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Abstract

The Cox statistic for non nested models is used to test the probit and logit specifications estimated by Bardasi and Monfardini (1997) for the occupational choice of the Italian workers among the private, the public and the self-employed options. Computation of different versions of test is performed, some of which rely on simulation of the pseudo-true value. The bootstrap technique is then applied to control for the actual properties of the test. The results of the testing procedure indicate the probit as the more adequate model, supporting previous evidence found against the IIA assumption imposed by the logit formulation.

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1 Introduction

The issue of specification testing is often neglected in the applied econometric literature of multinomial choice models. When the choice is represented by a discrete variable, as in many applications in labour economics or in the transport mode literature, two basic specifications are put forward: the multinomial logit (MNL) and the multinomial probit (MNP) one. The MNL model stems from the assumption of type 1 extreme value distribution of the error terms, while in the MNP model the errors are assumed to be normally distributed. It is well known that the two models are distinguished not only in as far as the functional form is concerned, but also in the flexibility they exhibit in terms of structure of the correlations between the different alternatives. However, the greater generality of the MNP model, which allows for the relaxation of the assumption of Independence of the Irrelevant Alternatives (IIA) hypothesis imposed by the MNL formulation, can be exploited by imposing a much greater computational burden in the derivation of the Maximum Likelihood estimates of the parameters.

Bardasi and Monfardini (1997) emphasized this trade-off and estimated both models, using the same information set, to explain the choice of the labour sectors of the Italian workers among the private, public and self-employment options. The main finding of their investigation is the strong rejection of the IIA hypothesis. This hypothesis can in fact be tested once the MNP specification is adopted, as in this framework it is possible to estimate the model both imposing a zero covariance parameter and releasing this constraint. Another important conclusion concerns the comparison of the two formulations. The results of the estimation of MNL and MNP models are found to be different in terms of estimated parameters of the utility regressions, i.e. of the deterministic component, and significance of the explanatory variables. Interestingly, such differences also emerge when the two models are estimated by imposing the same covariance matrix structure, indicating the effect of the pure functional form assumption.

In a recent review on the multinomial probit model, Weeks (1995) highlights that the lack of specification testing is a serious problem in this kind of modelling, as the misspecification of the stochastic component (i.e. of the error term) has an impact on the parameters of the deterministic component. More precisely, the mean equation parameters, which are generally the ones of interest for the analyst, are influenced by an incorrect assumption on the distribution of the underlying data generating process. The author points out the complexity of the estimation procedure for the MNP model as the cause of the neglection of specification analysis in multinomial choice models. In fact, the most widely applied procedures, Hausman's test (1978) and the nested logit approach of Hausman and McFadden (1984), provide tools for specification tests of the MNL model which exclude consideration of the MNP as an alternative.

The aim of this paper is to analyse in greater depth the estimation results of Bardasi and Monfardini (1997), and to discover if the evidence they found in favour of the MNP model as far as the appropriateness of the IIA assumption is concerned, can be confirmed by performing rigorous specification testing to compare the MNP and the MNL formulations. In particular, we adopt the classical non-nested approach based on the Cox test statistic (1961, 1962). This test statistic might well present two computational difficulties: this is because it involves the evaluation of an integral in its numerator that may not have a closed form solution, and it requires the evaluation of the pseudo-true value which may not be analytically computable.

Pesaran and Pesaran (1993, 1995) suggest how simulations can be used to overcome these difficulties. Pesaran and Pesaran (1993) also apply the simulated Cox statistic to test a binary logit model against a binary probit one, and show that in this case the integral in the numerator of the test is analytically computable. This result is also valid for MNL and MNP models, and the application of the Cox test to these models only requires the estimation through simulation of the pseudo-true value. Unfortunately, however, the performance in finite samples of the Cox test, both evaluated analytically or via simulation, is unknown. A broad Monte Carlo investigation on the issue of the test of MNL and

MNP models using the Cox's test is given in Weeks (1995). His results on the performance of different versions of the test show that the behaviour of the test in small samples could differ substantially from the asymptotic one. As pointed out by the author, a possible solution to this problem in the application of the test to real data is given by the bootstrap methodology (see, among many others, Hall (1992)), which allows the derivation of an approximation of the small sample distribution of the test.

In this paper, we perform the computation of a series of versions of the Cox test for the MNL and the MNP specifications for the two models estimated in Bardasi and Monfardini (1997) for the choice of the labour sector in Italy for men and women respectively. In order to make an investigation on the actual size of the test, we apply the "parametric" bootstrap technique by resampling from the "estimated" distribution of the dependent variable. The aim of the bootstrap experiment is twofold: firstly, it allows an evaluation of the ability of the finite sample distribution to represent a good approximation of the asymptotic one, secondly, it provides guidelines for rejecting the null hypothesis on the basis of the finite sample distribution of the test. The closer the bootstrap distribution comes to the asymptotic one, the more likely is that the decision to accept or reject the null taken by confronting the applied test with the asymptotic critical values will coincide with the decision based on the bootstrap critical values.

In the case of women, although our sample is quite large (2,563 observations) the empirical distribution is found to be quite "distant" from the asymptotic one, confirming the caution that has to be used in applications of the Cox test based on the asymptotic distribution. On the contrary, in the model for men, which is estimated on 4,790 observations, the bootstrap distribution appears to be fairly close to the asymptotic one. Therefore, it can be conjectured that the dimension of the sample must be very large in order to safely apply the Cox test without deriving its bootstrap distribution.

As far as the outcome of the test is concerned, our results generally support the MNP model. In particular, we find that in the case of women the logit model can be rejected in favour of the probit model, while this latter cannot be rejected in favour of the logit one (although it can be rejected in a direction other than that of the logit). In case of the men the logit model can be rejected in favour of the probit, while the probit model cannot be rejected. Moreover, when the MNL is confronted with a MNP estimated by imposing the constraint of the IIA assumption, in both male and female cases the MNP is preferable to the MNL, but with less favourable evidence, as the logit model is rejected in favour of the probit model, while the probit is rejected in a direction other than the logit. In other words, it is more difficult to discriminate between the two models when they only distinguish in the functional form, and the covariance structure is not relaxed in the probit formulation. This provides further evidence against the adequacy of imposing a zero covariance structure in our estimated models.

The paper is structured as follows: section 2 recalls the features of trinomial probit and logit models, section 3 describes how the form of the Cox test specialises in this case and presents different asymptotically equivalent versions of the test. In section 4 the results of the application of the test and of the bootstrap procedure are reported and commented on. Section 5 concludes.

2 The trinomial logit and probit models

In order to model the choice among discrete alternatives, it is usual to express the utility that individual i attaches to alternative j, U_{ij}, as the sum of a non-stochastic component, which is a linear function of individual and alternative specific observable characteristics $\underline{x}_i$, and an unobserved random component, ε_{ij}:

$$U_{ij} = \underline{x}_i'\beta_j + \varepsilon_{ij}, \tag{1}$$

$j = 1...J$. We will focus on the case $J = 3$. The parameter vectors β_j are unknown and they are the object of inference. Alternative j is chosen by individual i if $U_{ij} > U_{ik}$ for all $k \neq j$; the utility indicator U_{ij} is not observed, while the choice among the three alternatives is. Introducing the discrete variable y_i, with domain $\{1, 2, 3\}$, we put $y_i = j$ iff choice j is observed for individual i. Given this framework, what differentiates the probit model from the logit is the distributional assumption for the error term $\varepsilon_i = (\varepsilon_{i1}, \varepsilon_{i2}, \varepsilon_{i3})'$. In both models such a random vector is assumed to be independently and identically distributed across the N individuals belonging to the information set, i.e. we can write $\varepsilon_i = \varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3)' \quad \forall i = 1...N$. The probit model postulates for ε a trivariate normal distribution, with $E(\varepsilon_j) = 0, Var(\varepsilon_j) = \omega_j^2$, $Cov(\varepsilon_j \varepsilon_k) = \omega_{jk}$; $j = 1, 2, 3,\ j \neq k$. The logit model is characterized by the assumption that the ε_j have identical independent type 1 extreme value distribution, which implies $Var(\varepsilon_j) = \frac{\pi^2}{6}$, $Cov(\varepsilon_j \varepsilon_k) = 0$. Notice that the probit model can also be distinguished from the logit one on account of its capacity to postulate a covariance pattern among the error components of the utility indicators, which makes it more general.

