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Abstract

The expected value of a lotto ticket depends on the particular numbers selected on the payslip. This is because (in most lotto games around the world) the individual prizes are not fixed in advance; instead, a prize pool is divided between all the players with winning tickets. This means that if you choose a popular set of numbers, your share of the prize will be low, should you win. The expected value of every ticket would be the same if all players were to choose their numbers randomly and independently. However, previous studies show that the actual distribution of the numbers chosen by players is far from uniform. Many players select numbers which are lucky, or which have personal significance.

In this paper, I posit a group of "rational" players, who choose their numbers randomly, but specifically avoid the superstitious numbers. For these players, the expected value of a lotto ticket is higher, the greater the number of superstitious people, and the smaller the set of combinations that they play. Despite the low prize payout rates in most lotto games, I show that the expected value may even exceed the ticket price, in which case the purchase of lotto tickets by the rational players may be consistent with standard Expected Utility theory, even if they are risk averse.

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1. Introduction

Since its introduction less than twenty years ago, lotto has become the most popular lottery game in both Europe and the US. It offers players the chance to win extremely large prizes, at very long odds. The value of the prizes is determined by the level of sales. So as people buy more tickets, the prizes get bigger. In any given draw, there is also a small probability that no ticket will win the jackpot, in which case it is added to the jackpot for the next draw. This rollover boosts ticket sales further, so that the prizes become bigger still.

The success of lotteries in general, and lotto in particular, has vexed economists over the past century. The standard mechanism for modelling the demand for lotteries is to treat a lottery ticket as an uncertain prospect, and then to estimate its Expected Utility. Unfortunately, under the standard assumption that people are risk averse, no fair gamble of any kind would ever be accepted. Certainly, nobody would ever buy a lotto ticket, which typically returns as little as 50% of the price of the ticket to prizes, according to Expected Utility theory.

A variety of explanations have been proffered to account for the demand for lottery tickets. Friedman and Savage (1948) suggested abandoning the assumption that people are risk averse, at least over a certain range of income levels. Their solution was a utility curve that is first concave, then convex, and finally concave again. Kahneman and Tversky (1979) claimed that people tend to overestimate very small probabilities, and thus to overvalue lottery tickets. Conlisk (1993) presented a model which appends a tiny utility of gambling to an Expected Utility model.

All of these models explain the demand for certain types of lottery. However, in the case of lotto, there is a another, simpler explanation which may account for the purchase of tickets by at least some players. It stems from the "interactive" design of the game, which allows players to choose their own numbers, and from the fact that winning tickets are not necessarily unique.

In a typical version of lotto, players mark six numbers from a play grid which contains the numbers 1 through to 49.[1] In each draw, six winning numbers are randomly chosen. Anyone who matches their six numbers with the winning combination claims a share of the jackpot prize. If no ticket matches the winning

[1] Variants of 6/49 lotto are played around Europe, including France, Germany, Portugal, Spain, and the UK.

combination, the jackpot is "rolled over" to the next draw. There are usually several tiers of smaller prizes, for people who match 3, 4 or 5 numbers.[2]

Since players with winning tickets have to share the prize with other winners, the value of the individual prizes in each draw is not known in advance, but depends on the level of sales and the number of winners. This also means that the expected value of each lotto ticket depends on the number of other players who have the chosen the same combination.

Under the assumption that "all numbers are equal", so that players choose their numbers randomly and independently, the expected number of winners is the same for every combination. However, there is considerable evidence which suggests that some numbers, and combinations of numbers, on the play grid are much more popular than others. This means that there may be big differences in the expected value of lotto tickets with different combinations. So the expected value of some tickets may be considerably higher than the prize payout ratio, and possibly higher even than the price of the ticket.

To test this hypothesis, I replace the assumption that all numbers are equal, and acknowledge the fact that some (possibly most) people play numbers which are special to them, regardless of how popular those numbers might be. I postulate that there is a second group of people who choose their numbers randomly, but specifically avoid those combinations which are known to be special. Under this assumption, I can then calculate the expected value of a lotto ticket for these "rational" players.

2. Design of Lotto and Choice of Numbers

The expected value of a lotto ticket depends on the probability of matching the ticket's numbers with the winning numbers, the size of the prizes, and the number of other people with winning tickets (with whom the prizes are shared). The design of the game thus ensures that the expected value of each ticket is not simply the prize payout ratio.

