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Organizational Structure and Performance

BAUKE VISSER

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Abstract

The effect of organizational structure on performance is studied using a project selection framework in which heterogeneous, rational agents can reject or accept projects. Using the expected profits on accepted projects as a criterion, I determine the optimal ordering of agents within a given structure, and compare the performance of different structures. I discuss as well the performance from a welfare point of view, and from an informational point of view.

Keywords: Organizational Structure, Project Selection, Organizational Performance

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1 Introduction

That the internal structure of an organization affects its performance has been widely recognized. Chandler (1966) notes that the success of the multidivisional form was due to the seperation of operational and strategic decisions. He claimed that the organizational structure reflected the chosen corporate strategy, or, in his well-known phrase, that "Structure follows Strategy". His ideas led to a rich literature on the relationship between strategy and structure (see Caves (1980) and the references therein). There has, however, always been support for the opposite claim as well. That "Strategy follows Structure" has recently been put forward by, for example, Hammond (1994). He shows how the organizational structure influences the gathering and processing of information, and how the decisions taken at various levels interact to form the organizational choice. The organizational strucure screens or filters decision relevant information. In short, he shows how the structure of an organization sets its strategic decision-making agenda.

The relationship between, on the one hand, the way decentralized information is processed and communicated, the ordering in which decisions are taken, and, on the other hand, the choice and performance of an organization is an underdeveloped part of the theory of the firm.

This paper addresses these issues using the framework of the project selection model introduced by Sah and Stiglitz ((1985), (1986), (1988)). An organization consists of heterogeneous, rational agents who, given the limited information they possess, maximize the expected pay-off of the organization. Agents evaluate projects their organization may implement. Projects can either be good or bad. Good projects, once implemented by the organization give rise to a profit, while the organization incurs a loss on bad, accepted projects. Agents can either accept or reject a project. The structure of an organization fixes the sequence in which agents decide (who is first, who is next?), and the situation in which they decide (does an agent decide irrespective of the preceeding agents' decisions, or only if a specific series of decisions has already been taken?). An agent has knowledge that is partial and complementary to the knowledge other agents have, and this information cannot be communicated. Their decisions, though, partly reveal the information on which the decision is based. Since agents are rational their actions reflect the structure of the organization and the uncertainty ensuing from the privacy of information. Agents learn from the preceeding agents' decisions. Arguably, differences in organizational structure may lead to differences in actions taken.

I compare four simple archetypical organizational forms that are used throughout the business community to appraise the value of projects. The simplest is a single agent "organization": if this agent decides to accept a project, the organization accepts it, while if it rejects the project the organization rejects it. Secondly, a hierarchy. In this structure, if the first agent rejects a project it is rejected by the organization, while if it is accepted by the first agent it moves on to the second agent. Whether the organization will then implement the project depends on this agent's decision: acceptance by the second agent implies acceptance by the organization, and rejection by this agent leads to rejection by the organization. In a polyarchy, projects accepted by the first agent will be implemented by the organization, while those projects he rejects find their way to the second agent's desk. Whether the organization implements the projects then depends on the second agent's decision, just as in a hierarchy. In an omniarchy, finally, whatever the first agent's decision, the second agent considers the quality of the project as well. If the latter decides to accept the project, the organization accepts it, while rejection on the organizational level follows rejection by the second agent.

Projects are characterized by a profit or a loss that is made if accepted. Agents are defined in terms of an interval in which the value they attribute to a project lies. Agents may have intervals of differing lengths.

I address four questions:

- How do changes in the values of a parameters characterizing projects and agents affect the performance of a given organizational form?
- If agents are heterogeneous does their ordering matter? If so, what ordering of agents within a given organizational structure is the best?

What is the type of information needed to determine the optimal ordering?

- For given values of the parameters what organizational form maximizes the expected pay-off?

- For given values of the parameters what organizational form maximizes the difference between, on the one hand, the expected value of accepted, good projects, and, on the other hand, the expected value of accepted, but bad projects and rejected, but good projects?

Let me point out two major differences with the framework used by Sah and Stiglitz.

They model agents as fallible human beings, fallible in the sense that they make errors when evaluating the value of projects: agents are *parametrized* by a probability of rejecting good projects and a probability of accepting bad projects. That is, these probabilities are independent of both the organizational architecture and the agent's position within a given structure. Nor are they affected by the decisions other agents take. Differences in organizational architectures entail differences in the way these exogenously determined individual errors are *aggregated*, and their goal is to relate the best organizational form to the type of environment.

In my model the decision to accept or reject a project, and hence implicitly the probability that an agent accepts or rejects a project, *reflects* the overall structure of the organization, the agent's position within a given structure, and the characteristics of both the pool of projects and all the agents populating the organization. That is, individual probabilities determine and are determined by the overall organizational structure.

Consequently, in the Sah and Stiglitz framework, an omniarchy, *i.e.*, the organizational structure in which, whatever the first agent's decision, a second agent evaluates the project, performs as well as a single agent organization consisting of the second agent only. In the set-up studied here, however, the first agent has the information generating role of seperating apparently good from apparently bad projects which allows the second agent to apply two evaluation standards: one for good and one for bad projects. Thus, an omniarchy may perform better than a single agent.

The second difference with the Sah and Stiglitz papers is that I compare different organizational architectures not only in terms of the expected profits each generates, but as well taking into account the value of good projects that organizations reject. This entails an ordering of organizational forms which is rather different from the one implied by the standard of expected profits.

I show that, in terms of expected profits, an omniarchy, characterized by a double check on every project, performs uniformly the best, while a single agent organization is always performing the worst. The ordering of the polyarchy and the hierarchy, two organizational forms allowing for a second instance of screening conditional on the preceeding decision and on the basis of independent information depends on both the characteristics of the agents and on the type of environment. If agents are different but the possible losses X_2 are as large as the possible gains X_1, the polyarchy and the hierarchy are performing equally well, whereas in the case of identical agents the hierarchy performs better than the polyarchy in tough environments. The opposite holds for relatively friendly environments.

If, on the other hand the value of rejected but good projects is taken into account, the omniarchy stills performs better than any other organizational form. Interestingly, if agents are different but possible gains are as large as possible losses, the polyarchy performs uniformly better than both the hierarchy and the single agent organization. Indeed, the hierarchy may perform even worse than a single agent organization. If agents are identical, the polyarchy now performs better than the hierarchy for a set of parameters that is larger than when the standard of expected profits is used.

The paper shows that in order for the omniarchy to maximize performance the ordering of heterogeneous agents is crucial. Indeed, misallocating agents in this organizational form gives rise to an expected pay–off that is lower than the pay–off obtained in the polyarchy and the hieriarchy. Since the maximization of expected profits in these organizational forms does not require a specific ordering, they enjoy a certain advantage over the omniarchy.

In the next section I discuss the modelling assumptions I use throughout the paper. The succeeding sections address the above mentioned questions related to a specific organizational structure: section 3 analyses the single agent case, section 4 deals with the hierarchy, section 5 turns to polyarchies, while section 6 discusses omniarchies. Section 7 compares the performance of the four organizational architectures. Related literature is discussed in section 8. Section 9 concludes. Proofs can be found in the Appendix.

2 The Model

As noted in the introduction, the model used in this paper is a variant of the project selection model studied by Sah and Stiglitz ((1985), (1986), and (1988)). Analysis is limited to organizations in which at most two agents decide on the project's future. I study four different structures used to select projects: a single agent organization (S), a hierarchy (H), a polyarchy (P), and an omniarchy (O). Let me denote an organizational form by F, where $F \in \{S, H, P, O\}$. They are graphically represented in figure 1.

The nodes represent agents, an arrow indicates the flow of projects within the organization, while the labels on the arcs stand for the decision agents take: A stands for Acceptance, while R stands for Rejection. These are the only actions agents can take. From a theoretical point of view, the limitation to a binary action set may seem a strong assumption; as a characterization of real world approval systems, it seems to be acceptable. In a hierarchy, a project is accepted if and only if every manager approves the project; inspection stops as soon as bad review is received. In a polyarchy, a project is rejected if and only if every manager rejects the project; inspection stops when a good review is recieved. In an omniarchy, whatever Mr. i 's decision, the project is looked at as well from Ms. j 's perspective. The single agent "organization" has a straightforward interpretation. I write $F(i, j)$ for organizational form F in which Mr. i is the

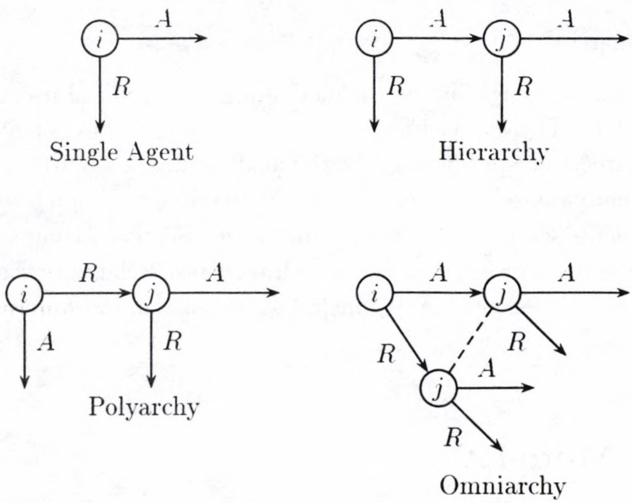

Figure 1: The four organizational forms studied

first to decide, and Ms. j the second. In particular, $S(i) := S(i, j)$ stands for the single agent organization consisting of Mr. i only.

Mr. i and Ms. j are team–member in the sense of Marschak and Radner (1972): given the information agents have they undertake that course of action that maximizes the expected pay–off of the organization as a whole. This implies the absence of any principal–agent problem.

Every project the organization considers is characterized by the tuple (θ^i, θ^j, V). θ^i can be interpreted as a signal Mr. i obtains, or as a value he atributes to the project. Similarily, θ^j is the signal or valuation of Ms. j . These signals are private and cannot be communicated. The lack of communication reflect high communcication costs due to, for example high costs of processing, transmitting, and interpreting specialized information. V, finally, is the value of a project if it is implemented. Neither Mr. i nor Ms. j observe V. An interpretation of this characterization would be to consider Mr. i and Ms. j as operating in different departments, having different competences. Note that this not imply that analysis is restricted to *organizations* that have at most two agents. In reality, specialized employees confine themselves to problems they are capable of solving: engineers decide on the technical feasability of, say, a new product, while the marketing department contemplates the marketability of the product. Projects or problems of a merely technical character will not pass through the latter department. Hence, the real limitation resides in the assumption that just two aspects of a project determine its profitability. The impossibility to communicate the value of θ^i and θ^j could reflect the high degree of specialization that is concomitant to the organization's divisional structure.

The characteristics (θ^i, θ^j) of the projects the enterprise considers are randomly drawn from two independent uniform distributions:

$$\begin{aligned} \theta^i &\sim U[-\theta, \theta] \\ \theta^j &\sim U[-\theta - h, \theta + h] \end{aligned} \tag{1}$$

where $\theta > 0$ and $h > -\theta$. The random variables have the same expected value, $E(\theta^i) = E(\theta^j) = 0$, but they may have different variances: $Var(\theta^j) > Var(\theta^i)$ if and only if $h > 0$. The value V of an implemented project depends on the sum of the values of the individual characteristics:

$$V = \begin{cases} X_1 & : \quad \theta^i + \theta^j > 0 \\ -X_2 & : \quad \theta^i + \theta^j \leq 0 \end{cases} \tag{2}$$

where $X_i > 0$ for $i = 1, 2$. This specification implies that what a project lacks in terms of, say, marketability may be made up by its ease and efficiency in production. It ensures as well that an accepted project that is profitable gives rise to a pay-off that is independent of the particular values of $\theta^i$and θ^j. The size of the pool of projects is irrelevant to what follows, and can be set equal to one. Given the set-up, ex ante half of the projects is good, while the other half is bad.

Assumption 1 specifies those elements that are common knowledge.

Assumption 1 *The organizational form, the distribution of $\theta^i$and of θ^j, the specification of V, as well as the rationality of the agents is common knowledge. Moreover, if agent i's decision preceedes j's decision, the former decision is common knowledge when j has to decide.*

An agent is rational in the sense that based on the information it has it takes whatever action gives rise to the highest expected pay-off. If an agent is indifferent it decides to reject the project. Given assumption 1. an agent's decision reflects the organizational structure, the distributions of the signals, and the rationality. This contrasts sharply with the actions taken by the agents that populate the organizations of Sah and Stiglitz. In the framework they use, an agent is characterized by its probability of accepting profitable projects and its probability of accepting inefficient projects. These probabilities are independent of both the organizational architecture and the agent's position within a given structure. Nor are they affected by the decisions other agents take. One could say that the agents in Sah and Stiglitz are individualistic, in that they do things by themselves, ignoring others, and in their own way, while the ones I model are organizational, in the sense that their actions reflects the presence of other agents and the organizational structure in which their actions are embedded.

A central feature of the optimal decision rules for both agents is the presence of a threshold value: for values of the signal larger than this threshold projects will be accepted by an agent, while for values smaller than or equal to this cut-off value projects will be rejected[1]. Throughout the paper, $E(\Pi|F(i,j))$ denotes the expected profits made on accepted projects in organization $F(i,j)$, and $E(\Pi|F) := \max(E(\Pi|F(i,j)), E(\Pi|F(j,i)))$. That is, the former expression denotes the expected profits of given organizational structure with given ordering of agents, while the latter stands for the expected profits of an organizational form which has arranged its agents in the best way.

Now I can restate the four questions mentioned in the introduction.

- How does the expected value of accepted projects, $E(\Pi|F(i,j))$ of an organizational form F respond to changes in the parameters (θ, h, X_1, X_2)?

- If the variances of the valuations are different for the agents, who

[1] I show in section 6.1 that there exist organizations as well in which projects are accepted if and only if their value is sufficiently low.

should be the first to evaluate a project? Does the ordering depend on the sign of h only, or does the value of h matter as well?

- Given values of (θ, h, X_1, X_2) which organizational form F maximizes the expected pay–off $E(\Pi|F)$?

- Given values of (θ, h, X_1, X_2) which organizational form F maximizes the difference between, on the one hand, the expected value of accepted, good projects, and, on the other hand, the expected value of accepted bad projects and rejected, but good projects?

3 The Single Agent Case

Before discussing organizations *in senso stricto*, I analyse the behaviour of a single agent whose expected pay–off is determined by the sum of the signals $\theta^i + \theta^j$ as specified in equation (2). I show that the agent with the largest variance ensures the highest pay–off, and derive the expression for expected profits $E(\Pi|S)$, as well as two comparative statics results.

