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Abstract

We derive the generating process of temporally aggregated variables, when the original variables follow a discrete time $VARIMAX$ process. We consider different temporal aggregation schemes, which are likely applied to generate the available data on many economic variables.

Key words: Temporal Aggregation, $VARIMA$ process, $VARIMAX$ process.
JEL Classification: C32, C43, C5.

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1 Introduction

It is quite common in econometrics to analyse temporally aggregated data, because the frequency of data collection is in general much lower than that of data generation. While the data generating process (DGP) of the temporally aggregated variables can be rather different from the original DGP, the usual aim of econometric studies is to make inference on the latter, in order to assess the reliability of a particular economic theory. Thus, it is important to determine what characteristics of the original DGP are invariant to temporal aggregation, and can therefore be tested with temporally aggregated data. As a more complete alternative, given the original DGP and the particular temporal aggregation scheme which has generated the available data, the theoretical temporally aggregated DGP could be derived. Its compatibility with the data would then provide an indirect check of the appropriateness of the hypothesised original DGP.

Dynamic economic models, at least in their reduced form representation, often imply that the variables are generated by a vector autoregressive model (VAR), possibly with moving average errors ($VARMA$), integrated variables ($VARIMA$), exogenous conditioning variables ($VARIMAX$), and particular restrictions on the coefficients. These generating processes can also provide an adequate statistical characterization for many time series, and their adoption in applied econometrics has steadily grown since the pioneering work of Sims (1980). A nice reconciliation between the economic and statistical justifications for these models can be found in Hendry and Mizon (1993), who suggest to evaluate economic models on the basis of their capacity to encompass a parsimonious and congruent statistical representation for the data under analysis.

Hence, in this paper we focus on the derivation of the DGP of the temporally aggregated variables when the original variables are generated by a model in the discrete time $VARIMAX$ class.[1] For the univariate $ARIMA$ case, Brewer (1973), Wei (1981) and Weiss (1984) have shown that the aggregated process is still of the $ARIMA$ type, and they have derived the order and the coefficients of its AR and MA components. A more

[1]Some recent references for the counterpart in continuous time are Christiano and Eichenbaum (1987), Bergstrom (1990), Marcet (1991) and Comte (1994).

detailed analysis of particular cases is presented in Campos *et al.* (1990) and Granger and Siklos (1995). But to deal with economic models the multivariate case has to be considered. With reference to this, Lütkepohl (1987, ch.6) has shown that the class of $VARIMA$ processes is closed with respect to temporal aggregation, and has proposed upper bounds for the order of the AR and MA components of the aggregated process.

In Section 2 we introduce an alternative method which lets us often derive more parsimonious representations of the aggregated process. Different temporal aggregation schemes are considered, and the whole analysis is conducted in the time domain, because this is the natural framework for economic models. In Section 3 we extend the discussion to $VARIMAX$ models, while Section 4 presents some concluding remarks and directions of further research. The proofs of the Propositions in the text are contained in the Appendix.

2 Temporal aggregation of a VARIMA process

In this Section we derive the DGP of the aggregated process, for different temporal aggregation schemes, when the original n dimensional process, $x = \{x_t\}_{t=1}^{\infty}$, evolves according to the system of difference equations

$$G(L)x_t = S(L)\varepsilon_{xt}, \tag{1}$$

where L is the lag operator, $G(L) = I - G_1L - G_2L^2 - ... - G_gL^g$, $S(L) = I - S_1L - S_2L^2 - ... - S_sL^s$, the roots of $|G(L)| = 0$ and $|S(L)| = 0$ lie outside the unit circle and are not common, the Gs and Ss are $n \times n$ matrices of coefficients, $\varepsilon_{xt} \sim i.i.d.(0, \Upsilon_x)$, and, for simplicity, the initial conditions are set equal to zero. In the final subsection we also consider the possibility that the variables are integrated.

2.1 Point-in-time sampling

In the case of point-in-time sampling at frequency k, the temporally aggregated process, x_k, is obtained by selecting only the k^{th} elements of x. Hence, $x_k = \{x_{tk-j}\}_{t=1}^{\infty}$, where j is an integer in the interval $[0, k-1]$, and this formulation lets us consider all the k possibly relevant subprocesses of x.

For example, the elements of a quarterly process can be obtained by selecting the $3^{rd}, 6^{th}, 9^{th}, ...$ elements of a monthly process. But, they could also consist of the $1^{st}, 4^{th}, 7^{th} ...$ or $2^{nd}, 5^{th}, 8^{th} ...$ elements of the same monthly process. We first assume that $j = 0$, and then show that the results that we obtain are invariant to the choice of j.

Proposition 1. If it is possible to determine an $n \times n$ polynomial matrix of degree $gk - g$ in the lag operator, $B(L)$, such that the coefficients of the lags which are not multiple of k in the product $B(L)G(L)$ are equal to zero, then the DGP of x_k is the $VARMA$ model:

$$C(Z)x_{kt} = H(Z)\varepsilon_{kxt}, \tag{2}$$

where $Z = L^k$ is a lag operator such that $Zx_{kt} = x_{t-k} = x_{kt-1}$, the degree in Z of $C(Z)$ and $H(Z)$ are reported in Table 1, and their coefficients and the variance covariance matrix of the white noise errors ε_{kx}, Υ_{kx}, are derived in the proof. □

In order to provide a sufficient condition for the existence of $B(L)$ and to determine its coefficients, we define the vectors of matrices

$$\underset{1\times gk-g}{B^v} = (B_1, B_2, ..., B_{gk-g}) \text{ and } \underset{1\times gk}{G^v} = (G_1, G_2, ..., G_g, 0, ..., 0)$$

and the matrix of matrices

$$\underset{gk-g\times gk}{G^m} = \begin{pmatrix} -I & G_1 & G_2 & ... & G_g & 0 & 0 & ... & 0 & 0 \\ 0 & -I & G_1 & ... & G_{g-1} & G_g & 0 & ... & 0 & 0 \\ 0 & 0 & -I & ... & G_{g-2} & G_{g-1} & G_g & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 0 & 0 & 0 & ... & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 & ... & G_g & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 & ... & G_{g-1} & G_g \end{pmatrix}.$$

We also name G_{-k}^v and G_{-k}^m the $1 \times gk - g$ vector and $gk - g \times gk - g$ matrix which are obtained by deleting the k^{th} columns of G^v and G^m. Then we have,

Proposition 2. If $|G_{-k}^m| \neq 0$, then $B(L)$ exists, its coefficients are the elements of $B^v = G_{-k}^v(G_{-k}^m)^{-1}$, and the coefficients of $C(Z)$ are the elements of $G_{-k}^v(G_{-k}^m)^{-1}G^m - G^v$. □

There can be cases where the $B(L)$ matrix in Proposition 1 does not exist, and the procedure to obtain the DGP of the temporally aggregated variables has to be modified. To this aim, it is useful to consider an alternative representation of (1), namely,

$$G^*(L)x_t = S^*(L)\varepsilon_{xt}, \tag{3}$$

where $G^*(L)$ is a diagonal matrix whose elements on the diagonal are all equal to the determinant of $G(L)$, $g(L)$, while $S^*(L) = G^a(L)S(L)$ and $G^a(L)$ is the adjoint matrix of $G(L)$. The degree in L of $G^*(L)$ and $S^*(L)$ are in general, respectively, gn and $s + g(n-1)$.

