Coalitions in International Monetary Policy Games

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Abstract

A well-known result from the analysis of monetary policy coordination of two countries is that coordination of the two policies pareto-dominates the outcome of the non-cooperative game. Hence, both countries will always have an incentive to form a Union when it is ensured that the other country joins it as well.

When analyzing coalition formation in international monetary policy coordination we have to consider at least three countries. We show in a n-country (symmetric) framework that the two-country result cannot be extended straightforwardly. In fact, the coalition formation might stop when three countries are in the coalition because it is then better for an individual country not to join the coalition. This result highlights the fact that the coalition formation process itself has positive spillovers for the *countries outside* through reducing the negative externalities created by non-coordinated policies. When there are three countries in the coalition it is better for a country outside to play an optimal response to the coalition's policy than to reduce the remaining externalities by joining the coalition.

1 Monetary policy coalitions

Coalitions in international economic policy coordination can be observed in a broad range of fields. OPEC might serve as an example, as well as trade blocs such as NAFTA, the European Union and the recently formed bloc in South East Asia. The EMU, especially the idea of a 'hard core EMU', is a putative example of a coalition in monetary policy at the international level. Proposals for 'Hard core EMU' and 'multispeed EMU' have one common idea: it might be optimal to form a monetary union where not all possible members are in the union.

Academic research has tried to explain the existence of policy coordination modelling a stabilization game which the governments play against each other after their economies suffered an exogenous shock. The equilibrium outcome is determined by the optimal strategies. These may be pareto-optimal or optimal in the sense of choosing an optimal response for each country individually. Correspondingly, the result could be a fully cooperative outcome or a Nash equilibrium. When more than two players are involved game-theoretical structures like coalition formation can evolve. Hence, a third possible outcome is the existence of a coalition implying partial coordination, where countries inside the coalition coordinate their policies and the countries outside play a non-cooperative Nash game against it. This is the possibility addressed in the current paper.

A similar problem structure underlies the literature on cartel formation in industrial organization. Non-coordinated behaviour of firms leads to a pareto-inferior market performance of the firms; the firms impose negative externalities on each other. It has been shown that explicit consideration of coalition formation in n-firm models creates additional aspects to the duopoly case[1]. The coalition formation itself imposes positive effects on the firms in the fringe through the more disciplined cartel behaviour. D'Aspremont et al. [6] have shown that there is an optimal size of the cartel where the cartel members prefer to stay in the cartel whereas the firms in the fringe prefer not to join the cartel. These results have been refined in various ways in the cartel literature and, hence, the methodological tools on cartel formation are well developed. It can be expected

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[1] For an overview of these results on collusion in industrial organization see e.g. the graduate textbook of Martin [10].

that the application of these tools to models of international policy coordination should give further insights on why and how coalitions in policy coordination are formed. However, these tools can only be applied in a framework with more than two countries.

Academic research in international monetary policy coordination has often been concerned with the question when policy coordination is beneficial. However, such benefits can be demonstrated in the context of two countries. Consequently, theory focussed on two country models. A partial justification of such an approach is found in Hamada [8], who in a symmetric setup with more than two countries compares the Nash, the one-country Stackelberg leadership and the fully cooperative outcome, giving equal weight to each country in the cooperative objective function. Since his results are not essentially different from the two country case, economists felt justified in interpreting each country in the two country model as a 'bloc' of countries that have (in the aggregate) symmetric relations (in terms of size, influence on world markets, etc.). However, such an approach neglects questions of coalition formation, questions which are typically not addressed even in models with more than two countries. A few recent contributions can be seen as trying to fill this gap. I shall discuss three models which serve as an example of the different directions taken in this literature, viz. Canzoneri [3] and Alesina and Grilli [1] while my own model in section 2 is based on Henderson and Canzoneri [5].

Canzoneri [3] examines exchange intervention policy in a two and three country setup in order to find the optimal exchange rate regime and the effects of a currency union on a non-union country. The government, whose ultimate aim is to smooth employment fluctuations[2], has to face two types of shock, goods market disturbances and financial market disturbances. In order to highlight exchange rate interventions governments are assumed to peg nominal interest rates. Canzoneri's policy recommendation in the two country case is to smooth exchange rate fluctuations if financial disturbances are large relative to goods market disturbances. Unfortunately the two country results cannot be used to indicate who should form a policy union in the sense of a fixed exchange rate regime. While financial disturbances within the union are not a source of conflict between union members, di-

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[2]In most other papers in international monetary policy coordination governments minimize a loss function of employment/output and inflation.

sturbances between a union member and a non-member can be, because the union transmits the volatile flows between one member country and the non-member to the second member country, although this country originally did not experience any disturbances vis á vis the non-member. The lesson is that the formation of a union, normally understood as a fixed exchange rate regime, can cause externalities for non-union countries. He *assumes* that two countries form a coalition in order to analyze the effect on the third country, rather than considering coalition formation in the sense of whether the third country would prefer to join the union, as well. Canzoneri's result is crucially dependent on asymmetries in the disturbances. In fact, we will show in a symmetric (and somewhat different) model that, up to three countries, it is always beneficial for all countries involved to join the coalition.

Alesina and Grilli [1] draw attention to questions of reputation and credibility. They investigate coalition formation in the sense of a 'multispeed Europe' in a five country model. Although the title of the paper implies international monetary policy issues, it highlights rather commitment technology. Countries differ in their degree of "conservativeness", i.e. in their emphasis on the objective of price stability relative to employment. They show that a coalition might include only two or three countries. The mechanism which drives this result is as follows. The degree of conservativeness of the Union central bank is supposed to be perfectly credible and it is supposed to be set by the most conservative government. A country outside the Union will join the Union if it can gain credibility and improve upon the Nash equilibrium. In that sense the paper is perfectly in line with the results established by Rogoff [11], who shows that a government might be better off appointing an (independent) conservative central banker in order to avoid the time-inconsistency "trap". Alesina and Grilli provide a theoretical argument for the current political discussions about the European central bank, its degree of independence and its influence on price stability in member countries. Unfortunately, the model is not very useful for our analysis since it lacks links between the economies which transmit spillovers. Since we use a static game we do not consider issues like credibility and reputation.

Canzoneri and Henderson [5] use a three country setup where two countries form a coalition by assumption and play Nash against a third country.

They show that the coalition outcome is pareto-better than the outcome of a non-cooperative Nash game if the outsider does not react to the changed policy of the coalition members. But choosing such a strategy means choosing a strategy which lacks credibility because in a Nash game the country outside the coalition will play an optimal response to the others' strategies which are now determined by the coalition's policy. Canzoneri and Henderson show that when assuming an optimal response of the non-member the result is not necessarily pareto-ranked to the non-cooperative Nash-outcome. This can be seen as "partial coordination might not pay off" in the debate whether coordination is beneficial. The result of Canzoneri and Henderson is essentially due to asymmetries in the model. In fact, we will show in this paper model that up to a coalition of three members it is always pareto-better to extend the coalition (countries want to join the coalition which reduces losses in- and outside). When more than three countries are in the coalition, no pareto-ranking of the coalition- extension (and thus of more coordination) is possible, since countries which join the coalition are worse off whereas countries which remain where they are, are better off.

The existing literature on coalition formation in international monetary policy coordination (including the literature on hard core EMU) is based almost exclusively on asymmetries in country size, shocks or central bank reputation. Reputational considerations are important, since lack of credibility with the private sector can make monetary policy ineffective. But reducing the EMU discussion to reputational considerations has one problem. If one wants to have the "toughest" country in the coalition (which *all* the models about hard core EMU do by assumption), another incentive apart from reputation is needed; one *must* model this for reasons of incentive compatibility of the model.

We will not consider questions of reputation and credibility since we will show in a somewhat more basic sense that partial coordination can exist without asymmetries and differences in reputation just by strictly selecting only optimal strategies (optimal for an individual country). The optimal response is rather applied to the choice whether to join the coalition or not. When a country is outside the coalition, it can do what it wants i.e. it will play an optimal response to the other's policies. What makes the result different from the usual Prisoner's dilemma is that when a country joins the

4

coalition (i.e. decides to internalize externalities it imposes on the other members) it can be sure that the other members keep in the coalition (at least in equilibrium) as well. This means that there is no cheating possible in the sense that the other members internalize the externalities they impose on a member. Thus, the Prisoner's Dilemma game is reduced.