Suppose Mr. i operates on his own. He accepts all projects characterized by a signal or valuation θ^i where θ^i satisfies $X_1 \Pr(\theta^i + \theta^j > 0) - X_2 \Pr(\theta^i + \theta^j \leq 0) > 0$. Given Mr. i 's knowledge of the distribution of θ^j , he accepts all those projects that he considers sufficiently good. That is, all those projects characterized by $\theta^i > \overline{\theta}^i$, where the threshold value $\overline{\theta}^i$ solves $X_1 \Pr(\overline{\theta}^i + \theta^j > 0) - X_2 \Pr(\overline{\theta}^i + \theta^j \leq 0) = 0$. Hence, Mr. i accepts all those projects with a valuation larger than $\overline{\theta}^i$, where $\overline{\theta}^i$ equals

$$\overline{\theta}^i = (\theta + h)\frac{k-1}{k+1} \tag{3}$$

where $k := X_2/X_1$. k is an important parameter in this model, and it should be noted that for finite and positive values of X_1 and X_2, $0 < k < \infty$. The higher the value of k the higher the loss X_2 relative to X_1. That is, the higher k the tougher the environment. The value of $\overline{\theta}^i$ may be larger than θ , in which case Mr. i rejects all projects; or it may be smaller than $-\theta$, leading Mr. i to accept all projects: if $\overline{\theta}^i \in (-\theta, \theta)$, than his valuation of the project determines the project's future. Clearly, the values θ and θ divide the range of possible threshold values in three areas, each with a corresponding type of decision, namely reject irrespective of valuation, accept irrespective of valuation, and finally decision depends on valuation. I call these types of decisions qualitatively different.

Lemma 1 *The threshold value $\overline{\theta}^i$ is larger than θ if and only if*

$$\frac{h}{\theta} \geq \frac{2}{k-1} \text{ and } k > 1 \tag{4}$$

The threshold value $\overline{\theta}^i$ is smaller than $-\theta$ if and only if

$$\frac{h}{\theta} \geq -\frac{2k}{k-1} \text{ and } k < 1 \tag{5}$$

For all other values of (θ, h, k) the threshold value $\overline{\theta}^i \in (-\theta, \theta)$.

The same kind of analysis applies in case of Ms. j being on her own. She accepts all those projects satisfying $\theta^j > \overline{\theta}^j$, where $\overline{\theta}^j$ equals

$$\overline{\theta}^j = \theta\frac{k-1}{k+1} \tag{6}$$

Lemma 2 specifies when Ms. j accepts or rejects all projects, and when her decision depends on the project's valuation.

Lemma 2 *The threshold value $\overline{\theta}^j$ is larger than $\theta + h$ if and only if*

$$\frac{h}{\theta} \leq \frac{2}{k+1} \text{ and } k > 1 \tag{7}$$

The threshold value $\overline{\theta}^j$ is smaller than $-\theta - h$ if and only if

$$\frac{h}{\theta} \leq -\frac{2k}{k+1} \text{ and } k < 1 \tag{8}$$

For all other values of (θ, h, k) the threshold value $\overline{\theta}^j \in (-\theta - h, \theta + h)$.

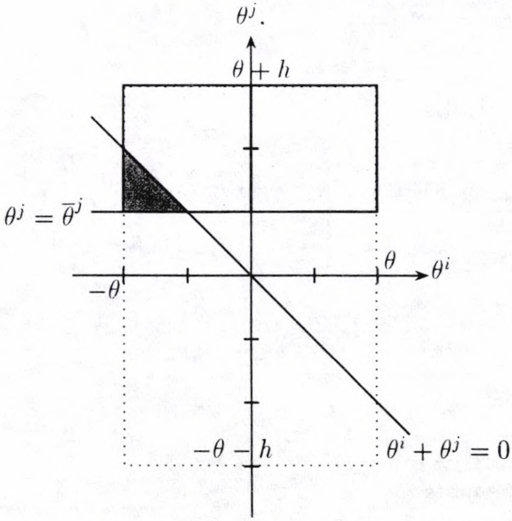

Figure 2: Profits and Losses in a Single Agent Organization

These lemmas show that the type of decision depends on the interplay of the three main parameters of the model, but, interestingly enough, in a special way as both h/θ and $k = X_2/X_1$ are ratios, the first capturing the diversity of the agents, and the second the diversity of the value of projects. Figure 2 illustrates the areas where good and bad projects are accepted when Ms. j forms organization. The area enclosed by the lines $\theta^j = \overline{\theta}^j$, $\theta^i = -\theta$, $\theta^j = \theta + h$, and $\theta^i = \theta$ contains all the projects with values (θ^i, θ^j) that Ms. j accepts if the parameter values equal (θ, h, k) satisfying the third restriction of lemma 2. The lightgrey area above the line $\theta^i + \theta^j = 0$ represents the area for which a profit will be made, while the grey area below and on the line $\theta^i + \theta^j = 0$ lead to a loss. Proposition 1 summarizes the expected profits for a single agent, $E(\Pi|S(i))$ and $E(\Pi|S(j))$.

Proposition 1 *$E(\Pi|S(i))$ equals*

$$\begin{cases} X_1 \dfrac{(2\theta - hk + h)^2}{8\theta(\theta + h)(k+1)} & : \begin{cases} k > 1,\ h \in \left(0. -\theta \dfrac{2k}{k-1}\right) & \text{or} \\ k < 1, h \in \left(0, \theta \dfrac{2}{k-1}\right) \end{cases} \\ 0 & : \ k > 1,\ h \geq \theta \dfrac{2}{k-1} \\ X_1 \dfrac{1-k}{2} & : \ k < 1\ h \geq -\theta \dfrac{2k}{k-1} \\ X_1 \dfrac{\theta - hk}{2(\theta(k+1))} & : \ h \leq 0 \end{cases}$$

while $E(\Pi|S(j))$ amounts to

$$\begin{cases} X_1 \dfrac{(2\theta + h(k+1))^2}{8\theta(\theta + h)(k+1)} & : \ h \geq 0 \\ 0 & : \ k > 1,\ h \leq -\theta \dfrac{2}{k+1}, \\ X_1 \dfrac{1-k}{2} & : \ k < 1,\ h \geq -\theta \dfrac{2k}{k+1}, \\ X_1 \dfrac{\theta + h(k+1)}{2(\theta + h)(k+1)} & : \begin{cases} k \leq 1,\ h \in \left(-\theta \dfrac{2}{k+1}, 0\right), & \text{or} \\ k \geq 1\ h \in \left(-\theta \dfrac{2k}{k+1}, 0\right) \end{cases} \end{cases}$$

Clearly, if an agent accepts no project at all, the expected pay–off equals zero, while if an agent accepts all projects, expected pay–off equals $X_1(1-k)/2$, which is strictly positive since $k < 1$ if an agent accepts all projects. Proposition 2 shows that the agent with the largest variability in his evaluations makes the highest profit.

Proposition 2 *In the single agent case, the agent with the largest variance ensures the highest profit: $E(\Pi|S(i)) > E(\Pi|S(j)) \iff h < 0$. Therefore*

$$E(\Pi|S) = \begin{cases} X_1 \dfrac{\theta - hk}{2\theta(k+1)} & : \ h < 0 \\ X_1 \dfrac{(2\theta + h(k+1))^2}{8\theta(\theta + h)(k+1)} & : \ h \geq 0 \end{cases} \tag{9}$$

The intuition for this result is that the agent knows its own valuation when deciding, and, in order to maximize expected pay-offs, it prefers a variable with a small variance to one with a high variance. Hence, the agent with the largest variance should form the organization. The following comparative statics results involving changes in the environment or changes in the agents' characteristics hold for such optimal single agent organizations. From equation 6 and figure 2 it can be seen that increasing the value of k (keeping X_1 fixed) has two opposing effects. First of all, it leads to an increase in the loss made in the area below the $\overline{\theta}^i + \overline{\theta}^j = 0$ line. Secondly, it shifts up the threshold value $\overline{\theta}^j$, which implies two opposing effects. On the one hand, this reduces the size of the surface where a loss is made, but on the other hand, it reduces as well the size of the area where a profit is made. Corollary 1 shows the net effect of an increase in k.

Corollary 1 *The expected profits of an optimal single agent organization decrease in k:*

$$\frac{\partial E(\Pi | S(j), h > 0)}{\partial k} < 0 \tag{10}$$

The expected profits of an optimal single agent organization grows with any increase in $|h|$:

$$\begin{aligned} \frac{\partial E(\Pi | S; h > 0)}{\partial h} &> 0 \\ \frac{\partial E(\Pi | S; h < 0)}{\partial h} &< 0 \end{aligned} \tag{11}$$

This corollary shows first of all that an increasingly thougher project environment leads to a decrease in expected profits. It shows as well that any increase in the degree of heterogeneity of the agents raises the expected profits. This can be understood by looking at figure 2. Given that $h > 0$, Ms. j will be in the organization. Her threshold value $\overline{\theta}^j$ can be either positive or negative, but in any event it falls within the interval $(-\theta, \theta)$. Therefore, any increase in h increases the size of the area where a profit is made, while leaving the surface of the area where a loss is made unaffected. The overall effect is an increase in expected profit.

4 Hierarchies

This section studies the behaviour of Mr. i and Ms. j in a hierarchy. Mr. i , who is the first to look at a project, accepts a project if and only if the expected value of an accepted project is larger than the expected value of a rejected project. Rejection of a project leads to zero profits for sure. Hence, i accepts the project if its expected value is positive. The expected pay–off is not only determined by i's signal, but as well by j's signal. Although Mr. i does not know the value of θ^j, he does know that j takes that action that maximizes expected value. Given that rejection by j leads to zero profits with certainty, j accepts whatever project will lead to an expected positive pay–off. Moreover, when j decides, she knows that i decided favourably on the project. Let me assume that i's decision rule is of the type Accept if and only if $\theta^i > \overline{\theta}^i$ and Reject if and only if $\theta^i \leq \overline{\theta}^i$. Then, it can be shown that there exists a threshold value $\overline{\theta}^j$ such that j accepts if and only if $\theta^j > \overline{\theta}^j$ and rejects if and only if $\theta^j \leq \overline{\theta}^j$. Moreover, given this cut–off rule used by j, i's best response is to apply a cut–off rule as well. In other words, the use of a cut–off rule by both Mr. i and Ms. j is consistent.

Let me therefore assume that i uses a cut–off rule. If j has to decide she knows that $\theta^i > \overline{\theta}^i$ and hence accepts all projects with a value θ^j that satisfies

$$X_1 \Pr(\theta^i + \theta^j > 0 |\, \theta^i > \overline{\theta}^i) - X_2 \Pr(\theta^i + \theta^j \leq 0 |\, \theta^i > \overline{\theta}^i) > 0 \qquad (12)$$

It is straightforward to see that Ms. j 's threshold value $\overline{\theta}^j$ satisfies

$$X_1 \Pr(\theta^i + \overline{\theta}^j > 0 |\, \theta^i > \overline{\theta}^i) - X_2 \Pr(\theta^i + \overline{\theta}^j \leq 0 |\, \theta^i > \overline{\theta}^i) = 0 \qquad (13)$$

and j accepts all those projects such that $\theta^j > \overline{\theta}^j$. This shows that $\overline{\theta}^j$ is a function of $\overline{\theta}^i$. This function can be considered Ms. j 's best response function. If $\overline{\theta}^i \geq \theta$, Mr. i rejects all projects, and Ms. j never receives a project. Her threshold value $\overline{\theta}^j$ can therefore take on any value. If, on the other hand, $\overline{\theta}^i \leq -\theta$, Mr. i accepts all projects. Ms. j's response to $\overline{\theta}^i < -\theta$ is therefore the same as the response to $\overline{\theta}^i = -\theta$. If $\overline{\theta}^i \in (-\theta, \theta)$, equation (13) amounts to

$$X_1 \int_{-\overline{\theta}^j}^{\theta} \frac{dx}{\theta - \overline{\theta}^i} - X_2 \int_{\overline{\theta}^i}^{-\overline{\theta}^j} \frac{dx}{\theta - \overline{\theta}^i} =$$

$$X_1 \frac{\theta + \overline{\theta}^j}{\theta - \overline{\theta}^i} - X_2 \frac{-\overline{\theta}^j - \overline{\theta}^i}{\theta - \overline{\theta}^i} = 0 \tag{14}$$

Therefore, $\overline{\theta}^j(\overline{\theta}^i)$ equals

$$\begin{cases} (-\infty, \infty + 1) & : \quad \overline{\theta}^i \geq \theta \\ -\overline{\theta}^i \frac{k}{k+1} - \theta \frac{1}{k+1} & : \quad \overline{\theta}^i \in (-\theta, \theta) \\ \theta \frac{k-1}{k+1} & : \quad \overline{\theta}^i \leq -\theta \end{cases} \tag{15}$$

Let me now turn to Mr. i . For a project to which he assigns a value of θ^i the expected pay–off equals the probability that j accepts a project that has been accepted by i times the expected value of such a project:

$$\Pr(\theta^j > \overline{\theta}^j)\left(X_1 \Pr(\theta^i + \theta^j > 0 |\ \theta^j > \overline{\theta}^j) - X_2 \Pr(\theta^i + \theta^j \leq 0 |\ \theta^j > \overline{\theta}^j)\right)$$

Consequently, i accepts those projects with a value $\theta^i > \overline{\theta}^i$, where $\overline{\theta}^i$ solves

$$\Pr(\theta^j > \overline{\theta}^j)\left(X_1 \Pr(\overline{\theta}^i + \theta^j > 0 | \theta^j > \overline{\theta}^j) - X_2 \Pr(\overline{\theta}^i + \theta^j \leq 0 | \theta^j > \overline{\theta}^j)\right) = 0 \tag{16}$$

Suppose $\overline{\theta}^j \geq \theta + h$, meaning that Ms. j rejects all projects that Mr. i accepts. As a consequence, Mr. i is indifferent between acceptance and rejection, and therefore rejects all projects, or $\overline{\theta}^i \geq \theta$. If, on the other hand, $\overline{\theta}^j \leq -\theta - h$, Ms. j accepts all projects, and Mr. i 's behaviour in case of $\overline{\theta}^i < -\theta - h$ is the same as in case of $\overline{\theta}^j = -\theta - h$. For $\overline{\theta}^j \in [-\theta - h, \theta + h)$ equation (16) becomes

$$\frac{\theta + h - \overline{\theta}^j}{2(\theta + h)}\left(X_1 \frac{\theta + h + \overline{\theta}^i}{\theta + h - \overline{\theta}^j} - X_2 \frac{-\overline{\theta}^i - \overline{\theta}^j}{\theta + h - \overline{\theta}^j}\right) = 0 \tag{17}$$

This amounts to

$$\overline{\theta}^i(\overline{\theta}^j) = \begin{cases} (\theta, \infty) & : \quad \overline{\theta}^j \geq \theta + h \\ -\overline{\theta}^j \frac{k}{k+1} - (\theta + h)\frac{1}{k+1} & : \quad \overline{\theta}^j \in (-\theta - h, \theta + h) \\ (\theta + h)\frac{k-1}{k+1} & : \quad \overline{\theta}^j \leq -\theta - h \end{cases} \tag{18}$$

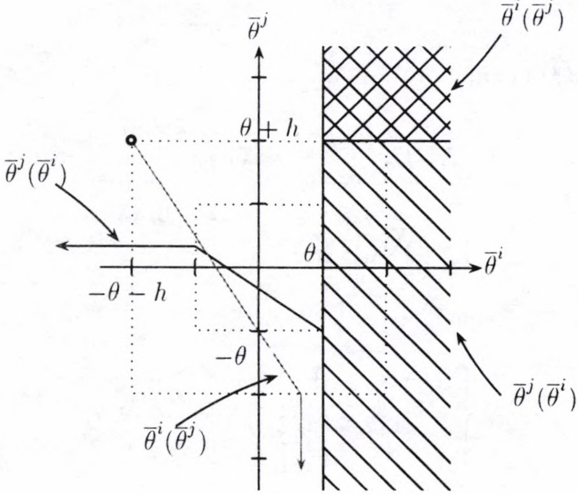

Figure 3: The best replies of i and j in a hierarchy

Equations (15) and (18) jointly determine the mutually consistent values of $\overline{\theta}^i$ and $\overline{\theta}^j$. They are graphically represented in figure 3. This graph implicitly assumes that $h > 0$, *i.e.*, the situation where Ms. j 's valuations have the highest variance. Figures 3 shows first of all that $(\overline{\theta}^i, \overline{\theta}^j) \in (\theta, \infty) \times (\theta + h, \infty)$ are consistent threshold values. Since these threshold levels are independent of the parameter k that characterizes the environment they do not interest me here. For these threshold values the organizational pay–off is equal to zero.