The probability of winning a prize is based on the size of the play grid, and the number of selections from the grid. To win the jackpot, for example, it is necessary to match all six numbers on your ticket with the winning numbers. If you have to choose six numbers between 1 and 49, then there are 13,983,816

[2] Sometimes an extra bonus number is drawn, in which case the prize tier immediately below the jackpot is for tickets matching five of the main numbers plus the bonus number.

possible combinations (this is $^{49}C_6$).[3] So the probability of winning the jackpot works out at about 1 in 14 million.

The total value of the prizes to be awarded in each draw is a fixed proportion of sales (the prize payout ratio) plus the rollover, if there is one. Usually, a fixed percentage of the total prize pool (excluding the rollover) is allocated to each tier.[4] If there is a rollover from the previous draw, this is added to the jackpot.

Because the prize money in each tier is shared between all the winning tickets in that tier,[5] the number of other winners enters the calculation of the expected value of a lottery ticket. But this requires some assumption on the numbers which people choose. The standard assumption is that players choose their numbers randomly and independently.

In reality, the numbers chosen by players are far from random.[6] People choose numbers which represent particular dates or events (such as birthdays), or personal lucky numbers. Some choose their combination of numbers such that they form a pattern on the play grid. Others make sure their numbers are not too close together (often in the misguided belief that this makes their choice of numbers more random).

Clotfelter and Cook (1989) analysed the choice of combinations on the (approximately) 5 million tickets purchased on a particular day for the 6/40 lotto game in Maryland. They found that the most popular 34% of combinations accounted for 73% of sales. The most popular combination was {1,2,3,4,5,6}, which was played more than 2,000 times the average rate.

Many lotto games now offer a "Quick Pick" (or "Lucky Dip") option, in which the lottery terminal selects random numbers for the player. Although some people choose their numbers in this way (Lucky Dip accounts for 10% - 15% of lotto sales in the UK), by far the majority of players still prefer to choose their own numbers.

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[3] ^nC_r is defined as $n!/r!(n-r)!$

[4] This is not necessarily the case, however. The UK National Lottery, for example, offers a fixed small prize (£10) to each winner in the lowest prize tier. The value of these prizes is subtracted from the prize pool before the rest of the money is distributed to the other tiers.

[5] With possible exceptions (see footnote 4).

[6] For evidence see, for example, Clotfelter and Cook (1989).

3. Calculating the Expected Value

In this section, I calculate the expected value of a ticket in a non-rollover draw for a simplified version of lotto, in which there is just one tier of prizes, so that all the prize money is channelled towards the jackpot. This simplification makes the analysis clearer. I discuss the implications of relaxing it in section 5.

To replace the standard assumption that all numbers are chosen randomly and independently, I make the following stylized assumption. Suppose that there are two types of lottery players. The first type, whom I will label "superstitious" players, choose numbers which are somehow special or meaningful. The second group of players, whom I will call "rational", choose their numbers quasi-randomly. That is, they pick their numbers at random from the set of possible combinations, *excluding* the combinations which are chosen by superstitious players.

I also assume that it is common knowledge (at least amongst the rational players) which combinations of numbers are chosen by the superstitious players. Thus the total set of combinations is split into two disjoint sets, one of which contains superstitious numbers, the other the rest of the numbers. Each rational player chooses their numbers randomly and independently from the set of non-superstitious numbers.

Under this assumption, it is possible to calculate the expected value of a lotto ticket for a rational player. It is not possible to work out the expected value of those tickets with superstitious numbers without making further assumptions, as the expected value will be different for each ticket, depending on the popularity of each individual combination. The point is that it is not necessary to know the distribution of the superstitious numbers in order to calculate the expected value of the tickets bought by the rational players.

In order to demonstrate the impact of assuming a skewed distribution, I first derive the expected value of a lotto ticket under the standard assumption that all numbers are chosen randomly and independently.

3.1 When All Numbers Are Equal

This result was worked out by Lim (1995).[7] Let Q be the number of tickets sold at one unit of currency each, and let r be the prize payout ratio. Then the total jackpot J is equal to rQ, assuming that there is no rollover. This jackpot is shared between the winning tickets.

[7] An approximation of the result was presented earlier by Cook and Clotfelter (1991,1993).

Let p be the probability of winning the jackpot. If the lotto game is 6/49 (as described above), then p is equal to $1/{}^{49}C_6$. It follows that the probability of a certain player *not* winning is equal to 1-p. The probability that there are no winners at all (i.e. the probability of a rollover) is then $(1-p)^Q$, under the assumption that the numbers on each ticket are chosen randomly and independently.

