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Abstract

The transmission of innovations to macroeconomic variables is traditionally studied concentrating on their permanent effects. Yet, there is no conclusive evidence that only the long-term dynamics should be of interest. In this paper we offer a method based on the notion of codependence to identify the structural innovations contributing to the stationary part of a vector of cointegrated (1,1) variables. To achieve this, we introduce the notion of *common cycles of order i* and of a *complete set of common cycles*, whose implications for economic fluctuations are fully derived and discussed.

*Without implicating, we would like to thank Renato Leoni and Massimiliano Marcellino for many useful discussions.

1 Introduction

In their seminal paper on the sources of fluctuations in the presence of trends common to a set of nonstationary variables, King, Plosser, Stock and Watson ([9], henceforth KPSW) offered a method to identify the influence of "structural" innovations on the dynamics of a set of variables, on the basis of an estimated VAR system. This method proved to generate quite challenging results for macroeconomists. A major conclusion reached by KPSW is in fact that a structural innovation satisfying the usual assumptions associated to a real productivity shock seems to have a much smaller contribution to economic fluctuations than advocated by a standard real business cycle model. This is a point which is even more forcefully proposed by Cochrane ([3]), who presents a very skeptical view on the existing evidence in the literature.

However, in spite of its novelty and interest, this method is restricted to the identification of "permanent" structural innovations, that is innovations having a nonstationary impact on the economic variables, as it concentrates on the common trends representation. Mellander, Vredin and Warne ([12], henceforth MVW) extending the work of KPSW have suggested a method to be applied to transitory innovations, but have been unable to implement it empirically, recognizing that the *"question about the nature of the transitory shocks deserve further study"* (MVW, p.376).

The issue is clearly of major importance for applied macroeconomics, since it hinges around important debates such as the one on short term neutrality of money (King and Watson, [10]), on the importance of short run price stickiness (Ball and Mankiw, [1]), or on the dynamics of the deviations from purchasing power parity (MacDonald and Taylor, [11]).

The purpose of this paper is to offer a method for the identification of an economic system's responses to transitory innovations, totally consistent with the data. The method we propose draws heavily on the notion of codependence offered by Gourieroux and Peaucelle ([7]) and Engle and Kozicky ([6]), and applied to a nonstationary environment by Vahid and Engle ([15] - VE1, [16], VE2). There is a close parallel between the notions of cointegration and codependence, and between the notions of common trends and common cycles. There is codependence among stationary series when at least one linear combination of them exists which is of smaller moving average order than others. As shown by VE1, a strong form of codependence (or *serial correlation common feature* as defined by Engle and Kozicki, [6]) among the first differences of cointegrated variables implies that a linear combination of the variables is a pure white noise, or, which is the same, that part of their stationary dynamics is generated by common shocks. In other words, these variables share *common cycles.*

In this paper we shall develop the concept of a *complete set of common cycles*, showing how it allows us to totally identify the dynamics generated by transitory innovations. This is so because, when the data exhibit a complete set of common cycles, the variables can be decomposed into two components, namely common trends and common cycles. Interestingly, the notion of a complete set of common cycles implies that all common cycles need not be defined for the same period.

To avoid ambiguities since the concept of "structure" has several meanings in economics and econometrics, it has to be stressed that the identification of structural innovations, be they permanent or transitory, by no means corresponds to identifying "the" structural model of an economic system, nor to characterizing these innovations from an economic point of view such as "money supply" shocks or "wage shocks", and so on.

The aim of an identification exercise is just to make explicit the dynamic effects of a given (and unobservable) disturbance on a set of variables when the empirical evidence provides testable restrictions which need to be taken into account. It is the task of economic theory to offer plausible explanations of these effects, through suitable economic reasoning and modelling. Once this is done, the identification exercise allows for the assessment of the relative importance of several economic innovations, the way these are suggested and labelled according to the theory, which is impossible to do directly from the estimated relationships.

The structure of the paper is as follows: Section 2 summarizes some results from KPSW and MVW, showing the implications for the identification of innovations (or, equivalently, on the system's response to innovations). In Section 3, we develop the tools related to codependence which shall be used later. In Section 4, we show how the results on the identification of shocks to the permanent and transitory components of the variables change when both cointegration and codependence are taken into account. The method allows us to understand the relative contribution of each shock to the dynamics of a given economic system. Concluding remarks follow.

2 The Representation of Economic Fluctuations

2.1 Reduced Form Representation

Let us denote the $(n \times 1)$ vector of nonstationary I(1) variables, whose dynamic evolution is of interest, as $\mathbf{x}_t$. We will assume that these variables are cointegrated of order (1,1) and that a stationary Wold representation exists for the first differences of the variables

$$\Delta \mathbf{x}_t = \mathbf{C}(L)\boldsymbol{\varepsilon}_t, \tag{1}$$

where $\mathbf{C}(L)$ is a $(n \times n)$ matrix polynomial in the lag operator L ($L^j z_t = z_{t-j}$), namely ($\mathbf{C}(L) = \mathbf{I} + \mathbf{C}_1 L + \mathbf{C}_2 L^2 + \ldots$), with $\sum_{j=0}^{\infty} j\,|\mathbf{C}_j| < \infty$, $\boldsymbol{\varepsilon}_t$ ~i.i.d.$(\mathbf{0}, \boldsymbol{\Sigma})$ and serially uncorrelated[1]. The matrix $\mathbf{C}(1) = \sum_{j=0}^{\infty} \mathbf{C}_j$ represents the long-run impact matrix and summarizes the long-run stochastic dynamics of the system. Its meaning is clear when deriving the corresponding expression for the levels of the variables by backward

[1]We will neglect for simplicity the presence of drift parameters $\boldsymbol{\mu}$, which translate into the presence of a deterministic time trend in the representation for the levels (cf. Stock and Watson, [13], p.1098), or of dummy variables as in MVW.

recursive substitution,

$$\mathbf{x}_t = \mathbf{x}_0 + \mathbf{C}(1)\sum_{j=0}^{t-1}\boldsymbol{\varepsilon}_{t-j} + \mathbf{C}^*(L)\boldsymbol{\varepsilon}_t \tag{2}$$

where $\mathbf{C}(1) = \sum_{j=1}^{\infty}\mathbf{C}_j$ and $\mathbf{C}_j^* = \sum_{i>j}\mathbf{C}_i$. In fact, assuming the presence of $0 < r < n$ vectors of cointegration arranged in a matrix $\boldsymbol{\alpha}$ of dimension $(n \times r)$ implies (by definition) that

$$\boldsymbol{\alpha}'\mathbf{C}(1) \;=\; \mathbf{0} \tag{3}$$

$$\mathrm{rank}[\mathbf{C}(1)] \;=\; n - r \equiv k \tag{4}$$

The restrictions implied by cointegration are detected (e.g. Johansen, [8]) from a finite order VAR(p) model for $\mathbf{x}_t = \mathbf{\Pi}_1\mathbf{x}_{t-1} + \ldots + \mathbf{\Pi}_p\mathbf{x}_{t-p} + \boldsymbol{\varepsilon}_t$, reparameterizing it into a vector error-correction model (VECM) to read

