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The regulation of entry and aggregate productivity*

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Abstract

The aim of this paper is to contribute to explaining differences in aggregate productivity between similar, industrialized countries such as the US and European Union (EU) member states. By introducing shifts in administrative entry cost and a firm technology adoption decision in a model of heterogeneous firms close to Hopenhayn (1992), it matches the following facts: higher entry cost is associated with (1) both lower labor and total factor productivity, (2) more capital-intensive production, and (3) lower firm turnover. Compared to previous studies of reallocation intensity and aggregate productivity, endogenizing capital intensity through technology choice leads to stronger results; higher equilibrium capital intensity acts as an entry barrier to new firms, and protects low-productivity incumbents. Notably, the very small differences in the administrative cost of entry as documented by Djankov, La Porta, Lopez-de-Silanes and Shleifer (2002) suffice to explain 10 to 20% of differences in TFP and the capital-output ratio between Europe and the US. To obtain this, both heterogeneity of firms and allowing for technology choice are crucial.

JEL codes: E22, G38, L11, L16, O33, O40

Keywords: growth theory, aggregate productivity, technology adoption, firm dynamics, entry and exit, reallocation, selection, regulation of entry

1 Introduction

The lag of Euro Area countries in labor and total factor productivity (TFP) relative to the Unites States is a topic of ongoing discussion in Europe, reflected in political projects (e.g. the Lisbon Agenda), commission reports (e.g. the Sapir Report 2004), and many academic papers (e.g. Blanchard 2004, Prescott 2004). What is stressed less often is that whereas Europe lags in these measures, in working hours, and in many measures of human capital, its economies

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Table 1: Country statistics, 5 large EU economies and US

	Firm turnover rate	Administrative entry cost	Y/L	K/Y
Germany	0.13	0.325	0.82	1.25
France	0.195	0.355	0.82	1.19
Italy	0.165	0.448	0.83	1.13
Netherlands	0.175	0.308	0.81	1.12
United States	0.22	0.017	1.00	1.00

Labor productivity (output per worker) and capital-output ratios are from Hall and Jones (1999). They are for 1988 and are expressed in PPP terms, relative to the US values. Firm turnover rates are the sum of average annual firm entry and exit rates over the 1990s and are from Bartelsman, Haltiwanger and Scarpetta (2004). The administrative cost of entry is the sum of direct payments and the cost of time spent on the procedure needed to establish a small business. It is expressed as a fraction of the country's per capita output and is from Djankov et al. (2002).

are much more capital intensive. While this could simply be attributed to relatively higher taxes on labor compared to capital and to stricter labor market regulation,[1] there is more to it: This paper shows that in a model of heterogeneous firms that adopt a technology upon entry, a small shift in administrative entry cost equivalent to those reported by Djankov, La Porta, Lopez-de-Silanes and Shleifer (2002) – corresponding to a small fraction of the total cost of the entry investment – can explain part of the differences in TFP, capital-output ratios, and turnover observed between European countries and the US. The effect largely arises because the equilibrium with higher administrative entry cost features a higher equilibrium investment at entry. This acts as an entry barrier, and protects low-productivity incumbents. The paper hence complements empirical studies on productivity and firm dynamics, and explores the implications of their results for the macro level. From a policy point of view, the analysis here complements recent theoretical and empirical work such as Blanchard and Giavazzi (2003) and Nicoletti and Scarpetta (2003) that stresses productivity effects of product and labor market regulation.

Table 1 summarizes data on labor productivity (measured as output per worker), capital-output ratios, administrative cost of entry, and firm turnover rates for the US and major EU economies. A salient feature of these indicators, stable over time and over measures of labor productivity, is that European labor productivity is below its US counterpart, despite higher

[1]It is well known that labor taxes in most European countries are higher than in the US; for an illustration of their impact see e.g. Prescott (2004). Moreover, effective tax rates on capital tend to be lower in European countries compared to the US (Chen, Mintz and Poschmann 2005). Stricter labor market regulation in Europe is well-documented, see e.g. Blanchard (2004) and references therein. All these factors should induce substitution towards more capital-intensive production in Europe.

capital-output ratios. As many papers have remarked, this must be due to differences in TFP, since differences in human capital are smaller than this gap (see e.g. Klenow and Rodriguez-Clare 1997, Hall and Jones 1999, Caselli 2005). As these countries are similarly developed, these TFP differences should not arise from differences in access to production technologies. Another stable feature is that Euro Area countries feature systematically higher administrative entry costs and systematically lower firm turnover rates than the US. Matching these patterns and evaluating the impact of small changes in administrative entry costs on aggregate productivity is the objective of this paper.

Any model used for addressing these aims needs to incorporate entry and exit, and, as shown below, firm heterogeneity. The importance of these processes has been stressed in the burst of empirical literature on the topic published in the last decade.[2] For example, Dwyer (1996) finds that productivity differs by a factor 3 between establishments in the 9th and the 2nd decile of the productivity distribution in the US textile sector. Foster, Haltiwanger and Krizan (2001) find that in the Census of Manufactures, more than a quarter of the increase in aggregate productivity between 1977 and 1987 was due to entry and exit. This is even more pronounced in the retail sector, as they find in their (2002) paper. The fact that this is the sector where the productivity divergence between Europe and the US is strongest (van Ark, Inklaar and McGuckin 2002) again suggests a role for firm dynamics. To take these insights into account, the model used matches many features of firm dynamics, by construction or by calibration.

Moreover, this paper introduces entrants' technology choice in a dynamic stochastic heterogeneous-firm model building on Hopenhayn (1992), thereby endogenizing part of the underlying productivity process affecting firms. This is the contribution of this paper from a modeling point of view. It is clear from the importance of entry and exit for aggregate productivity that entrants' choice of technology can have a large impact on aggregate productivity. Yet, existing heterogeneous-firm models such as Jovanovic (1982), Hopenhayn and Rogerson (1993), and Samaniego (2005) exogenously endow firms with a technology upon entry and then have their productivity evolve stochastically.[3] This restricts the changes of the productivity distribution

[2] Some extensive surveys of methods and results on firm-level dynamics for developed and developing countries are Baldwin (1995), Roberts and Tybout (1996), Sutton (1997), Haltiwanger (1997), Caves (1998), Foster, Haltiwanger and Krizan (2001, 2002), Bartelsman and Doms (2000), Tybout (2000), Scarpetta, Hemmings, Tressel and Woo (2002), Bartelsman et al. (2004), and references therein.

[3] An exception is Ericson and Pakes (1995). They address technology choice by allowing firms to influence their productivity by investments with a stochastic outcome. Though very rich, their model is also quite complex, therefore hard to aggregate, and thus more easily applied at a more disaggregated level.

that can arise; the formulation chosen here allows richer results.

Technology choice is modeled by letting entrants irreversibly choose a parameter determining expected productivity. After entry, a firm's productivity follows a Markov process that depends on this parameter.[4] The cost of the sunk entry investment is increasing and convex in the expected productivity of the technology chosen. Stochastically evolving firm productivity, optimal choice of technology at entry, and endogenous exit of unprofitable firms then yield a stationary distribution of firms over productivity levels. So, although firms constantly enter, exit, and change position within the distribution, the distribution itself and other aggregate variables do not change.

This stationary equilibrium of the model is calibrated to the US business sector, using static and dynamic moments of its firm distribution. Calibration allows imputing the parameters of the unobservable entry cost function (for a particular choice of functional form) and thereby makes comparative dynamics exercises possible.

The empirical contribution of the paper lies in the evaluation of the effect of small differences in administrative entry cost. These are modeled as a shift of the entry cost function by a constant for all technologies. Their effect is obtained by comparing stationary equilibria. It results that output loss exceeds the direct additional entry cost significantly already for small administrative entry costs such as those described in detail by Djankov et al. (2002). This is due to two effects: On the one hand, firms adopt better technologies in the new equilibrium, implying a larger entry investment and more capital-intensive production. On the other hand, high capital intensity acts as an entry barrier (an idea often found in the Industrial Organization literature, but rarely endogenized as here), depresses the number of firms, and allows less efficient firms to survive, reducing labor productivity and TFP. Numerically, it turns out that the second effect dominates, so output per capita falls. These results correspond to the patterns observed when comparing European countries to the US. Quantitatively, the reaction to an increase in the entry cost by less than ten percent of median firm output, and around one percent of average firm output, is sufficient for explaining 10 to 20% of differences in TFP and the capital-output ratio between Germany and the US.

Finally, it is shown that both technology choice and firm heterogeneity are needed to generate these results. In particular, without heterogeneity, it is hard to explain why there is so much

[4] Empirical research stresses firm-specific shocks as the main driver of firm level dynamics. Their importance can be inferred from the fact that rates of turnover differ across industries, but tend to be similar across countries for a given industry. Moreover, productivity differences within exceed those between industries (Foster et al. 2001). Within industries, Dhrymes (1991) and Dwyer (1996) find productivity levels to differ strongly even when technology used and the environment are very close, underlining the importance of firm-specific shocks.

entry in the data, despite high entry costs and very high exit rates for young firms.

To summarize, the contribution of this paper is to trace part of the difference in aggregate productivity between similarly developed economies to differences in administrative entry cost, while also matching qualitative patterns of relative capital-output ratios and firm turnover rates. This is particularly relevant in the light of the current European debate about its productivity lag with respect to the US, and about the possible role of regulation.

The paper is organized as follows. Section 2 describes the economy and characterizes optimal firm behavior. In section 3, an equilibrium for this economy is defined, a proof is sketched, and a solution algorithm outlined. (Formal statements of results and proofs are provided in the Appendix.) In section 4, the model is calibrated to the US business sector. The effect of raising entry cost is discussed in section 5. Section 6 concludes and indicates directions for further research.

2 The Model

2.1 The economy

The economy consists of a continuum of measure $\bar{N}$ of workers, of a continuum of active firms of endogenous measure μ, and of an unlimited pool of potential entrants. Active firms are indexed by i. Firms are risk-neutral and maximize the discounted value of expected profits. All individuals in the economy own equal shares in all firms; profits are redistributed to them.

