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On Technological Differences In Oligopolistic Industries

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Abstract

A two-stage game is studied in which firms choose their technology in the first stage and set quantities in the second. The choice is between high fixed/low marginal and low fixed/higher marginal costs. When quantities are chosen non-cooperatively, an increase in the number of firms in the market can lead to higher equilibrium profits for all firm types. If, instead, firms collude in the second stage they will tend to prefer technologies with higher fixed costs and subsequently lower marginal costs. The effect of an increase in competition on the colluding firms depends critically on this fixed cost and the corresponding equilibrium market structure.

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1 Introduction

This paper describes a simple model of duopoly and oligopoly with specific functional forms when firms face a choice of which technology to adopt. In this respect it differs from much of the existing literature on cost differences when firms' technologies (especially when asymmetric) are assumed to be exogenously determined. The choice of technology often leads to differentiated cost structures when firms compete for market shares. Given the possibility of different costs, I discuss the consequences of changes in the technologies, demand parameters and the level of competition on the firms' strategies and pay-offs. The analysis proceeds by using a two-stage game where firms first decide which cost structure (technology) to employ and then choose quantities, either using Cournot conjectures or collusively. Examples are used to highlight the results where useful and an examination is provided of the effects of changes in the exogenous parameters on the equilibrium outcomes.

One result of the paper is the potentially ambiguous effect of an increase in competition on the equilibrium pay-offs (profits). I demonstrate how an increase in the number of firms can cause a change in the choice of technologies employed which itself leads to higher profits for *all* firm types. When firms collude in the second stage, they are subject to the same form of commitment problem as colluding firms who over-invest in capacity[1]. If the fixed cost of obtaining lower marginal costs is itself low, this leads to an over-investment in the low marginal cost technology and smaller profits.

I start by describing the model for the duopoly case in functional form[2] and then demonstrate (using numerical examples) the results and

[1] See Phlips [5] for references to this literature.

[2] An analysis of two-stage duopoly games with general forms is given by Shapiro [8] who discusses conditions for uniqueness and existence as well as the differrent effects that the cost structures have on the rivals' behaviour. For references to other articles in the literature I refer the reader to Shapiro [ibid.].

potentially ambiguous outcomes that are possible when cost differences occur in an oligopoly. Thus by highlighting the conditions and results of endogenous technologies within a two-stage (non-cooperative and cooperative) game. I provide a different perspective to the existing literature.

2 Model

The market consists of firms with two possible cost structures called v and b where v denotes the cost structure with higher marginal. but lower fixed costs and b the reverse. Costs are defined by

$$C_i = F_i + c_i q_i \qquad i = 1, ..., n$$

for $q_i > 0$, $c_i > 0$ and $F_i \geq 0$. To simplify matters I assume that type v's fixed cost is normalised to 0 and therefore drop the subscript on F. We are then left with potential costs as follows[3]

$$\begin{aligned} C_b &= F + c_b q_b \\ C_v &= c_v q_v \end{aligned} \tag{1}$$

where $c_b < c_v$.

One simple interpretation of the different cost structures is as follows. Each firm has a choice of whether to employ a technology with a high capital cost outlay but then low labour and maintenance costs thereafter (type b) or to use a less capital intensive technology which requires more labour and therefore relatively higher marginal costs (type v). Clearly there can be other interpretations of the model.

The demand side of the industry is given by the inverse demand function

$$p = \begin{cases} \alpha - \beta Q & for \quad 0 \leq Q \leq \frac{\alpha}{\beta} \\ 0 & for \quad Q > \frac{\alpha}{\beta} \end{cases} \tag{2}$$

[3] This appears to imply that type v has constant returns to scale whilst type b has increasing returns to scale. This normalisation is for convinience and the results hold as long as the degree of returns to scale is greater for the firm with type b costs.

where p is market price, Q is total industry output. To simplify the algebra I normalise $\beta = 1$ so that $p = \alpha - Q$.