The probability that the jackpot is awarded (to at least one person) is thus $1-(1-p)^Q$. So the expected prize awarded to the Q tickets considered in aggregate is equal to the probability that the jackpot is awarded multiplied by the expected jackpot rQ:

$$\text{Total Expected Prize} = \left[1-(1-p)^Q\right]rQ.$$

By symmetry, the expected value of a single ticket is just a Q'th share of the total expected prize. This gives the following formula for the expected value of a lottery ticket in a non-rollover draw:[8]

$$EV = r\left[1-(1-p)^Q\right]. \qquad (1)$$

This expression is the product of the prize payout ratio and the probability that the jackpot is awarded (i.e. that there is no rollover to the next draw). To get a feeling for the importance of the separate components, consider a 6/49 lotto game with a 45% payout ratio. Suppose that 65 million tickets are sold.[9] Then the probability that the jackpot is awarded is equal to 0.990 (to 3 decimal places). So the expected value of a ticket works out at slightly less than the payout ratio, 0.45.

3.2 When Some Numbers Are More Equal Than Others

In order to fully specify the distribution, I must introduce two new variables: the proportion of tickets bought by players who are superstitious (s), and the proportion of the combinations of numbers which are chosen by superstitious players (n).

Suppose that the total number of combinations on the play grid is N. So for the lotto game described above, N is equal to 13,983,816. (Note that N is the reciprocal of p.) So the number of superstitious tickets is equal to sQ, and they cover nN of the possible combinations. This means that the rational players buy

[8] If there is a rollover R, the formula becomes: $EV = \left(r+\frac{R}{Q}\right)\left[1-(1-p)^Q\right]$.

[9] These figures describe (approximately) the UK National Lottery.

(1-s)Q tickets, and choose their numbers randomly and independently from the remaining (1-n)N combinations.

I will now calculate the expected value of a lotto ticket for a rational player, using the standard formula:

$$\mathrm{EV} = \sum_{i=1}^{\mathrm{N}} \left(\mathrm{p}_i \ \mathrm{EV}_i\right), \tag{2}$$

where i represents each of the N possible combinations, p_i is the probability that combination i wins, and EV_i is the expected value of a rational player's ticket given than combination i wins.

Without loss of generality, I will assume that the first nN combinations are the superstitious ones. So for the range of combinations from 1 to nN, EV_i will be equal to zero, since rational players never choose these numbers (by assumption). Note also that p_i is equal to p for all i, since every combination has an equal probability of winning. So equation (2) reduces to:

$$\mathrm{EV} = \mathrm{p} \sum_{i=n\mathrm{N}+1}^{\mathrm{N}} \mathrm{EV}_i. \tag{3}$$

Now suppose that a non-superstitious combination is the winning set in the draw. Denote this event by NSC. Let the probability that a rational player wins, given that a non-superstitious combination has come up, by $\mathrm{p}_{\mathrm{NSC}}$. Each rational player is choosing randomly from (1-n)N numbers. So the probability of any ticket chosen by a rational player winning is given by:

$$\mathrm{p}_{\mathrm{NSC}} = \frac{1}{(1-n)\mathrm{N}}.$$

The probability that a rational player loses, given that a non-superstitious combination of numbers has come up, is 1-$\mathrm{p}_{\mathrm{NSC}}$. Then the probability that there are no winners is equivalent to the probability that all the rational players lose (given that no superstitious player will have chosen this combination, by definition of it being non-superstitious). So the probability of no winners is given by:

$$\begin{aligned}\mathrm{P(no\ winners} \mid \mathrm{NSC)} &= \left(1-\mathrm{p}_{\mathrm{NSC}}\right)^{(1-s)\mathrm{Q}} \\ &= \left(1-\frac{1}{(1-n)\mathrm{N}}\right)^{(1-s)\mathrm{Q}},\end{aligned}$$

since the (1-s)Q rational players each choose their numbers randomly and independently. Then the probability that at least one ticket wins the jackpot is:

$$\text{P(jackpot awarded | NSC)} = 1 - \left(1 - \frac{1}{(1-n)\text{N}}\right)^{(1-s)\text{Q}}.$$

The total expected prize to the rational players, given that a non-superstitious combination wins, is the probability that the jackpot is awarded multiplied by the jackpot rQ.

$$\text{Total Expected Prize | NSC} = \left[1 - \left(1 - \frac{1}{(1-n)\text{N}}\right)^{(1-s)\text{Q}}\right] \text{rQ}.$$

By symmetry, the expected value of each ticket bought by a rational player is an equal share of this expected prize:

$$\begin{aligned}\text{Expected Value | NSC} &= \frac{\text{Total Expected Prize | NSC}}{(1-s)\text{Q}} \\ &= \frac{\text{r}}{1-s}\left[1 - \left(1 - \frac{1}{(1-n)\text{N}}\right)^{(1-s)\text{Q}}\right].\end{aligned}$$

