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PRODUCTION
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Abstract

Technological knowledge and organizational knowledge are distinguished in order to define economic competence formally. This definition is made operational in the simplest possible linear model of production. Productivity gains are shown to originate from changes in organizational capabilities that are made effective by changes in the assignment of complementary input factors. They have no connection with economies of scale or scope. A numerical application illustrates the role of internal organization as a source of productivity in the operation of unchanged quantities of inputs combined in fixed proportions.

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1 INTRODUCTION

To the economic historian who examines business practices, the performance of firms reflects not the outcome of a deterministic technology but rather the degree of success of what is commonly referred to as organizational capabilities. In the words of Chandler (1992):

> "[T]he key concept I use to explain the similarities in the beginnings and growth of modern industrial enterprises is that of organizational capabilities. (...) *These capabilities were the collective physical facilities and human skills as they were organized within the enterprise.*" (p. 487, added emphasis)

Can organizational capabilities be distinguished from input factors involved in production? This is the question that motivates the following investigation. A tautological answer is to define organizational capabilities as what constitutes the competence of the firm or, in more common words, its ability to transform inputs and sell the resulting outputs profitably. Past contributions to the theory of the firm offered some more pervasive answers in models of production. While no brief summary can do justice to the richness of the literature that concentrates on the theory of the firm, two main streams can be identified that give contrasted answers to the question. The first one is commonly referred to as the *technological view* of the firm. In a very stylized manner, it favours the derivation of formal propositions founded on some explicitly defined axiomatic framework. This approach describes production as a "black box" in which the organizational capabilities of the firm are merely reflected by the specification of the quantities of all the outputs that can be obtained from some given inputs in the commodity space. The second stream can be referred to as the *organizational view* of the firm. From a more realistic stance, it opens the "black box" in order to put emphasis on the way input factors are used to produce outputs, that is on the internal organization of production. In this perspective, the organizational capabilities of the firm can be viewed as knowledge about how to exploit the potential services of given resources in production.

The objective of this paper is to draw some distinctive features from the two traditions in order to recast some intuitively appealing insights offered in the latter into a standard axiomatic setting of the kind introduced by the former. The analysis proceeds as follows. In section 2, the concepts of technological knowledge and organizational knowledge are defined that lead to a formal expression of economic competence. The nature of the firm is characterized as a set of organizational capabilities. In section 3 a simple linear model of production is described that encapsulates the concepts defined in the previous section. In section 4 some propositions are derived from the linear model that shed light on some Penrosian claims on the growth of the firm. These propositions concern the change in the maximum output level that results from a change in the quantity of available input factors and/or in the organizational capabilities of the firm. In section 5 a numerical application is discussed. Section 6 concludes the paper.

Throughout the analysis, the terms capital (or labour) resources, capital (or labour) inputs and capital (or labour) factors are used indifferently. The terms *worker* and *machine* are used for the sake of clarity in examples or illustrations. They do not rule out the study of the production of services of any kind despite the connotation of manufacturing activities they carry along.

2 The Nature of the Firm

This section defines the concepts of technological knowledge and organizational knowledge that lead to the formal expression of economic competence.

Consider an hypothetical representative firm that operates in a static production economy. It has M possible existing or potential factors of production serving as inputs to be transformed into N goods or services defined as outputs. Both input and output markets are competitive. Throughout the analysis vectors and matrices are denoted by bold characters while sets are denoted by capital characters. A technology transforming inputs $\mathbf{x} \in R_+^M$ into outputs $\mathbf{y} \in R_+^N$ is represented by a production possibility set that describes all production plans in the commodity space that are technically feasible for the firm. The sets of all subsets in R_+^M and in R_+^N are denoted by $2^{R_+^M}$ and $2^{R_+^N}$, respectively. Following Shephard (1970), the definition of two distinct sets for input and output quantities can be derived from the production possibility set. This turns out to be convenient a notation for the purposes of the analysis below and authorises the standard neo-classical definition of technological knowledge to be formulated as follows.

DEFINITION 1: TECHNOLOGICAL KNOWLEDGE *is described by the input correspondence* $X(\mathbf{y})$ *and the output correspondence* $Y(\mathbf{x})$*, which are such that:*

$X : R_+^N \to 2^{R_+^M}$ *maps outputs into subsets* $X(\mathbf{y}) \subseteq R_+^M$ *of inputs. The set* $X(\mathbf{y})$ *is called the input set and it denotes all input vectors* $\mathbf{x} \in R_+^M$ *that yield output* $\mathbf{y} \in R_+^N$*; and*

$Y : R_+^M \to 2^{R_+^N}$ *maps inputs into subsets* $Y(\mathbf{x}) \subseteq R_+^N$ *of outputs. The set* $Y(\mathbf{x})$ *is called the output set and it denotes all output vectors* $\mathbf{y} \in R_+^N$ *that are obtainable from* $\mathbf{x} \in R_+^M$.

Technological knowledge refers to all physically possible transformations of inputs $\mathbf{x} \in R_+^M$ into outputs $\mathbf{y} \in R_+^N$ that are constrained by the intrinsic characteristics of these inputs and outputs. The definition is in line with the theory of the firm that concentrates on the physically *embodied* form of knowledge as identified with "artifacts", that is the physical resources in which technological knowledge is embedded, namely capital assets (machines) and labour assets (workers). In more general terms, this form of technological knowledge can also be referred to as books of blueprints.

Beyond the intrinsic characteristics of inputs and outputs that impact on the productive possibilities of the firm, the technological limits on the actions of the firm can be modelled as influenced by the economic environment. This environment may be captured by a parameter vector (or matrix) $\mathbf{z}$ of real numbers. The environment in which production occurs is determined by exogenous parameters, which may be variable parameters or constant parameters. The variable parameters may impact on the productivity of input factors, and one is interested in how different values of these parameters determine different instances of production possibilities. The constant parameters may also impact on the productivity of input factors, but these parameters are considered to be fixed for all instances of production possibilities. Accordingly, they are not included among the arguments of $\mathbf{z}$. The values taken by the parameters of $\mathbf{z}$ put some restrictions on the input and output sets. The economic content captured by the parameters in $\mathbf{z}$ is depending upon the *raison d'être* of the production model under scrutiny.[1] In the present analysis, $\mathbf{z}$ is meant to capture the source of differences in productivity associated with differences in the assignment of complementary input factors such as machines and workers for the purpose of producing some good(s) or service(s).

[1] For a series of possible interpretations and an extensive introduction to the general representation of production possibilities, see McFadden (1978, pp. 60-66).

In order to make clear what is meant by an assignment of complementary input resources, consider the down-to-earth problem faced by production managers that concerns the combination of heterogeneous capital assets with heterogeneous labour assets. More eloquently, the problem is one of assigning different types of machines to different types of workers. The reduction of the real-world complexity of the huge set of inputs to exactly two complementary sets allows the model to capture positive externalities in production as induced from the combined use of heterogeneous inputs imposed by the technological knowledge, while leaving the analysis tractable. Accordingly, in the following capital assets and labour assets will be described by two subsets of inputs, making a notational distinction necessary. Define the subsets X_K and X_L that partition the set of all possible existing or potential input factors. Indices $k \in \{1, \ldots, K\}$ and $l \in \{1, \ldots, L\}$ label qualitatively different types of capital resources and labour resources, respectively, as distinguished with respect to their intrinsic characteristics. Observe that $K + L = M$. Define also the set $X_{K \times L} = X_K \times X_L$, that is the set of all possible pairs of complementary inputs. In the case of capital inputs, intrinsic characteristics are physical or functional attributes. In the case of labour inputs, intrinsic characteristics are manual or intellectual attributes. These characteristics are assumed throughout to be of an objective kind.

Consider the simplest possible situation in which the production of a good is made possible by the combination of one unit of capital input (a machine) and one unit of labour input (a worker), that is a pair $(x_k, x_l) \in X_{K \times L}$ with $x_k = x_l = 1$. Changes in the environment summarized by $\mathbf{z}$ impose some specifications on the production technology. These can relate to the types of inputs or to the types of outputs (or both) and also to any other contextual variable. More specifically, the contribution of a given (type of) worker or of a given (type of) machine to production may first depend on which task is being performed by the same pair (x_k, x_l). Each input renders a different service when combined in the same pair in order to produce qualitatively different outputs. Second, the contribution of a given (type of) worker or of a given (type of) machine to production may also depend on which pair (x_k, x_l) is combined for the performance of a given task. Each input renders a different service when contributing to the performance of the same task but when assigned in different pairs. Third, other production instances may come up from changes in the environment that relate neither to input or output characteristics nor to the nature of the task but to features such as effort conventions, corporate culture or institutional parameters of all kinds.

Concentrating on the latter case, let the same pair (x_k, x_l) perform the same task in two different environments. If productivity is compared in the two instances, the measured outcomes resulting from the performance of a given task by the same (type of) worker combined with the same (type of) machine can vary from one environment to another. Anything that may result in a negative or positive externality onto the productivity of given inputs applies. The elements in the environment at the origin of differences in the measured outcomes can be control variables (e.g., reward structures or compensation packages) implemented by the administrative entity that defines the tasks to be performed and designs the production structure that assigns types of capital inputs to types of labour inputs. These elements can be traced back neither in differences in the types of machine or in the types of worker nor in the nature of the task. They are responsible for variations in the marginal contribution to production of unchanged inputs participating to an unchanged production structure. Whatever the rationale that may be put forward for the explanation of the phenomenon, these elements are observed to enhance the productivity of inputs, other things being equal.

