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Rules of Thumb and Local Interaction

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Abstract

This paper studies an economy where boundedly rational agents choose one of two technologies. Agents ask some of their neighbors about their experiences and, in addition, may receive an exogenous signal. Using this information, they apply an exogenous rule of thumb to determine the better option. The non-trivial behavior of the economy is generated by the explicitly modeled neighborhood relation. Considering various rules of thumb, the paper shows that it is not true that agents will infer the better technology almost surely. If they receive an exogenous signal in addition to communicating with some neighbors, the signal determines the steady state of the economy. In contrast, if there is no exogenous signal, it is shown that the economy has two stationary states: the better technology may die out or it may drive out the worse one. The latter result is in sharp contrast to previous articles where almost sure convergence to the better technology can be ensured by certain conditions.

Keywords: Technology choice; local interaction; bounded rationality.

JEL classification: C78; D80.

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Contents

1 Introduction

Recently some authors have studied simple learning rules called rules of thumb. This line of research includes Ellison and Fudenberg (1993, 1995), and Banerjee and Fudenberg (1994). The simplicity of rules of thumb basically follows from two assumptions: first, current experience of different agents is the only source of information, and, secondly, there is no strategic interaction among agents. Typically an individual asks some members of the population about their current experiences and then chooses the option which seems to be better. The question of interest is whether such a naive exogenously given learning rule can lead to a socially optimal decision. The answer usually found is positive: while the economy does not always converge to the better state, there are parameter values for which it almost surely does, the crucial parameter being the number of agents each decision maker talks to. In summary, it is argued in recent articles that naive myopic rules can lead to the optimal decision if each agent communicates with a certain number of other agents.

Studying rules of thumb is of interest for essentially three reasons. First, they seem to be more realistic than complicated fully rational learning rules, for example Bayes rule. Secondly, accounting for personal communication is important since it is often the major source of individual information acquisition. And finally, the aggregate behavior of the economy can *explicitly* be derived from the simple *individual* decision rules. This leads to microfoundations of aggregate behavior that go beyond the standard paradigm of a representative agent.[1]

While the present paper shares the view that it is interesting to analyze naive learning rules, it argues that the results found by Ellison and Fudenberg (1993, 1995) and Banerjee and Fudenberg (1994)

[1]Despite the fact that nobody has ever met a representative agent, this paradigm seems to survive all challenges. For a discussion of problems with this approach see Kirman (1992); Kirman (1994) provides a survey of alternative models.

are not robust. Crucial features of their frameworks are that there is a continuum of agents and that interaction between any two agents in the population is feasible. While the *continuum of agents* approach is a convenient modelling device in various economic contexts, one must not take for granted that it is always justified to use it. It needs to be shown that working with a continuum of agents is not too much of an abstraction, that is, that the results found do not crucially depend on this assumption. The other standard assumption of the above papers is that interaction is *global*: everybody can communicate with everybody else in the population with equal probability. There certainly are plenty of economic situations where this is a reasonable assumption. For example, interaction in centralized markets is extensively global since all market participants can interact with each other. However, assuming global interaction seems rather odd in a context where personal communication crucially matters. In Ellison and Fudenberg's (1993) example of a farmer's production technology choice, for instance, agents can observe the average payoff across the whole population. However, personal communication realistically is restricted to a small number of other individuals, such as, friends or neighbors.

Since both of the above assumptions can be questioned, this paper investigates the same or similar rules as Ellison and Fudenberg (1993, 1995) and Banerjee and Fudenberg (1994) in a set–up with a *countable infinity* of individuals and with *local* interaction. The structure of the economy follows from the application of the theory on interacting particle systems.[2] More precisely, each of the countably infinite number of individuals is assumed to be able to communicate with a large but finite number of other agents, who are his neighbors. The agent's decision problem is to infer from the current experiences of the neighbors which

[2]The systematic mathematical treatment of the models used can be found in Liggett (1985) and Durrett (1988). A nice informal introduction is presented by Durrett and Levine (1994b). Föllmer (1974) appears to have been the first who used such a model to describe economic phenomena. More recently, economists have shown increasing interest in this approach; see, for example, An and Kiefer (1993, 1994), Bak, Chen, Scheinkman, and Woodford (1993), Blume (1993), Durlauf (1993) and Foley (1994). The importance of this kind of approach for economic theory was emphasized by Brock (1991).

of two available technologies is better to use in the future. The explicit description of the neighborhood relation is a novelty in the literature. Moreover, the results found are in strict contrast to those derived by Ellison and Fudenberg (1993, 1995) and Banerjee and Fudenberg (1994). While they could always find conditions for almost sure convergence to the better state, this economy converges to the good state with a positive probability less than one. Hence the long run outcome can be "bad" when boundedly rational rules guide decisions in a locally interacting environment.

Before going into detail, a methodological remark should be made. The precise derivation of the macroeconomic behavior from individual decisions requires an extensive use of the mathematical techniques developed to study interacting systems. In order to avoid the technical details that are not necessary to follow the analysis, I am going to present the model and the proofs rather informally. Hence the emphasis is put on the economic interpretations. However, the appendix contains the formal description of the model and the theorems and lemmas that are used in the paper.

The rest of the paper is organized as follows. Section II outlines informally a model of local interaction. Section III considers various rules of thumb, namely, rules with and without an exogenous signal. In both cases the long run aggregate behavior of the economy is derived. Section IV compares the results of the previous section with the findings of other works on mouth–to–mouth communication and herd behavior. Finally, section V concludes.

2 A model of local interaction

There is a countable infinity of agents living on a d–dimensional integer lattice $\mathbb{Z}^d$ for some $d \geq 1$. An agent with address $x \in \mathbb{Z}^d$ is connected with his nearest neighbors, where the set of neighbors is defined by $\mathcal{N}(x) = \{y : |x - y| = 1\}$. This definition implies that an agent has $2d$ neighbors and that the neighborhood relation is symmetric: if y is a neighbor of

x then x is a neighbor of y. Each agent can take two possible actions denoted by 0 and 1. The state of an agent at a particular location is defined by his current action. More precisely, let $\eta : \mathbb{Z}^d \longrightarrow \{0,1\}$ be a map and denote the family of η by E, then $\eta(x)$ gives the state of agent x while η describes the state of the whole economy. Let E be also endowed with an ordering defined by $\eta \leq \zeta$ for $\eta, \zeta \in E$ if and only if $\eta(x) \leq \zeta(x)$ for all $x \in \mathbb{Z}^d$. In addition, equipping E with the standard σ–field, the evolution of the economy over time $t \in [0, \infty)$ is determined by the mapping $c : \mathbb{Z}^d \times E \longrightarrow \mathbb{R}_+$ such that the process at time t $\eta_t \in E$ will satisfy

$$P\left(\eta_{t+\Delta t}(x) \neq \eta_t(x) | \eta_t = \eta\right) = c(x, \eta)\Delta t + o(\Delta t) \tag{1}$$

$$P\left(\eta_{t+\Delta t}(x) \neq \eta_t(x), \eta_{t+\Delta t}(y) \neq \eta_t(y) | \eta_t = \eta\right) = o(\Delta t) \tag{2}$$

as $\Delta t \downarrow 0$ $\forall x, y \in \mathbb{Z}^d$ satisfying $x \neq y$, and $\forall \eta \in E$.

The revision rate $c(x, \eta)$ defines a *spin economy* as a continuous time strong Markov process η_t on the product space E. The Markov property implies that η_t is uniquely determined for any initial state η. This construction departs substantially from the traditional Markovian universe. Since agents always interact with the same neighbors, the decisions are naturally correlated. Therefore, the evolution of one individual decision is no longer Markovian. Such models are usually avoided in economic theory by either assuming random matching schemes or by introducing some noise which suppress the correlation among individuals. In contrast, our model of local interaction remains tractable despite the correlation across agents because the state of the whole economy η_t is a Markov process.

The definition of the process is rather informal because no argument is presented as to how η_t can be uniquely constructed from the revision rate such that η_t is right–continuous in and has a left limit at each $t \geq 0$. The formal arguments can be found in appendix A.

Several assumptions are made in order to maintain tractability of the analysis. *First*, the model is formulated as a continuous time Markov process where at any point in time only one agent has the opportunity to revise. The "lucky"person who can revise is chosen randomly from $\mathbb{Z}^d$ according to a Poisson process as stated by equations (1) and (2). *Secondly*, some regularities are imposed in order to obtain a "nicely" behaved Markov process. In particular, $c(x, \eta)$ is taken to be bounded in the sense that any single agent's effect on the entire economy be finite. This feature will be ensured by the assumption that the revision rate depends on the state of the nearest neighbors, each of them having a finite effect on the economy. It should be noted, however, that any agent is connected with all other agents through a chain of neighborhood structures. This means that any agent's decisions are affected either directly or indirectly by all decisions of other agents. The other important assumption on the revision rate is that it is spatially homogeneous, meaning that if the initial state of the economy is shifted then the time evolution shifts only in space. [This property is also called translation invariance.] The previous assumption is convenient since it makes agents identical in the sense that each individual follows the same decision rule. However, it is also clear that the results of any two decisions differ according to how the states of the agents in the two neighborhoods vary.

Having outlined the model, a couple of observations can be made. *First*, the lattice should be regarded as a *communication* rather than a spatial structure. Clearly, we are not living in a d–dimensional space, but we certainly communicate with a lot of people. Communication is likely to be more intensive before a decision is made. In the model, this is captured by the neighborhood structure. *Secondly*, the decision of an agent, which is defined by his revision rate, is clearly *boundedly rational*. This comes about because agents base their decisions only on current payoffs rather than on the expected present discounted value of future ones. However, as noted by Blume (1993), myopic rules may be fully rational if the mean waiting time between two revisions is large compared to the discount rate. In addition, bounded rationality also follows from the assumption that the rules of thumb are not optimal learning rules. They are simply given exogenously.[3] *Finally*, an *agent*

[3]However, such a rules could also be derived from optimizing behavior. Rosenthal (1993) presents examples of how players choose between different rules according to

cannot revise his decision at any time. After a decision, the time of waiting for a new decision opportunity is distributed exponentially with parameter one. The resulting commitment to a chosen technology may be justified by the presence of some decision costs (rather than by the laziness of the agents). For example, there may exist costs of collecting information, switching costs from one action to another, or adjustment costs of any other form. Alternatively, one might think of agents with an exponentially distributed lifetime, who decide once and for all at the beginning of their lives about the preferred action. When an agent dies, he then gives birth to one offspring, who can choose an action again.

In summary, all agents have i.i.d. Poisson "alarm clocks" [Copyright Lawrence E. Blume]. An agent located at a given site chooses one of the two possible actions and commits himself to stick to it until his alarm clock goes off. At this moment he looks at his neighbors and revises the previous decision according to some rule. The decision rule is defined by the revision rate. The main goal of the subsequent analysis of this spin economy is to characterize the long run macroeconomic behavior of the system. In particular, the interest is in determining the set of invariant distributions for the process η_t, and in deriving conditions for the ergodicity of this process.[4]

Finally, there is an important class of revision rates which shall play an important role in the sequel. Its formal definition goes as follows:

Definition 1 A spin economy with revision rate $c(x, \eta)$ is *attractive* if whenever $\eta \leq \zeta \in E$, then for all $x \in \mathbb{Z}^d$

$$\begin{aligned} c(x,\eta) &\leq c(x,\zeta) \qquad \text{if} \quad \eta(x) = \zeta(x) = 0, \qquad \text{and} \\ c(x,\eta) &\geq c(x,\zeta) \qquad \text{if} \quad \eta(x) = \zeta(x) = 1. \end{aligned}$$

Agents applying such a decision rule are more likely to adopt an action the more of their neighbors do it already. Since individuals attract the decisions of each other, the revision rate is called attractive. Alternatively,

their cost of use.

[4]A process is called ergodic if it converges to a unique invariant distribution for any initial η.

we can say that this decision rule incorporates popularity weighting, i.e. the more popular option is more likely to be adopted by the decision maker.

