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Non-Stationary Inflation

BILL RUSSELL
AND
ANINDYA BANERJEE

EUROPEAN UNIVERSITY INSTITUTE

Department of Economics

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Bill Russell[#]   Anindya Banerjee[†]

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ABSTRACT

Modern theories of inflation incorporate a vertical long-run Phillips curve and are usually estimated using techniques that ignore the non-stationary behaviour of inflation. Consequently, the estimates obtained are imprecise and are unable to distinguish between competing models of inflation and test the veracity of a vertical long-run Phillips curve. We estimate a Phillips curve model taking into account the non-stationary properties in inflation and identify a small but significant positive relationship between inflation and unemployment. The results provide some evidence that the trade-off between inflation and the unemployment rate in the short-run worsens as the mean rate of inflation increases.

[*]   [#]The Department of Economic Studies, University of Dundee. Correspondence: brussell@brolga.net. [†]Department of Economics, European University Institute. The final version of the paper was written while the first-named author was a Visiting Fellow at the Department of Economics of the European University Institute.

1.   INTRODUCTION

The insight of Friedman (1968) and Phelps (1967) that permanently higher inflation would not lead to a permanent reduction in the unemployment rate and that the long-run Phillips curve is vertical underpins the modern theoretical, empirical and policy literature on inflation. Their insight is both simple and powerful. Simple in the sense that the concept of a vertical long-run Phillips curve now appears to be 'common sense' and powerful in so far as this concept completely dominates modern macroeconomics.

One of the implications of a vertical long-run Phillips curve is that inflation may be non-stationary with multiple long-run rates of inflation. Indeed, the argument that the original Phillips curve 'broke down' in the late 1960s and early 1970s implies that the expected rate of inflation had changed due to a change in the long-run rate of inflation. Therefore, the 'breakdown' was due to a period where inflation was non-stationary. Subsequently, the issue of how rapidly expectations of inflation adjust to changes in the long-run rate of inflation came to be a crucial element of the debate surrounding adaptive and rational expectations.

To argue the converse, namely that inflation is stationary with a constant mean over the past five decades, would imply that (a) there has only been one short-run Phillips curve over this time (i.e. the curve where expected inflation matches the unique long-run rate of inflation); and (b) the long-run Phillips curve in a practical sense is a single point corresponding to a unique long-run rate of inflation and the long-run rate of unemployment. From this it follows that stationary inflation data cannot identify a vertical long-run Phillips curve as the data contain no relevant information concerning different long-run rates of inflation. Unless we are willing to abandon all the Phillips curve literature since Friedman and Phelps that incorporates a vertical long-run Phillips curve we must conclude that inflation is non-stationary.[1]

Paradoxically, the conclusion that inflation is non-stationary sits uncomfortably with most of the recent literature that estimates Phillips curve models of inflation. The papers by Batini, Jackson and Nickell (2000, 2005), Galí and Gertler (1999), Galí, Gertler and López-Salido

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[1]   The argument that inflation is non-stationary, by considering the implications of the converse, is dealt with in more detail in Russell (2006).

(2001, 2005), and Rudd and Whelan (2005) are particularly notable in this regard. This is because the techniques chosen to estimate these models are appropriate only if inflation is stationary with a constant mean. For example, Phillips curve models are frequently estimated using generalised method of moments (GMM) so as to allow for correlation between the forward looking variables and the error term. It is well documented (see for example Pesaran 1981, 1987, Stock, Wright, and Yogo 2002 and Mavroeidis 2004, 2005) that GMM is an unsuitable estimation technique if the data are non-stationary and will provide biased estimates leading to poor inference. These difficulties are exacerbated if, as we suspect, there are breaks in the data. Consequently, these estimation techniques are inappropriate for examining the validity of the competing standard models of inflation.[2]

In the following section we demonstrate the shortcomings of inflation models estimated by GMM when the data are non-stationary. We conclude that the behavioural emphasis placed on the estimates in terms of how expectations are formed and whether agents are forward or backward-looking in these models is misplaced. This does not imply that agents do not behave in the ways outlined in the standard theories, only that empirical inflation models estimated using GMM provide such imprecise and biased estimates that they are unhelpful when trying to distinguish between the competing models. Furthermore, we conclude that these estimated models lack the precision necessary to identify whether or not the long-run Phillips curve is indeed vertical.

Graph 1 shows United States quarterly CPI inflation for the period March 1952 to September 2004.[3] The long period of low inflation in the 1950s and early 1960s is brought to an end by increasing inflation towards the end of the 1960s. This is followed by the high inflation of the 1970s and early 1980s associated with the OPEC oil price increases and then two discrete reductions in inflation in the early 1980s and early 1990s. The first reduction is commonly referred to as the 'Volker deflation' and the second coincides with a large recession. These visual shifts in mean inflation can be shown more formally by a rolling 10-year regression of inflation on a constant in the form:

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[2]   The argument in this paper that GMM is an inappropriate method for estimating Phillips curve models due to the non-stationary properties of inflation data can be generalised to any estimation technique that is appropriate only for stationary data with a constant mean.

[3]   See the data appendix for details and sources of the data used in this paper.

$$\Delta p_t = \gamma + \phi_t \qquad\qquad (1)$$

where Δp_t is United States CPI inflation and γ is a constant. The estimated constant, $\hat{\gamma}$, along with the 5 per cent critical value are plotted in Graph 2 and shows the steady increase in the mean rate of inflation in the first half of the sample followed by a steady decline in the second half.

The changes in mean have not been confined to inflation. Graph 3 shows the United States unemployment rate for the period March 1952 to September 2004. We again see a general increase in the unemployment rate up until the early 1980s followed by a general decline thereafter. Similar to the above, we can show this more formally by a 10-year rolling regression of the unemployment rate on a constant similar to that estimated in equation (1). The estimated constant is shown in Graph 4.

How then can we examine Friedman and Phelps' original insight that the long-run Phillips curve is vertical? One way forward is to model inflation as an integrated process of order one which allows us to examine directly if there is a long-run relationship between inflation and the unemployment rate in the sense of Engle and Granger (1987).[4] In section 3 we estimate a two variable I(1) system containing inflation and the unemployment rate. We find that Friedman and Phelps were possibly not pessimistic enough as we find evidence for a small but significant *positive* slope to the long-run relationship between inflation and the unemployment rate. That is, persistently high inflation leads not only to unemployment returning to its long-run level but that the long-run unemployment rate is higher with higher inflation. Persistently high inflation leads to higher unemployment in the long-run.

Indeed Friedman (1977), in his Nobel lecture, conjectures that the long-run Phillips curve may have a positive slope. In that lecture he presents some graphical analysis based on 5-year averaging of inflation and the unemployment rate to support this conjecture. We undertake a similar analysis by plotting in Graph 5 the rolling means of inflation against unemployment (taken from Graphs 2 and 4). This graph shows a clear positive correlation

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[4]   We argue below that, although the 'true' statistical process may be stationary with a very frequently shifting mean, we can approximate this process as integrated of order 1.

between mean unemployment and inflation, suggesting that Friedman's graphical analysis in 1977 near the peak in inflation continues to show in our data which includes the long general decline in inflation since the start of the 1980s.

One might argue that the positive long-run relationship that we identify between inflation and the rate of unemployment is simply due to supply-side influences on the long-run rate of unemployment. The steady increase in the rate of unemployment up until the early 1980s simply reflects a steady worsening in the supply-side influences on unemployment. Similarly, the steady decrease thereafter reflects a steady improvement in the supply-side influences which are independent of, and not due to, changes in the mean rate of inflation. If this is the case then the positive correlation between the mean rate of inflation and these supply side influences must be regarded as extremely unlikely and therefore fortuitous.

Friedman (1977) argues that the positive slope to the long-run Phillips curve that he identifies may persist for many decades due to the greater uncertainty that is associated with higher inflation.[5] The uncertainty persists in the transition due to the time taken for new arrangements to develop and deal with the higher uncertainty.

Our preferred explanation for the positive long-run relationship between inflation and the rate of unemployment is based on Banerjee and Russell (2001a) and Banerjee, Cockerell and Russell (2001). These papers identify a negative long-run relationship between inflation and the markup of price on unit labour costs for a range of developed economies including the United States. This relationship implies that there is a positive relationship between inflation and the real wage relative to productivity.[6] Consequently, persistently high inflation is associated with a persistently high real wage relative to the level of productivity. If high real wages relative to productivity leads to higher unemployment then the positive long-run relationship between inflation and the rate of unemployment rate can be explained through

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[5] Friedman (1977) argues, in what he calls the 'long-long-run', that the Phillips curve will again be vertical. However, in the transition period that may extend for many decades, the long-run Phillips curve will have a positive slope. The slope persists due to the difficulty that economic agents have in fully adjusting to the new institutional and political arrangements that occur with changes in mean inflation.

