



# EUI WORKING PAPERS IN ECONOMICS

EUI Working Paper ECO No. 95/32

**Indeterminacy and Welfare Increasing Taxes  
in a Growth Model with Elastic Labour Supply**

ALESSANDRA PELLONI  
and  
ROBERT WALDMANN

© The Author(s). European University Institute.  
Digitised version produced by the EUI Library in 2020. Available Open Access on Cadmus, European University Institute Research Repository.

WP  
330  
EUR

European University Institute, Florence



EUROPEAN UNIVERSITY INSTITUTE



3 0001 0021 9345 8



**EUROPEAN UNIVERSITY INSTITUTE, FLORENCE**

**ECONOMICS DEPARTMENT**

**EUI Working Paper ECO No. 95/32**

**Indeterminacy and Welfare Increasing Taxes  
in a Growth Model with Elastic Labour Supply**

**ALESSANDRA PELLONI  
and  
ROBERT WALDMANN**

**WP 330  
EUR**

**BADIA FIESOLANA, SAN DOMENICO (FI)**

**All rights reserved.  
No part of this paper may be reproduced in any form  
without permission of the authors.**

**© Alessandra Pelloni and Robert Waldmann  
Printed in Italy in September 1995  
European University Institute  
Badia Fiesolana  
I - 50016 San Domenico (FI)  
Italy**



# Indeterminacy and Welfare Increasing Taxes in a Growth Model with Elastic Labour Supply \*

Alessandra Pelloni and Robert Waldmann  
European University Institute

July 1995

## Abstract

The inclusion of a labour/leisure choice in endogenous growth models has interesting and somewhat counter-intuitive effects. For some parameter values the economy is a stable dynamical system so the market equilibrium is indeterminate. Odd effects of policy appear. The long-run growth rate is increased by lump sum taxes used to fund wasteful government spending. Again for parameter values, such policy can increase the representative consumer's welfare.

---

\*We would like to thank Michele Boldrin, Roger Farmer, Akos Valentinyi and the participants in the 1995 Annual Meeting of the Society for Economic Dynamics and Control in Barcelona for helpful suggestions. The usual disclaimer applies. This work was partly financed by the Research Council of the European University Institute



# 1 Introduction

In this paper we study and draw some policy implications of a simple model of endogenous growth, where agents elastically supply labour. A first issue we address is if the market situation can be indeterminate in the model. We define indeterminate a situation in which there exists a whole interval of different equilibrium paths sharing the same initial condition.

Recently a number of papers has appeared that study the transitional dynamics in models of unbounded accumulation and focus in particular on the problem of indeterminacy of equilibria. One distinguishes between local indeterminacy that is the existence of a continuum of equilibria in a neighborhood of a balanced growth path and global indeterminacy that arises in models with multiple growth paths, the selection between which depends on control variables. We also have models with multiple balanced growth paths that do not exhibit indeterminacy i.e. whose asymptotic equilibrium is known when initial conditions are given.<sup>1</sup>

Formally for an equilibrium solution to be locally unique, i.e. for a balanced growth path to be determinate, the Jacobian matrix of the system evaluated at the balanced growth path must not have more eigenvalues with a strictly negative real part than there are state variables.

As is well known stable steady states are associated with a continuum of rational expectations equilibria driven by self-fulfilling beliefs. These are dynamic examples of what Shell (1977) labeled "sunspot equilibria" and have welfare properties much different from equilibria that depend only on fundamentals. In a growth framework the existence of a continuum of equilibria means that countries with identical tastes technology and initial endowments will converge to the same constant growth rate only in the very long run—while the level of income will remain different for ever. This seems an explanation deserving investigation for the differences in the growth patterns of the various countries observed in reality. The models suggest the possibility that appropriate policy interventions will in fact raise welfare, acting as selection devices for equilibria that differ during the transition interval. In fact cultural and institutional features in general may be seen as playing this same role.

An extremely simple example of indeterminacy in a growth model is offered by Benhabib and Rustichini (1994). They assume utility logarithmic over

---

<sup>1</sup>A leading example is the model by Azariadis and Drazen (1990) featuring thresholds externalities in human capital accumulation.



consumption while knowledge is the only capital good. In the social production function of knowledge there are constant returns to knowledge and strictly increasing returns to labour. As is known increasing returns can be accommodated in an equilibrium framework either through externalities or through monopolistic competition, the first way being often preferred, for its simplicity. This model has the additional controversial feature of an upward sloping labour demand. Although it is extremely difficult to provide intuitive explanations for the behaviour of dynamical systems, the following reasoning, although incomplete, may be useful to understand the key mechanism at work in producing the indeterminacy result in this model. Imagine starting from an equilibrium path and building another one by increasing consumption i.e. reducing the fraction of labour devoted to accumulate knowledge. To avoid violating a non-negativity constraint or the transversality condition it is necessary for the rate of growth of consumption to fall even more than the rate of growth of knowledge so the economy eventually returns to the balanced growth path. To begin with, this is possible, in equilibrium, only if the intertemporal elasticity of substitution of consumption is not too low. A further necessary condition for this decrease in the growing rate of consumption to be (privately) optimal is that the rate of return on capital decreases enough. The assumption of increasing returns to labour is necessary for this to happen in this model. Benhabib and Rustichini also consider the same model with leisure entering the utility function linearly. This makes for indeterminacy since now the reduction in the rate of capital accumulation can be brought about by a reduction in labour supply, which reduces the rate of return on capital. Benhabib and Farmer (1994) further the analysis of this issue studying a model in which utility is additively separable over the logarithm of consumption and a power function of labour supply. They assume an increasing returns Cobb-Douglas technology using labour and capital. They study two versions of the model: in the first the marginal productivity of capital is decreasing, so, asymptotically, the economy reaches a stationary state. In the second the social production function is linear in capital so the mode

l displays endogenous growth. For both versions a necessary condition for indeterminacy is that the exponent of labour in the social production function is greater than one, the difference between the two being positively related to the (absolute value) of the inverse of the labour supply elasticity.

Coming to investigations of the transitional dynamics in two capital goods models with increasing returns there are the studies, conducted with different methods, by Mulligan and Sala-i-Martin (1992), Chamley (1993), Caballé and Santos (1993), Benhabib and Perli (1994), Xie (1994) and Ladrón-de Guevara,

Ortigueira and Santos (1994).

The presence of a second capital good e.g. human capital makes for indeterminacy because, again starting from equilibrium, the reduction in the marginal productivity of physical capital necessary to justify the slowing down in its accumulation and in the rate of growth of consumption may be brought about by a simultaneous deceleration in the accumulation of human capital. As regards the Lucas (1988) model without externalities Caballé and Santos notice however that stability can be ruled out by the strict concavity of the optimization problem. In fact otherwise there would be a continuum of initial values of the controls converging optimally to the same steady state but then convex combinations of those would yield higher welfare.