3 Rules of thumb on a lattice

Following the literature on rules of thumb, it is assumed that the two states of the spin economy are associated with two technologies that have payoffs u^1 and u^0. The superscript 1 [0] refers to the state of an agent at time t, that is, if $\eta_t(x) = 1$ [$\eta_t(x) = 0$], the agent x uses the first [second] technology at time t. The agent has to decide which technology to use. The difference between the stochastic payoffs is given by

$$u^1 - u^0 = \theta + \epsilon_t^1 - \epsilon_t^0, \tag{3}$$

where θ is a nonnegative constant and the ϵ_t^i are normally distributed idiosyncratic shocks with zero mean and standard deviation σ, that is, $\epsilon_t^i \sim \mathcal{N}(0, \sigma)$. The expected payoff difference is θ. However, the users of both technologies face random shocks, which are independent across technologies and agents. This means that agents working with the same technology may well earn different profits at the same time.

As noted before, the long run behavior of the spin economy depends heavily on the decision rule applied. In the next two subsections some different rules of thumb are analyzed.

3.1 Rules of thumb with an exogenous signal

It is assumed that the set of available technologies is common knowledge, that is, each agent knows the existence of the two technologies even if he has only used one of them. At time zero an initial state η is set exogenously. At any time $t \geq 0$ an exogenous signal arrives at a randomly chosen location x. This signal tells agent x which technology is better. The signal reports, with probability α, that $\mathbf{E}(u^1) \geq \mathbf{E}(u^0)$ and with probability $1-\alpha$ that $\mathbf{E}(u^0) \geq \mathbf{E}(u^1)$. Individuals do not necessarily trust in the signal, therefore agent x follows it only with probability λ which is assumed to satisfy $0 < \lambda < 1$. However, with probability $1-\lambda$, he asks for the opinion of one of his neighbors. He does so because he thinks that a technology is more likely to be better today if it performed better yesterday, and if it performed better yesterday, it is more likely to be used by the neighbors. This conjecture results in imitation regardless of whether the imitated technology performs currently better in the neighborhood or not. Hence, I call this decision rule *pure imitation.* The probability of meeting an agent who is using the better technology initially is $P_{\mathcal{N}(x),1} = (1/2d)\sum_{y\in\mathcal{N}(x)} \eta(y)$, while the probability of meeting one using the worse technology is $P_{\mathcal{N}(x),0} = 1 - P_{\mathcal{N}(x),1}$.

According to the description above, the rate of revision for agent x can be written as

$$c(x,\eta) = \begin{cases} \alpha\lambda + \dfrac{1-\lambda}{2d} \displaystyle\sum_{y\in\mathcal{N}(x)} \eta(y) & \text{if} \quad \eta(x) = 0 \\ (1-\alpha)\lambda + \dfrac{1-\lambda}{2d} \displaystyle\sum_{y\in\mathcal{N}(x)} (1-\eta(y)) & \text{if} \quad \eta(x) = 1, \end{cases} \tag{4}$$

where $0 < \lambda < 1$. Moreover, it should be noted that the population is homogeneous in the sense that each agent applies the same decision rule where both α and λ are the same for all agents. However, the actual value of $c(x,\eta)$ varies from agent to agent since it depends on the particular neighborhood structure. The long run behavior of this economy is described by the following proposition:

Proposition 1 *The spin economy resulting from the rule of thumb with pure imitation and an exogenous signal is ergodic. It converges with exponential rate λ from any initial distribution to the product measure $\nu\{\eta : \eta(x) = 1\} = \alpha$.*

Proof: A rigorous proof of the proposition can be obtained by simple application of lemma 1 as given in appendix C. A less formal proof goes as follows. The assumption of spatial homogeneity is satisfied since $c(x,\eta)$ depends on x through η only. In addition, the above decision rule defines a spin economy in which an agent is more likely to switch to an alternative technology, the more of his neighbors use this technology. Such an economy is also called attractive. For an attractive, spatially homogeneous economy, theorem 7 in appendix B applies. It says that starting from the state where everybody uses the worse (better) technology, the joint probability of using the better one is increasing (decreasing) over time and has a limit in both cases. This implies that a spin economy is ergodic if and only if the two limits for these probabilities are the same.

To show this, let η^0 be the state in which everybody uses the worse technology at time 0, μ_t^0 the distribution of the process at time t starting from η^0, and δ^0 the pointmass on η^0. Since everybody uses the same technology, δ^0 is spatially homogeneous. Furthermore, the strong Markov property of η_t ensures that μ_t^0 has the same characteristic $\forall t \geq 0$. Roughly speaking, the micro behavior of the economy is compatible with the macro one for all $t \geq 0$ in such a way that the probability that an arbitrary agent uses the better technology is the same for any agent. Formally, the marginal distribution of μ_t^0 denoted by $\rho_t^0 \equiv \mu_t^0\{\eta : \eta(x) = 1\}$ is independent of x. Moreover, the spatial homogeneity also implies that $\varrho^0(x,y) \equiv \mu_t^0\{\eta : \eta(x) = 1, \eta(y) = 0\} = \mu_t^0\{\eta : \eta(x) = 0, \eta(y) = 1\}$ for any $x, y \in \mathbb{Z}^d$.

While the marginal distribution is constant *across agents*, it evolves over time. At any moment ρ_t changes because of the decision of one and only one agent whose "Poisson alarm clock" rang. Given the state of his neighbors $y \in \mathcal{N}(x)$ at t, the probability that agent x use the better technology at time $t + \Delta t$ is

$$\begin{aligned} \rho_{t+\Delta t}^0 &= (1-\rho_t^0)\left[(\alpha\lambda + (1-\lambda)\frac{\varrho^0(x,y)}{1-\rho_t^0}\right]\Delta t \\ &+ \rho_t^0\left[1 - \left((1-\alpha)\lambda + (1-\lambda)\frac{\varrho^0(x,y)}{\rho_t^0}\right)\Delta t\right] + o(\Delta t). \qquad (5) \end{aligned}$$

The *first term* on the right hand side is the probability that an agent using the worse technology switches to the better one in a short period of time. Looking at the details, we can see that $\alpha\lambda$ is the probability that the individual receives and follows a signal which favors the better technology. In addition, if he does not follow the signal, he still has a chance to meet someone using the better technology. If x is a worse technology user, this event occurs with probability $(1-\lambda)\varrho^0(x,y)/(1-\rho_t^0)$. The first term now simply is the sum of these probabilities multiplied by Δt. Let us turn now to the *second term* on the right hand side which is the probability that an agent using the better technology *does not switch* to the worse one in a short period of time. The wrong signal is followed with probability $(1-\alpha)\lambda$. Moreover, if the agent does not follow the signal, he may run into a neighbor who operates the worse technology. The likelihood that this happens is $(1-\lambda)\varrho^0(x,y)/\rho_t^0$ if the agent uses the better technology. Adding the two terms and multiplying by Δt provides the probability that a user of the better technology switches to the worse one in a short period of time. Since we are interested in the complement event, we subtract this expression from 1 which yields the second term in (5).

Rearranging equation (5) gives

$$\frac{\rho_{t+\Delta t}^0 - \rho_t^0}{\Delta t} = \alpha\lambda - \lambda\rho_t^0 + \frac{o(\Delta t)}{\Delta t}. \tag{6}$$

Taking the limit for $\Delta t \downarrow 0$ leads to

$$\dot{\rho}_t^0 = \alpha\lambda - \lambda\rho_t^0. \tag{7}$$

Since the ergodic distribution is invariant over time, it can be obtained by setting the left hand side of (7) equal to zero and solving the resulting equation for ρ_t^0. This obviously leads to the desired result.

It is also easy see that the arguments of equation (5) do not depend on the state from which the economy started from. Hence (5) remains valid if ρ_t^0 is replaced by ρ_t^1, which is defined the same way for η^1 as ρ_t^0 for η^0. This implies

$$\dot{\rho}_t^1 = \alpha\lambda - \lambda\rho_t^1. \tag{8}$$

The proof of the proposition is completed by noting that $\nu\{\eta : \eta(x) = 1\} = \lim_{t\to\infty} \rho_t^0 = \lim_{t\to\infty} \rho_t^1 = \alpha$ and that both (7) and (8) are simple differential equation that converge to the steady state with rate λ. □

This result is somewhat surprising since it asserts that *pure imitation may not matter in the long run* if there is a small exogenous probability that an agent revises his choice independently from his neighbors. In this case the effect of the neighbors on the decision of the agent vanishes in the long run. The exogenous signal or exogenous information is thus essential for the long run behavior of the system. In particular, *the economy converges almost surely to the steady state where the better technology is operated if and only if the exogenous signal almost surely is true.* This result implies that any institution that provides correct information globally might be more important for reaching the better state than the neighborhood communication. Imitation has no effect on the steady state, but it affects the transitional dynamics. If an agent puts more weight on the imitation (i.e. lowers λ), then he lowers the rate of convergence to the steady state and the economy needs more time to converge. Hence the speed of convergence has similar characteristics to that obtained by the traditional learning models. Namely, if agents rely less on their private signal, the less information is available at the aggregate.[Compare Vives (1993)]

I modify now the previous structure in order to analyze the robustness of the result. The most natural way to alter the pure imitation model is to assume that agent x receives the same exogenous signal as before, but that he gathers more information from the neighbors than in the case of pure imitation. As previously, he receives an exogenous signal and talks to a randomly chosen neighbor. However, now he asks not only which technology is used by this neighbor but also how the technology performs currently. Knowing the neighbor's payoff, he compares it with his return. If the randomly chosen neighbor operates the same technology as he does, nothing happens. In the other cases, he switches to the alternative opportunity if the neighbor earns more money than he currently does. Therefore this decision rule is called *imitation of success.* In order to formulate the revision rate, we need the following probabilities: As before, the probability of meeting somebody operating the better technology is $P_{\mathcal{N}(x),1}$. In addition to the previous rule of thumb, the probability that the payoff to agent x is lower than that of a randomly chosen $y \in \mathcal{N}(x)$ is

$$\begin{aligned} p &= P\left(\epsilon_t^1 - \epsilon_t^0 \geq \theta : \eta(x) = 0, \eta(y) = 1, y \in \mathcal{N}(x)\right) \\ &= 1 - \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2}\theta/\sigma}^{\infty} \exp\left(-\frac{x^2}{2}\right) dx, \end{aligned}$$

because $\epsilon_t^1 - \epsilon_t^0 \sim \mathcal{N}\left(0, \sigma/\sqrt{2}\right)$. Since $\theta \geq 0$, $p \geq 1/2$, it follows that the probability of meeting a neighbor who works with the better technology and whose technology is really performing better at this moment in time is $pP_{\mathcal{N}(x),1}$. Similarly, the probability of meeting someone with the worse technology that actually performs better at that time is $(1-p)(1-P_{\mathcal{N}(x),1})$.

Defining α and λ as before, the revision are of the afore described decision rule is given by

$$c(x,\eta) = \begin{cases} \alpha\lambda + \dfrac{p(1-\lambda)}{2d} \displaystyle\sum_{y \in \mathcal{N}(x)} \eta(y) & \text{if } \eta(x) = 0 \\ (1-\alpha)\lambda + \dfrac{(1-p)(1-\lambda)}{2d} \displaystyle\sum_{y \in \mathcal{N}(x)} (1-\eta(y)) & \text{if } \eta(x) = 1, \end{cases} \tag{9}$$

where $0 < \lambda < 1$.

Although we were not able to characterize the ergodic distribution explicitly, we found the following boundaries for the set of stationary measures:

Proposition 2 *The economy resulting from the rule of thumb with imitation of success and with an exogenous signal has a stationary distribution that satisfies the following inequality:*

$$\alpha \leq \nu\{\eta : \eta(x) = 1\} \leq 1 - \frac{1-\alpha}{1 + (2p-1)\dfrac{1-\lambda}{\lambda}}. \tag{10}$$

Proof: Since the set of stationary measures is a non–empty convex set by construction [as stated in the appendix B], it has to be shown only that the above inequalities hold. The proof is based on the idea that if there is a certain relation between the revision rates of two stochastic processes which are defined on the same probability space, then a similar relationship holds between their probability measures. This is stated formally in theorem 6 in appendix B. Intuitively, if η_t is more likely than ζ_t to reach state A, but is less likely than ζ_t to leave A, then the probability that η_t is in A is at least as high as the probability that ζ_t is in A.

