Essays in Macroeconomics and Macroeconometrics

Francesca Loria

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence, 12 June 2018
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Statement of inclusion of previous work (if applicable):

I confirm that chapter 1 was jointly co-authored with Knut Are Aastveit and Francesco Furlanetto and that I contributed 50% of the work. It draws upon a working paper “Has the Fed Responded to House and Stock Prices? A Time-Varying Analysis”, Working Paper 1/2017 (2017), Norges Bank, and Working Paper 1713, Banco de España.

I confirm that chapter 2 was jointly co-authored with Nicolás Castro Cienfuegos and I contributed 50% of the work.

Signature and Date: 22 February 2018
Abstract

This thesis investigates topics in macroeconomics and macroeconometrics.

Chapter 1, joint with Knut Are Aastveit and Francesco Furlanetto, uses a structural VAR model with time-varying parameters and stochastic volatility to investigate whether the Federal Reserve has responded systematically to asset prices and whether this response has changed over time. To recover the systematic component of monetary policy, the interest rate equation in the VAR is interpreted as an extended monetary policy rule responding to inflation, the output gap, house prices and stock prices. Some time variation is found in the coefficients for house prices and stock prices but fairly stable coefficients over time for inflation and the output gap. We find that the systematic component of monetary policy in the U.S. i) attached a positive weight to real house price growth but lowered it prior to the crisis and eventually raised it again and ii) only episodically took real stock price growth into account.

Chapter 2, joint with Nicolás Castro Cienfuegos, constructs a New Keynesian model with production linkages to study how monopolistic competition, sticky prices and production networks influence aggregate productivity, measured as the Solow residual. We show that, in the presence of production networks, measured TFP is a function not only of pure technology shocks, but also of sectoral markups and of the production network itself. In this case, monetary shocks and cost-push shocks can have a negative short-run impact on TFP through their effect on individual markups, which is stronger the greater the price stickiness.

Chapter 3 studies how large and small oil price shocks affect investments in the U.S., an oil producing country. I estimate a Bayesian Markov-switching VAR and compute regime dependent impulse responses. Small surprise increases in the oil price make investment decline while large oil price shocks have an ambiguous effect on total investment because non-oil investment falls while oil investment increases. A 25% oil price increase generates a 3% increase in aggregate investment and a 0.4% increase in GDP. A Markov-switching DSGE model is built to explain the empirical evidence I discover. If the ability to cover oil firms' fixed costs depends on the size of the oil price shock, the model reproduces well the impulse responses present in the data. I show that agents' expectations about switching oil price shock regime are crucial to deliver the outcome.
Acknowledgments

This Ph.D. thesis was made possible by the kind and generous acts of many people. While the account that follows is by no means exhaustive or fair I hope it will at least give an idea of how important it is in one’s life to be supported by friends and teachers on one’s journey of self- and world-discovery.

To start with, I cannot resist to think back at my first dive into the world of education. My parents had just moved to Mönchengladbach, a city in Western Germany. Rumor has it that on the first day of elementary school I felt a bit out of place for being the only non-blonde person in my class. The following days were not any better. I often left German or Math classes because I could not understand any of it. If the situation quickly changed it is thanks to the continuing support of my teacher Renate and of my wonderful classmates, in particular my blonde(!) and blue-eyed friend Lukas, which saw in my exoticness a hidden treasure they were eager to discover. Renate is the prototypical example of those passionate and devoted teachers that can make a difference in a student’s life. She never stopped believing that I could make it and did everything possible to make my school experience an enjoyable one. School became a happy place were to learn and grow together and that feeling hasn’t left me since then.

This feeling became even stronger at the German School of Rome, where I learned a lot about life and the world thanks to its wonderful teachers and, in particular, to my mentor Frau Sabbadini. They nurtured my brain and soul while making me realize that one’s curiosity and emotionality are gifts to take care of. I am also grateful to Francesco for driving the bus to school and for making sure I always had music keeping me company on the long way there. I owe a great deal to my “sodalis” Ludovica for always inspiring me with her tremendous intelligence and kindness. Studying and solving problem sets with you was pure joy.

A new chapter of my life started when I moved to Berlin for my undergraduate studies at Humboldt Universität zu Berlin. I warmly thank the German Academic Exchange
Service for supporting me and for organizing annual meetings at which I met other foreign students coming from German schools abroad. All of you are incredibly talented and kindhearted people and I wish to thank you for making me feel at home at a time were I had left my family and friends behind in Italy. As to the academic side of my Berlin experience, my biggest thank you goes to Lutz Weinke for being a wonderful mentor. He made me passionate about macroeconomics and monetary economics and taught me so very much about how to address questions in these areas with precision and rigour. I thank all other academic and non-academic staff for their support as well as my friends in Berlin for making my stay there a truly unforgettable one.

Following the advice of Lutz, I decided to take on my graduate studies at the Barcelona Graduate School of Economics. This year proved to be one of the most intense of my life. Thanks to the support of Derrick and Christian, two passionate fellow Jedi knights who were also lucky to have Lutz as their New Keynesian master, I was able to survive and eventually enjoy my studies there. For making me passionate about empirical macroeconomics and for their support throughout those difficult months I am indebted with Vasco Carvalho, Kristoffer Nimark and Barbara Rossi. For his exciting lectures and textbooks in monetary economics, essential guides throughout my graduate years, I thank Jordi Galí.

Inspired by the idea of further deepening my studies in empirical macro I embarked on this incredibly adventurous journey called “Ph.D.” at the European University Institute. Here I worked under the supervision of Fabio Canova. This is probably not the place where to explain at due length how important a supervisor is in a Ph.D. student’s life and how important Fabio has been in mine. Let me at least say, however, that Fabio was a lighthouse and a safe harbour during the most difficult times of this tempestuous journey. He never made me feel left alone and was generous enough to always empower me to succeed academically and to keep my head cold during times where I had to deal with the frustratingly typical “Ph.D. blues”. I am deeply grateful to him for his help and for his teachings which profoundly shaped my critical understanding and approach to empirical macroeconomics and inspired me to become a better researcher.

I got double lucky with my second advisor, Juan Dolado, who proved to be one of my fiercest supporters since early on. I am grateful to him for all the times he helped me preparing my presentations and revising my research papers as well as for the countless hours and gigabytes of music he sent me to keep rolling on this bumpy Ph.D. journey.

Needless to say, I thank all other EUI professors for their teachings. My biggest thank you goes to Evi Pappa for her personal support during the job market and for being an inspiration to this young woman in the very male populated field of macroeconomics.
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The Ph.D. writing process can be a very lonely one. Luckily enough, in the past years I met wonderful co-authors and men who have supported and stimulated me. They made me a better researcher and helped me in so many ways on a personal level too. For this and more I have to thank Carlos Montes-Galdón, Donghai Zhang, Francesco Furlanetto, Knut Are Aastveit, Mario Porqueddu, Nicolás Castro Cienfuegos, Nikolay Iskrev, Pedro Brinca and Shengliang Ou. I am particularly grateful to Pedro who has been since the very beginning a generous mentor and friend.

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This thesis is dedicated to my dog Coccolino as well as to grandparents: Elio, Mirella, Giovanni and Maddalena. Thank you for your tenderness, I miss you all very much.
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**Size Matters**

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Has the Fed Responded to House and Stock Prices? A Time-Varying Analysis

***

This is joint work with Knut Are Aastveit and Francesco Furlanetto and published as Norges Bank and Banco de España 2017 Working Paper¹.

1.1 Introduction

The length and the severity of the Great Recession generated considerable interest in the evolution of US monetary policy over the period that preceded the recent economic slump. However, while the financial nature of the Great Recession revived the debate on whether monetary policy should respond directly to asset prices (cf. Borio and Lowe, 2002, Cecchetti et al., 2002, Bernanke and Gertler, 2000, 2001, Christiano et al., 2010, Galí, 2014), less attention has been devoted to the measurement of the actual response of the Federal Reserve to asset prices in recent years.

In this paper we take an empirical approach and evaluate to what extent the Fed reacted to asset prices over the Great Moderation period until the beginning of the Great

¹This paper should not be reported as representing the views of Norges Bank or Banco de España. The views expressed are those of the authors and do not necessarily reflect those of Norges Bank or Banco de España. For useful comments we thank our discussants Simona Delle Chiaie and Sandra Eickmeier, Hilde Bjornland, Fabio Canova, Efrem Castelnuovo, Laurent Ferrara, Jordi Galí, Veronica Harrington, Massimiliano Marcellino, Christian Matthes, Elmar Mertens, Giorgio Primiceri, Juan Rubio Ramírez, Herman Van Dijk, one anonymous referee and seminar participants at the CEF conference in Bordeaux, Banque de France, Deutsche Bundesbank, the EEA conference in Geneva, the ESOBE conference in Venice, the IAAE conference in Milan, LUISS University in Rome and Norges Bank.
Recession. In particular, we consider whether stock prices and house prices entered the Fed’s reaction function with a positive and significant coefficient. Our key contribution is in providing time-varying estimates of the monetary policy response to asset prices by using a VAR model with time-varying parameters and stochastic volatility (TVP–SV–VAR, henceforth), following Primiceri (2005) and Cogley and Sargent (2005). More specifically, we interpret the interest rate equation in our VAR using five variables (interest rate, inflation rate, the output gap, house prices and stock prices) as an extended monetary policy rule in the spirit of Arias et al. (2015), Belongia and Ireland (2016a,b), Canova and Gambetti (2009) and Primiceri (2005), among others. This set-up allows us to track the systematic response to stock prices and house prices over our sample period, which goes from 1975:Q2 to 2008:Q4. As far as we know, the seminal contributions in this literature (cf. Bernanke and Gertler, 2000, and Rigobon and Sack, 2003) and the following extensions are all based on models with constant coefficients. Note that an alternative approach to using a TVP–SV–VAR is to estimate constant coefficient models for either rolling window samples or various sub-samples. However, the TVP–SV–VAR has the advantage of being more flexible as it jointly takes into account the information contained by all the variables in the full sample while at the same time explicitly modeling time variation in both the coefficients and the volatility.

Our main result is that the Fed responded to house prices and stock prices. While the response to stock prices was mild and episodic, the response to house prices was significant, from a statistical and economic point of view. We estimate the coefficient for house price growth to be about one third of the inflation coefficient in the policy rule. Moreover, we identify non-negligible time variation in the coefficients. The coefficient on stock prices is higher around the end of the 1980s, thus capturing a marked response to the stock market crash of 1987, whereas it is relatively low and stable in the last part of the sample. The coefficient on house price inflation exhibits more pronounced swings: we identify a lower response around the mid 1990s and also in the Pre–Great Recession period. Nevertheless, the coefficient is large, even in the pre–Great Recession period. Finally, the coefficients on inflation and the output gap and the interest rate smoothing term are relatively stable over time, with the exception of the mid 1990s.

While we do not find major evidence of time variation in the coefficients for inflation and the output gap, the use of a model with time-varying coefficients and stochastic volatility turns out to be crucial for detecting the Fed’s response to house price growth and to stock market returns. In fact, when we shut down time variation in the coefficients or stochastic volatility, the model does not find any response to house price growth. Moreover, the response to stock prices is estimated to be not statistically significant in a model with constant coefficients. Therefore, we conclude that having a model
with time-varying coefficients and stochastic volatility is important in order to analyze our research question. Notably, the finding of a significant response to house prices is robust to changing the order of the variables in our VAR.

This paper contributes to two strands of the literature. First, we obviously complement previous studies on the monetary policy response to stock prices. Bernanke and Gertler (2000) estimate Taylor-type rules with a GMM methodology for the US and Japan and find evidence of a very small response, always statistically insignificant and in some cases even negative. Rigobon and Sack (2003) estimate a VAR identified through heteroscedasticity and conclude that the response of monetary policy to stock prices in the US was positive and significant over the period 1985–1999. The same result emerges in Castelnuovo and Nisticò (2010), in an estimated dynamic stochastic general equilibrium (DSGE) model where monetary policy responds to fluctuations in the stock market, and in Bjørnland and Leitemo (2009), in a VAR identified using a combination of short-run and long-run restrictions. In contrast, a more recent literature argues that the Rigobon and Sack’s finding is confined to specific periods (around the 1987 stock market crash in Furlanetto (2011) and more generally around recession periods in Ravn, 2012). While those results rely on various forms of sample splitting, they may highlight some instability in the relationship between monetary policy and stock prices, thus calling for the use of a model with time-varying coefficients.

Interestingly, while several papers study the response of monetary policy to stock prices, the response to house prices is largely unexplored. A noteworthy exception is Finocchiaro and von Heideken (2013) who estimate the house price coefficient in a monetary policy rule and find evidence of a positive and significant response in the US in the context of a DSGE model. Bjørnland and Jacobsen (2013) provide evidence on the (conditional) response of interest rates to shocks originating in the stock market and in the housing sector but do not report the coefficients in the interest rate equation.

We contribute also to a second (and more recent) strand of literature that introduces asset prices into TVP-SV-VAR models. Prieto et al. (2016) use data on several financial variables (including house prices and stock prices) to investigate the time-varying transmission mechanism and the relative importance of various financial shocks. However, they do not consider the systematic component of monetary policy in their analysis. Galí and Gambetti (2015) study the time-varying response of stock prices to monetary policy shocks. While our model is similar, we focus on the opposite relationship, i.e. the response of monetary policy to stock prices.

Finally, we also contribute to the debate on the monetary policy stance in the pre-Great Recession period initiated by Taylor (2007, 2009) who argues that the interest was kept too low for too long prior to the crisis. Belongia and Ireland (2016b) estimate a
TVP-SV-VAR model with three variables (a measure of inflation, the interest rate and a measure of real economic activity) and find evidence of a lower response to inflation in recent years, thus supporting the Taylor evidence. Our model can be seen as an extension of their model to include asset prices in the analysis.

The paper proceeds as follows. Section 2 lays out the model and the details of the estimation. Section 3 presents our results and a sensitivity analysis. Section 4 relates our results to the debate on the monetary policy stance in the pre-Great Recession period. Finally, Section 5 concludes.

### 1.2 Econometric Model

To study how the Fed responded to asset prices in the pre-Great Recession period, we use the time-varying parameters and stochastic volatility VAR model à la Primiceri and Cogley and Sargent with the reduced form representation

\[
x_t = c_t + B_{1,t}x_{t-1} + \ldots + B_{p,t}x_{t-p} + u_t, \text{ } t = 1, \ldots, T,
\]

where \( x_t \) is a \( n \times 1 \) vector of endogenous variables, \( c_t \) is a \( n \times 1 \) vector of time-varying coefficients that multiply constant terms, \( B_{i,t}, i = 1, \ldots, p \) are \( n \times n \) matrices of time-varying coefficients, and \( u_t \sim MVN(0, \Omega_t) \), with \( \Sigma_t \) diagonal and \( A_t \), the contemporaneous (time-varying) coefficients matrix, lower triangular. In stacked form, the model is equal to:

\[
x_t = Z_t' B_t + A_t^{-1} \Sigma_t \varepsilon_t, \text{ where } Z_t' \equiv I_n \otimes [1, x_{t-1}', \ldots, x_{t-p}'].
\]

The time-varying parameters evolve according to:

\[
B_t = B_{t-1} + \nu_t, \quad \alpha_t = \alpha_{t-1} + \zeta_t, \quad \log \sigma_t = \log \sigma_{t-1} + \eta_t.
\]

It is assumed that the innovations in the model are jointly normally distributed with the following variance-covariance matrix:

\[
V \equiv Var \left( \begin{bmatrix} \varepsilon_t \\ \nu_t \\ \zeta_t \\ \eta_t \end{bmatrix} \right) = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{bmatrix}.
\]

---

2We follow the updated MCMC procedures suggested by Del Negro and Primiceri (2015). They retain most of the procedures in Primiceri (2005) except that sampling of stochastic volatilities is preceded by sampling of states for mixture component approximations to errors with log chi-square distributions.

3To check the validity of these assumptions we run a rolling window constant parameter VAR model. We find random-walk behaviour in the first difference of the coefficients, a result which substantiates the assumptions.
As a first pass, the structural representation is recovered via a recursive identification scheme. This identification strategy follows the seminal contributions by Christiano et al. (1999) in VAR models with constant coefficients and Primiceri (2005) in VAR models with time-varying coefficients. Notice that our results are robust to different orderings of the variables in the VAR and do not display any particular anomaly that would require a different identification scheme. For instance, we have never encountered counterfactual monetary policy rule coefficients using our recursive identification scheme. This is in line with the results in Arias et al. (2013) which show that structural VARs identified via recursive identification schemes, unlike those identified with restrictions on the sign of impulse responses, do not imply Taylor rule coefficients on inflation and real economic activity with the wrong (negative) sign.

The posterior distributions of $B_t$, $Q$, $A_t$, $S$ and $W$ are obtained via Gibbs sampling with standard prior assumptions as in Primiceri (2005)

$$
B_0 \sim N\left(\hat{B}_{OLS}, 4 \cdot V(\hat{B}_{OLS})\right), \quad (1.4)
$$
$$
Q \sim IW\left(k_Q^2 \cdot \tau \cdot V(\hat{B}_{OLS}), \tau\right), \quad (1.5)
$$
$$
A_0 \sim N\left(\hat{A}_{OLS}, 4 \cdot V(\hat{A}_{OLS})\right), \quad (1.6)
$$
$$
S_m \sim IW\left(k_S^2 \cdot (m + 1) \cdot V(\hat{A}_{m,OLS}), (m + 1)\right), \quad (1.7)
$$
$$
\text{log } \sigma_0 \sim N(\text{log } \hat{\sigma}_{OLS}, 4 \cdot I_n) \quad (1.8)
$$
$$
W \sim IW\left(k_W^2 \cdot (n + 1) \cdot I_n, (n + 1)\right), \quad (1.9)
$$

where $m = 1, \ldots, n - 1$, $\hat{A}_{m,OLS}$ is the $m$-th row of $\hat{A}_{OLS}$ and $\tau$ is the size of the training sample on which a time invariant VAR is estimated by OLS in order to calibrate the prior distributions described above. Following Primiceri (2005), we use the first 10 years of data as a training sample to calibrate the priors for estimation over the actual sample period, which starts in 1985:Q3. The selection of the hyperparameters also follows Primiceri (2005) in choosing $k_Q = 0.01$ and $k_S = 0.1$, with the sole exception of allowing for more time variation in the stochastic volatility by setting $k_W$ to 1 as opposed to 0.01 and by tuning the prior variance of $\text{log } \sigma_t$ to $4I_n$ instead of $I_n$. With these choices of hyperparameters, the priors are diffuse and uninformative and, in fact, the prior for

---

4The total number of Gibbs sampling iterations is set to 150,000 with a burn-in of 100,000 draws and convergence is checked by means of rolling variances plots. We keep the remaining 50,000 draws and use every 100th for inference. The results are basically identical if, more conservatively, we kept every 20th draw instead. In that case, if anything, we find that the coefficient on S&P 500 growth is significant with a magnitude of 0.02 not only around the 1987 financial crisis but also in 2007:Q3, i.e., prior to the onset of the Great Recession. Also, results are unaffected if we do or we do not truncate the autoregressive matrices to yield stationary draws. In all exercises, the number of stationary draws is always above $2/3$. 

---
the stochastic volatility of the model described in (1.8) is de facto flat. This choice of priors is conservative for the question we address in the sense that it does not restrict the amount of potential time variation in the volatility of the model and, thus, does not artificially blow up the time variation in the policy coefficients.

We consider quarterly data from 1975:Q2 to 2008:Q4. In particular, the vector $x_t = [\Pi_t \ Y_t \ \Delta H_t \ \Delta S&P_{500} \ FFR_t]'$ consists of $\Pi_t$, year-over-year percentage changes in the deflator for personal consumption expenditures (excluding food and energy), $\tilde{Y}_t$, the output gap measured as the percentage-point difference between actual real GDP and the US Congressional Budget Office estimate of real potential GDP, $\Delta H_t$, the percentage growth of the real Freddie Mac House price index, $\Delta S&P_t$, the percentage growth of the real S&P 500 index, and $FFR_t$, the federal funds rate. Asset prices are deflated by core PCE. All raw series are drawn from the FRED database.

A lag length of $p = 1$ is suggested by the BIC criterion obtained from OLS estimation of the constant parameters version of our model. This lag order has the fortunate by-product of facilitating the comparison with the macroeconomic literature on Taylor rules with interest rate smoothing.

The systematic component of monetary policy is recovered from the structural representation of our model

$$A_t x_t = A_t c_t + A_t B_{1,t} x_{t-1} + \Sigma_t \varepsilon_t$$

(1.10)

or, equivalently, and omitting constant terms

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\Pi_t \\
\tilde{Y}_t \\
\Delta H_t \\
\Delta S&P_t \\
\Delta R_t
\end{bmatrix} + \begin{bmatrix}
\sigma_1, t & 0 & 0 & 0 & 0 \\
0 & \sigma_2, t & 0 & 0 & 0 \\
0 & 0 & \sigma_3, t & 0 & 0 \\
0 & 0 & 0 & \sigma_4, t & 0 \\
0 & 0 & 0 & 0 & \sigma_5, t
\end{bmatrix} \begin{bmatrix}
\varepsilon_{\pi, t} \\
\varepsilon_{Y, t} \\
\varepsilon_{H, t} \\
\varepsilon_{SP, t} \\
\varepsilon_{R, t}
\end{bmatrix}
$$

(1.11)
Looking at the fifth row of \((1.1)\), we have

\[
a_{51,t} \Pi_t + a_{52,t} \tilde{Y}_t + a_{53,t} \Delta H_t + a_{54,t} \Delta S_500 + R_t \\
= ab_{51,t} \Pi_{t-1} + ab_{52,t} \tilde{Y}_{t-1} + ab_{53,t} \Delta H_{t-1} + ab_{54,t} \Delta S_500 + ab_{55,t} R_{t-1} + \sigma_{5,t} \varepsilon_t^R.
\]

(1.12)

Bringing \(R_t\) over to the left-hand side yields

\[
R_t = -a_{51,t} \Pi_t - a_{52,t} \tilde{Y}_t - a_{53,t} \Delta H_t - a_{54,t} \Delta S_500 \\
+ ab_{51,t} \Pi_{t-1} + ab_{52,t} \tilde{Y}_{t-1} + ab_{53,t} \Delta H_{t-1} + ab_{54,t} \Delta S_500 + ab_{55,t} R_{t-1} + \sigma_{5,t} \varepsilon_t^R, \tag{1.13}
\]

where the coefficient \(ab_{55,t}\) captures the degree of interest rate smoothing.

We will focus on the time evolution of the sum of the coefficients on the contemporaneous and lagged variables (e.g. \(-a_{51,t} + ab_{51,t}\) for \(\Pi_t\)) but we will also present results for the long-run coefficients (obtained by dividing the sum of coefficients by \((1 - ab_{55,t})\)) in order to represent the response of the interest rate to a permanent one percentage-point increase in the variables included in the VAR. The interest on the sum of coefficients and long-run coefficients follows the literature on TVP-SV-VAR models, starting with Primiceri (2005) and including Canova and Gambetti (2009), among others. These coefficients are viewed as the correct empirical benchmark for detecting violations of the so-called Taylor principle, derived by the theoretical literature. Moreover, two additional reasons justify the interest in long-run responses. The first is an economic one and deals with the fact that central banks might not observe precisely data from the current quarter and thus rather put substantial weight on data from the previous period. The second is an econometric one and it recognizes the fact that in autoregressive models the coefficients on the lags of a process can compensate each other. Concentrating on a single coefficient would thus be misleading if one seeks to explore the contribution of a given variable to changes in the interest rate path.

1.3 Results

In this section we present estimates for the time-varying coefficients on real house price and real stock price growth as well as for the coefficients on the standard objectives of monetary policy in the context of our baseline model. Later on we perform sensitivity analysis to discuss issues related to simultaneity and to the importance of time-varying parameters and stochastic volatility for our results.

The coefficients are reported along with the 16\%-84\% credibility intervals for the period ranging from 1985:Q3 to 2008:Q4 since we discard the training sample.
1.3.1 Baseline Estimation

The coefficient on the lagged interest rate finds a counterpart in the interest rate smoothing term in a Taylor-type monetary policy rule. Our median estimate is centered around 0.85, as shown in the top left panel of Figure 1.1. Incidentally, this value is fully in line with estimates in the DSGE literature. Indeed, the posterior mode estimate for the interest rate parameter is 0.81 in the seminal paper by Smets and Wouters (2007).

In the mid panel in Figure 1.1 we plot time-varying estimates for the sum of coefficients on current and lagged core PCE inflation and output gap. When compared with Taylor rule coefficients, our estimates appear to be particularly small. The coefficients generally considered standard in a Taylor (1993) rule are higher than 1.5 for the term on inflation and between 0.5 and 1 for the term on the output gap. However, those numbers ignore the role of the interest rate smoothing coefficient. To give some purely illustrative guidance on how to interpret our estimated coefficients, consider as an example the following Taylor-type rule with constant coefficients featuring interest rate smoothing:

$$R_t = (1 - \rho_R)(c + \omega_\pi \pi_t + \omega_y \bar{y}_t) + \rho_R R_{t-1},$$

where the parenthesis contains the inflation and output gap objectives and $c$ represents a constant, possibly related to the natural rate of interest. As highlighted by the previous equation, our estimated coefficients on inflation and on the output gap should be divided by $(1 - \rho_R)$ to recover average values for $\omega_\pi$ and $\omega_y$ of approximately 1.75 and 1.2 respectively.

The coefficients are fairly stable over time, with evidence of a gradual decrease and subsequent increase in the inflation coefficient from early 1992 to late 1998. The output coefficient features the inverse pattern and thus (partially) compensates for the decline in the inflation coefficient. While we find some time variation during the 1990s, the evidence for the 2000s favors more stable coefficients. Nevertheless, we find a smaller response to inflation and higher response to the output gap during the pre-Great Recession period. We will discuss further the implications of our results for the debate over the stance of monetary policy in the pre-Great Recession period in Section 4.

Next, we answer the main questions of interest to this paper, namely i) whether the Fed responded systematically to asset prices and ii) whether this response changed over time. The evidence shown in the bottom panel Figure 1.1 provides a positive answer to both questions. Indeed, the real S&P 500 growth coefficient is significant and equal to 0.02 around the 1987 financial crisis and otherwise (weakly) insignificant. The magnitude of the coefficient is in the ballpark of the one found by Rigobon and Sack (2003). Moreover, our results are in line with Furlanetto (2011), who shows that the positive and
significant coefficient found by Rigobon and Sack (2003) relies on the 1987 financial crisis period being present in the sample. A very similar value (around 0.025, once the estimate for the smoothing coefficient has been taken into account) is found also by Castelnuovo and Nisticò (2010) in an estimated DSGE model with overlapping generations using data on stock price growth as an additional observable variable. All in all, we conclude that stock price growth entered the central bank’s reaction function with a statistically significant coefficient only around the 1987 stock market crash.

