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Hiring Quality Labour

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Hiring Quality Labour

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Abstract

This paper presents an overlapping generations matching model in which firms have no reliable information about an applicant's past employment history. Labour demand will then be a function of the perceived job quality of applicants and of the number of workers who quit and were fired. Workers when deciding to quit take account of their chances to be re-employed. This interconnection of hiring and quitting decisions can generate a type of sunspot equilibrium (here randomisations over perfect foresight paths) because of a quit/quality externality. However, even if such externalities are ignored because quitters have lagged or static expectations, deterministic cycles can occur. The results of this paper may support the view that the unemployed are 'stigmatised' by firms, in the sense that if there are fewer unemployed then they find it harder to get a job.

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1. Introduction

Firms like to hire workers who they believe are productive. When they do not have any information about an individual's productivity they might turn to the market for information of the likely quality of the average job applicant. Hiring, when an individual's characteristics are known, has received much attention (see for example Waldman [1984], Greenwald [1986] and Gibbons and Katz [1991]). Gibbons and Katz for example assume that workers can be distinguished by the reason they are in the unemployment pool, e.g. plant closure or lay off. Firms can then offer lower wages to laid off workers. There might however be many situations in which firms are unable to tell the quality of an applicant or in which firms only receive unreliable information about an applicant's employment history. The reason might be that quits are difficult to distinguish from lay-offs because employers write good reports for someone who they want to get rid of. Alternatively in many labour markets job applicants are able to lie about their past employment history or are not required to report on it .

This paper develops a matching model in which employers (firms, managers) are unable to distinguish workers; they only observe a job applicant; they do not observe his or her reason for being in the applicant pool. Additional assumptions are that wages are fixed and that workers differ in both ability and job enjoyment and that these two are uncorrelated. Labour demand will then be a function of the perceived quality of applicants and of the number of workers who quit and where fired. Workers when deciding to quit take account of their chances to be re-employed. This interconnection of hiring and quitting decisions can have interesting consequences. A type of sunspot equilibrium (randomisations over perfect foresight path) can occur because of quit externalities. However, even if such externalities are ignored because quitters have static or lagged expectations deterministic cycles can occur. In all of these cases job matches may not be optimal and, more interestingly, hiring may not be at its optimum so that involuntary unemployment may exist and be endogenous to the market.

These results therefore distinguish this paper from the work of Jovanovic [1979] and derivative papers such as Jovanovic [1984] and Mc Laughlin [1991]. These models imply efficient matches and turnover. In the model of this paper quits are always voluntary from a worker's point of view. Quits are however not necessarily good from an employer's point of view. Equally, lay-offs are often not optimal from an employees point of views, but they are always optimal from an employers point of view. In` the models of the papers cited above matches are dissolved when they become inefficient. In this model employers can only lay-off a restricted number of workers, and workers can quit whether this is good for employers or not. Quits in this model are generally lower than in a model in which workers do not take account of their re-employment chances, that is of aggregate hiring.

In Mc Laughlin [1991] the quit lay-off distinction arises out of a censoring of wage revisions. Initial asymmetries in information create private incentives for revising the wage. If a separation results, then the side initiating the wage revision determines the turnover label, that is, the distinction between quits and lay-offs. Independent of who initiates a separation, all separations are efficient. This model departs strongly from this way of looking at dissolutions of wage contracts; the primary distinction is that this model does assume that ability and job enjoyment are uncorrelated and that wages are fixed. As a result quits and lay-offs may not be optimal to both sides of a contract and wages can not compensate for job dissatisfaction (or job satisfaction).

The reason for the possibility of multiple equilibria in quitting decisions in this model is easy to explain: higher quits may cause higher hiring based on the rational belief by employers that higher unemployment is a signal of a higher average quality of the unemployed. More intensive hiring of the unemployed may therefore cause badly matched workers to quit who would otherwise not have quit because they would have evaluated their chances of finding another job as too low. Multiple perfect foresight equilibria therefore may exist due to a quit/ quality externality. This result may support a view that the unemployed may be 'stigmatised' by firms in the sense that if there are fewer unemployed then they might find it harder to get another job. This type of hiring behaviour may however be fully rational in this model since higher unemployment may be a signal of the fact that there are "many good workers in the streets"[12], that is of higher turnover.

The point that multiple equilibria might occur due to quitting externalities was suggested by Hall [1988] in his discussion of Akerlof, Rose and Yellen's [1988] idea that if jobs are rationed then workers may stick to jobs they do not like because the chances of finding another job are low. It is unsatisfactory that Akerlof, Rose and Yellen present this idea within the context of a model in which (involuntary) unemployment and reemployment chances (i.e. hiring behaviour) are determined exogenously; they are not determined within the model. Their analysis also misses the important point, that quits are not necessarily a function of unemployment only, but of the probability to be re-hired, which depends on unemployment, quits and the perceived quality of applicants. This paper (apart from developing a very different model to that of Akerlof, Rose and Yellen) goes well beyond Hall's suggestion that 'thick market' theory (Diamond [1982]) should be applied to quitting decisions. It demonstrates how quitters' beliefs about employment opportunities become important within a model in which hiring and quitting decisions are interconnected and employment opportunities are endogenous to the model. In such a relatively complicated context it is shown how such simple constraints as not being able to tell an applicant's quality creates a quit/quality externality, thus making the expectations of quitters important. Sunspot equilibria (here multiple perfect foresight paths) may as a consequence exist.

In the next section the model will be described. Section 3 will then analyse hiring demand with respect to various different assumptions about the expectations of workers and managers. Section 4 will comment on several variations of the basic modelling set-up. Section 5 will conclude.

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[1]For an alternative analysis of the relationship between stigma and quits see for example Kirman and Waldmann [1992]. Note however that their model is very different to the one developed in this paper

[2]Pissarides [1992] on the other hand shows that multiple equilibria are a possibility if workers loose some of their skills during unemployment. The mechanism to achieve this result is a thin market externality that reduces the supply of jobs when the duration of unemployment increases.

2. The Model

This section describes the basic structure of the model. Section 4 will comments on relaxing some of the central assumptions.

2.1: Structural assumptions

•The <u>time structure</u> of the model is as follows: At the beginning of each period workers can quit their job, and they can be fired by employers. At the beginning of a period, after observing the level of transitory unemployment[3] T, employers hire from the unemployment pool. It is be assumed that employers can only hire form the (transitory) unemployment pool. In reality, a significant number of job switches are of course not via the unemployment pool, but directly from job to job[4], but this is not what will be modelled.

$|_t \rightarrow$ quit and fire $\rightarrow T_t \rightarrow$ hire $\rightarrow$ production $\rightarrow |_{t+1}$

$|_{t+1} \rightarrow$ quit and fire $\rightarrow T_{t+1} \rightarrow$ hire $\rightarrow$ production

[3] T is an observable, public variable. A transitorily unemployed worker is someone who quit, was fired, was unable to get a job the period before or is a young worker entering the job market. Transitory unemployment is therefore the pool of workers who are looking/ applying for a job and who do not have another job. A transitorily unemployed worker need however not be unemployed during any production period since s/he may be hired as soon as s/he enters the unemployment pool T. Transitory unemployment is described in more detail in section 2.2 below.

[4]Akerlof, Rose and Yellen [1988] quote a statistic (page 514, footnote 16) which states that in a typical month in 1977 in the USA 3.3 million employed workers searched for a job compared with 6.3 million unemployed workers (all of whom by definition looked for work). The statistic is taken form Carl Rosenfeld "The extend of job search by employed workers" in the Department of Labour, Special Report Force Report 202 (GPO, 1977). They use this statistic to emphasise the importance of job to job quits. We interpret this statistic as emphasising the importance job switches via unemployment, even if the unemployment spell is only short.

