**Economics Department** 

Speculative Noise Trading and Manipulation in the Foreign Exchange Market

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# Speculative Noise Trading and Manipulation in the Foreign Exchange Market

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abstract

We investigate the possibility that in the foreign exchange market an uninformed speculator finds it convenient to trade on noise in order to gain an informational advantage she can exploit in future. In a two-period model, we analyze the trade-off she faces between the cost of the "informational investment" and the profits this brings about. Our results give a possible explanation for the large volume of noise trading present in the foreign exchange market.

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#### 1 Introduction

A large component of transactions in securities markets derives from noise traders. According to Black (1986), noise traders are agents who sell and buy assets on the basis of irrelevant information. These speculators do not possess inside or fundamental information and trade irrationally on noise as though this gave them an edge. Despite its irrational nature, noise trading represents an important aspect of the functioning of securities markets, because it reduces the risk of market crashes and facilitates transactions among agents. Indeed, if all traders were rational it would not be convenient to gather information, because prices would be fully revealing. Conversely, when noise traders are present, rational speculators will gain profits at the expense of the irrational ones. As in this case prices will not be fully revealing, there would still be an incentive to gather information, so that in practice noise trading may be beneficial for the efficiency of securities markets.

Different types of behavior are associated with noise and liquidity trading.<sup>1</sup> These comprise hedging strategies, such as portfolio insurance and stop-loss orders, popular models of forecasting and trading, such as chart and technical analysis, and so forth. In particular, considering the market for foreign exchange, there is ample evidence of widespread use of chartism by traders. Indeed, on the basis of a survey conducted in the London foreign exchange market, Allen and Taylor (1990) point out that most traders consider chartism at least as relevant as fundamentalism in the formulation of exchange rate expectations in the short run. Likewise, Frankel and Froot (1987), using survey data in the foreign exchange market, find strong evidence of extrapolative expectations and attribute it to the utilisation of chartism by professional traders.

A problem with this description of noise trading in securities markets is its profitability. In fact, it is commonly believed that noise traders incur losses, because they buy when prices are high and sell when prices are low. Nevertheless, empirical studies question this thesis. Goodman (1979) compares the profitability of different forecasting techniques in the market for foreign exchange. In the seventies, the worst technically orientated forecasting technique was three times more profitable than the best fundamentally orientated one. Likewise, Schulmeister (1988) finds that large speculative profits of commercial banks in

<sup>&</sup>lt;sup>1</sup>Liquidity traders differ from noise traders in that their transactions are led by hedging needs and not by speculative reasons. Generally this distinction is lost in theoretical models of securities markets, while it represents an important element of our analysis.

the market for foreign exchange are due to the utilization of technical trading rules. Levich and Thomas (1993) confirm the thesis that chartism is profitable employing a bootstrap approach, while Menkhoff and Schlumberger (1995) extend previous results to a longer period.

Therefore, explaining the use of chartism and other forms of noise trading represents an important topic of research. Several explanations have been proposed. Frankel and Froot (1986) suggest that if traders learn slowly the fundamental value of a currency, chartists can dominate fundamentalists and lead the exchange rate on a bubble path. De Long, Shleifer, Summers and Waldmann (1990) show that if irrational traders can bear more risk than other traders, they can gain larger profits than risk-averse rational speculators. Froot, Scharfstein and Stein (1992) show that if traders have short horizons, they may find it convenient to trade on the basis of information completely unrelated to fundamentals. Palomino (1996) proves that in small markets in which investigate are not price takers, noise traders might hurt sophisticated traders more than are not price takers, noise traders might hurt sophisticated traders more than are not price takers, noise traders might hurt sophisticated traders more than a specific traders more traders more than a specific traders more traders more traders more traders more traders more traders more traders.

Our analysis suggests an alternative explanation, based on a particular mechanism of manipulation of expectations and exchange rates. Even if this analysis could be applied to other dealer markets, we refer principally to the foreign exchange market because it is there that most of the evidence on profitability of noise trading is concentrated. Moreover, it is in this market that the mechanism of manipulation we suggest it is most likely to take place.

Noise trading may be part of a valid strategy of manipulation and a source of speculative profits, because it can bring about an informational advantage. This happens in a dealer market when noise trading interferes with the learning process of dealers. In fact, assuming that dealers are competitive and cannot discinguish between informed and uninformed trades, noisy market orders placed by an uninformed speculator will reduce their ability to learn the fundamental information contained in the order flow. On the other hand, the uninformed speculator will be able to extract more of the fundamental information present in the flow of orders. Then, in subsequent periods she can use her informational advantage to recoup the losses incurred with the initial noisy market orders and also gain some profits.

In what follows we consider a model of the foreign exchange market with private information on the fundamental value of a foreign currency. This frame work permits studying the impact of noise trading on the learning processes of the market maker and the uninformed speculator and on the stochastic process.

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governing the exchange rate. In addition, we can analyze the trade-off between the cost of noisy market orders and the informational advantage they provide and study under which conditions they may be part of a profitable trading strategy of an uninformed speculator.

This paper is organized as follows. In the next section we show that an uninformed trader can gain an informational advantage placing random market orders in such a market. In the following section we study a two period formulation of the model and consider different hypotheses on the informational contain of the order flow of the dealer and the behavior of other market participants. We show both analytically and numerically under which circumstances manipulation takes place. A conclusion which summarizes the results of our analysis completes the paper, while an appendix gives detailed proofs of Propositions and Lemmas.

## 2 Noise in the Foreign Exchange Market

To analyze the effect of noise trading on the performance of the foreign exchange market, we consider a simple model based on the batch framework put forward by Kyle (1985). Despite the structure of the foreign exchange market differs from that of the auction market studied by Kyle, there are several reasons that justifies the use of the framework he proposed in the present context. First, this framework is elegant and powerful, since simple analytical solutions are easily derived and have intuitive interpretations. Second, it captures the lack of transparency of the foreign exchange market. In fact, in both the auction and in the foreign exchange markets dealers cannot observe all market orders and prices cannot immediately reveal all information contained in individual trades. This characteristic is fundamental for the functioning of any dealer market and consequently the batch framework, despite its abstraction, is a valid approximation for the study of noise trading in the foreign exchange market.

Now, suppose in the market for foreign exchange a representative dealer (market maker) transacts a foreign currency with a population of customers before some news on the fundamental value of the currency is announced. Such news could concern any macroeconomic variable affecting the value of the currency, such as the money supply.

We assume that trading takes place in a sequence of auctions or rounds of trading. Any round of trading is organized as follows. First market participants

place their market orders, which are batched and passed to the market maker. Then, since price competition enforces a zero-expected profit condition for the dealers, the market maker will set the exchange rate equal to the expected fundamental value given his information, according to a weak form efficiency condition.

We suppose that there are T rounds of trading before the fundamental value of the foreign currency,  $f_T$ , is announced. The fundamental value is given by the sum of T+1 independent and normally distributed random variables:  $f_T = f_0 + \sum_{t=1}^T \xi_t^f$ , where  $f_0 \propto N(0, \Sigma_0^f)$  and  $\xi^f \propto \mathbf{N}(\mathbf{0}, \sigma_f^2 \mathbf{I}_T)$ .

