Studies*, 56(3), 357–73.

Carroll, C. D. (1997): "Buffer-Stock Saving and the Life Cycle/Permanent Income Hypothesis," *The Quarterly Journal of Economics*, 112(1), 1–55.

Castañeda, A., J. Díaz-Giménez, and J.-V. Ríos-Rull (2003): "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111(4), 818–857.

Chatterjee, S., D. Corbae, M. Nakajima, and J.-V. Ríos-Rull (2007): "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, 75(6), 1525–1589.

Cooley, T. F., and E. C. Prescott (1995): "Economic Growth and Business Cycles," in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley, chap. 1, pp. 1–38. Princeton University Press, Princeton, New Jersey.

Cooper, R., and A. John (1988): "Coordinating Coordination Failures in Keynesian Models," *The Quarterly Journal of Economics*, 103(3), 441–463.

Diamond, P. A. (1971): "A model of price adjustment," *Journal of Economic Theory*, 3(2), 156–168.

Faig, M., and B. Jerez (2005): "A theory of commerce," *Journal of Economic Theory*, 122(1), 60–99.

Flodén, M. (2001): "The effectiveness of government debt and transfers as insurance," *Journal of Monetary Economics*, 48(1), 81–108.

Flodén, M., and J. Lindé (2001): "Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?," *Review of Economic Dynamics*, 4(2), 406–437.

Gourieroux, C., A. Monfort, and E. Renault (1993): "Indirect Inference," *Journal of Applied Econometrics*, 8(S), S85–118.

Guerrieri, V., and G. Lorenzoni (2015): "Credit Crises, Precautionary Savings, and the Liquidity Trap," Working paper.

Guvenen, F. (2011): "Macroeconomics with Heterogeneity: A Practical Guide," *Economic Quarterly*, 97(3), 255–327.

Guvenen, F., G. Kaplan, and J. Song (2014): "The Glass Ceiling and The Paper Floor: Gender Differences among Top Earners, 1981-2012," Working Paper 20560, National Bureau of Economic Research.

Guvenen, F., and A. A. Smith (2014): "Inferring Labor Income Risk and Partial Insurance From Economic Choices," *Econometrica*, 82(6), 2085–2129.

Hall, R. E. (2014): "What the Cyclical Response of Advertising Reveals about Markups and other Macroeconomic Wedges," Hoover Institution, Stanford University.

Hansen, G. D., and A. İmrohoroğlu (1992): "The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard," *Journal of Political Economy*, 100(1), 118–42.

Head, A., and A. Kumar (2005): "Price Dispersion, Inflation, and Welfare," *International Economic Review*, 46(2), 533–572.

Heathcote, J., K. Storesletten, and G. L. Violante (2009): "Quantitative Macroeconomics with Heterogeneous Households," *Annual Review of Economics*, 1(1), 319–354.

——— (2014): "Consumption and Labor Supply with Partial Insurance: An Analytical Framework," *American Economic Review*, 104(7), 2075–2126.

Hopkins, E., and R. M. Seymour (1996): "Price Dispersion: an Evolutionary Approach," Discussion paper.

——— (2002): "The Stability of Price Dispersion under Seller and Consumer Learning," *International Economic Review*, 43(4), 1157–1190.

Huggett, M. (1993): "The risk-free rate in heterogeneous-agent incomplete-insurance economies," *Journal of Economic Dynamics and Control*, 17(5-6), 953–969.

——— (1996): "Wealth distribution in life-cycle economies," *Journal of Monetary Economics*, 38(3), 469–494.

Huo, Z., and J.-V. Ríos-Rull (2013): "Paradox of Thrift Recessions," Discussion Paper 490, Federal Reserve Bank of Minneapolis Research Department Staff Report.

İmrohoroğlu, A. (1992): "The welfare cost of inflation under imperfect insurance," *Journal of Economic Dynamics and Control*, 16(1), 79–91.

İmrohoroğlu, A., S. İmrohoroğlu, and D. H. Joines (1995): "A Life Cycle Analysis of Social Security," *Economic Theory*, 6(1), 83–114.

Kaplan, G., and G. Menzio (2015): "The Morphology of Price Dispersion," *International Economic Review*.

——— (2016): "Shopping Externalities and Self-Fulfilling Unemployment Fluctuations," *Journal of Political Economy*, 3(124).