Mulligan and Sala-i-Martin (1992) calibrate the same model and report that computer experimentation has not found any balanced growth equilibrium which does not display saddle-path stability, even with mild externalities to human capital in the production of goods. Benhabib and Perli qualify this result showing that necessary conditions for local indeterminacy are i) the time discount factor is higher than the sum of the rate of population growth and of the parameter measuring the effectiveness of investment in human capital and ii) the intertemporal elasticity of substitution of consumption is greater than one by a difference inversely related to the size of the externality parameter. Finally a sufficient condition is that there are strictly increasing returns to human capital. Xie studies the same model under the restriction that the inverse of the elasticity of substitution of consumption is equal to the share of capital. This allows one to explicitly solve for the time paths of the variables and to show that, under increasing returns to human capital, i.e. if there is a continuum of equilibria, indexed by the initial fraction of non-leisure time devoted to goods production, a country with lower endowments of both kinds of capitals can overtake a country with greater endowments, if the investment in human capital is high enough. He also proves that an equilibrium associated with a lower initial fraction of non-leisure time devoted to goods production always gives higher utility. Chamley modifies the Lucas model assuming no externalities in the production of goods and assuming instead a spillover effect to the fraction of time devoted to "learning", i.e. to accumulating human capital, as opposed to "doing". He notices that if the necessary and sufficient condition for uniqueness of equilibrium he provides is violated indeterminacy may arise. This condition is formulated in terms of the intertemporal elasticity of substitution of consumption— a higher value of which is conducive to stability—the first and second derivatives of the learning function with respect to the individual and av-



erage time devoted to learning, and the learning time itself, all measured on the balanced growth path. The more concave the learning function the less likely stability, a necessary condition for which is the presence of the spill-over effect mentioned above. Benhabib and Perli also numerically study a version of the Lucas model with preferences specified as in Benhabib and Farmer. They also adopt more general Cobb-Douglas specifications for the production functions in the two sectors allowing the exponents of labour quality (human capital) to be different from those of labour quantity. Finally they consider external effects to human capital in both sectors and an external effect to labour in the education sector. In fact the exponent of labour in the state equation for human capital turns out to be the critical element in the analysis: if it is less than one there is only one, determinate, balanced growth path; if it is bigger than one there are either none or two balanced growth paths, the one with lower steady state growth being always determinate and the other indeterminate if the intertemporal elasticity of substitution of labour is high enough. In fact there is a tradeoff between the size of the exponent of labour in the state equation for human capital, capturing the external effect of labour and the size of the intertemporal elasticity of substitution of labour in the utility function, so stability obtains even if the latter is very low (i.e.-100) provided the former is very high (i.e. 0.6). However combinations of the two consistent with stability deliver unplausibly low rates of growth. So the basic result of the model is that strictly increasing returns to labour in the production of human capital deliver global indeterminacy in the sense that the initial endowments of the two capital stocks are not sufficient to select one optimal orbit.

To sum up it seems that what all these models have in common is that they find that a condition for indeterminacy is that there are increasing returns to labour and/or human capital in the production of at least one factor that can be accumulated whether it is physical capital, human capital or knowledge.<sup>2</sup> In this paper we are however able to present an example of indeterminacy in an economy with decreasing returns to labour and constant returns to capital. We show this in a simple one sector model where consumption and leisure are

---

<sup>2</sup>Ladron-de-Guevara et al.(1994) show that even without externalities it is possible to have a multiplicity of steady states with an additively separable utility function logarithmic in consumption and CES in leisure, in a two capital good model. In the only numerical example, built assuming a Cobb-Douglas technology, they study as regards the transition properties of the steady states it turns out that there are three (optimal) steady states, one unstable and two saddle point stable, and that which of these two is relevant as an attractor is determined by the initial endowments of human and physical capital. So theirs is not a sunspots model.



both highly intertemporally substitutable while the factors of production are highly complementary. A heuristic explanation is the following: starting from an optimal path if one attempts to construct a new equilibrium by increasing consumption then if the elasticity of substitution of consumption is more than one leisure will increase as well. If the elasticity of substitution between labour and capital is low the reduction in labour supply will cause a sharp decline in the interest rate. But with a high elasticity of substitution of consumption this can cause an even sharper decline in the rate of growth of consumption that could decline even more than the rate of growth of capital. Thus the economy would return to the balanced growth path.

We also consider some implications of the model for tax policy. In endogenous growth models, since policies have the potential to influence the growth rate in the long run, there is generally a much larger quantitative influence of policies on welfare than in the neoclassical model, where the growth rate is governed asymptotically by the exogenous rate of technical progress. Recent papers by Jones and Manuelli (1990), Lucas (1990), King and Rebelo (1990), Rebelo (1991), Jones, Manuelli and Rossi (1993), Stokey and Rebelo (1993), Roubini and Milesi-Ferretti (1994) and others have used endogenous growth models to look at both the positive and normative aspects of taxation. These authors use a similar basic framework, assuming that there is a "core" of capital goods which can be produced without the the direct or indirect contribution of non-reproducible factors. Indeed most analyses follow Lucas (1988) in permitting labour input to be reproducible. The Lucas model is variously extended for instance to allow for labour supply being elastic and/or for a commodity input entering the accumulation of human capital. In general externalities are ruled out so that the competitive and optimal allocations coincide in the absence of public interventions. These works focus in particular on flat-rate taxes, in particular on capital and labour income, whose proceeds are assumed to be rebated as lump-sum transfers. They try to determine how changes in the tax structure affect the division of production between consumption and investment, the division of time between leisure and income-directed activities and the division of income directed time between the production of goods and the accumulation of human capital.<sup>3</sup> It is generally recognized in the literature that if the social rate of return on investment exceeds the private return, subsidies to investment can raise the growth rate and levels of utility. However it has not been noticed

---

<sup>3</sup>Quantitative assessments of the size of the effects of tax reform towards the optimal tax structure on welfare and growth are then offered calibrating the model to U.S. data. In fact these quantitative conclusions differ widely.

that a lump-sum tax may in itself stimulate growth increasing welfare. In fact lump sum taxes have an impact on the allocation of resources because due to income effects they influence labour supply and therefore the rate of return on capital i.e. the rate of growth. The standard practice of evaluating all other taxes by assuming revenues are returned as a lump sum transfer is misleading in this case. In particular in our model a lump sum tax will increase growth, by making people work more even if the tax revenue is thrown away instead of being used to subsidize investment. Moreover we show that, for some parameter values, this can increase the representative consumer's welfare.

The paper has four sections the first of which is this introduction. The second presents a model of unbounded accumulation with leisure in the utility function. The third deals with tax policy. The fourth draws conclusions.

## 2 A Model

In this section we present a simple endogenous growth model with variable labour supply. All worker/consumers are identical and maximize the same CES intertemporal utility function, multiplicatively separable in consumption of the homogeneous good  $C$  and leisure  $l$ :

$$V = \int_0^{\infty} e^{-\rho t} \frac{C^{1-\sigma}}{1-\sigma} l^{1-\chi} dt \quad (1)$$

where either  $\sigma < 1, \chi < 1, \sigma + \chi > 1$  or  $\sigma > 1, \chi > 1$ . These restrictions insure strict quasi-concavity of the objective function. As is standard from Romer (1986) we assume that there is a continuum of competitive firms and that the production set at the firm level is convex in labour  $L$  and capital  $K$  but that average capital  $\bar{K}$  causes a labour augmenting spill-over which is taken as given by each firm, so that the social production function is linear in capital. With population normalized to one production  $Y$  is then given by equation 2

$$Y = F(L\bar{K}, K) = KF(L, 1) \equiv Kf(L) \quad (2)$$

$L = 1 - l$  is labour supplied. Equation 2 gives the instantaneous budget constraint

$$\frac{\dot{K}}{K} = \frac{Y - C}{K} = f(L) - \frac{C}{K} \quad (3)$$

The static first order condition for the choice between labour and leisure is equation 4

$$\frac{(1-L)(1-\sigma)}{C(1-\chi)} = \frac{1}{W} \quad (4)$$

The consumers consumption savings choice implies the Euler equation 5 along with a transversality condition that the present value at time zero of the capital stock at time  $t$  goes to zero.

$$\sigma \frac{\dot{C}}{C} + (1-\chi) \frac{\dot{L}}{1-L} = r - \rho \quad (5)$$

Plugging the time derivative of the log of equation 4 into equation 5 gives equation 6

$$(-1 + \chi + \sigma) \frac{\dot{C}}{C} + (1-\chi) \frac{\dot{W}}{W} = r - \rho = \sigma \frac{\dot{W}}{W} + (1-\sigma-\chi) \frac{\dot{L}}{1-L} \quad (6)$$

Profit maximization by firms and perfect competition give the wage and real interest rate.

$$W = Kf'(L) \quad (7)$$

and

$$r = F_2(L\bar{K}, K) = F_2(L, 1) \quad (8)$$

Note that equation 7 and equation 4 together imply that the ratio of consumption to capital is a monotonically decreasing function of labour supply.