[6] The markup, mu, on unit labour costs can be written in natural logarithms as $mu = (p - w) + (y - l)$ where p, w, y, and l are the price level, wage rate, output and labour input respectively. Decreases in the markup, therefore, correspond to an increase in the real wage relative to the level of productivity.

the effect of inflation on the real wage. This explanation is examined in section 4 where we estimate a 3 variable I(1) system consisting of inflation, the markup of price on unit labour costs and the rate of unemployment. Two long-run relationships are identified in the data. The first is the positive long-run relationship between inflation and the rate of unemployment and the second re-establishes the negative long-run relationship between inflation and the markup identified in our earlier work.

Our findings suggest that Friedman's (1977) conjecture was correct. Furthermore, not acknowledging explicitly the non-stationary nature of inflation has meant that the positive slope may have been masked by the imprecise estimates, obtained using techniques that are suitable only if the data are stationary with constant mean, or by the invalid imposition of a vertical Phillips curve.

It is a moot question whether the estimated relationships between inflation and the rate of unemployment and between inflation and the markup are truly 'long-run' in the sense used by economic theorists. It may be that after a number of decades (i.e. Friedman's 'long-long-run') of a constant high rate of inflation the relationship disappears. However, from a policy perspective the relationship appears to persist for enough time for it to be considered the 'long-run'. Certainly the persistence in the data easily satisfies the standard tests of the long-run in the sense of Engle and Granger (1987).

2. ESTIMATING THE HYBRID PHILLIPS CURVE

The mainstream literature that incorporates a vertical long-run Phillips curve can be understood in terms of the hybrid Phillips curve where inflation, Δp_t, in time t depends on expected inflation, Δp_{t+1}^e, conditioned on information available in time t, lagged inflation, Δp_{t-1}, and a 'forcing' variable, $u_t - u^*$.[7] The hybrid model can be written:

$$\Delta p_t = \delta_f \, \Delta p_{t+1}^e + \delta_b \, \Delta p_{t-1} + \delta_u \left(u_t - u^*\right) + \varepsilon_t \tag{2}$$

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[7] There are now a number of clear expositions of the New Keynesian Phillips curve and the hybrid Phillips curve. One such exposition is Henry and Pagan (2004).

where the error term, ε_t, represents the random errors of agents and inflation shocks, Δ is the change in the variable and lower case variables are in natural logarithms so that $\Delta p_t = p_t - p_{t-1}$. The 'forcing' variable represents excess demand and is measured in a variety of ways in the literature including the gap between real and potential output, real marginal costs, labour's share of income and, as shown here, the gap between the unemployment rate, u_t, and the long-run rate of unemployment, u^*.

The Friedman-Phelps Phillips curve and New Keynesian Phillips curve models are special cases of the hybrid Phillips curve. In the purely backward looking Freidman-Phelps model, agents hold adaptive expectations and so $\delta_f = 0$ and $\delta_b = 1$. In the purely forward-looking rational expectations New Keynesian models of Clarida, Galí and Gertler (1999) and Svensson (2000) $\delta_b = 0$ and $\delta_f = 1$. Finally, hybrid models assume that there are both forward and backward-looking price setting agents and that $\delta_f + \delta_b = 1$.

The hybrid Phillips curve has a number of appealing characteristics. First, the model embodies a vertical long-run Phillips curve if $\delta_f + \delta_b = 1$, and $0 < \delta_f, \delta_b < 1$ where inflation is independent of the real variables in the model in the long-run. Second, the hybrid model incorporates the observationally legitimate staggered price adjustment in contrast with instantaneous price adjustment in the New Keynesian models. Third, the hybrid model allows for both forward and backward looking elements that would appear consistent with how many firms and agents operate. The model, therefore, provides a simple way to choose between the three competing standard theories of the Phillips curve by identifying the proportion of firms that base their pricing decisions on past inflation and expected inflation. Finally, estimating the model allows us to empirically verify the validity of the vertical long-run Phillips curve if we find that $\delta_f + \delta_b = 1$.

These 'appealing characteristics' are somewhat tempered by the theoretical and practical difficulties associated with the estimation of these models. On a theoretical level, Phillips curve models that incorporate a vertical long-run Phillips curve make very strong statements concerning the statistical properties of the inflation data. If the model is true and $\delta_f + \delta_b = 1$ then inflation is non-stationary and empirical work should proceed by allowing for the

possibility that inflation is non-stationary.

Furthermore, the defining features of each model are the underlying behavioural assumptions concerning expectation formation and pricing behaviour and that these behavioural assumptions correspond to particular values of δ_f, δ_b and that $\delta_f + \delta_b = 1$. If these models are true then such important 'model-defining' behavioural assumptions should be present at all times. That is, the behaviour must be present not just over the entire sample but during sub samples as well. In this case, the estimated values for δ_f and δ_b should be stable over time and sum to 1. Unfortunately, what we find with inflation data is that there are long periods when inflation behaves as a stationary variable. Without estimating the model, we know that when inflation is stationary the sum of the coefficients $\delta_f + \delta_b$ must be less than 1. Therefore, the coefficients cannot be stable over time and this must question the behavioural significance of the estimated coefficients.

On a practical level, if we estimate Phillips curve models with non-stationary inflation data using techniques such as GMM that are suitable for stationary data then the estimates will lack precision and are likely to be biased.[8] Consequently, any estimates of δ_f and δ_b that are retrieved will be unable to distinguish between the competing theories and whether or not $\delta_f + \delta_b = 1$. Very small deviations in these imprecise estimates will lead to very different conclusions in terms of choosing between the models.

A number of conclusions follow from this discussion. First, unless we allow for the non-stationary nature of inflation, there can be no behavioural significance to particular estimated values of δ_f and δ_b unless we are happy to accept that the behaviour of economic agents is very unstable. Second, to estimate the long-run Phillips curve requires non-stationary

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[8]   Estimating Phillips curve models that incorporate expected inflation and a contemporaneous 'forcing' variable imply that expected inflation will be correlated with the error term in equation (2). The standard solution is to estimate the model using GMM where the instruments are usually lags of both inflation and the 'forcing' variable. Considerable work has focused on overcoming problems when estimating hybrid Phillips curves using GMM. Most of the focus in the empirical literature is on determining whether or not lags in inflation and the 'forcing' variable are suitable instruments for the variables. As inflation more closely approximates an integrated variable the instruments become better predictors of expected inflation but simultaneously more correlated with the error term.

inflation data so that $\delta_f + \delta_b = 1$. Finding $\delta_f + \delta_b < 1$ does not invalidate theories that incorporate a vertical long-run Phillips curve. All it implies is that the data are stationary and that it contains no relevant information concerning different long-run rates of inflation. Finally, estimation of the model should proceed under the assumption that the model is correct and therefore estimation should allow for the possibility that inflation is non-stationary.

These conclusions concerning estimating the hybrid Phillips curve can be demonstrated by estimating the model using GMM over the full sample between March 1952 and September 2004 and comparing the results with those obtained from a rolling 10-year estimation of the model. The results for the full sample are reported in Table 1. The last five columns provide critical values assuming that the data are stationary, CV_{prob}^{S}, a random walk, CV_{prob}^{RW}, and autoregressive with a coefficient on lagged inflation of 0.9, $CV_{prob}^{0.9}$. The degree of significance of the critical value is indicated by the subscript *prob*.[9] We find that the coefficients on expected and lagged inflation are significantly less than 1 while the sum of the coefficients is insignificantly different from 1. These results are similar to those found in the literature. However, the diagnostics indicate that the residuals are stationary but highly serially correlated and the *J*-test indicates that the instruments are not valid.

Leaving aside the fact that the diagnostic tests indicate that the model is poorly specified, the results are broadly supportive of the hybrid Phillips curve hypothesis. In particular, the behavioural underpinnings of the model are supported with expected inflation and forward looking agents having a large and possibly dominant role in the inflation process, there is a smaller proportion of backward looking agents and, finally, that the long-run Phillips curve is vertical.

