EUI WORKING PAPERS
ECO No. 2006/25

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EUI Working Paper **ECO** No. 2006/25

ISSN 1725-6704

Printed in Italy
European University Institute
Badia Fiesolana
I – 50016 San Domenico di Fiesole (FI)
Italy

http://www.iue.it/
http://cadmus.iue.it/dspace/index.jsp

SLOW MONEY DISSEMINATION*

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June 2006

Abstract

A model of limited participation in the asset market is developed, in which varieties of consumption bundles are purchased sequentially. By this, heterogeneity in money holdings and in the effective elasticity of substitution of consumers arises, which affects optimal markups chosen by oligopolistic firms. The model generates a short-term inflation-output trade off, although all firms can set their optimal price each period and no informational problems exist. The responses are persistent even after a one-time monetary shock due to an internal propagation mechanism that stems from the slow dissemination of newly injected money. Furthermore, a liquidity effect, countercyclical markups, procyclical profits and marginal costs after monetary shocks are obtained. The model is simple and tractable, such that analytical results for the linearized model can be derived.

Keywords: Limited Participation, Countercyclical Markups, Liquidity Effect Phillips Curve, Oligopolistic Competition

JEL-Codes: E31, E32, E51

*I want to thank Jordi Galí, Gernot Müller, Stefan Niemann and Stefano Gnocchi for helpful discussions and valuable comments as well as the European Investment Bank for financial support.

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1 Introduction

Monetary economics has seen a considerable variety of models labeled limited participation models over the last two decades. Because of their common ability to replicate well some empirical features, '[t]hese models represent a serious alternative to the sticky-price and sticky-wage Keynesian models that have been popular in recent policy analysis.', according to Williamson (2005). Originally, Grossman and Weiss (1983) had an infrequent adjustment of asset holdings by households in mind, combined with overlapping shopping sequences. In their model, consumers are assumed to make staggered money withdrawals, i.e. at any given time only half the population is at the bank. The notion that households do not adjust their assets each period is theoretically shown as optimal by Jovanovic (1982) and empirically supported by Christiano et al. (1996), who find that 'households do not adjust their financial assets and liabilities for several quarters after a monetary shock'. Households in the Grossman-Weiss model need money balances to finance their consumption, since they face a Cash-in-Advance constraint. A liquidity effect is obtained because falling interest rates are needed after an increase in the money supply, to induce higher money holdings by the agents currently at the bank. Furthermore, prices rise slowly after a positive monetary shock. However, the path of adjustment is not satisfying. The price level and the interest rate are reaching the new steady state in an oscillating manner. Because of the simplification of setting output exogenously and constant, no inflation-output trade off is modelled. Rotemberg (1984) uses the same timing structure, but introduces production and capital. He finds that after a monetary expansion, output increases and returns slowly to the steady state without oscillating. A Cobb-Douglas function for the combination of utility from both period's consumption is assumed. Therefore, households spend equal amounts of their cash holdings each period in between their visits to the asset market, such that one important aspect of intertemporal optimization of the households is not addressed. Also, because of perfect competition, the optimal markup is not considered.

The present paper tries to shed some light on these and other questions by using a model with oligopolistic competition in the goods market with arbitrary values for the elasticities of substitution between varieties and periods.[1] Analytical results are derived, which are not available for the Grossman-Weiss and Rotemberg models. In these models, the distribution of wealth over time is

[1] Note that oligopolistic competition here does not necessarily stem from few firms populating the economy, but from the fact that each consumer buys at a countable number of shops.

difficult to track because of the heterogenous agents. Hence, the models are limited to study the effects of one-time monetary shocks in a deterministic setting. Maintaining the heterogenous money holdings and the sequential shopping sequence of the models mentioned, I assume an ownership structure of the shops that mimics the slow dissemination of newly injected money throughout the economy, and leads to a model that can be analyzed with the standard tools for dynamic stochastic general equilibrium models.

A different and widely used solution to the problem of tractability was found by Lucas (1990), who lets household members pool their trade receipts at the end of the period. By this, a degenerate money distribution, and therefore tractability is reached. However, the paper and many follow-ups deal mainly with the liquidity effect and asset pricing implications, not with the inflation-output trade off. Because households undo the effects of monetary policy at the end of each period, only unanticipated monetary shocks have real effects, and last merely one period. Based on classical search models of money such as Kiyotaki and Wright (1989), papers like Shi (1997) use Lucas's solution to study the effects of monetary policy in this environment. Search models discard the Walrasian auctioneer and decentralize trading activities. Typically, potential buyers and sellers meet with a certain probability and engage in trade if their wants coincide. In an alternative to Lucas's method, Lagos and Wright (2005) assume a periodic access for all agents to a centralized market in a search model, where they choose the same money balances because of a restriction on the utility function. While these solutions overcome the non-tractability and therefore the unsuitability of studying monetary policy, a non-degenerate wealth distribution is also likely to have considerable effects and would therefore be interesting to study.

Thus, some new models re-introduce the heterogenous money holdings into different settings. Like Lagos and Wright, Williamson (2006) assumes a search and a centralized location. A random, periodic re-allocation of agents between both allows for a slow spreading of new central bank money, which can only be injected at the centralized location. Closed form solutions for the stochastic version can be derived in the case of no monetary interventions. Monetary shocks, anticipated or not, lead to distributional and persistent effects. The model of the present paper also features heterogenous money holdings, but goes back to the original setup of the Grossman-Weiss and Rotemberg models. As in their models, staggered money withdrawals are placed in a Walrasian environment. Shopping bundles of consumption goods takes time, in contrast to an implicit assumption taken in standard models of monopolistic competition. In the present model, consumers buy each variety of their bundles one after the other. If unexpected events occur during this sequence, the original plan is altered according to the new circumstances. Since it is unlikely that all consumers start and finish their sequences at the same points in time, these sequences overlap.

In the following section I will briefly discuss a benchmark model of oligopolistic competition with two households populating the economy. As is usual in these standard models, no real effects of monetary interventions will arise. The following assumption has to be made: New money injected into the system by the central bank reaches all agents in an identical way. Heterogenous shocks to money holdings cannot be studied with this model because these shocks would lead to explosive behavior or a violation of the Euler equation. Since all agents are informed about the monetary shock, prices will immediately rise by the same percentage as the money stock.

In section 3, homogeneity is replaced by limited participation. One period is divided into two; in each of these new periods both consumers are shopping, but only one of them is visiting the asset market. Hence, monetary injections reaches only this consumer. It seems realistic to assume that not all agents are benefiting from central bank actions in the same way, or as Williamson (2005) puts it: 'For example, when the Fed conducts an open market operation, the economic agents on the receiving end of this transaction typically are large financial institutions that are not directly connected to all other economics agents in the economy through exchange. (...) This difference will be important for short-run movements in interest rates, aggregate output, and the distribution of wealth across the population.' Additionally, the assumption that each shop belongs to one consumer together with a sequential opening of these shops, leads to a slow spreading of the newly injected money and keeps the model tractable. Because monetary injections take time to be distributed equally through second-round effects, unequal wealth levels arise which in turn affect the aggregate price elasticity firms are facing. The reason is the following: If consumers in the beginning of their shopping sequences have a larger weight due to monetary transfers to them via the asset market, aggregate elasticity is higher since these consumers spread the new income over the goods to follow in the sequence. This introduces strategic interaction in the price-setting behavior of firms and thereby lowers the optimal markup chosen. Hence, a short-term inflation-output trade off is reached. For monetary shocks, the model predicts therefore countercyclical markups, as empirically found by Rotemberg and Woodford (1999). Specifically, markups are countercyclical at the firm level, coinciding with evidence in the supermarket industry presented by Chevalier and Scharfstein (1996). These authors also confirm that prices are strategic complements. The Phillips Curve derived in section 4 displays inflation inertia via the internal propagation mechanism, i.e. even one-time shocks lead to long lasting responses. The impulse response functions developed in section 5 are empirically plausible. Output and profits rise, marginal costs (which correspond to wages) do so moderately while the interest rate falls (i.e. there is a liquidity effect) after a positive monetary shock; features that were found empirically by Christiano et al. (1997). In section 6, the general formulae for any given number of agents are developed. This number can also be seen as the free parameter describing the lags in the monetary system suggested in the last paragraph of Lucas (1990). Section 7 concludes.