The Sozio-Ökonomisches panel (a West German panel) as used by Winkelmann and Zimmermann [1992] for example shows that about half of job switches are job to job (less than one month of unemployment or direct job to job changes)

Although both statistics quoted demonstrate the importance of job to job switches they also emphasise the importance of job switches via unemployment.

In order to make the model tractable let us impose a two period overlapping generations structure. Any new generation enters the unemployment pool before (transitory) unemployment T is determined[5]. They may then be hired form the unemployment pool T_t. Those of the generation starting at t who were hired can at the beginning of period t+1 decide whether to quit or not. Quitting and firing decisions are taken independently. Those who quit then have to enter the unemployment pool which is composed of themselves,those who were fired, the new generation entering at t+1 and those who where not hired the period before. Before unemployment is determined those agents who entered the labour market in period 't-1' retire and those entering in period t+1 enter. Let us summarise this information for one generation:

$|_t \rightarrow T_t \rightarrow$ hire $\rightarrow$ produce $\rightarrow$ $\quad |_{t+1}$ $\quad \rightarrow T_{t+1} \rightarrow$ hire $\rightarrow$ produce $\rightarrow$ $\quad |_{t+2}$

$\rightarrow$ hired $\rightarrow$ produce $\rightarrow$ $\quad$ quit($\downarrow$)/fired($\downarrow$) $\quad \rightarrow$ produce $\rightarrow$

g_t enters T_t $\quad$ g_t retires

$\rightarrow$ not hired $\rightarrow$ unemployed $\rightarrow |_{t+1}$ $\quad \rightarrow T_{t+1} \rightarrow$ hired ($\Uparrow$) or unemployed

• Workers differ in ability and job enjoyment. Job enjoyment is match but not worker specific. Ability on the other hand is worker specific but not match specific. For simplicity it will be assumed that there are only two types: 'high' and 'low' ability workers. It is assumed that ability and job enjoyment are uncorrelated, that is, it is assumed that when a worker has high ability then s/he is good at all jobs, however s/he prefers some jobs to others. Equally, low ability workers are bad at all jobs but still prefer some jobs to others. An employer may therefore want a worker to stay but the worker does not like his or her job. Alternatively a worker may like his or her job but be undesirable. In terms of notation:

	productivity at all firms	job enjoyment at job k for wage w
worker (i)	x_i	e_{ik}

[5] The analysis would not change if one assumed that each period a fixed proportion of the young entered employment directly. Also see section 4 for further comments.

and e_{ik} is assumed to be a continuous variable and $x_i \in \{0,1\}$. In terms of enjoyment there is therefore a horizontal matching problem. Assume furthermore that unemployed workers are perfectly allocated to posted vacancies. Agents only get to know their job enjoyment after taking up a job. It is always worthwhile for an unemployed worker to take a job and for an employed worker to stick to his or her job when not expecting to get another one, that is, I assume that for all i and k, $w \geq -e_{ik}$, and that for an unemployed worker $w=0$ and $e_{ik}=0$. Unemployment during production periods is therefore involuntary. Should there be an excess demand of labour then labour is evenly distributed to announced vacancies. It will also be assumed that each generation has the same number of good and bad workers.

• Information structure: Workers do not know their job enjoyment before having worked in a job. Firms when hiring do not know any of the private characteristics of a job applicant, that is they neither know the age, ability or past job enjoyment of a worker nor do they know whether the worker quit or was fired (should s/he have had a job before). Instead of saying that employers do not know this information one can also assume that they do not want to or are not allowed to base their decisions on this information. That employers do not know somebody's age is an assumption that does not make much difference (see section 4). The other assumptions are of importance however[6] and do not, as a matter of fact, seem that implausible. Firing is often covered up as quits by both employer and employee (for various reasons), so that the distinction between them may be blurred. Further, a firm's hiring and firing policy is to some extend controlled and constrained by unions. In a big labour market it may also be either too costly or simply impossible to find out about a worker's past employment history, and it may be fairly easy for applicants to claim that they got laid off due to some reason unrelated to competence.

• Wages: For simplicity it is assumed that all workers receive the same wage w whatever the job or the quality of the worker, and independent

[6]This assumption is one of the main features which differentiates this paper from the asymmetric information literature ((see for example Waldman [1984], Greenwald [1986] and Gibbons and Katz [1991]), since in this literature some of the private characteristics of a worker such as his or her past wage or job assignment must be observable.

of whether there is unemployment or not. Naturally this is a stringent assumption. It would seem preferable to pay better workers more than worse workers. On the other hand a firm has the incentive to pay everybody the same low wage if it can get away with it.

It seems a fair generalisation that in most economies it is generally the case that labour demand is lower than labour supply (this of course ignores markets in specialised labour). Once one assumes (as I do) that firms must pay the same wage rate to everybody (they might have to do so because of union pressure) then one can easily explain that wages may be rigid downwards in times of unemployment. The reason may be that firms only hire applicants at very low wages since they have lower ability on average. The employed workforce on average has higher ability than the unemployed workforce so that the former can threaten to strike should firms try to implement lower wages. In this model therefore quality and not hiring and firing restrictions create 'insider power'. That wages are rigid upwards when labour demand is higher than labour supply seems slightly less plausible. One can however specify the model so that in Nash equilibrium this case never occurs.

• Since I want to concentrate on the hiring decisions of firms the <u>market structure</u> will be kept as simple as possible. In terms of assumptions this is supposed to mean that workers can apply for any job and have an equal chance to fill it; firms take input and output prices as given; there is a small number of firms and a large number of workers. The assumption of fixed wages is particularly appropriate for the partial labour market of low paid jobs. What I have in mind is, for example, the market for 'unskilled' labour in Germany, where metal workers can become miners, where the wages of metal workers are more or less the same as those of miners, and where the price of steel and coal are more or less fixed by world markets .

•<u>Magnitudes</u>: It will be assumed that at any period of time there are 2 generations of G workers each in the economy (G young workers and G old workers). Say that 2G is a measurable interval which is normalised to 1 (i.e. 2G=1). Assume a finite[7] number F of identical firms who

[7] In section 3.3 it will be assumed that F is very large.

maximise profits with respect to hiring a proportion of the available workers. Firms therefore hire fractions of an interval.

It is slightly counter-intuitive to define profit, technologies and costs over proportions of the workforce rather than over the number of workers employed. Since it was assumed that the size of the population remains constant this makes little difference however.

• Notation: Although the notation will be introduced and defined later in the text, here is a quick reference: let π^f_t be the profit to firm f at time t and let l_t be the proportion of the total workforce (2G) employed at firm f at time t. Let h^f_t be the proportion of workers firm f wants to hire at time t and let Q^f_t, r^f_t and M^f_t be the proportion of the total workforce who quit, retired and were fired at firm f at time t respectively. T_t denotes the proportion of total workforce (transitorily) unemployed (before hiring) at time t. The variable q_t denotes the probability that any non-retiring employed worker quits.

