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Economic Growth, (Re-)Distributive Policies, Capital Mobility and Tax Competition in Open Economies

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Economic Growth, (Re-)Distributive Policies, Capital Mobility and Tax Competition in Open Economies*

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1 Introduction

In this paper I analyze the trade-off between growth and redistribution policies in a two-country world with tax competition and varying degrees of capital mobility. In many policy discussions that address the issue of growth vs. redistribution, setting higher taxes for redistributive purposes is deemed to slow growth. Yet most developed and some developing countries redistribute a significant share of their GDP. Does this always lead to lower GDP growth? In the model developed in this paper it is argued that the experience of lower or higher GDP growth, when governments opt for redistribution, depends on who their opponents are when setting taxes in a non-cooperative environment. Furthermore, it is argued that the growth/redistribution trade-off problem depends on technology.

Barro and Sala-i-Martin (1990) show that the government may have room to stabilize the private return on capital and through this the growth rate. The trade-off between growth and redistribution for a closed economy has, for instance, been analyzed by Alesina and Rodrik (1994). They show that a government that cares about the non-accumulated factor of production experiences slower growth if it redistributes resources to that factor. Bertola (1993) derives similar results in a different framework. This vein of research suggests that in terms of growth the room for redistribution is limited for governments that wish to pursue redistributive policies.

The present paper extends the growth redistribution trade-off problem as formulated in Alesina and Rodrik (1991), (1994) to a two-country world. Each economy consists of two groups of agents, namely capital owners and workers. The workers never save, and supply labour inelastically.\textsuperscript{1} The capital owners do not work, accumulate capital and decide where to install their capital. To formulate the model in these terms keeps matters simple and allows one (1) to concentrate on the problem of growth and redistribution and (2) to relate to the literature

\textsuperscript{1}Bertola (1993) derives this behaviour for utility maximizing agents.
on majority voting on tax rates.2

Capital is internationally mobile, but this mobility is assumed to be in general less than perfect. The underlying forces governing the varying degrees of capital mobility are left unmodelled. Instead a convenient function is assumed to represent the net allocation of domestically owned capital in the foreign country. Autarky and perfect capital mobility are assumed to be limiting cases.

Like Alesina and Rodrik I assume that the governments tax the capital owners' wealth, but not the non-accumulated factor of production and that the wealth tax scheme represents a broader class of tax arrangements.3 That allows me to concentrate on problems associated with taxation of the accumulated factor of production in the growth process. By assumption expropriation of capital is ruled out for the governments.4 For open economies the following tax principles for capital income taxation are common.5

Under the 'residence principle' residents are taxed uniformly on their worldwide income regardless of the source of income (domestic or foreign), while non-residents are not taxed on income originating in the

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2See e.g. Romer (1975), Roberts (1977), Meltzer and Scott (1981), and Mayer (1984).

3The choice of tax base is not at all innocuous. As has been pointed out by Bertola (1991), (1993) and Alesina and Rodrik (1994) indirect taxation may lead to very different results as regards the growth redistribution trade-off. For instance, a capital income taxation cum investment subsidy tax scheme designed to equal a tax on consumption may guarantee higher growth for left-wing governments than right-wing ones as is shown in Rehme (1995a).

4Although a command optimum in this model would involve expropriation of capital even for a government maximizing the welfare of the capital owners, I rule it out since it is not very common in the real world. Modelling why and when expropriation may come about is clearly outside the scope of this paper.

5Razin and Yuen (1992) use an endogenous growth set-up in order to show that the residence principle is Ramsey efficient. This result seems to suffer from a time inconsistency problem since distortional capital or wage taxation may produce time inconsistent solutions. [Cf. Fischer (1980), Chamley (1985).] Capital taxation in economies with high capital mobility has received quite some attention recently in e.g. Chamley (1992), Canzonieri (1989), Roubini and Sala-i-Martin (1992), Gosh (1991) and Devereux and Shi (1991).
country.

Under the 'source principle' all types of income originating in a country are taxed uniformly, regardless of the place of residence of the income recipients.

If a country loses capital it may suffer in terms of welfare of capital owners or workers. Given the danger of losing capital, the source principle appears more suited as a tax principle, since the governments in a non-cooperative environment cannot perfectly monitor their residents' income or wealth. Therefore, in this paper the source principle for wealth taxation is adopted as a tax rule.

In the optimum the capital owners allocate their capital depending on the after-tax returns in the economies. For given public policies in the two economies, I show that in a market equilibrium the GDP and GNP growth rates depend crucially on the capital allocation decision and that in general GDP and GNP do not grow at the same rate. In particular, in this model GDP growth is a weighted average of GNP growth where the weights vary over time and depend on the after-tax returns.

I then conduct a public policy analysis. I assume that the governments in each country are taken to be of two types. They are either 'right-wing' and only care about the capital owners, or they are 'left-wing' caring about labour only. The governments' objectives reveal that a right-wing government wants to maximize GNP, whereas a left-wing government is concerned about redistribution, GDP and GDP growth. I show that in a closed economy the right-wing government acts growth maximizing.

Next, I assume that the governments have to choose taxes and redistribution non-cooperatively and engage in tax competition. The ob-

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7 Tax competition between fiscal authorities has been studied in numerous papers such as, for instance. Gordon (1983), Zodrow and Mieszokowski (1986), Wilson (1986), Wildasin (1988), Wilksas (1989). Gordon (1992), Bond and Samuelson
jectives also imply that the welfare maximizing governments implicitly compete for capital. The governments (left or right-wing) in each country (domestic and foreign) move simultaneously, but before the private sector.

For technologically similar economies it is shown that in the Nash Equilibria of the tax competition game there may be no room for redistribution for two left-wing governments. This result holds for sufficiently high capital mobility. For very low capital mobility the governments redistribute, but less than in the open economy. The intuition behind this result is the following: The left-wing governments face the trade-off between growth and redistribution. For the latter they need capital which is internationally mobile. They can only get more capital if they set a tax rate that approaches the one guaranteeing the maximum after-tax return. For redistribution they want to set higher taxes. Since it is capital that is being redistributed, tax competition causes a left-wing government to concentrate on securing high enough wages. By this the effects of the concern for wealth inequality are reduced. The result is driven by capital mobility and strategic interaction between two governments which have the same preferences.

If a left-wing and a right-wing government compete in taxes the strategic interaction is shown to be less. The reason for this is that the right-wing government is not concerned about redistribution. It just wishes to maximize the capital owners’ utility by securing them a maximum after-tax return on capital. Since the after-tax return determines growth, it maximizes GNP growth and by this it also attracts foreign capital. The lack of redistributive concern makes its problem a lot simpler. In the model it results in an extremely simple reaction function which the right-wing government possesses regardless of who its opponent may be. Given the fixed right-wing reaction function, the left-wing

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8 For instance, competition for capital has recently been analyzed by Sinn (1993).
9 For a model that studies the related problem of solving the trade-off between the provision of government consumption goods and growth in a Barro (1990) world see Devereux and Mansoorian (1992).
government knows it cannot attract foreign capital and as a consequence it chooses to redistribute, albeit less than in the closed economy, and experience lower GDP growth.

Is is then shown that as capital mobility increases the left-wing governments begin mimicking right-wing policies. From the argument above and given high capital mobility it follows that two competing left-wing governments will optimally set tax rates closer to the GNP growth maximizing one than under left-right competition.

Next, the paper addresses the issue of technological efficiency differences. I argue that as long as an efficiency gap can be maintained the efficient country will get more capital. This is especially true for an efficient left-wing government. If the gap is large enough it may be able to guarantee redistribution and higher GDP growth than a right-wing opponent. From this the aforementioned trade-off appears to be less important vis-à-vis opponents with other preferences. That suggests that the growth redistribution trade-off may not be a question of being right or left-wing (preferences), but rather a problem of being efficient or not (technology).

Finally, for some capital mobility this may have the surprising implication that a left-wing government may be better off in terms of GDP growth if it faces competition from another left-wing government. This goes with the cost of a reduction in or no redistribution. Competing against a right-wing government in turn allows for some redistribution in the optimum, at the cost of reduced GDP growth.

From the above one may conclude that high GDP growth and redistribution may be possible if a country is sufficiently efficient. Government preferences alone may not adequately explain the pattern of growth and redistribution in open economies with wealth tax competition, differences in strategic behaviour and varying degrees of factor mobility.

The paper is organized as follows: Section 2 presents the model set-up, derives the equilibrium in a closed economy and briefly presents the optimal policy choices as have been put forth in Alesina and Rodrik (1991). Section 6 formulates a game where governments with different
political objectives compete in wealth tax rates. The main results of this paper are stated in propositions. Section 7 concludes.

2 The Model

Consider a two-country world with a "domestic" and a "foreign" country. Let us denote variables in the foreign country by a (*). There are two types of many identical individuals in each country. In each country the types have the same rate of time preference, \( \rho \).\(^{10}\) One type of individuals owns capital and no labour and the other type owns labour, but no capital. Let us call the latter group 'workers' (\( W \)) and the former group 'capitalists' (\( k \)). Workers and capitalists are assumed to derive logarithmic utility from the consumption of a homogeneous, malleable good that is produced in the two countries.\(^{11}\) This assumes that foreign and domestic output, \( Y \) and \( Y^* \) are perfect substitutes in consumption.

Those who own capital, own shares of two representative firms. There are many firms assumed to be production units only. Following Barro (1990) aggregate production takes place according to

\[
Y_t = AK_t^\alpha G_t^{1-\alpha} L_t^{1-\alpha}, \quad \text{where} \quad 0 < \alpha < 1
\]

\[
K_t = \omega_t k_t + \phi^* (\omega_t^*) k_t^*
\]

\[
0 \leq \phi^* (\omega_t^*) \leq 1 - \omega_t^*, \quad \frac{\partial \phi^*}{\partial \omega_t^*} \leq 0, \quad \frac{\partial^2 \phi^*}{\partial \omega_t^*^2} \leq 0
\]

where \( Y_t \) is output produced in the home country, \( K_t \) the net overall domestically installed real capital stock, \( k_t \ (k_t^*) \) is the real capital stock owned by domestic (foreign) capitalists, \( G_t \) are public inputs to production and \( A \) is an efficiency index, which depends on cultural, institutional

\(^{10}\)So each type in e.g. the domestic country has the same time preference. One may justify this by assuming that each type belongs to an infinitely lived dynasty.

\(^{11}\)This specification is adopted for analytical convenience. The model can be extended to more general classes of utility functions without altering the qualitative results.
and technological development indicators. In this paper I focus on endogenous growth by eliminating all exogenous factors that may play a role in the growth process by assuming that $A$ is constant over time. Furthermore, I set $L_t = 1$, so that labour is supplied inelastically over time. The function $\phi^*(\omega^*_t)$ captures the fact that capital may be imperfectly mobile. Imperfect capital mobility may be due to many factors including, for instance, installation or transfer costs. However, in this model the exact influence of these factors on capital transfers is left unmodelled and for simplicity I will assume that $\phi^*$ depends on $\omega^*_t$, i.e. the allocation decision $\omega^*_t$ of the investors in the foreign country investing in the domestic country. Furthermore, the capital mobility functions are assumed to be symmetric so that $\phi^* = \phi^*$. The variable $\omega_t \in [0, 1]$ denotes the fraction of real capital at date $t$ owned by domestic capitalists installed in the home country. The fraction $\phi(\omega_t)$ is installed abroad by the domestic capitalists. Similar reasoning applies to the foreign capitalists and $\phi^*(\omega^*_t)$.

I will call the economies similar if $A = A^*$ since the countries may well be different in terms of institutional or cultural development. I will refer to the economies as being different if $A > A^*$, that is, if the countries are significantly different in their cultural, institutional or technological development. For instance, $A > A^*$ may capture the situation when one compares a developed Northern with a less-developed Southern country. I will also abstract from problems arising from depreciation of the capital stock. Note that by assumption installed foreign and domestic capital are perfect substitutes. This abstracts from the possibility that foreign capital may be a necessary input for domestic production. As it is the aim of this paper to model tax competition with the consequence of attracting capital, assuming complementarity would only exacerbate that competition, but would not change the results of this paper in any fundamental way.$^{12}$

$^{12}$The following is worth noting: I have assumed imperfect capital mobility and perfect substitution in consumption of the same good. Call it ectoplasm, cf. Barro and Sala-i-Martin (1995). Note that ectoplasm and its fruits are assumed to be edible (consumption good), the fruits can also be used as investment for growing more
2.1 The Public Sector

The governments in both countries redistribute and tax wealth at constant rates at each point in time. Let $\tau$ be the tax rate on real capital (wealth) which is held domestically by domestic investors and on capital installed in the home country. Thus, $\tau$ is levied on $\omega_t k_t$. The government also taxes the real foreign capital located in the home country, i.e. $\phi^*(\omega_t^*) k_t^*$. Analogous definitions hold for the foreign country. This way of taxing wealth means that the countries adopt the source principle as a tax rule which requires that all types of wealth present in a country be taxed uniformly, regardless of the place of residence of the owners of wealth. If capital is internationally mobile it makes sense to adopt this principle since governments in a non-cooperative environment cannot perfectly monitor their residents’ wealth.

Given the Barro-type production function I define the following government budget constraint, which is assumed to be balanced at each point in time

$$\tau K_t = G_t + \lambda \tau K_t. $$

The LHS depicts the tax revenues and the RHS public expenditures. The workers receive the fraction $\lambda$ of tax revenues, that is, $\lambda \tau K_t$, as transfers and $G_t$ is spent on public inputs to production. The parameter $\lambda$ is assumed to represent the degree of (capital) redistribution in the economy. Rearranging and taking into account that the domestic government may have two sources of tax revenues we contemplate the following budget constraint:

$$G_t = (1 - \lambda) \tau K_t. \quad (4)$$

We may note that $\tau$ is set by the government independently of other
factors in the economy which corresponds to the uniform taxation of wealth as required by the strict form of the source principle.\footnote{Differential taxation of foreigners and residents in the presence of perfect capital mobility has been taken up in Rehme (1995b). In contrast, in this paper the strict form will be assumed to hold. As shown in e.g. Razin and Sadka (1994) or Bovenberg (1994) the source principle entails a uniform taxation of residents’ and foreigners’ capital income. The model can in principle allow for discriminatory taxation at the expense of considerable technical complications. Of course, the question whether tax discrimination plays a major role in the equilibria below is of interest, but then all the equilibria found in this paper may be interpreted as and shown to be results about average tax rates in a model with discriminatory taxation.}

### 2.2 The Private Sector

The private sector is made up of many identical firms, workers and capital owners.