However, the poor model specification indicated by the diagnostic tests should not be overlooked. Estimating the hybrid Phillips curve on a rolling 10-year basis reveals the estimates of the model are poorly identified with the coefficients on expected and lagged

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[9]   The critical values shown in the table as CV^{RW} and $CV^{0.9}$ were calculated using Monte Carlo simulation techniques. That is, the same model with 209 observations is re-estimated 10,000 times and the histogram of the *t*-statistics used to compute the 95 per cent critical value.

inflation highly unstable. The three panels of Graph 6 report the rolling estimates of the coefficients on expected inflation, lagged inflation and the sum of the inflation coefficients. Also shown on the graphs are the critical values testing for the coefficients equal to 1 assuming that the data are stationary. We see that for much of the period the coefficients on expected and lagged inflation display large variation and that for much of the time they are significantly less than 1 based on critical values assuming that the data are stationary. We also see in the bottom panel that for nearly the entire sample the sum of the coefficients is insignificantly different from 1. However the imprecision of the estimates is evident in the graphs. We cannot reject that the sum of the coefficient is 1 even though the point estimates of the sum range between around zero and 2 ½.

These results draw attention to two issues. First, the estimates are very poorly determined with very large standard errors. Consequently, it is very difficult to reject hypotheses concerning the size of the estimates. We cannot be confident from these results that the long-run Phillips curve is vertical even though we cannot reject the hypothesis that the sum of the coefficients equals to one. It may be that the long-run Phillips curve has a small but economically important slope.

Second, placing a behavioural emphasis on these results in terms of the expectation formation or proportions of agents who are forward or backward looking is misplaced as the estimates are so inaccurate that we cannot reject most hypotheses. Furthermore, given the importance of the behavioural hypotheses in defining their respective models the behaviour should be stable over time. If the behavioural arguments are correct then it appears from the results in Graph 6 that agents appear to forget quickly and learn again only to forget again.

3.   THE LONG-RUN INFLATION-UNEMPLOYMENT RATE RELATIONSHIP

So far we have not addressed the issue of what type of non-stationarity is present in the inflation data. Two forms of non-stationarity present themselves. The first is to assume the monetary authorities respond to a series of shocks by making discrete changes in their implicit inflation target and this leads inflation to be stationary with shifting means. The usual way to proceed here would be to introduce as many shift dummies as necessary to 'render' inflation a stationary series. Typically this will not require a large number of shift dummies. Invariably this small number of dummies will not identify all the shifts in the

mean rate of inflation and the unidentified shifts will continue to be problematic when estimating inflation models. Furthermore, it is not easy to interpret the dummies in an economic sense other than they indicate shifts in the mean rate of inflation.

The second way to proceed is to assume that inflation shocks are very frequent and that the monetary authorities adjust the target rate of inflation at least partially in response to the shocks. This assumption is consistent with the view that central banks did not actively offset the increases in inflation during the 1970s by increasing nominal interest rates by more than the increase in expected inflation (for example see McCullum 2000). This suggests central banks accommodated the higher inflation and raised their implicit inflation target in the 1970s and may well have acted in a similar fashion when shocks served to reduce inflation at other times like in the early 1950s and 1990s. We therefore assume that, while the 'true' statistical process of inflation may be stationary around a very frequently shifting mean, this process can be approximated by an integrated process of order 1. In any case, as the number of shifts in means increases, a stationary process with shifting means converges in the limit on an integrated variable.

The hybrid Phillips curve model in equation (2) suggests that the inflation process can be considered in terms of four variables, namely inflation, expected inflation, the unemployment rate and the long-run unemployment rate. In the long-run expectations are realised and expected inflation equals actual inflation. Similarly, actual unemployment will equal the long-run unemployment rate. If we assume that the long-run rate of unemployment may not be fixed then this variable can vary in the long-run. In this case, the long-run relationship that implicitly underpins the hybrid Phillips curve when expectations are realised can be written:

$$\Delta p_t = \alpha_0 + \alpha_1 u_t^* + \varepsilon_t \tag{3}$$

In the standard models that incorporate a vertical long-run Phillips curve, the long-run coefficient α_1 is equal to zero. Alternatively, if $\alpha_1 \neq 0$ then the long-run Phillips curve is not vertical. The advantage of this approach over estimating the model using GMM is threefold. First, in estimating (3) we can allow for the possibility that the variables are non-stationary and in so doing improve the precision of the estimates. Second, in equation (3) the test of the

slope of the long-run relationship is whether or not α_1 is significantly different from zero. This is in contrast with equation (2) where the test is whether or not the sum of the leads and lags in inflation is significantly different from 1.[10] Third, estimating equation (3) allowing for non-stationary data allows us to either reject or accept the vertical long-run relationship. This is in contrast with the GMM estimates of the hybrid Phillips curve model which is unable to reject the vertical Phillips curve when the data are non-stationary.

However, these advantages do come with a disadvantage insofar as estimating equation (3) does not allow us to distinguish among the three competing standard theories of inflation. In particular, the estimates cannot distinguish between the proportions of forward and backward looking agents or how expectations are formed. Estimating equation (3) allowing for non-stationary data simply allows us to answer the question of the slope of the long-run Phillips curve and provide better estimates of the dynamics of inflation.

If we assume that inflation is an integrated variable of order 1, we can estimate a two variable I(1) system containing inflation and the unemployment rate. The system is conditioned on a predetermined business cycle variable which is measured as de-trended constant price gross domestic product (GDP) and a series of spike dummies for periods when the residuals of each estimated equation in the system are greater than 3 standard errors.

The results of estimating the inflation-unemployment rate I(1) system are provided in Table 2. The trace statistics for the number of cointegrating vectors indicate that the hypothesis of one long-run relationship between the variables is evident in the data. We find that the long-run coefficient, α_1, is significant and negative indicating a positive long-run relationship between inflation and the unemployment rate. The estimated long-run coefficient is – 2.714 and this implies that a 1 percentage point increase in annual inflation is associated with approximately a 0.37 of a percentage point increase in the rate of

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[10] Consider the case where $\delta_f + \delta_b > 1$ and $\delta_u < 0$ in the 'true' model in equation (2). If we solve for the long-run relationship then the coefficient on unemployment will be $\dfrac{\delta_u}{1-\left(\delta_f + \delta_b\right)} > 1$. If $\delta_f + \delta_b$ is only slightly larger than 1 then it will be difficult when estimating equation (2) not to accept the hypothesis that the coefficients sum to 1 even though in the true model the sum of the coefficients is greater than 1. This difficulty is due to the large standard errors associated with estimating equation (2) using GMM.

unemployment in the long run. This estimate can be compared with Graphs 2 and 4 where we see that mean inflation increased by around 7 percentage points between the start of 1962 and the peak in 1982 while the unemployment rate increased by about 3 percentage points over the same period. A 'crude' estimate of the long-run relationship from these figures would suggest a long-run coefficient of around 0.4 (i.e. 3 divided by 7).

4. EXPLAINING THE LONG-RUN INFLATION-UNEMPLOYMENT RATE RELATIONSHIP

An explanation of the positively sloping long-run Phillips curve is provided in a series of papers by us that estimate a negative long-run relationship between inflation and the markup of price on unit costs for a number of developed economies.[11] This suggests that higher inflation leads to a lower markup of price on unit costs in the long-run or, equivalently, a higher real wage relative to the level of productivity. If high real wages increase unemployment then high inflation will be associated with high unemployment and a positive sloping long-run Phillips curve.

To examine this proposition we estimate a three variable I(1) system that includes inflation, the markup of price on unit labour costs, and the rate of unemployment. We expect that the data will show two long-run relationships. The first is that identified in the previous section between inflation and the rate of unemployment. The second relationship follows Banerjee, Cockerell and Russell (2001) and is given by:[12]

$$\Delta p = \beta_0 + \beta_1 \, mu \qquad (4)$$

where mu is the markup of price on unit labour costs and β_1 is the parameter that measures the trade-off in the long-run between inflation and the markup. The markup is calculated as $p - ulc$, where the price level, p, is the consumer price index and ulc is a measure of unit

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[11] For example, see estimates of the long-run inflation-markup relationship provided in Banerjee, Cockerell and Russell (2001), and Banerjee and Russell (2001a, 2001b, 2004, 2005).