Equation 7 implies that the rate of growth of wages is described by equation 9

$$\frac{\dot{W}}{W} = \frac{\dot{K}}{K} + \frac{f''(L)\dot{L}}{f'(L)} \quad (9)$$

The instantaneous budget constraint 3, the first order condition for consumption and leisure 4 and the wage 7 imply equation 10:

$$\frac{\dot{K}}{K} = f(L) - \frac{(1-L)(1-\sigma)}{(1-\chi)} f'(L) \quad (10)$$

Which shows that the rate of growth of capital is a monotonically increasing function of labour supply. This means that if and only if labour supply  $L$  is fixed then the growth rate of capital and, as noted above, the consumption capital ratio are fixed and the economy experiences balanced growth. Define  $\tilde{L}$  as a (not necessarily unique) balanced growth labour supply.



Combining equations 6, 8, 9 and 10 we finally obtain equation 11 the law of motion of labour supply which relates the time derivative of  $L$  to  $L$ .

$$\left(-\sigma \frac{f''(L)}{f'(L)} + \frac{\sigma + \chi - 1}{(1-L)}\right) \dot{L} = \rho - r + \sigma \left(f(L) - \frac{(1-L)(1-\sigma)}{(1-\chi)} f'(L)\right) \quad (11)$$

This is not a simple or elegant equation, but it shows how to choose  $\rho$  to make any labour supply  $\underline{L}$  the balanced growth labour supply. With  $dL/dt$  equal to zero equation 11 becomes an equation which gives  $\rho$  as a function of  $\underline{L}$ . Furthermore note that this  $\rho$  is positive if  $\underline{L}$  is less than  $\frac{1-\sigma}{2-\chi-\sigma}$  and  $\sigma < 1$ . Also notice that if  $\underline{L}$  were equal or larger than  $\frac{1-\sigma}{2-\chi-\sigma}$  the term inside the parenthesis in the right-hand side of 11 would be equal or larger than the interest rate. But this would violate the transversality condition  $\rho > g(1-\sigma)$  where  $g \equiv (r - \rho) / \sigma$  indicates the asymptotic rate of growth. The condition can be then rewritten

$$r(\tilde{L}) > g(\tilde{L}) \quad (12)$$

and

$$\rho > r(\tilde{L})(1-\sigma) \quad (13)$$

Equation 11 is also useful when studying the dynamics of the economy near balanced growth labour supply  $\tilde{L}$ . It is fairly easy to determine the sign of  $d\dot{L}/dL$ . This is interesting, since if, for  $L$  just below  $\tilde{L}$ ,  $L$  is increasing and vice versa then the balanced growth path is dynamically stable. This would imply that the market equilibrium is not determined by initial state (capital stock) and that the market outcome could be affected by a sunspot.

To decide if the balanced growth path is stable and indeterminate first note that for all  $L$  the left hand side is a positive multiple of the time derivative of  $L$ . Call this multiple  $a(L)$ . At the balanced growth labour supply  $\tilde{L}$  the derivative  $a'(\tilde{L})$  does not affect the total derivative of equation 11 since it is multiplied by zero. This means that the effect of  $L$  on the time derivative of  $L$  is described by statement 14

$$\frac{d\dot{L}(\tilde{L})}{dL} \text{ signs as } -\frac{dr(\tilde{L})}{dL} + \sigma \left( f'(\tilde{L}) \frac{(2-\sigma-\chi)}{(1-\chi)} - \frac{(1-\tilde{L})(1-\sigma)}{(1-\chi)} f''(\tilde{L}) \right) \quad (14)$$

If the right hand side of statement 14 is negative then the balanced growth path is stable and indeterminate. That is the balanced growth path is stable and indeterminate if inequality 15 holds

$$-\frac{dr(\tilde{L})}{dL} + \sigma \left( f'(\tilde{L}) \frac{(2-\sigma-\chi)}{(1-\chi)} - \frac{(1-\tilde{L})(1-\sigma)}{(1-\chi)} f''(\tilde{L}) \right) < 0 \quad (15)$$

It is fairly clear that inequality 15 holds for some production functions and for some parameter values. It is necessary that the elasticity of substitution of capital and labour is very low so the wage falls sharply and the interest rate rises sharply as labour supply increases. It is also necessary that little leisure is consumed so that the final term in statement 15 is small. Finally it is necessary that  $\sigma$  is small that is that marginal utility of consumption declines only slowly as consumption increases.

It is not at all evident that all of these conditions can be met by an economically meaningful solution to the differential equation that is by a solution which satisfies the transversality condition. However that it is possible for inequalities 13 and 15 to hold as is shown below by giving a specific example of a stable balanced growth path which satisfies the transversality condition.

We consider a CES production function with elasticity of substitution of  $\frac{1}{1-\phi}$ .  $\phi$  must be less than one if the single firm is to face a concave programming problem. So we have:

$$Y = [\alpha(\bar{K}L)^\phi + (1-\alpha)K^\phi]^\frac{1}{\phi} \quad (16)$$

This implies equations 17, 18, 19 and 20.

$$W = f'(L)K = \alpha L^{\phi-1}K [\alpha L^\phi + 1 - \alpha]^\frac{1}{\phi}-1 \quad (17)$$

$$\frac{dW}{KdL} = f''(L) = \alpha(1-\alpha)(\phi-1)L^{\phi-2} [\alpha L^\phi + 1 - \alpha]^{-2+\frac{1}{\phi}} \quad (18)$$

$$r = (1-\alpha) [\alpha L^\phi + 1 - \alpha]^\frac{1}{\phi}-1 \quad (19)$$

$$\frac{dr}{dL} = \alpha(1-\alpha)(1-\phi)L^{\phi-1} [\alpha L^\phi + 1 - \alpha]^\frac{1}{\phi}-2 \quad (20)$$

Equation 11 now reads like

$$\dot{L} = \frac{\rho - [\alpha L^{-\phi} + 1 - \alpha]^\frac{1}{\phi}-1 \left[ (1-\alpha)(1-\sigma)L^{1-\phi} - \alpha\sigma\frac{(2-\sigma-\chi)}{(1-\chi)}L + \alpha\sigma\frac{(1-\sigma)}{(1-\chi)} \right]}{\frac{\sigma+\chi-1}{1-L} + \frac{(1-\phi)\sigma(1-\alpha)}{L[\alpha L^\phi + 1 - \alpha]}} \quad (21)$$

Inequality 15 becomes inequality 22

$$0 > \sigma \frac{(2-\sigma-\chi)}{(1-\chi)} [\alpha \tilde{L}^\phi + 1 - \alpha] + (1-\phi)(1-\alpha) \left[ \sigma \frac{(1-\tilde{L})(1-\sigma)}{(1-\chi)\tilde{L}} - 1 \right] \quad (22)$$

We study the case of  $\sigma < 1$ ,  $\phi < 0$ . Consider the partial derivative of the expression in 22 with respect to  $\tilde{L}$  (the balanced growth labour supply), calculated keeping  $\alpha, \sigma, \chi, \phi$  constant

$$\sigma \frac{(2-\sigma-\chi)}{(1-\chi)} \phi \alpha \tilde{L}^{\phi-1} - (1-\phi)(1-\alpha)\sigma \frac{(1-\sigma)}{(1-\chi)\tilde{L}^2} \quad (23)$$