The firms in each country operate in a perfectly competitive environment and act as profit maximizers. The firms are owned by domestic and foreign capital owners. Foreign and domestic capitalists rent capital to and demand shares of the representative domestic firm. The same holds for the foreign firm. The domestic capitalists’ assets are their shares of the firms. The shares of the domestic and foreign firms are collateralized one-to-one by physical capital. The markets for assets and physical capital are assumed to clear at each point in time. For convenience assume that the representative domestic firm faces a given path of the market clearing rental rate, \( \{r_t\} \), of domestically installable capital, \( K_t \).\footnote{The assumptions that assets are collateralized one-to-one and the rental rate of capital is - later on - uniform and for simplicity constant can easily be relaxed without altering the results. For a justification of these assumptions cf. Barro and Sala-i-Martin (1995), Chpt. 2.} Given perfect competition the firms in the domestic economy rent capital and hire labour in spot markets in each period in their country. We assume that foreign and domestic output are perfect substitutes in consumption and set the price of \( Y_t \) and \( Y_t^* \) equal to 1. Given constant returns to capital and labour, factor payments exhaust output. Profit maximization then entails that firms pay each factor of production its
marginal product
\[ r = \frac{\partial Y_i}{\partial K_i} = \alpha A[(1 - \lambda)\tau]^{1-\alpha}, \quad (5) \]
\[ w_t = \frac{\partial Y_i}{\partial L_t} \equiv \eta(\tau, \lambda)K_t = (1 - \alpha)A[(1 - \lambda)\tau]^{1-\alpha}K_t, \quad L_t = 1, \forall t. \quad (6) \]

Equation (5) implies an intra-country arbitrage which makes the return on foreign and domestic capital installed in each firm equal in the domestic country. The same, of course, applies to the foreign country. Notice that the return on capital is constant, whereas the wages vary over time depending on the evolution of the installed capital stock. It can be seen that the tax rate has a bearing on the marginal product of capital. Use the definitions given in (4), and (5), assume \( 0 < \omega_t \leq 1, 0 \leq \phi^*(\omega^*_t) < 1 \) and fixed for the home country and let \( E \equiv (1 - \alpha)A[(1 - \lambda)\tau]^{-\alpha} \). Then
\[ \frac{\partial r}{\partial \tau} = \alpha E(1 - \lambda) > 0, \quad \frac{\partial r}{\partial \lambda} = \alpha E(-\tau) < 0. \quad (7) \]

So redistribution has a negative effect on the return on capital and increases in the tax rates raise the rate of return. For the wages, \( (\eta K_t) \), we obtain the following relationships that are easy to verify
\[ \frac{\partial \eta}{\partial \tau} = (1 - \alpha)E(1 - \lambda) > 0, \quad \frac{\partial \eta}{\partial \lambda} = (1 - \alpha)E(-\tau) < 0. \quad (8) \]

Thus for given \( \omega_t, \omega^*_t \) an increase in \( \tau \) leads to a positive change in the rate of return and in wages. Redistribution lowers each of them.

The *workers* are assumed to derive a utility stream from consuming their wages and government transfers. They do not invest and are not taxed by assumption.\(^{15}\) Their intertemporal utility is given by
\[ \int_0^\infty \ln C^W_t e^{-\rho t} dt \quad \text{where} \quad C^W_t = \eta(\tau, \lambda)K_t + \lambda \tau K_t. \quad (9) \]

This assumption is reminiscent of growth models such as Kaldor (1956), where different proportions of profits and wages are saved. However, in\(^{15}\) Negative values for \( \lambda \) would be tantamount to wage taxes or taxes on human capital. In order to focus on the effects of capital taxation I will abstract from any effects of wage taxes on the economies.
Kaldorian models the growth rate determines the factor share incomes, whereas in endogenous growth models such as this one the direction is rather from factor shares to the growth rate. (On this distinction see, for example, Bertola (1993).)

The capitalists in each country cannot move, and choose how much to consume or invest. Each individual capitalist takes the paths of \((r, r^*, \tau, \tau^*)\) as given. Since the capital owners have the opportunity to invest in either country they have to determine where their capital is to be located, \(\omega_t\). Recall that we have assumed that imperfect capital mobility may prevail so that that less than \(1 - \omega_t\) of domestically owned capital is generally installed abroad. The capitalists have perfect foresight and maximize their intertemporal utility according to the following programme

\[
\max_{C_t^k, \omega_t} \int_0^\infty \ln C_t^k e^{-\rho t} dt \tag{10a}
\]

s.t. \[
\dot{k}_t = (r - \tau) \omega_t k_t + (r^* - \tau^*) \phi(\omega_t) k_t - C_t^k \tag{10b}
\]

\[
0 \leq \omega_t \leq 1 \tag{10c}
\]

\[
k(0) = k_0, \quad k(\infty) = \text{free.} \tag{10d}
\]

Equation (10b) is the dynamic budget constraint of the capitalists. So the capitalists earn income at home \(r \omega_t k_t\) and abroad, \(r^* \phi(\omega_t) k_t\). Furthermore, I assume that re-investment of profits earned in a country is costless in that particular country. The necessary first order conditions for this problem are given by (10b), (10c), (10d) and the following equations:

\[
\frac{1}{C_t^k} - \mu_t = 0 \tag{11a}
\]

\[
\mu_t (r - \tau) k_t + \mu_t (r^* - \tau^*) \phi'(\omega_t) k_t = 0 \tag{11b}
\]

\[
\dot{\mu}_t = \mu_t \rho - \mu_t [(r - \tau) \omega_t + (r^* - \tau^*) \phi(\omega_t)] \tag{11c}
\]

\[
\lim_{t \to \infty} k_t \mu_t e^{-\rho t} = 0. \tag{11d}
\]

where \(\mu_t\) is a positive co-state variable which can be interpreted as the instantaneous shadow price of one more unit of investment at date \(t\). Equation (11a) equates the marginal utility of consumption to the shadow
price of more investment, (11c) is the standard Euler equation which relates the costs of foregone investment (LHS) to the discounted gain in marginal utility (RHS), noting \( \frac{1}{C^t} = \mu_t \), and (11d) is the transversality condition for the capital stock which ensures that the present value of the capital stock approaches zero asymptotically. Equation (11b) describes the capitalists' capital allocation decision, which is dependent on the after-tax returns in the two countries, since

\[
\phi'(\omega_t) = -\frac{r - \tau}{r^* - \tau^*}.
\]

If we totally differentiate this expression we see that the allocation decision \( \omega_t \) is increasing in the ratio of the domestic to the foreign after-tax returns,

\[
\frac{d\omega_t}{d(\frac{r - \tau}{r^* - \tau^*})} = -\frac{1}{\phi''(\omega_t)} \geq 0, \text{ where } \phi''(\omega_t) < 0.
\]

Thus, if the domestic after-tax return increases, then the capitalists install more capital at home. To fix ideas and keep matters simple I will consider the following explicit \( \phi \) function:

\[
\phi(\omega_t) = \frac{z}{z + 1} - \frac{z}{z + 1} \omega_t^{1 + \frac{1}{z}}.
\]

This function obeys all the restrictions I have put on \( \phi \) earlier on.\(^\text{16}\) The parameter \( z \) measures the degree of capital mobility imperfection. If \( z \to \infty \) (0) we have perfect capital mobility (autarky).\(^\text{17}\) Then the

\(^{16}\)It is immediate that \( \phi', \phi'' \leq 0 \). In order to check if \( \phi(\omega) \leq (1 - \omega) - \phi(\omega) \) let \( d \equiv (1 - \omega) - \phi(\omega) \). Then \( d_\omega = -1 + \omega^{\frac{1}{z}} \) and \( d_{\omega^z} = \frac{1}{z}\omega^{\frac{1}{z} - 1} \) establishes that \( \omega_1 \) minimizes \( d \) for any \( z \in [0, \infty) \) so that \( d \geq 0 \). Now let \( m \equiv \frac{z}{z + 1} \) and note that \( m_z = \frac{1}{(z + 1)^2} \geq 0 \). Then \( m \to 0 \) (1) as \( z \to 0 \) (\( \infty \)). Suppose \( \omega < 1 \). Then for \( z \to 0 \) we have \( \omega^{1 + \frac{1}{z}} \to 0 \) and \( m \to 0 \) so that \( d \to 1 - \omega > 0 \). For \( z \to \infty \) we have \( \omega^{1 + \frac{1}{z}} \to \omega \) and \( m \to 1 \) so that \( d \to 0 \). From this I conclude that \( (1 - \omega) \geq \phi(\omega) \) for all \( z \in [0, \infty) \) and \( \omega \in [0, 1] \).

\(^{17}\)We may think of the exogenous \( z \) as reflecting the ex ante outcome of a bargaining process between two countries that has become legally binding. Then the fact that governments alter in office clearly matters, but I assume that the contract is binding so that, for instance, a government preferring autarky cannot shut its economy off if the contract specifies otherwise. This is a realistic specification for most countries that have entered some unilateral or multilateral trade agreements.
optimal decision rule is given by
\[ \omega = \min \left\{ \left( \frac{r - r^*}{\tau - r^*} \right)^z, 1 \right\}. \] (13)

This expression is increasing in the domestic after tax return for \( \frac{r - r^*}{\tau - r^*} < 1 \) and constant over time. If the after-tax return ratio is larger or equal to 1, then the investors leave their capital in their home country, \( \omega = 1, \phi(\omega) = 0 \). If the foreign after-tax return goes up at \( \frac{r - r^*}{\tau - r^*} \geq 1 \), the investors may shift their capital abroad. Then, depending on the after-tax returns in the two countries the growth rate of consumption follows in a standard way from (11c) and (13):
\[ \gamma_t \equiv \frac{C^k_t}{C^l_t} = (r - \tau)\omega + (r^* - \tau^*)\phi(\omega) - \rho. \] (14)

So consumption growth is increasing in the after-tax returns. Suppose the capitalists’ capital stock and their consumption grow at the same rate. Then this means that the capital stock of the capitalists grows at a rate which depends on the after-tax returns in the two countries. If the after-tax returns are such that \( \omega = 1 \), then all the growth of domestically owned capital takes place in the home country. If the foreign after-tax return is sufficiently high it becomes attractive for the domestic investor to shift capital abroad. Then the domestically owned capital stock grows at a rate that is a mixture of contributions of home and foreign investment. Finally, note that from (14) and (10b) the instantaneous consumption of the capitalists in steady state is given by \( C^k_t = \rho k_t \).

3 Market Equilibrium

The constancy of \( \tau, r^* \) implies constancy of \( r, r^* \) and \( \omega, \omega^* \) and hence \( \gamma, \gamma^* \). For the rest of the paper let the partial derivative of a variable \( x \) with respect to a variable \( y \) be denoted by \( x_y \).

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18 In section 3 it is shown that for constant tax rates we get balanced growth such that \( \gamma = \frac{C^k_t}{C^l_t} = \frac{k_t}{k_l} \).
3.1 Two-Country World

I will first look at the dynamic market equilibrium of the domestic economy in our two-country world given arbitrary tax rates. Let us define $\nu(t) \equiv \frac{\omega k_t}{K_t}$ and $\nu^*(t) \equiv \frac{\phi^* (\omega^*) k^*_t}{K^*_t}$ as the shares of domestic and foreign capital in overall installed capital. To save on notation let $\nu(0) \equiv \nu$ and $\nu^*(0) \equiv \nu^*$.

Divide (10b) by $k_t$, and use the fact that in steady state $\gamma_k$ is constant. Rearranging and taking time derivatives yields $\gamma = \gamma_k$ where $\gamma$ is given by (14) and constant. Substituting $\gamma$ for $\gamma_k$ in (10b) establishes $C^K_t = \rho k_t$ as the instantaneous consumption of the capital owners in steady state. Hence, in the open economy the domestic capitalists' consumption grows at the same, constant rate as their capital stock. The same holds for the foreign capitalists. In the two-country world with given $\omega, \omega^*$ the domestic resource constraint is given by $I_t = GDP_t - C^K_t - C^W_t - G_t$, i.e.

$$I_t = \dot{K}_t = (r - \tau)[\omega k_t + \phi^* k^*_t] + [\eta(\cdot) + \lambda \tau] K_t - C^K_t - C^W_t.$$ 

where I have defined $C^K_t$ as a residual and call $C^K_t$ the aggregate consumption of the domestic and foreign capitalists consuming the domestic output at date $t$. Then we use the binding constraint, $C^W_t = [\eta + \lambda \tau] K_t$. Recall $\gamma_k = \gamma$ and $\gamma^*_k = \gamma^*$. Then $K_t = \omega k_t + \phi^* k^*_t$ and the constancy of $\gamma, \gamma^*$ entails $k_t = e^{\gamma t} k_0$ and $k^*_t = e^{\gamma^* t} k^*_0$. Thus, $K_t = \omega e^{\gamma t} k_0 + \phi^* e^{\gamma^* t} k^*_0$ and $\dot{K}_t = \gamma \omega e^{\gamma t} k_0 + \gamma^* \phi^* e^{\gamma^* t} k^*_0$. Then the growth rate of the aggregate capital stock at any $t$ is given by

$$\Gamma_t \equiv \frac{\dot{K}_t}{K_t} = \frac{\gamma \omega e^{\gamma t} k_0 + \gamma^* \phi^* e^{\gamma^* t} k^*_0}{\omega e^{\gamma t} k_0 + \phi^* e^{\gamma^* t} k^*_0} = \nu(t) \gamma + \nu(t)^* \gamma^*.$$ 

Notice that this growth rate is a weighted average of the growth rates of the domestic and the foreign capitalists' capital stock and it is not constant over time. To see this we calculate

$$\frac{d\Gamma_t}{dt} = \frac{\left( \gamma^2 \omega k_0 e^{\gamma t} + \gamma^* \phi^* e^{\gamma^* t} \right) K_t - (\dot{K}_t)^2}{K_t^2} = (\gamma - \gamma^*)^2 \Delta.$$
where \( \Delta = \frac{\omega k_0 e^{\gamma t} k_0^* e^{-t}}{K_t} \). Thus, \( \Gamma_t \) is increasing over time, unless \( \gamma = \gamma^* \).