[12] The two long-run relationships of equations (3) and (4) cannot strictly be true since the markup approaches zero and the unemployment rate approaches infinity as inflation tends to infinity. This suggests the 'true' relationships are non-linear. However, over the small range of inflation as experienced in the United States over the last 50 years, the log-linear relationship appears to be a good approximation.

labour costs. The long-run model is again conditioned on a business cycle variable and a series of spike dummies that capture the sometimes erratic behaviour of the data.

In the standard Phillips curve models, inflation has no impact on the markup in the long run as the real wage will return to its constant long-run value on the vertical long-run Phillips curve. Thus $\beta_1 = 0$ in equation (4) and $\alpha_1 = 0$ in equation (3). In our more general model estimated here we expect that $\beta_1 < 0$ as in earlier work and $\alpha_1 > 0$ as estimated in section 3.

The results of estimating the three variable, inflation-unemployment rate-markup I(1) system are provided in Table 3. The trace statistic indicates that we can comfortably accept the hypothesis of 2 long-run relationships in the data and these are reported at the top of the table. The first long-run relationship between inflation and the unemployment rate is slightly larger but similar in magnitude to that reported above with a long-run coefficient of – 3.271. The second long-run relationship between inflation and the markup reports a long-run coefficient of 26.344 which is slightly larger but similar to estimates reported in earlier work.[13] Both long-run relationships are highly significantly different from zero leading us to conclude that there is a positive slope to the long-run Phillips curve.

The long-run estimates from the three variable I(1) system suggest that an increase in general inflation of around 5 percentage points as happened during the 1970s would be associated with an increase in unemployment in the long-run of around 1 ½ percentage points.

Graph 7 shows as a solid thick line the estimated long-run Phillips curve along with the actual combinations of inflation and the unemployment rate used in the estimation. The latter are shown by the scattering of symbols. The positive relationship is evident in the data and the long-run relationship.

The actual data on the graph are separated into 5 inflationary 'episodes' that are listed below the graph along with the identifying symbols and mean rates of inflation. We would expect

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[13]   The long-run inflation-markup coefficient can be re-parameterised and compared with earlier work. The re-parameterised value is 3.795. Earlier estimates for the United States reported in Banerjee and Russell (2001a, 2001b) of 0.62 using annual 'private sector' gross domestic product data and 0.46 using aggregate gross domestic product data and 6.602 using a measure of marginal cost markups and 2.526 using unit cost markups in Banerjee and Russell (2005).

the actual inflation and unemployment rate data to be strung out and clustered around a series of short-run Phillips curves associated with each inflationary episode. Unfortunately the short-run Phillips curves can not be identified directly from this analysis. As a proxy for these short-run Phillips curves, the trend lines for the data in each inflationary episode are shown as thin lines and identified with numbers 1 to 5 that correspond to each inflationary episode in the table.

If we consider the data in each episode we see that at the start of the sample we are in a period of low inflation with low unemployment rates (diamonds). This is followed by a period of increasing inflation and unemployment rates at the end of the 1960s and early 1970s (triangles) and then the high inflation and high unemployment of the 1970s (crosses). The deflation of the 1980s (squares) and the 1990s (dashes) see us again return to a low inflation and low unemployment environment on the graph. What is made clear by separating the data into inflationary episodes is that as inflation rises the short-run Phillips curve as proxied by the thin trend lines and the associated cluster of data not only moves upwards but also to the right. Importantly, this process is reversed with declining inflationary episodes and we see the short-run Phillips curve shifting not only down as expected but to the left. It is the apparent sideway shifts, rather than purely vertical shifts, of the short-run curve that justifies our result of a positive sloping long-run Phillips curve.

5.   CONCLUSION

This paper argues that modern theoretical models of inflation suggest that inflation is non-stationary. However, empirical work on identifying which of the competing theories 'best' describes the inflationary process appears to ignore resolutely the possibility that the inflation data may be non-stationary. Consequently, inflation models are typically estimated using techniques that are best suited for stationary data and the resulting estimates are likely to be biased and lack precision so that inference concerning which model is the 'best' is poor. The problem of poorly estimated coefficients makes it impossible to test properly whether or not the long-run Phillips curve is vertical as small deviations in the estimated inflation coefficients from 1 imply that the long-run curve has a slope.

To overcome this problem we explicitly acknowledge the non-stationary nature of inflation in our estimation of the long-run Phillips curve. We estimate that there is a small but

significant positive slope to the long-run Phillips curve. This slope implies that a 5 percentage point increase in the general rate of inflation leads to an increase in the unemployment rate in the long-run of around 1 ½ percentage points. Movements of this magnitude in the unemployment rate are likely to be important in both economic and political senses.

Finally, notice on Graph 7 that the proxies for the short-run Phillips curves become steeper with increases in the mean rates of inflation. The trend lines for the low inflation episodes numbered 1 and 5 are almost horizontal. As the mean rates of inflation increase the trend lines rotate in a clockwise direction and become steeper suggesting the short-run trade-off between inflation and unemployment diminishes. The very flat short-run Phillips curves at low mean rates of inflation suggests that the temptation for governments to use expansionary macroeconomic policy is great. However, the positive slope to the long-run Phillips curves argues strongly against such myopic behaviour and lends support to the idea of independent central banks that deliver low stable inflation.

6. REFERENCES

Banerjee, A., Cockerell, L. and B. Russell (2001). An I(2) An analysis of Inflation and the Markup, *Journal of Applied Econometrics*, Sargan Special Issue, vol. 16, pp. 221-240.

Banerjee, A. and B. Russell (2001a). The Relationship between the Markup and Inflation in the G7 Economies and Australia, *Review of Economics and Statistics*, vol. 83, No. 2, May, pp. 377-87.

Banerjee, A. and B. Russell. (2001b). Industry Structure and the Dynamics of Price Adjustment, *Applied Economics*, 33:17, pp. 1889-901.

Banerjee, A. and B. Russell, A Reinvestigation of the Markup and the Business Cycle, *Economic Modelling*, vol. 21, pp. 267-84.

Banerjee, A. and B. Russell (2005). Inflation and Measures of the Markup, *Journal of Macroeconomics*, vol. 27, pp. 289-306.

Batini, N., B. Jackson, and S. Nickell. (2000). Inflation Dynamics and the Labour Share in the UK, Bank of England External MPC Unit Discussion Paper: No. 2 November.

Batini, N., B. Jackson, and S. Nickell. (2005). An Open-Economy New Keynesian Phillips Curve for the U.K., *Journal of Monetary Economics*, vol. 52, pp. 1061-71.

Clarida, R., Gali, J. and Gertler, M. (1999). The Science of Monetary Policy: a New Keynesian Perspective, *Journal of Economic Literature*, vol. 37, pp. 1661-1707.

Engle, R.F. and C.W.J. Granger, (1987). Co-integration and Error Correction: Representation, Estimation, and Testing, *Econometrica*, vol. 55, no. 2, May, pp. 251-76.

Friedman, M. (1968). The role of monetary policy, *American Economic Review*, 58, 1 (March), pp. 1-17.

Friedman, M. (1977). Nobel Lecture: Inflation and Unemployment, *The Journal of Political Economy*, vol. 85, no. 3, pp. 451-72.

Gali, J., and M. Gertler (1999). Inflation Dynamics: A Structural Econometric Analysis,

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Journal of Monetary Economics, vol. 44, pp. 195-222.

Gali, J., Gertler M., and J.D. Lopez-Salido (2001). European Inflation Dynamics, *European Economic Review*, vol. 45, pp. 1237-1270.

Gali, J., Gertler M., and J.D. Lopez-Salido (2005). Robustness of the Estimates of the Hybrid New Keynesian Phillips Curve, *Journal of Monetary Economics*, vol. 52, pp. 1107-18.

Hansen, L.P. (1982). Large Sample Properties of Generalized Method of Moments Estimators, *Econometrica*, Vol. 49, pp. 1377-98.

Henry, S.G.B. and A.R. Pagan (2004). The Econometrics of the New Keynesian Policy Model: Introduction, *Oxford Bulletin of Economics and Statistics*, vol. 66, supplement, pp. 581-607.

Johansen, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*, Oxford: Oxford University Press.

Mavroeidis, S. (2004). Weak Identification of Forward-looking Models in Monetary Economics, *Oxford Bulletin of Economics and Statistics*, vol. 66, supplement, pp. 609-35.

Mavroeidis, S. (2005). Identification Issues in Forward-looking Models Estimated by GMM, with an Application to the Phillips Curve, *Journal of Money, Credit and Banking*, vol. 37, pp. 421-48.