The model is a two stage game as follows

1. Strategic Game: in the first stage the firms decide which cost structure to use, $c = (c_1, c_2, ..., c_n)$, $c_i = \{c_v, c_b\} \quad i = 1, 2, ..., n$. c is the vector of all firms strategies while c_i is each individual firms strategy.

2. Quantity Game: in the second stage, given the number of firms and their respective cost structures, output is chosen as a Cournot quantity setting game, $q = \{q_1, q_2, ..., q_n\}$, $\quad q_i \in [0, a-p-Q_{-i}] \quad i = 1, ..., n$, where Q_{-i} denotes the industry output excluding firm i.

3 Duopoly Equilibrium

To determine the overall equilibrium in the model I use the sub-game perfect Nash equilibrium concept where equilibrium is required in both stages to constitute an overall equilibrium in the game. Therefore I solve the model backwards starting with the quantity game in which there is a tuple $\{q, \pi\}_i$ representing the strategy and payoff to each firm, followed by the strategic game which has its strategy choice as $c_i \in \{c_v, c_b\} \quad i = 1, 2$.

3.1 Quantity Game

Given the two-firm structure, there are three possible interactions between the firms; (i) both firms are of type v, (ii) both firms are of type b, and (iii) there is one of each type. Table 1 below provides the results for each combination.

TABLE 1

Duopoly Strategies & Pay-offs

Structure	Quantity	Profit
$i, j = v$	$q_v = \frac{\alpha - c_v}{3}$	$\pi_v = \frac{(\alpha - c_v)^2}{9}$
$i, j = b$	$q_b = \frac{\alpha - c_b}{3}$	$\pi_b = \frac{(\alpha - c_b)^2}{9} - F$
$i = v$ $j = b$	$q_v = \frac{\alpha - 2c_v + c_b}{3}$ $q_b = \frac{\alpha - 2c_b + c_v}{3}$	$\pi_v = \frac{(\alpha - 2c_v + c_b)^2}{9}$ $\pi_b = \frac{(\alpha - 2c_b + c_v)^2}{9} - F$

where the subscripts represent the firm types. The equilibrium strategies in Table 1 are the intersection points of the two firms reaction functions. Note that in addition to the restrictions in (2), for both firms to produce positive outputs in the third case requires that

$$\alpha - 2c_i + c_j > 0 \qquad i = v, b \qquad i \neq j.$$

However, given that $c_v > c_b$ by definition, this reduces to $\alpha > 2c_v - c_b$.

3.2 Strategic Game

To determine what will be the optimal choice of the firms in the strategic game requires comparing the different pay-offs available which are depicted in the strategic game form in Table 2.

TABLE 2

Pay-offs in Strategic Stage

		Player 2	
		v	**b**
Player 1	**v**	π^1_{vv}, π^2_{vv}	π^1_{vb}, π^2_{vb}
	b	π^1_{bv}, π^2_{bv}	π^1_{bb}, π^2_{bb}

where the subscripts represent the strategies of firm 1 and 2 respectively i.e. π^1_{bv} means the profit to firm 1 when 1 has cost structure b and 2 has cost structure v. The firms having solved the game backwards will know what their pay-offs will be having made their choice of cost structure.

Because of the symmetry[4] in decision-making we can examine the choice facing firm 1. Given firm 2 plays **v** it must evaluate $\pi^1_{vv} - \pi^1_{bv}$ and the same when firm 2 plays **b**, $\pi^1_{vb} - \pi^1_{bb}$ to determine its best strategy. These comparisons result in

$$\pi^1_{vv} - \pi^1_{bv} \geq 0 \quad as \quad \Delta \cdot (\alpha - c_b) \leq \tfrac{9}{4}F \tag{3}$$
$$\pi^1_{vb} - \pi^1_{bb} \geq 0 \quad as \quad \Delta \cdot (\alpha - c_v) \leq \tfrac{9}{4}F \tag{4}$$

where $\Delta = c_v - c_b$. These conditions determine the possible equilibria in the strategic stage. For example, if the LHS of both conditions (3) and (4) are positive, the unique Nash equilibrium will be for both firms to have cost structure v, whereas if neither conditions are met, the unique Nash equilibrium is for both to choose b. Should only one hold, we will have the situation where there are two possible equilibria where one firm is of type v and the other of type b. This is the common problem when there are multiple equilibria and there is no way of telling which would be chosen were the game to be played[5].