Next, I wish to find \( \lim_{t \to \infty} \Gamma_t \) if \( \gamma > \gamma^* \) which is given by

\[
\lim_{t \to \infty} \Gamma_t|_{\gamma > \gamma^*} = \frac{\gamma \omega k_0 + \lim_{t \to \infty} \gamma^* \phi^* e^{(\gamma^* - \gamma)t} k_0^*}{\omega k_0 + \lim_{t \to \infty} \phi^* e^{(\gamma^* - \gamma)t} k_0^*} = \gamma.
\]

**Lemma 1** The GDP growth rate \( \Gamma_t \) is increasing over time, \( \frac{d\Gamma_t}{dt} > 0 \), for any \( \gamma \neq \gamma^* \). If \( \gamma > \gamma^* \), then \( \lim_{t \to \infty} \Gamma_t|_{\gamma > \gamma^*} = \gamma \).

So the GDP growth rate approaches the domestic GNP growth rate, if \( \gamma > \gamma^* \). Furthermore, at time 0 I get \( \Gamma_0 = \Gamma_Y \) from the production function and the fact that output is equal to GDP in equilibrium. Thus, GDP and the aggregate capital stock grow at the same rate. Also \( \Gamma_0 = C_t^W/C_t^W \) from the workers' budget constraint. Hence, at \( t = 0 \) the economy will be characterized by balanced growth for \( K_t, Y_t, C_t^W \), that is, \( \Gamma_0 = \Gamma_Y = \Gamma_{C_t^W} \).\(^{19}\)

From this it is clear that \( \omega, \omega^* \) play a crucial role in determining the equilibrium in our two-country world. But since the tax parameters are given arbitrarily at this stage one cannot say anything about the exact form of our equilibrium. It appears that arbitrary levels and combinations of tax rates would sustain multiple market equilibria. Economically, this suggests that one cannot say very interesting things about the economies unless more structure is being put on the way taxes are set, which will be the objective of the tax competition game I will contemplate below.

\(^{19}\)Recall that \( \gamma = \frac{C_t^k}{C_t^k} \). Thus, there is a crucial difference between GDP or GNP consumption growth of the capital owners. For instance, if we devide the resource constraint by \( K_t \) then we may get an expression \( \Gamma_t - (\gamma - \tau) = \frac{C_t^k}{K_t} \). Taking time derivatives yields \( \dot{\Gamma}_t = \frac{C_t^k}{K_t} \left( \frac{C_t^k}{C_t^k} - \Gamma_t \right) \). We know that \( \dot{\Gamma}_t \) evaluated at \( t = 0 \) depends on \( (\gamma - \gamma^*)^2 \) so that in general \( \Gamma_{C_t^k} \not\equiv \Gamma_0 \).
3.2 Closed Economy

For a closed economy $\omega = 1, K = k$. It is not difficult to see then that the market equilibrium is characterized by steady state, balanced growth, that is, $g_Y = g_k = g = \gamma_C w$ with $g = (r - \tau) - \rho$ where $r = \alpha A[(1 - \lambda)\tau]^{1-\alpha}$. The $(\tau, \lambda)$ combination that maximizes growth must satisfy $r_{\lambda} \leq 0$ and $r_{\tau} = 1$. From (7) we get that $r_{\lambda} < 0$ so that $\lambda = 0$ and from $r_{\tau} - 1 = 0$ we obtain $\hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\alpha}}$ as the growth maximizing $(\tau, \lambda)$ combination.

**Lemma 2** The $(\tau, \lambda)$ combination that maximizes growth in a closed economy is given by $\lambda = 0$ and $\hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\alpha}}$ where $\hat{\tau}$ solves $r_{\tau} = 1$.

The relationship between the wealth tax rate and growth in a closed economy can be read off from figure 1 below. At $\hat{\tau}$ the growth rate is maximal. If higher taxes - for example for wealth redistribution ($\lambda > 0$) - are levied, then $\tau > \hat{\tau}$ and the growth rate decreases. i.e. growth is traded off against redistribution at a point such as $\hat{\tau}$ with $\lambda > 0$.

The after-tax return is given by

$$r - \tau = \alpha A[(1 - \lambda)\tau]^{1-\alpha} - \tau = \tau \left[\alpha A[(1 - \lambda)\tau]^{-\alpha} - 1\right].$$

Substitution of the growth maximizing $(\tau, \lambda)$ combination establishes $\hat{\tau} - \hat{\tau} = \hat{\tau} \left(\frac{\alpha}{1-\alpha}\right)$. The effect of a marginal increase in efficiency on
the maximum growth rate is given by \( \frac{d(\hat{\gamma} - \hat{\gamma})}{dA} = \left( \frac{\alpha}{1-\alpha} \right) \frac{d\hat{\gamma}}{dA} \) where \( \frac{d\hat{\gamma}}{dA} = \hat{\gamma} \alpha^2 (1-\alpha)A^{-1} > 0 \). Hence,

Lemma 3 The maximum after-tax return is \( \hat{\gamma} - \hat{\gamma} = \hat{\gamma} \left( \frac{\alpha}{1-\alpha} \right) \). An increase in efficiency raises the maximum growth rate, the maximum after-tax return and the growth maximizing tax rate, that is, \( \frac{dj}{dA} > 0 \), \( j = \hat{\gamma}, (\hat{\gamma} - \hat{\gamma}), \hat{\gamma} \).

The result that a more efficient economy may have higher maximum growth corresponds to common economic intuition and is hardly surprising. The lemma will be useful below. In terms of Figure 1 we may think of an increase in \( A \) as an upward shift of the concave relationship between taxes and growth.

4 The Government

I will consider government objectives where each domestic government maximizes the welfare of its domestic clientele. The governments are assumed to take as their welfare measure the intertemporal utility of their clientele.\(^20\) In appendix A I show that a domestic right-wing (strictly

\(^{20}\)We know from the theory of optimal taxation that the government’s objective can be stated in terms of the indirect utility function. Note that the welfare function is a function of the government’s instruments and that this function need not necessarily coincide with the individual agents’ utilities as noted in e.g. Atkinson and Stiglitz (1989), chpt.12 and Diamond and Mirrlees (1971). For example, an earlier version of this paper assumed that the domestic right-wing government represented the interests of the (domestic and foreign) capital owners as a class. Then a right-wing government would be interested in GDP growth and would have an objective function very similar to a left-wing government’s one. Both right and left-wing governments would then compete in taxes for capital. But as governments are voted for by their national constituencies, this kind of class objective is inconsistent with a truly representative democracy. It is an interesting question whether pro-capital governments are truly and only representing their national voters (capital owners) in reality. The answer to that question is outside this model. Instead, I now assume that the governments really do represent the national capital owners’ interests only.
pro-capitalists) government \((r)\) has the following objective function

\[
V^r = \int_0^\infty \ln C_t^k e^{-\rho t} dt = \frac{\ln C_0^k}{\rho} + \frac{\gamma}{\rho^2}
\]

where \(C_0^k = \rho k_0, \gamma = (r - \tau)\omega + (r^* - \tau^*)\phi(\omega) - \rho. \tag{16}\]

Notice that the growth rate here is that of \(k_t\) and not \(K_t\). This follows from the fact that the right-wing government serves domestic capitalists only and is therefore concerned about GNP and not GDP.

Similarly, the welfare measure of a domestic left-wing (strictly pro-labour) government \((l)\) integrates to

\[
V^l = \int_0^\infty \ln C_t^l e^{-\rho t} dt = \frac{\ln C_0^l}{\rho} + \frac{1}{\rho} \int_0^\infty \Gamma_t e^{-\rho t} dt
\]

where \(C_0^l = (\eta(\tau, \lambda) + \lambda\tau) K_0, \Gamma_t = \nu(t)\gamma + \nu^*(t)\gamma^*, \nu(t) \equiv \frac{\omega k_0 e^{\tau t}}{K_t}, \nu^*(t) \equiv \frac{\phi^*(\omega^*) k_0^* e^{\tau^* t}}{K_t} \tag{17}\]

where I have used integration by parts to show that \(V^l\) depends on \(\Gamma_t\). Notice that the left-wing government is concerned about GDP and the growth rate of GDP, since wages depend on \(K_0\), the overall installed capital stock at \(t = 0\).

In appendix A I show that the two objective functions \(V^r\) and \(V^l\) in (16), (17) are increasing in \((k_0, \gamma)\) and continuous in tax rates for given \(\omega, \omega^*\). Thus, given everything else getting more domestic capital is in the interest of right and left-wing governments in this model. Furthermore, welfare of the workers is also increasing in \(k_0^*, \gamma^*\). So it turns out that each government's objective possesses the feature that is implicitly compatible with some other objective, namely that of increasing the growth rate of domestically owned capital. So any policy that generates higher domestic capitalists' capital growth is in the interest of both types of government since it raises each group's welfare. From this I conclude that in this model the objective to maximize the welfare of workers or capitalists and maximizing GNP growth are positively related, that is, they represent some - possibly the same - underlying preferences in different ways.
5 The Government's Problem in a Closed Economy

Suppose the government in the closed economy \((\omega = 1, \omega^* = 1)\) cares about both capitalists and workers. Respecting the right of private property, the government has to choose \(\tau\) and \(\lambda\) to solve the following intertemporal problem, which is equivalent to the model of Alesina and Rodrik:

\[
\max_{\tau, \lambda} (1 - \beta) V^r + \beta V^l \quad s.t. \quad \lambda \geq 0
\]

where \(V^r, V^l\) are given by the expressions in (16) and (17). The parameter \(\beta \in [0, 1]\) represents the welfare weight attached to the two groups in the economy. If \(\beta = 1, (0)\) the government cares about the workers (capitalists) only. I will call

\[\beta = 1, (0) \text{ - a left-wing (right-wing) government.}\]

The constancy of \(\beta\) may be justified by interpreting \(\beta\) as reflecting the socio-economic institutions in an economy. Then the fact that governments alternate in office becomes less of an issue since institutional features are usually constant for very long periods of time.

The condition \(\lambda \geq 0\) restricts the governments so that even a right-wing government does not tax workers. In that sense even a right-wing government is 'nice' to the workers. A negative \(\lambda\) would effectively amount to a tax on wages. At this stage let \(\beta \in [0, 1]\). Then the solution to the government's problem is presented in appendix B and is given by:

If \(\beta \rho \geq [(1 - \alpha)A]^{\frac{1}{\alpha}}\) then:

\[
\hat{\tau} = \beta \rho, \quad \hat{\lambda} = 1 - \frac{[(1 - \alpha)A]^{\frac{1}{\alpha}}}{\beta \rho}.
\]

If \(\beta \rho < [(1 - \alpha)A]^{\frac{1}{\alpha}}\) then:
\[ \hat{\tau}[1 - \alpha(1 - \alpha)A^{\hat{\tau} - \alpha}] = \beta(1 - \alpha). \]

A right-wing government, \( \beta = 0 \), is only concerned about growth in this model. It chooses the growth maximizing \((\tau, \lambda)\) combination. Recall that by Lemma 2 the growth maximizing tax rate with \( \lambda = 0 \) is given by

\[ \hat{\tau} = [\alpha(1 - \alpha)A]^{\frac{1}{\hat{\alpha}}}. \]

From (20) and (18), we see that \( \hat{\tau} > \hat{\tau} \) when \( \lambda \geq 0 \) so that growth is not maximized. This can be visualized using Figure 1. Thus, a redistributing government \((\lambda > 0)\) trades off growth against redistribution at a point such as \( \hat{\tau}, \lambda > 0 \). It is interesting to note that \( \beta \) is inversely related to the growth rate. Another implication is that there is a wide range of values where no wealth redistribution takes place. Note that if \( \rho \) is a lot lower than \( \beta \), i.e. capitalists and workers are patient, then even a left-wing government might not redistribute as can be seen in (19). But unless \( \rho = 0 \) a left-wing government will set higher taxes than a right-wing one and have lower growth - in this model.