We assume that in any period t the market maker cannot observe the fundamental value of the foreign currency,  $f_t = f_0 + \sum_{\tau=1}^t \xi_{\tau}^f$ , but that he receives a total order flow,  $\Delta x_t$ , given by the following expression:

$$\Delta x_t = a_t(f_t - s_{t-1}^m) + \epsilon_t^l,$$

 $\Delta x_t = a_t(f_t - s_{t-1}^m) + \epsilon_t^l,$  where  $a_t > 0$  and  $s_{t-1}^m$  is the exchange rate set in the previous round of trading.

This assumption implies that the order flow contains both noise and signal components. In fact, random market orders, placed by liquidity traders, follow: white noise process,  $\{\epsilon_t^l\}_{t=1}^T$ , of variance  $\sigma_l^2$  and independent of the fundamental value. Simultaneously, informative market orders are placed by an agent that possesses information on the fundamental value,  $f_t$ , and arbitrages against the undervaluation (overvaluation) of the foreign exchange,  $f_t - s_{t-1}^m$ . In practice, since it is very unlikely that individual investors possess a better knowledge of the fundamental value than professional dealers, this agent might well became central bank that has superior information on its current and future monetary policy. As suggested in the literature (Edison (1993)), central banks are often active in the market for foreign exchange with sterilised intervention operations aimed at signalling changes in the fundamentals of exchange rates.

Trading in the foreign exchange market is not conducted as in the auction. market described here. However, our market maker can represent a typical dealer in the foreign exchange market. By trading with his customers and with other dealers and by observing brokered inter-dealer transactions, this dealer is able to derive some estimate of the aggregate flow of orders placed by external customers. Then, according to this interpretation, we could assume that the quantity  $\epsilon_t^l$  contains both a measure of liquidity trading and an error term of the estimate of the total flow of orders. For simplicity we abstract from this second component.

The weak form efficiency condition implies that the exchange rate is equal to the expected fundamental value of the foreign currency, given the history of the order flow,  $s_t^m = E[f_t | \Delta x_1, \dots, \Delta x_t]$ . The application of the projection theorem for Normal random variables immediately gives a recursive expression for the exchange rate:

 $s_t^m = s_{t-1}^m + \lambda_t^{mf} \Delta x_t, \tag{1}$ 

with  $s_0^m = s_0$ . If one defines  $\sum_{t=1}^{mf}$  as the conditional variance of the fundamental value in period t-1 given the history of the order flow,  $\{\Delta x_k\}_{k=1}^{t-1}$ , the liquidity coefficient,  $\lambda_t^{mf}$ , is:

$$\lambda_t^{mf} = \frac{a_t(\sum_{t-1}^{mf} + \sigma_f^2)}{a_t^2(\sum_{t-1}^{mf} + \sigma_f^2) + \sigma_l^2}.$$
 (2)

Then, the conditional variance of the fundamental value in period t is:

$$\Sigma_t^{mf} = (1 - a_t \lambda_t^{mf}) (\Sigma_{t-1}^{mf} + \sigma_f^2), \tag{3}$$

where  $\Sigma_0^{mf} = \Sigma_0^f$ .

Suppose, now, that an uninformed speculator enters in the foreign exchange market. She cannot observe the fundamental process,  $\{f_t\}_{t=0}^T$ , but can place unpredictable random market orders. In this respect, let us introduce the following Assumption.

**Assumption 1** In any round of trading t the speculator can commit to use one from a set of noise trading technologies,  $\mathcal{T}$ . Any element  $\mathbf{t}$  of  $\mathcal{T}$  will place at time t a random market order  $b_t\eta_t$ , where  $\{b_t\}_{t=1}^T$  are positive constants and  $\{\eta_t\}_{t=1}^T$  is a white noise process of variance  $\sigma_l^2$ , independent of the fundamental value and of the liquidity trading process.

In practice we assume that the speculator may access a series of automatic trading programmes, based on the processing of information unrelated to the innovations in the fundamental value of the foreign currency. This is the case for investors using chart and technical analysis: their transactions are dictated by patterns observed in past exchange rates, which in efficient markets are not related to future changes in the fundamentals.<sup>2</sup>

When a noisy trading technology t is used the order flow becomes:

$$\Delta x_t = a_t(f_t - s_{t-1}^m) + b_t \eta_t + \epsilon_t^l.$$

<sup>&</sup>lt;sup>2</sup>Notice that a mechanism of commitment is necessary to avoid problems of time consistency. This mechanism can be designed using some form of portfolio delegation as suggested by Biais and Germain (1997).

Thus, in equation (2), the liquidity coefficient,  $\lambda_t^{mf}$ , is now equal to:

$$\lambda_t^{mf} = \frac{a_t(\Sigma_{t-1}^{mf} + \sigma_f^2)}{a_t^2(\Sigma_{t-1}^{mf} + \sigma_f^2) + \sigma_{ut}^2},\tag{4}$$

where  $\sigma_{ut}^2 = \sigma_l^2(1+b_t^2)$ . Because of the reduction in  $\lambda_t^{mf}$ , which measures the informativeness of the order flow, the conditional variance of  $f_t$  in equation (3) rises for any given value of  $\Sigma_{t-1}^{mf}$ . This shows that the uninformed speculator can condition the learning process of the market maker: by injecting noise in the order flow she can determine the market maker uncertainty on the fundamental value. Indeed, it is immediate to prove the following Proposition.

**Proposition 1** For any choice of  $\{a_t\}_{t=1}^T$ , the degree of uncertainty of the market maker on the fundamental value,  $\Sigma_t^{mf}$ , is governed by the noise injected in the order flow by the speculator. In particular, if the fundamental value follows a random walk, that is if  $\sigma_f^2$  is strictly positive, the speculator is able to increase this uncertainty across time.

Within the batch framework, customers cannot directly observe the order flow of the dealer. Anyway, the value of  $\Delta x_t$  can be determined in equilibrium observing the exchange rate customers will pay or receive for one unit of the foreign currency in the round t of trading (customers can simply invert equation (1) to obtain  $\Delta x_t$ ). In other words, a speculator can "test the water" and obtain some useful information on the activity of the foreign exchange market by placing a market order with a dealer. On the contrary, posted quotes are generally only indicative and never represent firm values, so that they do not constitute an instrument of diffusion of information in the market.