Proposition 3. The DGP of x_k is the $VARMA$ model:

$$C^*(Z)x_{kt} = H^*(Z)\varepsilon_{kxt}, \tag{4}$$

the degree in Z of $C^*(Z)$ and $H^*(Z)$ can be obtained from Table 1 after substituting g with gn and s with $s+g(n-1)$, and their coefficients and the variance covariance matrix of the white noise errors ε_{kx}, Υ^*_{kx}, are derived in the proof. □

There can also be intermediate situations where to obtain the aggregated DGP it is not necessary to reparameterize the original DGP as in (3), but it is sufficient to increase the degree in L of the matrix $B(L)$ in Proposition 1. This can also determine an increase in the order of the aggregate components in Table 1.

In order to identify the most parsimonious representation for the temporally aggregated DGP, the following strategy can be adopted. As a first step it has to be checked whether the condition in Proposition 1 can be satisfied. If it can, then Proposition 1 is applied to derive the aggregated DGP. Otherwise, the degree in L of $B(L)$ is increased up to $gkn - gn$, and it is verified whether it is possible to determine $B(L)$ so that the coefficients of the lags which are not multiple of k in the product $B(L)G(L)$ are equal to zero. In this case, a properly modified version of Proposition 1 can be applied. When this second step also fails, the original DGP is reparameterized as in (3) and Proposition 3 is exploited to derive the temporally aggregated DGP.

If we now consider the other point-in-time temporally aggregated processes $x_k = \{x_{tk-j}\}_{t=1}^{\infty}$, $j \in [1, k-1]$, their DGP is obtained by multiplying both sides of (2) or (4) by L^j. Thus both the orders and the coefficients of the AR and MA components are invariant to the choice of j.

The presence of a constant or of a deterministic trend in the *DGP* of x does not change the conclusions because these deterministic regressors are simply transferred into the *DGP* for x_k, even if with different coefficients. More complicate models with time varying parameters and non *i.i.d.* errors can be handled in a similar manner. Once a proper $B(L)$ matrix is found, the characteristics of the resulting temporally aggregated *DGP* can be studied. In these cases the choice of j can also affect the resulting *DGP*.

Finally, it can be worthwhile noticing that an aggregated *VARMA* model might result from temporal aggregation of a non *VARMA* model or of different *VARMA* models, and this problem is the counterpart of aliasing in the frequency domain analysis of time series, see, e.g., Koopmans (1974).[2]

2.2 Average sampling

From an economic point of view, point-in-time sampling seems suited for stock variables but not for flow variables, whose aggregated values are represented by partial sums of the original data. Moreover, there are cases where partial weighted averages of the original observations are analysed, and we now have to deal with these more general situations. Hence, we introduce average sampling, which can be thought of as a two step procedure. In the first step a linear filter, $\omega(L) = (\omega_0 + \omega_1 L + ... + \omega_{k-1} L^{k-1})$, is applied to the elements of x in order to obtain a new process x^*, $x^* = \{\omega(L)x_t\}_{t=k}^{\infty}$. In the second step, point in time sampling is applied to x^*, i.e., only the k^{th} elements of x^* are retained and they are used to construct the process x_k, $x_k = \{x^*_{tk-j}\}_{t=1}^{\infty}$, where j is an integer in the interval $[0, k-1]$.

This formulation lets us consider all the subprocesses of x that can be obtained by linearly aggregating and then selecting its elements. For example, if all the weights are equal to one or one over k, then the elements of x_k are, respectively, non overlapping partial sums and averages of those of x. If instead the weights are all equal to zero except one, the different possibilities of point-in-time sampling are obtained. We have already seen that the *DGP* of a point-in-time sampling temporally aggregated process is invariant to the choice of j, so that average sampling, which boils down

[2]Consider for example the two $VAR(1)$ processes $x_t = Ax_{t-1} + \varepsilon_t$ and $x_t = -Ax_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim i.i.d.(0, I)$. After point in time sampling with $k = 2$ both of them become $x_{kt} = A^2 x_{kt-1} + \varepsilon_{kt}$, where $\varepsilon_{kt} \sim i.i.d.(0, I + AA')$. This issue has interesting implications for the estimation of missing observations, see Marcellino (1995a).

to point-in-time sampling from x^*, is also invariant to this choice, and from now on we assume $j = 0$, for ease of notation.

Proposition 4. If it is possible to determine an $n \times n$ polynomial matrix of degree $gk - g$ in the lag operator, $B(L)$, such that the coefficients of the lags which are not multiple of k in the product $B(L)G(L)$ are equal to zero, then the *DGP* of the average sampling temporally aggregated process x_k is the *VARMA* model:

$$C(Z)x_{kt} = P(Z)\varepsilon_{kxt} \tag{5}$$

where the degree in Z of $C(Z)$ and $P(Z)$ are reported in Table 1, and their coefficients and the variance covariance matrix of the white noise errors ε_{kx}, Υ_{kx}, are derived in the proof. □

Notice that the aggregated *AR* component is still of order g. Moreover, it is independent of the weighting scheme and therefore, in particular, it is equal to that for point-in-time sampling. This result is due to the equality of the *AR* components in the *DGP* of x and x^*. A further implication of such an equality is that the sufficient condition for the existence of $B(L)$ in Proposition 2 is valid also for average sampling.

When the condition in Proposition 4 can not be satisfied, we have to reparameterize the original *DGP* as in (3). Then,

Proposition 5. The *DGP* of x_k is the *VARMA* model:

$$C^*(Z)x_{kt} = P^*(Z)\varepsilon_{kxt}, \tag{6}$$

the degree in Z of $C^*(Z)$ and $P^*(Z)$ can be obtained from Table 1 after substituting g with gn and s with $s+g(n-1)$, and their coefficients and the variance covariance matrix of the white noise errors ε_{kx}, Υ^*_{kx}, are derived in the proof. □

The *AR* component is still independent of the weighting scheme and equal to that for point-in-time sampling, when the latter is obtained from Proposition 3.