In any case, each pair (x_k, x_l) involves some embodied technological knowledge as defined by subsets of intrinsic characteristics, namely physical facilities (for capital assets) and human skills (for labour assets). When the assignment of input factors in the performance of an unchanged task remains constant, the embodied dimension of technological knowledge that is supported by the combined assets in use remains constant also. Accordingly, every variable parameter that impacts on the productivity of input factors belongs to a dimension of knowledge that is not embodied in the factors of production at work. If the rise in productivity cannot be explained by technological changes – i.e., changes in the volumes of capital and labour inputs – it must be explained by changes in organizational capabilities – i.e., changes in the ability to use given capital and labour resources. Whence the following tentative definition.

Definition 2: Organizational knowledge *is described by the set of pairs of capability parameters*

$$Z(\mathbf{y}) = \{z_{kl} : z_{kl} \in [0,1]\} \cup \{z_{lk} : z_{lk} \in [0,1]\},$$

where $k \in \{1, \ldots, K\}$ *and* $l \in \{1, \ldots, L\}$, *that are attached (i) to each input type* $k \in \{1, \ldots, K\}$ *when combined with some input type* $l \in \{1, \ldots, L\}$, *and (ii) to each input type* $l \in \{1, \ldots, L\}$ *when combined with some input type* $k \in \{1, \ldots, K\}$, *respectively, in the task* $(k, l) = (l, k)$ *for the production of some output* $\mathbf{y} \in Y(\mathbf{x})$.

$$
\begin{array}{c|ccc|}
\tilde{X}_K(\mathbf{y}) & 1 & \cdots l \cdots & L \\
\hline
1 & z_{11} & \cdots & z_{1L} \\
\cdots & & & \\
k & \cdots & z_{kl} & \cdots \\
\cdots & & & \\
K & z_{K1} & \cdots & z_{KL} \\
\hline
\end{array}
$$

Figure $1(a)$: **capability parameters** attached to each input type $k \in \{1, \ldots, K\}$ when combined with some input type $l \in \{1, \ldots, L\}$ in the productive task $(k, l) = (l, k)$. The capability attached to capital input k when combined with labour input l is z_{kl}.

$$
\begin{array}{c|ccc|}
\tilde{X}_L(\mathbf{y}) & 1 & \cdots k \cdots & K \\
\hline
1 & z_{11} & \cdots & z_{1K} \\
\cdots & & & \\
l & \cdots & z_{lk} & \cdots \\
\cdots & & & \\
L & z_{L1} & \cdots & z_{LK} \\
\hline
\end{array}
$$

Figure $1(b)$: **capability parameters** attached to each input type $l \in \{1, \ldots, L\}$ when combined with some input type $k \in \{1, \ldots, K\}$ in the productive task $(l, k) = (k, l)$. The capability attached to labour input l when combined with capital input k is z_{lk}.

The consideration of technological knowledge as given in Definition 1 and of organizational knowledge as given in Definition 2 motivates a tentative definition of economic competence, as follows.

DEFINITION 3: ECONOMIC COMPETENCE *for the production of some given* $\mathbf{y} \in Y(\mathbf{x})$ *is a relation* $\tilde{X}(\mathbf{y})$ *such that*

$$\tilde{X}(\mathbf{y}) = \{(x_k, z_{kl}) : z_{kl} = z_{kl}(x_k)\} \cup \{(x_l, z_{lk}) : z_{lk} = z_{lk}(x_l)\},$$

for all $(x_k, x_l) \in X_{K \times L}(\mathbf{y})$, *that is the union of the subsets of* $X_K(\mathbf{y}) \times [0,1]$ and $X_L(\mathbf{y}) \times [0,1]$ *containing the pairs* (x_k, z_{kl}) *and* (x_l, z_{lk}), *respectively.*

In other words, economic competence is defined as the union of the graphs of a mapping from $X_K(\mathbf{y})$ to $[0,1]$ that is such that $x_k \to z_{kl}$ and of a mapping from $X_L(\mathbf{y})$ to $[0,1]$ that is such that $x_l \to z_{lk}$. The scalars z_{kl} and z_{lk} are proxies for embodied knowledge as involved in some production activity and can be interpreted as the respective degrees of membership of the objective characteristics supported by each member in the pair (x_k, x_l) to the set $\tilde{X}(\mathbf{y})$ when combined together in the same task. Each input factor $x_k \in X_K(\mathbf{y})$ and $x_l \in X_L(\mathbf{y})$ is modelled as used – or exploited – in a given production instance to some degrees z_{lk} and z_{kl} that describe the range $[0,1]$ in all possible assignments to a complementary input factor $x_l \in X_L(\mathbf{y})$ and $x_k \in X_K(\mathbf{y})$, respectively. In substance, economic competence is the observed ability to exploit the intrinsic characteristics supported by input factors identified by the pair $(x_k, x_l) \in X_{K \times L}(\mathbf{y})$ involved in the production of some output vector $\mathbf{y} \in Y(\mathbf{x})$. This definition is an attempt to give some formal content to the following claim by Penrose (1959) that may be made operational in models of production:

> "Strictly speaking, it is never *resources* themselves that are the "inputs" in the production process, but only the *services* that the resources can render. The services yielded by resources are a function of the way in which they are used - exactly the same resource when used for different purposes or in different ways and in combination with different types or amounts of other resources provides a different service or set of services. (...) [I]t is largely in this distinction that we find the source of the uniqueness of each individual firm" (p. 25, original emphasis).

Capability parameters capture differences in the services that can be rendered by given resources $\mathbf{x} \in R_+^M$ involved in the production of some output $\mathbf{y} \in R_+^N$. For clarity, the terms in the vector $\mathbf{z}$ are obtained as displayed in Table 1 above. The production technology of the firm transforming inputs $\mathbf{x} \in R_+^M$ into outputs $\mathbf{y} \in R_+^N$ are now represented by the output correspondence $Y(\mathbf{x}, \mathbf{z})$ or by the input correspondence $X(\mathbf{y}, \mathbf{z})$. The elements of the capability vector $\mathbf{z}$ can take different values in the range bounded by the null vector and the unit vector. That is, $(0, \ldots, 0) \leq \mathbf{z} \leq (1, \ldots, 1)$. Different values for the elements of $\mathbf{z}$ yield different output quantities $\mathbf{y}(\mathbf{z}) \in Y(\mathbf{x}, \mathbf{z})$ that fall between the no production situation $0 \in Y(\mathbf{x}, 0)$ and the technologically determined output situation $\mathbf{y} \in Y(\mathbf{x})$. That is, $0 \leq \mathbf{y}(\mathbf{z}) \leq \mathbf{y}$.

Differences in capabilities are reflected by a distribution of weights attached to each input involved in production. These weights do not refer to differences in the scale of inputs but rather to differences in the ability to exploit the technological knowledge supported by unchanged resources. Differences in the economic competence for the production of some given $\mathbf{y} \in Y(\mathbf{x})$ in some particular activity result in differences both in the composition and in the quantities of the obtained output bundle. Some positive weights $\mathbf{z} \geq (0, \ldots, 0)$ result in some positive level of output $\mathbf{y}(\mathbf{z}) \geq 0$. For some positive weights $\mathbf{z} \leq (1, \ldots, 1)$ the obtained level of output $\mathbf{y}(\mathbf{z}) \leq \mathbf{y}$ falls short of the technologically determined level of output that would be obtained with $\mathbf{z} = (1, \ldots, 1)$. This accounts for organizational slack.

Note that changes in the values taken by the capability parameters do not imply any change in the ratios of capital inputs to labour inputs. Simply, capability parameters express the idea that the capital-*service* and the labour-*service* – rather than capital itself and labour itself – are involved in production. The quantities of complementary inputs are regarded as technologically determined while the values of capability parameters are regarded as organizationally fixed. Only the scaling in units of output can vary as a result of differences in organizational capabilities from one production instance to another, other things being equal. Accordingly, only some distinctive endowments in organizational capabilities characterise differences among firms that operate the same quantities of inputs for the production of some perfectly substitutable outputs.

