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Endogenous Timing in a Duopoly Model with Incomplete Information

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Abstract

Two timing games which endogenize the order of moves in duopoly are applied for a quantity setting model with incomplete information. The first timing game is such that firms have commit to a quantity in one out of two periods. This timing game exhibits multiple equilibria. This is in contrast to previous results in the literature which establish a unique timing equilibrium under restricted assumptions. The second timing game is such that firms announce their timing decision in advance without committing to an action. This timing game reduces the number of equilibria and supports a Cournot equilibrium for a wide range of parameters.

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Contents

1 Introduction 1

2 The Model 5

3 Endogenous Timing with Action Commitment 8

4 Trembling Hand Perfectness of the Sequential Equilibria 15

5 Endogenous Timing with Observable Delay 21

6 Concluding Remarks 28

Appendix 30

The Signalling Stackelberg Equilibrium 30

Existence of Type III and IV Equilibria 32

References 35

1 Introduction

In many situations, it seems perfectly reasonable that firms have some leeway in timing their business decisions and will use this leeway strategically. In theoretical oligopoly analysis the simplest way of reflecting this is to model firms as moving simultaneously or sequentially. Models of simultaneous choice include the Cournot and the Bertrand model; Stackelberg and price leader models are based on sequential moves of the firms. Usually the order of moves in oligopoly models is assumed to be exogenous, but there is a growing literature[1] which establishes conditions and criteria under which firms agree endogenously whether to play a simultaneous or a sequential move game. The results of such models of endogenous timing are the benchmark for the robustness of results obtained from models in which firms are restricted to a certain timing structure.

Particularly interesting is the problem of endogenous timing in the context of uncertainty[2]. Under perfect information, and with downward sloping reaction functions, there is an incentive for moving first. The Stackelberg leader earns a higher profit than the Stackelberg follower. Also the profit in a simultaneous move Cournot equilibrium is strictly preferred to the profit of a Stackelberg follower. However, in a game with incomplete information, the preferences for moving first and second might be changed. Gal-Or (1987), in

[1] See Hamilton and Slutsky (1990) and Van Damme and Hurkens (1993, 1995) for a general analysis of games with perfect information.

[2] Albæk (1990, 1992), Spencer and Brander (1992), Mailath (1993) and Daughety and Reinganum (1994).

a quantity setting model with a privately informed Stackelberg leader and an uninformed follower, showed that the leader might earn lower expected profits than in a Cournot model with complete information. Due to the distortion in the equilibrium action of the privately informed player which typically arises in signalling games, it is possible that moving first is a disadvantage.

Mailath (1993) analyses a model of endogenous timing allowing for such a first-mover disadvantage. A firm with superior information can choose between either moving first and acting as a Stackelberg leader, or moving second and playing a simultaneous Cournot game with the uninformed firm. The uninformed firm is restricted to moving second. Intuitively, the choice of the informed firm should depend on whether the first mover disadvantage exists or not, but Mailath shows that this intuition will not hold. Even when moving first is disadvantageous for the informed firm, playing the simultaneous Cournot game is not an equilibrium. The informed firm moves first, regardless of its private information.

From an empirical point of view Mailath's restriction, that only the informed firm is allowed to move first, might be justifiable. Mailath gives an example of a scenario in which this possibility is important. A patent holding firm must choose a new capacity at the end of the life of the patent. If the capacity is chosen just before the expiration of the patent, then the incumbent is able to act like a Stackelberg leader; if the incumbent waits, then it moves simultaneously with the uninformed firm. In such a scenario both the superior information as well as the timing flexibility of only the incumbent firm are plausible. There are, however, equally plausible situations in which both firms have the possibility of moving first or second. Consider, for example, entry into a new market. Due to a deregulation, it is known that the new market will exist in a certain amount of time. Because of its experience, a firm which operates in a related line of business has some idea of what the demand in the market will be. In contrast to this, a new firm which starts the business from the beginning will be uncertain about demand. In a scenario of this kind the asymmetric information is justified, but both firms have leeway in their timing decision.

From a theoretical point of view, the question is whether or not Mailath's result is robust to a relaxation of the restriction on the uninformed firm's timing choice. This is the first problem addressed in this paper. Mailath's model is extended for the case in which both firms are allowed to produce their quantity first or second. At the end of his paper, Mailath claims that in this case the only change is to introduce one more equilibrium: in this equilibrium the uninformed firm plays Stackelberg leader and the informed firm follower. If this were true, the model would still have some predictive power, because simultaneous Cournot equilibria could be ruled out. However, this claim is wrong. There are further equilibria with the uninformed firm moving first, and some types of the informed firm playing Cournot and others Stackelberg follower.

The multitude of equilibria calls for the application of equilibrium refinements. Already, Mailath's restricted analysis exhibits many sequential equilibria. Therefore, he requires the sequential equilibria to pass the equilibrium refinement $D1$ (Cho and Kreps (1987)). Imposing this refinement indeed reduces the set of sequential equilibria drastically. This explains how Mailath establishes his strong result. In the cases discussed in this paper, $D1$ has no power. The equilibria of the extended game derived in this paper do survive $D1$ because the uninformed firm moves simultaneously with, or before, the informed firm. In a similar context, Daughety and Reinganum (1994) suggest trembling hand perfectness (Selten (1975)) as an additional equilibrium refinement. While Daughety and Reinganum apply the criterion successfully, it turns out that trembling hand perfectness does not reduce the number of equilibria in this model. The timing equilibria are robust to trembles in firms' timing decisions.

The second novelty of this paper is to analyse a different timing game. In Mailath (1993) firms can make their timing decision only by committing to a quantity. When moving first, they have to make their choice without knowing what the rival firm is doing. This timing game is also applied, e.g. by Albæk (1992), Van Damme and Hurkens (1993, 1995) and Daughety and Reinganum (1994). In the alternative timing game, firms announce their timing decision in a pre-production stage without specifying the quantity. Firms are committed to this timing decision and produce in the sequence which resulted from the announcements. Applications of this game can be found, e.g. in Albæk (1990) and Spencer and Brander (1992). Hamilton and Slutsky (1990) call the first game "the extended game with action commitment", and second game "the extended game with observable delay". Their terminology is used in this paper.

The extended game with observable delay has not been applied yet in a context of incomplete information. For the present model, it turns out that timing results differ significantly to those of the extended game with action commitment. For a wide range of parameters the timing game with observable delay supports a simultaneous Cournot equilibrium involving all types of the informed firm. This outcome cannot be sustained as an equilibrium in the timing game with action commitment.

In the next section the general model is outlined. In Section 3, endogenous timing with action commitment is analysed: Mailath's (1993) analysis is extended for the case that also the uninformed firm has the flexibility to move first or second. Trembling hand perfectness of the timing equilibria is examined in Section 4. In Section 5, the results for the extended game with observable delay are given. Section 6 is the conclusion.