Timing is as follows. Time is discrete and the horizon infinite. In every period, the following events and actions occur. All active firms pay a fixed operating cost, and then learn their new productivity level. Based on this, they choose output and employment, and the wage adjusts to clear the labor market. Firms also decide whether they will be active next period; i.e. incumbents decide whether to exit and potential entrants whether to enter. Firms that decide to enter choose a technology. They then receive a draw from the distribution of entrants' productivity at the start of the next period.

Production entails a strictly positive fixed operating cost c^f per period.[5] Active firms then sell their output at a constant price normalized to 1. They produce according to the stochastic production function

$$q_{it} = f(s_{it}, n_{it}), \tag{1}$$

[5]A fixed cost is necessary to ensure positive exit; otherwise instead of exiting, firms could cut production to zero and wait for better times. It can also be thought of as the cost of foregoing an outside option.

where s_{it} denotes the realization at time t of the stochastic process driving its productivity, and n_{it} the amount of labor it employs at time t. $f(\cdot)$ has standard properties summarized in Assumption 1.

Assumption 1 *$f(\cdot)$ is twice continuously differentiable and strictly increasing in both arguments. It is concave in n. Moreover, an Inada condition holds, i.e. $\lim_{n\to 0} f_n(\cdot) = \infty$, and $\lim_{n\to\infty} f_n(\cdot) = 0$.*[6] *The cross-derivatives of employment with the productivity state, $f_{ns}(\cdot)$, is positive.*

The production technology thus has decreasing returns at the firm level. As a consequence, firm size is a well-defined concept, and a firm size distribution arises.

Employment n_{it} can be adjusted costlessly every period. Firms hire labor on a competitive labor market. Denote aggregate labor demand by N_t. Neither the model nor results are much affected by labor supply elasticity, so assume that labor supply is inelastic at $\bar{N}$. Then the wage ω_t is a function of aggregate labor demand only.

The idiosyncratic productivity shock s follows a first-order Markov process. Specifically, assume that

Assumption 2 *s_{it} follows an AR(1) process with firm-specific constant v_i:*

$$s_{it} = v_i + \rho s_{i,t-1} + \epsilon_{it}, \qquad 0 < \rho < 1, \tag{2}$$

where ϵ is distributed normally with mean zero and variance $\sigma^2 > 0$. It is independent both across firms and over time.

Here, think of $v_i \in \mathbb{R}_+$ as the technology that firm i operates; it determines expected lifetime productivity. Because $\rho < 1$, s_{it} is stationary and mean-reverting. Denote the p.d.f. of s_{it} for a given v_i conditional on $s_{i,t-1}$ by $g_{v_i}(s_t|s_{t-1})$ and its conditional distribution function by $G_{v_i}(s_t|s_{t-1})$. Moreover, a firm's production facilities break down with an exogenous probability δ, forcing the firm to exit. This ingredient is necessary to fit the fact that although empirically, the exit hazard is higher for small plants, there are still some large plants that exit.

Entrants that start producing in period t and have technology v_i draw their initial productivity s_t^0 from a p.d.f. $h_{v_i}(s^0)$. For concreteness, suppose

Assumption 3 *$s^0 \sim N(\bar{s}^0, \sigma_e^2)$, with $\bar{s}^0 = \kappa\frac{v_i}{1-\rho}$, and $\kappa, \sigma_e^2 > 0$.*

[6] Derivatives of functions of several variables are indicated by subscripts.

The constant κ serves to calibrate entrants' relative to incumbents' productivity, while v_i is entrants' choice variable. From period $t+1$ on, entrants follow the process (2). Among surviving firms there is a selection effect: since low-productivity firms exit to avoid the fixed operating cost, average productivity is higher than implied by the asymptotic mean of (2). Hence, Assumption 3 implies that entrants expect to start with a realization of their productivity state below average productivity of incumbents, unless κ is much larger than 1. As a consequence, young firms are more likely to exit, and the hazard rate declines in age. Hence, by Assumptions 2 and 3, the structure of the productivity process captures the features of persistence and mean reversion of productivity, learning/selection, low productivity of entrants, and declining hazard rates found in the data (see e.g. the surveys by Caves 1998, Bartelsman and Doms 2000).

The final element to specify is entrants' technology choice. Due to Assumptions 2 and 3, technology v_i determines expected productivity over a firms' lifetime. Concretely, for any v' and v with $v' > v$, the unconditional distribution functions $g_{v'}(s)$ and $h_{v'}(s^0)$ first-order stochastically dominate $g_v(s)$ and $h_v(s^0)$, respectively.

Technology v_i is irreversibly chosen upon entry at a cost given by the entry investment function $c^e(v_i)$. This function gives the investment c^e (in units of output) that a firm has to make to enter the market with technology v_i. Ruling out a scrap value of the firm on exit, this investment is irreversible and sunk. Moreover, the menu of technologies available and the associated costs do not change over time.

The shape of $c^e(v_i)$ is governed by the following assumption.

Assumption 4 *The entry investment cost function $c^e(v)$ is positive for all v and strictly increasing and convex in v.*

This assumption is needed to ensure that an equilibrium with choice of a finite level of technology exists. (Exact conditions are given in Proposition 7 in the Appendix.) Intuitively, the entry investment cost function must have such a shape that the marginal cost of a better technology exceeds its marginal value from some v on. Since firm value is increasing in v (shown below), the cost function also has to be increasing, and steeper than the value function from some v on. Later, an exponential specification is chosen for tractability; it is simple enough to make its parameters identifiable just from calibration.

Assumption 4 allows several economic interpretations. The production technology is embodied in a fixed factor that firms acquire upon entry. Now it could be that there are information costs about this factor that increase in its efficiency; or the life cycle of the technology could matter, with less competition in more advanced, younger products; or there could be decreasing returns in the production of the technology.

Optimal choice of technology also means that the technological frontier is endogenous in this model. While a little more advanced technologies are available, adopting them is not optimal, while the prohibitive cost of much more advanced technologies can be seen as equivalent to non-availability.

The last missing element of the description of the economy is the firm distribution. To track firms' cross-sectional distribution, define $\mu_t(v, s)$ as the mass of firms that have technology v and productivity state s in period t. Denote the set of all v with V, that of all s with S, and the $V \times S$ state space with Σ. Then $\mu_t(\Sigma)$ is a measure of the total size of the industry. The distribution $\mu_t(\cdot)$ is common knowledge.

Since all units with the same v are independently affected by the same stochastic process, this number of units is large, and there is no aggregate uncertainty, the evolution of the cross-sectional distribution can be characterized by the underlying probability distribution, and aggregate variables follow a deterministic path given an initial distribution of firms.[7] With respect to the firm distribution $\mu(\cdot)$ this means that although the identity of firms with any s is random, their measure is deterministic. In a stationary equilibrium as focussed on here, aggregate variables are constant, so the time subscript can be dropped.

2.2 Firm behavior

Firms' individual state variables are v and s; they have a static control n and dynamic controls that consist of the entry and exit decisions and choice of technology. They take three types of decisions: Potential firms decide whether to enter, incumbents decide whether to exit, and active firms maximize current profits.

The incumbent's problem: The problem for an active firm is to maximize current profits. By the properties of f, this yields the firm's labor demand function $n(s_i, \omega)$. It is continuous, increasing in s, and decreasing in the wage ω. The employment choice uniquely determines firm output q and profits π, which are strictly increasing in v and in s, and strictly decreasing in ω.

Aggregating over firms yields aggregate labor demand

$$N(\mu(\cdot), \omega) = \int_\Sigma n(s, \omega)\, \mathrm{d}\mu(v, s) \tag{3}$$

[7]Formally, this follows from the Glivenko-Cantelli Theorem (see e.g. Billingsley 1986). For a more thorough discussion, see Feldman and Gilles (1985) and Judd (1985).

and similarly aggregate output. Since all the firm labor demand curves are decreasing in the wage, this is the case for aggregate labor demand, too. In addition, for given $\mu(\cdot)$, aggregate labor demand N and output Y are uniquely determined as functions of the wage. So for any firm distribution $\mu(\cdot)$, imposing labor market clearing implies a unique wage. In the following, for brevity, decisions of the firm can then be written as functions of the wage only, although fundamentally, they depend on the whole firm distribution.

Discounting profits by a common discount factor $\beta \in (0,1)$, the value of an incumbent is

$$W(v_i, s_i, \omega) = \pi(s_i, \omega) + \max\left\{0,\ \beta(1-\delta)\, E[W(v_i, s_i', \omega)|s_i]\right\}, \tag{4}$$

where primes denote next-period values. The max operator indicates a firm's option to exit if staying has negative expected value. By standard arguments, a unique value function exists, is continuously differentiable in v, strictly increasing in v and s, and strictly decreasing in the wage. (See also Lemma 1 and Corollary 2 in the Appendix.)

Exit: A firm exits when the expected value from continuing is smaller than the value of exiting W_x: $E[W(v_i, s_i', \omega)|s_i] < W_x$. With W strictly increasing in s, for given v and ω, this is the case for s strictly smaller than some unique exit trigger $s_x(v_i, \omega)$ given by

$$E[W(v_i, s_i', \omega)|s_x] = 0 \tag{5}$$

that is strictly decreasing in v_i and increasing in ω. Firms with more productive technologies can endure lower levels of the productivity shock before being forced to exit, whereas higher wages decrease firm value and therefore raise s_x. From Assumption 2, $G_v(s_x|s) > 0$ for all v and s, so there always is a strictly positive measure of firms that exit.

Entry and Technology Choice: Entrants compare costs and benefits of entry, and choose v_i to maximize the expected net present value of entry. Benefits correspond to the expected value of a firm with technology v_i, costs are given by the entry investment $c^e(v_i)$. The net value of entry W^e at the optimal choice then is

$$W^e(\omega_t) = \max_{v_i}\{E[W(v_i, s_i^0, \omega)|v_i] - c^e(v_i)\}, \tag{6}$$

where the expectation is over the initial draw of s^0 conditional on the v chosen.