2 A Standard Two-Consumer Model

I will develop a two-consumer, two-firm model of oligopolistic competition as a reference model in order to make clear which assumptions in section 3 drive the results. This model is built on the standard monopolistic-competition Dixit and Stiglitz (1977) model, with two agents instead of a representative household, and two firms instead of a continuum. The stocks of each firm are owned by one of the two consumers, such that the households earn the profits of 'their' firm. Furthermore, consumers have to obey a Cash-in-Advance (CIA) constraint. Note that the oligopolistic structure stems mainly from the assumption that each consumer buys at two different firms, which does not have to translate literally into two firms populating the economy.

2.1 Setup

Households Each consumer maximizes the following standard utility function. The subscript $i = 1, 2$ denotes the consumer.

$$E_t(\overline{U}_i) = E_t \left(\sum_{t=0}^{\infty} \beta^t \frac{\overline{C}_{i,t}^{1-\sigma}}{1-\sigma} \right)$$

The consumption aggregate $\overline{C}_i$ combines the consumption goods of the two firms for each consumer:

$$\overline{C}_{i,t} = \left[{C_{i,t}^{a}}^{\frac{\gamma-1}{\gamma}} + {C_{i,t}^{b}}^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} \qquad \gamma > 1$$

where C_i^j is the current consumption of consumer $i = 1, 2$ at store $j = a, b$. Households underlie a Cash-in-Advance constraint

$$M_{i,t} \geq P_t^a C_{i,t}^a + P_t^b C_{i,t}^b, \tag{1}$$

where P_t^j is the current price of the good of firm j. Visible from the intertemporal budget constraint, households can acquire the cash needed for consumption in the same period (asset market opens first, $M_{i,t}$ is beginning of period cash)

$$M_{i,t} + B_{i,t} = (1 + i_{t-1})B_{i,t-1} + \Pi_{t-1}^i + S_t, \tag{2}$$

where Π_t^i is the nominal revenue $Y_t^j P_t^j$ of a firm, whose stocks are owned by household i. Here and in what follows, I assume that consumers do not take into account their ownership of a firm while shopping at that particular firm. $B_{i,t}$ are bonds bought by household i, which cost one unit of the currency and earn the interest rate i_t between time t and $t + 1$. S_t are this period's nominal transfers, which are the same for each household. The equation implies that at the beginning of each period, households have to decide how to divide available resources between money holdings needed for shopping and savings, i.e. bonds. Money cannot be transferred as cash between periods.[2] Current income from business activity cannot be used for current consumption while the cash injections can be used contemporaneously, as in Lucas (1982).[3] As normal, the central bank can either set the money supply via S_t or the nominal interest rate i_t, where the respective other variable has to adjust accordingly.

Furthermore, a transversality condition has to be obeyed

$$\lim_{T\to\infty} \Pi_{t=0}^T \beta^t \overline{C}_{i,t}^{-\sigma} B_{i,t-1} \geq 0, \tag{3}$$

where $\overline{C}_{i,t}^{-\sigma}$ is the marginal utility of period t consumption and $B_{i,t-1}$ are bond holdings at the beginning of the period.

[2]See section 3 for a longer discussion of this point. Here it is clearly not optimal to carry over in the case of positive nominal interest rates.

[3]In order to avoid the inflation-tax effects normally present in CIA models, the firms are 'de-personalized', i.e. their costs do not depend on the CIA constraint of the owner. Hence, no real effects of this restriction arises. See also footnote 4.

Producers The two producers maximize the following utility function (the focus of the model is on the optimal markup, such that no labor market is modelled to keep the model as simple as possible)

$$U_t^j = Y_t^j P_t^j - \frac{N_t^{j1+\rho}}{1+\rho} P_t^{jC},$$

where N_t^j is the labor used to produce output Y_t^j (labor enters as a pure utility cost), P_t^{jC} is the price with which real costs are converted into nominal costs and ρ measures the disutility of working. In order to stay close to the standard models and avoid distortions due to the CIA constraint, this will be set to the current price index $\overline{P}_t$[4]

$$\overline{P}_t = \left[P_t^{a1-\gamma} + P_t^{b1-\gamma} \right]^{\frac{1}{1-\gamma}}. \tag{4}$$

Technology is linear in labor with a coefficient of technology A_t common to both firms

$$Y_t^j = A_t N_t^j.$$

2.2 First-Order Conditions

Optimizing the above stated problems leads to a set of first-order conditions for households and firms.

Households Division of demand between the two consumption goods is governed, as usual, by the elasticity of substitution γ and the relative price. Since there are only two firms in the economy, the demand function is written in a slightly non-standard form. I use the explicit demand schedule facing a producer who takes her influence on the price level into account, because both producers play a Cournot game. The different schedule comes about from the fact that purchases of one good reduce consumers' remaining resources. This income effect vanishes in a model with an continuum of firms, because consumers spend zero income on each variety. Demand for good a by consumer i is given

$$C_{i,t}^a = \left[\left(\frac{P_t^a}{P_t^b} \right)^\gamma + \frac{P_t^a}{P_t^b} \right]^{-1} \frac{M_{i,t}}{P_t^b}, \tag{5}$$

[4] Alternative specifications do not change the qualitative results, but do complicate the analysis. Forward looking costs would distort the pricing decision in the presence of expected inflation and would therefore give rise to real effects. While this could be undone by introducing a subsidy, this would make it difficult to see where the differences between this and the next section arise.

and vice versa for good b.[5] The intertemporal consumption path is characterized by a normal Euler equation relating consumption today with consumption tomorrow

$$E_t \left(\frac{\overline{C}_{i,t+1}}{\overline{C}_{i,t}} \right)^{\sigma} = \beta(1+i_t) E_t \frac{\overline{P}_t}{\overline{P}_{t+1}}. \tag{6}$$

The budget constraint (2), the transversality condition (3) and the CIA constraint (1) complete the description of household's optimal behavior.

Producers Resulting from equation (5), producer a faces the following aggregate demand schedule

$$C_1^a + C_2^a = \left[\left(\frac{P_t^a}{P_t^b} \right)^{\gamma} + \frac{P_t^a}{P_t^b} \right]^{-1} \frac{M_{1,t} + M_{2,t}}{P_t^b}, \tag{7}$$

which is valid symmetrically for producer b. The optimal price setting for firm a and the real marginal costs MC_t^a are given by the formulae

$$P_t^a = \left[\frac{P_t^a(\gamma - 1)}{MC_t^a \overline{P}_t} - \gamma \right]^{\frac{1}{1-\gamma}} P_t^b, \qquad MC_t^a = \frac{Y_t^{a\rho}}{A_t^{1+\rho}}. \tag{8}$$

2.3 Equilibrium

Equilibrium requires market clearing, i.e. the complete amount of both varieties produced is sold to both agents

$$Y_t^a = C_1^a + C_2^a \qquad Y_t^b = C_1^b + C_2^b.$$

Because in this section households are identical, including the amount of transfers received from the government, there will be no borrowing nor lending between them in equilibrium. Combined with the fact that in this closed economy without investment net savings, i.e. bonds, have to be zero, private bond holding of each agent are zero, too. Hence, equation (6) merely determines the interest rate.

[5] One can rewrite this condition also in the standard form using the price index (4) as

$$C_{i,t}^j = \frac{P_t^j}{\overline{P}_t}^{-\gamma} \frac{M_{i,t}}{\overline{P}_t}.$$

In this case one has to make sure to account for the effect of P_t^j on $\overline{P}_t$ in firms' optimization processes.

Furthermore, both households consume the same amount of resources from each firm

$$C_1^a = C_2^a, \qquad\qquad C_1^b = C_2^b.$$

The resulting price is[6]

$$P_t^j = \frac{\gamma + 1}{\gamma - 1} MC_t^j \overline{P}_t,$$

which by the symmetry assumption of $\overline{P}_t = P_t^j$ leads to the standard result of

$$\frac{1}{MU} = MC_t \tag{9}$$

with MU being the constant markup $\frac{\gamma+1}{\gamma-1}$.

Results Equation (8) leads under symmetry assumptions to the following result:

$$Y_t = 2A_t^{\frac{1+\rho}{\sigma+\rho}} \left(\frac{\gamma - 1}{\gamma + 1}\right)^{\frac{1}{\rho+\sigma}}.$$

From this equation it is obvious that output depends only on technology and deep parameters. Combined with the Euler equation (6), it becomes clear that the real interest rate stays constant after a change in the nominal interest rate, i.e. $\beta(1 + i_t) = \frac{P_{t+1}}{P_t}$. Inflation jumps once, and is zero thereafter. Thus, the model exhibits neutrality and superneutrality with respect to monetary policy. As was already visible from equation (9), optimal (and realized) markups stay constant, a feature that has been empirically rejected.[7]

2.4 Heterogeneity

One can argue that monetary policy is unlikely to affect all people in the economy in the same way and at the same time. For example, businesses taking credits for large investment projects are benefiting more from a decrease in interest rates than people who are taking small credits or are even savers. Hence, it would be interesting to explore the effects of policy actions on different groups of agents, or as Williamson (2005) writes: 'Indeed, these distributional effects may be very important for how

[6]Note that if the firms were not to take their influence on the general price level into account, the optimal markup would be $\frac{\gamma}{\gamma-1}$, which would not change the conclusions drawn in this section.