The definition of organizational knowledge as a *disembodied* environmental factor that impacts on the productive possibilities of the firm as determined by the *embodied* dimension of knowledge in input factors can be interpreted in the light of the recent literature that specializes on the management of productive operations. An example is Bohn and Jaikumar (1992) who define technological knowledge as the understanding of the effects of the input variables on a process of production. If this process is restricted to a task (k, l) performed by the pair (x_{kl}, x_{lk}) of complementary input factors, the case $z_{kl}\, z_{lk} = 0$ describes a state of no understanding (no task is performed) and the case $z_{kl}\, z_{lk} = 1$ describes a state of complete understanding (the embodied knowledge is fully exploited in the performance of the task). Capability parameters can be interpreted in the same manner as the levels of participation that are attached in behavioural studies to the qualities (such as intelligence or patience) describing what psychologists call the behavioural profile of individuals when involved in various social environments. The representation of these levels of participation in mathematical terms is detailed in Aubin (1993, pp. 197-201). In the present analysis, qualities are the intrinsic characteristics of input factors $\mathbf{x}$ and social environments become instances of various output bundles $\mathbf{y} \in Y(\mathbf{x})$. This means that technological knowledge sets the upper physical boundaries to the productive possibilities offered by the transformation of input factors. And this means that, within these boundaries, organizational knowledge captures differences in productivity that are observed among a population of firms operating the same quantities of inputs for the production of substituable outputs, an idea that is best expressed in Eliasson (1994). The definition of organizational knowledge connects also to what Cohen and Levinthal (1994) term an absorptive capacity, namely the ability to exploit new technological developments. The reason is simply that the set of capital and labour inputs as indexed by $k = 1, \ldots, K$ and $l = 1, \ldots, L$, respectively, were assumed above to describe all existing or potential factors of production to serve as inputs. Some organizational capabilities may relate to inputs that are not available and even unknown to the firm. Accordingly, they capture the ability to exploit new technological knowledge embodied by resources that will be made available in a future period only. Finally, observe that the definition of economic competence can easily be extended to the industry level in a way that borrows from a contribution by Zimmermann (1995). To see this, assume that firms in an industry differ only in the organizational capabilities that enable them to transform some given quantities with some common technology. Then the *total* economic competence involved in the production of some output $\mathbf{y} = \sum_j \mathbf{y}^j$ in the industry is the envelope of the economic competences involved in the production of $\mathbf{y}^j \in Y^j(\mathbf{x})$ by individual firms $j \in \{1, \ldots, J\}$. Formally, total economic competence is

$$\cup_j \tilde{X}^j(\mathbf{y}^j) = \left\{(x_k, z_{kl}), (x_l, z_{lk}) : z_{kl} = \sup\{z_{kl}^j\} \text{ and } z_{lk} = \sup\{z_{lk}^j\}\right\},$$

for all $(x_k, x_l) \in X_{K \times L}(\mathbf{y})$, which is the union of *public* economic competence with the *private* economic competence. Public economic competence is

$$\cap_j \tilde{X}^j(\mathbf{y}^j) = \left\{(x_k, z_{kl}), (x_l, z_{lk}) : z_{kl} = \inf\{z_{kl}^j\} \text{ and } z_{lk} = \inf\{z_{lk}^j\}\right\},$$

for all $(x_k, x_l) \in X_{K \times L}(\mathbf{y})$, which is the (disembodied) knowledge shared by all firms. And private economic competence is simply $\cup_j \tilde{X}^j(\mathbf{y}^j) \setminus \cap_j \tilde{X}^j(\mathbf{y}^j)$, which is the (disembodied) knowledge that is not shared by all firms in the industry.

The analysis that follows integrates the definitions above in a linear model of production that takes explicit account of the choice of some particular combination of complementary factors out of a range of alternatives.

3 A Linear Model

The definition of economic competence above presupposes some variable utilisation of inputs in production. In the single output case, this utilisation is optimal when expressed by a production function, which in turn presupposes a physical maximization of output from given inputs. However, optimality is by no means a matter of course and results from some organizational design of productive activities that connect to the down-to-earth manner in which inputs are transformed in workshops and carried on the market.

The observed economic competence of a given firm can be modelled as dependent upon the organization of production that happens to be selected from a range of alternative assignments of inputs. To do that, in this section the simplest possible linear model of production is defined that incorporates explicitly the capability parameters attached to the complementary input factors in various assignments. The three following assumptions describe the model formally.

ASSUMPTION $A.1$: *There exists a class $\{A^i\}_{i=1}^{I}$ of possible activities (or productive processes) that are completely specified by the quantities of each of the inputs which they consume and each of the outputs which they produce when carried on at unit level, such that*

$$A^i : \sum_{m=1}^{M} \alpha^i_m x_m \rightarrow \sum_{n=1}^{N} \beta^i_n y_n,$$

where $\mathbf{x} = (x_1, \ldots, x_M) \in R^M_+$, $\mathbf{x}^i = (\alpha^i_1 x_1, \ldots, \alpha^i_M x_M) \in R^M_+$, $\mathbf{y} = (y_1, \ldots, y_N) \in R^N_+$, $\mathbf{y}^i = (\beta^i_1 y_1, \ldots, \beta^i_N y_N) \in R^N_+$ *and where* $\alpha^i = (\alpha^i_1, \ldots, \alpha^i_m) \in R^M_+$, $\beta^i = (\beta^i_1, \ldots, \beta^i_n) \in R^N_+$ *are such that*

$$\begin{array}{llll} \sum_{i=1}^{I} & \beta^i_m & > 0, \;\; m & = 1, \ldots, M, \quad (i) \\ \sum_{m=1}^{M} & \beta^i_m & > 0, \;\; i & = 1, \ldots, I, \quad (ii) \\ \sum_{i=1}^{I} & \alpha^i_n & > 0, \;\; n & = 1, \ldots, N, \quad (iii) \\ \sum_{n=1}^{N} & \alpha^i_n & > 0, \;\; i & = 1, \ldots, I. \quad (iv) \end{array}$$

The constraint (i) asserts that each output is producible, (ii) asserts that each activity produces at least one output, (iii) asserts that each input is required by at least one activity, (iv) asserts that each activity uses at least one input. This standard assumption asserts that an activity converts certain commodities into certain other commodities in technologically determined ratios between outputs and inputs. The class of activities describes all particular methods of production that constitute technological knowledge as given in Definition 1. In general, production may be thought of as the joint operation of several activities at various levels, that is as a program of activities. The firm can use activity A^1 at level u^1, activity A^2 at level u^2, and so on, where the parameter vector $\mathbf{u} = (u^1, \ldots, u^I)$ denotes the intensity level at which each activity is run (on this see Färe (1988), pp. 43-49).

ASSUMPTION $A.2$: *There exists a class $\{\mathbf{z}^i\}_{i=1}^I$ of vectors of capability parameters attached to each capital input type k (and, symmetrically, to each labour input type l) when combined with some complementary labour input type l (capital input k) for the performance of the productive task $(k,l) = (l,k)$ in activity A^i, such that*

$$\mathbf{z}^i \equiv (z_{11}^i, \ldots, z_{1L}^i, \ldots, z_{K1}^i, \ldots, z_{KL}^i, z_{11}^i, \ldots, z_{1K}^i, \ldots, z_{L1}^i, \ldots, z_{LK}^i),$$

where $z_{kl}^i \in [0,1]$, $z_{lk}^i \in [0,1]$ and the subscripts $k \in \{1, \ldots, K\}$, $l \in \{1, \ldots, L\}$ index two subsets of input factors that partition the set of input types in the economy.

This assumption forces the standard linear model above to capture the impact on production of organizationally determined values that are taken by capability parameters. For a given positive quantity $\mathbf{x} \in X(\mathbf{y})$ of input factors, let $a_{kl}(\mathbf{z}^i)$ be the contribution to output obtained in the i-th activity from some fraction x_{kl} of some capital input x_k as combined with some fraction x_{lk} of some labour input x_l.

ASSUMPTION $A.3$: *There is complementarity in production, that is*

$$a_{kl}(\mathbf{z}^i) = a_{lk}(\mathbf{z}^i) \begin{cases} \geq 0 & \text{if } z_{kl}^i \, z_{lk}^i > 0, \\ = 0 & \text{if } z_{kl}^i \, z_{lk}^i = 0, \end{cases}$$

for all $A^i \in \{A^i\}_{i=1}^I$, for all nonnegative quantities $x_{kl} \leq x_k$, $x_{lk} \leq x_l$, and for all $z_{kl}^i, z_{lk}^i \in [0,1]$.

From Definition 2, we know that the parameters z_{kl}^i and z_{lk}^i can be interpreted as proxies for the organizational capabilities of the firm in the operation of tasks $(k,l) = (l,k)$ in the i-th activity by pairs of complementary inputs types. The expression of $a_{kl}(\mathbf{z}^i)$ emphasizes the complementarity in production as resulting from some organizational capabilities and not from some intrinsic attributes that define the type of input resources. The simplest possible linear model of production that is parametrically defined in $\mathbf{z}$ can now be written as

$$X(\mathbf{y}, \mathbf{z}) = \{\mathbf{x}(\mathbf{z}) : \mathbf{x}(\mathbf{z}) \geq \mathbf{u}\alpha, \ \mathbf{u} \in R_+^I\}, \tag{1}$$

for all $\mathbf{y} \in R_+^N$ such that $\mathbf{u}\beta \geq \mathbf{y}$, or equivalently as

$$Y(\mathbf{x}, \mathbf{z}) = \{\mathbf{y}(\mathbf{z}) : \mathbf{u}\beta \geq \mathbf{y}(\mathbf{z}), \ \mathbf{u} \in R_+^I\}, \tag{2}$$

for all $\mathbf{x} \in R_+^M$ such that $\mathbf{x} \geq \mathbf{u}\alpha$. Here α and β are the $I \times M$-dimensional input matrix and the $I \times N$-dimensional output matrix, respectively, that describe the M inputs by means of which the activities in $\{A^i\}_{i=1}^I$ produce the N outputs, and $\mathbf{z}$ is the competence matrix in which each column is the competence vector $\mathbf{z}^i$ as defined above. A coefficient α^i_m of α denotes the quantity of the m-th input used by the i-th activity at unit intensity and a coefficient β^i_n of β denotes the n-th output quantity of the i-th activity at unit intensity.