In order to prove the inequalities, define two processes ζ'_t and ζ''_t with the following revision rates:

$$c'(x,\zeta') = \begin{cases} \alpha\lambda + \dfrac{p(1-\lambda)}{2d} \displaystyle\sum_{y\in\mathcal{N}(x)} \zeta'(y) & \text{if} \quad \zeta'(x) = 0 \\ (1-\alpha)\lambda + \dfrac{p(1-\lambda)}{2d} \displaystyle\sum_{y\in\mathcal{N}(x)} (1-\zeta'(y)) & \text{if} \quad \zeta'(x) = 1, \end{cases}$$

and

$$c''(x,\zeta'') = \begin{cases} \alpha\lambda + (2p-1)(1-\lambda) + \dfrac{(1-p)(1-\lambda)}{2d} \displaystyle\sum_{y\in\mathcal{N}(x)} \zeta''(y) & \text{if } \zeta''(x) = 0 \\ (1-\alpha)\lambda + \dfrac{(1-p)(1-\lambda)}{2d} \displaystyle\sum_{y\in\mathcal{N}(x)} (1-\zeta''(y)) & \text{if } \zeta''(x) = 1. \end{cases}$$

It is easy to see that there is a certain relation between these two revision rates, namely, if $\zeta' \leq \eta \leq \zeta''$, then

$$c'(x,\zeta') \leq c(x,\eta) \leq c''(x,\zeta'') \qquad \text{if} \quad \eta(x) = \zeta(x) = 0, \qquad \text{and}$$
$$c'(x,\zeta') \geq c(x,\eta) \geq c''(x,\zeta'') \qquad \text{if} \quad \eta(x) = \zeta(x) = 1.$$

Theorem 6 in appendix B implies that if $\zeta' \leq \eta \leq \zeta''$ then $\mu'_t \leq \mu_t \leq \mu''_t$ for all $t \geq 0$ where μ'_t, μ_t, μ''_t are the distributions of the corresponding processes at time t. It follows that if ζ'_t (ζ''_t) is ergodic then its ergodic distribution is the lower (upper) bound for the set of stationary measures for η_t

To prove the proposition, we can follow similar arguments as in the proof of the previous proposition [rigorous result can be obtained again by lemma 1 in appendix C] because the revision rates are spatially homogeneous and attractive. Let us consider the process ζ'_t first. The distribution of ζ'_t at time t is given by $\mu^{i\prime}_t$ where $i = \{0,1\}$ refers to the initial state which is either η^0 or η^1. Let $\rho^{i\prime}_t \equiv \mu^{i\prime}_t\{\eta : \eta(x) = 1\}$ which is independent of x due to the spatial homogeneity for all $t \geq 0$. Given a state at, if the "Poisson alarm clock" of one agent rings and t, the conditional probability that agents use the better technology at time

$t + \Delta t$ is

$$\begin{aligned} \dot{\rho}^{i'}_{t+\Delta t} &= (1-\rho^{i'}_t)\left[\alpha\lambda + p(1-\lambda)\frac{\varrho^{i'}(x,y)}{1-\rho^{i'}_t}\right]\Delta t \\ &+\rho^{i'}_t\left[1-\left((1-\alpha)\lambda + p(1-\lambda)\frac{\varrho^{i'}(x,y)}{\rho^{i'}_t}\right)\Delta t\right] + o(\Delta t), \quad (11) \end{aligned}$$

where $\varrho^{i'}(x,y)$ is defined similarly to $\varrho^{0}(x,y)$ in the proof of proposition 1. Rearranging, dividing by Δt and taking the limit as $\Delta t \downarrow 0$ results in

$$\dot{\rho}^{i'}_t = \alpha\lambda - \lambda\rho^{i'}_t. \quad (12)$$

By the arguments in the previous proposition, it follows immediately that the ergodic distribution of the process ζ'_t is given by

$$\nu'\{\zeta' : \zeta'(x) = 1\} = \alpha. \quad (13)$$

We can redo the above exercise for ζ''_t which gives the following equations for $i = \{0, 1\}$

$$\begin{aligned} \dot{\rho}^{i''}_{t+\Delta t} &= (1-\rho^{i''}_t)\left[\alpha\lambda + (2p-1)(1-\lambda) + (1-p)(1-\lambda)\frac{\varrho^{i''}(x,y)}{1-\rho^{i''}_t}\right]\Delta t \\ &+\rho^{i''}_t\left[1-\left((1-\alpha)\lambda + (1-p)(1-\lambda)\frac{\varrho^{i''}(x,y)}{\rho^{i''}_t}\right)\Delta t\right] + o(\Delta t), \quad (14) \end{aligned}$$

where $\varrho^{i''}(x,y)$ is defined similarly to $\varrho(x,y)$ in the proof of proposition 1. As $\Delta t \downarrow 0$, this turns out to be

$$\dot{\rho}^{i''}_t = \alpha\lambda + (2p-1)(1-\lambda) - [\lambda + (2p-1)(1-\lambda)]\rho^{i''}_t. \quad (15)$$

It follows directly that the unique invariant distribution of the process ζ''_t is given by

$$\nu''\{\zeta'' : \zeta''(x) = 1\} = \frac{\alpha\lambda + (2p-1)(1-\lambda)}{\lambda + (2p-1)(1-\lambda)}. \quad (16)$$

Having found the ergodic distributions $\nu\{\zeta' : \zeta'(x) = 1\}$ and $\nu\{\zeta'' : \zeta''(x) = 1\}$, the proof of the proposition clearly follows. □

Not surprisingly, the result implies that if an agent imitates the successful neighbors, he reaches a state which is at least as good as the state he can reach by pure imitation. In addition, similar to the case of pure imitation, the exogenous signal has a crucial influence on the long run behavior of the economy, namely, *the economy converges almost surely to the steady state where everybody operates the better technology if and only if the exogenous signal is true almost surely.* If the probability, α, that the signal is true, is close to 1, the long run equilibrium will thus be close to the situation where every agent uses the better technology with probability one. However, if α is small, because the institutions providing information for the public operate badly, then the ergodic distribution of the process is bounded away from 1.

As in the previous rule of thumb example, the exogenous signal turned out to be important, since in both cases its has an essential role in determining the long run outcome. In particular, the role of the exogenous signal becomes more important if the economy is very noisy. If $\sigma \uparrow \infty$, then $p \to 1/2$, hence $\nu\{\eta : \eta(x) = 1\} \to \alpha$, that is, in a high noise regime the intuitive result is that the exogenous signal is the only reliable piece of information, governing the long run behavior of the economy.

3.2 Rules of thumb without an exogenous signal

Until up this point, individuals have benefited from the exogenous signal. It is natural to ask whether there is a significant difference between the regimes with or without an exogenous signal. As we will see, without an exogenous signal a qualitatively different economy results.

I assume now that there is a *finite set of agents at time* 0 who have got the idea for a superior technology. One can think that this set of agents is able to produce new ideas while the remaining part of the economy is only able to adopt these ideas. However, the agents that have ideas are also not sure whether the new technology is really better than the prevailing one. A neighbor using an old technology might convince him that the new idea is not good. After the initial date an agent switches to the alternative technology if and only if he talks to a neighbor who earns more than he does. The probability that a worse technology user meets a neighbor that uses the better technology and has a higher than him is $pP_{\mathcal{N}(x),1}$. Similarly, with probability $(1-p)(1-P_{\mathcal{N}(x),1})$, he meets a neighbor that uses the worse technology and more successful than him.

This rule of thumb results in the revision rate

$$c(x,\eta) = \begin{cases} \dfrac{p}{2d} \displaystyle\sum_{y \in \mathcal{N}(x)} \eta(y) & \text{if } \quad \eta(x) = 0 \\ \dfrac{1-p}{2d} \displaystyle\sum_{y \in \mathcal{N}(x)} (1 - \eta(y)) & \text{if } \quad \eta(x) = 1. \end{cases} \tag{17}$$

This process is clearly non–ergodic. If each agent uses the worse technology, there is no chance to observe how the better technology performs, thus no agent wants to switch. Of course, the same is true for the state where everybody uses the better technology. In this respect, this model is very interesting because we can look for conditions under which the economy converges the better state if initially the number of agents using the better technology is small. Our main interests are to look at the survival probability of the new idea, to determine the limit distribution of the process, and to understand how it spreads out.

Proposition 3 *In the spin economy resulting from the rule of thumb with imitation of success and without an exogenous signal, the probability that the better technology survives is given by*

$$P_{k_0}(\tau = \infty) = 1 - \left(\frac{1-p}{p}\right)^{k_0} \tag{18}$$

where k_0 is the number of agents originated the new idea, $\tau \equiv \inf\{t : A_t = \varnothing\}$ and $A_t \equiv \{x : \eta_t(x) = 1\} \subset \mathbb{Z}^d$.

Proof: Let consider the process A_t and A_t^c where and $A_t^c \equiv \{x : \eta_t(x) = 0\} \subset \mathbb{Z}^d$. Clearly, no revision takes place in the interior of either A_t or A_t^c since a revision requires communication between agents having a different opinion. Let $\partial A_t \equiv \{(x,y) : |x-y| = 1,\ x \in A_t, y \notin A_t\}$ and $|A_t| \equiv \{\#x : x \in A_t\}$. If a revision opportunity has arrived for agent $v \in \partial A_t$ for the $(n+1)$th time and v changes his technology, then the transition between $|A_t|_{n+1}$ and $|A_{t'}|_n$ for some $t' < t$ is given by

$$|A_t|_{n+1} = \begin{cases} |A_{t'}|_n + 1 & \text{with probability} \quad p \\ |A_{t'}|_n - 1 & \text{with probability} \quad 1-p, \end{cases} \tag{19}$$

which is an asymmetric random walk. Let $\tau_m = \inf\{t : |A_t| \geq m\}$ and $P_{k,m} \equiv P(\tau_m < \tau : k = |A_0| \leq m)$. If the economy is in a state where the probability that the better technology survives, is $P_{k,m}$, then at the next transition it goes either with probability p to $P_{k+1,m}$ or with probability $1-p$ to $P_{k-1,m}$. The law of total probability implies that

$$P_{k,m} = pP_{k+1,m} + (1-p)P_{k-1,m} \tag{20}$$

with boundary conditions $P_{0,m} = 0$ and $P_{m,m} = 1$. The first boundary condition comes from the assumption that if nobody uses the better technology, nobody can adopt it. The second one follows from the definition of P_k. The above equation is well known in probability theory as a difference equation for the gambler's ruin problem.

The rest of the proof is purely algebraic. Equation (20) implies

$$P_{k+1,m} - P_k = \frac{1-p}{p}\left(P_{k,m} - P_{k-1,m}\right) = \left(\frac{1-p}{p}\right)^k \left(P_{1,m} - P_{0,m}\right). \tag{21}$$

Taking the sum of the series of equations (21) for $k = 0, ..., m-1$ and using the boundary conditions yields

$$1 = P_{1,m}\sum_{k=0}^{m-1}\left(\frac{1-p}{p}\right)^k = P_{1,m}\frac{1-[(1-p)/p]^m}{1-(1-p)/p}. \tag{22}$$

Moreover, using (21) with the boundaries gives

$$1 - P_{k_0} = \sum_{k=k_0}^{m-1}(P_{k+1,m} - P_{k,m}) = P_{1,m}\sum_{k=0}^{m-1}\left(\frac{1-p}{p}\right)^k - P_{1,m}\sum_{k=0}^{k_0}\left(\frac{1-p}{p}\right)^k. \tag{23}$$

Combining (22) and (23) yields

$$P_{k_0}(\tau_m < \tau) = P_{k_0,m} = 1 - \frac{[(1-p)/p]^{k_0} - [(1-p)/p]^m}{1-[(1-p)/p]^m}. \tag{24}$$

As $m \uparrow \infty$, $\tau_m \uparrow \infty$ implying that $P_{k_0,m}(\tau_m < \tau) \rightarrow P_{k_0}(\tau = \infty)$. The limit of the right hand side of (24) yields the desired formula. □

This result says that there is positive probability that new idea always survives [$p > 1-p$ by assumption]. However, this probability is always less than one. This is bad news because it means that similar to the cases with an exogenous signal, *simple rules of thumb do not lead to a good long run outcome with probability one.* In addition, it is interesting to observe that the probability of spreading out is an increasing function of the number of agents being able to originate new ideas at time 0.