2 The Model

There are two quantity setting firms, with q_1 and q_2 denoting firms' quantities. The inverse demand function is assumed to be linear:

$$p = a - q_1 - q_2. \tag{1}$$

The demand intercept, a, might take three different values: a^l, a^m and a^h, where

$$a^l < a^m < a^h, \quad a^l > a^h/2 \tag{2}$$

(note that in a model with only two types, the first mover disadvantage does not occur; see the Appendix). The restriction $a^l > a^h/2$ is to ensure (without loss of generality) that the equilibrium quantities derived below are positive. Let firm 1 be the firm which is informed about the state of demand. The uninformed firm, firm 2, has prior beliefs on the states of demand

$$prob.(a = a^l) = \rho^l, \tag{3}$$

$$prob.(a = a^m) = \rho^m, \tag{4}$$

$$prob.(a = a^h) = \rho^h. \tag{5}$$

which are common knowledge. Firms produce at equal constant marginal cost. Price is measured net of marginal cost, so notation for cost is not needed. There is no entry or exit.

Firms must decide when to produce their output. Their choice is restricted to one out of two possible periods, $t \in \{1, 2\}$. The way in which firms choose their production period is given by two extended timing games: the extended game with action commitment and the extended game with observable delay. The analysis in Section 3 applies the extended game with action commitment: a firm can move first only by committing to an output. When doing so, the firm does not know what its rival is doing. When waiting, i.e. producing in period 2, a firm can observe the other firm's period 1 action, which is either a quantity or also the decision to wait. The equilibrium outcomes are obtained by checking each potential equilibrium separately: does either firm deviate from a proposed equilibrium given the rival's time and quantity decision?

In the extended game with observable delay (which is applied in Section 5), firms first announce at which periods they will produce their output. At this stage, firms need not yet specify the quantity, however, firms are committed to the timing decision. Both firms know the timing choice of the rival firm when producing the output. Production is in the sequence which results from the timing announcements. Subgame perfection implies that for each timing order there is only one possible outcome. If one firm announces $t = 1$ production, while the second chooses $t = 2$, a Stackelberg equilibrium results. If both firms announce the same period, a Cournot game is played. For equilibrium analysis of the extended game with observable delay the profits for all possible timing decisions have to be derived, and then the timing outcome is obtained by comparing across profits of the Cournot and Stackelberg equilibria.

The equilibrium concept for these games with incomplete information is that of sequential equilibrium as introduced by Kreps and Wilson (1982). A sequential equilibrium requires the specification of strategies and beliefs in and off the equilibrium path. The system of beliefs has to be consistent with firms' strategy profiles.

3 Endogenous Timing with Action Commitment

Maintaining the restriction that the uninformed moves second, Mailath proves the following theorem.

Theorem 1. (Mailath (1993)). *The only equilibrium outcomes that satisfy D1 are the separating Stackelberg outcome and the revealing equilibria (when they exist) in which either a^m produces $q_1^m = a^m/3$ in period 2 or a^m produces $q_1^m = a^m/3$ in period 2.*

The equilibria not eliminated by $D1$ (Cho and Kreps (1987)) all yield the quantities of the separating Stackelberg equilibrium. The revealing equilibria of Theorem 1 involve period 2 production by one type of firm 1, so that the lack of production at $t = 1$ reveals the type to firm 2. The level of production in these equilibria is the level from the separating equilibrium. Generically (see the Corollary in Mailath (1993)) the separating Stackelberg outcome is the only timing equilibrium satisfying $D1$.

In this section, the constraint on firm 2's choice is relaxed. To begin with, note that giving also firm 2 the possibility of moving first does not have any impact on the existence of the equilibria in Theorem 1. Fix firm 1's behaviour. Firm 2's reaction at $t = 2$ is optimal, so it cannot be profit increasing to move first. But, when switching to $t = 1$, it loses the information gained by observing firm 1's decisions at $t = 1$. Thus, firm 2 is worse off by deviating from period 2 production and does not deviate from any of the equilibria in Theorem 1.

When firm 2 has timing flexibility, there are four additional potential types of equilibria:

I. the non-signalling Stackelberg equilibrium, with firm 2 as the Stackelberg leader and all types of firm 1 as followers,

II. the Cournot equilibrium, with firm 2 and all types of firm 1 moving at $t = 1$,

III. equilibria with firm 2 moving at $t = 1$, one type of firm 1 playing Cournot at $t = 1$ and the other two types playing Stackelberg follower at $t = 2$,

IV. equilibria with firm 2 moving at $t = 1$, two types of firm 1 playing Cournot at $t = 1$ and the other type playing Stackelberg follower at $t = 2$.

In all these cases, firm 2 moves before or simultaneously with firm 1. Thus, if equilibria of type I–IV exist, they will depend on firm 2's priors only.

At the end of his paper, Mailath makes the following claim: *"Finally, observe that if both firms have the option of choosing in either period 1 or 2, then the only change is to introduce one additional equilibrium. In this equilibrium, firm 2 chooses in period 1 and firm 1 chooses in period 2."* This claim is wrong in the sense that there are more equilibria; equilibria of type

III and IV also exist. Consider all four types of equilibria in turn. Denote prior mean beliefs by $\bar{a} = \rho^l a^l + \rho^m a^m + \rho^h a^h$ and let $a \in \{a^l, a^m, a^h\}$ denote the true state of demand.

Case I. First consider the case in which the uninformed firm, firm 2, acts as the Stackelberg leader. Firm 2 maximizes a Stackelberg leader profit of

$$\pi_2 = (\bar{a} - \frac{\bar{a} - q_2}{2} - q_2)q_2. \tag{6}$$

Solving for q_2 gives $q_2 = \bar{a}/2$, and firm 1's optimal response to this is $q_1 = (2a - \bar{a})/4$. These quantities yield the following profits (all profit expressions of firm 2 are expected profits)

$$\pi_1 = \left(\frac{2a - \bar{a}}{4}\right)^2. \tag{7}$$

$$\pi_2 = \frac{(\bar{a})^2}{8}. \tag{8}$$

Now check whether this timing configuration is consistent with sequential equilibrium. Given firm 2's output decision, firm 1 cannot gain by moving first. Firm 1 would have to produce the same quantity at $t = 1$ which does not change its profits. Firm 2 is worse off when defecting to $t = 2$ production. It would lose the first mover advantage and would not improve its level of information. Since neither firm has an incentive to deviate, the equilibrium with firm 2 as the Stackelberg leader is a sequential equilibrium in endogenous timing.