$$\Delta\mathbf{x}_t = \mathbf{\Pi}_1^*\Delta\mathbf{x}_{t-1} + \ldots + \mathbf{\Pi}_{p-1}^*\Delta\mathbf{x}_{t-p-1} + \mathbf{\Pi}\mathbf{x}_{t-p} + \boldsymbol{\varepsilon}_t + \boldsymbol{\varepsilon}_t \tag{5}$$

where $\mathbf{\Pi}_i^* = -\mathbf{I} + \mathbf{\Pi}_1 + \ldots + \mathbf{\Pi}_i$ and $\mathbf{\Pi} = -(\mathbf{I} - \mathbf{\Pi}_1 - \ldots - \mathbf{\Pi}_p)$. The presence of cointegration is such that the $(n \times n)$ matrix $\mathbf{\Pi}$ is of reduced rank r and can be expressed as the outer product of the two $(n \times r)$ matrices of rank r

$$\mathbf{\Pi} = \boldsymbol{\beta}\boldsymbol{\alpha}' \tag{6}$$

where $\boldsymbol{\beta}$ is a matrix of loadings representing the impact of the stationary combinations $\boldsymbol{\alpha}'\mathbf{x}_{t-1}$ on $\Delta\mathbf{x}_t$ (cf. Johansen, [8]).

2.2 Structural Form Representation

Let us consider that economic variables are the outcome of the dynamic propagation of economically interpretable shocks through the system. Some shocks are termed *permanent* since they are the only to contribute to the long-run dynamics of the variables $\mathbf{x}_t$ and others are termed *transitory*, since they contribute just to the short-run dynamics. Therefore, we will assume, following the analysis by Blanchard and Quah ([2]) and KPSW that there exists a data generating process such that

$$\Delta\mathbf{x}_t = \mathbf{\Gamma}(L)\boldsymbol{\eta}_t, \tag{7}$$

where $\boldsymbol{\eta}_t$ is a $(n \times 1)$ vector of underlying structural shocks, unobserved random variables characterized by zero mean, identity variance–covariance matrix[2] and serial uncorrelation, with $\boldsymbol{\Gamma}(L) = \boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1 L + \ldots$. In fact, (1) can be seen as a reduced form representation in which the $\boldsymbol{\varepsilon}_t$ vector represents a mixture of $\boldsymbol{\eta}_t$.

In order for (7) to be identified we need to adopt the following restrictions

$$\begin{aligned} \boldsymbol{\varepsilon}_t &= \boldsymbol{\Gamma}_0 \boldsymbol{\eta}_t && (8) \\ \boldsymbol{\Gamma}(1) &= (\tilde{\boldsymbol{\Gamma}}_g | \mathbf{0}) && (9) \end{aligned}$$

where $\boldsymbol{\Gamma}_0$ is an invertible matrix and $\tilde{\boldsymbol{\Gamma}}_g$ is a $(n \times k)$ matrix [3]. Pairwise comparison between (1) and (7) shows that

$$\mathbf{C}_i \boldsymbol{\Gamma}_0 = \boldsymbol{\Gamma}_i \qquad \forall i > 1 \tag{10}$$

and hence

$$\mathbf{C}(L)\boldsymbol{\Gamma}_0 = \boldsymbol{\Gamma}(L) \qquad \text{and} \qquad \mathbf{C}(1)\boldsymbol{\Gamma}_0 = \boldsymbol{\Gamma}(1) \tag{11}$$

Again, we can write the structural form representation for the levels which provides a clear interpretation on the nature of the shocks. Let us partition the vector $\boldsymbol{\eta}_t$ into two subvectors $\boldsymbol{\eta}_{1t}$ $(k \times 1)$ and $\boldsymbol{\eta}_{2t}$ $(r \times 1)$, respectively. Correspondingly, the matrix $\boldsymbol{\Gamma}_0$ can be partitioned by column as $\boldsymbol{\Gamma}_0 = (\boldsymbol{\Gamma}_{0g} | \boldsymbol{\Gamma}_{0s})$ of suitable dimensions. We have,

$$\begin{aligned} \mathbf{x}_t &= \boldsymbol{\Gamma}(1) \sum_{j=0}^{t-1} \boldsymbol{\eta}_{t-j} + \boldsymbol{\Gamma}^*(L)\boldsymbol{\eta}_t \\ &= \mathbf{x}_0 + \tilde{\boldsymbol{\Gamma}}_g \sum_{j=0}^{t-1} \boldsymbol{\eta}_{1,t-j} + \boldsymbol{\Gamma}^*(L)\boldsymbol{\eta}_t && (12) \end{aligned}$$

[2] We prefer to adopt this convention (as in MVW), rather than leaving the diagonal variance-covariance matrix unrestricted as in KPSW, since the latter unloads a need for normalization onto the $\boldsymbol{\Gamma}_0$ matrix. In fact, assuming the presence of a shock with standard deviation of σ and impact of γ, is equivalent to assuming the presence of a shock with unit variance and impact of $\gamma\sigma$.

[3] $\tilde{\boldsymbol{\Gamma}}_g$ is subject (as in KPSW or MVW) to identifying restrictions necessary in any VAR model to interpret the results: in this case we will assume that its top $(k \times k)$ submatrix is lower triangular.

with $\mathbf{\Gamma}(1)$ and $\mathbf{\Gamma}^*(L)$ defined analogously as $\mathbf{C}(1)$ and $\mathbf{C}^*(L)$. The expression (12) can be written in the more familiar common trend representation (Stock and Watson, [13]) which highlights the contributions of the elements of $\boldsymbol{\eta}_t$ to the dynamics of $\mathbf{x}_t$. We have

$$\mathbf{x}_t = \mathbf{x}_0 + \tilde{\mathbf{\Gamma}}_g \boldsymbol{\tau}_t + \mathbf{\Gamma}^*(L)\boldsymbol{\eta}_t \tag{13}$$

where

$$\boldsymbol{\tau}_t = \boldsymbol{\tau}_{t-1} + \boldsymbol{\eta}_{1t} = \boldsymbol{\tau}_0 + \sum_{j=0}^{t-1} \boldsymbol{\eta}_{1,t-j}$$

is the common trend component which is nonstationary $I(1)$ and of size $k < n$. From (13) it is clear, that the first term is $I(1)$ and the second is $I(0)$. Thus the subvector $\boldsymbol{\eta}_{1t}$ contains *the innovations contributing to the permanent component* $\boldsymbol{\tau}_t$ *of* $\mathbf{x}_t$ (sometime called the *permanent innovations)* while $\boldsymbol{\eta}_{2t}$ contains *the innovations not contributing to the permanent component* $\boldsymbol{\tau}_t$ *of* $\mathbf{x}_t$ (sometime called the *temporary innovations).* The latter denomination is somewhat misleading since the transitory $I(0)$ component $\mathbf{\Gamma}^*(L)\boldsymbol{\eta}_t$ is determined by both $\boldsymbol{\eta}_{1t}$ and $\boldsymbol{\eta}_{2t}$[4]. The short-run dynamics is clearly determined by all structural shocks, so that if we want to obtain the dynamic multipliers of $\boldsymbol{\eta}_t$ to $\mathbf{x}_t$, we need to identify all the $\mathbf{\Gamma}_j$ matrices ($j \geq 0$). On the basis of (10), the problem then reduces itself to the study of the matrix $\mathbf{\Gamma}_0$, since the matrices $\mathbf{C}_j$ can be estimated.