Both the trinomial probit (TNP) and the trinomial logit (TNL) models in the form outlined above suffer from an identification problem deriving from the uninformativeness of the observed choice on the level of the utilities. It is convenient, therefore, to reparametrize the model in differenced utilities or, equivalently, to set one of the mean equation parameters β_j equal to zero. Putting $u_{ij} = U_{ij} - U_{i3}$ (the normalization is arbitrary) and particularizing the notation the two models to which we will refer in the rest of the paper are as follows:

- TNL:

$$\begin{aligned} u_{i1}^l &= \underline{x}_i'\delta_1 + \eta_{i1} \\ u_{i2}^l &= \underline{x}_i'\delta_2 + \eta_{i2} \\ u_{i3}^l &= 0 \end{aligned} \qquad \begin{pmatrix} \eta_{i1} \\ \eta_{i2} \end{pmatrix} \sim i.i.d.\ Logistic\left(\underline{0}, \begin{bmatrix} \frac{\pi^2}{3} & 0 \\ 0 & \frac{\pi^2}{3} \end{bmatrix}\right)$$

- TNP:

$$\begin{aligned} u_{i1}^p &= \underline{x}_i'\alpha_1 + v_{i1} \\ u_{i2}^p &= \underline{x}_i'\alpha_2 + v_{i2} \\ u_{i3}^p &= 0 \end{aligned} \qquad \begin{pmatrix} v_{i1} \\ v_{i2} \end{pmatrix} \sim i.i.d.\ N\left(\underline{0}, \begin{bmatrix} 1 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}\right)$$

where η_{ij} and $\upsilon_{ij}, j = 1,2$, are obtained by subtracting to the corresponding error terms ε_{ij} in (4.1) the error component of the third utility, ε_{i3}, which gives the logistic distribution in the first case, and the normalization $\sigma_1^1 = 1$ has to be imposed because the scale of the vector $\upsilon_i = (\upsilon_{i1}, \upsilon_{i2})'$ is not identified.[1] Collecting the parameters as: $\delta = (\delta_1', \delta_2')'$, $\alpha = (\alpha_1', \alpha_2', \sigma_{12}, \sigma_2^2)'$, and denoting by $P_{ih}^j = \Pr\{y_i = j\} = \Pr\left\{u_{ij}^h > u_{ik}^h, \forall k \neq j\right\}$, $h = l, p$, the selection probability implied by the two models, their loglikelihood functions are given by:

- TNL(δ):

$$\begin{aligned} \overline{L}_l(\delta) &= \frac{1}{N}\sum_{i=1}^{N} l_l^i(\delta) \\ &= \frac{1}{N}\sum_{i=1}^{N}\left(m_{i1}\ln P_{il}^1(\delta) + m_{i2}\ln P_{il}^2(\delta) + m_{i3}\ln P_{il}^3(\delta)\right), \end{aligned} \tag{2}$$

where:

$$\begin{aligned} P_{il}^j(\delta) &= \frac{\exp(\underline{x}_i'\delta_j)}{\sum_{k=1}^{3}\exp(\underline{x}_i'\delta_k)}, \qquad j = 1,2,3, \quad \delta_3 = 0, \\ m_{ij} &= \begin{cases} 1 & if \quad y_i = j \\ 0 & otherwise \end{cases} \end{aligned}$$

- TNP(α):

$$\begin{aligned} \overline{L}_p(\alpha) &= \frac{1}{N}\sum_{i=1}^{N} l_p^i(\alpha) \\ &= \frac{1}{N}\sum_{i=1}^{N}\left(m_{i1}\ln P_{ip}^1(\alpha) + m_{i2}\ln P_{ip}^2(\alpha) + m_{i3}\ln P_{ip}^3(\alpha)\right), \end{aligned} \tag{3}$$

[1]See Bardasi and Monfardini (1997) for a more detailed discussion of the identification problem in the probit model.

with:

$$P^1_{ip}(\alpha) = \int_{-\infty}^{\frac{x_i'\alpha_1 - x_i'\alpha_2}{\sqrt{1+\sigma_2^2-2\sigma_{12}}}} \int_{-\infty}^{x_i'\alpha_1} \varphi(z_1, z_2; \rho_1)\, dz_1 dz_2,$$

$$P^2_{ip}(\alpha) = \int_{-\infty}^{\frac{x_i'\alpha_2 - x_i'\alpha_1}{\sqrt{1+\sigma_2^2-2\sigma_{12}}}} \int_{-\infty}^{\frac{x_i'\alpha_2}{\sigma_2}} \varphi(z_1, z_2; \rho_2)\, dz_1 dz_2,$$

$$P^3_{ip}(\alpha) = \int_{-\infty}^{-x_i'\alpha_1} \int_{-\infty}^{\frac{-x_i'\alpha_2}{\sigma_2}} \varphi(z_1, z_2; \rho_3)\, dz_1 dz_2,$$

where $\varphi(z_1, z_2; \rho)$ is the bivariate normal density function of two random variables having zero mean, unit variance and correlation coefficient ρ, with: $\rho_1 = \frac{1-\sigma_{12}}{\sqrt{1+\sigma_2^2-2\sigma_{12}}}$, $\rho_2 = \frac{\sigma_2^2-\sigma_{12}}{\sqrt{1+\sigma_2^2-2\sigma_{12}}}$, $\rho_3 = \frac{\sigma_{12}}{\sigma_2^2}$.

Both the loglikelihood functions above have to be maximized numerically, but the probit one is complicated by the presence of bivariate integrals with no closed form solution inside the loglikelihood function itself. This is not a major problem in this bivariate case (and in the trivariate one), as it is possible to insert the numerical solutions of the integrals inside the numerical maximization procedure using, for example, the existing routines in Gauss.

3 The Cox test for TNL and TNP models

The testing procedure proposed by Cox (1961, 1962) is a generalization of the likelihood ratio test for the non-nested case. The non-nested testing approach has the advantage of leading to tests which are powerful against the specific alternative considered. On the other hand, the nesting approach, based on the formulation of a general model from which the model under scrutiny can be derived as a special case, will in general lead to tests having "some" power against different directions of departure from the null hypothesis. Anyway, for the example in which we are interested, the non-nested approach seems the more natural one to choose, as we have two estimated alternative models available. The procedure consists in taking the two models in turn as the null hypothesis and in checking the validity of the one model against the evidence provided by the other, which is taken as the alternative hypothesis. If one model can be rejected and the other cannot, this will support the latter.

The application of the Cox test is usually limited in the econometric practice because it is likely to involve, in many cases, some computational difficulties which can be, however, overcome by resorting to simulation techniques. Different recent studies propose a range of simulation-based methods to evaluate the test statistic: see Pesaran and Pesaran (1993,1995), Weeks (1995), Monfardini (1995). For the case we analyse in this paper, the form of the likelihood function is such that the above-mentioned computational problems are partially solved. In fact, the expected value which characterizes the numerator of the test statistic can be solved analytically. The second source of computational difficulty is the evaluation of the pseudo-true value, i.e. the expected value of the parameter of the alternative model under the null one. The estimation of the pseudo-true value can be obtained using simulations. Another possible solution is simply to use the maximum likelikood estimator of the parameter under the alternative which is asymptotically equivalent to the estimated pseudo-true value under the null, as proposed by White (1982). We assume the regularity conditions set out in the same article, needed for the asymptotic properties of Pseudo Maximum Likelihood Estimators (PMLE), to hold.

Under these conditions, Cox's statistic has an asymptotic standard normal distribution. The test can be given a one-sided interpretation, motivated by the fact that under the alternative hypothesis the distribution is shifted to the left, or a two-sided interpretation, if one wants to take into account departures from the null model in directions other than the alternative model.

Depending on which model is taken as the null hypothesis and the choice of the estimator for the pseudo-true value and for the variance, we can particularize the different forms of the Cox test statistic as described in the following two sections.