This is always negative for  $\phi < 0$ . So we want to choose  $\rho$  in such a way as to make  $\tilde{L}$  as big as possible. Recall that  $\tilde{L}$  can take any positive value less than  $\frac{1-\sigma}{2-\chi-\sigma}$  for some positive  $\rho$ , if  $\sigma$  is less than one. As  $\tilde{L}$  goes to  $\frac{(1-\sigma)}{(2-\chi-\sigma)}$  from below inequality 22 becomes inequality 24:

$$0 > \frac{\alpha\sigma(2-\sigma-\chi)}{1-\chi} \left( \frac{1-\sigma}{2-\chi-\sigma} \right)^\phi + (1-\alpha) \left[ (1-\phi)(\sigma-1) + \frac{\sigma(2-\sigma-\chi)}{1-\chi} \right] \quad (24)$$

This inequality can hold for negative values of  $\phi$ , for some utility functions. For instance as  $\chi$  goes to  $1-\sigma$  and  $\sigma$  goes to zero, it holds if  $\phi$  is less than  $-\frac{\alpha}{1-\alpha}$ .

This shows how it is possible to construct a model with a stable indeterminate balanced growth path, utility a time separable Cobb-Douglas function of consumption and leisure and production given by a CES production function of capital and labour augmented by a spillover.

We now show that the conditions  $\sigma < 1, \phi < 0$  are in fact necessary for stability. First we prove that, no matter what the values of the other parameters are,  $\phi \geq 0$  precludes stability. In fact

$$\sigma \frac{(2-\sigma-\chi)}{(1-\chi)} \left[ \alpha \tilde{L}^\phi + 1 - \alpha \right] + (1-\phi)(1-\alpha) \left[ \sigma \frac{(1-\tilde{L})(1-\sigma)}{(1-\chi)\tilde{L}} - 1 \right] \geq \sigma \frac{(2-\sigma-\chi)}{(1-\chi)} (1-\alpha) - (1-\phi)(1-\alpha) \quad (25)$$

since the suppressed terms are both positive. But the expression on the right-hand side of inequality 25 is always positive for  $\phi \geq 0$ , since  $\sigma \frac{(2-\sigma-\chi)}{(1-\chi)} > 1$ .<sup>4</sup> So we need only to examine the case in which  $\sigma > 1, \phi < 0$ . With a CES production function  $\tilde{L}$  is implicitly defined by

$$(1-\alpha)(1-\sigma)\tilde{L}^{1-\phi} - \alpha\sigma \frac{(2-\sigma-\chi)}{(1-\chi)} \tilde{L} + \alpha\sigma \frac{(1-\sigma)}{(1-\chi)} = \rho \left[ \alpha + (1-\alpha)\tilde{L}^{-\phi} \right]^{-\frac{1}{\phi}+1} \quad (26)$$

Notice that the derivative with respect to  $\tilde{L}$  of the left hand side is always negative while the derivative with respect to  $\tilde{L}$  of the right hand side is always positive for  $\sigma > 1, \phi < 0$ . So the two curves intersect only once— $\tilde{L}$  is unique.

<sup>4</sup>In fact  $\sigma \frac{(2-\sigma-\chi)}{(1-\chi)} > 1$  implies  $\chi > \frac{\sigma(2-\sigma)-1}{\sigma-1} = 1-\sigma$ , which is always true.



We also have that the higher is  $\rho$  the lower is  $\tilde{L}$ . So we consider  $\tilde{L}$  when  $\rho = 0$ , i.e. the solution to

$$\tilde{L} = \frac{\alpha\sigma^{\frac{(1-\sigma)}{(1-\chi)}}}{\alpha\sigma^{\frac{(2-\sigma-\chi)}{(1-\chi)}} + (1-\alpha)(1-\sigma)\tilde{L}^{-\phi}} \quad (27)$$

The second term in the denominator is increasing in  $\phi$  so that as  $\phi$  goes to minus infinity  $\tilde{L}$  goes to its upper bound that is again  $\frac{1-\sigma}{2-\chi-\sigma}$ . But inequality 37 cannot hold if  $\sigma$  is more than one. This completes the proof. Notice that since as is clear from equation 21 the time derivative of labour supply is a continuous function of labour supply, for  $0 < L < 1$ , there cannot be multiple balanced growth paths all locally unstable. So given our proof that the restrictions  $\phi > 0$  and/or  $\sigma > 1$  are sufficient to rule out local indeterminacy we know that these restrictions rule out global indeterminacy as well.

In conclusion we have shown that a model with one good produced under constant returns to capital and decreasing returns to labour, i.e. featuring a downward sloping labour demand can have a stable balanced growth path and that for indeterminacy we need a low elasticity of substitution of labour and capital and an intertemporal elasticity of consumption less than one i.e. that the marginal utility of consumption declines slowly with consumption.

Stability means that if the economy is near balanced growth and labour supply drops slightly while consumption increases slightly then labour supply returns to its balanced growth level and the consumption capital ratio returns to its balanced growth level.

Such stability implies indeterminacy. The transversality condition may imply that the economy converges to a balanced growth path, but this does not determine either the current consumption level or the current labour supply. Instead consumption and labour supply can take a range of values each consistent with utility maximization and perfect foresight. As is well known this in turn means that economically irrelevant publicly observable variables (sunspots) can effect the economy even if all agents are rational utility maximizers with rational expectations. This is of some interest especially since the effects of a sunspot are permanent. The long run expected value of labour supply and of the consumption capital ratio are not affected by the sunspot, but the level of consumption and the capital stock are affected.

A sunspot can lead rational agents to supply less labour and to consume more. This will lower their wealth at every future time and the difference between their wealth at each time and what it would have been were there no

such sunspot will never diminish. A low level of labour supply implies a low interest rate and high consumption capital ratio. This means that the capital stock grows slowly. Eventually the growth rate of capital will converge to the higher rates which would have occurred without the sunspot, but the growth rates can never cross. This means that differences in the capital stock at each time will widen more and more slowly but never contract.

### 3 Growth Maximizing Public Policy

The decentralized solution to the model described above is not optimal, since firms and investors ignore the positive externality caused by investment. Since all citizen-worker-consumers are identical optimal lump sum taxation is feasible. A government could tax its citizens a lump sum according to a deterministic function of time which would exactly equal the revenues it wished. If tax revenues are not returned to consumers, the reduction in their wealth causes them to supply more labour which increases the interest rate and the growth rate. This leads to a rather counterintuitive point. Suppose the government is unable to observe who works and who owns capital so that it is not capable to subsidize investment or work. Instead the tax revenue is used to finance the provision of goods that do not affect the marginal utility of private consumption or the production possibilities of the private sector. This assumption is useful to isolate the effects of taxation from those of government expenditure. It turns out that the growth maximizing policy is to fund pure waste via a poll tax.

Consider the model modified to allow government consumption (assumed to be pure waste) and with a Cobb-Douglas production function—i.e. with  $\phi$  equal to zero— so the instantaneous budget constraint becomes equation 28

$$\dot{K} = Y - C - G = L^\alpha K^{1-\alpha} \bar{K}^\alpha - C - G \quad (28)$$

The government chooses  $G$  or, to be more exact, the government taxes each citizen in the population (normalized to one)  $G$  and throws the proceeds away. We consider simple policies of the form

$$G = \tau \bar{K} = \tau K \quad (29)$$

where  $\tau$  is a constant and  $\bar{K}$  is used to remind the reader that the taxes are considered to be lump sum by each taxpayer.