Since the speculator will observe ex-post the realisation of her random market order,  $\Delta x_t^s$ , she can extract at the end of period t the total market order of the other customers:  $\Delta y_t = \Delta x_t - \Delta x_t^s$ . In this way, she can form her own expectations of the fundamental value,  $s_t^s = E[f_t | \Delta y_1, \dots, \Delta y_t]$ , which in parallel with equation (1) respects the following recursion:

$$s_t^s = s_{t-1}^s + \lambda_t^{sf} \Delta y_t, \tag{5}$$

with  $s_0^s = s_0$ . If one defines  $\sum_{t=1}^{sf}$  as the conditional variance of the fundamental value  $f_{t-1}$  given the history of the other customers' order flow up to period t-1,  $\{\Delta y_k\}_{k=1}^{t-1}, \lambda_t^{sf}$  respects a formulation similar to equation (2):

$$\lambda_t^{sf} = \frac{a_t(\sum_{t-1}^{sf} + \sigma_f^2)}{a_t^2(\sum_{t-1}^{sf} + \sigma_f^2) + \sigma_l^2},\tag{6}$$

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Obviously a recursion for the conditional variance similar to equation (3) holds:

$$\Sigma_t^{sf} = (1 - a_t \lambda_t^{sf})(\Sigma_{t-1}^{sf} + \sigma_f^2), \tag{7}$$

where  $\Sigma_0^{sf} = \Sigma_0^f$ . Then, studying equations (3) and (7) it is immediate to verify the following Proposition.

**Proposition 2** For any choice of  $\{a_t\}_{t=1}^T$ , if the speculator employs a noisy trading technology in period 1,  $b_1 > 0$ , then the degree of uncertainty of the speculator on the fundamental value is smaller than that for the market maker in any round of trading: that is,  $\forall t, \Sigma_t^{sf} < \Sigma_t^{mf}$ .

Proposition 2 implies that an uninformed speculator can exploit the uncertainty of the market maker on the information possessed by his clients to derive more precise information on the fundamental value of the foreign currency. She just needs to inject some noise in the order flow in the first round of trading and hence observe the exchange rates set by the market maker in the following ones. This is sufficient to gain an informational advantage in the first period and to conserve it until the uncertainty on the fundamental value is resolved. The informational gain is obtained reducing the information contained in the order flow observed by the market maker, that is reducing its signal-to-noise ratio.

In order to analyze how the speculator can use her informational advantage we now consider a two period formulation of this model.

#### 3 A Two Period Formulation

In the remaining of the paper we suppose that T=2, so that there are only two rounds of trading. This hypothesis permits isolating a first period, in which the speculator invests in the acquisition of an informational advantage, from a second one, in which she uses it to gain speculative profits. A multi-period formulation of this model is not analytically tractable, but if manipulation is viable when only two rounds of trading are possible, then trading over several periods would facilitate it.

Let us assume the speculator is risk-neutral. Then, at any round of trading t, t = 1, 2, she maximises the expected value of her trading profits,  $E[\pi^s|I_{ts}]$ , where  $I_{ts}$  is her information set. We indicate with  $f_2$  the fundamental value of

the foreign currency in period 2, with  $s_t^m$  the exchange rate fixed by the market maker and with  $\Delta x_t^s$  the market order of the speculator at time t. As the fundamental value is publicly announced at the end of period 2, the speculator will realise the following total profits:  $\pi^s = \sum_{t=1}^2 (f_2 - s_t^m) \Delta x_t^s$ .

Since the speculator does not possess any superior information on the fundamental value of the foreign currency, she might find it useful to gain an informational advantage by using some noisy trading technology, **t**, in period 1. As she does not have an incentive to preserve an informational advantage beyond period 2, no further noise trading will be considered.<sup>3</sup>

Hence, suppose that in period 1 the speculator chooses a noise trading technology  $\mathbf{t}$  and places a random market order,  $b_1\eta_1$ . Given that the total market order the dealer observes in period 1 is  $\Delta x_1 = a_1(f_1 - s_0) + b_1\eta_1 + \epsilon_1^l$ , the dealer sets the following exchange rate in accordance to the weak-form efficiency condition:

$$s_1^m = s_0 + \lambda_1^{mf} \Delta x_1, \quad \text{where} \quad \lambda_1^{mf} = \frac{a_1 \Sigma_1^f}{a_1^2 \Sigma_1^f + \sigma_{u1}^2},$$

$$\Sigma_1^f = \Sigma_0^f + \sigma_f^2 \quad \text{and} \quad \sigma_{u1}^2 = \sigma_l^2 (1 + b_1^2).$$
(8)

Thus, considering that the speculator cannot predict her random market order before selecting the technology t, the profits she expects from trading in the first period are given by:

$$E[(f_2 - s_1^m)\Delta x_1^s | I_{s1}] = -b_1^2 \lambda_1^{mf} \sigma_l^2.$$
(99)

Moreover, if one defines  $s_1^s$  as the expected fundamental value conditional on the information at the end of period 1, this is given by:

$$s_1^s = \lambda_1^{sf} \Delta y_1, \quad \text{where} \quad \lambda_1^{sf} = \frac{a_1 \Sigma_1^f}{a_1^2 \Sigma_1^f + \sigma_l^2}$$
 (10)

and  $\Delta y_1 = \Delta x_1 - b_1 \eta_1$ . Then, we can prove the following Lemma.

**Lemma 1** Suppose the speculator places a market order,  $b_1\eta_1$ , according to the noise trading technology t in the first period. Then, her expected second period

<sup>&</sup>lt;sup>3</sup>Instead, in a multi-period formulation the speculator could find it profitable to use a trading strategy which comprises a noise component in all, but the last, rounds of trading. Therefore, our conclusion that noise trading might be part of a profitable trading strategy would be reinforced.

profits, conditional on her first period information set, are given by the following expression:

 $E[\pi_2^s|I_{s1}] = a_1 \Sigma_1^f (\lambda_1^{sf} - \lambda_1^{mf}) \left( \frac{(1 - a_2 \lambda_2^{mf})^2}{4\lambda_2^{mf}} \right), \tag{11}$ 

where  $\lambda_2^{mf}$  is the liquidity coefficient for period 2.

Since  $(\lambda_1^{sf} - \lambda_1^{mf})$  and  $b_1^2 \lambda_1^{mf} \sigma_l^2$  are both increasing in  $b_1$  corollary of Lemma 1 is the following Proposition.

**Proposition 3** The speculator in period 1 faces a trade-off between second period profits and first period losses.

Indeed, in order to acquire an informational advantage the speculator has to inject noise in the order flow. Given that in period 1 she does not have more information than the dealer and that this provides liquidity to the market with a transaction cost, the speculator faces a cost for her "informational investment". The cost for this investment is increasing in the noise injected in the order flow, and so are the profits the speculator can obtain in the second round of trading. In fact, these are increasing in  $a_1\Sigma_1^f(\lambda_1^{sf}-\lambda_1^{mf})$ , which measures the informational advantage she acquires in the first period.