3.1 Testing probit *versus* logit

The testing problem has the form:

$$\begin{cases} H_0 : TNP(\widehat{\alpha}) \\ H_1 : TNL(\widehat{\delta}) \end{cases}$$

where $\widehat{\alpha}$ and $\widehat{\delta}$ are the (pseudo) maximum likelihood estimators maximizing (3) and (2) respectively, $plim_{H_0}\ \widehat{\alpha} = \alpha_0$ and $plim_{H_0}\ \widehat{\delta} = \delta_{\alpha_0}$. The first distinction between two possible versions is to use the PMLE $\widehat{\delta}$, or an estimate of the pseudo-true value, $\delta_{\widehat{\alpha}}$ inside the expected value in the numerator:

$$\begin{aligned} S_p(\widehat{\delta}) &= \sqrt{N}\frac{\overline{L}_p(\widehat{\alpha}) - \overline{L}_l(\widehat{\delta}) - E_{\widehat{\alpha}}\left[\overline{L}_p(\widehat{\alpha}) - \overline{L}_l(\widehat{\delta})\right]}{V_{\widehat{\alpha}}^{\frac{1}{2}}} = \sqrt{N}\frac{\phi_p(\widehat{\delta})}{V_{\widehat{\alpha}}^{\frac{1}{2}}}, \quad (4) \\ S_p(\delta_{\widehat{\alpha}}) &= \sqrt{N}\frac{\overline{L}_p(\widehat{\alpha}) - \overline{L}_l(\widehat{\delta}) - E_{\widehat{\alpha}}\left[\overline{L}_p(\widehat{\alpha}) - \overline{L}_l(\delta_{\widehat{\alpha}})\right]}{V_{\widehat{\alpha}}^{\frac{1}{2}}} = \sqrt{N}\frac{\phi_p(\delta_{\widehat{\alpha}})}{V_{\widehat{\alpha}}^{\frac{1}{2}}}. \end{aligned}$$

The second distinction concerns the choice for the estimator of the variance in the denominator. Pesaran and Pesaran (1995), propose two analytical estimators, we denote by $V_{\widehat{\alpha}}^{a}$ and $V_{\widehat{\alpha}}^{c}$,[2] whose expressions are:

$$\begin{aligned} V_{\widehat{\alpha}}^{a}(\widehat{\delta}) &= \frac{1}{N}\sum_{t=1}^{N}(\hat{b}_{p,i} - \bar{b}_p)^2 - \Psi'_{p,N}(\hat{\alpha},\hat{\delta})F_{p,N}(\hat{\alpha})^{-1}\Psi_{pN}(\hat{\alpha},\hat{\delta}), \\ V_{\widehat{\alpha}}^{c}(\widehat{\delta}) &= \frac{1}{N-1}\sum_{t=1}^{N}(\hat{b}_{p,i} - \bar{b}_p)^2, \end{aligned}$$

where:

$$\hat{b}_{p,i} = l_p^i(\hat{\alpha}) - l_l^i(\hat{\delta}),$$

[2]This estimator ignores the variance due to the uncertainty of the estimated parameters under the null hypothesis.

$$
\begin{aligned}
\bar{b}_p &= \frac{1}{N}\sum_{i=1}^{N}\hat{b}_{p,i}, \\
\Psi_{p,N}(\hat{\alpha},\hat{\delta}) &= \frac{1}{N}\sum_{i=1}^{N}\hat{b}_{p,i}\frac{\partial l_p^i(\alpha)}{\partial\alpha}_{|\alpha=\hat{\alpha}}, \\
F_{p,N}(\hat{\alpha}) &= \frac{1}{N}\sum_{i=1}^{N}\frac{\partial l_p^i(\alpha)}{\partial\alpha}\frac{\partial l_p^i(\alpha)}{\partial\alpha'}_{|\alpha=\hat{\alpha}},
\end{aligned}
$$

Moreover the two expressions in (4) can be evaluated substituting $\widehat{\delta}$ for $\delta_{\widehat{\alpha}}$, in this case it is convenient to adopt the differentiated notation: $V_{\widehat{\alpha}}^{a'}(\delta_{\widehat{\alpha}})$, $V_{\widehat{\alpha}}^{c'}(\delta_{\widehat{\alpha}})$.

As far as the practical computation of test is concerned, it is important to notice that for the case under scrutiny, the expected value which appears in the numerator of (4) does have an analytical soultion, i.e. we have, considering for example $S_p(\widehat{\delta})$: [3]

$$
E_{\widehat{\alpha}}\left[\overline{L}_p(\widehat{\alpha})-\overline{L}_l(\widehat{\delta})\right]=\frac{1}{N}\sum_{t=1}^{N}\left(P_{ip}^1(\widehat{\alpha})\ln\frac{P_{ip}^1(\widehat{\alpha})}{P_{il}^1(\widehat{\delta})}+P_{ip}^2(\widehat{\alpha})\ln\frac{P_{ip}^2(\widehat{\alpha})}{P_{il}^2(\widehat{\delta})}+P_{ip}^3(\widehat{\alpha})\ln\frac{P_{ip}^3(\widehat{\alpha})}{P_{il}^3(\widehat{\delta})}\right)
$$

What is instead impossible to compute analytically is the estimated pseudo-true value $\delta_{\widehat{\alpha}}$. As indicated by Pesaran and Pesaran (1993), this quantity can be evaluated by simulation, averaging over H replication of the PMLE $\widehat{\delta}^h(\widehat{\alpha})$ obtained using simulated observations for the dependent variable y_i^h, independently drawn from the model under H_0 and in correspondence of the PMLE $\widehat{\alpha}$.[4] The simulated pseudo true value is then given by:

$$
\delta_{\widehat{\alpha}}^H=\frac{1}{H}\sum_{h=1}^{H}\widehat{\delta}^h(\widehat{\alpha}).
$$

To sum up, combining all the computational possibilities, we can identify the following versions of the Cox test statistic which are all asymptotically equivalent:

[3] The analogous quantity for $S_p(\delta_{\widehat{\alpha}})$ is obtained by substituting $\widehat{\delta}$ for $\delta_{\widehat{\alpha}}$.

[4] See Monfardini (1995) for further details and for some properties of the simulated pseudo-true value.

$$S_p^a(\widehat{\delta}),\ S_p^c(\widehat{\delta}),\ S_p^a(\delta_{\widehat{\alpha}}^H),\ S_p^c(\delta_{\widehat{\alpha}}^H),\ S_p^{a'}(\delta_{\widehat{\alpha}}^H),\ S_p^{c'}(\delta_{\widehat{\alpha}}^H), \tag{5}$$

where the first two do not resort to any simulation, the second two use the simulated pseudo-true value only in the numerator, while the third two use it in both numerator and denominator.

3.2 Testing logit *versus* probit

The testing problem has the form:

$$\begin{cases} H_0 : TNL(\widehat{\delta}) \\ H_1 : TNP(\widehat{\alpha}) \end{cases}$$

where $\widehat{\alpha}$ and $\widehat{\delta}$ are defined as above, $plim_{H_0}\widehat{\delta} = \delta_0$ and $plim_{H_0}\ \widehat{\alpha} = \alpha_{\delta_0}$. All the quantities presented in the above section, for example $\phi_l(\widehat{\alpha})$, $V_{\widehat{\delta}}^a(\widehat{\alpha})$, etc., are redefined by simply changing the role of the two models. This leads to the identification of the asymptotically equivalent test statistics:

$$S_l^a(\widehat{\alpha}),\ S_l^c(\widehat{\alpha}),\ S_l^a(\alpha_{\widehat{\delta}}^H),\ S_l^c(\alpha_{\widehat{\delta}}^H),\ S_l^{a'}(\alpha_{\widehat{\delta}}^H),\ S_l^{c'}(\alpha_{\widehat{\delta}}^H), \tag{6}$$

where the simulated pseudo true value involves drawing from the logit model and is given by:

$$\alpha_{\widehat{\delta}}^H = \frac{1}{H}\sum_{h=1}^{H}\widehat{\alpha}^h(\widehat{\delta}).$$

4 Application to the estimated TNL and TNP models

4.1 The observed results

The estimated models to which we refer are reported in Table A.1 and Table A.2 (TNP and TNL models respectively, for the occupational choice of men) and in Table A.3 and Table A.4 (explaining the occupational choice of women). These results are obtained in Bardasi and Monfardini (1997). In both cases the probit model is estimated by imposing $\sigma_2^2 = 1$, resulting in the estimation of the correlation coefficient between the differenced utilities, say ρ_{12}. The very significant value of the estimated correlation coefficient for both male and female models made the TNP preferable to the TNL, as it led to the rejection of the IIA assumption embodied in the TNL formulation ($\rho_{12} = 0$). It is thus interesting to investigate the outcome of the comparison of the two models on the basis of the "adjusted" likelihood ratio test represented by the Cox's statistic and to find out if the TNP model will be confirmed as the favoured one.