In section 2 we will develop a n-country framework of a standard model in international policy coordination, namely the one in Canzoneri and Henderson [5]. We will show in section 3 that the coalition formation process creates positive spillovers on the non-members through the more disciplined policy of the coalition members. A stability condition, similar to the one used for cartels in D'Aspremont et al. [6] gives a clear result: there exists an optimal coalition size of three countries.

2   The underlying economy

The basic models used in monetary policy cooperation can be roughly divided into two types: those with and without inherent dynamics. This subsection presents a typical model without dynamics. An example of a model of this type is that in Canzoneri and Henderson [5][3]. The model here is slightly modified. There are no asymmetries in the economy size and the three country model of Canzoneri and Henderson is extended to the n country case. The n country setup makes the analytical calculations of the equilibrium more complicated. But we need an n country setup where the number of countries n is variable since we will analyze coalition formation depending on n and the coalition size.

All variables represent deviations of actual values from zero-disturbance equilibrium values and are expressed in terms of logarithms except for the interest rate. The individual country's economy is described by a standard model of monetary policy. We will describe an n-country setup. The country's variables are indexed by i; the domestic country is i; $j = 1 \ldots n, j \neq i$ denote the foreign countries. Only some parameters are indexed by i and thus allowed to vary over countries. In particular the openness of a country, to be denoted β, and the weights in the policymakers' loss functions

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[3]Canzoneri and Henderson [4] provide a simplified version of this model without international capital markets.

might be different for different countries.

Each country is specialized in the production of one good. Output y_i increases in employment l_i and decreases with some (world) productivity disturbance x (independently distributed with mean 0). The following equation describes the **supply of the domestic good**, based on the production function.

$$y_i = (1 - \alpha)l_i - x \tag{1}$$

with α between 0 and 1.

Profit-maximizing firms hire labour up to the point at which real wages $(w - p)$ are equal to the marginal product of labour. **Labour demand** is therefore determined by:

$$w_i - p_i = -\alpha l_i - x \tag{2}$$

Monetary policy is effective because of contractually fixed nominal wages. Home wage setters set w at the beginning of the period so as to fix employment at a full-employment level ($l_i = 0$) if disturbances are zero and expectations are fulfilled. They minimize the expected deviation of actual employment from full-employment by **setting the nominal wage**:

$$w_i = m_i^e \tag{3}$$

with m_i^e the expected money supply deviation and w_i the deviation from the full-employment wage-level[4]. Actual labour demand might differ due to unexpected disturbances. It is assumed that the wage setters (e.g. unions) guarantee that labour demanded is always supplied.

The **consumer price index** q_i is a weighted average of the home country's and the foreign countries' price levels where all foreign countries are weighted equally (according to the structure of the demand equation below). β_i is the fraction of imported goods in total consumption (for further explanation see below).

$$q_i = (1 - \beta_i)p_i + \beta_i \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} (e_{ij} + p_j) \tag{4}$$

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[4]Equations 1, 2 and 8 give $m = w + n$. Home wage setters solve the optimization problem $min_w E[n^2] = min_w E[(m - w)^2]$. This is obviously minimized by setting w equal to m^e.

p_i is the price of domestic output and e_{ij} is the nominal exchange rate, i.e. the price of the currency of country j in terms of the domestic currency. From the definition of exchange rates it follows that $e_{ij} = -e_{ji}$, as well as $e_{ij} = e_{ih} - e_{hj}$. The price index can transmit spillovers: monetary policy abroad affects the exchange rate and the price level abroad. The domestic price index is affected through the share of the price of imported goods. An increase in the price of a foreign good or a rise in the exchange rate causes a rise in prices in the home country.

The **real exchange rate** z, i.e. the relative price of the foreign good in terms of the domestic good, is defined as:

$$z_{ij} = (e_{ij} + p_j - p_i) \tag{5}$$

Contractionary monetary policy in the home country improves the terms of trade, lowers the price of imports and thus lowers inflation. Abroad, the price of imports is increased, thus causing inflation. If all policymakers perform anti-inflationary policy, monetary policy will have a negative externality.

The **demand for the good in the home country** is:

$$y_i = \delta \sum_{\substack{j=1 \\ j \neq i}}^{n} z_{ij} + (1 - \beta_i)\epsilon y_i + \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_j \epsilon y_j - (1 - \beta_i)\nu r_i - \frac{1}{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_j \nu r_j + u \tag{6}$$

Demand for the domestic good rises with y_h, $h = 1, \ldots, n$; residents of each country increase spending by the same fraction ϵ of increases in income y_h. u is a demand shock. The average propensity to import, β_i is equal by assumption to the marginal propensity to import. The parameter β_i is often referred to as 'openness'. A β of zero means that no foreign goods are consumed while a β of one means that no domestic good is consumed. The demand for all goods decreases with expected real interest rates, r_i. The residents in each country spend the amount ν less for each percentage point increase in the expected real interest rate.

The demand for goods includes two possibilities to transmit spillovers: the real exchange rate and the demand for domestic goods in other countries. A fall in the real exchange rate with any other country will have a negative effect on the demand for the domestic good since the relative price of this good abroad has increased. A negative demand shock abroad will affect the demand for the domestic good negatively, as well.

The **expected real interest rate** is:

$$r_i = i_i - q_i^e + q_i \tag{7}$$

where i_i is the nominal interest rate and q_i^e is the expected value of the consumer price index tomorrow based on the information available today.

Goods market equilibrium is obtained by equating demand and supply. The **market equilibrium for money** is realized when the money supply satisfies a simple Cambridge equation:

$$m_i = p_i + y_i \tag{8}$$

International capital mobility and perfect substitutability of bonds give an additional condition:

$$i_i = i_j + e_{ij}^e - e_{ij} \tag{9}$$

for all $i, j = 1, \ldots, n$. Only with this condition will private agents hold positive amounts of both bonds. Clearly, the capital market provides a source for spillovers through the exchange rate.

The capital market assumption needs some further consideration with respect to monetary policy coalitions. It is not exactly determined what capital market arrangement one has in mind when speaking of coalitions in monetary policy. If we speak of a currency union, in general terms we mean a common currency or a fixed exchange rate regime. Then, equation (9) is reduced to equality of the interest rates. We could imagine that the government pegs the interest rate and can then use the money supply to stabilize the exchange rate. Monetary policy is then constrained. In the union we have to stabilize $(n - 1)$ exchange rates, but we have n money supplies. This leaves one money supply free for optimizing the common loss function. The question is then which country can set the money supply 'freely'. The other countries will have to adapt their money supplies so as to peg their exchange rates.

Another possibility would be to have – as in Canzoneri and Henderson [4] – no international capital markets. The only available assets are money. But then we have a kind of restriction on the propensities to import. If there is no capital mobility, trade must always be balanced. Since natural rates

8

of output are the same in all countries, the average propensities to import must match each other. The same average propensities for all countries would do this job, but some asymmetric constellations as well (e.g. every country trades with its partner to the 'left' twice as much as with all other countries). For the time being, we will use the assumptions underlying equation (9): a floating exchange rate regime and perfect international capital markets.

2.1 Policymakers' objectives

Welfare is captured in the policy objective function. This is typically a Tinbergen type quadratic loss function over the deviation of macroeconomic variables (employment and inflation[5]) from some target values (natural rate of employment and often zero inflation).

The objective function is:

$$L_i = \frac{1}{2}(\sigma_i l_i^2 + q_i^2) \tag{10}$$

The parameter σ_i denotes the relative weight the policymaker gives to the objective 'full-employment'. A low σ_i denotes a 'conservative' monetary authority for whom price stability is the ultimate goal. Policymakers minimize this function subject to the restrictions arising from the economy.

2.2 Reduced form of the economy's behaviour

Equations (1) to (9) for all countries $i = 1, \ldots n$ determine the constraints for the policymaker's optimization problem. Essentially, the whole system consists of equations (1) to (4), (6) to (8) each actually representing n equations and equations (5) and (9) which can be reduced respectively to $n-1$ nominal and $n-1$ real exchange rates. This gives $9n-2$ equations with as many variables which have to be solved simultaneously. The money supply m_i is free as an instrument for optimizing the loss function. The equations are the restrictions for the optimization problem. For small n this can be solved e.g. with the Lagrange method. For bigger n the

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[5]A rise in the level q of the CPI is the same thing as inflation if we assume that q_{-1} equals zero.

capacity limits of a computer are easily reached if we cannot reduce the number of equations which restrict the optimization problem.