Suppose the left-wing government redistributes wealth in the optimum. Then (18) holds with \( \lambda > 0 \). I will now check under what conditions we have \( \gamma > 0 \) with \( \lambda > 0 \). If (18) holds then \((1 - \lambda)\hat{\tau} = [(1 - \alpha)A]^{\frac{1}{\hat{\beta}}} \). Let \( \Theta \equiv (1 - \alpha).A \), then

\[ r = \alpha A[(1 - \lambda)\hat{\tau}]^{1-\alpha} = \alpha A\Theta^{\frac{1-\alpha}{\alpha}} = (\frac{\alpha}{1 - \alpha}) \Theta^{\frac{1}{\hat{\beta}}}. \]

From (18) we have \( \hat{\tau} = \rho \geq \Theta^{\frac{1}{\hat{\beta}}} \) and for \( \gamma > 0 \) we have \( r - \hat{\tau} - \rho > 0 \). So \( \hat{\tau} = \rho \) has to satisfy

\[ \hat{\tau} > \Theta^{\frac{1}{\hat{\beta}}} \land (\frac{\alpha}{1 - \alpha}) \Theta^{\frac{1}{\hat{\beta}}} > 2\hat{\tau} \iff \hat{\tau} (\frac{\alpha}{1 - \alpha}) \Theta^{\frac{1}{\hat{\beta}}} > \Theta^{\frac{1}{\hat{\beta}}} 2\hat{\tau} \iff \alpha > \frac{2}{3}. \]

Notice that from (1) \( \alpha \) measures the elasticity of output w.r.t. private capital. Thus, private capital \( k_i \) has to be sufficiently more important in production than public inputs \( G_i \). Furthermore, for an increase in \( A \) one finds \( \frac{d\lambda}{dA} < 0 \) so that \( \lambda \) would be lower in the new optimum. Hence,

**Lemma 4** For a redistributing \((\lambda > 0)\), left-wing government \( \gamma > 0 \) only if \( \alpha > \frac{2}{3} \). An increase in efficiency makes the left-wing government redistribute less wealth given its optimal policy, that is, \( \frac{d\lambda}{dA} < 0 \).
This is an interesting implication of the model in Alesina and Rodrik (1991), which they do not discuss. The lemma is clearly empirically relevant and testable. For example, it entails that an increase in efficiency causes the left-wing government to redistribute less wealth in the optimum and place more weight on growth. Finally, we may note that the optimal tax rates are non-zero.\footnote{This is due to the assumption that \( \lambda \) is non-negative and labour supply is inelastic. As has been shown by Jones, Manuelli and Rossi (1993a) and Jones, Manuelli and Rossi (1993b) and in contrast to, for instance, Chamley (1986) this leads to non-zero tax rates on capital income.}

In order to concentrate on the distributional conflict I will only look at the border cases of \( \beta = 1 \) ('left-wing'), and \( \beta = 0 \) ('right-wing') in the next sections.

6 Tax Competition in a Two-Country World with Capital Mobility

In this section I investigate what happens to the optimal choices of tax rates and redistribution parameters if these choices have to be made in a two-country world with capital mobility and countries cannot coordinate their policies. This is a relevant question for countries where full tax harmonization may not be possible. There is a possibility then that countries engage in tax competition.\footnote{For a similar point see, for instance, Bovenberg (1994).} I will look for a Nash Equilibrium of the game described below. The strategies of the two governments are the choices of \( \tau, \lambda \) and \( \tau^*, \lambda^* \) and only pure strategies are considered. For the formulation of the game I employ the following

Assumptions:

1. There is no uncertainty. Perfect knowledge about all the parameters, objective functions, the strategies and the sequence of moves prevails.
2. All agents act non-cooperatively.

3. The governments move simultaneously.

4. The private sector, that is, the workers and the capitalists move simultaneously.

5. The governments move before the private sector.

6. At each point in time the agents are confronted with the same problem.

7. Agents remember at $t$ only what they have done at date 0.

8. $k_0 = k_0^*$, i.e. both economies have the same initial capital stock. (Unless stated otherwise.)

9. $A = A^*$, i.e. the economies are equally efficient or similar. (Unless stated otherwise.)

10. $\rho = \rho^*$, i.e. the agents' time preference rates are equal across countries.

Assumption (5.) defines a game the solution to which is called a *Ramsey Equilibrium*. This is similar to a *Stackelberg Leadership Solution*, where the governments are the Stackelberg leaders. Assumption (6.) defines a *repeated game* and (7.) means that the information structure is *open-loop*. If the capitalists can invest in a global environment it makes sense to assume that they have the same rate of time preference.

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23To formulate the distributional conflict between capitalists and workers in a closed economy as a dynamic game has, for example, been done in Lancaster (1973), Pohjola (1983), Basar, Haurie and Ricci (1985), Mehrling (1986), Haurie and Pohjola (1987), Shimonura (1991), de Zeeuw (1992), or Seierstad (1993).

24The justification for assuming this information structure may lie in the fact that democratic governments of either political leaning may constantly be reminded of their pre-election promises so that the outcome of the game in the first stage provides a benchmark for their decisions at time $t$. If the governments could remember the whole history of the game, time inconsistency issues would emerge. Modelling problems of time inconsistency and assuming appropriate trigger strategies for a *closed loop* information structure is beyond the scope of this paper. Thus it is implicitly assumed that governments commit themselves to their decisions. How this commitment is
From the assumptions about the game the following should be noted: Given the timing of moves and the assumption on the information structure the game is reduced to a two-stage game. First the public sector (Stackelberg leader) moves and then the private sector (Stackelberg follower) moves. For our game this means that given the optimal capital allocation decision of the capitalists (see equation (13)), that is, \( \omega \) and \( \omega^* \), the governments decide on the tax rates and redistribution. Given the tax rates and \( \lambda, \lambda^* \) the private sector decides on where to invest. Solving backwards in this way requires a domestic government to maximize (16) or (17) with respect to its instruments taking its opponent’s choices of \((\tau^*, \lambda^*)\) as given. I will consider each possible match between a right and a left-wing government, under the assumption that the economies are similar \((A = A^*)\) or significantly different \((A > A^*)\).

6.1 The Right-wing Government’s Problem

Consider the domestic right-wing government’s problem. From the closed economy solution we know that it will set \( \lambda = 0 \). Using (16) it has to solve

\[
\max_{V^r} \quad \text{s.t.} \quad \tau^* \text{ given, } \lambda = 0.
\]

I will suppress the time subscript \(0\) for \(C^k, k\) (cf. (16)) in what is to follow. Then the FOC from (16) involves

\[
\frac{C^k_\tau}{C^k \rho} + \frac{\gamma_\tau}{\rho^2} = 0. \tag{21}
\]

Note that \(C^k = \rho k\) in steady state so that \(C^k_\tau = 0\). For the growth rate I find

\[
\gamma_\tau = (r - \tau)\omega_\tau + (r_\tau - 1)\omega + (r^* - \tau^*)\phi_\tau.
\]

Let us define \(\Sigma_1 \equiv \frac{r - r_\tau}{r - \tau}, \Omega^1_\tau = \frac{r - 1}{r - r_\tau} \) and \(\Omega^2_\tau = \frac{r^* - 1}{r^* - \tau^*}\). Then from (13) one gets

\[
\omega_\tau = z\Sigma^1_1 \Omega^1_\tau, \quad \phi_\tau = -z\Sigma^2_1 \Omega^2_\tau.
\]

References for dynamic games are e.g. Petit (1990) and Basar and Olsder (1995).
The domestic capitalists’ FOC then directly implies \( \phi(r^* - r^*) + \omega_r(r - r^*) = 0 \) and so \( \gamma_r = (r_r - 1) \omega \). Similar definitions for \( \lambda \), i.e. \( \Omega^1, \Omega^2, \), establish that \( \gamma_\lambda = r_\lambda \omega \). So the solution to (21) is given by \( \tau = \hat{\tau} = [\alpha(1 - \alpha)A]^{1/2} \) and is identical to the solution of the right-wing government’s problem in a closed economy. Note that the optimal tax rate is independent of the degree of capital mobility \( (z) \), the efficiency in the foreign country \( A^* \) and the opponent government’s tax choice.

**Proposition 1** For any \( A, A^*, z \) and \( r^* \) a domestic right-wing government will always choose \( \tau = \hat{\tau} = [\alpha(1 - \alpha)A]^{1/2} \), i.e. the growth maximizing tax rate, irrespective of who its opponent is.

This means that the best thing the right-wing governments can do is to pursue their national GNP growth maximizing policies. It also implies that in the optimum the right-wing governments have a completely fixed reaction function since \( \hat{\tau} \) is independent of any foreign policy instruments.

### 6.2 The Left-wing Government’s Problem

Given Nash behaviour the domestic left-wing government has to take the policy choices of its opponent as given. Then the left-wing government’s problem is given by

\[
\max_{\tau, \lambda} \int_0^\infty \ln C_t^W e^{-\nu t} dt \\
\text{s.t. } C_t^W = (\eta(\tau, \lambda) + \lambda \tau) K_t; \ K_t = \omega k_0 e^{\gamma t} + \phi k_0^* e^{\gamma^* t}; \ 
\lambda \geq 0; \ \tau^*, \lambda^* \text{ given.}
\]  

(22)

This problem is not easily solved in general unless restrictions are put on how the opponent behaves or one rules out certain consumption paths. I will state these restrictions as I go along.

### 6.3 Left-wing vs. Right-wing

In this section I analyze the domestic left-wing government’s problem if it has to compete in tax instruments with a foreign right-wing government.
Assume that the economies are similar, that is, \( A = A^* \). From Proposition 1 we know that the right-wing government will always set \( \tau = \hat{\tau} \). Then under common knowledge the left-wing government's problem in (22) is equivalent to

\[
\max_{\tau, \lambda} V^l \quad \text{s.t.} \quad \tau^* = \hat{\tau}^*, \quad \lambda \geq 0
\]

where \( V^l \) is given by (17). Since the foreign right-wing government always sets \( \tau^* = \hat{\tau}^* \), and given \( A = A^* \), the domestic left-wing government cannot guarantee a higher after-tax return than the foreign right-wing government. But then it must be that \( \omega^* = 1 \) and so \( \phi^*(\omega^*) = 0 \) and \( \Gamma_t = \gamma \). Thus, the GDP and GNP growth rate will be the same for the domestic left-wing government and it will not be able to attract any foreign capital. One may then verify that the FOC is given by\(^{25}\)

\[
V^l_\tau = \frac{C^W_\tau}{\rho C^W_0} + \frac{\gamma_\tau}{\rho^2} = 0, \quad (23)
\]

\[
V^l_\lambda = \lambda \left( \frac{C^W_\lambda}{\rho C^W_0} + \frac{\gamma_\lambda}{\rho^2} \right) = 0. \quad (24)
\]

As in the closed economy the left-wing government wants to set \( \tau \geq \hat{\tau} \) for either maximization of wages or redistribution. But any tax rate \( \tau > \hat{\tau} \) makes \( \gamma_\tau \) in (23) negative since \( \tau_\tau < 1 \) for \( \tau > \hat{\tau} \). Recall that \( C^W_0 = (\eta + \lambda r)K_0 \) and notice that

\[
\Gamma_{|\phi^*=0} = \gamma = (r - \tau)\omega + (\tau^* - \hat{\tau}^*)\phi(\omega) - \rho
\]

since \( \omega < 1 \) as \( \tau^* - \hat{\tau}^* \geq r - \tau \). From this I get that

\[
\gamma_\tau = (r_\tau - 1)\omega + (r - \tau)\omega_\tau + (\tau^* - \hat{\tau}^*)\phi_\tau. \quad (25)
\]

Let \( \Sigma_1 \equiv \frac{r - r_\tau}{r_\tau - r}, \quad \Omega^1_\tau = \frac{r_\tau - 1}{r_\tau - r}, \quad \Omega^2_\tau = \frac{r - r_\tau}{r_\tau - r}, \quad \Omega^1_\lambda = \frac{r_\lambda}{r_\tau - r}, \quad \Omega^2_\lambda = \frac{-r_\lambda}{r_\tau - r}. \) Then

\[
\omega_\tau = z \Sigma^1_1 \Omega^1_\tau, \quad \phi_\tau = -z \Sigma^1_1 \Omega^2_\tau, \quad \omega_\lambda = z \Sigma^2_1 \Omega^1_\lambda, \quad \phi_\lambda = -z \Sigma^2_1 \Omega^2_\lambda. \quad (26)
\]

Some algebra reveals that \( \phi_\tau(\tau^* - \hat{\tau}^*) + (r - \tau)\omega_\tau = 0 \). Hence \( \gamma_\tau = (r_\tau - 1)\omega \) and analogously \( \gamma_\lambda = r_\lambda \omega \). Substituting the expressions

\(^{25}\)The \( \lambda(\cdot) \) expression enters because of complementary slackness for \( \lambda \geq 0 \).
above in (23) and (24) yields for an interior solution

\begin{align}
\frac{\eta_\tau + \lambda}{\eta + \lambda \tau} + \frac{\omega_\tau}{\omega} &= - \frac{r_\tau - 1}{\rho} \omega, \\
\frac{\eta_\lambda + \tau}{\eta + \lambda \tau} + \frac{\omega_\lambda}{\omega} &= - \frac{r_\lambda}{\rho} \omega.
\end{align}

(27)

(28)

Multiplying these equations by the inverses of \(r_\tau - 1\) and \(r_\lambda\) resp., setting the resulting equations equal, some rearranging and noting that \(r_\tau \eta_\lambda = r_\lambda \eta_\tau\) establishes\(^{26}\)

\[\tau = \left[\frac{1}{\rho} \frac{(1 - \alpha) A}{\beta}\right]^{\frac{1}{\alpha}}.\]

(29)

This is the same result as in the closed economy case. So again \(\tau > \hat{\tau}\).