To establish if manipulation on the part of the speculator takes place, we need to see which noise trading technology she selects in period 1. This technology will be chosen maximizing the expected value of her total profits,  $\pi^s$ , that is:

$$b_{1} = \operatorname{argmax} E[\pi^{s}(b_{1})|I_{1s}], \text{ where}$$

$$E[\pi^{s}(b_{1})|I_{s1}] = -b_{1}^{2}\lambda_{1}^{mf}\sigma_{l}^{2} + a_{1}\Sigma_{1}^{f}(\lambda_{1}^{sf} - \lambda_{1}^{mf})\left(\frac{(1 - a_{2}\lambda_{2}^{mf})^{2}}{4\lambda_{2}^{mf}}\right). \quad (12)$$

Unfortunately, a simple analytical solution for this optimisation problem does not exist, since  $\lambda_1^{mf}$  and  $\lambda_2^{mf}$  depend on  $b_1$  in a complicated way, and unless we impose some restrictions on the parameters of the model, some numerical procedure will be necessary to find the optimal value of  $b_1$ . Anyway, there is an important case in which an analytical solution exists.

#### 3.1 Sporadic Central Bank Intervention

As we already suggested, central banks often intervene in the market for foreign exchange. In effect, sterilised intervention may be employed to signal changes

in the monetary policy and condition market expectations and exchange rates.<sup>4</sup> However, this kind of intervention is generally sporadic and concentrated in short periods of time, so that there always exists a spell of time between this intervention and the release of data on monetary aggregates. In other words, we refer to situations in which a sterilised operation is carried out a day before some announcement on the monetary growth. We can then introduce the following Assumption.

**Assumption 2** In the second round of trading the dealer receives market orders only from the speculator and the liquidity traders, in that  $a_2 = 0$ .

Then, the following Proposition is proved in the Appendix.

Proposition 4 Under the conditions of Assumption 2 there exists a unique le near Nash equilibrium of the foreign exchange market with manipulation on the part of an uninformed speculator. The manipulation mechanism of the specular tor comprises the use of a noisy trading technology t in the first period, and any informative market order in the second one.

In equilibrium, the market orders of the speculator and the exchange rates are as follows:

$$\Delta x_1^s = b_1 \eta_1, \quad \Delta x_2^s = b_2 (s_1^s - s_1^m),$$
 (13)

$$s_1^m = s_0 + \lambda_1^{mf} \Delta x_1, \quad s_2^m = s_1^m + \lambda_2^{mf} \Delta x_2,$$
 (14)

where:

$$b_{1} = \left(\frac{a_{1}^{2}\Sigma_{1}^{f} + \sigma_{l}^{2}}{16a_{1}^{2}\Sigma_{1}^{f} + 15\sigma_{l}^{2}}\right)^{1/2}, \qquad b_{2} = \frac{4(a_{1}^{2}\Sigma_{1}^{f} + \sigma_{l}^{2})}{a_{1}\Sigma_{1}^{f}}, \tag{15}$$
$$\lambda_{1}^{mf} = \frac{a_{1}\Sigma_{1}^{f}(16a_{1}^{2}\Sigma_{1}^{f} + 15\sigma_{l}^{2})}{16(a_{1}^{2}\Sigma_{1}^{f} + \sigma_{l}^{2})^{2}}, \qquad \lambda_{2}^{mf} = \frac{a_{1}\Sigma_{1}^{f}}{8(a_{1}^{2}\Sigma_{1}^{f} + \sigma_{l}^{2})}, \tag{16}$$

$$\lambda_1^{mf} = \frac{a_1 \Sigma_1^f (16a_1^2 \Sigma_1^f + 15\sigma_l^2)}{16(a_1^2 \Sigma_1^f + \sigma_l^2)^2}, \qquad \lambda_2^{mf} = \frac{a_1 \Sigma_1^f}{8(a_1^2 \Sigma_1^f + \sigma_l^2)}, \tag{16}$$

$$s_1^s = s_0 + \lambda_1^{sf} \Delta y_1, \qquad \lambda_1^{sf} = \frac{a_1 \Sigma_1^f}{a_1^2 \Sigma_1^f + \sigma_l^2}.$$
 (17)

This Proposition suggests that foreign exchange intervention stimulates trading in the market for foreign exchange, because a speculator enters in the

<sup>&</sup>lt;sup>4</sup>See Dominguez and Frankel (1993) for a general discussion on foreign exchange policy and a detailed analysis of data on foreign exchange intervention. See also Vitale (1997) for a formal analysis of the signalling effect of sterilised intervention.

market in order to acquire information from the central bank activity and gain speculative profits. On turn, her action increases the volatility of exchange rates. Consider, in fact, that the variance of the exchange rate conditional on  $f_t$  is  $(\lambda_1^{mf})^2\sigma_{u1}^2$  in period 1 and  $(\lambda_2^{mf})^2\sigma_l^2$  in period 2. These values are larger when  $b_1 > 0$ , that is when the speculator enters in the market. In effect, central bank intervention is often accompanied by a rise in the volume of transactions and the volatility of exchange rates and it is a least reassuring that the present model captures this aspect of the functioning of the foreign exchange market.

#### 3.2 Comparative Static

To complete our analysis of the effect of noise trading in the foreign exchange market, we can consider a short comparative static exercise. In this respect let us introduce the following Lemma.

**Lemma 2** Under the conditions of Assumption 2 the profits the speculator expects to gain in the second round of trading are twice as much the losses expected in the first period. The total expected profits of the speculator conditional on her first period information set are given by the following expression:

$$E[\pi^s \mid I_{s1}] = \frac{a_1 \sigma_l^2 \Sigma_1^f}{16(a_1^2 \Sigma_1^f + \sigma_l^2)}.$$
 (18)

This Lemma permits proving the following Proposition, which links the speculator profits to the parameters of the model.

**Proposition 5** Under the conditions of Assumption 2 the expected profits of the speculator are increasing in the volume of liquidity trading  $(\sigma_l^2)$  and the uncertainty on the fundamental value  $(\Sigma_0^f)$ . They are increasing (decreasing) in the intensity of trading of the insiders,  $a_1$ , if this is smaller (larger) than  $(\sigma_l^2/\Sigma_1^f)^{1/2}$ .

In effect, a larger volume of liquidity trading implies a more aggressive trading strategy for the speculator, since her trading intensities,  $b_1$  and  $b_2$ , are larger in both periods (see Table 1, in which we report the signs of the derivatives of the various coefficients characterizing the equilibrium of the market with

respect to the parameters of the model).<sup>5</sup> Indeed, the increased volume of liquidity trading reduces the unitary cost of her random market order in the first period and permits her to hide better her market order in the second period. The larger noise-to-signal ratio of the order flow in both periods also explains why we have a more liquid (smaller values of  $\lambda_t^{mf}$ , for t=1,2) and less efficient (larger  $\Sigma_t^{mf}$ , for t=1,2) market.

On the other hand, a larger initial uncertainty on the fundamental value,  $\Sigma_0^f$ , forces the speculator to inject less noise in the order flow in the first round of trading and to be less aggressive in the second one as well ( $b_t$  is smaller for t=1 and 2). Hence, the order flow is more informative and the liquidity of the market falls ( $\lambda_t^{mf}$  is larger for t=1 and 2). Despite a less aggressive trading strategy, the speculator still gains larger profits. In effect, the unconditional profits of the speculator in the second period correspond to the expected losses of the liquidity traders, which are, for a given  $\sigma_t^2$ , increasing in  $\lambda_2^{mf}$ . Note that more informative order flow does not mean in this case that the market is more efficient. In fact, given the larger initial uncertainty, the conditional variances of the fundamental value ( $\Sigma_t^{mf}$  for t=1 and 2) augment.