Denote the measure of entrants using technology v by $M(v)$. Optimal choice of technology v_i^* requires[8]

$$\partial E[W(v_i^*, s_i^0, \omega)|v_i^*]/\partial v_i = c^{e\prime}(v_i^*). \tag{7}$$

Since the solution v_i^* to (7) is a function of the aggregate variable ω only, all entrants in a given period adopt the same technology, so the i subscript on $v^*(\omega)$ can be dropped. At the same time, under free entry, entry occurs ($M(v^*) > 0$) until

$$E[W(v^*, s_i^0, \omega)|v^*] = c^e(v^*) \tag{8}$$

in equilibrium. This also implies that the value of exit W_x is zero – starting a new firm after exit will yield zero net value. Since a strictly positive measure of firms exits every period, $M(v^*)$ also must be strictly positive for the firm productivity distribution to be stationary, as considered in the following. Equation (8) hence holds with equality. The wage adjusts to ensure this, and is thus determined at the extensive margin. The solution to the system of (7) and (8) then is a pair $(v^*, \omega^*) \in \mathbb{R}_+^2$.

Cross-sectional distribution: Firms' choices determine the cross-sectional distribution of firms over v and s. In a stationary state, $\mu(v, s)$ evolves according to

$$\mu'(v, s) = \int_{s_x(v,\omega)}^{\infty} (1-\delta)\mu(v, u)\, g_v(s|u)\, \mathrm{d}u + M(v)\, h_v(s) \quad \text{for any}(v, s). \tag{9}$$

The integral captures the evolution of continuing firms, while the last term accounts for entry.

3 Stationary Equilibrium

In this section, a stationary equilibrium is defined, its determination is sketched, and an algorithm for finding it is given. More detailed proofs are provided in the Appendix. Note that this is the equilibrium of an industry. It can also be interpreted as general equilibrium of an economy with consumer preferences that are linear in consumption.

Define a *stationary competitive equilibrium* as real numbers $v^*, \omega^*, M^*, s_x^*, N^*$, and functions $\mu^*(v, s), W(v, s, \omega^*)$ such that:

(i) entry is optimal: v^* and ω^* satisfy (7) and (8) if $M^* > 0$, and $E[W(v, s_i^0, \omega)|v] < c^e(v)$ for all v otherwise;

[8] Lemma 5 in the Appendix shows that the expected value of entry is differentiable in v and that the problem is concave, so the first order condition is sufficient for an optimum.

(ii) exit is optimal: s_x^* satisfies (5);

(iii) firm value $W(v, s, \omega)$ is given by (4) for all v, s, ω;

(iv) markets clear: $\bar{N} = N^* = N(\mu^*, \omega^*)$; and

(v) the firm distribution is stationary: $\mu' = \mu = \mu^*$ from (9) given M^*, s_x^* and v^*.

The restriction to stationary equilibria does not allow considering dynamic changes of the distribution. However, it does allow the analysis of processes within the distribution and the comparison of stationary equilibria (as in the comparative dynamics exercise to follow), which is sufficient for obtaining interesting results.

Existence of a competitive equilibrium intuitively follows from the following argument.[9] It has been shown in several contexts, starting with Lucas and Prescott (1971), that equilibria in similar models of industry evolution maximize net discounted surplus in the industry.[10] This objective is continuous. It is bounded above under the assumption that very advanced technologies are prohibitively costly, i.e. the net value of entry goes to minus infinity as v goes to infinity, since then equilibrium v^* is finite. Hence, without loss of generality, the domain of (v^*, M^*) can be restricted to a compact subset X of $\mathbb{R}^2$. The feasible set then is the set of all (v^*, M^*) such that equilibrium conditions (i) to (v) hold. Finding a competitive equilibrium then corresponds to maximizing a bounded and continuous objective on the compact set X. By Weierstrass's Theorem, an allocation that maximizes net discounted surplus, and hence a competitive equilibrium, exist.

Intuitively, equilibrium can be imagined as follows. Figure 1 shows the cost and value of entry around the equilibrium as functions of v. Equations (7) and (8) jointly determine the optimal technology v^* and the equilibrium wage ω^*. By optimality condition (7), the slopes of the two curves have to be equal in equilibrium. Under the assumptions imposed on $c^e(v)$, this occurs for a finite v^*. Proposition 7 in the Appendix shows that it is also unique. "Very advanced" here need not be defined too precisely, the empirical justification being that only finite productivity has been observed in reality. The economic mechanism for the determination of (v^*, ω^*) is as follows. If the value of entry exceeds the cost of entry at any v, there is excess demand for entry, driving the wage up until the net value of entry is zero. If the cost of entry exceeds its value at all v, and there is exit, then the wage needs to drop to clear the labor market.

[9] This argument has already been outlined by Hopenhayn (1992) in a very similar context.

[10] This can be measured as consumer surplus minus costs of production. Since there are no distortions in the model, the Welfare Theorems apply.

In this way, v and the wage adjust until there is equilibrium entry, i.e. there is a pair (v^*, ω^*) where the value of entry schedule is tangent to the entry cost curve, whereas it lies below it for all other v. The analysis of comparative dynamics will be conducted in this framework.

A crucial intermediate result to be used for evaluating the impact of administrative entry cost is that the derivative of the value of entry with respect to the technology v is negatively related to the wage. This is shown in Lemma 6 in the Appendix. Intuitively, firm value rises in v, but an increase in the wage shortens expected firm lifetime and thereby the benefits from a higher v, reducing W_v.

For illustration, Figure 2 shows the benchmark firm distribution resulting from the calibration in the next section, with productivity relative to average productivity on the x-axis. The mode lies at 0.68; the distribution is heavily skewed to the right because of exit. The exit threshold lies at 0.31. This means that in their year of exit, exiting firms are only about a third as productive as the average firm.

Using the equilibrium conditions, a stationary equilibrium can be found by applying an algorithm that consists of the following steps. First, obtain the firm labor demand functions $n(s_i, \omega)$. Then, obtain expressions for firm value $W(v_i, s_i, \omega)$ and for the exit trigger $s_x(v_i, \omega)$ (equilibrium condition (ii)). In the numerical implementation, this is done by value function iteration, discretizing the state space S into a grid of 800 points. The boundaries of the grid influence results if set too narrowly. Therefore, they are expanded until results are not affected anymore. Using the exit condition (5), a firm productivity transition matrix P_x incorporating exit follows from this. The equilibrium pair (v^*, ω^*) satisfies the two equilibrium conditions (7) and (8); it can be obtained by only a few iterations on these two equations. Then the stationary firm distribution (equilibrium condition (v)) is given up to a multiplicative constant corresponding to the number of firms by the ergodic distribution $\mu = (I - P_x^T)^{-1}\mu^0$ of a stochastic process with transition matrix P_x and initial state μ^0, where μ^0 is a vector capturing the distribution of entrants over S, I is the identity matrix, and the superscript T denotes the transpose of a matrix. Finally, equating labor demand and supply (equilibrium condition (iv)) yields the number of firms.

4 Benchmark economy

In this section, functional forms and parameters are chosen to fit the US business sector. Given these choices, model quantities resulting from calibration then uniquely determine the parameters of the entry cost function via the optimal v condition (7) and the free entry condition (8).

The production function has to satisfy Assumption 1. A natural choice for its functional form is $f(s,n) = e^s n^\alpha$, $0 < \alpha < 1$,. In particular, it ensures decreasing returns at the firm level and therefore well-defined firm size, since $\alpha < 1$. For the entry investment, only one value is observed in a stationary equilibrium. This is because all firms choose the same technology v^*, as shown before. However, an entry investment cost function needs to be specified for evaluating the impact of an increase in administrative entry cost. This problem can be solved in a simple way. It is sufficient to calibrate only one value $c^e \equiv c^e(v^*)$ of the entry investment function. v^* can be normalized to unity for the benchmark economy because it just scales the level of productivity and output of the economy, but does not influence the shape of the productivity distribution or any ratios. Then choosing the simple functional form $c^e(v) = k_1 e^{k_2 v}$, $k_1 > 0$, $k_2 > 1$ for the entry investment cost function implies that the parameters k_1 and k_2 are pinned down by the equilibrium conditions (7) and (8) as $k_2 = W_v(v^*, \omega^*)/W(v^*, \omega^*)$ and $k_1 = W(v^*, \omega^*)e^{-k_2 v^*}$.[11]

To ensure comparability with statistics from firm-level data, the time period is set to one year. As conventional in the literature, labor's share of revenue α is set to 0.64. To match a real annual interest rate of 4%, β is set to 0.96. The remaining parameters $\rho, \sigma^2, \delta, \bar{s}^0, \sigma_e^2$, and the ratio of the equilibrium entry investment c^e to the fixed operating cost c^f together determine the shape, location and truncation point of the firm productivity distribution. Since these parameters have interacting effects, they cannot be calibrated individually. Instead, they are calibrated jointly to fit a set of data moments of equal size. This fit is very nonlinear in the parameters; so a genetic algorithm following Dorsey and Mayer (1995) is used to find the best fit. Given these parameters, the levels of c^e and c^f determine the number and size of firms. They are fixed to match the average firm size of 26.4 reported by Bartelsman, Scarpetta and Schivardi (2003) (BSS) for US the US business sector.[12]

The following static and dynamic characteristics of the firm distribution are chosen as targets: the firm turnover rate, the productivity of firms that entered within the last 10 periods relative to the average firm, their productivity relative to exiting firms, the proportion of firms below average productivity, the relative size of entrants, and their probability of still being active four years after entry. The measures of the relative productivity of entrants allow anchoring the mean of entrants' productivity distribution. Given the functional form of the production function and the choice of α close to a third, their relative size is quite a different target. It is close to

[11] Due to the local nature of the calibration, these values are valid only locally, allowing experiments where v^* does not change much.