This linear model is compatible with the neo-classical axiomatic framework. It inherits all properties from the parent output and input correspondences as defined in the previous section. Concentrating now for expositional purposes on the simplest possible situation in which a single output is produced, observe (i) that there exists a production function $F(\mathbf{x}) = \sup\{y \in R^1_+ : y \in Y(\mathbf{x}, \mathbf{z})\}, \mathbf{x} \in R^M_+$, and (ii) that $F(\mathbf{x})$ is homogeneous of degree 1 in $\mathbf{x}$, and (iii) that $F(\mathbf{x})$ is homogeneous of degree 1 in $\mathbf{z}$. The latter two properties of homogeneity in input quantities and in capability parameters are related to the nature of technological and organizational knowledge as described in Definitions 1 and 2, respectively.[2] Their significance can be made clear by considering two attributes of any economic good that are commonly identified in the theory of externalities, namely rivalry and excludability. These attributes can be represented by two axes that define four categories as follows. By definition, a private good is both rivalrous and excludable while a public good is both nonrival and nonexcludable. Goods that are rivalrous and nonexcludable can be referred to as weak private goods. They are defined as rivalrous goods that may be excludable in principle but at some prohibitively high cost. Goods that are nonrivalrous and excludable can be named weak public goods. A common example for this category is the patented process of production or design of a product that can be replicated at virtually no cost.[3]

Both technological knowledge and organizational knowledge can be located in this typology. First, (embodied) technological knowledge – as characterised by Definition 1 – is a set of related inputs and outputs that belong to the category of private goods. Rivalry and excludability apply because the definition of production sets above implicitly states that the physical facilities that describe some capital type and the human skills that describe some labour type are inherently tied to a physical object (some piece of machinery or some human body). The economic consequence is a replication argument. For a given competence matrix $\mathbf{z}$, if all capital and labour inputs are changed by the same proportion, then the level of output is changed by the same proportion also. Second, (disembodied) organizational knowledge – as characterized

[2]The proof of those claims are standard and can be obtained from the author upon request.

[3]The definitions of rivalry and excludability follow Romer (1990).

by Definition 2 – can be interpreted as a set of weak public goods, that is as a set of (possibly completely) excludable and also nonrivalrous factors of production. The excludability assumption is meant to guarantee the possibility that firms may differ in their organizational capabilities. The nonrivalry attribute is assumed to hold because disembodied knowledge is not tied to any physical object by Definition 2. The economic interpretation is straightforward. If the capital and labour inputs **x** together with all the capability parameters as represented in the competence matrix **z** are increased by the same proportion, then the output is more than proportionally increased. More formally from the properties of homogeneity given in (*iii*) and (*iv*) above one obtains

$$Y(\lambda\mathbf{x}, \lambda\mathbf{z}) > \lambda Y(\mathbf{x}, \mathbf{z})$$

where $\lambda > 0$ must be such that $\lambda z^i_{kl} \leq 1$ and $\lambda z^i_{lk} \leq 1$ for all parameters z^i_{kl} and z^i_{lk} in **z** for the inequality to make economic sense. In Penrosian terms, this says that the "*productive opportunity*" (1959, p. 31) – represented here by $Y(\mathbf{x}, \mathbf{z})$– of some given resources **x** available to the firm is conditioned by some organizational knowledge **z** that refers to the ability to extract some potential services embodied in capital and labour assets when combined together in productive tasks. For some given organizational knowledge, the exact replication of input factors does not imply any change in the organization of production and therefore results in constant returns to scale. For some given amount of input factors, a proportional increase in organizational capabilities as represented by the capability parameters leads to the same replication argument and accordingly to constant returns to scale also. With an increase in the physical medium supporting technological knowledge and in the disembodied organizational knowledge that describes the firm capabilities in organizing this medium in production, output increases more than proportionally.

When the size of the firm is measured by the quantity of output that is obtained from the transformation of some given input resources involved in production, the inverse relationship between the input and output correspondences says that the study of the growth of the firm is equivalent to the examination of $Y(\mathbf{x}, \mathbf{z})$.

4 Implications for the Growth of the Firm

The objective of this section is to derive some propositions from the linear model that was defined above that support the following claims by Penrose (1959):

"[S]o long as expansion can provide a way of using the services of its resources more profitably than they are being used, a firm has an incentive to expand; or alternatively, so long as any resources are not used fully in current operations, there is an incentive for a firm to find a way of using them more fully." (p.67).

The question of comparative statics that is examined hereafter is what happens when a change occurs in input factors, on the one hand, and in economic competence, on the other hand. In order to be consistent with the terms used in the preceding sections, the two phenomena are referred to as *technological change* and as *organizational change*, respectively. The analysis concentrates on the transformation of inputs into a single output by one single activity studied in isolation. Accordingly, superscripts that refer to this particular activity can are omitted hereafter for clarity, as would be the case in a single productive process situation. For a given quantity $\mathbf{x}$ of available resources, the organizational problem is now to select one assignment – out of all possible assignments respecting the proportions specified by the activity – that maximises the sum of contributions $a_{kl}(\mathbf{z}) = a_{lk}(\mathbf{z})$ to production. Recall that complementarity in the production of $y \in R^1_+$ imposes not only that $X_K(y, \mathbf{z}) \neq \emptyset$ and $X_L(y, \mathbf{z}) \neq \emptyset$ but also that $X_K(y, \mathbf{z}) \cup X_L(y, \mathbf{z}) = X(y, \mathbf{z})$. Accordingly, the problem in the allocation of input factors boils down to matching the two sets $X_K(y, \mathbf{z})$ and $X_L(y, \mathbf{z})$, by combining fractions x_{kl} of some capital input x_k with some fraction x_{lk} of some labour input x_l. The outcome $a_{kl}(\mathbf{z}) = a_{lk}(\mathbf{z})$ of any pair (x_{kl}, x_{lk}) involved in the performance of the task $(k, l) = (l, k)$ is set out in the $K \times L$-dimensional performance matrix $\mathbf{a}(\mathbf{z})$. The assignment problem can be recasted into linear programming (LP) terminology to give

$$F(\mathbf{x}, \mathbf{z}) = \max_{x_{kl}, x_{lk}} \sum_{k=1}^{K} \sum_{l=1}^{L} a_{kl}(\mathbf{z}), \tag{3}$$

such that

$$\begin{aligned} &\textstyle\sum_{k=1}^{K} x_{lk} \leq x_l, \quad l = 1, \ldots, L, \\ &\textstyle\sum_{l=1}^{L} x_{kl} \leq x_k, \quad k = 1, \ldots, K, \\ &x_{kl}, x_{lk} \geq 0. \end{aligned}$$

Technological change: The following proposition relates to the first part of the Penrosian quotation above, which says that so long as expansion can provide a way of using the services of its resources more profitably than they are being used, a firm has an incentive to expand. The term *expansion* is interpreted as an increase in output that is made possible by the integration of additional inputs. That is, changes occur in $\mathbf{x}$ only.

Under scrutiny is the direction of productivity gains (and *a fortiori* profitability gains, other things being equal) that result from an expansion in available input resources $\mathbf{x}$, for some unchanged capability vector $\mathbf{z}$.

PROPOSITION 1: *Under assumptions A1 – A3, $F(., \mathbf{z})$ has*

(i) decreasing differences in $\mathbf{x}$ on X_K and on X_L, that is

$$F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'',\mathbf{z}) - F(\mathbf{x}+\mathbf{x}',\mathbf{z}) \leq F(\mathbf{x}+\mathbf{x}'',\mathbf{z}) - F(\mathbf{x},\mathbf{z}), \tag{4}$$

if either $(\mathbf{x}',\mathbf{x}'') \in X_K \times X_K$ or $(\mathbf{x}',\mathbf{x}'') \in X_L \times X_L$, *and*

(ii) increasing differences in $\mathbf{x}$ on $X_K \times X_L$, that is

$$F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'',\mathbf{z}) - F(\mathbf{x}+\mathbf{x}',\mathbf{z}) \geq F(\mathbf{x}+\mathbf{x}'',\mathbf{z}) - F(\mathbf{x},\mathbf{z}), \tag{5}$$

if either $(\mathbf{x}',\mathbf{x}'') \in X_K \times X_L$ or $(\mathbf{x}',\mathbf{x}'') \in X_L \times X_K$.

Proof: in Appendix.

This proposition says that the maximum output $F(., \mathbf{z})$ resulting from the optimal assignment of complementary factors $\mathbf{x} \in R_+^{K+L}$ involved in the activity under scrutiny displays (i) decreasing differences in these inputs when the additional factors $\mathbf{x}'$ and $\mathbf{x}''$ that are introduced in production both belong to the subset of capital inputs or to the subset of labour inputs, and (ii) increasing differences when they do not belong to the same subset of inputs. The former case can be interpreted as a particular instance of the law of diminishing marginal productivity that is not rooted in the unavailability of some particular resources but in the fact that they do not belong to complementary sets of input factors. In the latter case, everything happens as if the changing available amount of complementary input factors were exploited in an increasingly efficient way by an unchanged set of organizational capabilities. This means that the integration of additional complementary resources provides a way of using the services of productive resources more profitably, other things being equal. In any case, technological change captures an extention of the boundaries that describe the physical possibilities of transforming resources. These possibilities are rooted in the embodied attributes of capital and labour inputs. Changes within these boundaries are of the organizational kind.

Organizational change: The following proposition relates to the second part of the Penrosian quotation above, which says that so long as any resources are not used fully in current operations, there is an incentive for a firm to find a way of using them more fully. In the present analysis, the *way of using resources more fully* is interpreted as an increased ability to exploit a given set of resources, that is as a rise in the capability parameters attached to the inputs involved in production. In this case, changes occur in $\mathbf{z}$ only. Accordingly, the analysis concentrates on the characterisation of changes in the scale of production as a result of changes in the amount $\mathbf{z}$ of organizational capabilities, for some unchanged amount of inputs $\mathbf{x}$ operated in fixed proportions. To save notational clutter, denote by $Z_K = \{\mathbf{z} : z_{lk} = 0\}$ the set of competence vectors for which all capability parameters attached to type-l input are zero. And denote by $Z_L = \{\mathbf{z} : z_{kl} = 0\}$ the set of competence vectors for which all capability parameters attached to some type-k input are zero, all $k \in \{1, \ldots, K\}$ and all $l \in \{1, \ldots, L\}$.