Finally, an increase in the quality of the external signal, measured by  $a_1$ , has different effects according to its starting value. If  $a_1$  is smaller than a threshold value,  $(\sigma_t^2/\Sigma_1^f)^{1/2}$ , we have the same effect of an increment of the uncertainty on the fundamental value, with the unique difference given by the efficiency of the market. which now rises. On the other hand, when  $a_1$  is larged any increment of the signal quality forces the market maker to increase the liquidity of the market  $(\lambda_t^{mf})$  is smaller for t=1 and 2) reducing the expected profits of the speculator in the second period. This means that the speculator will trade less aggressively in the first round of trading, injecting less noise in the order flow, and will expect smaller total profits.

#### 3.3 Continuous Central Bank Intervention

Despite the presence of central banks in the foreign exchange market is sporadic, it might be interesting to establish if manipulation takes place when sterilised

<sup>&</sup>lt;sup>5</sup>The sign +/- in the Table indicates that the derivative of the coefficient changes sign from positive to negative when the parameter crosses a threshold value. The signs of the derivatives of the liquidity coefficients, and the trading intensities of the speculator are calculated deriving analytical expressions for the derivatives. The same applies to the first efficiency indicator,  $\Sigma_1^{mf}$ , while for the second period conditional variance,  $\Sigma_2^{mf}$ , a simple numerical analysis has been carried out.

intervention is continuous. As we said, when  $a_2$  is positive we are not able to provide a closed form solution for the optimisation problem of the speculator and a numerical procedure will be required. Nevertheless, general qualitative results may be obtained studying equation (12). In particular, the following Lemma provides a useful intuitive result.

**Lemma 3** For any noise trading technology selected by the speculator in period 1, her expected profits are decreasing in  $a_2$ .

In fact, an increase in the intensity of trading of the central bank,  $a_2$ , will force a reduction of the market order of the speculator in the second round of trading with a contraction of her expected profits. This Lemma immediately leads to the following important Proposition.

**Proposition 6** For any choice of  $a_1$ , there exists a minimum value  $\underline{a}_2$  such that for  $a_2 \geq \underline{a}_2$ ,  $E[\pi^s(b_1)|I_{s_1}]$  is negative for any noise trading technology  $\mathbf{t}$ . As a consequence, the set of points  $(a_1, a_2)$  such that no manipulation takes place is non-empty.

Indeed, as suggested by this Proposition, an aggressive trading rule of the central bank can always force the speculator out of the market. As an increase in  $a_2$  reduces the value of the "informational investment", the speculator will not find it convenient to meet the initial cost for its acquisition and there will not be any manipulation.

As an explicative example consider Figure 1. Here, we present the graph of the expected profits of the speculator for two different trading rules of the central bank when  $\sigma_l = \Sigma_0^f = 1$ , and  $\sigma_f = 0$ . In the left panel  $a_1 = 10a_2$ , while in the right one  $a_1 = a_2/2$ . As it is clear from the two panels, only when the central bank trades much less aggressively in the second period than in the first one, the speculator can expect profits when entering in the market. For  $a_1 < a_2$ , instead, the profits she can gain in the second round of trading cannot cover the losses she incurs in the first one and the speculator does not enter in the market at all. In synthesis, an active central bank can severely limit manipulation of the kind discussed here.

#### 4 Conclusion

In this paper we have considered a possible reason which may induce uninformed speculators to trade on the basis of irrelevant information in the market for foreign exchange. We suggest that noise trading may be used to manipulate expectations and exchange rates in order to gain an informational advantage and hence a profit opportunity. To investigate this opportunity, we have developed a two auction model of the foreign exchange market, in which an uninformed speculator can exploit the impossibility of the market maker to distinguish between informed and uninformed clients to manipulate his expectations and the exchange rates and hence gain speculative profits. The main results of our analysis are as follows.

- 1. Injecting noise in the order flow of the dealer, a rational uninformed speculator interferes with the learning process of the market maker and determines his degree of uncertainty on the fundamental value of a foreign currency. The possibility of reducing the resiliency of the market, that is the speed of convergence of the exchange to its fundamental value, permits the speculator to gain and preserve an informational advantage with respect to the dealer.
- 2. The acquisition of this informational advantage corresponds to an "informational investment", because the dealer provides liquidity to the market at a cost and hence placing random market orders is expensive. On the other hand, the informational advantage yields profits from future informed trading. The solution of the trade-off between these costs and profits determines the optimal volume of speculative noise trading.
- 3. The equilibrium of the foreign exchange market with sporadic central banks intervention clearly indicates that speculative noise trading may be part of a profitable trading strategy in the foreign exchange market. In effect, the practice of giving spurious signals to other market participants to condition their expectations is common among professional traders. Investigating the market for foreign exchange, Lyons (1995) finds that a large component of the order flow of a dealer does not contain information. Because most transactions in the foreign exchange market are between professional traders, we can interpret this as evidence of attempts of manipulation.
- 4. When central bank intervention is continuous, the scope for manipulation can be severely limited, because the monetary authorities have the faculty of excluding uninformed speculator from the market. This also suggests that in other dealer markets, in which informed agents act strategically, the kind of manipulation discussed here is less likely to emerge.

# 5 Appendix

#### Proof of Proposition 1.

We will prove that for  $\sigma_f^2 > 0$  there exists a value  $\Sigma^*$  for the conditional variance of the fundamental value at time t-1,  $\Sigma_{t-1}^{mf}$ , such that if this is smaller than or equal to  $\Sigma^*$ , then the conditional variance of the fundamental value in the following round of trading,  $\Sigma_t^{mf}$ , is greater than  $\Sigma_{t-1}^{mf}$ . We will also see that the value of  $\Sigma^*$  is positive and increasing in the intensity of noise trading,  $b_t$ .