It is important to identify these matrices because they allow us to obtain the structural impulse-response functions, since they are defined as

$$\frac{\partial \Delta x_{i,t+h}}{\partial \eta_{jt}} = \gamma_{ij}^h$$

where γ_{ij}^h is the $i, j - th$ element of the matrix $\mathbf{\Gamma}_h$.

[4]As a matter of terminology, we will refer in what follows to $\boldsymbol{\eta}_{1t}$ and $\boldsymbol{\eta}_{2t}$ as innovations to the permanent, respectively, transitory components of $\mathbf{x}_t$.

2.3 Cointegration and the Identification of the Structural Innovations

In this subsection we will recall the formulas for the identification of shocks to the permanent and transitory components, as derived by KPSW and MVW under the hypothesis of the presence of just cointegration. We will see in section 4 that considering the presence of codependence alters substantially these results.

Recall that the conditions (8) and (9) ensure that there is no other structural innovation vector $\boldsymbol{\eta}_t^*$ giving rise to the same reduced form (1). Hence, let us consider the expression linking structural and reduced-form shocks

$$\boldsymbol{\Gamma}_0^{-1}\boldsymbol{\varepsilon}_t = \boldsymbol{\eta}_t \Longleftrightarrow \begin{pmatrix} \mathbf{G} \\ \mathbf{K} \end{pmatrix} \boldsymbol{\varepsilon}_t = \begin{pmatrix} \boldsymbol{\eta}_{1t} \\ \boldsymbol{\eta}_{2t} \end{pmatrix} \qquad (14)$$

where the $(k \times n)$ matrix $\mathbf{G}$ represents the inverse mapping from $\Re^n$ to $\Re^k$, disentangling the shocks contributing to the permanent component and the $(r \times n)$ matrix $\mathbf{K}$ represents the inverse mapping from $\Re^n$ to $\Re^r$, disentangling the shocks contributing just to the transitory component.

When considering just the implications of cointegration, the expression for $\mathbf{G}$ can be found under the hypothesis of an identity variance-covariance matrix for $\boldsymbol{\eta}_t$ by following MVW. The matrix $\mathbf{G}$ is given by [5]

$$\mathbf{G} = (\tilde{\boldsymbol{\Gamma}}_g'\tilde{\boldsymbol{\Gamma}}_g)^{-1}\tilde{\boldsymbol{\Gamma}}_g'\mathbf{C}(1). \qquad (16)$$

[5] In fact, from (11) and (14) we have

$$\begin{cases} \tilde{\boldsymbol{\Gamma}}_g\boldsymbol{\eta}_{1t} = \mathbf{C}(1)\boldsymbol{\varepsilon}_t \\ \mathbf{G}\boldsymbol{\Sigma}\mathbf{G}' = \mathbf{I}_k \end{cases} \qquad (15)$$

we have that

$$\begin{aligned} \tilde{\boldsymbol{\Gamma}}_g\tilde{\boldsymbol{\Gamma}}_g' &= \mathbf{C}(1)\boldsymbol{\Sigma}\mathbf{C}(1)' \\ \tilde{\boldsymbol{\Gamma}}_g'\tilde{\boldsymbol{\Gamma}}_g\tilde{\boldsymbol{\Gamma}}_g'\tilde{\boldsymbol{\Gamma}}_g &= \tilde{\boldsymbol{\Gamma}}_g'\mathbf{C}(1)\boldsymbol{\Sigma}\mathbf{C}(1)'\tilde{\boldsymbol{\Gamma}}_g \\ (\tilde{\boldsymbol{\Gamma}}_g'\tilde{\boldsymbol{\Gamma}}_g)^{-1}\tilde{\boldsymbol{\Gamma}}_g'\mathbf{C}(1)\boldsymbol{\Sigma}\mathbf{C}(1)'\tilde{\boldsymbol{\Gamma}}_g(\tilde{\boldsymbol{\Gamma}}_g'\tilde{\boldsymbol{\Gamma}}_g)^{-1} &= \mathbf{I}_k \end{aligned}$$

There is another way to see how the common trend representation arises. Considering

$$\mathbf{x}_t = \mathbf{x}_0 + \mathbf{C}(1) \sum_{j=0}^{t} \boldsymbol{\varepsilon}_{t-j} + \mathbf{C}^*(L)\boldsymbol{\varepsilon}_t,$$

and noticing that $\mathbf{C}(1)$ is of reduced rank k, we can rewrite it as $\tilde{\mathbf{\Gamma}}_g\mathbf{G}$ and hence represent $\mathbf{x}_t$ as

$$\mathbf{x}_t = \mathbf{x}_0 + \tilde{\mathbf{\Gamma}}_g\mathbf{G} \sum_{j=0}^{t} \boldsymbol{\varepsilon}_{t-j} + \mathbf{C}^*(L)\boldsymbol{\varepsilon}_t.$$

Then, from (13), it is clear that

$$\boldsymbol{\tau}_t \equiv \mathbf{G} \sum_{j=0}^{t} \boldsymbol{\varepsilon}_{t-j} \equiv \sum_{j=0}^{t} \boldsymbol{\eta}_{1,t-j}$$

which highlights in a more direct way the nature of the common trends as random walks from zero mean, unit variance, uncorrelated random shocks, and the fact that the matrix $\mathbf{G}$ acts as to map $\boldsymbol{\varepsilon}_t \subset \Re^n$ into $\boldsymbol{\tau}_t \subset \Re^k$.

Still under the hypothesis of cointegration alone, the matrix $\mathbf{K}$ can be derived form the condition of orthogonality between $\boldsymbol{\eta}_{1t}$ and $\boldsymbol{\eta}_{2t}$, that is,

$$E(\boldsymbol{\eta}_{1t}\boldsymbol{\eta}_{2t}') = E(\mathbf{G}\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\mathbf{K}') = \mathbf{G\Sigma K}' = \mathbf{0}$$

which shows that $\mathbf{G}$ and $\mathbf{K}$ are orthogonal to each other relative to the metric defined by $\mathbf{\Sigma}$. Substituting the expression for $\mathbf{G}$ we get

$$(\tilde{\mathbf{\Gamma}}_g'\tilde{\mathbf{\Gamma}}_g)^{-1}\tilde{\mathbf{\Gamma}}_g'\mathbf{C}(\mathbf{1})\mathbf{\Sigma K}' = \mathbf{0} \tag{17}$$

Recalling (6) and the result documented in Yoo ([17]) that $\mathbf{C}(\mathbf{1})\boldsymbol{\beta} = \mathbf{0}$, a solution for $\mathbf{K}$ can be found (cf. MVW p.376) by substitution in (17)

$$(\tilde{\mathbf{\Gamma}}_g'\tilde{\mathbf{\Gamma}}_g)^{-1}\tilde{\mathbf{\Gamma}}_g'\mathbf{C}(\mathbf{1})\boldsymbol{\beta} = \mathbf{0}$$

Hence, a solution for $\mathbf{K}$ (taking only cointegration into account) takes the form

$$\mathbf{K} = \mathbf{Q}'\boldsymbol{\beta}'\mathbf{\Sigma}^{-1} \tag{18}$$

where $\mathbf{Q}$ is a nonsingular $(r \times r)$ matrix, chosen as to fulfill the last requirement, namely the identity variance covariance matrix of $\boldsymbol{\eta}_{2t}$, that is,

$$\mathbf{K\Sigma K}' = \mathbf{I}$$

or

$$\mathbf{Q}'\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\beta}\mathbf{Q} = \mathbf{I} \Leftrightarrow (\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\beta}))^{-1} = \mathbf{QQ}' \tag{19}$$

that is, $\mathbf{Q}$ can be uniquely obtained from the Cholesky decomposition of $(\boldsymbol{\beta}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\beta})^{-1}$. Remark that the lower triangularity of $\mathbf{Q}$ does not entail establishing a hierarchy among the structural errors, since the matrix $\mathbf{K}$ is not lower triangular.