2.2: When to quit

The only decision workers have to take is whether to quit. At the beginning of each period agents who are employed can either quit and apply for a new job or stay at the same job as before; they might also be fired. Unemployed workers and agents who will retire do not face a decision problem. Firms only fire low ability workers. Low ability workers face a probability 'p_m' of not being fired. Good workers are never fired (for them $p_m=1$). The probability of being fired depends thus on the type, i.e. $p_m(x_i)$ (where the type is either a good or a bad worker). The job enjoyment level workers can expect in other jobs is the mean enjoyment level over all jobs $E(e_{ip})$. Assume that there are enough different jobs so that somebody's present job does not influence the mean $E_p(e_{ip})$ (i.e. the number of different jobs is 'large'). The expected return of quitting depends on the expected hiring intensity P_t which depends on how many workers will quit and on how many of the unemployed workers firms hire. Call the probability of being re-employed for someone in the applicant pool at time t 'P_t' (the hiring intensity at time t). Define

$$P_t \quad =(H_t/T_t) \quad \text{if } H_t<T_t \qquad (1)$$
$$\quad = 1 \quad \text{if otherwise} \qquad (1)$$

where H_t is aggregate fraction of the total workforce firms want to hire (i.e., aggregate hiring demand) and T_t is the 'transitory unemployment' rate at time t which is composed of the aggregate fraction of quits (Q_t), of lay-offs (M_t), of new workers G and of the unemployed from the previous period ($(1-P_{t-1})G$):

$$T_t \quad \equiv G+ Q_t+M_t-q_tM_t+(1-P_{t-1})G$$
$$\quad = 1+ Q_t+M_t-q_tM_t-{}^1/_2P_{t-1} \qquad (2)$$

remembering that $G={}^1/_2$ and taking account of the fact that quitting and lay off decisions are taken independently so that one has to subtract q_tM_t from Q_t and M_t. Transitory unemployment T_t is thus the fraction of workers looking for work in period t after quits, lay-offs and retirements occurred and before firms hire. It therefore is the pool of workers who are applying for a job and who do not yet have another job. Note that a transitorily unemployed worker need not be unemployed during any production period since hiring in this model only takes place between production periods. The variables H_t, Q_t, q_t and M_t shall be derived below.

Since workers take their quitting decision before unemployment is observed and before firms hire, one has to make an assumption on the way workers form their expectations. In this paper I shall only consider two different beliefs/ expectations assumptions:

<u>Assumption (Static expectations)</u>: $P^e_t=P_{t-1}$ (3)

<u>Assumption (Perfect foresight)</u>: $P^e_t=(H_t/T_t)^e= H_t/T_t$ (4)

With static or lagged expectations workers believe that their chances of being re-hired are the same as they were the period before (i.e. when they entered the labour market).

For workers to have perfect foresight one requires that they forecast both H_t and T_t. In section 3.4, below, it will be shown that T_t

can be expressed as a function of H_t and P_{t-1} only so that P_t will be a function of H_t and P_{t-1}. Firms on the other hand will be able to determine hiring demand on the basis of quits q_t and P_{t-1}. One then needs to solve for a perfect foresight Nash equilibrium in q_t and H_t and in property 2 it will be shown that such a symmetric Nash equilibrium will always exist (for the cases I shall consider).

Firing and quitting decisions are taken independently. Workers therefore have to take their quitting decisions uncertain of whether they will be fired, which seems plausible. This requires that workers take their quitting decision on the basis of whether their expected utility from staying at the same job is lower than their expected utility from quitting. Now let person i's utility at job k be $w+e_{ik}$ and let Ee_{ip} be the expected job enjoyment to worker i of any job p. Worker i will then quit his or her job at firm f if the following inequality holds:

$$P^e_t(w+Ee_{ip})\geq$$

$$\geq p_m(x_i)(w+e_{if})+(1-p_m(x_i)(P^e_t(w+Ee_{ip}))+(1-p_m(x_i)(1-P^e_t)\bullet 0))$$

Noting that $p_m(x_i)>0$ one can then define the following 'quit' function:

$$q_i(e_{ik}, P^e_t) = 1 \quad \text{if} \quad [P^e_t(w+Ee_{ip})-(w+e_{ik})]\geq 0 \qquad (5)$$
$$= 0 \quad \text{otherwise} \qquad (5)$$

It can be seen from equation (5) that one's type does not influence ones quitting decision, since job enjoyment and ability are uncorrelated. Note furthermore that the specification I adopted implies that all workers applying for a job have the same chances of being re-hired, independent of whether they have been fired, of whether they were unemployed, of whether they quit or of whether they are young workers entering the unemployment pool. This is of course not entirely realistic, but simplifies the model tremendously. I shall now assume[8] that job

[8] This assumption leads to the easiest functional form by far. Economically it is not that implausible since it allows that workers like or dislike their jobs and that the (dis)utility of work (including the wage) always outweighs the disutility of being unemployed. It also implies that nobody would ever quit his or her job when the probability of getting another job is zero.

enjoyment is distributed uniformly on the interval [-w,+w]. It then follows that

$$\mathrm{Prob}(e_{ik} \leq x) = [(x+w)/2w] \equiv \Phi(x) \tag{6}$$

so that $\Phi(P^e_t(w+Ee_{ip})-w) = (([P^e_t(w+Ee_{ip})-w]+w)/2w) = {}^1/_2 P_t^e$. The probability of quitting for any non-retiring and employed worker then is.

$$q_t = {}^1/_2 P_t^e \tag{7}$$

To see what is going on intuitively note that at the most half of the non-retiring employed workers can quit, so if $P_t^e=1$ then $q_t={}^1/_2$. Equally if $P_t^e=0$ then nobody wants to quit, so that $q_t=0$. It follows that $q:[0,1]\rightarrow[0,{}^1/_2]$.

Now, in this overlapping generations model $G={}^1/_2$ workers enter the working population at the beginning of each period and at any period there are only 2G workers in the economy. It must therefore be the case that $({}^1/_2/T_{t-1})$ of the workers hired per firm (from the applicant pool) are young workers, that is that the number of non-retiring employed wokers per firm $(l^f_{t-1}-r^f_{t-1})$ equals ${}^1/_2(h^f_{t-1}/T_{t-1})$. It follows that the fraction of workers per firm who quit in period t is

$$Q^f_t = {}^1/_2 q_t (h^f_{t-1}/T_{t-1}) \tag{8}$$

where h^f_t is endogenous to the model and will be solved for in the next section.

In the above model workers quit because they dislike their job a lot and because they evaluate their chances of being re-hired as high. Since it was assumed that a worker's productivity is independent of his or her job enjoyment it follows that a worker's quitting decision is independent of his or her productivity- this can be seen in equation (5). However quitting and firing decisions overlap so that a proportion of the workers who are fired also quit simultaneously.

2.3: Firing and Hiring

Let us now pin down the firm's decision problem. Firms are identical and 'large'. They are managed by well matched and high ability workers belonging to the older generation. These workers shall be called managers. Managers cannot tell the quality of an applicant, nor can they tell his or her entry date into the labour market. They therefore only observe an unemployed worker, but not his or her age[9], ability or job enjoyment. Managers can however tell the quality (but not the job enjoyment) of their employed workforce. Firms can fire workers after production has occurred and before hiring, on the basis of this observation.

Firms have two decisions to take, to hire and to fire. They maximise profits which depend on the number of workers employed l_t^f and on their average quality/ability. The number of employed workers l_t^f equals the number of workers newly hired h_t^f and the workers which remain from the previous period rem^f_{t-1}, i.e. those workers who neither retired, nor quit nor were fired. The average quality of someone newly hired is the same as that of someone in the applicant pool T_t (since hiring in this model only goes on from the applicant pool). The average quality of those in h_t^f and in rem_{t-1}^f will therefore be denoted by a^T_t and a^r_t respectively.

Managers observe the performance of their workers during production (or at the end of it) and so can identify their quality (but not their job enjoyment). If they could, they would therefore throw out all their bad (non-retiring) workers (since the workers with low ability have zero productivity). However, this would in general be clearly unrealistic[10]; firms often face rather stringent government[11] or union

[9] This assumption can be relaxed, see section 4.
[10] Nonetheless, this case will briefly be discussed in section 4.
[11] Emerson [1988] contains a brief review of firing restrictions in Europe, the US and Japan. In several European countries such as Germany, Sweden and Italy work councils or trade unions have to be consulted before someone is to be dismissed. In Germany, if a work council does not consent a dismissal then the dismissed worker can resort to the courts. One could imagine that work councils and unions are also guided by the number of dismissals brought before it. This would be one way of justifying assumption M.