The outcome of the computation of the different versions of the test presented in the above section is contained in Table 1. At this stage, i.e. before running the bootstrap experiment, the values in Table 1 are to be compared with the theoretical critical values, relative to the asymptotic standard normal distribution. Starting from the models for women, we firstly observe a discrepancy in the decision based on the superscript "*a*" and "*c*" statistics when the null model is the TNP one: the first set of statistics would lead to rejection of the TNP in direction other than the TNL one when the test is given a two-sided interpretation (critical values equal to ± 1.96 at a 5% level), while the second set would lead to the acceptance of the TNP regardless the choice of giving to the test a one-sided (critical value equal to -1.64 at a 5% level) or a two-sided nature. When the two hypotheses are reversed, on the contrary, all the statistics indicate a sharp rejection of the TNL model. For the models for men, the conclusions of the test are less ambiguous, as all the statistics support the acceptance of the TNP null model and rejection of the TNL null one, both in the one-sided and in the two sided interpretation. This provides general evidence in favour of the TNP model, which, however, will have to be supported by the bootstrap experiment.

Before analysing the outcome of the bootstrap, there are some further observations than can be made on the results of the application of the test to the available data.

It is interesting to remark that there is some diversity between the statistics evaluated in the PMLE and the corresponding ones using the simulated pseudo-true value (cf., for example, rows 1) and 3) of Table 1). The latter has been obtained with $H = 100$, which should be a value high enough for the simulated pseudo-true value to be very close to its analytical counterpart (the convergence follows from the application of the Weak Law of Large Numbers). Moreover, the sample size in both cases is greater that 2,000, a value for which some simulation experiments in Weeks (1995) show that the simulated pseudo-true value settles down for H greater than 50. These observations justify the conjecture that the diversity between the statistics using the PMLE and the ones using the simulated pseudo-true value can be taken as a rough measure of distance in the WALD testing procedure spirit, according to which the difference between the two quantities, i.e. $\widehat{\delta} - \delta_{\widehat{\alpha}}$ and $\widehat{\alpha} - \alpha_{\widehat{\delta}}$, will be close to zero when the null model is true. To be more precise, we can refer to Table 2, which displays the disaggregated quantities involved in the computation of the different test statistics, and compare rows 3) with 7) or 4) with 8). It can be noticed that, while substituting the simulated pseudo-true value for the PMLE produces very little change when the null model is the TNP one, for both women and men, the differences are considerable when the null is the TNL model. Although this pattern should be subjected to a rigorous WALD non-nested test procedure, as indicated by Gourieroux, Monfort, Trognon (1983), it can nevertheless be inferred that this test would probably provide some further evidence in favour of the TNP model.

In Table 3 the TNL model is compared with the TNP model estimated by imposing the IIA hypothesis constraint, $\rho_{12} = 0$. In other words, here the two competing models only differ in the distributional assumption of the error terms. In this case, both models are rejected for both women and men, when the two-sided interpretation of the test is adopted. This provides evidence that it is more difficult to distinguish between the two models on the basis of the available data once the generality of the TNP model is not exploited to specify the correlation between the differenced utilities. However, the TNP model could still be chosen as the preferred one, given that it is rejected in a direction other than the TNL alternative, while the TNL model is rejected in the TNP direction.

Table 1. Results of the application of the Cox's test.

	$H_0: TNP(\widehat{\alpha}),\ H_1: TNL(\widehat{\delta})$			$H_0: TNL(\widehat{\delta}),\ H_1: TNP(\widehat{\alpha})$		
		Women	**Men**		**Women**	**Men**
1)	$S_p^a(\widehat{\delta}) = \frac{\sqrt{N}\phi_p(\widehat{\delta})}{V^a_{\widehat{\alpha}}(\widehat{\delta})^{\frac{1}{2}}}$	3.1831	-1.5588	$S_l^a(\widehat{\alpha}) = \frac{\sqrt{N}\phi_l(\widehat{\alpha})}{V^a_{\widehat{\delta}}(\widehat{\alpha})^{\frac{1}{2}}}$	-14.2189	-36.3435
2)	$S_p^c(\widehat{\delta}) = \frac{\sqrt{N}\phi_p(\widehat{\delta})}{V^c_{\widehat{\alpha}}(\widehat{\delta})^{\frac{1}{2}}}$	1.4052	-1.1624	$S_l^c(\widehat{\alpha}) = \frac{\sqrt{N}\phi_l(\widehat{\alpha})}{V^c_{\widehat{\delta}}(\widehat{\alpha})^{\frac{1}{2}}}$	-13.2073	-30.9371
3)	$S_p^a(\delta^H_{\widehat{\alpha}}) = \frac{\sqrt{N}\phi_p(\delta^H_{\widehat{\alpha}})}{V^a_{\widehat{\alpha}}(\widehat{\delta})^{\frac{1}{2}}}$	4.4199	-0.4871	$S_l^a(\alpha^H_{\widehat{\delta}}) = \frac{\sqrt{N}\phi_l(\alpha^H_{\widehat{\delta}})}{V^a_{\widehat{\delta}}(\widehat{\alpha})^{\frac{1}{2}}}$	-7.6672	-12.0130
4).	$S_p^c(\delta^H_{\widehat{\alpha}}) = \frac{\sqrt{N}\phi_p(\delta^H_{\widehat{\alpha}})}{V^c_{\widehat{\alpha}}(\widehat{\delta})^{\frac{1}{2}}}$	1.9512	-0.3632	$S_l^c(\alpha^H_{\widehat{\delta}}) = \frac{\sqrt{N}\phi_l(\alpha^H_{\widehat{\delta}})}{V^c_{\widehat{\delta}}(\widehat{\alpha})^{\frac{1}{2}}}$	-7.1217	-10.2260
5)	$S_p^{a'}(\delta^H_{\widehat{\alpha}}) = \frac{\sqrt{N}\phi_p(\delta^H_{\widehat{\alpha}})}{V^a_{\widehat{\alpha}}(\delta^H_{\widehat{\alpha}})^{\frac{1}{2}}}$	3.9726	-0.5046	$S_l^{a'}(\alpha^H_{\widehat{\delta}}) = \frac{\sqrt{N}\phi_l(\alpha^H_{\widehat{\delta}})}{V^a_{\widehat{\delta}}(\alpha^H_{\widehat{\delta}})^{\frac{1}{2}}}$	-29.8545	-24.9636
6)	$S_p^{c'}(\delta^H_{\widehat{\alpha}}) = \frac{\sqrt{N}\phi_p(\delta^H_{\widehat{\alpha}})}{V^a_{\widehat{\alpha}}(\delta^H_{\widehat{\alpha}})^{\frac{1}{2}}}$	1.8581	-0.3652	$S_l^{c'}(\alpha^H_{\widehat{\delta}}) = \frac{\sqrt{N}\phi_l(\alpha^H_{\widehat{\delta}})}{V^a_{\widehat{\delta}}(\alpha^H_{\widehat{\delta}})^{\frac{1}{2}}}$	-24.4033	-19.4179

Table 2. Quantities involved in the computation of the test.