Kaplan, G., G. Menzio, L. Rudanko, and N. Trachter (2016): "Relative Price Dispersion: Evidence and Theory," .

Kaplan, G., B. Moll, and G. L. Violante (2016): "Monetary Policy According to HANK," Working Paper 21897, National Bureau of Economic Research.

——— (2017): "Monetary Policy According to HANK," Working paper.

Kaplan, G., and K. Pytka (2016): "General Equilibrium Theory of Shopping, Consumption, and Price Dispersion," .

Kaplan, G., and G. L. Violante (2010): "How Much Consumption Insurance beyond Self-Insurance?," *American Economic Journal: Macroeconomics*, 2(4), 53–87.

——— (2014): "A Model of the Consumption Response to Fiscal Stimulus Payments," *Econometrica*, 82(4), 1199–1239.

Krueger, A. B., and A. I. Mueller (2012): "The Lot Of The Unemployed: A Time Use Perspective," *Journal of the European Economic Association*, 10(4), 765–794.

Krueger, D., K. Mitman, and F. Perri (2016): "Macroeconomics and Household Heterogeneity," in *Handbook of Macroeconomics*, vol. 2.

Krusell, P., and A. A. Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), 867–896.

Lahkar, R. (2011): "The dynamic instability of dispersed price equilibria," *Journal of Economic Theory*, 146(5), 1796 – 1827.

Lippi, F., S. Ragni, and N. Trachter (2015): "Optimal monetary policy with heterogeneous money holdings," *Journal of Economic Theory*, 159(PA), 339–368.

Loecker, J. D., and J. Eeckhout (2017): "The Rise of Market Power and the Macroeconomic Implications," Discussion paper.

Matsuyama, K. (1999): "Playing Multiple Complementarity Games Simultaneously," Discussion Papers 1240, Northwestern University, Center for Mathematical Studies in Economics and Management Science.

——— (2002): "Explaining Diversity: Symmetry-Breaking in Complementarity Games," *American Economic Review*, 92(2), 241–246.

McCormick, G. (1976): "Computability of global solutions to factorable nonconvex programs: Part I — Convex underestimating problems," *Mathematical Programming*, 10(1), 147–175.

Moen, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105(2), 385–411.

Mortensen, D. (2003): *Wage Dispersion: Why Are Similar Workers Paid Differently?* The MIT Press.

Mortensen, D. T. (2005): "A comment on "Price dispersion, inflation, and welfare" by A. Head and A. Kumar," *International Economic Review*, 46(2), 573–578.

Mortensen, D. T., and C. A. Pissarides (1994): "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, 61(3), 397–415.

Nekarda, C. J., and V. A. Ramey (2013): "The Cyclical Behavior of the Price-Cost Markup," Working Paper 19099, National Bureau of Economic Research.

Petrosky-Nadeau, N., E. Wasmer, and S. Zeng (2016): "Shopping time," *Economics Letters*, 143(C), 52–60.

Prescott, E. C. (1975): "Efficiency of the Natural Rate," *Journal of Political Economy*, 83(6), 1229–36.

Pytka, K. (2016): "Lotteries in Equilibrium Price Dispersion," .

——— (2017): "Shopping Effort in Self-Insurance Economies," .

Rocheteau, G., P.-O. Weill, and T.-N. Wong (2015): "A Tractable Model of Monetary Exchange with Ex-post Heterogeneity," Working Paper 21179, National Bureau of Economic Research.

Rogerson, R. (1988): "Indivisible labor, lotteries and equilibrium," *Journal of Monetary Economics*, 21(1), 3–16.

Ríos-Rull, J.-V. (1996): "Life-Cycle Economies and Aggregate Fluctuations," *The Review of Economic Studies*, 63(3), 465–489.

Samuelson, P. (1947): *Foundations of Economic Analysis*. Harvard University Press, Cambridge, Mass.

Scheinkman, J. A., and L. Weiss (1986): "Borrowing Constraints and Aggregate Economic Activity," *Econometrica*, 54(1), 23–45.

Stroebel, J., and J. Vavra (2016): "House Prices, Local Demand, and Retail Prices," Working Paper 1-030, Kilts Booth Marketing series.

Tourin, A. (2013): *Introduction to Finite Differences Methods*pp. 201–212. Springer New York, New York, NY.

Uhlig, H. (1996): "A Law of Large Numbers for Large Economies," *Economic Theory*, 8(1), 41–50.