MVW call for further analysis about the interpretation of the nature of the shocks contributing to the temporary component. In what follows we will argue that in the presence of codependence such an interpretation can be provided, and that the solution proposed by MVW for $\mathbf{K}$ must be modified.

Another way of looking at (18) is by recalling that the decomposition of $\mathbf{A}(1) = \boldsymbol{\beta}\boldsymbol{\alpha}'$ is not unique, since we can choose a nonsingular $\mathbf{Q}$ such that $\boldsymbol{\Pi} = \boldsymbol{\beta}\mathbf{Q}\mathbf{Q}^{-1}\boldsymbol{\alpha}'$. In the present case we are able to provide a meaningful normalization based on the need for normalizing the variance-covariance matrix of the structural shocks hence choosing the only $\boldsymbol{\beta}^*$ such that $\boldsymbol{\beta}^{*\prime}\boldsymbol{\Sigma}^{-1}\boldsymbol{\beta}^* = \mathbf{I}$.

The derivation of the matrix $\boldsymbol{\Gamma}_0$ follows in a straightforward way. In fact, recall that from $\boldsymbol{\varepsilon}_t = \boldsymbol{\Gamma}_0\boldsymbol{\eta}_t$ we get $\boldsymbol{\Sigma} = \boldsymbol{\Gamma}_0\boldsymbol{\Gamma}_0'$, and hence

$$\begin{aligned} (\mathbf{H}|\mathbf{J}) \equiv \boldsymbol{\Gamma}_0 = \; & \boldsymbol{\Sigma}(\boldsymbol{\Gamma}_0')^{-1} \equiv \boldsymbol{\Sigma}(\mathbf{G}'|\mathbf{K}') \\ \Rightarrow \; & \begin{cases} \mathbf{H} = \boldsymbol{\Sigma}\mathbf{G}' = \boldsymbol{\Sigma}\mathbf{C}(\mathbf{1})'\tilde{\boldsymbol{\Gamma}}_g(\tilde{\boldsymbol{\Gamma}}_g'\tilde{\boldsymbol{\Gamma}}_g)^{-1} \\ \mathbf{J} = \boldsymbol{\Sigma}\mathbf{K}' = \boldsymbol{\beta}\mathbf{Q} = \boldsymbol{\beta}^* \end{cases} \end{aligned}$$

from which the impulse response functions and the confidence intervals around them can be constructed following the procedure suggested by MVW.

3 Codependence and Common Cycles

The idea of codependence was first suggested by Gourieroux and Peaucelle ([7]) as a way to extend the concept of cointegration to stationary variables. In fact, we have codependence among a set of stationary variables when the order of a moving average of the vector stationary process is decreased by taking a linear combination of the variables.

VE1 were the first to embed the notion of codependence (called cofeature by Engle and Kozicki [6]) into a nonstationary environment and then offered the notion of *common cycles* later extended to the definition of *non-synchronous common cycles (*VE2, building on the concept of Scalar Component Models suggested by Tiao and Tsay, [14]) as the consequence of the presence of codependence in the stationary part.

In the case of a common trend representation, the idea of codependence would be applied to the stationary part $\mathbf{C}^*(L)\boldsymbol{\varepsilon}_t$, an MA($\infty$) as such, but a linear combination of which might be of a smaller order. In what follows we propose to use the notion of *common cycles of order i*, where $i \geq 0$

3.1 Common Cycles of Order 0

A common cycle of order 0 is what VE1 call common cycle, and can usefully be defined starting from the definition of *Scalar Component Model (Serial Correlation Common Feature* in the terminology of VE1).

Definition 1 *Let us consider again the Wold MA representation of the reduced form model*

$$\Delta \mathbf{x}_t = \mathbf{C}(\mathbf{L})\boldsymbol{\varepsilon}_t.$$

We have a Scalar Component Model of order $(0,0)$ *in the terminology of Tiao and Tsay ([14]) if there exists a* $(n \times s_0)$ *matrix* $\tilde{\boldsymbol{\alpha}}_0$ *such that*

$$\tilde{\boldsymbol{\alpha}}_0'\mathbf{C}(L) = \tilde{\boldsymbol{\alpha}}_0'(\mathbf{I} + \mathbf{C}_1 L + \mathbf{C}_2 L^2 + \dots) = \tilde{\boldsymbol{\alpha}}_0'$$

that is, $\tilde{\boldsymbol{\alpha}}_0'$ *forms a basis for the intersection of the left nullspaces of* $\mathbf{C}_1$, $\mathbf{C}_2$, *etc. since it sets them all to zero.*

Definition 2 *The matrix* $\tilde{\boldsymbol{\alpha}}_0'$ *is called cofeature matrix of order* 0.

A different way of looking at the SCM(0,0) is to consider the common trend representation (in structural form)

$$\begin{aligned}\mathbf{x}_t &= \mathbf{x}_0 + \tilde{\boldsymbol{\Gamma}}_g \sum_{j=0}^{t-1} \boldsymbol{\eta}_{t-j} + \boldsymbol{\Gamma}^*(L)\boldsymbol{\eta}_t \\ &= \mathbf{x}_0 + \tilde{\boldsymbol{\Gamma}}_g \boldsymbol{\tau}_t + \mathbf{c}_t \end{aligned} \tag{20}$$

we have $\tilde{\boldsymbol{\alpha}}_0'\boldsymbol{\Gamma}^*(L) \equiv \tilde{\boldsymbol{\alpha}}_0'\mathbf{C}^*(L) = \mathbf{0}$ or, which is the same, $\tilde{\boldsymbol{\alpha}}_0'\mathbf{c}_t = \mathbf{0}$, so that $\tilde{\boldsymbol{\alpha}}_0'$ spans the nullspace of $\mathbf{c}_t$ and hence the rank of $\mathbf{c}_t$ is equal to $n - s_0$. The implication is that

$$\tilde{\boldsymbol{\alpha}}_0'\boldsymbol{\Gamma}_j^* = \mathbf{0} \qquad \text{and} \qquad \tilde{\boldsymbol{\alpha}}_0'\mathbf{C}_j^* = \mathbf{0} \qquad \forall\, j \geq 0 \tag{21}$$

Note that the matrix $\tilde{\boldsymbol{\alpha}}_0'$ has opposite characteristics to the $\boldsymbol{\alpha}'$ cointegration matrix. In fact, the latter transforms a nonstationary $I(1)$ vector $\mathbf{x}_t$ into lower dimension ($r < n$) combinations which are stationary. The former, instead, transforms $\mathbf{x}_t$ into lower dimension ($s_0 < n$) combinations of pure nonstationary elements (without cyclical components). As we will see, there might be intermediate cases of other combinations where the nonstationary component is preserved, but in which the stationary component $\mathbf{c}_t$ has a lower order MA representation.