Case II. Next consider the Cournot equilibrium with all types of firm

1 and firm 2 moving at $t = 1$. Firm 2 maximizes

$$\pi_2 = (\bar{a} - q_1 - q_2)q_2 \tag{9}$$

with respect to q_2. This yields an equilibrium output of $q_2 = \bar{a}/3$. Given q_2, firm 1 of type a produces $q_1 = (3a - \bar{a})/6$. Profits in this alleged equilibrium are

$$\pi_1 = \left(\frac{3a - \bar{a}}{6}\right)^2, \tag{10}$$

$$\pi_2 = \left(\frac{\bar{a}}{3}\right)^2. \tag{11}$$

Given firm 2's action, firm 1 cannot gain by defecting from this equilibrium. However, firm 2 has an incentive to delay. Fix firm 1's quantity decision at $t = 1$. Firm 2, by waiting, can produce according to the realized quantity of firm 1, which strictly increases its profits. In addition, firm 2 can also infer the state of demand from firm 1's action at this stage, because the different types of firm 1 produce different quantities. When defecting to $t = 2$ production, firm 2's optimal response to $q_1 = (3a - \bar{a})/6$ is $q_2 = (3a + \bar{a})/12$. Thus, by defecting from the equilibrium and waiting, firm 2 gets an expected profit of

$$\pi_2^{def} = \rho^l \left(\frac{3a^l + \bar{a}}{12}\right)^2 + \rho^m \left(\frac{3a^m + \bar{a}}{12}\right)^2 + \rho^h \left(\frac{3a^h + \bar{a}}{12}\right)^2. \tag{12}$$

By Jensen's inequality, the defection payoff $\pi_2^{def} \geq (\bar{a})^2/9$, with the equality holding for any $\rho = 1$. Hence, firm 2 has an incentive to delay and the Cournot equilibrium at $t = 1$ fails to be an equilibrium outcome in endogenous timing.

Case III. In this case, one type of firm 1 plays simultaneously with firm 2 at $t = 1$, while the other types produce at $t = 2$. Denote the type of firm 1 which moves first by a', and the two types moving second by a'' and a'''. Firm 2 has to maximize the following expression

$$\pi_2 = \rho'(a' - q_1' - q_2)q_2 + \rho''(a'' - \frac{a'' - q_2}{2} - q_2)q_2 + \rho'''(a''' - \frac{a''' - q_2}{2} - q_2)q_2. \quad (13)$$

In equilibrium, $q_2 = \bar{a}/(2 + \rho')$ and $q_1 = (a(2 + \rho') - \bar{a})/(2(2 + \rho'))$. These quantities give equilibrium profits of

$$\pi_1 = \left(\frac{a(2 + \rho') - \bar{a}}{2(2 + \rho')}\right)^2. \quad (14)$$

$$\pi_2 = \frac{(\bar{a})^2(1 + \rho')}{2(2 + \rho')^2}. \quad (15)$$

Note that if $\rho' = 1$ and the state of demand is $a = a'$, one obtains the full information Cournot equilibrium profits $\pi_2 = \pi_1' = (a')^2/9$. If $\rho'' = 1$ and $a = a''$, one gets the Stackelberg equilibrium profits $\pi_2 = (a'')^2/8$, $\pi_1'' = (a'')^2/16$. Similarly, the Stackelberg equilibrium for a''' is obtained for $\rho''' = 1$.

Consider defection from this equilibrium. Given that firm 2 produces q_2 at $t = 1$, there is no incentive for firm 1 to deviate. Firm 2, by defecting, can observe whether or not type a' moved first, and can appropriately update its priors. If there is period 1 production, firm 2 puts all weight on a' in its beliefs. Its optimal reaction to q_1 is then $q_2^{def} = (a'(2 + \rho') + \bar{a})/(4(2 + \rho'))$. If firm 2 observes no $t = 1$ production, it puts all weight on ρ'' and ρ''', such that its mean beliefs are $(\rho''a'' + \rho'''a''')/(\rho'' + \rho''')$. However, with these two types firm 2 now plays Cournot rather than Stackelberg leader. Thus, firm 2's expected defection profit is

$$\pi_2^{def} = \rho' \frac{(a')^2(2+\rho')^2 - (\bar{a})^2}{8(2+\rho')^2} + \frac{(\rho'' a'' + \rho''' a''')^2}{9(\rho'' + \rho''')}. \tag{16}$$

Type III equilibria exist when π_2 as in (15) is larger than π_2^{def}. Assume firm 2's priors are such that there is only a small weight on a', the type moving first, i.e. ρ' is small. Now, as $\rho' \to 0$, $\pi_2 - \pi_2^{def} \to (\rho'' a'' + \rho''' a''')^2/72 > 0$, and adhering to the equilibrium is more profitable than defecting from it. Thus type III equilibria exist. While $\rho' \approx 0$ is a sufficient condition, it is far from being necessary (see the Appendix). Note also that, for ρ' small, any type constellation can be an equilibrium, no matter whether a^l, a^m or a^h moves first. In other words, there are three equilibria of type III.

Case IV. Two types of firm 1 play simultaneously, with firm 2 at $t = 1$ and the remaining type produces at $t = 2$ as a Stackelberg follower. Denote the types moving first by a' and a'', and the type which moves second by a'''. Firm 2 maximizes

$$\pi_2 = \rho'(a' - q_1' - q_2)q_2 + \rho''(a'' - q_1'' - q_2)q_2 + \rho'''(a''' - \frac{a''' - q_2}{2} - q_2)q_2. \tag{17}$$

One gets equilibrium quantities of $q_2 = \bar{a}/(3-\rho''')$ for firm 2 and $q_1 = (a(3-\rho''') - \bar{a})/(2(3-\rho'''))$ for firm 1 of type a. These quantities give profits of

$$\pi_1 = \left(\frac{a(3-\rho''') - \bar{a}}{2(3-\rho''')}\right)^2, \tag{18}$$

$$\pi_2 = \frac{(\bar{a})^2(2-\rho''')}{2(3-\rho''')^2}. \tag{19}$$

Given firm 2's behaviour, firm 1 cannot gain by deviating from the above equilibrium. Firm 2, when defecting, can observe whether or not a' or a'' moved first. If there is period 1 production, firm 2 can infer the state of demand since these types produce different quantities. Then, $q_2^{def} = (a(3-\rho''')+\bar{a})/(4(3-\rho'''))$. If there is no period 1 production, firm 2 knows that type a''' produces at $t = 2$. By defecting, firm 2 loses the possibility to commit itself to the Stackelberg leader quantity: with probability ρ''' it now plays Cournot at $t = 2$ and no longer Stackelberg leader with a'''. Thus, firm 2's expected profit from defecting is

$$\pi_2^{def} = \rho' \frac{(a')^2(3-\rho''')^2 - (\bar{a})^2}{8(3-\rho''')^2} + \rho'' \frac{(a'')^2(3-\rho''')^2 - (\bar{a})^2}{8(3-\rho''')^2} + \rho''' \frac{(a''')^2}{9}. \tag{20}$$

Assume that the priors held by firm 2 are such that small weight is on the types moving first (a', a''). As $\rho''' \to 1$, $\pi_2 - \pi_2^{def} \to (a''')^2/72 > 0$, so firm 2 adheres to the equilibrium and type IV equilibria are shown to exist. The sufficient condition, $\rho''' \approx 1$, clearly overstates the necessary conditions for the existence of type IV equilibria (see the Appendix). As under III, there are three equilibria of type IV: any type of firm 1 can move first or second.