Rearrange (27) to obtain

\[\frac{\eta_\tau + \lambda}{\eta + \lambda \tau} \left[\frac{z}{r - \tau} + \frac{\omega}{\rho}\right]^{-1} = - (r_\tau - 1).\]

(30)

Letting \(\tau \to \hat{\tau}^+\) the expression \(r_\tau - 1\) will be negative and so the RHS positive. From this one immediately gets that \(\tau \to \hat{\tau}\) if \(z \to \infty\). Turning to redistribution I rearrange (28), use (29) and simplify in order to check

\(^{26}\)To see that this is true let \(a_1 \equiv \frac{\eta_\tau + \lambda}{\eta + \lambda \tau}\) and \(a_2 \equiv \frac{\eta_\lambda + \tau}{\eta + \lambda \tau}\). Then

\[(27): \frac{a_1}{r_\tau - 1} + \frac{\omega_r}{\omega(r_\tau - 1)} = - \frac{1}{\rho} \omega, \quad (28): \frac{a_2}{r_\lambda} + \frac{\omega_\lambda}{\omega r_\lambda} = - \frac{1}{\rho} \omega.\]

Thus we get \(\frac{a_1}{r_\tau - 1} + \frac{\omega_r}{\omega(r_\tau - 1)} = \frac{a_2}{r_\lambda} + \frac{\omega_\lambda}{\omega r_\lambda}\). The definitions of \(\Omega_\tau^1, \Omega_\lambda^1\) imply \(\frac{\omega_r}{\omega(r_\tau - 1)} = \frac{\omega_\lambda}{\omega r_\lambda}\). So we are left with \(\frac{a_1}{r_\tau - 1} = \frac{a_2}{r_\lambda}\). Multiplication of both sides by \((\eta + \lambda \tau)\) yields \((\eta_\tau + \lambda)r_\lambda = (\eta_\lambda + \tau)(r_\tau - 1)\). From (7) and (8) we have \(r_\tau \eta_\lambda = r_\lambda \eta_\tau\) so that \(\eta_\lambda + \lambda r_\lambda = \tau r_\tau - \tau\) which is identical to equation (B3) in appendix B. Solving yields (29).
whether $V_\lambda' > 0$ evaluated at $\lambda = 0$. The condition for this is\footnote{Again let $a_2 \equiv \frac{a_{s+\gamma}^{-\lambda}}{y_{s+\lambda}}$. Using (26) it is true for (28) that $a_2 > \frac{1}{\alpha} \left( \frac{\omega_1 + r_1 \omega_1}{\rho_1 \omega_1} \right) = -r_1 \left( \frac{\omega_1 + r_1 \omega_1}{\rho_1 \omega_1} \right)$ and $\frac{a_2}{r_1} = -r_1 \left( \frac{\omega_1 + r_1 \omega_1}{\rho_1 \omega_1} \right)$. Evaluating at $\lambda = 0$ implies $\frac{a_2}{r_1} = \frac{\omega_1 + r_1 \omega_1}{\rho_1 \omega_1}$. \textit{Recall} $E \equiv (1 - \alpha).A[(1 - \lambda)\lambda]^{-\alpha}$ and $\eta = E\tau, r_1 = -\sigma E, \eta_1 = -\sigma E$ from (6), (7), and (8). At $\lambda = 0$ this implies $a_2 = \frac{\tau r(1 - \alpha)}{E \eta E \sigma}. \text{Now substitute for } E$ from (29). $\lambda = 0$ implies $E = 1$ and so $a_2 = \frac{\tau r(1 - \alpha)}{E \eta E \sigma}$. Thus, for $V_\lambda' > 0$ we must have (31).} 

\begin{align*}
(1 - \alpha)A^{-\frac{1}{\alpha}} > \left[ \frac{z}{\rho - \tau} + \frac{\omega}{\rho} \right]
\end{align*}

(31)

The LHS is clearly positive. Suppose the workers are impatient\footnote{Note that extreme patience $\rho \to 0$, too, causes the left-wing government to mimic a right wing one's policy. This is similar to the closed economy case.} ($\rho$ large) and $z \to 0$ (near autarky) then it is clearly possible that $V_\lambda' > 0$. However, since $z \in [0, \infty)$ there must be a $z^0$, where $V_\lambda' \leq 0$. Thus, there exists a $z > z^0$ where $\lambda = 0$. Refer to the tax rate chosen for $z \in [0, z^0]$ as $r_0^0$ and the one chosen for $z > z^0$ as $r_1$. Recall that $\Gamma_1 = \gamma$ for $\phi^* = 0$. Identify the GDP growth rate, $\Gamma$, associated with $r_0$ as $r_0$ and the one chosen for $r_1$ as $r_1$. Recall that $r_0$ is increasing over time and in the limit equal to $r_1$. But then $r_0 < r_1$ for $r_1$ and $r_0$ and all $t$. From the argument about $z \to \infty$ if follows that $r = \hat{r}$. But any optimal $r$ will be a function of $z$. With $r = \hat{r}$ as $z \to \infty$ it may happen that in the limit $\omega = 0$ and so $U^W \to -\infty$ which cannot be optimal. Thus, I conclude that if $z \to \infty$ a domestic left-wing government will definitely set $r = \hat{r}$ in the optimum. For this case define $\omega_2$ and $\Gamma^2$. Then $\omega_2 = 1$ and $\Gamma^2 = \Gamma^*$. Consequently $\Gamma^0 < \Gamma^1 < \Gamma^2$ where $\Gamma^2 = \Gamma^*$. From these arguments I obtain\footnote{Notice that the proposition establishes that there will be steady state growth in the domestic economy, which may not be the case for the foreign right-wing government's economy. $\Gamma^*$ and so GDP increase over time. I will interpret this as being...}
Proposition 2 For left-wing, right-wing competition and \( A = A^* \) we have

1. For low degrees of capital mobility, \( z \in [0, z^0] \) the domestic left-wing government sets \( \tau > \tilde{\tau} \) and \( \lambda > 0 \). Then \( \omega^0 < \omega^1 \) and \( \Gamma^0 < \Gamma^1 \).

2. If \( z > z^0 \), there will be no redistribution, \( \lambda = 0 \). The government just maximizes wages. Then \( \omega^0 < \omega^1 < 1 \) and \( \Gamma^1 < \Gamma^*_t \).

3. If \( z \to \infty \), i.e. capital mobility is very perfect, the left-wing government will mimic the right-wing government and will choose \( \tau = \tilde{\tau} \). Then \( \omega^2 = 1 \) and \( \Gamma^2 = \gamma = \Gamma^*_t \) and constant.

4. \( \Gamma^0 < \Gamma^1 < \Gamma^2 \) where \( \Gamma^2 = \Gamma^*_t \).

Comparing the solutions with \( \lambda = 0 \) for the left-wing government in the closed economy, (18), and the open economy case, (30), I find that wages will be lower when opening the economies up. Define \( \tau_{\lambda=0,\omega=1} \) as the tax rate that the left-wing government sets in the closed economy and \( \tau_{\lambda=0,\omega\leq1} \) as the one in the two-country world. Then, if \( z > 0 \), one will observe \( \tau_{\lambda=0,\omega=1} > \tau_{\lambda=0,\omega\leq1} \) so that wage maximization by a domestic left-wing government may be adversely affected by a high degree of capital mobility in the optimum. I will express this as

Corollary 1 For a domestic left-wing government facing a right-wing foreign government we have that \( \tau_{\lambda=0,\omega=1} \geq \tau_{\lambda=0,\omega\leq1} \). Thus, wages will be lower in an open than in a closed economy.

Two important features of Proposition 2 merit attention. First, left-wing governments will not redistribute in equilibrium if capital mobility is high. The reason for this is that the effects of the concern for inequality is competed away by fear of losing capital. Capital is good to left-wing really good for the foreign workers. However, the main focus of this section is on the optimal behaviour of the domestic government given the optimal behaviour of the foreign government.
governments for redistributive reasons and for wages. Facing tax competition the left-wing government is better off if it puts more emphasis on securing high wages instead of redistribution, if capital mobility is high. Also, with perfect capital mobility the after-tax returns are equal across countries, capital is in fact indifferent where to go.\textsuperscript{30} This is so since in a situation where both countries are equally efficient \textit{and} perfect capital mobility prevails both governments will optimally act as a right-wing government would by setting the GNP growth maximizing tax rates.

Second, since the right-wing government chooses a rather inflexible tax profile, the left-wing government can take this into account and will choose to redistribute if capital mobility is not high. This appears to be due to a lack of strategic interaction. The left-wing government knows that it cannot influence the choice of the right-wing government and that is cannot attract foreign capital. Its best response is to take the optimal choice of the right-wing government as given and then solve its tax, redistribution problem.\textsuperscript{31}

6.4 Left-wing vs. Left-wing

Now the domestic left-wing government's problem is to choose tax instruments when facing a foreign left-wing government. Again, assume $A = A^*$. Since capital is good for left-wing governments the best the domestic left-wing government can do is to find optimal policies for given tax choices of its left-wing opponent. Then the domestic left-wing government's problem in (22) is given by

$$\max_{\tau, \lambda} V^{t} \quad s.t. \quad \tau^{*} \text{ given, } \lambda \geq 0.$$ 

\textsuperscript{30}If $z \to \infty$ the solution to the capitalists' problem in (11b) takes a 'bang-bang' form. That case is analyzed in Rehme (1995b).

\textsuperscript{31}Note that this is still an outcome of a game. The right-wing government uses the $\omega$ reaction function of the second stage of the game. So even though the game collapses in the first stage (fixed reaction), the optimal tax choice of the right-wing government is still the result of a sequential (two-stage) game.
Maximization involves setting the derivatives $\frac{\partial V}{\partial \tau}$ and $\frac{\partial V}{\partial \lambda}$ equal to zero for finding the optimum. I will now restrict the analysis to steady state paths with $\gamma = \gamma^*$. So $\tau, \lambda$ must solve $\left(\frac{\partial V}{\partial j}\right)_{\gamma = \gamma^*} = 0$ where $j = \tau, \lambda$. In appendix C it is shown that under the steady state restriction, $\gamma = \gamma^*$, the following FOC must be satisfied

$$\tau : \frac{\eta + \lambda}{\eta + \lambda \tau} + \frac{\omega_r k_0 + \phi^* r_k}{\delta_0} + \frac{\gamma_r \nu + \gamma_r^* \nu^*}{\rho} = 0 \quad (32)$$

$$\lambda : \frac{\lambda}{\eta + \lambda \tau} + \frac{\omega_\lambda k_0 + \phi^* r_k}{\delta_0} + \frac{\gamma_\lambda \nu + \gamma_\lambda^* \nu^*}{\rho} = 0 \quad (33)$$

where $\nu = \frac{\omega _c}{\delta_0}$, $\nu^* = \frac{\phi^* r_k}{\delta_0}$ and the $\lambda()$ expression enters because of complementary slackness for $\lambda \geq 0$. The first two expressions on the LHS in (32) and (33) represent the effects of changes in the wage rate. The third expression shows how the growth rate, weighted by its contribution to the overall capital stocks, reacts to changes in policy. Notice that the steady state assumption implies that $\Gamma = \gamma = \gamma^*$ so that in an equilibrium GDP and GNP grow at the same rate.

From above (see equation (26)) we know that $\gamma_r = (r_r - 1) \omega$. For evaluation of $\gamma_r^*$ let $\Sigma_1 \equiv \frac{r_r - r}{r - r}, \Sigma_2 \equiv \frac{r_r - r}{r - r}, \Omega_r \equiv \frac{r_r - 1}{r - r}$ and $\Omega_\lambda \equiv \frac{r_r - 1}{r - r}$. Then

$$\omega_r^* = -z \Sigma_2 \Omega_r^1, \quad \phi^*_r = z \Sigma_2^{-1} \Omega_r^1, \quad \phi^*_\lambda = z \Sigma_2^{-1} \Omega_\lambda^1. \quad (34)$$

and $\omega_r^*(r_r - r^*) + \phi^*_r(r - r) = 0$ so that

$$\nu \gamma_r = \nu (r_r - 1) \omega \quad \text{and} \quad \nu^* \gamma_r^* = \nu^* (r_r - 1) \phi^*. \quad (35)$$

Let $a_1 \equiv \frac{\eta + \lambda}{\eta + \lambda \tau}, a_2 \equiv \frac{\eta + \lambda}{\eta + \lambda r}, b_1 \equiv \frac{\omega r k_0 + \phi^* r_k}{\delta_0}$ and $b_2 \equiv \frac{\omega_\lambda k_0 + \phi^* r_k}{\delta_0}$. Assume an interior solution for $\lambda$ exists. From (32) and (33) it is true that

$$\tau : \frac{a_1 + b_1}{r - 1} = -\frac{\nu \omega + \nu^* \phi^*}{\rho} \quad \text{and} \quad \lambda : \frac{a_2 + b_2}{r \lambda} = -\frac{\nu \omega + \nu^* \phi^*}{\rho}.$$ 

Setting these equations equal yields

$$(a_1 + b_1) r_{\lambda} = (a_2 + b_2) (r_r - 1).$$
It is not difficult, but cumbersome to verify that \( b_1 r_\lambda = b_2 (r_\tau - 1) \).\footnote{The equality may be verified using (26) and (34)}

Thus, we are left with expressions that yield the same result as in the closed economy case, i.e. \( \tau = \frac{[(1-\alpha)\lambda]^{\frac{1}{\lambda}}}{1-\lambda} \). (See footnote 26.) Since I need this result later on I make it

**Lemma 5** Under left-right or left-left competition, for a \((\tau, \lambda)\) solution with \( \lambda \geq 0 \) the condition \( \tau = \frac{[(1-\alpha)\lambda]^{\frac{1}{\lambda}}}{1-\lambda} \) has to be satisfied.

Given everything else two left-wing governments would like to set \( \tilde{r} > \hat{r} \). But higher taxes mean that \( b_1 \) is definitely negative since \( r_\tau < 1 \) for \( \tau > \hat{r} \). So it follows that taxes, and hence wages, must be smaller in the open economy than in the closed one. (Cf. Corollary 1 in the previous section.) Notice that

\[
\begin{align*}
\frac{b_1}{K_0} &= \frac{\omega_r k_0 + \phi_r^* k_0^*}{K_0} \\
&= \frac{z}{r-\tau} \left( \frac{r-\tau}{r-\tau^*} \right) \frac{k_0}{K_0} + \frac{z}{r-\tau} \left( \frac{r^*-\tau^*}{r-\tau} \right) \frac{k_0^*}{K_0} \\
&= \frac{z}{r-\tau} \left( \frac{r-\tau}{r-\tau^*} \right) \frac{k_0^*}{K_0} + \frac{z}{r-\tau} \left( \frac{r^*-\tau^*}{r-\tau} \right) \frac{k_0^*}{K_0}
\end{align*}
\]

so that we can rearrange equation (32) as follows

\[
\begin{align*}
\frac{\alpha_1 \left[ \frac{z}{r-\tau} \left( \frac{r-\tau}{r-\tau^*} \right) \frac{k_0}{K_0} + \left( \frac{r^*-\tau^*}{r-\tau} \right) \frac{k_0^*}{K_0} \right]^{-1}}{\frac{\nu w + \nu^* \phi^*}{\rho}} &= -(r_\tau - 1)
\end{align*}
\]

Let \( \tau^0 \) solve this equation. If \( k_0 = k_0^* \), our problem is completely symmetric in terms of strategy spaces (action sets), agents etc. Since the strategies are continuous variables and symmetric we are in fact contemplating a symmetric game. One can then apply the following theorem (see Lemma 6 in Dasgupta and Maskin (1986) or Theorem 5.10 in Rasmusen (1989), p.127, presented here)
Theorem 1 (Symmetric Equilibrium Theorem) Every symmetric game that has an equilibrium has a symmetric equilibrium; that is, every game in which players' actions sets are identical at each point in time has an equilibrium in which mixing probabilities (perhaps equal to one) are identical.