[12] Using data of the US Small Business Administration (SBA) on firm size distributions (available on http://www.sba.gov/advo/research/data.html) yields a similar result.

the relative skewness of the two distributions. Similarly, the proportion of firms below average productivity says something about the skewness of the distribution of active firms. Finally, the overall turnover rate and entrants' survival rate mainly help calibrate the entry investment, the fixed cost, and the exogenous breakdown rate. To evaluate the calibration, the investment rate, the productivity spread between the 85^{th} and 15^{th} percentile of the productivity distribution, the employment-weighted firm turnover rate, and the fraction of entrants that exit after only one period of operation are used. Moreover, the implied exit hazard can be compared to the data.

Bartelsman et al. (2004) (BHS) report average yearly firm turnover of 22% for the US for the 1990ies. This is higher than older estimates, e.g. from Cable and Schwalbach (1991), yet firm turnover has been rising in almost all countries on which there is data, probably due to deregulation. These and other sources also find that other developed countries tend to have slightly lower turnover rates. Estimates of entrants' relative productivity agree that both in Census of Manufactures (Foster et al. 2001) and in LRD (Haltiwanger 1997) data, the mean of the distribution of entrants' productivity is on average slightly below that for incumbents, while Baily, Hulten and Campbell (1992) and Bartelsman and Dhrymes (1998) both show that the variance of the distribution is high. Employing a variety of measures, Foster et al. (2001) settle on a value of around 99% for average productivity of firms that entered over the last 10 years relative to that of incumbents. They also report that these firms are 13% more productive than exiting firms. The relative size of entrants (in terms of employment) is 28% according to BSS, and their probability of still being active four years after entry is 63% according to BHS. Finally, the firm productivity distribution is very skewed, 88% of firms have below average productivity.

Table 2: Parameter assignments

standard		firm dynamics	
α	0.64	ρ	0.94
β	0.96	σ^2	0.30
		κ	1.00
		σ_e^2	0.70
		δ	0.050
$c^e(v)$		Costs in benchmark economy (% of avg firm output)	
k_1	4.38e-22	Fixed cost	9.8%
k_2	69.3	Entry cost	123%

Table 3: Benchmark economy versus target statistics

	data (US)	model
Firm turnover rate	22%	22.7%
Average firm size	24.6	26.4
TFP entrants/incumbents	99%	99.5%
TFP entrants/exiter	113%	105.2%
Relative size of entrants	28%	28.4%
Four-year survival rate of entrants	63%	62.3%
Firms below average TFP	88%	85.8%
other statistics:		
Investment rate	14.4%	14.0%
Productivity spread	3-4	3.88
Employment-weighted firm turnover rate	7%	8.3%
Fraction one-year firms	23%	24.5%

Finally adopted parameters are shown in Table 2. Target statistics are summarized in the left and resulting statistics for the model economy in the right column of Table 3. The calibration fits target moments quite closely. Only the productivity of entrants relative to exiting firms is slightly on the low side.

The adopted parameter values are also reasonable. As generally found in the empirical literature, firm productivity is very persistent, as indicated here by a ρ of 0.94. The consensus in the literature does not specify what "very persistent" means. On the one hand, Campbell (1998) uses a random walk, and Hopenhayn and Rogerson (1993) find a 5-year autocorrelation coefficient of 0.93 in a panel of firms from the LRD, both implying more persistence than the value used here. On the other hand, empirical studies using state of the art dynamic panel methods (e.g. Blundell and Bond 1999) tend to find lower coefficients than these two papers. Dwyer (1996) explores the topic in detail for one industry and shows that the productivity process at the firm level is not easy to extract from a panel. In this sense, a ρ of 0.94 certainly is in the right ballpark. There is not much direct evidence on what values σ^2 should take on. For example, Cooper and Haltiwanger (2002) find a variance of the idiosyncratic shock of 0.0785 in a balanced panel, i.e. conditional on firms surviving the whole 17 years of their sample. Since not taking into account exit drives down the estimate of the variance, the unconditional variance must be higher. So the value of 0.3 used here is plausible. The variance of entrant's productivity in the first period is more than twice that of incumbents. Other papers such as Hopenhayn and Rogerson (1993) that adopt a uniform distribution of entrants' productivity do not report a comparable statistic. However, the model implies that, although there are less entrants in higher productivity deciles, the distribution of entrants over productivity deciles is reasonably flat, and entrants enter all deciles (see Figure 3). This fits the data well. Overall, adopted parameter values thus seem reasonable.

Calibrated values for the parameters of the entry investment function, k_1 and k_2, are hard to interpret by themselves. However, the implied entry investment of 1.23 times average firm output seems reasonable, just as fixed operating costs of 10% of average firm output. In line with this, the investment rate, measured as the entry cost of new firms in a period divided by aggregate output, comes close to its empirical counterpart. This is despite the restriction to investment only upon entry, and although it was not targeted in the calibration.

Other quantities that have not been targeted also fit rather well. The employment-weighted firm turnover rate of 8.3% is quite close to the data value of 7%. The fraction of firms exiting after only one year also fits the data well. Finally, the productivity spread between the 85$^{\text{th}}$ and the 15$^{\text{th}}$ decile, although possibly a bit high at 3.88, fits well with reported values of 3 to 4 (see e.g. Dhrymes 1991, Dwyer 1996). Hence, the calibration fits well in both the targeted dimensions and in supplementary ones.

Next, it would be desirable to compare the exit hazard (Figure 4) implied by firms' life cycles in the model to that in the data. Unfortunately, reported series of this variable are usually too short or presented in such a way as not to be directly comparable with the measures obtained here.[13] However, the model clearly matches the empirical pattern that exit hazards decline in size and in age. The same holds true for the growth rate of surviving entrants (Figure 6). It is close to the doubling of employment by surviving entrants within a few years reported by BSS.To summarize, given that the model is very parsimonious and only few parameters have been calibrated, the calibration fits rather well. The next section explores the effect of adding administrative entry costs on aggregate productivity .

5 The effect of administrative entry cost

As illustrated in the introduction (see Table 1), the US has higher per capita output than other OECD members despite a lower capital-output ratio. This still holds when taking into account differences in human capital and in hours worked. Usually, this is "explained" by its higher TFP. This section explores how the present model can generate part of these differences. To fix ideas,

[13]Both remarks apply to the hazards reported by BSS. For example, they exclude firms that live only one year although there are many of them in the data.

think in terms of a comparison of the US and Germany, using data on aggregates from the Penn World Tables for 1988 (Summers and Heston 1991). (Hence the comparison is not affected by German reunification.) For that year, and similarly in other periods, the German capital-output ratio is 25% higher than that of the US, while output per worker is 18.2% lower. Human capital per worker is also 19.8% lower, so this explains only part of the gap. Using a capital share of one third, a growth accounting exercise yields German TFP of 8.8% less than that of the US (Hall and Jones 1999). While many factors could be used to explain this discrepancy, the scope of this section is to illustrate how the present model can resolve some of it. In passing, it also generates a lower firm turnover rate (13% in Germany). Although it is well-known that higher entry cost is associated with higher capital-output ratios when comparing industries,[14] cross-country studies have not yet addressed this connection. Moreover, even the cross-industry literature usually focusses on the effect of capital intensity on subsequent entry without taking into account its endogeneity.

In an influential article, Djankov, La Porta, Lopez-de-Silanes and Shleifer (2002) publish meticulously gathered data on administrative entry barriers in 85 countries. They describe the minimum cost needed to meet official requirements to legally operate a small industrial or commercial firm. This fits with the characteristics of entrants in the benchmark economy. The cost corresponds to 47% of per capita output in the average country, close to zero (0.5%) in the country with the lowest cost (United States), and 463% in the country with the highest cost (Dominican Republic). In Germany, it is 32.5%, close to the values for other continental European OECD members. Djankov et al. also relate these costs to other variables such as measures of corruption and conclude in favor of the public choice view that entry regulation benefits politicians and bureaucrats without necessarily increasing welfare. Yet more can be said. The consequences of entry regulation do not stop at its direct cost; through its effect on entry, technology choice, and aggregate productivity, the cost in terms of lost output can be several times the direct cost.

This section explores the consequence of imposing an additional entry cost of 30% of per capita output on entrants, regardless of the v chosen. This amounts to an upward shift of $c^e(v)$ by 1.1% (9.5%) of the output of the average (median) firm, or by 0.9% of the entry investment in the benchmark economy. Since this change is small, the parameters of the entry investment cost function imputed in the calibration can be used to find the new stationary equilibrium with this additional cost.

[14] For a survey, see Roberts and Tybout (1996).

From an economic point of view, the following channels are at work (for an illustration, see Figure 7): Start in a stationary equilibrium with technology v^* and wage ω^*. The administrative entry barrier then uniformly increases the cost of the entry investment for all technologies v. It thus corresponds to an upward shift of the entry investment cost function $c^e(v)$. At the old wage, this would make entry unprofitable for all v. Since exit continues, labor demand would fall. To reequilibrate the labor market, the new equilibrium wage has to be lower, such that there is entry again. Because the wage is negatively related to the derivative of the value of entry with respect to v (Lemma 6), this fall in ω raises the slope of the value of entry schedule at every v. Together with the condition in Proposition 7 that the net value of entry goes to zero as v goes to infinity, i.e. $c^e(\cdot)$ rises faster than the value of entry schedule, this implies that the new equilbrium technology v^* has to lie to the right of the old one. Intuitively, the marginal cost of adopting a better technology has remained constant, while the fall in the wage increases the marginal benefits of doing so, implying a higher v^* in the new equilibrium. Hence, the new equilibrium features a higher entry investment, net of the administrative cost, and a lower wage. Aggregating the entry investment over firms, this implies a larger capital stock. At the same time, output falls slightly because a distortion has been introduced. Hence, the capital-output ratio rises. With output per capita lower, and investment higher, it is also clear that TFP must be lower. Finally, firm turnover falls due to the higher entry investment and because the higher v and the lower wage depress the exit threshold.