The model might also be viewed as providing an explanation for why there are differences between economies in the long run. In the present rule of thumb environment, it is always a matter of luck whether an economy is successful or not. The luck in terms of probabilities can be increased or decreased by initial conditions. However, even economies with the same initial state can end up in different long run steady states. This uncertainty disappears if and only if there is no noise in the economy, that is, $\sigma \downarrow 0$ provided that initially only a finite number of agents uses the better technology. However, if *an infinite number of agents use the superior technology, then it almost surely drives out the worse one* [$\lim_{k_0 \to \infty} P_{k_0}(\tau = \infty) = 1$] provided $\sigma < \infty$.

Now we can characterize the stationary distribution of the process and understand how the number of agents using the better technology changes over time. To do this we recall two important mathematical results. The first one is called the "complete convergence theorem" and the second one the "general shape theorem". They can be found in Durrett (1988). Applying these theorems to our case, we obtain the following proposition:

Proposition 4 *The finite dimensional distribution of the spin economy resulting from the rule of thumb with the imitation of success and without an exogenous signal converges to a set of stationary distributions given by*

$$\eta_t \Longrightarrow \delta^0 P(\tau < \infty) + \delta^1 P(\tau = \infty),$$

where δ^0 and δ^1 denote the point masses on the states η^0 and η^1, respectively. Moreover, if the better technology survives, the set of agents using the better technology grows linearly, that is

$$\lim_{t \to \infty} \frac{|A_t|}{t^d} = C,$$

where C is a constant independent of A_t.

Proof: The proof is lengthy and it follows immediately from the results of Chapter 3 and Chapter 11c in Durrett (1988), therefore it is not presented here. □

The proposition says that the process has two stationary distributions: the point masses on the states η^0 and η^1. In economic terms this means that the better technology either dies out or survives. In addition, if the process survives, it grows linearly. The rate of growth depends positively on the dimension of the lattice. Thus, the richer the structure of the communication across agents, the faster the new technology spreads out. The convergence also depends on C. This constant is easy to determine for the one dimensional case. If the process starts from a finite interval $[a_1, a_2]$, then the two endpoints independently follows an asymmetric random walk. Therefore, by the law of large numbers $(a_2 - a_1)/t \rightarrow 2(2p-1)$ almost surely on the event $\{\tau = \infty\}$ implying $C = 2(2p-1)$.

It is also interesting to look at the case where the two technologies are equally good, that is, $\theta = 0$ implying $p = 1/2$. Then there is nothing to learn, although agents do not know this. The behavior of this economy is described by the following proposition:

Proposition 5 *In the spin economy resulting from the rule of thumb with imitation of success and without an exogenous signal, there is a positive probability that the two technologies coexist provided that the two technologies are equally efficient, the initial state is random and $d \geq 3$.*

Proof: The proof follows Durrett (1988, Chapter II). Formally, we have to prove that

$$P(\eta_t(x) \neq \eta_t(y)) > 0 \;\; \forall x, y \in \mathbb{Z}^d, x \neq y, \; \forall t \geq 0, \tag{25}$$

given that at $t = 0$ each agent is assigned to the worse technology with probability $q_0 > 0$ and to the better one with probability $1 - q_0$ independently from the other agents.

To prove this, we construct the dual of the original process. Let $X_s^{x,t}$ be the process that traces the origin of the opinion of agent x at time t. Let $X_0^{x,t} = x$, that is, the process for $s = 0$ just provides the address of the agent of interest. If agent x has last changed his opinion at some time $t - s_1$, then he has adopted the technology used by one of his neighbors $v_1 \in \mathcal{N}(x)$, that is, $X_{s_1}^{x,t} = v_1$. At some earlier time $t - s_2$, where $s_2 > s_1$,

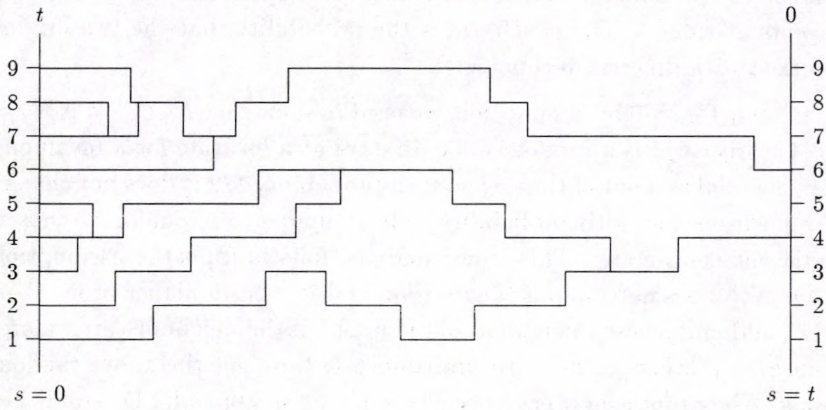

Figure 1: The dual of the technology adoption process

v_1 has switched to the technology operated successfully by $v_2 \in \mathcal{N}(v_1)$, that is, $X_{s_2}^{x,t} = v_2$. Working backwards until $t = s$, $X_t^{x,t} = v$ shows who originated the opinion of agent x at time 0. Notice that along such a path all agents use the same technology. If v used the worse technology at time 0, then all agents supplied by $X_s^{x,t}$ work with the worse option. Figure 1 shows one possible realization of the dual process for a subset of agents in one dimension. We see that the opinion of agent 4 and 6 at time 0 was adopted by all nine agents. For example, agent 5 has adopted the opinion of agent 4, who has earlier followed agent 5, who has looked at agent 6 and so on until time 0.

The construction of $X_s^{x,t}$ and $X_s^{y,t}$ implies — as it is also suggested by the figure — that agents x and y are using different technologies at time t, if and only if their opinions were induced by different agents having used different technologies at time zero. Therefore, equation (25) can be rewritten in the form:

$$P(\eta_t(x) \neq \eta_t(y)) = P\left(X_t^{x,t} \neq X_t^{y,t}\right)(q_0(1-q_0) + (1-q_0)q_0) \; \forall t \geq 0. \qquad (26)$$

The probability in (26) has two terms. The first term on the right hand side is the probability that x and y trace their opinions back to different agents at time 0. The next term is the probability that the two origins worked with different technologies.

Since $q_0 > 0$ by assumption, we need to show that $P(X_t^{x,t} \neq X_t^{y,t}) > 0$. Clearly, $X_s^{x,t}$ is a random walk. It stays at a location for a mean one exponential amount of time, then with probability 1/2 it does not change its position and with probability $1/4d$ it jumps to a randomly chosen adjacent coordinate. This transition rule follows from the assumption that $X_s^{x,t}$ does not jump if the decision maker earns a higher profit than the randomly chosen neighbor and it jumps if the neighbor earns more. Since both events occur with probability 1/2, we get the above random walk. The proof is now easy because lemma 2 in appendix D establishes that $X_s^{x,t}$ behaves like a simple symmetric random walk, that is, it is transient for $d \geq 3$. Since two transient random always avoid each other with positive probability [$P(X_t^{x,t} \neq X_t^{y,t}) > 0$], we have proved our claim. □

The result asserts that two equally efficient technologies may coexist forever. There is no force in the economy which would almost surely drive out one of the two technologies since they are equally good. However, it might seem surprising that coexistence can occur even if q_0 is small, that is, if only a small fraction of the whole society uses technology 0. In this case the assumption of the countable infinity of agents and of the neighborhood structure ensures that even unpopular technologies may survive.

Clearly, the above defined rule without an exogenous signal does not utilize all the information available in this environment. Instead of talking to one neighbor, he could also visit all of his neighbors. The sample averages would contain more information than a payoff realization of a single neighbor. For this reason, it is assumed now that agent x travels around and asks all neighbors about how much money they made. After having finished the trip, he computes the averages for the two different technologies [including himself]. He then chooses the technology which earns more on average at this particular moment. Of course, if nobody operates the alternative technology, he cannot sample anybody who does something else as he does something different from him. The decision rule just described results in the following revision rate:

$$c(x,\eta)=\begin{cases} 0 & \text{if } \sum_{y\in\mathcal{N}(x)}\eta(y)=0 \text{ and } \eta(x)=0 \\ P\left(\overline{u}^1\geq\overline{u}^0\right) & \text{if } \sum_{y\in\mathcal{N}(x)}\eta(y)>0 \text{ and } \eta(x)=0 \\ 0 & \text{if } \sum_{y\in\mathcal{N}(x)}(1-\eta(y))=0 \text{ and } \eta(x)=1 \\ P\left(\overline{u}^1<\overline{u}^0\right) & \text{if } \sum_{y\in\mathcal{N}(x)}(1-\eta(y))>0 \text{ and } \eta(x)=1, \end{cases} \tag{27}$$

where $P\left(\overline{u}^1\geq\overline{u}^0\right)$ $[P\left(\overline{u}^1<\overline{u}^0\right)]$ is the probability that the mean payoff to the agents using the better [worse] technology exceeds the payoff to agent x using the worse [better] technology. This construction results a complicated revision rate since for example an agent using the worse technology $[\eta(x)=0]$ the distribution of the average payoff difference is given by

$$\left(\overline{\epsilon}^1-\overline{\epsilon}^0\right)\sim\mathcal{N}\left[0,\sigma\left(\frac{1}{1+\sum_{y\in\mathcal{N}(x)}(1-\eta(y))}+\frac{1}{\sum_{y\in\mathcal{N}(x)}\eta(y)}\right)^{\frac{1}{2}}\right].$$

Nevertheless, it is still possible to characterize qualitatively the asymptotic behavior of the process by using coupling techniques as stated by the following proposition:

Proposition 6 *In the spin economy resulting from the rule of thumb with imitation of success by sampling and without an exogenous signal, the better technology always dies out with positive probability if initially a finite number of agents use the better technology, i.e.* $\exists\varepsilon>0$ *such that* $P(\tau_\eta<\infty)\geq\varepsilon$ *where* $A_t^\eta=\{x:\eta_t(x)=1\}\subset\mathbb{Z}^d$, $|A_t^\eta|=\{\#x:x\in A_t^\eta\}$ *and* $\tau_\eta=\inf\{t:A_t^\eta=\varnothing\}$.

Proof: It is first proven that a process ζ_t exists on E such that if $\eta\leq\zeta$, then $\eta_t\leq\zeta_t$ with probability one for all $t\geq0$. Moreover, it is shown that the better technology dies out with strictly positive probability in ζ_t. The proof is completed by noting that $\varepsilon\equiv P(\tau_\zeta<\infty)\leq P(\tau_\eta<\infty)$

[otherwise $P(\eta_t \leq \zeta_t | \eta \leq \zeta) = 1$ would be violated for some t].

According to Theorem 6 in appendix B, it suffices to construct a process ζ_t with a revision rate satisfying the condition of the theorem. Let the following probabilities be considered

$$\begin{aligned} p_s &= \sup_{\eta \in E} \left\{ P(\overline{u}^1 - \overline{u}^0 \geq 0) : \eta(x) = 0, \sum_{y \in \mathcal{N}(x)} \zeta(y) > 0 \right\} \\ p_i' &= \inf_{\eta \in E} \left\{ P(\overline{u}^0 - \overline{u}^1 > 0) : \eta(x) = 1, \sum_{y \in \mathcal{N}(x)} (1 - \zeta(y)) > 0 \right\}. \end{aligned}$$

Since the sample size is finite $[2d + 1 < \infty]$, and both $0 < P(\overline{u}^1 - \overline{u}^0 \geq 0) < 1$ and $0 < P(\overline{u}^0 - \overline{u}^1 > 0) < 1$ for all finite d, we can conclude that the probabilities p_i' and p_s exist satisfying $0 < p_i' \leq p_s < 1$. Using these probabilities, ζ_t shall be defined by the revision rate

$$c'(x, \zeta) = \begin{cases} p_s \sum_{y \in \mathcal{N}(x)} \zeta_t(y) & \text{if} \quad \zeta(x) = 0 \\ p_i \sum_{y \in \mathcal{N}(x)} (1 - \zeta(y)) & \text{if} \quad \zeta(x) = 1, \end{cases} \tag{28}$$

where $p_i = p_i'/(2d)$. This revision rate is well defined for ζ_t since it is nonnegative, spatially homogeneous and depends only on the opinion of finitely many other neighbors. Comparing the revision rate for η_t in equation (27) with the one for ζ_t in (28), it is easy to verify that the revision rate for ζ_t satisfies the desired condition. That is, whenever $\eta \leq \zeta$ for $\eta, \zeta \in E$, then for all $x \in \mathbb{Z}^d$

$$\begin{aligned} c(x, \eta) &\leq c'(x, \zeta) \quad \text{if} \quad \eta(x) = \zeta(x) = 0, \quad \text{and} \\ c(x, \eta) &\geq c'(x, \zeta) \quad \text{if} \quad \eta(x) = \zeta(x) = 1. \end{aligned}$$

Therefore, $\eta_t \leq \zeta_t \; \forall t \geq 0$ with probability one if the initial state satisfies $\eta \leq \zeta$.