Alternatively, one can interpret this firing restriction as being the consequence of imperfect monitoring; the bad performance of low ability workers restrictions which limit the number of workers they can fire. A reasonable way to model these restrictions is to assume that firms can at the most lay off a fixed proportion of their non-retiring workers ($l^f_{t-1} - r^f_{t-1}$) each period. I shall thus assume that:

Assumption M $\quad M^f_t \leq \alpha(l^f_{t-1} - r^f_{t-1})$ for some fixed, small α (9)

Since low quality workers are assumed to have zero productivity this constraint must always be binding. In this model firing is thus taken to be a very different variable to hiring, since only bad workers are fired, whereas hiring goes on from a pool of unemployed workers. This formulation of firing restrictions (which ignores adjustment costs) is briefly discussed in section 4.

Firms then, in effect, only have one decision to take: to hire. They therefore maximise:

$$\pi^f_t \quad = \pi(h^f_t; rem^f_{t-1}; a^T_t, a^r_t) \qquad (10)$$

subject to: $h^f_t \in [0; T_t - \Sigma_{-f}(h^j_t)]$ (11)

In order to solve this maximisation problem firms need to determine rem^f_{t-1} a^T_t and a^r_t. It is the purpose of the remaining subsections to do so.

2.3.1: Determining the size of the Workforce:

When determining their profits firms need to know the size and quality of their workforce. Their workforce in any production period is composed of two groups of workers, those newly hired (h^f_t) and those that remained from the previous period (rem^f_{t-1}), that is those that did not quit, nor retire, nor were fired.

The number of non-retiring workers per firm ($l^f_{t-1} - r^f_{t-1}$) is identical in this model to the number of young workers hired by the firm in period t-1. Since $G = 1/2$ workers enter the working population each period and in every period there are only 2G workers in the economy it must be the

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is then observed with probability ($1-p_m$) and high ability workers never perform badly, which means that they are never fired.

case that $({}^1/_2/T_{t-1})$ of the workers hired per firm are young workers (since hiring only goes on from the applicant pool T_{t-1}). The number of non-retiring wokers per firm $(l^f_{t-1}-r^f_{t-1})$ thus equals ${}^1/_2(h^f_{t-1}/T_{t-1})$. The number of workers remaining from the previous period is then:

$$\begin{aligned} rem^f_{t-1} &\equiv l^f_{t-1}-r^f_{t-1}-Q^f_t-M^f_t+q_tM^f_t \\ &= {}^1/_2(h^f_{t-1}/T_{t-1}) - {}^1/_2q_t(h^f_{t-1}/T_{t-1}) - {}^1/_2\alpha(1-q_t)(h^f_{t-1}/T_{t-1}) \\ &= {}^1/_2(1-q_t)(1-\alpha)(h^f_{t-1}/T_{t-1}) \end{aligned} \tag{12}$$

where the second line follows directly from the definitions of $(l^f_{t-1}-r^f_{t-1})$; Q^f_t and M^f_t.

Most cases that will be analysed below will assume that a symmetric Nash equilibrium was played in period t-1, i.e. all firms took identical hiring decisions in period t-1. In this case the fraction of non-retiring workers is identical to (in this two period overlapping generations framework) the hiring intensity (or probability of a worker in T_{t-1} being hired) times the population entering the labour force in period t-1 which equals a half, i.e.

$$(l^f_{t-1}-r^f_{t-1}) = {}^1/_2(h^f_{t-1}/T_{t-1}) = {}^1/_2(P_{t-1}/F) \tag{13}$$

where F is the number of firms.

2.3.2: Quality

All employed workers took their quitting decisions independent of their type; $Q^f_t = {}^1/_2q_t(P_t^e)(h^f_{t-1}/T_{t-1})$ nevertheless is a biased sample since firing and quitting decisions overlap (i.e. they are taken independently). Note however that one can also consider ${}^1/_2q_t(P_t^e)(h^f_{t-1}/T_{t-1})$ an unbiased sample and assume that effective firing only amounted to

$$M^f_t{}^* \equiv M^f_t - M^f_t q_t = {}^1/_2(1-q_t)\alpha(h^f_{t-1}/T_{t-1}) \tag{14}$$

bad workers fired (i.e. those that did not quit as well). The non-retireing workers have average ability of a half, quitters have average ability and those effectively fired have zero ability. Given this way of looking at layoffs it is easy to determine the average quality of those hired and of those remaining from the previous period at any firm.

The average quality of the workers in rem^f_{t-1} (those who neither quit, nor were layed off, nor retired) equals the proportion of good workers in rem^f_{t-1} (w.r.t. 2G) divided by the proportion of workers in rem^f_{t-1} (w.r.t. 2G). Recall that the productivity x_i of low and high ability workers is 0 and 1 respectively and that the population consists of an equal number of good and bad workers. Half of those workers who remained from last period and were not effectively layed off, have high ability (i.e. half of $(1-q_t)(h^f_{t-1}/T_{t-1})$ have zero productivity). It then follows that the average ability of someone remaining from the previous period equals:

$$a^r_t \quad =[(1/2)(1/2)(1-q_t)(h^f_{t-1}/T_{t-1})]/[1/2(1-q_t)(1-\alpha)(h^f_{t-1}/T_{t-1})]$$
$$=1/2(1/(1-\alpha)) \qquad (15)$$

which is constant and $a^r_t=a^r \in (1/2,1)$ if $\alpha \in (0,1/2)$, which should be the case.

The average ability of those newly hired is equally easy to define. It equals the proportion of good workers in T_t (w.r.t. 2G) divided by the proportion of workers in T_t (w.r.t. 2G). Now, note that the ability of the workers in M^*_t is zero and that half of those in $(T-M_t^*)$ must be of high ability and the other half of low ability. The reason is that $(T-M_t^*)$ equals $(1+Q_t+1/2P_{t-1})$, all of which have average ability. The average ability 'a^T_t' of those in T_t (that is of those newly hired) then is:

$$a^T_t= \ a^T(T_t, q_t, P_{t-1})=[0\bullet M_t^* + 1\bullet 1/2 \bullet(T-M_t^*) +0\bullet 1/2\ (T-M_t^*)]/T$$

$$=1/2 -1/2\ (M_t^*/T_t)$$
$$=1/2 -1/2\ ((1/2(1-q_t)\alpha P_{t-1})/T_t) \qquad (16)$$

One can therefore see that the average ability of someone in rem^f_{t-1} is always higher than that of someone in T_t. Firms therefore prefer few workers to quit. This then completes the basic set-up of the model

3. Hiring Quality Labour

In this section I shall discuss the dynamics of hiring demand with respect to different assumptions about how far sighted managers and workers are. Section 3.1 proves that if workers have static expectations then the quality of the unemployed is a function of the past hiring intensity P_{t-1} only and that the quality of the unemployed is decreasing in P_t. Sections 3.2 and 3.3 contain examples in which managers are 'myopic' and 'far sighted' respectively and in which workers have static expectations. In section 3.2 it will be shown that in the simple situation in which firms are myopic and workers have lagged expectations hiring can proceed along a deterministic cycle. Section 3.3 discusses some of the problems that arise once managers are more 'far sighted'. Section 3.4 proves that if workers have perfect foresight and if managers are 'myopic' then a symmetric perfect foresight equilibrium always exists. Section 3.5 will give an example of this case and demonstrate the possibility of multiple perfect foresight equilibria. In all sections, except section 3.3, I will only consider symmetric Nash equilibria (that is, I will assume that $(h^f_{t-1}/T_{t-1}) = (P_{t-1}/F)$).