[7]Again, see Rotemberg and Woodford (1999) for a longer discussion of this point.

monetary policy works, if not *the* reason we should care about monetary policy.' The model in this section is not suitable for analyzing the implications of assumptions of this type. The easiest way to model heterogeneity would be to consider 'partial helicopter drops', i.e. only one agent is receiving additional money from the central bank.[8] In order to let both agents fulfill the Euler equation (6), one has to allow for inter-household borrowing and lending. However, a monetary gift to only one person introduces a wealth effect that, as usual in this kind of heterogenous-agents (or countries) model, would let individual bond holdings explode, even though they are in zero net supply. Since explosive behavior should be ruled out, this model is not able to show the implications of heterogeneity in the out-of-steady-state income distribution or money holdings. This and other questions can be addressed with the model of the following section.

3 A Sequential-Purchases Two-Agent Model

In the above section, it is assumed that all actions are done simultaneously, although in discrete-time models one period is assumed to be of considerable length. Agents aggregate the consumption goods they buy during one period and choose their consumption paths by relating these bundles across periods. It is crucial that it takes virtually no time to acquire these goods. This implies on the one hand that while deciding on the money spent on each good, all prices of the goods in the consumption bundle are known. On the other hand, all households receive their income and identical transfers at the same moment in time, implying that they can calculate equilibrium good prices while buying bonds. As a result of all this, any changes in income (including monetary shocks) lead to an instantaneous adjustment of prices, and therefore no real effects can be observed.

Only with the assumption of instantaneous purchases it can be justified that periods for all agents start and end at the same time. If one is to assume that it takes some time to acquire a consumption bundle, it is unlikely that consumers start and stop their shopping sequences at the same dates. To account for this point, I am dividing one period into two and introduce overlapping shopping sequences. Instead of visiting both shops simultaneously, the two consumers now buy at one shop in the first period and at the other in the second period. Both goods still enter the same consumption bundle. Each shop is serving two customers, who are in different stages of their shopping sequence. Seen from the

[8] Varying the interest rate could only have an effect if both agents face different interest rates. I chose to discuss the more intuitive alternative, which leads to the same conclusions.

firm's perspective, this is equivalent to an economy with a representative consumer, but uncertainty about the current stage of the shopping sequence of this consumer. Consumers start their shopping sequences after having received their income and having adjusted their financial positions. Hence, I assume that one consumer receives income from business activity, bonds and transfers in, say, even periods, while the other receives income in odd periods. This aspect of the model is close to Grossman and Weiss (1983) and Rotemberg (1984). The assumption that consumers do not adjust their assets instantaneously is empirically supported by Christiano et al. (1996). Graphic 1 shall help explain the difference. In the upper half, people buy at both shops during one period, i.e. simultaneously. It does not matter in which order they shop or who starts first. One could imagine that the shops open one after the other, but since all actions in the period are compressed to one single point in time, this does not play any role. In the lower half, shopping sequences overlap. Now it matters when which shop opens and who is buying there. Therefore, the length of the periods is cut in half with only one shop being open in each 'new' period. The arrows show the dates when the agents visit the bank and therefore between which points in time they can save by buying bonds. The limitation to a finite number of shops introduces strategic complementarity between the prices of the firms. Empirical evidence for this complementarity was found by Chevalier and Scharfstein (1996) in the sector of supermarkets.

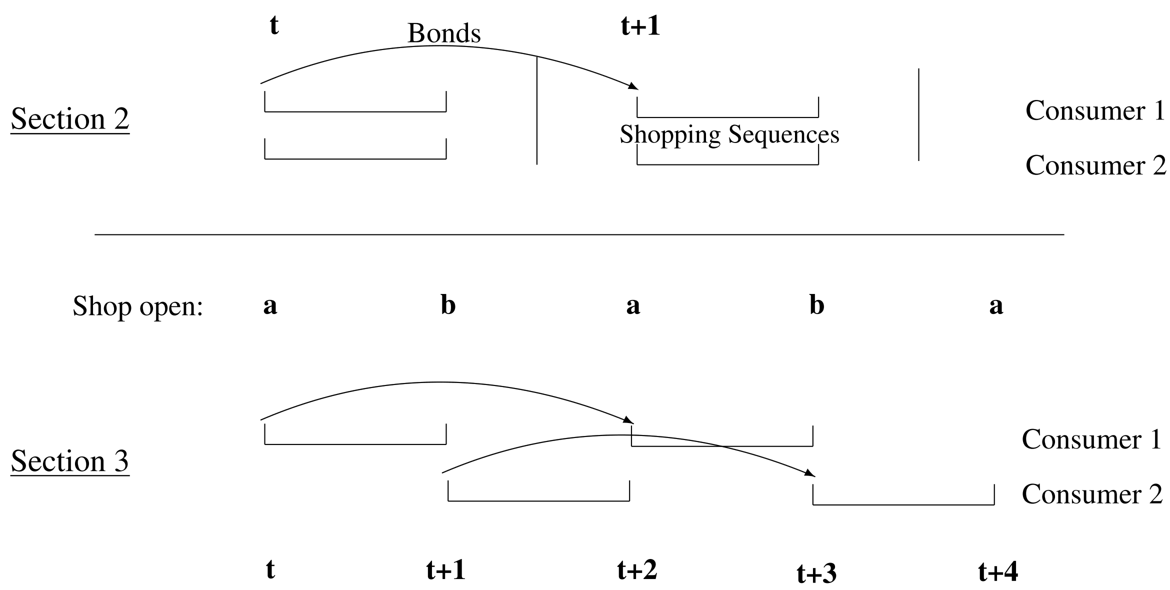


Figure 1: Difference in timing

3.1 Notation

Since now there is only one shop open each period, the time index is enough to distinguish between the two firms. The firm which is open in t hands its profits over to its owner, the consumer who starts shopping in time $t+1$. Furthermore, I will use the subscripts $i = 1, 2$ to indicate consumers in the following way: 1 denotes the consumer that is at the beginning of its shopping sequence, while consumer 2 is at the end of this sequence. Hence, a person with the subscript 1 in t will have the subscript 2 in $t+1$ and again 1 in $t+2$. Decisions on bonds and the division of money spent on both goods are made at the beginning of the sequence, hence by the person with the index 1.

3.2 Setup

Households Most of the setup is similar to the one described in section 2. A CES-utility function is to be maximized

$$E_t(\overline{U}_{1,t}) = E_t\left(\sum_{l=0}^{\infty} \beta^l \frac{\overline{C}_{1,t+2l}^{1-\sigma}}{1-\sigma}\right),$$

where the consumption bundle $\overline{C}_1$ is acquired over the course of two periods, such that the counter increases in steps of two. This consumption aggregate $\overline{C}_1$ is defined as in section 2, but since the consumer buys sequentially, she does not know the price of the good to be purchased in the next period when starting her shopping sequence in this period. Hence, she maximizes the expected value of the bundle

$$E_t(\overline{C}_{1,t}) = E_t\left[\left(C_{1,t}^{\frac{\gamma-1}{\gamma}} + C_{2,t+1}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}\right]. \qquad \gamma > 1 \tag{10}$$

The Cash-in-Advance constraint in the new notation is

$$M_{1,t} \geq P_t C_{1,t} + P_{t+1} C_{2,t+1} \equiv \overline{C}_t \overline{P}_t, \tag{11}$$

while the budget constraint also remains mostly unchanged

$$M_{1,t} + B_t = (1 + i_{t-2}) B_{t-2} + \Pi_{t-1} + S_t \tag{12}$$

where Π_{t-1} is the revenue of the firm, whose stocks are owned by the household with index 1 in t. Since the asset market is visited only every second period, interest is earned on bonds bought two periods ago. The transversality condition is still valid.

$$\lim_{T\to\infty} \Pi_{t=0}^{T} \beta^t \overline{C}_t^{-\sigma} B_{t-1} \geq 0. \tag{13}$$

Producers At the producer side, nothing changed. The two producers maximize the same utility function

$$U_t = Y_t P_t - \frac{N_t^{1+\rho}}{1+\rho} P_t^C.$$

P_t^C is again the price that converts real to nominal costs, since labor enters only as utility costs. This will later be set to P_t, such that comparisons can easily be made with section 2.[9] Technology is also the same as in section 2: $Y_t = A_t N_t$.