The definition of common cycle of order 0 (or synchronous in VE1) follows.

Definition 3 (Common Cycle of Order 0) *If the rank of* $\mathbf{c}_t$ *is* $n-s_0$, *then* $\mathbf{c}_t = \mathbf{F}\tilde{\mathbf{c}}_{0t}$ *where* $\mathbf{F}$ *is a* $(n \times n-s_0)$ *matrix and the vector* $\tilde{\mathbf{c}}_{0t}$ *includes the lower dimension* $(n - s_0) < n$ *common cycles of order* 0.

Let us examine some of the implications of this (fairly restrictive) class of common cycles in detail.

For the reduced form, we have the result by VE1 (Proposition 2)

$$\mathbf{C}_0^* \equiv \sum_{j>0} \mathbf{C}_j \equiv (\mathbf{I} - \mathbf{C}(1)) \Rightarrow \tilde{\boldsymbol{\alpha}}_0'(\sum_{j>0} \mathbf{C}_j) = \mathbf{0} \Leftrightarrow \tilde{\boldsymbol{\alpha}}_0'\mathbf{C}(1) = \tilde{\boldsymbol{\alpha}}_0'$$

showing that $\tilde{\boldsymbol{\alpha}}_0'$ are the eigenvectors associated to unit eigenvalues of $\mathbf{C}(1)$[6].For the structural form we have

$$\begin{aligned} \tilde{\boldsymbol{\alpha}}_0'\boldsymbol{\Gamma}_0^* &= \mathbf{0} \Leftrightarrow \tilde{\boldsymbol{\alpha}}_0'(\sum_{j>0} \boldsymbol{\Gamma}_j) = \mathbf{0} \Leftrightarrow \tilde{\boldsymbol{\alpha}}_0'\boldsymbol{\Gamma}_0 = \tilde{\boldsymbol{\alpha}}_0'\boldsymbol{\Gamma}(1) \\ &\Leftrightarrow \begin{cases} \tilde{\boldsymbol{\alpha}}_0'\boldsymbol{\Gamma}_{0g} = \tilde{\boldsymbol{\alpha}}_0'\tilde{\boldsymbol{\Gamma}}_g \neq \mathbf{0} \\ \tilde{\boldsymbol{\alpha}}_0'\boldsymbol{\Gamma}_{0s} = \mathbf{0} \end{cases} \end{aligned}$$

Summing up, $\tilde{\boldsymbol{\alpha}}_0'\mathbf{c}_t = \mathbf{0}$ has a powerful meaning in terms of the constraints imposed on the structural form.

1. it makes the combination of instantaneous effects of shocks to the permanent component equal to the same combination of their long term impact;
2. it makes the combination of aggregate delayed effects of shocks to the permanent component equal to zero;
3. it makes the combination of aggregate delayed effects of shocks to the temporary component equal to zero.

A Special Case: $s_0 = k$. This case arises when the matrix $\tilde{\boldsymbol{\alpha}}_0'$ is of order $(n \times k)$. By analogy with the usual notation for the cointegration space and the space orthogonal to it which are denoted by $sp(\boldsymbol{\alpha})$, respectively, $sp(\boldsymbol{\alpha}_\perp)$, we will denote

the space spanned by the cofeature vectors as $sp(\tilde{\boldsymbol{\alpha}}_0) \subset \Re^k$, and

[6]Put it differently, the presence of common cycles of order 0, implies that $\mathbf{C}(1)$ has r zero eigenvalues (with $\boldsymbol{\alpha}$ associated eigenvectors), s_0 unit eigenvalues and $k - s_0$ eigenvalues different from 1 or from 0.

the space orthogonal to it as $sp(\tilde{\boldsymbol{\alpha}}_{0\perp}) \subset \Re^{n-k}$.

Then we have the following proposition which shows that $\tilde{\boldsymbol{\alpha}}_0 \subset sp(\boldsymbol{\alpha}_\perp)$, and that $\boldsymbol{\alpha} \subset sp(\tilde{\boldsymbol{\alpha}}_{0\perp})$:

Proposition 4 *We have*

$$\tilde{\boldsymbol{\alpha}}'_{0\perp}\mathbf{C}(1)\boldsymbol{\Gamma}_0 \equiv \tilde{\boldsymbol{\alpha}}'_{0\perp}\boldsymbol{\Gamma}(1) = \mathbf{0}$$

since $\tilde{\boldsymbol{\alpha}}'_{0\perp}\tilde{\boldsymbol{\Gamma}}_g = 0$.

Proof. The cofeature space is spanned by $\tilde{\boldsymbol{\alpha}}_0$ (the eigenvectors associated with the unit eigenvalues of $\mathbf{C}(1)$. The orthogonal space to it is associated with the zero eigenvalues of $\mathbf{C}(1)$ and hence belongs to the cointegration space. Hence $\tilde{\boldsymbol{\alpha}}_{0\perp} = \boldsymbol{\alpha}\mathbf{Q}$, with $\mathbf{Q}$ nonsingular $(r \times r)$ matrix, and $\tilde{\boldsymbol{\alpha}}_0 = \boldsymbol{\alpha}_\perp\mathbf{P}$, with $\mathbf{P}$ nonsingular $(k \times k)$ matrix. ■

Remark that, since the cofeature vectors of order 0 are the eigenvectors associated to the unit eigenvalues of $\mathbf{C}(1)$, when $s_0 = k$, the matrix $\mathbf{C}(1)$ is idempotent. Hence there are k eigenvalues equal to 1 and r eigenvalues equal to 0, with $r + k = n$. As a consequence, note that starting from $\mathbf{x}_t = \mathbf{C}(1)\sum_{j=0}^{\infty}\boldsymbol{\varepsilon}_{t-j} + \mathbf{C}^*(L)\boldsymbol{\varepsilon}_t$, premultiplying by $\mathbf{C}(1)$, and simplifying we get

$$\mathbf{C}(1)(\mathbf{x}_t - \mathbf{C}^*(L)\boldsymbol{\varepsilon}_t) = (\mathbf{x}_t - \mathbf{C}^*(L)\boldsymbol{\varepsilon}_t)$$

which shows how the presence of cofeature of order 0 is associated to the presence of unit eigenvalues when subtracting from $\mathbf{x}_t$ the short term dynamics $\mathbf{C}^*(L)\boldsymbol{\varepsilon}_t$.