[Table 1 about here]

As for point-in-time sampling, there can be intermediate situations where the aggregated *DGP* can be obtained by premultiplication of the original DGP by a $B(L)$ matrix whose degree in L is larger than $gk - g$. To determine the most parsimonious $VARMA$ representation of the average sampling temporally aggregated variables, the strategy in the former subsection can be adopted.

It is valuable pointing out that there can be particular cases where the coefficients of the predicted highest lags in the AR and MA components in the DGP of x_k turn out to be zero. Hence, the order of the components in Table 1 are more properly upper bounds.[3]

The method to derive the temporal aggregated DGP that we have proposed can be seen as an extension to the multivariate case of that in Brewer (1973). Actually, for the univariate case the results in Propositions 1 and 4 coincide, respectively, with those in Propositions 3 and 5, and they are equal to those in Brewer (1973), Wei (1981) and Weiss (1984). But these authors consider only point-in-time and average sampling with unit weights and $j = 0$, while we have shown that the choice of j and of the weights is irrelevant as long as all the weights are different from zero and only the order of the aggregated AR and MA components is of interest. Moreover, the results for the multivariate case with reference to the order of the aggregated components turn out to be equal to those for the univariate case when the conditions in Propositions 1 and 4 are satisfied, as can be verified from Table 1.

This is in general no longer true when we have to reparameterize the original model as in (3) to aggregate it. In this case, the order of the AR component, gn, coincides with that in Lütkepohl (1987, ch. 6), while that of the MA component is still often lower. We think that a major advantage of our method with respect to Lütkepohl's one is just that it provides a more parsimonious representation of the aggregated process for a large range of cases.[4]

[3]This happens, for example, when $G(L) = G(L^k)$, or when $G(L)$ can be factored into $G^{**}(L)\omega(L)$. In the latter case the aggregated AR components for point-in-time and average sampling can be different. Stram and Wei (1986) provide conditions for the reduction in the order not to take place in the univariate case. It is also *a priori* possible that a singular variance covariance matrix for the original errors is transformed into a non singular variance covariance matrix for the temporally aggregated errors, and viceversa.

[4]Lütkepohl's procedure requires to apply a particular deterministic selection matrix to a reparameterized version of the original DGP. Its details are not reported to save space. It can also be applyed to mixed sampling, which is considered in the next subsection, and similar comments apply. On the other hand, Lütkepohl's method can be simply

2.3 Mixed sampling

Up to now we have assumed that the same temporal aggregation scheme is applied to all the variables. However, there can be cases where a different aggregation scheme is required for different variables. Imagine, for example, that flow and stock variables such as consumption and wealth are jointly analysed, or that we only have averaged data for a variable and end of period data for another one. We refer to these situations as mixed sampling, and in this subsection we study how the DGP for this type of temporally aggregated variables can be obtained.

To start with, let us substitute the $\omega(L)$ operator in the first step of average sampling with the $n \times n$ diagonal matrix

$$\Omega(L) = \begin{pmatrix} \omega_1(L) & 0 & ... & 0 \\ 0 & \omega_2(L) & ... & 0 \\ ... & & & \\ 0 & 0 & ... & \omega_n(L) \end{pmatrix},$$

where $\omega_i(L)x_i = (\omega_{i0}x_i + \omega_{i1}Lx_i + \omega_{i2}L^2x_i + ... + \omega_{ik-1}L^{k-1}x_i)$ leads to the desired aggregation of the i^{th} variable in x, x_i^*, for $i = 1, ..., n$. Hence, we have $x^* = \{\Omega(L)x_t\}_{t=k}^{\infty}$ and we wish to determine the DGP of $x_k = \{x_{tk}^*\}_{t=1}^{\infty}$.

Proposition 6. If it is possible to determine an $n \times n$ polynomial matrix of degree $g^{**}k - g^{**}$ in the lag operator, $B(L)$, such that the coefficients of the lags which are not multiple of k in the product $B(L)G^{**}(L)$ are equal to zero, where $G^{**}(L)$ is the AR component in the DGP of x^* and g^{**} its degree in L, then the DGP of the mixed sampling temporally aggregated process x_k is the $VARMA$ model:

$$Q(Z)x_{kt} = R(Z)\varepsilon_{kxt}. \tag{7}$$

The degree in Z of $Q(Z)$ and $R(Z)$ can be obtained from the column of Table 1 which is referred to point-in-time sampling, after substituting g and s with g^{**} and s^{**}, where s^{**} is the degree in L of the MA component in the DGP of x^*, $S^{**}(L)$. g^{**}, s^{**}, the coefficients of $Q(Z)$ and $R(Z)$, and the variance covariance matrix of the white noise errors ε_{kx}, Υ_{kx}, are derived in the proof. □

modified to deal with aggregation over agents, while our proposal is specific for temporal aggregation.

If the DGP of x is substituted with that of x^*, the sufficient condition for the existence of $B(L)$ in Proposition 2 can be also applied to mixed sampling.

When the condition in Proposition 6 is not satisfied,

Proposition 7. The DGP of x_k is the $VARMA$ model:

$$C^*(Z)x_{kt} = R^*(Z)\varepsilon_{kxt}, \tag{8}$$

the degree in Z of $C^*(Z)$ and $R^*(Z)$ can be obtained from the column of Table 1 which is referred to point-in-time sampling, after substituting g with gn and s with $s + g(n-1) + k - 1$, and their coefficients and the variance covariance matrix of the white noise errors ε_{kx}, Υ^*_{kx}, are derived in the proof. □

Notice that the AR component in (8) is independent of the choice of the mixed sampling weighting scheme and, therefore, it is equal to that for average and point-in-time sampling. In general, this is not true for Proposition 6. Moreover, when $k > g+1$ and all the weighting schemes are different, aggregation of the reparameterized original DGP leads to a more parsimonious aggregated DGP.

To conclude, also in this case there can be intermediate situations where an increase in the degree in L of $B(L)$ lets the aggregated DGP to be derived without reparameterizing the original model. When $k < g+1$, application of the strategy of Section 2.1 yields the most parsimonious representation for the DGP of the mixed sampling temporally aggregated variables.

2.4 Integrated variables

Up to now we have dealt with stationary processes, but the methods that we have discussed can be also applied when the variables are integrated of order d, $I(d)$. Actually, we have not used the hypothesis on the roots of the AR component in the proofs of Propositions 1, 4, and 6, which therefore are valid even for explosive processes.

A similar result holds for Propositions 3, 5 and 7. Actually, when the variables are $I(d)$, we can assume that their DGP is the $VARIMA(g, d, s)$ model

$$\begin{aligned} G(L)(1-L)^d x_t &= S(L)\varepsilon_{xt} \text{ or} \\ G^{***}(L)x_t &= S^{***}(L)\varepsilon_{xt}, \end{aligned} \tag{9}$$

where $G^{***}(L)$ is a diagonal matrix whose terms are $g(L)(1-L)^d$, and $S^{***}(L) = G^a(L)S(L)$. Hence, we can apply Propositions 3, 5 and 7 under the assumption that the original DGP is (9) instead of (3).