3.3 First-Order Conditions

Due to the different timing assumptions, some differences in the first order conditions arise. Notably, since shopping is now sequential, households plan their purchases at each store based on *expected* future relative prices of competitors. Due to this different consumption behavior, price setting of firms is also affected.

Households At time t, the household which is at the beginning of its shopping sequence has to decide how much cash to hold for this sequence, and how much to put into her account, resulting in an Euler equation. This equation is quite standard, except that it is more convenient that the household directly maximizes over cash holdings, a certain variable, in contrast to consumption that is uncertain because the price of the second good purchased in $t+1$ is unknown as of time t.

$$M_{1,t}^{-\sigma} = \frac{E_t \left[M_{1,t+2}^{-\sigma} E_{t+2} \left(\overline{P}_{t+2}^{\sigma-1} \right) \right]}{E_t \left(\overline{P}_t^{\sigma-1} \right)} \beta(1+i_t), \tag{14}$$

where i_t is the going interest rate, which is earned on bonds between t and $t+2$. In deriving this equation, I already made use of the Cash-in-Advance constraint that can be expressed as

$$\overline{C}_{1,t} \leq \frac{M_{1,t}}{\overline{P}_t}. \tag{15}$$

[9] The model focuses mainly on the optimal markup. If nominal marginal costs were to be described in a more detailed way via a labor market, this assumption would stem from the labor market frictions that Christiano et al. (1997) found necessary to limit the initial impact of monetary shocks on nominal marginal costs.

This equation holds with equality if no cash is carried over from period $t+1$ to $t+2$, i.e. resources are transferred to period $t+2$ only via the interest bearing bonds. In the appendix, this is proven to be an optimal behavior for increases in the money supply that increase prices.[10] However, it turns out that this equality does not have to be the case for price decreases, which gives rise to an interesting possible extension of the model. The model as described from here on is mainly valid for increases in the money supply, whereas decreases would trigger an asymmetric, different response. This asymmetry is in line with conventional wisdom about different dynamics during times of inflations and deflations. After unexpected price decreases, consumers are delaying purchases, i.e. they carry cash over between periods. Extending the model also for this option could prove a fruitful exercise that highlights the asymmetric effects of monetary policy.

Turning back to the setup of the model, in order for the definition $P_t C_{1,t} + P_{t+1} C_{2,t+1} \equiv \overline{C}_t \overline{P}_t$ in equation (11) to hold, the price index $\overline{P}_t$ has to be defined in a more complicated manner. This is the case because it involves P_{t+1}, which is unknown at the time of the decision on how much to consume in period t. Hence, to make the price index consistent with the optimal consumption chosen in t, based on $E_t\{P_{t+1}\}$, and the realized consumption in $t+1$, it has to involve this expectation and the realized P_{t+1}[11]

$$\overline{P}_t = \left[1 + \left(E_t\left\{P_{t+1}^{\frac{1-\gamma}{\gamma}}\right\} P_{t+1}^{\frac{\gamma-1}{\gamma}}\right)^{\gamma-1}\right]^{\frac{\gamma}{1-\gamma}} \left[1 + \left(E_t\left\{P_{t+1}^{\frac{1-\gamma}{\gamma}}\right\}\right)^{\gamma} P_t^{\gamma-1}\right] P_t. \tag{16}$$

Having decided how much cash $M_{1,t}$ to hold at the beginning of period t for shopping in t and $t+1$, the consumer now has to decide how to divide the cash between the two goods. As argued above, the CIA constraint will be binding in the analyzed cases. Hence, the household will spend all remaining cash in $t+1$ on the good purchased in that period:

$$C_{2,t+1} = \frac{M_{2,t+1}}{P_{t+1}}. \tag{17}$$

Knowing this, one can also solve for the consumption of the good in period t by maximizing the expected value of the consumption bundle (10), resulting in a kind of second Euler equation

$$C_{1,t} = (R_t + 1)^{-1} \frac{M_{1,t}}{P_t}, \tag{18}$$

[10] Also Rotemberg (1984) argues that acting in such a way is optimal as long as a positive interest rate prevails, which is also the case here.

[11] See Green (1964), chapter 3, on a more general discussion on the requirements of a price index.

with the expected transformed price ratio R_t equal to

$$R_t = P_t^{\gamma-1}\left[E_t\left(P_{t+1}^{\frac{1-\gamma}{\gamma}}\right)\right]^{\gamma}.$$

Again, the budget constraint (12) and the transversality condition (13) complete the description of households' optimal behavior.

Producers Since shopping periods overlap, i.e. at each point of time one consumer is at the beginning of her sequence (equation 18), and another one at the end (equation 17), total demand facing a producer at time t is

$$C_t = (R_t+1)^{-1}\frac{M_{1,t}}{P_t} + \frac{M_{2,t}}{P_t}. \tag{19}$$

Note, that people at the beginning of their shopping sequence have a higher elasticity of substitution than the people further down the sequence (the consumer in the last period of the sequence just spends all her money). Hence, when setting its price, the firm faces a trade off between extracting more profits from the costumers with a low elasticity, and loosing profits from the customers at the beginning of the sequence, who might substitute to firms that come later in the row. The optimal price is implicitly given by

$$\frac{\frac{M_{2,t}}{M_{1,t}}(R_t+1)^2+\gamma R_t+1}{(\gamma-1)R_t} = \frac{A_t^{1+\rho}}{Y_t^{\rho}}, \tag{20}$$

which corresponds the standard formula Markup=1/Marginal Costs. Steady-state markup is $MU = (\gamma+3)/(\gamma-1)$. Note that the firm is taking household expectations R_t as given, such that the game played between the producers does not change compared to section 2. Instead of turning into a Stackelberg leader, the individual firm does not assume that its price setting affects people's expectations of future prices, hence the shops still play a Cournot game.

3.4 Equilibrium

Equilibrium requires market clearing in each period

$$Y_t = C_{1,t} + C_{2,t}. \tag{21}$$

In contrast to section 2, money holdings across agents are not identical anymore, which is crucial for the results. Like Grossman and Weiss (1983) and Rotemberg (1984), I make the simplifying assumption that inter-household borrowing and lending is not possible. Although this would influence money holdings and therefore probably also the results, it would contradict the structure of the model, in which consumers are not going to the bank during their shopping sequence. Hence, consumers currently at the bank do not engage in borrowing and lending with the consumers not at the bank. Together with the fact that aggregate savings in a closed economy without investment have to be zero, this leads to the same result as in section 2, namely that the interest rate adjusts in such a way that buying bonds is not optimal for households. In this respect, the Euler equation (14) again only determines the interest rate. Due to these restrictions, it is possible to find a reduced form representation of the linearized system in the next section.

4 Linearized System

An advantage of this model is the possibility of finding an analytical solution. In a first step, money holdings are substituted out to find two equations summarizing the dynamics, plus one to determine the equilibrium interest rate. A monetary policy rule and a process for technology complete the description. Below, these equations will be derived for the model of two (kinds of) agents, as presented in the preceding section. An analytical solution, i.e. expressions for the variables as functions of the states, the exogenous variables and the shocks, is derived in section 4.1 and simulated in section 5. The formulae for any given number of agents is then given in section 6.

Equation 1: Effects of Monetary Policy An equation for the effects of monetary policy can be found by combining the log-linearized versions of equations (12), (15), (17), (18), (20), and (21), substituting out consumption and money holdings. The following equation emerges[12]

$$\pi_t + \frac{\gamma+3}{4}\rho\hat{y}_t + \Delta y_t = s_t, \tag{22}$$

where y_t and $\hat{y}_t$ are expressed in percentage deviations from their steady-state values, π_t is current inflation and s_t is $\frac{S_t}{M_1^{St.St.}}$.[13] $\hat{y}_t$ is the output gap, defined here as actual output minus the level of output that would prevail in an economy with equal money holdings, given by $\frac{\rho+1}{\rho}a_t$. Since firms can adjust prices at every period, the flex-price equilibrium is always reached. This implies, contrary to

[12] Alternatively, one could also develop a formula describing the process of updating expectations: $E_t(\pi_{t+1}) = E_{t-1}(\pi_t) + \frac{\gamma+3}{\gamma-1}\rho\hat{y}_t + \frac{2}{\gamma-1}s_t$.