3.1: Workers with static expectations

The case in which workers have static expectations ($P_t^e = P_{t-1}$) is fairly easy to analyse as will be shown in this section. In sections 3.2 and 3.3 examples will then be given which will illustrate the consequences of static expectations by workers.

<u>Property 1</u>: Assume that a symmetric equilibrium was played in period t-1 and that workers have static expectations, then:

a) P_{t-1} alone determines the average quality of the unemployed in T_t, and $\partial T_t/\partial P_{t-1} < 0$ for $P_{t-1} \in (0,1)$.

b) The quality of the unemployed is a decreasing function of P_{t-1}.

<u>Proof</u>: a) The transitory unemployment rate was defined in equation 2:

$$T_t \quad = 1 + Q_t + M_t - q_t M_t - {}^1\!/\!_2 P_{t-1}$$
$$= 1 - {}^1\!/\!_2(1-\alpha)(1-q_t)P_{t-1}$$

Static expectations of workers ($P_t^e = P_{t-1}$) imply that $q_t = {}^1\!/\!_2 P_{t-1}$ so that $T_t = 1 - {}^1\!/\!_2(1-\alpha)(1 - {}^1\!/\!_2 P_{t-1})P_{t-1} \equiv T(P_{t-1})$ (P1)

Differentiating T_t with respect to P_{t-1} yields:

$\partial T_t/\partial P_{t-1}=-{}^1/_2(1-\alpha)(1-P_{t-1})<0.$

b)The average ability of those in the applicant pool T_t is

$a^T(T_t, q_t,P_{t-1})={}^1/_2 -{}^1/_2 (({}^1/_2(1- q_t)\alpha P_{t-1})/T_t)$

Part (a) of the proof implies that $T_t=T(P_{t-1})$ and static expectations imply that $q_t={}^1/_2P_{t-1}$. It follows that the ability of the unemployed

$a^T(T_t, q_t,P_{t-1})=a(T(q(P_{t-1}),P_{t-1}), q(P_{t-1}),P_{t-1}) \equiv a^{T*}(P_{t-1})$ (P2)

is a function of the the past hiring intensity P_{t-1} only. Now

$(\partial a^{T*}{}_t/\partial P_{t-1})=-{}^1/_2[{}^1/_2\alpha(1-P_{t-1})T_t - {}^1/_2(1-q_t)\alpha P_{t-1}(\partial T_t/\partial P_{t-1})]$

so that $a^{T*}{}'(P_{t-1})<0$ for $P_{t-1}\in(0,1)$. Q.E.D.

Observe that the maximisation problem of firms becomes much simpler in the case in which the expectations of workers are static and a symmetric Nash equilibrium was played in period t-1. In section 2.3.2 it was shown that the quality of the employed ($rem^f{}_{t-1}$) is constant and in property 1 it was shown that the quality of those in T_t is a function of P_{t-1} only, i.e. $T_t=T(P_{t-1})$. Note furthermore that $rem^f{}_{t-1}=(1-T_t)/F$. It follows that the quality of those employed ($rem^f{}_{t-1}$ and $h^f{}_t$), is determined by P_{t-1} and $h^f{}_t$ only. The two variables which determine profit in the case in which workers have static expectations are therefore $h^f{}_t$ and P_{t-1}, i.e.

$$\begin{aligned}\pi^f{}_t &= \pi(h^f{}_t;\ rem^f{}_{t-1};a^T{}_t,\ a^r{}_t)\\ &= \pi(h^f{}_t;\ (1-T(P_{t-1}))/F;\ a^{T*}(P_{t-1}),\ a^r{}_t)\\ &= \pi^*(h^f{}_t;\ P_{t-1}) \qquad (17)\end{aligned}$$

subject to: $h^f{}_t\in[0;\ T(P_{t-1})-\Sigma_{-f}(h^j{}_t)]$ (18)

where $T(P_{t-1})$ and $a^{T*}(P_{t-1})$ were defined in equation (P1) and (P2) above.

3.2 ...who are hired by 'myopic' managers

Consider the bench-mark case in which managers are 'myopic'. 'Myopic' managers maximise one period decision problems.They therefore hire without taking into account the effect their hiring decisions have on future demand and quality. Managers therefore do not care about anything beyond their retirement. In section 3.3 'far sighted' managers shall be considered

Consider the following profit function :

$$\pi_t \quad =(rem^f_{t-1}a^r+a^T_t h^f_t)^{\gamma}-w(rem^f_{t-1}+h^f_t) \qquad (19)$$

which equals

$$\pi_t \quad =(^1/_4\ (1-q_t)P_{t-1}/F+a^{T*}(P_{t-1})h^f_t)^{\gamma}-w(rem^f_{t-1}+h^f_t) \qquad (19')$$

given the definitions of T_t, rem^f_{t-1} $a^{T*}{}_t$ and $a^r{}_t$ and the assumption that a symmetric equilibrium was played in period t-1. In order to understand the expression $[^1/_4\ (1-q_t)P_{t-1}/F]$ in economic terms note that the fraction of non retiring workers hired per firm (if a symmetric equilibrium was played the period before) is $^1/_2P_{t-1}/F$. Half of these are of high ability and q_t of these are expected to quit.

<u>Constant returns</u>:
The unconstrained first order condition in the case of constant returns to average efficiency labour units (set γ=1 in equation (19')) is

$$\partial\pi(.)/\partial h^f_t= a^{T*}(P_{t-1}) - w \qquad (20)$$

In the case in which every firm wants to hire the maximum amount possible labour is allocated symmetrically. A feasible symmetric Nash equilibrium then consists in hiring T_t/F in the case of $a^T(P_{t-1}) > w$, i.e.:

$$h^*(P_{t-1}) \quad \begin{cases} = T_t/F & \text{if } a^{T*}(P_{t-1}) > w \\ \in [0,T_t/F] & \text{if } a^{T*}(P_{t-1}) = w \\ = 0 & \text{if } a^{T*}(P_{t-1}) < w \end{cases} \qquad (21)$$

Since $a^{T*\prime}(P_{t-1})<0$ one can re-express the constraint $a^{T*}(P_{t-1})=w$ in terms of a function $P_{t-1} = p(w)$. The solution in equation (21) can be rewritten as follows

$$P_t \quad \begin{cases} = 1 \quad \text{if} & \text{if } P_{t-1} < p(w) \\ \in [0,1] & \text{if } P_{t-1} = p(w) \\ = 0 & \text{if } P_{t-1} > p(w) \end{cases} \qquad (21)$$

The graphical solution of constant returns to scale is depicted in figure 1 below. As can be seen there is one solution in which the hiring intensity P_t will be constant and and another solution in which the hiring intensity cycles between 0 and one. Which of the two solutions will prevail depends on both initial values and to which solution hiring intensity will tend. In the case of constant returns to scale, unless the case of $a^{T*}(P_{t-1})= w$ occurs, hiring intensity will cycle between zero and one.

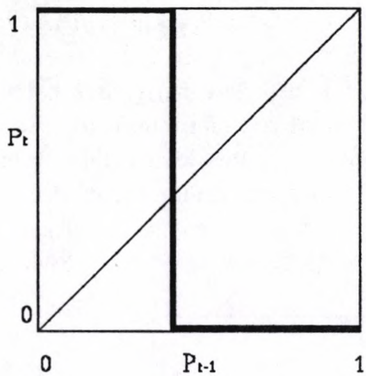

Figure 1

Intuitively cycles occur because firms hire all workers when unemployment is high; this leads to lower quality of the unemployed the following period (property 1) because firms can fire many workers and no un-hired workers remain (and this even though quits will be high due to static expectations). It follows that that firms hire fewer workers in the following period.