Secondly, the figure shows that for $h > 0$ the intersection of the best response functions is always to the right of the line $\overline{\theta}^i = \theta$ and in between the lines $\overline{\theta}^j = \theta + h$ and $\overline{\theta}^j = -\theta - h$. Hence, if $h > 0$, *i.e.*, if Ms. j has the higher variance, her cut–off value falls within the interval $(-\theta - h, \theta + h)$. Hence, her decision depends on the signal she receives. Whether the threshold value $\overline{\theta}^i$ of Mr. i falls within the interval $(-\theta, \theta)$ depends on the position of the intersection of the best response functions $\overline{\theta}^i(\overline{\theta}^j)$ and $\overline{\theta}^j(\overline{\theta}^i)$. The following lemma specifies the conditions under which Mr. i 's decision is affected by the signal he obtains. It tells as well under which conditions Ms. j 's actions are independent of the signals she receives in case $h < 0$.

Lemma 3 *The threshold value* $\overline{\theta}^i$ *is smaller than* θ . *It is smaller than or equal to* $-\theta$ *if and only if*

$$\frac{h}{\theta} \geq \frac{2k}{k+1} \tag{19}$$

The threshold value $\overline{\theta}^j$ *is smaller than* $\theta + h$. *It is smaller than or equal to* $-\theta - h$ *if and only if*

$$\frac{h}{\theta} \leq -\frac{2k}{3k+1} \tag{20}$$

Two observations are in order. First of all, the fact that $\overline{\theta}^i < \theta$ and that $\overline{\theta}^j < \theta + h$ shows that the mere introduction of a second agent evaluating the projects excludes the possibility that an agent rejects all projects. Remember that in the single agent case Mr. i rejects all projects if $k > 1$ and h is sufficiently positive, while Ms. j rejects for $k > 1$ and h sufficiently negative. Secondly, note that it is impossible that both agents accept all projects irrespective of their valuations. The precise values for $\overline{\theta}^i$ and $\overline{\theta}^j$ that satisfy equations (15) and (18) are given in proposition 3.

Proposition 3 *The following threshold values for agent* i *and* j *are consistent:*

$$\begin{cases} \overline{\theta}^i = (\theta + h)\frac{k-1}{k+1} & \overline{\theta}^j \leq -\theta - h & : \ -\theta < h \leq -\theta\frac{2k}{3k+1} \\ \overline{\theta}^i = -\theta\frac{1}{2k+1} - h\frac{1+k}{2k+1} & \overline{\theta}^j = -\theta\frac{1}{2k+1} + h\frac{k}{2k+1} & : \ -\theta\frac{2k}{3k+1} < h < \theta\frac{2k}{k+1} \\ \overline{\theta}^i \leq -\theta & \overline{\theta}^j = \theta\frac{k-1}{k+1} & : \ h \geq \theta\frac{2k}{k+1} \\ \overline{\theta}^i \geq \theta & \overline{\theta}^j \geq \theta + h & : \ \forall(\theta, h, k) \end{cases} \tag{21}$$

The expected profit $E(\Pi|H(i,j))$ can now be calculated. It equals

$$X_1 \iint\limits_{\Theta_H^{ij}} \Pr(\theta^i + \theta^j > 0)\, d\theta^i d\theta^j - X_2 \iint\limits_{\Theta_H^{ij}} \Pr(\theta^i + \theta^j \leq 0)\, d\theta^i d\theta^j \tag{22}$$

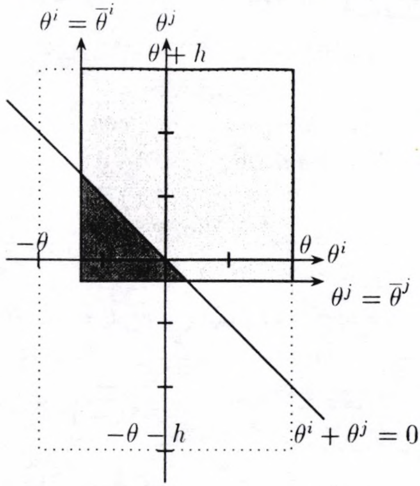

Figure 4: Profits and Losses in a Hierarchy

where $\Theta_H^{ij} = \{(\theta^i, \theta^j) \,|\, \theta^i \in (\overline{\theta}^i, \theta],\ \theta^j \in (\overline{\theta}^j, \theta + h]\}$. Figure 4 illustrates this expression.

The graph has been drawn under the assumption that $\overline{\theta}^i \in (-\theta, \theta)$ and $\overline{\theta}^j \in (-\theta + h, \theta + h)$, and, in particular $h \in (\theta 2k/(k+1))$. The area enclosed by the lines $\theta^j = \overline{\theta}^j$, $\theta^i = \overline{\theta}^i$, $\theta^j = \theta + h$, and $\theta^i = \theta$ contains all the projects with values (θ^i, θ^j) that a hierarchy accepts if the parameter values equal (θ, h, k), and satisfy the just mentioned restriction. The lightgrey area above the line $\theta^i + \theta^j = 0$ represents the area for which a profit will be made, while the grey area below and on the line $\theta^i + \theta^j = 0$ lead to a loss.

Proposition 4 *The expected profit of a hierarchy $E(\Pi|H(i,j))$ equals*

$$E(\Pi|H(i,j)) = \begin{cases} X_1 \dfrac{\theta - hk}{2\theta(k+1)} & : \quad -\theta < h \leq -\theta \dfrac{2k}{3k+1} \\ X_1 \dfrac{(1+k)(2\theta + h)^2}{8\theta(\theta + h)(2k+1)} & : \quad -\theta \dfrac{2k}{3k+1} < h < \theta \dfrac{2k}{k+1} \\ X_1 \dfrac{\theta + h(k+1)}{2(\theta + h)(k+1)} & : \quad h \geq \theta \dfrac{2k}{k+1} \end{cases} \tag{23}$$

Note that for both $h \in (-\theta, -\theta\, 2k/(3k+1))$ and for $h \geq \theta\, 2k/(k+1)$, the hierarchy consists of just one agent who evaluates "actively" projects; the other merely accepts all projects that arrive at its desk. It would be a mistake, though, to say that therefore the hierarchy turns into a single agent organization, since the precise parameter values for which just one agent is "active", and for which the other agent accepts all projects, does depend on the overall organizational structure.

The expressions for expected profits given above allow me to analyse an important issue, namely, the optimal ordering of agents with given characteristics within the hierarchy. Mr. i, who is characterized by valuations ranging from $-\theta$ to θ, should he be the first to evaluate a project or the second given that Ms. j 's valuations belong to the interval $[-\theta - h, \theta + h]$? The expressions I obtained until now are based on the assumption that Mr. i is the first to decide. Making Ms. j the first to consider projects simply means rewriting the above expressions, in particular the ones stating the expected profits, where θ should be replaced by $\theta' := \theta + h$, and h replaced by $h' := -h$. The following proposition can be shown to hold.

Proposition 5 *The ordering of heterogeneous agents does not affect the expected pay–off of a hierarchy.*

An important consequence of this result is that no knowledge regarding the agents' characteristics is required to find the optimal positioning of agents within a hierarchy. In other words, informational requirements in order to find the optimal ordering of a given pair of agents in a hierarchy are absent.

This result reflects the symmetry of the organizational maximization problem. Although Mr. i 's and Ms. j 's decision problems have been formulated in terms of finding a cut–off rule, they are equivalent to a single maximization problem; the best response functions $\overline{\theta}^i(\overline{\theta}^j)$ and $\overline{\theta}^j(\overline{\theta}^i)$ can be obtained as the first order conditions ensuing from a maximization of equation 22 with respect to $\overline{\theta}^i$ and $\overline{\theta}^j$, respectively. This problem is clearly symmetric in the agents' positions.

Just as the section on the single agent organization, this section concludes with a comparative statics result.

Corollary 2 *An increase in k leads to an decrease in the expected profits a hierarchy obtains, whatever the value of (θ, h).*

$$\frac{\partial E(\Pi|H)}{\partial k} < 0 \tag{24}$$

while any increase in the heterogeneity of the agents leads to an increase in the expected profits:

$$\begin{gathered}\frac{\partial E(\Pi|H; h>0)}{\partial h} > 0 \\ \frac{\partial E(\Pi|H; h<0)}{\partial h} < 0\end{gathered} \tag{25}$$

Although the *ordering* of heterogeneous agents does not determine the performance of a hierarchy, the *degree* of heterogeneity does affect the expected pay–off. Indeed, the corollary shows that the organization benefits from any increase in differences between agents.

5 Polyarchy

In this section I discuss the behaviour and performance of a polyarchy in which the same Mr. i and Ms. j operate within the same project–environment as in the last section. Just as in a hierarchy, Mr. i accepts a project if and only if the expected pay–off of acceptance is higher than the expected pay–off or rejection. The expected pay–off of rejection no longer equals zero, but equals the expected pay–off of a project that has been rejected by Mr. i but *accepted* by Ms. j . When Ms. j sits down to consider a project, she knows that it has been rejected by Mr. i so she knows that $\theta^i \leq \overline{\theta}^i$. Hence, she accepts a project if and only her valuation $\theta^j > \overline{\theta}^j$, where $\overline{\theta}^j$ solves

$$X_1 \Pr(\theta^i + \overline{\theta}^j > 0 |\ \theta^i \leq \overline{\theta}^i) - X_2 \Pr(\theta^i + \overline{\theta}^j \leq 0 |\ \theta^i \leq \overline{\theta}^i) = 0 \tag{26}$$

Suppose $\overline{\theta}^i \geq \theta$, *i.e.*, Mr. i rejects all projects. Ms. j 's reaction to $\overline{\theta}^i > \theta$ and to $\overline{\theta}^i = \theta$ are identical. In case $\overline{\theta}^i \leq -\theta$, Ms. j receives no projects at all, as all are directly accepted by Mr. i . Hence $\overline{\theta}^j$ can be freely chosen. Finally, for $\overline{\theta}^i \in (-\theta, \theta)$, equation 26 can be rewritten as

$$X_1 \frac{\overline{\theta}^i + \overline{\theta}^j}{\theta + \overline{\theta}^i} - X_2 \frac{-\overline{\theta}^j + \theta}{\theta + \overline{\theta}^i} = 0 \tag{27}$$

Therefore, in a polyarchy $\overline{\theta}^j(\overline{\theta}^i)$ equals

$$\begin{cases} \theta \frac{k-1}{k+1} & : \quad \overline{\theta}^i \geq \theta \\ -\overline{\theta}^i \frac{1}{1+k} + \theta \frac{k}{1+k} & : \quad \overline{\theta}^i \in (-\theta, \theta) \\ (-\infty, \infty) & : \quad \overline{\theta}^i \leq -\theta \end{cases} \tag{28}$$

Mr. i 's decision rule can now be stated as follows.

$$\text{Mr. } i \text{ accepts if and only if } \theta^i \text{ satisfies}$$
$$X_1 \Pr(\theta^i + \theta^j > 0) - X_2 \Pr(\theta^i + \theta^j \leq 0) >$$
$$\Pr(\theta^j > \overline{\theta}^j)\left(X_1 \Pr(\theta^i + \theta^j > 0 |\, \theta^j > \overline{\theta}^j) - X_2 \Pr(\theta^i + \theta^j \leq 0 |\, \theta^j > \overline{\theta}^j)\right)$$

For every value of $\overline{\theta}^j$, lemma 4 states Mr. i 's best response $\overline{\theta}^i(\overline{\theta}^j)$.

Lemma 4 *Mr. i 's best response function $\overline{\theta}^i(\overline{\theta}^j)$ takes on the following form*

$$\begin{cases} [\theta, \infty) & : \quad \overline{\theta}^j \leq -\theta - h \\ -\overline{\theta}^j \frac{1}{1+k} + (\theta + h)\frac{k}{1+k} & : \quad \overline{\theta}^j \in (-\theta - h, \theta + h) \\ (\theta + h)\frac{k-1}{k+1} & : \quad \overline{\theta}^j \geq \theta + h \end{cases} \tag{29}$$

Equations (28) and (29) are drawn in figure 5 for the case where $h > 0$.

Note that in case of $h > 0$ the intersection is always to the right of the line $\overline{\theta}^i = -\theta$ and in between the lines $\overline{\theta}^j = \theta + h$ and $\overline{\theta}^j = -\theta - h$. Hence, Ms. j 's action always depends on the signal she receives, while Mr. i 's approval of a project depends on his valuation if and only if the intersection falls within the inner rectangular marked by the dotted lines. Lemma 5 specifies the conditions under which decisions depend on the signals agents obtain. Hence, similar to a hierarchy, there are parameter values such that decisions are taken irrespective of the agents' signal. Contrary to a hierarchy, however, projects are not unconditionally accepted but possibly

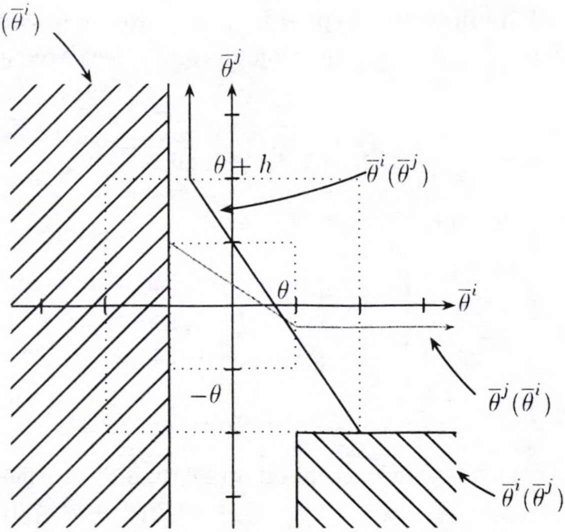

Figure 5: The best replies of i and j in a polyarchy

rejected irrespective of the, possibly high, valuation. The interest of this observation is that, although Mr. i can make the organization accept a project by accepting it himself, he will never do so independent of his private valuation.