Numerical results (Table 4) show that some of these effects are sizeable. While the firm turnover rate drops only slightly, the entry investment and thereby the capital-output ratio increase significantly. Welfare effects are strong, too; the loss in consumption is several times the direct burden imposed by the administrative cost. Per capita output falls despite the increase in v. Theoretically, this is not surprising, since the administrative entry cost acts as a distortion that should lower welfare. More concretely, there are two effects counteracting the effect of the increase in v. First, the productivity difference between entering and exiting firms increases by almost as much as the equilibrium technology. This implies that the rise in v does not translate into an improvement of the whole firm distribution; the lower tail, protected by the higher equilibrium entry investment, does not shift up. Secondly, due to the higher equilibrium entry investment, there are less firms, with higher employment on average. With decreasing returns to scale at the firm level, this reduces output per worker. These two negative effects taken together dominate the positive effect of the increase in v. Due to the higher entry investment, TFP falls by even more than output per worker.

Table 4: Effects of introducing administrative entry cost of 30% of per capita output (benchmark economy = 100)

Equilibrium technology v^*	100.07
Wage ω^*	99.66
Entry investment	105.54
Aggregate output	99.8
Capital-output ratio	105.4
Aggregate TFP	98.98
Consumption	98.9
Consumption loss /exogenous cost increase	6.85
Firm turnover rate	99.15
Average firm size	100.47
number of firms	99.54
TFP entrants/incumbents	100
TFP entrants/exiter	100.05
Relative size of entrants	100.32
Four-year survival rate of entrants	100.70
Firms below average TFP	100.08

Comparing Germany and the US, the reaction of the capital-output ratio, output per worker, TFP, the relative size of entrants, and entrants' survival rate all quantitatively fit the evidence. Only the increase in average firm size does not fit observed patterns. However, average firm size in the US is far higher than in most other countries, which is probably due to effects of market size and geography that are not captured here. Quantitatively, the change in the capital-output ratio represents around one fifth and that in TFP around one tenth of the differences in the two variables between Germany and the US. Hence, the model helps explain a non-negligible portion of differences in the capital-output ratio and in TFP by taking into account the effect of a small, but well-measured difference between the two countries.

For intuition, the result can be interpreted as an endogenous, equilibrium amplification of entry barriers. High capital-output ratios are often interpreted as such at the industry level. They also act in this way in the model; a higher equilibrium entry investment discourages entry and protects low-productivity firms. What is novel is that they arise endogenously as an equilibrium outcome in a competitive industry, in response to just a small shift in entry costs, and without strategic interaction.

5.1 Both heterogeneity and technology choice matter

The model used here differs from most other macroeconomic models in two dimensions, the heterogeneity of firms and technology choice upon entry. Table 5 shows that both of these are necessary for obtaining the results on the capital-output ratio and TFP.

With a fixed technology v as in the original Hopenhayn (1992) model, an increase in administrative entry cost can only have a direct effect. The indirect effect through an equilibrium adjustment of firms' entry investment is ruled out by assumption. As a result, entry barriers change little. Moreover, only the wage and the number of firms can adjust. Table 5 shows that aggregate output drops; it falls a bit more than with optimal choice of v because fixing v means shutting down one margin of optimization. The consumption loss does not exceed the direct burden of the administrative cost much. Aggregate productivity falls only very little. The capital-output ratio *falls* slightly because the number of firms declines more than output. It cannot change much more, with v fixed. Turnover does not change much either. Hence, the model with fixed technology both has a smaller response of productivity to the rise in entry cost and barely generates movements of the capital-output ratio and the turnover rate that fit the patterns shown in Table 1.

Similar remarks apply to a model with technology choice, but homogeneous firms. (The homogeneous firm model used here is described in detail in the appendix.) Turnover is exogenously fixed by construction (by imposing an exogenous exit probability of 11% for all firms). The same holds for all measures related to the firm distribution, which is degenerate here. More importantly, technology choice reacts far less to the rise in entry cost. The reason is that with stochastic productivity s, i.e. heterogeneous firms, firm value is convex in s. Due to Jensen's inequality, the expectation of the value of the firm is then higher than the value at the expected s. Since expected s increases in the technology chosen at entry, firms are prepared to pay higher entry costs when productivity is stochastic. This also means that they react more to changes in entry cost. In economic terms: Even if the average entrant has below-average productivity and a large initial exit hazard, there is a small probability that the firm will become very efficient and make large profits. This warrants paying even a large entry cost. This effect is absent from homogeneous-firm models, causing them to underestimate willingness to pay for entry. So a heterogeneous-firm model shows much better why firms enter even if entry cost is high and probability of success low, as often observed in the literature on entry (see e.g. Geroski 1995). As a result, in the homogeneous firm model, aggregate productivity (TFP) falls only slightly. The rise in efficiency of the technology is more than compensated by the increase in spending

Table 5: Effects of introducing administrative entry cost of 30% of per capita output, 3 specifications (respective benchmark economy = 100)

	optimal v	fixed v	homogeneous firms
Equilibrium technology v^*	100.07	100.00	100.00
Wage ω^*	99.66	99.65	99.75
Entry investment	105.54	100.00	100.92
Aggregate output	99.8	99.65	99.74
Capital-output ratio	105.4	99.39	100.00
Aggregate TFP	98.98	99.67	99.51
Consumption	98.9	99.68	99.80
Consumption loss /exogenous cost increase	6.85	1.92	1.12
Firm turnover rate	99.15	100.00	exog.
Average firm size	100.47	100.97	101.17
number of firms	99.54	99.04	98.84
TFP entrants/incumbents	100.00	100.00	100.00
TFP entrants/exiter	100.05	100.00	100.00
Relative size of entrants	100.32	100.00	100.00
Four-year survival rate of entrants	100.70	99.46	exog.
Firms below average TFP	100.08	100.00	100.00

on it. The fall is smaller than with heterogeneous firms also because the effect of protection for unproductive firms due to the higher entry investment is absent here.

To summarize, considering slight shifts in entry cost, as caused by administrative entry costs, in conjunction with technology choice by firms and idiosyncratic shocks, allows to explain 10 to 20% of observed differences in TFP and capital-output ratios and a small proportion of differences in firm turnover and output per worker between similar countries such as European Union member states and the US. These effects arise because a smaller number of firms, higher average firm size, and the protection of less efficient firms due to the higher equilibrium entry investment dominate the effect of choosing a more efficient technology. For this result, both heterogeneity of firms and technology choice are crucial elements.

6 Conclusion and Directions for Further Research

Differences in total factor productivity are a puzzle, particularly between similarly developed countries. This paper has analyzed the effect of small shifts in entry cost in a dynamic stochastic model of heterogeneous firms with technology adoption. Results fit the observed patterns qualitatively, and correspond to 10 to 20% of observed differences: a country with lower entry cost has higher productivity and output despite lower capital intensity, and it has higher firm turnover. Notably, the consumption loss caused by increasing administrative entry cost is a multiple of the direct burden of the regulation. Via technology choice, the administrative cost encourages endogenous formation of entry barriers in the form of high capital intensity. These results are relevant for ongoing discussions in Europe about a productivity gap compared to the US, particularly with regard to the role of regulation.

The model proposed here differs from other models of industry evolution by allowing entrants to choose technology, modeled as a parameter affecting their expected productivity after entry and over their life. To be able to conduct the analysis of changes in entry cost, an algorithm for finding a stationary equilibrium with positive entry and exit was presented. By calibrating the model, I have obtained parameters of the entry cost function needed for the comparative dynamics exercises leading to above-mentioned results.

The present approach fits in both with the recent theoretical (e.g. Veracierto 2003, Samaniego 2005) and empirical (see fn. 1) literature on heterogeneous firms and productivity outcomes, and with the literature on (de)regulation and productivity (see e.g. Blanchard and Giavazzi 2003). These works suggest that considering the heterogeneity of firms should offer many more opportunities for interesting future research.

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Appendix

A Formal Statements of Results and Proofs

A firm's individual state variables are v and s; the aggregate state is completely described by $\mu(\cdot)$. However, the aggregate state that is relevant for firm decisions is the wage ω. Therefore, to describe the firm's actions, it is sufficient to consider instead of μ (with domain $\Sigma = V \times S$) only a reduced aggregate state space Ω of positive, finite ω. This is sufficient because ω is uniquely determined by μ (from equation (3)). The total state space the firm considers (including individual and aggregate state variables) then is $Z = V \times S \times \Omega$. Since it results below (Proposition 7) that the optimal v is always finite, Z can be taken to be a compact subset of $\mathbb{R}^3$.

Then, for any point of the state space, a firm's labor demand, output and profits, and aggregate labor demand and output can be obtained by static optimization. Firm value then is given by the functional equation

$$W(v, s, \omega) = \sup_{x \in \{0,1\}} \left\{ \pi(s, \omega) + (1 - x)\beta \left[E_s W(v, s', \omega) | s \right] \right\}, \tag{10}$$

where x is the value taken on by the exit policy function $X(v, s, \omega)$ ($x = 1$ means exit), and $\pi(\cdot)$ is the profit function resulting from static optimization.

