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# Can Waste Improve Welfare?\*

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## Abstract

In endogenous growth models with a capital spillover, the market outcome is not Pareto efficient since agents ignore the positive externalities caused by investment. This makes it natural to conclude that taxes on investment or subsidies to consumption will impose first order welfare costs. In fact this is not true in a very simple model of endogenous growth with an infinite lived representative consumer who supplies labour elastically. We present such a model in which, for all parameter values, either a small tax on capital income whose proceeds are thrown away causes increased welfare, or a small marginal subsidy to consumption causes increased welfare. We also show that for a broad range of parameters values, a lump sum tax whose proceeds are also thrown away will increase growth and welfare.

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# 1 Introduction

In endogenous growth models there can be a much larger quantitative influence of policies on welfare than in the neoclassical model, since policies have the potential to influence the growth rate in the long run. The welfare impact of distortionary taxation on output growth rates can far exceed Harberger measures.

As is well known, in the path-breaking Romer (1986) paper endogenous growth is driven by externalities to capital accumulation, so that the social rate of return on investment exceeds the private return. Under this assumption subsidies to investment can raise the growth rate and levels of utility, as shown for instance by Romer (1986) and (1989), Barro and Sala-i-Martin (1992) and (1995), chap 4. What has not been noticed is that, under the same assumption, a tax on capital income may stimulate growth, thus increasing welfare. In this paper, we consider a one sector infinite horizon model of endogenous growth, where there's a spillover to capital so the social production function is in fact linear in capital, which creates the potential for unbounded accumulation. The basic difference in assumptions with respect to the models in the above mentioned papers is that labour supply is explicitly analysed. In our model both capital taxes and lump sum taxes have an impact on the allocation of resources, because they influence labour supply and therefore the rate of return on capital and therefore the rate of growth. In particular either capital taxes or lump sum taxes will increase growth, by making people work more, even if the tax revenue is thrown away. These surprising conclusions do not require implausible parameter values. In fact, in our model for any parameter values, either a small amount of waste financed by lump sum taxes or a small amount of waste financed by capital taxes causes increased growth. Moreover we show that this can increase the representative consumer's welfare. We also show that when a tax on capital income does not increase growth and welfare, growth and welfare will be increased by a consumption subsidy financed by a lump-sum tax.

The effects of tax policies depend on whether the market equilibrium of the model is stable. As is well known, if the market equilibrium is stable it is also indeterminate.<sup>1</sup> In fact multiple equilibria make it difficult to study

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<sup>1</sup>Recently there has been a renewed interest in indeterminacy, or alternatively said in the existence of a continuum of equilibria in dynamic economic models, which means that "sunspots" and "animal spirits" can matter. See among others Benhabib and Farmer (1996) and (1994), Benhabib and Perli (1994), Benhabib and Rustichini (1994), Boldrin and Rustichini (1994), Caballé e Santos (1994), Galí (1994), Xie (1994), Chamley (1993).



the effect of taxes because outcomes are indeterminate with or without taxes. In such cases we simply assume that, with or without taxes, the economy is always on its balanced growth path. If the market equilibrium is stable, capital taxes increase growth and lump sum taxes and consumption subsidies reduce growth. If the equilibrium is unstable, capital taxes reduce growth and lump sum taxes and consumption subsidies increase growth. The connection between stability and the effects of tax policies can be explained using a simple graphical analysis. This analysis makes it clear why it is always possible to find a tax and waste policy which increases growth. Each of the policies which can increase growth can also increase welfare. Simple algebra shows that capital taxes and consumption subsidies always increase the representative consumer's welfare when such policies increase growth. Some simple numerical analysis shows that lump sum taxes will increase welfare for a broad region of the parameter space. Thus, in our model, it is always possible to improve welfare using either a tax on capital income or a consumption subsidy, and for plausible specifications of tastes and technology it is possible to achieve the same using a lump sum tax.

Such counterintuitive effects have not, to our knowledge, being noted in the literature before. Recent papers analysing the effects of taxation in endogenous growth models, e.g. Romer (1986) and (1989), Lucas (1990), King and Rebelo (1990), Rebelo (1991), Barro and Sala-i-Martin (1992), Jones, Manuelli and Rossi (1993), Stokey and Rebelo (1993), Roubini and Milesi-Ferretti (1994) do not consider them.

The paper is organized as follows: the model is spelled out in section 2, where a general condition for tastes and technology is given that implies indeterminacy. An example of an economy with an indeterminate market outcome is presented in Pelloni and Waldmann (1997). In section 3 we consider public policy, in particular a income tax policy with proceeds thrown away (subsection 3.1 and appendix A), the same with proceeds returned lump sum (subsection 3.2 and appendix B), the same with proceeds used to subsidize labour (subsection 3.3 and appendix C), a lump sum tax with proceeds thrown away (subsection 3.4 and appendix D) and a consumption subsidy financed through a lumpsum tax (subsection 3.5). Section 4 draws conclusions.

## 2 A Model

We present a simple endogenous growth model with variable labour supply. All worker/consumers are identical and maximize the same CES inter-temporal



utility function, multiplicatively separable in consumption of the homogeneous good  $C$  and the amount of labour they supply,  $L \in [0, 1]$ :

$$V = \int_0^\infty e^{-\rho t} \frac{C^{1-\sigma}}{1-\sigma} h(L) dt \quad (1)$$

where  $h(L)$  is twice differentiable and the following two conditions must hold for utility to be increasing in consumption and decreasing in labour:

$$h(L) > 0 \quad (2)$$

$$(1 - \sigma)h'(L) < 0 \quad (3)$$

while the following two conditions must hold for concavity:

$$(1 - \sigma)h''(L) < 0 \quad (4)$$

$$\frac{\sigma}{(\sigma - 1)} h''(L) h(L) > (h'(L))^2 \quad (5)$$

We assume that there is a continuum of competitive firms. As is standard from Romer (1986) we assume that the production set at the firm level is convex in labour  $L$  and capital  $K$  but that average capital  $\bar{K}$  causes a labour augmenting spill-over which is taken as given by each firm, so that the social production function is linear in capital. With population normalized to unity production  $Y$  is then given by equation 6

$$Y = F(L\bar{K}, K) = KF(L, 1) \equiv Kf(L) \quad (6)$$

Profit maximization by firms and perfect competition give the wage and real interest rate.

$$W = Kf'(L) \quad (7)$$

and

$$r = f(L) - f'(L)L \quad (8)$$

Equation 9 gives the instantaneous budget constraint consumers face:

$$\frac{\dot{K}}{K} = r + \frac{WL}{K} - \frac{C}{K} \quad (9)$$

The static first order condition for the choice between labour and leisure is:

$$C = \frac{(\sigma - 1)h(L)W}{h'(L)} \quad (10)$$

The consumers consumption savings choice implies:

$$r - \rho = \sigma \frac{\dot{C}}{C} - \frac{h'(L)}{h(L)} \frac{\dot{L}}{L} = \left( -\sigma \frac{h''(L)}{h'(L)} + (\sigma - 1) \frac{h'(L)}{h(L)} \right) \frac{\dot{L}}{L} + \sigma \frac{\dot{W}}{W} \quad (11)$$

where the second equality is obtained by differentiating the first order condition for consumption and leisure 10.