While our results on the response to stock prices are in line with the previous literature, even though the methodology is different, we uncover some new findings when we investigate the response to real house price growth, which we plot in the bottom panel of Figure 1.1. The estimated coefficient is significant and roughly equal to 0.1, which is about one third of the inflation coefficient. Such a high response to house prices is estimated over most of our sample with the important exception of the second part of the 1990s where we identify a lower response to house prices (and to PCE inflation). The coefficient gradually decreases from early 2004 and starts rising again back to its previous level in the year prior to the onset of the financial crisis in 2007:Q4. Notably, the decline in the response to house price inflation during the pre–Great Recession period is substantially larger than the decline in the response to consumer price inflation.

We believe that our results on the Fed’s conduct are open to different interpretations. On the one hand, we find that the Fed has on average responded to fluctuations in house prices and that this response has on average been quantitatively important. On the other hand, the response declined somewhat sharply precisely in the period when house prices were growing most, i.e. the pre–Great Recession period.

A positive and significant response to house prices from the monetary policy authority to the best of our knowledge has never been found in the VAR literature. As far as we know, only one study provides estimates of the monetary policy response to house prices: Finocchiaro and von Heideken (2013) estimate a DSGE model with nominal loans and collateral constraints in which they embed a monetary policy rule with a direct response to house prices. The mean of their estimated coefficient for the US is 0.36 which, once adjusted for the estimated degree of interest rate smoothing (0.71 in their case), corresponds to the average response over our sample period.

It is important to highlight that our model features a time-varying constant and, thus, a time-varying unconditional mean $E(x_t) = (I - B_{1,t})^{-1} c_t$. The latter may be related to a measure of the natural nominal rate of interest, which is often considered a fixed number in the literature on Taylor rules. In the top right panel of Figure 1.1 we plot the evolution of the time-varying constant as estimated in our model. It peaks at around 8 percent in 1993 and the declines substantially until the beginning of the new century. Since then it
has been reasonably stable, reaching its highest value around 5.75 percent in 2006. As for the other parameters, most time variation seems to be concentrated over the 1990s. Finally, a word of caution regarding these estimates. Fluctuations in the constant in the VAR may reflect fluctuations in the natural rate of interest but also shifts in the inflation target. Our model is unable to disentangle these two sources of variation.

Next, we report in Figure 1.2 the long-run coefficients which, as in Canova and Gambetti (2009) and Primiceri (2005) among others, measure the total increase in the federal funds rate that would follow a permanent one percentage point increase in the respective variable. Not surprisingly, the dynamics of the long-run coefficients are very similar to the ones presented previously for the sum of coefficients. The only difference lies in the magnitude of the coefficient, which is amplified by the interest rate smoothing term.

While the focus of our paper is the systematic component of monetary policy, an interesting by-product of our analysis is the impulse responses to a monetary policy shock. In order to obtain the impulse responses for the data in terms of levels, we apply the standard practice of cumulating the impulse responses over horizons at each point in time. The magnitude of the impulse responses can thus be interpreted as a percentage change in a given variable in levels following a 1 percentage point increase in the federal funds rate. In Figure 1.3 we see that a contractionary monetary policy shock has a negative effect on real economic activity as in Galí and Gambetti (2015) and Prieto et al. (2016). Moreover, as in the latter papers and in Primiceri (2005), we do not find a price puzzle in the inflation response over the entire sample period. Turning to the response of stock prices, the latter increase permanently in response to a contractionary monetary policy shock, which is fully in line with the results reported in Galí and Gambetti (2015) for the post-1985 period. Clearly, a positive response of stock prices following a monetary policy shock is at odds with what the “conventional” view would predict, but it can be rationalized in the presence of a bubble component driving stock prices as pointed out theoretically by Galí (2014) and shown empirically by Galí and Gambetti (2015). Finally, a contractionary monetary policy shock unambiguously lowers house prices over the entire sample period, consistent with several previous papers summarized in Williams (2015). Note that the estimated effect is particularly small, at the lower bound of the estimates reviewed in Williams (2015). This is not so surprising, however, since in our model the systematic response of monetary policy largely undoes the direct effect of a monetary policy shock on house prices. We conclude that, according to our model, monetary policy surprises are not effective in curbing house prices.
1.3. RESULTS

Figure 1.1 – Top Panel: Degree of Interest Rate Smoothing and Interest Rate Trend. Mid Panel: Sum of Coefficients on Inflation and Output Gap. Bottom Panel: Sum of Coefficients on Real House Price Inflation and S&P 500 Growth.
Figure 1.2 – Long-Run Coefficients. *Top Panel:* Inflation and Output Gap, *Bottom Panel:* Real House Price Inflation and S&P 500 Growth.
Figure 1.3 – Impulse Responses to a Monetary Policy Shock. *Top Panel*: Interest Rate, *Mid Panel*: PCE Prices and Output Gap, *Bottom Panel*: House Prices and Stock Prices.
1.3.2 Sensitivity Analysis

In this section we perform robustness checks on the estimates for the systematic response of monetary policy to house prices and stock prices.

In the first exercise we change the ordering of the variables in the econometric model. In our baseline we have restricted the impact response of house prices and stock prices to a monetary policy shock since our main focus is on the response of monetary policy to financial variables. Here, we change the vector $x_t$ from $x_t = [\Pi_t \tilde{Y}_t \Delta H_t \Delta S & P_{500}^t FFR_t]'$ to $x_t = [\Pi_t \tilde{Y}_t FFR_t \Delta H_t \Delta S & P_{500}^t]'$, thus imposing that monetary policy does not respond to house prices and stock prices within the quarter. The results, reported in the top panel of Figure 1.4, point towards a level shift in the coefficients on house prices and S&P 500 but do not change our main result that the FED responded to house prices and that it did so in a time-varying fashion. The response to stock prices is now slightly larger and statistically significant over almost the entire sample period. These results show that different impact responses of variables to shocks (as determined by a different order of the variables) do not have large effects on the estimates for the systematic component of monetary policy. Since the choice of the order is necessarily arbitrary when dealing with fast moving variables such as interest rates, stock prices and (to a lesser extent) house prices, it is reassuring that the main patterns are confirmed.

In a second exercise we use the order $x_t = [\Pi_t \tilde{Y}_t \Delta H_t \Delta S & P_{500}^t FFR_t]'$, where we recognize that stock prices are a fast moving variable but maintain the assumption that house prices do not respond within a quarter to monetary policy shocks, as in the baseline. Not surprisingly, the results presented in the mid panel of Figure 1.4 are an intermediate case between the alternative model presented in the top panel and the baseline model.

In a third exercise we extend the sample to 2015:Q2 by using the Wu and Xia (2016) shadow rate as the monetary policy tool. As is clearly evident from the results reported in the bottom panel of Figure 1.4, we can not overturn the conclusion that the coefficients on house prices and stock prices were positive and significant prior to the onset of the Great Recession and around the 1987 stock market crash, respectively.

The last three exercises are devoted to understanding the role of time variation and stochastic volatility in our econometric model. As becomes evident from Figure 1.5, it is crucial to account for both channels. Indeed, shutting down one or both of them would lead a researcher to mistakenly infer that the FED did not care about house prices in its conduct of monetary policy. While the response to stock prices is preserved in the absence of stochastic volatility, this is not the case for house prices. To detect a positive response to house prices, both time variation and stochastic volatility are needed. This
result justifies the use of a model with time variation and stochastic volatility to analyze our research question, rather than a simple constant coefficient VAR, even if the estimated amount of time variation is rather limited (although non-negligible). Note that a limited amount of time variation in the estimated coefficients is in keeping with the previous literature using this kind of model (cf. Belongia and Ireland (2016b), Canova and Gambetti (2009) and Primiceri (2005)).
Figure 1.4 – Sum of Coefficients on Real House Price Inflation and S&P 500 Growth. Top Panel: Reordered Model with $x_t = [\Pi_t \tilde{Y}_t FFR_t \Delta H_t \Delta S&P_{500}^{t-1}]$ (Top Panel), Mid Panel: Reordered Model with $x_t = [\Pi_t \tilde{Y}_t \Delta H_t FFR_t \Delta S&P_{500}^{t-1}]$. Bottom Panel: Model Extended to 2015:Q2, Shadow Rate as Monetary Tool.
Figure I.5 – Sum of Coefficients on Real House Price Inflation and S&P 500 Growth. Top Panel: Model with No Stochastic Volatility. Mid Panel: Model with No Time Variation in $A_t$ and $B_{1,t}$. Bottom Panel: Model with No Stochastic Volatility and No Time Variation in $A_t$ and $B_{1,t}$. 
1.4 Debate on the Evolution of US Monetary Policy

In this section we discuss the evolution of US monetary policy in the pre–Great Recession period. In particular, we contribute to the debate between Taylor (2007, 2009) and Bernanke (2015) through the lenses of our model that includes house prices and stock prices.

In recent years, Taylor argued that the FOMC policy has been “too low for too long” compared to the interest rate path prescribed by his rule (Taylor, 2007, 2009). Bernanke (2015) shows that when using i) real–time data, ii) a modified Taylor rule with a coefficient of 1 for the output gap and iii) PCE inflation instead of GDP inflation, the “Great Deviation” pointed out by Taylor does not emerge.

Belongia and Ireland (2016b) estimate a three variable TVP-SV-VAR model and find evidence of i) declining coefficients in their model’s estimated policy rule, pointing to a shift in the Fed’s emphasis away from stabilizing inflation during the period 2000–2007, and ii) large expansionary monetary policy shocks during the period 2003:Q3–2004:Q2, seen as evidence of a more discretionary policy and thus supporting Taylor’s argument.

Our model with house prices and stock prices confirms some limited changes in the systematic component of monetary policy. We find evidence of a slightly lower response to inflation (but not to the output gap) in the pre–Great Recession period. Moreover, we also find a decline in the response to house prices that points to a less aggressive reaction of monetary policy to economic conditions. However, we find only relatively small changes in the non–systematic component of monetary policy. More specifically, we consider the fitted interest rate implied by our model:

\[
\hat{R}_t = -\hat{a}_{51,t}\Pi_t - \hat{a}_{52,t}\tilde{Y}_t - \hat{a}_{53,t}\Delta H_t - \hat{a}_{54,t}\Delta S&P_{500}^t + \hat{ab}_{51,1}\Pi_{t-1} + \hat{ab}_{52,1}\tilde{Y}_{t-1} + \hat{ab}_{53,1}\Delta H_{t-1} + \hat{ab}_{54,1}\Delta S&P_{500}^{t-1} + \hat{ab}_{55,1}\hat{R}_{t-1}.
\]

The left panel of Figure 1.7 shows that the implied VAR interest rate fits well with the actual interest rate movements. We find some expansionary monetary policy shocks during the period 2003–2004, but we do not detect a “Great Deviation” from the historical rule. In Figure 1.6 we plot the posterior median of the standard deviation of monetary policy shocks and we note a period of relatively high volatility at the beginning of the new century. However, this volatility declines rapidly from 2003 and reaches its minimum in 2004 before increasing again. Moreover, the expansionary shocks estimated over the period 2003–2004 are fairly balanced by a series of contractionary shocks in 2006–2007.

Overall, we confirm previous results in Belongia and Ireland (2016b), as we identify a shift in Federal Reserve policy away from inflation and house price inflation stabilization.
and some departures from rule-like behavior. However, our reading of the results is that these changes are quantitatively small and that this evidence is not sufficient to conclude that the interest rate was too low for too long.

Next, Orphanides (2001) stresses that real-time policy recommendations may differ considerably from those obtained with ex-post revised data. However, Croushore and Evans (2006) show that accounting for data revisions has only a modest effect quantitatively on the recursively identified monetary policy shock measures and impulse responses obtained from standard VARs, supporting the common approach of using ex-post revised data in structural VARs for monetary policy analysis. Nevertheless, to ensure that our estimated time-varying parameters are also relevant in real time, we plot in the right panel of Figure 1.7 the implied interest rate path based on the estimated TVP-SV-VAR parameters, replacing the ex-post values for core PCE inflation and the output gap with their real-time counterparts. We obtained the real-time data vintages from the Federal Reserve Bank of Philadelphia’s Real-Time Dataset for Macroeconomists (RTDSM), described in Croushore and Stark (2001). The bottom panel of Figure 1.7 shows that also when using real-time data the implied VAR interest rate fits well with the actual interest rate movements. In keeping with the argument in Bernanke (2015), the use of real-time data reduces further the role of unsystematic policy in recent years.

To sum up, in the case of both real-time data and ex-post data, we do not find evidence of a “Great Deviation” from the path prescribed by the systematic component estimated by our model, in line with Bernanke’s arguments.

![Figure 1.6](image_url)  
**Figure 1.6** – Standard Deviation of Monetary Policy Shocks Over Time.

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5Note that real-time data vintages for PCE inflation are only available from 1996:Q2 onwards.
Chapter 1. Has the Fed Responded to House and Stock Prices? A Time-Varying Analysis

1.5 Conclusion

The main contribution of this paper is to provide evidence on the time-varying response of monetary policy to stock prices and house prices. We find that the response to stock price fluctuations has been small and episodic, in keeping with the previous literature. Our main result is that we find a significant response to house prices, both in economic and statistical terms. While the response to house prices declines somewhat in the pre-Great Recession period, our evidence shows that the Fed considers variables other than inflation and real economic activity in its estimated reaction function. Our analysis has no normative implications for whether such a response to asset prices (and house prices in particular) was optimal, insufficient or excessive. Nevertheless, we believe it is interesting to document that it was substantial.

One direction for future research is to further take into account the simultaneity in the determination of house prices, stock prices and interest rates. In this paper we have shown that our results hold when imposing different orders. However, it would be interesting to explore alternative identification schemes, perhaps building on previous attempts to deal with the simultaneity problem in the context of models with constant coefficients (cf. Bjørnland and Leitemo, 2009, D’Amico and Farka, 2011, Rigobon and Sack, 2003). Bringing these insights into models with TVP-SV-VAR seems an interesting avenue for future research.
References


CHAPTER I. HAS THE FED RESPONDED TO HOUSE AND STOCK PRICES? A TIME-VARYING ANALYSIS


A New Keynesian Perspective on Total Factor Productivity via Production Networks

This is joint work with Nicolás Castro Cienfuegos.

2.1 Introduction

This paper seeks to study the effect of monopolistic competition, sticky prices and production networks on both the steady state level and the business cycle movements of the total productivity of factors (TFP). To address these questions we provide a multi-sector New Keynesian model in which sectors are related to each other by input-output production linkages. The model is calibrated to the 2015 Bureau of Economic Analysis (BEA) input-output (I-O) tables and features monopolistic competition as well as à la Calvo price frictions.

An aggregate production function is derived to show that (i) the level of the Solow residual is a function of technology, cost-push shocks, sectoral markups and the production network, (ii) monetary shocks can have a short run impact on TFP through their effect on individual markups and (iii) the presence of production networks as defined by the input-output linkages can amplify the gap between Solow’s residual and true technology. In a nutshell, what we show is that, in the presence of production networks,

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1For their valuable feedback we thank Árpád Ábrahám, Fabio Canova, Vasco Carvalho, Daniel Dias, Juan Dolado, João Duarte, Leonardo Melosi, Evi Pappa as well as participants of the ECB’s DG Research internal seminar, EUI Macro Working Group, 17th Annual Meeting of the Portuguese Economic Journal.
measured TFP and pure productivity shocks are not the same (i.e., $TFP \neq z^p$). More specifically, we derive that

$$Y = f(z^p, L, K, X) \neq z^p g(L, K) h(X)$$

(2.1)

$$\Rightarrow \frac{Y}{h(X)} = \frac{f(z^p, L, K, X)}{h(X)} = \frac{TFP(z^p, X, \Omega, \mu) g(L, K)}{h(X)} \neq z^p g(L, K) = C$$

(2.2)

where $Y$ is gross output, $C$ is net, value-added output (i.e., GDP), $L$ is labor, $K$ is capital, $X$ are intermediate inputs, $f(\cdot)$ is a function that maps production inputs to output, $h(\cdot)$ a function that links intermediate inputs to final output, $\Omega$ describes the input-output production linkages and $\mu$ stands for the (sectoral) price markups. On the other hand, in the absence of production networks

$$\frac{Y}{h(X)} = \frac{f(z^p, L, K, X)}{h(X)} = \frac{z^p g(L, K) h(X)}{h(X)} = C,$$

(2.3)

so that $TFP = z^p$ is independent of the production network channel.

In the presence of production networks, the key mechanism works as follows. Firms within each sector face monopolistic competition, which results in sector-level markups. The inefficiencies produced by these markups are amplified by the network structure creating losses in aggregate productivity. Moreover, pricing frictions modeled as in Calvo (1983) lead to a misalignment from optimal pricing and thus affect the aggregate measure of productivity. The greater the price stickiness, the more elastic are markups to exogenous shocks and, thus, the more responsive is TFP. Finally, heterogeneity in firm’s staggered price setting also acts as an amplification device.

Our main findings are that monopolistic competition negatively affects TFP and that the presence of networks amplifies this effect. As to the steady state TFP level, we find that (i) for a given level of average price markup, more connected networks reduce TFP more and (ii) for a given network, the loss in TFP can vary steeply with the average markup in the economy. With respect to the dynamic behavior of TFP, we find that it depends crucially on the presence of networks. Indeed, without production networks TFP only varies with productivity shocks. However, in the presence of networks, TFP is also affected by monetary policy and cost-push shocks. From a qualitative standpoint, the way markups respond to shocks is key in understanding whether TFP is negatively or positively affected. In particular, in our model an increase in markups makes the economy less efficient and this inefficiency is amplified differently by network type.
At least since Solow (1957) aggregate productivity change has been a central piece in debates related to economic growth and researchers have invested a non-negligible amount of time and effort in trying to understand better what determines the growth rate of aggregate productivity and its fluctuations around that rate (e.g., Lucas, 1988) or Kydland and Prescott (1982) are seminal papers related to this). In most of these efforts the analysis has been done at the aggregated level, abstracting from the fact that modern economies are composed by complex networks of input and output relations between firms and consumers. This approach can be attributed to the work of Hulten (1978), where under perfect competition and a set of regular assumptions the aggregate impact of sectoral productivity shocks is shown to equal the share of that sector’s sales on total output. As an implication, the shares of sectoral sales are sufficient statistics for the impact of idiosyncratic productivity shocks on aggregate productivity. Under these conditions there is no need to model a disaggregated and more realistic economy since the (appropriately) weighted sum of sectoral productivity shocks is a sufficient statistic of the aggregate impact of these shocks. It then follows that the weighted sum of sectoral shocks can be recast as an aggregate shock. A more extreme version of this argument is made in Lucas (1981) by arguing that when the level of disaggregation of an economy increases, the law of large numbers implies that idiosyncratic shocks of different firms or sectors should average out. Under this view only aggregate shocks would matter to explain fluctuations in aggregate productivity.

However, a relatively new branch of the macroeconomic literature has pointed out the importance of idiosyncratic shocks in order to explain aggregate fluctuations. On the one hand there’s a granularity argument initially proposed by Gabaix (2011): The fat tail of the firm–size distribution invalids the law of large numbers argument because the idiosyncratic shocks to the larger firms of the economy do not average out in the aggregate. On the other hand, Acemoglu et al. (2012) show how production networks can amplify sectoral shocks depending on the structure of the input–output linkages. Since then other papers have followed this route. For example, Acemoglu et al. (2016) explore how demand–side and supply–side shocks propagate in an economy with network structures, and Carvalho et al. (2017) measure the aggregate effects on the Japanese economy of an exogenous disruption of the production network due to the 2011 earthquake. A multi-sector New Keynesian model is used by Pasten et al. (2016) and Pasten et al. (2017) to respectively quantify the real effects of monetary shocks in economies with production networks and to study the implications of sectoral shocks on GDP volatility.

Contrary to the aforementioned papers, we do not focus on explaining how aggregate fluctuations can arise from microeconomic shocks, but we rather try to characterize how

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2See Carvalho (2014) for a comprehensive review of the literature.
the Solow residual reacts to different types of shocks. In this sense our work, is closer to the one of Basu and Fernald (2002) where the authors show that aggregate productivity can differ from aggregate technology due to the presence of different gaps. Our model is also related to Basu (1995) since it yields monetary-induced productivity fluctuations in addition to the mechanisms of imperfect competition and input–output linkages. We believe our model represents a more realistic framework to be mapped into the data since we introduce capital into the analysis and the input–output linkages can be directly calibrated by using the I-O tables published by the U.S. Bureau of Economic Analysis.

From an empirical standpoint, there is ample evidence that measured productivity is not strictly exogenous. For instance, Evans (1992) finds that money, interest rates and government spending Granger-cause measured TFP, Mankiw (1989) examines data from the early 1940s and finds that the surge in TFP from 1939 to 1944 is likely a demand-driven response to the military buildup of World War II, Hall (1988) and Hall (1989) both discard invariance properties of the Solow residual in favour of noncompetitive forces affecting the latter. A similar result is established in Caballero and Lyons (1992) who find evidence of external increasing returns in sectoral data. Finally, Levinsohn and Petrin (2003) use intermediate goods as a proxy for productivity.

Studies of resource misallocation like the ones of Hsieh and Klenow (2009) and Jones (2011) are also related to our work. Although we do not provide measures of marginal products of labor and capital as in Hsieh and Klenow (2009) we do identify the factors that lower total productivity, which would allow a country-by-country comparison of these factors. As in Jones (2011), we use a multi-sector economy to explain how production networks can affect the total productivity of an economy. However, our paper deviates from Jones (2011) in two important aspects. First, we introduce imperfect competition to the analysis, allowing productivity to depend on the sectoral markups of the economy. This channel can be important even if the markups are low due to the amplification of the network structure. Second, our model can be used to analyze the behavior of productivity not only in a static but also in a dynamic dimension by studying the responses of aggregate productivity to different types of shocks.

The remaining structure of this paper is the following. Section 3.3 develops a multi-sector New Keynesian model with production networks. Contrary to the usual one-sector New Keynesian model, the framework of Section 3.3 requires to introduce sectoral price gaps as state-variables of the model. The model converges to the benchmark case only under homogeneity of price stickiness across sectors and no presence of idiosyncratic shocks. This implies different optimal dynamics for the variables of the model and hence, for aggregate productivity. Section 2.3 derives an expression for the aggregate
productivity as a function of the variables of the model and characterizes the conditions under which aggregate productivity coincides with the aggregate technology level. The results of Section 2.3 are then used in Section 2.4 to analyze how productivity behaves dynamically under different types of shocks. It also reports how the level of productivity changes for different configurations of the economy. Finally, Section 3.5 concludes and presents future research avenues.

2.2 A New Keynesian Model with Production Networks

The theoretical model builds up on the work of Castro Cienfuegos (2018).

2.2.1 Households

The economy is composed of a representative household with preferences over aggregate consumption and leisure. Infinitely lived, it maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - L_t^{1+\varphi}}{1-\sigma} \right),$$

where $\beta$ is the time discount factor, $\sigma$ the inverse elasticity of substitution and $\varphi$ the inverse Frisch elasticity of labor supply. $L_t$ denotes employment and $C_t$ is a Cobb-Douglas aggregator of $N$ sectoral consumption bundles

$$C_t := \prod_{i=1}^{N} C_{i,t}^{\alpha_i}.$$  

Each sectoral bundle is defined as a CES aggregator composed by the products of a continuum of firms between $[0, 1]$, so

$$C_{i,t} := \left( \int_0^1 C_{i,j,t}^{\epsilon} \, dj \right)^{\frac{1}{\epsilon-1}},$$

where $C_{i,j,t}$ is the quantity of good produced by firm $j$ in industry $i$ at time $t$ and $\epsilon$ is the constant elasticity of substitution.

The problem of the household implies the following optimal demands for the composite good of sector $i$ and individual good $j$ of sector $i$

$$C_{i,t} = \alpha_i C_t P_t,$$
where \( P_t \) is the consumer price index defined by 
\[
P_t := \prod_{i=1}^{N} \left( \frac{P_{i,t}}{\alpha_i} \right).
\] (2.9)

The budget constraint of the representative household is 
\[
\sum_{i=1}^{N} \int_{0}^{1} P_{i,j,t} C_{i,j,t} dj +Q^B_t B_t + \sum_{i=1}^{N} P_t Q_t \left[ K_{i,t+1} - (1 - \delta^k) K_{i,t} \right] \leq B_{t-1} + \sum_{i=1}^{N} (W_t L_{i,t} + Z_t K_{i,t} + A_{i,t}),
\] (2.10)
where \( P_{i,j,t} \) is the price charged by firm \( j \) in sector \( i \), \( B_t \) denotes holdings of one-period discount bonds, \( Q^B_t \equiv \frac{1}{1+i_t} \) is the unitary price of the bond and \( i_t \) is the net nominal interest rate, \( L_{i,t} \) is labor supplied to sector \( i \), \( W_t \) is the wage rate, \( Q_t \) is the unitary price of capital, \( K_{i,t} \) is capital supplied to sector \( i \), \( Z_t \) is the rental rate of capital, and \( A_{i,t} = \int_{0}^{1} A_{i,j,t} dj \) denotes dividends from all firms in sector \( i \). The budget constraint can be recast as 
\[
P_tC_t + Q^B_t B_t + P_t Q_t \left[ K_{t+1} - (1 - \delta^k) K_t \right] \leq B_{t-1} + W_t L_t + Z_t K_t + A_t
\] (2.11)
since \( \sum_{i=1}^{N} P_{i,t} C_{i,t} = P_tC_t = Z^C_t \) and where \( L_t, K_t \) and \( A_t \) are the sum of supplied sectoral labor, capital and dividends, respectively.