To finish the proof, we have to show that ζ_t dies out with positive probability. Let $A_t^\zeta = \{x : \zeta_t(x) = 1\} \subset \mathbb{Z}^d$, $|A_t^\zeta| = \{\#x : x \in A_t\}$ and $\partial A_t^\zeta = \{(x, y) : |x - y| = 1, \; x \in A_t^\zeta, y \notin A_t^\zeta\}$. So, the number of agents using the better technology evolves over time according to a birth–and–death process. During any short time interval $|A_t^\zeta|$ increases by rate $p_s|\partial A_t^\zeta|$ and decreases by rate $p_i|\partial A_t^\zeta|$. Under such a transition rules the process $|A_t^\zeta|_n$ becomes an asymmetric random walk with transition probabilities:

$$|A_t^\zeta|_{n+1} = \begin{cases} |A_s^\zeta|_n + 1 & \text{with probability } \dfrac{p_s}{p_s + p_i} \\ |A_s^\zeta|_n - 1 & \text{with probability } \dfrac{p_i}{p_s + p_i} \end{cases} \tag{29}$$

for some $s < t$, where n refers the nth change of $|A_s^\zeta|_n$. However, we know already that starting from a finite set $|A_0^\zeta|$ the probability that such a process dies out is

$$P(\tau_\zeta < \infty) = \left(\frac{p_i}{p_s}\right)^{|A_0^\zeta|}.$$

Since $p_i < p_s$, $P(\tau_\zeta < \infty)$ is clearly positive. This implies that the better technology in η_t also dies out with a strictly positive probability. □

As we see, spending some money on a trip to visit the neighbors may increase the probability of success. Unfortunately, it will be still bounded away from one implying that the economy might not converge to the better state. This implies that our previous result remained robust against altering the decision rule.

One may naturally ask how robust are our the results about the impossibility of the almost sure convergence in a spin economy. The next theorem establishes an impossibility condition for a broader class of decision rules.

Theorem 1 *Let $c(x, \eta)$ be a spatially homogeneous revision rate. Suppose that $c(x, \eta) = 0$ if and only if $\eta(y) = 0$ $\forall y \in \mathcal{N}(x)$ and for any $x \in \mathbb{Z}^d$. Then starting from a finite number of better technology users, the better technology dies out with positive probability.*

Proof: To prove the theorem, two processes ζ_t' and ζ_t'' will be constructed satisfying $P(\eta_t \le \zeta_t' | \eta \le \zeta') = 1$ for all $t \ge 0$. It follows that if if the process ζ'_t dies out with positive probability, then so does η_t.

Let a_s and a'_i be defined as

$$\begin{aligned} a_s &= \sup_{\eta \in E} \{c(x,\eta) : c(x,\eta) > 0, \eta(x) = 0\} \\ a'_i &= \inf_{\eta \in E} \{c(x,\eta) : c(x,\eta) > 0, \eta(x) = 1\}. \end{aligned}$$

Since $0 < c(x,\eta) < \infty$ by assumption, a'_i, a_s satisfying $0 < a'_i \leq a_s < \infty$ exist. Let ζ'_t be defined by

$$c'(x,\zeta') = \begin{cases} a_s \sum_{y \in \mathcal{N}(x)} \zeta'_t(y) & \text{if} \quad \zeta'(x) = 0 \\ a_i \sum_{y \in \mathcal{N}(x)} (1 - \zeta'(y)) & \text{if} \quad \zeta(x)' = 1, \end{cases} \tag{30}$$

where $a_i = a'_i/2d$. Since $a_i < a_s$ and $c'(x,\zeta') = 0$ if and only if $c(x,\eta) = 0$, $c'(x,\zeta')$ satisfies the condition of Theorem 6 in appendix B implying $P(\eta_t \leq \zeta'_t | \eta \leq \zeta') = 1$. Therefore, we have to show that if $|A_0^{\zeta'}| = k_0 < \infty$ then $\zeta'_t \rightarrow \zeta^{0\prime}$ with positive probabilityt where $|A_t^{\zeta'}| = \{\#x : x \in \mathbb{Z}^d, \eta(x) = 1\}$ and $\zeta^{1\prime} = \{\zeta' : \zeta'(x) = 1 \ \forall x \in \mathbb{Z}^d\}$. However, by the arguments of proposition 6, $|A_0^{\zeta'}|$ turns out to be an asymmetric random walk. It survives with a probability given by

$$P_{k_0}(\tau_{\zeta'_t} = \infty) = 1 - \left(\frac{a_i}{a_s}\right)^{k_0}$$

where k_0 is the number of better technology users at time zero. Since $a_i < a_s$ by construction, $P_{k_0}(\tau_{\zeta'_t} < \infty) > 0$, which proves the theorem. □

The above theorem describes qualitatively the convergence behavior of any decision rule which satisfies some relative weak conditions. On one hand, spatial homogeneity of the revision rate is required which was assumed throughout the paper and discussed already. On the other hand, the revision rate has to satisfy two further conditions. *First*, an agents cannot change his opinion without talking to a neighbor who is doing something else as he does. This means that if all of his neighbors share his view, he can talk to nobody having different opinion, therefore he continues doing the same action as before $[c(x, \eta) = 0]$. *Secondly*, if at least one of his neighbors has different opinion as he has, he may adopt it $[c(x, \eta) > 0]$. Recall, the positivity of the revision rate does not express necessity, it expresses possibility. If the decision maker does not meet or sample a neighbor who has different opinion, he continues working with the previous technology.

It is easy to construct examples for which the above theorem does and does not hold. It holds for example if an agent takes samples of size $n \leq 2d$ and he adopts the technology which performed better on average in his sample. More generally, if an agent is more likely to adopt the more popular technology, and he adopts a technology with positive probability provided that at least one of his neighbors uses it, the above theorem describes the technology adoption process. However, the theorem also holds for decision rules without popularity considerations. For example, if each agent asks his neighbors sequentially, he stops at the first one who uses different technology and he adopts it provided its payoff is higher than that of his technology, the conditions of the theorem are satisfied. Clearly, the popularity of the two different options does not matter in this decision algorithm. It does only matter whether one neighbor is doing something else as the decision maker or not. In contrast for example, if an agent adopts a technology if and only if at least k of his neighbors using the alternative technology achieve a higher payoff than he does, then the theorem does not apply.

4 Discussion

4.1 Rules of thumb and word–of–mouth communication

The importance of analyzing simple naive learning rules has recently been popularized by Ellison and Fudenberg (1993, 1995), and Banerjee and Fudenberg (1994). Seemingly, the heart of these models are some simple assumptions, namely, that agents guide their decisions by observing other individuals, that there is no strategic interaction, and finally that agents can revise choices as many times as the probabilistic structure allows. These frameworks provide conditions for almost sure convergence to the better state. Although the model of this paper clearly satisfies these three assumptions,[5] the results exhibit the striking difference that almost sure convergence does not occur. In this sense, *none of the results of the three closely related papers can be replicated in our framework except for trivial cases.* It will be now discussed now what the origins of this difference are.

Our model differs from the previous ones in three important aspects. *First*, in the present model there is only a countable infinity of interacting agents while in the other models a continuum of agents is assumed. This assumption makes it possible to analyze the behavior of one agent. Clearly, this is not possible in a continuum of agent model where the probability that one individual [one point on the line] undertakes an action is zero. *Secondly*, our individuals can communicate only with their nearest neighbors unlike in the other models, where an agent can interact with anybody else in the population. This assumption formulates explicitly a locally interacting setting which essentially generates the differences. *Finally*, it is also an important difference that our agents are distributed in a communication space represented by the points on d–dimensional integer lattice. As we have seen in proposition 5, the the parameter d plays an important role in determining the long run behavior of the economy.

As we have shown, under the above assumptions the better technology dies out with positive probability if initially finitely many agents are using the inferior technology. It might well happen that the finite number of better technology user observe only worse technology user who earn higher payoff than they do. Since such an event occurs with positive probability in finite time, the better technology may die out. It should be noted that without local interaction this problem does not arise at all. If agents can observe anybody else in the population, they would observe

[5]The assumption of randomly arriving revision opportunities can be regarded as a limit of the case when a fraction of the individuals revise their decision in any period.

the performance of bad technologies with probability one. This comes about because the superior technology is used by finitely many agents while the inferior one by infinitely many ones.

Having discussed the differences in the structure of the models and the main result of this paper, we can compare our results with those of the related works. Since these papers focus on decision problems without an exogenous signal, we also concentrate on this case in the comparison.[6] Ellison and Fudenberg (1993) consider the simple decision problem that agents have to choose one of two available technologies having an expected payoff difference θ. The random shock to the payoff difference is uniformly distributed on the interval $[-\sigma, \sigma]$ and is the same for everybody. Agents change their opinion whenever the average payoff difference is larger than a weight m multiplied by the relative popularity of the alternative technology. Ellison and Fudenberg show that a carefully determined m ensures the almost sure convergence to the better state. We can replicate this result in our setup by assuming that the payoff realizations are publicly available information. Intuitively, this is the same assumption as allowing agents to observe the average payoffs in a continuum of agent framework, which is the assumption of Ellison and Fudenberg or to observe any other individual in the population. Since this decision rule is not related directly to the rules of thumb analyzed before, we state our claim formally:

Proposition 7 *Assuming that the payoff realization is publicly revealed, the spin economy of the rule of thumb with popularity weighting is ergodic if $m \leq \sigma - \theta$. Its unique invariant distribution is the product measure*

$$\nu\{\eta : \eta(x) = 1\} = \frac{1}{2} + \frac{\theta}{2(\sigma - m)}. \tag{31}$$

Furthermore, the economy converges from any initial distribution with

[6] However, it should be mentioned that a continuum of agent model with global interaction and a framework of local interaction with countable infinity of individuals give similar results if an exogenous signal is present. For example, the long run outcome of the pure imitation case would be the same in both models.

exponential rate

$$\frac{\sigma - m}{\sigma}. \tag{32}$$

If $\sigma - \theta < m < \sigma + \theta$, the spin economy converges with probability one to the state where everybody is using the better technology. If $m \geq \sigma + \theta$, the economy is non-ergodic.

Proof: See Appendix E.

These results are essentially the same as derived by Ellison and Fudenberg, except that they do not determine the rate of convergence. Clearly, there exists an optimal m, notably $m = \sigma - \theta$, for which the economy converges to the better technology with probability one. However, this is trivial because if $m = \sigma - \theta$, then an agent switches to the worse technology if and only if at least one of his neighbors uses it. On the other hand, if he operates the worse technology, he changes with a strictly positive rate to the better one even if no neighbor uses it. [This follows from equation (E.13) in appendix E.] Since the effect of the neighbors on the revision rate in either direction is the same, the result of convergence is not surprising. We may also notice that the economy converges faster if $m < \sigma - \theta$, that is, it is then easier to reach a worse than a better state.

Ellison and Fudenberg (1995) considered a slightly different framework. The payoff difference is assumed to face both aggregate and idiosyncratic shocks and agents try to infer the better technology by taking samples. Therefore agents adopt the alternative technology with positive probability only if they sample such a technology. Ellison and Fudenberg concluded that if the sample size is small, the economy converges to a consensus where everybody uses the same technology. Furthermore, if the sample size is too large, the economy fluctuates between the two options. However, at an intermediate sample size the economy converges to the better technology with probability one. This finding is comparable with our impossibility result of theorem 1 because such a decision rule satisfies the condition of the theorem. As was shown, coexistence never occurs and starting from a finite set the economy never converges to consensus with probability one. These results are in sharp contrast to Ellison and Fudenberg. However, starting from an infinite set a broad class of decision rules without popularity consideration can lead to the better state. Ellison and Fudenberg also look at the case where the expected payoff difference is zero. The comparison of their result with our in proposition 5 also highlights the difference between a model with a countable infinity of agents and with explicit neighborhood structure, on the one hand, and, one the other hand, a model with a continuum of agents and global interaction. Ellison and Fudenberg found that if only a small fraction of agents revises their decisions at any period and agents sample only one other agent, then the economy converges to consensus. In contrast, if the sample size is at least two, the two technologies will coexists. In contrast, in our framework coexistence of two equally good technologies always occurs with positive probability if the number of neighbors is at least six. The intuition of Ellison and Fudenberg, namely, that small samples make adopting a product with a very little market share less likely, does not work here. Although the intuition turns out to be valid, the explicitly formulated neighborhood structure does not allow that this small probability [small q_0 in our model] vanishes in the long run.