McCullum (2000). Alternative Monetary Policy Rules: A Comparison with Historical Settings for the United States, the United Kingdom, and Japan, *Federal Reserve Bank of Richmond Economic Quarterly*, vol. 86, Winter, pp. 49-79.

Perron, P., (1997). Further Evidence on Breaking Trend Functions in Macroeconomic Variables. *Journal of Econometrics*, 80 no. 2, 355-385.

Pesaran, M.H. (1981). Identification or Rational Expectations Models, *Journal of Econometrics*, vol. 16, no. 3, pp. 375-98.

Pesaran, M.H. (1987). *The Limits to Rational Expectations*, Blackwell, Oxford.

Phelps, E.S. (1967). Phillips curves, expectations of inflation, and optimal unemployment

over time, *Economica*, 34, 3 (August), pp. 254-81.

Rudd, J. and K. Whelan (2005). New Tests of the New-Keynesian Phillips Curve, *Journal of Monetary Economics*, vol. 52, pp. 1167-81.

Russell, B. (2006). Non-stationary Inflation and the Markup: an Overview of the Research and some Implications for Policy, forthcoming in Dundee Discussion Papers, Department of Economic Studies, University of Dundee

Stock, J., J. Wright, and M. Yogo (2002). GMM, Weak Instruments, and Weak Identification, *Journal of Business and Economic Statistics*, vol. 20, pp. 518-30.

Svennson, L.E.O. (2000). Open Economy Inflation Targeting, *Journal of International Economics*, vol. 50, pp. 155-83.

A1 DATA APPENDIX

The CPI data and unemployment rate data were downloaded from the United States of America, Bureau of Labour Studies. The national accounts data was downloaded from the National Income and Product Account tables from the United States of America, Bureau of Economic Analysis. The data for March 1952 to September 2004 was downloaded on 25 November 2004. Except where indicated, the data are quarterly and seasonally adjusted. The mnemonics are from the databases that they were downloaded from.

Table A1: Sources and details of the data manipulation

| *Variable*	*Details*
Consumer price inflation	The monthly CPI is the US city average, all items, 1982-84=100, ID: CUSSR0000SA0. The derived quarterly data is the average of the monthly data. CPI inflation is the change in the natural logarithm of the quarterly CPI multiplied by 400 to give the annualised rate.
Unemployment rate	The unemployment rate is the number of people over 16 years as a percentage of the non-institutionalised civilian population, ID: LNS14000000. The derived quarterly data is the average of the monthly data.
Gross domestic product (GDP) at constant prices	Constant price GDP at 2000 prices, Table 1.1.6, line 1.
Gross domestic product (GDP) implicit price deflator at factor cost	Nominal GDP at factor cost is nominal GDP (Table 1.1.5, line 2) plus subsidies (Table 1.10, line 10) less taxes (Table 1.10, line 9). GDP implicit price deflator is nominal GDP at factor cost divided by constant price GDP.
Total labour compensation	Wages, salaries and supplements, Table 1.10, line 2.
Unit labour costs	Calculated as total labour compensation divided by constant price GDP.
Markup	Calculated as the consumer price index divided by unit labour costs.
De-trended Variables	The variables are de-trended using broken linear trends. The breaks are identified using the augmented Perron (1997) unit root test which allows for the presence of an endogenous change in the level and slope of the trend function. Having identified the break (or breaks by using the technique sequentially) the data is regressed on a constant and trend as well as a 'short' trend, shift constant, and spike dummy for each break in trend. The short trend is zero for each period up to the break and then a linear trend thereafter. The shift dummy is zero up until the break and one thereafter. The 95 % critical value is - 3.13. Unemployment Rate: There is a break in trend and constant in June 1978 (test statistic -4.1). Business cycle (measured as de-trended natural logarithm of constant price GDP): The test identifies two breaks in the trend and constant. September 1963 (test statistic – 4.8). December 1979 (test statistic – 5.6).