The theorem establishes that if the game has any Nash Equilibria at least one of them must be symmetric. For what is to follow I will only consider symmetric equilibria. (In appendix D I show that under the assumption $\gamma = \gamma^*$ all Nash Equilibria of the game are indeed unique and symmetric.) Symmetry entails $T^0 = T^* = T^0$ with $T > T^*$. Then it must be that $r - T = r^* - T^* \leq \hat{r} - \hat{r}^*$ so that $\omega = 1, \phi^* = 0, (\omega^* = 1)$ and $K_0 = k_0$. This modifies $b_1$ and leaves $a_1$ as it is. Now $T_0^0$ would have to solve

$$V'_T|_{r^0=r^*} = a_1 + b'_1 + c'_1$$

$$= \frac{\eta + \lambda}{\eta + \lambda r} + z \left(\frac{r^* - 1}{r - T} \left(\frac{k + k^*}{k}\right) + \frac{r^* - 1}{\rho}\right) = 0.$$ 

Now rearrange this equation to obtain

$$\frac{\eta + \lambda}{\eta + \lambda r} \left[\frac{z}{r - T} \left(\frac{k + k^*}{k}\right) + \frac{1}{\rho}\right]^{-1} = -(r^* - 1).$$ 

(37)

If $z \to \infty$ then $r^0 \to \hat{r}$ and if $z \to 0$ then $r^0 \to \hat{r}$, where $\hat{r}$ is the left-wing government's tax choice in the closed economy. This last result already tells us that for low enough capital mobility $\lambda > 0$. Proceeding as before I impose symmetry on (33), rearrange it, use (29), simplify and check whether $V'_T > 0$ evaluated at $\lambda = 0$. Then the condition for $V'_T > 0$

33 Under the assumption of $\gamma = \gamma^*$, that is, balanced growth, I show that the after-tax returns must be equal and that this together with the FOC in (32) and (33) implies uniqueness and symmetry. An alternative procedure would have been to say at the beginning of this section that with $k_0 = k_0^*$ the game is symmetric and that I only consider symmetric equilibria. The justification for proceeding as above is that requiring $\gamma = \gamma^*$ restricts the analysis to balanced growth paths and requires equal after-tax returns, which does not automatically require symmetry or uniqueness in tax parameters. So the adopted route is slightly more general.
becomes

$$[(1 - \alpha)A]^\frac{1}{\rho} > \left[\left(\frac{z}{r - \tau}\right)\left(\frac{k + k^*}{k}\right) + \frac{1}{\rho}\right].$$  \hspace{1cm} (38)$$

The derivation is identical to the one presented in footnote 27 for the condition \( \lambda > 0 \) under left-right competition. It seems clear that as \( z \) becomes infinite, \( \lambda \) will definitely be set equal to zero and with \( z \to 0 \) one may get a positive \( \lambda \). Furthermore, substituting (29) in (31) and solving for \( z \) establishes that there must be a \( z > 0 \), call it \( z^3 \), where \( V^*_l < 0 \) and so \( \lambda = 0 \). Given the symmetry of the problem, it follows that both left-wing governments set the same tax rate so that \( \omega = \omega^* = 1 \) and \( \Gamma = \Gamma^* \). Hence,

**Proposition 3** If \( A = A^* \) and two left-wing governments compete in taxes then

1. For \( z > z^3 > 0 \) there will no redistribution \( \lambda = \lambda^* = 0 \).
2. The governments set the same tax rates so that \( \omega = \omega^* \) and \( \Gamma = \Gamma^* \).
3. If \( z \to \infty \), then \( \tau \to \hat{\tau} \) and the left-wing governments act like right-wing ones.

### 6.5 Comparison of Left-Right with Left-Left Tax Competition

Suppose we consider a domestic left-wing government that faces either another foreign left-wing government or a foreign right-wing government. Comparing the conditions for redistribution, i.e. (38) of section 6.3 and (31) in 6.4, I find that the condition for redistribution is weaker if the domestic government faces a foreign right-wing government. To see this consider the RHS of (38) and the RHS of (31). Notice that the LHS’s of (38) and (31) are equal. Then for any \( z > 0 \) one obtains by comparison of the RHS’s that

$$\left[\left(\frac{z}{r - \tau}\right)\left(\frac{k + k^*}{k}\right) + \frac{1}{\rho}\right] > \left[\frac{z}{r - \tau} + \frac{\omega}{\rho}\right].$$
I conclude from this that the marginal utility of redistribution is reduced if a left-wing government competes in taxes with another left-wing government.

One may check that exactly the same condition applies for a comparison of the optimal tax choices, namely (30) and (37). So I conclude that a domestic left-wing government will set lower taxes when competing with a foreign left-wing government than when confronting a right-wing government. The domestic left-wing government will only choose the same tax rates if \( z = 0 \) or \( z \rightarrow \infty \). Let us call the domestic left-wing government’s tax choice \( \tau^4 \) when it faces a foreign right-wing opponent and \( \tau^5 \) when confronted with a foreign left-wing government. Similarly, define \( \Gamma^4 \), \( \Gamma^5 \), \( \lambda^4 \) and \( \lambda^5 \). Then the results may be stated as

**Proposition 4** If \( A = A^* \) and the domestic left-wing government faces either a foreign right-wing or a foreign left-wing government, then for any \( z \in [0, \infty) \) we have \( \tau^5 \geq \tau^4 \), \( \lambda^4 \leq \lambda^5 \) and \( \Gamma^5 \leq \Gamma^4 \).

Thus, the proposition establishes that in two equally efficient economies a domestic left-wing government competing in tax parameters will redistribute at least as much when facing a right-wing as when facing a left-wing government. Also, it taxes wealth at least as much when facing a right-wing as when facing a left-wing government. Thus, for a left-wing government it matters a lot who it competes with in taxes. The comparison between the two possible opponents suggests that we may observe more GDP growth at the expense of less redistribution if the domestic left-wing government faces a foreign left-wing government. Alternatively, one may observe more redistribution and less GDP growth if the domestic left-wing government competes with a foreign right-wing government.

The reason for this result lies in the fact that in this model the right-wing government guarantees that \( \omega^* = 1 \) so that the left-wing government has no chance to attract foreign capital. The strategic interaction between two left-wing governments is more intense because each government may get some capital off its left-wing opponent. This is in
conflict with more redistribution (higher \( \lambda \)), but may be worthwhile in terms of wages.

If \( z \to 0 \) the strategic element in finding the optimal tax rate becomes less important. Then it makes sense that a left-wing government sets the same taxes as in the closed economy. On the other hand, if capital mobility is nearly perfect, \( z \to \infty \), the left-wing government optimally mimics a right-wing one for fear of losing capital.

6.6 Different Technological Efficiency

The results so far only apply under the assumption of equal efficiency. In contrast, suppose now that the domestic government has a more efficient technology so that \( \bar{A} > A^* \). For the closed economy we know from Lemma 3 that \( \tilde{\tau} > \bar{\tau}^* \) and \( \tilde{\tau} - \tilde{\tau} > \bar{\tau}^* - \bar{\tau}^* \) if \( \bar{A} > A^* \). Proposition 1 tells us that two right-wing governments will always choose national GNP growth maximizing policies. Then it is clear that the right-wing government with a more efficient economy will get more capital and experience higher GDP growth.

The other constellations of the game, i.e. left-right and right-right competition, are not easily analyzed. I will only make a few observations on the nature of possible equilibria in this section. Take the left-wing government. Before imposing symmetry I have shown in section 6.4 for a domestic left-wing government that the optimal choice of \( \tau \) is obtained by solving equation (36), which I present again for convenience

\[
a_1 \left[ \frac{z}{r - \tau} \left( \frac{r - \tau}{r^* - \tau^*} \right)^z \frac{k_0}{K_0} + \left( \frac{r^* - \tau^*}{r - \tau} \right)^{z+1} \frac{k_0^*}{K_0} \right] + \frac{\nu \omega + \nu^* \phi^*}{\rho}
\]

\[
= -(r - 1)
\]

where \( a_1 = \frac{\nu \omega + \lambda \phi}{\eta^* + \lambda^* \eta} \). For any foreign vs. domestic after-tax return combinations and \( A - A^* = \epsilon \) with \( \epsilon \) very small notice that as \( z \to \infty \) we have that \( \tau \to \tilde{\tau} \) and \( \lambda = 0 \). So I conclude that for slight efficiency differences and very high degrees of capital mobility, the domestic, more efficient economy with a left-wing government has higher GDP growth than its right-wing opponent.
Proposition 5 If $z \to \infty$ and $A - A^* = \epsilon$, $\epsilon > 0$ and small, a domestic left-wing government will have higher GDP growth than a foreign right-wing government, $\Gamma > \Gamma^* = 0$.

It is interesting to note that a situation where $A > A^*$ and $z \to \infty$ is really bad for the workers in the inefficient, foreign country, because as $z \to \infty$ we may get $\omega^* = 0, \omega = 1$ and $(\eta^* + \lambda^* \tau^*) K^* = 0$ and so $U(CW^*) \to -\infty$. This means that it is really crucial for the workers under a left-wing government to have an efficient economy if capital mobility is very high. Since the left-wing government only represents domestic workers in this model (a form of left-wing nationalism), the policy of a domestic left-wing government with an efficient economy may cause the foreign workers to starve in the inefficient economy.

Next, notice that for very high differences in $(A, A^*)$ it may well be possible that a domestic left-wing government redistributes wealth, grows more than and gets at least as much capital as its right-wing counterpart, even if capital mobility is very low. Suppose $z \approx 0$, then one gets approximately the same solution as for the closed economy case. From Lemma 5 we know that the solution involves $\tau = \frac{[(1 - \alpha)A]^\frac{1}{\alpha}}{1 - \lambda}$. Lemma 4 tells us that for positive GNP growth and wealth redistribution we must have that private capital is more important than public inputs, that is, $\alpha > \frac{2}{3}$, which I assume to hold now. I will now check whether it is possible for a domestic left-wing government to have $\gamma \geq \gamma^*$ with $\lambda > 0$, if the foreign government is right-wing with an inefficient economy. Now $\gamma \geq \gamma^*$ and $A > A^*$ involves

$$r - r - \rho \geq r^* - r^* - \rho \iff r - r \geq \frac{\alpha}{1 - \alpha} r^*$$

(see Lemma 3).

Notice that $r = \frac{\alpha}{1 - \alpha} [(1 - \alpha)A]^\frac{1}{\alpha}$ under Lemma 5 so that for $\gamma \geq \gamma^*$ we must have

$$\frac{\alpha}{1 - \alpha} [(1 - \alpha)A]^\frac{1}{\alpha} - \alpha(1 - \alpha)A^*]^\frac{1}{\alpha} \geq r.$$

For redistribution ($\lambda > 0$) we need $\tau > [(1 - \alpha)A]^\frac{1}{\alpha}$. Now if $P \geq x$ and $x > Q$ then $Px > Qx$ so that

$$\frac{\alpha}{1 - \alpha} [(1 - \alpha)A]^\frac{1}{\alpha} - \alpha(1 - \alpha)A^*]^\frac{1}{\alpha} \tau > \tau [(1 - \alpha)A]^\frac{1}{\alpha}$$
Assuming Lemma 4 holds we have $\alpha > 2 \frac{2}{3}$ which means that the LHS is smaller than 0.946. If we let $A = x A^*$, $x > 1$, the inequality holds if $x > 1.056$ so that in this model an efficiency advantage of, say, 6 percent in a world of very low capital mobility ($z \approx 0$) is enough for a left-wing government to be able to redistribute wealth and grow at least at the same rate as its right-wing opponent’s economy.

**Proposition 6** If $z \approx 0$ and $\alpha > \frac{2}{3}$, then for some $A > A^*$ a domestic left-wing government may redistribute and grow at least at the same rate as its right-wing opponent’s economy, that is, $\gamma \geq \gamma^*$ and $\lambda > 0$ and $\Gamma \geq \Gamma^*$.

For all other degrees of capital mobility exact solutions are difficult to obtain. One may conjecture that there should always be $(z, A)$ combinations that guarantee that a domestic government redistributes wealth and has higher GDP growth so that the solutions should be somewhere between those found in Propositions 5 and 6. From this I conclude that it is important for the workers, as well as the capital owners to live in an efficient economy, because this is good in terms of welfare and GDP growth and may mitigate the effects of tax competition in a non-cooperative environment.

## 7 Conclusion

Employing the framework of a simple growth model with distributional conflicts seems to imply that if one taxes wealth, the growth rate is reduced by redistribution. This is the argument presented for example in Alesina and Rodrik (1994) and Bertola (1993) and would suggest that redistribution always implies lower growth.
If one extends the growth redistribution trade-off problem to a two-country world with varying degrees of capital mobility and introduces non-cooperative behaviour, by which governments compete in wealth tax rates using the source principle, the possibility of losing capital features saliently in the optimal decisions of a government that wishes to redistribute.

It is shown that in a situation where the opponent’s economy is equally efficient and capital mobility is sufficiently high, no redistribution may take place in the optimum for two left-wing governments. This may hold even though both care about redistribution. The intuitive reason for this is that capital is good for left-wing governments. Losing capital reduces wages and the utility loss of a government incurred by a drop in wages outweighs the utility gain derived from redistribution. However, the workers are compensated for this by higher wages.