Consider that by the application of the projection theorem we have:

$$\Sigma_t^{mf} = \frac{\sigma_{ut}^2(\Sigma_{t-1}^{mf} + \sigma_f^2)}{a_t^2(\Sigma_{t-1}^{mf} + \sigma_f^2) + \sigma_{ut}^2}.$$

From this follows that  $\Sigma_t^{mf} < \Sigma_{t-1}^{mf}$  is equivalent to:

$$a_t^2 (\Sigma_{t-1}^{mf})^2 + a_t^2 \sigma_f^2 \Sigma_{t-1}^{mf} - \sigma_f^2 \sigma_{ut}^2 > 0.$$

For  $\sigma_f^2=0$  this inequality is always satisfied, so that the uncertainty on the fundamental value is decreasing over time. For  $\sigma_f^2>0$ , instead the inequality holds for  $\Sigma_{t-1}^{mf}>\Sigma^*$ , where this threshold value is given by:

$$\Sigma^* = -\frac{1}{2}\sigma_f^2 + \frac{1}{2}\sqrt{\sigma_f^2 \left(\sigma_f^2 + \frac{4\sigma_{ut}^2}{a_f^2}\right)}.$$

Because  $\Sigma^*$  is increasing in  $\sigma^2_{ut}$  and hence in  $b_t$  the proof is completed.  $\square$ 

#### Proof of Proposition 2.

Consider the inequality  $\Sigma_t^{mf} > \Sigma_t^{sf}$ . From the definition of the conditional variances for the market maker and the speculator we have that:

$$\Sigma_{t}^{mf} = \frac{\Sigma_{t-1}^{mf} + \sigma_{f}^{2}}{1 + \frac{a_{t}^{2}}{\sigma_{t-1}^{2}}(\Sigma_{t-1}^{mf} + \sigma_{f}^{2})}, \qquad \Sigma_{t}^{sf} = \frac{\Sigma_{t-1}^{sf} + \sigma_{f}^{2}}{1 + \frac{a_{t}^{2}}{\sigma_{t}^{2}}(\Sigma_{t-1}^{sf} + \sigma_{f}^{2})}.$$

So that the inequality among the variances is equivalent to:

$$\sum_{t-1}^{mf} - \sum_{t-1}^{sf} + a_t^2 (\sum_{t-1}^{mf} + \sigma_f^2) (\sum_{t-1}^{sf} + \sigma_f^2) \left( \frac{1}{\sigma_l^2} - \frac{1}{\sigma_{ut}^2} \right) > 0.$$

For t=1, note that  $\Sigma_0^{mf}=\Sigma_0^{sf}=\Sigma_0^f$ ; while for  $b_1>0$   $\sigma_{u1}^2>\sigma_l^2$ . Thus,  $\Sigma_1^{mf}>\Sigma_1^{sf}$ . Then, for  $t=2,\cdots,T$  the proof that  $\Sigma_t^{mf}>\Sigma_t^{sf}$  follows by induction. In fact, if  $\Sigma_{t-1}^{mf}>\Sigma_{t-1}^{sf}$  we have that  $\Sigma_t^{mf}>\Sigma_t^{sf}$  for any value of  $b_t$ .  $\square$ 

#### Proof of Lemma 1.

We start this proof by analyzing the game which the dealer and the speculator play

in the second period. Suppose the expected fundamental value of the speculator at the end of period 1 is  $s_1^s$  and that the pricing rule of the market maker for the second period is:

$$s_2^m = s_1^m + \lambda_2^{mf} \Delta x_2, \tag{19}$$

where  $s_t^m$  is the exchange rate in period t and  $\Delta x_2$  is the total order flow received by the market maker in period 2. Hence, it is immediate to see that:

$$\Delta x_2^s = b_2(s_1^s - s_1^m), \quad \text{where} \tag{20}$$

$$b_2 = \frac{1 - a_2 \lambda_2^{mf}}{2\lambda_2^{mf}}. (21)$$

Then, it is simple to verify that in the second period the expected profits of the speculator are:

$$E[\pi_2^s|I_{2s}] = \frac{[(1 - a_2\lambda_2^{mf})(s_1^s - s_1^m)]^2}{4\lambda_2^{mf}}.$$
 (22)

Thus, the expected value of these profits, conditional on the speculator information set at the end of period 1, is given by:

$$E[\pi_2^s)|I_{s1}] = \frac{(1 - a_2 \lambda_2^{mf})^2}{4\lambda_2^{mf}} E[(s_1^s - s_1^m)^2 | I_{s1}]. \tag{23}$$

In order to evaluate this expectation, consider that in period 1 if the speculator places a random market order  $b_1\eta_1$ , the dealer receives the following total market order,  $\Delta x_1 = a_1(f_1 - s_0) + b_1\eta_1 + \epsilon_1^l$ . He can use this observation of the order flow to form an expectation of the fundamental value,  $f_1$ :

$$s_1^m = s_0 + \lambda_1^{mf} \Delta x_1$$
, where  $\lambda_1^{mf} = \frac{a_1 \Sigma_1^f}{a_1^2 \Sigma_1^f + \sigma_{u_1}^2}$ . (24)

Observing the exchange rate at the end of the first round of trading the speculator can recover the value of the total market order,  $\Delta x_1$ , from equation (24). Thus, she is able to form a more efficient expectation of  $f_1$ :

$$s_1^s = s_0 + \lambda_1^{sf} \Delta y_1, \quad \text{where} \quad \lambda_1^{sf} = \frac{a_1 \Sigma_1^f}{a_1^2 \Sigma_1^f + \sigma_l^2}$$
 (25)

and  $\Delta y_1 = \Delta x_1 - b_1 \eta_1$ . In period 2 the market maker observes a new value of the order flow. Considering the expressions for  $\Delta x_2$  and for  $s_1^s$ , this observation of the order flow becomes:

$$\Delta x_2 = a_2(f_2 - s_1^m) + a_1 b_2 \lambda_1^{sf} f_1 - b_2 s_1^m + b_2 \lambda_1^{sf} \epsilon_1^l + \epsilon_2^l.$$
 (26)

Then, applying the projection theorem one obtains that the exchange rate in the second round of trading is given by equation (19), since the expected value of  $\Delta x_2$ .

given the information set of the dealer at time 1 is null. In other words, the strategies of the two agents (in equations (19) and (20)) are mutually consistent.

Considering the expressions of  $s_1^m$ ,  $s_1^s$  and those of  $\lambda_1^{mf}$  and  $\lambda_1^{ml}$ , we can derive the following expectations:

$$E[(s_1^s)^2|I_{s1}] = a_1 \Sigma_1^f \lambda_1^{sf}, \quad E[(s_1^m)^2|I_{s1}] = a_1 \Sigma_1^f \lambda_1^{mf}, \quad E[s_1^s s_1^m|I_{s1}] = a_1 \Sigma_1^f \lambda_1^{mf}.$$

Plugging these expectations in the right hand side of equation (23) one can obtain the following expression:

$$E[\pi_2^s|I_{s1}] = a_1 \Sigma_1^f (\lambda_1^{sf} - \lambda_1^{mf}) \left( \frac{(1 - a_2 \lambda_2^{mf})^2}{4\lambda_2^{mf}} \right). \quad \Box$$
 (27)

#### Proof of Proposition 4.

To prove the Proposition we just need to show that the speculator finds it useful to employ a noise trading technology t in period 1 and then recoup the initial losses this generates in the second round of trading. Note that only unpredictable market orders in period 1 are useful, because otherwise the predictable part of  $\Delta x_1^s$  would be filtered out from the total order flow, without affecting its informativeness, but increasing the expected losses of the speculator in period 1. Furthermore, considering that  $a_2 = 0$  the speculator maximizes with respect to  $b_1$  the following expression:

$$E[\pi^s|I_{s1}] = -b_1^2 \lambda_1^{mf} \sigma_l^2 + \frac{1}{4\lambda_2^{mf}} E[(s_1^s - s_1^m)^2 | I_{s1}]. \tag{28}$$