Graph 1: United States CPI Inflation
March 1952 – September 2004
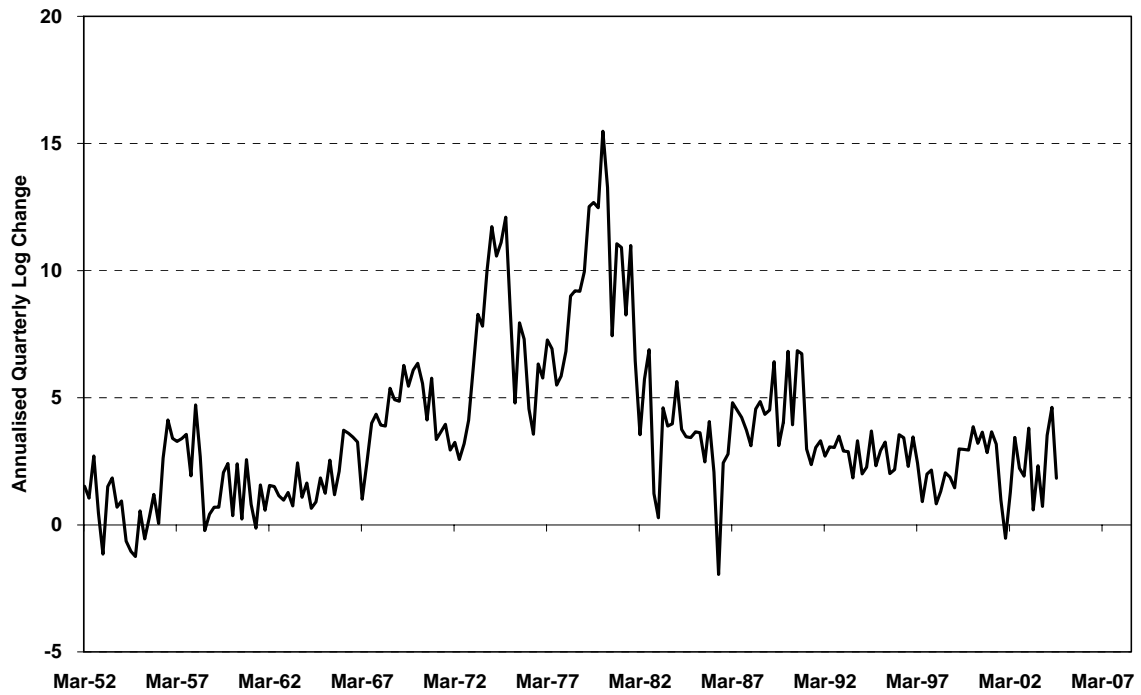
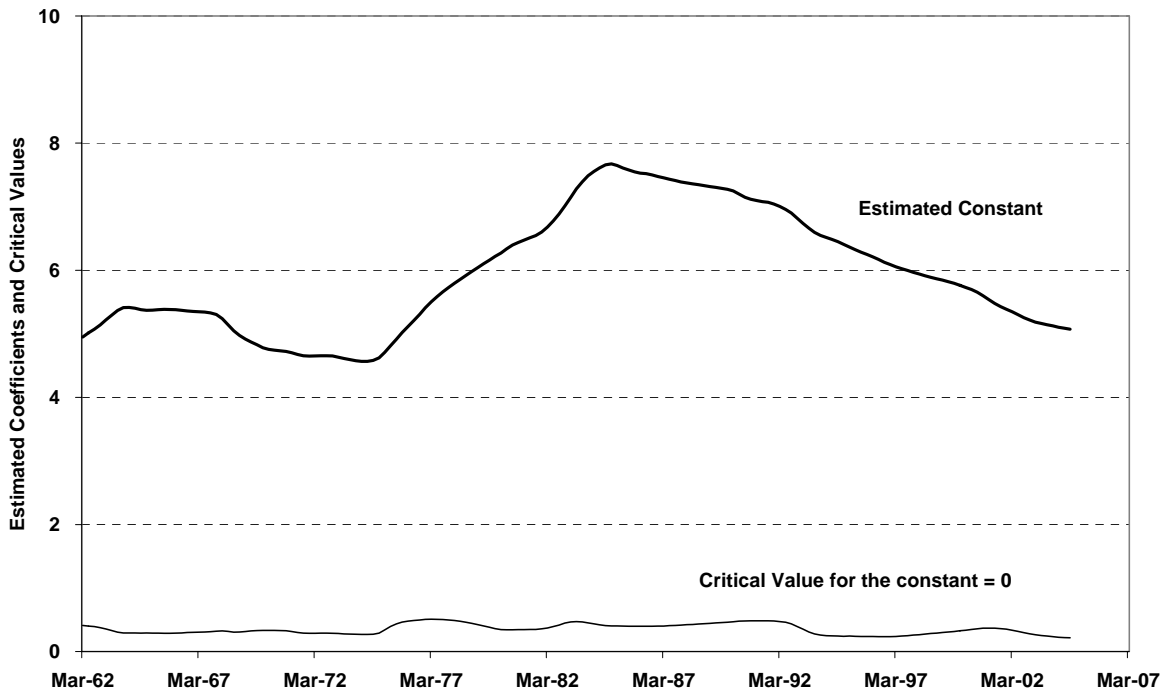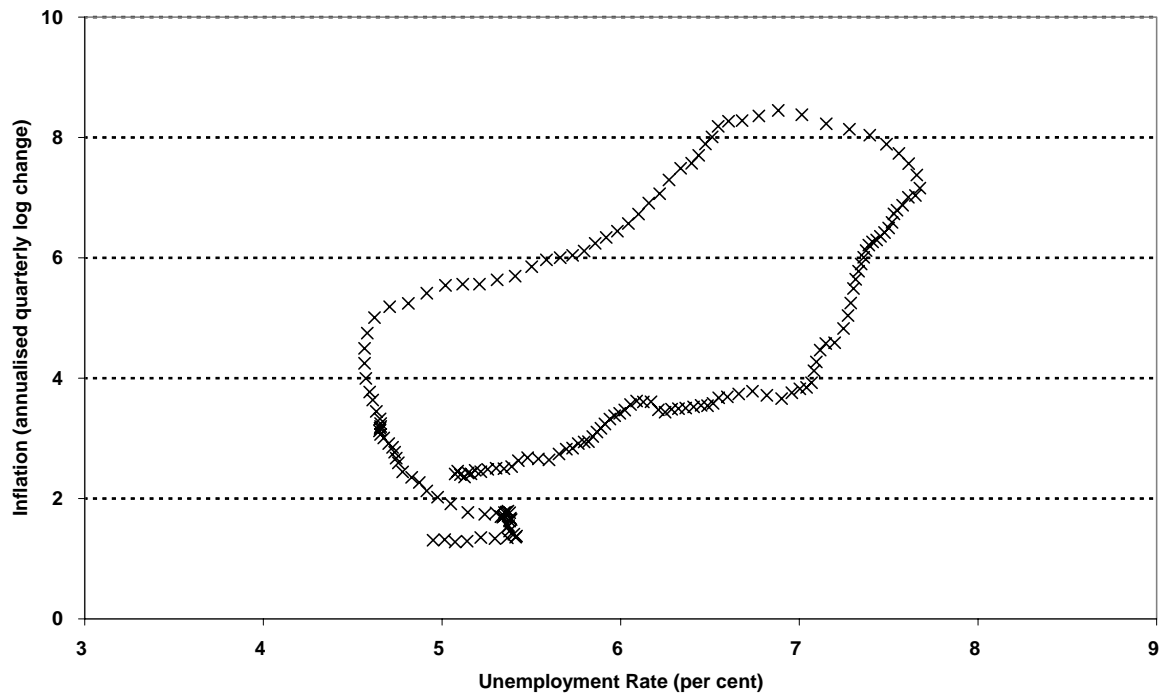## Graph 2: Estimated Constant from Rolling 10 Year CPI Inflation Regression
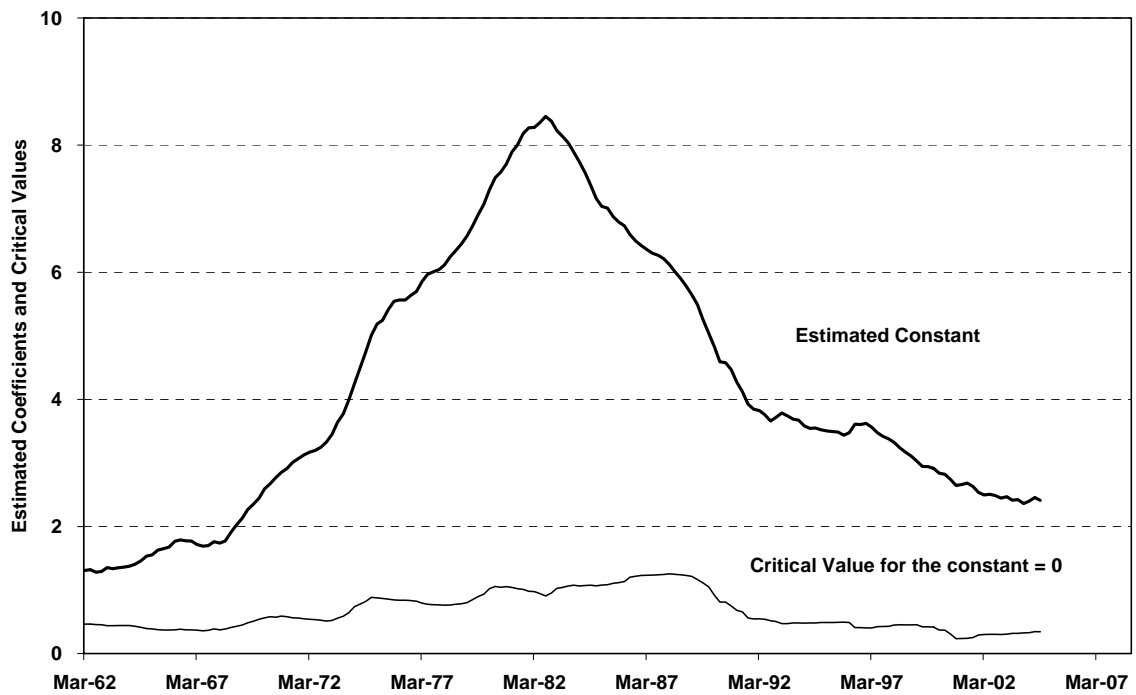
Graph 3: United States Unemployment Rate