Examining the strategies $c = (c_v, c_v)$ as a possible equilibrium we can see from the conditions that the larger are the cost differences between types or the larger is the demand intercept, the less likely it is that the conditions will hold and therefore the firms will want to both have cost structure b (the one with lower marginal costs).

3.3 Duopoly Example

To aid exposition and to demonstrate the possible outcomes we can examine a simple numerical example. Inverse demand is given by $p = 10 - Q$. Costs are

$$C_b = 2 + 2q_b$$
$$C_v = 2.5q_v$$

Given these costs the payoff matrix will be

[4] By symmetry I mean that $\pi^1_{bv} = \pi^2_{vb}$.

[5] Due to the symmetry we cannot even resort to *Pareto dominance* or another similar concept to determine which equilibrium is more likely (Binmore [1], ch. 7).

		Player 2	
		v	**b**
Player 1	**v**	$\underline{6.25}, \underline{6.25}$	$\underline{6.03}, 5.44$
	b	$5.44, \underline{6.03}$	$5.11, 5.11$

hence the overall equilibrium is for both firms to choose cost structure v[6].

If we change the numbers slightly by increasing the marginal costs of type v from 2.5 to 2.7 (and therefore increasing Δ) we see that neither condition (3) nor (4) are met and the resulting payoff matrix is

		Player 2	
		v	**b**
Player 1	**v**	$5.92, 5.92$	$4.84, \underline{6.41}$
	b	$\underline{6.41}, 4.84$	$\underline{5.11}, \underline{5.11}$

with the unique Nash equilibrium being $c = (c_b, c_b)$ as might have been expected as we raised type v's marginal cost. Lastly taking type v's marginal costs to be the intermediate value of 2.6 we have two possible Nash equilibria where both firms would like to be the only firm with cost structure b.

		Player 2	
		v	**b**
Player 1	**v**	$6.08, 6.08$	$\underline{5.14}, \underline{6.22}$
	b	$\underline{6.22}, \underline{5.14}$	$5.11, 5.11$

4 Oligopoly

In this section I examine the situation when there are n-firms in the market operating under similar conditions as before. The main differences

[6] I have underlined the best response of each firm given the other firms strategy is held fixed. Where both strategies are underlined together indicates a Nash equilibrium.

between the n-firm oligopoly and duopoly are that industry output is now given by

$$Q = \sum_{i=1}^{k} q_i + \sum_{j=k+1}^{n} q_j$$

where n is the number of firms in the market, k is the number of firms with cost structure v and $(n-k)$ firms with cost structure b.

4.1 Quantity Game

Given the above we can see that the industry is characterised by a combination of the two firm types (for example, 4-firms: 1 of type v and 3 of type b) in $n+1$ possible combinations. For four firms this implies the following possible combinations of quantities and profits.

TABLE 3
Example of 4-firm Oligopoly

Type	v	0	1	2	3	4
	b	4	3	2	1	0
Quantity	v		q^v_{13}	*etc.*		q^v_{40}
	b	q^b_{04}	q^b_{13}			
Profit	v		π^v_{13}	*etc.*		π^v_{40}
	b	π^b_{04}	π^b_{13}			

There is a certain amount of symmetry involved as we restrict the different cost structures to only two types. This implies that when there are 3 firms with type v and 1 with type b, the three type-v firms are identical.