	$H_0: TNP(\widehat{\alpha}),\ H_1: TNL(\widehat{\delta})$			$H_0: TNL(\widehat{\delta}),\ H_1: TNP(\widehat{\alpha})$		
		Women	**Men**		**Women**	**Men**
1)	$\phi_p(\widehat{\delta})$	0.0048	-0.0039	$\phi_l(\widehat{\alpha})$	-0.0449	-0.1034
2)	$\overline{L}_p(\widehat{\alpha}) - \overline{L}_l(\widehat{\delta})$	0.0227	0.0312	$\overline{L}_l(\widehat{\delta}) - \overline{L}_p(\widehat{\alpha})$	-0.0227	-0.0312
3)	$E_{\widehat{\alpha}}\left[\overline{L}_p(\widehat{\alpha}) - \overline{L}_l(\widehat{\delta})\right]$	0.0179	0.0351	$E_{\widehat{\delta}}\left[\overline{L}_l(\widehat{\delta}) - \overline{L}_p(\widehat{\alpha})\right]$	0.0222	0.0722
4)	$V^a_{\widehat{\alpha}}(\widehat{\delta})^{\frac{1}{2}}$	0.0731	0.1724	$V^a_{\widehat{\delta}}(\widehat{\alpha})^{\frac{1}{2}}$	0.1627	0.1968
5)	$V^c_{\widehat{\alpha}}(\widehat{\delta})^{\frac{1}{2}}$	0.1751	0.2312	$V^c_{\widehat{\delta}}(\widehat{\alpha})^{\frac{1}{2}}$	0.1751	0.2312
6)	$\phi_p(\delta^H_{\widehat{\alpha}})$	0.0066	-0.0012	$\phi_l(\alpha^H_{\widehat{\delta}})$	-0.0242	-0.0342
7)	$E_{\widehat{\alpha}}\left[\overline{L}_p(\widehat{\alpha}) - \overline{L}_l(\delta^H_{\widehat{\alpha}})\right]$	0.0161	0.0324	$E_{\widehat{\delta}}\left[\overline{L}_l(\widehat{\delta}) - \overline{L}_p(\alpha^H_{\widehat{\delta}})\right]$	0.0015	0.0030
8)	$V^a_{\widehat{\alpha}}(\delta^H_{\widehat{\alpha}})^{\frac{1}{2}}$	0.0860	0.1664	$V^a_{\widehat{\delta}}(\alpha^H_{\widehat{\delta}})^{\frac{1}{2}}$	0.0418	0.0947
9)	$V^c_{\widehat{\alpha}}(\delta^H_{\widehat{\alpha}})^{\frac{1}{2}}$	0.1839	0.2299	$V^c_{\widehat{\delta}}(\alpha^H_{\widehat{\delta}})^{\frac{1}{2}}$	0.0511	0.1218

Table 3. Results of the application of the Cox's test. (TNP with constrained correlation)

	$H_0: TNP(\widehat{\alpha}_{\rho_{12}=0}),\ H_1: TNL(\widehat{\delta})$			$H_0: TNL(\widehat{\delta}),\ H_1: TNP(\widehat{\alpha}_{\rho_{12}=0})$		
		Women	**Men**		**Women**	**Men**
1)	$S_p^a(\widehat{\delta}) = \frac{\sqrt{N}\phi_p(\widehat{\delta})}{V^a_{\widehat{\alpha}}(\widehat{\delta})}$	13.4119	9.2764	$S_l^a(\widehat{\alpha}) = \frac{\sqrt{N}\phi_l(\widehat{\alpha})}{V^a_{\widehat{\delta}}(\widehat{\alpha})}$	-12.1926	-14.4130
2)	$S_p^c(\widehat{\delta}) = \frac{\sqrt{N}\phi_p(\widehat{\delta})}{V^c_{\widehat{\alpha}}(\widehat{\delta})}$	6.0144	5.6068	$S_l^c(\widehat{\alpha}) = \frac{\sqrt{N}\phi_l(\widehat{\alpha})}{V^c_{\widehat{\delta}}(\widehat{\alpha})}$	-11.1626	-12.7530

4.2 The bootstrap results

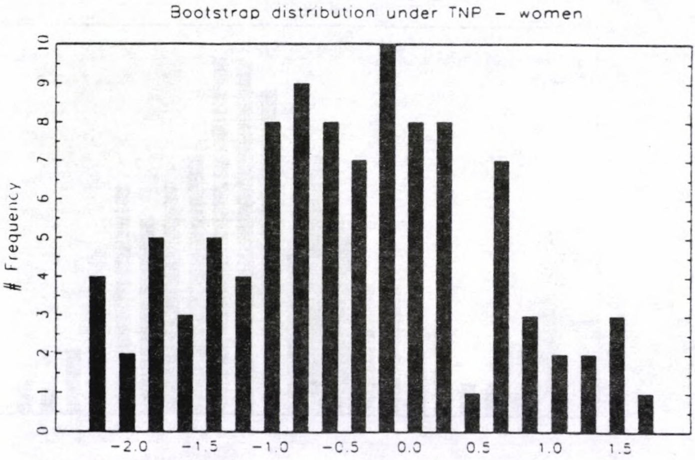
The experiment consists in repeatedly drawing from the null model samples of simulated observations for the dependent variable $y_i^b, b = 1...B$, in correspondence of the PMLE of the parameters of the null model and for fixed $\underline{x}_i$. This leads to the evaluation of B replications of the Cox's test statistic, denoted by superscript "b" in the following tables, which can be used to approximate its finite sample distribution. The number of the bootstrap samples B has been set equal to 100. This number is limited by the complexity of the estimation problem, requiring numerical maximization of the loglikelihood function for both models.[5] Moreover, due to these computational time constraints, the bootstrapping has been limited to the versions of the test not requiring further simulations in their computation, i.e. to the statistics in rows 1) and 2) of Table 1.

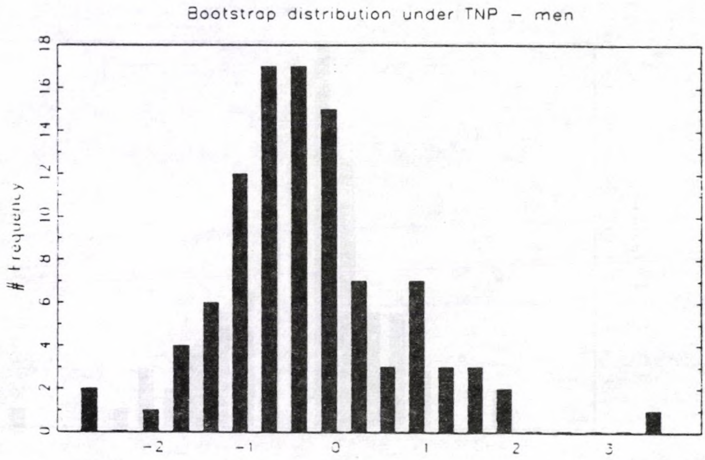
The bootstrap results can be exploited for two purposes. Firstly, they allow for an evaluation of the distance of the finite sample distribution from the asymptotic one, which can provide some guidance for any future applications of the Cox test to TNP and TNL models. More precisely, if the bootstrap distribution were to be judged as a very good approximation of the theoretical standard normal one, this would encourage future applications of the Cox test based on the standard normal critical values to these models. It is anyway important to emphasize that

[5] A replication of the test statistic, involving the estimation of both models, requires about 40 minutes on a Pentium 150 MHz machine.

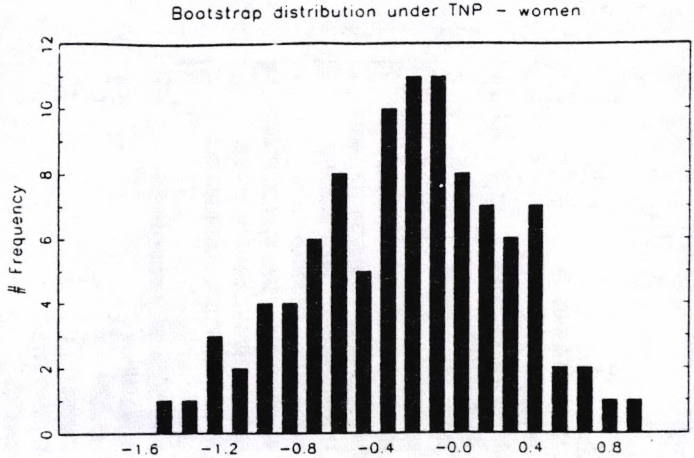
it is not be possible to draw a general conclusion, as the results obtained are relative to a particular information set (data available, chosen explanatory variables, sample size etc...). With respect to this point, from Table 4 it can be noticed that in the female case the bootstrap distribution exhibits substantial differences from the asymptotic one, as indicated by the number of values falling in the left and right tails according to the standard normal percentiles. The rejection frequency of the one-sided test is double the expected one when the null is the TNP model, and four times the expected one when the null is the TNL model. The particularly bad performance of the test under the logit model was also found by Weeks in a simulation experiment with sample size $N = 2,000$. For the model for women we have $N = 2,653$. Another emerging feature is the small variance, compared with the theoretical value (i.e. one) of the distribution of the superscript "c" version of the test (cf. rows 5) with 10)) under the TNP model, which could explain the "low" values of the same statistic observed in Table 1 (rows 2), 4), 5), left section). On the contrary, in the case of men the reported indicators suggest that the bootstrap distribution is fairly close to the standard normal, under both null hypotheses. In this case, we have a fairly large sample size, i.e. $N = 4,790$. This suggests that the asymptotic theory starts to apply for very large samples, and that caution has to be used in taking decisions based on the asymptotic critical values in applied modelling exercises. These observations are also confirmed by the inspection of the following graphs, reporting the bootstrap distributions derived in our experiment.