The third model closely related paper is presented by Banerjee and Fudenberg (1994). Their model differs from the two setups discussed previously because now rational agents in the Bayesian sense have to choose between the two technologies. Which technology is better depends on the states of the world. Agents are in different states of the world and achieve the efficient outcome, that is, adopt the technology that performed better in this state of the world. Banerjee and Fudenberg have shown that the economy converges to the efficient outcome if each person in the population is equally likely to be sampled. The argument is based on the feasibility of proportional sampling which clearly fails to be satisfied in a spin economy. In such a framework only the nearest neighbors can be sampled. However, we could reconsider their Bayesian agent in our framework. A Bayesian has some priors about which option is better and determines his posterior beliefs by asking the neighbors about their payoffs. Then he chooses the technology which has a higher expected payoff according to his posterior beliefs. The probability that his posterior belief favors a technology, typically depends on how many agents use currently the two technologies in the neighborhood. We could use our impossibility theorem to show that both technologies may die out implying that there is no almost sure convergence to any of the two states.

4.2 Herd behavior

The model of this paper is also related to the herding or cascade literature. In these models rational agents have to choose sequentially one of two available options. The decisions are irreversible: if an agent has already undertaken an action, he cannot change his mind. The information of any individual consists of the observation of all decisions made in the past and the individual's private information. Herding arises naturally in such environment, that is, agents neglect their private information and simply copy the decision their predecessors.[7] The results on herding imply that even rational agents may decide for the worse option. However, one can certainly find examples for irreversible decisions or cases in which it is feasible to observe the whole history, there are several situations in which it is not the case. Agents often cannot observe the whole history and do not know the sequence of the past decisions. Moreover, it is easy to find real world examples for strong "once and for all" commitments which finally do not last forever.[8]

Our framework offers a possibility of how herding may emerge in a stochastic environment if agents can revise their decisions after a while. Let us consider a situation where agents simple imitate one of their randomly chosen neighbors. [They may do so because of some rational reasons exogenous to the analysis.] Such a model is known as the voter model in the theory of interacting particle systems and was extensively

[7] The herding and cascades idea was recently analyzed in a formal model by Banerjee (1992) and Bikhchandani, Hirshleifer, and Welch (1992) and generalized by Lee (1993) and Smith and Sørensen (1994)

[8] Orlean (1995) also argues that the assumption of sequential decisions is strong and proposes a model to relax it.

reviewed by Liggett (1985, Chapter V) and by Durrett (1988, Chapter II). The behavior of the voter model crucially depends on the process $X_s^{x,t}$ which describes how a particular opinion reaches agent x between time $t - s$ and time t as in proposition 5. In this case, $X_s^{x,t}$ is a simple symmetric random walk which is well known to be recurrent in one and two dimensions and transient in higher dimensions.[9] Hence, it follows that the society herds on one opinion almost surely if the number of neighbors is less than four. However, if there are at least six, then the two different opinions may coexists with positive probability. This implies that the imitation of the others is not sufficient to obtain herding in the present setup.

Finally, it is also useful to compare our results with those of Kirman (1993). He analyzes the behavior of agents who imitate the others with a given probability. However, each agent may choose an option independently from the others [called mutation] with some small likelihood. The ergodic distribution of this economy turns out to depend on the small probability of mutation. This is a similar result as obtained in proposition 1 where the ergodic distribution is determined by the exogenous signal. However, as proposition 5 suggests, steady fluctuations between two states can be generated without mutation if countable agents communicate in a virtually spatial environment.

5 Concluding remarks

Recently several authors [Ellison and Fudenberg (1993, 1995), Banerjee and Fudenberg (1994)] have found that under certain conditions naive learning rules can almost surely lead to an optimal decision. This paper has argued that their attractive result crucially depends on the framework they used. Our analysis suggests that Ellison and Fudenberg's finding changes qualitatively if one removes the assumptions about both

[9] A proof is presented for example by Durrett (1991, Chapter III) who also also tells the following very intuitive observation of Kakutani: "A drunk man will find his way home but a drunk bird may get lost forever". Essentially, this mechanism works in neighborhood communication too.

the continuum of agents and the global interaction. The setup of this paper has considered a countable infinity of agents with local interaction where communication is possible only with nearest neighbors. The analysis of two different rules of thumb suggests that if an imperfect *exogenous signal* is available about the better option, then the long run outcome is essentially determined by this signal. In particular, the economy converges almost surely to the better state if and only if the exogenous signal is true with probability one. As noted previously, the two frameworks deliver similar results in this case. In contrast, if the *exogenous signal is absent*, we have demonstrated above that *none of the results of the previous papers can be replicated in this framework for other than trivial cases.*[10] Every outcome occurs with positive probability, that is, the good technology may either die out or may drive out the worse one, implying that there is no almost sure convergence to the better state.

This result suggests that given neighborhood communication is important and agents use simple exogenous rules for their decisions, the aggregate outcome might be a matter of luck only. Some good ideas may die out while some bad ones may be adopted by the whole society. There are famous real world examples for this phenomenon such as the selection between competing typewriter machines or video systems of different quality.

Fortunately, societies are sometime able to discover ideas again, a famous example being the early history of genetics. Mendel's idea about how characteristics are inherited was of so little interest to the public years age that even he stopped his research on the topic. It took fifty years until Morgan popularized the issue. Another, more recent example is provided by the history of neural networks. There was a certain interest in studying artificial neurons in the 1960s, but, in 1969 two researchers

[10] A similar results were obtained recently by other authors as well. Durrett and Levine (1994a) have shown about *coexistence and survival of different strategies* in the context of evolutionary biology that the prediction of the models in which individuals have an equal probability of interaction may differ considerable from the prediction of models where it is not the case. Ellison (1993) has demonstrated that the *convergence behavior* of a model with local interaction is different from that of a model with global interaction.

demonstrated some learning difficulties in such assemblies. As a result the interest in the subject died relatively fast, twenty years later it turned out that the previous difficulties can be resolved.

Appendix

A The formal definition of the spin economy

Give a two element set $W = \{0, 1\}$ the discrete topology and $E = W^{\mathbb{Z}^d}$ the product topology, where $\mathbb{Z}^d$ for some $d \geq 1$ denotes the d-dimensional integer lattice. W is compact in the discrete topology, E is compact in the product topology, and both are separable and metrizable. The state of the economy is given by the mapping $\eta : \mathbb{Z}^d \longrightarrow W$. Consequently, the state of the economy is denoted by $\eta \in E$ while the state of agent $x \in \mathbb{Z}^d$ is given by $\eta(x)$.

Let $C(E)$ be the set of of all real valued continuous functions f on E, regarded as a Banach space under the sup norm $\|f\| = \sup_{\eta \in E} |f(\eta)|$. Moreover, let E be endowed with a measurable structure given by the σ–algebra of Borel sets. The set of all probability measures on these Borel sets is denoted by $\mathcal{P}(E)$. Give $\mathcal{P}(E)$ the topology of weak convergence, that is, $\mu_n \rightarrow \mu$ in $\mathcal{P}(E)$ if and only if $\int f d\mu_n \rightarrow \int f d\mu$ for all $f \in C(E)$. $\mathcal{P}(E)$ is compact in this topology because E is compact.

Definition 2 Let $D_E[0, \infty)$ be the set of all functions η_t on $[0, \infty)$ with values in E which are right continuous and have left limit at all $t \geq 0$. Moreover, let $\mathcal{F}$ be the smallest σ–algebra on $D_E[0, \infty)$ relative to which all the mappings η_t are measurable and $\mathcal{F}_s$ the smallest σ–algebra on $D_E[0, \infty)$ relative to which all the mappings $\eta_{s'}$ for $s' \leq s$ are measurable. A *Markov process* on E is a collection of probability measures $\{P^\eta, \eta \in E\}$ with the following properties:

(a) $P(\eta_t \in D_E[0, \infty) | \eta_0 = \eta) = P^\eta(\eta_t \in D_E[0, \infty)) = 1$ for all $t \geq 0$ and $\eta \in E$,

(b) for all $A \in \mathcal{F}$ and $t \geq 0$ the mapping $\eta \longmapsto P^\eta(\eta_t \in A)$ from E to $[0, 1]$ is measurable,

(c) $P^\eta(\eta_{s+s'} \in A|\mathcal{F}_s) = P^\eta(\eta_{s+s'} \in A|\eta_s)$ for every $\eta \in E$ and $A \in \mathcal{F}$ for all $s' > 0$.

Definition 3 A one parameter family $\{S(t) : t \geq 0\}$ of bounded linear operators on $C(E)$ is called a *semigroup* if $S(0)$ equals the identity operator on $C(E)$ and $S(t+s) = S(t)S(s)$ $\forall s, t \geq 0$. It is called *contraction semigroup* if $\|S(t)f\| \leq \|f\|$ for all $f \in C(E)$ and $t \geq 0$. In addition, a semigroup is said to be a *Markov semigroup* if it satisfies the conditions:

(a) $S(t)f \in C(E)$ (Feller property),
(b) $S(t)f : [0, \infty) \longrightarrow C(E)$ is right continuous at all $t \in [0, \infty)$ and $\forall f \in C(E)$,
(c) $S(t+s)f = S(t)S(s)f$ $\forall f \in C(E)$ and $\forall\, s, t \geq 0$,
(d) $S(t)1 = 1$ $\forall t \geq 0$,
(e) $S(t)f \geq 0$ for all nonnegative $f \in C(E)$,

A semigroup is *strongly continuous* if $\lim_{t\to 0} S(t)f = f$ $\forall f \in C(E)$.

Definition 4 A linear operator Ω on $C(E)$ is a linear mapping whose domain $\mathcal{D}(\Omega) \subset C(E)$ and whose range $\mathcal{D}(\Omega) \subset C(E)$. Ω is said to be closed if its graph $\mathcal{G}(\Omega) = \{(f, \Omega f) : f \in \mathcal{D}(\Omega)\}$ is a closed subset of $C(E) \times C(E)$. The generator of a semigroup $S(t)$ on $C(E)$ is a linear operator Ω defined by

$$\Omega f = \lim_{t\to 0} \frac{S(t)f - f}{t}$$

for all $f \in C(E)$ for which this limit exists. Finally, $D \subset \mathcal{D}(\Omega)$ for a closed linear operator Ω is said to be a core for Ω if Ω is the closure of its restriction to D.

Definition 5 Let a shift transformation ω_y be defined for $x, y \in \mathbb{Z}^d$ by $\omega_y\eta(x) = \eta(y+x)$ and for all f on E by $\omega_y f(\eta) = f(\omega_y\eta)$. The revision rate $c(x, \eta)$ is said to be *translation invariant* if $c(x+y, \omega_y\eta) = c(x, \eta)$ for all $y \in E$. In addition, $\mu \in \mathcal{P}(E)$ is called translation invariant denoted by $\omega_y\mu = \mu$ if $\int f d[\omega_y\mu] = \int \omega_y f d\mu = \int f d\mu$ for all $y \in E$.

Theorem 2 *Let* $\eta_x(y) = \eta(y)$ *for* $x \neq y$ *and* $\eta_x(y) = 1 - \eta(y)$ *for* $x = y$ $\forall\, x, y \in \mathbb{Z}^d$. *Let* $c : \mathbb{Z}^d \times E \longrightarrow \mathbb{R}_+$ *be a continuous mapping which satisfies*

(a) translation invariance,

(b) $\sup_{x \in \mathbb{Z}^d} c(x, \eta) < \infty$,

(c) $M = \sup_{x \in \mathbb{Z}^d} \sum_{u \in \mathbb{Z}^d} \sup_{\eta \in E} |c(x, \eta) - c(x, \eta_u)| < \infty$.