[13] The steady-state employed here is a stationary economy with no trending variables.

the standard Calvo-model, that the markup mu_t is the inverse of marginal costs, i.e. the negative of marginal costs in the linearized form, in each period. Due to this fact marginal costs are

$$mc_t = \rho y_t - (\rho + 1)a_t = \rho \hat{y}_t = -mu_t. \tag{23}$$

Note that marginal costs here are costs for labor only, since capital is missing. Furthermore, profits are $Y_t MU_t$, linearized $y_t + mu_t$. It is visible from equation (22) that monetary injections are likely to raise nominal GDP, where the size of the effect depends on y_{t-1}, and the division between inflation and real output depends on current and past expectations, see next equation.

Equation 2: Phillips Curve In order to derive the Phillips Curve, the same mentioned equations can be used, resulting in

$$\pi_t = \frac{\gamma + 3}{4} \rho \hat{y}_t - \Delta y_t + \frac{\gamma - 1}{2} \left[E_t(\pi_{t+1}) - E_{t-1}(\pi_t) \right]. \tag{24}$$

While the output gap is standard in Phillips-Curves of different models, the negative impact of GDP-growth on inflation is not. For a discussion of the Phillips Curve, see section 4.1.

Equation 3: Interest Rate In order to determine the interest rate one can log-linearize equation (14), using (12) and (16) to substitute out the money holdings to get:

$$r_t = \frac{1 - \sigma}{2} E_t(\pi_{t+3}) + (1 - \sigma) E_t(\pi_{t+2}) + \frac{1 + \sigma}{2} E_t(\pi_{t+1}) +$$

$$+ \sigma \pi_t + \sigma [E_t(\Delta y_{t+1}) + \Delta y_t] + \sigma [E_t(\Delta s_{t+2} + \Delta s_{t+1})],$$

with r_t being the percentage deviation of the interest rate from its steady-state $\bar{i} = 1/\beta - 1$.[14]

Equation 4: Exogenous Variables To close the model, a monetary policy rule and a process for technology have to be assumed. Since I mainly want to present the internal propagation mechanism, I keep these equations as simple as possible; namely AR(1) processes with coefficients $0 < \eta_s, \eta_a < 1$, the monetary policy shock ε and the shock to technology υ:

[14] Although the percentage deviation of a variable measured in percentage points is a bit awkward, by this we do not have to choose a value for β.

$$
\begin{aligned}
s_t &= \eta_s s_{t-1} + \varepsilon_t \\
a_t &= \eta_a a_{t-1} + \upsilon_t. \qquad (25)
\end{aligned}
$$

4.1 Solution

Once reduced to the two equations describing the dynamics of the model, (22) and (24), the endogenous variables π_t and y_t can be expressed as functions of the states and shocks only. However, since the intrinsic state variables M_1 and M_2 were substituted out because they are not directly observable, other variables have to fulfill the role of state variables. It turns out that yesterdays values of output y_{t-1} and the exogenous variables s_{t-1}, a_{t-1} summarize all necessary information. The former is needed because M_1 depends on last periods revenue, while the latter variables are important for building expectations, required to determine M_2. Hence, the following guess can be postulated:

$$
\begin{aligned}
y_t = &\quad \lambda_{yy} y_{t-1} + \lambda_{ys} s_{t-1} + \lambda_{ya} a_{t-1} + \lambda_{y\varepsilon} \varepsilon_t + \lambda_{y\upsilon} \upsilon_t \\
\pi_t = &\quad \lambda_{\pi y} y_{t-1} + \lambda_{\pi s} s_{t-1} + \lambda_{\pi a} a_{t-1} + \lambda_{\pi\varepsilon} \varepsilon_t + \lambda_{\pi\upsilon} \upsilon_t, \qquad (26)
\end{aligned}
$$

where expectations of future shocks are set to zero, according to the assumption of AR(1) processes for the exogenous variables. This dynamic system is verified and solved by using the method of undetermined coefficients. The stability of the system is governed by the value for λ_{yy}, for which a quadratic equation is obtained. For $\gamma > 1$ and $\rho > 0$ - as assumed in section 3 - both solutions to this equation are positive, with one solution being greater and the other lesser than one. The first root is therefore discarded, and the solution to the equation using the second root is given below together with the coefficients for the effects of past output and monetary policy.

$$
\begin{aligned}
\lambda_{yy} &= \frac{(\gamma+3)^2\rho + 8(\gamma-1) - \sqrt{(\gamma+3)\rho[(\gamma+3)^3\rho - 64(\gamma-1)]}}{2[(\gamma+3)\rho+4](\gamma-1)} && > 0; < 1 \\
\lambda_{\pi y} &= \frac{4 + \lambda_{yy}[(\gamma+3)\rho - 4]}{4 + 2(\gamma-1)(1-\lambda_{yy})} && > 0 \\
\lambda_{\pi s} &= 2\eta_s \frac{2(\gamma-1)\lambda_{\pi y} + (\gamma+3)\rho - 4}{4(\gamma-1)\lambda_{\pi y} + 4(\gamma+3)\rho + (\gamma-1)(1-\eta_s)[(\gamma+3)\rho+4]} && < \eta_s
\end{aligned}
$$

$$\lambda_{y\varepsilon} = \frac{4(1-\lambda_{\pi s})}{(\gamma+3)\rho+4} \qquad > 0$$
$$\lambda_{ys} = \frac{4(\eta_s-\lambda_{\pi s})}{(\gamma+3)\rho+4} \qquad > 0$$
$$\lambda_{\pi\varepsilon} = \frac{4(\gamma-1)\lambda_{\pi y}+(\gamma-1)\lambda_{\pi s}[(\gamma+3)\rho+4]+2[(\gamma+3)\rho-4]}{4(\gamma-1)\lambda_{\pi y}+4(\gamma+3)\rho}.$$

The effects of technology can be seen from the following coefficients

$$\lambda_{\pi a} = \eta_a \frac{(\gamma+3)(\rho+1)[(\gamma-1)\lambda_{\pi y}-4]}{(\gamma-1)\{[(\gamma+3)\rho+4](1-\eta_a)+4\lambda_{\pi y}\}+4(\gamma+3)\rho}$$
$$\lambda_{\pi\upsilon} = \frac{(\gamma-1)[(\gamma+3)\rho(\lambda_{\pi a}-\lambda_{\pi y})+4\lambda_{\pi a}-(\gamma+3)\lambda_{\pi y}]-(\gamma+3)(1+\rho)[(\gamma+3)\rho-4]}{(\gamma^2-9)\rho+4(\gamma+1)-2(\gamma-1)^2\lambda_{\pi y}}$$
$$\lambda_{ya} = \frac{(\gamma+3)(\rho+1)\eta_a-4\lambda_{\pi a}}{(\gamma+3)\rho+4}$$
$$\lambda_{y\upsilon} = \frac{(\gamma+3)(\rho+1)-4\lambda_{\pi\upsilon}}{(\gamma+3)\rho+4}.$$

Note that the effects $\lambda_{ys}, \lambda_{ya}, \lambda_{\pi s}, \lambda_{\pi a}$ of yesterdays exogenous variables a_{t-1} and s_{t-1} disappear if their AR(1) processes are reduced to white noise, i.e. $\eta_a = \eta_s = 0$. It can also be seen from $\lambda_{y\varepsilon} > 0$ that expansionary monetary shocks unambiguously have a positive effect on current output, while $\lambda_{\pi y} > 0$ shows that past economic activity has a positive effect on inflation. Furthermore, positive monetary shocks have a stimulating effect on output also in the following period, see $\lambda_{ys} > 0$. The sign of the effect of monetary shocks on current inflation, $\lambda_{\pi\varepsilon}$, depends on ρ and γ. To evaluate the dynamics of the Phillips Curve (24) of this model, I compare the implications of the curve and the solution of the model in this section with the critique of the New Keynesian Phillips Curve in Mankiw (2001). He names three 'failures of the New Keynesian Phillips Curve': the inability to generate inflation persistence, the possibility to generate disinflationary booms and the impulse-response functions after monetary shocks.

The New Keynesian Phillips Curve has problems in generating the empirically observed inflation persistence. In the present model, the state variable y_{t-1} can be replaced by π_{t-1} by reformulating the dynamic system (26). Having done so, it is evident that the effect of past inflation on current inflation is equal to λ_{yy}. This coefficient is smaller than one but larger than zero, such that inflation persistence also exists with the internal propagation mechanism only, no autocorrelated shocks or similar dynamics have to be assumed. One should keep in mind that this is a stylized model, which lacks any other mechanisms often employed for slow adjustment of real and nominal variables. However, the model suffers from another criticism of the New Keynesian Phillips Curve in Mankiw (2001), namely inflationary recessions.[15] If the central bank announces inflationary measures, today's output falls for common values of ρ and γ.[16] Higher values of ρ lower the effects of inflation on output, while a low γ decreases the effects of expected inflation on current inflation. Both effects minimize inflationary recessions after announcements of inflationary measures. However, one should consider that the effects of announced monetary policy in this model stems from a change in behavior of the households due to the announcement, namely a different decision on cash balances. This change in behavior is likely to be small in low-inflation environments, where most consumers do not actively follow the announcements of the central bank. Furthermore, for large announcements, it is probable that households increase the frequency of visits to the asset market, such that announced measures could end up not having any real effect.