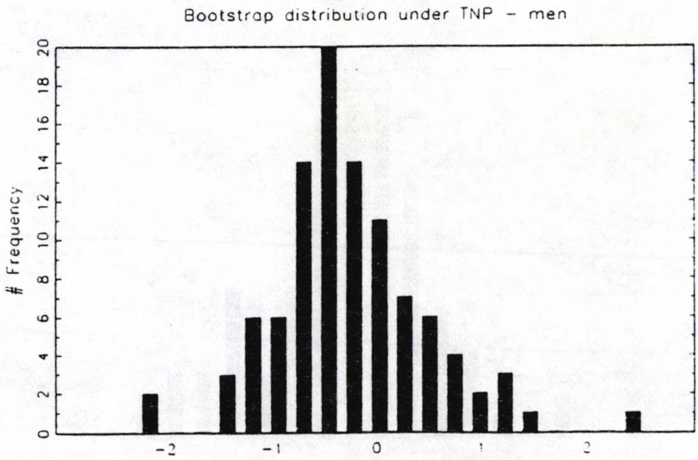
Bootstrap distribution of S^a under TNP. 100 replications.



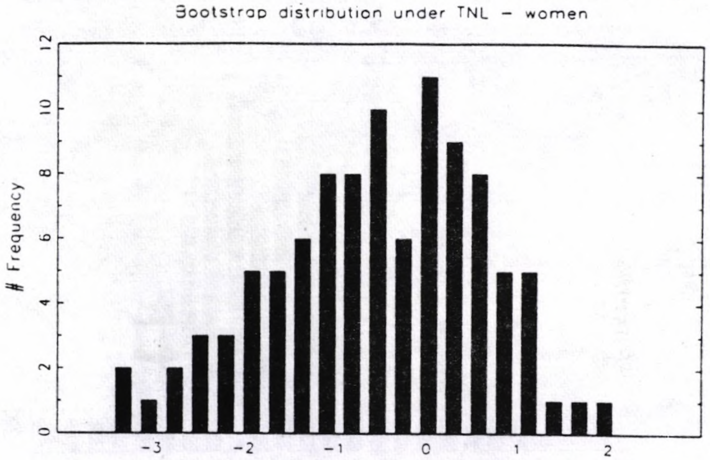


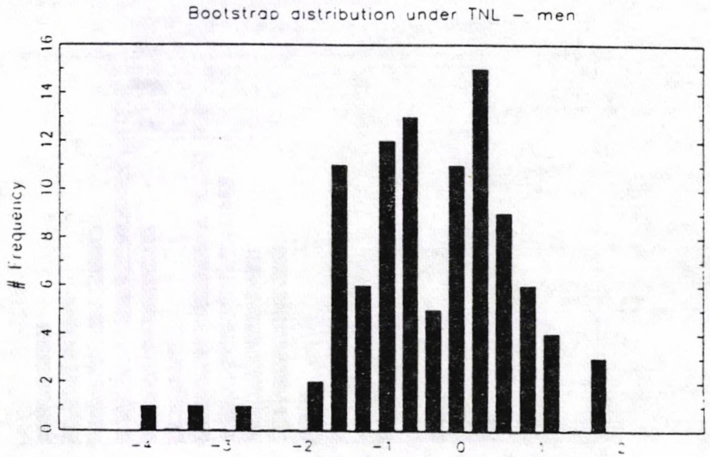
Bootstrap distribution of S^c under TNP. 100 replications.



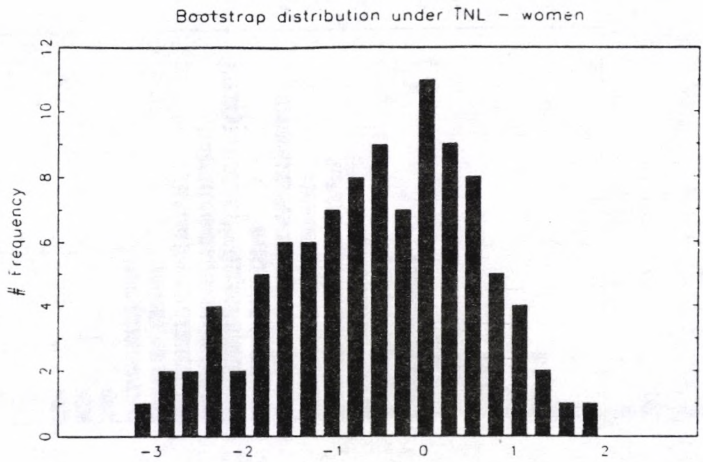


Bootstrap distribution of S^a under TNL. 100 replications.





Bootstrap distribution of S^c under TNL. 100 replications.



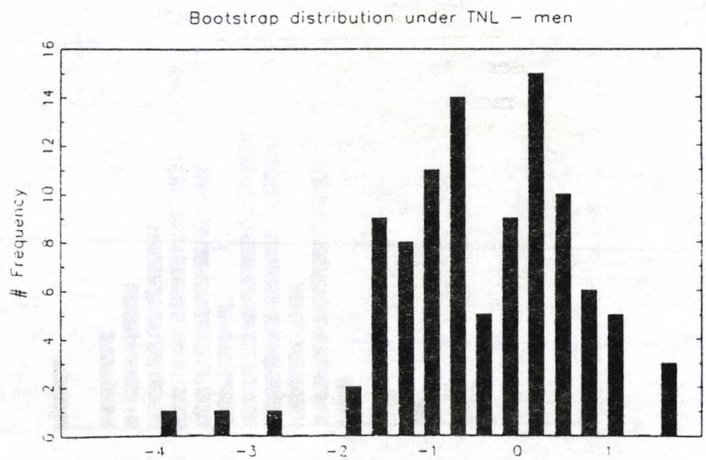


Table 4. Bootstrap results: finite sample distribution indicators.

100 replications.

	$H_0: TNP(\widehat{\alpha}),\ H_1: TNL(\widehat{\delta})$			$H_0: TNL(\widehat{\delta}),\ H_1: TNP(\widehat{\alpha})$		
		Women	**Men**		**Women**	**Men**
1)	$\#S_p^a(\widehat{\delta})^b < -1.64$	11	6	$\#S_l^a(\widehat{\alpha})^b < -1.64$	21	6
2)	$\#S_p^a(\widehat{\delta})^b < -1.96$	5	2	$\#S_l^a(\widehat{\alpha})^b < -1.96$	12	3
3)	$\#S_p^a(\widehat{\delta})^b > 1.96$	0	1	$\#S_l^a(\widehat{\alpha})^b > 1.96$	1	0
4)	$mean(S_p^a(\widehat{\delta})^b)$	-0.4097	-0.3145	$mean(S_l^a(\widehat{\alpha})^b)$	-0.5617	-0.3838
5)	$SD(S_p^a(\widehat{\delta})^b)$	0.9449	0.9767	$SD(S_l^a(\widehat{\alpha})^b)$	1.1558	1.0012
6)	$\#S_p^c(\widehat{\delta})^b < -1.64$	0	2	$\#S_l^c(\widehat{\alpha})^b < -1.64$	18	5
7)	$\#S_p^c(\widehat{\delta})^b < -1.96$	0	2	$\#S_l^c(\widehat{\alpha})^b < -1.96$	11	3
8)	$\#S_p^c(\widehat{\delta})^b > 1.96$	0	1	$\#S_l^c(\widehat{\alpha})^b > 1.96$	1	0
9)	$mean(S_p^c(\widehat{\delta})^b)$	-0.2359	-0.2437	$mean(S_l^c(\widehat{\alpha})^b)$	-0.5291	-0.3761
10)	$SD(S_p^c(\widehat{\delta})^b)$	0.5097	0.7406	$SD(S_l^c(\widehat{\alpha})^b)$	1.0903	0.9655