In balanced growth, as above, the growth rate of consumption and of the capital stock are the same and so the instantaneous budget constraint implies equation 30

$$\tau = \tilde{L}^\alpha - \tilde{c} - \frac{(1-\alpha)\tilde{L}^\alpha - \rho}{\sigma} \quad (30)$$

Where  $\tilde{c}$  is the balanced growth consumption capital ratio. Equation 30 has total derivative equation 31

$$d\tau = \left( \frac{\alpha-1}{\sigma} + 1 \right) \alpha \tilde{L}^{\alpha-1} d\tilde{L} - d\tilde{c} \quad (31)$$

As noted above  $c$  is monotonically decreasing in  $L$  and so  $\tilde{c}$  is monotonically decreasing in  $\tilde{L}$  according to equation 32

$$\frac{d\tilde{c}}{d\tilde{L}} = \frac{\alpha(1-\sigma)}{(1-\chi)} \left[ (\alpha-1)\tilde{L}^{\alpha-2} - \alpha\tilde{L}^{\alpha-1} \right] \quad (32)$$

Plugging equation 32 into equation 31 gives the effect of government waste on balanced growth path labour supply equation 33 <sup>5</sup>

$$0 < \frac{d\tilde{L}}{d\tau} = \frac{\tilde{L}^{1-\alpha}}{\alpha} \left[ \frac{\alpha-1}{\sigma} + 1 + \frac{\alpha(1-\sigma)}{(1-\chi)} + \frac{(1-\sigma)}{(1-\chi)} (1-\alpha)\tilde{L}^{-1} \right]^{-1} \quad (33)$$

This means that an increase in government spending increases the balanced path growth rate. Note that the economy is unstable. This means that the only way to satisfy the transversality condition is to jump immediately to the balanced growth path so the economy is always in balanced growth and the policy of tax and waste increases the growth rate as soon as it is introduced. An anticipated future increase in tax and waste policies has a similar effect, since the effect of the policy is a pure income effect.

Interestingly it is possible to find parameter values for which the welfare maximizing policy involves taxing and wasting.

<sup>5</sup>The sign of  $\frac{d\tilde{L}}{d\tau}$  is clear if we consider that

$$\begin{aligned} \frac{\alpha-1}{\sigma} + 1 + \frac{\alpha(1-\sigma)}{(1-\chi)} + \frac{(1-\sigma)}{(1-\chi)} (1-\alpha)\tilde{L}^{-1} &> \\ \frac{\alpha-1}{\sigma} + 1 + \frac{\alpha(1-\sigma)}{(1-\chi)} + \frac{(1-\sigma)}{(1-\chi)} (1-\alpha) &= \\ \frac{\alpha}{\sigma} + \frac{(1-\sigma)(\sigma+\chi-1)}{\sigma(1-\chi)} &> 0 \end{aligned}$$



Define the balanced path growth rate to be  $g$ , then the effect of a tax and waste policy on the growth rate is given by equation 34

$$\frac{dg}{d\tau} = \frac{(1-\alpha)\alpha\tilde{L}^{\alpha-1}}{\sigma} \frac{d\tilde{L}}{d\tau} = \frac{1-\alpha}{\frac{\sigma(1-\sigma)}{1-\chi} \left[ \frac{(1-\alpha)}{\tilde{L}} + \alpha \right] + \alpha + \sigma - 1} \quad (34)$$

recall the consumers objective function which for constant  $L = \tilde{L}$  and consumption growing at rate  $g$  becomes equation 35

$$V = \int_0^\infty e^{(-\rho+g(1-\sigma))t} \frac{C_0^{1-\sigma}}{1-\sigma} (1-\tilde{L})^{1-\chi} dt \quad (35)$$

Integrating and considering equations 4 and 7, equation 35 becomes equation 36

$$V = K_0^{1-\sigma} \frac{\tilde{L}^{(1-\sigma)(\alpha-1)}}{(1-\sigma)} \frac{(1-\tilde{L})^{2-\chi-\sigma}}{(\rho-g(1-\sigma))} \left( \frac{\alpha(1-\sigma)}{1-\chi} \right) \quad (36)$$

The derivative of the log of  $V$  (assuming  $\sigma$  less than one) with respect to  $\tau$  is:

$$\frac{d \log V}{d\tau} = \left[ \frac{(1-\sigma)(\alpha-1)}{\tilde{L}} - \frac{2-\chi-\sigma}{(1-\tilde{L})} + \frac{(1-\sigma)\frac{\alpha(1-\alpha)\tilde{L}^{\alpha-1}}{\sigma}}{\epsilon} \right] \frac{d\tilde{L}}{d\tau} \quad (37)$$

where  $\epsilon \equiv \rho - g(1-\sigma) > 0$  and we have used the fact that, since  $g = \frac{(1-\alpha)\tilde{L}^{\alpha-\rho}}{\sigma}$ ,  $\frac{dg}{d\tilde{L}} = \frac{\alpha(1-\alpha)\tilde{L}^{\alpha-1}}{\sigma}$ . To show that the expression inside the parenthesis can be positive note that the parameters of the model can be chosen to give any desired (positive) value of  $\tilde{L}$  less than  $\frac{1-\sigma}{2-\sigma-\chi}$  and any positive value of  $\epsilon$ , with the two linked in the following way:

$$\alpha\tilde{L}^\alpha + \epsilon = \frac{\alpha(1-\sigma)(1-\tilde{L})\tilde{L}^{\alpha-1}}{(1-\chi)} \quad (38)$$

Equation 38 can be derived by considering the expressions for the balanced rate of growth of consumption and capital  $g_c = g_k = g$  — equations 5 and 10.  $g_k$  is calculated by substituting in equation 10 the expression for  $C/K$  obtained by equations 4 and 7. With a Cobb-Douglas technology we then get:

$$g_c = \frac{(1-\alpha)\tilde{L}^{\alpha-\rho}}{\sigma} \\ g_k = \tilde{L}^\alpha - \frac{\alpha(1-\sigma)(1-\tilde{L})\tilde{L}^{\alpha-1}}{(1-\chi)}$$

but since  $\rho = (1-\sigma)g_c + \epsilon$  we can rewrite the first equation as  $g_c = (1-\alpha)\tilde{L}^\alpha - \epsilon$ . Equating  $g_c$  and  $g_k$  then implies equation 38.

As  $\epsilon$  goes to zero from above and  $\tilde{L}$  goes to  $\frac{1-\sigma}{2-\sigma-\chi}$  the first term inside the parenthesis in equation 37 goes to  $(2-\chi-\sigma)(\alpha-1)$ , the second term goes to

$-(2 - \chi - \sigma)^2(1 - \chi)^{-1}$  while the third goes to plus infinity. This means that starting with no tax the introduction of a tax can lead to a gigantic increase in welfare. In general it is possible to find parameter values such that a tax and waste policy improves welfare compared to market equilibrium without a government. For example for  $\sigma = \alpha = \frac{1}{2} = \tilde{L}$ , the coefficient of  $\frac{d\tilde{L}}{d\tau}$  in equation 37 is  $\frac{5(2-\sqrt{2})}{2\sqrt{2}}$ .

An attempt to provide an intuitive explanation for this result is the following: in this model due to externalities to capital accumulation the rental rate of capital is inefficiently low. A lump-sum tax reducing disposable income induce people to work more thus increasing the rate of return on capital and in a sense internalizing the externalities. This has a positive effect on welfare. For the overall effect to be positive this indirect substitution effect must outweigh the negative direct effect on welfare caused by the subtraction of resources from the private sector.