Returning to the original expected value formula in equation (3), EV_i is the same for all values of i within the range of non-superstitious numbers, and is equal to Expected Value | NSC. So the expected value of a lotto ticket for a rational player is:

$$\text{EV} = \text{p}(1-n)\text{N}\frac{r}{1-s}\left[1 - \left(1 - \frac{1}{(1-n)\text{N}}\right)^{(1-s)\text{Q}}\right].$$

Finally, substituting p = 1/N gives:[10]

$$\text{EV} = \text{r}\left(\frac{1-n}{1-s}\right)\left[1 - \left(1 - \frac{\text{p}}{1-n}\right)^{(1-s)\text{Q}}\right]. \tag{4}$$

[10] If there is a rollover R, the formula becomes:

$$\text{EV} = \left(\text{r} + \frac{\text{R}}{\text{Q}}\right)\left(\frac{1-n}{1-s}\right)\left[1 - \left(1 - \frac{\text{p}}{1-n}\right)^{(1-s)\text{Q}}\right].$$

4. Examining the Results

I shall first look at the expected value of a specific lotto game, for different values of n and s. I will then generalise the results.

4.1 Specific Example

Again, consider a 6/49 lotto game with a 45% prize payout ratio, for which 65 million tickets are sold in each draw. Using the notation above, p = 1/13,983,816, r = 0.45, and Q = 65,000,000. The next table shows the expected value of a rational player's lotto ticket (which costs one unit of currency) for different values of s and n in a non-rollover week.

		Proportion of players who play superstitious numbers (s)								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	0.1	0.45	0.50	0.56	0.64	0.75	0.88	**1.06**	**1.30**	**1.63**
	0.2	0.40	0.45	0.51	0.58	0.68	0.81	0.99	**1.24**	**1.59**
Proportion	0.3	0.35	0.39	0.45	0.52	0.61	0.73	0.91	**1.16**	**1.53**
of	0.4	0.30	0.34	0.38	0.45	0.53	0.64	0.81	**1.06**	**1.46**
combinations	0.5	0.25	0.28	0.32	0.37	0.45	0.55	0.70	0.95	**1.36**
which are	0.6	0.20	0.22	0.26	0.30	0.36	0.45	0.58	0.81	**1.24**
superstitious	0.7	0.15	0.17	0.19	0.22	0.27	0.34	0.45	0.64	**1.06**
(n)	0.8	0.10	0.11	0.13	0.15	0.18	0.22	0.30	0.45	0.81
	0.9	0.05	0.06	0.06	0.07	0.09	0.11	0.15	0.22	0.45

Table 1: Expected value of a ticket in a non-rollover week

When all numbers were assumed to be equal, the expected value of a lotto ticket was equal to 0.45 (after rounding to two decimal places). Now, allowing for the fact that not all numbers are equal, the table shows that it is the relationship between the two variables, s and n, which determines by how much the expected value differs from the payout ratio. When s and n are equal, whatever value they actually take, the expected value is still approximately equal to the payout ratio (the difference is only apparent beyond the second decimal place).

The expected value to a rational player of a ticket increases, the higher is the proportion of superstitious players, and the lower is the proportion of superstitious numbers. And if there are enough superstitious players choosing few enough combinations, the expected value may even rise above the price of the ticket (these are the highlighted cells in the table). The flip side is that if there are relatively few superstitious players choosing from a large range of numbers, then the expected value to rational players may be considerably less than the payout ratio.

One interesting implication from this analysis, from an economist's point of view, is that if the distribution of numbers is sufficiently skewed, then the purchase of lotto tickets by the rational players may be consistent with standard Expected Utility theory, even if the players are risk averse. However, as the table shows, a very skewed distribution is required. And of course, Expected Utility theory still has nothing to say about why the majority of players, the "superstitious" ones, continue to play lotto.

The following example illustrates by how much the expected value may vary, depending on the skew of the distribution of the numbers chosen. Suppose that 80% of the players are superstitious, and choose from 40% of the possible combinations of numbers. Then a lotto ticket is a good gamble for a rational player: its expected value is 1.06, slightly higher than the price of the ticket. So even risk averse players might be attracted to the game.

But now suppose that only 40% of players are superstitious, and choose from 80% of the combinations. Then the expected value of a lotto ticket is only 0.15. Appropriate advice to rational players in this case would be for them to swallow their pride, and consult their nearest clairvoyant to choose their numbers.