The instantaneous budget constraint 9 and the first order condition 10 imply:

$$\frac{\dot{K}}{K} = f(L) + (1 - \sigma) \frac{h(L)}{h'(L)} f'(L) \quad (12)$$

Differentiating 7 considering 12, and substituting in 11 we get:

$$\left( -\sigma \frac{h''(L)}{h'(L)} - (1 - \sigma) \frac{h'(L)}{h(L)} + \sigma \frac{f''(L)}{f'(L)} \right) \frac{\dot{L}}{L} = r - \rho - \sigma \frac{\dot{K}}{K} = r - \rho - \sigma \left( f(L) + (1 - \sigma) \frac{h(L)}{h'(L)} f'(L) \right) \quad (13)$$

Balanced growth  $\tilde{L}$  is then given by

$$\rho = (1 - \sigma) f(\tilde{L}) - \left( \tilde{L} + \sigma(1 - \sigma) \frac{h(\tilde{L})}{h'(\tilde{L})} \right) f'(\tilde{L}) \quad (14)$$

The transversality condition that the present value at time zero of the capital stock as time goes to infinity, goes to zero implies that  $\rho > g(1 - \sigma)$  where  $g$  indicates the asymptotic rate of growth. In this case, that is with no taxes we have  $g = (r(\tilde{L}) - \rho) / \sigma$  with  $r(\tilde{L})$  being the asymptotic rate of interest. The transversality condition can then be rewritten as  $r(\tilde{L}) > g$  or as  $\rho > (1 - \sigma)r(\tilde{L})$ . By substituting in this last inequality the value for  $\rho$  given by 14 and rearranging we get 15 which we will use later.

$$\tilde{L} < (\sigma - 1) \frac{h(\tilde{L})}{h'(\tilde{L})} \quad (15)$$

The coefficient of  $\tilde{L}$  in 13 is always negative. In fact, 5 can be rewritten as  $\sigma \frac{h''(L)}{h'(L)} + (1 - \sigma) \frac{h'(L)}{h(L)} > 0$ . The balanced growth path is then locally indeterminate if and only if:

$$B(\tilde{L}) \equiv \left( \tilde{L} + \sigma(1 - \sigma) \frac{h(\tilde{L})}{h'(\tilde{L})} \right) f''(\tilde{L}) + \left( \sigma + \sigma(1 - \sigma) \left( 1 - \frac{h(\tilde{L})h''(\tilde{L})}{(h'(\tilde{L}))^2} \right) \right) f'(\tilde{L}) < 0 \quad (16)$$



In 16 the coefficient of  $f'(\tilde{L})$  is positive both for  $\sigma < 1$  and for  $\sigma > 1$ ; in fact 5 implies

$$1 + (1 - \sigma)\left(1 - \frac{h(\tilde{L})h''(\tilde{L})}{(h'(\tilde{L}))^2}\right) > 1 + 1 - \sigma - \frac{(1 - \sigma)^2}{\sigma} = \frac{1}{\sigma} \quad (17)$$

A necessary condition for stability is therefore that the coefficient of  $f''(\tilde{L})$  in 16 is positive. From 15 it's easy to infer that for  $\sigma > 1$  we have  $\tilde{L} + \sigma(1 - \sigma)\frac{h(\tilde{L})}{h'(\tilde{L})} = \tilde{L} - \sigma\frac{C}{W} < 0$ . Therefore as noted in Pelloni and Waldmann (1997) for  $\sigma > 1$  any balanced growth equilibrium is locally unstable. Since  $\tilde{L}$  is a continuous function of  $L$  local instability implies global instability. Thus, balanced growth equilibrium is unique for  $\sigma$  greater than one.

However it is fairly clear that inequality 16 will hold for some production and utility functions, for  $\sigma$  less than one, so that stability obtains. For this it is necessary i) that the elasticity of substitution of capital and labour is very low so that the wage falls sharply and the interest rate rises sharply as labour supply increases and ii) that  $\sigma$  is small so that the marginal utility of consumption declines only slowly as consumption increases. That it is possible for inequality 16 to hold is shown in Pelloni and Waldmann (1997) by giving a specific example of a stable balanced growth path which satisfies the transversality condition.

A heuristic explanation of the importance of a high elasticity of consumption ( $\sigma < 1$ ) for stability is the following: for stability it is necessary that if the initial consumption to capital ratio is slightly higher than balanced growth consumption to capital ratio then the rate of growth of consumption is lower than the rate of growth of capital. High initial consumption can imply a low rate of consumption if leisure is a normal good (as it must be if  $\sigma < 1$ ) so high initial consumption implies low initial labour supply. If the elasticity of substitution between labour and capital is low the reduction in labour supply will cause a sharp decline in the interest rate. But with a high elasticity of substitution of consumption this can cause an even sharper decline in the rate of growth of consumption which can decline even more than the rate of growth of capital. Thus the economy can return to the balanced growth path.

A simple diagrammatic analysis can help in clarifying the issue of the stability of equilibrium. Figures 1 and 2 show, as functions of  $L$ , the  $\frac{K}{K}$  curve, that is, the budget constraint, and the  $\frac{r - \rho}{\sigma}$  curve—which equals  $\frac{C}{C}$  if  $L = \tilde{L}$  is constant. At balanced growth  $\tilde{L}$  the  $\frac{K}{K}$  curve intersects the  $\frac{r - \rho}{\sigma}$  curve. If the  $\frac{K}{K}$  curve cuts the  $\frac{r - \rho}{\sigma}$  curve from above, as in figure 1, the balanced growth path is stable, while if the  $\frac{K}{K}$  curve cuts the  $\frac{r - \rho}{\sigma}$  curve from below, as in figure 2, the

balanced growth path is unstable. This is clear from inspection of equation 13, given that coefficient of  $\dot{L}$  in the equation is always negative.<sup>2</sup>

### 3 Implications for Public Policy

In this section we show that in the model described above, that extends the Romer (1986) model only by allowing for elastic labour supply many counterintuitive effects of policy appear. First policies of taxing capital income or lump sum and throwing away the revenue can be welfare increasing. Moreover even if investment is inefficiently low, if the government taxes citizens that consume less than average and subsidize citizens that consume more than average, the result can be an increase in the rate of growth and in welfare. The same effect would occur if the government forced a higher consumption to capital ratio by decree by e.g. threatening to punish consumers who consume too little. The analysis of stability in the preceding section is used to show that the signs of the effects of tax policies depend on whether the no-tax equilibrium is stable or unstable. In particular, if the equilibrium is stable, taxing capital income (a little) and destroying the proceeds improves welfare, while if the equilibrium is unstable subsidizing consumption (a little) improves welfare.