Each firm selects its output using the profit maximisation process as before although now it is faced by $(n-1)$ other firms. We can examine this for the case of one firm i. Firstly note that the two *boundary* cases (i.e. all firms of one type) lead to the standard symmetric oligopolistic outcome.

$$\begin{aligned} q_i &= \tfrac{\alpha - c_i}{(n+1)} \qquad\qquad i = v, b \\ \pi_v &= \tfrac{(\alpha - c_v)^2}{(n+1)^2} \\ \pi_b &= \tfrac{(\alpha - c_b)^2}{(n+1)^2} - F \end{aligned} \tag{5}$$

The situation for an *interior* solution has the firm maximise profits considering all possibilities. Its profit function is one of the following[7]

$$\pi_v = (p - c_v)q_v = \left(\alpha - \left(q_v + \sum_{k-1} q_v + \sum_{n-k} q_b\right) - c_v\right) q_v$$

$$\pi_b = (p - c_b)q_b = \left(\alpha - \left(q_b + \sum_{k} q_v + \sum_{n-k-1} q_b\right) - c_b\right) q_b - F$$

depending on its type. The first-order conditions which are the same for all firms of the same type

$$\frac{\partial \pi_i}{\partial q_i} = 0 = \alpha - 2q_i - (k-1)q_i - (n-k)\, q_j - c_i \qquad i = 1.\dots.k$$

$$\frac{\partial \pi_j}{\partial q_j} = 0 = \alpha - 2q_j - (n-k-1)\, q_j - kq_i - c_j \qquad j = k+1.\dots.n.$$

Solving the n simultaneous first-order conditions leads to the following quantity (and subsequent profit) schedules

$$\begin{array}{ll} q_v = \frac{\alpha-(n-k+1)c_v+(n-k)c_b}{n+1} & \pi_v = \left(\frac{\alpha-(n-k+1)c_v+(n-k)c_b}{n+1}\right)^2 \\ q_b = \frac{\alpha-(k+1)c_b+kc_v}{n+1} & \pi_b = \left(\frac{\alpha-(k+1)c_b+kc_v}{n+1}\right)^2 - F \end{array} \qquad (6)$$

which constitute a Nash equilibrium, $q = (q_i, q_{-i})$ for the n-firms (of type v or b) where $k \neq 0$. We can also note that at the limits (when $k \to n$ or $k \to 0$) we have the *boundary* solutions

$$\lim_{k\to n} q_v = \frac{\alpha-c_v}{n+1}$$

$$\lim_{k\to 0} q_b = \frac{a-c_b}{n+1}.$$

4.2 Strategic Game

Given the possible strategies and pay-offs in the second stage (quantity) we can examine the first stage strategies to see if possible equilibria exist

[7]Note that

$$Q = \sum_k q_i + \sum_{n-k} q_j$$
$$Q = q_i + \sum_{k-1} q_i + \sum_{n-k} q_j \quad typev$$
$$Q = \sum_k q_i + q_j + \sum_{n-k-1} q_j \quad typeb$$

for the game as a whole. To do this I note that the profit for each firm can be written as a function of k, the number of firms with cost structure v, as $\pi_i(k) \quad i = 1, ..., n$. In effect k becomes the parameter which can be used as the reference point for the equilibrium determination.

TABLE 4

Example Oligopoly Payoff Table

		All Firms (including 1)				
		k = n	**k = n − 1**	**k = n − 2**	...	**k = 0**
Firm 1	**v**	$\pi_v(n)$	$\pi_v(n-1)$	$\pi_v(n-2)$	...	
	b		$\pi_b(n-1)$	$\pi_b(n-2)$	...	$\pi_b(0)$

For the strategies (c_i, c_{-i}) to be an equilibrium requires that neither firm i, nor any of the other firms $(-i)$ would wish to deviate from the chosen cost structure. These strategies are illustrated in Table 4 by the different values of k. The corresponding conditions to determine whether a particular k is an equilibrium for a given value of n are[8]

$$\pi_v(k) \;>\; \pi_b(k-1) \tag{7}$$

$$\pi_b(k) \;>\; \pi_v(k+1) \tag{8}$$

for the *interior* cases. The *boundary* cases are more straightforward as in each case only one condition is required, for example, if all firms have cost structure v, for it to be a Nash equilibrium we require that $\pi_v(n) > \pi_b(n-1)$ (i.e. (condition 7)) for all firms.