Using the results obtained by Clotfelter and Cook for the Maryland game (see Section 2), if 73% of tickets are purchased by superstitious players, choosing from 34% of the combinations, then the expected value of the rational players' tickets, if they choose randomly and independently from the remaining combinations, is 0.94. This is only slightly less than the price of the ticket.

Finally, consider a rollover draw. Suppose that the value of the rollover is 9 million, and that sales rise to 75 million.[11] Under the standard assumption that all numbers are chosen randomly and independently, the expected value of a ticket rises to 0.57. Table 2 shows the expected value for different values of s and n under the alternative assumption of a skewed distribution.

		Proportion of players who play superstitious numbers (s)								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	0.1	0.57	0.64	0.72	0.83	0.97	**1.16**	**1.42**	**1.79**	**2.30**
	0.2	0.51	0.57	0.65	0.75	0.88	**1.06**	**1.32**	**1.68**	**2.23**
Proportion	0.3	0.44	0.50	0.57	0.66	0.78	0.95	**1.20**	**1.56**	**2.14**
of	0.4	0.38	0.43	0.49	0.57	0.68	0.83	**1.06**	**1.42**	**2.02**
combinations	0.5	0.32	0.36	0.41	0.47	0.57	0.70	0.91	**1.26**	**1.88**
which are	0.6	0.25	0.28	0.33	0.38	0.46	0.57	0.75	**1.06**	**1.68**
superstitious	0.7	0.19	0.21	0.24	0.28	0.34	0.43	0.57	0.83	**1.42**
(n)	0.8	0.13	0.14	0.16	0.19	0.23	0.28	0.38	0.57	**1.06**
	0.9	0.06	0.07	0.08	0.09	0.11	0.14	0.19	0.28	0.57

Table 2: Expected value of a ticket in a rollover week

[11] Again, these figures are relevant for the UK National Lottery.

In a rollover week, it requires a smaller difference between the proportion of players who are superstitious and the proportion of numbers that they play to ensure that the expected value for the rational players is greater than the ticket price. So the implication that a lotto ticket might be attractive to Expected Utility-maximizing risk averse players is considerably more plausible in a rollover week than in a normal week.

4.2 Generalizing the Results

Although the results described above relate to a specific example, they can all be generalized. In order to do this, it is first worth examining the separate components of equation (4).

4.2.1 Examining the Components of Expected Value

The expected value formula is the product of three terms. The first term is the prize payout ratio. The second is the ratio between 1-n and 1-s. This is the proportion of numbers available to the rational players divided by the proportion of players who are rational.

The final term, in square brackets, is the probability that the jackpot is awarded given that a non-superstitious combination wins. It is usually very close to one, reflecting the fact that the probability of a rollover is very small. This can be verified by examining the order of magnitude of the variables in the expression: in a typical lotto game, N (the reciprocal of p) and Q will be very large, generally both measured in hundreds of thousands, if not millions. The extreme values of p and Q tend to dominate the expression.

The only circumstances in which this third term might be significantly less than one is when there are few rational players choosing between a large set of numbers, i.e. when the value of s is high and n is low. Then the probability that nobody wins the jackpot when a non-superstitious combination comes up becomes significantly greater than zero, and the probability that the jackpot is awarded is accordingly significantly less than one.

4.2.2 Evaluating the Expected Value

First, consider the expected value of a ticket for a rational player whenever s is equal to n. The second term in equation (4) reduces to one, and since the term in square brackets will always be very close to one, the expected value will be approximately equal to the value of the payout ratio.

Next, look at how the expected value (EV) of a ticket changes as s and n change. Intuitively, the more people known not to play your numbers, the fewer people you might have to have to share the jackpot with, should you win. So the partial derivative with respect to s (∂EV/∂s) should be positive. And as more combinations are available to rational players, fewer tickets are likely to win when any given non-superstitious combination comes up. So the partial derivative with respect to n (∂EV/∂n) should be negative. The numbers in Table 1 support both these arguments.

However, if you look at the separate components of expected value in equation (4), the impact of changes in n and s is ambiguous. As s increases, the second term (the ratio of 1-n and 1-s) also increases, but the third term (the probability that the jackpot will be awarded given that a non-superstitious combination wins) decreases. While as n increases, the second term decreases, but the third term increases. So the expected value is not necessarily always increasing with s and decreasing with n.

In order to determine the conditions under which $\partial EV/\partial s > 0$ and $\partial EV/\partial n < 0$, it is necessary to examine the partial derivatives of EV with respect to these two variables. This exercise reveals that the following conditions are sufficient to ensure the required signs of the partial derivatives:[12] ∂EV/∂s is always positive, and ∂EV/∂n is always negative, provided (1-n)N and (1-s)Q are both greater than one. These conditions are equivalent to requiring that there is more than one rational player choosing from more than one non-superstitious combination of numbers. So it is reasonable to assume that these conditions are satisfied.