The reduced form in the case of asymmetric countries ($\beta_i \neq \beta_j$ for at least one pair $\{i, j\}$) is – for n countries – analytically calculable only for very specific cases. Fortunately, in the symmetric case ($\beta_i = \beta_j$ for all $i, j = 1, \ldots, n$) the reduced form can be analytically calculated.

But it is not only considerations of analytical tractability which make it preferable to analyze the case of a symmetric setup. Our model is symmetric although 'real world' countries often differ in their structure and should hence be modelled asymmetrically. If it can be shown that even in a symmetric setup coalitions are an equilibrium outcome, coalition formation does not depend on differences in the countries' structures. Hence, the equilibrium can have asymmetric features although the model structure is symmetric. It is often argued that real world coalitions are based on hegemonic or at least asymmetric structures. This is for example one of the justifications for a hard core EMU: if countries differ too much, especially in the degree of conservativeness of their central banks, the 'core' coalition members will not allow them to join the coalition. But the existence of an asymmetric equilibrium in a symmetric model means in the context of the hard core EMU discussion that policy should not only focus on the reduction of differences as it is in the 'convergence hypothesis' for the EU, i.e. countries have to come closer before forming a coalition. If, on the contrary, it can be shown that there is no incentive in a symmetric environment to form a coalition which does not include all countries, it is justified to focus on asymmetry issues.

Following these considerations we restrict our attention to a completely symmetric structure. This refers to the economies' structure as well as to the type of exogenous shock, where we restrict ourselves to examining the case of a productivity shock x which affects all countries in the same way[6].

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[6]The case of a demand shock (i.e. $u \neq 0$) is different. In this model a symmetric demand shock is neutral, i.e. mathematically it is ruled out when calculating the reduced form. Things probably change when either the country setup or the shock is asymmetric. But the asymmetric country setup unfortunately is analytically not solvable.

The reduced forms for l_i, m_i are[7]:

$$l_i = m_i \tag{11}$$

$$q_i = \lambda m_i - \kappa \sum_{\substack{j=1 \\ j \neq i}} m_j + x \tag{12}$$

with:

$$\lambda = \alpha + \frac{\beta(1-\alpha)\left(1 - \epsilon(1 - \frac{n}{n-1}\beta)\right)}{\delta n + \nu(1 - \frac{n}{n-1}\beta)^2} \quad > 0$$

$$\kappa = \frac{(\lambda - \alpha)}{n-1} > 0 \tag{13}$$

I will briefly explain the reduced form. Each country's employment rises one for one with the domestic money supply. Employment rises one for one with output (according to equation (1)). The price level rises with the money supply. The reason is that the price of the domestic good rises, since real wages have to match the increase in employment. The exchange rate depreciates. Consequently, the consumer price level, which is a weighted sum of the domestic good price and the prices of the imported goods, rises. This is reflected in equation (12) because λ is positive.

A symmetric world productivity disturbance gives rise to a stabilization game. Without policy intervention a negative disturbance $(x > 0)$ would have no effect on employment. According to equation (1), output decreases and the domestic good price level increases, since employment only remains constant if the real wage falls, i.e. the price of domestic goods rises. There is no change in the real exchange rate. Consequently, the consumer price index rises. Each policymaker – facing a loss function which increases in the square of employment and CPI deviations – now has an incentive to contract the money supply a little bit in order to lower inflation. He accepts the small loss from reducing employment below the full employment level in favour of the significant gain from lowering inflation. Contractionary monetary policy leads to an appreciation of the exchange rates of the domestic currency and thus creates inflation abroad, since the price index abroad contains the relative prices of imported goods as well. This externality is reflected in the negative coefficient (κ is positive) of

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[7]The reduced form is derived in Appendix A. The signs of the coefficients are determined in Appendix A, as well.

foreign monetary policy in equation (12). If all policymakers contract money supplies, they impose a negative externality on each other. The result is a competitive appreciation which leads to a contractionary bias in the losses. The exchange rate in the end remains unchanged but all policymakers have contracted too much with respect to their optimal money supply. This could be avoided if all countries coordinated on a less contractionary monetary policy.

3 A non-cooperative game with coalition formation

In the following the model is solved taking a possible coalition formation into account. A coalition is defined as a subset of countries which optimize a common loss function. This common loss function is a weighted average of the individual countries' loss functions.

$$\mathcal{L} = \sum_{j=1}^{k} \alpha_j L_j$$

The weights are denoted α_j and $\sum_{j=1}^{k} \alpha_j = 1$. One could determine the relative weights over some kind of bargaining process. But, as all members have the same economic structure, there is no obvious reason why the result of such a bargaining process should be unequal weights. For the time being, we assume $\alpha_j = \overline{\alpha} = \frac{1}{k}$ for all $j = 1, \ldots, k$.

First, the reaction functions of the countries outside the coalition and of the coalition itself will be determined. The equilibrium is the intersection of the reaction functions. It is dependent on n, the number of countries, and k, the number of coalition members. In subsection 3.2 the equilibrium losses are analyzed with respect to a change of n and k. This is extended to an analysis of the 'stability' of the coalition, using an algorithm drawn from the industrial organization literature.

3.1 The optimal strategies and the equilibrium

We assume that k out of the n countries are members of the coalition C. We order the countries such that countries $1, \ldots, k$ are in the coalition and

12

countries $k+1, \ldots, n$ are outside the coalition.

The countries outside the coalition In order to solve the policymaker's optimization problem when he is outside the coalition, we calculate the Nash strategy. We replace n_i and q_i in the loss function by the reduced form equations. This function is minimized with respect to m_i subject to given strategies of the other countries $m_j = \overline{m_j}$ for all $j \neq i$. The symmetric setup implies that all countries have the same degree of conservativeness σ. Since we have a symmetric structure in every respect, we can assume that all countries outside the coalition have the same optimal money supply m_{nc}^*. We can derive the money supply of a non-member as a function of the coalition's money supply[8]:

$$m_{nc}^* = \theta \sum_{j=1}^{k} \overline{m}_j - \vartheta x \tag{14}$$

with:

$$\theta = \frac{\lambda \kappa}{\sigma + \lambda^2 - \lambda \kappa (n - k - 1)} > 0$$
$$\vartheta = \frac{\theta}{\kappa} > 0$$

Equation (14) deserves some attention. The optimal policy outside the coalition depends positively on coalition policy i.e. the money supplies of a non-member and a coalition member are strategic complements[9]. This means that a less contractionary monetary policy of the coalition members triggers a less contractionary response from the non-members. The reasoning is as follows: the coalition creates less competitive appreciation for the non-members by contracting less. Hence, the countries outside the coalition also need to contract less, because they face less 'imported' inflation. We will refer to this result later.

The behaviour of the coalition In order to solve the optimization problem of the coalition members we have to clarify a further element of

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[8]The results are derived in Appendix B.1.

[9]Strategic complements imply upward sloping reaction functions, see Bulow et al. [2]. The reaction function of a non-member is upward sloping since θ is positive.

13

the structure of the game. The coalition can be involved in a Nash game or in a Stackelberg game with the non-members.

The coalition outcome represented by a Nash equilibrium where the players play cooperatively within the coalition cannot be achieved without a commitment technology since the coalition members are off their individual Nash reaction functions. We have to assume that the coalition members can enter into a binding agreement that is known about by all players. But then, it is not clear why the coalition would not use this commitment technology to behave as a Stackelberg leader in a Stackelberg game and realize a Stackelberg leader profit.

However, the Stackelberg concept gives in general a time-inconsistent result, that is the Stackelberg leader would ex post like to change his strategy and, hence, does not play an optimal response. Only a structural difference in the *timing* of the decision making could explain such behaviour – which is a problem since we have a static model. One could argue that a Stackelberg structure is reasonable in so far as the coalition has to announce its policy at an early stage because all its members have to coordinate on the optimal policy. It sets its money supply before the non-members react or it can credibly commit itself to its monetary policy. But this creates a problem when looking at the coalition *formation* process. There is no obvious reason in the model structure for the Stackelberg leadership of a single country. Hence, when there is only one country in the coalition we should get the non-cooperative Nash equilibrium as the outcome. But assuming a Stackelberg leadership for the coalition implies assuming a Stackelberg leadership for the single country at the 'early' stage of the coalition formation. And this again has to be explained by another structural difference which we have not modelled.