March 1952 – September 2004
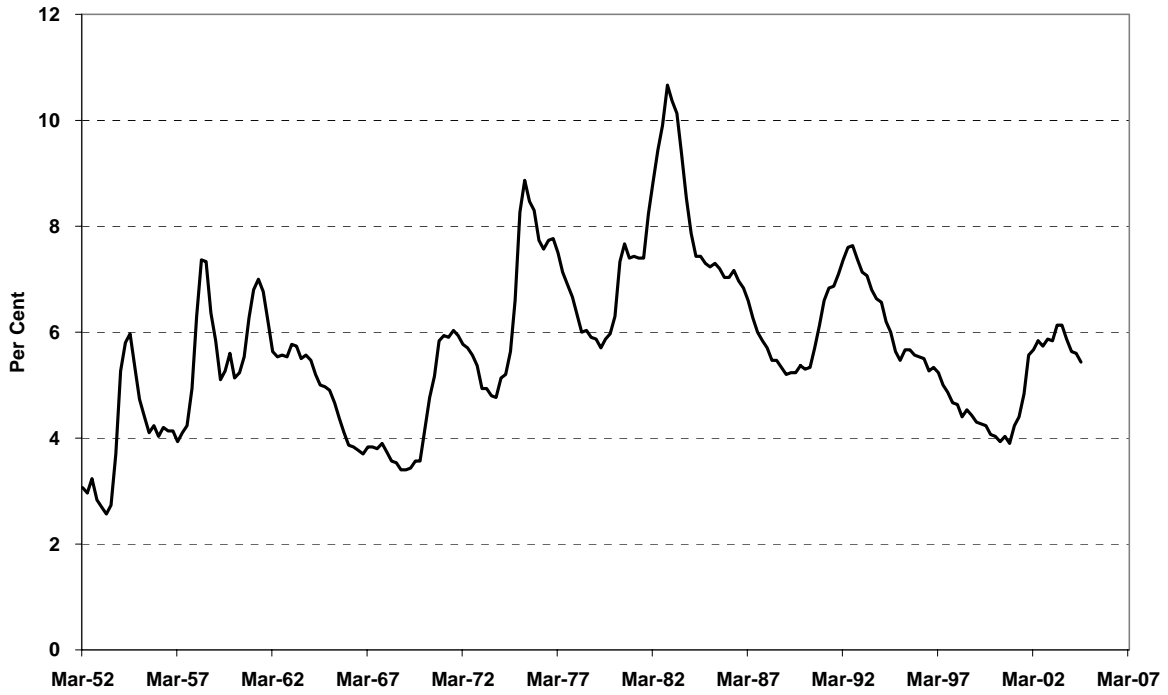
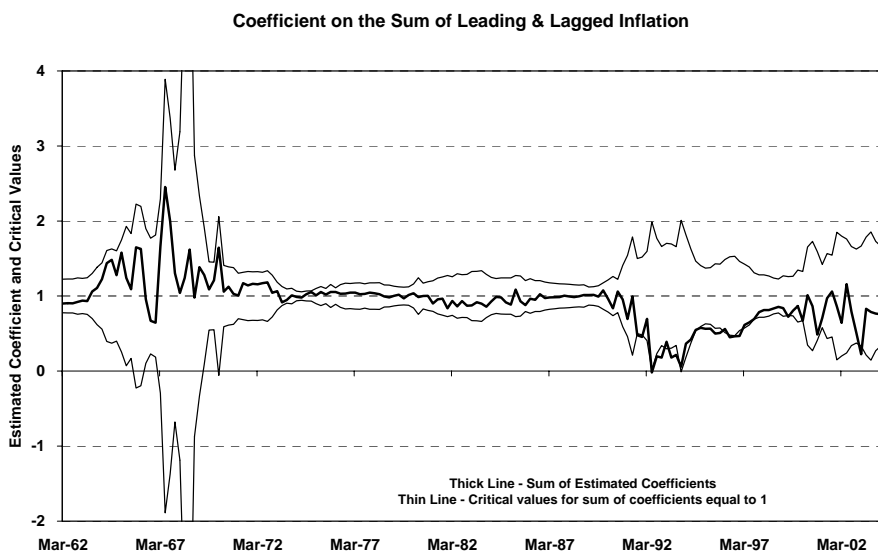**Graph 4: Estimated Constant from Rolling 10 Year Unemployment Rate Regression**
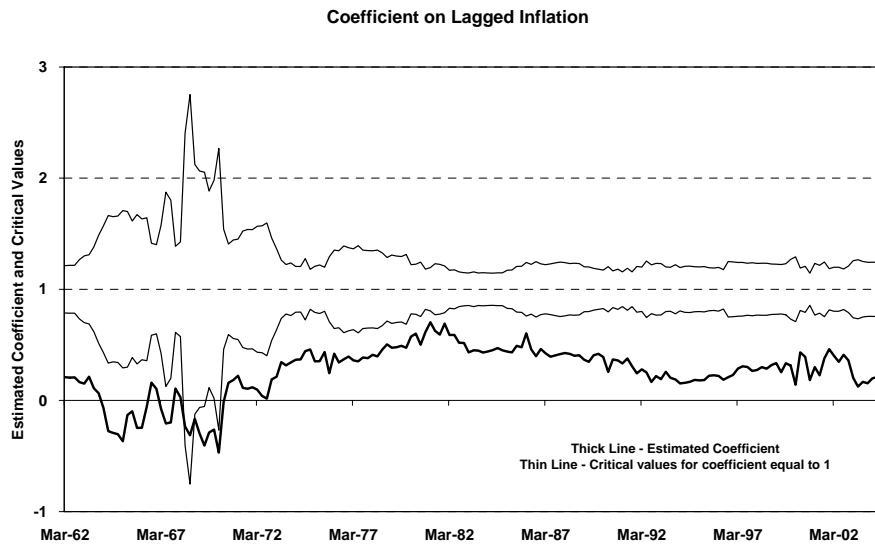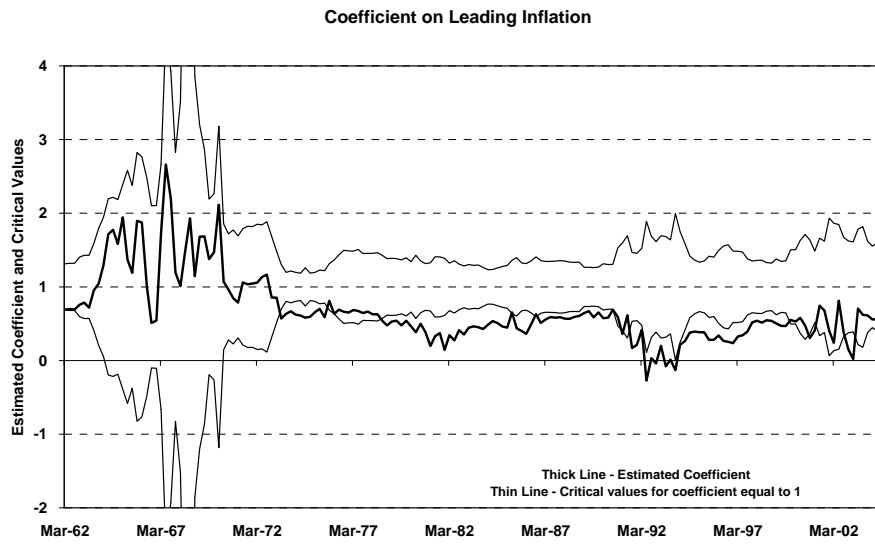
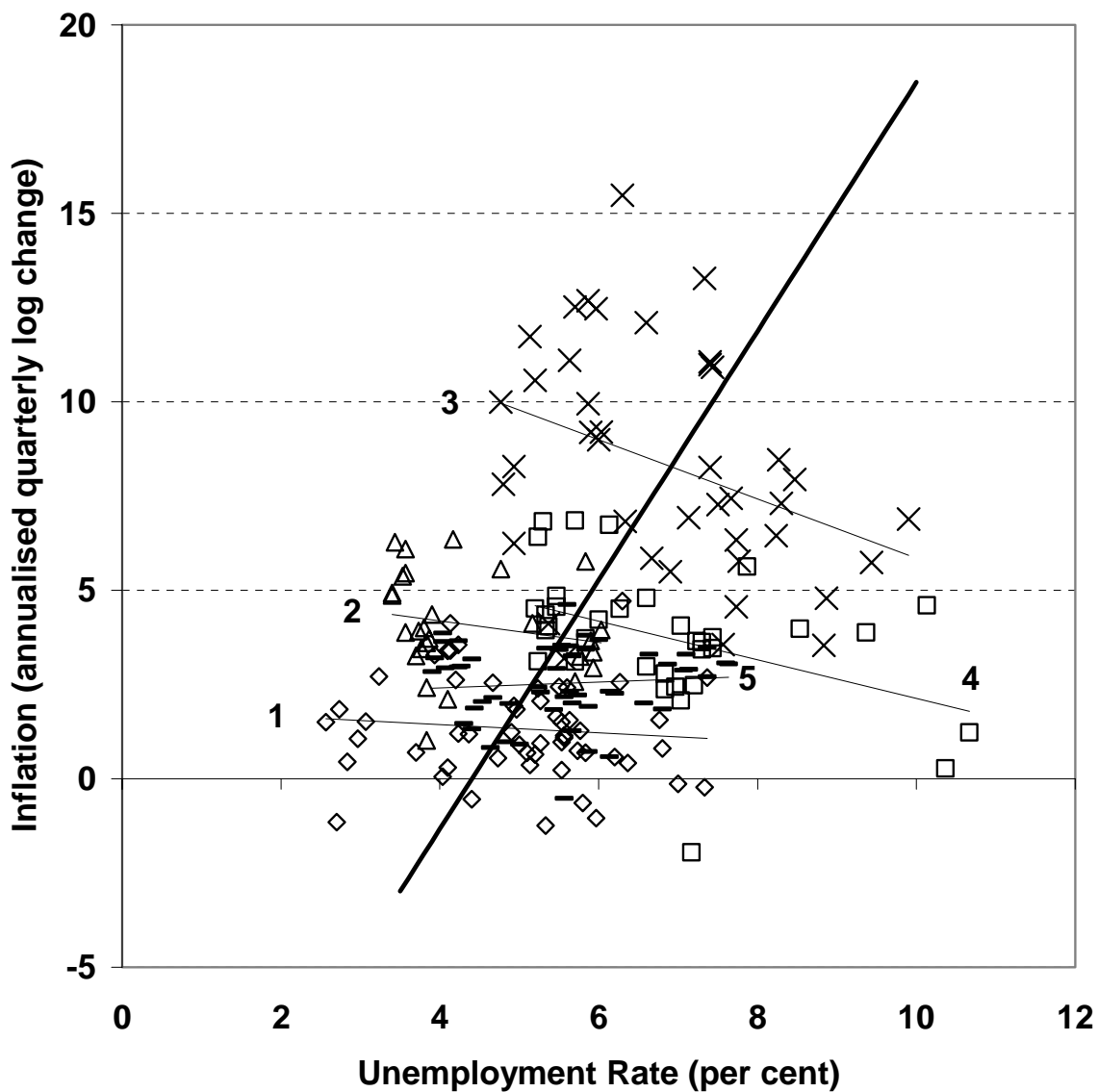**Graph 5: Mean Inflation and Unemployment Rates**

Notes: The mean values of inflation and the unemployment rate are the estimated constants from the 10-year rolling regressions reported in Graphs 2 and 4.