The representative household maximizes expected lifetime utility 
\[
\max_{\{C_t, L_t, B_t, K_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} \right)
\] (2.12)
subject to the budget constraint (2.11). The associated first order conditions are given by 
\[
w_t - p_t = \sigma c_t + \varphi \ell_t,
\] (2.13)  
\[
c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho),
\] (2.14)  
\[
c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(E_t\{r_{t+1}\} - \rho).
\] (2.15)
where lower case variables denote log levels (i.e., \( x_t = \log(X_t) \)).

\[^3\text{Note that the wage rate and the real rental rate of capital are the same across sectors, we thus impose a perfectly competitive labor and capital market across sectors. Also, since capital producers choose investment so as to maximize } Q_t I_t - I_t, \text{ the capital price will be equal to } 1.\]
2.2.2 Firms

Within each sector \( i = 1, \ldots, N \) there is a continuum of atomistic infinitely lived firms indexed by \( j \in [0,1] \). Each firm \( j \) within a sector \( i \) produces with the same CRS production function,

\[
Y_{i,j,t} = e^{z_{i,t}^p} \left( K_{i,j,t}^\gamma L_{i,j,t}^{1-\gamma} \right)^{\delta} \prod_{i'=1}^{N} X_{i,j,i',t}^{\omega_{i,i',t}},
\]

(2.16)

where \( z_{i,t}^p \) is a pure productivity shock, \( K_{i,j,t} \) is capital, \( L_{i,j,t} \) is labor and \( X_{i,j,i',t} \) is a CES composite good made from products of firms in industry \( i' \),

\[
X_{i,j,i',t} := \left( \int_0^1 X_{i,j,i',j',t} \, dj' \right)^{1/\epsilon_f}.
\]

(2.17)

The total cost of producing a quantity \( Y_{i,j,t} \) is given by

\[
C(Y_{i,j,t}) = W_t L_{i,j,t} + Z_t K_{i,j,t} + \sum_{i'=1}^{N} \int_0^1 P_{i',j',t} X_{i,j,i',j',t} \, dj',
\]

(2.18)

so profits in terms of aggregate units of consumption are

\[
A_{i,j,t} = \frac{P_{i,t} Y_{i,j,t} - C(Y_{i,j,t})}{P_{i,t}}.
\]

Concurrent, an individual firm \((i,j)\) faces a total demand composed by consumption demand (2.8) and the sum of input demands from all firms \( j' \) of all other sectors \( i' \), \( X_{i',j',i,j,t} \). Total input demand faced by firm \((i,j)\) is given by \( \sum_{i'=1}^{N} \int_0^1 X_{i',j',i,i,j,t} \, dj' \) so that the goods market clearing condition reads

\[
Y_{i,j,t}^d = C_{i,t} + \sum_{i'=1}^{N} \int_0^1 X_{i',j',i,j,t} \, dj'.
\]

(2.19)

A firm \((i,j)\) then has to choose its optimal price \( P_{i,j,t} \) in order to maximize profits \( A_{i,j,t} \) subject to the individual demand \( Y_{i,j,t}^d \). It can be shown that optimality requires an optimal mix of inputs given by

\[
X_{i,j,i',t} = \left( \frac{P_{i',t}}{P_{i,j,t}} \right)^{\epsilon_f} X_{i,j,i',t}, \text{ where } P_{i',t} \text{ is a production price index defined by } P_{i',t} := \left( \int_0^1 P_{i',j',t} \, dj' \right)^{1/\epsilon_f}.
\]

The symmetry of the problem implies that the input demand faced by firm \((i,j)\) also takes that form. Then total demand for firm \((i,j)\) is

\[
Y_{i,j,t}^d = C_{i,t} \left( \frac{P_{i,t}}{P_{i,j,t}} \right)^\epsilon + \sum_{i'=1}^{N} X_{i',i,t} \left( \frac{P_{i,t}}{P_{i,j,t}} \right)^{\epsilon_f},
\]

(2.20)

where, given the large number of firms within each sector \( i \), the quantities \( C_{i,t} \) and \( X_{i',i,t} \) and prices \( P_{i,t} \) and \( P_{i',t} \) are taken as exogenously given by firm \((i,j)\).
Firms within a particular sector face a probability \( 1 - \theta_i \) of resetting their prices each period and solve the following optimization problem

\[
\max_{P_{i,j,t}} \mathbb{E}_t \left\{ \sum_{m=0}^{\infty} \Lambda_{t,m+1} \beta_i^m \left[ \frac{P_{i,j,t} Y_{i,j,t+m|t} - C(Y_{i,j,t+m|t})}{P_{t+m}} \right] \right\}
\]

s.t. \( C(Y_{i,j,t+m|t}) = W_{t+m|t} L_{i,j,t+m|t} + Z_{t+m|t} K_{i,j,t+m|t} + \sum_{i' t+m X_{i',i,t+m|t}} \)

\[
Y_{i,j,t+m|t} = C_{i,t+m} \left( \frac{P_{i,t+m}}{P_{i,j,t}} \right)^\epsilon + \sum_{i' t+m} \left( \frac{P_{i',t+m}}{P_{i,j,t}} \right)^\epsilon' \]

In the following, we will assume that \( \epsilon = \epsilon' \) and, thus, \( P_{i,t} = P_{i,t}^p \). A log-linearization of the first order condition of a typical firm in sector \( i \) around the zero-inflation flexible-price steady state yields

\[
p_{i,t}^* = (1 - \beta \theta_i) \sum_{m=0}^{\infty} (\beta \theta_i)^m \mathbb{E}_t \left\{ \mu_{i,t+m}^* + \psi_{i,t+m} \right\} ,
\]

where \( \mu_{i,t}^* := \log M_{i,t} \) and \( \psi_{i,t+m} := \log \Psi_{i,t+m} \) are the log-markup and log-marginal cost of a firm that last reset its price in period \( t \).

We can express the log-optimal markup in terms of a cost-push shock \( \mu_{i,t}^* := \mu_i + z_{i,t}^{cp} \) where \( z_{i,t}^{cp} := (\mu_{i,t}^* - \mu_i) \). It is assumed that the cost-push shocks follows the process

\[
z_{t+1}^{cp} = \Lambda_{t} z_{t}^{cp} + u_{t+1}^{cp},
\]

where \( u_{t+1}^{cp} \) is i.i.d. with mean 0 and the notation in bold \( x \) stands for the column stacked vector of sectoral variables \( x_{i,t} \).

At the sectoral level, the Dixit–Stiglitz price index

\[
P_t \equiv \left[ \int_0^1 P_t(i)^{1-\epsilon} \, di \right]^{1/1-\epsilon}
\]

follows the law of motion

\[
\pi_{i,t} = (1 - \theta_i) \left( p_{i,t}^* - p_{i,t-1} \right).
\]
Using this condition together with \((2.24)\) it is possible to derive the sectoral pricing equation which, in stacked form, reads

\[
\pi_t = \beta \mathbb{E}_t \{\pi_{t+1}\} - \Lambda \hat{\mu}_t + \Delta z^{cp}_t, \tag{2.28}
\]

where \(\Lambda\) is a \(N \times N\) diagonal matrix with elements \(\lambda_i := \frac{(1-\theta_i)(1-\beta \theta_i)}{\theta_i}\) in the diagonal and \(\hat{\mu}_t = \mu_t - \mu\) are the stacked sectoral (log) deviations of the average markup from their steady state levels\(^4\).

### 2.2.3 Monetary Policy

The monetary authority is assumed to set the interest rate according to the Taylor rule

\[
i_t = \rho + \phi_\pi \pi_t + \phi_c \hat{c}_t + z^m_t, \tag{2.29}
\]

where \(\rho\) is the household’s time discount rate, \(\phi_\pi\) and \(\phi_c\) can be respectively thought of as the weights on inflation and on value-added GDP in deviations from its steady state level, and \(z^m_t\) is a monetary policy shock.

### 2.2.4 Model Solution

The objective of this section is to produce the rational expectations system of the model\(^5\). To this end, we express the solutions of the sticky price equilibrium as a system of four sets of variables: (i) the aggregate consumption gap \(\hat{c}_t\), (ii) the sectoral inflation rates \(\pi_t\), (iii) aggregate capital gap \(\hat{k}_{t+1}\), and (iv) the relative sectoral price gaps \(\hat{p}_t^R\) defined as sectoral prices in deviations from the economy’s average price level \(\bar{p}_t := \hat{p}_t - 1\hat{p}_t\). The system is hence composed of \(2N + 2\) variables.

First, to obtain the aggregate consumption gap \(\hat{c}_t\), we combine the Taylor rule described by \((2.29)\) with the Euler equation \((2.14)\) (written in deviations from steady state) and obtain

\[
\hat{c}_t = \frac{\sigma}{\sigma + \phi_c} \mathbb{E}_t \{\hat{c}_{t+1}\} + \frac{1}{\sigma + \phi_c} \alpha' \mathbb{E}_t \{\pi_{t+1}\} - \frac{\phi_\pi}{\sigma + \phi_c} \alpha' \pi_t - \frac{1}{\phi_c + \sigma} z^m_t. \tag{2.30}
\]

\(^4\)Notice that since we assume an equal and constant elasticity of substitution \(\epsilon\) steady state markups are the same across sectors.

\(^5\)We solve the latter using the Gensys algorithm developed by Sims (2002).

\(^6\)Relative sectoral price gaps ensure the existence of a solution to the rational expectations system.
Second, the sectoral inflation rates \( \pi_t \) can be obtained by solving for sectoral markup gaps and using equation (2.28). This produces sectoral New Keynesian Phillips Curves

\[
\pi_t = E_t[\pi_{t+1}] s + \frac{\delta}{1 + \varphi (\delta \gamma + 1 - \delta)} (\sigma (1 - \gamma) + \varphi + \gamma) \Lambda \hat{c}_t - \frac{\gamma \delta (1 + \varphi)}{1 + \varphi (\delta \gamma + 1 - \delta)} \Lambda \hat{k}_t
\]

\[
-\Lambda \left[ I - \Omega + \frac{\delta \varphi}{1 + \varphi (\delta \gamma + 1 - \delta)} \mathbf{1} \left( (1 - \gamma) \kappa' (I - \Omega) - \left( 1 + \frac{1}{\varphi} \right) \alpha' \right) \right] \hat{p}_t
\]

\[
-\Lambda \left[ I + \mathbf{1} \delta (1 - \gamma) \frac{\varphi}{1 + \varphi (\delta \gamma + 1 - \delta)} \kappa' \right] \mathbf{z}^p_t
\]

\[
+\Lambda \left[ I - 1 \left[ \left( \delta (1 - \gamma) \frac{\varphi}{1 + \varphi (\delta \gamma + 1 - \delta)} \left( \kappa' + \gamma \delta (\kappa^k)' + \gamma \delta (\kappa^k)' \right) \right) \times (I - X')^{-1} X' + \gamma \delta (\kappa^k)' \right] \right] \mathbf{z}^p_t
\]

\[
(2.31)
\]

where \( \Omega \) is the input–output matrix, \( X \) is a matrix collecting the sectoral intermediate goods to sectoral output ratios and the parameters \( \kappa_i := L_i / L \) and \( \kappa^k_i := K_i / K \) respectively indicate the ratios of sectoral labor and sectoral capital to their aggregates. It can be shown that under symmetry and homogeneity of price stickiness (i.e., \( \hat{p}_t = 1 \hat{p}_t \) and \( \Lambda = \lambda I \)) prices are no longer present on the Phillips curve.

Third, to describe the evolution of the aggregate capital gap \( \hat{k}_{t+1} \) we equate the nominal rental rate of capital of the firm’s problem to the real rate of capital of the household’s problem, which yields

\[
\hat{k}_{t+1} = \mathbb{E}_t \{ \hat{c}_{t+1} \} - \frac{\phi_c}{1 - \beta (1 - \delta^k)} \hat{c}_t - \frac{\phi_\pi}{1 - \beta (1 - \delta^k)} \alpha' \pi_t + \frac{1}{1 - \beta (1 - \delta^k)} \alpha' \mathbb{E}_t \{ \pi_{t+1} \}
\]

\[
- \frac{1}{1 - \beta (1 - \delta^k)} \mathbf{z}^m_t - (\kappa^k)' \left( I + (I - X')^{-1} X' \right) \mathbb{E}_t \{ \mathbf{z}^{cp}_{t+1} \}
\]

\[
(2.32)
\]

where \( \kappa^k \) and \( X \) are respectively capturing the vector of the steady state sectoral capital to aggregate capital ratios and the matrix of the ratios of demanded sectoral intermediate goods to sectoral outputs.

Finally, log-prices have been defined relative to the aggregate price level in each moment of time, i.e. \( p_{i,t} := \log \frac{P_{i,t}}{P_t} \). Given the definition of the relative price vector, we can write the law of motion using the identity \( \hat{p}_{i,t} = \hat{p}_{i,t-1} + \pi_{i,t} - \pi_t \). The law of motion of relative prices is then given by

\[
\hat{p}_t = \hat{p}_{t-1} + (I - 1 \alpha') \pi_t.
\]

(2.33)
2.2.5 Shocks Structure

All shocks are assumed to follow AR(1) processes with i.i.d. mean zero innovations. The monetary policy shock thus evolves according to

\[ z^m_t = \rho_m z^m_{t-1} + \varepsilon^m_t. \]  

(2.34)

The sectoral productivity and cost-push shocks (\( z^p_t \) and \( z^{cp}_t \) respectively) have a common and an idiosyncratic component

\[ z^p_t = 1 z^p_{t-1} + 1 \varepsilon^p_t + \tilde{z}^p_t + \varepsilon^p_t \quad \text{and} \quad z^{cp}_t = 1 z^{cp}_{t-1} + 1 \varepsilon^{cp}_t + \tilde{z}^{cp}_t + \varepsilon^{cp}_t. \]

The common components evolve according to

\[ z^p_t = \rho^p p z^p_{t-1} + \varepsilon^p_t \quad \text{and} \quad z^{cp}_t = \rho^{cp} p z^{cp}_{t-1} + \varepsilon^{cp}_t, \]

whereas the idiosyncratic components follow the processes

\[
\begin{align*}
\tilde{z}^p_t &= \begin{bmatrix}
  z^p_{1,t} \\
  z^p_{2,t} \\
  \vdots \\
  z^p_{N,t}
\end{bmatrix} = \begin{bmatrix}
  \rho^p_{1,p} z^p_{1,t-1} \\
  \rho^p_{2,p} z^p_{2,t-1} \\
  \vdots \\
  \rho^p_{N,p} z^p_{N,t-1}
\end{bmatrix} + \begin{bmatrix}
  \varepsilon^p_{1,t} \\
  \varepsilon^p_{2,t} \\
  \vdots \\
  \varepsilon^p_{N,t}
\end{bmatrix} = \Lambda^p \tilde{z}^p_{t-1} + \varepsilon^p_t, \\
\tilde{z}^{cp}_t &= \begin{bmatrix}
  z^{cp}_{1,t} \\
  z^{cp}_{2,t} \\
  \vdots \\
  z^{cp}_{N,t}
\end{bmatrix} = \begin{bmatrix}
  \rho^{cp}_{1,p} z^{cp}_{1,t-1} \\
  \rho^{cp}_{2,p} z^{cp}_{2,t-1} \\
  \vdots \\
  \rho^{cp}_{N,p} z^{cp}_{N,t-1}
\end{bmatrix} + \begin{bmatrix}
  \varepsilon^{cp}_{1,t} \\
  \varepsilon^{cp}_{2,t} \\
  \vdots \\
  \varepsilon^{cp}_{N,t}
\end{bmatrix} = \Lambda^{cp} \tilde{z}^{cp}_{t-1} + \varepsilon^{cp}_t.
\end{align*}
\]

The vectors of shocks and innovations are thus given by \( z_t \equiv [z^m_t \ z^p_t \ z^{cp}_t \tilde{z}^p_t \tilde{z}^{cp}_t]' \) and \( \varepsilon_t \equiv [\varepsilon^m_t \ \varepsilon^p_t \ \varepsilon^{cp}_t \ \varepsilon^p_t \ \varepsilon^{cp}_t]' \). The common and idiosyncratic components are assumed to be uncorrelated. This is standard in the econometric literature of production networks (see Foerster et al., 2011) and allows us to separate the two components in the variance-covariance matrix of the innovations \( \Sigma_{\varepsilon} \) which thus reads

\[
\Sigma_{\varepsilon} = \begin{bmatrix}
  \sigma^2_m & 0 & 0 & 0 & 0 \\
  0 & \sigma^2_p & 0 & 0 & 0 \\
  0 & 0 & \sigma^2_{cp} & 0 & 0 \\
  0 & 0 & 0 & \Sigma_p & 0 \\
  0 & 0 & 0 & 0 & \Sigma_{cp}
\end{bmatrix}.
\]
The variance-covariance matrices of the innovations to the idiosyncratic productivity and cost-push shocks are described by

\[
\Sigma_p = \begin{bmatrix}
\sigma_{1,p}^2 & 0 & \ldots & 0 \\
0 & \sigma_{2,p}^2 & \ldots & \vdots \\
\vdots & \vdots & \ddots & 0 \\
0 & 0 & \ldots & \sigma_{N,p}^2
\end{bmatrix}, \quad \Sigma_{cp} = \begin{bmatrix}
\sigma_{1,cp}^2 & 0 & \ldots & 0 \\
0 & \sigma_{2,cp}^2 & \ldots & \vdots \\
\vdots & \vdots & \ddots & 0 \\
0 & 0 & \ldots & \sigma_{N,cp}^2
\end{bmatrix}.
\]

### 2.2.6 Production Networks

When studying the static and dynamic determinants of TFP, we consider two prominent production network types studied in the literature\(^7\).

The first is known as the “star network” and represents the case where only one sector supplies intermediate inputs to itself and to the other sectors in the economy. Figure 2.1 displays such an intermediate goods trade structure.

![Figure 2.1 – Star Network. One sector supplies intermediate goods to the whole economy.](image)

To get a sense of how the input–output matrix and the consumption weights would look like in such an economy suppose that (i) there are \(N = 4\) sectors in the economy, (ii) sector 4 is the central supplier and (iii) the intermediate goods share is \((1 - \delta) = 0.5\). In this case,

\[
\Omega = \begin{bmatrix}
0 & 0 & 0 & 0.5 \\
0 & 0 & 0 & 0.5 \\
0 & 0 & 0 & 0.5 \\
0 & 0 & 0 & 0.5
\end{bmatrix}, \quad \alpha = \begin{bmatrix}
1/3 \\
1/3 \\
1/3 \\
0
\end{bmatrix}
\]

and

\[
\alpha^t \mathcal{H} \equiv \alpha^t (I - \Omega)^{-1} = \begin{bmatrix}
1/3 & 1/3 & 1/3 & 1
\end{bmatrix}.
\]

\(^7\)The illustrations are borrowed from Acemoglu et al. (2012).
where the matrix $H := (I - \Omega)^{-1}$ is known as the Leontief inverse. The typical element $h_{i,j}$ of this matrix shows the percentage effect of a 1% increase in productivity in sector $j$ on output in sector $i$ by taking into account all indirect effects at work in the model. The entries of the vector $\alpha' H$ gives a sense of how much idiosyncratic shocks occurring in one specific sector are amplified at the aggregate level (e.g., to aggregate total factor productivity). In this particular example, for instance, an idiosyncratic productivity shock occurring in sector 4 would be more amplified compared to other sectors (since $0.5 > 1/3$).

The second frequently studied network type is the so-called “roundabout network” which we report in Figure 2.2.

![Figure 2.2 - Roundabout Network](image)

Figure 2.2 – Roundabout Network. Every sector supplies intermediate goods to the other sectors in equal amounts.

To set ideas suppose again that $N = 4$ and $(1 - \delta) = 0.5$, then

$$\Omega = \begin{bmatrix} 0.125 & 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 & 0.125 \\ 0.125 & 0.125 & 0.125 & 0.125 \end{bmatrix}, \quad \alpha = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

and

$$\alpha' H \equiv \alpha' (I - \Omega)^{-1} = \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}$$

In this example it is straightforward to see that an idiosyncratic shock would be equally amplified independently of the sector where it originates. Moreover, compared to the previous examples, idiosyncratic shocks occurring in the first three sectors would experience higher amplification (since $0.5 > 0.3$) while disturbances bursting in the fourth sector would be less magnified (due to $0.5 < 1$).
2.3 Stairway to Total Factor Productivity

To derive an expression for total factor productivity we start from the optimal sectoral input choice of a representative firm in sector $i$

$$\max_{\{L_{i,t}, K_{i,t}, X_{i,i'}t\}} P_{i,t}Y_{i,t} - W_t L_{i,t} - Z_t K_{i,t} - \sum_{i'=1}^{N} P_{i',t}X_{i,i',t}$$ (2.35)

s.t.

$$Y_{i,t} = e^{z_{i,t}} \left(K_{i,t}^{-1}\gamma\right)^{\delta} \prod_{i'=1}^{N} X_{i,i',t}$$ (2.36)

$$P_{i,t} = P_{i,t}(Y_{i,t})$$ (because of monopolistic competition) (2.37)

The sectoral FOCs of the for optimal input demands imply

$$L_{i,t} = \delta (1 - \gamma) \frac{1}{M_{i,t}} \frac{P_{i,t}Y_{i,t}}{W_t} = \delta (1 - \gamma) \frac{1}{M_{i,t}} \frac{C_t}{W_t} D_{i,t}$$, (2.38)

$$K_{i,t} = \delta \gamma \frac{1}{M_{i,t}} \frac{P_{i,t}Y_{i,t}}{Z_t} = \delta \gamma \frac{1}{M_{i,t}} \frac{C_t}{Z_t} D_{i,t}$$, (2.39)

$$X_{i',i,t} = \omega_{i',i} \frac{1}{M_{i',t}} \frac{P_{i',t}Y_{i',t}}{P_{i,t}} = \bar{\omega}_{i',i,t} \frac{D_{i',t}}{D_{i,t}} Y_{i,t}$$, (2.40)

where the second equalities follow from the definition of Domar weights. The latter are defined as the ratio of the value of sectoral gross output to the sum of value added in all sectors, i.e., $D_{i,t} \equiv \frac{P_{i,t}Y_{i,t}}{C_t}$. Using the expression for optimal intermediate goods (2.40) in the market clearing condition we have

$$Y_{i,t} = C_{i,t} + \sum_{i'=1}^{N} X_{i',i,t} = C_{i,t} + \sum_{i'=1}^{N} \omega_{i',i} \frac{1}{M_{i',t}} \frac{P_{i',t}Y_{i',t}}{P_{i,t}}$$, (2.41)

which makes it possible to express a condition pinning down the Domar weights

$$D_{i,t} = P_{i,t} \frac{C_{i,t}}{C_t} + \sum_{i'=1}^{N} \omega_{i',i} \frac{1}{M_{i',t}} D_{i',t} = \alpha_i + \sum_{i'=1}^{N} \bar{\omega}_{i',i,t} D_{i',t}$$, (2.42)

or, equivalently, in stacked form

$$D_t = \left( I - \bar{\Omega}_t \right)^{-1} \alpha$$. (2.43)

Let $\kappa_{i,t}^L := L_{i,t}/L_t$ and $\kappa_{i,t}^K := K_{i,t}/K_t$ denote the share of sectoral labor over total labor...
and the share of sectoral capital over total capital, respectively. We can use the previous
equations to write these shares in terms of the Domar weights,

\[ \kappa_{\ell}^{i,t} = \frac{\delta (1 - \gamma) \frac{1}{M_{i,t}} \frac{C_i}{W_t} D_{i,t}}{\sum_{i'=1}^N \delta (1 - \gamma) \frac{1}{M_{i',t}} \frac{C_{i'}}{W_{t'}} D_{i',t}} \]

and analagously,

\[ \kappa_{k}^{i,t} = \frac{\frac{1}{M_{i,t}} D_{i,t}}{\sum_{i'=1}^N \frac{1}{M_{i',t}} D_{i',t}}. \]  

Given these shares, we can express sectoral labor and capital in terms of the aggregate
variables, \( L_{i,t} = \kappa_{\ell}^{i,t} L_t \) and \( K_{i,t} = \kappa_{k}^{i,t} K_t \), so we can write sectoral output as

\[ Y_{i,t} = e^{z_{i,t}} (L_{i,t})^\delta (1 - \gamma) (K_{i,t})^\delta \gamma \prod_{i'=1}^N \omega_{i,i',t} \]

or equivalently, in log terms, and using the fact that \( \kappa_{\ell}^{i,t} = \kappa_{k}^{i,t} = \kappa_{i,t} \)

\[ y_{i,t} = \left[ \log (\kappa_{i,t}) + \sum_{i'=1}^N \omega_{i,i',t} \log \left( \frac{\bar{\omega}_{i,i',t} D_{i,t}}{D_{i',t}} \right) \right] + \delta (1 - \gamma) \ell_t + \delta \gamma k_t + \sum_{i'=1}^N \omega_{i,i',t} y_{i',t} + z_{i,t}. \]

In stacked form, this reads

\[ y_t = (I - \Omega)^{-1} (a_t + z_t^p + \delta (1 - \gamma) 1_\ell_t + \delta \gamma 1_k_t). \]  

Next, we express sectoral output as a function of aggregate value-added production, \( C_t \). Using the Domar weights definition we obtain

\[ c_t = \alpha' (y_t + p_t - d_t) \]

\[ = \alpha' \left[ (I - \Omega)^{-1} a_t + p_t - d_t + (I - \Omega)^{-1} z_t^p \right] + \delta (1 - \gamma) \alpha' (I - \Omega)^{-1} 1_\ell_t + \delta \gamma \alpha' (I - \Omega)^{-1} 1_k_t \]
It can be shown that the term $\alpha'(p_t - d_t)$ is actually equal to $(I - X')^{-1}X'z_{tp}$. Then, aggregate value-added production can be expressed as

$$C_t = TFP_t \cdot L_{t}^{\alpha_{a,\ell}} \cdot K_{t}^{\alpha_{a,k}}$$

where

$$\alpha_{a,\ell} = (1 - \gamma)$$
$$\alpha_{a,k} = \gamma$$
$$TFP_t = \exp \left\{ \alpha'H [a_t + z_{tp}'] + \alpha' (I - X')^{-1}X'z_{tp}' \right\}$$

(2.48)

$$H := (I - \Omega)^{-1}$$

A few comments about this result. First, starting from a sectoral CRS value-added production we just showed that the aggregate value-added production also exhibits CRS. Second, the aggregate measure of TFP is composed of three terms (i) $a_t$, a vector of weighted markup indices per sector, (ii) $z_{tp}$, the vector of productivity shocks and (iii) $z_{tp}$, the vector of cost-push shocks. Last, the first two terms, $a_t$ and $z_{tp}$, depend on the network structure through the consumption index weighted Leontief matrix $\alpha'H$ (illustrated in Section 2.2.6), which amplifies their effects.