Theorem 2. *If both firms have the option of choosing in either period 1 or period 2, then equilibria additional to those in Theorem 1 have firm 2 moving at $t = 1$ and i) all types of firm 1 playing Stackelberg follower at $t = 2$, ii) one type of firm 1 playing Cournot at $t = 1$ and the other types producing at*

$t = 2$, iii) two types of firm 1 producing at $t = 1$ and the other type producing at $t = 2$.

Since type III and type IV equilibria allow for all combination of types of firm 1, there are seven equilibria additional to those in Theorem 1.

4 Trembling Hand Perfectness of the Sequential Equilibria

Daughety and Reinganum (1994) suggest trembling hand perfectness (Selten (1975)) as an additional equilibrium refinement. Trembling hand perfect equilibrium follows a different route than belief-based refinements. Here, out-of-equilibrium messages are treated in such a way that each firm takes into account that the rival firm might make uncorrelated mistakes which lead to this unexpected event. Sequential equilibria are trembling hand perfect if they are robust to such small perturbations of the equilibrium strategies. Here the robustness of the timing equilibria to small trembles in both firm's timing decisions is checked. Trembles in quantity decisions are not considered.

Equilibria of Theorem 1. It is straightforward to check that the equilibria in Theorem 1 are robust to trembles. Take the separating Stackelberg equilibrium and assume that firm 2 trembles. With a probability of ϵ, firm 2 will produce at $t = 1$ according to its priors. Both firms' output will vary in ϵ, but for ϵ small both firms will approximately produce their equilibrium quantities. Then, firm 1 strictly prefers to move first and there is no incentive to defect. If firm 1 trembles, it moves second with probability ϵ. Unless ϵ is not very big, firm 2 cannot gain defecting to $t = 1$ production. For the revealing equilibria of Theorem 1, the same argument holds because type a^m or a^l produce exactly the separating equilibrium quantity at $t = 2$. Thus, the equilibria in Theorem 1 are trembling hand perfect.

Equilibria of Theorem 2, trembles by firm 1. Next check the equilibria of Theorem 2 and consider first trembles by firm 1. If some type of firm 1, which moves first, trembles such that it moves second with a small probability ϵ, firm 2 does not defect. The opposite is true, the incentives of firm 2 to move first are increased because it plays Stackelberg leader with a higher probability. If a type which moves second trembles, this changes equilibrium outputs in ϵ because, with probability ϵ, firm 2 now plays Cournot rather than Stackelberg leader with these types. However, as ϵ, small both firms play roughly their equilibrium quantities. So for a small enough ϵ, firm 2 will adhere to the equilibrium. Thus, trembles by firm 1 do not reduce the number of equilibria in Theorem 3.2.

Equilibria of Theorem 2, trembles by firm 2, case I. Take the Stackelberg equilibrium with the uninformed firm as the Stackelberg leader. With probability $1 - \epsilon$, firm 2 produces its equilibrium output $q_2 = \bar{a}/2$ and, with probability ϵ, it trembles. If firm 1 adheres to the equilibrium, it gets the equilibrium profit with probability $1 - \epsilon$. In the event of a tremble, firm 2 is still uninformed and plays Cournot, with firm 1 at $t = 2$. In this case, firm

1 of type a produces $(3a - \bar{a})/6$. Hence, if firm 2 trembles, adhering to the equilibrium gives firm 1 a profit of

$$\pi_1^{adh} = (1-\epsilon)\left(\frac{2a-\bar{a}}{4}\right)^2 + \epsilon\left(\frac{3a-\bar{a}}{6}\right)^2. \tag{21}$$

Defecting from this equilibrium, firm 1's out-of-equilibrium action is to produce the equilibrium quantity $q_1 = (2a - \bar{a})/4$, but in period 1. With probability $1 - \epsilon$ nothing changes in comparison to the equilibrium: firm 2 still produces the Stackelberg leader quantity at $t = 1$, so firm 1 gets the same profit. But, in the event of a tremble, firm 2 observes firm 1's quantity at $t = 1$. Assuming that firm 2 assigns probability 1 that this out-of-equilibrium message was sent by type a (which maximizes firm 1's incentive to defect), it produces $q_2 = (2a + \bar{a})/8$. So firm 1's defection payoff is

$$\pi_1^{def} = (1-\epsilon)\left(\frac{2a-\bar{a}}{4}\right)^2 + \epsilon\left(\frac{4a^2-(\bar{a})^2}{32}\right). \tag{22}$$

Comparing the equilibrium profit π_1^{adh} to the defection payoff π_1^{def}, adhering to the equilibrium is preferred if $144a^2 - 192a\bar{a} + 68\bar{a}^2 > 0$. Since $2a < \bar{a} < a/2$, this inequality holds and there is no profitable deviation for any type of firm 1. Thus, the non-signalling Stackelberg equilibrium is robust to trembles in the timing decision.

Case II. The Cournot equilibrium at $t = 1$, constituted by all types of firm 1 and firm 2, is not a sequential equilibrium, so trembling hand perfectness does not have to be checked.

Case III. In a sequential equilibrium of case III, type a' produces first and types a'' and a''' move second. Firm 2 plays the equilibrium action with probability $1-\epsilon$, and with ϵ it trembles. Equilibrium quantities will vary in ϵ, but since ϵ is small this effect can be ignored. If there is no tremble, types a'' and a''' receive their equilibrium profits. With probability ϵ, firm 2 observes that type a' did not produce at $t = 1$ and accordingly updates its posteriors. Denote firm 2's posterior mean beliefs by $\tilde{a} = (\rho''a'' + \rho'''a''')/(\rho'' + \rho''')$. At $t = 2$, firm 2 plays a Cournot quantity of $\tilde{a}/3$. Thus, by adhering to the equilibrium, a'' and a''' get a profit of

$$\pi_1^{adh} = (1-\epsilon)\left(\frac{a(2+\rho') - \bar{a}}{2(2+\rho')}\right)^2 + \epsilon\left(\frac{3a - \tilde{a}}{6}\right)^2. \tag{23}$$