3.2 Common Cycles of Higher Order

The extension to the concept of cofeatures of higher order (giving rise to non-synchronous common cycles in the terminology of VE2) is fairly straightforward, and more adherent to the original idea by Gourieroux and Peaucelle ([7]). Let us start from the following

Definition 5 *From the Wold MA representation of the reduced form model, the Scalar Component Model of order* $(0, i)$ *in the terminology of Tiao and Tsay ([14]) if there exists a* $(n \times s_i)$ *matrix* $\tilde{\boldsymbol{\alpha}}_i$ *such that*

$$\tilde{\boldsymbol{\alpha}}_i'\mathbf{C}(L) \equiv \tilde{\boldsymbol{\alpha}}_i'(\mathbf{I}+\mathbf{C}_1L+\mathbf{C}_2L^2+\ldots) = \tilde{\boldsymbol{\alpha}}_0'(\mathbf{I}+\mathbf{C}_1L+\mathbf{C}_2L^2+\ldots+\mathbf{C}_iL^i)$$

that is, the cofeature matrix $\tilde{\boldsymbol{\alpha}}_i'$ *of dimension* $(n \times s_i)$ *forms a basis for the intersection of the left nullspaces of* $\mathbf{C}_{i+1}$, $\mathbf{C}_{i+2}$, *etc. since it sets them all to zero.*

Definition 6 *The matrix* $\tilde{\boldsymbol{\alpha}}_i$ *is called cofeature matrix of order* i.

It is then straightforward, extending the results previously obtained for $\tilde{\boldsymbol{\alpha}}_0$, to see that any of the following is true

$$\tilde{\boldsymbol{\alpha}}_i'\mathbf{C}_j^* = \mathbf{0}\ \forall j \geq i \tag{22}$$

$$\tilde{\boldsymbol{\alpha}}_i'\mathbf{C}_j = \mathbf{0}\ \forall j > i \tag{23}$$

$$\tilde{\boldsymbol{\alpha}}_i'\boldsymbol{\Gamma}_j^* = \mathbf{0}\ \forall j \geq i \tag{24}$$

$$\tilde{\boldsymbol{\alpha}}_i'\boldsymbol{\Gamma}_j = \mathbf{0}\ \forall j > i \tag{25}$$

Let us spell out in detail the implications for the structural form of (24). We have

1. $$\tilde{\boldsymbol{\alpha}}_i'\sum_{j=0}^{i}\boldsymbol{\Gamma}_{jg} = \tilde{\boldsymbol{\alpha}}_i'\tilde{\boldsymbol{\Gamma}}_g$$

   that is, the combination of impacts up to lag i bearing on the permanent component is equal to the combination of long-run impacts. No relevant long-run dynamics is added for the combination after i.

2. $$\tilde{\boldsymbol{\alpha}}_i'\sum_{j=0}^{i}\boldsymbol{\Gamma}_{js} = \mathbf{0}$$

   that is, the combination of impacts up to lag i bearing on the transitory component is equal to zero.

It is instructive to write the common trend representation as

$$\begin{aligned} \mathbf{x}_t &= \tilde{\mathbf{\Gamma}}_g \boldsymbol{\tau}_t + \mathbf{c}_t \equiv \tilde{\mathbf{\Gamma}}_g \boldsymbol{\tau}_t + \mathbf{\Gamma}^*(L)\boldsymbol{\eta}_t \\ &\equiv \tilde{\mathbf{\Gamma}}_g \boldsymbol{\tau}_t + \mathbf{\Gamma}^{*i}(L)\boldsymbol{\eta}_t + \bar{\mathbf{\Gamma}}^{*i}(L)\boldsymbol{\eta}_t \equiv \tilde{\mathbf{\Gamma}}_g \boldsymbol{\tau}_t + \mathbf{c}_t^i + \bar{\mathbf{c}}_t^i \end{aligned} \quad (26)$$

where the matrix polynomial $\mathbf{\Gamma}^*(L)$ is divided into two parts, $\mathbf{\Gamma}^{*i}(L) \equiv (\mathbf{\Gamma}_0^* + \mathbf{\Gamma}_1^* L + \ldots + \mathbf{\Gamma}_{i-1}^* L^{i-1})$ and $\bar{\mathbf{\Gamma}}^{*i}(L) \equiv (\mathbf{\Gamma}_i^* L^i + \mathbf{\Gamma}_{i+1}^* L^{i+1} + \ldots)$ to visualize immediately the meaning of common cycles of higher order (the upper bar indicates the part of the polynomial which is set to zero by the cofeature matrix). The two terms $\mathbf{c}_t^i$ and $\bar{\mathbf{c}}_t^i$, in fact, represent the stationary part affecting, respectively, up to (and not including) period i in the future, and from period i on. The presence of cofeature of order i, therefore, shows that the influence of the shocks starting from period i draws on a smaller number of shocks. In fact, the matrix $\tilde{\boldsymbol{\alpha}}_i$ is such that $\tilde{\boldsymbol{\alpha}}_i' \mathbf{\Gamma}^{*i}(L) \neq \mathbf{0}$, but $\tilde{\boldsymbol{\alpha}}_i' \bar{\mathbf{\Gamma}}^{*i}(L) = \mathbf{0}$, so that $\tilde{\boldsymbol{\alpha}}_i'$ is defined as spanning the left nullspace of $\bar{\mathbf{\Gamma}}^{*i}(1)$, (intersection of the left nullspaces of $\mathbf{\Gamma}_i^*, \mathbf{\Gamma}_{i+1}^*, \ldots$), establishing a linear dependence among the columns of $\bar{\mathbf{\Gamma}}^{*i}(1)$. Hence, in reference to (26), we can derive the following

Proposition 7 *Given that $\bar{\mathbf{\Gamma}}^{*i}(L)$ has reduced rank, it is possible to write*

$$\bar{\mathbf{c}}_t^i = \bar{\mathbf{\Gamma}}^{*i}(L)\boldsymbol{\eta}_t = \mathbf{F}^{(i)} \tilde{\mathbf{\Gamma}}^{*i}(L)\boldsymbol{\eta}_t \equiv \mathbf{F}^{(i)} \tilde{\mathbf{c}}_t^i$$

where $\mathbf{F}^{(i)}$ is an $(n \times n - s_i)$ matrix and $\tilde{\mathbf{\Gamma}}^{*i}(L) = (\tilde{\mathbf{\Gamma}}_i^* L^i + \tilde{\mathbf{\Gamma}}_{i+1}^* L^{i+1} + \ldots)$. The matrices $\tilde{\mathbf{\Gamma}}_j^*$, $j \geq i$ are each of order $(n - s_i \times n)$.

Proof. It parallels the argument of VE1 p.344. ■

Assuming that there are no common cycles up to order $i - 1$, this proposition means that, starting form period i, the n shocks $\boldsymbol{\eta}_t$ do not exert their influence independently of each other.

Definition 8 (Common Cycles of Order i) *The common cycles of order i are defined as the $(n - s_i \times 1)$ vector $\tilde{\mathbf{c}}_t^i$.*

Note that, unlike the case of the decomposition implied by common trends into shocks contributing to the permanent and transitory components, since $n - s_i > n - k$, the contribution of the two kinds of shocks to the common cycle cannot be explicitly derived.