Throughout this section we suppress the argument of  $f$ , and  $h$  and their derivatives in order to simplify the notation.

#### 3.1 Tax on Capital, Proceeds Thrown Away

We will now analyze the steady state effects on growth and welfare of a tax on capital, whose proceeds are thrown away or used to pay for useless expenditures. The budget constraint of each agent becomes

$$\frac{\dot{K}}{K} = r(1 - \tau_k) + \frac{WL}{K} - \frac{C}{K} \quad (18)$$

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<sup>2</sup>Notice that both the  $\frac{r-\rho}{\sigma}$  curve and the  $\frac{\dot{K}}{K}$  curve always slope up. In fact for the former this is immediate since the rate of interest is increasing in labour. For the latter differentiating the right hand side of 9 with respect to labour we find:

$$(1 - \sigma) \frac{h}{h'} f'' + \sigma \left( 1 + (1 - \sigma) \left( 1 - \frac{hh''}{(h')^2} \right) \right) f' > 0$$

as we know from 17 and the conditions for concavity of  $f$  and the utility function.



In steady state the Euler equation is:

$$\sigma \frac{\dot{C}}{C} = r(1 - \tau_k) - \rho \quad (19)$$

Equating the rates of growth of capital and consumption we get the following implicit expression for the steady state level of labour:

$$\frac{\rho}{\sigma} = r(1 - \tau_k) \left( \frac{1 - \sigma}{\sigma} \right) + \frac{W}{K} \left( (\sigma - 1) \frac{h}{h'} - \tilde{L} \right) \quad (20)$$

Assuming the economy is always in balanced growth, the value function is, given 10 and 7:

$$V = \int_0^\infty e^{(-\rho+g(1-\sigma))t} \frac{C_0^{1-\sigma}}{1-\sigma} h dt = \int_0^\infty e^{(-\rho+g(1-\sigma))t} \frac{K_0^{1-\sigma}}{1-\sigma} \left( (\sigma - 1) \frac{h}{h'} f' \right)^{1-\sigma} h dt \quad (21)$$

where  $g$  is again the steady state rate of growth. Integrating and noticing that  $\rho - g(1 - \sigma) = \frac{\rho}{\sigma} - \frac{r(1-\tau_k)-\rho}{\sigma}(1 - \sigma) = \frac{W}{K} \left( (\sigma - 1) \frac{h}{h'} - \tilde{L} \right)$ , where for the last equality we have used 20, 21 can be rewritten as:

$$V = \frac{h^{2-\sigma} K_0^{1-\sigma} \left( \frac{(\sigma-1)}{h'} \right)^{1-\sigma}}{(1-\sigma)(f')^\sigma \left( (\sigma-1) \frac{h}{h'} - \tilde{L} \right)} dt \quad (22)$$

Since  $\frac{\log((1-\sigma)V)}{(1-\sigma)}$  is an increasing transformation of  $V$ , in general for the introduction of a tax  $\tau$  to be beneficial, starting from a no tax equilibrium, we need:

$$\frac{d \log((1-\sigma)V)}{(1-\sigma) d\tau} \Big|_{\tau=0} = \frac{\partial(\log(1-\sigma)V)}{(1-\sigma) \partial \tau} \Big|_{\tau=0} + \frac{\partial L}{\partial \tau} \frac{\partial \log((1-\sigma)V)}{(1-\sigma) \partial L} \Big|_{\tau=0} > 0 \quad (23)$$

To keep the notation simple we suppress the notation  $|_{\tau=0}$  below. All derivatives are to be evaluated at the no tax market equilibrium. That is, we consider the first order effects of small taxes.

When  $V$  is expressed as a function of  $K_0$  and  $\tilde{L}$  as in 22 the partial derivative of  $V$  with respect to  $\tau_k$  is zero. To calculate whether introducing the tax can be beneficial, we just calculate  $\frac{1}{(1-\sigma)} \frac{\partial \log((1-\sigma)V)}{\partial L} \frac{dL}{d\tau_k}$ . In appendix A we show that, starting from the no tax equilibrium, an increase in labour supply causes increased welfare if and only if  $\sigma$  is less than one.

Therefore the tax on capital increases welfare if  $\sigma$  is less than one and the tax causes increased labour supply. Differentiating 20 we obtain

$$\frac{dL}{d\tau_k} = \frac{r(\sigma - 1)}{B(\tilde{L})} \quad (24)$$

We recall that  $B(\tilde{L})$  being negative is the condition 16 for indeterminacy. So if the economy is stable the tax will increase labour supply, while if it is unstable it will increase labour supply if and only if  $\sigma$  is bigger than one. This implies, given what we know of the effect on welfare of increasing labour supply, that the tax will increase welfare if and only if the economy is stable. Notice that in this case the tax will increase the growth rate as well. In fact when  $\sigma$  is less than one, leisure and the ratio between consumption and capital must move together.<sup>3</sup> So the tax will reduce consumption at time 0 and leisure at all times. But since from 21 we can see that  $V$  is an increasing function of  $\frac{C_0}{K_0}$ ,  $1 - \tilde{L}$  and  $g$ , given  $K_0$ , we obtain the result that, if the tax increases welfare, then it increases growth.

To show that when the economy is unstable the tax will decrease growth as well as welfare we calculate

$$\begin{aligned} \frac{dg}{d\tau_k} &= \frac{-r}{\sigma} - \frac{\tilde{L}f''}{\sigma} \frac{dL}{d\tau_k} = \\ &= -\frac{r \left( \tilde{L} + (1 - \sigma) \frac{h}{h'} \right) f'' + \sigma \left( 1 + (1 - \sigma) \left( 1 - \frac{hh''}{(h')^2} \right) \right) f'}{\sigma B(\tilde{L})} \end{aligned} \quad (25)$$

$B(\tilde{L})$  is positive when the economy is unstable. We know from 15 that  $\tilde{L} + (1 - \sigma) \frac{h}{h'} < 0$  and from 17 that  $\left( 1 + (1 - \sigma) \left( 1 - \frac{hh''}{(h')^2} \right) \right) > 0$ . So we can conclude that  $\frac{dg}{d\tau_k}$  is negative when the economy is unstable. This proves the assertion and also confirms that when, on the contrary, the equilibrium is stable the tax will increase growth. Summing up, a capital income tax can induce people to work more and when the economy is stable this will actually increase the after tax rate of return on capital. When this happens there is a positive effect on welfare, as the effect of increased growth outweighs the direct cost of the tax.