Lemma 1 *For v^* finite, there is a unique firm value function W that satisfies (10). The exit policy function X is single-valued and lets firms attain the supremum in (10).*

Proof. Proof is by applying Theorem 9.12 from Stokey and Lucas (1989). Assumption 9.1 trivially holds. Since the expectation of s is finite and v^* is assumed finite, total returns are bounded, and Assumption 9.2 holds. Conditions (a) and (b) of Theorem 9.12 are also fulfilled if v^* is finite. In Proposition 7 below it is verified that this is generally true when Assumption 4 holds. ■

Corollary 2 *The firm value function W is continuous, strictly increasing in v and in s, and strictly decreasing in ω. For given v, it is bounded.*

This follows from the properties of the profit function; by Theorems 9.7 and 9.11 in Stokey and Lucas (1989) they carry over to the value function. Boundedness then follows from the fact that $E(s'|s)$ is well-defined and finite for all s

Corollary 3 *For $c^f > 0$ and under Assumption 2, there is a unique exit trigger $s_x(v, \omega) \equiv \{s \text{ s.t.} E[W(v, s', \omega)|s] = 0\}$. Hence, the exit policy function X is single-valued; it takes value* 1 *(exit) for $s < s_x$ and value* 0 *for $s \geq s_x$. The exit trigger $s_x(\cdot)$ is strictly decreasing in v, strictly increasing in ω, and continuous in both.*

Proof. Firms exit whenever the expected value of continuing is smaller than the value of exiting:

$$E[W(v, s', \omega)|s] < 0, \tag{11}$$

where the value of exit is zero due to the zero net value of entry condition (8). Since $E(s'|s)$ increases in s by Assumption 2, and because firm value increases in s by Corollary 2, the left-hand side (LHS) of (11) is strictly increasing in s. Moreover, given any $c^f > 0$, there is an s

so low that expected value of continuing is negative, and an s so high that it is positive. Then there is a unique s_x such that an equality replaces the inequality in (11). Firms exit whenever $s < s_x$. The properties of s_x follow from the properties of the value function. ■

To ensure that the condition for optimal technology choice (7) is well-defined, it is necessary to show that the value function is differentiable with respect to v. For this, it first has to be shown that expected firm life is finite. This is also crucial for a stationary equilibrium.

Lemma 4 *Given the specification of the stochastic process for s in (2), the lifetime T of a firm is finite for all v with probability* 1*. It has a well-defined pre-entry expectation $\bar{T}$ that is the same for all firms.*

Proof. Proof is easiest by reasoning in terms of the properties of Markov processes. Define the set $S_x = \{s \in S : s < s_x\}$. Once a firm draws an $s \in S_x$, it exits, so S_x is an ergodic set. Because ϵ has positive variance, there are $s \geq s_x$ such that $G_v(s_x|s) > 0$, i.e. with a positive probability of exiting in the next period. Hence, the set $\{s \in S : s \geq s_x\}$ is transient. Then, by Theorem 5.6 in Doob (1953), s can remain outside S_x for a finite time only with probability 1. Moreover, the probability of remaining in the transient set decreases at a geometric rate. As a consequence, expected firm life is finite with probability 1. This implies that it has a well-defined expectation $\bar{T}$. As all firms have the same v, it is the same for all firms. ■

Lemma 5 *If v^* is finite, Assumption 4 holds, and very advanced technologies are prohibitively costly, i.e. $lim_{v\to\infty} c^e(v) - E[W(v, s^0, \omega)|v] = \infty$, then the expected value of entry W^e is continuously differentiable in v in a neighborhood of v^*, with $W^e_v > 0$ for all v.*

Proof. In any period, the probability of surviving beyond the following period is given by the probability $1 - G_v(s_x(v, \omega)|s)$ of drawing an $s' > s_x(v, \omega)$ next period. Firm value can then be expressed as a sum of profits, weighted by conditional survival probabilities. This sum has a finite number of bounded elements because firm lifetime is finite with probability 1 by Lemma 4. Value of a firm given a current productivity $s_0 \geq s_x(v, \omega)$ then is

$$W(v, s_0, \omega) = \pi(s_0, \omega) + \beta E[\pi(s_1, \omega)|s_0] + \sum_{t=2}^{T} \beta^t \mathcal{P}\{s_{t-1} \geq s_x(v, \omega)|s_0\}\, E[\pi(s_t, \omega)|s_0], \quad (12)$$

and expected net value of entry is

$$W^e(v, \omega) = E[W(v, s^0, \omega)|v] = E[\pi(s^0, \omega) + \beta\pi(s_1, \omega) + \sum_{t=2}^{T} \beta^t \mathcal{P}\{s_{t-1} \geq s_x(v, \omega)|v\}\, \pi(s_t, \omega)] - c^e(v). \quad (13)$$

Expected gross value of entry increases in v because the exit probability falls in v and because higher v raises the probability of high draws of s^0. Since expected lifetime is well-defined and finite, $c^e(v)$ can be decomposed into a sum of discounted, annualized payments, weighted by survival probabilities. Expected net entry value then is the expectation of a weighted sum of net period returns, i.e. profits net of the entry investment. By the assumption that very advanced technologies are prohibitively costly, by the fact that W^e is continuous and monotonically increasing in v, and by $\lim_{v\to-\infty} c^e(v) > 0 > -cf = \lim_{v\to-\infty} W^e(v)$, both expected net value of entry and these net period returns are concave in v. Then, expected value of entry is differentiable with respect to v by Theorem 9.10 in Stokey and Lucas (1989). The derivative is positive by Corollary 2. Moreover, the firm's technology choice problem is concave, and the first order condition (7) is sufficient. ■

For comparative statics, it is necessary to know how W_v^e and ω interact. Unfortunately, general statements about second derivatives of value functions are hard to make, but the next result establishes that W_v^e falls in the wage.

Lemma 6 *W_v^e falls in ω.*

Proof. Write expected gross value of entry as

$$W^e(v,\omega) = \int_S h_v(s^0)\, W(v,s^0,\omega)\,\mathrm{d}s^0. \tag{14}$$

Its derivative with respect to v is

$$\frac{\partial W^e(v,\omega)}{\partial v} = \int_S h_v(s^0)\,\frac{\partial W(v,s^0,\omega)}{\partial v}\,\mathrm{d}s^0 + \int_S \frac{\partial h_v(s^0)}{\partial v} W(v,s^0,\omega)\,\mathrm{d}s^0. \tag{15}$$

Now consider an $\omega' > \omega$. The second integral becomes smaller because W decreases in ω. The first integral is a weighted average of W_v for $s^0 \geq s_x(v,\omega)$ (continue), which is positive, and for $s^0 < s_x(v,\omega)$, which is zero. Increasing the wage puts more weight on the second term, hence the first integral decreases in ω, too. As a result, W_v^e falls in ω. ■

The central result for a unique equilibrium then is:

Proposition 7 *Equilibrium condition (i) is fulfilled by a unique finite pair (v^*,ω^*) if very advanced technologies are prohibitively costly, i.e. $\lim_{v\to\infty}\{c^e(v) - E[W(v,s^0,\omega)|v]\} = \infty$.*

Proof. Properties of the expected value of entry are closely related to incumbents' value. By Lemma 5 and Assumption 3, the expected value of entry is continuous, strictly increasing, and differentiable in v, and continuous and strictly decreasing in ω. Hence, the LHS of (7) is well-defined.

Equilibrium existence has been shown in the main text. Finiteness of v^* follows from the prohibitive cost of very advanced technologies. Uniqueness of w^* given v^* follows from the properties of the value of entry. Given v^*, the RHS of both (7) and (8) is constant; the LHS is strictly decreasing in ω; ω^* is the wage that solves both of them. Uniqueness of the (v^*,ω^*) pair can be established by the following argument: Suppose that there are two optimal pairs (v_1,ω_1) and (v_2,ω_2), with $v_1 < v_2$ and $\omega_1 \neq \omega_2$. Each is then associated to a value function $E[W(v,s^0,\omega_i)|v]$, $i = 1,2$. Inspection (in the framework of Figure 1) shows that these have to cross. However, since expected entry value is strictly decreasing in ω, this is not possible: changing ω shifts the value function, and value functions for different wages do not cross. Hence, the optimum pair (v^*,ω) is unique and finite. To rule out in general a continuum of pairs (v^*,ω^*) with v^* an interval and a unique ω^*, the assumption on the cost function would have to be strengthened. This could be achieved for instance by assuming that the entry cost function is "more convex" than the value function in the sense that if (7) holds at $(\tilde{v},\omega^*)$, the net value of entry is negative for all $v > \tilde{v}$, given ω^*. ■

With an expression for the exit trigger, and v^* and ω^* consistent with positive entry in hand, it remains to determine a firm distribution μ consistent with a stationary equilibrium. In this, there are two crucial ingredients. First, as shown in the main text, all entrants in a given period adopt the same technology. For a stationary equilibrium, clearly, this is be constant over time so that we can fix v at v^* and consider $\mu(s)$. Second, there is a one-to-one mapping from the exit trigger s_x to entry mass M. This follows from the fact that with stationary $\mu(s)$, the total measure of firms has to be constant, and hence the measure of exiting firms $\mu(s < s_x)$ has to equal the measure of entering firms M. Since expected firm life is finite (Lemma 4), this can be achieved. The firm distribution in a stationary equilibrium then is a fixed point of the operator T defined by

$$(T\mu)(s) = \int_{s_x(\mu)}^{\infty} (1-\delta)\mu(u)\, g_{v^*}(s|u)\,\mathrm{d}u + M\, h_{v^*}(s), \tag{16}$$

i.e. a μ such that $(T\mu)(s') = \mu(s')$. Fixed-point arguments as given in Stokey and Lucas (1989) do not apply easily in this case because, due to entry and exit, the transition function for $\mu(s)$ is not monotone: Every period, low-productivity firms perish and are replaced by more productive ones, with only the remaining firms' productivity following a monotone process. However, the conditions for the existence of a unique stationary equilibrium with positive entry and exit derived in Hopenhayn (1992, equation (12)) carry over exactly to the present case. The result that v^* is finite and the assumption that the firm is a price taker in input markets are sufficient for this.