If a right-wing and a left-wing government compete in taxes, the right-wing government will optimally pursue its domestically preferred policy (GNP growth maximization in this model) and not be influenced by its opponent’s tax choice. From this the right-wing government’s reaction to any opponent’s tax choice is very unresponsive in this model. Since the right-wing government guarantees the maximum after-tax return for its capital owners when both economies are equally efficient, a competing left-wing government is unable to attract foreign capital. It is then optimal for the left-wing government to redistribute at least as much as it would when competing with another left-wing government. It is shown that as capital mobility increases, tax competition intensifies and the left-wing governments begin to mimic right-wing policies.

If the countries are technologically different, i.e. one country is more efficient than another one, then more capital will locate in the efficient country. If the efficient country wishes to redistribute, it can afford to do so at the expense of losing some capital. Hence, the amount of redistribution depends on who the opponent is and on the efficiency gap that distinguishes it from its opponents. From this last point one may conclude that policies that are geared to make an economy more efficient are
in the interest of both workers and capital owners. This holds especially true for workers with a left-wing government.

Finally, in the model one would observe left-wing governments to behave differently in the optimum when facing different opponents. If the opponent is left-wing (same preferences) it will choose higher GDP growth and higher wages at the cost of reduced redistribution. If it confronts a right-wing government it will redistribute at least as much at the expense of lower GDP growth. This result again hinges on the intensity of strategic interaction and the degree of capital mobility.

In this paper it is argued that high GDP growth and redistribution may be possible with a large enough efficiency gap or low enough capital mobility. Government preferences alone may then not adequately explain the pattern of redistribution and GDP growth in open economies with wealth tax competition, differences in strategic behaviour and varying degrees of factor mobility.

Several caveats apply. I have considered wealth taxes as a tax base. Other tax base choices may change the results in a two-country world considerably. (See, for instance Rehme (1995a).) I have abstracted from questions of time inconsistency. If governments could condition on the whole history of the game the outcome might well be different. I have not analyzed the effects of tariffs on capital flight. It is quite likely that a country that experiences capital outflows will set up tariffs. It would also be desirable to use a less aggregated set-up when investigating the trade-off problem. In reality workers own capital and some of the well capital endowed work. These and other problems provide room for further extensions of this model and for more research on the trade-off between growth and redistribution.
Appendix

A The left and right-wing welfare measures

By assumption \( \tau, \tau^* \) and so \( \omega, \omega^* \) and \( \gamma, \gamma^* \) are constant. For the right-wing government the welfare integral is given by \( V^r = \int_0^t \ln C_i^k e^{-pt} \). Let \( t \to \infty \) and use integration by parts. For this define \( v_2 = \ln C_i^k \), and \( dv_1 = e^{-pt}dt \). Recall that \( C_i^k = \rho k_t \). Then \( dv_2 = C_i^k / C_i^k = \gamma \) and constant in steady state, and \( v_1 = -1 / \rho \ e^{-pt} \). Then

\[
\int_0^\infty \ln C_i^k e^{-pt} dt = -\frac{1}{\rho} \left[ \ln C_i^k e^{-pt} \right]_0^\infty + \frac{1}{\rho} \int_0^\infty \gamma e^{-pt} dt
\]

\[= \frac{\ln C_0^k}{\rho} - \frac{1}{\rho^2} \left[ \gamma e^{-pt} \right]_0^\infty\] (A1)

where \( C_0 = \rho k_0 \). Evaluation at the particular limits yields the expression of \( V^r \) in (16).

The left-wing government’s welfare integral is given by \( V^l = \int_0^t \ln C_i^k e^{-pt} \). Now let \( t \to \infty \) and define \( v_2 = \ln C_i^W \), and \( dv_1 = e^{-pt}dt \). Recall \( C_i^W = (\eta + \lambda \tau)K_t \). Then \( dv_2 = C_i^W / C_i^W = \Gamma_t \), and \( v_1 = -1 / \rho \ e^{-pt} \). Thus,

\[
\int_0^\infty \ln C_i^W e^{-pt} dt = -\frac{1}{\rho} \left[ \ln C_i^W e^{-pt} \right]_0^\infty + \frac{1}{\rho} \int_0^\infty \Gamma_t e^{-pt} dt\] (A2)

which is equivalent to the expression for \( V^l \) in (17). From (15)

\[
\Gamma_t = \frac{\gamma \omega \gamma^t k_0 + \gamma^* \phi \gamma^* t k_0^*}{\omega e^{\gamma t} k_0 + \phi^* e^{\gamma^* t} k_0^*}
\]

which depends on time so that one would have to evaluate \( \int_0^\infty \Gamma_t e^{-pt} dt \) if one wanted to find a definite solution.

Next turn to the properties of \( V^l \) and \( V^r \) with respect to \( k_t, k_t^* \). First note that \( \frac{d \ln C_i^k}{\delta_j} \geq 0 \), for \( i = W, k \), \( j = k_t, k_t^* \) and \( \forall t \). For two
paths \( k_{1t} > k_{2t} \) for all \( t \), one has \( \ln C_{1t}^i > \ln C_{2t}^i \) for \( i = W, k \). But then welfare must also be higher, that is, \( \int_0^\infty \ln C_{1t}^i \text{e}^{-pt} dt > \int_0^\infty \ln C_{2t}^i \text{e}^{-pt} dt \).

Similarly, for \( k_{1t}^* > k_{2t}^* \) one obtains \( \int_0^\infty \ln C_{1t}^W \text{e}^{-pt} dt > \int_0^\infty \ln C_{2t}^W \text{e}^{-pt} dt \) and \( \int_0^\infty \ln C_{1t}^k \text{e}^{-pt} dt = \int_0^\infty \ln C_{2t}^k \text{e}^{-pt} dt \) since \( \frac{d\ln C_{1t}^k}{dk_{1t}^*} = 0 \). So increases in \( k_t \) raise the welfare of capitalists and workers and increases in \( k_t^* \) raise the workers’ welfare only. Since \( k_t = k_0 \text{e}^{\gamma t} \) and \( k_t^* = k_0^* \text{e}^{\gamma^* t} \) increases in \( k_0, k_0^* \) and in \( \gamma, \gamma^* \) increase \( k_t, k_t^* \) and hence the workers’ welfare. Increases in \( k_0, \gamma \) increase the capital owners’ welfare. Finally, since \( \frac{d\gamma}{dv}, \frac{d\gamma^*}{dv} \) where \( v = \tau, \tau^*, \lambda, \lambda^* \) exist, the welfare measures are continuous in the tax parameters.

\section{Derivation of the optimal solutions}

The government solves

\[ \max_{\tau, \lambda} (1 - \beta) V\tau + \beta V\lambda \quad \text{s.t.} \quad \lambda \geq 0 \]

The first order conditions for \( \tau \) and \( \lambda \) are given by

\[ \beta \frac{\eta_\tau + \lambda}{(\eta + \lambda \tau) \rho} + \frac{\gamma_\tau}{\rho^2} = 0, \quad \lambda \left( \beta \frac{\eta_\lambda + \tau}{(\eta + \lambda \tau) \rho} + \frac{\gamma_\lambda}{\rho^2} \right) = 0 \]

Concentrating on an interior solution for \( \lambda \), simplifying and rearranging one obtains

\[ \beta \frac{\eta_\tau + \lambda}{(\eta + \lambda \tau)} = -\frac{\gamma_\tau}{\rho}, \quad \beta \frac{\eta_\lambda + \tau}{(\eta + \lambda \tau)} = -\frac{\gamma_\lambda}{\rho} \]  

(B1)

Notice that \( \gamma_\tau \) must be negative for the first equation to hold, so in the optimum \( \tau > \hat{\tau} \) by Lemma 2. Division of these two equations by one another yields

\[ \frac{\eta_\tau + \lambda}{\eta_\lambda + \tau} = \frac{\gamma_\tau}{\gamma_\lambda} \]  

(B2)

and must hold in an optimum with \( \lambda > 0 \). Since \( \gamma_\lambda = r_\lambda \) and \( \gamma_\tau = r_\tau - 1 \) I get \( (\eta_\tau + \lambda)r_\lambda = (\eta_\lambda + \tau)(r_\tau - 1) \) which upon multiplying out becomes

\[ \eta_\tau r_\lambda + \lambda r_\lambda = r_\tau \eta_\lambda + r_\tau \tau - \eta_\lambda - \tau. \]  

(B3)
Notice that $r_{\lambda} \eta_r = r_{\tau} \eta_{\lambda}$ and that $\eta = \frac{1-\alpha}{\alpha} r$. Then $\lambda r_{\lambda} = r_{\tau} \tau - \frac{1-\alpha}{\alpha} r_{\lambda} - \tau$ and so

$$\left(\lambda + \frac{1-\alpha}{\alpha}\right) r_{\lambda} = r_{\tau} \tau - \tau \iff \left(\lambda + \frac{1-\alpha}{\alpha}\right) = \frac{r_{\tau} \tau - \tau}{r_{\lambda}}$$

Recall $r_{\tau} = \alpha E(1-\lambda), r_{\lambda} = \alpha E(-\tau)$ where $E = (1-\alpha)A[(1-\lambda)\tau]^{-\alpha}$ as in (7). Then $\frac{r_{\tau}}{r_{\lambda}} = -\frac{\alpha E(1-\lambda)}{\alpha E r} = -(1-\lambda)$. So for above

$$\lambda + (1-\lambda) + \frac{1-\alpha}{\alpha} = -\frac{\tau}{r_{\lambda}} \iff \frac{r_{\lambda}}{\alpha} = -\tau$$

which means that $E = 1$ and so

$$\tau = \frac{[(1-\alpha)A]^\frac{1}{\alpha}}{1-\lambda}. \quad (B4)$$

For the first order condition for $\tau$ with $E = 1$ note that $\eta = (1-\alpha)A[(1-\lambda)\tau]^{1-\alpha} = E[(1-\lambda)\tau] = [(1-\alpha)A]\frac{1}{\tau}$. Furthermore, $\eta_r = (1-\alpha)(1-\lambda), \, r_r = \alpha(1-\lambda)$. Then from (B4) we have $\lambda = 1 - \frac{[(1-\alpha)A]^\frac{1}{\tau}}{\tau}$ so that

$$\eta + \lambda \tau = [(1-\alpha)A]\frac{1}{\tau} + \tau \left(1 - \frac{[(1-\alpha)A]^\frac{1}{\tau}}{\tau}\right) = \tau.$$

Then the first order condition for $\tau$ becomes

$$\beta \frac{\eta_r + \lambda}{(\eta + \lambda \tau)} = -\frac{\gamma_r}{\rho} \iff \frac{\eta_r + \lambda}{\tau} = -\frac{\gamma_r}{\beta \rho} \iff \frac{\eta_r + \lambda}{\gamma_r} = -\frac{\tau}{\beta \rho}.$$ 

For an interior solution $\tau$ must solve this equation, but the solution must also satisfy (B2). Let $D \equiv \frac{\tau}{\beta \rho}$ and note that $\frac{\eta_r + \lambda}{\gamma_r} = -D = \frac{\eta_r + \lambda}{\gamma_r}$ and so $\frac{\gamma_r}{\eta_r + \lambda} = -1/D = \frac{\gamma_r}{\eta_r + \lambda}$ and hence $\frac{\eta_r + \lambda}{\gamma_r} = -D = \frac{\gamma_r}{\eta_r + \lambda} = -1/D$. Thus, $-D = -D, \, D = 1$ so that the solution satisfies $\tau = \beta \rho$. So one gets

$$\tau = \beta \rho \quad \text{and} \quad \lambda = 1 - \frac{[(1-\alpha)A]^\frac{1}{\tau}}{\beta \rho} \quad (B5)$$

which is equation (18). So $\lambda \geq 0$ only if $\beta \rho \geq [(1-\alpha)A]^\frac{1}{\tau}$. 

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Suppose \( \lambda = 0 \), then the first order condition for \( \tau \) becomes

\[
\frac{\eta_r}{\eta} = \frac{-r_r - 1}{\beta \rho} \iff \frac{(1 - \alpha)E}{\tau E} = \frac{-\alpha E - 1}{\beta \rho} \iff (1 - \alpha)\beta \rho = \tau - \alpha \tau E
\]

so that the solution with \( \lambda = 0 \) is given by

\[
(1 - \alpha)\beta \rho = \tau \left[ 1 - \alpha(1 - \alpha) A \tau^{-a} \right]
\]

which holds only if \( \beta \rho < \left(1 - \alpha\right)\frac{1}{A} \). For the right-wing (\( \beta = 0 \)) government the first order conditions are given by

\[
\frac{\gamma_r}{\rho^2} = 0, \quad \lambda \left( \frac{\gamma_\lambda}{\rho^2} \right) = 0.
\]

Since \( \gamma_\lambda = r_\lambda < 0 \) we have \( \lambda = 0 \) and because \( \gamma_r = r_r - 1 = 0 \) we have \( \tau = \hat{\tau} \).

**C The left-wing government’s problem**

Recall the left-wing government’s problem in (22), that is,

\[
\max_{\tau, \lambda} \int_0^\infty \ln C_i^{W} e^{-\rho t} dt
\]

\[
\text{s.t. } C_i^{W} = (\eta(\tau, \lambda) + \lambda \tau) K_t, \quad K_t = \omega k_0 e^{\gamma t} + \phi^* k^* e^{\gamma^* t}, \quad \lambda \geq 0.
\]

I look for constant policy parameters in the optimum. To this end employ the Leibniz Rule and differentiate through the integral

\[
\int_0^\infty \left( \frac{\partial C_i^{W}}{\partial \tau} \frac{1}{C_i^{W}} \right) e^{-\rho t} dt = 0, \quad \lambda \left( \int_0^\infty \left( \frac{\partial C_i^{W}}{\partial \lambda} \frac{1}{C_i^{W}} \right) e^{-\rho t} dt \right) = 0 \quad \text{(C1)}
\]

where the expression for \( \lambda \) enters because of complementary slackness.