### 2.4 Static and Dynamic Determinants of TFP

We study how the steady state level and the dynamic responses of TFP are influenced by (i) the type of production networks, (ii) the degree of monopolistic competition and (iii) price stickiness. In all simulations, the economy is calibrated at a quarterly frequency to the U.S. economy’s 14 main sectors and results are reported along with those for the posterchild theoretical examples of the star and of the roundabout network. As to the latter, we construct the I-O matrix $\Omega$, depicted in Figure 2.3, using the summary-level BEA 2015 “make” and “use” tables (excluding the government sector) according to the procedure outlined in Pasten et al. (2016).

The consumption shares $\alpha$ for the 14–sectors U.S. economy are obtained as the ratio of a sector’s personal consumption expenditures (as reported in the “use” tables) to the total aggregated level. We use the latter as weights to obtain the sectoral degrees of price stickiness as a weighted sum of the price stickiness parameters computed by Pasten et al. (2016) at the more granular 350 sectors level. The same price stickiness parameters are used for the theoretical network types (star and roundabout) so as to facilitate comparison. Finally, the deep parameters of the model are set to standard values used in the

---

8The calibration does not change substantially if previous years are considered.
DSGE literature (see Galí (2008)), as reported in Table 2.1.

![Input-Output Matrix, BEA 2002, U.S. Economy’s 14 Main Sectors.](image)

**Figure 2.3** – Input-Output Matrix, BEA 2002, U.S. Economy’s 14 Main Sectors.

<table>
<thead>
<tr>
<th>Deep Parameters Parameterization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$  Subjective Discount Factor</td>
</tr>
<tr>
<td>$\delta$ Labor and Capital Share</td>
</tr>
<tr>
<td>$\delta^k$ Capital Depreciation</td>
</tr>
<tr>
<td>$\gamma$ Capital Coefficient</td>
</tr>
<tr>
<td>$\sigma$ Inverse Intertemporal Elasticity of Substitution</td>
</tr>
<tr>
<td>$\epsilon$ Elasticity of Substitution Between Goods</td>
</tr>
<tr>
<td>$\varphi$ Inverse Frisch Elasticity of Labor Supply</td>
</tr>
<tr>
<td>$\theta$ (Heterogeneous) Sectoral Price Stickiness</td>
</tr>
<tr>
<td>$\phi_n$ Inflation Taylor Rule Coefficient</td>
</tr>
<tr>
<td>$\phi_c$ Value Added GDP Taylor Rule Coefficient</td>
</tr>
</tbody>
</table>

We start by inspecting the determinants of TFP levels by tracking how steady state TFP varies with the degree of monopolistic competition prevailing in the economy. We do so by letting the steady state gross markup $M_i = M = \left(\frac{\epsilon}{\epsilon - 1}\right)$ range from 10% to 50% which respectively correspond to $\epsilon = 11$ and $\epsilon = 3$, the highest and lowest values for elasticities of substitution across goods documented by Nakamura and Steinsson (2010). Moreover, we consider four production network cases, namely (i) no network, (ii) star...
network, (iii) roundabout network, (iv) main 14 U.S. economy sectors network, as well as two intermediate goods shares (i) \( (1 - \delta) = 0.5 \) and (ii) \( (1 - \delta) = 0.6 \). We will pay special attention to the case of \( \delta = 0.5 \) since according to Christiano (2015) a typical firm sells about one-half of its output to other firms and materials purchases from other firms account for roughly half of the firm’s input costs.

Figure 2.4 shows the percentage deviation of steady state TFP from its frictionless level\(^9\) as a function of the economy’s average gross markup.

Four results stand out. First, for a given level of average price markup, more connected networks amplify inefficiencies, reducing TFP. Second, for a given network, the loss in TFP produced by monopolistic competition can vary steeply with the average markup. Third, the inefficiency introduced by the presence of monopolistic competition is stronger the larger the share of intermediate goods in the economy. This is because, in this model, monopolistic competition reduces TFP by inducing an inefficiently low level of intermediate goods trade. Thus, the higher the intermediate goods share in

\[^9\text{The latter could be obtained if, e.g., an employment subsidy was in place. In this case } (1 - \tau_i) = 1/\mathcal{M}_i = \frac{\varepsilon - 1}{\varepsilon} \text{ and monopolistic competition would not have any impact on steady state TFP.}\]
production the more sub-optimal is the production of the latter and the less efficient the economy. Clearly, in the absence of production networks this channel is inactive and monopolistic competition does not influence TFP. Last, for the 14-sectors calibrated U.S. economy with $\delta = 0.5$, even a relatively small average price markup of 15% over marginal costs corresponds to a level of TFP on average 25% lower than the one obtained under perfect competition.

This exercise is intended as an illustration of the channels playing a role in influencing TFP levels. For instance, the model could be calibrated to study this type of distortion within and/or across countries in different periods of time. In this context, it is important to note that estimations of average markups vary quite a lot, sometimes even exceeding 50% (Hall, 1988 estimates an average markup of 180% at the one-digit level). This not only highlights the importance of good data for markups but most importantly suggests that sizeable differences in incomes across countries could potentially emerge from different degrees of monopolistic competition.

Moving to the dynamic determinants of TFP, the parameterization of the shocks’ parameters is reported in Table 2.2. In order to make the IRFs comparable we set the autocorrelation to 0.9 and the annualized standard deviation to 1% on an annual basis for all shocks.

<table>
<thead>
<tr>
<th>Table 2.2 – Shocks’ Parameters Parameterization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autocorrelation of Shocks</td>
</tr>
<tr>
<td>Standard Deviation of Shocks (= 1% on annual basis)</td>
</tr>
</tbody>
</table>

One of our main findings is that the response of TFP to a monetary and a cost-push shock closely tracks the response of markups to these disturbances. In particular, as stressed above, whenever markups increase then the inefficiency in the economy increases too since firms use a sub-optimal low level of intermediate inputs and thus produce a level of output which is lower than in the frictionless case. Prior to showing the impulse responses we can thus predict how TFP will move by inspecting what happens to markups. First, in response to a positive monetary policy shock we have that, due to the presence of price stickiness, the decrease in prices does not compensate the decrease in nominal marginal costs (i.e., real marginal costs decrease) so that markups rise and, thus, TFP decreases:

$$\uparrow \mathcal{M} = \frac{P \downarrow}{MC \downarrow}, \quad \text{“Short Run”: } TFP \downarrow$$
Second, following a positive price markup shock real marginal costs decrease, markups increase and TFP thus decreases:

\[ \uparrow M = \frac{P}{MC} \downarrow, \quad \text{“Short Run”: } TFP \downarrow \]

Last, a positive technology shock engineers a drop in real marginal costs which corresponds to a rise in markups:

\[ \uparrow M = \frac{P}{MC} \downarrow, \quad \text{“Short Run”: } TFP \uparrow \]

This increase in markups is, however, contrasted by a direct increase in productivity. For a technology shock this pure productivity channel turns out to be the quantitatively most important one. Thus, positive productivity shocks in this economy exert a positive effect on TFP. In all cases markups eventually revert to their steady state levels so that shocks affect TFP only temporarily.

In the special case of no networks and CRS we have \( \Omega = 0, \delta = 1 \) and \( \pi = 0 \). In this case, it is possible to show that \( \hat{a}_t = 0 \) which means that neither monetary policy nor cost-push shocks would have an effect on aggregate TFP. It is straightforward to see why this is the case. In the presence of no production networks the goods market clearing condition reads \( Y_{i,t} = C_{i,t} \). That is, gross sectoral output is equal to net sectoral output and \( Y_{i,t} = e^{\varepsilon_{i,t}} \left( K_{i,t}^{\gamma} L_{i,t}^{1-\gamma} \right)^{\delta} \) and TFP is hence composed of the pure productivity shock only.

These mechanisms become evident in Figure 2.5 which illustrates the impulse response of TFP to a monetary policy shocks for different network types and for the 14-sector U.S. economy calibrated heterogeneous price stickiness (from a minimum of 0.2834 in the “agriculture, forestry, fishing, and hunting” sector to a maximum of 0.9722 in the “other services, except government” sector). Even though in New Keynesian models featuring production networks it is found that monetary policy shocks have real effects independently of the presence and the type of intermediate goods trade across sectors (see Pasten et al., 2016), measured TFP does not move in the absence of production networks. In the network economies, in contrast, TFP is negatively affected by the monetary policy shock due to its negative impact on markups. In particular, a 1% increase in the nominal interest rate on an annual basis leads to a drop in TFP from roughly 3.5% in the calibrated U.S. economy and to around 3% in the star and roundabout economy. We investigate the role of price stickiness in Figure 2.6 where we consider three different cases: (i) heterogeneous price stickiness as parameterized in the baseline, (ii) heterogeneous and low price stickiness (i.e., the baseline \( \theta \) is multiplied
by 0.1) and, finally, (iii) homogeneous price stickiness¹⁰ (θᵢ = θ = 0.8199, ∀ᵢ which is an average of case (i)). Consistently with previous findings of the effect of heterogeneous price stickiness on real effects, homogeneous price stickiness tends to have a smaller effect on TFP compared to the heterogeneous counterpart. Moreover, in keeping in line with the standard New Keynesian model, lower price stickiness dampens the effects of monetary policy shocks. Finally, only in the case of heterogeneous and medium–high price stickiness do network economies feature substantial and different TFP responses. Indeed, in the case of homogeneous price stickiness the IRFs do not vary across network types and in the heterogeneous but low price stickiness case they are almost undistinguishable.

The effect of a common and an idiosyncratic productivity shock originating in the manufacturing sector (which features a price stickiness of θ₅ = 0.5245) are shown in Figure 2.7 and Figure 2.8 respectively. Several results are worth commenting on. First, the productivity shock is the only shock to which TFP reacts in the absence of production networks. This is actually a “true” technology shock, so it is by construction positively correlated with aggregate TFP. We can see the effect of the network amplification mechanism at work: Economies with networks present a bigger impact response. In terms of magnitudes, a 1% common productivity shock on an annual basis leads to an average 2.25% increase in TFP for the network economies and to a 1% increase in the no networks case. The idiosyncratic productivity shock produces smaller effects, ranging from less than 0.1% in the absence of networks, 0.2% in the calibrated U.S. economy, 0.1% in the roundabout economy to a bit less than 0.45% in the star economy. In Figure 2.9 we compute the same IRFs in an economy with 50 sectors, with price stickiness parameters ranging in an equispaced interval from 0.5 to 0.8. In this case, the importance of idiosyncratic shocks tends to decrease. This is true for both the no networks and for the roundabout economy. In the case of the star economy, the idiosyncratic shock occurs in the central sector of the economy so that the contribution of this idiosyncratic shock doesn’t go away at the aggregate level. This is related to the outdegree of the central sector in this economy (see Acemoglu et al., 2012) and reflected in the entry of the manufacturing sector being the biggest one in magnitude in the amplification vector αᴴ (this is by construction since we make this sector the central one in the star economy), along the lines of the discussion in Section 2.2.6.

Another interesting piece of evidence comes from the impulse responses to a common and to an idiosyncratic cost–push shock originating in the manufacturing sector, illustrated in Figure 2.10 and Figure 2.11 respectively. Again, TFP doesn’t move in the ab-

¹⁰The homogeneous price stickiness parameter corresponds to the consumption shares weighted average of the (heterogeneous) sectoral degrees of price stickiness.
sence of intermediate goods trade across sectors. In the network economies however, this shock has a negative effect on TFP since it leads to a direct increase in markups which in turn make the economy less efficient. In the case of a common shock the TFP decrease is of about 0.1%, 0.5% and 0.7% for the calibrated, roundabout and star economy respectively. In the idiosyncratic case the magnitudes are small for the roundabout and calibrated case, being around 0.1%, whereas of non-negligible magnitude for the star network which features a TFP drop of roughly 0.02% after one and two quarters. The effects of an idiosyncratic shocks in a roundabout economy converge to zero as \( N \) increases while they are slightly milder for the star network, as becomes evident from Figure 2.12.

Finally, in Appendix 2.6, we explore how the IRFs vary according to (i) the elasticity of substitution \( \epsilon \) (and, thus, average markup in the economy), (ii) the share of intermediate goods \( (1 - \delta) \) and (iii) the degree of price stickiness. As to the first, we find that the milder is monopolistic competition, the more dampened is the drop in TFP in the case of the roundabout and star network and the more pronounced is an increase in TFP in the calibrated sectors U.S. economy following a cost-push shocks. The impulse response of TFP to other shocks is barely affected when varying \( \epsilon \). As to the second, the higher is the intermediate goods share \( 1 - \delta \), the stronger is the impact on TFP. These two sensitivity checks thus provide a picture which is consistent with the results and arguments presented for steady state TFP. As to the last, a higher degree of price stickiness is associated with a milder effect of productivity shocks for price markup shocks to less negative and, in some cases, even positive responses.
Figure 2.5 – IRF to Monetary Policy Shock.

Figure 2.6 – IRF to Monetary Policy Shock – Different $\theta$. 
2.4. Static and Dynamic Determinants of TFP

**Figure 2.7** – IRF to Common Productivity Shock.

**Figure 2.8** – IRF to Idiosyncratic Productivity Shock.
Figure 2.9 – IRF to Idiosyncratic Productivity Shock. $N = 50$ Sectors.

Figure 2.10 – IRF to Common Price-Markup Shock.
2.4. Static and Dynamic Determinants of TFP

**Figure 2.11** – IRF to Idiosyncratic Price-Markup Shock.

**Figure 2.12** – IRF to Idiosyncratic Price-Markup Shock. \( N = 50 \) Sectors.
2.5 Concluding Remarks

In this paper we have provided a framework to quantify the effect of monopolistic competition, sticky prices and production networks on aggregate TFP.

As to its static dimension, we find that for a given level of average price markup, more connected networks amplify inefficiencies, reducing steady state TFP. Also, for a given network, the loss in TFP produced by monopolistic competition can vary steeply depending on the average markup.

Our model also sheds a light on the dynamic behavior of TFP. With no networks, TFP only varies with productivity shocks whilst in the presence of networks, TFP varies with all the types of shocks introduced here. Contractionary monetary policy and positive cost-push shocks decrease TFP. The presence of networks can amplify idiosyncratic productivity shocks so as to potentially induce large aggregate effects. Price-markup shocks decrease TFP, especially if they occur in sectors with large out-degrees. Idiosyncratic price-markup shocks can have large aggregate effects.

Our future work involves different tasks. First, we can use this model to decompose the main factors of TFP changes in the U.S. economy. This would first involve constructing a correct measure of sectoral price gaps. Our framework could also be used to explain the differences in TFP levels across countries, by quantifying the relative importance of monopolistic competition and production networks in each economy. As to business cycle TFP movements, we plan to analyze whether our theoretical prediction that contractionary monetary policy shocks have a negative impact on aggregate TFP also holds in the data.
References


2.6 Appendix

We perform three sensitivity checks. First, we analyze how the elasticity of substitution affects the response of TFP to common and idiosyncratic cost-push shocks from Figure 2.13 to 2.18. Second, from Figure 2.19 to 2.23, we study how the IRFs are affected by varying the share of intermediate goods in the production function. Finally, we explore how results are affected by varying the degree of price stickiness in the simulation exercises. More specifically, in Figures 2.24 to 2.27 we show how the IRFs change if the price stickiness parameters across sectors are (i) heterogeneous, (ii) heterogeneous and low, (iii) homogeneous.

Figure 2.13 – IRF to Common Price-Markup Shock, Star Economy. Varying $\epsilon$. 
Figure 2.14 – IRF to Idiosyncratic Price-Markup Shock, Star Economy. Varying $\epsilon$.

Figure 2.15 – IRF to Common Price-Markup Shock, Roundabout Economy. Varying $\epsilon$. 
Figure 2.16 – IRF to Idiosyncratic Price-Markup Shock, Roundabout Economy. Varying $\epsilon$.

Figure 2.17 – IRF to Common Price-Markup Shock, 14 Sectors Calibrated U.S. Economy. Varying $\epsilon$. 

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Figure 2.18 – IRF to Idiosyncratic Price-Markup Shock, 14 Sectors Calibrated U.S. Economy. Varying $\epsilon$.

Figure 2.19 – IRF to Monetary Policy Shock. Varying Intermediate Goods Shares.
Figure 2.20 – IRF to Common Productivity Shock. Varying Intermediate Goods Shares.

Figure 2.21 – IRF to Idiosyncratic Productivity Shock. Varying Intermediate Goods Shares.
Figure 2.22 – IRF to Common Price-Markup Shock. Varying Intermediate Goods Shares.

Figure 2.23 – IRF to Idiosyncratic Price-Markup Shock. Varying Intermediate Goods Shares.
Figure 2.24 – IRF to Common Productivity Shock. Varying Degrees of Price Stickiness.

Figure 2.25 – IRF to Idiosyncratic Productivity Shock. Varying Degrees of Price Stickiness.
Figure 2.26 – IRF to Common Price-Markup Shock. Varying Degrees of Price Stickiness.

Figure 2.27 – IRF to Idiosyncratic Price-Markup Shock. Varying Degrees of Price Stickiness.
Chapter 3

The Effect of Oil Price Shocks on U.S. Investment: Size Matters

***

This work is single-authored and has been used as my job market paper.1


“At the most basic level, oil and natural gas are just primary commodities, like tin, rubber, or iron ore. Yet energy commodities are special, in part because they are critical inputs to a very wide variety of production processes of modern economies. They provide the fuel that drives our transportation system, heats our homes and offices, and powers our factories. Moreover, energy has an influence that is disproportionate to its share in real gross domestic product (GDP) largely because of our limited ability to adjust the amount of energy we use per unit of output over short periods of time.”

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1I am indebted to my supervisor Fabio Canova for invaluable guidance and support. For their time and precious feedback I am grateful to Árpád Ábrahám, Christian Bayer, Régis Barnichon, Alessia Campolmi, Nicolás Castro Cienfuegos, Daniel Dias, Juan Dolado, Reinhard Ellwanger, Francesco Franco, Gabriela Galassi, Matteo Gatti, Andreas Lageborg, Helmut Lütkepohl, Leonardo Melosi, Evi Pappa, William B. Peterman, Anna Rogantini Picco, Hernán Daniel Seoane, Matthew D. Shapiro, Alejandro Vicondoa, Robert Vigfusson and Antonio Villanacci. In particular, I thank Junior Maih for fruitful discussions and guidance on Markov-switching DSGE models and the RISE toolbox. I also wish to thank participants of the EUI Pre-Job Market Forum and of the 3rd EUI Alumni Conference in Economics. All errors are my own.

“Small oil price declines may be expansionary through usual channels, but really big
debits set in motion a process of forced deleveraging among producers that can be
a significant drag on the world economy [...]”

3.1 Introduction

Between 2014Q2 and 2016Q1 the real price of oil declined sharply, by around 68% (see
Figure 3.1). There has been a debate about the effect that this sharp decline in global oil
prices has on U.S. economic growth. In retrospect, the U.S. economy grew on average at
1.8% (in annualized rates) between 2012Q1 and 2014Q2, the period prior to the oil price
decline. This figure increased to 2.2% in the 2014Q3–2016Q1 period.

![Graph showing West Texas Intermediate (WTI) Crude Oil Price - CPI Deflated]

Figure 3.1 – The real price of oil is defined as the West Texas Intermediate (WTI) crude oil price,
divided by the U.S. CPI deflator. It is measured on a log scale and multiplied by 100 so that its
swings can be read as percentage changes. The shaded grey bars are NBER dated U.S. recessions.

According to Baumeister and Kilian (2017) this mild growth increase occurred because
while consumption surged, investment did not pick up much, due to a fall in oil-related
investment. This pattern is not isolated and in fact similar to the one observed in the 1986 great oil price collapse (see Kilian and Edelstein, 2007).

The recent oil price drop thus calls into question whether the transmission of oil price shocks to the U.S. economy is linear. After all, the conventional view predicts that oil price drops (booms) are unconditionally expansionary (contractionary), a view which is at odds with the observed mild increase in U.S. economy activity. In this paper I investigate why oil price shocks might propagate nonlinearly in the economy. The idea is that the size of oil price shocks matters for the sign of the response of investment in an oil producing economy. Indeed, investment decisions in the oil sector may depend on whether the increase in revenues induced by higher oil prices is large enough to cover the high fixed costs characteristic of the oil business (e.g., costs associated with well exploration, operating oil extraction platforms, pipelines construction and maintenance). For instance, Casassus et al. (2005) provide evidence of the importance of considering fixed costs in the oil sector. They also find two oil price regimes associated with “far-from investment” and “near-to-investment” regions in which oil prices are respectively lower- and higher-than-average. Appert and Favennec (2015) report that fixed costs in the oil sector represent up to 80% of total costs of processing a tonne of crude oil, an estimate in line with Kellogg (2014), who sets the constant component of rig costs to two thirds of a well’s total drilling costs. Stevens (1998) claims that “[t]he cost structure of most oil-industry operations, with their heavy weighting towards fixed or sunk costs, means that the “bygones rule” is supreme” while Al-Sahlawi (2014) reports that in the oil industry “the exploration and development stage is part of the overall production operation” so that “production costs have large fixed costs”.

Hence, an unexpected oil price increase may lead to a drop in non-oil investment due to lower aggregate demand but also to an increase in oil investment, if the oil price hike is large enough to more than compensate for the fixed costs. Aggregate investment may then increase or decrease in response to large unexpected oil price hikes. In fact, despite the small size of the oil sector relative to the aggregate economy, oil prices are an important production input and consumption good. Hitzemann and Yaron (2016) find that oil shocks induce welfare costs in the order of 2% of consumption certainty equivalent, a magnitude comparable to the cost of temporary business cycle shocks. The authors suggest that the effect is largely driven by the high persistence of such shocks. My idea is that spillover effects might amplify the impact of oil price shocks. Baqee and Farhi (2017) find that negative shocks to the oil and gas industry have a significantly larger

---

2The authors examined and excluded alternative explanations based on imperfect passthrough of the oil price to retail fuel prices, credit-constrained consumers, uncertainty about future gasoline prices, costly factor inputs reallocation and petroleum trade balance effects.
impact on output than negative shocks to other, less crucial industries. Thus, although
the oil and gas industry is small in terms of GDP share, the impact of oil price shocks
might be macroeconomically relevant, even in the absence of financial or demand side
frictions.

To provide evidence on the issue I run a Markov-switching VAR on oil prices and se-
lected categories of private nonresidential fixed investment, covering the period from
1970Q1 to 2016Q4. I show that while all U.S. investment categories decline following a
small surprise increase in the oil price, large oil price shocks increase investment in the
oil and oil-related sectors. This implies that total investment initially declines and event-
ually increases. In particular, after one year, a 25% oil price surprise increase generates
a 3% increase in total investment and a 0.4% increase in GDP. This pattern is similar
to the one documented by Baumeister and Kilian (2017) for the recent oil price decline.
Restricting the analysis to the sample prior to the shale oil revolution preserves the qual-
itative features of my findings. This nonlinearity is thus not a recent phenomenon.

I develop a DSGE model of an oil producing economy to explain the empirical find-
ings. The model features households which consume non-oil goods and oil. Non-oil
goods are produced by monopolistically competitive firms which use capital, labor and
oil in production and set prices in a staggered fashion. Oil is produced by perfectly com-
petitive firms using capital and labor. A monetary authority sets the nominal interest
rate according to a Taylor rule. I introduce a Markov-switching process in both the oil
price shock volatility and the oil firms’ fixed costs to capture the notion that the ability
to cover fixed costs varies with firm profits. I show that agents’ expectations about the
possibility of switching regime is crucial to obtain the nonlinearity observed in the data.
Intuitively, this happens because agents anticipate that in a regime where the oil price
shock is smaller and, thus, expected oil firm profits are lower, fixed costs are relatively
larger. This induces them to increase oil investment in the large oil price shock regime,
in which fixed costs are easier to cover. This channel is so important that it more than
compensates the negative effect coming from a fall in oil demand following the surprise
increase in oil prices, which is dominant in the small oil price shock regime. In a coun-
terfactual experiment, the GDP gain due to increased oil investment is confirmed to be
around 0.3%.

3.1.1 Contribution to Related Literature

Academic research has extensively investigated the effects of oil price changes on con-
sumer spending. It has been shown that this effect is quantitatively important\(^3\). Indeed,

\[^3\text{See, e.g., Hamilton (2003) and Edelstein and Kilian (2009) and, more recently, Baumeister and Kilian (2017), Gelman et al. (2016) and Ready (2016).}\]
the Energy Information Administration (EIA) estimated that lower gasoline prices resulted in extra $700 in households’ pockets. This line of research, however, has so far disregarded the effect of oil prices on investment, which is the focus of this paper. An exception is the literature which investigates the effects of commodity price booms on resource rich economies⁴.

There is a line of research arguing in favour of a nonlinearity in the relationship between oil prices and macroeconomic activity⁵. However, differently from this paper, it focuses on whether the sign of oil price shocks matters⁶. Several mechanisms have been proposed. Hamilton (1988) suggests that oil price shocks are relative price shocks, which induce reallocations of production inputs. As these reallocations are not frictionless factors become unemployed. This mechanism amplifies the depressing effect of oil price increases and dampens the stimulating force of oil price decreases⁷. Bernanke et al. (1997) argue that asymmetric responses are due to how monetary policy responds to oil price shocks. To the extent that monetary authorities respond to the inflationary pressures associated with surprise increases in the price of oil by raising the interest rate, the economic contraction associated with higher oil prices will be reinforced. According to real options theory, uncertainty about future oil prices might delay firms’ investment and production activities as they prefer to “wait and see” how the uncertainty reveals before making investment decisions⁸. There are issues with these proposed mechanisms. As noted by Baumeister and Kilian (2017), these interpretations do not fit the recent oil price drop as they predict that unexpected oil price declines are expansionary or, worse, followed by recessions.

One contribution of this paper is to propose an alternative mechanism that goes a long way to explain the pattern present in the data.

The paper is organized as follows. Section 3.2 lays out the statistical model and presents empirical results on the nonlinear transmission of oil price shocks to U.S. investment. A Markov-switching DSGE model of an oil producing economy which can explain the empirical findings is presented in Section 3.3. The conclusions are in Section 3.5.

⁶An exception is Herrera (2015) in which they find no evidence that the sign and size of an oil price shock matter for the response of U.S. stock returns.
⁷See also Herrera and Karaki (2015) or Herrera et al. (2017) for more recent studies.
3.2 Does Size Matter?