To analyse defection by a'' or a''', the out-of-equilibrium action to be examined is the production of $q_1 = (a(2+\rho') - \bar{a})/(2(2+\rho'))$ at $t = 1$. Assuming firm 2 correctly identifies that this message was sent by type a (which maximizes the incentive to defect), it produces $q_2 = (a(2+\rho') + \bar{a})/(4(2+\rho'))$. Thus, from defecting, types a'' and a''' get a profit of

$$\pi_1^{def} = (1-\epsilon)\left(\frac{a(2+\rho') - \bar{a}}{2(2+\rho')}\right)^2 + \epsilon\frac{a^2(2+\rho')^2 - (\bar{a})^2}{8(2+\rho')^2}. \tag{24}$$

Types a'' and a''' adhere to the equilibrium if $\pi_1^{adh} > \pi_1^{def}$. Recall that $\rho' \approx 0$ is a sufficient condition for the existence of type III equilibria. Now, as $\rho' \to 0$, the term $\pi_1^{adh} > \pi_1^{def}$ simplifies to $144a^2 - 192a\tilde{a} + 68\tilde{a}^2 > 0$ as above for case I. Since this condition holds, there are type III equilibria which survive the trembling hand criterion. Note that, in contrast to case I, for some types and some priorsthere might be equilibria which are indeed erased by applying tremblinghand perfect equilibrium. To complete the analysis one would have to check whether or not a'' and a''' have a profitable defection *and* that a' could not profitably play this out of equilibrium action. However, since focus is here on the existence of equilibria, it is sufficient to show that, for $\rho' \approx 0$, type III equilibria survive the trembling hand criterion.

Case IV. Finally, consider the equilibria in which only one type of firm 1, a''', produces at $t = 2$. Ignoring the impact which firm 2's trembles have on the equilibrium quantities, for type a''' with probability $1 - \epsilon$ nothing changes in comparison to the non-trembling equilibrium. With probability ϵ, firm 2 delays its production until $t = 2$. Then, firm 2 observes no period one production, puts all weight on type a''' and plays Cournot with this type. Thus, if firm 2 trembles, type a''' has an equilibrium profit of

$$\pi_1^{adh} = (1 - \epsilon)\left(\frac{a'''(3 - \rho''') - \bar{a}}{2(3 - \rho''')}\right)^2 + \epsilon\frac{(a''')^2}{9}. \tag{25}$$

When defecting from this equilibrium, type a''' produces the equilibrium quantity $(a'''(3-\rho''')-\bar{a})/(2(3-\rho'''))$, but at $t = 1$. If firm 2 does not tremble, it plays its equilibrium strategy and profits are the same as in equilibrium. In the event of a tremble, after observing type a''', firm 2 puts all weight on a''' (assuming a' and a'' could not gain by switching to this quantity). Firm 2 produces $q_2 = (a'''(3 - \rho''') + \bar{a})/(4(3 - \rho'''))$. From defecting, type a''' gets a profit of

$$\pi_1^{def} = (1 - \epsilon)\left(\frac{a'''(3 - \rho''') - \bar{a}}{2(3 - \rho''')}\right)^2 + \epsilon\,\frac{(a''')^2(3 - \rho''') - \bar{a}^2}{8(3 - \rho''')}. \tag{26}$$

Firm 1 of type a''' will adhere to the equilibrium if π_1^{adh} is larger than π_1^{def}. This is the case if $\bar{a} > a'''(3 - \rho''')/3$. Using $\rho''' \to 1$, a sufficient condition for the existence of type IV equilibria, this simplifies to $a''' > \frac{2}{3}a'''$. As explained for case III, there might be equilibria which are not trembling hand perfect, in any case, there are type IV equilibria which are trembling hand perfect.

Theorem 3. *The equilibria in Theorem 1 and 2 are trembling hand perfect.*

Why does trembling hand perfection reduce the number of equilibria in Daughety and Reinganum (1994)? The Daughety and Reinganum model differs from the model in this paper in a number of features: a priori, none of the two firms is informed: both firms first have to decide whether or not to buy information about an uncertain demand parameter, which is the slope of the demand curve, and not, as in Mailath (1993) and in this paper, the demand intercept. But this difference alone does not lead to differences in results. Given the decision to buy this information, they play a timing game equivalent to the extended game with action commitment. In particular, in the case in which one firm decides to buy information while the other decides not to, their analysis is equivalent to the one in this paper.

Daughety and Reinganum introduce only two types, so the first mover disadvantage does not exist. This simplifies the timing decision of the informed firm. Further, they introduce two severe parameter restrictions. First, they restrict the differences between the two types in such a way that signalling distortions cannot arise. Put another way, this means that the full information quantities are separating. Second. the uninformed firm's prior beliefs about the two types are assumed to be equal. i.e. both states of demand have a prior probability of 0.5. These restrictions are by no means without loss of generality.

Due to these assumptions and simplifications, Daughety and Reinganum do not obtain equilibria similar to those of type III and type IV in the notation of Section 3. Dropping these assumptions. one can show that there are more equilibria. In these equilibria the uninformed firm moves first, while one type of the informed firm moves first and the other moves second. Since there are two types of the informed firm there exist two equilibria. Furthermore. it is also due to the parameter restrictions that Daughety and Reinganum manage to erase the non-signalling Stackelberg equilibrium. In their model, one type has a profitable defection. As shown above. this is not the case in the more general model of this paper.

5 Endogenous Timing with Observable Delay

In the extended game with observable delay firm 1's timing decision is an information set for firm 2. To begin with. beliefs are assumed to be such that if firm 1 chooses $t = 1$, firm 2 carries forward its prior beliefs; and if firm 1 announces $t = 2$, then firm 2 puts all weight on a^h. Later in this section, this assumption is relaxed and the impact that other beliefs have on equilibrium behaviour is analysed.