<u>Decreasing returns</u> :
The unconstrained hiring function for decreasing returns is given by

$$h_t^f=[w/\gamma]^{1/(\gamma-1)}\,[1/a^{T*}_t]^{\gamma/(\gamma-1)} - 1/4(1-q_t)P_{t-1}/Fa^{T*}_t \quad (22)$$

so that the Nash equilibrium in constrained hiring takes the following form:

$$h^*_t \begin{cases} = T_t/F & \text{if } h^f_t > T_t/F \\ = h^f_t & \text{if } h^f_t \in [0, T_t/F] \\ = 0 & \text{if } h^f_t < 0 \end{cases} \qquad (23)$$

It can easily be verified that h^f_t is a decreasing function of P_{t-1}. As above one can then re-express the solution in terms of a function $p(P_{t-1})$. I.e.

$$P_t \begin{cases} = 1 \quad \text{if} & \text{if } P_{t-1} < h^{-1}(T_t/F) \\ \in [Fh(P_{t-1})/T_t] & \text{if } P_{t-1} = h^{-1}(T_t/F) \\ = 0 & \text{if } P_{t-1} > h^{-1}(T_t/F) \end{cases} \qquad (24)$$

The graph of this solution is depicted in figure 2 below. Again the results are similar to the case of constant returns to scale. If managers and workers behave as depicted in this section then hiring demand is either constant or cycles. However, in the case of decreasing returns other, smaller cycles in hiring intensity can also occur, depending on the curvature of the function $h^f_t(P_{t-1})$.

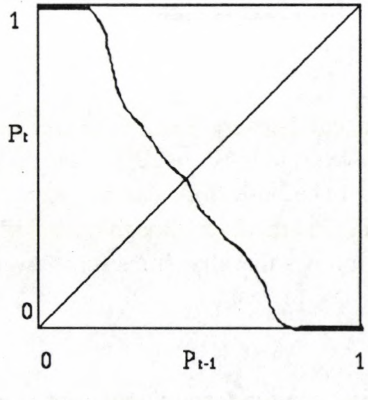

Figure 2

• <u>Wages</u>

Let me briefly comment on the effect of higher wages on labour demand (two economies are compared which are equal in all aspects apart form the wage rate). In both cases, constant and decreasing returns, it is easy to observe the effects of lower wages: for lower wages (w"<w) iso-profit curves shift up and to the right, that is labour demand increases for every P_{t-1}. As a consequence the feasibility constraint Γ becomes binding for higher past hiring intensities P_{t-1}, that is for lower quality of

the unemployed. It also follows that the stationary solution of lower wages implies a higher hiring intensity.

3.3: ...who are hired by 'far sighted' managers

This section shall have another look at the consequences of static expectations of workers, but this time managers maximise a two period problem. Managers analyse the constant returns to scale technology of section 3.2 (i.e. equation (19') with γ=1). The interpretation of the two period case is as follows: the best matched workers with high quality become managers after one period. They then determine hiring for their second life time, but also care about the period after they retire (i.e. their retirement period). Beyond this period they do not care. Managers are therefore 'far sighted' in the sense that when deciding to hire they take into account the effect this has on the firm once they are already retired. This two period problem brings out all the dynamic elements of the model (and the problems associated with them).

This small extensions makes the analysis a lot more complicated, however. The complication arises because once one lets managers choose their maximising hiring rate one needs to give up the symmetry assumption in the profit function for t+1 (since one first needs to show that in such a situation a perfect foresight symmetric Nash equilibrium exists. The model therefore looses much of its simplicity. In order to keep the problem 'manageable', I will therefore assume that managers only consider the direct effect of their hiring decision on company profit (that is, I will assume that managers set $[\partial P_t/\partial h^f_t]=0$). Managers maximise the following objective function with respect to h^f_t and h^f_{t+1}:

$$\pi^*(h^f_t;\, P_{t-1})+\beta\pi(h^f_{t+1};\, rem^f_t; a^T_{t+1},\, a^r_t)$$

$$= \left(\tfrac{1}{4}\,(1-q_t)P_{t-1}/F+a^{T*}(P_{t-1})h^f_t\right)-w(rem^f_{t-1}+h^f_t)\,+$$

$$+\,\beta\left(\tfrac{1}{4}(1-q_{t+1})h^f_t/T_t+a^T_{t+1}h^f_{t+1}-w(\tfrac{1}{2}(1-q_{t+1})(1-\alpha)h^f_t/T_t+h^f_{t+1})\right) \quad (25)$$

subject to the standard constraint in equation (11) for t and t+1.

Note that second period decision making (the maximisation problem w.r.t. h^f_{t+1}) was discussed in sections 3.2 (given the assumption that a symmetric Nash equilibrium is played in the first period). The analysis of myopic decision making also applies to first period decision making (the maximisation problem w.r.t. h^f_t) in the special case in which $\beta=0$. By continuity one can therefore look at the myopic case as a bench-mark case for small βs. First period decision making is a lot more complex than second period decision making because managers now also need to consider the consequences of their hiring behaviour on future profit.

Maximising this two period profit function with respect to h^f_t one then obtains the following unconstrained first order condition:

$$a(P_{t-1})-w+\beta k=0 \qquad (26)$$

where

$$k \quad =\tfrac{1}{2}((\tfrac{1}{2}-w)(1-q_{t+1})+w\alpha(1-q_{t+1}))/T_t \quad (27)$$

having set $[\partial P_t/\partial h^f_t]$ equals to zero. In order to understand the economics behind 'k' note that $(1/(2T_t))$ is the proportion of non retiring workers; of these half will have high ability and q_{t+1} of them are expected to quit. Beyond this there is a wage gain which is due to the fraction α of the non-retireing workers who will be fired (taking account of the fact that quits and lay offs may overlap since they are decided simultaneously). A symmetric and feasible Nash equilibrium would look as follows:

$$h_t^{f*} \quad \begin{array}{ll} = T_t/F & \text{if } a(P_{t-1})-w+\beta k > 0 \\ \in [0,T_t/F] & \text{if } a(P_{t-1})-w+\beta k = 0 \\ = 0 & \text{if } a(P_{t-1})-w+\beta k < 0 \end{array} \qquad (28)$$

One now needs to show that a perfect foresight symmetric Nash equilibrium exists. In this regard note that T_t is a function of P_{t-1} only because of Property 1 and q_{t+1} is a function of P_t because it was assumed that workers have static expectations. Hence k is a correspondence of P_t and P_{t-1} only. Note that optimal and feasible h_t^f (i.e h_t^{f*}) is therefore a function of expected P_t (i.e.P_t^e) and P_{t-1} only. I.e. $h_t^{f*}=h^*(P_t^e, P_{t-1})$. Now sum over the h_t^{f*} and divide by T_t (which again is a function of P_{t-1}). One therefore obtains a convex valued upper hemi continuous correspondence $P_t=P(P_t^e,P_{t-1})$ which is defined over the intervals $P:[0,1]\times P_{t-1}\rightarrow[0,1]$ and this for every P_{t-1}. P(.) is a convex valued correspondence because h(.) is a convex valued correspondence, because T_t is continuous in P_{t-1} and because T_t is bounded below by $^1/_2(1+\alpha)>0$. Hence for every $P_{t-1}\in[0,1]$ one can invoke the Kakutani fixed point theorem[12] which implies that there exists a fixed point $P_t=(P_t)^e$. A perfect foresight symmetric Nash equilibrium thus always exists.

Beyond existence of equilibrium one would of course also like to characterise equilibrium and know how it differs from 'myopic' decision making. If hiring occurs at all then k is always positive. This implies that 'far sighted' managers hire more workers than 'myopic' managers. This is fairly obvious. The chance of multiple perfect foresight path seems remote however since k is decreasing in P_t. This implies that the more mangers expect others to hire the less they hire themselves, and vice versa. This occurs because more intensive hiring of other managers encourages workers to quit (since they have lagged expectations), thus reducing the individual return to hiring.