PROPOSITION 2: *Under assumptions* $(A1 - A3)$, $F(\mathbf{x}, .)$ *has*

(i) *decreasing differences in* $\mathbf{z}$, *that is*

$$F(\mathbf{x}, \mathbf{z} + \mathbf{z}' + \mathbf{z}'') - F(\mathbf{x}, \mathbf{z} + \mathbf{z}') \leq F(\mathbf{x}, \mathbf{z} + \mathbf{z}'') - F(\mathbf{x}, \mathbf{z}), \tag{6}$$

if either $(\mathbf{z}', \mathbf{z}'') \in Z_K \times Z_K$ *or* $(\mathbf{z}', \mathbf{z}'') \in Z_L \times Z_L$; *and*

(ii) *increasing differences in* $\mathbf{z}$, *that is*

$$F(\mathbf{x}, \mathbf{z} + \mathbf{z}' + \mathbf{z}'') - F(\mathbf{x}, \mathbf{z} + \mathbf{z}') \geq F(\mathbf{x}, \mathbf{z} + \mathbf{z}'') - F(\mathbf{x}, \mathbf{z}), \tag{7}$$

if either $(\mathbf{z}', \mathbf{z}'') \in Z_K \times Z_L$ *or* $(\mathbf{z}', \mathbf{z}'') \in Z_L \times Z_K$.

Proof: in Appendix.

This proposition asserts that the direction of changes in productivity gains is dependent upon whether or not changes in economic competence simultaneously affect the use of complementary inputs. Changes in productivity gains are negative (positive) when improvements in organizational capabilities concern the use of substitutable (complementary) working factors. There are increasing returns to economic competence when some innovation results in a more efficient use of at least two available and effectively working resources that belong to complementary sets of inputs. It follows that improvements in the ability to use more fully some unchanged complementary resources in production lead to increasing gains in profitability, other things being equal. The main result of the analysis is simply that these changes in productivity gains do not relate to (dis)economies of scale or scope. They stem from complementarities in the use of resources in production.

5 A Numerical Example

The objective of this section is to apply the definitions and propositions offered in the previous sections by examining two cases of a numerical example that illustrate the propositions on organizational change. Consider a single-process production system that involves only two pairs of complementary factors. This is represented by

$$\sum_{m=1}^{4} \alpha_m x_m = y,$$

where $\mathbf{x} = (x_1, x_2, x_3, x_4) = (1, 1, 1, 1)$, $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (1, 2, 1, 2)$, with $x_1, x_2 \in X_K$ and $x_3, x_4 \in X_L$. Formally, the organizational problem consists in finding

$$\begin{array}{ll} F(\mathbf{x}, \mathbf{z}) = \max_{x_{kl}, x_{lk}} \sum_k \sum_l a_{kl}(\mathbf{z}), & \\ \text{such that} & \\ \sum_{k=1}^{2} x_{lk} \leq x_l, & l = 3, 4, \\ \sum_{l=3}^{4} x_{kl} \leq x_k, & k = 1, 2, \\ x_{kl}, x_{lk} \geq 0, & \end{array}$$

where

$$a_{kl}(\mathbf{z}) = a_{lk}(\mathbf{z}) \begin{cases} = z_{kl}\alpha_k x_k + z_{lk}\alpha_l x_l & \text{if } z_{kl}\, z_{lk} > 0, \\ = 0 & \text{if } z_{kl}\, z_{lk} = 0. \end{cases}$$

Different values taken by the capability parameters attached to each input factor give different productive instances. From the proof of propositions 1 and 2 in the appendix, we know that the continuous LP problem above is strictly equivalent to a discrete LP problem. This means that the organizational problem boils down to a choice between two possible sets of productive tasks (k, l), all k, all l, that describe alternative assignment types – say, Type A and Type B – of input factors, where

$$\begin{aligned} \text{Type } A &= \{(1,3), (2,4)\}, \\ \text{Type } B &= \{(1,4), (2,3)\}. \end{aligned}$$

In assignment Type A, capital input 1 and labour input 3 are assigned to each other while capital input 2 and labour input 4 are assigned to each other. The choice in assignment types (i.e., two collections of disjoint pairs of complementary inputs) does not imply any change in the input ratios which are technologically determined for the productive activity by the vector α. Shifts in assignment types (from A to B or from B to A) as a result of changes in capability parameters are recombinations of the same input factors in the same proportions into different productive tasks that give the maximum level of output. Two cases can be considered in turn that give decreasing differences and increasing differences, respectively, in the maximum amount of output $F(\mathbf{x},\mathbf{z})$ obtained from the transformation of some unchanged quantities of capital and labour inputs.

Case 1: decreasing differences The capability parameters of the firm are displayed in Tables $1(a)$.

$\tilde{X}_K(y)$	3	4
1	0.1	**0.1**
2	0.8	0.5

$\tilde{X}_L(y)$	1	2
3	**0.1**	0.1
4	0.5	0.6

Table $1(a)$: capability parameters attached to each input type $k \in \{1,2\}$ when combined with some input type $l \in \{3,4\}$, and to each input type $l \in \{3,4\}$ when combined with some input type $k \in \{1,2\}$, in the productive tasks $(l,k)=(k,l)$. For example, the value taken by the capability parameter attached to capital input 1 when combined with labour input 3 is $z_{13}=0.1$, and the value taken by the capability parameter attached to labour input 4 when combined with capital input 2 is $z_{42}=0.6$. Incremental changes occur in z_{14} and z_{31} (in bold characters).

Consider positive changes in the values taken by parameters z_{14} and in z_{31}, other things remaining equal. The optimal choice of assignment type for given values taken by the capability parameters results in output levels that are displayed in Table $1(b)$. The production function $F(\mathbf{x},\mathbf{z})$ is characterized by decreasing differences in $\mathbf{z}$, as expected.

z_{14} \ z_{31}	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	1.40$_B$	1.40$_B$	1.40$_B$	1.40$_B$	*1.40*	1.45$_A$	1.50$_A$	1.55$_A$	1.60$_A$	1.65$_A$
0.2	1.45$_B$	1.45$_B$	1.45$_B$	1.45$_B$	1.45$_B$	*1.45*	1.50$_A$	1.55$_A$	1.60$_A$	1.65$_A$
0.3	1.50$_B$	1.50$_B$	1.50$_B$	1.50$_B$	1.50$_B$	**1.50$_B$**	*1.50*	**1.55$_A$**	1.60$_A$	1.65$_A$
0.4	1.55$_B$	1.55$_B$	1.55$_B$	1.55$_B$	1.55$_B$	1.55$_B$	1.55$_B$	*1.55*	1.60$_A$	1.65$_A$
0.5	1.60$_B$	1.60$_B$	1.60$_B$	1.60$_B$	1.60$_B$	**1.60$_B$**	1.60$_B$	**1.60$_B$**	*1.60*	1.65$_A$
0.6	1.65$_B$	1.65$_B$	1.65$_B$	1.65$_B$	1.65$_B$	1.65$_B$	1.65$_B$	1.65$_B$	1.65$_B$	*1.65*
0.7	1.70$_B$	1.70$_B$	1.70$_B$	1.70$_B$	1.70$_B$	1.70$_B$	1.70$_B$	1.70$_B$	1.70$_B$	1.70$_B$
0.8	1.75$_B$	1.75$_B$	1.75$_B$	1.75$_B$	1.75$_B$	1.75$_B$	1.75$_B$	1.75$_B$	1.75$_B$	1.75$_B$
0.9	1.80$_B$	1.80$_B$	1.80$_B$	1.80$_B$	1.80$_B$	1.80$_B$	1.80$_B$	1.80$_B$	1.80$_B$	1.80$_B$
1.0	1.85$_B$	1.85$_B$	1.85$_B$	1.85$_B$	1.85$_B$	1.85$_B$	1.85$_B$	1.85$_B$	1.85$_B$	1.85$_B$

Table $1(b)$: $F(\mathbf{x},\mathbf{z})$ displays decreasing differences in the capability vector $\mathbf{z}$. Subscripts A or B refer to the assignment type. Numbers in italic give the maximum output quantities indifferently obtained with type A or type B assignments. For an example of decreasing differences, observe the change in output that results from a change in z_{14} from 0.3 to 0.5 together with a change in z_{31} from 0.6 to 0.8 (in bold characters). One finds $1.60 - 1.60 \leq 1.55 - 1.50$.