Substituting from (6) into (7)-(8) to determine the conditions we have the following, starting with condition (7)

$$\begin{aligned} \frac{(\alpha-(n-k+1)c_v+(n-k)c_b)^2}{(n+1)^2} > \frac{(\alpha-kc_b+(k+1)c_v)^2}{(n+1)^2} - F \quad &\Leftrightarrow \\ \frac{\Delta n(-2\alpha+2c_v+2c_bk-2c_vk-c_bn+c_vn)}{(n+1)^2} > -F \quad &\Leftrightarrow \\ 2(\alpha-c_v) < \frac{F(n+1)^2}{\Delta n} - (2k-n)\Delta & \end{aligned} \tag{9}$$

[8] These conditions are conceptually equivalent to those of internal and external stability which are used in the cartel literature, see Selten [7] and d'Aspremont *et. al.* [2], for example.

and

$$\begin{aligned} \frac{(\alpha-(k+1)c_b+kc_v)^2}{(n+1)^2} - F > \frac{(\alpha-(n-k)c_v+(n-k-1)c_b)^2}{(n+1)^2} & \Leftrightarrow \\ \frac{\Delta n(2\alpha-2c_b-2c_bk+2c_vk+c_bn-c_vn)}{(n+1)^2} > F & \Leftrightarrow \\ 2(\alpha-c_b) > \frac{F(n+1)^2}{\Delta n} - (2k-n)\Delta & \end{aligned} \tag{10}$$

where $\Delta = (c_v - c_b)$ as before. (9) and (10) can be combined to give the following single condition for a particular industry structure (i.e. a value of k) to be a Nash equilibrium

$$(\alpha - c_v) < \frac{r(n)F}{\Delta} - \Delta(k - n/2) < (\alpha - c_b) \tag{11}$$

where $r(n) = \frac{(n+1)^2}{2n}$. By using condition (11) we can test whether a particular industry structure will be a Cournot-Nash equilibrium for the overall game. In addition we can examine (11) to see what effect changes in the parameters will have on the equilibrium cost structures adopted by the firms.

4.3 Analysis of Equilibrium Conditions

A graphical analysis of the condition is given in Figure 1 which shows the two outer constraints $(\alpha - c_i)$ and the middle term as the downward sloping curve in k-space[9]. This is to be interpreted as follows; where the point on the curve is above a particular value of k. that value of k is a Cournot-Nash equilibrium of the overall game as it satisfies condition (11) implying that it is in no firms interest to deviate from its particular cost structure.

If we denote the middle expression of (11) by $X(F, n, k, \Delta)$ we can see the following,

$$\frac{\partial X}{\partial F} = \frac{r(n)}{\Delta} > 0 \tag{12}$$

$$\frac{\partial X}{\partial n} = \frac{\Delta^2 n^2 + F(n^2-1)}{2\Delta n^2} > 0 \tag{13}$$

which can be interpreted as follows;

[9] There is a slight abuse of notation here as k is a discrete variable and therefore $X(\cdot)$ is really a series of points rather than a curve.

- (12) shows that an increase in the fixed cost of type b shifts the X-curve upwards implying, *ceteris paribus*, that more firms will be of cost structure v in equilibrium. This is not surprising as the benefit of having the lower marginal cost is offset by the higher fixed cost.

- (13) is the effect of an increase in competition (the number of firms) on possible equilibria. As the increase in competition leads to necessarily lower market shares for all firms, the benefit of having the lower marginal cost associated with the fixed cost decreases which leads to higher values of k. Clearly the type b firms need to cover their fixed costs before reaping the benefits of the lower marginal costs.

- An increase in the demand intercept, α, will shift upwards the outer constraints band implying that a lower value of k is more likely in equilibrium. This is expected as the larger is the overall market the higher will be the benefits from being type b and having the lower marginal cost.

4.4 Competition Effects

In order to see how an increase in competition affects the industry structure we can examine the Cournot-Nash equilibria of the overall games to see how they change as n, the number of firms increases.