The derivatives in the brackets are given by

\[
\frac{\partial C_i^{W}}{\partial \tau} \frac{1}{C_i^{W}} = \frac{(\eta_\tau + \lambda) K_t}{(\eta + \lambda \tau) K_t} + \frac{\partial K_t}{\partial \tau} \frac{1}{K_t}
\]

\[
\frac{\partial C_i^{W}}{\partial \lambda} \frac{1}{C_i^{W}} = \frac{(\eta_\lambda + \tau) K_t}{(\eta + \lambda \tau) K_t} + \frac{\partial K_t}{\partial \lambda} \frac{1}{K_t}.
\]

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Now let $\Delta_1 \equiv \frac{\eta + \lambda}{\eta + \lambda \tau}$ and $\Theta_1 \equiv \frac{\eta + \gamma}{\eta + \lambda}$ and notice that $\Delta_1, \Theta_1$ are constant. For the change in $K_t$ I obtain

$$\frac{\partial K_t}{\partial \tau} = \Delta_2 + \Delta_3 = \frac{(\omega_{\tau} + \gamma_{\tau} \omega t) e^{\gamma_{\tau} t} k_0}{\omega k_0 e^{\gamma_{\tau} t} + \phi^* e^{\gamma_{\tau} t} k_0^*} + \frac{(\phi^* + \gamma^* \phi^* t) e^{\gamma_{\tau} t} k_0^*}{\omega k_0 e^{\gamma_{\tau} t} + \phi^* e^{\gamma_{\tau} t} k_0^*}$$

$$\frac{\partial K_t}{\partial \lambda} = \Theta_2 + \Theta_3 = \frac{(\omega_{\lambda} + \gamma_{\lambda} \omega t) e^{\gamma_{\lambda} t} k_0}{\omega k_0 e^{\gamma_{\lambda} t} + \phi^* e^{\gamma_{\lambda} t} k_0^*} + \frac{(\phi^* + \gamma^* \phi^* t) e^{\gamma_{\lambda} t} k_0^*}{\omega k_0 e^{\gamma_{\lambda} t} + \phi^* e^{\gamma_{\lambda} t} k_0^*}$$

where I have defined $\Delta_2 \equiv \frac{(\omega_{\tau} + \gamma_{\tau} \omega t) e^{\gamma_{\tau} t} k_0}{\omega k_0 e^{\gamma_{\tau} t} + \phi^* e^{\gamma_{\tau} t} k_0^*}$ and $\Delta_3, \Theta_2, \Theta_3$ analogously. Notice that these expressions are not constant, but depend on time in a complex way. With the definitions I reformulate the optimality conditions in (C1) as

$$\tau: \int_0^\infty (\Delta_1 + \Delta_2 + \Delta_3) e^{-\rho t} dt = 0 \quad \text{(C2)}$$

$$\lambda: \lambda \left( \int_0^\infty (\Theta_1 + \Theta_2 + \Theta_3) e^{-\rho t} dt \right) = 0. \quad \text{(C3)}$$

By Lemma 1 $\Gamma_t$ is increasing over time and approaches $\max(\gamma, \gamma^*)$. In what is to follow I will concentrate on steady state paths. For steady state growth $\gamma = \gamma^*$. Let us focus on the optimality condition for $\tau$. Imposing $\gamma = \gamma^*$ entails $K_t = \omega k_0 e^{\gamma t} + \phi^* k_0^* e^{\gamma t}$ and so $\Delta_2 = \frac{(\omega_{\tau} + \gamma_{\tau} \omega t) e^{\gamma_{\tau} t} k_0}{K_0}$ and $\Delta_3 = \frac{(\phi^* + \gamma^* \phi^* t) e^{\gamma_{\tau} t} k_0^*}{K_t} = \frac{(\phi^* + \gamma^* \phi^* t) k_0^*}{K_0}$. Then (C2) becomes

$$\int_0^\infty \Delta_1 e^{-\rho t} dt + \int_0^\infty \left( \frac{(\omega_{\tau} + \gamma_{\tau} \omega t) k_0}{K_0} \right) e^{-\rho t} dt + \int_0^\infty \left( \frac{(\phi^* + \gamma^* \phi^* t) k_0^*}{K_0} \right) e^{-\rho t} dt = 0.$$

For the evaluation of each integral notice that

$$\int_0^\infty e^{-\rho t} dt = \left[ -\frac{1}{\rho e^{\rho t}} \right]_0^\infty = \frac{1}{\rho}, \quad \int_0^\infty te^{-\rho t} dt = \left[ -\frac{1}{\rho^2 e^{\rho t}} - \frac{t}{\rho e^{\rho t}} \right]_0^\infty = \frac{1}{\rho^2}.$$

Then it is not difficult to verify that

$$\int_0^\infty \Delta_1 e^{-\rho t} dt = \frac{\Delta_1}{\rho}.$$
\[
\int_0^\infty \left( \frac{(\omega + \gamma \cdot \omega \tau)}{K_0} \right) e^{-\rho t} dt = \frac{\omega k_0}{\rho K_0} + \frac{\gamma \cdot \omega k_0}{\rho^2 K_0}
\]
\[
\int_0^\infty \left( \frac{\phi + \gamma \cdot \phi \tau}{K_0} \right) e^{-\rho t} dt = \frac{\phi k_0}{\rho K_0} + \frac{\gamma \cdot \phi k_0}{\rho^2 K_0}.
\]

From (C1) and (C2) I may now express the FOC for \( \tau \) under the steady state condition \( \gamma = \gamma^* \) as

\[
\int_0^\infty \left( \frac{\partial C_i^{CW}}{\partial \tau} \frac{1}{C_i^{CW}} \right) e^{-\rho t} dt = 0 \iff \Delta_1 + \frac{\omega \tau k_0 + \phi^* k_0^*}{\rho K_0} + \frac{\gamma \tau \omega k_0 + \gamma^* \phi k_0^*}{\rho^2 K_0} = 0
\]

which is equivalent to the expression in (32). Analogous reasoning establishes that for an interior solution for \( \lambda \)

\[
\int_0^\infty \left( \frac{\partial C_i^{CW}}{\partial \lambda} \frac{1}{C_i^{CW}} \right) e^{-\rho t} dt = 0 \iff \Theta_1 + \frac{\omega \lambda k_0 + \phi^* k_0^*}{\rho K_0} + \frac{\gamma \lambda \omega k_0 + \gamma^* \phi k_0^*}{\rho^2 K_0} = 0
\]

which corresponds to equation (33) in the text.

D Symmetry and Uniqueness of Nash Equilibria in the Left-Left Tax Competition Game when \( \gamma = \gamma^* \)

If \( \gamma = \gamma^* \), then

\[
(r - \tau) \omega + (r^* - \tau^*) \phi(\omega) - \rho = (r^* - \tau^*) \omega^* + (r - \tau) \phi^*(\omega^*) - \rho
\]

has to be satisfied. Rearranging and cancelling \( \rho \) implies

\[
\left( \frac{r - \tau}{r^* - \tau^*} \right) [\omega - \phi^*] = \omega^* - \phi.
\]

(D1)
Now let \( \Sigma_1 \equiv \left( \frac{r - \tau}{\tau^* - \tau^*} \right) \) and recall

\[
\omega = \Sigma_1^z, \quad \phi = \frac{z}{z + 1} \left( 1 - \Sigma_1^{z+1} \right),
\]

\[
\omega^* = \Sigma_1^{-z}, \quad \phi^* = \frac{z}{z + 1} \left( 1 - \Sigma_1^{-z-1} \right).
\]

Any Nash Equilibrium in \((\tau, \lambda, \tau^*, \lambda^*)\) must be such that either \( \Sigma_1 < 1 \) or \( \Sigma_1 > 1 \) or \( \Sigma_1 = 1 \). Suppose \( \Sigma_1 < 1 \). Then \( \omega^* = 1 \) by the optimal behaviour of the capital owners, equation (13), and so \( \phi^* = 0 \). Then (D1) reduces to

\[
\Sigma_1 \omega = 1 - \phi
\]
\[
\Sigma_1^{z+1} = 1 - \frac{z}{z + 1} + \frac{z}{z + 1} \Sigma_1^{z+1}
\]
\[
(z + 1) \Sigma_1^{z+1} = (z + 1) - z + z \Sigma_1^{z+1}
\]
\[
\Sigma_1^{z+1} = 1
\]

which can only be satisfied if \( \Sigma_1 = 1 \), hence, contradicting our assumption that \( \Sigma_1 < 1 \). Next, suppose \( \Sigma_1 > 1 \). Then \( \omega = 1 \) and \( \phi = 0 \) by equation (13). From (D1) this implies that

\[
\Sigma_1 (1 - \phi^*) = \Sigma_1^{-z}
\]
\[
\Sigma_1 - \frac{\Sigma_1 z}{z + 1} + \frac{\Sigma_1 z}{z + 1} \Sigma_1^{-(z+1)} = \Sigma_1^{-z}
\]
\[
(z + 1) \Sigma_1 - \Sigma_1 z + z \Sigma_1^{-z} = (z + 1) \Sigma_1^{-z}
\]
\[
\Sigma_1^{z+1} = 1
\]

which again can only be satisfied if \( \Sigma_1 = 1 \), contradicting the assumption that \( \Sigma_1 > 1 \). Therefore, we must have \( \Sigma_1 = 1 \) in equilibrium if \( \gamma = \gamma^* \); that is, the after-tax returns must be equal. It is not difficult to see that \( \Sigma_1 = 1 \) indeed satisfies the condition \( \gamma = \gamma^* \). So I conclude that any Nash Equilibrium in \((\tau, \lambda, \tau^*, \lambda^*)\) that satisfies \( \gamma = \gamma^* \) must satisfy \( r - \tau = r^* - \tau^* \).

Given symmetry with respect to technology, preferences, action sets etc. the after-tax functions \((r - \tau, r^* - \tau^*)\) are symmetric. Given that \( r - \tau \) first (strictly) increases and then (strictly) decreases in \( \tau \) we may
have two tax rates $\tau$ that sustain the same after-tax return as can be seen from Figure 1. Thus, in order to get uniqueness I must check whether only one of the tax parameters will be chosen in equilibrium. To this end I will distinguish two cases.

First, suppose an interior solution with $\lambda \geq 0, \lambda^* \geq 0$ would be Nash Equilibrium. Then the FOC in (32), (33) and Lemma 5 tell us that

$$\tau = \frac{[(1-\alpha)A]^{\frac{1}{\delta}}}{1-\lambda} \quad \text{and} \quad \tau^* = \frac{[(1-\alpha)A^*]^{\frac{1}{\delta}}}{1-\lambda^*}$$

must hold in any possible Nash Equilibrium with $\lambda \geq 0, \lambda^* \geq 0$. Since $r = \alpha A[(1-\lambda)\tau]^{1-\alpha}$ and $r(1-\lambda) = [(1-\alpha)A]^{\frac{1}{\delta}}$ it is not difficult to see that

$$r = \frac{\alpha}{1-\alpha} [(1-\alpha)A]^{\frac{1}{\delta}} = r^*.$$

So Lemma 5 implies $r = r^*$ and under the condition $\gamma = \gamma^*$, that is, $\Sigma_1 = 1$, this implies that $\tau = \tau^*$ and so $\lambda = \lambda^*$. From this I conclude that any Nash Equilibrium with $\lambda \geq 0, \lambda^* \geq 0$ satisfying $\gamma = \gamma^*$ must be symmetric. Furthermore, from the optimality condition of Lemma 5 and for $\lambda \geq 0$ we must

$$0 \leq \lambda = 1 - \frac{[(1-\alpha)A]^{\frac{1}{\delta}}}{\tau} \Leftrightarrow \tau \geq [(1-\alpha)A]^{\frac{1}{\delta}} > [\alpha(1-\alpha)A]^{\frac{1}{\delta}} = \hat{\tau}$$

so that $\tau > \hat{\tau}$ and so by symmetry $\tau^* > \hat{\tau}$. But since then either after-tax return function is (strictly) decreasing in $\tau$ or $\tau^*$ the tax rates chosen in equilibrium would have to be unique. Hence, in any Nash Equilibrium with $(\lambda, \lambda^* \geq 0)$ and $\gamma = \gamma^*$, the equilibrium must be symmetric and unique.

Next, I have to check whether any equilibrium with $\lambda = 0, \lambda^* = 0$ will be symmetric and unique under the assumption $\gamma = \gamma^*$. In this case equation (32) with $\lambda = \lambda^* = 0$ must be analyzed. Now $\gamma = \gamma^*$ requires

$$r - \tau = r^* - \tau^*$$

which allows for two symmetric and two asymmetric equilibria. (See Figure 1.) I will now show that the condition of equal after-tax returns
allows for a unique equilibrium only. In equilibrium the FOC in (32) must be satisfied for an interior solution in \( \tau \). Under the condition \( \gamma = \gamma^* \) and so equal after-tax returns

\[
V_\tau(\lambda = \lambda^* = 0) = \frac{\eta_\tau}{\eta} + z \left( \frac{r_\tau - 1}{r - \tau} \right) \left( \frac{k + k^*}{k} \right) + \frac{(r_\tau - 1)}{\rho} = 0
\]

where \( \eta_\tau / \eta > 0 \) for all \( \tau \in [0,1] \). Suppose \( \tau < \hat{\tau} \) would hold in equilibrium. Since \( r_\tau - 1 > 0 \) for \( \tau < \hat{\tau} \), marginal utility \( V_\tau \) would be positive which cannot be the case in equilibrium. Thus, any equilibrium combination \((\tau, \tau^*)\) must be such that \( \tau, \tau^* \geq \hat{\tau} \). But if \( \gamma = \gamma^* \) in equilibrium the equilibrium \((\tau, \tau^*)\) combination must be unique and symmetric. This follows since the after-tax functions are symmetric and so only \( \tau = \tau^* \) satisfy \( r - \tau = r^* - \tau^* \). Also, for \( \tau = \tau^* > \hat{\tau} \) the after-tax functions are (strictly) decreasing so that given the parameters of the model, any equilibrium combination must be unique.

Hence, any Nash Equilibrium in \((\tau, \lambda, \tau^*, \lambda^*)\) that satisfies \( \gamma = \gamma^* \) must be unique and symmetric.

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