### 3.2 Tax on Capital, Proceeds Returned Lump Sum.

It is not easy to give an intuitive explanation of the results described above. It is somewhat easier to explain the slightly less surprising result that, when the no-tax balanced growth path equilibrium is stable, a small tax on capital whose proceeds are returned as a lump sum increases growth and welfare. In

<sup>3</sup>Differentiating 10 we get:

$$\frac{\partial C/K}{\partial L} = (\sigma - 1) \frac{h}{h'} f'' + (\sigma - 1) \left( 1 - \frac{hh''}{(h')^2} \right) f'$$

which can be easily signed when  $\sigma < 1$ , given 2, 3 and 4.



this case, since tax revenues are returned to consumers, the budget constraint is unaffected by the tax and so is given by 9, the marginal rate of substitution between consumption and leisure equals their relative price if 10 holds, while the Euler equation is given by 19. As above, we assume that the economy is always in balanced growth.<sup>4</sup> Equating the rates of growth of capital and consumption gives:

$$\frac{(1 - \tau_k^l)r - \rho}{\sigma} = f + (1 - \sigma)\frac{h}{h'}f' \quad (26)$$

where  $\tau_k^l$  is the tax on capital income whose proceeds are rebated equally to each consumer. Figures 3 and 4 show the effect of a tax on capital whose proceeds are rebated. This  $\frac{r-\rho}{\sigma}$  is replaced by the  $\frac{(1-\tau_k^l)r-\rho}{\sigma}$  curve down while the  $\frac{K}{K}$  curve is not affected. Figure 3 shows that, if the balanced growth path is stable, the tax and transfer policy causes increased growth, while figure 4 shows that, if the balanced growth path is unstable the tax and transfer policy causes reduced growth.

In appendix B we show that the tax and transfer policy increases welfare if and only if it increases growth. Thus if the balanced growth path is stable, the tax and transfer policy increases growth and welfare. Indeed, as shown in appendix B, taxing capital and returning the proceeds lump sum yields higher welfare than a equal tax whose proceeds are wasted. A comparison with the tax and waste policy described above makes it easy to understand why a small tax and transfer policy increases welfare when it increases growth. Discarding the revenues of the tax on capital would shift the  $\frac{K}{K}$  curve down which would give lower growth for the same  $L$  (as well as a smaller effect on balanced growth  $\tilde{L}$ ). The static first order condition is not affected by the tax on capital income, so the  $C/K$  ratio is given by the same function of  $L$  in either case. This means that a tax and rebate lump sum policy which implies the same balanced growth  $\tilde{L}$  as a tax and waste policy yields higher welfare. Since we know that a small tax and waste policy improves welfare if it increases growth, a tax and rebate lump sum policy definitely improves welfare when it increases growth.

### 3.3 Tax on Capital, Proceeds Returned as a Subsidy to Labour

The results above show that policies which increase labour supply can cause increased growth and welfare. It is tempting to guess that such beneficial effects

<sup>4</sup>We also assume that the balanced growth path is unique or, at least, that small changes in taxes do not cause the economy to jump to a different balanced growth path

of a tax would be increased if the tax revenues were used to subsidize labour income. In particular, one might suspect that the policy of taxing capital to subsidize labour might increase growth even if the balanced growth path is unstable. In our model this is not true. In fact, as is shown in appendix C, a tax capital used to subsidize labour causes increased growth and welfare if and only if the balanced growth path is stable, that is, if and only if a tax capital and waste policy causes increased growth and welfare. Furthermore, as is shown in appendix C, for the same tax on capital, higher growth and welfare is obtained if the revenues are wasted than if they are returned as a subsidy to labour. This result surprised us, but it can be understood if one recalls that a condition for balanced growth is that the income and substitution effects of increased after tax wages cancel.

### 3.4 Lump-Sum Taxes

In this section we consider the effects of wasteful spending funded by a lump sum (poll) tax. That is we consider a tax in which all consumers are required to pay an equal sum which depends only on aggregate variables so they take their tax liabilities as given. This is very interesting for two reasons.

Any effect of lump sum taxes on Pareto efficiency is interesting, because it is often maintained, by analogy with a Walrasian model, that lump sum taxes do not affect efficiency. Therefore it is a common practice to assume that revenues are rebated lump sum when considering the effects of taxes. In a model with externalities, the income effect of lump sum taxes can cause more efficient (Pareto improving) individual choices even if there is no substitution effect. This means that lump sum taxes can improve the welfare of a representative agent as is shown below.

In particular, in this model, it is interesting to note that waste funded by a lump sum tax causes increased growth if and only if the balanced growth path is unstable. This means that positive effects of waste on growth in this model do not require any particular parameter values. For any parameter values, either waste funded by a tax on capital or waste funded by a poll tax cause increased growth.

In this section, we assume that the government taxes each citizen in the population  $\tau_a$  and throws the proceeds away. We consider a policy of the form

$$\tau_a \bar{K} = \tau_a K \quad (27)$$



this means that the government taxes each taxpayer a constant percentage  $\tau_a$  of the *average* capital stock  $\bar{K}$ , whose path the agents, being atomistic, take as a variable beyond their control. This makes the policy a lump sum tax. Notice that the only equation affected by the introduction of the tax is the budget constraint, while equations 10 and 11 remain the same. The balanced growth labour supply now is given by:

$$\frac{\rho}{\sigma} = r\left(\frac{1-\sigma}{\sigma}\right) + \frac{W}{K} \left( (\sigma-1)\frac{h}{h'} - \tilde{L} \right) + \tau_a \quad (28)$$

By differentiating 28 we get

$$\frac{\partial L}{\partial \tau_a} = \frac{\sigma}{B(\tilde{L})} \quad (29)$$

from this we see that the tax will raise the balanced growth labour supply if and only if the system is unstable, as is shown in figures 5 and 6.

In appendix D we show that a small lump sum tax and waste policy can increase welfare for a wide range of parameter values, which are generally considered palusible. Basically, it is necessary that the marginal utility of consumption not decline very quickly in consumption, that the elasticity of substitution of capital and labour is low and that the elasticity of labour supply is low. A heuristic explanation of these results is the following. If utility is very concave in consumption increased growth is not very important since consumers will soon be virtually satiated in any case. Thus the short term cost of the tax and waste policy would outweigh the beneficial effect on growth. In appendix D we give examples with  $\sigma$  as high as 2 in which the lump sum tax and waste policy improves welfare. It is necessary for labour and capital to be poor substitutes so that a small increase in  $\tilde{L}$  causes a large increase in  $r$  and therefore in growth. Finally a low elasticity of labour supply implies that the tax and waste policy causes a small increase in  $\tilde{L}$  with only small (short run) utility costs. The required value of  $\sigma$  is considered plausible by many economists and is, in fact, widely used for calibration purposes. The low elasticities of substitution of capital and labour and of labour supply are not usually assumed but are strongly supported by empirical evidence

A similar policy — incentives to consume — causes increased welfare whenever the economy is unstable.

### 3.5 A Consumption subsidy

A third counterintuitive result on tax issues is that a small consumption subsidy increases growth and welfare in this model whenever the balanced growth path is unstable. Therefore, for any parameter values in this model, either a policy of funding waste with a capital income tax or an incentive to consume improves growth and welfare.