I identify regimes of large and small oil price shocks and study the regime-dependent effect of oil price shocks on several U.S. private nonresidential fixed investment (PNFI henceforth) categories using a Markov-switching Bayesian vector autoregressive model (MS-BVAR)\(^9\). The main advantage of a Bayesian treatment vis-à-vis a frequentist approach is that it makes the estimation problem computationally simpler and gives exact results in small sample rather than relying on asymptotic approximations. This statistical model has a long-standing tradition in empirical macroeconomics starting with Hamilton (1989), Hamilton (1990) and Kim and Nelson (1999), and more recently revived by Sims and Zha (2006) and Sims et al. (2008) in its Bayesian setting. It is particularly suitable to study data whose mechanisms are subject to (discrete) regime shifts and allow the parameters of the model to vary with the regime. This enables me to derive regime-dependent responses of U.S. investment categories to oil price shocks, where the regimes are characterized by large and small oil price volatility.

Since my interest is in the size of an oil price shock, I choose to not model changes in oil volatility using GARCH or stochastic volatility approaches. While those models can be particularly useful to study the effect of a shock to the volatility of the oil price - an “oil price uncertainty shock”\(^{10}\) - as in Kellogg (2014), Jo (2014) and Elder and Serletis (2010), they are unsuited to answer the question of whether the size of a first-moment oil price shock has different effects. In a Markov-switching framework, uncertainty shocks would be shocks that lead to switches from a large to a small oil price shock regime, instead of oil price shocks occurring in a given regime. This is an important distinction since I am interested in the size of an oil price shock and not in how uncertainty affects investment decisions.

3.2.1 The Statistical Model

The Markov-switching VAR model captures the joint dynamic process of a \(n\)-dimensional time series vector \(y_t\) with \(t = 1, 2, \ldots, T\) and of an unobservable regime variable \(s_t = i\) with \(i = 1, 2, \ldots, M\). The unobservable regime variable \(s_t\) is assumed to follow a Markov process governed by a \(M \times M\) transition matrix \(Q\) whose rows sum to 1. If the vector of

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\(^9\)See Hamilton (2016) for a survey of the literature studying regime changes and for a summary of the available methods.

\(^{10}\)See Serletis (2012) for a review of the literature on oil price uncertainty shocks.
parameters $\Theta$ is assumed to be regime-specific the joint density can be partitioned as

$$p\left(y_t | \Theta, Q \right) = \prod_{t=1}^{T} \left( \sum_{s_t=1}^{M} p\left(y_t | s_t, Y_{t-1}, \Theta \right) \times p\left(s_t | Y_{t-1}, \Theta, Q \right) \right),$$

(3.1)

where $Y_{t-1}$ is the history of $y_t$ for a given transition matrix $Q$. The characterization of the model in (3.1) is completed by the specification of $Pr(y_t | s_t)$. I choose a vector autoregressive model with regime-dependent intercepts, autoregressive coefficients and volatilities. Its structural form is given by

$$A_0(s_t)y_t = \nu(s_t) + A_1(s_t)y_{t-1} + \ldots + A_p(s_t)y_{t-p} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, I), \quad t = 1, \ldots, T \quad (3.2)$$

where $A_0(s_t)$ is a $n \times n$ invertible matrix under regime $s_t$, $A_i(s_t)$ is a $n \times n$ dimensional matrix which contains the coefficients for lag $i = 1, \ldots, p$ and regime $s_t$, $\nu(s_t)$ is a $n$-dimensional vector collecting the regime $s_t$ dependent intercepts. In its reduced form it reads

$$y_t = c(s_t) + B_1(s_t)y_{t-1} + \ldots + B_p(s_t)y_{t-p} + u_t, \quad u_t \sim \mathcal{N}(0, \Sigma(s_t)), \quad t = 1, \ldots, T \quad (3.3)$$

where $c(s_t)$ are regime-dependent intercepts, $B_i(s_t)$ are $n \times n$ reduced form matrices of autoregressive coefficients for regime $s_t$ and $\Sigma(s_t)$ is the regime $s_t$ specific covariance matrix.

The MS-BVAR uses quarterly data from 1970Q1 to 2016Q4\textsuperscript{12} and as in Edelstein and Kilian (2009) $y_t = \left[ \Delta \hat{P}_{o,t}, \Delta I_{s,t} \right]^\prime$, where $\Delta \hat{P}_{o,t}$ is the log-difference of the real oil price and $\Delta I_{s,t}$ is the log-difference of a given real investment category $s$, expressed in annualized terms. The oil prices series is the West Texas Intermediate (WTI) crude oil price\textsuperscript{13}. As to the investment series, I rotate categories of private fixed non-residential investment in structures and equipment. Their average shares over the sample period are reported in Table 3.1. I also consider a MS-BVAR with either one of three broad investment categories, namely i) oil sector investment, defined as investment in mining exploration, shafts, and wells structures (investment in mining and oilfield machinery equipment is not available at quarterly level), ii) non-oil sector investment, defined as the sum of investment in commercial and healthcare, manufacturing and other structures and equip-

\textsuperscript{12} All datasets are drawn from the FRED database of the Federal Reserve Bank of St. Louis.

\textsuperscript{13} The WTI crude oil price refers to oil extracted from U.S. wells sent to Oklahoma via pipelines. Most of crude oil contracts are around the Brent crude oil price which refers to oil from the North Sea. Using the Brent instead of the WTI does not affect the main results since the two series are nearly identical with the exception of the 2010Q4–2014Q4 period in which the Brent was slightly higher but followed similar patterns as the WTI.
investment and, iii) “oil-like” sector investment, defined as investment in categories linked to the oil sector.

<table>
<thead>
<tr>
<th>Investment Category</th>
<th>% Share</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Structures Total</strong></td>
<td>29.49%</td>
</tr>
<tr>
<td>Commercial and Healthcare</td>
<td>12.88%</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>4.84%</td>
</tr>
<tr>
<td>Mining Exploration, Shafts, and Wells</td>
<td>5.32%</td>
</tr>
<tr>
<td>Others</td>
<td>6.44%</td>
</tr>
<tr>
<td><strong>Equipment Total</strong></td>
<td>70.51%</td>
</tr>
<tr>
<td>Information Processing</td>
<td>22.41%</td>
</tr>
<tr>
<td>Industrial</td>
<td>16.95%</td>
</tr>
<tr>
<td>Transportation</td>
<td>15.86%</td>
</tr>
<tr>
<td>Others</td>
<td>15.28%</td>
</tr>
<tr>
<td><strong>Selected Categories Total</strong></td>
<td>100%</td>
</tr>
<tr>
<td>Non–Oil</td>
<td>39.45%</td>
</tr>
<tr>
<td>Oil</td>
<td>5.32%</td>
</tr>
<tr>
<td>Oil–Like</td>
<td>55.23%</td>
</tr>
</tbody>
</table>

Table 3.1 – Average shares of private nonresidential fixed investment (PNFI) categories to total structures and equipment PNFI from 1970Q1 to 2016Q4.

Nominal variables are deflated by the CPI\(^\text{14}\) and by population growth to ensure consistency with the theoretical model. The VAR follows a \(M = 2\) regimes Markov process and model dynamics are described by \(p = 4\) lags.\(^\text{15}\) The BVAR prior is a diffuse Sims and Zha (1998) Normal–Wishart prior whereas a Dirichlet prior is assumed for the Markov process. Further details on prior specification and estimation of the MS–BVAR are in Appendix 3.A.

Following Kilian and Edelstein (2007), Kilian (2008), Blanchard and Gali (2008) and Gelman et al. (2016) among others, oil price shocks are identified assuming that oil prices do not respond within a quarter to investment. While a small system might confuse the origin of spillovers, restricting the model to be small comes at no loss of generality if the

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\(^\text{14}\)Due to the high correlation with the CPI, results are unchanged if GDP deflator or PPI are used instead.

\(^\text{15}\)The marginal likelihood for the model favours a two-regime vis–à–vis a three regime MS–BVAR. As to the lag length, the marginal likelihood of the model – computed from the randomly permuted MCMC output following the methods described in Frühwirth–Schnatter (2006) – is generally higher for \(p = 4\). Moreover, given the history dependent nature of investment decisions a lag length of at least a year is desirable and standard in the literature.
researcher is interested in estimating consistently the average effects of oil price shocks on macroeconomic aggregates, see Kilian (2008). As pointed out by Kilian (2009), a real oil price shock is predetermined as it can be written as a weighted average of structural demand and supply shocks in the crude oil market, which are predetermined. To confirm this finding I run the Kilian (2009) VAR on my sample and report the historical decomposition of the WTI into oil supply, aggregate demand and oil-specific/precautionary demand shocks. As evident from Figure 3.2 the real oil price is indeed driven by all shocks at all times. Finally, since I identify only oil price shocks the fact that the other shocks are mongrels has no effect on my analysis.

![Historical Decomposition of (Log) Real Oil Price](image)

**Figure 3.2** – Historical decomposition of the log of the real price of oil into an oil supply shock (blue), an aggregate demand shock (red) and an oil-specific/precautionary demand shock (yellow) based on the Kilian (2009) VAR. The shaded grey bars are NBER dated U.S. recessions.

The assumption that the oil price is predetermined with respect to U.S. investment within a quarter is defendable on several grounds. The WTI crude oil price (i.e., the price of crude oil produced in the U.S.) historically followed the Brent Blend crude oil price,
with rare exceptions. The Brent is the price of oil produced in the North Sea and makes up for about two thirds of all globally traded crude contracts and like the WTI petroleum it is considered as a “sweet” crude oil. This comovement is likely driven by the fact that if U.S. oil producers try to sell their extracted crude oil to refineries at a higher price than the Brent, then the latter would buy oil elsewhere. In fact, when the gap between the WTI and Brent crude oil price widened significantly for the first time in 2011, U.S. petroleum prices followed the Brent price since most U.S. refiners, located on the Gulf Coast, had better access to seaborne oil reserves rather than to the landlocked Cushing WTI crude oil hub. In this sense, macroeconomic conditions in the U.S. are unlikely to drive WTI crude oil prices, at least within a quarter, and especially so if one considers that oil producers were not allowed to export their crude oil until recently. A recent study by Rondina (2017) supports the view that oil price shocks are essentially exogenous to the U.S. economy.

3.2.2 Large and Small Oil Price Shock Regimes

In my MS-BVAR, regimes are identified by changes in the VAR autoregressive coefficients and in the volatility of the shocks. When referring to a small/large oil price shock I thus mean a shock occurring in a regime associated with small/large oil price swings, as captured by a different transmission mechanism and by a low/high volatility of the oil price shock.

Figure 3.3 presents percentage changes in the oil price along with the smoothed mean large oil price shock regime probabilities estimated in a MS-BVAR with oil investment. Oil price swings exceeding one standard deviation of the real oil price series are defined as large and small otherwise. Five major oil pricing events fall under the large oil price shock regime:

16Refining capacities in Cushing, Oklahoma could not keep up with the increased supply of unconventional oil due to inadequate transportation infrastructure. The WTI oil price fell below the Brent due to this excess supply. The spread has closed since then following improvements in oil transportation infrastructure from and to Cushing. Please refer to Kilian (2016) for a detailed analysis of this episode.
17Indeed the Energy Policy and Conservation Act of 1975 did not allow U.S. producers to sell their oil to U.S. refineries until December 18th 2015, when the congress lifted the export ban.
18The major oil price episodes to which the VAR assigns high probabilities of being in a large oil price shock regime are essentially the same across VARs with different investment categories, with the largest exception of the VAR with aggregate investment (see Appendix 3.B) which also identifies as large oil price shock regimes the 1978–1979 Iranian revolution, the 1980–1981 Iran–Iraq War outbreak (which lead to sizeable output reductions in both countries), the 1997–1998 East Asian crisis (episode during which the recession in Thailand, Korea and other East Asian countries reduced the demand for oil, leading to the lowest real oil price since 1974) and the 2002–2003 Venezuela oil strike (an attempt by the Venezuelan opposition to President Hugo Chávez to force a new presidential election).
19See Hamilton (2011) and Baumeister and Kilian (2016a) for a more detailed exposition and discussion.

II. The 1985–1986 great price collapse: Saudi Arabia and other OPEC members give up their attempts to stabilize oil prices at a high level to preserve their market share.\(^{20}\)

III. The 1990–1991 Iraq invasion of Kuwait: Oil output is compromised in Iraq and Kuwait.\(^{21}\) The large price increase was also due to a huge rise in precautionary oil

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\(^{20}\)More precisely, mostly Saudi Arabia cut production unilaterally. Most other OPEC countries cheated on their quota which led to cartel instability.

\(^{21}\)Readers passionate about photography might have vivid memory of Sebastião Salgado’s photographs of Kuwait’s oil fields set ablaze by the Iraqi army published in his monograph “Kuwait: A Desert on Fire".

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inventories which followed as a consequence of the conflict (see Kilian and Murphy, 2014).


V. The 2014–2015 outbreak of the “shale revolution”: Surge in U.S. oil supply drives down the price of oil. The exact causes of the oil price decline are still debated. Other factors might have been increased OPEC production and relatively weak global economic growth (see Baumeister and Kilian, 2016b).

Notice that the large oil price shock regime includes episodes in which the oil price was driven by both supply and demand factors. In this sense, it can be safely assumed that the effects of large and small oil price shocks I identify are not tainted by the endogeneity problem raised by Kilian (2009). Indeed, if one found large oil price shocks to be largely demand driven then finding a positive response of investment to large oil price shocks and a negative response to small oil price shocks might not come at a surprise. In that case a more sophisticated identification than the one used here, would be needed. To see this consider the following two scenarios.

In a first scenario, a positive oil price shock is caused by a supply disruption. In this case, total investment is expected to decrease in response to a small oil price increase because both non-oil investment and oil investment decrease. A large oil price increase, on the other hand, would cause investment in the oil sector to increase and investment in the non-oil sector to fall. This is because depressed consumer demand due to higher gasoline prices drags down non-oil investment, while the oil price increase is so large that the oil company can cover the fixed costs and invest more. To the extent that extra investment in the oil sector creates extra demand of intermediate inputs from other sectors spillover effects would be present and other sectors would also increase their investments. The effect on aggregate investment would then be ambiguous. In the second scenario, a positive oil price shock is due to an unexpected surge in global demand. In this case, it is rather unlikely that investment in the non-oil sector would decrease. While on one hand consumers have less money to spend due to higher retail gasoline prices, demand for goods is booming. So aggregate investment might go up in response to both a small and a large oil price shock. Hence, if large oil price shocks are mainly demand-driven then, by construction, they will lead to an increase in investment.

Thus, the observed nonlinearity in the response of aggregate investment to a large and a small oil price shock could be driven by shocks of different sizes having a different composition of supply and demand factors. I look into this concern by computing impulse responses not only of aggregate investment but also of individual investment.
categories. If the sign of the responses to the same shock differs across categories it can be concluded that it is not the composition of oil price shocks which is driving the nonlinearity in the response but rather the size of the shock. This is because since all categories are hit by the same oil price shock any difference in their responses has to be driven by other factors than the sources of the shock.

Note that if large and small oil price shocks did not have a different impact on U.S. investment the MS-VAR would not detect the different regimes in the first place. The nonlinearity studied in this paper is thus “endogenously” selected by the statistical model. In addition, one might wonder whether the current analysis neglects the fact that the level at which the oil price shock occurs matters. It turns out that, in a VAR in levels, the same regimes are detected and the impulse responses display the same pattern.

3.2.3 The Effects of Oil Price Shocks on Investment

Following Ehrmann et al. (2003) I compute impulse responses without taking into account that the regime changes when propagating the shock over horizons\(^2\). This assumption is not restrictive in light of the fact that the estimated mean probability of staying in a regime conditional on being in that regime is around 95\% for the large oil price shock regime and 98\% for the small oil price shock regime. IRFs for the variables in levels are obtained by cumulating the impulse responses to the variables in growth rates.

A large oil price shock corresponds to a 25\%–30\% impact increase in the price of oil, whereas a small oil price shock is associated with little more than an almost 10\% increase, as evident from Figure 3.4. The very persistent, almost permanent nature of the oil price shock is in line with work by Aguiar-Conraria and Wen (2007), Blanchard and Gali (2008), Acurio-Vásconez (2016) and Gelman et al. (2016) as well as with survey evidence from Anderson et al. (2013) who cannot reject the null hypothesis of a random walk in consumer expectations of future gasoline prices. This feature is crucial since agents would not alter investment decisions unless price changes are perceived as permanent. Indeed, the decision to invest requires that expected profits are large enough to be willing to give up current consumption.

\(^2\)Another approach would be the one by Krolzig (2006) where the existence of a Markov chain in influencing the propagation of the shock is explicitly acknowledged.
A novel piece of evidence on the role of small and large oil price shocks on U.S. investment is presented in Figure 3.5. Non-oil investment responds negatively to unexpected oil price increases, irrespectively of the size of the oil price shock (see Figure 3.5a). Indeed, a small positive oil price shock contracts non-oil investment by about 1%, starting two quarters after its occurrence. Similarly, a large oil price shock leads to an immediate drop of 10–12% in the first year followed by a persistent decline of little more than 8%. Oil investment, on the contrary, decreases after a small oil price shock but increases after a large oil price shock (see Figure 3.5b). In particular, after a large oil price surprise hike, oil investment increases up to 4% after four quarters and plateaus at 2%–3% growth thereafter. The difference in the sign of the investment response to large oil price disruptions is consistent with the presence of fixed costs (a distinctive feature of oil sector’s investment projects). The negative response to a small shock can be rationalized by depressed demand of oil from both households and from the non-oil producing sector, or from capital left to depreciate.
Other investment categories are found to exhibit similar responses as investment in the oil sector (see Figure 3.5c). These are investments in information processing equipment, industrial equipment and transportation equipment - which account for 55.23% of aggregate investment. As shown in Appendix 3.C, the oil sector relies on these investment goods (e.g., in the form of services requiring computer and other electronic equipment, oil extraction machinery and pipeline and truck transportation). As shown by Baumeister and Kilian (2017) for the recent oil price decline, the traditional transportation sector (airlines, trains, etc.) is only partially affected by variations in oil prices, likely reflecting its ability to hedge against fluctuations in oil prices via futures markets.

Thus, while the share of oil in aggregate investment is small, investment spillover from the oil sector to the aggregate economy might occur because of network effects whereby increased demand from the oil sector to other sectors leads to increased investment in the latter. Similar spillover effects have been first pointed out by Aguiar-Conraria and Wen (2007) and more recently documented by Baqaee and Farhi (2017). Feyrer et al. (2017) also find evidence of spillover effects from oil to non-oil producing states and from oil to non-oil producing industries.

Interestingly, while each category responds negatively to a small oil price shock (see Appendix 3.B), the sum of the oil-like responding categories exhibits a positive response, thus reflecting an aggregation bias. Aggregation biases are pervasive in macroeconomics. For instance, Kilian and Edelstein (2007) showed that aggregation of investment categories lead to mistakenly find an asymmetry between IRFs to positive and negative oil price shocks.

It is interesting to see how the patterns in the investment categories carry over to the response of total investment (see Figure 3.5d). The fact that total investment does not move after a small unexpected oil price increase can be explained in at least two ways. Either investment adjustment costs are larger at an aggregate level or the positive response of the oil-like responding sectors, due to an aggregation bias, is offsetting the negative response in the non-oil and oil sector. Somewhat intriguing is the response to a large surprise increase in the price of oil. Total investment drops by 2% in the second to fourth quarter and eventually increases in the medium term. This is because non-oil investment decreases quickly and considerably while oil investment picks up considerably only with a year delay.

The magnitude of this effect is anything but negligible considering that private fixed non-residential investment totals 95% of gross private domestic investment which, in turn, makes up some 14% of GDP. The large increase in the price of oil can thus be translated to a 0.4% increase in GDP in annualized terms. Interestingly, Baumeister and Kilian (2016a) find a similar magnitude using back-of-the-envelope calculations and focusing
on the latest oil price decrease episode. They argue that had investment in the oil sector not dropped GDP growth after 2014Q2 would have been higher by 0.3 percentage points after a year.

**Figure 3.5** – Impulse responses, along with 68% credibility intervals, of real private nonresidential fixed investment categories for the small oil price shock regime (blue) and for the large oil price shock regime (red).
3.2.4 Robustness

In the last decade, the advent of technologies involving horizontal drilling and hydraulic fracturing (also known as “fracking”) made it economically attractive to extract oil and gas from vast and dormant oil resources. Unlike vertical drilling, the exploitation of shales usually features high activity in the first few months, followed by a sharp decline. This entails that i) to maintain output at a stable level oil wells have to be continuously drilled and ii) production is very responsive to price changes and characterized by shorter investment payback periods. This revolution in U.S. oil production has certainly had an impact on U.S. investment patterns and their response to oil price shocks. Indeed, when I limit the sample in 2009Q4 (see Figure 3.6a) investment does not respond on impact as under the baseline period but rather after roughly one year and shows a more persistent response at longer horizons. This is in line with the notion that investment in unconventional oil is more responsive to oil price movements than the one in conventional oil. Still, the qualitative patterns I document remain.

If the price of oil was indeed predetermined with respect to U.S. investment as I assume, then any additional variable I could include in the VAR would not be contemporaneously correlated with it. Thus, ignoring variables that influence investment decisions will not bias the effect of shocks to the real price of oil on U.S. investment. I explore whether this is the case by including as an extra variable in the MS-VAR with oil and investment the Gilchrist and Zakrajišek (2012) credit spread (i.e., the yield spread between private and government debt instruments of similar maturity) or the financial and macroeconomic uncertainty measures of Jurado et al. (2015) and Ludvigson et al. (2015). The credit spread reflects investor sentiment about the credit market and expectations about future economic activity; the uncertainty measures capture economic sentiment. Including the credit spread and uncertainty measures as a third variable in my VARs (see Figures 3.6b, 3.6c, 3.6d and 3.6e) does not affect my results. Finally, reversing the order of oil prices and U.S. investment in the VAR does not alter the results (see Figure 3.6f). Also, the IRFs for the main investment categories are quantitatively the same if instead of a set of bivariate VARs a four variable VAR with the real oil price and the three main investment categories (non-oil, oil and oil-like) is considered. If anything, the aggregation bias which leads to a misleadingly positive response of oil-like investment in response to a small oil price shock is correctly captured as being negative.


This robustness exercise has been run with $p = 2$ lags due to the shorter data availability of the credit spread which starts 1973Q1 in and ends in 2016Q2.

While the conclusions extend to all investment categories, the sensitivity analysis reported in includes only the oil investment category due to space constraints.
Figure 3.6 – Impulse responses, with 68% credibility intervals, of oil investment in robustness exercises for the small oil price shock regime (blue) and for the large oil price shock regime (red).
### 3.3 A Markov-Switching Model of an Oil Producing Economy

In this section, I build a DSGE model to explain the evidence found in Section 3.2. The model features a closed, oil producing economy. There are two production sectors, one produces non-oil goods and the other oil. The non-oil sector is monopolistically competitive, sets prices in a staggered fashion and assembles intermediate goods varieties to produce a final consumption good with labor, capital and oil. The oil sector produces oil with labor and capital only. Households can invest in both sectors and consume the non-oil final good as well as oil. There is a monetary authority setting the interest rate according to a Taylor rule. I assume that a Markov process governs the volatility of the oil price as well as the oil firms’ fixed costs, to capture the notion that the ability to cover fixed costs varies with firm profits.

Oil price changes affect macroeconomic activity through three channels. First, production costs of non-oil final goods are directly affected by oil price movements, since oil is used as an intermediate good in production. Second, oil prices affect the representative household’s disposable income, since oil is part of the consumption basket. Third, the presence of price rigidities amplifies the real effects of oil price shocks as in Blanchard and Gali (2008), Blanchard and Riggi (2013), Natal (2012) and Plante (2014).

The equations of model and the steady state are presented in Appendix 3.D.

#### 3.3.1 Households

The representative household derives utility from current and past consumption as well as from leisure

$$
E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t - bC_{t-1})^{1-\sigma} \exp \left( (\sigma - 1) \frac{N_t^{1+\varphi}}{1+\varphi} \right)}{1-\sigma} \right), \tag{3.4}
$$

where $\beta \in (0, 1)$ is the subjective discount factor, $\sigma > 0$ is the coefficient of relative risk aversion (or inverse elasticity of intertemporal substitution), $\varphi > 0$ is the inverse Frisch elasticity of labor supply and $b \in [0, 1)$ regulates the strength of external habit formation. Thus, as in Smets and Wouters (2007) households take the level of consumption prevailing in the previous period $C_{t-1}$ as given. The term $N_t$ denotes hours worked and $C_t$ is a constant elasticity of substitution (CES, henceforth) aggregate of non-oil consumption goods $C_{no,t}$ and oil consumption $O_{c,t}$ as in Bodenstein et al. (2011), Natal (2012) and Plante (2014)

$$
C_t \equiv \left[ (1 - \omega_c) (C_{no,t})^{\frac{\nu_c-1}{\nu_c}} + \omega_c (O_{c,t})^{\frac{\nu_c-1}{\nu_c}} \right]^{\frac{\nu_c}{\nu_c-1}}, \tag{3.5}
$$
where the quasi-share parameter $\omega_c \in [0, 1]$ determines the share of oil consumed out of total consumption expenditures and $\nu_c > 0$ is the elasticity of substitution between non-oil goods and oil consumption. Notice that equating crude oil to refined oil does not come at a big loss. Indeed, according to the U.S. Energy Information Administration (EIA) gasoline price at the pump is composed of 20% taxes, 12% distribution and marketing, 20% refining and 49% crude oil.

The non-oil consumption good $C_{no,t}$ is itself a CES aggregate composed of consumption goods produced by a continuum of monopolistic competitive intermediate goods firms $i \in [0, 1]$, as described in Section 3.3.2. More specifically, the bundle of the non-oil good is given by

$$C_{no,t} = \left( \int_0^1 C_{no,i,t} d\pi(i) \right)^{\frac{1}{\varepsilon}} \varepsilon^{-1},$$

where $C_{no,i,t}$ is the quantity of good produced by firm $i$ in the non-oil industry at time $t$ and $\varepsilon$ is the constant elasticity of substitution between non-oil consumption goods.

Oil $O_{c,t}$ is produced by a representative firm which operates in an environment of flexible prices and perfect competition as in Stevens (2015). The production of oil is outlined in Section 3.3.3.

The optimal sectoral demand schedules are

$$C_{no,t} = (1 - \omega_c) \frac{1}{\nu_c} \left( \hat{P}_{no,t} \right)^{-\nu_c} C_t,$$

$$O_{c,t} = \omega_c \frac{1}{\nu_c} \left( \hat{P}_{o,t} \right)^{-\nu_c} C_t,$$

where $\hat{P}_{no,t} \equiv \frac{P_{no,t}}{\tilde{P}}$ and $\hat{P}_{o,t} \equiv \frac{P_{o,t}}{\tilde{P}}$ are respectively the real non-oil and oil price.