Hence, it is not clear which of the two concepts should be chosen. We will perform the analysis for both structures. We will see that the results do not differ very much qualitatively. Though the results of the Stackelberg game are not robust to changes in the values of the model parameters, the basic argument of the existence of a stable coalition which does not include all countries remains valid in both games.

The Cooperation-Nash equilibrium In the Cooperation-Nash[10] game the coalition solves its optimization problem subject to a given money supply of the non-members. We exploit the symmetry assumption $m_{j,c}^* = m_c^*$ for all $j = 1, \ldots, k$. This gives a coalition member's reaction function which depends on the non-members' money supply. Through equating the reaction functions we obtain the equilibrium of the Nash game with a coalition[11] as:

$$m_c^* = -\rho x \tag{15}$$
$$m_{nc}^* = -\omega x \tag{16}$$

with:

$$\rho = > 0$$
$$\omega = \rho \kappa (n-k) k \theta + (n-k) \vartheta > 0$$

The equilibrium policies in both games are linear functions of the shock x. If the shock is zero, the optimal policies are zero, as well, since there is no need for a stabilization game. If the shock is negative, i.e. $x > 0$, the optimal policy for all countries is a contractionary monetary policy since ρ and ω are positive, respectively.

The coalition eliminates the negative externalities which the member countries impose on each other. The coalition members conduct a less contractionary, and thus less deflationary, policy. But if the coalition countries contract less, the inflation in the non-member countries is lower as well, since the currency of a coalition member appreciates less against *all* currencies. We have seen already that the non-member country has now the possibility of contracting less. It increases employment somewhat without having an inflation as high as it would be without the influence of the coalition policy. This clearly improves the loss function of the non-member. It means that the *coalition formation process produces positive spillovers* for non-members. In the following, we will analyze whether the process of coalition formation might 'stop' at a certain point, since the spillovers from the coalition formation process might be high enough that a country prefers to stay outside.

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[10]The notation of a *Cooperation-Nash equilibrium* and a *Cooperation-Stackelberg equilibrium* is an adaptation from Canzoneri and Henderson [5], Chapter 3.

[11]ρ is quite a long expression which can be checked in Appendix B.1.

The Cooperation-Stackelberg equilibrium In the Cooperation-Stackelberg game the coalition solves its optimization problem under explicit consideration of the non-members' reaction functions. The optimization problem yields the money supplies in the Stackelberg equilibrium[12]:

$$m_c^* = -\varphi x \tag{17}$$

$$m_{nc}^* = -\phi x \tag{18}$$

with:

$$\varphi = \frac{(1 + (n-k)\theta)(\lambda + \kappa - k\kappa(1 + (n-k)\theta))}{\sigma + (\lambda + \kappa - k\kappa(1 + (n-k)\theta))^2} > 0$$

$$\phi = (\kappa k\varphi + 1)\vartheta > 0$$

Again, the equilibrium policies are linear functions of the shock x. If the shock is zero, the optimal policies are zero, as well, since there is no need for a stabilization game. If the shock is negative, i.e. $x > 0$, the optimal policy for all countries is a contractionary monetary policy since φ and ψ are positive, respectively. The interpretation of the Nash game applies accordingly.

3.2 The stability of coalitions in equilibrium

The parameters n, the number of countries, and k, the number of coalition members, are of specific interest with respect to coalition formation. The values of the optimal policies and the losses in equilibrium are dependent on n and k. The decision whether a country would like to join the coalition or whether it would like to leave it, can change when n and k change.

The factors ρ and ω and φ and ϕ, respectively, in the optimal policy are quite complicated and it is not easy to analyze how the model parameters affect the outcome. One possible approach is to perform numerical simulations with specific values for the model parameters whilst varying n and k. I will report the results for a simulation where n varies from 3 to 22 and k varies from 1 to 22[13].

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[12]The analytical part can be checked in Appendix B.2.

[13]The parameter values were: $\alpha = 0.5$, $\beta = 0.5$, $\epsilon = 0.8$, $\nu = 0.05$, $\sigma = 1$ and $\delta = 0.3$. A robustness analysis was performed; the results did not change qualitatively in the Cooperation-Nash game. On the Cooperation-Stackelberg game see Appendix C.

First, the loss functions for coalition members and non-members are analyzed with respect to n and k. Then, a stability condition adapted from the cartel formation literature in industrial organization[14] is used. The loss function of a non-member is denoted by $L_{nc}(n, k)$. If it joins the coalition (and no other country changes from one group to another), it will have the loss $L_c(n, k+1)$. If $L_{nc}(n, k)$ is smaller than $L_c(n, k+1)$, the country has no incentive to join the coalition – the coalition is called "externally stable". A similar condition holds for the coalition members. If $L_c(n, k)$ is smaller than $L_{nc}(n, k - 1)$, the country has no incentive to leave the coalition. The coalition is called "internally stable". If both conditions are fulfilled, the coalition is stable, with size k[15]. If only external stability is fulfilled, the coalition is still stable in some sense, since it is possible that countries which join the coalition are committed to stay in. The commitment can arise from reputational considerations or from a formal international contract.

3.2.1   The Cooperation-Nash game

The loss functions   The losses in equilibrium are determined through equilibrium policies:

$$L_c = \sigma m_c^{*^2} + (\lambda m_c^* - \kappa(k-1)m_c^* - \kappa(n-k)m_{nc}^* + x)^2$$
$$L_{nc} = \sigma m_{nc}^{*^2} + (\lambda m_{nc}^* - \kappa k m_c^* - \kappa(n - k - 1)m_{nc}^* + x)^2$$

The losses varying with k and n are shown in figures 1 and 2. Losses decrease for all values of n with the number of coalition members. The explanation is as follows: the more members a coalition has, the more externalities are internalized. As we have seen above the coalition formation process has a positive externality for non-members: the less contractionary coalition policy evokes a less contractionary optimal policy on the part of

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[14]The stability condition used here is the one proposed by D'Aspremont et al. [6].

[15]One could interpret this condition as an entry or exit condition. However, entry or exit decisions would be better analyzed in an explicit game with sequential entry or exit decision stages. The algorithm here assumes that only the country under consideration takes a decision; all other countries remain in their 'group'. If the result is stability, there is no problem since no one actually will change. But if the result is instability, this algorithm might give an incorrect signal since all members of a group will take the decision to change and not only the country under consideration.

the non-member. The country outside the union will be able to increase employment without increasing inflation. This will lower the losses for both parties.
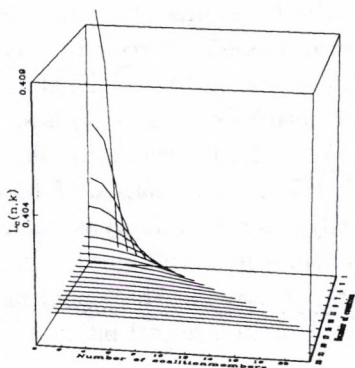

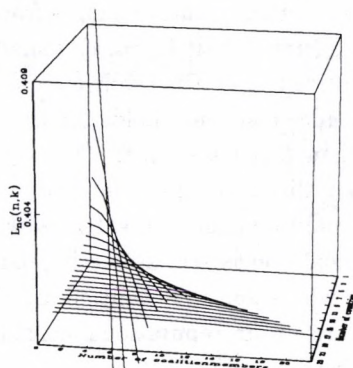Figure 1: Lossfunction of a coalition member

Figure 2: Lossfunction of a country outside the coalition

The stability of the equilibrium   The stability analysis gives the following result (for a graphic illustration see figure 3). The coalition is internally stable only for $k = 2$ and $k = 3$; this is true for all values of n. When the coalition size exceeds three, each coalition member individually could gain by leaving the coalition. The coalition is externally stable for all constellations where three or more countries are in the coalition. When the coalition size is one (in fact, then there is no coalition) or two, a country outside could reduce its losses by joining the coalition. In other words, if *three* of the n countries are in the coalition, no country outside has an incentive to join the coalition. The coalition members do not want to leave the coalition in this situation either. Hence, there exists an equilibrium with a stable coalition which is not joined by all countries. This stable coalition size is three for all n.