The second objective of the bootstrap analysis is the comparison of the observed values with the values of the test statistics calculated from the bootstrap samples. In particular, through the bootstrap distribution it is possible to evaluate the $P-value$, or significance level, associated with the observed realization of the statistic, say, with general notation, $\widehat{S}$. To be more precise, following Davidson and MacKinnon (1996), we denote the $P-value$ as: $p(\widehat{S}) = \Pr_{H_o(\theta_0)}\left\{S < \widehat{S}\right\}$, where the probability depends on the process generating the data under the null hypothesis, characterised by the parametric value θ_0, and on the sample size N. In the parametric case we consider, the bootstrap samples are generated from the null model by replacing the unknown θ_0 with its estimate $\widehat{\theta}$. This leads to the definition of the bootstrap $P-value$ as: $p^{boot}(\widehat{S}) = \Pr_{H_o(\widehat{\theta})}\left\{S < \widehat{S}\right\}$. In practice, however, what can be computed is only an approximation of $p^{boot}(\widehat{S})$, given the finite number of replications performed, i.e. $S^b, b = 1...B$, in order to derive the finite sample distribution of the test statistic S. Supposing that a high accuracy is reached in this approximation, the discrepancy between $p^{boot}(\widehat{S})$ and $p(\widehat{S})$ will depend only on the fact that we estimate θ_0 by $\widehat{\theta}$. Davidson and MacKinnon point out that the bootstrap $P-value$, $p^{boot}(\widehat{S})$, is likely to be an accurate estimate of the actual one, $p(\widehat{S})$, if the test statistic S is nearly pivotal (i.e. its distribution does not depend "much" on the value of θ), and the estimate $\widehat{\theta}$ is close to θ_0. In particular, they show that when the estimator $\widehat{\theta}$ is root-N consistent and is an extremum estimator satisfying first order conditions in the interior of the parameter space (as we assume for our maximum likelihood estimation problem), the error in the bootstrap $P-value$ is $O(N^{-\frac{3}{2}})$.

With the above considerations in mind, inspection of Tables 5-A) and B), in which the ranking of the observed statistics with respect to the observed ones are reported, leads to the following observations. In the female case (Table 5-A)), despite the previous finding of quite a wide discrepancy between the finite sample distribution of the test and the asymptotic one, the indications provided by Table 1 are confirmed. Rows 1) and 2) show that under both null models the observed values of the Cox's statistics are extreme, giving strong evidence against the TNL model but also showing that the TNP is not ideal, as it would be rejected by a two-tailed test. The previously observed evidence in favour of the TNP model is also confirmed in the case of men, where the ranking of the realizations of the statistic is not extreme under the probit model (about the 7% of the simulated value are lower than the observed one), but still is under the logit one.

In rows 3) and 4) we report the numerator of the Cox's test, which is not standardized in its variance, but is in its expected value, and the first term of the numerator, a statistic which is not standardized in variance nor in mean (cf. expressions in (4)). The idea is to find out if the bootstrap $P-value$ derived by these non-standardized quantities differs from the one derived from the standardized statistics, according to the intuition that the distribution of the latter should be closer to pivotal. It is interesting to remark that, while the results of row 3) in both tables, which refer to the finite sample distribution of the numerator of the Cox statistic, are in agreement with the bootstrap $P-values$ observed for the standardized test, some discrepancy with the standardized statistic is observed in row 4), in particular for men. This provides an indication on the importance of performing the bootstrap experiment after standardizing "at least" the numerator of the test statistic.

A final remark concerns the power of the test. We have decided to neglect it in this bootstrap analysis as the Cox non-nested test is designed to have high power against the considered alternative. This property is also evident in the Monte Carlo experiments by Weeks, in which the probability of rejecting the false alternative is around 90% when the true model is the TNP one, and around 76% when the true model is the TNL for the test S^a.[6]

Table 5-A). Bootstrap results: comparison with observed values.

Women model

	$H_0 : TNP(\widehat{\alpha}),\ H_1 : TNL(\widehat{\delta})$		$H_0 : TNL(\widehat{\delta}),\ H_1 : TNP(\widehat{\alpha})$	
1)	$\#\ S_p^a(\widehat{\delta})^b < 3.1831$	100	$\#\ S_l^a(\widehat{\alpha})^b < -14.2189$	0
2)	$\#\ S_p^c(\widehat{\delta})^b < 1.4052$	100	$\#\ S_l^c(\widehat{\alpha})^b < -13.2073$	0
3)	$\#\ \phi_p(\widehat{\delta})^b < 0.0048$	100	$\#\ \phi_l(\widehat{\alpha})^b < -0.0449$	0
4)	$\#\ \left[\overline{L}_p(\widehat{\alpha}) - \overline{L}_l(\widehat{\delta})\right]^b < 0.0227$	97	$\#\ \left[\overline{L}_l(\widehat{\delta}) - \overline{L}_p(\widehat{\alpha})\right]^b < -0.0227$	0

Table 5-B). Bootstrap results: comparison with observed values.

Men model

	$H_0 : TNP(\widehat{\alpha}),\ H_1 : TNL(\widehat{\delta})$		$H_0 : TNL(\widehat{\delta}),\ H_1 : TNP(\widehat{\alpha})$	
1)	$\#\ S_p^a(\widehat{\delta})^b < -1.5588$	7	$\#\ S_l^a(\widehat{\alpha})^b < -36.3450$	0
2)	$\#\ S_p^c(\widehat{\delta})^b < -1.1624$	7	$\#\ S_l^c(\widehat{\alpha})^b < -30.9371$	0
3)	$\#\ \phi_p(\widehat{\delta})^b < -0.0039$	6	$\#\ \phi_l(\widehat{\alpha})^b < -0.1034$	0
4)	$\#\ \left[\overline{L}_p(\widehat{\alpha}) - \overline{L}_l(\widehat{\delta})\right]^b < 0.0312$	38	$\#\ \left[\overline{L}_l(\widehat{\delta}) - \overline{L}_p(\widehat{\alpha})\right]^b < -0.0312$	0

[6]The latter result is not size-corrected, while the test appears to seriously over-reject the true logit model.

5 Conclusions

This paper is an example of how the new computational possibilities opened up by the availability of powerful computers make feasible the application of classical testing procedures, whose usefulness could not be entirely exploited in the past. Our aim is to compare with each other two different trinomial choice models, a probit and a logit one, separately estimated for women and men to explain Italian workers' occupational choice among the private, the public and the self-employed options in Bardasi and Monfardini (1997). Having the two estimated models at our disposal makes the Cox's test procedure for non-nested models a natural choice for the investigation. We evaluate different versions of the Cox statistic proposed in the literature, some of which resort to the use of simulations for the estimation of the pseudo-true value. Then, given that the reference distribution of the Cox statistic is only valid asymptotically, we perform a parametric bootstrap experiment in order to control for the actual size properties of the testing procedure in our finite sample application. An important conclusion that can be derived from the results of the bootstrapping is that the size of the sample needs to be very large for the finite sample distribution to approach the asymptotic one reasonably well. As far as the comparison of the two models is concerned, the results of the non-nested testing process suggests that the probit model is the preferred one on the basis of the available data. This evidence is stronger for the case of men than for the case of women. In the female case, the logit model can be rejected in favour of the probit one, and the probit cannot be rejected in favour of the logit but can be rejected in an alternative direction other than the logit. In the male case, the logit model can be rejected while the probit cannot. The preference for the probit model indicated by this specification test supports the conclusions of Bardasi and Monfardini (1997), which show that the IIA hypothesis imposed by the logit formulation is strongly rejected on the basis of the available data.