The household maximizes utility subject to the budget constraint

$$P_{no,t}C_{no,t} + P_{o,t}O_{c,t} + P_{no,t}I_{no,t} + P_{o,t}I_{o,t} + Q_tB_t = B_{t-1} + W_tN_t + Z_{no,t}K_{no,t} + Z_{o,t}K_{o,t} + D_{no,t} + D_{o,t},$$

where I have assumed that the price of investment is the same as the price of consumption goods and $B_t$ denotes holdings of one-period nominally riskless bonds, $Q_t$ is the unitary price of the bond and $R_t$ the gross nominal interest rate set by the central bank, $N_{no,t}$ and $N_{o,t}$ are hours worked supplied to the non-oil and oil sector, $W_t$ is the common nominal wage rate under the assumption of perfectly competitive labor markets and $D_{s,t} = \int_0^1 D_{s,i,t} d\pi(i), s = \{no,o\}$ are the dividends from holding shares of firms in the non-oil and oil sector. The household is also subject to a no Ponzi game condition

$$\lim_{t \to \infty} E_t \{ B_t \} \geq 0, \forall t.$$

I assume that investment is subject to adjustment costs à la Christiano et al. (2005).
As in Arezki et al. (2016) the adjustment costs are on sectoral investment so as to exclude intratemporal reallocation of capital between the two sectors, a feature considered plausible given the sectoral specificity of capital. The immobility of capital across sectors is a channel which Hamilton (1988) suggested could account for nonlinearities in the effects of positive vs. negative oil price shocks. Non-oil and oil capital accumulation evolve according to

\[ K_{no,t+1} = I_{no,t} \left[ 1 - \frac{\kappa_{no}}{2} \left( \frac{I_{no,t}}{I_{no,t-1}} - 1 \right)^2 \right] + (1 - \delta) K_{no,t}, \quad (3.10) \]

and

\[ K_{o,t+1} = I_{o,t} \left[ 1 - \frac{\kappa_{o}}{2} \left( \frac{I_{o,t}}{I_{o,t-1}} - 1 \right)^2 \right] + (1 - \delta) K_{o,t}. \quad (3.11) \]

To make the model conformable to the setup used in the empirical analysis I assume that investment is decided one period in advance. This is consistent with the assumption that the oil price does not respond contemporaneously to shocks affecting current investment and with the fact that investment barely moves on impact following an oil price shock.

### 3.3.2 Non-Oil Production

The final non-oil consumption good is produced by good-packers which operate in an environment of perfect competition. They assemble different varieties of the intermediate goods \( Y_{no,i,t} \) according to the CES technology.

\[ Y_{no,t} \equiv \left( \int_0^1 Y_{no,i,t}^{\frac{\varepsilon}{\varepsilon-1}} di \right)^{\frac{\varepsilon-1}{\varepsilon}} \quad (3.12) \]

subject to the following demand schedule for intermediate goods

\[ Y_{no,i,t} = \left( \frac{P_{no,i,t}}{P_{no,t}} \right)^{-\varepsilon} Y_{no,t}, \quad (3.13) \]

which can be derived from the good packers profit maximization problem. I assume that the elasticity of substitution across non-oil consumption goods is the same as the elasticity of substitution across non-oil intermediate goods varieties.

Individual varieties \( Y_{no,i,t} \) are produced by monopolistic competitive firms according to a Cobb–Douglas production function in which first capital \( K_{no,i,t} \) and oil \( O_{no,i,t} \) are bundled together and then combined with labor inputs \( N_{no,i,t} \) before being turned into a non-oil intermediate good in the spirit of Backus and Crucini (2000), Balke et al. (2010)
and Kim and Loungani (1992)

\[
X_{no,i,t} = \left[ \omega_k \left( K_{no,i,t} \right)^{\nu_{no}-1} + (1 - \omega_k) \left( O_{no,i,t} \right)^{\nu_{no}-1} \right]^{\frac{\nu_{no}}{\nu_{no}-1}}, \quad (3.14)
\]

\[
Y_{no,i,t} = A_{no,t} \alpha_{no,i,t} N_{no,i,t}^{1-\alpha_{no}}, \quad (3.15)
\]

where \( \alpha_{no} \in [0, 1] \) is the share of the bundle of capital and oil inputs, \( \omega_k \in [0, 1] \) is the quasi-share parameter which governs the weight of capital purchases in the firm’s capital–oil bundle \( X_{no,t}, \nu_{no} > 0 \) represents the elasticity of substitution between capital and oil in production and \( A_{no,t} \) is a non-oil sector specific, total factor productivity shock.

Intermediate goods firms set prices in a staggered fashion à la Calvo (1983), according to which firms can optimally reset prices in a given periods with probability \( \theta \). Since the production function features constant returns to scale, the pricing decision of an intermediate non-oil goods firm which last reset its price in period \( t \) can be written as a function of sectoral and aggregate variables only:

\[
\max_{P^*_{no,t}} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} Q_{t,t+k} \theta^k \left[ P^*_{no,t} Y^d_{no,t,k} - C \left( Y_{no,t+k} \right) \right] \right\}, \quad (3.16)
\]

subject to the optimal demand schedule derived above

\[
Y_{no,t+k} = \left( \frac{P^*_{no,t}}{P_{no,t+k}} \right)^{-\varepsilon} Y_{no,t+k} \quad (3.17)
\]

and where \( Q_{t,t+k} = \beta^k \mathbb{E}_t \left\{ \frac{\hat{\lambda}_{i+k} P_t}{\lambda_t P_{t+k}} \right\} \) is the stochastic discount factor.

Calvo Pricing implies the following inflation dynamics

\[
1 = \left( 1 - \theta \right) \left( \frac{\Pi^*_{no,t}}{\Pi_{no,t}} \right)^{1-\varepsilon} + \theta \left( \Pi_{no,t} \right)^{\varepsilon-1} \quad (3.18)
\]

while the optimal price setting equation can be represented in recursive form as

\[
\frac{P^*_{no,t}}{P_t} \nu_{1,t} = \mathcal{M}_{no} \nu_{2,t}, \quad (3.19)
\]

\[
\nu_{1,t} = Y_{no,t} + \theta \mathbb{E}_t \left\{ Q_t \Pi^\varepsilon_{no,t+1} \nu_{1,t+1} \right\} \quad (3.20)
\]

and

\[
\nu_{2,t} = Y_{no,t} \mathcal{M}_{no,t} + \theta \mathbb{E}_t \left\{ Q_t \Pi_{t+1} \Pi^\varepsilon_{no,t+1} \nu_{2,t+1} \right\}. \quad (3.21)
\]
3.3.3 Oil Production

Oil is produced by a continuum of perfectly competitive firms. I do so because Bornstein et al. (2017) find that the response of the economy to demand and supply shocks is similar both in qualitative and quantitative terms when, instead, firms are assumed to behave as a cartel or competitive fringe. This conforms with the notion that, historically, the U.S. oil price was pegged to the Brent price so that U.S. producers took the oil price as given.

Prices in the oil sector are assumed to be flexible for two main reasons. First, I want to avoid that monetary policy channels are a predominant force in decisions made by the oil sector. Second, this allows me to have an exogenous oil price process which is crucial since a model with endogenous oil prices is unable to deliver the oil price persistence observed in the data.

The representative firm produces oil using a bundle of labor and capital

\[ X_{o,t} = \left[ \alpha_o \left( K_{o,t} \right)^{\nu_o-1} + (1 - \alpha_o) \left( N_{o,t} \right)^{\nu_o-1} \right]^{\frac{\nu_o}{\nu_o-1}}, \]  

(3.22)

\[ O_t = A_{o,t}X_{o,t}, \]  

(3.23)

where \( \alpha_o \in [0, 1] \) is the quasi-share parameter which governs the importance of capital purchases in the oil firm’s capital-labor bundle, \( \nu_o > 0 \) represents the elasticity of substitution between capital and labor in oil production and \( A_{o,t} \) is an oil sector specific TFP shock.

The growth rate of the real oil price is assumed to evolve exogenously according to the Markov-switching AR(1) process

\[ \Delta \log \left( \hat{P}_{o,t} \right) = \rho_{\hat{P}_o}(s_t)\Delta \log \left( \hat{P}_{o,t-1} \right) + \epsilon_t \hat{P}_o, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_{\hat{P}_o}(s_t)), \]  

(3.24)

where \( \rho_{\hat{P}_o}(s_t) \) and \( \sigma_{\hat{P}_o}(s_t) \) follow a two regimes Markov chain \( s_t \) of small and large oil price shock volatility, in the spirit of what was found in the empirical analysis in Section 3.2.

Given that the oil price evolves exogenously, the oil firm derives its optimal input demands by minimizing costs for a given demanded oil quantity according to

\[ \min_{\{N_{o,t}, K_{o,t}\}} \quad W_tN_{o,t} + Z_{o,t}K_{o,t} + F(s_t)K_{o,t} - P_{o,t}O_t \]  

(3.25)

s.t. \[ O_t = A_{o,t} \left[ \alpha_o \left( K_{o,t} \right)^{\nu_o-1} + (1 - \alpha_o) \left( N_{o,t} \right)^{\nu_o-1} \right]^{\frac{\nu_o}{\nu_o-1}}, \]  

(3.26)

where \( F(s_t) \) are fixed costs, assumed to vary according to the Markov chain \( s_t \) governing the volatility of the oil price shock. This is a reduced form device that, to a first approximation, allows fixed costs to be large or small depending on the size of the oil price shock.
To the extent that the size of the oil price is a measure of current (and expected future) profits, the setup captures the notion that fixed costs can be more easily covered when firm revenues are higher. Alternatively, this reduced form device captures the idea that when the oil price shock is large firms have easier access to credit in order to finance new investment projects or boost existing ones. I follow Cooper and Haltiwanger (2006) and Miao and Wang (2014) in assuming that fixed costs are proportional to the capital stock. This ensures that fixed costs are non-negligible even for large enough firms. As already pointed out in Section 3.1, oil sector’s fixed costs are estimated to be substantial (66%-80% of total costs) and thus important for oil investment decisions. Note that the formulation we employ is observationally equivalent to having a tax on capital, which reduces the real rate of return and thus discourages investment.

The cost minimization problem results in the input demand schedules

\[
[N_{o,t}] : \quad W_t = (1 - \alpha_o) \hat{P}_{o,t} \left( \frac{X_{o,t}}{N_{o,t}} \right)^{\frac{1}{\nu_o}},
\]

\[
[K_{o,t}] : \quad Z_{o,t} + F(s_t) = \alpha_o \hat{P}_{o,t} \left( \frac{X_{o,t}}{N_{o,t}} \right)^{\frac{1}{\nu_o}}.
\]

where \(W_t\) and \(Z_{o,t}\) are respectively the real wage and real rate of return on oil capital. The fixed costs thus introduce a state dependent wedge in the optimal capital demand decision.

### 3.3.4 Monetary Policy

The central bank is assumed to follow the monetary policy reaction function

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left[ \left( \frac{\Pi_{no,t}}{\Pi_{no,ss}} \right)^{\phi_x} \left( \frac{Y_{VA}^t}{Y_{VA}^{ss}} \right)^{\phi_y} \right]^{1-\rho_R} \Xi_t,
\]

where \(R = \frac{\Pi}{P}\) is the steady state gross return on bonds, \(\Pi_{no}\) is steady state core inflation, \(Y_{VA}^t\) is steady state value-added output, value-added output is defined as \(Y_{VA}^t = C_t + I_t = P_{o,t}O_t + P_{no,t}Y_{no,t}^A\), where \(Y_{no,t}^A = Y_{no,t} - \frac{P_{o,t}}{P_{no,t}}O_{no,t}\) and \(\Xi_t\) is a monetary policy shock. The degree with which the monetary authority smooths the interest rate is \(\rho_R\) whereas the strength with which it reacts to deviations of core inflation and value-added output from their steady state is governed by \(\phi_x\) and \(\phi_y\) respectively. The feature that monetary policy responds to the sticky component of the price index is not only in line with the conduct of U.S. monetary policy, which targets core inflation (see Bernanke, 2015), but also desirable from a welfare perspective, as shown by Aoki (2001).
3.3.5 Market Clearing and Feasibility

The market clearing and feasibility conditions of the model economy are the following.

Bonds Market Clearing  \[ B_t = B_{t-1} = 0. \]  \hspace{1cm} (3.30)

Goods and Aggregate Market Clearing

\[ Y_{no,t} = C_{no,t} + I_{no,t}, \]  \hspace{1cm} (3.31)

\[ O_t = O_{c,t} + O_{no,t} + I_{o,t}, \]  \hspace{1cm} (3.32)

\[ Y_{tVA} = C_t + I_t. \]  \hspace{1cm} (3.33)

Definition of Aggregate Labor, Capital and Investment

\[ N_t = N_{no,t} + N_{o,t}, \]  \hspace{1cm} (3.34)

\[ K_t = K_{no,t} + K_{o,t}, \]  \hspace{1cm} (3.35)

\[ I_t = \hat{P}_{no,t} I_{no,t} + \hat{P}_{o,t} I_{o,t}. \]  \hspace{1cm} (3.36)

Aggregate Inflation

Using the definition of the aggregate price index one obtains

\[ \Pi_t = \left[ (1 - \omega_c) \nu_c \left( \Pi_{no,t} \hat{P}_{no,t-1} \right)^{1-\nu_c} + \omega_c \nu_c \left( \Pi_{o,t} \hat{P}_{o,t-1} \right)^{1-\nu_c} \right]^{1/(1-\nu_c)}. \]  \hspace{1cm} (3.37)

Output Aggregation

Equilibrium in the non-oil goods market

\[ \int_0^1 Y_{no,t} di = Y_{no,t} \int_0^1 \left( \frac{P_{no,i,t}}{P_{no,t}} \right)^{-\varepsilon} di = Y_{no,t} S_{no,t} \]

\[ \Rightarrow \quad Y_{no,t} S_{no,t} = A_{no,t} X_{no,t}^{\alpha_{no}} Y_{no,t}^{1-\alpha_{no}}. \]  \hspace{1cm} (3.38)

where \( S_{no,t} \) denotes the relative price dispersion which can be recast in recursive form as

\[ S_{no,t} = (1 - \theta) \left( \frac{\Pi_{no,t}}{\hat{P}_{no,t}} \right)^{-\varepsilon} + \theta (\Pi_{no,t})^\varepsilon S_{no,t-1}. \]  \hspace{1cm} (3.39)

3.3.6 Calibration

The values of the parameters used in the simulations are in Table 3.2. The subjective discount factor \( \beta \) is calibrated to 0.993 in order to match an average annualized interest rate of 3%. Following Smets and Wouters (2007) I set \( \sigma \), which governs intertemporal
substitution of the consumption-hours bundle, to 1.39 and the exponent on labor in the utility function $\varphi$ to 1.92. The quasi-share parameter $\omega_c$ is calibrated so as to match a steady state personal consumption expenditures energy share $\left(\frac{P_o O_c}{C}\right)$ of 3.5% $^{26}$. In line with Natal (2012), I set the elasticity of substitution between non-oil consumption goods $\nu_c$ to 0.3, which intermediate between 0.4, obtained by simulated method of moments in Bodenstein et al. (2011), and 0.25, calibrated in Plante (2014), and well inside the range of the estimates reported by Kilian and Murphy (2014). The depreciation rate of capital $\delta$ is 0.025, as standard in the literature. The elasticity of substitution between non-oil intermediate goods is chosen to be the same as the one between non-oil consumption goods $\varepsilon$ and equal to 6 (implying a steady state price markup of 20%), whereas the price stickiness parameter is set to 0.75 (implying an average price duration of four quarters) as in, e.g., Natal, 2012 and Plante, 2014). The quasi-share parameter $\omega_k$ targets a steady state energy share in production $\left(\frac{P_o O_{no}}{P_{no} Y_{no}}\right)$ of 2.8%. I obtain this statistic by computing the shares of mining and utilities in non-energy production over the period from 1970 to 2015 via the Use Tables Input–Output Accounts Data provided by the Bureau of Economic Analysis. The elasticity of substitution between capital and oil $\nu_{no}$ is taken from Backus and Crucini (2000) and set to 0.09. Arezki et al. (2016) set the capital share of oil production to 0.49 and the labor share to 0.13, devoting the remaining 0.38 to oil reserves. Since I abstract from oil reserves, I distribute 1/3 of this share to capital and 2/3 to labor so that the first is roughly 0.68. As to the monetary policy parameters, the degree of interest rate smoothing $\rho_R$ is chosen to be 0.8 whereas the Taylor rule coefficients on inflation and value-added output are respectively set to 1.5 and 0.5/4. Core steady state inflation is set to 1 in order to focus on a non-inflationary steady state.

The remaining parameters are estimated by impulse response matching (IRM henceforth) of non-oil and oil investment to an oil price shock. They are the habit persistence $b$, the elasticity of substitution between oil capital and labor in oil production $\nu_o$ and the investment adjustment costs in the non-oil and oil sector, $\kappa_{no}$ and $\kappa_o$ respectively. To match the IRFs one needs a fair degree of habit persistence, a less than unitary elasticity of capital and labor in oil production (i.e., factor inputs are rather complements than substitutes), small investment adjustment costs in the non-oil sector and mild investment adjustment costs in the oil sector.

As to the parameters associated with regime switches, the transition probabilities of the Markov chain $s_t$ are set equal to the median estimates of the transition probabilities in the Markov–switching VAR: The probability to move from the large to the small oil

$^{26}$If, alternatively, one loosely interpreted oil consumption as general energy consumption from gasoline, gas, etc. the average share would be lower and around 6%. Calibrating $\omega_c$ to match this value does not change any of the qualitative features of the results I present.
price shock regime is 0.04948, whereas the probability of switching from a small to a large oil price shock regime is 0.0226. The regime-specific standard deviations $\sigma_{P_o}(s_t)$, autoregressive parameters $\rho_{P_o}(s_t)$ and fixed costs $F(s_t)$ are estimated by matching the impulse response of oil prices and oil investment to an oil price shock. Given the ergodic regime probabilities, the fixed costs of 0.125% percent of capital in the small oil price shock regime and of 0% in the large oil price shock regime result in an ergodic mean in which fixed costs correspond roughly to 62% of production costs, a value in line with Kellogg (2014) and Appert and Favennec (2015).

These parameters are estimated via IRM since they are either important for the dynamics (in particular, $b$, $\rho_{P_o}(s_t)$ and $\sigma_{P_o}(s_t)$) or because there is no (consensus) value in the literature ($\nu_o$, $\kappa_{no}$, $\kappa_o$ and $F(s_t)$). Indeed, an estimation of the degree of substitutability between capital and labor, of investment adjustment and of fixed costs in the oil sector is itself an important byproduct of the analysis.
### 3.3. A MARKOV–SWITCHING MODEL OF AN OIL PRODUCING ECONOMY

**Table 3.2 – Model Calibration**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Target/Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Household</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta \in (0, 1) ) Subjective discount factor</td>
<td>0.993</td>
<td>3% nominal interest rate</td>
</tr>
<tr>
<td>( b \in [0, 1) ) External consumption habit formation</td>
<td>0.8</td>
<td>Non-Oil &amp; Oil Investment IRM</td>
</tr>
<tr>
<td>( \sigma &gt; 0 ) Governs intertemporal substitution of the consumption–hours bundle</td>
<td>1.39</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>( \varphi &gt; 0 ) Inverse Frisch elasticity of labor supply</td>
<td>1.92</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>( \omega_e \in [0, 1] ) Governs oil consumption share</td>
<td>0.6825</td>
<td>( \frac{\hat{p}<em>o q</em>{no}}{\hat{p}<em>o q</em>{no} + q_{no}} = 0.035 )</td>
</tr>
<tr>
<td>( \nu_c &gt; 0 ) Elasticity of substitution btw. non-oil consumption goods and oil</td>
<td>0.3</td>
<td>Natal (2012)</td>
</tr>
<tr>
<td>( \varepsilon &gt; 0 ) Elasticity of substitution btw. non-oil consumption goods</td>
<td>6</td>
<td>Standard</td>
</tr>
<tr>
<td>( \delta \in [0, 1] ) Capital depreciation rate</td>
<td>0.025</td>
<td>Standard</td>
</tr>
<tr>
<td>( \kappa_{no} &gt; 0 ) Non-oil investment adjustment costs</td>
<td>0.25</td>
<td>Non-Oil &amp; Oil Investment IRM</td>
</tr>
<tr>
<td>( \kappa_o &gt; 0 ) Oil investment adjustment costs</td>
<td>2</td>
<td>Non-Oil &amp; Oil Investment IRM</td>
</tr>
<tr>
<td><strong>Firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varepsilon &gt; 0 ) Elasticity of substitution btw. non-oil intermediate goods</td>
<td>see above</td>
<td></td>
</tr>
<tr>
<td>( \theta \in [0, 1] ) Price stickiness</td>
<td>0.75</td>
<td>Standard</td>
</tr>
<tr>
<td>( \alpha_{no} \in [0, 1] ) Capital share in non-oil production</td>
<td>0.30</td>
<td>Standard</td>
</tr>
<tr>
<td>( \omega_x \in [0, 1] ) Capital share in capital–oil bundle</td>
<td>0.9985</td>
<td>( \frac{\hat{p}<em>o q</em>{no}}{\hat{p}<em>o q</em>{no} + q_{no}} = 0.028 )</td>
</tr>
<tr>
<td>( \nu_{no} &gt; 0 ) Elasticity of substitution btw. capital and oil</td>
<td>0.09</td>
<td>Backus and Crucini (2000)</td>
</tr>
<tr>
<td>( \alpha_o \in [0, 1] ) Capital share in oil production</td>
<td>0.68</td>
<td>Arezki et al. (2016)</td>
</tr>
<tr>
<td>( \nu_o &gt; 0 ) Elasticity of substitution btw. capital and labor</td>
<td>0.886</td>
<td>Non-Oil &amp; Oil Investment IRM</td>
</tr>
<tr>
<td>( F(s_t = S) ) Fixed cost (small oil price shock regime)</td>
<td>0.00125</td>
<td>Non-Oil &amp; Oil Investment IRM</td>
</tr>
<tr>
<td>( F(s_t = L) ) Fixed cost (large oil price shock regime)</td>
<td>0</td>
<td>Non-Oil &amp; Oil Investment IRM</td>
</tr>
<tr>
<td><strong>Monetary Policy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_R \in [0, 1] ) Interest rate smoothing coefficient</td>
<td>0.8</td>
<td>Standard</td>
</tr>
<tr>
<td>( \phi^*_y &gt; 0 ) Taylor rule value-added output coefficient</td>
<td>1.5</td>
<td>Standard</td>
</tr>
<tr>
<td>( \phi^*_y &gt; 0 ) Taylor rule value-added output coefficient</td>
<td>0.5/4</td>
<td>Standard</td>
</tr>
<tr>
<td>( \Pi_{no} \geq 1 ) Steady state non-oil inflation</td>
<td>1</td>
<td>Standard</td>
</tr>
<tr>
<td><strong>Oil Price Shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho_{\hat{p}_o} (s_t = S) \in \mathbb{R} ) Autoregressive coefficient (small oil price shock regime)</td>
<td>0.2780</td>
<td>Oil Price IRM</td>
</tr>
<tr>
<td>( \rho_{\hat{p}_o} (s_t = L) \in \mathbb{R} ) Autoregressive coefficient (large oil price shock regime)</td>
<td>-0.2003</td>
<td>Oil Price IRM</td>
</tr>
<tr>
<td>( \sigma_{\hat{p}_o} (s_t = S) &gt; 0 ) Standard deviation (small oil price shock regime)</td>
<td>8.4370</td>
<td>Oil Price IRM</td>
</tr>
<tr>
<td>( \sigma_{\hat{p}_o} (s_t = L) &gt; 0 ) Standard deviation (large oil price shock regime)</td>
<td>28.0272</td>
<td>Oil Price IRM</td>
</tr>
</tbody>
</table>
3.4 The Nonlinearity between Oil Prices and Investments

I compute a first order approximation of the Markov-switching model and compute generalized regime specific IRFs to an oil price shock, taking into account that when agents are in one regime at a particular IRF horizon, they form expectations about the possibility of switching to the other regime as the horizon evolves. Note that the fact that when computing the impulse responses in the empirical part, I do not consider that the regime might have changed is not at odds with what done here. This is because the regime-specific IRFs of the empirical model already capture agents’ expectations about moving to another regime when taking action in one regime.

**Figure 3.7** – Theoretical (circled) and empirical (straight lines, along with shaded 68% credibility intervals) impulse responses of real oil price, non-oil and oil investment for the small oil price shocks regime (blue) and for the large oil price shock regime (red).
Regime-dependent impulse responses to a small and a large oil price shock are in Figure 3.7. The model replicates two key facts. First, following a small oil price surprise increase there is a drop in investment, both in the oil and the non-oil sector. This is due to the fact that a demand channel, through which there is a drop in oil demand from both households and firms and a drop in consumer’s disposable income, is prevailing. Depressed demand then lowers investment in both sectors. Second, the response of investment in the oil and non-oil sector to an unexpected large oil price hike is asymmetric. Indeed, while non-oil investment drops, there is a surge in oil investment. This is because under a large oil surprise hike the dominant effect comes from higher oil prices, which increase wages and rates of return on capital investment. This channel is weaker in the small oil price shock regime due to the presence of fixed investment costs.

When there are large oil price shocks, this effect is dominating because agents expect that they might switch to the small oil price shock regime where they would incur relatively higher fixed costs and, thus, lower rates of return on their oil capital investment. Hence, when the surprise increase in the price of oil is large enough, investment becomes more profitable as fixed costs are relatively low and agents overinvest in this regime to compensate for the fact that the relatively high fixed cost in the other regime will prevent them to do the required investments.

Next, I show that this mechanism, whereby the anticipation of the possibility of ending up in a regime where fixed costs are more difficult to cover, is indeed the key mechanism driving the nonlinearity in oil investment.

I compute the response of oil investment in a model where i) the transition probability of moving from a large to a small oil price shock regime is set from roughly 5% to 0% (circled lines) and ii) the transition probability of moving from a small to a large oil price shock regime is set from around 2% to 0% (squared lines).