Lemma 5 *The threshold value $\overline{\theta}^i$ is larger than $-\theta$. It is larger than or equal to θ if and only if*

$$\frac{h}{\theta} \geq \frac{2}{k+1} \tag{30}$$

The threshold value $\overline{\theta}^j$ is larger than $-\theta - h$. It is larger than or equal to $\theta + h$ if and only if

$$\frac{h}{\theta} \leq -\frac{2}{k+3} \tag{31}$$

In other words, the mere introduction of a second agent ensures that nor Mr. i nor Ms. j accepts a project independent of his or her valuation. This should be compared to the single agent case in which the whole pool of projects may get accepted without evaluation, and to the hierarchy in which projects may get unconditionally accepted, but not rejected. Moreover, with increasing k, the area of the parameter space in which a project may get accepted by any agent at all shrinks until it becomes infinitesimally small for $k \to \infty$.

The threshold values $\overline{\theta}^i$ and $\overline{\theta}^j$ that satisfy equations (28) and (29) are given by proposition 6.

Proposition 6 *The following threshold values for agent i and j are consistent:*

$$\begin{cases} \overline{\theta}^i = (\theta + h)\frac{k-1}{k+1} & \overline{\theta}^j \geq \theta + h & : \; -\theta < h \leq -\theta\frac{2}{k+3} \\ \overline{\theta}^i = \theta\frac{k}{k+2} + h\frac{k+1}{k+2} & \overline{\theta}^j = \theta\frac{k}{k+2} - h\frac{1}{k+2} & : \; -\theta\frac{2}{k+3} < h < \theta\frac{2}{k+1} \\ \overline{\theta}^i \geq \theta & \overline{\theta}^j = \theta\frac{k-1}{k+1} & : \; h \geq \theta\frac{2}{k+1} \end{cases} \tag{32}$$

In a polyarchy, pay–offs are determined by the projects accepted by either Mr. i or Ms. j . That is, $E(\Pi|P(i,j))$ equals

$$X_1 \iint\limits_{\Theta_P^i} \Pr(\theta^i + \theta^j > 0)\, d\theta^i d\theta^j - X_2 \iint\limits_{\Theta_P^i} \Pr(\theta^i + \theta^j \leq 0)\, d\theta^i d\theta^j + \tag{33}$$

$$X_1 \iint\limits_{\Theta_P^{ij}} \Pr(\theta^i + \theta^j > 0)\, d\theta^i d\theta^j - X_2 \iint\limits_{\Theta_P^{ij}} \Pr(\theta^i + \theta^j \leq 0)\, d\theta^i d\theta^j$$

where $\Theta_P^i = \{(\theta^i,\theta^j) \,|\, \theta^i \in (\overline{\theta}^i, \theta],\, \theta^j \in [-\theta - h, \theta + h]\}$, and $\Theta_P^{ij} = \{(\theta^i,\theta^j) \,|\, \theta^i \in [-\theta, \overline{\theta}^i),\, \theta^j \in (\overline{\theta}^j, \theta + h]\}$. Figure 6 illustrates equation (33). The light grey area represents all those (θ^i, θ^j) where a profit is made on accepted projects, while the grey indicate the area where a polyarchy incurs a loss. The graph has been drawn under the assumption that $0 < h < \theta\frac{2}{k+1}$.

Proposition 7 *The expected profit $E(\Pi|P(i,j))$ in a polyarchy equals*

$$\begin{cases} X_1 \dfrac{\theta - hk}{2\theta(k+1)} & : \; -\theta < h \leq -\theta\dfrac{2}{k+3} \\ X_1 \dfrac{h^2 k(k+1) + 8\theta(\theta + h)}{8\theta(\theta + h)(k+2)} & : \; -\theta\dfrac{2}{k+3} < h < \theta\dfrac{2}{k+1} \\ X_1 \dfrac{\theta + h(k+1)}{2(\theta + h)(k+1)} & : \; h \geq \theta\dfrac{2}{k+1} \end{cases} \tag{34}$$

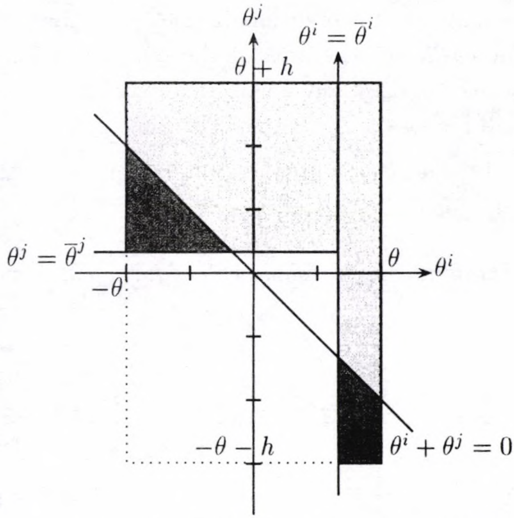

Figure 6: Profits and Losses in a Polyarchy

Just as in a hierarchy, the ordering of heterogeneous agents does not affect organizational performance.

Proposition 8 *The ordering of heterogeneous agents does not affect the expected pay–off of a polyarchy.*

The intuition is once again the symmetry of the organizational decision problem. Note that, in a polyarchy, the organizational decision problem of maximizing the value of accepted projects is equivalent to minimizing the value of rejected projects. Since the probability that a project is rejected is the product of the probability that Mr. i rejects and the probability that Ms. j rejects, the minimization problem is clearly symmetric in the positioning of the agents. Hence, the related maximization problem is symmetric as well.

Corollary 3 shows that for all parameter values, increasing heterogeneity contributes positively to the organization's performance. Moreover, it shows that a tougher project selection environment decreases the performance of a hierarchy.

Corollary 3 *Any increase in the heterogeneity of the agents contributes positively to the performance of a polyarchy:*

$$\begin{aligned} \frac{\partial E(\Pi|P;h>0)}{\partial h} &> 0 \\ \frac{\partial E(\Pi|P;h<0)}{\partial h} &< 0 \end{aligned} \tag{35}$$

while any increase in the ratio k leads to a decrease in expected profits:

$$\frac{\partial E(\Pi|P)}{\partial k} < 0 \tag{36}$$

6 Omniarchy

The last organizational architecture I discuss is the omniarchy. In this organizational structure, whatever Mr. i 's decision, Ms. j evaluates the project as well. When analyzing a project, she knows the decision Mr. i took. Just as in the preceeding sections, Ms. j 's decision rule is of the threshold type, but, since the projects that reach her have either been rejected or accepted by Mr. i , there is a threshold value for θ^j corresponding to either decision. Let $\overline{\theta}^{jR}$ stand for the threshold value in case i rejected a project, and let $\overline{\theta}^{jA}$ denote the cut–off value if i accepted a project. Since the expected profit of a project rejected by j equals zero, j accepts projects with a positive expected pay–off. As a consequence, the relationship between $\overline{\theta}^i$ and $\overline{\theta}^{jA}$ is the same as the relationship between $\overline{\theta}^i$ and $\overline{\theta}^j$ in a hierarchy, while the relationship between $\overline{\theta}^i$ and $\overline{\theta}^{jR}$ coincides with the relationship between $\overline{\theta}^i$ and $\overline{\theta}^j$ in case of a polyarchy. They are repeated here for the sake of convenience.

$$\overline{\theta}^{jR}(\overline{\theta}^i) = \begin{cases} \theta\frac{k-1}{k+1} & : \ \overline{\theta}^i \geq \theta \\ -\overline{\theta}^i\frac{1}{k+1} + \theta\frac{k}{k+1} & : \ \overline{\theta}^i \in (-\theta, \theta) \\ (-\infty, \infty) & : \ \overline{\theta}^i \leq -\theta \end{cases} \tag{37}$$

$$\overline{\theta}^{jA}(\overline{\theta}^i) = \begin{cases} (-\infty, \infty) & : \ \overline{\theta}^i \geq \theta \\ -\overline{\theta}^i\frac{k}{k+1} - \theta\frac{1}{k+1} & : \ \overline{\theta}^i \in (-\theta, \theta) \\ \theta\frac{k-1}{k+1} & : \ \overline{\theta}^i \leq -\theta \end{cases} \tag{38}$$

Intuitively, if Mr. i's threshold level is such that he accepts some projects, while he rejects others, then projects that he accepted should meet a less severe test on Ms. j's desk than projects that Mr. i has rejected. That is, if $\overline{\theta}^i \in (-\theta, \theta)$, one would expect $\overline{\theta}^{jR} > \overline{\theta}^{jA}$. It is straightforward to show that this holds for equations (37) and (38). Mr. i 's decision rule can be stated as Accept if and only if θ^i satisfies

$$\Pr(\theta^j > \overline{\theta}^{jA}) \left(X_1 \Pr(\theta^i + \theta^j > 0 | \theta^j > \overline{\theta}^{jA}) - X_2 \Pr(\theta^i + \theta^j \leq 0 | \theta^j > \overline{\theta}^{jA}) \right)$$

is larger than

$$\Pr(\theta^j > \overline{\theta}^{jR}) \left(X_1 \Pr(\theta^i + \theta^j > 0 | \theta^j > \overline{\theta}^{jR}) - X_2 \Pr(\theta^i + \theta^j \leq 0 | \theta^j > \overline{\theta}^{jR}) \right)$$

Lemma 6 shows that this decision rule is still of the threshold type.

Lemma 6 *The decision rule of agent i takes on the following form. If $\theta^i > \overline{\theta}^i$, where $\overline{\theta}^i$ is defined as*

$$\begin{cases} \overline{\theta}^i = -\frac{1}{1+k}\left(\overline{\theta}^{jR} - k(\theta + h)\right) & : \begin{array}{l} \overline{\theta}^{jR} \in (-\theta - h, \theta + h) \text{ and} \\ \overline{\theta}^{jA} \leq -\theta - h; \end{array} \\ \overline{\theta}^i = -\frac{1}{1+k}\left(\theta + h + k\overline{\theta}^{jA}\right) & : \begin{array}{l} \overline{\theta}^{jR} \geq \theta + h \text{ and} \\ \overline{\theta}^{jA} \in (-\theta - h, \theta + h); \end{array} \\ \overline{\theta}^i = -\frac{1}{1+k}\left(\overline{\theta}^{jR} + k\overline{\theta}^{jA}\right) & : \begin{array}{l} \overline{\theta}^{jR}, \overline{\theta}^{jA} \in (-\theta - h, \theta + h) \text{ and} \\ \overline{\theta}^{jR} > \overline{\theta}^{jA}; \end{array} \\ \overline{\theta}^i = \frac{k-1}{k+1}(\theta + h) & : \begin{array}{l} \overline{\theta}^{jR} \geq \theta + h \text{ and} \\ \overline{\theta}^{jA} \leq -\theta - h; \end{array} \end{cases} \tag{39}$$

the project will be accepted; all projects are rejected (i.e., $\overline{\theta}^i \geq \theta$) if

$$\begin{cases} \overline{\theta}^{jR}, \overline{\theta}^{jA} \geq \theta + h \text{ or} \\ \overline{\theta}^{jR}, \overline{\theta}^{jA} \leq -\theta - h \text{ or} \\ \overline{\theta}^{jR} = \overline{\theta}^{jA} \in (-\theta - h, \theta + h). \end{cases} \tag{40}$$

Finally, for $\theta^i < \overline{\theta}^i$, and $\overline{\theta}^i$ defined as follows

$$\begin{cases} \overline{\theta}^i = -\frac{1}{1+k}\left(\overline{\theta}^{jA} - k(\theta+h)\right) & : \quad \begin{array}{l} \overline{\theta}^{jR} \leq -\theta - h \text{ and} \\ \overline{\theta}^{jA} \in (-\theta-h, \theta+h); \end{array} \\ \overline{\theta}^i = -\frac{1}{1+k}\left(\theta + h + k\overline{\theta}^{jR}\right) & : \quad \begin{array}{l} \overline{\theta}^{jA} \geq \theta + h \text{ and} \\ \overline{\theta}^{jR} \in (-\theta-h, \theta+h); \end{array} \\ \overline{\theta}^i = -\frac{1}{1+k}\left(\overline{\theta}^{jA} + k\overline{\theta}^{jR}\right) & : \quad \begin{array}{l} \overline{\theta}^{jR}, \overline{\theta}^{jA} \in (-\theta-h, \theta+h) \text{ and} \\ \overline{\theta}^{jR} < \overline{\theta}^{jA}; \end{array} \\ \overline{\theta}^i = \frac{k-1}{k+1}(\theta+h) & : \quad \begin{array}{l} \overline{\theta}^{jR} \leq -\theta - h \text{ and} \\ \overline{\theta}^{jA} \geq \theta + h; \end{array} \end{cases} \tag{41}$$

the project is accepted as well.

Although Mr. i still uses a threshold value, it is not necessarily the case that only those projects with a sufficiently *large* value will be accepted. Indeed, if $\overline{\theta}^{jR} < \overline{\theta}^{jA}$, and $\overline{\theta}^{jx} \in (-\theta - h, \theta + h)$ (for $x \in \{A, R\}$, then Mr. i accepts projects only if their valuations are sufficiently low.

The next step in the analysis is to find the vector of consistent threshold values $(\overline{\theta}^i, \overline{\theta}^{jR}, \overline{\theta}^{jA})$. Since there are now three values to be determined, graphical methods, like the ones used in the preceeding sections are not very revealing.

Equations (37), (38), and (41) can never lead to a consistent set of threshold values, as equations (37) and (38) have been derived on the basis of the assumptions that Mr. i accepts if and only if $\theta^i > \overline{\theta}^i$, while the threshold values defined by (41) are such that a project is accepted if and only if $\theta^i < \overline{\theta}^i$. In section 6.1, I derive consistent threshold values under the assumption that Mr. i accepts if and only if $\theta^i \leq \overline{\theta}^i$. The idea is to determine Ms. j 's reaction function $\overline{\theta}^{jR}(\overline{\theta}^i)$ and $\overline{\theta}^{jA}(\overline{\theta}^i)$ under the assumption that Mr. i accepts if and only if $\theta^i \leq \overline{\theta}^i$.