In order to find  $\lambda_2^{mf}$  we need to investigate the determination of the liquidity coefficient in the second period. Using the projection theorem once again, we find that:

$$\lambda_{2}^{mf} = \frac{Hb_{2}}{Fb_{2}^{2} + \sigma_{l}^{2}},$$
where
$$H = [a_{1}\Sigma_{1}^{f} - (\sigma_{l}^{2} + a_{1}^{2}\Sigma_{1}^{f})\lambda_{1}^{mf}]\lambda_{1}^{sf},$$

$$F = [a_{1}^{2}\Sigma_{1}^{f} + \sigma_{l}^{2} - a_{1}(2\sigma_{l}^{2} + a_{1}^{2}\Sigma_{1}^{f})\lambda_{1}^{mf}](\lambda_{1}^{sf})^{2}.$$
(29)

Since  $b_2 = 1/(2\lambda_2^{mf})$ , it easy to prove that equation (29) becomes:

$$\lambda_2^{mf} = \frac{1}{2\sigma_l} \sqrt{2H - F}.$$

Then, it is just a question of algebra to verify that 2H - F is equal to  $a_1 \Sigma_1^f (\lambda_1^{mf} - \lambda_1^{sf})$ , Thus, we obtain that:

$$\lambda_2^{mf} = \frac{\sqrt{a_1 \Sigma_1^f (\lambda_1^{sf} - \lambda_1^{mf})}}{2\sigma_l}.$$
 (30)

It is then immediate to insert  $\lambda_2^{mf}$  in the expected second period profits and verify that the speculator maximises the following expression:

$$\mathcal{V}(b_1) = \left\{ -b_1^2 \lambda_1^{mf} \sigma_l^2 + \frac{\sigma_l}{2} \sqrt{a_1 \Sigma^f (\lambda_1^{sf} - \lambda_1^{mf})} \right\}.$$

Let us take the first derivative of  $\mathcal{V}$  with respect to  $b_1$ . Then, considering that

then immediate to insert 
$$\lambda_2^{mf}$$
 in the expected second period profits and verify the speculator maximises the following expression: 
$$\mathcal{V}(b_1) = \left\{ -b_1^2 \lambda_1^{mf} \sigma_l^2 + \frac{\sigma_l}{2} \sqrt{a_1 \Sigma^f (\lambda_1^{sf} - \lambda_1^{mf})} \right\}.$$
 is take the first derivative of  $\mathcal{V}$  with respect to  $b_1$ . Then, considering that 
$$\frac{\partial b_1^2 \lambda_1^{mf} \sigma_l^2}{\partial b_1} = 2b_1 \lambda_1^{mf} \sigma_l^2 \left( \frac{a_1^2 \Sigma_1^f + \sigma_l^2}{a_1^2 \Sigma_1^f + \sigma_{u1}^2} \right),$$
 
$$\frac{\partial \sigma_l \sqrt{a_1 \Sigma_1^f (\lambda_1^{sf} - \lambda_1^{mf})}}{\partial b_1} = \frac{b_1 \lambda_1^{mf} \sigma_l^2}{a_1^2 \Sigma_1^f + \sigma_{u1}^2} \frac{a_1 \sigma_l \Sigma_1^f}{\sqrt{a_1 \Sigma_1^f (\lambda_1^{sf} - \lambda_1^{mf})}},$$
 betain that: 
$$\frac{\partial \mathcal{V}}{\partial b_1} = \frac{2b_1 \lambda_1^{mf} \sigma_l^2}{a_1^2 \Sigma_1^f + \sigma_{u1}^2} \left[ \frac{a_1 \sigma_l \Sigma_1^f}{4\sqrt{a_1 \Sigma_1^f (\lambda_1^{sf} - \lambda_1^{mf})}} - (a_1^2 \Sigma_1^f + \sigma_l^2) \right].$$
 Provided A root of the first derivative of  $\mathcal{V}$  corresponds to a root of the following equations: 
$$a_1 \sigma_l^2 \Sigma_1^f = a_1 c_1 (2^2 \Sigma_1^f + a_2^2)$$

we obtain that:

$$\frac{\partial \mathcal{V}}{\partial b_1} = \frac{2b_1\lambda_1^{mf}\sigma_l^2}{a_1^2\Sigma_1^f + \sigma_{u1}^2} \left[ \frac{a_1\sigma_l\Sigma_1^f}{4\sqrt{a_1\Sigma_1^f(\lambda_1^{sf} - \lambda_1^{mf})}} - (a_1^2\Sigma_1^f + \sigma_l^2) \right].$$

$$\frac{a_1 \sigma_l^2 \Sigma_1^f}{\lambda_s^{sf} - \lambda_l^{mf}} = 16(a_1^2 \Sigma_1^f + \sigma_l^2)^2. \tag{35}$$

Now, considering that

$$\lambda_1^{sf} - \lambda_1^{mf} = \frac{a_1 b_1^2 \sigma_l^2 \Sigma_1^f}{(a_1^2 \Sigma_1^f + \sigma_l^2)(a_1^2 \Sigma_1^f + \sigma_{n1}^2)} .$$

a root of equation (31) corresponds to a root of the following one:

$$(a_1^2 \Sigma_1^f + \sigma_{u1}^2) = 16b_1^2 (a_1^2 \Sigma_1^f + \sigma_l^2).$$

This quadratic equation possesses two distinct roots, symmetric around zero. We can take the positive one. We do not need to check for the second derivative, because for  $b_1 \downarrow 0$  the negative part of the derivative goes to zero faster than the positive one, so that we have a positive maximum. Thus, in the first round of trading the speculator employs a noise trading technology t, where:

$$b_1 = \left(\frac{a_1^2 \Sigma_1^f + \sigma_l^2}{16a_1^2 \Sigma_1^f + 15\sigma_l^2}\right)^{1/2}.$$

Finally, following forward the solution of the model already sketched completes the proof and gives a full characterization of the equilibrium of the market. In fact, the exchange rate set in the first round of trading is given by equation (24),

$$s_1^m = s_0 + \lambda_1^{mf} \Delta x_1,$$

where, plugging the expression of  $b_1$  in  $\lambda_1^{mf}$ , the liquidity coefficient is:

$$\lambda_1^{mf} = \frac{a_1 \Sigma_1^f (16 a_1^2 \Sigma_1^f + 15 \sigma_l^2)}{16 (a_1^2 \Sigma_1^f + \sigma_l^2)^2}.$$