To summarize, I have shown that whenever the proportion of players who are superstitious is equal to the proportion of the combinations that they play, the expected value of a lotto ticket for a rational player is approximately (actually slightly less than) the prize payout ratio. The expected value increases as the proportion of superstitious players increases, and decreases as the proportion of numbers which are superstitious increases.

5. Allowing For Multiple Tiers Of Prizes

In the previous two sections, the prize structure of the lotto game under analysis contained just one tier. In fact, games are typically designed with four or five prize tiers, as described at the start of the paper. The natural extension of the analysis above would be to take this feature into account.

Under the standard assumption that all numbers are selected randomly and independently, this is a straightforward exercise. Denote the number of tiers by T. Let the probability of winning a prize in tier t be p_t. Suppose that the total prize pool is split between the tiers in fixed proportions j_t (where $0 < j_t < 1$ and

[12] The proof, which is complicated, is included as Appendix One.

$\Sigma j_t = 1$). Then the expected value of a ticket is just the sum of the expected values of winning in each of the prize tiers.

$$EV = r \sum_{t=1}^{T} \left[1 - (1 - p_t)^Q\right] j_t .$$

However, when I assume a skewed distribution, it is no longer possible to extend the analysis to multiple prize tiers. This is because it is not possible to calculate the expected number of winners in the lower tiers.

To illustrate the problem, consider again a lotto game in which players choose six numbers from 49. As usual, the top prize is shared between the players who match all six of their numbers to the winning set. Note that, depending on whether or not the winning combination is a superstitious set of numbers, the jackpot will be shared either between superstitious players or between rational players. But it can never be shared between players from both groups.

Now consider the prize tier for players who match (exactly) five numbers out of six. Let J_5 be the total value of the Match 5 prize. Then the expected value for a rational player who matches five numbers may be calculated as follows:

$$\text{Expected Prize} \mid \text{Match } 5 = \sum_{m=0}^{Q-1} \text{P(exactly } m \text{ other winners)} \frac{J_5}{m+1} .$$

The problematic term in this equation is the expression for the probability that there are exactly m other winners. When the other winners are known to be all rational players, as is the case for the jackpot prize, it is possible to calculate this expression for any number of other winners. However, the set of combinations which match exactly five out of six numbers[13] may include some superstitious and some non-superstitious combinations. Because the distribution of the superstitious combinations is not uniform, the probabilities of the number of other winners depends on what the winning combination actually is. So it is impossible to calculate the expected value of a lotto ticket for a rational player, without making further assumptions on the distribution of the superstitious combinations.

[13] There are 258 different combinations for a Match 5 prize (assuming that there is no bonus number).

6. Conclusion

In this paper, I assumed that lotto players can be divided into two groups: a group of "superstitious" people who choose special numbers on their tickets, and a group of "rational" players who choose their numbers randomly, avoiding the superstitious numbers. I then calculated the expected value of a lotto ticket for the rational players.

For a simplified version of lotto, with a single tier of prizes, the results show that when the proportion of players who are superstitious is the same as the proportion of combinations that they choose, then the expected value of a ticket for a rational player is approximately equal to the prize payout ratio of the game. The expected value rises when there are more superstitious people playing a smaller range of combinations. It is even possible that the expected value exceeds the ticket price, particularly when there is a rollover draw. In this case, the purchase of lotto tickets by rational players may be consistent with standard Expected Utility theory, even if the players are risk averse.

This analysis cannot be extended to allow for several tiers of prizes, without making further assumptions on the choice of numbers by the superstitious players.

A1. Appendix One

In Section 3.2, the expected value of a lotto ticket for a rational player was calculated as:

$$\mathrm{EV} = \mathrm{r}\left(\frac{1-n}{1-s}\right)\left[1-\left(1-\frac{\mathrm{p}}{1-n}\right)^{(1-s)\mathrm{Q}}\right], \tag{4}$$

where r is the prize payout ratio, s is the proportion of tickets bought by players who are superstitious, n is the proportion of the combinations of numbers which are chosen by superstitious players, p is the probability of winning the jackpot, and Q is the number of tickets sold at one unit of currency each.

This appendix contains the proof that the partial derivative of expected value with respect to s ($\partial\mathrm{EV}/\partial s$) is positive, and the partial derivative with respect to n ($\partial\mathrm{EV}/\partial n$) is negative, provided that $(1-n)\mathrm{N} > 1$ and $(1-s)\mathrm{Q} > 1$.