In this example, increases in capability parameters reflect changes in the ability to perform productive tasks that cannot possibly appear together in the same assignment type. In particular, changes in z_{14} express some increased ability in the performance of the task $(1,4)$ in assignment Type B. Changes in z_{31} account for some increased ability in the performance of the task $(1,3)$ that appear in assignment Type A only. Contributions to knowledge in the performance of the task $(1,4)$ have no effect on productivity when the maximisation of output imposes the choice of assignment Type A in production. In other words, with Type A, since task $(1,4)$ does not appear no type-1 capital factor is combined with type-4 labour factor (i.e., $x_{14} = 0$). This in turn implies that increases in z_{14} have no effect on the total product (i.e., $z_{14}x_{14} = 0$). By the same token, contributions to knowledge in the performance of the task $(1,3)$ has no effect on productivity when assignment Type B is selected. That is, with Type B, $x_{31} = 0$ so that $z_{31}x_{31} = 0$. In effect, everything happens in the model as if successive increases in capability parameters affected the same – and *a fortiori* substitutable – organizational capability (either z_{14} or z_{31}), leading to decreasing differences for $F(\mathbf{x}, \mathbf{z})$ as predicted by Proposition $2(i)$.

Case 2: increasing differences The capability parameters of the firm are displayed in Table $2(a)$.

$\tilde{X}_K(y)$	3	4
1	**0.1**	0.1
2	0.8	0.5

$\tilde{X}_L(y)$	1	2
3	**0.1**	0.1
4	0.5	0.6

Table $2(a)$: capability parameters attached to each input type $k \in \{1,2\}$ when combined with some input type $l \in \{3,4\}$, and to each input type $l \in \{3,4\}$ when combined with some input type $k \in \{1,2\}$, in the productive tasks $(l,k) = (k,l)$. For example, the value taken by the capability parameter attached to capital input 1 when combined with labour input 4 is z_{14}= 0.1, and the value taken by the capability parameter attached to labour input 4 when combined with capital input 2 is z_{42}= 0.6. Incremental changes occur in z_{13} and z_{31} (in bold characters).

Consider now positive changes in the values taken by the parameters z_{13} and z_{31}, other things remaining equal. The optimal choice of assignment type for given values taken by the capability parameters results in output levels that are displayed in Table 2(b). In this case the production function $F(\mathbf{x}, \mathbf{z})$ is characterized by increasing differences in $\mathbf{z}$.

z_{13} \ z_{31}	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	1.40_B	1.40_B	1.40_B	1.40_B	*1.40*	1.45_A	1.50_A	1.55_A	1.60_A	1.65_A
0.2	1.40_B	1.40_B	$\mathbf{1.40_B}$	*1.40*	$\mathbf{1.45_A}$	1.50_A	1.55_A	1.60_A	1.65_A	1.70_A
0.3	1.40_B	1.40_B	*1.40*	1.45_A	1.50_A	1.55_A	1.60_A	1.65_A	1.70_A	1.75_A
0.4	1.40_B	*1.40*	$\mathbf{1.45_A}$	1.50_A	$\mathbf{1.55_A}$	1.60_A	1.65_A	1.70_A	1.75_A	1.80_A
0.5	*1.40*	1.45_A	1.50_A	1.55_A	1.60_A	1.65_A	1.70_A	1.75_A	1.80_A	1.85_A
0.6	1.45_A	1.50_A	1.55_A	1.60_A	1.65_A	1.70_A	1.75_A	1.80_A	1.85_A	1.90_A
0.7	1.50_A	1.55_A	1.60_A	1.65_A	1.70_A	1.75_A	1.80_A	1.85_A	1.90_A	1.95_A
0.8	1.55_A	1.60_A	1.65_A	1.70_A	1.75_A	1.80_A	1.85_A	1.90_A	1.95_A	2.00_A
0.9	1.60_A	1.65_A	1.70_A	1.75_A	1.80_A	1.85_A	1.90_A	1.95_A	2.00_A	2.05_A
1.0	1.65_A	1.70_A	1.75_A	1.80_A	1.85_A	1.90_A	1.95_A	2.00_A	2.05_A	2.10_A

Table 2(b): $F(\mathbf{x}, \mathbf{z})$ **displays increasing differences** in the capability vector $\mathbf{z}$. Subscripts A or B refer to the assignment type. Numbers in italic give the maximum output quantities indifferently obtained with type A or type B assignments. For an example of increasing differences, observe the change in output that results from a change in z_{13} from 0.2 to 0.4 together with a change in z_{31} from 0.3 to 0.5 (in bold characters). One finds $1.55 - 1.45 \geq 1.45 - 1.40$.

By contrast with the previous example, increases in capability parameters now reflect changes in the ability to perform productive tasks that can only appear altogether in the same assignment type. This is made very clear by the fact that changes in z_{13} and z_{31} express some increased ability in the performance of the same task $(1,3)$ in assignment Type A. However, observe that simultaneous changes in, say, z_{13} and z_{42} or in z_{24} and z_{31} would lead also to increasing differences in the maximum output level, as predicted by Proposition 2(ii).

In the two cases of the numerical example above, efficiency in production implies that more productive assignment types – that is, organizational choices – tend to replace less productive ones through cumulative improvements in capabilities. Impediments of all kinds that prevent a shift in assignment types would make improvements in organizational capabilities ineffective. Observe that these shifts are discrete and make obsolete those organizational capabilities that were working in some abandoned assignment type because of some improvement in other capabilities. For a given technology as specified by some fixed input ratios, the direction of shifts in assignment types (from A to B or from B to A) depend on the initial distribution of capabilities and on the directions of their improvements. In all cases, optimal changes in assignment types require some investments in capabilities before their becoming effective in production. Observe that a more complex example including more than two tasks in production would be associated some assignment types with overlapping tasks. In that case shifts in assignment types can result both from investments in "working" capabilities (i.e., capabilities attached to tasks that are effectively performed in production) and from investments in "maturing" capabilities (i.e., capabilities that may become effective in some other assignment type).

The interpretation of this example at the industry level is straightforward. To see this, assume that the costs incurred in improving capabilities are nonzero and increasing in the elements of $\mathbf{z}$. Consider the situation in which organizational knowledge is fully excludable. Then decreasing (increasing) differences for $F(\mathbf{x}, \mathbf{z})$ in $\mathbf{z}$ are equivalent to diminishing (increasing) returns to scale in knowledge investments. Other things (i.e. market size in particular) being equal, in a competitive output market the directions of investments in capabilities would operate as a selection mechanism. This means that firms represented in case 2 would grow at the expense of firms represented in case 1. The market share of firms represented in case 2 would increasingly grow with positive changes in organizational capabilities at the expense of the market share of firms represented in case 1. This would come out as a consequence of differences in the directions of investments in capabilities only.

6 Final Remarks

The numerical example illustrates two interesting features of the model that emphasise the role of internal organization as a source of productivity gains in the operation of unchanged quantities of inputs combined in fixed proportions. First, the state of organizational capabilities command the choice of organization type in production. As an empirical implication, this means that two firms that happen to be identical in all aspects but in their endowments of capabilities (same inputs, same proportions, same types of outputs) may find it profitable to adopt different types of organization in production. Second, the discrete character of changes in the organization of production forces some capabilities involved in some abandoned assignment type to become useless in the adopted assignment type. Symmetrically it makes some cumulative investments in capabilities effective that had no impact on productivity with another organization format. In other words that are common in the literature specializing on innovations, positive changes in organizational capabilities are competence destroying (some capabilities become useless) or competence enhancing (some capabilities become effective) after a change in the organization of production. The empirical implication is that a firm that invests in training or research and development with a view to improve productive efficiency will not benefit from these efforts unless it can reassign unchanged input factors on the shop-floor.

No attempt was made to model the observed actions by business organizations that may contribute to enhance organizational knowledge. Arguably, the definitions and propositions above can be used as building blocks for the examination of these practices in further research.

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7 Appendix

First, observe that the specific form that was chosen for $a_{kl}(\mathbf{z})$ in order to introduce complementarity in the linear model, namely

$$a_{kl}(\mathbf{z}) = a_{lk}(\mathbf{z}) \begin{cases} = z_{kl}\alpha_k x_k + z_{lk}\alpha_l x_l & \text{if} \quad z_{kl}\, z_{lk} > 0, \\ = 0 & \text{if} \quad z_{kl}\, z_{lk} = 0, \end{cases}$$

is a natural consequence of the homogeneity of degree 1 of $F(\mathbf{x}, \mathbf{z})$ in $\mathbf{x}$ together with Euler's theorem, since

$$z_{kl}x_k \frac{\partial \sum_{m=1}^{M} \alpha_m x_m}{\partial x_k} + z_{lk}x_l \frac{\partial \sum_{m=1}^{M} \alpha_m x_m}{\partial x_l} = z_{kl}\alpha_k x_k + z_{lk}\alpha_l x_l,$$

for all $k \in \{1, \ldots, K\}$, all $l \in \{1, \ldots, L\}$. Second, observe that the maximum value of the objective function in the LP problem in (3) is attained with all $x_{kl} = 0$ or $x_{kl} = x_k$ and with all $x_{lk} = 0$ or $x_{lk} = x_l$ (the proof is available in Dantzig (1963), p. 318). This means that the fractions x_{kl} and x_{lk} disappear from the solution. The following (continuous) linear programming problem

$$F(\mathbf{x}, \mathbf{z}) = \max_{x_{kl}, x_{lk}} \sum_{k=1}^{K} \sum_{l=1}^{L} a_{kl}(\mathbf{z}),$$

for some given productive activity, is therefore equivalent to the (discrete) assignment problem that consists in solving

$$F(\mathbf{x},\mathbf{z}) = \max_{\{(k_s,l_s),\ldots,(k_S,l_S)\}} \sum_{s=1}^{S} a_{k_s l_s}(\mathbf{z});$$

where $S = \inf(K, L)$. The maximum is taken over all possible assignments or collections of disjoint pairs $\{(k_s,l_s),\ldots,(k_S,l_S)\}$, all $k \in \{1,\ldots,K\}$, all $l \in \{1,\ldots,L\}$. Then one can now adapt the proofs of two theorems by Shapley (1962) to the present problem.