Note as well that in certain cases an omniarchy turns into a hierarchy or a polyarchy; if, for example, Ms. j rejects all projects, whatever her valuation, *i.e.*, if $\overline{\theta}^{jR} \geq \theta + h$, then a project rejected by Mr. i implies rejection by the organization as a whole. That is, an omniarchy with

$\overline{\theta}^{jR} \geq \theta + h$ equals a hierarchy. The idea is then to check whether consistent threshold values for a hierarchy can be reconciled with $\overline{\theta}^{jR}$ as a best response to $\overline{\theta}^{i}$ can satisfy $\overline{\theta}^{jR}(\overline{\theta}^{i}) \geq \theta + h$. In this way, all consistent threshold values for $\overline{\theta}^{jR} \geq \theta + h$ and/or $\overline{\theta}^{jA} \leq -\theta - h$ can be found. The remaining cases can then be dealt with by analysing the consistency for $\overline{\theta}^{i} \leq -\theta$, $\overline{\theta}^{i} \in (-\theta, \theta)$, and $\overline{\theta}^{i} \geq \theta$.

Proposition 9 *If i accepts if and only if $\theta^i > \overline{\theta}^{i}$, and j accepts if and only if $\theta^j > \overline{\theta}^{jx}$, where $x \in \{A, R\}$, then the following values are consistent for $\left(\overline{\theta}^{i}, \overline{\theta}^{jR}, \overline{\theta}^{jA}\right)$*[2]*:*

$$
\begin{array}{ll}
\left((\theta + h)\frac{k-1}{k+1}, \theta + h, -\theta - h\right) & : \begin{cases} k \leq 1, \; h \in \left(-\theta, -\theta\frac{2}{k+3}\right) \\ k > 1, \; h \in \left(-\theta, -\theta\frac{2k}{3k+1}\right) \end{cases} \\
\left(-\theta\frac{1}{2k+1} - h\frac{1+k}{2k+1}, \theta + h, -\theta\frac{1}{2k+1} + h\frac{k}{2k+1}\right) & : h \in \left(-\theta\frac{2k}{3k+1}, -\theta\frac{1}{k+1}\right) \\
\left(-\theta, \theta + h, \theta\frac{k-1}{k+1}\right) & : h \geq \theta\frac{2k}{k+1} \\
\left(\theta, \theta\frac{k-1}{k+1}, \theta + h\right) & : h \in \left(-\theta, -\theta\frac{2}{k+1}\right] \\
\left(\theta, \theta\frac{k-1}{k+1}, -\theta - h\right) & : h \geq \theta\frac{2}{k+1} \\
\left(\theta\frac{k}{k+2} + h\frac{k+1}{k+2}, \theta\frac{k}{k+2} - h\frac{1}{k+2}, -\theta - h\right) & : h \in \left(-\theta\frac{2}{k+3}, -\theta\frac{k}{k+1}\right] \\
\left(\theta, \theta\frac{k-1}{k+1}, \theta\frac{k-1}{k+1}\right) & : \begin{cases} k \leq 1, \; h > -\theta\frac{2k}{k+1} \\ k > 1, \; h > -\theta\frac{2}{k+1} \end{cases} \\
\left(0, \theta\frac{k}{k+1}, -\theta\frac{1}{k+1}\right) & : \begin{cases} k \leq 1, \; h > -\theta\frac{k}{k+1} \\ k > 1, \; h > -\theta\frac{k}{k+1} \end{cases}
\end{array}
\tag{42}
$$

Finally, for $h > \theta\frac{2}{k+1}$, $\overline{\theta}^{i} \geq \theta, \overline{\theta}^{jR} = \theta\frac{k-1}{k+1}$, $\overline{\theta}^{jA} \leq -\theta\frac{k+3}{k+1}$ is a consistent triple of cut-off values, whereas for $h > \theta\frac{2k}{k+1}$, a consistent triple of threshold values equals $\overline{\theta}^{i} \leq -\theta, \overline{\theta}^{jR} \geq \theta\frac{3k+1}{k+1}$, and $\overline{\theta}^{jA} = \theta\frac{k-1}{k+1}$ is .

For many parameter values different vectors of threshold values are consistent. Indeed, there are three different ways in which different vectors of threshold values can be consistent. The most trivial example is the one given in footnote 2; for, say, $h \geq \theta\frac{2}{k+1}$, $(\theta, \theta\frac{k-1}{k+1}, -\theta - h)$ is a consistent

[2] It should be noted that if $(\overline{\theta}^{i}, \overline{\theta}^{jR}, \overline{\theta}^{jA})$ equals, say, $(\theta, \theta\frac{k-1}{k+1}, -\theta - h)$ this means that $\overline{\theta}^{i} \geq \theta$ and $\overline{\theta}^{jA} \leq -\theta - h$, which does not imply any difference in either i's or j's behaviour.

threshold value, but so is, for every $\epsilon_1, \epsilon_2 > 0$, $(\theta + \epsilon_1, \theta\frac{k-1}{k+1}, -\theta - h - \epsilon_2)$. Mr. i 's threshold value can be increased by any number without affecting in any way Mr. i 's behaviour and Ms. j's best reply. That $\overline{\theta}^{jA}$ can be decreased holds for the same reason. No change in expected pay-off is implied.

The second type can be illustrated by, say $h > \theta\frac{2}{k+1}$, implying that $\overline{\theta}^i = \theta, \overline{\theta}^{jR} = \theta\frac{k-1}{k+1}$, and $\overline{\theta}^{jA} \leq -\theta\frac{k+3}{k+1}$ is a consistent triple of cut-off values. Here, $\overline{\theta}^{jA}$ can take on any value satisfying $\overline{\theta}^{jA} = (-\theta - h, -\theta\frac{k+3}{k+1}]$. Note once again that such differences are inconsequential as far as individual behaviour given the consistent threshold values is concerned; since $\overline{\theta}^i \geq \theta$, all projects will be rejected, and Ms. j therefore never applies, as it were, $\overline{\theta}^{jA}$. Once again, expected profits are not affected by such changes.

And finally the third type of multiple consistent threshold values. For $h \geq \theta\frac{2k}{k+1}$ and $k \geq 1$, for example, there are four different vectors of consistent threshold values: $(-\theta, \theta + h, \theta\frac{k-1}{k+1})$, $(\theta, \theta\frac{k-1}{k+1}, -\theta - h)$, $(\theta, \theta\frac{k-1}{k+1}, \theta\frac{k-1}{k+1})$, and $(0, \theta\frac{k}{k+1}, -\theta\frac{1}{k+1})$. Three of these vectors imply behaviour that is qualitatively different one from the other, *i.e.*, behaviour that is or is not guided by the signal the agent obtains; in the first case, Mr. i accepts all projects, and Ms. j accepts some of these; in the second and third case, Mr. i rejects all projects, while Ms. j accepts some of these; in the fourth case, Mr. i accepts on average one out of two projects, while whatever his choice, Ms. j accepts some and rejects others. Let me call two vectors of threshold levels qualitatively different if the implied behaviour of the agents is qualitatively different.

Corollary 4 *In an omniarchy, for almost every vector of parameters* (θ, h, k) *there are at least two qualitatively distinct vectors of consistent cut-off values* $(\overline{\theta}^i, \overline{\theta}^{jR}, \overline{\theta}^{jA})$. *The only exception is the vector* $(\theta, h, k) = (\theta, -\frac{1}{2}\theta, 1)$.

Qualitatively different consistent threshold values may give rise to differences in organizational performance. I now derive expressions for the expected profit in an omniarchy. The general expression for $E(\Pi|O(i, j))$

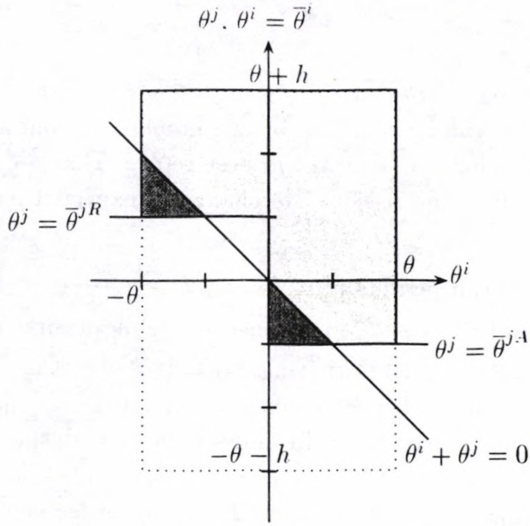

Figure 7: Profits and Losses in an Omniarchy

amounts to

$$X_1 \iint\limits_{\Theta_O^{ijA}} \Pr(\theta^i + \theta^j > 0)\, d\,\theta^i d\,\theta^j - X_2 \iint\limits_{\Theta_O^{ijA}} \Pr(\theta^i + \theta^j \leq 0)\, d\,\theta^i d\,\theta^j + \tag{43}$$
$$X_1 \iint\limits_{\Theta_O^{ijR}} \Pr(\theta^i + \theta^j > 0)\, d\,\theta^i d\,\theta^j - X_2 \iint\limits_{\Theta_O^{ijR}} \Pr(\theta^i + \theta^j \leq 0)\, d\,\theta^i d\,\theta^j$$

where $\Theta_O^{ijA} = \{(\theta^i, \theta^j)\,|\,\theta^i \in (\overline{\theta}^i, \theta],\ \theta^j \in (\overline{\theta}^{jA}, \theta + h]\,\}$, and analogously $\Theta_O^{ijR} = \{(\theta^i, \theta^j)\,|\,\theta^i \in [-\theta, \overline{\theta}^i),\ \theta^j \in (\overline{\theta}^{jR}, \theta + h]\,\}$. For $\overline{\theta}^i \in (-\theta, \theta)$, and $\overline{\theta}^{jR}, \overline{\theta}^{jA} \in (-\theta - h, \theta + h)$, figure 7 represents the accepted, good or bad, projects.

Because of the abundance of consistent threshold values, I limit attention to two special cases. I either assume that the possible loss on a project is as large as its possible profit, *i.e.*, $k = 1$, or I assume that Mr. i and Ms. j have the same characteristics, *i.e.*, $h = 0$.

First the case where agents are identical, or $h = 0$. Proposition 9 shows that there are just two possible vectors of consistent threshold values: either $(\theta, \theta\frac{k-1}{k+1}, \theta\frac{k-1}{k+1})$ or $(0, \theta\frac{k}{k+1}, -\theta\frac{1}{k+1})$. Proposition 10 below shows that the expected profits of an omniarchy applying the latter set of threshold values dominates for all k and θ the expected profits of an omniarchy applying the former.

Proposition 10 *The expected profit of an omniarchy for $h = 0$ and with $(\overline{\theta}^i, \overline{\theta}^{jR}, \overline{\theta}^{jA}) = (\theta, \theta\frac{k-1}{k+1}, \theta\frac{k-1}{k+1})$ equals*

$$E(\Pi|O(i,j); (\theta, \theta\frac{k-1}{k+1}, \theta\frac{k-1}{k+1})) = \frac{X_1}{2}\frac{1}{k+1} \tag{44}$$

The expected pay-off of an omniarchy with $h = 0$ and with $(\overline{\theta}^i, \overline{\theta}^{jR}, \overline{\theta}^{jA})$ equal to $(0, \theta\frac{k}{k+1}, -\theta\frac{1}{k+1})$ amounts to

$$E(\Pi|O(i,j); (0, \theta\frac{k}{k+1}, -\theta\frac{1}{k+1})) = \frac{X_1}{4}\frac{k+2}{k+1} \tag{45}$$

The latter set of threshold values gives rise to the highest expected profit.

A direct corollary of this proposition is that any increase in the ratio k leads to a decrease in expected profits:

Corollary 5

$$\frac{\partial E(\Pi|O(i,j); (0, \theta\frac{k}{k+1}, -\theta\frac{1}{k+1}))}{\partial k} \tag{46}$$

That is, even in an omniarchy, implying a second test of each and every project, increasing the ratio k leads to a decrease in expected profits.

I now limit attention to the case where $k = 1$. As agents can now be heterogeneous, an important question that should be addressed is who should be the first to look at a project, Mr. i or Ms. j ? As there are multiple vectors of consistent cut-off values possible for every segment of the line $k = 1$, except for $h = -\frac{1}{2}\theta$, a choice has to be made from among these vectors. For every pair of parameters (θ, h) and for either ordering of the agents I take the vector of threshold values that maximizes the expected profit. Then, for every pair (θ, h), I compare the expected profits of these vectors.

Proposition 11 *In an omniarchy, the ordering of heterogeneous agents matters. For $k = 1$, the agent with the smallest variance should be the first to analyze a project in order to maximize expected pay-offs.*

This proposition shows that, contrary to both a hierarchy and a polyarchy, but like a single agent organization, the ordering of heterogeneous agents matters in an omniarchy. The intuition is that, although the second agent's cut-off rule reflects the overall organizational structure and in particular the decision taken by the first agent, in a certain sense, it is the second agent who decides. The first agent merely signals information it has. When the second agent has to decide on the project, it knows its own valuation, but is uncertain about the other agent's. Hence, just as in a single agent case, the second agent prefers a more informative signal to a less informative one. Therefore, the first agent should be the one with the smallest variance.

The effect of increasing differences between agents on organizational performance is positive for any vector (θ, h), as the following corollary shows.

Corollary 6 *For any vector of parameters (θ, h), and for $k = 1$, an increase in the difference between agents has a positive effect on the performance of a hierarchy.*

6.1 Acceptance for low valuations

As I noted above, that part of Mr. i 's best response which requires the acceptance of relatively low valued projects cannot lead to consistent threshold values since until now the best reply functions of Ms. j are based on the assumption that Mr. i *rejects* for low values. That is, equations (41) cannot lead to consistent threshold values since equations (37) and (38) are based on the assumption that Mr. i accepts if and only if $\theta^i > \overline{\theta}^i$. What, however, would happen if the cut-off values Ms. j applies are based on the assumption that projects with a sufficiently *low* value have been accepted by Mr. i ? That is, what if Mr. i were known to accept if and only if $\theta^i \leq \overline{\theta}^i$ for some $\overline{\theta}^i$?. The best reply cut-off rule $\overline{\theta}^{jA}(\overline{\theta}^i)$ derived in the preceeding section, and applied by Ms. j in case of projects accepted by Mr. i would now be *identical* to the cut–off rule applied by Ms. j in case of projects *rejected* by Mr. $i$when acceptance requires $\theta^i \leq \overline{\theta}^i$. Ms. j still accepts only those projects with a sufficiently high value θ^j, and lemma 6, stating $\overline{\theta}^i$ as a function of $(\overline{\theta}^{jR}, \overline{\theta}^{jA})$, therefore still applies. Since Ms. jś reaction function is now based on the assumption that Mr. i acceptes if and only if $\theta^i \leq \overline{\theta}^i$ part (39) no longer leads to consistent threshold values: indeed, only parts (40) and (41) may give rise to consistent values.