As Figure 3.8 indicates, while the estimated transition probabilities are small, they are crucial for agents’ expectations and, thus, oil investment decisions. In case i), agents living in a large oil price shock regime do not expect to be hit by a small oil price shock, where fixed costs are more difficult to cover. This leads them to not increase investments in the large oil price shock regime and to decrease investments by more than in the baseline model in the small oil price shock regime. In case ii), agents in the small oil price shock regime do not expect to end up in a regime where it is possible to cover the fixed costs. This induces agents to increase investments much more in the large oil price shock regime compared to the baseline. This is due to the fact that the ergodic regime probability of being in the small oil price shock regime becomes higher when the transition probability of moving from a small to a large oil price shock regime is set to zero.
In Figure 3.8 I calculate the GDP gain due to the fact that oil investment increases following a large oil price shock. The gain is computed as follows. First, I compute the difference between the IRFs of oil investment following a large oil price shock in the original model and the same IRF in a counterfactual model, where the fixed costs channel is switched off. Next, I multiply this difference by the steady state ratio of oil investment to value added GDP in the model \( \frac{\hat{P}_oI_o}{Y_{VA}} \), equal to 5.99%. Even though not targeted by calibration, this share is close to the historical 5.06% share of oil investment to GDP, computed by multiplying the historical share of oil investment to PNFI, equal to 5.32% (as reported in Table 3.1), and multiplying it by the PNFI share in GDP (roughly equal to 95%). The GDP gain can be as high as approximately 0.3% after the first year, which is consistent with the back-of-the-envelope calculation for the GDP gain made in Section 2 and in Baumeister and Kilian (2017).
To evaluate the performance of the model I also compare the empirical IRFs to the IRFs obtained by running the VAR of Section 3.2 on simulated data. This is an important cross-check since, as shown by Chari et al. (2005), statistical models might fail to approximate well the process which generates the simulated data and so the theoretical and empirical IRFs might diverge substantially. I simulate as many data points for the oil price growth, non-oil investment growth and oil investment growth series as are available in the empirical analysis. The data are simulated using, in each point in time over the simulation horizon, the same transition probabilities identified by the VAR of Section 3.2. The simulated data is then run through the same VAR, which gives the estimated regime probabilities and regime-dependent impulse responses respectively depicted in Figure 3.10 and 3.11. As it is evident, the empirical model captures well the time dependent probabilities of being in a large oil price shock regime (with the exception of very short-lived ones like the one before 2005). Most importantly, the theoretical and empirical IRFs not only match qualitatively in that they exhibit the same nonlinearity but also quantitatively they are fairly similar.
Figure 3.10 – Time evolution of smoothed mean regime probability of being in a large oil price shock regime used in simulation (blue) and estimated by empirical model (red).

Figure 3.11 – Median impulse responses along with 68% credibility intervals of i) empirical model for small (blue) and large (red) oil price shock regime and ii) empirical model run on model simulated data for small (cyan) and large (pink) oil price shock regime.
3.5 Conclusion

I conclude that the effect of oil price shocks on U.S. investment is nonlinear. In particular, I show that the size of an oil price shock matters for the sign of the response of U.S. investment to oil price shocks, both in an empirical and in a theoretical model.

First, I estimate a Markov-switching VAR using data on oil prices and U.S. investment and compute regime-dependent impulse responses. I find that while small oil price shocks always lead to a decline in investment, large oil price shocks have ambiguous effects. While large oil price shocks lead to a decline in non-oil investment, they induce the oil sector and oil-related sectors to invest more. This leads aggregate investment to initially decline but eventually increase. A 25% large oil price increase generates a 3% annualized increase in aggregate investment, which entails a 0.4% increase in GDP. These findings challenge the conventional wisdom that oil price increases are unconditionally contractionary, at least as far as the investment channel is concerned.

Second, I build a DSGE model of an oil-producing economy with two production sectors. The non-oil sector produces a final consumption good with labor, capital and oil. The oil sector produces oil with labor and capital only. Households can invest in both sectors and consume the non-oil final good as well as oil. A monetary authority sets the interest rate according to a Taylor rule. The oil price and the oil firms’ fixed costs follow a Markov chain which varies according to regimes of high and low oil price volatility to capture the notion that the ability to cover fixed costs varies with firm profits. I show that to match the empirical IRFs agents have to expect that while being in the large oil price shock regime they might end up in the small oil price shock regime in which fixed costs are more difficult to cover. In a counterfactual analysis, the model confirms that the GDP gain due to increased oil investment is around 0.3%, as calculated in the empirical analysis.

An interesting extension of the current analysis would be to investigate whether the size of other shocks also matters for the transmission to key macroeconomic aggregates. For instance, due to rational inattention agents might only pay attention to fiscal and monetary shocks of sizeable magnitude. This question constitutes a promising research avenue.
References


Blanchard, O. J. and J. Galí (2008). The Macroeconomic Effects of Oil Shocks: Why are the 2000s so Different from the 1970s?


REFERENCES


Appendix 3.A  Details of Empirical Model

Prior and Posterior  The joint prior over \((\Theta, Q, S_T)\) is partitioned into a conditional Sims and Zha (1998) prior for the regime-dependent parameters as in Sims et al. (2008) and a Dirichlet prior for the Markov process:

\[
Pr (\Theta, Q, S_T) = Pr (\Theta) \cdot Pr (Q) \cdot Pr (s_0|\Theta, Q) \cdot \prod_{t=1}^{T} Pr (s_t|\Theta, Q, S_{t-1}),
\]  

where \(s_0\) is the initial state with prior \(1/M\) and \(S_{t-1}\) the previous time period path of states.

This prior specification entails the following conditional posteriors (see Sims et al., 2008)

\[
Pr (S_T|Y_T, \Theta, Q) \propto Pr (s_t|S_T), \forall t
\]  

\[
Pr (Q|Y_T, \Theta, S_T) \propto \prod_{i=1}^{M} p_{i,m}^{n_{i,m}+\alpha_{i,m}},
\]  

\[
Pr (\Theta|Y_T, S_T, Q) \propto \mathcal{N} \left( \hat{\Theta}, \hat{\Sigma} \right).
\]

The prior for the BVAR part of the model is a diffuse Sims and Zha (1998) Normal-Wishart prior with the exception of prior tightness in the sum-of-coefficients needed to ensure stationary draws in some specifications\(^{27}\). In particular, the prior tightness is chosen so as to ensure nonexplosive credibility intervals while preserving the median estimate. The hyperparameters and their chosen values are listed in Table 3.3.

<table>
<thead>
<tr>
<th>Table 3.3 – Sims and Zha (1998) Normal-Wishart Prior Hyperparameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>(\lambda_0)</td>
</tr>
<tr>
<td>(\lambda_1)</td>
</tr>
<tr>
<td>(\lambda_2)</td>
</tr>
<tr>
<td>(\lambda_3)</td>
</tr>
<tr>
<td>(\lambda_4)</td>
</tr>
<tr>
<td>(\lambda_5)</td>
</tr>
<tr>
<td>(\lambda_6)</td>
</tr>
</tbody>
</table>

\(^{27}\) These tighter prior choices are supported by higher marginal likelihoods vis-à-vis their diffuse counterparts.

\(^{28}\) The limit case \(\lambda_5 = \infty\) implies unit root in each equation and cointegration is ruled out.

\(^{29}\) If the dummy–initial–observations prior is imposed then weight is placed on a model where variables...
The Dirichlet prior for the Markov-switching process is chosen so as to reflect the
number of large and small oil price swings occurrences observed in the data\textsuperscript{30}. In par-
ticular, there are 34 episodes in which the quarter-on-quarter change in the price of oil
was above one standard deviation (which we classify as large) and 153 episodes in which
the oil price swings were small.

**Regime Identification**  There is a identification problem intrinsic to MS-VAR models
related to the interchangeability in the labeling of the states\textsuperscript{31}. This interchangeability
can be cured by adding some prior beliefs to the model about certain characteristics of
the regimes, see McCulloch and Tsay (1994). To this end, I add the prior that the regimes
of the Markov-switching process differ in the variance of the oil price shock\textsuperscript{32}. In partic-
ular, the states are ranked in ascending order so that the second regime corresponds to
episodes featuring mostly large oil price shocks.

**Posterior Mode Finding and Sampling**  The posterior mode of the model is found via
the Expectation–Maximization (EM) algorithm, an iterative ML estimation technique de-
signed for models where time series evolve according to some latent stochastic vari-
able\textsuperscript{33}. Since the posterior distribution is proportional to the likelihood of the model
this maximization procedure is equivalent to finding the posterior mode of the posterior
distribution. Following Sims et al. (2008) I iterate on the following five steps\textsuperscript{34}:

I. Maximize over intercepts $c(s_t)$.

II. Maximize over autoregressive coefficients $B_i(s_t), i = 1, ..., 4$.

III. Maximize over the covariance matrix $\Sigma(s_t)$.

IV. Maximize over the transition matrix $Q$.

The EM algorithm does not provide the posterior distribution of the parameters.

---

\textsuperscript{30}Robustness checks, available upon request, show that results hold through for other prior durations.

\textsuperscript{31}This is because the indices of the Markov chain $s_t$ can be permuted without changing the law of the
process (see Krolzig (1997) and Frühwirth-Schnatter (2006)).

\textsuperscript{32}The exact same MS–BVAR coefficients and transition probabilities are estimated if the states are iden-
tified on a Markov-switching process based on the intercept of the oil price equation instead.

\textsuperscript{33}As shown by Hamilton (1990), the latter can be used in conjunction with the BLHK filter to obtain the
maximum likelihood estimate of the model.

\textsuperscript{34}Starting values are randomly drawn. In all applications convergence is achieved after less than 50
iterations.
Thus, to draw from the posterior distribution I use a Gibbs sampler\textsuperscript{35} which cycles through the conditional densities using five steps\textsuperscript{36}:

1. **Draw the state-space for the Markov process** $s_t$. This step uses the Baum-Lindgren-Hamilton-Kim (BLHK) filter and smoother\textsuperscript{37} described in Krolzig (1997). Draws are obtained via the forward-filter-backward-sampling algorithm\textsuperscript{38}, see Frühwirth-Schnatter (2006).

2. **Draw the Markov transition matrix** $Q$. Conditional on the other parameters, this takes a draw from a Dirichlet posterior.

3. **Regression step update.** Conditional on the state-space and the transition matrix of the Markov process obtained in the previous two steps, estimate a set of $M = 2$ regressions, one for each regime\textsuperscript{39}.

4. **Draw the regression coefficients** $B_i(s_t), i = 1, ..., 4$ and $\Sigma$. Draw the regime-dependent intercepts and autoregressive coefficients from a Normal distribution and the regime-dependent covariance matrices from an Inverse-Wishart distribution.

\textsuperscript{35}The initialization step consists in estimating a BVAR, non-regime switching model. More specifically, $M$ VAR regressions are based on a K-means clustering of the time series with $M$ clusters. The initial choice of $Q$ is based on the results from the K-means clustering of the data.

\textsuperscript{36}To sample from the posterior distribution I discard the initial 2000 draws and keep the remaining 3000 for inference.

\textsuperscript{37}In a nutshell, the filter calculates how likely it is that an observation at time $t$ has been generated by the VAR in regime $i$ conditional on parameter values and the data.

\textsuperscript{38}The algorithm essentially allows sampling from the full conditional posterior distribution of the hidden state $s_t$.

\textsuperscript{39}This is done using a GLS estimator with dummy variables corresponding to each regime. In particular, the dummies take the value of unity if the regime $m$ has been drawn for $s_t$ by the Gibbs sampler in the previous step, see Krolzig (1997) for details.
Appendix 3.B  Results by Investment Category

Figure 3.12 – (Left) Smoothed mean regime probability of large real oil price shock regime (right axis, black) along with large and small real real oil price swings (left axis, red and blue respectively). (Middle and Right) Impulse responses, along with 68% credibility intervals, of the real oil price and of investment categories for the small oil price shock regime (blue) and for the large oil price shock regime (red).
Appendix 3.C  Spillover Effects

Spillover effects are likely behind i) the decrease of oil investment in response to a small oil price shock and ii) the increase of investment in information processing, industrial and transportation equipment in response to a large oil price shock. To assess this channel ideally one would need to map investment categories to industries producing those investment goods. Unfortunately, investment categories cannot be linked to NAICS codes and it is thus impossible to have a one-to-one mapping to the BEA industry classifications.\footnote{One attempt in this direction is made by House et al. (2017) which use a BEA provided mapping to go from very detailed industry/product annual data on production of capital goods and input usage in the NBER Productivity Database to the more aggregated investment categories in the BEA detail tables.}

An attempt to establish the existence of sectoral linkages is to inspect the detailed 2007 BEA Input-Output tables and document if the oil industry is indeed an important supplier to the manufacturing sector and an important demander of intermediate goods from sectors related with industrial, transportation and information processing equipment.

It is evident from Table 3.4 that the oil sector is indeed an important supplier to the manufacturing sector. For instance, manufacturing related sectors (e.g., petrochemical, asphalt paving mixture and plastics materials manufacturing) demand around 20% of $1 oil and gas extraction gross output. Concurrently, Table 3.5 reveals that indeed, industries related with transportation (e.g., pipeline and truck transportation), industrial machinery (e.g., derricks – lifting devices supporting drilling equipment) and services which require computers and other digital electronic equipment (e.g., management of companies and enterprises and legal services) are indeed suppliers to the oil sector, providing roughly 15% of $1 worth of oil and gas drilling.

As a further experiment to examine the presence of spillover effects consists I estimate a constant parameter version of the empirical model presented in Section 3.2.1 in which I include real oil prices as well as the three main investment categories, i.e., non-oil, oil and “oil-like” investment. I document that investment shocks occurring in both the “non-oil” and “oil-like” category are found to explain a substantial portion of the forecast error variance of the oil investment series at both short and long horizons.
Table 3.4 – Industries Demanding from Oil and Gas Drilling and Extraction in the U.S.

<table>
<thead>
<tr>
<th>Industry (Commodity)</th>
<th>Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil and gas extraction</td>
<td>13.600978</td>
</tr>
<tr>
<td>Drilling oil and gas wells</td>
<td>12.147115</td>
</tr>
<tr>
<td>Natural gas distribution</td>
<td>3.902063</td>
</tr>
<tr>
<td>Petroleum refineries</td>
<td>3.275674</td>
</tr>
<tr>
<td>Petrochemical manufacturing</td>
<td>2.071590</td>
</tr>
<tr>
<td>Other petroleum and coal products manufacturing</td>
<td>1.765019</td>
</tr>
<tr>
<td>Pipeline transportation</td>
<td>1.306175</td>
</tr>
<tr>
<td>Other basic organic chemical manufacturing</td>
<td>1.182006</td>
</tr>
<tr>
<td>Asphalt paving mixture and block manufacturing</td>
<td>1.106203</td>
</tr>
<tr>
<td>Electric power generation, transmission, and distribution</td>
<td>0.961973</td>
</tr>
<tr>
<td>Plastics material and resin manufacturing</td>
<td>0.960188</td>
</tr>
<tr>
<td>Asphalt shingle and coating materials manufacturing</td>
<td>0.938266</td>
</tr>
<tr>
<td>Synthetic rubber and artificial and synthetic fibers and filaments manufacturing</td>
<td>0.831892</td>
</tr>
<tr>
<td>Fertilizer manufacturing</td>
<td>0.771965</td>
</tr>
<tr>
<td>Other basic inorganic chemical manufacturing</td>
<td>0.666529</td>
</tr>
<tr>
<td>Air transportation</td>
<td>0.628389</td>
</tr>
<tr>
<td>Water transportation</td>
<td>0.628052</td>
</tr>
<tr>
<td>Truck transportation</td>
<td>0.61058</td>
</tr>
<tr>
<td>Lime and gypsum product manufacturing</td>
<td>0.590610</td>
</tr>
<tr>
<td>Wet corn milling</td>
<td>0.588253</td>
</tr>
<tr>
<td>Carbon and graphite product manufacturing</td>
<td>0.564881</td>
</tr>
<tr>
<td>Industrial gas manufacturing</td>
<td>0.564425</td>
</tr>
<tr>
<td>Grain farming</td>
<td>0.544010</td>
</tr>
<tr>
<td>Rail transportation</td>
<td>0.484261</td>
</tr>
<tr>
<td>Cement manufacturing</td>
<td>0.451596</td>
</tr>
<tr>
<td>Polystyrene foam product manufacturing</td>
<td>0.439044</td>
</tr>
<tr>
<td>Paperboard mills</td>
<td>0.424493</td>
</tr>
<tr>
<td>Plastics bottle manufacturing</td>
<td>0.421058</td>
</tr>
<tr>
<td>Iron, gold, silver, and other metal ore mining</td>
<td>0.420788</td>
</tr>
<tr>
<td>Alumina refining and primary aluminum production</td>
<td>0.420460</td>
</tr>
</tbody>
</table>

Note: $1 of oil and gas drilling and extraction gross output supplies the share of gross output of each of the commodities listed in the table.
### Table 3.5 – Industries Supplying to Oil and Gas Drilling and Extraction

<table>
<thead>
<tr>
<th>Industry (Commodity)</th>
<th>Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil and gas extraction</td>
<td>41.878878</td>
</tr>
<tr>
<td>Drilling oil and gas wells</td>
<td>37.089740</td>
</tr>
<tr>
<td>Management of companies and enterprises</td>
<td>1.827461</td>
</tr>
<tr>
<td>Other support activities for mining</td>
<td>1.519633</td>
</tr>
<tr>
<td>Legal services</td>
<td>1.463781</td>
</tr>
<tr>
<td>Nonresidential maintenance and repair</td>
<td>1.085720</td>
</tr>
<tr>
<td>Petroleum refineries</td>
<td>0.991999</td>
</tr>
<tr>
<td>Wholesale trade</td>
<td>0.984379</td>
</tr>
<tr>
<td>Commercial and industrial machinery and equipment rental and leasing</td>
<td>0.823878</td>
</tr>
<tr>
<td>Pipeline transportation</td>
<td>0.727327</td>
</tr>
<tr>
<td>Architectural, engineering, and related services</td>
<td>0.572299</td>
</tr>
<tr>
<td>Monetary authorities and depository credit intermediation</td>
<td>0.521972</td>
</tr>
<tr>
<td>Other real estate</td>
<td>0.489221</td>
</tr>
<tr>
<td>Petrochemical manufacturing</td>
<td>0.465107</td>
</tr>
<tr>
<td>Cutting and machine tool accessory, rolling mill, and other metalworking machinery manufacturing</td>
<td>0.424061</td>
</tr>
<tr>
<td>Lessor of nonfinancial intangible assets</td>
<td>0.392979</td>
</tr>
<tr>
<td>Iron and steel mills and ferroalloy manufacturing</td>
<td>0.369740</td>
</tr>
<tr>
<td>Services to buildings and dwellings</td>
<td>0.348444</td>
</tr>
<tr>
<td>Insurance carriers</td>
<td>0.283661</td>
</tr>
<tr>
<td>Electric power generation, transmission, and distribution</td>
<td>0.277458</td>
</tr>
<tr>
<td>Mining and oil and gas field machinery manufacturing</td>
<td>0.230286</td>
</tr>
<tr>
<td>Truck transportation</td>
<td>0.227171</td>
</tr>
<tr>
<td>Natural gas distribution</td>
<td>0.223363</td>
</tr>
<tr>
<td>Nondepository credit intermediation and related activities</td>
<td>0.219895</td>
</tr>
<tr>
<td>Securities and commodity contracts intermediation and brokerage</td>
<td>0.215865</td>
</tr>
<tr>
<td>Other financial investment activities</td>
<td>0.192588</td>
</tr>
<tr>
<td>Accounting, tax preparation, bookkeeping, and payroll services</td>
<td>0.188469</td>
</tr>
<tr>
<td>Wired telecommunications carriers</td>
<td>0.185847</td>
</tr>
<tr>
<td>All other chemical product and preparation manufacturing</td>
<td>0.179639</td>
</tr>
<tr>
<td>Other petroleum and coal products manufacturing</td>
<td>0.156574</td>
</tr>
</tbody>
</table>

Note: $1 of oil and gas drilling and extraction gross output demands the share of gross output of each of the commodities listed in the table.
### Table 3.6 – Forecast Error Variance Decomposition

#### Shock to Real Oil Price

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Real Oil Price</th>
<th>Non-Oil Investment</th>
<th>Oil-Like Responding Investment</th>
<th>Oil Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>99.880773</td>
<td>0.000016</td>
<td>0.000543</td>
<td>0.118669</td>
</tr>
<tr>
<td>2</td>
<td>92.639879</td>
<td>0.002534</td>
<td>0.000592</td>
<td>7.356995</td>
</tr>
<tr>
<td>3</td>
<td>86.058658</td>
<td>0.002340</td>
<td>0.009662</td>
<td>13.929340</td>
</tr>
<tr>
<td>4</td>
<td>84.274930</td>
<td>0.023182</td>
<td>0.086540</td>
<td>15.615348</td>
</tr>
<tr>
<td>5</td>
<td>83.498919</td>
<td>0.032308</td>
<td>0.108679</td>
<td>16.360094</td>
</tr>
<tr>
<td>20</td>
<td>83.371724</td>
<td>0.106807</td>
<td>0.151678</td>
<td>16.309730</td>
</tr>
</tbody>
</table>

#### Shock to Non-Oil Investment

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Real Oil Price</th>
<th>Non-Oil Investment</th>
<th>Oil-Like Responding Investment</th>
<th>Oil Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000000</td>
<td>74.155899</td>
<td>4.919835</td>
<td>21.824256</td>
</tr>
<tr>
<td>2</td>
<td>26.252505</td>
<td>47.872688</td>
<td>2.51081</td>
<td>23.357726</td>
</tr>
<tr>
<td>3</td>
<td>34.005317</td>
<td>42.945209</td>
<td>2.52162</td>
<td>20.520811</td>
</tr>
<tr>
<td>4</td>
<td>33.458283</td>
<td>42.92164</td>
<td>2.560896</td>
<td>21.058657</td>
</tr>
<tr>
<td>5</td>
<td>32.718946</td>
<td>38.742672</td>
<td>2.665987</td>
<td>25.872395</td>
</tr>
<tr>
<td>20</td>
<td>32.059485</td>
<td>38.809682</td>
<td>3.12533</td>
<td>20.638280</td>
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</tbody>
</table>

#### Shock to Oil-Like Responding Investment

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Real Oil Price</th>
<th>Non-Oil Investment</th>
<th>Oil-Like Responding Investment</th>
<th>Oil Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>99.997683</td>
<td>0.022315</td>
</tr>
<tr>
<td>2</td>
<td>24.134982</td>
<td>3.857194</td>
<td>67.749747</td>
<td>4.258277</td>
</tr>
<tr>
<td>3</td>
<td>18.898281</td>
<td>6.380861</td>
<td>49.887490</td>
<td>24.633365</td>
</tr>
<tr>
<td>4</td>
<td>15.235095</td>
<td>7.17739</td>
<td>42.070168</td>
<td>34.976998</td>
</tr>
<tr>
<td>5</td>
<td>14.498140</td>
<td>12.082908</td>
<td>40.361942</td>
<td>33.057730</td>
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<tr>
<td>20</td>
<td>14.074982</td>
<td>14.352996</td>
<td>38.054028</td>
<td>33.517994</td>
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</table>

#### Shock to Oil Investment

<table>
<thead>
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<th>Real Oil Price</th>
<th>Non-Oil Investment</th>
<th>Oil-Like Responding Investment</th>
<th>Oil Investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>100.000000</td>
</tr>
<tr>
<td>2</td>
<td>2.946736</td>
<td>0.008345</td>
<td>0.024804</td>
<td>97.027415</td>
</tr>
<tr>
<td>3</td>
<td>3.324780</td>
<td>0.086547</td>
<td>0.045436</td>
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<td>0.12703</td>
<td>0.072152</td>
<td>95.195604</td>
</tr>
</tbody>
</table>

Note: Results come from constant parameter Bayesian VAR with diffuse priors and Cholesky identification scheme.
Appendix 3.D  Details of Theoretical Model

3.D.1 Operational Model Equations

The operational model has to feature as many equations as there are endogenous variables in the theoretical model. Moreover, in the code it has to hold that capital and investment are predetermined variables so that $K_{s,t+1} \Rightarrow K_{s,t}$ and $I_{s,t+1} \Rightarrow I_{s,t}$, for $s = \{no,o\}$. Finally, to ensure stationary, I express the operational model in terms of real variables (e.g., $Z_t/P_t = Z_t$) and sectoral to aggregate price ratios denoted by a hat (e.g, $P_{o,t}/P_t \equiv \hat{P}_{o,t}$).