7.1 Proposition 1

Under scrutiny is the impact on $F(\mathbf{x},\mathbf{z})$ of a change in the quantity of input factors $\mathbf{x}$.

Proof of Proposition 1(i) **(decreasing differences in x)** WLG consider the case $(\mathbf{x}',\mathbf{x}'') \in X_K \times X_K$ with $\mathbf{x}' = (0,\ldots,0,x'_{k'},0,\ldots 0)$, and $\mathbf{x}'' = (0,\ldots,0,x''_{k''},0,\ldots 0)$, where $k', k'' \in \{1,\ldots,K\}$ and $x'_{k'}, x''_{k''} > 0$. That is, the input factors $\mathbf{x}$ to be assigned optimally by pairs in production are augmented by $\mathbf{x}'$ and $\mathbf{x}''$. The proof proceeds by induction on the number $\lambda(K,L) = \inf(K, L-1)$.

<u>Case 1</u>: $\lambda(K,L) = \inf(K, L-1) = K = 0$ is ruled out by assumption $A.3$ (complementarity).

<u>Case 2</u>: $\lambda(K,L) = \inf(K,L-1) = L-1 = 0 \Rightarrow L = 1$ and $K \geq 1$. Because this case was seen as "*easy, though not wholly trivial*" (p. 46) in the proof of the corresponding Theorem 1 by Shapley (1962), the discussion was omitted. It can proceed as follows. In that case the $K \times L$-dimensional performance matrix $\mathbf{a}(\mathbf{z})$ collapses to a K-dimensional vector $a(\mathbf{z}) = (a_{11}(\mathbf{z}), a_{21}(\mathbf{z}),\ldots,a_{K1}(\mathbf{z}))$ with $K \geq 1$. The problem consists of assigning a single worker type, say, type-1 worker in quantity x_1, to one of the K machine types available in quantities $x_{k\in\{1,\ldots,K\}}$ that yields the highest possible level of output. Because the parameters that describe the competence vector remain constant, for clarity $\mathbf{z}$ is ommited in the notation of this paragraph. One obtains

$$\begin{aligned}
F(\mathbf{x}) &= \sup\{a_{k1}\}_{k=1}^{K}, \\
F(\mathbf{x}+\mathbf{x}') &= \sup\{F(\mathbf{x}), a_{k'1}\}, \\
F(\mathbf{x}+\mathbf{x}'') &= \sup\{F(\mathbf{x}), a_{k''1}\}, \\
F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') &= \sup\{F(\mathbf{x}), a_{k'1}, a_{k''1}\},
\end{aligned}$$

where $k', k'' \in \{1, \ldots, K\}$. Two subcases arise. First, $F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') = F(\mathbf{x}) \Rightarrow F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') = F(\mathbf{x}+\mathbf{x}') = F(\mathbf{x}+\mathbf{x}'')$. Therefore $F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'')-F(\mathbf{x}+\mathbf{x}') = F(\mathbf{x}+\mathbf{x}'')-F(\mathbf{x})$, and Proposition 1($i$) holds. Second, $F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') > F(\mathbf{x})$ leads to other 4 subcases:

<u>(i) *either*</u> $a_{k'1} > F(\mathbf{x})$ and $a_{k''1} \leq F(\mathbf{x})$, then $F(\mathbf{x}+\mathbf{x}') = F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') = a_{k'1}$ and $F(\mathbf{x}+\mathbf{x}'') = F(\mathbf{x}) = a_{k1}$. Therefore $F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'')-F(\mathbf{x}+\mathbf{x}') = F(\mathbf{x}+\mathbf{x}'')-F(\mathbf{x})$, and Proposition 1($i$) holds;

<u>(ii) *or*</u> $a_{k'1} > F(\mathbf{x})$ and $a_{k''1} \geq F(\mathbf{x})$, then $F(\mathbf{x}+\mathbf{x}') = a_{k'1}$ and $F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') = \sup\{a_{k'1}, a_{k''1}\}$. If $\sup\{a_{k'1}, a_{k''1}\} = a_{k'1}$ then $F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') - F(\mathbf{x}+\mathbf{x}') = 0$ and $F(\mathbf{x}+\mathbf{x}'') - F(\mathbf{x}) \geq 0$. If $\sup\{a_{k'1}, a_{k''1}\} = a_{k''1}$ then $F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') - F(\mathbf{x}+\mathbf{x}'') = 0$ and $F(\mathbf{x}+\mathbf{x}') - F(\mathbf{x}) > 0$. Therefore $F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') - F(\mathbf{x}+\mathbf{x}') \leq F(\mathbf{x}+\mathbf{x}'') - F(\mathbf{x})$, and Proposition 1($i$) holds;

<u>(iii) *or*</u> $a_{k''1} > F(\mathbf{x})$ and $a_{k'1} \leq F(\mathbf{x})$: symmetric to subcase ($i$);

<u>(iv) *or*</u> $a_{k''1} > F(\mathbf{x})$ and $a_{k'1} \geq F(\mathbf{x})$: symmetric to subcase ($ii$).

<u>Case 3</u>: $\lambda(K, L) = \inf(K, L-1) = L-1 > 0 \Rightarrow L \geq 2$ and $K \geq 1$. Now the two capital inputs $\mathbf{x}'$ and $\mathbf{x}''$ can be simultaneously introduced in production. Three subcases arise. The first two are trivial, the third one is of more interest. First, only $x'_{k'}$ appears in the output maximizing assignment. One finds $F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') = F(\mathbf{x}+\mathbf{x}')$ and $F(\mathbf{x}+\mathbf{x}'') = F(\mathbf{x})$, and Proposition 1($i$) holds. Second, only $x''_{k''}$ appears in the output maximizing assignment. One finds $F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') = F(\mathbf{x}+\mathbf{x}'')$ and $F(\mathbf{x}+\mathbf{x}') = F(\mathbf{x})$ and Proposition 1(i) holds. Third, both $x'_{k'}$ and $x''_{k''}$ appear in the output maximizing assignment. In this subcase the following is more or less immediate from Shapley (1962). The induction hypothesis says that Proposition 1(i) holds at rank $\lambda(K, L)$, that is for

$$\text{either} \begin{cases} \lambda(K,L) = L-1, \\ \lambda(K,L) \leq K, \end{cases} \text{or} \begin{cases} \lambda(K,L) = K, \\ \lambda(K,L) \leq L-1, \end{cases}$$

when input factors are augmented by capital inputs only (i.e., the "capital" version). Suppose now that Proposition 1(i) is false for the "capital" version for all $(\mathbf{x}', \mathbf{x}'') \in X_K \times X_K$, i.e. that

$$F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') - F(\mathbf{x}+\mathbf{x}') > F(\mathbf{x}+\mathbf{x}'') - F(\mathbf{x}), \tag{8}$$

and look for a contradiction. Let $l', l'' \in \{1, \ldots, L\}$ be the indices of the labour inputs $x_{l'}$ and $x_{l''}$ paired with $x'_{k'}$ and $x''_{k''}$, respectively, in the output maximizing assignment of the augmented quantity of inputs $\mathbf{x}+\mathbf{x}'+\mathbf{x}''$. The corresponding vectors $(\mathbf{x}_{l'}, \mathbf{x}_{l''}) \in$

$X_L \times X_L$ are such that $\mathbf{x}_{l'} = (0,\ldots,0,x_{l'},0,\ldots 0)$, and $\mathbf{x}_{l''} = (0,\ldots,0,x_{l''},0,\ldots 0)$. Now $F(\mathbf{x}+\mathbf{x}'+\mathbf{x}'') = F(\mathbf{x}-\mathbf{x}_{l'}-\mathbf{x}_{l''}) + a_{k'l'} + a_{k''l''}$ together with (8) implies that

$$F(\mathbf{x}-\mathbf{x}_{l'}-\mathbf{x}_{l''}) + a_{k'l'} + a_{k''l''} - F(\mathbf{x}+\mathbf{x}') > F(\mathbf{x}+\mathbf{x}'') - F(\mathbf{x}). \qquad (9)$$

Moreover

$$\begin{cases} F(\mathbf{x}+\mathbf{x}') & = F(\mathbf{x}-\mathbf{x}_{l'}) + a_{k'l'} \\ F(\mathbf{x}+\mathbf{x}'') & = F(\mathbf{x}-\mathbf{x}_{l''}) + a_{k''l''} \end{cases}$$

together with (9) implies that

$$F(\mathbf{x}-\mathbf{x}_{l'}-\mathbf{x}_{l''}) + a_{k'l'} + a_{k''l''} - [F(\mathbf{x}-\mathbf{x}_{l'}) + a_{k'l'}] > [F(\mathbf{x}-\mathbf{x}_{l''}) + a_{k''l''}] - F(\mathbf{x}).$$

Eventually, one obtains $F(\mathbf{x}) - F(\mathbf{x}-\mathbf{x}_{l''}) > F(\mathbf{x}-\mathbf{x}_{l'}) - F(\mathbf{x}-\mathbf{x}_{l'}-\mathbf{x}_{l''})$, which says that Proposition $1(i)$ does not hold for the "labour" version (when input factors are augmented by labour inputs only) at rank $\lambda(L-2,K)$. Now observe that $\lambda(L-2,K) = \min(L-2,K-1) = \min(L-1-1,K-1) = \min(K-1,L-1-1) = \lambda(K,L)-1$. This implies in turn that Proposition $1(i)$ does not hold for the "capital" version at rank $\lambda(K,L)-1$, contradicting the induction hypothesis. Therefore Proposition $1(i)$ holds for the "capital" version at rank $\lambda(K,L)+1$, as required.□

Proof of Proposition $1(ii)$ **(increasing differences in x)** The adaptation of the proof of the Theorem 2 by Shapley (1962, p. 47) for the proof of Proposition $1(ii)$ strictly follows the adaptation above of the proof of the Theorem 1 by Shapley (1962, p. 46) for the proof of Proposition $1(i)$ above. Considering the case $(\mathbf{x}',\mathbf{x}'') \in X_K \times X_L$, the proof now proceeds by induction on the number $\lambda(K,L) = \inf(K,L)$. In order to avoid redundancy it is omitted here and can be obtained from the author upon request.□

7.2 Proposition 2

One can now build on Proposition 1 in order to consider the impact on $F(\mathbf{x},\mathbf{z})$ of a change in capability parameters in $\mathbf{z}$.