## 4 Conclusions

In this paper we have proposed a model with one good produced from capital and labour that can have a stable balanced growth path. If the economy is in the proximity of the balanced growth path and consumption and leisure increase then leisure eventually returns to its balanced growth level and the consumption capital ratio returns to its balanced growth. Such stability implies indeterminacy. Consumption and labour supply can take a continuum of values each consistent with utility maximization and perfect foresight. As is well known this in turn means that economically irrelevant publicly observable variables (sunspots) can effect the economy even if all agents are rational utility maximizers with rational expectations. This is of some interest especially since the effects of a sunspot are permanent. The long run expected value of labour supply and of the consumption capital ratio are not affected by the sunspot, but the level of consumption and the capital stock are affected.

Coming to tax policy we have shown that when labour supply is elastic a lump sum tax will increase growth, by making people work more, even if the tax revenue is not used in incomethrown away instead of being used to subsidize investment. Moreover we have shown that for some parameter values a lump sum tax will increase welfare even if the tax revenue is thrown away.

## References

- [1] Azariadis C. and A. Drazen (1990). "Thresholds in Economic Development," *Quarterly Journal of Economics*, 105, 501–526.
- [2] Benhabib J. and R. Farmer (1994). "Indeterminacy and Increasing Returns," *Journal of Economic Theory*, 63, 19–41.
- [3] Benhabib J. and R. Perli (1994). "Uniqueness and Indeterminacy: On the Dynamics of Endogenous Growth and Increasing Returns," *Journal of Economic Theory*, 63, 113–42.
- [4] Benhabib J. and A. Rustichini (1994). "Introduction to the Symposium on Growth, Fluctuations and Sunspots: Confronting the Data," *Journal of Economic Theory*, 63, 1, 1–19.
- [5] Caballè J. and M. Santos (1993). "On Endogenous Growth with Physical and Human Capital," *Journal of Political Economy*, 101, 1042–1067.
- [6] Chamley C. (1993). "Externalities and Dynamics in Models of 'Learning or Doing'," *International Economic Review*, 34, 3, 583–609.
- [7] Jones L. and R. Manuelli (1990). "A Convex Model of Equilibrium Growth: Theory and Policy Implications," *Journal of Political Economy*, 98, 1008–38.
- [8] Jones L., R. Manuelli and P. Rossi (1993). "Optimal Taxation in Models of Endogenous Growth," *Journal of Political Economy*, 101, 485–517.
- [9] King R. and S. Rebelo (1990). "Public Policy and Economic Growth: Developing Neoclassical Implications," *Journal of Political Economy*, 98, 5, 2, S126–S150.
- [10] Ladrón-de-Guevara A., S. Ortigueira and M. Santos (1994). "Equilibrium Dynamics in Two-Sector Models of Endogenous Growth," ITAM Discussion Paper, Centro de Investigación Económica, Instituto Tecnológico Autónomo de México.
- [11] Lucas R. (1988). "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22, 3–42.
- [12] Lucas R. (1990). "Supply-Side Economics: an Analytical Review," *Oxford Economic Papers*, 42, 293–316.



- [13] Mulligan C. and X. Sala-i-Martin (1993). "Transitional Dynamics in Two-Sector Models of Endogenous Growth," *Quarterly Journal of Economics*, 108, 736–774.
- [14] Rebelo S. (1991). "Long-Run Policy Analysis and Long-Run Growth," *Journal of Political Economy*, 99, 500–21.
- [15] Romer P. (1986). "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, 94, 1002–37.
- [16] Roubini N. and G.M. Milesi-Ferretti (1994). "Optimal Taxation of Human and Physical Capital in Endogenous Growth Models," NBER Working Paper # 4882.
- [17] Shell K. (1977). "Monnaie et Allocation Intertemporelle," Mimeo, Séminaire d'Econométrie Roy-Malinvaud, Centre Nationale de la Recherche Scientifique, Paris.
- [18] Stokey N. and S. Rebelo (1993). "Growth Effects of Flat-Rate Taxes," NBER Working Paper # 4426.
- [19] Xie D. (1994). "Divergence in Economic Performance: Transitional Dynamics with Multiple Equilibria," *Journal of Economic Theory*, 63, 1, 97–113.





# EUI WORKING PAPERS

EUI Working Papers are published and distributed by the  
European University Institute, Florence

Copies can be obtained free of charge  
– depending on the availability of stocks – from:

The Publications Officer  
European University Institute  
Badia Fiesolana  
I-50016 San Domenico di Fiesole (FI)  
Italy

**Please use order form overleaf**





# Publications of the European University Institute

## Department of Economics Working Paper Series

To Department of Economics WP  
 European University Institute  
 Badia Fiesolana  
 I-50016 San Domenico di Fiesole (FI)  
 E-mail: [publish@datacomm.iue.it](mailto:publish@datacomm.iue.it)  
 Italy

From Name .....

Address .....

.....

.....

.....

(Please print)

- Please enter/confirm my name on EUI Economics Dept. Mailing List
- Please send me a complete list of EUI Working Papers
- Please send me a complete list of EUI book publications
- Please send me the EUI brochure Academic Year 1996/97

Please send me the following EUI ECO Working Paper(s):

No, Author .....

Title: .....

No, Author .....

Title: .....

No, Author .....

Title: .....

No, Author .....

Title: .....

Date ..... Signature .....

**Working Papers of the Department of Economics  
Published since 1993**

**ECO No. 93/1**  
Carlo GRILLENZONI  
Forecasting Unstable and Non-Stationary  
Time Series

**ECO No. 93/2**  
Carlo GRILLENZONI  
Multilinear Models for Nonlinear Time  
Series

**ECO No. 93/3**  
Ronald M. HARSTAD/Louis PHILIPS  
Futures Market Contracting When You  
Don't Know Who the Optimists Are

**ECO No. 93/4**  
Alan KIRMAN/Louis PHILIPS  
Empirical Studies of Product Markets

**ECO No. 93/5**  
Grayham E. MIZON  
Empirical Analysis of Time Series:  
Illustrations with Simulated Data

**ECO No. 93/6**  
Tilman EHRBECK  
Optimally Combining Individual  
Forecasts From Panel Data

**ECO NO. 93/7**  
V́ctor GÓMEZ/Agustín MARAVALL  
Initializing the Kalman Filter with  
Incompletely Specified Initial Conditions

**ECO No. 93/8**  
Frederic PALOMINO  
Informed Speculation: Small Markets  
Against Large Markets

**ECO NO. 93/9**  
Stephen MARTIN  
Beyond Prices Versus Quantities

**ECO No. 93/10**  
José María LABEAGA/Angel LÓPEZ  
A Flexible Demand System and VAT  
Simulations from Spanish Microdata

**ECO No. 93/11**  
Maozu LU/Grayham E. MIZON  
The Encompassing Principle and  
Specification Tests

**ECO No. 93/12**  
Louis PHILIPS/Peter MØLLGAARD  
Oil Stocks as a Squeeze Preventing  
Mechanism: Is Self-Regulation Possible?