In the Cournot equilibrium at $t = 1$, firm 2 produces $q_2 = \bar{a}/3$ according to its priors. Firm 1 of type a has an output of $q_1 = (3a - \bar{a})/6$. These output decisions lead to equilibrium profits of

$$\pi_1 = \left(\frac{3a - \bar{a}}{6}\right)^2. \tag{27}$$

$$\pi_2 = \left(\frac{\bar{a}}{3}\right)^2. \tag{28}$$

If the sequential move equilibrium, with firm 1 as the Stackelberg leader, results from the timing decisions, firm 1's output is a second information set for firm 2. The attention is restricted to the separating equilibrium as derived in the Appendix. The signalling Stackelberg equilibrium profits are, for the high state of demand,

$$\pi_1^h = \frac{(a^h)^2}{8}. \tag{29}$$

$$\pi_2^h = \frac{(a^h)^2}{16}. \tag{30}$$

for the medium state of demand:

$$\frac{(a^m)^2}{9} \leq \pi_1^m < \frac{(a^m)^2}{8}. \tag{31}$$

$$\frac{(a^m)^2}{9} \geq \pi_2^m > \frac{(a^m)^2}{16}. \tag{32}$$

and for the low state of demand:

$$\frac{(a^l)^2}{10} < \pi_1^l < \frac{(a^l)^2}{8}. \tag{33}$$

$$\frac{(a^l)^2}{7} > \pi_2^l > \frac{(a^l)^2}{16}. \tag{34}$$

Denote firm 1's Stackelberg leader profit by π_{1L} and let firm 2's expected profit be $\pi_{2F} = \pi_2^h \rho^h + \pi_2^m \rho^m + \pi_2^l \rho^l$. Note that the medium and the low demand type of firm 1 have to produce less output and receive lower profits than in a model with complete information. Strikingly, this distortion can lead to a first-mover disadvantage for the low demand type: the informed Stackelberg leader earns a lower profit, and the uninformed Stackelberg follower earns a higher profit, than in a Cournot equilibrium with complete information (Gal-Or (1987)). For some parameters, the profit of the Stackelberg follower is even higher than the profit of a Stackelberg leader with complete information.

The Cournot equilibrium at $t = 2$ requires firm 2 to have all weight on a^h and so firm 2 produces $q_2 = a^h/3$. Firm 1 of type a chooses an output of $q_1 = (3a - a^h)/6$. These quantities give equilibrium profits of

$$\pi_1 = \left(\frac{3a - a^h}{6}\right)^2, \tag{35}$$

$$\pi_2 = \left(\frac{a^h}{3}\right)^2. \tag{36}$$

In the equilibrium with the uninformed firm 2 as the Stackelberg leader, firm 2 produces $q_2 = a^h/2$, while firm 1 of type a chooses $q_1 = (2a - a^h)/4$. Equilibrium profits are thus

$$\pi_1 = \left(\frac{2a - a^h}{4}\right)^2, \tag{37}$$

$$\pi_2 = \frac{(a^h)^2}{8}. \tag{38}$$

The profits of the different Cournot and Stackelberg equilibria are summarized in Figure 1. In the upper right cell of the matrix (the Stackelberg signalling equilibrium) the profits depend on beliefs and types. They have to be obtained from (29)–(34).

		Firm 2: *Move first*	Firm 2: *Move second*
Firm 1	*Move first*	$\frac{(\bar{a})^2}{9}$; $\frac{(3a-\bar{a})^2}{36}$	π_{2F} ; π_{1L}
	Move second	$\frac{(a^h)^2}{8}$; $\frac{(2a-a^h)^2}{16}$	$\frac{(a^h)^2}{9}$; $\frac{(3a-a^h)^2}{36}$

Figure 1: The payoff matrix for the timing game with observable delay.

Now take firm 1's timing decision. If firm 2 moves first, then firm 1 has to choose between playing Cournot at $t = 1$ and being Stackelberg follower. From (27) and (37):

$$\begin{aligned} \left(\frac{3a-\bar{a}}{6}\right)^2 &> \left(\frac{2a-a^h}{4}\right)^2 \\ \bar{a} &< \frac{3}{2}a^h. \end{aligned} \tag{39}$$

which holds for all types and any prior beliefs held by firm 2. Thus firm 1 chooses $t = 1$ if firm 2 moves first.

If firm 2 moves second, the comparison is between the separating Stackelberg leader profit (as in (29), (31) and (33)) and the profit of the Cournot equilibrium at $t = 2$, as in (35). For high demand, firm 1's condition for a preference for $t = 1$ is

$$\frac{(a^h)^2}{8} > \left(\frac{3a^h - a^h}{6}\right)^2$$
$$\frac{a^h}{\sqrt{8}} > \frac{a^h}{3}. \tag{40}$$

so the high demand type prefers to be Stackelberg leader. For the medium demand type, take the lowest possible profit from (31), $\pi_1^m = (a^m)^2/9$. For this to be higher than the Cournot profit at $t = 2$,

$$\frac{(a^m)^2}{9} > \left(\frac{3a^m - a^h}{6}\right)^2 \tag{41}$$

must hold. Using $a^m = \frac{3}{4}a^h$ (which is required to get the lower bound in (31) (see the Appendix)), inequality (41) becomes

$$\frac{a^h}{4} > \frac{\frac{5}{4}a^h}{6}. \tag{42}$$

and so Stackelberg leadership is preferred by the medium demand type also. For the low demand type, the lower bound of (33) is $\pi_1^l > (a^l)^2/10$. The condition is now

$$\frac{(a^l)^2}{10} > \left(\frac{3a^l - a^h}{6}\right)^2. \tag{43}$$

For the lower bound to hold, it must be that $a^l = \frac{9}{16}a^h$ (see the Appendix). Plugging this into (43)

$$\frac{\frac{9}{16}a^h}{\sqrt{10}} > \frac{\frac{11}{16}a^h}{6} \tag{44}$$

results and firm 1 chooses $t = 1$ for low demand. Thus, for all types of firm 1, the Stackelberg leader profit is larger than the Cournot profit at $t = 2$, i.e. the optimal choice is $t = 1$. Firm 1 has a dominant strategy to move first.

Now consider firm 2's timing decision. Given firm 1's dominant strategy, firm 2 has to choose between the Cournot equilibrium at $t = 1$ and playing Stackelberg follower at $t = 2$. Whether or not it is more profitable to announce $t = 1$ or $t = 2$ depends on firm 2's priors as well as on the existence of the first mover disadvantage. Firm 2 will play $t = 1$ if its expected profit from playing Cournot is larger than the expected profit as a Stackelberg follower, i.e. if $\pi_{2F} > (\bar{a})^2/9$. Firm 2 will choose $t = 2$ if the inequality is reversed. So, a first result is that, depending on whether $\pi_{2F} > (\bar{a})^2/9$ and given beliefs as specified, the Cournot equilibrium or the Stackelberg signalling equilibrium emerge.