Proof of Proposition $2(i)$ **(decreasing differences in z)** WLG, let $(\mathbf{z}',\mathbf{z}'') \in Z_K \times Z_K$ with $\mathbf{z}' = (0,\ldots,0,z_{k'l'},0,\ldots 0)$, and $\mathbf{z}'' = (0,\ldots,0,z_{k''l''},0,\ldots 0)$, where k', $k'' \in \{1,\ldots,K\}$, l', $l'' \in \{1,\ldots,L\}$ and $z_{k'l'} \geq 0$, $z_{k''l''} \geq 0$ are such that $z_{k'l'} + z_{k'l'} \leq 1$ and $z_{k''l''} + z_{k''l''} \leq 1$, and for the particular case in which $k' = k'' = k$ and $l' = l'' = l$,

$z_{kl} + z_{k'l'} + z_{k''l''} \leq 1$ for the change in the capability parameters to make economic sense. Four cases arise. Case 1: $z_{k'l'} = 0$ and $z_{k''l''} = 0$. $z_{k'l'}x_{kl} = z_{k'l'}x_{kl} = 0 \Rightarrow F(\mathbf{x}, \mathbf{z} + \mathbf{z}' + \mathbf{z}'') = F(\mathbf{x}, \mathbf{z} + \mathbf{z}') = F(\mathbf{x}, \mathbf{z} + \mathbf{z}'') = F(\mathbf{x}, \mathbf{z})$, and Proposition $2(i)$ holds. Case 2: $z_{k'l'} = 0$ and $z_{k''l''} > 0$. $z_{k'l'}x_{kl} = 0 \Rightarrow F(\mathbf{x}, \mathbf{z} + \mathbf{z}') = F(\mathbf{x}, \mathbf{z})$ and $F(\mathbf{x}, \mathbf{z} + \mathbf{z}' + \mathbf{z}'') = F(\mathbf{x}, \mathbf{z} + \mathbf{z}'')$, and Proposition $2(i)$ holds. Case 3: $z_{k'l'} > 0$ and $z_{k''l''} = 0$. $z_{k''l''}x_{kl} = 0 \Rightarrow F(\mathbf{x}, \mathbf{z} + \mathbf{z}'') = F(\mathbf{x}, \mathbf{z})$ and $F(\mathbf{x}, \mathbf{z} + \mathbf{z}' + \mathbf{z}'') = F(\mathbf{x}, \mathbf{z} + \mathbf{z}')$, and Proposition $2(i)$ holds. Case 4: $z_{k'l'} > 0$ and $z_{k''l''} > 0$. Considering changes in the performance of tasks (k', l') and (k'', l'') that are induced by a change in organizational knowledge by the addition of $\mathbf{z}'$ and/or $\mathbf{z}''$ to $\mathbf{z}$, one obtains $a_{k'l'}(\mathbf{z} + \mathbf{z}') = z_{k'l'}\alpha_{k'}x_{k'} + z_{k'l'}\alpha_{k'}x_{k'} + z_{l'k'}\alpha_{l'}x_{l'}$. This implies that $F(\mathbf{x}, \mathbf{z} + \mathbf{z}') = F(\mathbf{x} + \mathbf{x}', \mathbf{z})$, where

$$\begin{cases} \mathbf{x}' = (0, \ldots, 0, z_{k'l'}x_{k'}, 0, \ldots, 0) & \in X_K \quad \text{if } z_{k'l'} = 0, \\ \mathbf{x}' = (0, \ldots, 0, \frac{z_{k'l'}}{z_{k'l'}}x_{k'}, 0, \ldots, 0) & \in X_K \quad \text{if } z_{k'l'} > 0. \end{cases}$$

Symmetrically, $a_{k''l''}(\mathbf{z} + \mathbf{z}'') = z_{k''l''}\alpha_{k''}x_{k''} + z_{k''l''}\alpha_{k''}x_{k''} + z_{l''k''}\alpha_{l''}x_{l''}$ implies that $F(\mathbf{x}, \mathbf{z} + \mathbf{z}'') = F(\mathbf{x} + \mathbf{x}'', \mathbf{z})$, where

$$\begin{cases} \mathbf{x}'' = (0, \ldots, 0, z_{k''l''}x_{k''}, 0, \ldots, 0) & \in X_K \quad \text{if } z_{k''l''} = 0, \\ \mathbf{x}'' = (0, \ldots, 0, \frac{z_{k''l''}}{z_{k''l''}}x_{k''}, 0, \ldots, 0) & \in X_K \quad \text{if } z_{k''l''} > 0. \end{cases}$$

Eventually, $F(\mathbf{x}, \mathbf{z} + \mathbf{z}' + \mathbf{z}'') - F(\mathbf{x}, \mathbf{z} + \mathbf{z}') = F(\mathbf{x} + \mathbf{x}' + \mathbf{x}'', \mathbf{z}) - F(\mathbf{x} + \mathbf{x}', \mathbf{z})$, and $F(\mathbf{x}, \mathbf{z} + \mathbf{z}'') - F(\mathbf{x}, \mathbf{z}) = F(\mathbf{x} + \mathbf{x}'', \mathbf{z}) - F(\mathbf{x}, \mathbf{z})$, where $(\mathbf{x}', \mathbf{x}'') \in X_K \times X_K$. Accordingly, Proposition $1(i)$ applies to yield $F(\mathbf{x}, \mathbf{z} + \mathbf{z}' + \mathbf{z}'') - F(\mathbf{x}, \mathbf{z} + \mathbf{z}') \leq F(\mathbf{x}, \mathbf{z} + \mathbf{z}'') - F(\mathbf{x}, \mathbf{z})$, as required.□

Proof of Proposition $2(ii)$ (increasing differences in z) WLG consider the case $(\mathbf{z}', \mathbf{z}'') \in Z_K \times Z_L$ with $\mathbf{z}' = (0, \ldots, 0, z_{k'l'}, 0, \ldots 0)$, and $\mathbf{z}'' = (0, \ldots, 0, z_{l''k''}, 0, \ldots 0)$, where $k', k'' \in \{1, \ldots, K\}$, $l', l'' \in \{1, \ldots, L\}$ and $z_{k'l'} \geq 0$, $z_{l''k''} \geq 0$ are such that $z_{k'l'} + z_{k'l'} \leq 1$ and $z_{k''l''} + z_{l''k''} \leq 1$, for the change in the capability parameters to make economic sense. Then the proof proceeds as in the preceding paragraph (for decreasing differences) and one finds $F(\mathbf{x}, \mathbf{z} + \mathbf{z}' + \mathbf{z}'') - F(\mathbf{x}, \mathbf{z} + \mathbf{z}') = F(\mathbf{x} + \mathbf{x}' + \mathbf{x}'', \mathbf{z}) - F(\mathbf{x} + \mathbf{x}', \mathbf{z})$, and $F(\mathbf{x}, \mathbf{z} + \mathbf{z}'') - F(\mathbf{x}, \mathbf{z}) = F(\mathbf{x} + \mathbf{x}'', \mathbf{z}) - F(\mathbf{x}, \mathbf{z})$, where

$$\begin{cases} \mathbf{x}' = (0, \ldots, 0, z_{k'l'}x_{k'}, 0, \ldots, 0) & \in X_K \quad \text{if } z_{k'l'} = 0, \\ \mathbf{x}' = (0, \ldots, 0, \frac{z_{k'l'}}{z_{k'l'}}x_{k'}, 0, \ldots, 0) & \in X_K \quad \text{if } z_{k'l'} > 0, \end{cases}$$

and

$$\begin{cases} \mathbf{x}'' = (0, \ldots, 0, z_{l''k''}x_{l''}, 0, \ldots, 0) & \in X_L \quad \text{if } z_{l''k''} = 0, \\ \mathbf{x}'' = (0, \ldots, 0, \frac{z_{l''k''}}{z_{l''k''}}x_{l''}, 0, \ldots, 0) & \in X_L \quad \text{if } z_{l''k''} > 0. \end{cases}$$

Accordingly, Proposition 1(ii) applies to yield $F(\mathbf{x}, \mathbf{z}+\mathbf{z}'+\mathbf{z}'') - F(\mathbf{x}, \mathbf{z}+\mathbf{z}') \geq F(\mathbf{x}, \mathbf{z}+\mathbf{z}'') - F(\mathbf{x}, \mathbf{z})$, as required.$\square$

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