Graph 6: GMM Rolling 10-Year Estimation of Phillips Curve Model

Coefficient on Leading Inflation

Thick Line - Estimated Coefficient
Thin Line - Critical values for coefficient equal to 1

Coefficient on Lagged Inflation

Thick Line - Estimated Coefficient
Thin Line - Critical values for coefficient equal to 1

Coefficient on the Sum of Leading & Lagged Inflation

Thick Line - Sum of Estimated Coefficients
Thin Line - Critical values for sum of coefficients equal to 1

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Graph 7: United States Phillips Curve

	Inflation Episodes	Symbol	Mean Inflation
1	March 1952 to September 1965	Diamond	1.3
2	December 1965 to June 1972	Triangle	4.1
3	September 1972 to September 1982	Cross	8.3
4	December 1982 to June 1991	Square	3.8
5	September 1991 to September 2004	Dash	2.5

Table 1: Estimates of the United States Hybrid Phillips Curve
March 1952 to September 2004

Variable	Estimated Coefficient	Standard Error	t Statistic	$CV_{5\%}^{S}$	$CV_{2.5\%}^{RW}$	$CV_{97.5\%}^{RW}$	$CV_{2.5\%}^{0.9}$	$CV_{2.5\%}^{0.9}$
Δp_{t+1}	0.7055	0.0988	7.14	1.96	- 0.36	2.78	- 0.39	2.59
Δp_{t-1}	0.2946	0.0822	3.59	1.96	- 0.37	2.75	- 0.32	3.09
UE_{t}	- 0.0238	0.0567	- 0.42	1.96				
δ	- 0.0305	0.1140	- 0.27	1.96				

Testing if coefficients are insignificantly different from 1

	Estimated Coefficient	Standard Error	t Statistic	$CV_{5\%}^{S}$	$CV_{5\%}^{RW}$	$CV_{5\%}^{0.9}$
$\delta_f - 1$	- 0.2945	0.0988	- 2.98	- 1.64	- 2.23	- 2.14
$\delta_b - 1$	- 0.7054	0.0822	- 8.58	- 1.64	- 2.37	- 2.66
$\delta_f + \delta_b - 1$	0.0001	0.0303	0.00	- 1.64	- 0.72	- 0.90

Diagnostic Tests

Lagrange multiplier test of serial correlation: $\chi_1^2 = 173.09$, p-value 0.0000; $\chi_4^2 = 202.85$, p-value 0.0000. ADF test of residuals: t statistic = - 8.17, $CV_{1\%}$ = - 2.58. J Test (Hansen 1992) of instrument validity: 12.43, p-value 0.0060, $CV_{1\%}$ $\chi_1^2 = 6.64$ and $CV_{5\%}$ $\chi_1^2 = 3.84$. $\overline{R}^2 = 0.79$. SEE = 1.3951.

Notes: Estimated model: $\Delta p_t = \delta_f \Delta p_{t+1}^e + \delta_b \Delta p_{t-1} + \delta_U UE_t + \delta + \varepsilon_t$

Instruments: constant, three lags of Δp and three lags of UE. Number of usable observations, 207.

Data are quarterly and the unemployment rate is demeaned before estimation with a broken linear trend. Further details of the data are provided in the data appendix.

Table 2: I(1) Inflation-Unemployment Rate Model

Normalised Cointegrating Vector	u	Δp
Long-run Relationship	- 2.714 (0.333)	1 (0.160)
Adjustment Coefficients	0.038 [8.2]	0.034 [1.3]

Standard errors reported as ( ), *t*-statistics reported as [ ]. The adjustment coefficients are the values with which the long-run relationship enters each equation of the system. The long-run relationship, or dynamic error correction term is therefore: $ECM_t \equiv \Delta p_t - 2.714 u_t$.

Likelihood ratio tests on the long-run relationship (a) test of coefficient on inflation is zero is rejected, $\chi_1^2 = 31.41$, p-value = 0.00, (b) test of coefficient on the unemployment rate is zero is rejected, $\chi_1^2 = 50.65$, p-value = 0.00, and (c) test of the coefficient on the trend is zero is accepted $\chi_1^2 = 0.00$, p-value = 1.00.

Predetermined Variables: de-trended constant price GDP, spike dummies for periods where residuals are greater than 3 standard errors – March 1954, December 1958, March 1975, September 1980, December 1981, June 1986.

Testing for the Number of Cointegrating Vectors

Estimated trace statistic for the null hypothesis $H_0 : r = 0$ is 59.16 {13.31}, $H_0 : r = 1$ is 0.17 {2.71}. Numbers in { } are the relevant 90 per cent critical values from Table 15.3 of Johansen (1995). Statistics computed with 3 lags of the core variables chosen by the significance of the last dynamic terms. The sample is December 1952 to September 2004 and has 208 observations with 194 degrees of freedom.

System Diagnostics for the Mode

(a) Tests for Serial Correlation

Ljung-Box (52) $\chi^2 (198) = 211.744$, p-value = 0.24

LM(1) $\chi^2 (4) = 9.143$, p-value = 0.06

LM(4) $\chi^2 (4) = 9.441$, p-value = 0.05

(b) Test for Normality: Doornik-Hansen Test for normality: $\chi^2 (4) = 5.941$, p-value = 0.20.

Table 3: I(1) Inflation-Unemployment Rate-Markup Model

Normalised Cointegrating Vector	u	Δp	mu
Long-run Relationship 1	- 3.271 (0.376)	1 (0.158)	
Adjustment Coefficients	0.030 [7.5]	- 0.005 [- 0.2]	- 0.000 (- 3.9)
Long-run Relationship 2		1 (0.176)	26.344 (5.088)
Adjustment Coefficients	0.014 [2.2]	0.026 [0.7]	0.001 [5.7]

Standard errors reported as ( ), *t*-statistics reported as [ ]. The adjustment coefficients are the values with which the long-run relationship enters each equation of the system. The two long-run relationship, or dynamic error correction term are therefore: $ECM\,1_t \equiv \Delta p_t - 3.271 u_t$ and $ECM\,2_t \equiv \Delta p_t + 26.344\,mu_t$.

Likelihood ratio tests (a) Long-run Relationship 1 - test of coefficient on inflation is zero is rejected, $\chi_1^2 = 26.43$, p-value = 0.00, and the markup is zero is rejected, $\chi_1^2 = 19.28$, p-value = 0.00 (b) Long-run Relationship 2 - test of coefficient on the inflation is zero is rejected, $\chi_1^2 = 12.06$, p-value = 0.00, and (c) test of the coefficient on the trend is zero is accepted $\chi_1^2 = 1.58$, p-value = 0.45.

Predetermined Variables: de-trended constant price GDP, and spike dummies for periods where residuals are greater than 3 standard errors – March 1954, March 1956, March 1958, March 1975, June 1978, September 1980, December 1981, March 1982, December 1982, June 1986, and March 2000.

Testing for the Number of Cointegrating Vectors

Estimated trace statistic for the null hypothesis $H_0 : r = 0$ is 102.26 {26.70}, $H_0 : r = 1$ is 35.85 {13.31}, $H_0 : r = 2$ is 2.01 {2.71}. Numbers in { } are the relevant 90 per cent critical values from Table 15.3 of Johansen (1995). Statistics computed with 4 lags of the core variables chosen by the significance of the last dynamic terms. The sample is March 1953 to September 2004 and has 207 observations with 182 degrees of freedom.

System Diagnostics for the Model

(a) Tests for Serial Correlation

Ljung-Box (51) $\chi^2 (426) = 426.949$, p-value = 0.48

LM(1) $\chi^2 (9) = 12.295$, p-value = 0.20

LM(4) $\chi^2 (9) = 12.358$, p-value = 0.19

(b) Test for Normality: Doornik-Hansen Test for normality: $\chi^2 (6) = 7.126$, p-value = 0.31.