Next, check whether beliefs different from those specified above lead to further equilibria. Denote the mean beliefs following $t = 1$ announcement by $\tilde{a}$ and the mean beliefs following $t = 2$ announcement by $\hat{a}$. Assume that firm 2 plays $t = 1$. Firm 1 prefers moving first if playing Cournot at $t = 1$ (with firm 2 holding beliefs of $\tilde{a}$) is more profitable than playing Stackelberg follower (with firm 2 holding beliefs of $\hat{a}$), that is if

$$\left(\frac{3a - \tilde{a}}{6}\right)^2 > \left(\frac{2a - \hat{a}}{4}\right)^2 \tag{45}$$

or

$$\tilde{a} < \frac{3}{2}\hat{a}. \tag{46}$$

Firm 1 prefers moving second if inequality (46) is reversed. Note that this inequality is independent of the type of firm 1 a, so there are no equilibria in which different types of firm 1 announce different periods of production. Since all types choose the same period, it must be that either $\tilde{a}$ or $\hat{a}$ is equal to prior beliefs $\bar{a}$. Above, it was assumed that $\tilde{a} = \bar{a}$ and $\hat{a} = a^h$. Clearly, $\bar{a} < \frac{3}{2}a^h$, i.e. (46) holds and firm 1 chooses $t = 1$ as derived. However, if $\tilde{a} > \frac{3}{2}\hat{a}$ firm 2 playing Stackelberg leader and firm 1 follower is an equilibrium. If, for example, $\tilde{a} = a^h$ and $\hat{a} = \bar{a}$ and if, in addition, $a^h > \frac{3}{2}\bar{a}$, then the non-signalling Stackelberg equilibrium is the equilibrium of the extended game with observable delay.

One possible timing equilibrium remains: the Cournot equilibrium at $t = 2$. It is easy to see that this is not an equilibrium. Firm 2 holds the same posteriors when firm 1 plays $t = 2$, whether as a Stackelberg leader or in the Cournot equilibrium. But, in this case firm 2 does strictly better by being Stackelberg leader than by playing Cournot. Thus, both firms playing $t = 2$ is not an equilibrium.

This completes the analysis of the timing game with observable delay.

Theorem 4. *In the extended game with observable delay, the Cournot game at* $t = 1$ *is the timing equilibrium if* $\pi_{2F} > \frac{(\bar{a})^2}{9}$*; if the inequality is reversed, the Stackelberg equilibrium with firm 1 as the Stackelberg leader is the equilibrium. The Stackelberg equilibrium with firm 2 as Stackelberg leader can be sustained as a timing equilibrium if prior beliefs satisfy* $a^h > \frac{3}{2}\bar{a}$*. There are no other equilibria.*

For a wide range of parameters. both firms choose $t = 1$ and the Cournot equilibrium is the timing equilibrium. In particular. if the signalling distortions in the Stackelberg equilibrium are not too strong (e.g. if a^m and a^l produce a quantity higher than the perfect information Cournot equilibrium), the Cournot equilibrium results, independently of firm 2's priors. Even if parameters are such that the incentives for firm 2 to choose $t = 2$ are at their maximum (i.e. if $\pi_2^m = (a^m)^2/9$ and $\pi_2^l > (a^l)^2/9$) prior beliefs of $\rho^h \geq 0.13$ are sufficient to make firm 2 still choose $t = 1$. Put another way. for the signalling Stackelberg outcome to be an equilibrium under endogenous timing, a sufficient signalling distortion and high weights on the low and medium demand types ($\rho^l + \rho^m > 0.87$) are required. Also, the timing equilibrium with firm 2 as the Stackelberg leader holds for a limited range of parameters only. Prior beliefs have to satisfy $\frac{a^h}{2} < \bar{a} < \frac{2}{3}a^h$ as a necessary condition.

6 Concluding Remarks

In this paper endogenous timing in a duopoly with a privately-informed firm was examined. Due to a greater generality in comparison to previous papers, unfortunately, the model has multiple equilibria. There are Stackelberg equilibria with either the informed or the uninformed firm moving first, and equilibria in which the uninformed firm plays Cournot with some types of the informed firm and Stackelberg leader with others. The strong result obtained by Mailath (1993) and Daughety and Reinganum (1994), suggesting that there is a unique timing equilibrium with the privately informed firm as the Stackelberg leader, cannot be restated in this model.

The paper also explores what happens if firms may first announce in which period they wish to produce without committing themselves to a specific output. Such a timing game reduces the number of equilibria. For a wide range of parameters the outcome is the Cournot equilibrium at stage one, constituted by all types of the informed firm and the uninformed firm. Ironically, this is the only timing outcome which cannot be sustained as an equilibrium under the extended game with action commitment (see also Albæk (1992)). The timing game obviously has a big impact on the sequencing outcome. Unfortunately, most authors are not explicit about the impact that the specification of the extended game has on the sequencing outcome, though, a priori, neither of the timing games is preferable to the other. They are relevant in different situations (see Hamilton and Slutsky (1990) and Spencer and Brander (1992)).

A theoretical parallel to the two timing games can be found in the prices-versus-quantities literature. Singh and Vives (1984) analyse a game in which firms first simultaneously choose either price or quantity competition and afterwards compete contingent on the chosen type of competition. This is similar to the extended game with observable delay. Klemperer and Meyer (1986) develop a model in which duopolists simultaneously commit to either a concrete quantity or a concrete price. Given this choice, the remaining prices and quantities are determined to clear all markets. This is similar to the extended game with action commitment.

Appendix

The Signalling Stackelberg Equilibrium

In this section firm 1 is restricted to producing at $t = 1$, and firm 2 at $t = 2$. This Stackelberg game, with the informed firm as the Stackelberg leader and the uninformed firm as the follower, constitutes a signalling game: the output of firm 1 may signal some information about the state of demand to firm 2. The focus is on the separating equilibrium[3]. In a separating equilibrium firm 2's inferences about the state of demand are correct. Consider the output choice of each type of firm 1 in turn.

Type a^h cannot do better than to produce its full information output $q_1^h = a^h/2$ in a separating equilibrium. After observing q_1^h, firm 2 produces $q_2^h = a^h/4$. These quantities give equilibrium profits of

$$\pi_1^h = \frac{(a^h)^2}{8}. \quad (47)$$

$$\pi_2^h = \frac{(a^h)^2}{16}. \quad (48)$$

Type a^m has to choose q_1^m such that type a^h does not defect from its equilibrium quantity q_1^h. That is, q_1^m has to solve the incentive compatibility

[3]The Separating equilibrium is the only equilibrium which survives the equilibrium refinement $D1$ (Cho and Kreps (1987)). See Mailath (1993).

constraint

$$\left(\frac{2a^h - a^m - q_1^m}{2}\right) q_1^m = \frac{(a^h)^2}{8}. \tag{49}$$