Finally, concerning the response of output, inflation, marginal costs, markup and the interest rate to shocks, I refer to section 5, where impulse response functions for the model in this paper are plotted. While the responses of the variables are generally consistent with the empirical findings, the hardest point to fulfill is the slow and gradual effects of monetary shocks on inflation and output. Autocorrelated monetary shocks, as often used in the literature, low values for γ or increasing the number of agents generate hump-shaped impulse response functions.

[15] Since the model in this paper deals mainly with inflationary episodes, I focus on the equivalence of disinflationary booms for announced inflations.

[16] Note that the expectations of future shocks in (26) were set to zero. Hence, they do not appear there, but in (24).

5 Simulation

In this section I calculate the impulse-response functions to further check the dynamic implications of the model. In section 5.1 I will simulate the model for $n = 2$ agents, whereas the model with three agents in section 5.2 allows for richer dynamics, i.e. agents react to shocks by adjusting their expectations and re-optimizing their plans in the middle of their shopping sequence. The general formulae for any given n are given in section 6.

5.1 Simulation for n=2

I use equations (22) to (25) to simulate the model with the parameter values $\gamma = 16$ (which implies a steady-state markup of 26%)[17], $\rho = 0.1$ (close to constant marginal costs) and $\sigma = 3$ (which merely determines the size of the interest rate reaction).[18] The steady-state markup and the value for σ are not as controversial as the value for ρ, for which values between zero and infinity are used in the literature. The value used here is rather low and implicitly implies a labor supply elasticity of 10%. Raising this value shortens the reactions by lowering the response of output while increasing the effect on inflation. Whereas otherwise the responses do not change qualitatively, this result seems to strengthen the point of Christiano et al. (1997) that labor market frictions (like predetermined wages that would dampen the initial impact on inflation) are probably needed if the model is to be completed with a labor market.[19]

In the left panel of figure 2, a one-time monetary injection of 1% to total money supply takes place (which corresponds to $s_1 = 3/2$, zero otherwise).[20] The horizontal axis shows periods, whilst the vertical axis depicts percentage deviations from the steady-state. Note that the interest rate got scaled down by the factor 10 to fit in the same picture; a 0.1 percent shift in the graph corresponds to a movement of one percent. In the right panel an autocorrelation of $\eta_s = 0.3$ in the monetary policy rule (25) with $\varepsilon_1 = 3/2$ is assumed. In the left panel of figure 3, the one-percent shock takes place twice, i.e. both agents receive the same positive monetary injection ($s_1 = s_2 = 3/4$, zero otherwise).[21] It is noteworthy, that even for one-time monetary injections, the internal propagation

[17]Steady-state markup is $\frac{\gamma+3}{\gamma-1}$. Because of the oligopoly, the markup is generally relatively high for given values of γ.

[18]The function was calculated using the Matlab codes of Uhlig (1997).

[19]With a labor elasticity of 1 ($\rho = 1$), an initial effect on output around 0.18% is reached after the same shock. Lower values of γ increase this value.

[20]s is the percentage shock to MR_1. For a 1% shock to total money supply $MR_1 + MR_2$, s has to be adjusted accordingly.

[21]To stay with the picture of two agents with alternating trips to the bank, this would imply that both agents receive the same amount of money at the same time, but one agent 'picks up' the money one period later. The second shock is therefore anticipated one period ahead, which indirectly leads to the initial rise in the interest rate.

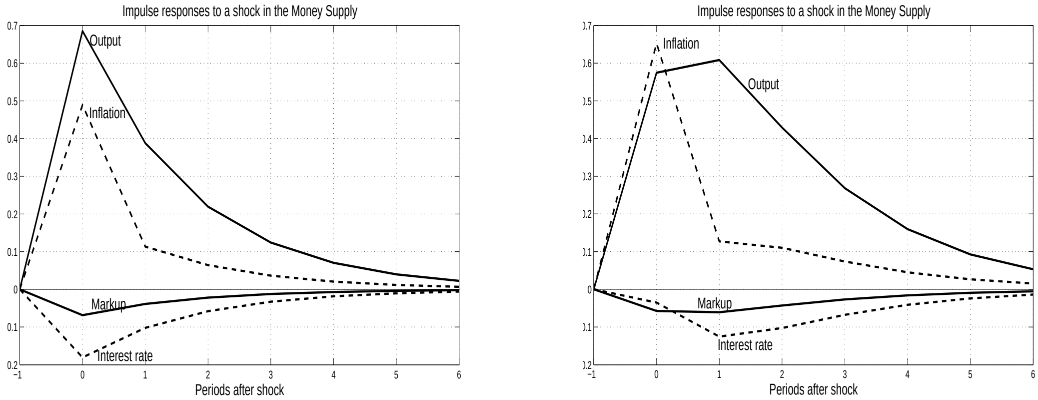


Figure 2: n=2. On the left no autocorrelated monetary policy rule, on the right $\eta_s = 0.3$

mechanism already prolongs the responses. The model displays a liquidity effect, except on impact for the two subsequent shocks and very high autocorrelations of the monetary injections. Hump-shaped responses are obtained for autocorrelated monetary shocks and $n = 3$. Low values for γ lower the initial response of inflation, such that no initial reaction with a subsequent hump-shaped response can be reached for $\gamma = 7$, see the right panel of figure 3. Output would in this case rise initially to 1.2% because of strong substitution effects. However, in this case the steady-state markup is unrealistically high at 66%. Generally, the impulse-response functions are in line with qualitative empirical facts. The markup falls and moves countercyclical, both features are empirical observations which are difficult to reproduce in the standard new Keynesian framework. Real marginal costs in the linearized form are given by $mc_t = -mu_t$ (mu being the markup). Hence, they increase mildly after a positive monetary shock. Because of the lack of capital, procyclical marginal costs would equal wages after the introduction of a labor market. Linearized profits are $y_t + mu_t$ and procyclical as well, since output rises more than the markup falls. Thus, the model coincides with the observations of Christiano et al. (1997), who find that after a positive monetary shock output and profits rise, wages increase mildly and the interest rate falls.

5.2 Simulation for n=3

The linearized first-order equations for the case of $n = 3$ agents are developed from the general formulae from section 6, with the optimal consumption for each agent already inserted. They are given as

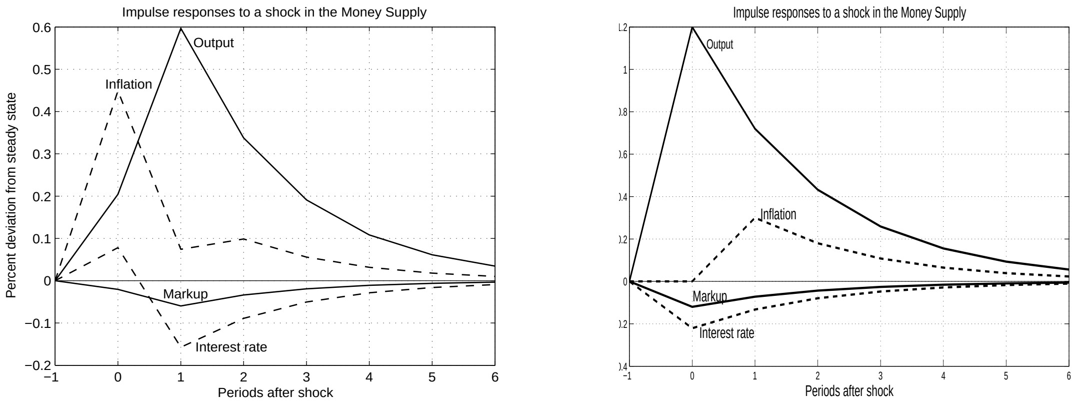


Figure 3: n=2. On the left two subsequent shocks; on the right $\gamma = 7$, one shock