Notice that $B(L)$ and $B^*(L)$ are equal to those for the stationary case multiplied by $(1 + L + L^2 ... + L^{k-1})^d$. Thus, the AR components of the aggregated process are $C(Z)(1-Z)^d$ or $C^*(Z)(1-Z)^d$, and x_k are still $I(d)$. Moreover, d need not be an integer number, so that also fractional integration, see, e.g., Hosking (1981), is preserved through temporal aggregation.

The case where the variables are not only integrated but also cointegrated is examined in details in Marcellino (1995b). If the condition in Propositions 1, 4 and 6 is satisfied, then no further modifications are required. Otherwise, the original DGP can be transformed into an equivalent stationary restricted $VARMA$ process, as in Mellander et al. (1992), whose representation with a diagonal AR component substitutes (3) in Propositions 3, 5, and 7. In both cases, the cointegration rank and vectors are invariant to temporal aggregation.

3 Temporal aggregation of a VARIMAX process

In this Section we derive the generating process of x_k, when x_k is obtained by means of one of the three temporal aggregation schemes, and the DGP of the original process x is

$$G(L)x_t = F(L)y_t + S(L)\varepsilon_{xt}, \tag{10}$$

where y is an r dimensional vector of exogenous variables, $F(L) = F_0 - F_1 L - ... - F_f L^f$, the Fs are $n \times r$ matrices and, for simplicity, the relevant initial conditions are set equal to zero.

If the values of y were known for every period, we could simply follow the approach in the former Section, namely, premultiply both sides of (10) by a proper matrix, $B(L)$. Unfortunately, the values of y are also in general not known for time periods which are not multiple of k, so that many terms in the product $B(L)F(L)y_t$ are unknown. Hence, we have to explicitly state a DGP also for y and a fairly general specification is the $VARIMA$ model

$$M(L)y_t = D(L)\varepsilon_{yt}, \tag{11}$$

where $M(L) = I - M_1L - ... - M_mL^m$, $D(L) = I - D_1L - ... - D_dL^d$, the Ms and Ds are $r \times r$ matrices, the roots of $|M(L)| = 0$ lie outside or on the unit circle and are not in common with those of $|D(L)| = 0$, which lie outside the unit circle, while $\varepsilon_{yt} \sim i.i.d.(0, \Upsilon_y)$ and, for simplicity, it is assumed that they are uncorrelated with ε_{xt-i} for all i and that the relevant initial conditions are equal to zero.

Thus, we focus on the effects of temporal aggregation on the joint process $\{x_t, y_t\}$.

3.1 Point-in-time sampling

It is convenient to rewrite (10) and (11) as

$$\overline{G}(L)z_t = \begin{pmatrix} G(L) & -F(L) \\ 0 & M(L) \end{pmatrix} \begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} S(L) & 0 \\ 0 & D(L) \end{pmatrix} \begin{pmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \end{pmatrix} = \overline{S}(L)\varepsilon_{zt}. \tag{12}$$

Then, we have

Proposition 8. If it is possible to determine an $(n+r) \times (n+r)$ matrix $\overline{B}(L)$, which can be partitioned into

$$\overline{B}(L) = \begin{pmatrix} \underset{n\times n}{B_1(L)} & \underset{n\times r}{B_2(L)} \\ \underset{r\times n}{0} & \underset{r\times r}{B_3(L)} \end{pmatrix},$$

and is such that the coefficients of the lags which are not multiple of k in the product $\overline{B}(L)\overline{G}(L)$ are equal to zero, then the DGP of the point-in-time temporally aggregated process $\{x_{tk}, y_{tk}\}$ is the $VARIMAX$ model:

$$\overline{C}(Z)z_{kt} = \overline{H}(Z)\varepsilon_{kzt}. \tag{13}$$

$\overline{C}(Z)$ can be partitioned into

$$\overline{C}(Z) = \begin{pmatrix} \underset{n\times n}{C_1(Z)} & \underset{n\times r}{C_2(Z)} \\ \underset{r\times n}{0} & \underset{r\times r}{C_3(Z)} \end{pmatrix},$$

where the degree in Z of $C_1(Z)$, $C_2(Z)$, and $C_3(Z)$ are still g, f, and m. The required degree in L of $B_1(L)$, $B_2(L)$, and $B_3(L)$, the degree in Z of $\overline{H}(Z)$, the coefficients of $\overline{C}(Z)$ and $\overline{H}(Z)$, and the variance covariance matrix of the white noise errors ε_{kz}, Υ_{kz}, are derived in the proof. □

In order to provide a sufficient condition for the existence of $\overline{B}(L)$, we define the vectors

$$B_1^v = B^v \ , \quad G_2^v = G^v,$$

$$\underset{1\times gk-g+f-m}{B_2^v} = (B_{21}, B_{22}, ..., B_{2gk-g+f-m}), \qquad \underset{1\times gk-g+f}{G_2^v} = (G_{21}, G_{22}, ..., G_{2gk-g+f}),$$

$$\underset{1\times mk-m}{B_3^v} = (B_{31}, B_{32}, ..., B_{mk-m}), \qquad \underset{1\times mk}{G_3^v} = (M_1, M_2, ..., M_m, 0, ..., 0),$$

where the i^{th} column of G_2^v is the coefficient of L^i in $B_1(L)F(L)$, and the matrices

$$G_1^m = G^m,$$

$$\underset{gk-g+f-m\times gk-g+f}{G_2^m} = \begin{pmatrix} -I & M_1 & M_2 & ... & M_m & 0 & 0 & ... & 0 & 0 \\ 0 & -I & M_1 & ... & M_{m-1} & M_m & 0 & ... & 0 & 0 \\ 0 & 0 & -I & ... & M_{m-2} & M_{m-1} & M_m & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 0 & 0 & 0 & ... & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 & ... & M_m & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 & ... & M_{m-1} & M_m \end{pmatrix},$$

$$\underset{mk-m\times mk}{G_3^m} = \begin{pmatrix} -I & M_1 & M_2 & ... & M_m & 0 & 0 & ... & 0 & 0 \\ 0 & -I & M_1 & ... & M_{m-1} & M_m & 0 & ... & 0 & 0 \\ 0 & 0 & -I & ... & M_{m-2} & M_{m-1} & M_m & ... & 0 & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 0 & 0 & 0 & ... & 0 & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 & ... & M_m & 0 \\ 0 & 0 & 0 & ... & 0 & 0 & 0 & ... & M_{m-1} & M_m \end{pmatrix}.$$

We name G_{2-k}^v, G_{3-k}^v, G_{2-k}^m, and G_{3-k}^m the $1 \times gk - 2g + f$ and $1 \times mk - k$ vectors and $gk - g + f - m \times gk - 2g + f$ and $mk - m \times mk - k$ matrices which are obtained by deleting the k^{th} columns of G_2^v, G_3^v, G_2^m, and G_3^m. G_{2-k}^{v*} and G_{2-k}^{m*} are the vectors and matrices which result from deleting only the first m k^{th} columns of G_2^v and G_2^m. For simplicity, we state and prove the condition for $m \leq g$, and a similar result can be obtained by increasing the order of B_2^v, G_2^v, and G_2^m as it is indicated in the proof of Proposition 8.