A1.1 The partial derivative with respect to *s*

Taking the partial derivative of EV with respect to s gives:

$$\frac{\partial \mathrm{EV}}{\partial s} = \mathrm{r}\ \frac{1-n}{(1-s)^2}\left\{1+\left(1-\frac{1}{(1-n)\mathrm{N}}\right)^{(1-s)\mathrm{Q}}\left[\ln\left(1-\frac{1}{(1-n)\mathrm{N}}\right)^{(1-s)\mathrm{Q}}-1\right]\right\}. \tag{A1}$$

Making the substitution:

$$\mathrm{V} = \left(1-\frac{1}{(1-n)\mathrm{N}}\right)^{(1-s)\mathrm{Q}},$$

equation (A1) simplifies to:

$$\frac{\partial \mathrm{EV}}{\partial s} = \mathrm{r}\ \frac{1-n}{(1-s)^2}\left\{1+\mathrm{V}[\ln \mathrm{V}-1]\right\}. \tag{A2}$$

Since $\mathrm{r} > 0$ and n and s are only defined in the range [0,1], the partial derivative with respect to s is positive provided that $n \neq 1$, $s \neq 1$, and the term in curly brackets is positive.

Denote the term in curly brackets by $y(\mathrm{V})$:

$$y(\mathrm{V}) = 1+\mathrm{V}[\ln \mathrm{V}-1]$$

Then the first two derivatives of y with respect to V are:

$$\frac{dy}{dV} = \ln V, \text{ and}$$

$$\frac{d^2y}{dV^2} = \frac{1}{V}.$$

So $y(V)$ has a minimum at $V = 1$; $y(1) = 0$ and y is positive for all other values of V. V is the probability that there are no winners given that a non-superstitious number wins. So $V \neq 1$ provided that $(1\text{-}s)Q \neq 0$. Also, to ensure that the function V is defined, it is necessary that $(1\text{-}n)N \neq 0$.

So the conditions $(1\text{-}n)N > 1$ and $(1\text{-}s)Q > 1$ are sufficient to ensure that $y(V) > 0$, $n \neq 1$ and $s \neq 1$. These conditions are thus sufficient to ensure that the partial derivative of expected value with respect to s is positive.

A1.2 The partial derivative with respect to *n*

The partial derivative of expected value with respect to n is:

$$\frac{\partial EV}{\partial n} = \frac{r}{1-s}\left\{\left(1 - \frac{1}{(1-n)N}\right)^{(1-s)Q-1}\left[1 + \frac{(1-s)Q-1}{(1-n)N}\right] - 1\right\}. \qquad \text{(A3)}$$

Making the substitutions:

$$V = (1-n)N$$
$$W = (1-s)Q - 1,$$

equation (A3) simplifies to:

$$\frac{\partial EV}{\partial n} = \frac{r}{1-s}\left\{\left(1 - \frac{1}{V}\right)^{W}\left[1 + \frac{W}{V}\right] - 1\right\}.$$

Define $y(V,W)$ as follows:

$$y(V,W) = \left(1 - \frac{1}{V}\right)^{W}\left[1 + \frac{W}{V}\right]. \qquad \text{(A4)}$$

Then the expression for $\partial EV/\partial n$ simplifies further to:

$$\frac{\partial EV}{\partial n} = \frac{r}{1-s}\{y(V,W) - 1\}. \qquad \text{(A5)}$$

Since $r > 0$ and s is only defined over the range $[0,1]$, the value of the first term in equation (A5) is greater than zero provided $s \neq 1$. The assumption that

(1-s)Q > 1 is sufficient to ensure that this is the case. Then $\partial EV/\partial n$ is negative if and only if $y(V,W) < 1$.

Below, I prove that $y(V,W) < 1$, provided that $(1-n)N > 1$ and $(1-s)Q > 1$, considering separately the three cases W < V, W = V, and W > V. Note that, using the definitions of V and W, the conditions $(1-n)N > 1$ and $(1-s)Q > 1$ simplify to V > 1 and W > 0.