Presented in Table 5 are the unique Nash equilibria for the game with parameter values set as follows; $\alpha = 10$, $c_v = 3$, $c_b = 2$ and $F = 2$.

TABLE 5

Example Strategies and Pay-offs[10]

n	k^*	q_v	q_b	Q	π_v	π_b	P
3	0		2.00	6.00		2.00	4.00
4	1	0.80	1.80	6.20	0.64	1.23	3.80
5	*2*	*0.67*	*1.67*	*6.33*	*0.44*	*0.78*	*3.67*
6	*4*	*0.71*	*1.71*	*6.29*	*0.51*	*0.94*	*3.71*
7	5	0.63	1.63	6.38	0.39	0.64	3.63

We can see from the table that as n increases, profits to both firm types do not necessarily decrease as we might expect[11]. In fact, when n goes from 5 to 6, and k^* changes from 2 to 4, equilibrium profits increase for both firm types. Total industry output and price change marginally ($\pm 1\%$) although individual firm profits change by 13% and 17% respectively for types v and b and even though there is one less firm of type b, total industry profits have increased.

The example above demonstrates how an increase in competition causes the firms in the industry to restructure themselves, in this particular case the number of type v firms changes from 2 out of 5 to 4 out of 6 leading to an increase in profits for both firm types.

5 Collusive Outcomes

It is interesting to see how the equilibrium conditions and results are affected by the possibility that firms will collude in the second stage of the game. This analysis is complementary, and in some respects analogous to the literature on semi-collusion[12] in R&D, capacity and investment, although extending the results to the case of cost (technology) differences. Using the previous methodology I extend the work of Schmalensee [6] by (i) using a two-stage game in which the choice of technology is made in the first stage and then firms collude in the second; (ii) including demand parameters to see how they affect the outcomes when they change, (iii) allowing for several firms of either cost type (rather than a leader and follower fringe environment), and (iv) introducing fixed costs into the analysis. Schmalensee provided a detailed numerical analysis by comparing various collusion technologies and solution concepts. He used bargaining theory to determine how the colluding firms will select a point on the profit possibility frontier that will itself have been determined by the collusive technology chosen. However, in his paper firms were simply endowed with their technology and the low cost firms were always at an advantage as there was no fixed cost or minimum output level to consider.

[11] This result is robust to various values for the exogenous parameters.

[12] See Phlips [5] for references.

In this section I assume that the technology choice is made, knowing that firms will collude in the second stage. All firms adopt the *Proportional Reduction* (PR) approach to determining the quantities that each will produce in the collusive equilibrium. This requires that firms maintain their non-cooperative Cournot market shares with respect to the total collusive quantity. In some respects the PR approach will be inefficient from the firms' (and a welfare maximiser's) perspectives as all firms will produce positive quantities in equilibrium, rather than one firm with cost type b producing everything. Had only one firm produced, the joint-profit maximising point could have been reached as the economies of scale would have been greater, although this would necessary involve side-payments which I believe to be unrealistic in many markets[13]. The PR approach is also intuitively in accordance with the credible threat strategy proposed by Osborne [3] where firms increase their outputs proportionately (relative to their market share) in response to cheating. However one must recognise that the collusive gains under PR could be substantially less than other methods when costs differ greatly and in this respect, PR could be considered the lower-bound to the potential benefits from collusion.

Having chosen the PR technology to determine the profit possibility frontier I assume that firms maximise the generalised Nash product to locate the exact quantities produced by the firms. In this respect it is as if the firms agree on a scaling factor from their non-cooperative Cournot-Nash quantities. They effectively say to each other: we will all produce $q_i = r_i q_i^c$ where $i = b, v$, superscript c denotes the Cournot quantity and $r_i \in [0, 1]$. In order to determine q_i and r_i I maximise the generalised Nash product using the number of each firm type as the bargaining power. This can be written as

$$\max_{q_v q_b} \left(\pi_v^t - \pi_v^c\right)^k \left(\pi_b^t - \pi_b^c\right)^{(n-k)}$$

s.t.