Proposition 9. If $m = g$, $|G^m_{-k}| \neq 0$, $|G^m_{2-k}| \neq 0$, and $|G^m_{3-k}| \neq 0$, then $\overline{B}(L)$ exists and it is $B^v_1 = G^v_{-k}(G^m_{-k})^{-1}$, $B^v_2 = G^v_{2-k}(G^m_{2-k})^{-1}$, $B^v_3 = G^v_{3-k}(G^m_{3-k})^{-1}$. If it is $m < g$, G^v_{2-k} and G^m_{2-k} have to be substituted with G^{v*}_{2-k} and G^{m*}_{2-k}. □

The coefficients of the components $C_1(Z)$, $C_3(Z)$, and $C_2(Z)$ are, respectively, those in the k^{th} columns of the vectors $G^v_{1-k}(G^m_{1-k})^{-1}G^m_1 - G^v_3$, $G^v_{3-k}(G^m_{3-k})^{-1}G^m_3 - G^v_3$, and $G^v_{2-k}(G^m_{2-k})^{-1}G^m_2 - G^v_2$ or $G^{v*}_{2-k}(G^{m*}_{2-k})^{-1}G^{m*}_2 - G^{v*}_2$. When the condition $|G^{m*}_{2-k}| \neq 0$ is not satisfied, the matrices G^{v*}_{2-k} and G^{m*}_{2-k} can be obtained by deleting m different k^{th} columns of G^v_2 and G^m_2. In this case, if the proper determinant is different from zero, the highest lag of y_k in the DGP of x_k will be larger than m.[5]

We now consider an equivalent representation of the system (10), (11) which is useful when the condition in Proposition 8 cannot be satisfied. If the expression for y_t in (11) is substituted in (10), the $VARIMA$ process

$$\begin{aligned} \widehat{G}(L)z_t &= \begin{pmatrix} G(L) & -F(L)(I - M(L)) \\ 0 & M(L) \end{pmatrix} \begin{pmatrix} x_t \\ y_t \end{pmatrix} = \\ &= \begin{pmatrix} S(L) & F(L)D(L) \\ 0 & D(L) \end{pmatrix} \begin{pmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \end{pmatrix} = \widehat{S}(L)\varepsilon_{zt} \end{aligned} \tag{14}$$

is obtained, see, e.g., Lütkephol (1991).

Hence, we can reparameterize the original DGP of $\{x_t, y_t\}$ as in (14), and then apply the method in Section 2.1 or 2.4 to obtain the DGP of the aggregated process $\{x_{kt}, y_{kt}\}$. Unfortunately, it is then difficult to determine in general whether the resulting $VARIMA$ model still admits a $VARIMAX$ representation.[6]

Finally, as for the $VARIMA$ case, there can be intermediate situations where an aggregated $VARIMAX$ process can be obtained by increasing the degree in L of $\overline{B}(L)$ in Proposition 8.

3.2 Average sampling

The generating process of $\{x_t, y_t\}$ is still represented by (12) but this time temporal aggregation requires the application to all the elements of the

[5] The order of the AR and MA components, and of the highest lag of y_k in the DGP of x_k should be considered as upper bounds because, in particular cases, lower orders could be obtained.

[6] Tiao and Wei (1976) analyse a bivariate model and notice that temporal aggregation often destroys the $VARIMAX$ structure.

filter $\omega(L) = \omega_0 + \omega_1 L + ... + \omega_{k-1} L^{k-1}$ in a first step, which leads to the process $\{x_t^*, y_t^*\}$. Then, point in time sampling at frequency k from $\{x_t^*, y_t^*\}$ determines the desired temporally aggregated process $\{x_{kt}, y_{kt}\}$.

The first step generates the system

$$\begin{pmatrix} G(L) & -F(L) \\ 0 & M(L) \end{pmatrix} \begin{pmatrix} x_t^* \\ y_t^* \end{pmatrix} = \begin{pmatrix} S(L) & 0 \\ 0 & D(L) \end{pmatrix} \begin{pmatrix} \omega(L)\varepsilon_{xt} \\ \omega(L)\varepsilon_{yt} \end{pmatrix}. \tag{15}$$

Notice that the matrix of coefficients in the left hand side is independent of the choice of $\omega(L)$ and is equal to that in (12). The consequence is that the matrix $\overline{B}(L)$ which is required for the second step, point-in-time sampling from $\{x_t^*, y_t^*\}$, is exactly equal to that in the former subsection. Hence, when such a matrix exists, we can apply Proposition 8 with (15) as the original DGP in order to derive the $VARIMAX$ DGP of the average sampling temporally aggregated process $\{x_{kt}, y_{kt}\}$. When it does not exist, we can reparameterize (15) as in (14), and then apply the method in Section 2.1 or 2.4.

In both cases, the resulting aggregated AR component is equal to that for point-in-time sampling and, more generally, it is independent of the weights, while there are differences in the MA component.

3.3 Mixed sampling

When a different temporal aggregation scheme is applied to the variables under analysis, it is convenient, as a first step, to premultiply both sides of (12) by a diagonal matrix whose terms are given by the product of the different aggregation schemes. It is obtained that

$$\Omega(L)\overline{G}(L)z_t = \Omega(L)\overline{S}(L)\varepsilon_{zt}. \tag{16}$$

with

$$\underset{n+r\times n+r}{\Omega(L)} = \begin{pmatrix} \prod_{i=1}^{n+r} \omega_i(L) & & 0 \\ & ... & \\ 0 & & \prod_{i=1}^{n+r} \omega_i(L) \end{pmatrix},$$

where $\omega_i(L) = (\omega_{i0} + \omega_{i1} L + \omega_{i2} L^2 + ... + \omega_{ik-1} L^{k-1})$ leads to the desired aggregation of the i^{th} variable in z, z_i^*.