Suppose that the government subsidizes consumption at the rate  $\tau_c$  paying for the policy with a lump sum tax, whose revenue is equal to  $\tau_c \bar{C}$ . The budget constraint of consumers becomes:

$$\frac{\dot{K}}{K} = r + \frac{WL}{K} - \frac{C(1-\tau_c)}{K} - \frac{\tau_c}{K} \bar{C} \quad (30)$$

and in equilibrium is not affected. Nor is the Euler equation affected. The only effect is on the intra temporal choice between consumption and leisure which becomes

$$C = \frac{(\sigma - 1)hW}{(1 - \tau_c)h'} \quad (31)$$

The balanced growth labour supply is then given by:

$$\frac{\rho}{\sigma} = r\left(\frac{1-\sigma}{\sigma}\right) + \frac{W}{K} \left( \frac{(\sigma-1)h}{(1-\tau_c)h'} - \tilde{L} \right) \quad (32)$$

this implies that

$$\frac{\partial L}{\partial \tau_c} = \frac{\sigma(\sigma-1)\frac{h}{h'}f'}{B(\tilde{L})} \quad (33)$$

The denominator of 33 is positive if the economy is unstable, the numerator is the ratio  $\sigma C/K$  which is, of course, always positive, so the subsidy will increase labour supply and growth if and only if the equilibrium is unstable. We will now show that this increase is welfare enhancing.

We have in this case:

$$V(\tau_c) = \frac{h^{2-\sigma} K_0^{1-\sigma} \left( \frac{(\sigma-1)}{(1-\tau_c)h'} f' \right)^{1-\sigma}}{(1-\sigma) \left( \frac{(\sigma-1)}{(1-\tau_c)h'} \frac{h}{h'} - \tilde{L} \right) f'} \quad (34)$$

and

$$\frac{\partial \log(V(1-\sigma))}{(1-\sigma)\partial \tau_c} = \frac{\sigma \frac{h}{h'} - \tilde{L}}{(\sigma-1)\frac{h}{h'} - \tilde{L}} \quad (35)$$



that is

$$\frac{d \log(V(1-\sigma))}{(1-\sigma)d\tau_c} = \frac{\sigma \frac{h}{h'} - \tilde{L}}{(\sigma-1)\frac{h}{h'} - \tilde{L}} - \frac{\left( \frac{(2-\sigma)h'}{h} + \frac{(\sigma-1)h''}{h'} - \sigma \frac{f''}{f'} + \frac{\left(1+(1-\sigma)(1-\frac{hh''}{(h')^2})\right)}{((\sigma-1)\frac{h}{h'} - \tilde{L})} \right) f' \frac{h}{h'}}{\left( \frac{\tilde{L}}{\sigma} + (1-\sigma)\frac{h}{h'} \right) f'' + \left(1+(1-\sigma)(1-\frac{hh''}{(h')^2})\right) f'} > 0 \quad (36)$$

To prove the inequality we note that the denominators of both fractions are positive, so the inequality is equivalent to:

$$\begin{aligned} & \left( \sigma \frac{h}{h'} - \tilde{L} \right) \left( \left( \frac{\tilde{L}}{\sigma} + (1-\sigma)\frac{h}{h'} \right) f'' + \left(1+(1-\sigma)(1-\frac{hh''}{(h')^2})\right) f' \right) \\ & - \left( (\sigma-1)\frac{h}{h'} - \tilde{L} \right) \left( \frac{(2-\sigma)h'}{h} + \frac{(\sigma-1)h''}{h'} - \sigma \frac{f''}{f'} + \frac{\left(1+(1-\sigma)(1-\frac{hh''}{(h')^2})\right)}{((\sigma-1)\frac{h}{h'} - \tilde{L})} \right) f' \frac{h}{h'} \\ & = -\frac{\tilde{L}^2 f''}{\sigma} > 0 \end{aligned} \quad (37)$$

which inequality always holds.

Notice that the benefit of the consumption subsidy occurs because of the increase in consumption as a function of  $L$ . The same outcome could occur if the new higher consumption to capital ratio were imposed in some other way, such as, via a law mandating high consumption.<sup>5</sup>

The mechanism driving the result is similar to that behind the lump sum tax. Either increased consumption or a lump sum tax and waste policy reduces  $\frac{K}{K}$  as a function of  $L$  without affecting  $\frac{r-p}{\sigma}$  as a function of  $L$ . However the present policy has the advantage that there is no waste of resources.

## 4 Conclusions

In this paper we have proposed a model with one good produced from capital and labour that can have a stable balanced growth path.

We show that many unexpected effects of tax policy appear. We show that a small amount of capital taxation, contrary to the received opinion, will be both growth and welfare increasing whenever the balanced growth path is stable. This is true whether the proceeds are thrown away, rebated lump-sum or used to subsidize labour.

<sup>5</sup>Although this might not be the unique possible outcome, given such a law.

The effect of lump sum taxation is also surprising. It turns out that when the economy is unstable the rate of growth is increased by a policy of pure waste funded via a poll tax, and that for a broad range of parameters values this will increase welfare as well.

Finally, when the economy is unstable, even if saving is inefficiently low if the government taxes citizens that consume less than average a small amount and subsidize citizens that consume more than average, the result will be an increase in the rate of growth and in welfare.

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## A Labour Supply and Welfare

In this appendix we prove that welfare is increasing in  $L$ , at the no-tax balanced growth  $\tilde{L}$ , if and only if  $\sigma > 1$ . We notice that, for any kind of tax  $\tau$  the effect of  $L$  on  $V$  is the same, that is:

$$\frac{\partial \log((1-\sigma)V)}{(1-\sigma)\partial L} = \frac{1}{(1-\sigma)} \left( \frac{(2-\sigma)h'}{h} + \frac{(\sigma-1)h''}{h'} - \sigma \frac{f''}{f'} + \frac{(1+(1-\sigma)(1-\frac{hh''}{(h')^2}))}{((\sigma-1)\frac{h}{h'} - \tilde{L})} \right) \quad (38)$$

It is possible to sign this effect. In fact the term inside the square brackets is positive for all values of  $\sigma$ , which means that increasing the balanced growth labour supply increases welfare, if and only if  $\sigma$  is less than one. This can be seen as follows.

For  $\sigma > 1$ , the sum of the first two terms is positive if

$$\frac{(2-\sigma)h'^2 + (\sigma-1)h''h}{hh'} > 0 \quad (39)$$

Since the denominator is positive we want the numerator positive as well and we have indeed:

$$(2-\sigma)h'^2 + (\sigma-1)h''h > (2-\sigma)h'^2 + \frac{(\sigma-1)^2}{\sigma}h'^2 = \frac{h'^2}{\sigma} \quad (40)$$

The inequality in 40 stems from 5. Also from 17 we know that the numerator of the fourth term in 38 is always positive, while from 15 we have that the denominator is positive.