- **Household**

  $\hat{\lambda}_t = (C_t - bC_{t-1})^{-\sigma} \exp \left( (\sigma - 1) \frac{N_{t+1}^{1+\varphi}}{1+\varphi} \right)$  \hspace{1cm} (1 - Lagrange Multiplier)

  $W_t = (C_t - bC_{t-1}) N_t^{\varphi}$  \hspace{1cm} (2 - Intratemporal Optimality)

  $Q_t = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \Pi_{t+1}^{-1} \right\}$  \hspace{1cm} (3 - Euler Equation)

  $Q_t = \frac{1}{R_t}$  \hspace{1cm} (4 - No Arbitrage)

  $\hat{\lambda}_t Q_{no,t} = \beta \mathbb{E}_t \left[ \hat{\lambda}_{t+1} (Z_{no,t+1} + (1 - \delta) Q_{no,t+1}) \right]$  \hspace{1cm} (5 - Non-Oil Capital FOC)

  $\hat{\lambda}_t Q_{o,t} = \beta \mathbb{E}_t \left[ \hat{\lambda}_{t+1} (Z_{o,t+1} + (1 - \delta) Q_{o,t+1}) \right]$  \hspace{1cm} (6 - Oil Capital FOC)

  $\mathbb{E}_t \left\{ \hat{\lambda}_{t+1} \hat{P}_{no,t+1} \right\} = \mathbb{E}_t \left\{ \hat{\lambda}_{t+1} Q_{no,t+1} \left[ 1 - \frac{\kappa_{no}}{2} \left( \left( \frac{I_{no,t+1}}{I_{no,t}} - 1 \right)^2 + 2 \frac{I_{no,t+1}}{I_{no,t}} \left( \frac{I_{no,t+1}}{I_{no,t}} - 1 \right) \right) \right] \right\}$

  $+ \beta \mathbb{E}_t \left\{ \hat{\lambda}_{t+2} Q_{no,t+2} \kappa_{no} \left( \frac{I_{no,t+2}}{I_{no,t+1}} - 1 \right) \left( \frac{I_{no,t+2}}{I_{no,t+1}} - 1 \right) \right\}$  \hspace{1cm} (7 - Non-Oil Investment FOC)

  $\mathbb{E}_t \left\{ \hat{\lambda}_{t+1} \hat{P}_{o,t+1} \right\} = \mathbb{E}_t \left\{ \hat{\lambda}_{t+1} Q_{o,t+1} \left[ 1 - \frac{\kappa_o}{2} \left( \left( \frac{I_{o,t+1}}{I_{o,t}} - 1 \right)^2 + 2 \frac{I_{o,t+1}}{I_{o,t}} \left( \frac{I_{o,t+1}}{I_{o,t}} - 1 \right) \right) \right] \right\}$

  $+ \beta \mathbb{E}_t \left\{ \hat{\lambda}_{t+2} Q_{o,t+2} \kappa_o \left( \frac{I_{o,t+2}}{I_{o,t+1}} - 1 \right) \left( \frac{I_{o,t+2}}{I_{o,t+1}} - 1 \right) \right\}$  \hspace{1cm} (8 - Oil Investment FOC)

  $C_{no,t} = (1 - \omega_c) \nu_c \left( \hat{P}_{no,t} \right)^{-\nu_c} C_t$  \hspace{1cm} (9 - Non-Oil Goods Consumption)

  $O_{c,t} = \omega_c \nu_c \left( \hat{P}_{o,t} \right)^{-\nu_c} C_t$  \hspace{1cm} (10 - Oil Goods Consumption)
• Non-Oil Firm

\[ Y_{\text{no},t} S_{\text{no},t} = A_{\text{no},t} X_{\text{no},t}^{\alpha_{\text{no}} N_{\text{no},t}^{1-\alpha_{\text{no}}}} \]  

(11 - Non-Oil Production Function)

\[ X_{\text{no},t} = \left[ \omega_k (K_{\text{no},t}) \frac{\nu_{\text{no}} - 1}{\nu_{\text{no}}} + (1 - \omega_k) (O_{\text{no},t}) \frac{\nu_{\text{no}} - 1}{\nu_{\text{no}} - 1} \right] \frac{\nu_{\text{no}}}{\nu_{\text{no}} - 1} \]  

(12 - Non-Oil Production Capital-Oil Bundle)

\[ W_t = (1 - \alpha_{\text{no}}) \hat{P}_{\text{no},t} \frac{Y_{\text{no},t}}{M_{\text{no},t} N_{\text{no},t}} \]  

(13 - Non-Oil Labor Demand)

\[ Z_{\text{no},t} = \alpha_{\text{no}} \omega_k \hat{P}_{\text{no},t} \frac{Y_{\text{no},t}}{M_{\text{no},t} K_{\text{no},t}} \frac{\nu_{\text{no}} - 1}{\nu_{\text{no}}} \]  

(14 - Non-Oil Capital Demand)

\[ \hat{P}_{\text{no},t} = \alpha_{\text{no}} (1 - \omega_k) \hat{P}_{\text{no},t} \frac{Y_{\text{no},t}}{M_{\text{no},t} O_{\text{no},t}} \frac{\nu_{\text{no}} - 1}{\nu_{\text{no}}} \]  

(15 - Non-Oil Intermediate Oil Demand)

\[ \mathcal{M}_{\text{no},t} = \frac{W_t}{(1 - \alpha_{\text{no}}) A_{\text{no},t} \left( \frac{X_{\text{no},t}}{N_{\text{no},t}} \right)^{\alpha_{\text{no}}}} \]  

(16 - Non-Oil Real Marginal Cost)

• Oil Firm

\[ X_{\text{o},t} = \left[ \omega_k (K_{\text{o},t}) \frac{\nu_{\text{o}} - 1}{\nu_{\text{o}}} + (1 - \omega_k) (O_{\text{o},t}) \frac{\nu_{\text{o}} - 1}{\nu_{\text{o}} - 1} \right] \frac{\nu_{\text{o}}}{\nu_{\text{o}} - 1} \]  

(17 - Oil Capital-Labor Bundle)

\[ W_t = (1 - \alpha_{\text{o}}) \hat{P}_{\text{o},t} \frac{X_{\text{o},t}}{N_{\text{o},t}} \]  

(18 - Oil Labor Demand)

\[ Z_{\text{o},t} + F(s_t) = \alpha_{\text{o}} \hat{P}_{\text{o},t} \frac{X_{\text{o},t}}{N_{\text{o},t}} \]  

(19 - Oil Capital Demand)

• Price Setting

\[ \Pi_{\text{no},t} \mathcal{V}_{1,t} = \mathcal{M}_{\text{no}} \mathcal{V}_{2,t} \]  

(20 - Non-Oil Optimal Price Ratio)

\[ \mathcal{V}_{1,t} = Y_{\text{no},t} + \theta \mathbb{E}_t \{ Q_1 \Pi_{\text{no},t+1} \mathcal{V}_{1,t+1} \} \]  

(21 - VI)

\[ \mathcal{V}_{2,t} = Y_{\text{no},t} \mathcal{M}_{\text{no},t} + \theta \mathbb{E}_t \{ Q_1 \Pi_{t+1} \Pi_{\text{no},t+1} \mathcal{V}_{2,t+1} \} \]  

(22 - V2)

\[ 1 = (1 - \theta) \left( \frac{\Pi_{\text{no},t}}{\hat{P}_{\text{no},t}} \right)^{1-\varepsilon} + \theta (\Pi_{\text{no},t})^{\varepsilon - 1} \]  

(23 - Non-Oil Price Inflation)

\[ S_{\text{no},t} = (1 - \theta) \left( \frac{\Pi_{\text{no},t}}{\hat{P}_{\text{no},t}} \right)^{-\varepsilon} + \theta (\Pi_{\text{no},t})^{\varepsilon} S_{\text{no},t-1} \]  

(24 - Non-Oil Price Dispersion)

\[ \Pi_{\text{o},t} = \frac{\hat{P}_{\text{o},t}}{\hat{P}_{\text{o},t-1}} \Pi_t \]  

(25 - LoM Oil Relative Price)
\[ \Pi_t = \left( 1 - \omega_c \right)^{\nu_c} \left( \Pi_{no,t} \hat{P}_{no,t-1} \right)^{1 - \nu_c} \left( \Pi_{o,t} \hat{P}_{o,t-1} \right)^{1 - \nu_c} \]

(26 - Aggregate Price Inflation)

- **Monetary Policy**

\[ \frac{R_t}{R_t} = \left( \frac{R_{t-1}}{\Pi_{no,ss}} \right)^{\rho_R} \left( \frac{\Pi_{no,t}}{\Pi_{no,ss}} \right)^{\phi_{\pi}} \left( \frac{Y_t}{Y_{VA}} \right)^{\phi_y} \left( 1 - \rho_R \right) \Xi_t \]

(27 - Monetary Policy)

- **Market Clearing**

\[ Y_{VA}^V = C_t + I_t \]

(28 - Aggregate Resource Constraint)

\[ Y_{no,t} = C_{no,t} + I_{no,t} \]

(29 - Non-Oil Market Clearing)

\[ O_t = O_{c,t} + O_{no,t} + I_{o,t} \]

(30 - Oil Market Clearing)

\[ N_t = N_{no,t} + N_{o,t} \]

(31 - Aggregate Labor)

\[ K_t = K_{no,t} + K_{o,t} \]

(32 - Aggregate Capital)

\[ I_t = \hat{P}_{no,t} I_{no,t} + \hat{P}_{o,t} I_{o,t} \]

(33 - Aggregate Investment)

- **Capital Accumulation**

\[ K_{no,t+1} = I_{no,t} \left[ 1 - \frac{K_{no}}{2} \left( \frac{I_{no,t}}{I_{no,t-1}} - 1 \right) \right] + (1 - \delta) K_{no,t} \]

(34 - Non-Oil Capital Accumulation)

\[ K_{o,t+1} = I_{o,t} \left[ 1 - \frac{K_{o}}{2} \left( \frac{I_{o,t}}{I_{o,t-1}} - 1 \right) \right] + (1 - \delta) K_{o,t} \]

(35 - Oil Capital Accumulation)

- **Shocks**

\[ \log(A_{no,t}) = (1 - \rho_{A_{no}}) \log(A_{no,ss}) + \rho_{A_{no}} \log(A_{no,t-1}) + \epsilon_{A_{no,t}}^{\Pi}, \quad \epsilon_{A_{no,t}}^{\Pi} \sim \mathcal{N}(0, \sigma_{A_{no}}) \]

(36 - Non-Oil TFP Shock)

\[ \log(A_{o,t}) = (1 - \rho_{A_o}) \log(A_{o,ss}) + \rho_{A_o} \log(A_{o,t-1}) + \epsilon_{A_{o,t}}^{\Pi}, \quad \epsilon_{A_{o,t}}^{\Pi} \sim \mathcal{N}(0, \sigma_{A_o}) \]

(37 - Oil TFP Shock)

\[ \log(\Xi_t) = (1 - \rho_{\Xi}) \log(\Xi_{ss}) + \rho_{\Xi} \log(\Xi_{t-1}) + \epsilon_{\Xi_t}, \quad \epsilon_{\Xi_t} \sim \mathcal{N}(0, \sigma_{\Xi}) \]

(38 - Monetary Policy Shock)

\[ \Delta \log \left( \hat{P}_{o,t} \right) = \rho_{\hat{P}_o}(s_t) \Delta \log \left( \hat{P}_{o,t-1} \right) + \epsilon_{\hat{P}_{o,t}}^{\hat{P}_o}, \quad \epsilon_{\hat{P}_{o,t}}^{\hat{P}_o} \sim \mathcal{N}(0, \sigma_{\hat{P}_o}(s_t)) \]

(39 - Real Oil Price Growth)
3.D.2 Steady State Relationships

I focus on the non-inflationary steady state $\Pi_{no} = 1$. The analytical steady state is available for the case where the oil sector features a Cobb–Douglas production function. I use that steady state as an initial guess for the steady state solver.

\[
\begin{align*}
A_{no} &= 1 \quad \text{(SS.1)} \\
A_o &= 1 \quad \text{(SS.2)} \\
\Xi &= 1 \quad \text{(SS.3)} \\
\Pi_{no} &= 1 \quad \text{(SS.4)} \\
\Pi_o &= 1 \quad \text{(SS.5)} \\
\Pi &= 1 \quad \text{(SS.6)} \\
Q &= \beta \quad \text{(SS.7)} \\
R &= \frac{1}{\beta} \quad \text{(SS.8)} \\
S_{no} &= 1 \quad \text{(SS.9)} \\
\hat{P}_o \text{ and } \hat{P}_{no} \text{ solved numerically} \quad \text{(SS.10, SS.11)} \\
Q_{no} &= \hat{P}_{no} \quad \text{(SS.12)} \\
Q_o &= \hat{P}_o \quad \text{(SS.13)} \\
\Pi^* &= \hat{P}_{no} \quad \text{(SS.14)} \\
\mathcal{MC}_{no} &= \frac{\hat{P}_{no}}{M_{no}} \quad \text{(SS.15)} \\
W &= \left( \frac{(1 - \alpha_o)A_o \left( \frac{\alpha_o}{1 - \alpha_o} \right)^{\alpha_o}}{\left( \frac{1}{\beta} - 1 + \delta \right)^{\alpha_o}} \right)^{\frac{1}{1 - \alpha_o}} \hat{P}_o \quad \text{(SS.16)} \\
Z_{no} &= \left( \frac{1}{\beta} - 1 + \delta \right) Q_{no} \quad \text{(SS.17)} \\
Z_o &= \left( \frac{1}{\beta} - 1 + \delta \right) Q_o \quad \text{(SS.18)} \\
N &= \left( \frac{(1 - b\beta)W}{\phi \psi} \right)^{\frac{1}{\nu - 1}} \quad \text{(SS.19)} \\
\frac{C_{no}}{O_c} &= \left( \frac{1 - \omega_c}{\omega_c} \right)^{\nu_c} \left( \frac{\hat{P}_{no}}{\hat{P}_o} \right)^{-\nu_c} \quad \text{(SS.20)} \\
\frac{O_{no}}{K_{no}} &= \left( \frac{1 - \omega_k Z_{no}}{\omega_k \hat{P}_o} \right)^{\nu_{no}} \quad \text{(SS.21)}
\end{align*}
\]
\[
\frac{X_{no}}{K_{no}} = \left( \omega_k + (1 - \omega_k) \left( \frac{O_{no}}{K_{no}} \right)^{\frac{\nu_{no} - 1}{\nu_{no}}} \right) \left( \frac{\nu_{no} - 1}{\nu_{no}} \right) \quad (SS.22)
\]
\[
\frac{X_{no}}{O_{no}} = \left( \omega_k \left( \frac{K_{no}}{O_{no}} \right)^{\frac{\nu_{no} - 1}{\nu_{no}}} + (1 - \omega_k) \right) \left( \frac{\nu_{no} - 1}{\nu_{no}} \right) \quad (SS.23)
\]
\[
X_k = \frac{\alpha_n \omega_k}{1 - \alpha_n} W \quad (SS.24)
\]
\[
X_o = \frac{\alpha_n (1 - \omega_k)}{1 - \alpha_n} \frac{W}{P_o} \quad (SS.25)
\]
\[
K_o = \frac{\alpha_o}{1 - \alpha_o} \frac{W}{Z_o} \quad (SS.26)
\]
\[
N = \frac{N_{no}}{N bless}\]

\[
N_{no} \text{ solved numerically} \quad (SS.27)
\]
\[
N_o = N - N_{no} \quad (SS.28)
\]
\[
K_o = \frac{K_o}{N_o} N_o \quad (SS.29)
\]
\[
X_{no} = \left( \frac{X_{no}}{K_{no}} \right)^{\frac{1}{\nu_{no}}} X_k \quad (SS.30)
\]
\[
K_{no} = \frac{X_{no}}{K_{no}} \quad (SS.31)
\]
\[
O_{no} = \frac{O_{no}}{K_{no}} K_{no} \quad (SS.32)
\]
\[
Y_{no} = A_{no} X_{no}^{\alpha_{no}} N_{no}^{1 - \alpha_{no}} \quad (SS.33)
\]
\[
O = A_o K_o \quad (SS.34)
\]
\[
I_{no} = \delta K_{no} \quad (SS.35)
\]
\[
I_o = \delta K_o \quad (SS.36)
\]
\[
C_{no} = Y_{no} - I_{no} \quad (SS.37)
\]
\[
O_c = O - O_{no} - I_o \quad (SS.38)
\]
\[
C = \tilde{P}_{no} C_{no} + \tilde{P}_o O_c \quad (SS.39)
\]
\[
I = \tilde{P}_{no} I_{no} + \tilde{P}_o I_o \quad (SS.40)
\]
\[
Y^{VA} = C + I \quad (SS.41)
\]
\[
K = K_{no} + K_o \quad (SS.42)
\]
\[
\nu_1 = \frac{Y_{no}}{1 - \beta \theta} \quad (SS.43)
\]
\[
\nu_2 = \frac{Y_{no} MC_{no}}{1 - \beta \theta} \quad (SS.44)
\]
\[
\hat{\lambda} = (1 - b) C - \psi N^\varphi)^{-\sigma} \quad (SS.45)
\]
• To derive $\hat{P}_o$, $\hat{P}_{no}$ and $W$ note that in the oil sector we have perfect competition, so

$$\hat{P}_o = MC_o = \frac{W^{1-\alpha_o}Z_o^{\alpha_o}}{(1-\alpha_o)A_o \left(\frac{\alpha_o}{1-\alpha_o}\right)^{\alpha_o}} = \frac{W^{1-\alpha_o} \left(\frac{1}{\beta} - 1 + \delta\right)^{\alpha_o} \left(\hat{P}_o\right)^{\alpha_o}}{(1-\alpha_o)A_o \left(\frac{\alpha_o}{1-\alpha_o}\right)^{\alpha_o}}$$

$$\Leftrightarrow W = \left(\frac{(1-\alpha_o)A_o \left(\frac{\alpha_o}{1-\alpha_o}\right)^{\alpha_o}}{\left(\frac{1}{\beta} - 1 + \delta\right)^{\alpha_o}}\right)^{1-\alpha_o} \hat{P}_o$$

Concurrently, we have that $\frac{\hat{P}_{no}}{M_{no}} = MC_{no}$ and so

$$\frac{\hat{P}_{no}}{M_{no}} = \frac{W}{(1-\alpha_{no})A_{no} \left(\frac{X}{N_{no}}\right)^{\alpha_{no}}}$$

$$= \frac{W}{(1-\alpha_{no})A_{no} \left[\frac{X_{no}}{K_{no}} \left(\frac{X_{no}}{K_{no}}\right)^{\nu_{no} - 1} \frac{\nu_{no} - 1}{K_{no} \left(\frac{X_{no}}{K_{no}}\right)^{\nu_{no} - 1} N_{no}}\right]^{\alpha_{no}}}$$

$$= \frac{W}{(1-\alpha_{no})A_{no} \left[\left(\frac{X_{no}}{K_{no}}\right)^{\frac{1}{\nu_{no}}} K_{no} \left(\frac{X_{no}}{K_{no}}\right)^{\frac{\nu_{no} - 1}{\nu_{no}}} N_{no}\right]^{\alpha_{no}}}$$

where $\frac{X_{no}}{K_{no}} = \left(\omega_k + (1-\omega_k) \left(\frac{O_{no}}{K_{no}}\right)^{\nu_{no} - 1}\right)^{\nu_{no}}$ and $\frac{K_{no} \left(\frac{X_{no}}{K_{no}}\right)^{\nu_{no} - 1} N_{no}}{\frac{X_{no}}{K_{no}}} = X_k = \frac{\alpha_{no} \omega_k W}{1-\alpha_{no} Z_{no}}$ and $\frac{O_{no}}{K_{no}} = \left(\frac{1-\omega_k \frac{Z_{no}}{\omega_k}}{\frac{\hat{P}_{no}}{\hat{P}_o}}\right)^{\nu_{no}} = \left(\frac{1-\omega_k \left(\frac{1}{\beta} - 1 + \delta\right)}{\omega_k} \hat{P}_{no}\right)^{\nu_{no}}$}$\frac{1}{\hat{P}_o} - \frac{1}{\hat{P}_{no}}$.

which gives

$$\hat{P}_{no} = \frac{MC_{no} \hat{W}(\hat{P}_o)}{(1-\alpha_{no})A_{no} \left[\left(\omega_k + (1-\omega_k) \left(\frac{1-\omega_k \left(\frac{1}{\beta} - 1 + \delta\right)}{\omega_k} \hat{P}_{no}\right)^{\nu_{no}}\right)^{\nu_{no} - 1} \frac{\nu_{no} - 1}{\alpha_{no} \omega_k W(\hat{P}_o)} \frac{1}{\hat{P}_o} - \frac{1}{\hat{P}_{no}}\hat{P}_{no}\right]}^{1-\alpha_{no}}$$

$$= \frac{\mathcal{M}_{no} \left(\frac{1-\alpha_{no}}{A_{no} \left(\frac{\alpha_{no}}{1-\alpha_{no}}\right)^{\alpha_{no}} \frac{1}{\hat{P}_o} - \frac{1}{\hat{P}_{no}}\hat{P}_{no}\right]}^{1-\alpha_{no}}}{\mathcal{M}_{no} \left(\frac{1-\alpha_{no}}{A_{no} \left(\frac{\alpha_{no}}{1-\alpha_{no}}\right)^{\alpha_{no}} \frac{1}{\hat{P}_o} - \frac{1}{\hat{P}_{no}}\hat{P}_{no}\right]}^{1-\alpha_{no}}$$

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From the definition of the aggregate price index

\[ P = \left[ (1 - \omega_c)\nu_c P_{no}^{1-\nu_c} + \omega_c \nu_c P_o^{1-\nu_c} \right]^{\frac{1}{1-\nu_c}} \]

\[ \Leftrightarrow \hat{P}_{no} = \left( \frac{1 - \omega_c \nu_c \left( \hat{P}_o \right)^{1-\nu_c}}{(1 - \omega_c \nu_c)} \right) \]

(SS.48)

(SS.47) and (SS.48) are two equations in two unknowns \((\hat{P}_{no}, \hat{P}_o)\) which can be solved numerically.

- To derive \(N\) and \(N_{no}\), start from the static labor-leisure choice

\[ \mathcal{W} = (1 - b)CN^\phi \]

\[ \Leftrightarrow N = \left( \frac{\mathcal{W}}{(1 - b)C} \right)^{\frac{1}{\phi}} \]

and obtain \(C\) from

\[ C = \hat{P}_{no} \frac{C_{no}}{N_{no}} N_{no} + \hat{P}_o \frac{O_c}{N_o} (N - N_{no}) \]

where

\[ \frac{C_{no}}{N_{no}} = \frac{Y_{no}}{N_{no}} - \frac{I_{no}}{N_{no}} = A_{no} \left( \frac{X_{no}}{N_{no}} \right)^{\alpha_{no}} - \frac{\delta K_{no}}{N_{no}} \]

where

\[ \frac{X_{no}}{N_{no}} = \left( \frac{X_{no}}{K_{no}} \right)^{\frac{1}{\nu_{no}}} X_k \]

\[ \frac{K_{no}}{N_{no}} = \frac{X_{no}}{K_{no}} \]

and

\[ \frac{O_c}{N_o} = \frac{O}{N_o} - \frac{O_{no} N_{no}}{N_o} N_{no} - \delta \frac{K_o}{N_o} \]

\[ = A_o \left( \frac{K_o}{N_o} \right)^{\alpha_o} - \frac{O_{no} N_{no}}{N_o} N_{no} - \delta \frac{K_o}{N_o} \]

so that

\[ C = \hat{P}_{no} \frac{C_{no}}{N_{no}} N_{no} + \hat{P}_o \left( \frac{O}{N_o} - \delta \frac{K_o}{N_o} \right) (N - N_{no}) - \hat{P}_o \frac{O_{no}}{N_{no}} N_{no} \]
and

\[
N = \left\{ \frac{W}{(1 - b) \left[ \hat{P}_n \frac{C_n}{N_n} N_n + \hat{P}_o \left( \frac{O}{N_o} - \delta \frac{K_o}{N_o} \right) (N - N_no) - \hat{P}_o \frac{O_{no}}{N_no} N_no} \right]^{\frac{1}{\nu_n}}} \right\}
\]

- To obtain non-oil sectoral labor \( N_{no} \), use the market clearing conditions

\[
C_{no} = Y_{no} - \delta K_{no}
\]

\[
O_c = O - O_{no} - \delta K_o
\]

which can be rewritten as

\[
C_{no} = A_{no} \left[ \frac{X_{no}}{K_{no}} \right]^{\frac{1}{\nu_{no}}} \alpha_{no} N_{no} - \delta \left[ \frac{K_{no}}{X_{no}} \right]^{\frac{1}{\nu_{no}}} \alpha_{no} N_{no} - \delta \left[ \frac{K_{no}}{X_{no}} \right]^{\frac{1}{\nu_{no}}} \alpha_{no} N_{no}
\]

Next, we have

\[
O_c = A_o \left( \frac{K_o}{N_o} \right)^{\alpha_o} (N - N_{no}) - \frac{O_{no}}{N_no} N_{no} - \delta \frac{K_o}{N_o} (N - N_{no})
\]

\[
= A_o \left( \frac{K_o}{N_o} \right)^{\alpha_o} (N - N_{no}) - \left[ \frac{O_{no}}{X_{no}} \right]^{\frac{1}{\nu_{no} - 1}} \frac{O_{no}}{N_no} \left( \frac{X_{no}}{O_{no}} \right)^{\frac{\nu_{no} - 1}{\nu_{no}}} N_{no} - \delta \frac{K_o}{N_o} (N - N_{no})
\]

where

\[
\frac{O_{no}}{X_{no}} \left( \frac{X_{no}}{K_{no}} \right)^{\frac{\nu_{no} - 1}{\nu_{no}}} N_{no} = X_{no} = \frac{\alpha_{no}(1 - \omega_k) W}{1 - \alpha_{no}} \frac{1}{\hat{P}_o}
\]

\[
\frac{X}{O_{no}} = \left( \omega_k \left( \frac{K_{no}}{O_{no}} \right)^{\frac{\nu_{no} - 1}{\nu_{no}}} + (1 - \omega_k) \right)
\]

We divide the equations for the two consumption goods to obtain an equation for
\[ N_{no} \]

\[
\frac{C_{no}}{O_c} = \frac{A_{no} \left( \frac{X}{K_{no}} \right)^{\frac{1}{\nu_{no}}} \mathcal{X}_k \right)^{\alpha_{no}} N_{no} - \delta \left( \frac{K_{no}}{X} \right)^{\frac{\nu_{no} - 1}{\nu_{no}}} \mathcal{X}_k \right)^{\alpha_{no}} N_{no} - \delta \frac{K_{o}}{N_{o}} (N - N_{no})
\]

\[
\frac{C_{no}}{O_c} \left\{ A_{o} \left( \frac{K_{o}}{N_{o}} \right)^{\alpha_{o}} N - A_{o} \left( \frac{K_{o}}{N_{o}} \right)^{\alpha_{o}} N_{no} - \left( \frac{O_{no}}{X} \right)^{\frac{\nu_{no} - 1}{\nu_{no}}} \mathcal{X}_o \right)^{\alpha_{no}} N_{no} - \delta \frac{K_{o}}{N_{o}} (N - N_{no}) \right\}
\]

\[
= A_{no} \left[ \left( \frac{X}{K_{no}} \right)^{\frac{1}{\nu_{no}}} \mathcal{X}_k \right)^{\alpha_{no}} N_{no} - \delta \left( \frac{K_{no}}{X} \right)^{\frac{\nu_{no} - 1}{\nu_{no}}} \mathcal{X}_k \right)^{\alpha_{no}} N_{no}
\]

\[
\frac{C_{no}}{O_c} \left\{ \delta \frac{K_{o}}{N_{o}} N_{no} - A_{o} \left( \frac{K_{o}}{N_{o}} \right)^{\alpha_{o}} N_{no} - \left( \frac{O_{no}}{X} \right)^{\frac{\nu_{no} - 1}{\nu_{no}}} \mathcal{X}_o \right)^{\alpha_{no}} N_{no} \right\} - A_{no} \left[ \left( \frac{X}{K_{no}} \right)^{\frac{1}{\nu_{no}}} \mathcal{X}_k \right)^{\alpha_{no}} N_{no}
\]

\[
+ \delta \left( \frac{K_{no}}{X} \right)^{\frac{\nu_{no} - 1}{\nu_{no}}} \mathcal{X}_k \right)^{\alpha_{no}} N_{no} = C_{no} \left( A_{o} \left( \frac{K_{o}}{N_{o}} \right)^{\alpha_{o}} N - \delta \frac{K_{o}}{N_{o}} N \right)
\]

Equations (SS.49) and (SS.49) are two equations in two unknowns \((N\) and \(N_{no}\)) which can be solved numerically.