Appendix

List of variables

Variable	Description
age:	age of individual at 31.12.1993
age2:	age squared
assets:	dummy=1 if the individual owns risky assets (shares. fondi comuni. etc.)
blue:	dummy=1 if the individual is a blue-collar worker
burden:	no. of non-earners/no. of total members in the family
child6:	no. of children aged less than 6 in the family
comune1:	dummy=1 if the ind. lives in a *comune* with less than 20.000 inhab.
comune2:	dummy=1 if the ind. lives in a *comune* with 20.000 to 40.000 inhab.
comune3:	dummy=1 if the ind. lices in a *comune* with 40.000 to 500.000 inhab.
educ2:	dummy=1 if the individual has at most the primary education level
educ3:	dummy=1 if the individual has at most the high school education level
educ4:	dummy=1 if individual has a degree or a post-graduated education level
North-West:	dummy=1 if the individual lives in the North-West of Italy
North-East:	dummy=1 if individual lives in the North-East of Italy
Centre:	dummy=1 if individual lives in Central Italy
headfam:	dummy=1 if the individual is the head of the family
house:	dummy=1 if the ind. owns at least 50% of the house in which he lives
manager:	dummy=1 if the individual is a manager or a top manager
married:	dummy=1 if the individual is married or livetoghether with a patner
mobil:	dummy=1 if the ind. has changed job at least 3 times in his work. career
nrecip:	total number of income recipients in the family
otherfi:	total annual amount of all other family income. excluding the earnings of the individual in million Lit.
otherfi2:	otherfi squared
otherpi:	total annual amount of all other personal income of the individual. excluding the earings from the main activity. in million Lit.
otherpi2:	otherpi squared
prevexp:	years of total work experience at the time of starting the present job
prevexp2:	prevexp squared
pwagepr:	predicted hourly wage in the private sector (logarithm of thousand Lit.)
pwagepr2:	pwagepr squared

pwagepu:	predicted hourly wage in the public sector (logarithm of thousand Lit.)
pwagepu2:	pwagepu squared
runrate:	regional male/female unempl. rate at the time of starting the present jo
sectagric:	dummy=1 if the ind. works in agricolture
sectbank:	dummy=1 if the ind. works in the banking and insurance sectors
secteduc:	dummy=1 if the ind. works in the education and health sectors
tenure:	number of total years worked in the present job or activity
tenure2:	tenure squared
wealth:	total amount of wealth at 31.12.1993. in billion Lit. (value of immovables. land. shares. etc.)
wealth2:	wealth squared

Table A.1. Unconstrained covariance probit estimates. Men [7]

Loglikelihood=-3713.80, 4790 obs.
Stars denote unsignificance at 5% level.

Variable	Coeff.	S.E.		Coeff.	S.E.
Private			*Public*		
age	0.1921	0.0177		0.0790	0.0161
age2/1000	-2.010	0.1990		-0.7700	0.1864
educ2	0.1922	0.0675		0.4763	0.0714
educ3	0.7631	0.0872		0.2855	0.0864
educ4	0.2135*	0.1325		0.2169*	0.1729
North-West	0.8446	0.0731		-0.4947	0.0729
North-East	0.6379	0.0740		-0.3959	0.0735
Centre	0.3624	0.0666		-0.1714	0.0638
otherfi	0.0040	0.0017		0.0048	0.0018
wealth	-0.6721	0.1410		-1.1916	0.1445
wealth2	0.0689	0.0345		0.0955	0.0141
house	0.0944*	0.0531		0.1950	0.0537
nrecip	0.0139*	0.0344		-0.1326	0.0371
otherpi	0.0172	0.0026		0.0040*	0.0027
otherpi2/1000	-0.3221	0.0228		-0.0200*	0.0252
runrate	-6.2733	0.9550		2.7385	0.9532
pwagepr	-33.4478	1.6529			
pwagepr2	6.0308	0.3192			
pwagepu				4.4792	2.2099
pwagepu2				-0.4880*	0.4349
constant	40.4622	1.9298		-10.5067	2.7610
ρ_{12}^{**}				-0.9531	0.0165

[7] The outcome self-employment is the comparison group.

Table A.2. Logit estimates. Men

Loglikelihood= -3863.32, 4790 obs.

Stars denote unsignificance at 5% level.

Variable	Coeff.	S.E.		Coeff.	S.E.
Private			*Public*		
age	0.3447	0.0325		0.1652	0.0342
age2	-3.6157	0.3724		-1.7940	0.3885
educ2	0.7379	0.1300		0.9570	0.1489
educ3	1.7760	0.1612		1.0501	0.1830
educ4	0.8305	0.2516		0.1585*	0.3346
North-West	0.9341	0.1504		-0.5463	0.1532
North-East	0.6307	0.1530		-0.5443	0.1540
Centre	0.4730	0.1377		-0.1073*	0.1345
otherfi	0.0175	0.0031		0.0209	0.0034
wealth	-2.8402	0.2391		-3.6989	0.2535
wealth2	0.2821	0.0378		0.2972	0.0293
house	0.3509	0.1025		0.5174	0.1085
nrecip	-0.1060*	0.0659		-0.3009	0.0737
otherpi	0.0378	0.0056		0.0208	0.0061
otherpi2	-0.5854	0.0588		-0.1418	0.0628
runrate	-13.5971	1.8668		-1.0045*	1.8924
pwagepr	-54.7612	2.3371			
pwagepr2	10.1066	0.4611			
pwagepu				-17.0210	3.6780
pwagepu2				3.7984	0.7381
constant	65.2070	2.7849		15.3602	4.6158

Table A.3. Uncostrained covariance probit estimates. Women

Loglikelihood=-1903.33, 2653 obs.

Stars denote unsignificance at 5% level.

Variable	Coeff.	S.E.		Coeff.	S.E.
Private			*Public*		
age	0.0540	0.0239		0.1381	0.0234
age2/1000	-0.8339	0.2938		-1.4533	0.2846
educ2	-0.0934*	0.0941		0.4276	0.1124
educ3	-0.0550*	0.1162		1.1054	0.1624
educ4	-0.1857*	0.2105		0.6166	0.2759
married	-0.2387	0.0829		-0.1441*	0.0854
North-West	0.9199	0.1139		-0.5068	0.1124
North-East	0.6782	0.1151		-0.4930	0.1131
Centre	0.6160	0.1048		-0.3688	0.1007
otherfi	0.0081	0.0033		0.0078	0.0031
otherfi2/1000	-0.0409	0.0192		-0.0255*	0.0185
wealth	-0.7030	0.1869		-1.3060	0.1725
wealth2	0.1004	0.0266		0.1371	0.0311
house	0.0497*	0.0764		0.2420	0.0778
nrecip	-0.0332*	0.0463		-0.1354	0.0455
otherpi	0.0490	0.0110		-0.0009*	0.0085
otherpi2/1000	-0.7474	0.2450		0.0540*	0.0909
pwagepr	-11.3891	1.7656			
pwagepr2	1.9119	0.3875			
pwagepu				-16.3414	2.7955
pwagepu2				3.6323	0.5784
headfam	-0.4310	0.1175		0.0924*	0.1224
runrate	-2.2180	0.5798		-1.3808	0.5727
constant	14.9424	1.9844		14.9858	3.4443
ρ_{12}^{**}				-0.8326	0.0452

Table A.4. Logit estimates. Women

Loglikelihood= -1963.48, 2653 obs.

Stars denote unsignificance at 5% level.

Variable	Coeff.	S.E.		Coeff.	S.E.
Private			*Public*		
age	0.1583	0.0487		0.1976	0.0498
age2/1000	-2.2787	0.5942		-2.3550	0.5953
educ2	0.1516*	0.1940		0.7909	0.2249
educ3	1.0280	0.2276		2.5376	0.2964
educ4	0.4947*	0.4023		1.6113	0.4920
married	-0.7849	0.1799		-0.7442	0.1832
North-West	1.1487	0.2422		-0.5650	0.2355
North-East	0.6558	0.2427		-0.7760	0.2322
Centre	0.6716	0.2155		-0.5463	0.2075
otherfi	0.0324	0.0071		0.0316	0.0070
otherfi2/1000	-0.1291	0.0430		-0.1163	0.0419
wealth	-3.0058	0.3531		-3.5698	0.3325
wealth2	0.3955	0.0743		0.4293	0.0674
house	0.2422*	0.1577		0.4412	0.1618
nrecip	-0.1919	0.0945		-0.2822	0.1007
otherpi	0.1034	0.0214		0.0497	0.0169
otherpi2/1000	-1.1178	0.3826		-0.2721*	0.1750
pwagepr	-18.4103	2.7279			
pwagepr2	3.3284	0.6032			
pwagepu				-31.2194	4.8038
pwagepu2				6.5027	0.9921
headfam	-0.8562	0.2498		-0.3404	0.2483
runrate	-6.0766	1.1326		-5.5242	1.1189
constant	23.0202	3.1000		34.5199	5.9513

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