A1.2.1 When W < V

First, note that $y(V,W) < 1$ if and only if $\ln y(V,W) < 0$. From equation (A4),

$$\ln y(V,W) = W\ln\left(1-\frac{1}{V}\right)+\ln\left(1+\frac{W}{V}\right). \qquad (A6)$$

The Maclaurin's expansion for $\ln(1 + x)$ is:

$$\ln(1+x) = \sum_{j=1}^{\infty}\frac{(-1)^{j+1}x^j}{j} = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots \qquad (-1 < x < 1).$$

This can be used to expand the logarithmic terms in equation (A6), under the assumptions that V > 1 and W/V < 1:

$$\ln y(V,W) = \sum_{j=1}^{\infty}\left\{(-1)^{j+1}\frac{1}{j}\left(\frac{W}{V}\right)^j - \frac{W}{jV^j}\right\}$$

$$= \sum_{k=1,3,5,\ldots}\frac{W}{V^{k+1}}\left\{\frac{1}{k+2}\frac{1}{V}\left(W^{k+1}-1\right)-\frac{1}{k+1}\left(W^k+1\right)\right\}.$$

By rearranging the expression in curly brackets, it can be shown that:

$$\frac{1}{k+2}\frac{1}{V}\left(W^{k+1}-1\right)-\frac{1}{k+1}\left(W^k+1\right)<0 \Leftrightarrow \frac{W^{k+1}-1}{W^k+1}<\frac{k+2}{k+1}V. \qquad (A7)$$

Now it can easily be shown that, for any x,

$$\frac{x^{n+1}-1}{x^n+1}<x \Leftrightarrow x>-1.$$

Using this result, and the assumption that V > W > 0, we have:

$$\frac{W^{k+1}-1}{W^k+1}<W<V<\frac{k+2}{k+1}V.$$

Putting this result back into equation (A7), it follows that the term in curly brackets is negative. Since every term in the expansion of $\ln y(V,W)$ is negative, $\ln y(V,W)$ must be negative, which gives the result that $y(V,W) < 1$.

A1.2.2 When W = V

In this case, equation (A4) simplifies to:

$$y(\mathrm{V}) = 2\left(1 - \frac{1}{\mathrm{V}}\right)^{\mathrm{V}}.$$

Taking the natural log of y and using the Maclaurin's expansion for $\ln(1 + x)$ gives:

$$\begin{aligned}\ln y(\mathrm{V}) &= \mathrm{V}\ln\left(1 - \frac{1}{\mathrm{V}}\right) + \ln 2 \\ &= (\ln 2 - 1) - \sum_{j=1}^{\infty} \frac{1}{(j+1)\mathrm{V}^{j}} .\end{aligned}$$

Since (ln 2 - 1) is negative, and the summation term is positive for all j, ln y is negative. Again, this ensures that $y(\mathrm{V,W}) < 1$.

A1.2.3 When W > V

Unfortunately, the Maclaurin's expansion cannot be used when W > V, since W/V is not less than one, and so ln (1+W/V) cannot be expanded. The proof is obtained by evaluating the partial derivative of y with respect to V or W. This, combined with the results obtained above, is sufficient to prove that $y(\mathrm{V,W}) < 1$.

From equation (A4), the partial derivative of y with respect to V is:

$$\frac{\partial y}{\partial \mathrm{V}} = \frac{\mathrm{W}(\mathrm{W}+1)}{\mathrm{V}^2(\mathrm{V}-1)}\left(1 - \frac{1}{\mathrm{V}}\right)^{\mathrm{W}}.$$

The assumptions that V > 1 and W > 0 ensure that $\partial y/\partial \mathrm{V} > 0$.

Now consider any two values V_1 and W_1, such that $\mathrm{W}_1 > \mathrm{V}_1$. From the result obtained in Section A1.2.2, we know that $y(\mathrm{W}_1,\mathrm{W}_1) < 1$. We also know that, as the value of V falls from W_1 to V_1 (keeping W fixed), $y(\mathrm{V,W})$ also decreases, since $\partial y/\partial \mathrm{V} > 0$. So:

$$y(\mathrm{V}_1,\mathrm{W}_1) < y(\mathrm{W}_1,\mathrm{W}_1) < 1.$$

Since this result holds for any W_1 and V_1, such that $\mathrm{W}_1 > \mathrm{V}_1$, it follows that $y(\mathrm{V,W}) < 1$. (This result can also be obtained by showing that the partial derivative $\partial y/\partial \mathrm{W} < 0$, in which case $y(\mathrm{V}_1,\mathrm{W}_1) < y(\mathrm{V}_1,\mathrm{V}_1) < 1$ when $\mathrm{W}_1 > \mathrm{V}_1$.)

I have now established that $y(\mathrm{V,W}) < 1$, under the three cases W < V, W = V, and W > V. This implies that $\partial \mathrm{EV}/\partial n < 0$, provided that $(1\text{-}n)\mathrm{N} > 1$ and $(1\text{-}s)\mathrm{Q} > 1$.

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