I will give a brief explanation why this result is possible even without asymmetries. When a country decides whether to join a coalition or not, two factors are involved. The country balances the gains from entering the coalition against the costs of giving up an optimal policy 'against' the

Gains from "changing the group" (n=22)
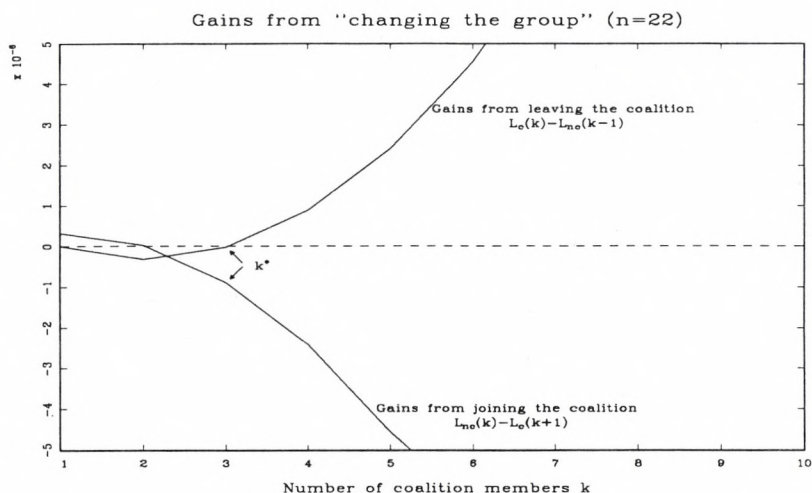
Figure 3: External stability $(L_{nc}(k) - L_c(k+1) \leq 0)$ and internal stability $(L_c(k) - L_{nc}(k-1) \leq 0)^a$

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[a] Negative "gains from changing the group" imply that changing does not pay and, hence, the group is stable. The convex graph shows the internal stability of the coalition where only coalition sizes of three or less are stable (negative gains from leaving). The concave graph shows the external stability where only coalition sizes of three or more are stable (negative gains from joining). Therefore, only a coalition of three countries fulfils both stability criteria.

coalition. The gains from entering arise from the elimination of competitive appreciations against the countries in the coalition. This is reached through a less contractionary monetary policy. The gains from staying outside are given by the possibility of carrying out an optimal policy against the coalition. But the coalition – by contracting less – evokes a less contractionary policy from the non-members. As the size of the coalition is increased, the optimal amount of contraction declines and so the non-member countries contract less, as well. When the coalition has reached a certain size, the optimal response of the non-member is already less contractionary to a certain extent. There is not much to gain by joining the coalition. The countries then prefer to stay outside, as they are no longer engaged in an inflationary appreciation of any considerable extent.

19

The critical size of the coalition does not vary over n[16]. This means that the share of the coalition size relative to the total number of countries converges to zero for a large number of countries. This result is in line with similar results in the cartel literature, e.g. D'Aspremont et al. [6], where in a game where the two strategic variables are complements the stable cartel size is always three. The crucial mechanism which might explain this result is created by the complementarity of the strategies, i.e. reducing the coalition's losses reduces the non-members' losses as well. Each additional coalition member lowers the coalition's losses by internalizing negative externalities. But it affects the non-members' losses in the same way. In this way we can explain as well the result that non-members are better off than coalition members (in the Nash and in the Stackelberg game).

This has to be seen in contrast to models with strategic substitutes which have a stable coalition size with a 'balanced' share of players in a coalition and outside (see e.g. Martin [9] who determines the optimal cartel size in a quantity setting oligopoly). The players divide profits in their respective groups and increasing the coalition's profits implies reducing the merger's profits. This will induce more players to join the coalition than in a game with strategic complements.

Coalition formation process   We will now have a more detailed look at the coalition formation process. For $k = 1$, i.e. the coalition consists of one country which is the same as if there were no coalition, the coalition is externally not stable. Hence, countries have an incentive to join the coalition. But there is a free-rider problem since the countries outside the coalition have lower losses than the coalition members for any combination of n and k (see figure 4). Hence, every country would like the others to join the coalition rather than going ahead itself[17]. But, still, if the others don't "move" (which is the underlying assumption of our stability condition) it is better to join the coalition than to stay outside. Hence, the coalition

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[16]This result even holds when we make β dependent on the number of trading partners n. The 'perfectly symmetric case' of $\beta = \frac{1}{n}$ is only a special case of a fixed β which can take all values between 0 and 1.

[17]In game theoretic terms, we have no dominant strategy. In the stable equilibrium we have in fact multiple Nash equilibria since all possible combinations where three countries are in the coalition constitute stable equilibria. This is a common problem and the question is, which equilibrium will finally be chosen.

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formation can take place up to the size of three. The countries which are then still outside (for reasons which are not determined in the model and probably cannot be determined in a symmetric model) have somehow been 'smarter'.
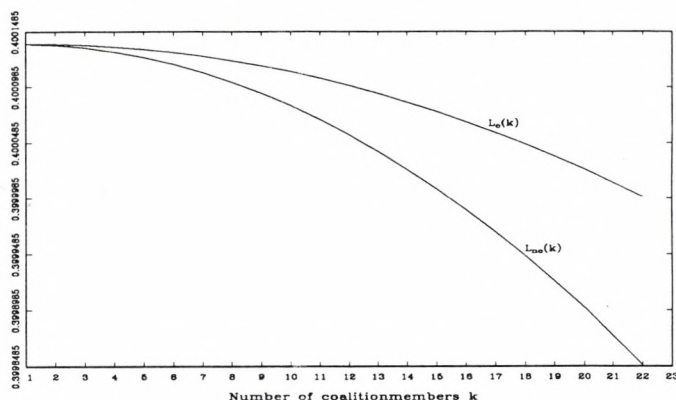

Figure 4: Loss functions inside and outside the coalition (for $n = 22$, k from 1 to 22)

3.2.2 The Cooperation-Stackelberg game

The loss functions and the equilibrium behaviour in the model where the coalition takes a Stackelberg leader position are qualitatively and quantitatively almost identical to the results of the Nash game. Therefore, we will focus here only on the main aspects. The results of the Stackelberg game are discussed in detail in Appendix C.

The Stackelberg game has *one* stable coalition size for each n. This stable coalition size is either two or three.

The losses in the Stackelberg equilibrium are for all parties involved always less than the losses in the Nash equilibrium. Additionally, the non-members have lower losses than the coalition members. The strategic reasoning behind this is that the Stackelberg leader can be never worse off than in the Nash equilibrium. He could always pick his Nash strategy and, hence, realize the Nash losses. Therefore, if he deviates from the Nash equilibrium money supply he does so because he is able to lower his losses

21

by choosing another money supply. The Stackelberg followers (the non-members) have lower losses than in the Nash game because the coalition imposes fewer externalities on them. Since we have strategic complements, the lower coalition money supply triggers a lower optimal money supply from the non-members. These results are in line with general results found by Dowrick [7] where the Stackelberg equilibrium is pareto superior to the Nash equilibrium in games with strategic complements. Though there are some quantitative differences between the Cooperation-Nash and the Cooperation-Stackelberg outcomes, they are very small in size. This may explain why we are still dealing with a optimal coalition size of two or three and not four or five.

4   Conclusions

The exercise in this paper has shown that – in the framework of a standard international policy coordination model – the explicit possibility of coalition formation gives results different from the ones often assumed for coordination models with more than two countries. The existence of a stable coalition size which does not include all countries has two important implications.

First, *assuming* that only some countries join a coalition and reducing the resulting two blocs to a two-country-model might be misleading. In particular, if the number of countries in the coalition bloc is higher than the stable coalition size, the resulting strategies are not optimal in the sense that some countries will prefer to leave the coalition. In fact, 'more' coordination is not always pareto-better than 'less' coordination if we allow for different coalition sizes (and hence, for different degrees of coordination).

Second, in the discussion of asymmetric real world structures like 'hardcore EMU' it is not enough to focus only on asymmetric features, such as central bank preferences. Asymmetric results may be produced by forces which evolve in a symmetric model, merely from the spillover effects of monetary policy, as well.

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A Deriving the reduced form

We calculate the reduced form of the economy in two steps. First, we derive the reduced form for employment, then the reduced form for the CPI.

Reduced form for l_i The reduced form for employment can be easily derived. We substitute equation (3) into (2):

$$p_i = m_i^e + \alpha l_i + x \tag{19}$$

In order to simplify the analysis we assume that the expected money supply (more precisely, its deviation) for wage-setters is $m_i^e = 0$. Substituting (1) and (19) into (8) yields the following expression for employment:

$$\mathbf{l_i} = \mathbf{m_i} - \mathbf{m_i^e} = \mathbf{m_i} \tag{20}$$

Thus, employment changes one for one with the domestic money supply and is not affected by monetary policy abroad.