$$q_i = s_i Q$$

[13] A notable exception to the possibility of maintaining side payments in a cartel is in the auction literature when ***settling-up***, as it is known, takes place immediately after an auction. This meeting of parties after the auction is when side-payments are distributed.

$$q_i \geq 0 \qquad i = v, b$$

where s_i is each firms Cournot-Nash market share given by

$$s_i = \frac{q_i^c}{Q^c} = \frac{q_i^c}{kq_v^c + (n-k)q_b^c} \qquad i = v, b.$$

Q (without superscript) is the total collusive quantity. π^t denotes the collusive profit and the actual profit functions are those discussed in Section 4.

Unfortunately, as the objective function is non-linear in q_i and the number of parameters is large (a, c_v, c_b, F, k, n) it is not possible to obtain explicit functions for the quantities (Schmalensee [6] had the same problem even though he normalised everything down to the ratio $R = \frac{c_v}{c_b}$) and I therefore give a numerical analysis of the results[14]. Partial results are given in summary form in Table 6 which uses selected values for the cost and demand parameters as the number of firms changes.

In order to provide an approximate *measure* of the cost difference between firm types we can examine the Total Cost functions given earlier in (1). If we define q^* as the output level when total costs are equal between firm types, we can determine its value given the cost parameters, as follows

$$\begin{aligned} c_v q_v &= F + c_b q_b \\ q^* &= \frac{F}{c_v - c_b} \end{aligned}$$

Then by setting the costs as $c_v = 3$ and $c_b = 2$, $F = q^*$ and we can use q^* as a gauge of the cost difference as well as the fixed cost.

[14] The analysis was performed using *Mathematica* where I maximised the objective function subject to the PR constraints. This lead to several possible solutions for quantities (local maxima) which I substituted back into the objective function to determine those which provided the global maximum.

TABLE 6

Selected Equilibrium Values

		Collusive					Cournot					
F	**n**	**k***	π_v^t	π_b^t	$\prod^t$	**CS**^{t}	**k***	π_v^c	π_b^c	$\prod^c$	**CS**^{c}	**%ΔCS**
2	3	0		3.33	10.0	8.00	0		2.00	6.00	18.0	106
	4	0		2.00	8.00	8.00	1	0.64	1.24	4.36	19.2	140
	5	0		1.20	6.00	8.00	2	0.44	0.78	3.22	20.1	151
	6	0		0.67	4.00	8.00	4	0.51	0.94	3.92	19.8	147
	7	0		0.29	2.00	8.00	5	0.39	0.64	3.23	20.3	153
2.5	3	0		2.83	8.50	8.00	1	1.56	2.56	6.69	16.5	106
	4	1	1.56	2.14	7.99	7.64	2	1.00	1.50	5.00	18.0	135
	5	2	1.28	1.70	7.67	7.39	4	1.00	1.50	5.50	18.0	143
	6	3	1.08	1.40	7.45	7.20	6	1.00		6.00	18.0	150
	7	5	1.20	1.55	9.09	6.79	7	0.77		5.36	18.8	176
3	3	2	3.33	4.24	10.9	6.98	3	3.06		9.19	13.8	111
	4	4	3.06		12.2	6.12	4	1.96		7.84	15.7	156
	5	5	2.45		12.2	6.12	5	1.36		6.81	17.0	177
	6	6	2.04		12.2	6.12	6	1.00		6.00	18.0	194
	7	7	1.75		12.2	6.12	7	0.77		5.36	18.8	207

$\%\Delta CS$ denotes the % change in consumer surplus when comparing the collusive and Cournot equilibria and $\prod^i$ is the total industry profit, $i = t, c$.