That only part (40) and (41) can lead to consistent cut–off values can be seen as follows as well. Suppose $-\theta \leq \underline{\alpha}^i < \overline{\alpha}^i \leq \theta$, where $\underline{\alpha}^i$ and $\overline{\alpha}^i$ are two signals obtained by Mr. i . What are the necessary conditions on $\overline{\theta}^{jR}$ and $\overline{\theta}^{jA}$ for it to be possible that the project with $\underline{\alpha}^i$ will be accepted by i, but the project with $\overline{\alpha}^i$ will be rejected? Suppose the omniarchy is such that all projects rejected by Mr. i are rejected by Ms. j as well, but that a project accepted by i may or may not be accepted by j. That is, $\overline{\theta}^{jR} \geq \theta + h > \overline{\theta}^{jA} > -\theta - h$, or the omniarchy is equivalent to a hierarchy. A project Mr. i rejects is rejected for sure by the organization, while a project he accepts may get accepted by the organization. Therfore, since Mr. i is rational, he does not want to reject an apparently more profitable project with valuation $\overline{\alpha}^i$ and at the same time accept a seemingly less profitable project characterized by $\underline{\alpha}^i$. The same line of reasoning applies to the polyarchy, and indeed to any omniarchy in which $\overline{\theta}^{jR} > \overline{\theta}^{jA}$. That is, part (39) , which is based on $\overline{\theta}^{jR} > \overline{\theta}^{jA}$ cannot be consistent with Mr. i accepting relatively low valued projects.

Consequently, if Mr. i accepts only those projects θ^i with $\theta^i \leq \overline{\theta}^i$ the best reply cut–off rules are as follows:

$$\overline{\theta}^{jA}(\overline{\theta}^i) = \begin{cases} \theta \frac{k-1}{k+1} & : \quad \overline{\theta}^i \geq \theta \\ -\overline{\theta}^i \frac{1}{1+k} + \theta \frac{k}{1+k} & : \quad \overline{\theta}^i \in (-\theta, \theta) \\ (-\infty, \infty) & : \quad \overline{\theta}^i \leq -\theta \end{cases} \tag{47}$$

$$\overline{\theta}^{jR}(\overline{\theta}^i) = \begin{cases} (-\infty, \infty) & : \quad \overline{\theta}^i \geq \theta \\ -\overline{\theta}^i \frac{k}{1+k} - \theta \frac{1}{1+k} & : \quad \overline{\theta}^i \in (-\theta, \theta) \\ \theta \frac{k-1}{k+1} & : \quad \overline{\theta}^i \leq -\theta \end{cases} \tag{48}$$

Mr. i accepts if and only if $\theta^i \leq \overline{\theta}^i$, where $\overline{\theta}^i = \overline{\theta}^i(\overline{\theta}^{jR}, \overline{\theta}^{jA})$ satisfies

$$\begin{cases} \overline{\theta}^i = -\frac{1}{1+k}\left(\overline{\theta}^{jA} - k(\theta+h)\right) & : \begin{cases} \overline{\theta}^{jR} \leq -\theta - h \text{ and} \\ \overline{\theta}^{jA} \in (-\theta - h, \theta + h); \end{cases} \\ \overline{\theta}^i = -\frac{1}{1+k}\left(\theta + h + k\overline{\theta}^{jR}\right) & : \begin{cases} \overline{\theta}^{jA} \geq \theta + h \text{ and} \\ \overline{\theta}^{jR} \in (-\theta - h, \theta + h); \end{cases} \\ \overline{\theta}^i = -\frac{1}{1+k}\left(\overline{\theta}^{jA} + k\overline{\theta}^{jR}\right) & : \begin{cases} \overline{\theta}^{jR}, \overline{\theta}^{jA} \in (-\theta - h, \theta + h) \text{ and} \\ \overline{\theta}^{jR} < \overline{\theta}^{jA}; \end{cases} \\ \overline{\theta}^i = \frac{k-1}{k+1}(\theta+h) & : \begin{cases} \overline{\theta}^{jR} \leq -\theta - h \text{ and} \\ \overline{\theta}^{jA} \geq \theta + h; \end{cases} \end{cases} \tag{49}$$

or all projects are accepted (*i.e.*, $\overline{\theta}^i \geq \theta$) for

$$\begin{cases} \overline{\theta}^{jR}, \overline{\theta}^{jA} \geq \theta + h \textit{ or} \\ \overline{\theta}^{jR}, \overline{\theta}^{jA} \leq -\theta - h \textit{ or} \\ \overline{\theta}^{jR} = \overline{\theta}^{jA} \in (-\theta - h, \theta + h). \end{cases} \tag{50}$$

The following proposition can now be seen to hold.

Proposition 12 *Suppose* $(\overline{\theta}^i, \overline{\theta}^{jR}, \overline{\theta}^{jA}) = (\overline{\theta}^{i*}, \overline{\theta}^{jR*}, \overline{\theta}^{jA*})$ *is a consistent vector of threshold values solving equations (37), (38), (39), and (40) for a particular vector* (θ, h, k) *under the assumption that* i *accepts if and only if* $\theta^i > \overline{\theta}^i$. *Then the vector* $(\overline{\theta}^i, \overline{\theta}^{jR}, \overline{\theta}^{jA}) = (\overline{\theta}^{i*}, \overline{\theta}^{jA*}, \overline{\theta}^{jR*})$ *is a consistent vector of threshold values solving equations (47), (48), (49), and (50) under the assumption that* i *accepts if and only if* $\theta^i \leq \overline{\theta}^i$. *The converse of this proposition holds as well.*

An example may help to illustrate. Suppose that the $(\overline{\theta}^i, \overline{\theta}^{jR}, \overline{\theta}^{jA}) = (5, 12, 7)$ is a vector of consistent threshold values for some vector of parameters (θ, h, k), and assume that a project, characterized by $(\theta^i, \theta^j) = (9, 8)$ is analyzed by the organization. Let me compare the organizational decision this project encounters when *(i)* Mr. i accepts a project if and only if $\theta^i > \overline{\theta}^i$, and the threshold values equal $(\overline{\theta}^i, \overline{\theta}^{jR}, \overline{\theta}^{jA}) = (5, 12, 7)$ with the situation *(ii)* in which Mr. i accepts if and only if $\theta^i \leq \overline{\theta}^i$, and the cut–off values equal $(\overline{\theta}^i, \overline{\theta}^{jR}, \overline{\theta}^{jA}) = (5, 7, 12)$. In *(i)*, $9 = \theta^i > \overline{\theta}^i = 5$, and so the project will be accepted by Mr. iThen Ms. j decides to accept the project as $8 = \theta^j > \overline{\theta}^{jA} = 7$, so the project will be accepted by the organization as a whole. In *(ii)* on the other hand, Mr. i will reject the project since $9 = \theta^i > \overline{\theta}^i = 5$. Now it arrives as a rejected project on Ms. jś desk, which she now compares with her threshold value $\overline{\theta}^{jR} = 7$. Hence, she accepts the project. The organization as a whole reacts in the same way in both cases.

This implies a very strong form of multiplicity of consistent threshold values. A direct corollary of this result is that proposition 9, which gives the consistent threshold values under the assumption that Mr. i accepts if and only if $\theta^i > \overline{\theta}^i$, is applicable as well when Mr. i accepts if and only if $\theta^i \leq \overline{\theta}^i$ once the values for $\overline{\theta}^{jR}$ and $\overline{\theta}^{jA}$ have been interchanged. This, in turn, has an important implication.

Corollary 7 *The expected profit of an omniarchy in case of a vector of consistent threshold values* $(\overline{\theta}^i, \overline{\theta}^{jR}, \overline{\theta}^{jA}) = (\overline{\theta}^{i*}, \overline{\theta}^{jR*}, \overline{\theta}^{jA*})$, *and where i accepts if and only if $\theta^i > \overline{\theta}^i$ is equal to the expected profit of an omniarchy in case of* $(\overline{\theta}^i, \overline{\theta}^{jR}, \overline{\theta}^{jA}) = (\overline{\theta}^{i*}, \overline{\theta}^{jA*}, \overline{\theta}^{jR*})$, *and where i accepts if and only if $\theta^i \leq \overline{\theta}^i$.*

This equivalence in terms of expected profits makes the choice of a particular vector of consistent threshold values and of a particular acceptance rule used by Mr. i on the basis of the ensuing profits impossible.

7 A Comparison

In the preceeding sections I studied in detail the behaviour of rational agents within given organizational structures. For a given vector of parameters characterizing the agents and the projects the organization faces I determined each agent's best response, consistent threshold values, the ensuing organization pay–off, and the optimal ordering of heterogeneous agents. Left fixed was the organizational architecture. It became clear, though, that different organizations respond in diverse ways to changes in the environment. In this section I compare the performance of different organizational forms, using two standards, the first being the expected profits made on accepted projects. The second standard takes as well the expected value of good but rejected projects into account. As the presence of good, but neglected projects represents a loss from a welfare perspective, this measure reflects in a more accurate way an organization's contribution to social welfare. These results are summarized in four propositions. Section 7.1 gives the results when organizations are compared on the basis of expected profits, while section 7.2 states two results when the comparison taking the neglected but good projects into account.

The analysis sofar has shown that the ordering of heterogenous agents is inconsequential as far as organizational performance is concerned in case of a hierarchy and a polyarchy. An omniarchy, however, requires ordinal information concerning the agents characteristics. The latter then is more demanding, informationally speaking, than the former two. Indeed, the omniarchy's superior performance may vanish partly or wholy if no such ordinal information is available. This observation is the subject of section 7.3.

7.1 A comparison based on expected profits

Proposition 13 *When agents are identical ($h = 0$), the expected pay-off induces the following ordering of organizational forms:*

$$\begin{cases} E(\Pi|O) > E(\Pi|P) > E(\Pi|H) > E(\Pi|S) & : \quad k < 1 \\ E(\Pi|O) > E(\Pi|H) = E(\Pi|P) > E(\Pi|S) & : \quad k = 1 \\ E(\Pi|O) > E(\Pi|H) > E(\Pi|P) > E(\Pi|S) & : \quad k > 1 \end{cases} \tag{51}$$

These results are in line with what one would intuitively expect. The omniarchy performs the best for every value of k as it requires a double check on every project. Similarily, the single agent organization performs the worst as it implies just a single check of any project's quality. That a hierarchy performs better than a polyarchy reflects the hierarchy's second check of a project accepted by the first agent. This is beneficial in the face of large possible losses.

Proposition 14 *If the possible losses and gains on an accepted project are identical.* i.e.. *if* $k = 1$. *if agents are optimally allocated within organizations, and if the value of* θ *remains fixed the ordering of organizations is as follows:*

$$\begin{cases} E(\Pi|O) > E(\Pi|H) = E(\Pi|P) = E(\Pi|S) & : \quad h \in (-\theta, -\frac{1}{2}\theta] \\ E(\Pi|O) > E(\Pi|H) = E(\Pi|P) > E(\Pi|S) & : \quad h \in (-\frac{1}{2}\theta, \theta) \\ E(\Pi|O) > E(\Pi|H) = E(\Pi|P) = E(\Pi|S) & : \quad h \geq \theta \end{cases} \tag{52}$$

The main interest of this proposition is that it shows that the polyarchy and the hierarchy perform equally well if the possible losses on accepted projects are as high as their possible gains.

7.2 A comparison taking rejected, but good projects into account

Until now I have limited attention to the performance of organizational forms from the perspective of the organization itself. Agents maximized expected pay–offs of *accepted* projects, and different architectures were compared using expected pay–offs of such projects as a criterion for ordering them. That is, the value of rejected, but profitable projects played no role in the analysis sofar. This captures well the interests of a firm, at least when it operates in isolation. From society's point of view, however, rejected profitable projects represent a welfare loss. Indeed, from this perspective the best organizational form is the one that accepts all profitable projects but rejects all bad projects. And even from the vantage point of the firm rejected profitable projects merely reflect the structurally determined incompleteness of information due to the limited number of agents appraising projects.

Figure 8 represent the single agent, the hierarchy, the polyarchy, and the omniarchy, respectively. In each figure, the white area represents the inefficient projects that have been rejected, the light grey area the profitable projects that have been accepted. the grey area the inefficient projects that have been accepted. and the dark gray area the profitable

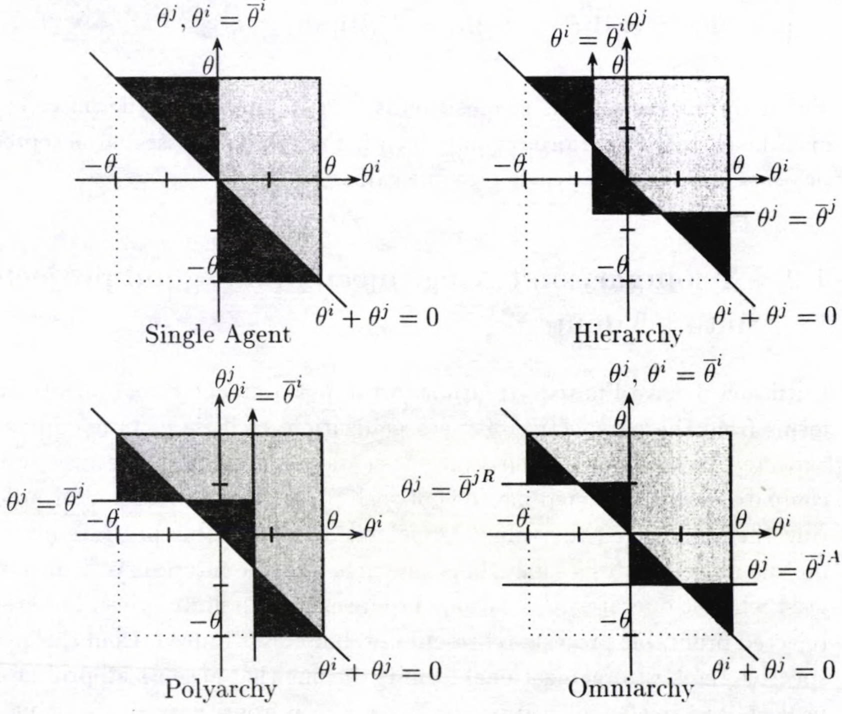

Figure 8: Four types of projects in the four organizations studied

projects that have been rejected. The figures assume that agents are identical, or $h = 0$.

Graphically, then, the index I use in this section takes the size of the dark gray into account as well. The larger it is, the more good projects have been rejected. D_F is calculatad as the difference between, on the one hand, the expected value of accepted projects, and, on the other hand, the expected value of the sum of accepted but bad projects and rejected but good projects for a given organizational form $F \in \{S, H, P, O\}$. The two propositions that follow analyze the behaviour of D for two special cases: the heterogeneous agents case ($h = 0$), and the case where the absolute value of possible profits and losses are identical ($k = 1$).