Households

Euler Equation $\quad (\sigma - 1)E_t\{\overline{p}_{t+3} - \overline{p}_t\} = \sigma(E_t\{mr_{1,t+3}\} - mr_{1,t}) - r_t$

with Price Index $\quad \overline{p}_t = p_t + p_{t+1} + p_{t+2}$

Money Holdings $\quad mr_{1,t} = p_{t-1} + y_{t-1} + s_t$

$$mr_{2,t} = mr_{1,t-1} - \tfrac{\gamma-1}{6}E_{t-1}\{\pi_{t+1} + 2\pi_t\}$$

$$mr_{3,t} = mr_{2,t-1} - \tfrac{\gamma-1}{2}E_{t-1}\{\pi_t\}$$

Pricing

$$30mr_{1,t} + 12mr_{2,t} - 42mr_{3,t} - 7(7\gamma + 11)mc_t = (\gamma - 1)E_t\{\pi_{t+2} + 25\pi_{t+1}\}$$

Market Clearing

$$3y_t = mr_{1,t} + mr_{2,t} + mr_{3,t} + \frac{\gamma - 1}{3}E_t\{\pi_{t+2}\} + \frac{\gamma - 1}{2}E_t\{\pi_{t+1}\} - 3p_t$$

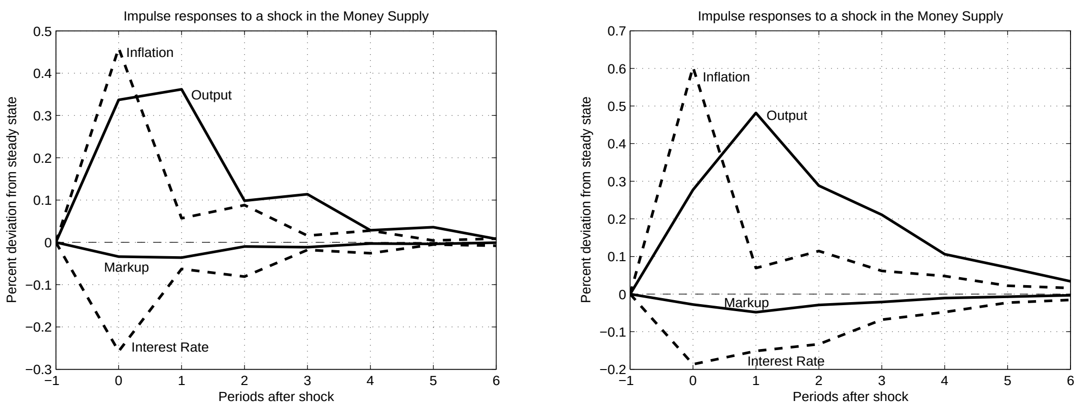


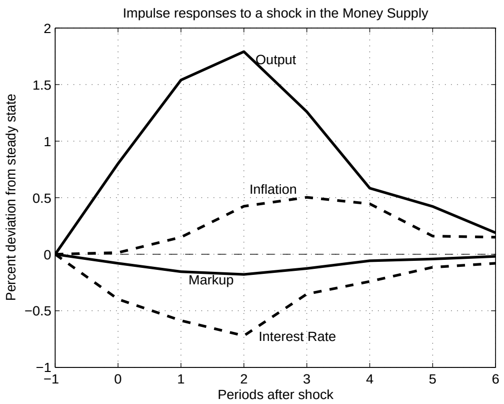
Figure 4: n=3. On the left no autocorrelated monetary policy rule, on the right $\eta_s = 0.3$

with marginal costs mc defined as in (23). The transversality condition also has to be obeyed. Steady-state markup is given by $\frac{\gamma+11/7}{\gamma-1}$. The equations for the money holdings can be used to substitute out these variables in the pricing and the market clearing equations, such that we get a system of two equations

$$(\gamma-3)\Big[E_t\{25\pi_{t+1}+2\pi_{t+2}\}+E_{t-1}\{2\pi_{t+1}-17\pi_t\}-7E_{t-2}\{\pi_t+2\pi_{t-1}\}\Big]$$
$$=30(\pi_{t-1}+\Delta y_{t-1}+\Delta s_t)+42(\pi_{t-2}+\Delta y_{t-2}+\Delta s_{t-1})+7mc_t \tag{27}$$

$$3(\pi_t+\Delta y_t)+2(\pi_{t-1}+\Delta y_{t-1})+\pi_{t-2}+\Delta y_{t-2}$$
$$=\frac{\gamma-1}{6}\Big[E_t\{2\pi_{t+2}+3\pi_{t+1}\}-E_{t-1}\{\pi_{t+1}+5\pi_t\}-E_{t-2}\{\pi_t+2\pi_{t-1}\}\Big] \tag{28}$$

As before, one could label the first one 'Effects of monetary policy' and the second 'Phillips Curve', but because of their complicated structure this is less obvious. For the simulation, I use the following parameter values: Coefficient of relative risk aversion as before $\sigma = 3$, elasticity of substitution $\gamma = 12$, coefficient of marginal disutility of labor again $\rho = 0.1$. As before, these values have been chosen to match standard values (σ) or to be close to standard values for resulting variables (the chosen γ implies a steady-state markup of 23%). The impulse response function for a shock of 1% the total money supply (i.e. $s_1 = 2$, zero otherwise) without any autocorrelated shock is depicted on the left panel of figure 4, while in the right panel $\eta_s = 0.3$ and $\varepsilon_1 = 2$. As can be seen, increasing the number of firms makes the responses longer lasting, but smaller in size. Output increases in a more or less hump-shaped manner before returning over time to the steady-state values. For illustrative purposes, an alternative value $\gamma = 4.5$ is chosen in figure 5. While this generates a very high steady state markup of 73%, the inflation response is lower on impact. On the left, all three agents receive the same 1% shock one after another. On the right, only one agents receives the monetary transfer.



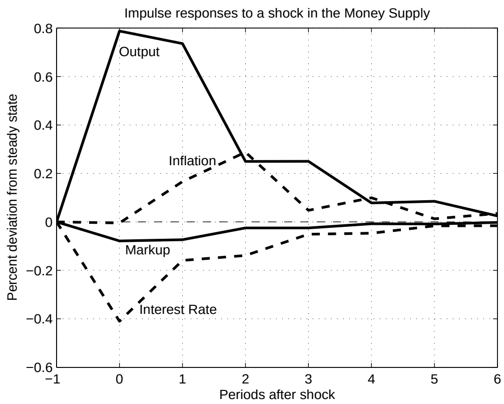


Figure 5: n=3, $\gamma = 4.5$. On the left three subsequent shocks; on the right one shock

6 General Case

In this section I explore the general case for any given number n of agents in the economy, which could also be seen as the free parameter that determines the length of a whole period asked for in the last paragraph of Lucas (1990). The structure of the model is maintained, in particular the overlapping purchases and the assumptions about the timing of shop openings (at each point in time, one shop opens).

Households The utility function is the same as in the preceding section

$$E_t(\overline{U}_{1,t}) = E_t\left(\sum_{l=t}^{\infty} \beta^l \frac{\overline{C}_{1,l+n}^{1-\sigma}}{1-\sigma}\right).$$

The Cash-in-Advance constraint is

$$M_{1,t} \geq \sum_{i=0}^{n-1} P_{t+i} C_{i+1,t+i} = \overline{C}_{1,t}\overline{P}_t, \tag{29}$$

while the budget constraint also remains mostly unchanged

$$M_{1,t} + B_t = (1 + i_{t-n})B_{t-n} + \Pi_{t-1} + S_t. \tag{30}$$

Again, the household receives last period's profits $\Pi_{t-1} = Y_{t-1}P_{t-1}$ from 'his' shop. Household i's expected value of the consumption bundle while buying the good of shop j is

$$E(\overline{C}_{1,t-i+1}) = E\left[\left(\sum_{l=1}^{i-1} C_{l,t-i+l}^{\frac{\gamma-1}{\gamma}} + \sum_{j=i}^{n} C_{j,t+j-i}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}\right], \qquad \gamma > 1 \tag{31}$$

where the goods 1 to $i-1$ already have been purchased and the consumption bundle consists of goods purchased between time $t-i+1$ and $t+n-i$. Maximizing (31) with respect to $C_{i,t}$ leads to

$$\frac{\partial E[\overline{C}_{1,t-i+1}]}{\partial C_{i,t}} = E\left[\overline{C}_{1,t-i+1}^{-\frac{1}{\gamma}}\left(C_{i,t}^{-\frac{1}{\gamma}} + \sum_{j=i+1}^{n} C_{j,t+j-i}^{-\frac{1}{\gamma}} \frac{\partial C_{j,t+j-i}}{\partial C_{i,t}}\right)\right] \equiv 0.$$

Solving this equation verifies homothetic preferences:

$$C_{i,t} \equiv \Psi_{i,t} M_{i,t}, \tag{32}$$

where $M_{i,t}$ is the cash remaining of agent i at time t. Recursive solution starts at the last shop n:

$$C_{n,t} = \frac{M_{n,t}}{P_t}, \tag{33}$$

and goes back from n until shop i. Solving leads to

$$\Psi_{i,t} = \left[\Phi_{i,t}^{\gamma} P_t^{\gamma} + P_t\right]^{-1}, \tag{34}$$

with

$$\Phi_{i,t} = E\left[\sum_{j=i+1}^{n}\left(\Psi_{j,t+j-i}\prod_{k=i+1}^{j-1}\Psi_{k,t+k-i}\Phi_{k,t+k-i}^{\gamma}P_{k,t+k-i}^{\gamma}\right)^{\frac{\gamma-1}{\gamma}}\right], \tag{35}$$

and the definitions

$$\Phi_{n,t} \equiv 0, \qquad \prod_{k=i+1}^{i}\Psi_{k,t+k-i}\Phi_{k,t+k-i}^{\gamma}P_{k,t+k-i}^{\gamma} \equiv 1. \tag{36}$$

The law of motion for all money holdings, except for M_1 determined in equation (30), is

$$M_{i,t} = M_{i,t-1} - C_{i,t-1}P_{t-1}. \tag{37}$$

Note that the index for each person is increased each period, since it denotes the time since she opened her shop.