B Homogeneous firm model

The production function is

$$y_i = e^{s_i} n^{\alpha} = e^{s} n^{\alpha}, \tag{17}$$

where $s_i = s$ for all i, i.e. all firms have the same, constant productivity. The optimal choice of n then is

$$n = \left(\frac{\alpha e^{s}}{\omega}\right)^{\frac{1}{1-\alpha}} \tag{18}$$

for all firms, yielding firm output

$$y = \left(\frac{\alpha}{\omega}\right)^{\frac{\alpha}{1-\alpha}} e^{s\frac{1}{1-\alpha}}. \tag{19}$$

Defining the number of firms as B, aggregate output then is $Y = By$. Labour supply is $\bar{N}$. The wage has to clear the labor market: $Bn = \bar{N}$. This implies a wage of

$$\omega = \alpha \left(\frac{B}{\bar{N}}\right)^{1-\alpha} e^{s}. \tag{20}$$

Current profits are

$$\pi(s) = e^{s\frac{1}{1-\alpha}} \left(\frac{1}{\omega}\right)^{\frac{\alpha}{1-\alpha}} \tilde{\alpha} - c^{f}, \tag{21}$$

where $\tilde{\alpha} = \alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}} > 0$ and c^f is the fixed cost. With an exogenous exit probability of δ for each firm each period, firm value then is

$$V(s) = \frac{\pi}{\rho}, \tag{22}$$

where $\rho = 1 - \beta(1-\delta)$. Firms choose s upon entry, at cost $c^e(s) = k_1 e^{k_2 s} + k_3$. Both the entry cost function and V are convex in s. The two central equilibrium conditions then are: Optimal choice of s (FOC)

$$V_s(s) = \frac{\pi_s}{\rho} = c^{e\prime}(s), \tag{23}$$

where subscripts indicate derivatives, and the condition that net value of entry has to be zero in equilibrium (NEC)

$$V(s) = c^e(s). \tag{24}$$

This system can be solved for optimal e^s and equilibrium ω, e.g. by substituting the FOC into the NEC. This yields

$$e^s = \left[\frac{c^f/\rho + k_3}{k_1(k_2(1-\alpha)-1)}\right]^{\frac{1}{k_2}} \tag{25}$$

and

$$\omega = \left[e^{s\,\frac{1}{1-\alpha}-k_2}\frac{\tilde{\alpha}}{\rho k_1 k_2(1-\alpha)}\right]^{\frac{1-\alpha}{\alpha}} \tag{26}$$

or

$$\omega = \left[e^{s\,\frac{1}{1-\alpha}}\frac{\tilde{\alpha}}{c^f + \rho c^e}\right]^{\frac{1-\alpha}{\alpha}}. \tag{27}$$

From this follow n, aggregate labor demand, and the number of firms $B = \bar{N}\left(\frac{\omega}{\alpha e^s}\right)^{\frac{1}{1-\alpha}}$. Then aggregate output is $Y = \bar{N}^{\alpha} B^{1-\alpha} e^s$.

To calibrate the model, set α, β to standard values 0.64 and 0.96, $\delta, \bar{N}$ directly to desired values 0.11 and 17mn (yielding 22% turnover rate of firms), and set k_1, k_2, and c^f to match the capital-output ratio and fixed cost of the benchmark economy of the heterogeneous firm model. I normalize s to have the same aggregate output in both models. Output in the heterogenous firm model is $Y_{het} = \bar{N}^{\alpha}(M\bar{s})^{1-\alpha}$, so to achieve the same level of output, the right standardization is ${e^s}_{hom} = \bar{s}^{1-\alpha}$. Then the wage follows from the target for n, which gives a relation between c^e and c^f. Given the target for $k/y = c^e/y = \frac{c^e}{e^s n^{\alpha}}$, this c^e is fixed, so that c^f can be inferred from the previous relationship. With all this in hand, the value function of the firm can be calculated, and k_1 and k_2 follow by using the value and the marginal value (wrt s) of entry at equilibrium.

The models differ significantly in the c^e/c^f ratio, which is lower in the homogeneous firm case. In the heterogenous firm case, there is a small probability of becoming one of the most productive firms, reaping large profits. Technically, the value function is convex in s and v, so $E[V(s)] > V(E(s))$ by Jensen's inequality, so that idiosyncratic uncertainty increases the value of entry and thus the willingness to bear entry cost. The case here is a bit more complicated because in the benchmark calibration, c^e/c^f is lower in the homogeneous firm case not because of lower c^e, but because of c^f. But this is also clear: In the heterogeneous firm model, fixed costs ensure exit. In the homogeneous firm case, exit is exogenous. Higher fixed costs do however have the effect of decreasing the number of firms in the homogeneous firm case. This has to happen because standardization is chosen such as to equalize output, and the skewed distribution of output across firms in the heterogeneous firm case implies that more firms are needed to produce the same output.

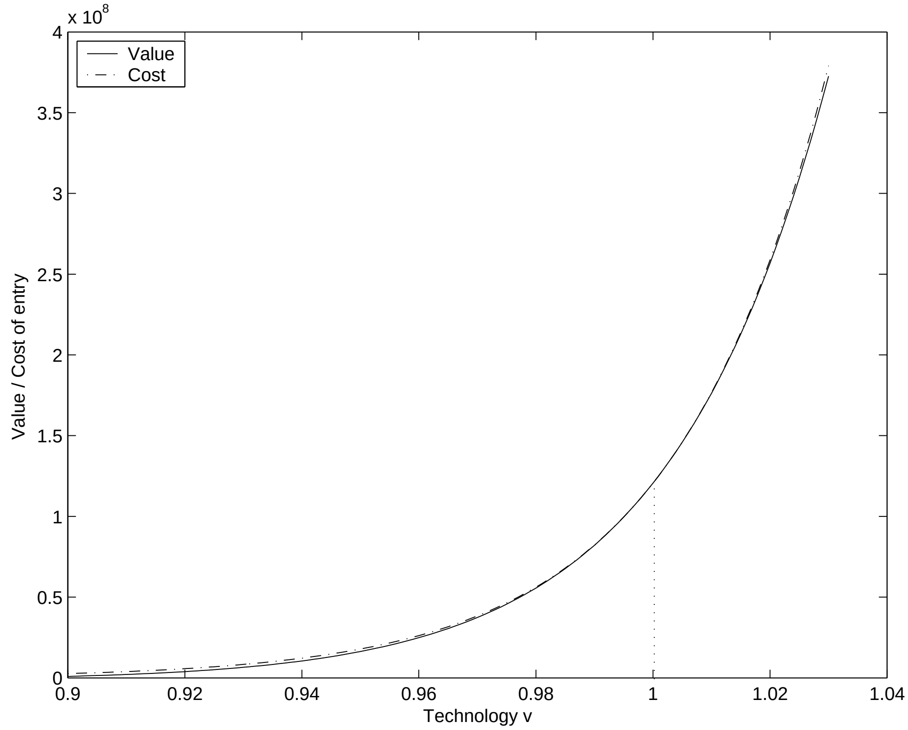

Figure 1: The value function and the entry cost function around the optimum ($v^* = 1$)