3.3 The Relationship between Common Cycles of Different Orders

Of course, nothing precludes the data from exhibiting common cycles of different orders. In other words, there may be more than one non-zero s_i, the detection of which is left to the estimation and testing procedure. Nevertheless, we can derive some theoretical characteristics that common cycles of different orders must obey to. First, we can propose the following:

Lemma 9 *The spaces spanned by $\tilde{\boldsymbol{\alpha}}_i'$ and $\tilde{\boldsymbol{\alpha}}_j'$ with $i \neq j$ have zero intersection.*

Proof. W.l.o.g. assume that $i < j$. Recall that $\tilde{\boldsymbol{\alpha}}_i'$ spans the intersection of the left nullspaces of $\mathbf{C}_h^*$, $h \geq i$, while $\tilde{\boldsymbol{\alpha}}_j'$ spans the intersection of the left nullspaces of $\mathbf{C}_h^*$, $h \geq j$.

$$\begin{cases} \tilde{\boldsymbol{\alpha}}_i'\mathbf{C}_l^* = \mathbf{0} \\ \tilde{\boldsymbol{\alpha}}_j'\mathbf{C}_l^* \neq \mathbf{0} \end{cases} \text{for } i \leq l \leq j.$$

■

Corollary 10 $\bigcup_{i=0}^{\infty} sp(\tilde{\boldsymbol{\alpha}}_i')$ *has dimension* $\sum_{i=0}^{\infty} s_i$.

We then get the following theorem which generalizes Theorem 18 in VE1:

Theorem 11 *The dimension of the union of the spaces spanned by the cofeature vectors of all order is equal to k.*

Proof. It follows from the definition of a cofeature vector, which is orthogonal to any cointegrating vector. The union of spaces spanned by vectors orthogonal to the cointegrating vectors is equal to the orthogonal complement ($\boldsymbol{\alpha}_\perp \in \Re^k$) to the cointegrating space. ■

3.4 The Complete Set of Common Cycles

By definition of cointegration space, it must be $\sum_{i=0}^{\infty} s_i = k$. For all practical purposes, one may expect that there exists a finite h such that $\sum_{i=0}^{h} s_i = k$.

In such an instance, let us denote by $\tilde{\boldsymbol{\alpha}}'_*$ the $(k \times n)$ matrix obtained by stacking all cofeature matrices[7]

$$\tilde{\boldsymbol{\alpha}}'_* \equiv \begin{pmatrix} \tilde{\boldsymbol{\alpha}}'_0 \\ \tilde{\boldsymbol{\alpha}}'_1 \\ \vdots \\ \tilde{\boldsymbol{\alpha}}'_h \end{pmatrix}. \tag{27}$$

Again, we will present the results for the structural form, recalling that the results apply also to the reduced form. The matrix $\tilde{\boldsymbol{\alpha}}'_*$ has the property that

$$\begin{cases} \tilde{\boldsymbol{\alpha}}'_* \boldsymbol{\Gamma}^*_{h-1} \neq \mathbf{0} & \\ \tilde{\boldsymbol{\alpha}}'_* \boldsymbol{\Gamma}^*_i = \mathbf{0} & \forall\, i \geq h \end{cases}.$$

We are now able to define a *complete set of common cycles* $\tilde{\mathbf{c}}^{\Lambda}_t$. Let us rewrite (26) as

$$\mathbf{x}_t = \tilde{\boldsymbol{\Gamma}}_g \boldsymbol{\tau}_t + \mathbf{c}_t \equiv \tilde{\boldsymbol{\Gamma}}_g \boldsymbol{\tau}_t + \boldsymbol{\Gamma}^*_{\Lambda}(L)\boldsymbol{\eta}_t + \bar{\boldsymbol{\Gamma}}^*_{\Lambda}(L)\boldsymbol{\eta}_t \equiv \tilde{\boldsymbol{\Gamma}}_g \boldsymbol{\tau}_t + \mathbf{c}^{\Lambda}_t + \bar{\mathbf{c}}^{\Lambda}_t \tag{28}$$

with an obvious extension of notation with respect to (26). It is clear that, next to the common trends element, there is a part, $\mathbf{c}^{\Lambda}_t$, which does not disappear upon premultiplication by $\tilde{\boldsymbol{\alpha}}'_*$, whereas $\bar{\mathbf{c}}^{\Lambda}_t$ does. We then offer the following definition of a *complete set of common cycles.*

Definition 12 *The complete set of common cycles is defined as an $(n - k \times 1)$ vector $\tilde{\mathbf{c}}^{\Lambda}_t$ representing combinations of the original shocks giving rise to $\bar{\mathbf{c}}^{\Lambda}_t$ through the $(n \times n - k)$ matrix $\mathbf{F}^{(\Lambda)}$:*

$$\bar{\mathbf{c}}^{\Lambda}_t = \mathbf{F}^{(\Lambda)} \tilde{\mathbf{c}}^{\Lambda}_t$$

$\tilde{\mathbf{c}}^{\Lambda}_t$ is called a *complete set*, because the corresponding stacked cofeature matrix defines all the available space as belonging to $\Re^k$.

[7] It is not necessary that all orders between 0 and h are represented.

4 Codependence and Identification of Structural Innovations

We are now in the position of showing how the identification of structural shocks is achieved when the restrictions implied by both cointegration and codependence are taken into account. Recall the expression (14) linking structural and reduced-form shocks

$$\boldsymbol{\Gamma}_0^{-1}\boldsymbol{\varepsilon}_t = \boldsymbol{\eta}_t \Longleftrightarrow \begin{pmatrix} \mathbf{G} \\ \mathbf{K} \end{pmatrix} \boldsymbol{\varepsilon}_t = \begin{pmatrix} \boldsymbol{\eta}_{1t} \\ \boldsymbol{\eta}_{2t} \end{pmatrix}$$

with the $(k \times n)$ matrix $\mathbf{G}$ pertaining to the permanent component of $\mathbf{x}_t$ and the $(r \times n)$ matrix $\mathbf{K}$ pertaining to the transitory component. Let us see now how the presence of a complete set of common cycles, i.e $\sum_{i=0}^{h} s_i = k$, modifies the procedures outlined in section 2.3 to derive $\mathbf{G}$ and $\mathbf{K}$. Since $\boldsymbol{\Gamma}_0$ is not known, we will make use of the implications of cofeature matrices on the reduced form.

4.1 The Identification of Permanent Innovations

We have already obtained an identifying expression for the matrix $\mathbf{G}$ given in 2.3 above starting from the reduced rank of $\mathbf{C}(1)$ and its decomposability as $\widetilde{\boldsymbol{\Gamma}}_g\mathbf{G}$. The existence of a complete set of common cycles implies that $\widetilde{\boldsymbol{\alpha}}'_*\mathbf{C}(1) = \widetilde{\boldsymbol{\alpha}}'_*\mathbf{C}_\Lambda$ so that, by substituting, we get

$$\mathbf{G} = (\widetilde{\boldsymbol{\alpha}}'_*\widetilde{\boldsymbol{\Gamma}}_g)^{-1}\widetilde{\boldsymbol{\alpha}}'_*\mathbf{C}(1) = (\widetilde{\boldsymbol{\alpha}}'_*\widetilde{\boldsymbol{\Gamma}}_g)^{-1}\widetilde{\boldsymbol{\alpha}}'_*\mathbf{C}_\Lambda \tag{29}$$

Since the cofeature matrix $\widetilde{\boldsymbol{\alpha}}'_*$ does not alter the long-run properties of the system, the expression for $\mathbf{G}$ is equivalent to the one in MVW, although here the presence of codependence is explicitly taken into account.