Reduced form of q_i Deriving the reduced form for the CPI takes a bit longer. We substitute equation (5) into (4):

$$q_i = p_i + \frac{1}{n-1}\beta \sum_{\substack{j=1 \\ j \neq i}}^{n} z_{ij} \tag{21}$$

We will now express the two terms on the right-hand side of this equation in terms of the money supplies. First, we express p_i in terms of m_i by substituting (20) into (19):

$$p_i = \alpha m_i + x \tag{22}$$

Next, we express $\frac{1}{n-1}\sum_{\substack{j=1 \\ j \neq i}}^{n} z_{ij}$ in terms of m_i. We sum (6) from $j = 1, j \neq i$ to n, which means that we obtain double sums on the right hand side, divide it by $(n-1)$ and subtract this equation from itself (i.e. (6) for country i). After collecting terms this yields:

$$
\begin{aligned}
\Big(y_i \ - \ & \frac{1}{n-1}\sum_{\substack{j=1 \\ j \neq i}}^{n} y_j\Big)\Big(1 - (1-\beta)\epsilon + \frac{1}{n-1}\beta\epsilon\Big) \\
= \ & \delta\frac{n}{n-1}\sum_{\substack{j=1 \\ j \neq i}}^{n} z_{ij} - \Big(r_i - \frac{1}{n-1}\sum_{\substack{j=1 \\ j \neq i}}^{n} r_j\Big)\Big((1-\beta)\nu - \frac{1}{n-1}\beta\nu\Big)
\end{aligned} \tag{23}
$$

23

We sum (7) for all countries $j, j \neq i$, divide it by $(n-1)$ and subtract it from (7) for country i. Note that $z_{ij}^e = 0$ which excludes speculative bubbles (for an explanation, see Canzoneri and Henderson [5])[18]. Together with equation (9) and equation (5) we obtain:

$$r_i - \frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} r_j = \left(\frac{\beta n}{n-1} - 1\right)\frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} z_{ij} \tag{24}$$

Using equations (1) and (20) (for all countries $j = 1,\ldots,n$ for the left-hand side of equation (23) and equation (24) for the right-hand side, then solving for the z_{ij}'s gives:

$$\frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} z_{ij} = \frac{(1-\alpha)\left(1-(1-\beta)\epsilon + \frac{1}{n-1}\beta\epsilon\right)}{\delta n + \left(1-\frac{\beta n}{n-1}\right)\left((1-\beta)\nu - \frac{1}{n-1}\beta\nu\right)}\left(m_i - \frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} m_j\right) \tag{25}$$

Substituting equations (25) and (22) into (21) we now obtain the reduced form for q_i:

$$q_i = \underbrace{\left(\alpha + \frac{\beta(1-\alpha)\left(1-\epsilon(1-\frac{n}{n-1}\beta)\right)}{\delta n + \nu(1-\frac{n}{n-1}\beta)^2}\right)}_{\lambda}m_i$$

$$- \underbrace{\frac{\beta(1-\alpha)\left(1-\epsilon(1-\frac{n}{n-1}\beta)\right)}{\delta n + \nu(1-\frac{n}{n-1}\beta)^2}}_{\kappa}\frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} m_j + x \tag{26}$$

Setting the first coefficient to λ and the second to κ, the reduced form for q_i can be rewritten as:

$$\mathbf{q_i} = \lambda\mathbf{m_i} - \kappa\sum_{\substack{j=1 \\ j\neq i}}^{n}\mathbf{m_j} + \mathbf{x}$$

---

[18]Furthermore, note that $z_{ij} = -z_{ji}$. This gives:

$$\sum_{\substack{j=1 \\ j\neq i}}^{n} z_{ij} - \frac{1}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n}\sum_{\substack{l=1 \\ l\neq j}}^{n} z_{jl} = \frac{n}{n-1}\sum_{\substack{j=1 \\ j\neq i}}^{n} z_{ij}$$

24

The sign of the coefficients We will show that the signs of the coefficients λ and κ in equation (26) are positive.

- $\lambda = \alpha + \dfrac{\beta(1 - \alpha)\left(1 - \epsilon(1 - \frac{n}{n-1}\beta)\right)}{\delta n + \nu(1 - \frac{n}{n-1}\beta)^2}$

  The first term of λ is positive, because α is positive; the denominator of the second term of λ is positive, because δ and ν are positive. Hence, λ is positive if the nominator of the fraction is positive. Since β is positive and α is positive and smaller than one, we have to show that:

$$1 - \epsilon\left(1 - \frac{n}{n-1}\beta\right) \;>\; 0$$

$$\underbrace{\frac{1}{\epsilon}}_{\epsilon < 1} > 1 \;>\; \underbrace{1 - \frac{n}{n-1}\beta}_{>0}$$

- $\kappa = \dfrac{\beta(1 - \alpha)\left(1 - \epsilon(1 - \frac{n}{n-1}\beta)\right)}{\delta n + \nu(1 - \frac{n}{n-1}\beta)^2} \dfrac{1}{n-1}$ can be rewritten as $\dfrac{\lambda - \alpha}{n-1}$.

  This fraction is positive for λ is α plus a positive term and n is larger than 2.

B Solving the equilibrium with a coalition

The countries $j = 1, \ldots, k$ are members of the coalition C, the countries $i = k + 1, \ldots, n$ are not in the coalition. The equilibrium is derived in three steps:

- The reaction function of a country outside the coalition

- The reaction function of a coalition member

- The equilibrium through equating the reaction functions

We distinguish two types of behaviour of the coalition against the non-members. The Cooperation-Nash equilibrium implies that the coalition members cooperate (i.e. minimize a joint loss function) amongst the members and then play a Nash game against the non-members. The Cooperation-Stackelberg equilibrium implies a Stackelberg behaviour of the coalition against the non-members.

25

B.1 The Cooperation-Nash equilibrium

The optimization problem which has to be solved by the monetary authority of a country can be summarized as follows. Outside the coalition L_i is minimized with respect to the own money supply; in the coalition L_i is minimized with respect to the money supplies of all coalition members.

$$min_{m_i,m_j} L_i = \frac{1}{2}\left(\sigma m_i^2 + (\lambda m_i - \kappa \sum_{\substack{j=1 \\ j \neq i}}^{n} m_j + x)^2\right)$$

The derivatives are, respectively:

$$\frac{\partial L_i}{\partial m_i} = (\sigma + \lambda^2)m_i - \lambda\kappa \sum_{\substack{j=1 \\ j \neq i}}^{n} m_j + \lambda x$$

$$\frac{\partial L_i}{\partial m_j} = -\lambda\kappa m_i + \kappa^2 \sum_{\substack{j=1 \\ j \neq i}}^{n} m_j - \kappa x$$

The reaction function of a country outside the coalition A country which is not in the coalition sets its own money supply so as to minimize its losses. It takes the other's money supplies as given (Nash conjectures).

$$min_{m_i} L_i \qquad s.t. \ m_j = \overline{m_j} \ \ \forall j \neq i$$

The first order condition $\frac{\partial L_i}{\partial m_i} = 0$ gives $m_{i,nc}^*$ as a function of all other money supplies.

The symmetric setup implies that all countries have the same degree of conservativeness σ. Hence, we can assume that all countries outside the coalition have the same optimal money supply m_{nc}^*. We can write the money supply of a non-member as a function of the coalition's money supply:

$$m_{nc}^* = \underbrace{\frac{\lambda\kappa}{\sigma + \lambda^2 - \lambda\kappa(n-k-1)}}_{\theta} \sum_{j=1}^{k} \overline{m}_{j,c} - \underbrace{\frac{\lambda}{\sigma + \lambda^2 - \lambda\kappa(n-k-1)}}_{\vartheta} x \qquad (27)$$

26

The reaction function of a coalition member The coalition solves its optimization problem subject to a given money supply of the non-members:

$$min_{m_j \in C} \mathcal{L} = \sum_{j=1}^{k} \frac{1}{k} L_j \qquad \text{s.t. } m_i = \overline{m}_{i,nc} \quad \forall i = k+1, \dots, n$$

The first order condition gives:

$$\frac{\partial \mathcal{L}}{\partial m_j} = \frac{1}{k} \frac{\partial L_j}{\partial m_j} + \sum_{\substack{h=1 \\ h \neq j}}^{k} \frac{1}{k} \frac{\partial L_h}{\partial m_j} = 0$$

Together with the symmetry assumption for the coalition money supplies $m_{j,c}^* = m_c^*$ for all $j = 1, \dots, k$ we obtain the coalition member's reaction function dependent on the non-members' money supplies.