Solving this expression for q_1^m, one gets two solutions. Only the lower solution is incentive compatible: $q_1^m = (2a^h - a^m - ((a^h - a^m)(3a^h - a^m))^{1/2}/2$. It is straightforward to see that the full information equilibrium output cannot be incentive compatible: $q_1^m = a^m/2$ solves (49) if and only if $a^m = a^h/3$. However, by assumption, $a^m > a^h/2$. Thus, incentive compatibility requires $q_1^m < a^m/2$. This establishes an upper bound on q_1^m. A lower bound is obtained for $a^m = \frac{3}{4}a^h$. For this value, the full information Cournot output $q_1^m = a^m/3$ solves the incentive compatibility constraint (49). Hence, $a^m/3 \leq q_1 < a^m/2$, which implies $a^m/3 \geq q_2 > a^m/4$. Using the upper and the lower bound of the equilibrium quantities, the range of separating equilibrium profits for the medium type of demand can be limited to

$$\frac{(a^m)^2}{9} \leq \pi_1^m < \frac{(a^m)^2}{8}, \tag{50}$$

$$\frac{(a^m)^2}{9} \geq \pi_2^m > \frac{(a^m)^2}{16}. \tag{51}$$

Incentive compatibility for the low demand type requires

$$\left(\frac{2a^m - a^l - q_1^l}{2}\right) q_1^l = \pi_1^m. \tag{52}$$

Solving (52) explicitly gives an expression which depends on the actual realization of π_1^m. As an upper bound one can again use $q_1^l < a^l/2$. For a small $3a^m - 4a^l$, $q_1^l < a^l/3$. The lowest possible value of q_1^l is obtained for $\pi_1^m = (a^m)^2/9$ (which requires $a^m = \frac{3}{4}a^h$) and $a^l = \frac{3}{4}a^m$. For these values

$q_1^l = (135 - 9\sqrt{97})a^l/162 \approx 0.286a^l$ and consequently $q_2^l \approx 0.357a^l$. Using the upper and the lower bound of the equilibrium quantities, the range of equilibrium profits is

$$\frac{(a^l)^2}{10} < \pi_1^l < \frac{(a^l)^2}{8}, \tag{53}$$

$$\frac{(a^l)^2}{7} > \pi_2^l > \frac{(a^l)^2}{16}. \tag{54}$$

The beliefs of firm 2 which support this separating equilibrium are the following. If $q_1 > q_1^m$, firm 2 puts all weight on a^h in its posteriors. If $q_1^l < q_1 \leq q_1^m$, the posteriors have probability one on a^m and, if $q_1 \leq q_1^l$, posterior beliefs are such that the low demand type has probability one. The equilibrium quantities, together with the specified beliefs, yield a sequential equilibrium.

Existence of Type III and IV Equilibria

For the proof of the existence of type III and type IV equilibria it was required above that the types of firm 1 which move first in equilibrium receive a low weight in firm 2's priors. In this appendix it is shown that the assumptions $\rho^l \approx 0$ and $\rho^l + \rho^{m} \approx 0$ are sufficient, but by no means necessary for the existence of type III and IV equilibria respectively.

Start with type III. Figure 2 shows the impact the parameter values have on the choice between adhering to and defecting from the equilibria. Whether $\pi_2^{adh} - \pi_2^{def}$ is positive or not depends both on ρ^l and the demand parameters.

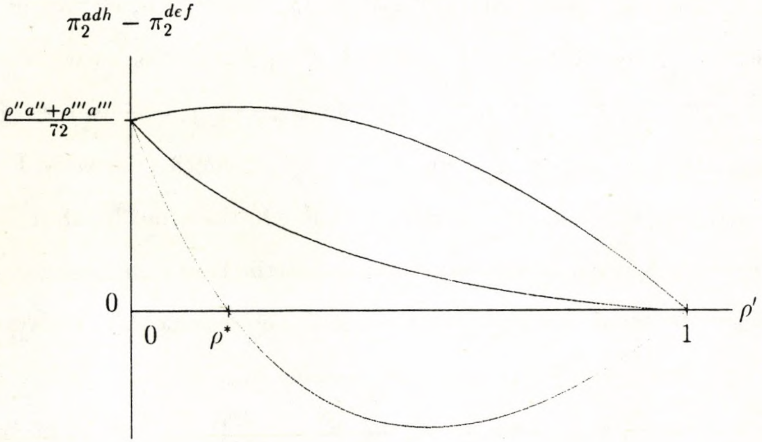

Figure 2.

In the Section 3. it was shown that. for $\rho' \to 0$. $\pi_2^{adh} - \pi_2^{def} \to (\rho''a'' + \rho'''a''')/72$. The intuition behind this is that firm 2 expects to play Stackelberg leader with a'' and a''' with a probability of approximately 1. When defecting, firm 2 plays Cournot with a'' and a'''. Thus firm 2's expected loss from defecting is exactly the difference between the Stackelberg leader and the Cournot profit played with a'' and a'''. This determines the intercept on the vertical axis in Figure 2. If $\rho' = 1$, firm 2 expects to play Cournot with a' at $t = 1$. Defecting to $t = 2$ does not change profits. given firm 1's equilibrium behaviour. So. expected gains or losses from defecting are zero if $\rho' = 1$. This determines $\pi_2^{adh} - \pi_2^{def} = 0$ for $\rho' = 1$.

The demand parameters determine the shape of the $\pi_2^{adh} - \pi_2^{def}$ curve.

When defecting, firm 2 can observe whether type a' moved at $t = 1$ or not. If there has been period 1 production, firm 2 puts all weight on a'; if not, firm 2 has all weight on a'' and a''' such that its mean beliefs are $(\rho''a'' + \rho'''a''')/(\rho'' + \rho''') = \tilde{a}$. Now for a small $a' - \tilde{a}$ firm 2's gain in information from defecting is small. The upper concave curve in Figure 2 represents the extreme case in which $a' = \tilde{a}$. On the other hand, if $a' - \tilde{a}$ is big, firm 2 might gain from defection because the gain in information might outweigh the losses from playing Cournot. The lower convex curve results from the parameter constellation maximizing firm 2's incentives to defect: $a'/2 \approx \tilde{a}$ (according to the restriction $a' > a^h/2$). All other ratios of a' and $\tilde{a}$ yield curves which lie between these two extremes, for example, the intermediate case, where $\frac{2}{3}a' = \tilde{a}$.

For $a' = \tilde{a}$, firm 2 is clearly better off adhering to the equilibrium for all values of ρ'. Simulations show that this is also the case for $\frac{2}{3}a' \leq \tilde{a} \leq \frac{3}{2}a'$. Even for the parameter constellation that maximizes the incentives to defect, there is a ρ^*, such that, for all $\rho' < \rho^*$, firm 2 does not have a profitable defection and type III equilibria still exist. Using numerical simulations, one can show that $\rho^* \approx 0.16$.

The same argument holds for type IV equilibria and a similar picture can be drawn. There is only one difference. In case IV there are two types of firm 1 moving first. When defecting, firm 2 can infer the state of demand because the different types produce different quantities. So, in contrast to III, there is an incentive for firm 2 to defect, even if $\rho' + \rho'' \approx 1$ (as follows from

Jensen's inequality). So, as $\rho' + \rho'' \rightarrow 1$, the $\pi_2^{adh} - \pi_2^{def}$ curve is non-positive – independent of the demand parameters.

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