4.2 The Identification of Transitory Innovations

The other submatrix $\mathbf{K}$ can now be derived. Let us first establish the following

Lemma 13 *The matrix* $\mathbf{C}_\Lambda$ *is invertible.*

Proof. We build our proof on two basic results:

1. The matrix $\tilde{\boldsymbol{\alpha}}'_*$ defines the left nullspace of $\overline{\mathbf{C}}_\Lambda$, i.e.

$$\tilde{\boldsymbol{\alpha}}'_*\mathbf{C}_\Lambda \neq \mathbf{0}; \qquad \tilde{\boldsymbol{\alpha}}'_*\overline{\mathbf{C}}_\Lambda = \mathbf{0}; \tag{30}$$

2. The matrix $\boldsymbol{\alpha}$ defines the left nullspace of $\mathbf{C}(1)$, i.e.

$$\boldsymbol{\alpha}'\mathbf{C}(1) \equiv \boldsymbol{\alpha}'\mathbf{C}_\Lambda + \boldsymbol{\alpha}'\overline{\mathbf{C}}_\Lambda = \mathbf{0}. \tag{31}$$

Because of the orthogonality between $\tilde{\boldsymbol{\alpha}}_*$ and $\boldsymbol{\alpha}$, we can deduce that given the second expression in (30), $\boldsymbol{\alpha}'\overline{\mathbf{C}}_\Lambda$ cannot be equal to $\mathbf{0}$, so that $\boldsymbol{\alpha}'\mathbf{C}_\Lambda$ must be $\neq \mathbf{0}$. Hence

$$\begin{pmatrix} \tilde{\boldsymbol{\alpha}}'_* \\ \boldsymbol{\alpha}' \end{pmatrix} \mathbf{C}_\Lambda \neq \mathbf{0}$$

which establishes the result given that the matrix $\binom{\tilde{\boldsymbol{\alpha}}'_*}{\boldsymbol{\alpha}'}$ spans $\Re^n$. ■

On the basis of this crucial result we can then prove the following

Theorem 14 *In the presence of cointegration and codependence*

$$\mathbf{K} = \mathbf{P}\boldsymbol{\alpha}'(\mathbf{C}'_\Lambda)^{-1}\boldsymbol{\Sigma}^{-1}$$

where $\mathbf{P}$ *is a* $(r \times r)$ *invertible matrix chosen as to normalize the variance covariance matrix of* $\boldsymbol{\eta}_{2t}$ *to be the identity matrix.*

Proof. The orthogonality condition between $\boldsymbol{\eta}_{1t}$ and $\boldsymbol{\eta}_{2t}$ is expressed as

$$\mathbf{G}\boldsymbol{\Sigma}\mathbf{K}' \equiv (\tilde{\boldsymbol{\alpha}}'_*\tilde{\boldsymbol{\Gamma}}_g)^{-1}\tilde{\boldsymbol{\alpha}}'_*\mathbf{C}_\Lambda\boldsymbol{\Sigma}\mathbf{K}' = \mathbf{0}_{(k\times n-k)}$$

from which the result follows (up to a rotation matrix $\mathbf{P}$) given the orthogonality between $\tilde{\boldsymbol{\alpha}}_*$ and $\boldsymbol{\alpha}$. ■

A Special Case: Common Cycles of Order 0. In the case of k common cycles of order 0, recall that $\tilde{\boldsymbol{\alpha}}_0'\mathbf{C}(1) = \tilde{\boldsymbol{\alpha}}_0'$, so that the relevant orthogonality condition $\mathbf{G\Sigma K}' = \mathbf{0}$ takes on a simpler form

$$\mathbf{G\Sigma K}' \equiv (\tilde{\boldsymbol{\alpha}}_0'\tilde{\boldsymbol{\Gamma}}_g)^{-1}\tilde{\boldsymbol{\alpha}}_0'\boldsymbol{\Sigma}\mathbf{K}' = \mathbf{0}_{(k\times n-k)}$$

from which

$$\mathbf{K} = \mathbf{P}\boldsymbol{\alpha}'\boldsymbol{\Sigma}^{-1}.$$

It is to be stressed that the identification of $\mathbf{K}$ is achieved without having to resort to any a priori assumption on the behavior of the economy. In other words, the data will tell us in each instance whether a complete set of common cycles is achievable. Some of the evidence produced in the literature do point to the existence of such complete sets[8].

The derivation of the matrix $\boldsymbol{\Gamma}_0$ follows again also in the presence of cointegration and codependence.

$$\boldsymbol{\Gamma}_0 = (\mathbf{H}|\mathbf{J}), \text{ where } \begin{cases} \mathbf{H} = \boldsymbol{\Sigma}\mathbf{G}' = \boldsymbol{\Sigma}\mathbf{C}_\Lambda'\tilde{\boldsymbol{\alpha}}_*(\tilde{\boldsymbol{\Gamma}}_g'\tilde{\boldsymbol{\alpha}}_*)^{-1} \\ \mathbf{J} = \boldsymbol{\Sigma}\mathbf{K}' = (\mathbf{C}_\Lambda)^{-1}\boldsymbol{\alpha}\mathbf{P}' \end{cases}$$

This will enable us to derive the correct impulse response functions for both the shocks to the permanent and to the transitory components, with suitable modifications for deriving the confidence intervals around them. Only when no common cycles are detected, does the procedure suggested by MVW seem appropriate.

[8]For example, VE1 find in their first example (two variables) one common trend and one common cycle of order zero; and in their second (five variables) three common trends and two common cycles of order zero. Engle and Issler ([5]) find six common trends and two common cycles of order zero for eight variables, while Engle and Issler ([4]) find two common trends and one common cycle for three variables.

5 Concluding Remarks

In this paper, we have shown how the codependence characteristics obtained from an estimated VAR system could be used, and under which conditions, to address the issue of the impact of structural innovations on the system.

In order to fulfill this objective, we defined two crucial notions : the notion of *common cycle of order i* on the one hand, and the notion of a *complete set of common cycles* on the other hand. We then proved that, when such a set exists (a common occurrence in practice), and taking into account the restrictions provided by cointegration, it is possible to fully identify the contribution of innovations to the system, be they innovations to the non stationary part of the system or, more importantly, the innovations contributing only to the stationary part. In other words, the impulse-response functions of the endogenous variables to *all* innovations are then obtained.

The discussion presented here has been entirely theoretic, with the purpose of focusing on some analytical implications of the procedure. The implementation of the analysis starting from an estimated VAR model taking into account the set of restrictions imposed by the presence of cointegration and common cycles will allow for a better understanding of a macroeconomic system. In particular, it will allow to effer new insights on the relative importance and the dynamic impact of permanent innovations (customarily labelled "real") and transitory ("monetary") innovations.

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