Then, plugging equation (31) in equation (30) we find that the liquidity coefficient in period 2 is given by the following expression:

$$\lambda_2^{mf} = \frac{a_1\Sigma_1^f}{8(a_1^2\Sigma_1^f + \sigma_l^2)}.$$

Thus, in the second period, considering  $\lambda_2^{mf}$ , the speculator places the following market order:

$$\Delta x_2^s = b_2(s_1^s - s_1^m),$$
 where  $b_2 = rac{4(a_1^2 \Sigma_1^f + \sigma_l^2)}{a_1 \Sigma_1^f},$ 

and  $s_1^s$  is given by equation (25). Finally, the dealer sets the exchange rate according to equation (19):

$$s_2^m = s_1^m + \lambda_2^{mf} \Delta x_2. \ \Box$$

#### Proof of Lemma 2.

From the expression for  $\lambda_1^{sf} - \lambda_1^{mf}$  we find that:

$$\frac{\sigma_l}{2} \sqrt{a_1 \Sigma_1^f (\lambda_1^{sf} - \lambda_1^{mf})} = \frac{a_1 \sigma_l^2 \Sigma_1^f}{8(a_1^2 \Sigma_1^f + \sigma_l^2)}.$$

On the other side, plugging the equilibrium value of  $b_1$  in  $b_1^2 \lambda_1^{mf} \sigma_l^2$  one derives the following expression:

$$b_1^2 \lambda_1^{mf} \sigma_l^2 = \frac{a_1 \sigma_l^2 \Sigma_1^f}{16(a_1^2 \Sigma_1^f + \sigma_l^2)}.$$

Form which the expression for the total expected profits in equilibrium,  $E[\pi^s|I_{s1}]$ , is immediate.  $\square$ 

#### Proof of Proposition 5.

Considering the envelope theorem this result is immediate. It is in fact simple to calculate the following derivatives:

$$\frac{\partial E[\pi^s | I_{s1}]}{\partial \sigma_l^2} = \frac{a_1^3 (\Sigma_1^f)^2}{16(a_1^2 \Sigma_1^f + \sigma_l^2)^2}.$$

$$\frac{\partial E[\pi^s | I_{s1}]}{\partial \sigma_l^2} = \frac{a_1 \sigma_l^4}{16(a_1^2 \Sigma_1^f + \sigma_l^2)^2}.$$

$$\frac{\partial E[\pi^s | I_{s1}]}{\partial a_1} = \frac{\sigma_l^2 [\sigma_l^2 - a_1^2 (\Sigma_1^f)^2]}{16(a_1^2 \Sigma_1^f + \sigma_l^2)^2}.$$

#### Proof of Lemma 3.

To prove this Lemma we need to show that  $(1 - a_2 \lambda_2^{mf})^2 / 4 \lambda_2^{mf}$  is decreasing in  $a_2$ . Therefore, we need to calculate the liquidity coefficient  $\lambda_2^{mf}$ . In accordance with the projection theorem:

$$s_2^m = s_1^m + \lambda_2^{mf} \Delta x_2$$
, where  $\lambda_2^{mf} = \frac{\cos(f_2, \Delta x_2 | I_{m1})}{\sin(\Delta x_2 | I_{m1})}$ ,

in that  $E[\Delta x_2|I_{m1}]=0$ . Calculating these variance and covariance we find that:

$$\lambda_2^{mf} = \frac{(1 - a_1 \lambda_1^{mf}) P \Sigma_1^f - \lambda_1^{mf} \lambda_1^{sf} \sigma_l^2 b_2 + a_2 \sigma_f^2}{(1 - a_1 \lambda_1^{mf}) P^2 \Sigma_1^f + (\lambda_1^{sf})^2 (1 - \lambda_1^{ml}) \sigma_l^2 b_2^2 - \lambda_1^{mf} \lambda_1^{sf} P \sigma_l^2 + \sigma_l^2 + a_2^2 \sigma_f^2}, \quad (32)$$

where  $\lambda_1^{ml} = \sigma_l^2 \lambda_1^{mf} / a_1 \Sigma_1^f$  and  $P = a_1 \lambda_1^{sf} b_2 + a_2$ . Considering that  $b_2 = (1 - a_2 \lambda_2^{mf}) / 2 \lambda_2^{mf}$  we have that:

$$\lambda_2^{mf} = \frac{1}{a_2 + 2b_2}.$$

Substituting this expression in equation (32) and rearranging gives the following equation in  $b_2$ :

$$Ab_{2}^{2} + Bb_{2} + C = 0, \text{ where}$$

$$A = a_{1} \Sigma_{1}^{f} (\lambda_{1}^{sf} - \lambda_{1}^{mf}) > 0,$$

$$B = a_{2} \left\{ 2\sigma_{f}^{2} + \Sigma_{1}^{f} [\lambda_{1}^{sl} + (1 - a_{1}\lambda_{1}^{mf})] \right\} > 0,$$

$$C = -\sigma_{l}^{2},$$

and  $\lambda_1^{sl} = \sigma_l^2 \lambda_1^{sf}/a_1 \Sigma_1^f$ . Then, taking the positive root of this equation we obtain:

$$b_2 = \frac{\Delta - B}{2A}, \qquad (33)$$

$$\lambda_2^{mf} = \frac{A}{a_2 A - B + \Delta}, \tag{34}$$

where  $\Delta = \sqrt{B^2 - 4AC}$ . From these two equations it is immediate to check that:  $(1 - a_2 \lambda_2^{mf}) > 0$ , and that:

$$\begin{split} \frac{\partial (1-a_2\lambda_2^{mf})}{\partial a_2} &=& \frac{A(B^2-\Delta^2)}{\Delta (a_2A-B+\Delta)^2} < 0, \\ \frac{\partial b_2}{\partial a_2} &=& \frac{B(B-\Delta)}{2a_2A\Delta} < 0. \end{split}$$

Since  $(1 - a_2 \lambda_2^{mf})^2 / 4 \lambda_2^{mf} = b_2 (1 - a_2 \lambda_2^{mf}) / 2$ , an increase in  $a_2$  reduces the expected second period profits of the speculator, given the noise trading technology  $\mathbf{t}$ .  $\square$ 

#### Proof of Proposition 6.

This proof is straightforward. Consider the expression for  $b_2$  in equation (33): for  $a_2 \uparrow +\infty \ \Delta \downarrow B$ . Therefore,  $b_2 \downarrow 0$  and so do the expected second period profits of the speculator for any choice of the noise trading technology t.  $\square$ 

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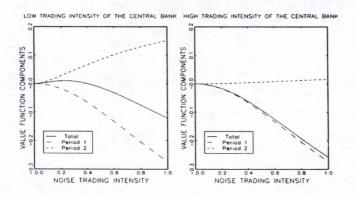
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Table 1: Comparative Static

Signs of Derivatives of the Characteristics of the Market						
	$b_1$	$\lambda_1^{mf}$	$b_2$	$\lambda_2^{mf}$	$\Sigma_1^{mf}$	$\Sigma_2^{mf}$
$\sigma_l^2$	+	-	+	-	+	+
$\Sigma_0^f$	-	+	-	+	+	+
$a_1$	_	+/-	-/+	+/-	-	-

Figure 1: Expected Profits of the Speculator as Functions of  $b_1$ 



The continuous line refers to the total expected profits, while the dotted ones refer to the expected first period losses and second period profits.  $\Sigma_0^f = \sigma_l = 1$ ,  $\sigma_f = 0$ .



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