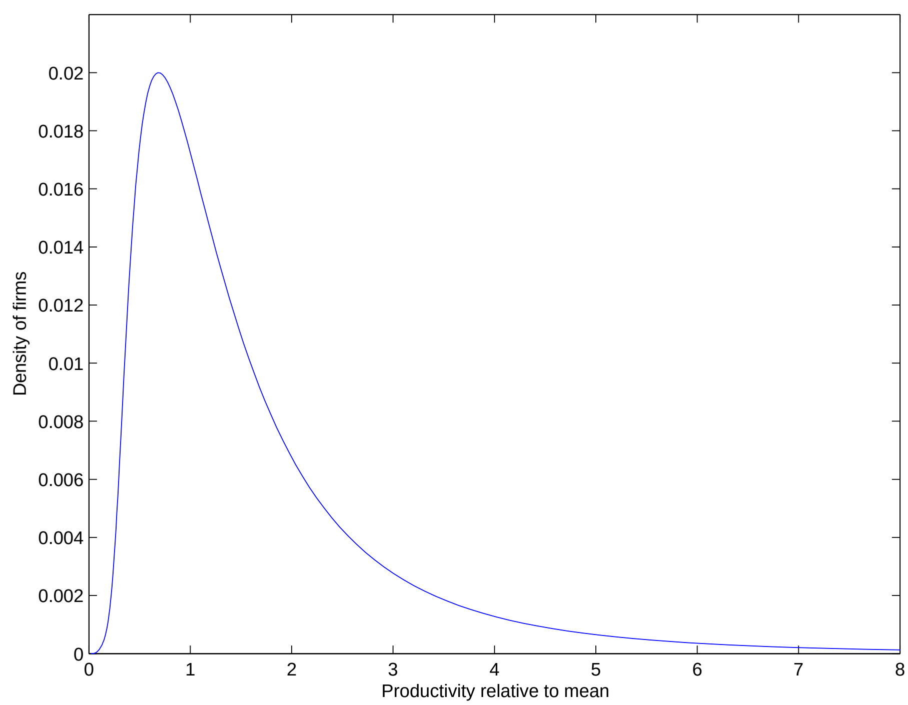

Figure 2: The firm distribution in the benchmark economy

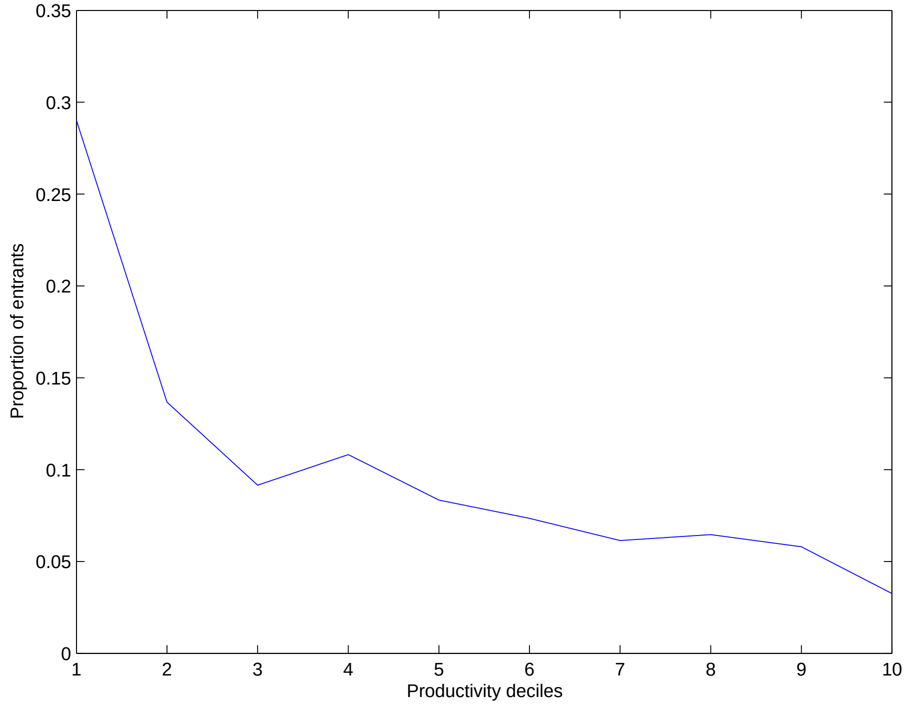

Figure 3: Distribution of entrants over productivity deciles

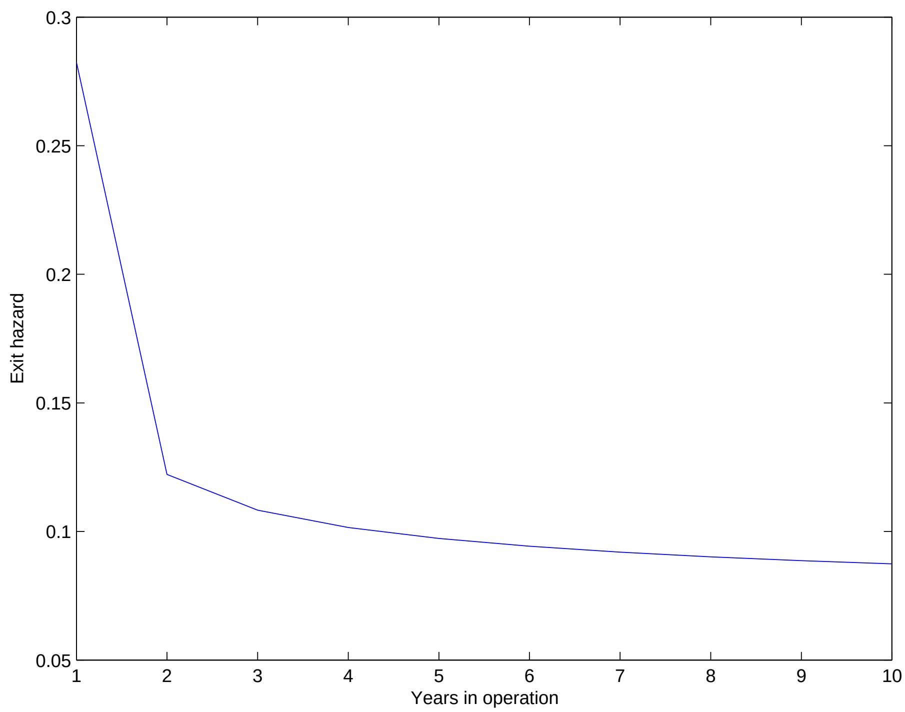

Figure 4: Exit hazard: probability of exiting conditional on being active x years after entry

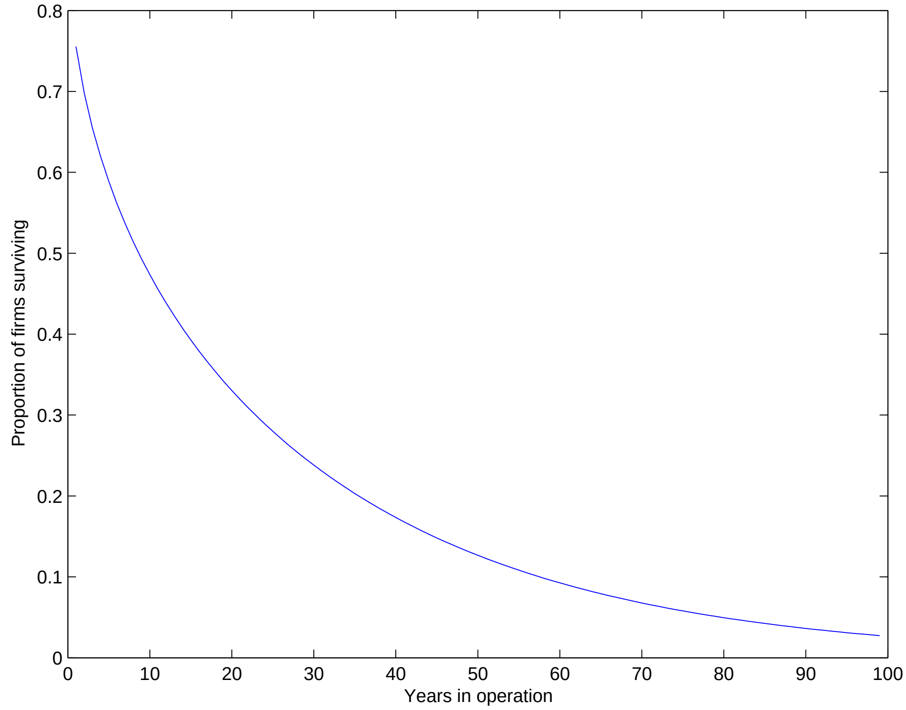

Figure 5: Proportion of firms surviving x years after entry

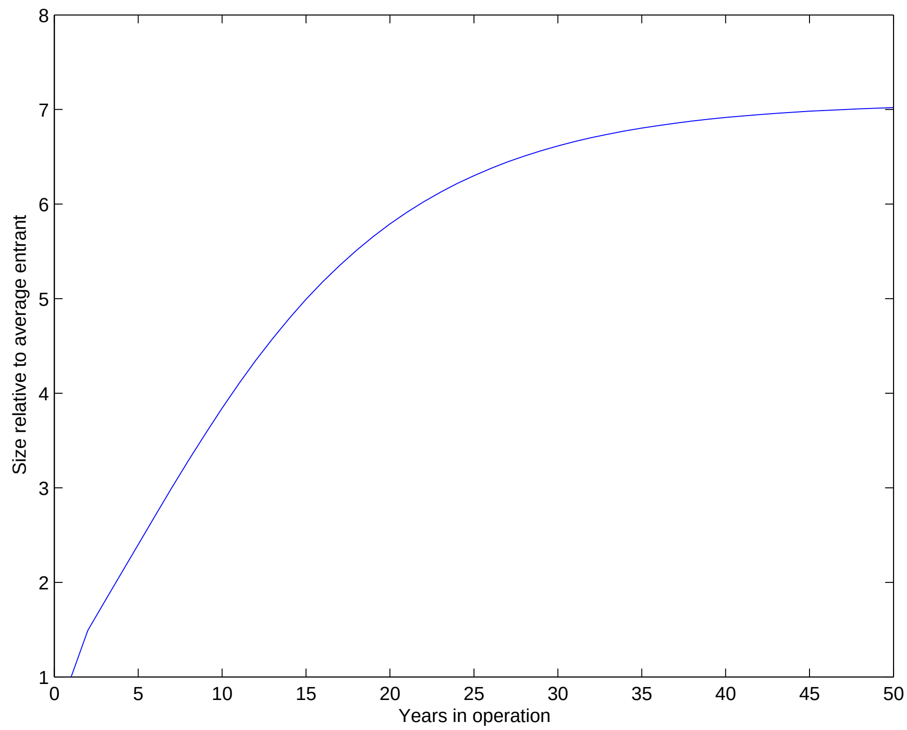

Figure 6: Average size of entrants after x years of activity relative to size at entry

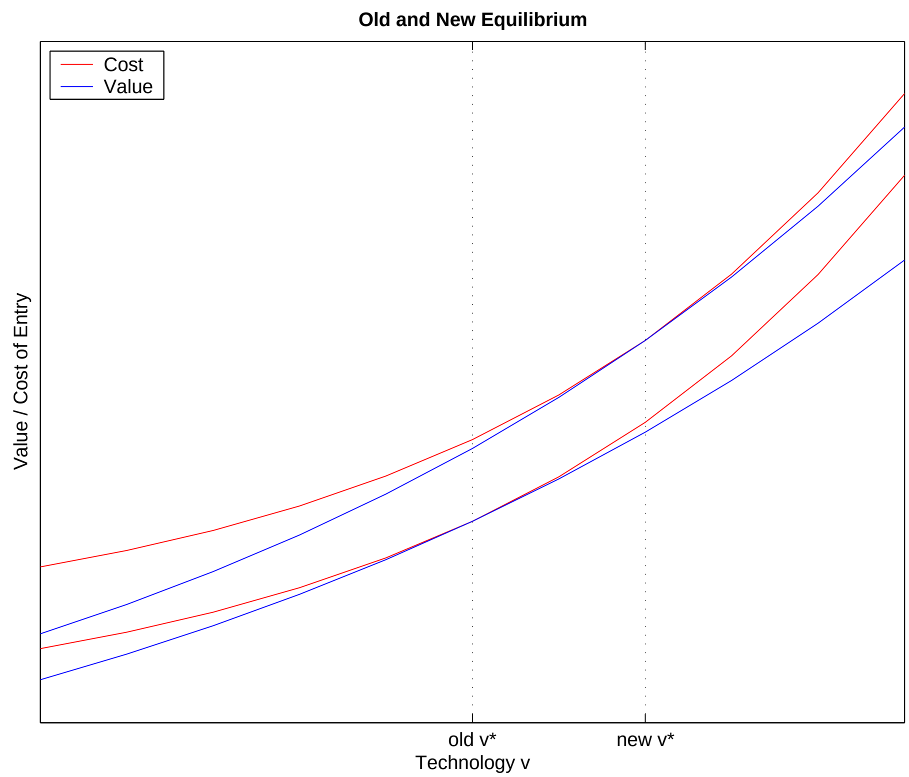

Figure 7: Upward shift in entry cost: old and new equilibrium