3.4 Workers with perfect foresight who are hired by 'myopic' managers

This section contains a proof that equilibrium always exists when workers have perfect foresight and when managers are 'myopic'. 'Myopic' managers solve the one period maximisation problem of equations (10) and (11). As is to be expected the results for the case of perfect foresight of workers are much weaker and a lot less straight forward than that in which workers have static expectations. However perfect foresight also yields more interesting results (e.g., multiple perfect foresight path as will be demonstrated in section 3.5). Note that the proof below of the existence of a symmetric perfect foresight equilibrium equilibrium only applies to the case in which firms maximise one period problems (managers are myopic).

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[12] In Border [1985, p. 72] Kakutani's theorem is stated as follows: Let K be a compact and convex subset of $\mathbf{R}^m$ and let γ be closed or upper hemi-continuous correspondence from K into K with non-empty convex compact values. Then γ has a fixed point.

Property 2: Assume that workers have perfect foresight of workers and that managers are 'myopic' and maximise profit functions which are concave in h_t^f. Then, when assuming that a symmetric equilibrium was played in period t-1, there always exists a perfect foresight equilibrium such that $P_t=(P_t)^e$.

Proof: 'Myopic' managers maximise a profit function $\pi^f_t=\pi(h^f_t;\ rem^f_{t-1};\ a^T_t,\ a^r_t)$ subject to $h^f_t\in[0;\ T_t-\Sigma_{-f}(h^j_t)]$.Given the assumption that a symmetric equilibrium was played in period t-1 and because of eq. 15 the profit function simplifies to $\pi(h^f_t,\ (1-T_t)/F,\ a^T_t,\ const.)$. One can now invoke the theory of the maximum[13] which together with the assumption that $\pi(.)$ is concave in h_t^f assures that h_t^f is a convex valued[14] and upper hemi-continuous correspondence of T_t and a_t. By definition (eq. (16)) the quality of applicants is a function of the past hiring intensity P_{t-1}, of quits q_t and of T_t. Hiring h^f_t is therefore a convex valued correspondence of q_t and P_{t-1}; and T_t. i.e. $h^f_t=h(q_t,T_t\ P_{t-1})$ and.$h:[0,1/2]\times T_t\times[0,1]\rightarrow[0,T_t]$

Quits, given perfect foresight of workers, take the functional form $q_t=1/2P_t^e$. Transitory unemployment equals:

$$\begin{aligned} T_t &\equiv G+Q_t+M_t-q_tM_t+(1-P_{t-1})G \\ &=1-1/2(1-\alpha)(1-q_t)P_{t-1} \\ &=1-1/2(1-\alpha)(1-1/2(P_t)^e)P_{t-1} \\ &\equiv \phi((P_t)^e,P_{t-1}) \end{aligned}$$

and note that $\phi:[0,1]\times[0,1]\rightarrow[G(1+\alpha),2G]$ so that the function can never become zero. It follows that

$$\begin{aligned} h^f_t &=h(q_t,T_t\ P_{t-1}) \\ &=h(1/2(P)^e,\phi((P_t)^e,P_{t-1}),\ P_{t-1}) \end{aligned}$$

13See for example Border [1985] theorem 12.1. The theorem says: Let G be a subset of $\mathbf{R}^m$ and Y be a subset of $\mathbf{R}^k$ and let γ be a compact valued correspondence from G into Y. Let f be a continuous function of Y into **R**. Define a correspondence μ from G into Y by $\mu(x)=\{y\in\gamma(x)$: y maximises f on $\gamma(x))$. If γ is continuous at x, then μ is closed and upper-hemi continuous at x and f is continuous at x. Furthermore μ is compact-valued.

14 The convex valuedness follows from the assumption that $\pi(.)$ is concave in h_t^f and that the constraint is convex. To see why this is so consider the following argument: Let h* maximise $\pi(.)$ at T_t and a_t. Let h^ be another value which maximises $\pi(.)$ at T_t and a_t, i.e. $\pi(h^*,.)=\pi(h^\wedge,.)$. It now follows that any convex combination $h=th^*+(1-t)h^\wedge$ for $0<t<1$, is feasible and must also be a maximising choice since
$\pi(th^*+(1-t)h^\wedge,.)\geq\pi(h^*,.)$ by the concavity of $\pi(.)$ in h^f_t. The upper hemi-continuous policy correspondence in h^f_t must therefore be convex valued.

$\equiv h^*((P_t)^e,P_{t-1})$

and that $h^*:[0,1]\times[0,1]\rightarrow[0,T_t]$. Now sum over all the h^f_t to get H_t and then divide by T_t to get P_t, i.e.

$$P_t \quad =\Sigma_f(h^*((P_t)^e,P_{t-1}))/\phi((P_t)^e,P_{t-1})$$
$$\equiv P((P_t)^e,P_{t-1})$$

Note that $P:[0,1]\times P_{t-1}\rightarrow[0,1]$ and this for each P_{t-1}. Note further that P(.) is a a convex valued and upper hemi-continuous correspondence because h*(.) and ϕ(.) are a convex valued and upper hemi-continuous correspondences and because ϕ(.) is bounded below by $^1/_2(1+\alpha)>0$. Hence for every $P_{t-1}\in[0,1]$ one can invoke the Kakutani fixed point theorem[15] which implies that there exists a fixed point $P_t=(P_t)^e$. A perfect foresight equilibrium thus always exists. QED.

Note that equilibrium need not be unique and, in fact, in the next section I shall give an example where multiple perfect foresight symmetric Nash equilibria exist .

3.5 Multiple perfect foresight paths in quitting decisions

This section contains an example in which multiple perfect foresight equilibria exist due to a quit externality. The intuition driving the examples is as follows: if workers expect hiring to be high, then many workers might quit because they believe that it will be easy to get another job. Higher quits increase the average quality of someone in the applicant pool (for any P_{t-1}) which in turn leads managers to hire more workers (given P_{t-1}) - the belief of workers that hiring would be high which induced them to quit was therefore self fulfilling. Equally if workers expect hiring to be low then they will not quit because they are unlikely to be re-employed. This means that fewer workers will quit which will have a negative effect on the average quality of the unemployed. Low quality of the unemployed will induce managers to hire fewer workers so that the expectations of workers were self-fulfilling.

Firms maximise profit with respect to hiring taking (q_t,T_t,P_{t-1}) as given (which implies that they also take a_t as given). In perfect foresight equilibrium workers take their quitting decisions, given their

[15]Border [1985]. The theorem was stated in a previous footnote.

expectation of P_t and $(P_t)^e=P_t$. One therefore has to show that the expectations of quitters are correct given that firms will maximise profit given their quitting decisions. Myopic managers maximise the constant returns profit function:

$$\pi_t=\left(rem^f_{t-1}a^r_t+a^T_t h^f_t\right)-w(rem^f_{t-1}+h^f_t) \tag{19}$$

subject to equation (11) and taking (q_t,T_t,P_{t-1}) as given .