Wolinsky, A. (1986): "True Monopolistic Competition as a Result of Imperfect Information," *The Quarterly Journal of Economics*, 101(3), 493–511.

A

Appendix

I. Data

II. Proofs for Subsection IV.B

A. Proof of Theorem (5).

Proof Lemma (4) implies that every retailer yields the same level of profits and that the highest quoted price is equal to ζ. The profit of retailers charging ζ comes only from captive consumers. The probability that a visiting buyer is captive is equal to $\int \frac{m_t(x)(1+s_t(x))}{D}\left(1-\frac{2s_t(x)}{1+s_t(x)}\right)$. Then the profit is equal to:

$$S(\zeta)=\theta^{-\alpha}\sum_{t=1}^{T}\int \frac{\theta^{1-\alpha}m_t(x)(1+s_t(x))}{D}\left(1-\frac{2s_t(x)}{1+s_t(x)}\right)(\zeta-1)d\mu_t(x). \tag{A.1}$$

Table A.1: Deciles of weekly earnings

10%	20%	30%	40%	50%	60%	70%	80%	90%
250.00	360.00	461.53	570.00	675.00	807.69	961.53	1192.30	1538.46

Profits of retailers charging different prices p must be equal to $S(\zeta)$ (otherwise those prices were not be chosen):

$$theta^{-\alpha}\sum_{t=1}^{T}\int\frac{\theta^{1-\alpha}m_t(x)(1+s_t(x))}{D}\left(1-\frac{2s_t(x)}{1+s_t(x)}G(p)\right)(p-1)d\mu_t(x)=S(\zeta).$$

The distribution of quoted prices given by (3.2) is the unique dispersed equilibrium given household search strategies. ∎

III. Statistical Tables

IV. Proof of Lemma 10

$$\frac{\partial\overline{q}}{\partial\overline{p}}\bigg|_{q=\overline{q}}=-\frac{\partial_{\overline{p}}\partial_q\mathcal{W}(q)}{\partial_q^2\mathcal{W}(q)}\bigg|_{q=\overline{q}}.\tag{A.2}$$

The formula for the integral of the one-price lottery is:

$$\int pdF^q(p)=\frac{(1-q)((\overline{p}-1)\log(p-1)+p)}{2q},\tag{A.3}$$

and the expected price from one draw is given by:

$$\int_{\underline{p}}^{\overline{p}}pdF^q(p)=\frac{(q-1)(\overline{p}-1)\left((q+1)\log\left(\frac{(1-q)(\overline{p}-1)}{q+1}\right)-(q+1)\log(\overline{p}-1)+2q\right)}{2q(q+1)}.$$

Similarly, the formula for the integral of the two-price lottery is:

$$\iint\min\{p_1,p_2\}dF^q(p_1)dF^q(p_2)=\frac{(1-q)^2\left(-\frac{(\overline{p}-1)^2}{p-1}-2(\overline{p}-1)\log(p-1)+p\right)}{4q^2}$$

Table A.2: Statistical table for $G(p)$

				$\frac{\Psi_{(+)}}{D}$			
$G(p)$	.01	.1	.3	.5	.7	.9	.99
.99	1.000	.998	.991	.980	.958	.847	.337
.975	.999	.994	.981	.953	.892	.694	.166
.95	.999	.989	.958	.912	.809	.531	.095
.9	.998	.978	.921	.832	.684	.359	.045
.85	.997	.967	.888	.771	.592	.273	.035
.8	.996	.958	.855	.717	.518	.215	.025
.75	.995	.947	.823	.670	.459	.187	.025
.7	.994	.938	.795	.623	.418	.158	.015
.65	.993	.928	.772	.589	.376	.139	.015
.6	.992	.919	.744	.556	.351	.120	.015
.55	.991	.910	.720	.529	.326	.110	.015
.5	.990	.901	.702	.502	.301	.100	.015
.45	.989	.892	.678	.475	.276	.091	.015
.4	.988	.882	.660	.455	.260	.081	.015
.35	.987	.873	.641	.434	.251	.081	.015
.3	.986	.866	.627	.414	.235	.072	.005
.25	.985	.857	.608	.401	.226	.072	.005
.2	.984	.849	.594	.387	.210	.062	.005
.15	.983	.840	.580	.374	.201	.062	.005
.1	.982	.833	.566	.360	.193	.062	.005
.05	.981	.826	.552	.347	.185	.053	.005
.025	.981	.822	.543	.340	.176	.053	.005
.01	.980	.820	.543	.333	.176	.053	.005
$\underline{p}$	.980	.818	.538	.333	.176	.053	.005