Producers Firm t, the only one selling at time t, maximizes utility:

$$\Pi_t = P_t \sum_{j=1}^{n} C_{i,t} - \left(\frac{Y_t}{A_t}\right)^{1+\rho} P_t^C,$$

where $\left(\frac{Y_t}{A_t}\right)^{1+\rho}$ are pure labor utility costs and P_t^C is the price which converts real to nominal costs, later set to P_t. Using equation (32), we get the following:

$$\Pi_t = P_t \sum_{i=1}^{n} M_{i,t}\Psi_{i,t} - \left(\frac{Y_t}{A_t}\right)^{1+\rho} P_t^C.$$

Plugging in the appropriate values leads to

$$\frac{\sum_{i=1}^{n}(\Phi_{i,t}^{\gamma}P_t^{\gamma-1}+1)^{-2}(\gamma\Phi_{i,t}^{\gamma}P_t^{\gamma-1}+1)M_{i,t}}{\sum_{i=1}^{n}(\Phi_{i,t}^{\gamma}P_t^{\gamma-1}+1)^{-2}\Phi_{i,t}^{\gamma}P_t^{\gamma-1}(\gamma-1)M_{i,t}} = \frac{A_t^{1+\rho}}{Y_t^{\rho}} \tag{38}$$

which equals $Markup = 1/MC$. This equation can be used together with equations (29), (30), (32)-(37), and a monetary policy rule to to simulate the economy or to derive a reduced form for the linearized version, as done in section 5 for $n = 3$.

7 Conclusion

With the present setup of the model, several empirical observations can be replicated: 1) a short-term inflation-output trade off after a monetary injection can be reached, although all firms can freely chose their optimal prices each period 2) empirical plausible impulse-response functions for output and inflation after monetary injections 3) a liquidity effect, and 4) a countercyclical markup at the firm level after monetary shocks, which would also imply procyclical wages. The model generates a microfounded, internal propagation mechanism which does not rely on capital or sticky prices, but on the slow spreading of newly inserted money. This can be seen as a way of describing the effects of central bank actions in reality, where only parts of the population benefit through first-round effects, while others are affected indirectly and later. The underlying friction of limited participation of consumers is empirically supported by Christiano et al. (1996). A reduced form description of the linearized system including a Phillips Curve can be obtained, without assuming a degenerate money distribution.

As stated, after monetary shocks optimal markup falls. Strategic complementarity is important in this model. Each firm wants to maintain a higher markup, but would suffer too large a drop in sales if it raised prices first, because customers substitute away to other firms. As other firms slowly adjust their prices, each firm can raise prices itself only gradually, thereby limiting coming pricing increases of competitors and so on. This effect arises due to the sequential structure of the model. In discrete time models with symmetry assumptions on firms, this process of reacting to other firms' price adjustments is done instantaneously. Price setters calculate their own optimal price knowing that all firms are alike. Hence, other firms' price increases are completely anticipated before setting their own price, and the new steady state is reached instantaneously. This could be seen as an unrealistic feature, since most firms react to other firms' observed price setting behavior, and do not increase prices relying on the belief that all other firms will make identical price increases at the same time. Only if all firms adjust at the same time will costumers not have the possibility of substituting to a cheaper competitor, who did not yet adjust. Hence, with the present model, deeper insights into collusion incentives, dynamic oligopolies and the role of coordination devices could possibly be gained, which in turn can be used to, e.g., study the mechanisms present during the introduction of the Euro.

Appendix: Carrying over Cash between Periods

This appendix shows when it is optimal to not carry over cash in between periods. First note that it could only be optimal to carry over resources in the form of cash if unforseen shocks happen after the agent has visited the bond market, because with positive nominal interest rates it is clearly optimal to carry over resources with interest-bearing bonds. Hence, the shock has to take place in $t+1$. In the following I will derive the optimality condition for the two-shop case, which also gives intuition for the general case with n agents.

Carrying over cash in between periods is not optimal if the following condition holds

$$\frac{\partial U_t}{\partial M_{2,t+1}} > \beta E_{t+1}\left\{\frac{\partial U_{t+2}}{\partial M_{1,t+2}}\right\}, \tag{39}$$

where purchases of period t were already done because of the above argument, i.e. the decision considered takes place in $t+1$. $M_{2,t+1}$ is the money used for purchases in $t+1$. In order for carrying cash over not to be optimal, the marginal utility of increasing these purchases has to be higher than the expected marginal utility of increasing expenditure in the coming periods. Since $\frac{M_{1,t+2}}{\overline{P}_{t+2}} = \overline{C}_{1,t+2}$, the following derivative of the RHS of the above formula can be taken

$$E_{t+1}\left\{\frac{\partial U_{t+2}}{\partial M_{1,t+2}}\right\} = E_{t+1}\left\{\overline{P}^{\sigma}_{t+2}\right\}.$$

Considering the LHS of (39), it has to be taken into account that the decision to carry over money only affects consumption of good $C_{2,t+1}$, such that the consumption bundle $\overline{C}_t$ looks like

$$\overline{C}_t = \left[C_{1,t} + \left(\frac{M_{2,t+1}}{P_{t+1}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}.$$

Hence, using the formulae from section 3 leads to the following marginal utility

$$\frac{\partial U_t}{\partial M_{2,t+1}} = \left(\frac{M_{1,t}}{\overline{P}_t}\right)^{\frac{1}{\gamma}-\sigma}\left(\frac{M_{2,t+1}}{P_{t+1}}\right)^{-\frac{1}{\gamma}}\frac{1}{P_{t+1}}.$$

Inserting all this into formula (39) leads to the following condition for carrying cash over to the next period not to be optimal

$$\left[\frac{M_{1,t}}{M_{2,t+1}}\frac{P_{t+1}}{\overline{P}_t}\right]^{\frac{1}{\gamma}} = \left[\frac{\overline{C}_{1,t}}{C_{2,t+1}}\right]^{\frac{1}{\gamma}} > \beta\frac{P_{t+1}}{\overline{P}_{t+2}}\left[\frac{M_{1,t}/\overline{P}_t}{M_{1,t+2}/\overline{P}_{t+2}}\right]^{\sigma}.$$

As stated above, I consider only increases in the money supply at $t+1$. $\overline{C}_{1,t} > C_{2,t+1}$ since $\overline{C}_{1,t}$ includes additional to $C_{2,t+1}$ also $C_{1,t}$. Remember that the discrete-number-of-agents indexes of section 3 are a sum, not an average. Hence, the LHS is larger than one. On the RHS, $\beta < 1$ and $P_{t+1} \leq \overline{P}_{t+2}$, since the prices increase monotonically after a positive monetary shock (even if prices jumped immediately to the new steady state, this would be true). Furhtermore, the consumer does not visit the asset market where he receives higher income until period $t+2$, but prices start rising already in $t+1$. Therefore $M_{1,t}/\overline{P}_t \leq M_{1,t+2}/\overline{P}_{t+2}$, i.e. the consumption of the agent who did not receive the monetary transfer first drops and picks up over time. Hence, the RHS is less than one and the condition is fulfilled.

While the necessary conditions are more complicated to derive for the general case with n shops, it should be intuitive that it is not optimal to carry over cash from one period to the next if prices are rising, as after a monetary expansion. Rather, it would be optimal to transfer money from coming periods to today, which is ruled out by the Cash-in-Advance constraint.

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