Table 6 summarises the overall game by showing the profit functions determined in the second stage and the equilibrium value of k from the first stage. It has several interpretations. One can see that when F is relatively low ($F = 2$), all firms in the collusive equilibrium will be type b, even though their profits are strictly increasing in k. This result is analogous to over-investment in industry capacity in similar two-stage games (Osborne, M.J. and C. Pitchik [4] and Phlips [5]) where firms use their investment in capacity (choice of production technology) as a pre-commitment to the collusive second stage[15]. Additionally, when the number of firms increases (i.e. for $n = 6, 7, ...$), it can be that the firms would earn greater profits

[15] In fact one can interpret F as the cost of installing capacity (although in this case it is the appropriate technology) and therefore when it is low and quantities are determined cooperatively, we should expect firms to precommit themselves.

had they been competing in the second stage due to the different industry structure which prevails in equilibrium. This situation changes when F is high ($F = 3$) as all firms adopt technology v earning strictly greater profits.

When comparing the collusive and Cournot equilibria, we can see that the structure of the industry (k^*) is more stable as F changes under Cournot competition. That is, when firms compete and F increases, the equilibrium values of k change more gradually.

When quantities are set cooperatively and for particular values of the exogenous parameters, there will be an F when it starts to pay the colluding firms to have $k^* > 0$, i.e. a mixture of firm types. This turning point is analogous to the cost of installing capacity in that literature when the cost is large enough that even though capacity (being type b here) acts as a form of commitment, it would be out-of-equilibrium behaviour for firms to install the excess capacity (be of type b).

In the collusive equilibria with high or low costs, consumer surplus is the same when the number of firms increases[16], because both quantities and prices remain unchanged. As might be expected under Cournot competition, as well as being absolutely higher in all cases, consumer surplus increases as the number of firms increases. Total surplus (not shown) depends on the change in industry profits and is closely linked to the appropriate values of k^*. When all firms choose technology b in the collusive equilibria (low F), industry profits decline rapidly as the number of firms increases due to the fixed cost of each additional firm. This will obviously cause a decline in total surplus as consumer surplus does not change.

6 Conclusions

Asymmetric oligopoly models allow us to examine the effect of differing technologies on economic performance. In a two-stage game using functional forms, I have examined the conditions required for a particular market structure to be an equilibrium when firms firstly choose a technology to

[16] With the exception of $F = n = 3$.

adopt and then compete for market shares. The effects of changes in the degree of competition and other parameters were analysed and numerical analysis demonstrated how an increase in competition can provide the right motivation to restructure and potentially increase the firms profits.

When firms collude in the second stage, the analysis shows that too many firms may opt for the high fixed/low marginal cost structure in stage 1 in order to commit themselves for the quantity sharing in the second stage. As the number of firms in the industry increases, the negative effect of 'over-investment' in technology worsens and colluding firms may find that the over-investment leads to inferior outcomes than would have occurred if they had competed.

Future research could incorporate (i) repeating the second stage of the game with suitable discount factors, and (ii) giving the technology a fixed life span (T) and (iii) issues relating to entry, which have been ignored in this paper. When combined, (i) and (ii) may lead to a better understanding of the duration and frequency of change in both technologies and market structures.

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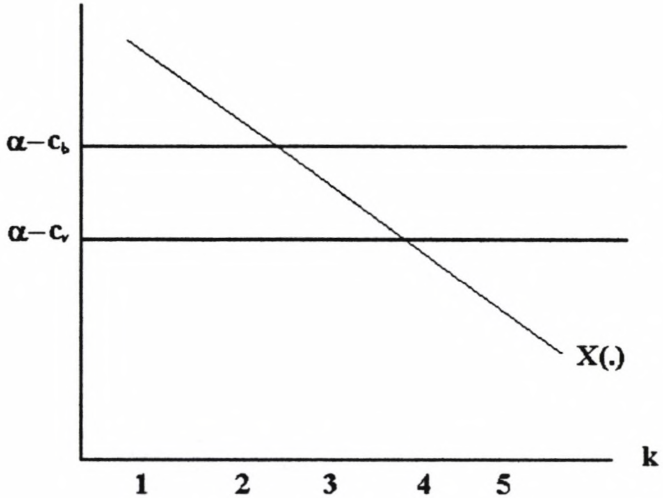
Figure 1



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