Moreover, let $C'(E) \subset C(E)$ *be the collection of functions depending only on finitely many coordinates,*

$$\Omega f(\eta) = \sum_{x \in \mathbb{Z}^d} c(x, \eta)(f(\eta_x) - f(\eta)) \quad \textit{and}$$

$$\epsilon = \inf_{x \in \mathbb{Z}^d, \eta \in E} |c(x, \eta) - c(x, \eta_x)| < \infty.$$

Then, the closure $\overline{\Omega}$ *of* Ω *is a Markov generator of a Markov semigroup and* $C'(E)$ *is a core for* $\overline{\Omega}$. *Moreover, if* $f \in C'(E)$, *then* $S(t)f \in C'(E)$ $\forall t \geq 0$ *and*

$$\|S(t)f\| \leq \exp[(M - \epsilon)t]\|f\|. \tag{A.1}$$

Finally, if $f \in \mathcal{D}(\Omega)$, *it follows that* $S(t)f \in \mathcal{D}(\Omega)$ *and* $d/(dt)S(t)f = \Omega S(t)f = S(t)\Omega f$.

Proof: See Liggett (1985) Theorem 2.9. and Theorem 3.9. in Chapter I, Ethier and Kurtz (1986) Chapter I.

Theorem 3 *Suppose* $\{S(t), t \geq 0\}$ *is a Markov semigroup on* $C(E)$. *Then there exists a unique Markov process* η_t *such that*

$$\mathbf{E}[f(\eta_t)|\eta_0 = \eta] = S(t)f(\eta) \tag{A.2}$$

for all $f \in C(E)$, $\eta \in E$, *and* $t \geq 0$. *Moreover, let* μ *be the initial distribution associated with* η, *then the finite dimensional distribution of the process is given by*

$$\mathbf{E}[f(\eta_t)] = \int S(t)f(\eta)\mu(d\eta). \tag{A.3}$$

Proof: Ethier and Kurtz (1986) Proposition 1.6 and Theorem 2.7. in Chapter IV.

Definition 6 Suppose $\{S(t), t \geq 0\}$ is a Markov semigroup. Given $\mu \in \mathcal{P}(E)$, $\mu S(t) \in \mathcal{P}(E)$ is defined by $\int f d[\mu S(t)] = \int S(t) f d\mu$ $\forall f \in C(E)$.

$\mathcal{I}(E)$ is called the set of stationary distributions if $\mathcal{I}(E) = \{\mu : \mu \in \mathcal{P}(E),\ \mu S(t) = \mu\, \forall t \geq 0\}$.

Definition 7 The Markov process with semigroup $S(t)$ is said to be ergodic if $\mathcal{I}(E) = \{\nu\}$ is a singleton and $\lim_{t\to\infty} \mu S(t) = \nu\ \forall\, \mu \in \mathcal{P}(E)$.

Definition 8 The partial order on E is defined by the relation that $\eta \leq \zeta$ $\forall \eta, \zeta \in E$ if and only $\eta(x) \leq \zeta(x)\ \forall x \in \mathbb{Z}^d$. In addition, $\{(\eta,\zeta) \in E \times E : \eta \leq \zeta\}$ is a closed subset of $E \times E$ endowed with the product topology. Two probability measures μ_1 and μ_2 on E are said to be ordered $\mu_1 \leq \mu_2$ if $\int f d\mu_1 \leq \int f d\mu_2$ for all continuous monotone increasing functions on E.

B Theorems about the ergodic properties of a spin economy

Theorem 4 *$\mathcal{I}(E)$ is a non empty compact convex subset of $\mathcal{P}(E)$. Moreover,*

(a) $\mu \in \mathcal{I}(E)$ if and only if

$$\int S(t) f d\mu = \int f d\mu$$

for all $f \in C(E)$,

(b) if $\nu = lim_{t\to\infty} \mu S(t)$ exists for some $\mu \in \mathcal{P}(E)$ then $\nu \in \mathcal{I}$,

(c) if Ω is a Markov generator of $S(t)$ and $f \in D \subset \mathcal{D}(\Omega)$ where D is dense in $\mathcal{D}(\Omega)$, then

$$\mathcal{I}(E) = \left\{\mu \in \mathcal{P}(E) : \int \Omega f d\mu = 0\ \forall f \in D\right\}.$$

Proof: see Liggett (1985) Proposition 1.8. and 2.13. in Chapter III.

Theorem 5 *Suppose μ_1 and μ_2 are probability measures on E. A necessary and sufficient condition for $\mu_1 \leq \mu_2$ is that there exists a probability measure ν on $E \times E$ which satisfies*

$$\nu\{(\eta,\zeta) : \eta \in A\} = \mu_1(A) \quad \textit{and} \quad \nu\{(\eta,\zeta) : \zeta \in A\} = \mu_2(A)$$

for all Borel sets $A \in E$ and $\nu\{(\eta_t, \zeta_t) : \eta_t \leq \zeta_t | \eta \leq \zeta\} = 1$.

Proof: See Liggett (1985) Theorem 2.4 in Chapter II.

Theorem 6 *Suppose $c_1(x,\eta)$, $c_2(x,\zeta)$ are the revision rates for the Markov processes η_t and ζ_t such that whenever $\eta \leq \zeta$,*

$$\begin{aligned} c_1(x,\eta) &\leq c_2(x,\zeta) \qquad \textit{if} \quad \eta(x) = \zeta(x) = 0, \qquad \textit{and} \\ c_1(x,\eta) &\geq c_2(x,\zeta) \qquad \textit{if} \quad \eta(x) = \zeta(x) = 1. \end{aligned}$$

Then $\forall\, \eta, \zeta \in E$ satisfying $\eta \leq \zeta$ and $\forall t \geq 0$

$$P(\eta_t \leq \zeta_t | \eta_0 = \eta, \zeta_0 = \zeta) = 1.$$

Moreover, if μ_1 and μ_2 are probability measures on E associated with η and ζ and satisfying $\mu_1 \leq \mu_2$, then

$$\mu_1 S_1(t) \leq \mu_2 S_2(t)\ \forall t \geq 0.$$

Proof: See Liggett (1985) Theorem 1.5. and Corollary 1.7. in Chapter III.

Theorem 7 *Let $\eta^i \equiv \{\eta : \eta(x) = i\ \forall x \in \mathbb{Z}^d\}$. Moreover, let δ^0 and δ^1 be the point masses at η^0 and η^1 and $S(t)$ be the Markov semigroup for an attractive spin economy. Then*

(a) $\delta^0 S(s) \leq \delta^1 S(t)$ for $0 \leq s \leq t$,
(b) $\delta^1 S(s) \geq \delta^1 S(t)$ for $0 \leq s \leq t$,
(c) $\delta^0 S(s) \leq \mu S(t) \leq \delta^1 S(t)$ for all $t \geq 0$ and $\mu \in \mathcal{P}(E)$,
(d) $\underline{\nu} = lim_{t\to\infty} \delta^0 S(t)$ and $\overline{\nu} = lim_{t\to\infty} \delta^1 S(t)$,
(e) if $\mu \in \mathcal{P}(E)$, $t_n \to \infty$, and $\nu = lim_{n\to\infty} \mu S(t_n)$, then $\underline{\nu} \leq \mu \leq \overline{\nu}$.

Furthermore, the attractive spin economy is ergodic if and only if $\underline{\nu} = \overline{\nu}$.

Proof: see Liggett (1985) Theorem 2.3. and Corollary 2.4. in Chapter III.

C A lemma for ergodicity of a special spin economy

Lemma 1 *Suppose that the revision rate for a spin economy has the form:*

$$c(x,\eta) = \begin{cases} a + \frac{c}{2d} \sum_{y\in\mathcal{N}(x)} \eta(y) & \text{if } \eta(x) = 0 \\ b + \frac{c}{2d} \sum_{y\in\mathcal{N}(x)} (1-\eta(y)) & \text{if } \eta(x) = 1 \end{cases}$$

such that $a \geq 0$, $b \geq 0$, $a+b > 0$, $c \geq 0$. This spin economy is ergodic. It converges with exponential rate $a+b$ to the product measure

$$\nu\{\eta : \eta(x) = 1\} = \frac{a}{a+b}.$$

Proof: This spin economy is clearly attractive. Therefore by Theorem 7 in the previous appendix it is sufficient to show that $\nu = \lim_{t\to\infty} \delta^0 S(t) = \lim_{t\to\infty} \delta^1 S(t)$ where δ^0 and δ^1 denote the point masses at η^0 and at η^1.

Since $c(x,\eta)$ depends on x through η only, this economy is translation invariant. In addition, the two point masses are also trivially translation invariant. Therefore, the semigroup property the distribution of the process at time t given by $\mu_t^i = \delta^i S(t)$ is also translation invariant for all $t \geq 0$ and $i = 0, 1$. This implies that $\rho_t^i = \delta^i S(t)\{\eta : \eta(x) = 1\}$ is independent from x. Let us pick up an arbitrary agent $v \in \mathbb{Z}^d$ and let $f(\eta) = \eta(v)$ implying that f is monotone in η. The generator of the process, which is given in Appendix A, can be written as

$$\Omega f(\eta) = c(v,\eta)(1 - 2\eta(v)). \tag{C.4}$$

Writing the revision rate in closed form

$$c(v,\eta) = a - (a-b)\eta(v) + (1-2\eta(v))\frac{c}{2d} \sum_{y\in\mathcal{N}(v)} \eta(y), \tag{C.5}$$

and taking into account that $(\eta(v))^k = \eta(v)$ and $(1-2\eta(v))^{2k} = 1$, the generator of the process can after some manipulation be shown to equal

$$\Omega f(\eta) \quad = \quad a - (a+b)\eta(v) + \frac{c}{2d} \sum_{y\in\mathcal{N}(v)} \Big(\eta(y) - \eta(v)\Big). \tag{C.6}$$

Let us consider the case that the economy starts from the state η^0, then

$f(\eta) = \eta^0(v)$ at time zero. Integrating (C.6) with respect to $\delta^0 S(t)$ yields

$$\begin{aligned} \int \Omega f[d\delta^0 S(t)] &= a - (a+b) \int f d[\delta^0 S(t)] \\ &+ \frac{c}{2d} \sum_{y \in \mathcal{N}(v)} \left(\int \eta^0(y) d[\delta^0 S(t)] - \int f d[\delta^0 S(t)] \right). \end{aligned} \quad \text{(C.7)}$$

The two parts of the last term in equation (C.7) refer to the same probability measures at different locations. However, due to the translation invariance of $\delta^0 S(t)$ the last term in equation C.7 vanishes. Moreover, from theorem 2 in appendix A it follows that equation (C.7) can equivalently be transformed to

$$\frac{d}{dt}\rho_t^0 = \int \Omega f d[\delta^0 S(t)] = a - (a+b)\rho_t^0. \quad \text{(C.8)}$$

Since $d/dt\rho_t^0 = 0$ for any stationary distribution, which is stated in theorem 4 in appendix B, it follows that

$$\lim_{t \to \infty} \rho_t^0 = \frac{a}{a+b}. \quad \text{(C.9)}$$

Clearly, the same algebra could be done for the process starting at η^1 since the generator is the same for both cases. Therefore $\nu = \lim_{t\to\infty} \delta^0 S(t) = \lim_{t\to\infty} \delta^1 S(t)$.

The statement that the rate of convergence is $a+b$ follows directly from (C.7), which is a simple differential equation. □

D The lemma for the proof of proposition 5

Lemma 2 *Let X_n be the location of a random walker on $\mathbb{Z}^d$ after the nth step. At each step he stays where he is with probability $1/2$ and jumps to each of the $2d$ adjacent coordinates with probability $1/4d$. This random walk is transient for $d \geq 3$, that is, the random walker returns to his initial location with probability less than one.*

Proof: Let P_n be the probability that $X_0 = X_n$ for a given coordinate.

A random walk is transient if and only if $\sum_{n=1}^{\infty} P_n < \infty$. [For a proof see for example Durrett (1991, Chapter III).] Therefore, it will be proved that our random walk satisfies this condition.