Table A.3: Statistical table for $G(p)$

				$\frac{\Psi_{(+)}}{D}$			
$F(p)$	.01	.1	.3	.5	.7	.9	.99
.99	1.000	.998	.986	.960	.875	.464	.055
.975	.999	.994	.972	.912	.750	.321	.035
.95	.999	.987	.944	.845	.626	.225	.025
.9	.998	.976	.897	.744	.493	.167	.015
.85	.997	.965	.855	.677	.418	.139	.015
.8	.996	.954	.818	.623	.376	.120	.015
.75	.995	.943	.786	.576	.335	.100	.015
.7	.994	.934	.758	.542	.310	.100	.015
.65	.993	.923	.734	.515	.293	.091	.015
.6	.992	.914	.711	.488	.276	.081	.015
.55	.991	.904	.688	.468	.260	.081	.015
.5	.990	.895	.669	.448	.243	.072	.015
.45	.989	.886	.655	.428	.235	.072	.015
.4	.988	.879	.636	.414	.226	.072	.005
.35	.987	.870	.622	.401	.218	.062	.005
.3	.986	.862	.608	.387	.210	.062	.005
.25	.985	.855	.594	.380	.201	.062	.005
.2	.984	.848	.580	.367	.193	.062	.005
.15	.983	.838	.571	.360	.193	.062	.005
.1	.982	.833	.557	.347	.185	.053	.005
.05	.981	.826	.548	.340	.185	.053	.005
.025	.981	.822	.543	.340	.176	.053	.005
.01	.980	.820	.538	.333	.176	.053	.005
$\underline{p}$	.980	.818	.538	.333	.176	.053	.005

and the expected price from the minimum from two draws is given by:

$$\iint_{(p_1,p_2)\in[\underline{p},\overline{p}]^2} \min\{p_1,p_2\}dF^q(p_1)dF^q(p_2) = \frac{(1-q)(\overline{p}-1)}{2q^2(q+1)}\Big\{-\big(q^2-1\big)\log\left(-\frac{(q-1)(\overline{p}-1)}{q+1}\right) + \big(q^2-1\big)\log(\overline{p}-1)+2q\Big\}$$

The expected price difference is defined as:

$$\mathcal{W}(q) = \int_{\underline{p}}^{\overline{p}} p dF^q(p) - \iint_{(p_1,p_2)\in[\underline{p},\overline{p}]^2} \min\{p_1,p_2\}dF^q(p_1)dF^q(p_2).$$

The maximizer $\overline{q}$ of $\mathcal{W}(q)$ is the solution to:

$$\partial_q \mathcal{W}(q) = 0,$$

where:

$$\partial_q \mathcal{W}(q) = \frac{(\overline{p}-1)\left((-q^2+q+2)\log\left(\frac{(1-q)(\overline{p}-1)}{q+1}\right)+(q^2-q-2)\log(\overline{p}-1)+2q(q+2)\right)}{2q^3(q+1)}.$$

Next I make a guess that $\overline{q} \approx .6348158$

$$\partial_q \mathcal{W}(q)\Big|_{q=.6348158} = 1.306013887361414\times 10^{-7}(\overline{p}-1).$$

$$\partial_{\overline{p}}\partial_q \mathcal{W}(q)\Big|_{q=.6348158} = 1.306013887361414 \times 10^{-7}.$$

Table A.4: $\partial_q \mathcal{W}(q)$.

	q				
$\overline{p}$	.1	.25	.6348158	.75	.9
1.1	.027	.019	1.306×10^{-8}	−.009	−.034
10	2.447	1.744	1.175×10^{-6}	−.803	−3.034
42	11.149	7.944	5.354×10^{-6}	−3.656	−13.821
1986	539.781	384.624	2.592×10^{-4}	−177.02	−669.174

Table A.5: $\partial_{\overline{p}}\partial_q \mathcal{W}(q)$.

q	.1	.25	.6348158	.75	.9
$\partial_{\overline{p}}\partial_q \mathcal{W}(q)$	.272	.019	1.306×10^{-8}	−.089	−.339