$$m_c^* = \frac{\kappa\lambda - \kappa^2(k-1)}{\sigma + \lambda^2 + \kappa^2(k-1)^2 - 2\kappa\lambda(k-1)} \sum_{i=k+1}^{n} \overline{m}_{i,nc} \\ - \frac{\lambda - \kappa(k-1)}{\sigma + \lambda^2 + \kappa^2(k-1)^2 - 2\kappa\lambda(k-1)} x \tag{28}$$

The equilibrium Replacing the non-members' money supply in equation (28) with equation (27) gives the equilibrium money supply of a coalition member:

$$m_c^* = -\frac{(\lambda - \kappa(k-1))(\sigma + \lambda^2 + \kappa\lambda)}{\underbrace{(\sigma + \lambda^2)\eta + \kappa^3\lambda(k-1)(n-1) + \kappa^2\lambda^2(k(n-k) - 2(n-1))}_{\rho}} x$$

$$\eta = \sigma + \lambda^2 + \kappa^2(k-1)^2 - \kappa\lambda(k+n-3)$$

and the equilibrium money supply of a non-member:

$$m_{nc}^* = -\frac{\lambda(\kappa k\rho + 1)}{\underbrace{\sigma + \lambda^2 - \kappa\lambda(n-k-1)}_{\omega}} x$$

The sign of the coefficients We will show that the signs of the coefficients θ, ϑ, ρ and ω are positive.

27

- $\theta = \dfrac{\lambda\kappa}{\sigma + \lambda^2 - \lambda\kappa(n - k - 1)}$

The nominator of θ is positive because λ and κ are positive. Hence, θ is positive if the denominator is positive.

$$\sigma + \lambda^2 - \lambda\kappa(n - k - 1) \;>\; 0$$

$$\sigma + \lambda^2 \;>\; \frac{\lambda(\lambda - \alpha)}{n - 1}(n - k - 1)$$

$$\sigma + \lambda^2 \;>\; \lambda^2 - \alpha\lambda - \frac{\lambda(\lambda - \alpha)}{n - 1}k$$

$$\sigma \;>\; -\lambda\underbrace{(\alpha + \frac{\lambda - \alpha}{n - 1}k)}_{>0 \text{ because } \lambda > \alpha}$$

- $\vartheta = \dfrac{\lambda}{\sigma + \lambda^2 - \lambda\kappa(n - k - 1)}$ can be rewritten as $\dfrac{\theta}{\kappa}$ and, since θ and κ are positive, is positive.

- $\rho = \dfrac{(\lambda - \kappa(k - 1))\,(\sigma + \lambda^2 + \kappa\lambda)}{(\sigma + \lambda^2)\eta + \kappa^3\lambda(k - 1)(n - 1) + \kappa^2\lambda^2\,(k(n - k) - 2(n - 1))}$

with $\eta = \sigma + \lambda^2 + \kappa^2(k - 1)^2 - \kappa\lambda(k + n - 3)$

The nominator of ρ is positive because λ and κ are positive and

$$\lambda - \kappa(k - 1) \;>\; 0$$

$$\lambda - \lambda\frac{k - 1}{n - 1} + \alpha\frac{k - 1}{n - 1} \;>\; 0$$

$$\underbrace{\lambda\frac{n - k}{n - 1}}_{\geq 0 \text{ since } n \geq k} + \alpha\frac{k - 1}{n - 1} \;>\; 0$$

The denominator of ρ is positive for all feasible values of k that is, k between 1 and n. The proof is as follows.

- o We will distinguish two cases: the first case where $\lambda \geq 2\alpha$ and the second case where $\lambda < 2\alpha$.

- o First case: $\lambda \geq 2\alpha$.
  We can rewrite the denominator of ρ as:

$$\underbrace{\sigma(\eta + \lambda^2)}_{\tau_1} + \underbrace{\kappa^3\lambda(k - 1)(n - 1) + \kappa^2\lambda^2(k(n - k) - 2(n - 1))}_{\tau_2}$$

28

τ_1 can be written as:

$$\sigma\Big(\sigma + \frac{1}{(n-1)^2}\Big(\underbrace{\lambda(\lambda-2\alpha)}_{\geq 0 \text{ for } \lambda\geq 2\alpha}\underbrace{\big(n(n-k)+k(k-1)\big)}_{>0 \text{ since } 1\leq k\leq n}$$

$$+\underbrace{\lambda\alpha(n(3n-k-4)}_{>0 \text{ for } n>2}+k+1)+\alpha^2(k-1)^2\Big)\Big)$$

τ_1 is positive for all feasible values of k and n when $\lambda \geq 2\alpha$.

τ_2 can be written as:

$$\frac{\lambda}{(n-1)^2}\big(\underbrace{\lambda^2\alpha n(n-k)}_{\geq 0 \text{ since } k\leq n}+\underbrace{\lambda\alpha^2 n(k-2)}_{\geq 0 \text{ for } k\geq 2}+\underbrace{\alpha^2 k(\lambda-\alpha)}_{>0 \text{ since } \lambda>\alpha}+\alpha^3\big)$$

τ_2 is positive for all $k \geq 2$ as well as for $k = 1$ where we get:

$$\frac{\lambda}{(n-1)^2}\lambda\alpha(n-1)(\lambda n-\alpha)$$

which is positive since $\lambda > \alpha$ and $n \geq 3$.

Hence, the denominator is positive for all k between 1 and n when $\lambda \geq 2\alpha$.

○ Second case: $\lambda < 2\alpha$.

We can interpret the denominator of ρ as a convex[19] parabola in k. If the parabola is downward sloping in $k = n$ all feasible values of k are left of the minimum, on the monotonically decreasing part of the parabola. Hence, the necessary and sufficient condition that the denominator of ρ is positive is that it takes a positive value in the lowest feasible point, $k = n$.

The derivative of the parabola in $k = n$ can be written as:

$$\frac{\partial \text{ denominator}}{\partial k}\Big|_{k=n} = \underbrace{-\frac{\lambda\alpha}{n-1}(n\lambda+\alpha)}_{\text{intercept} <0}+\underbrace{(\lambda-2\alpha)}_{<0 \text{ for } \lambda<2\alpha}\sigma$$

This expression can be interpreted as a linear function in σ with a negative intercept. Since σ can take any non-negative value a sufficient condition for a negative derivative is that $\lambda < 2\alpha$.

The value of the denominator in $k = n$ is:

$$\frac{1}{n-1}\underbrace{\big(\sigma^2(n-1)+\sigma\lambda(\lambda-\alpha)+\sigma\lambda^2(n-1)\big)}_{>0 \text{ since } \lambda>\alpha}+\frac{1}{n-1}\underbrace{\lambda\alpha^2(\lambda-\alpha)}_{>0} > 0$$

---

[19]The coefficient of k^2 is $\sigma\kappa^2 \geq 0$.

Hence, for $\lambda < 2\alpha$, the denominator of ρ is always positive.

- $\omega = \dfrac{\lambda(\kappa k \rho + 1)}{\sigma + \lambda^2 - \kappa\lambda(n - k - 1)}$

  is positive because the nominator is a sum of positive expressions and the denominator is positive (for proof see above, for θ).

B.2 The Cooperation-Stackelberg equilibrium

The optimization problem which has to be solved by the monetary authority of a country can be summarized as follows. Outside the coalition L_i is minimized with respect to the own money supply; in the coalition L_i is minimized with respect to the money supplies of all coalition members. The coalition takes explicitly account of the reaction of the non-members which is mathematically solved by replacing the non-members' money supplies with their reaction functions.