(16) can then be rewritten as

$$\begin{pmatrix} G^{**}(L) & -F^{**}(L) \\ 0 & M^{**}(L) \end{pmatrix} \begin{pmatrix} x_t^* \\ y_t^* \end{pmatrix} = \begin{pmatrix} S^{**}(L) & 0 \\ 0 & D^{**}(L) \end{pmatrix} \begin{pmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \end{pmatrix}, \tag{17}$$

and the second step consists in applying point–in-time sampling to (17), which is still a $VARIMAX$ model. Hence, the approach in Section 3.1 can be adopted to obtain the DGP of the mixed sampling temporally aggregated variables.

4 Conclusions

The adoption of temporally aggregated data in empirical analysis renders it important to study the relationship between the DGPs of the original and aggregated variables, and this is the leading theme of this paper. We have considered three main types of temporal aggregation, point-in-time, average and mixed sampling, under the assumption that the original DGP belongs to the $VARIMA$ or $VARIMAX$ type of processes in discrete time. Hence, we have provided concise formulae to determine the DGP of the temporally aggregated variables for each of these cases.

The original and aggregated DGPs can be rather different, and this suggests that when theoretical economic models are confronted with real data, some attention should be paid to the relationship between the theoretical data generating frequency and their actual observation frequency. In particular, when economic models imply that the variables are generated by a $VARIMAX$ process, as it is often the case, and explicitly state their hypothesised generating frequency, the results in this paper can be applied to obtain the theoretical aggregated generating model. If the latter is compatible with the available aggregated data, then the original model is corroborated.

As an alternative, it can be analysed whether some particular characteristics of the original DGP are invariant to temporal aggregation and can therefore be tested with aggregated data. This is also relevant when the original generating frequency is left unspecified or when the particular aggregation scheme that has generated the available data is uncertain. In a related paper, Marcellino (1995b), we have studied the effects of temporal aggregation on such characteristics as common trends, common cycles, Granger noncausality and different notions of exogeneity. The main result is that only those features which are related to the long run, such as the cointegration rank and vectors, are in general invariant to temporal aggregation, while there can be substantial modifications in the other characteristics, and the methods in the present paper permit the determination of these modifications. Such a result reinforces the idea that it is important to match the theoretical and actual generating frequency of the data before testing an economic model.[7]

Thus, an interesting subject of future research can be the analysis of the effects of temporal aggregation on a more general original DGP and for other temporal aggregation schemes, e.g., non linear models and average sampling with time varying weights could be considered.

[7]Marcellino (1995b) also presents theoretical and empirical examples which highlight the practical relevance of the temporal aggregation issue.

Appendix

Proof of Proposition 1.

Premultiplication of both sides of (1) by $B(L)$ leads to

$$\begin{gathered}(I - B_1 L - ... - B_{gk-g} L^{gk-g})(I - G_1 L - ... - G_g L^g) x_t = B(L) S(L) \varepsilon_{xt} \text{ or} \\ (I - C_1 L^k - C_2 L^{2k} - ... - C_g L^{kg}) x_t = N(L) \varepsilon_{xt} \text{ or,} \\ (I - C_1 Z - C_2 Z^2 - ... - C_g Z^g) x_{kt} = u_{xt}.\end{gathered} \tag{18}$$

Thus, the AR component in the DGP of x_k is still of order g, and its coefficients are those which are not equal to zero in the product $B(L)G(L)$.

The autocovariance function of the hypothesised aggregated MA component is:

$$\Gamma_k(j) = cov(u_{xt}, u_{xt-jk}) = \begin{matrix} \sum_{i=0}^{\alpha-jk} N_{i+jk} \Upsilon_x N_i', & \text{for } j \in N : \alpha \geq jk \\ 0 & \text{for } j \in N : \alpha < jk \end{matrix},$$

with $\alpha = gk - g + s$, $N(L) = (I - N_1 L - ... - N_\alpha L^\alpha)$, $N_0 = I$. Actually, this is the autocovariance function of an $MA(h)$ process, where h is the highest value of j such that $\Gamma_k(j) > 0$. The value of h depends on g, s, and k and the different possibilities are summarised in Table 1. The coefficients of the MA component $H(Z) = (I - H_1 Z - ... - H_h Z^h)$ and Υ_{kx} are the solutions of the nonlinear system:

$$\Gamma_k(j) = \sum_{i=0}^{h-j} H_{i+j} \Upsilon_{kx} H_i', \quad \text{for } j = 0, 1, ..., h,$$

with $H_0 = I$.

Hence, we have fully characterised the DGP of the point-in-time temporally aggregated variables x_k, which is the $VARMA$ model

$$C(Z) x_{kt} = H(Z) \varepsilon_{kxt}. \ \square$$

Proof of Proposition 2.

The coefficient of L^i in the product $B(L)G(L)$ is given by $\sum_{h,j} B_h G_j$ for all h and j such that $h + j = i$, $i > 0$. Hence, this coefficient coincides with the i^{th} column of the $1 \times gk$ vector $B^v G^m - G^v$.

$B(L)$ has to be such that the coefficients which are not referred to a multiple of L^k in $B(L)G(L)$ are equal to zero. These coefficients can be grouped in the $1 \times gk - g$ vector $B^v G^m_{-k} - G^v_{-k}$ and, therefore, the elements of $B(L)$ have to satisfy the linear system

$$B^v G^m_{-k} - G^v_{-k} = 0.$$

If $|G^m_{-k}| \neq 0$, then the former system can be solved and it is $B^v = G^v_{-k}(G^m_{-k})^{-1}$. The coefficients of the aggregated AR component, $C(Z)$, are those in the k^{th} columns of the vector $G^v_{-k}(G^m_{-k})^{-1}G^m - G^v$. □

Proof of Proposition 3.

We wish to show that for the representation of the process in (3) there always exists a $B^*(L)$ matrix of degree $gkn - gn$ in L such that the coefficients of the lags which are not multiple of k in the product $B^*(L)G^*(L)$ are zero. If this is true, then we can apply Proposition 1 to completely characterise the DGP of x_k.

Given that $g(L)$ is a scalar polynom of degree gn in L, it can always be factored into

$$g(L) = \prod_{i=1}^{gn}(1 - \gamma_i L).$$

Let us introduce a scalar polynom of degree $gkn - gn$ in L, $b(L)$, with

$$b(L) = \prod_{i=1}^{gn}(\sum_{j=0}^{k-1} \gamma_i^j L^j).$$

It turns out that

$$b(L)g(L) = \prod_{i=1}^{gn}(1 - \gamma_i^k L^k) = c(L^k) = c(Z).$$

Thus, $B^*(L)$ is a diagonal matrix whose terms on the diagonal are all equal to $b(L)$. The autoregressive component of the aggregated DGP, $C^*(Z)$, is then of degree gn in Z, it is also diagonal and its terms on the diagonal are all equal to $c(Z)$.