For  $\sigma < 1$ , the sum of the terms in  $h$ ,  $h'$  and  $h''$  in 38, divided by the positive term  $((\sigma-1)\frac{h}{h'} - \tilde{L})$  is

$$\begin{aligned} & \left( (\sigma-1)\frac{h}{h'} - \tilde{L} \right) \left( \frac{(2-\sigma)h'}{h} - \frac{(1-\sigma)h''}{h'} \right) + \left( 1 + (1-\sigma)(1 - \frac{hh''}{(h')^2}) \right) = \\ & \sigma(2-\sigma) + \sigma(\sigma-1)\frac{hh''}{(h')^2} + \tilde{L} \left( \frac{(\sigma-2)h'}{h} + \frac{(1-\sigma)h''}{h'} \right) > 0 \end{aligned} \quad (41)$$

The inequality is immediate from 2, 3 and 4.



## B Tax on Capital Returned Lump Sum

In the case of a tax on capital income whose proceeds are returned lump sum, 22 becomes 42

$$V = \frac{h^{2-\sigma} K_0^{1-\sigma} \left( \frac{(\sigma-1)}{h'} f' \right)^{1-\sigma}}{(1-\sigma) \left( \left( (\sigma-1) \frac{h}{h'} - \tilde{L} \right) f' - r\tau_k^l \right)} dt \quad (42)$$

as  $\rho - g(1-\sigma) = -r\tau_k^l + \left( -(1-\sigma) \frac{h}{h'} - \tilde{L} \right) f'$ . When  $V$  is expressed as 42 then

$$\frac{\partial \log((1-\sigma)V)}{(1-\sigma)\partial \tau_k^l} = \frac{r}{(1-\sigma) \left( -(1-\sigma) \frac{h}{h'} - \tilde{L} \right) f'} \quad (43)$$

while differentiating 26 we have

$$\frac{dL}{d\tau_k^l} = \frac{-r}{B(\tilde{L})} \quad (44)$$

this is positive if and only if the system is stable. We can write

$$\begin{aligned} \frac{d \log(V(1-\sigma))}{(1-\sigma)d\tau_k^l} &= \frac{r}{(1-\sigma) \left( -(1-\sigma) \frac{h}{h'} - \tilde{L} \right) f'} - \\ &\frac{r}{(1-\sigma)B(\tilde{L})} \left( \frac{(2-\sigma)h'}{h} + \frac{(\sigma-1)h''}{h'} - \sigma \frac{f''}{f'} + \frac{(1+(1-\sigma)(1-\frac{hh''}{(h')^2}))}{(-(1-\sigma)\frac{h}{h'} - \tilde{L})} \right) \end{aligned} \quad (45)$$

If the system is unstable 45 has the same sign as 46, if it is stable the opposite sign

$$\begin{aligned} &\left( \tilde{L} + \sigma(1-\sigma) \frac{h}{h'} \right) \frac{f''}{(1-\sigma)} + \left( 1 + (1-\sigma)(1 - \frac{hh''}{(h')^2}) \right) \frac{\sigma f'}{(1-\sigma)} \\ &+ \left( (1-\sigma) \frac{h}{h'} + \tilde{L} \right) \left( \frac{(2-\sigma)h'}{h} - \frac{(1-\sigma)h''}{h'} - \sigma \frac{f''}{f'} + \frac{(1+(1-\sigma)(1-\frac{hh''}{(h')^2}))}{(-(1-\sigma)\frac{h}{h'} - \tilde{L})} \right) \frac{f'}{(1-\sigma)} \\ &= \tilde{L}f'' + \frac{\tilde{L}f'}{(1-\sigma)} \left( \frac{(\sigma-2)h'}{h} + \frac{(1-\sigma)h''}{h'} \right) < 0 \end{aligned} \quad (46)$$

So we conclude that the program is welfare increasing if the economy is stable and welfare decreasing if it is unstable. Coming to the comparison with the

benchmark case in which the proceeds of the tax on capital are thrown away we have that returning them is welfare increasing. In fact:

$$\frac{d \log((1-\sigma)V)}{(1-\sigma)d\tau_k^L} - \frac{d \log((1-\sigma)V)}{(1-\sigma)d\tau_k} = \frac{\frac{r}{1-\sigma}}{\left(- (1-\sigma)\frac{h}{h'} - \tilde{L}\right) f'} - \frac{\sigma r \partial \log((1-\sigma)V)}{B(\tilde{L})(1-\sigma)\partial L} > 0 \quad (47)$$

## C Tax on Capital Used to Subsidize Labour

Given the result that a tax on capital which causes increased labour supply can improve welfare, one might imagine that it would be still better to use the revenues to subsidize employment. In fact, this policy is never superior both to taxing capital and wasting the proceeds and to doing nothing. We show that the policy of using a tax on capital to subsidize labour increases growth and welfare if and only if the balanced growth path is stable, that is, if and only if the tax and waste policy increases growth and welfare. Further we show that, if the balanced growth path is stable, a tax capital and waste policy yields higher welfare than a tax capital to subsidize labour policy.

When the tax on capital is coupled with a subsidy on labour  $s_w$ , with the government running a balanced budget, we have  $W s_w L = r \tau_k^s K$ . 10 becomes

$$\frac{(\sigma-1)h(l)}{Ch'} = \frac{1}{(1+s_w)W} = \frac{1}{\left(1 + \frac{r\tau_k^s K}{WL}\right)W} \quad (48)$$

The Euler condition in steady state is still 19, while the rate of growth of capital is given by 9. In steady state labour supply is given by

$$\frac{\rho}{\sigma} = r \left( \frac{1-\tau_k^s-\sigma}{\sigma} \right) + \frac{W}{K} \left( (\sigma-1)\frac{h}{h'} \left( 1 + \frac{r\tau_k^s K}{WL} \right) - \tilde{L} \right) \quad (49)$$

To calculate the effects on welfare we have, given 49,  $\rho = \frac{r(1-\tau_k^s)-\rho}{\sigma}(1-\sigma) = r\tau_k^s \left( (\sigma-1)\frac{h}{h'L} - 1 \right) + \left( (\sigma-1)\frac{h}{h'} - \tilde{L} \right) f'$ . Given 48 this allows us to write

$$(1-\sigma)V(\tau_k^s) = \frac{hK_0^{1-\sigma} \left( \frac{(\sigma-1)h}{h'} \left( 1 + \frac{r\tau_k^s}{f'L} \right) f' \right)^{1-\sigma}}{r\tau_k^s \left( (\sigma-1)\frac{h}{h'L} - 1 \right) + \left( (\sigma-1)\frac{h}{h'} - \tilde{L} \right) f'} dt \quad (50)$$

then

$$\frac{\partial \log(1-\sigma)V(\tau_k^s)}{(1-\sigma)\partial \tau_k^s} = -\sigma \frac{r}{(1-\sigma)\tilde{L}f'} \quad (51)$$



Differentiating 49 we find

$$\frac{dL}{d\tau_k^s} = \frac{-r \left(1 + \sigma(1 - \sigma) \frac{h}{h'L}\right)}{B(\tilde{L})} \quad (52)$$

If the economy is stable the denominator of 52 will be negative and the numerator negative as well. If it is unstable and  $\sigma > 1$ , both will be positive, while if it is stable and  $\sigma < 1$  the sign of the numerator is ambiguous.