The reaction function of a country outside the coalition   A country outside the coalition has to solve the same problem as above. Hence, the reaction function of a non-member is as in equation (27):

$$m_{nc}^* = \theta \sum_{j=1}^{k} \overline{m}_{j,c} - \vartheta x$$

The reaction function of a coalition member   The coalition solves its optimization problem subject to the reaction functions of the non-members which are dependent on the coalition's money supply:

$$min_{m_j \in C}\ \mathcal{L} = \sum_{j=1}^{k} L_j \qquad \text{s.t. } m_i = \theta \sum_{j=1}^{k} m_{j,c} - \vartheta x \quad \forall\, i = k+1, \ldots, n$$

The first order condition together with the symmetry assumption for the coalition money supplies $m_{j,c}^* = m_c^*$ for all $j = 1, \ldots, k$ gives the coalition member's money supply. Since this is already independent of the non-members' money supplies it is the equilibrium money supply of a coalition member.

$$m_c^* = -\underbrace{\frac{(1 + (n-k)\theta)(\lambda + \kappa - k\kappa(1 + (n-k)\theta))}{\sigma + (\lambda + \kappa - k\kappa(1 + (n-k)\theta))^2}}_{\varphi} x \qquad (29)$$

30

The equilibrium Replacing the coalition members' money supply in equation (27) with equation (29) gives the equilibrium money supply of a non-coalition member:

$$m_{nc}^* = - \underbrace{(\kappa k \varphi + 1) \vartheta}_{\phi} x \qquad (30)$$

The sign of the coefficients We will show that the signs of the coefficients φ and ϕ are positive.

- $\varphi = \dfrac{(1 + (n-k)\theta)(\lambda + \kappa - k\kappa(1 + (n-k)\theta))}{\sigma + (\lambda + \kappa - k\kappa(1 + (n-k)\theta))^2}$

  The denominator of φ is positive since it is the sum of the positive σ and a squared expression. The nominator of φ is positive if:

$$\lambda + \kappa - k\kappa(1 + (n-k)\theta) > 0$$
$$\frac{\sigma(\lambda \frac{n-k}{n-1} + \alpha \frac{k-1}{n-1}) + \frac{1}{n-1}(\lambda\alpha(\lambda n - \alpha))}{\sigma + \lambda^2 - \lambda\kappa(n-k-1)} > 0$$

  The denominator of the left-hand expression is positive as proved above for θ, the nominator is positive since $\lambda > \alpha$ and $n > 3$.

- $\phi = (\kappa k \varphi + 1)\vartheta$ is positive since it is the sum of positive expressions.

C   Results of the Cooperation-Stackelberg game

The results of the simulation[20] of the Cooperation-Stackelberg game – qualitatively similar to the Nash case – are summarized in the following.

The loss functions The losses in equilibrium are determined through equilibrium policies:

$$
\begin{aligned}
L_c &= \sigma m_c^{*2} + (\lambda m_c^* - \kappa(k-1)m_c^* - \kappa(n-k)m_{nc}^* + x)^2 \\
L_{nc} &= \sigma m_{nc}^{*2} + (\lambda m_{nc}^* - \kappa k m_c^* - \kappa(n-k-1)m_{nc}^* + x)^2
\end{aligned}
$$

---

[20]The numerical simulations were performed for the same parameter values as in the Cooperation-Nash game: $\alpha = 0.5$, $\beta = 0.5$, $\epsilon = 0.8$, $\nu = 0.05$, $\sigma = 1$ and $\delta = 0.3$.

31
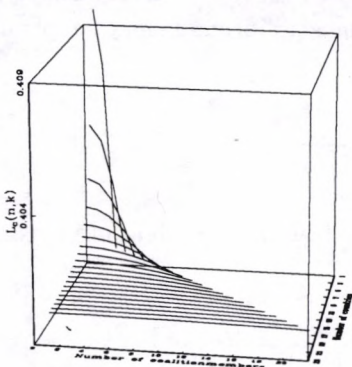
Figure 5: Lossfunction of a coalition member
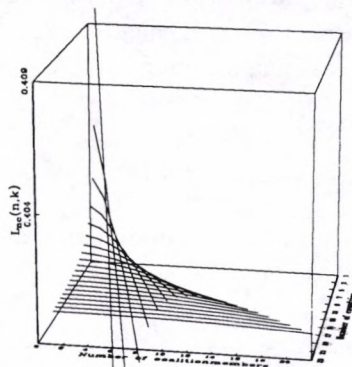
Figure 6: Lossfunction of a country outside the coalition

The losses varying with k and n are shown in figures 5 and 6. The shape of the resulting figures is almost identical to the corresponding figures of the Cooperation-Nash game. Losses decrease for all values of n with the number of coalition members. The explanation is as above: the more members a coalition has, the more externalities are internalized. This lowers losses for the countries inside and outside the coalition.

The stability of the coalition Each n has one stable coalition size, as in the Cooperation-Nash game. The coalition is internally stable for all k smaller or equal to the stable coalition size; it is externally stable for all $k \geq k^*$. Hence, the graphic illustration of the "gains from changing the group" is the same than in figure 3.

The stable coalition size is two for all n smaller than seven. For $n \geq 7$ the stable coalition size switches to three. However, the results are not perfectly robust against changes of the parameter values. The results of the sensitivity analysis are shown in table 1.

---

[21]When increasing the number of countries above 170 the results switch between a stable coalition size of two, three or no stable coalition size at all. The latter cases are all cases where the stability criterion is fulfilled with equality but not with strict inequality. For two reasons the lacking robustness of the results for higher n is not a serious-problem: first, a total number of countries beyond 150 will probably not be any 'real-world-constellation' the model might be faced with. Second, there is a strong

Variation of		Stable coalition size k* =	
	$n \leq 5$	$n = 6$	$n \geq 7$
Default values	2	2	3^{21}
β $\beta < 0.6$	2	2	3
$\beta \geq 0.6$	2	3	3
δ $\delta < 0.3$	2	3	3
$\delta \geq 0.3$	2	2	3
ϵ $\epsilon < 0.6$	2	3	3
$\epsilon \geq 0.6$	2	2	3
ν	2	2	3
α $\alpha = 0$	2	2	2
$0 < \alpha < 1$	The lower α the higher the n where k^* switches to 3.		
$\alpha = 1$	3	3	3
σ $\sigma \geq 1$	The higher σ the higher the n where k^* switches to 3.		

Table 1: Sensitivity analysis of the Cooperation-Stackelberg outcome

The coalition formation process The coalition formation process is similar to the one in the Cooperation-Nash game. For $k = 1$ we have the non-cooperative Nash equilibrium. It is externally unstable and, hence, additional countries will join the coalition until the stable coalition size is reached. The non-members have lower losses than the Stackelberg leader, i.e. the coalition. This result is – for models of duopolies in industrial organization – shown to be generally valid in Stackelberg games with strategic complements by Dowrick [7]. Hence, every country would like the other ones to go ahead with the coalition formation. This again might be an obstacle to getting the coalition formation started at all.

Comparison of Cooperation-Stackelberg and Cooperation-Nash outcome The Stackelberg leader, i.e. the coalition, has lower losses than in the Nash game since he can always realize the Cooperation-Nash losses. The simulation results indeed show that the Stackelberg leader money supplies are always higher (less contractionary) than the Nash money sup-

---

indication that the cases of no stable coalition are due to imprecisions in the calculation programs and that there is actually a stable coalition for these cases. Hence, the basic argument of the existence of a stable coalition which does not comprise all countries – may it be two or three – is still valid.
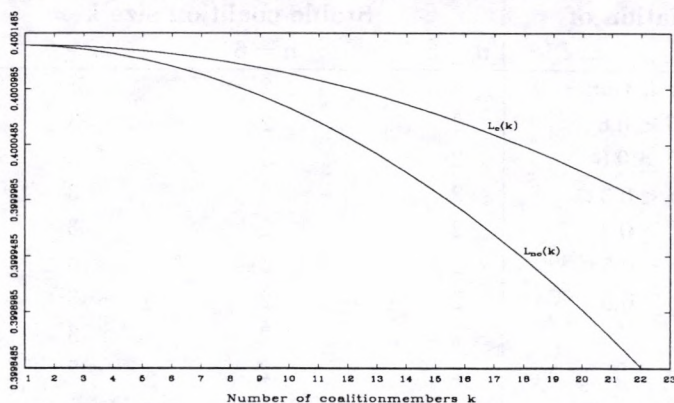
Figure 7: Loss functions inside and outside the coalition
(for $n = 22$, k from 1 to 22)

plies for the same n and k. However, the money supplies are only slightly higher than in the Nash game. In particular, for very small and very high k the results of the Nash and of the Stackelberg game are very close to each other.

Dowrick [7] shows in a general framework of strategic complements that the Stackelberg outcome is pareto superior to the Nash outcome. In other words, not only the Stackelberg leader but the Stackelberg follower improves upon its Nash outcome, too. In our model the Stackelberg money supplies of the non-members are less contractionary than in the Cooperation-Nash game since the non-members react with a higher money supply on the higher money supply of the coalition. Additionally, the losses of the non-members improve more upon the Cooperation-Nash equilibrium than the losses of the coalition.

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34

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