The unconstrained first order condition then is almost as in the previous section:

$$(\partial\pi_t/\partial h^f_t)= a^T(T_t, q_t,P_{t-1})-w \tag{29}$$

and $T_t=1-{}^1/_2(1-\alpha)(1-q_t)P_{t-1}$ assuming that a symmetric equilibrium was played in period t-1. Hence the first order condition is only a function of q_t and P_{t-1}. Let $a^T(1-{}^1/_2(1-\alpha)(1-q_t)P_{t-1}, q_t,P_{t-1}) \equiv a^{T\wedge}(q_t,P_{t-1})$ and note that the following parametric assumptions describe various Nash equilibrium paths:

$$a^{T\wedge}(q_t={}^1/_2, P_{t-1}=1) = w \qquad \text{and } P_t^e=P_t=1 \tag{30}$$
$$a^{T\wedge}(q_t=0, P_{t-1}=1) < w \qquad \text{and } P_t^e=P_t=0 \tag{31}$$
$$a^{T\wedge}(q_t, P_{t-1}=0) > w \qquad \text{and } P_t^e=P_t=1 \tag{32}$$

Assume that at a=w managers hire the highest possible amount (i.e. $P_t=1$). Then the above equations describe a case in which quits may make quitting desirable. In order to see this note first of all that wherever the economy starts, (P_0), the hiring intensity at the next stage (P_1) has to be either zero or one. Now if $P_{t-1}=0$ then the average quality of the workforce at time t must be so high that by assumption hiring will be at its maximum (since $a^{T\wedge}(q_t,P_{t-1}=0) > w$). That is at time t $P_t=1$. Now if $P_{t-1}=1$ then workers can make two perfectly consistent forecasts. If they expect $P_t=1$ then they consider their chances to be re-employed to be high which means that many workers will quit. This will mean that the quality of the average unemployed will be high enough for firms to hire the maximum amount they can hire (since by assumption $a^{T\wedge}(q_t={}^1/_2 P_t={}^1/_2,P_{t-1}=1) = w$ and at a=w managers hire $h^*(P_{t-1})=T_t/F$). If on the other hand workers expect that times will be bad (they expect $P_t=0$) then they will not quit. This implies that the quality of the unemployed in period t will be so low that manager will not hire (since by assumption $a^{T}\hat{}(q_t=0P_t=0,P_{t-1}=1) < w$).

This example thus described a simple case in which multiple perfect foresight path exist due to a quit/ quality externality. That there might be multiple equilibria in quitting decisions was suggested by Hall [1988] in a different context. The mechanism by which this quit externality functions here is an aggregate quality externality which may be invoked by the self-enforcing prophecies of workers.

4. Relaxing the Central Assumptions

- <u>All workers have the same productivity</u>

  Assume that workers all have the same productivity. Firms, according to this model, then have no reason to fire workers, but even if they do fire some, all workers in the unemployment pool will always have the same productivity. Much of the intricacy in a firm's decision problem arises because it wants to keep track of the quality of the workers hired and thus of profits. If all workers have the same quality then firms will always hire an amount of workers corresponding to the difference between their optimal employment rate and the number of workers who remain from the previous period. Further, the optimal employment rate of each firm should be constant in this model, so that, on average, there should be a constant average rate of transitory unemployment (due to a horizontal matching problem) and a constant average rate of long term unemployment, or no long term unemployment at all (due to a constant optimal employment rate per firm). Much -of the complication, but also the interest in the model thus stems from the assumption that workers differ in productivity.

- <u>All bad workers can be fired</u>

  If all bad workers could be fired, firms would (in this model) at the end of their production period get rid of all their bad workers, since they have zero productivity. This would imply that there always is the maximum amount of bad workers in the applicant pool T_t. The average ability of someone in the applicant pool is therefore constant if quits are constant. If quits are constant hiring is constant. Whether the average amount of quits is reduced by this possibility depends on hiring (that is on the profit function). One cannot rule out multiple perfect foresight path as in section 3.5. (However in order to fully consider this case one would have to slightly re-define ability as in the present definition a^r tends to infinity as a tends to 1)

- Firms can distinguish workers by age.

  As a consequence there would be two different hiring markets, that of the young and that of the old. As the model is set up it is never worth hiring the old before the young. As a consequence firms would hire all young workers. Older workers would only be hired if hiring demand is positive, given the average ability of the older unemployed. In this case quits would be reduced since chances to be re-employed would then be much lower. Technically therefore little would change if one assumed that firms could distinguish the young and the old.

- Workers live for more than two periods.

  This assumption makes the model a lot easier to work with. There would be three major consequences of dropping this assumption: first, the unemployment level would no longer be an unambiguous signal of of the quality of the unemployed in the simple way it is in this model. However, firms could tell the quality of the average unemployed via the size of the unemployment pool T_t and via the quality of the workforce in T_{t-1} (which they observed in their workforce if P_{t-1} was positive). The second consequence of dropping the two period assumption would be that the maximisation problem of employers would become somewhat more complicated, although the underlying logic would remain the same. A firm's maximisation problem could then be simplified if one assumed that firms can tell the age of workers. For each age group there would then be a separate unemployment pool as described above. The third consequence of dropping this assumption would be that the optimisation problem of workers would become a lot more complicated. Again, the set-up could be simplified if workers could be distinguished be age, but even then, the optimisation problem would be a lot more complicated than it is in this model. It is exactly for this reason that the two period assumption was made.

- Turnover and adjustment costs

  In the above model I did not assume adjustment or turnover costs. It seems however that they could be introduced without too much complication. One should however bear in mind that the way firing restrictions were modelled in this paper already imposes a constraint on firing by firms. However, in many countries there are undoubtedly significant costs to dismissing a worker (see Emerson [1988]) which underlines the importance of adjustment cost considerations[16].

- Flexible wages

  Wages could have been assumed partially flexible. This would then have slightly off-set the excess demand or supply for labour which might exist and, more importantly, it would have reduced the effects which exist in the model. Partially flexible wages would thus add little apart from making the model more complicated. One could also have introduced the assumption that workers who do not quit and are not thrown out after the first period receive a higher wage. This would then have account for the fact that wages increase with tenure and would also have reduced the quit rate.

5 Conclusions

This paper analysed the dynamics of an OLG matching model in which firms take account of quitting decisions and workers take account of hiring behaviour. This leads to the quality of the employed (and unemployed) workforce being endogenous to the economy. The driving force behind hiring behaviour is that firms do not have any reliable

16 Adjustment costs can be modelled in various ways: quadratic, linear, etc. Recent articles by Bentolila and Bertola [1990] and Bertola [1990] argued that linear adjustment costs are a better way to model adjustment costs then a quadratic characterisation.

information about the past employment history of an applicant so that their hiring behaviour depends on the expected ability of an applicant.

The model identifies situations in which the (transitory) unemployment rate and the hiring intensity of firms is an unambiguous signal of the average quality of the unemployed. This feature of the model simulates at least two real labour market features: First, that employers might under certain conditions be fully rational in believing that higher turnover implies a higher average quality of an applicant and lower turnover a lower quality. Second, that employers obtain signals from the market concerning the average quality of the unemployed.

Quits may be below their 'optimal' level since workers take account of their re-employment chances. Quits in this paper depend on the expected hiring intensity and are only indirectly related to unemployment. Given lagged or static expectations of workers quits are negatively related to (transitory) unemployment. In the case in which workers have perfect foresight, however, quits may be associated with high unemployment (although in this case hiring intensity must also be high). One should note that the case just described only describes one of the possible equilibrium paths.

The model draws a clear distinction between firing and quits, mainly because it was assumed that personal productivity and personal job enjoyment are not correlated. This implies that job matches may not be efficient, as in the efficient turnover literature. This seems a realistic feature of the model.

It was shown (section 3.2 and 3.3) that labour demand may proceed along a two period deterministic cycle and that it may be stationary. Which of these solutions occur depends on functional forms and parametric assumptions. Once one assumes that workers have perfect foresight then multiple perfect foresight equilibria in quitting decisions may occur (this was shown in section 3.5). The mechanism by which quitting decisions may become indeterminate is an aggregate quit/ quality externality. The model thus describes a case in which the forecasts of workers about hiring may become self enforcing.

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