The order of the MA component, h^*, corresponds to the highest j such that $\Gamma^*_k(j) > 0$, with

$$\Gamma^*_k(j) = cov(u_{xt}, u_{xt-jk}) = \begin{matrix} \sum_{i=0}^{\alpha-jk} N^*_{i+jk}\Upsilon_x N^{*\prime}_i, & \text{for } j \in N : \alpha \geq jk \\ 0 & \text{for } j \in N : \alpha < jk \end{matrix},$$

where $\alpha = gkn - gn + s + g(n-1)$, $u_{xt} = N^*(L)\varepsilon_{xt}$, $N^*(L) = (I - N_1^*L - N_2^*L^2 - ... - N_\alpha^*L^\alpha) = B^*(L)G^*(L)$, $N_0^* = I$. The different possible values of h^* are actually those in Table 1 when g and s are substituted with gn and $s + g(n-1)$. Finally, the coefficients of the MA component and Υ_{kx}^* are the solutions of the nonlinear system

$$\Gamma_k^*(j) = \sum_{i=0}^{h^*-j} H_{i+j}^* \Upsilon_{kx}^* H_i^{*\prime}, \quad \text{for } j = 0, 1, ..., h^*,$$

with $H_0^* = I$. □

Proof of Proposition 4.

To derive the generating process of x_k, we premultiply both sides of (1) before by $\omega(L) = (\omega_0 + \omega_1 L + ... + \omega_{k-1}L^{k-1})$ and then by $(I - B_1L - ... - B_{gk-g}L^{gk-g})$. It follows that:

$$\begin{gathered}(I - B_1L - ... - B_{gk-g}L^{gk-g})(I - G_1L - ... - G_gL^g)\omega(L)x_t = \\ = B(L)S(L)\omega(L)\varepsilon_{xt} \text{ or} \\ (I - C_1L^k - C_2L^{2k} - ... - C_gL^{kg})x_t^* = M(L)\varepsilon_{xt} \text{ or,} \\ (I - C_1Z - C_2Z^2 - ... - C_gZ^g)x_{kt} = u_{xt}.\end{gathered} \tag{19}$$

Thus, the aggregated AR component is still of order g and it is independent of the weighting scheme.

The autocovariance function of the hypothesised aggregated MA component is:

$$\Gamma_k(j) = cov(u_{xt}, u_{xt-jk}) = \begin{array}{ll} \sum_{i=0}^{\alpha-jk} M_{i+jk}\Upsilon_x M_i', & \text{for } j \in N : \alpha \geq jk \\ 0 & \text{for } j \in N : \alpha < jk \end{array}$$

where $\alpha = gk - g + s + k - 1$, $M(L) = (I - M_1L - ... - M_\alpha L^\alpha)$, $M_0 = I$. This is the autocovariance function of an $MA(p)$ process, where p is the highest value of j such that $\Gamma_k(j) > 0$. The actual value of p depends on g, s, and k and the different possibilities are summarised in Table 1. The coefficients of the MA component $P(Z) = (I - P_1Z - ... - P_pZ^p)$ and Υ_{kx} are the solutions of the nonlinear system:

$$\Gamma_k(j) = \sum_{i=0}^{p-j} P_{i+j}\Upsilon_{kx}P_i', \quad \text{for } j = 0, 1, ..., p.$$

with $P_0 = I$.

Hence, we have fully characterised the DGP of the average sampling temporally aggregated process x_k, which is the $VARMA$ model

$$C(Z)x_{kt} = P(Z)\varepsilon_{kxt}. \ \square$$

Proof of Proposition 5.

The demonstration for the AR component is equal to that in the proof of Proposition 3, when x is substituted with x^*.

The order of the MA component, p^*, is equal to the highest j such that $\Gamma_k^*(j) > 0$, with

$$\Gamma_k^*(j) = cov(u_{xt}, u_{xt-jk}) = \begin{array}{ll} \sum_{i=0}^{\alpha-jk} M_{i+jk}^* \Upsilon_x M_i^{*\prime}, & \text{for } j \in N : \alpha \geq jk \\ 0 & \text{for } j \in N : \alpha < jk \end{array},$$

where $\alpha = gkn - gn + s + k - 1 + g(n-1)$, $u_{xt} = M^*(L)\varepsilon_{xt}$, $M^*(L) = (I - M_1^*L - M_2^*L^2 - ... - M_\alpha^*L^\alpha) = B^*(L)G^*(L)\omega(L)$, $M_0^* = I$. The different possible values for p^* are actually those in Table 1 when g and s are substituted with gn and $s + g(n-1)$. Finally, the coefficients of the MA component and Υ_{kx}^* are the solutions of the nonlinear system

$$\Gamma_k^*(j) = \sum_{i=0}^{p^*-j} P_{i+j}^* \Upsilon_{kx}^* P_i^{*\prime}, \quad \text{for } j = 0, 1, ..., p^*,$$

with $P_0^* = I$. $\square$

Proof of Proposition 6.

Premultiplying both sides of (1) by $\Omega(L)$, we obtain:

$$\Omega(L)G(L)x_t = \Omega(L)S(L)\varepsilon_{xt}. \tag{20}$$

However, in this formulation x_i^* depends on $\omega_i(L)x_j$ with $i, j = 1, ..., n$ and $j \neq i$. But we want x_i^* to depend on x_j^* and not on $\omega_i(L)x_j$. Therefore, we have to premultiply both sides of (20) by another diagonal matrix, $\Omega_{xx}(L)$, with

$$\Omega_{xx}(L) = \begin{pmatrix} \prod_{i\neq 1}\omega_i(L) & 0 & ... & 0 \\ 0 & \prod_{i\neq 2}\omega_i(L) & ... & 0 \\ ... & & & \\ 0 & 0 & ... & \prod_{i\neq n}\omega_i(L) \end{pmatrix},$$

so that

$$G^{**}(L)x_t^* = \Omega_{xx}(L)\Omega(L)G(L)x_t = \Omega_{xx}(L)\Omega(L)S(L)\varepsilon_{xt} = S^{**}(L)\varepsilon_{xt}. \tag{21}$$

In general, the degree in L of $G^{**}(L)$, g^{**}, is $g+(k-1)(n-1)$, while that of $S^{**}(L)$, s^{**}, is $s+(k-1)n$. A lower degree is obtained when some of the weighting schemes are equal, the extreme cases being average sampling, where all the weighting schemes are equal and $\Omega_{xx}(L)=I$, and point-in-time sampling, where they are all equal to one and $\Omega_{xx}(L)=\Omega(L)=I$.

Once the $VARMA$ DGP of x^* is obtained, that of x_k can be derived by applying Proposition 1 with (21) instead of (1) as the original DGP. □

Proof of Proposition 7.