Considering the condition 23, assuming the system is unstable and simplifying,  $\frac{d((1-\sigma)\log V(\tau_k^s))}{(1-\sigma)d\tau_k^s}$  is seen to have the same sign as

$$-\frac{\sigma}{(1-\sigma)} \left(1 + (1-\sigma)\left(1 - \frac{hh''}{(h')^2}\right)\right) - \frac{1}{(1-\sigma)} \left(\frac{\tilde{L}}{\sigma} + (1-\sigma)\frac{h}{h'}\right) \left(\frac{(2-\sigma)h'}{h} - \frac{(1-\sigma)h''}{h'} + \frac{1 + (1-\sigma)\left(1 - \frac{hh''}{(h')^2}\right)}{((\sigma-1)\frac{h}{h'} - \tilde{L})}\right) \quad (53)$$

this means that for the tax program to be beneficial we want this expression to be positive. However this is impossible. In fact, 53 can be rewritten as

$$\frac{-\frac{\tilde{L}^2}{\sigma} \left(\frac{(2-\sigma)h'}{h} - \frac{(1-\sigma)h''}{h'}\right)}{(1-\sigma) \left((1-\sigma)\frac{h}{h'} + \tilde{L}\right)} < 0 \quad (54)$$

Again we have that the policy will be welfare increasing if and only if the system is stable.

Coming to the effect of the tax on growth we have

$$dg/d\tau_k^s = \frac{d}{d\tau_k^s} \left(\frac{r(1-\tau_k^s) - \rho}{\sigma}\right) = \frac{-r}{\sigma} + \frac{r \left((1-\sigma)\frac{h}{h'} + \tilde{L}\right) \tilde{L}f''}{\sigma B(\tilde{L})} = \frac{-r}{B(\tilde{L})} \left(1 + (1-\sigma)\left(1 - \frac{hh''}{(h')^2}\right)\right) f' \quad (55)$$

so again the tax will boost growth if and only if the system is stable.

Another interesting fact is that in this case it is better to throw away the proceeds from the tax on capital than to use them to subsidize labour, as we have:

$$\frac{d \log((1-\sigma)V(\tau_k^s))}{(1-\sigma)d\tau_k^s} - \frac{d \log((1-\sigma)V(\tau_k))}{(1-\sigma)d\tau_k} = -\sigma \frac{r}{(1-\sigma)\tilde{L}f'} +$$

$$\frac{\partial \log((1-\sigma)V)}{(1-\sigma)\partial L} \frac{-r\left(1+\sigma(1-\sigma)\frac{h}{h'L}\right) - r(\sigma-1)}{B(\tilde{L})} =$$

$$-\sigma \frac{r}{(1-\sigma)\tilde{L}f'} + \frac{\partial \log((1-\sigma)V)}{(1-\sigma)\partial L} \frac{r\sigma\left((\sigma-1)\frac{h}{h'L} - 1\right)}{B(\tilde{L})} < 0 \quad (56)$$

## D Welfare Effects of a Lump Sum Tax

Here we show that a policy of funding waste via a poll tax can increase the representative consumer's welfare for a broad range of parameter values which are considered plausible by many economists.

Using a procedure analogous to that seen in the case of capital taxation we arrive to:

$$V = \frac{h^{2-\sigma} K_0^{1-\sigma} \left(\frac{(\sigma-1)}{h'} f'\right)^{1-\sigma}}{(1-\sigma) \left(\tau_a + \left((\sigma-1)\frac{h}{h'} - \tilde{L}\right) f'\right)} dt \quad (57)$$

the condition 23 for the tax to be welfare increasing becomes in this case, using 38 and 29:

$$\frac{d \log(V(1-\sigma))}{(1-\sigma)d\tau_a} = \frac{1}{(\sigma-1) \left((\sigma-1)\frac{h}{h'} - \tilde{L}\right) f'} +$$

$$\frac{\sigma}{(1-\sigma)B(\tilde{L})} \left( \frac{(2-\sigma)h'}{h} + \frac{(\sigma-1)h''}{h'} - \sigma \frac{f''}{f'} + \frac{\left(1 + (1-\sigma)\left(1 - \frac{hh''}{(h')^2}\right)\right)}{\left((\sigma-1)\frac{h}{h'} - \tilde{L}\right)} \right) > 0 \quad (58)$$

where the first fraction measures  $\frac{\partial \log((1-\sigma)V)}{(1-\sigma)\partial \tau_a}$ . This, given 15, is immediately seen to be negative if  $\sigma$  is less than one and positive if  $\sigma$  is greater than one. This is enough to know that if the economy is stable the tax has a negative impact on welfare. In fact  $\frac{\partial \log((1-\sigma)V)}{(1-\sigma)\partial \tau_a}$  is negative and the effect working through the induced change in  $\tilde{L}$  is negative as well since the tax will reduce labour supply when the equilibrium is stable, which has a negative impact on welfare when  $\sigma < 1$ . So we study what happens when the equilibrium is unstable. We first consider the case of  $\sigma > 1$ . The denominator of both fractions in 58 is positive so the condition for the tax to be welfare increasing can be expressed, after some simplifying, as:

$$\left(\tilde{L}\left(\frac{1}{\sigma} - \sigma\right) + (1-\sigma)^2 \frac{h}{h'}\right) f'' + \left((\sigma-1)\frac{h}{h'} - \tilde{L}\right) \left(\frac{(2-\sigma)h'}{h} + \frac{(\sigma-1)h''}{h'}\right) f' > 0 \quad (59)$$



When  $\sigma < 1$ , the denominator of the first fraction in 58 is negative while that of the second fraction is positive so the condition for the tax to be welfare increasing is 59 reversed.

We consider the following specification for tastes

$$h(L) = (1 - L)^{1-\chi} \quad (60)$$

On the technology side we consider a CES production function with a labour augmenting spill-over to capital and elasticity of substitution of  $\frac{1}{1-\phi}$ .  $\phi$  must be less than one for the production possibilities set to be convex. So we have:

$$Y = A \left( \alpha (\bar{K}L)^\phi + (1 - \alpha) K^\phi \right)^{\frac{1}{\phi}} \quad (61)$$

where  $A$  is a scale factor and  $\alpha \in (0, 1)$ .