**ECO No. 93/13**  
Pieter HASEKAMP  
Disinflation Policy and Credibility: The  
Role of Conventions

**ECO No. 93/14**  
Louis PHILIPS  
Price Leadership and Conscious  
Parallelism: A Survey

**ECO No. 93/15**  
Agustín MARAVALL  
Short-Term Analysis of Macroeconomic  
Time Series \*

**ECO No. 93/16**  
Philip Hans FRANSES/Niels  
HALDRUP  
The Effects of Additive Outliers on Tests  
for Unit Roots and Cointegration

**ECO No. 93/17**  
Fabio CANOVA/Jane MARRINAN  
Predicting Excess Returns in Financial  
Markets

**ECO No. 93/18**  
Iñigo HERGUERA  
Exchange Rate Fluctuations, Market  
Structure and the Pass-through  
Relationship

**ECO No. 93/19**  
Agustín MARAVALL  
Use and Misuse of Unobserved  
Components in Economic Forecasting

**ECO No. 93/20**  
Torben HOLVAD/Jens Leth  
HOUGAARD  
Measuring Technical Input Efficiency for  
Similar Production Units:  
A Survey of the Non-Parametric  
Approach

\*out of print

**ECO No. 93/21**  
Stephen MARTIN/Louis PHLIPS  
Product Differentiation, Market Structure  
and Exchange Rate Passthrough

**ECO No 93/22**  
F. CANOVA/M. FINN/A. R. PAGAN  
Evaluating a Real Business Cycle Model

**ECO No 93/23**  
Fabio CANOVA  
Statistical Inference in Calibrated Models

**ECO No 93/24**  
Gilles TEYSSIÈRE  
Matching Processes in the Labour Market  
in Marseilles. An Econometric Study

**ECO No 93/25**  
Fabio CANOVA  
Sources and Propagation of International  
Business Cycles: Common Shocks or  
Transmission?

**ECO No. 93/26**  
Marco BECHT/Carlos RAMÍREZ  
Financial Capitalism in Pre-World War I  
Germany: The Role of the Universal  
Banks in the Financing of German  
Mining Companies 1906-1912

**ECO No. 93/27**  
Isabelle MARET  
Two Parametric Models of Demand,  
Structure of Market Demand from  
Heterogeneity

**ECO No. 93/28**  
Stephen MARTIN  
Vertical Product Differentiation, Intra-  
industry Trade, and Infant Industry  
Protection

**ECO No. 93/29**  
J. Humberto LOPEZ  
Testing for Unit Roots with the k-th  
Autocorrelation Coefficient

**ECO No. 93/30**  
Paola VALBONESI  
Modelling Interactions Between State and  
Private Sector in a "Previously" Centrally  
Planned Economy

**ECO No. 93/31**  
Enrique ALBEROLA ILA/J. Humberto  
LOPEZ/Vicente ORTOS RIOS  
An Application of the Kalman Filter to  
the Spanish Experience in a Target Zone  
(1989-92)

**ECO No. 93/32**  
Fabio CANOVA/Morten O. RAVN  
International Consumption Risk Sharing

**ECO No. 93/33**  
Morten Overgaard RAVN  
International Business Cycles: How  
much can Standard Theory Account for?

**ECO No. 93/34**  
Agustín MARAVALL  
Unobserved Components in Economic  
Time Series \*

**ECO No. 93/35**  
Sheila MARNIE/John  
MICKLEWRIGHT  
Poverty in Pre-Reform Uzbekistan:  
What do Official Data Really Reveal? \*

**ECO No. 93/36**  
Torben HOLVAD/Jens Leth  
HOUGAARD  
Measuring Technical Input Efficiency for  
Similar Production Units:  
80 Danish Hospitals

**ECO No. 93/37**  
Grayham E. MIZON  
A Simple Message for Autocorrelation  
Correctors: DON'T

**ECO No. 93/38**  
Barbara BOEHNLEIN  
The Impact of Product Differentiation on  
Collusive Equilibria and Multimarket  
Contact

**ECO No. 93/39**  
H. Peter MØLLGAARD  
Bargaining and Efficiency in a  
Speculative Forward Market

\*\*\*



**ECO No. 94/1**  
Robert WALDMANN  
Cooperatives With Privately Optimal  
Price Indexed Debt Increase Membership  
When Demand Increases

**ECO No. 94/2**  
Tilman EHRBECK/Robert  
WALDMANN  
Can Forecasters' Motives Explain  
Rejection of the Rational Expectations  
Hypothesis?

**ECO No. 94/3**  
Alessandra PELLONI  
Public Policy in a Two Sector Model of  
Endogenous Growth \*

**ECO No. 94/4**  
David F. HENDRY  
On the Interactions of Unit Roots and  
Exogeneity

**ECO No. 94/5**  
Bernadette GOVAERTS/David F.  
HENDRY/Jean-François RICHARD  
Encompassing in Stationary Linear  
Dynamic Models

**ECO No. 94/6**  
Luigi ERMINI/Dongkoo CHANG  
Testing the Joint Hypothesis of Rational-  
ity and Neutrality under Seasonal Coin-  
tegration: The Case of Korea

**ECO No. 94/7**  
Gabriele FIORENTINI/Agustín  
MARAVALL  
Unobserved Components in ARCH  
Models: An Application to Seasonal  
Adjustment \*

**ECO No. 94/8**  
Niels HALDRUP/Mark SALMON  
Polynomially Cointegrated Systems and  
their Representations: A Synthesis

**ECO No. 94/9**  
Mariusz TAMBORSKI  
Currency Option Pricing with Stochastic  
Interest Rates and Transaction Costs:  
A Theoretical Model

**ECO No. 94/10**  
Mariusz TAMBORSKI  
Are Standard Deviations Implied in  
Currency Option Prices Good Predictors  
of Future Exchange Rate Volatility?

**ECO No. 94/11**  
John MICKLEWRIGHT/Gyula NAGY  
How Does the Hungarian Unemploy-  
ment Insurance System Really Work? \*

**ECO No. 94/12**  
Frank CRITCHLEY/Paul  
MARRIOTT/Mark SALMON  
An Elementary Account of Amari's  
Expected Geometry

**ECO No. 94/13**  
Domenico Junior MARCHETTI  
Procyclical Productivity, Externalities  
and Labor Hoarding: A Reexamination of  
Evidence from U.S. Manufacturing

**ECO No. 94/14**  
Giovanni NERO  
A Structural Model of Intra-European  
Airline Competition

**ECO No. 94/15**  
Stephen MARTIN  
Oligopoly Limit Pricing: Strategic  
Substitutes, Strategic Complements

**ECO No. 94/16**  
Ed HOPKINS  
Learning and Evolution in a  
Heterogeneous Population

**ECO No. 94/17**  
Berthold HERRENDORF  
Seigniorage, Optimal Taxation, and Time  
Consistency: A Review

**ECO No. 94/18**  
Frederic PALOMINO  
Noise Trading in Small Markets \*

**ECO No. 94/19**  
Alexander SCHRADER  
Vertical Foreclosure, Tax Spinning and  
Oil Taxation in Oligopoly

**ECO No. 94/20**  
Andrzej BANIAK/Louis PHILIPS  
La Pléiade and Exchange Rate Pass-  
Through

**ECO No. 94/21**  
Mark SALMON  
Bounded Rationality and Learning;  
Procedural Learning

\*out of print

**ECO No. 94/22**  
Isabelle MARET  
Heterogeneity and Dynamics of  
Temporary Equilibria: Short-Run Versus  
Long-Run Stability

**ECO No. 94/23**  
Nikolaos GEORGANTZIS  
Short-Run and Long-Run Cournot  
Equilibria in Multiproduct Industries

**ECO No. 94/24**  
Alexander SCHRADER  
Vertical Mergers and Market Foreclosure:  
Comment

**ECO No. 94/25**  
Jeroen HINLOOPEN  
Subsidising Cooperative and Non-  
Cooperative R&D in Duopoly with  
Spillovers

**ECO No. 94/26**  
Debora DI GIOACCHINO  
The Evolution of Cooperation:  
Robustness to Mistakes and Mutation

**ECO No. 94/27**  
Kristina KOSTIAL  
The Role of the Signal-Noise Ratio in  
Cointegrated Systems

**ECO No. 94/28**  
Agustín MARAVALL/Víctor GÓMEZ  
Program SEATS "Signal Extraction in  
ARIMA Time Series" - Instructions for  
the User