$X_0 = X_n$ if and only if the random walker goes as many steps forward in any direction as backward. Let $[x] \equiv \{$the largest integer $\leq x\}$ For a given sequence of steps ($\sum_{i=1}^{d} n_i \leq [n/2]$), we can compute the probability that after n steps the walker returns his original location. We also have to take into account that in some steps $(n - 2\sum_{i=1}^{d} n_i)$ he did not move. However, to determine P_n, all such possible sequences should be considered. Using combinatorial arguments we get

$$\begin{aligned} P_n &= \sum_{\sum_{i=1}^{d} n_i \leq [n/2]} \frac{n!}{(n_1!n_2!\cdots n_d!)^2(n-2\sum_{i=1}^{d} n_i)!}\left(\frac{1}{4d}\right)^{2\sum_{i=1}^{d} n_i}\left(\frac{1}{2}\right)^{n-2\sum_{i=1}^{d} n_i} \\ &= \sum_{k=0}^{[n/2]} \sum_{\sum_{i=1}^{d} n_i = k} \frac{n!}{(n_1!n_2!\cdots n_d!)^2(n-2k)!}\left(\frac{1}{4d}\right)^{2k}\left(\frac{1}{2}\right)^{n-2k} \\ &= \sum_{k=0}^{[n/2]} \binom{n}{2k}\left(\frac{1}{2}\right)^{n} \sum_{\sum_{i=1}^{d} n_i = k} \frac{(2k)!}{(n_1!n_2!\cdots n_d!)^2}\left(\frac{1}{2d}\right)^{2k} \\ &= \left(\frac{1}{2}\right)^{n} + \sum_{k=1}^{[n/2]} \binom{n}{2k}\left(\frac{1}{2}\right)^{n} \sum_{\sum_{i=1}^{d} n_i = k} \frac{(2k)!}{(n_1!n_2!\cdots n_d!)^2}\left(\frac{1}{2d}\right)^{2k}. \quad \text{(D.10)} \end{aligned}$$

The last part of the second term is the probability that a simple symmetric random walk on $\mathbb{Z}^d$ returns to its initial position after $2k$ steps. This probability is of order $k^{-d/2}$ as $k \to \infty$ [See Durrett (1991, Chapter III)], that is, it is approximately $Ck^{-d/2}$, where C is some constant. Therefore equation (D.10) can be rearranged as follows:

$$\begin{aligned} P_n &\approx \left(\frac{1}{2}\right)^{n} + C\sum_{k=1}^{[n/2]} \binom{n}{2k}\left(\frac{1}{2}\right)^{n}\left(\frac{1}{k}\right)^{\frac{d}{2}} \\ &= \left(\frac{1}{2}\right)^{n} + \frac{4Cn^{1/2}}{(n+1)(n+2)} \sum_{k=1}^{[n/2]} \binom{n+2}{2k+2}\left(\frac{1}{2}\right)^{n+2} \frac{(2k+1)(2k+2)}{n^{1/2}k^{3/2}}\left(\frac{1}{k}\right)^{\frac{d-3}{2}}. \end{aligned}$$

If $d \geq 3$, we obtain

$$
\begin{aligned}
P_n &\leq \left(\frac{1}{2}\right)^n + \frac{4Cn^{1/2}}{(n+1)(n+2)} \sum_{k=1}^{[n/2]} \binom{n+2}{2k+2} \left(\frac{1}{2}\right)^{n+2} \frac{(2k+1)(2k+2)}{n^{1/2}k^{3/2}} \\
&\leq \left(\frac{1}{2}\right)^n + \frac{4Cn^{1/2}}{(n+1)(n+2)} \sum_{k=1}^{[n/2]} \binom{n+2}{2k+2} \left(\frac{1}{2}\right)^{n+2} \\
&\leq \left(\frac{1}{2}\right)^n + \frac{4Cn^{1/2}}{(n+1)(n+2)} \\
&\leq \left(\frac{1}{2}\right)^n + \frac{4C}{n^{3/2}}. \qquad \text{(D.11)}
\end{aligned}
$$

However, this means that

$$
\sum_{n=0}^{\infty} P_n < \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^n + \frac{4C}{n^{3/2}}\right] < \infty. \qquad \text{(D.12)}
$$

Thus, our random walk behaves similarly to the classic simple symmetric random walk, that is, it is transient for $d \geq 3$ and recurrent for $d \leq 2$.□

E The proof of proposition 7

Before beginning the proof some notation has to be introduced. According to the definitions of the decision rule, the revision probabilities can be written as $P_{x,01}\left(u^1 - u^0 \geq m_{01}\right)$ if agent x uses the worse technology and is thinking switching to the other one, and $P_{x,10}\left(u^1 - u^0 < m_{10}\right)$ if the agent uses the better technology. m_{01} and m_{10} denote the weight on the relative popularity on the better and worse technology, respectively. It follows that

$$
\begin{aligned}
P_{x,01} &= P\left(u^1 - u^0 \geq m_{01}\right) = 1 - P\left(\epsilon < m_{01} - \theta\right) = \frac{\sigma + \theta - m_{01}}{2\sigma}, \\
P_{x,10} &= P\left(u^0 - u^1 > m_{10}\right) = P\left(\epsilon < m_{10} - \theta\right) = \frac{\sigma - \theta - m_{10}}{2\sigma}.
\end{aligned}
$$

The parameters of the popularity weighting obey the following functional forms:

$$
m_{01} = m\left(1 - \frac{1}{d} \sum_{y \in \mathcal{N}(x)} \eta(y)\right)
$$

$$m_{10} \;=\; m\left(1 - \frac{1}{d}\sum_{y\in\mathcal{N}(x)}(1-\eta(y))\right),$$

where $m \geq 0$ is a constant set by agent x and measured in the same units as the payoffs are. m represents the strength of the popularity weighting. If $m = 0$, agents rely entirely on the observation of the last period's payoffs. Furthermore, if exactly half of the neighbors use a technology, the popularity has no informational content. In this case m_{01} and m_{10} become zero and we get a decision rule which behaves as if agents used the information on the payoffs only. So far the decision rule has the same characteristic as that in Ellison and Fudenberg (1993). Now, the revision rate can be written as

$$c(x,\eta) = \begin{cases} \dfrac{\sigma - m + \theta}{2\sigma} + \dfrac{m}{2d\sigma}\displaystyle\sum_{y\in\mathcal{N}(x)}\eta(y) & \text{if} \quad \eta(x) = 0 \\ \dfrac{\sigma - m - \theta}{2\sigma} + \dfrac{m}{2d\sigma}\displaystyle\sum_{y\in\mathcal{N}(x)}(1-\eta(y)) & \text{if} \quad \eta(x) = 1. \end{cases} \tag{E.13}$$

Notice that this revision rate is not correctly defined since m could be set so high that $c(x,\eta)$ becomes negative which is not possible by assumption. For this reason, I impose the following constraint: if $\exists\, \eta \in E$ for a given $\overline{m}$ such that $c(x,\eta,\overline{m}) = 0$, then $c(x,\eta',\overline{m}) = 0\ \forall \eta' \geq \eta$ satisfying $\eta' \in E$. In addition, if $\nexists\, \eta \in E$ for a given $\overline{m}$ such that $c(x,\eta,\overline{m}) \geq 0$, then $c(x,\eta,\overline{m}) = 0\ \forall\, \eta \in E$.

The form of the revision rate reveals that we need the assumption on the neighbor–independent publicly available information about the payoff realization.

Proof: If $m \leq \sigma - \theta$, the revision rate attractive, spatially homogeneous and has no absorbing state. Hence we can use the arguments and method of Proposition 1. Given a state at t, if the "Poisson alarm clock" rings, the conditional probability that agents use the better technology at time $t + \Delta t$ is

$$\rho^i_{t+\Delta t} \;=\; (1-\rho^i_t)\left[\left(\frac{\sigma+\theta}{2\sigma} + \frac{m}{2\sigma}\left(2\frac{\varrho^i(x,y)}{1-\rho^i_t} - 1\right)\right)\Delta t\right]$$

$$+\rho_t^i \left[1 - \left(\frac{\sigma-\theta}{2\sigma} + \frac{m}{2\sigma}\left(2\frac{\varrho^i(x,y)}{\rho_t^i} - 1\right)\right)\Delta t\right] + o(\Delta t), \quad \text{(E.14)}$$

where ρ_i^t is the marginal of μ_t^i and $\varrho^i(x,y) = \mu_t^i\{\eta : \eta(x) = 1, \eta(y) = 0\} = \mu_t^i\{\eta : \eta(x) = 0, \eta(y) = 1\}$ for any $x, y \in \mathbb{Z}^d$. Recall, i is either 0 or 1 depending on whether the initial state is η^0 or η^1. Rearranging and taking the limit $\Delta t \downarrow 0$ yield the following pair of differential equations for $i = 0$ or $i = 1$:

$$\dot{\rho_t^i} = \frac{\sigma - m + \theta}{2\sigma} - \frac{\sigma - m}{\sigma}\rho_t^i. \quad \text{(E.15)}$$

If one sets the left hand side equal to zero and solves the resulting equation for ρ_t^i, the statement about the ergodic distribution and about the rate of convergence follows directly.

If $\sigma - \theta < m < \sigma + \theta$, the result can be obtained by applying simple coupling technique. Let ζ_t a process on E with the revision rate

$$c'(x,\zeta) = \begin{cases} \dfrac{\sigma - m + \theta}{2\sigma} + \dfrac{m}{2d\sigma} \displaystyle\sum_{y \in \mathcal{N}(x)} \eta(y) & \text{if} \quad \eta(x) = 0 \\ \dfrac{m}{2d\sigma} \displaystyle\sum_{y \in \mathcal{N}(x)} (1 - \eta(y)) & \text{if} \quad \eta(x) = 1. \end{cases} \quad \text{(E.16)}$$

Comparing the revision rate of η in (E.13) with that of ζ in (E.16), we can observe that if $\zeta \leq \eta$ and $\sigma - \theta < m < \sigma + \theta$, then

$$\begin{aligned} c'(x,\zeta) &\leq c(x,\eta) \quad \text{if} \quad \eta(x) = 0 \quad \text{and} \\ c'(x,\zeta) &\geq c(x,\eta) \quad \text{if} \quad \eta(x) = 1. \end{aligned}$$

This is due to the assumption that if $\exists \eta \in E$ for a given $\overline{m}$ such that $c(x,\eta,\overline{m}) = 0$, then $c(x,\eta',\overline{m}) = 0$ $\forall \eta' \geq \eta$ satisfying $\eta' \in E$. Moreover, if $\nexists \eta \in E$ for a given $\overline{m}$ such that $c(x,\eta,\overline{m}) \geq 0$, then $c(x,\eta,\overline{m}) = 0$ $\forall \eta \in E$.

The state ζ^1 is an absorbing state. Moreover, according to Theorem 6 in Appendix B, if $\zeta \leq \eta$, then $\zeta_t \leq \eta_t$ $\forall t \geq 0$ with probability one. Hence, it is sufficient to show that ζ_t is ergodic and converges with probability one to the state ζ^0. Then η_t is also ergodic and converges to

η^1. Now given a state at time t, the conditional probability that an agent uses the better technology at time $t + \Delta t$ is

$$\begin{aligned}\rho^{0\prime}_{t+\Delta t} &= (1-\rho^{0\prime}_t)\left[\frac{\sigma+\theta}{2\sigma} + \frac{m}{2\sigma}\left(2\frac{\varrho^{i\prime}(x,y)}{1-\rho^{0\prime}_t} - 1\right)\right]\Delta t \\ &+\rho^{0\prime}_t\left[1-\left(\frac{\sigma-\theta}{2\sigma} + \frac{m}{2\sigma}\left(2\frac{\varrho^{i\prime}(x,y)}{1-\rho^{0\prime}_t} - 1\right)\right)\Delta t\right] + o(\Delta t), \quad \text{(E.17)}\end{aligned}$$

where $\rho^{0\prime}_t$ and $\varrho^{i\prime}(x,y)$ are defined for ζ_t in the usual way. Rearranging (E.17) and taking the limit $\Delta t \downarrow 0$ provide

$$\dot{\rho}^{0\prime}_t = \frac{\sigma-m+\theta}{2\sigma} - \frac{\sigma-m+\theta}{\sigma}\rho^{0\prime}_t. \quad \text{(E.18)}$$

Setting the left hand side of the equation equal to zero and solving it for $\rho^{0\prime}_t$ deliver the desired result.

If $\sigma+\theta < m$, both state becomes absorbing. In this case the process is non–ergodic. □

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