Premultiplying both sides of (3) by $\Omega(L)$ we get:

$$\begin{array}{c} \Omega(L)G^*(L)x_t = \Omega(L)S^*(L)\varepsilon_{xt} \text{ or} \\ G^*(L)x_t^* = T^*(L)\varepsilon_{xt}. \end{array} \tag{22}$$

Hence, we can apply Proposition 2 with (22) instead of (3) as the original DGP. □

Proof of Proposition 8.

If both sides of (12) are premultiplied by $\overline{B}(L)$, it is obtained that

$$\begin{array}{c} \begin{pmatrix} B_1(L)G(L) & -B_1(L)F(L)+B_2(L)M(L) \\ 0 & B_3(L)M(L) \end{pmatrix} \begin{pmatrix} x_t \\ y_t \end{pmatrix} = \\ = \begin{pmatrix} B_1(L)S(L) & B_2(L)D(L) \\ 0 & B_3(L)D(L) \end{pmatrix} \begin{pmatrix} \varepsilon_{xt} \\ \varepsilon_{yt} \end{pmatrix}, \end{array} \tag{23}$$

and

$$\overline{B}(L)\overline{G}(L) = \overline{C}(L) = \begin{pmatrix} C_1(Z) & C_2(Z) \\ 0 & C_3(Z) \end{pmatrix}.$$

It is immediate that $B_1(L)$ must be equal to $B(L)$ which implies $C_1(Z)=C(Z)$, and the AR component in the DGP of x_k is still of order g. Similarly, $B_3(L)$ is required to have degree $mk-m$ in L so that $C_3(Z)$, which is the AR component in the DGP of y_k, is of degree m.

The determination of the required degree of $B_2(L)=(B_{21}L+B_{22}L^2+\ldots+B_{2\beta}L^{\beta})$ and of the degree of $C_2(Z)$ is instead slightly more complicate because some subcases must be considered. We discuss in details two of them, and the other ones can be dealt with in a similar manner. In the first subcase, it is assumed that $g \leq f$, $f-g<k$, $g \geq m$. The degree in L of $B_1(L)F(L)$ is $gk-g+f$ and β has to be equal to $gk-g+f-m$, so that the degree of $-B_1(L)F(L)+B_2(L)M(L)$ is $gk-g+f$. The number

of matrices of coefficients in $B_1(L)F(L)$ and $B_2(L)M(L)$ whose power in L is not a multiple of k is $gk - g + f - g$, which is smaller than the number of matrices in $B_2(L)$, $gk - g + f - m$. Therefore, under the maintained existence hypothesis, it is possible to choose the elements of $B_2(L)$ in such a way that all the terms in $-B_1(L)F(L) + B_2(L)M(L)$ whose power in L is not a multiple of k have zero coefficients. But there would still be $(g-m)nr$ degrees of freedom in the choice of the elements of $B_2(L)$. Thus, further restrictions are needed for $B_2(L)$ to be univocally determined, and we assume that it is possible to equate to zero the matrices of coefficients of the $g-m$ highest power in L^k in $-B_1(L)F(L)+B_2(L)M(L)$. It follows that the degree in L of $C_2(L^k) = -B_1(L)F(L)+B_2(L)M(L)$ is km, that of $C_2(Z)$ is m, and this is also the highest lag of y_k in the DGP of x_k.

If instead it is still $g \leq f$, $f-g<k$, but $g<m$, then the number of matrices of coefficients in $B_1(L)F(L)$ and $B_2(L)M(L)$ whose power in L is not a multiple of k, $gk - g + f - g$, is larger than the number of matrices in $B_2(L)$, $gk-g+f-m$. In this case we are short of $(m-g)nr$ conditions and to find these we have to increase the degree in L of $B_2(L)$ by $(m-g)k$.[8] The degree of $B_2(L)$ becomes $mk - g + f - m$, that of $C_2(L^k)$ mk, and the highest lag of y_k in the DGP of x_k is again m.

For other combinations of g, f, m, and k the required degree of $B_2(L)$ can be determined as in the former cases, while it turns out that the highest lag of y_k in the DGP of x_k is still m.

The order of $\overline{H}(Z)$, the MA component in the DGP of z_k, is equal to the highest lag of L^k among those in $B_1(L)S(L)$, $B_2(L)D(L)$, and $B_3(L)D(L)$. Actually, it can be easily shown that this value, h, is such that the autocovariance function of the aggregated error process is different from zero for lower lags than h, and equal to zero for higher ones. The coefficients of $\overline{H}(Z)$ and Υ_{zk} can be determined as in Proposition 1, namely, by solving the nonlinear system which is obtained by equating the autocovariance function of the aggregated error process to that of an $MA(h)$ process. □

Proof of Proposition 9.

The proof of the existence of $B_1(L)$ and $B_3(L)$ and of the formula for their coefficients is equal to that in Proposition 2. $B_2(L)$ has to be such that

[8] Notice that an increase in the degree of $B_2(L)$ determines also an increase in the number of terms of $B_2(L)M(L)$ which are not a multiple of L^k, and therefore have to disappear. Thus, it is not enough to increase the degree of $B_2(L)$ by exactly $m-g$.

the coefficients which are not referred to a multiple of L^k in $-B_1(L)F(L)+B_2(L)M(L)$ are equal to zero. If $m=g$ and we group these coefficients in the $1 \times gk-2g+f$ vector $B_2^v G_{2-k}^m - G_{2-k}^v$, the elements of $B_2(L)$ have to satisfy the linear system

$$B_2^v G_{2-k}^m - G_{2-k}^v = 0.$$

When $|G_{2-k}^m| \neq 0$, such a system can be solved and it is $B_2^v = G_{2-k}^v (G_{2-k}^m)^{-1}$.

If $m<g$, then the number of rows in G_{2-k}^m is larger than that of columns. Hence, G_{2-k}^m has to be substituted with the square matrix G_{2-k}^{m*}. $|G_{2-k}^{m*}| \neq 0$ implies $B_2^v = G_{2-k}^{v*}(G_{2-k}^{m*})^{-1}$. □

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Table 1: DGP of x_k when x is VARMA(g,s) and B(L) exists.

Point-in-time sampling	Average sampling
$VARMA(g, g-1-q)$ for $qk < g-s \leq (q+1)k$ $q = 0, 1, ..., g-1$	$VARMA(g, g-q)$ for $qk < g-s+1 \leq (q+1)k$ $q = 0, 1, ..., g$
$VARMA(g, g)$ for $g = s$	$VARMA(g, g)$ for $g = s$
$VARMA(g, g+q)$ for $qk \leq s-g < (q+1)k$ $q = 0, 1, ...$	$VARMA(g, g+1+q)$ for and $qk \leq s-1-g < (q+1)k$ $q = 0, 1, ...$

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