**ECO No. 94/29**  
Luigi ERMINI  
A Discrete-Time Consumption-CAP  
Model under Durability of Goods, Habit  
Formation and Temporal Aggregation

**ECO No. 94/30**  
Debora DI GIOACCHINO  
Learning to Drink Beer by Mistake

**ECO No. 94/31**  
Víctor GÓMEZ/Agustín MARAVALL  
Program TRAMO "Time Series  
Regression with ARIMA Noise, Missing  
Observations, and Outliers" -  
Instructions for the User

**ECO No. 94/32**  
Ákos VALENTINYI  
How Financial Development and  
Inflation may Affect Growth

**ECO No. 94/33**  
Stephen MARTIN  
European Community Food Processing  
Industries

**ECO No. 94/34**  
Agustín MARAVALL/Christophe  
PLANAS  
Estimation Error and the Specification of  
Unobserved Component Models

**ECO No. 94/35**  
Robbin HERRING  
The "Divergent Beliefs" Hypothesis and  
the "Contract Zone" in Final Offer  
Arbitration

**ECO No. 94/36**  
Robbin HERRING  
Hiring Quality Labour

**ECO No. 94/37**  
Angel J. UBIDE  
Is there Consumption Risk Sharing in the  
EEC?

**ECO No. 94/38**  
Berthold HERRENDORF  
Credible Purchases of Credibility  
Through Exchange Rate Pegging:  
An Optimal Taxation Framework

**ECO No. 94/39**  
Enrique ALBEROLA ILA  
How Long Can a Honeymoon Last?  
Institutional and Fundamental Beliefs in  
the Collapse of a Target Zone

**ECO No. 94/40**  
Robert WALDMANN  
Inequality, Economic Growth and the  
Debt Crisis

**ECO No. 94/41**  
John MICKLEWRIGHT/  
Gyula NAGY  
Flows to and from Insured  
Unemployment in Hungary



**ECO No. 94/42**  
Barbara BOEHNLEIN  
The Soda-ash Market in Europe:  
Collusive and Competitive Equilibria  
With and Without Foreign Entry

**ECO No. 94/43**  
Hans-Theo NORMANN  
Stackelberg Warfare as an Equilibrium  
Choice in a Game with Reputation Effects

**ECO No. 94/44**  
Giorgio CALZOLARI/Gabriele  
FIORENTINI  
Conditional Heteroskedasticity in  
Nonlinear Simultaneous Equations

**ECO No. 94/45**  
Frank CRITCHLEY/Paul MARRIOTT/  
Mark SALMON  
On the Differential Geometry of the Wald  
Test with Nonlinear Restrictions

**ECO No. 94/46**  
Renzo G. AVESANI/Giampiero M.  
GALLO/Mark SALMON  
On the Evolution of Credibility and  
Flexible Exchange Rate Target Zones

\* \* \*

**ECO No. 95/1**  
Paul PEZANIS-CHRISTOU  
Experimental Results in Asymmetric  
Auctions - The 'Low-Ball' Effect

**ECO No. 95/2**  
Jeroen HINLOOPEN/Rien  
WAGENVOORT  
Robust Estimation: An Example

**ECO No. 95/3**  
Giampiero M. GALLO/Barbara PACINI  
Risk-related Asymmetries in Foreign  
Exchange Markets

**ECO No. 95/4**  
Santanu ROY/Rien WAGENVOORT  
Risk Preference and Indirect Utility in  
Portfolio Choice Problems

**ECO No. 95/5**  
Giovanni NERO  
Third Package and Noncooperative  
Collusion in the European Airline  
Industry

**ECO No. 95/6**  
Renzo G. AVESANI/Giampiero M.  
GALLO/Mark SALMON  
On the Nature of Commitment in Flexible  
Target Zones and the Measurement of  
Credibility: The 1993 ERM Crisis

**ECO No. 95/7**  
John MICKLEWRIGHT/Gyula NAGY  
Unemployment Insurance and Incentives  
in Hungary

**ECO No. 95/8**  
Kristina KOSTIAL  
The Fully Modified OLS Estimator as a  
System Estimator: A Monte-Carlo  
Analysis

**ECO No. 95/9**  
Günther REHME  
Redistribution, Wealth Tax Competition  
and Capital Flight in Growing  
Economies

**ECO No. 95/10**  
Grayham E. MIZON  
Progressive Modelling of  
Macroeconomic Time Series: The LSE  
Methodology

**ECO No. 95/11**  
Pierre CAHUC/Hubert KEMPF  
Alternative Time Patterns of Decisions  
and Dynamic Strategic Interactions

**ECO No. 95/12**  
Tito BOERI  
Is Job Turnover Countercyclical?

**ECO No. 95/13**  
Luisa ZANFORLIN  
Growth Effects from Trade and  
Technology

**ECO No. 95/14**  
Miguel JIMÉNEZ/Domenico  
MARCHETTI, jr.  
Thick-Market Externalities in U.S.  
Manufacturing: A Dynamic Study with  
Panel Data

**ECO No. 95/15**  
Berthold HERRENDORF  
Exchange Rate Pegging, Transparency,  
and Imports of Credibility

\*out of print



**ECO No. 95/16**  
Günther REHME  
Redistribution, Income cum Investment  
Subsidy Tax Competition and Capital  
Flight in Growing Economies

**ECO No. 95/17**  
Tito BOERI/Stefano SCARPETTA  
Regional Dimensions of Unemployment  
in Central and Eastern Europe and Social  
Barriers to Restructuring

**ECO No. 95/18**  
Bernhard WINKLER  
Reputation for EMU - An Economic  
Defence of the Maastricht Criteria

**ECO No. 95/19**  
Ed HOPKINS  
Learning, Matching and Aggregation

**ECO No. 95/20**  
Dorte VERNER  
Can the Variables in an Extended Solow  
Model be Treated as Exogenous?  
Learning from International Comparisons  
Across Decades

**ECO No. 95/21**  
Enrique ALBEROLA-ILA  
Optimal Exchange Rate Targets and  
Macroeconomic Stabilization

**ECO No. 95/22**  
Robert WALDMANN  
Predicting the Signs of Forecast Errors

**ECO No. 95/23**  
Robert WALDMANN  
The Infant Mortality Rate is Higher  
where the Rich are Richer

**ECO No. 95/24**  
Michael J. ARTIS/Zenon G.  
KONTOLEMIS/Denise R. OSBORN  
Classical Business Cycles for G7 and  
European Countries

**ECO No. 95/25**  
Jeroen HINLOOPEN/Charles VAN  
MARREWIK  
On the Limits and Possibilities of the  
Principle of Minimum Differentiation

**ECO No. 95/26**  
Jeroen HINLOOPEN  
Cooperative R&D Versus R&D-  
Subsidies: Cournot and Bertrand  
Duopolies

**ECO No. 95/27**  
Giampiero M. GALLO/Hubert KEMPF  
Cointegration, Codependence and  
Economic Fluctuations

**ECO No. 95/28**  
Anna PETTINI/Stefano NARDELLI  
Progressive Taxation, Quality, and  
Redistribution in Kind

**ECO No. 95/29**  
Ákos VALENTINYI  
Rules of Thumb and Local Interaction

**ECO No. 95/30**  
Robert WALDMANN  
Democracy, Demography and Growth

**ECO No. 95/31**  
Alessandra PELLONI  
Nominal Rigidities and Increasing  
Returns

**ECO No. 95/32**  
Alessandra PELLONI/Robert  
WALDMANN  
Indeterminacy and Welfare Increasing  
Taxes in a Growth Model with Elastic  
Labour Supply













© The Author(s). European University Institute.

Digitised version produced by the EUJ Library in 2020. Available Open Access on Cadmus, European University Institute Research Repository.