Proposition 15 *If agents are identical, $h = 0$, the following ordering of organizational forms is induced by the index D*[3]*:*

$$\begin{cases} D_O > D_P > D_H > D_S & : \quad k \in (0, 1.8) \\ D_O > D_P = D_H < D_S & : \quad k = 1.8 \\ D_O > D_H > D_P > D_S & : \quad k > 1.8 \end{cases} \tag{53}$$

This proposition shows that the finer approximation as implied by an omniarchy leads to an improvement relative to the other organizational forms when measured by the index D. The single agent organization performs the worst throughout the whole range of k. The polyarchy is still performing better than the hierarchy for low values, and has even extended the set of values for which this is the case. For high values of k, the hierarchy is still performing better than the polyarchy.

Proposition 16 *If the possible losses and gains on a project are identical, i.e., if $k = 1$, if agents are optimally allocated within organizations using the expected profits on accepted projects as the standard, and if the value of θ remains fixed, the following holds:*

[3]The value $k = 1.8$ is an approximation.

$$
\begin{cases}
D_O > D_P = D_S = D_H & : \quad h \in \left(-\theta, -\frac{1}{2}\theta\right] \\
D_O > D_P > D_S > D_H & : \quad h \in \left(-\frac{1}{2}\theta, -\frac{7}{20}\theta\right) \\
D_O > D_P > D_H > D_S & : \quad h \in \left(-\frac{7}{20}\theta, \frac{7}{13}\theta\right) \\
D_O > D_P > D_S > D_H & : \quad h \in \left(\frac{7}{13}\theta, \theta\right) \\
D_O > D_P = D_S = D_H & : \quad h \geq \theta
\end{cases} \tag{54}
$$

The omniarchy is still performing the best. Interestingly, this proposition shows that in the presence of heterogeneous agents the polyarchy is always performing better than the hierarchy. Indeed, the hierarchy may perform even worse than the single agent.

7.3 Informational Problems

Suppose that the persons that analyze projects have not decided themselves on the organizational form, and suppose that they have not decided either on the order in which they decide. Rather, this has been decided by somebody else. An example could be some departmental organization deciding on the implementation of some new product. The organizational structure may have been laid down a long time ago by head quarters, while the positions are filled by a board of people that does not analyze the products itself. In such a situation the persons that actually analyze the projects are likely to have more detailed information about their own characteristics and the characteristics of the projects they are to analyze. In particular, head quarters and the board may not know which agent has the larger interval for its valuations. Hence, the board may want to decide to set up a decisional structure which performance is independent of the exact ordering of the agents, or which is robust to wrongly positioned agents.

The following proposition shows the omniarchy is not robust to such misplacements. It may therefore be preferable to set up an organizational structure like the hierarchy or the polyarchy that does not require a particular ordering for maximum performance. In particular, the proposition says that if the agents have been allocated well half of the time, and if the possible loss on a project is equal to the possible gain, an omniarchy always performs worse than a hierarchy and a polyarchy.

Proposition 17 *If the agents in an omniarchy have been positioned well half of the time, and if $k = 1$, then the expected profits of an omniarchy are lower than the expected pay-offs of a hierarchy or a polyarchy for every value of h.*

8 Related Literature

Part of the existing literature on the internal structure of the firm and its effect on firm behaviour follows the Sah and Stiglitz framework of fallible agents. Koh ((1992), (1993a), (1993b), (1994a), and (1994b)) extends results obtained by Sah and Stiglitz by introducing the problem of moral hazard, by comparing the speed with which decisions are taken within hierarchies and polyarchies, by discussing the effect of variable evaluation costs on the relative performance of hierachies and polyarchies, by introducing a continuous signal model in a commitee framework, and by populating hierarchies and polyarchies with more than two agents, respectively. Ioannides (1987), relying on results obtained by Shannon and Moore, notes that the screening quality of an organization can be arbitrarily improved by replacing an agent by a replica of the original organizational structure as a whole. Perfect decisions can be obtained by judiciously arranging an infinite amount of error–prone decision–makers. He discusses as well the introduction of incentives. Both Bull and Ordover (1987) and Hendrikse (1992) discuss the relationship between the choice of the internal structure and the market environment. Sobel (1992), finally, looks at the best way an erratic human should count to thousand.

Other contributions assume full rationality on behalf of the agents populating organizations. In the context, Marschak and Radner (1972) study the role and value of decentralized information processing and communication within organizations whose agents form a team *i.e.*, whose preferences coincide. Carter (1995) is a recent contribution to this line of research.

Both Radner (1993), and Bolton and Dewatripont (1994) model the structure of an organization as a network of information processing units.

The organizational goal is to process a repeatedly arriving batch of information. Radner measures the performance of a network by the processing time and number of units required, while Bolton and Dewatripont focus on the costs of processing and communicating information.

Vassilakis (forthcoming) relates the time needed to develop a new product to the organization's architecture. Some organizational forms give rise to procrastination, cycling, and back–tracking, impeding a swift introduction of a new product, while others are conducive to solving problems concomitant to the development of new products quickly.

The present paper is related as well to the literature on social learning. Social learning refers to situations in which a number of selfish agents decide sequentially on the basis of both private information and information revealed by previously taken decisions. As the pay–off agents obtain are left unaffected by succeeding agents' decisions, agents do not take the informational impact of their actions into account when deciding. Typically, one decision is "good", while the other is "bad", but the latter becomes the choice of rational agents, given the information revealed by previous decisions. Since an agent decides whatever the preceeding agents' decisions, this situation is related to the omniarchy studied in this paper. The main questions asked in these papers are of a statistical nature: what is the probability that the population of agents makes the wrong decision, what is the probability that this population, after a series of wrong decisions, breaks out of this situation, etc.? (see for example Bikhchandani, Hirshleifer, and Welch (1992), and Vives (1996)). Gul and Lundholm (1995) address the question what ordering arises if heterogeneous agents control the moment of decision. This issue is related to the optimal ordering of heterogeneous agents within given organizational structures studied here. Gale (1996) argues that the literature on social learning pays undue attention to the statistical aspects involved, at the cost of ignoring the value created by sequential decision structures. In this light, the present paper can be viewed as a contribution that tries to redress this disequilibrium.

9 Conclusion

In this paper, I discuss the effect organizational structure can have on organizational performance in a project selection framework by analysing four different organizational architectures; a single agent organization. a hierarchy, a polyarchy, and an omniarchy. The agents' choices reflect the overall organizational structure, and, in particular, the informational content of other agents' actions. Agents maximize the expected profits made on accepted projects. An omniarchy, characterized by a double check on every project, performs uniformly the best, while a single agent organization is always performing the worst. The ordering of the polyarchy and the hierarchy, two organizational forms allowing for a conditional second instance of screening on the basis of independent information depends on both the characteristics of the agents and on the type of environment. If agents are different, the polyarchy and the hierarchy are performing equally well, whereas in the case of agent are identical the hierarchy performs better than the polyarchy in tough environments. The opposite holds for relatively friendly environments.

I compare as well the performance of these organizations relative to the ideal organization that accepts all good projects and rejects all bad ones. Once again, the omniarchy performs best, and the single agent organization the worst. The ordering of the polyarchy and the hierarchy in case of identical agents, however, is reversed, with now the polyarchy better approximating the ideal organization for tough project environments.

This should, however, not be taken as evidence that the omniarchy is the best structure when the goal is to select projects, as I showed it to have certain drawbacks relative to the other organizational forms. Firstly, I show that the optimal allocation of agents within a given organizational structure requires more detailed information about the agents' characteristics in an omniarchy than in a hierarchy or in a polyarchy. Secondly, the omniarchy suffers from the multiplicity of consistent individual decision rules, which may even give rise to the same expected profit. This complicates the choice of a specific set of individual decision rules. Thirdly, on average an omniarchy employs more time than the other organizations in evaluating projects. Although this causes no problem in the setting studied here, a thorough, but slow project selection procedure may well have negative effects on firm performance when operating in a competitive environment. The trade–offs these disadvantages imply constitute an interesting field for further research.

A Proofs

Proof of Lemma 1 Compare the value of $\overline{\theta}^i$ as determined by (3) with the boundaries of the interval of $\overline{\theta}^i \in (-\theta, \theta)$, taking into account the restriction that $h > -\theta$.

Proof of Lemma 2 Similar to the proof of lemma 1.

Proof of Proposition 1 In general, the profit expression equals

$$X_1 \frac{1}{2}\left(Prob(A \mid Good) - kProb(A \mid Bad)\right)$$

since the probability that a project is good equals a half. If j is the only agent, lemma 2 applies. For $h \leq -\theta 2/(k+1)$ and $k > 1$, no project is accepted, and hence $E(\Pi|S(j)) = 0$, while for $h \geq -\theta 2k/(k+1)$ and $k < 1$, all projects are accepted. Therefore $E(\Pi|S(j)) = X_1(1-k)/2$. Finally, if $\overline{\theta}^j \in (-\theta + h, \theta + h)$, the profit expression depends on the the sign of h. For $h > 0$, figure 2 shows that the expected profits equal the difference between $X_1 \frac{1}{2}\left(\frac{(\theta - \overline{\theta}^i)(\theta + h + \theta) - \frac{1}{2}(\theta - \overline{\theta}^i)^2}{2\theta(\theta + h)}\right)$ on the one hand and $X_2 \frac{1}{2}\left(\frac{(\theta - \overline{\theta}^i)(-\overline{\theta}^i + \theta + h) - \frac{1}{2}(\theta - \overline{\theta}^i)^2}{2\theta(\theta + h)}\right)$ on the other. This equals $X_1 \frac{(2\theta + h(k+1))^2}{8\theta(\theta + h)(k+1)}$. For $h \in \left(-\theta \frac{2}{k+1}, 0\right)$ and $k \leq 1$, or for $h \in \left(-\theta \frac{2k}{k+1}, 0\right)$ and $k \geq 1$, the expression amounts to

$$X_1 \frac{1}{2}\left(\frac{2(\theta - \overline{\theta}^i)(\theta + h) - \frac{1}{2}(\theta - \overline{\theta}^i)^2}{2\theta(\theta + h)} - k\frac{\frac{1}{2}(\theta + h - \overline{\theta}^i)^2}{2\theta(\theta + h)}\right)$$

which boils down to the expression $X_1 \frac{\theta + h(k+1)}{2(\theta + h)(k+1)}$. The expressions for $E(\Pi|S(i))$ can be obtained in the same fashion.

Proof of Proposition 2 This can easily be checked with the aid of proposition 1.

Proof of Lemma 3 First the case where $h > 0$. Figure 3 suggests to look at the order in which $\overline{\theta}^i(\overline{\theta}^j)$ and $\overline{\theta}^j(\overline{\theta}^i)$ cut the line $\overline{\theta}^i = -\theta$. Hence. $\overline{\theta}^i \leq -\theta$ if and only if $\overline{\theta}^j(-\theta) - \overline{\theta}^{i^{-1}}(-\theta) \geq 0$. This is equivalent to $\overline{\theta}^i \leq -\theta$ if and only if $h \geq \theta\frac{2k}{k+1}$. The same kind of analysis applies to the case where $h < 0$. Here, $\overline{\theta}^j \leq -\theta - h$ if and only if $\overline{\theta}^{j^{-1}}(-\theta - h) - \overline{\theta}^i(-\theta - h) \leq 0$. This is equivalent to $\overline{\theta}^j \leq -\theta - h$ if and only if $h \leq -\theta\frac{2k}{3k+1}$

Proof of Proposition 3 From lemma 3 it is clear that $\overline{\theta}^j \leq -\theta - h$ for $h \in (-\theta, \leq -\theta\frac{2k}{3k+1}]$. Hence, $\overline{\theta}^i(\overline{\theta}^j = -\theta - h) = (\theta + h)\frac{k-1}{k+1}$. For $-\theta\frac{2k}{3k+1} < h < \theta\frac{2k}{k+1}$, $\overline{\theta}^i$ and $\overline{\theta}^j$ solve the pair of equations $\overline{\theta}^i = \overline{\theta}^i(\overline{\theta}^j)$ and $\overline{\theta}^j = \overline{\theta}^j(\overline{\theta}^i)$. The solution is $\overline{\theta}^i = -\theta\frac{1}{2k+1} - h\frac{k+1}{2k+1}, \overline{\theta}^j = -\theta\frac{1}{2k+1} + h\frac{k}{2k+1}$. Finally, for $h \geq \theta\frac{2k}{k+1}$ $\overline{\theta}^i \leq -\theta$ and so $\overline{\theta}^j$ equals $\overline{\theta}^j(\overline{\theta}^i = -\theta - h) = (\theta + h)\frac{k-1}{k+1}$.

Proof of Proposition 4 $\overline{\theta}^j \leq -\theta - h$ for $h \in (-\theta, -\theta\frac{2k}{3k+1}]$. Hence, profits equal the level of profits made in a single agent organization by i for $h < 0$. For $h \geq \theta\frac{2k}{k+1}$, or $\overline{\theta}^i \leq \theta$, profits equal the profts made in single agent organization consisting of agent j only for $h > 0$. Finally, for $h \in (-\theta\frac{2k}{3k+1}, \theta\frac{2k}{k+1})$, use figure 4 to write

$$E(\Pi|H(i,j)) = \frac{X_1}{2}\frac{(\theta - \overline{\theta}^i)(\theta + h - \overline{\theta}^j) - \frac{k+1}{2}(-\overline{\theta}^j - \overline{\theta}^i)^2}{2\theta(\theta + h)}$$

which equals

$$X_1\frac{(1 + k)(2\theta + h)^2}{8\theta(\theta + h)(2k + 1)}$$

Proof of Proposition 5 Consider lemma 7.

Lemma 7 *Suppose $\theta > 0$ and $h > -\theta$, and define θ' and h' as $\theta' := \theta + h$ and $h' := -h$, respectively. Then, $\theta' > 0$ and $h' > -\theta'$. Moreover,*

$$h \gtreqless \theta\frac{2k}{k+1} \Longleftrightarrow h' \lesseqgtr -\theta'\frac{2k}{3k+1}$$

Proof Suppose $h \leq \theta \frac{2k}{k+1}$. Then $\theta = \theta + h \leq \theta \frac{3k+1}{k+1}$. Moreover, $h' = -h \geq -\theta \frac{2k}{k+1}$. Therefore, $h' \geq -\theta' \frac{2k}{k+1} \frac{k+1}{3k+1} = -\theta' \frac{2k}{3k+1}$. The same kind of reasoning shows that $h \geq \theta \frac{2k}{k+1} \iff h' \leq -\theta' \frac{2k}{3k+1}$.

This lemma establishes that if both $\overline{\theta}^i \in (-\theta, \theta)$ and $\overline{\theta}^j \in (-\theta - h, \theta + h)$, then both $\overline{\theta}^{i'} \in (-\theta', \theta') = (-\theta - h, \theta + h)$ and $\overline{\theta}^{j'} \in (-\theta' - h', \theta' + h') = (-\theta, \theta)$. This implies that before and after the switch of positions of Mr. i and Ms. j within the hierarchy, the expected profit is determined by the middle entry of equation (23), which is independent of θ and h. Therefore, pay-offs are left unaffected by the swapping of positions. Now suppose that $h \geq \theta \frac{2k}{k+1}$. Hence, $h' \leq -\theta \frac{2k}{3k+1}$. The pay-off comparison therefore equals the difference between the bottom entry of equation (23) with (θ, h) equal to (θ, h) and the top entry equation (23) with (θ, h) equal to $(\theta', h') = (\theta + h, -h)$. It is a matter of a simple calculus to show that these expressions are identical.