Using the fact that  $L^\phi = \frac{1-\alpha}{\alpha} \frac{S_L}{1-S_L}$ , 59 becomes, after rearranging:

$$\begin{aligned} & \left( (1 - \sigma)(\phi - 1)(1 - S_L) \left( \frac{2 - \sigma - \chi}{1 - \chi} + \frac{1}{\sigma} \right) + \frac{(2 - \sigma - \chi)^2}{1 - \chi} \right) \tilde{L}^2 \\ & - \left( (1 - \sigma)(\phi - 1)(1 - S_L) \left( \frac{2 - \sigma - \chi}{1 - \chi} + \frac{1}{\sigma} + \frac{1 - \sigma}{1 - \chi} \right) + \frac{(2 - \sigma - \chi)(1 - \sigma)}{1 - \chi} \right) \tilde{L} \\ & + (\phi - 1)(1 - S_L) \frac{(1 - \sigma)^2}{(1 - \chi)} < 0 \end{aligned} \quad (62)$$

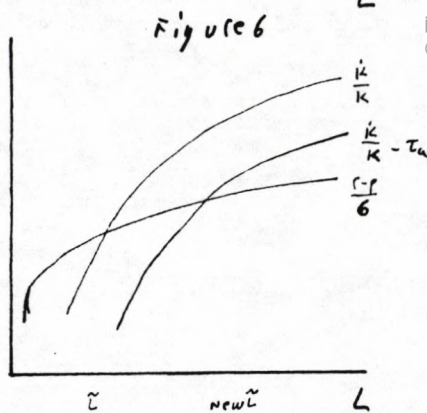
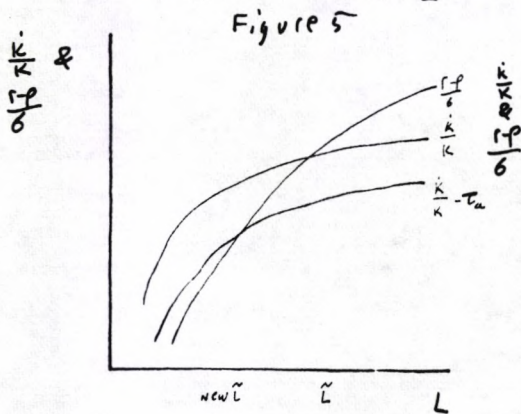
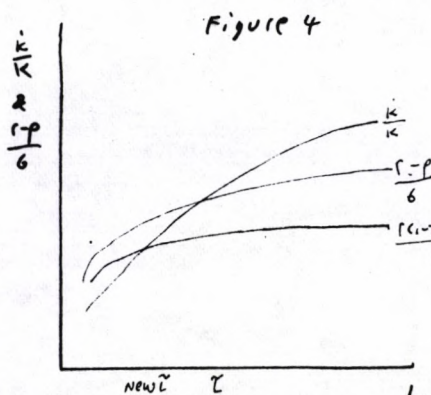
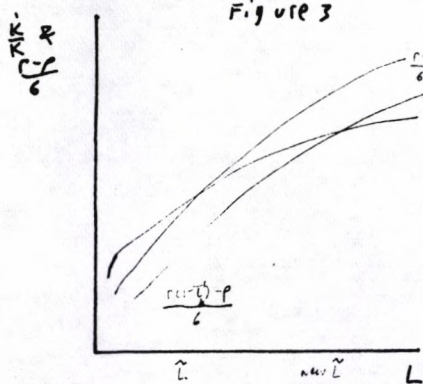
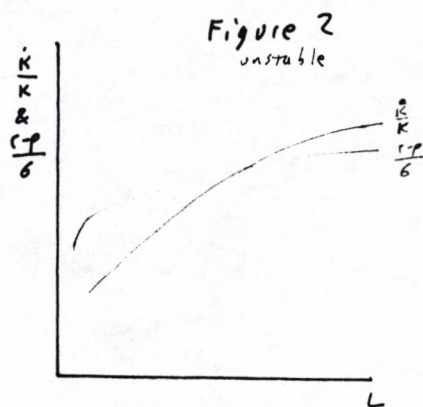
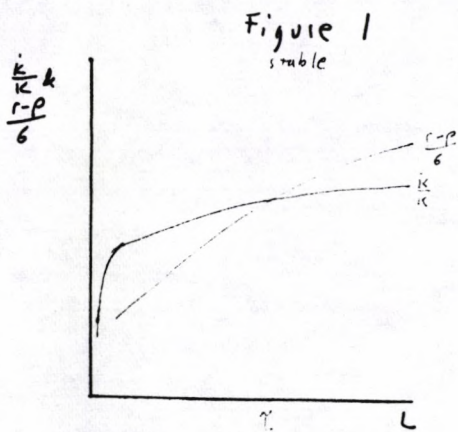
By substituting the balanced growth labour supply in 62 we see by simple calculations that a lump sum tax will be beneficial for a very broad range of parameter values which are considered plausible by many economists. For example focusing on the case of  $\sigma > 1$ , if we assume  $S_L = 0.6, v = 3, \phi = -10$ , the tax will be beneficial for  $\sigma$  as high as 1.9 provided  $\chi$  is high enough (7.1 or higher).<sup>6</sup> For the same but  $\phi = -5$ , the tax will be beneficial for  $\sigma$  as high as 1.7 again provided  $\chi$  is high enough (7.1 or higher). For  $S_L = 0.6, v = 2.5, \phi = -10$ , the tax is welfare increasing for  $\sigma$  as high as 2.1, provided  $\chi$  is higher than 8.1 etc.

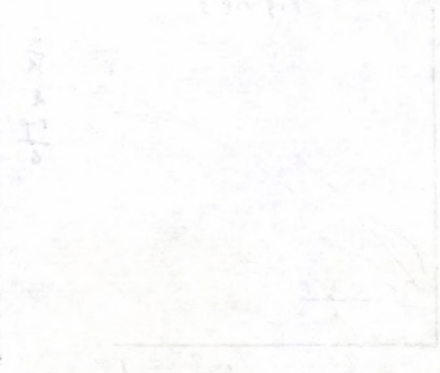
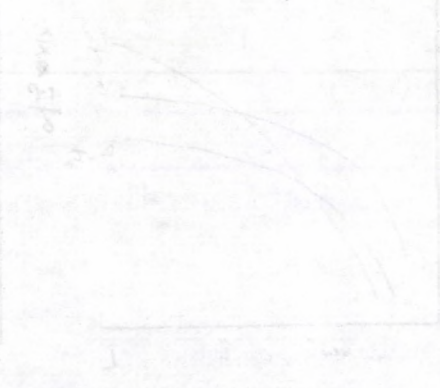
The standard practice of evaluating all other taxes by assuming revenues are returned as a lump sum transfer is misleading in this case. A lump-sum tax reducing disposable income induces people to work more thus increasing the rate of interest and in a sense internalizing the externalities. This has a

<sup>6</sup>Notice that since we have  $r = \frac{\rho v}{v - \sigma}$ ,  $\sigma$  cannot be greater than  $v$ .

positive effect on welfare. For the overall effect to be positive this indirect substitution effect must outweigh the negative direct effect on welfare caused by the subtraction of resources from the private sector.











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