Proof of Lemma 4 I discuss the cases (I) $\overline{\theta}^j > \theta + h$, (II) $\overline{\theta}^j \in (-\theta - h, \theta + h)$, and (III) $\overline{\theta}^j < -\theta - h$ in turn.

(I) First the case where $\overline{\theta}^j > \theta + h$, or $\Pr(\theta^j > \overline{\theta}^j) = 0$. Mr. i 's decision rule becomes Accept if and only if θ^i satisfies $X_1 \Pr(\theta^j > -\theta^i) - X_2 \Pr(\theta^j \leq -\theta^i) > 0$. This is equivalent to Accept if and only if $\theta^i > \overline{\theta}^i := \frac{k-1}{k+1}(\theta + h)$.

(II) Now assume $\overline{\theta}^j \in (-\theta - h, \theta + h)$. I split this case up in three subcases: (1) $\theta^i \geq \theta + h$, (2) $\theta^i \leq -\theta - h$, and (3) $\theta^i \in (-\theta - h, \theta + h)$.

(II.1) If $\theta^i \geq \theta + h$, $\Pr(\theta^j > \theta^i) = 1 = \Pr(\theta^j > -\theta^i | \theta^j > \overline{\theta}^{jR})$. Mr. i 's decision rule becomes Accept if and only if θ^i satisfies $X_1 > \Pr(\theta^j > \overline{\theta}^j) X_1$. Hence, for $\theta^i \geq \theta + h$ the project is accepted.

(II.2) If $\theta^i \leq -\theta - h$, the decision rule amounts to Accept if and only if θ^i satisfies $-X_2 > -\Pr(\theta^j > \overline{\theta}^j) X_2$. Hence, for $\theta^i \leq -\theta - h$ the project is rejected.

(II.3) If $\theta^i \in (-\theta - h, \theta + h)$, two cases obtain: (a) $-\theta^i \leq \overline{\theta}^j$ and (b) $-\theta^i > \overline{\theta}^j$. In case of (a), $\Pr(\theta^j > -\theta^i | \theta^j > \overline{\theta}^{jR}) = 1$, and the decision rule boils down to Accept if and only if

$$\left(\frac{\theta + h + \theta^i}{2(\theta + h)}\right) - k\left(\frac{-\theta^i + \theta + h}{2(\theta + h)}\right) > \left(\frac{\theta + h - \overline{\theta}^j}{2(\theta + h)}\right)$$

or, $\theta^i > -\overline{\theta}^j \frac{1}{k+1} + \frac{k}{k+1}(\theta + h)$. It is straightforward to show that $-\overline{\theta}^j \frac{1}{1+k} + \frac{k}{k+1}(\theta + h) \in (-\overline{\theta}^j, \theta + h)$. Hence, for $\theta^i \in (-\overline{\theta}^j, -\overline{\theta}^j \frac{1}{k+1} + \frac{k}{k+1}(\theta + h)]$ a project will be rejected, while for $\theta^i \in (-\overline{\theta}^j \frac{1}{k+1} + \frac{k}{k+1}(\theta + h), \theta + h)$ a project will be accepted.

In case of (b), $\Pr(\theta^j > -\theta^i | \theta^j > \overline{\theta}^{jR}) \in (0, 1)$, and the decision rule boils down to Accept if and only if $-k(\theta + h) > k\overline{\theta}^j$, which is false since $\theta + h > -\overline{\theta}^j$ by assumption. Hence for $-\theta^i > \overline{\theta}^j$ a project is rejected. In summary, if $\overline{\theta}^j \in (\theta + h, -\theta - h)$, then the decision rule amounts to Accept if and only if $\theta^i > \overline{\theta}^i := -\frac{\overline{\theta}^j}{k+1} + \frac{k}{k+1}(\theta + h)$.

(III) Finally the case where $\overline{\theta}^j < -\theta - h$ in turn. The decision rule becomes $X_1 \Pr(\theta^j > \theta^i) - X_2 \Pr(\theta^j \leq \theta^i) > X_1 \Pr(\theta^j > \theta^i) - X_2 \Pr(\theta^j \leq \theta^i)$ which never holds, and every project is therefore rejected: $\overline{\theta}^i \geq \theta$.

Proof of Lemma 5 Thi proof is similar to the proof of lemma 3.

Proof of Proposition 6 The proof is similar to the proof of proposition 3.

Proof of Proposition 7 I use figure 6 to write

$$E(\Pi | P(i,j)) X_1 \frac{h^2 k(k+1) + 8\theta(\theta + h)}{8\theta(\theta + h)(k+2)}$$

For $h \in (-\theta, -\theta \frac{2}{k+3})$, $\overline{\theta}^j \geq \theta + h$, and the profit expression equals the one of the single agent i for negative values of h, while for $h \geq \theta \frac{2}{k+1}$, $\overline{\theta}^i \geq \theta$, and so the profits equal the profits obtained by a single agent j, for $h > 0$.

Proof of Proposition 8 Consider lemma 8:

Lemma 8 *Suppose $\theta > 0$ and $h > -\theta$. Define $\theta' := \theta + h$ and $h' := -h$. Then,*

$$\begin{aligned} h \gtreqless \theta \tfrac{2}{k+1} &\iff h' \lesseqgtr -\theta' \tfrac{2}{k+3} \\ h \leq \theta \tfrac{2}{k+1} &\iff h' \geq -\theta' \tfrac{2}{k+3} \end{aligned}$$

Proof The proof is identical to the proof of lemma 7.

The proof of proposition 8 is now similar to the proof of proposition 5. The main difference is the fact that in the case of a polyarchy, profits for $h \in (-\theta\frac{2}{k+3}, \theta\frac{2}{k+1})$ do depend on θ and h.

Proof of Lemma 6 For $\overline{\theta}^{jR} \geq \theta + h$ an omniarchy turns into a hierarchy, which has been discussed in section 4. For $\overline{\theta}^{jA} \leq -\theta - h$ it becomes a polyarchy, and the proof of $\overline{\theta}^i(\overline{\theta}^{jR}, \overline{\theta}^{jA})$ is equivalent to the proof of $\overline{\theta}^i(\overline{\theta}^j)$ in case of a polyarchy. Attention will therefore be limited to those cases where $\overline{\theta}^{jA} > -\theta - h$ and $\overline{\theta}^{jR} < \theta + h$.

(I) Suppose $\overline{\theta}^{jR} < -\theta - h$, and assume either (1) $\overline{\theta}^{jA} \geq \theta + h$ or (2) $\overline{\theta}^{jA} \in (-\theta - h, \theta + h)$.

(I.1) For $\overline{\theta}^{jA} \geq \theta + h$, Mr. i 's decision problem becomes Accept if and only if θ^i satisfies $0 > X_1 \Pr(\theta^j > \theta^i) - X_2 \Pr(\theta^j \leq \theta^i)$. This is equivalent to Accept if and only if $\theta^i < \overline{\theta}^i := \frac{k-1}{k+1}(\theta + h)$.

(I.2) $\overline{\theta}^{jA} \in (-\theta - h, \theta + h)$. This case I split up in (a) $\theta^i \geq \theta + h$, (b) $\theta^i \leq -\theta - h$, and (c) $\theta^i \in (-\theta - h, \theta + h)$.

(I.2.a) $\theta^i \geq \theta + h$, and therefore $\Pr(\theta^j > \theta^i) = \Pr(\theta^j > -\theta^i | \theta^j > \overline{\theta}^{jA}) = 1$. Therefore, the decision rule amounts to Accept if and only if

$$\left(\frac{\theta + h - \overline{\theta}^{jA}}{2(\theta + h)}\right) X_1 > X_1$$

which is false. So, if $\theta^i \geq \theta + h$ a project will be rejected.

(I.2.b) Analogously, if $\theta^i \leq -\theta - h$, a project will be accepted.

(I.2.c) For $\theta^i \in (-\theta - h, \theta + h)$, two subcases should be distinguished: (i) $-\theta^i > \overline{\theta}^{jA}$ and (ii) $-\theta^i \leq \overline{\theta}^{jA}$.

(I.2.c.i) $-\theta^i > \overline{\theta}^{jA}$. The decision rule becomes Accept if and only if $\overline{\theta}^{jA} > -\theta - h$. Since this is false by assumption, a project with $-\theta^i > \overline{\theta}^{jA}$ will be rejected.

(I.2.c.ii) For $-\theta^i \leq \overline{\theta}^{jA}$, a project will only be accepted if

$$\left(\frac{\theta + h - \overline{\theta}^{jA}}{2(\theta + h)}\right) X_1 > X_1 \left(\frac{\theta + h + \theta^i}{2(\theta + h)}\right) - k \left(\frac{-\theta^i + \theta + h}{2(\theta + h)}\right)$$

which amounts to Accept if and only if $\theta^i < -\frac{1}{k+1}\overline{\theta}^{jA} + \frac{k}{k+1}(\theta + h)$. It is easy to show that $-\frac{1}{k+1}\overline{\theta}^{jA} + \frac{k}{k+1}(\theta + h) \in (-\overline{\theta}^{jA}, \theta + h)$.

In summary, for $\overline{\theta}^{jR} \leq -\theta - h$ and $\overline{\theta}^{jA} \in (-\theta - h, \theta + h)$ a project is accepted if and only if $\theta^i < \overline{\theta}^i := -\frac{1}{k+1}\overline{\theta}^{jA} + \frac{k}{k+1}(\theta + h)$.

(II) Suppose $\overline{\theta}^{jA} \geq \theta + h$ and $\overline{\theta}^{jR} \in (-\theta - h, \theta + h)$. The cases where $\theta^i \geq \theta + h$ or $\theta^i \leq -\theta - h$ can be handled in the same way as is done in (I.2.a) and (I.2.b), respectively; in the first case, a project will be rejected, while in the second case it will be accepted. Let me therefore limit attention to $\theta^i \in (-\theta - h, \theta + h)$. This is similar to case (I.2.c); hence, for $\overline{\theta}^{jA} \geq \theta + h$ and $\overline{\theta}^{jR} \in (-\theta - h, \theta + h)$, a project will be accepted if and only if $\theta^i < \overline{\theta}^i := -\frac{1}{1+k}(\theta + h + k\overline{\theta}^{jR})$.

(III) The case that remains to be studied is $\overline{\theta}^{jR}, \overline{\theta}^{jA} \in (-\theta - h, \theta + h)$. I split this case up in three subcases: (1) $\theta^i \geq \theta + h$, (2) $\theta^i \leq -\theta - h$, and (3) $\theta^i \in (-\theta - h, \theta + h)$.

(III.1) For $\theta^i \geq \theta + h$ $\Pr(\theta^j > -\theta^i | \theta^j > \overline{\theta}^{jR}) = \Pr(\theta^j > -\theta^i | \theta^j > \overline{\theta}^{jA}) = 1$, and the decision rule becomes Accept if and only if θ^i satisfies

$$\left(\frac{\theta + h - \overline{\theta}^{jA}}{2(\theta + h)}\right) X_1 > \left(\frac{\theta + h - \overline{\theta}^{jR}}{2(\theta + h)}\right) X_1$$

which is equivalent to $\overline{\theta}^{jA} < \overline{\theta}^{jR}$. Hence, if $\overline{\theta}^{jA} < \overline{\theta}^{jR}$, a project with $\theta^i \geq \theta + h$ will be accepted, while if $\overline{\theta}^{jA} \geq \overline{\theta}^{jR}$, a project satisfying $\theta^i \geq \theta + h$ will be rejected.

(III.2) Reasoning in the same for $\theta^i \leq -\theta - h$, I obtain that a project will be rejected if $\overline{\theta}^{jA} \leq \overline{\theta}^{jR}$, while it will be accepted if $\overline{\theta}^{jA} > \overline{\theta}^{jR}$.

(III.3) I split this up in four subcases: (a) $-\theta^i \leq \min(\overline{\theta}^{jA}, \overline{\theta}^{jR})$, (b) $\overline{\theta}^{jA} < -\theta^i \leq \overline{\theta}^{jR}$, (c) $\overline{\theta}^{jR} < -\theta^i \leq \overline{\theta}^{jA}$, and (d) $\overline{\theta}^{jR}, \overline{\theta}^{jA} < -\theta^i$.

(III.3.a) If $-\theta^i \leq \min(\overline{\theta}^{jA}, \overline{\theta}^{jR})$, then $\Pr(\theta^j > \theta^i | \theta^j > \overline{\theta}^{jR}) = \Pr(\theta^j >$-$\theta^i | \theta^j > \overline{\theta}^{jA}) = 1$, which is equivalent to case (III.1). Therefore, if $\overline{\theta}^{jA} < \overline{\theta}^{jR}$, a project with $\theta^i \geq \theta + h$ will be accepted, while if $\overline{\theta}^{jA} \geq \overline{\theta}^{jR}$, a project satisfying $\theta^i \geq \theta + h$ will be rejected.

(III.3.b) $\overline{\theta}^{jA} < -\theta^i \leq \overline{\theta}^{jR}$, and therefore $\overline{\theta}^{jA} < \overline{\theta}^{jR}$. Hence, $\Pr(\theta^j > -\theta^i | \theta^j > \overline{\theta}^{jR}) = 1 > \Pr(\theta^j > -\theta^i | \theta^j > \overline{\theta}^{jA}) > 0$. The decision rule equals

Accept if and only if θ^i satisfies

$$\left(\frac{\theta+h-\overline{\theta}^{jA}}{2(\theta+h)}\right)\left(\left(\frac{\theta+h+\theta^i}{\theta+h-\overline{\theta}^{jA}}\right)-k\left(\frac{-\theta^i-\overline{\theta}^{jA}}{\theta+h-\overline{\theta}^{jA}}\right)\right)>\left(\frac{\theta+h-\overline{\theta}^{jR}}{2(\theta+h)}\right)$$

which is equivalent to Accept if and only if $\theta^i > -\frac{1}{k+1}\left(\overline{\theta}^{jR}+k\overline{\theta}^{jA}\right)$. It is easy to show that $-\frac{1}{k+1}\left(\overline{\theta}^{jR}+k\overline{\theta}^{jA}\right)\in(-\overline{\theta}^{jR},-\overline{\theta}^{jA})$. Therefore, for $\theta^i\in(-\overline{\theta}^{jR},-\frac{1}{k+1}\left(\overline{\theta}^{jR}+k\overline{\theta}^{jA}\right)]$ a project will be rejected, while a project will be accepted if and only if $\theta^i\in(-\frac{1}{k+1}\left(\overline{\theta}^{jR}+k\overline{\theta}^{jA}\right),-\overline{\theta}^{jA})$.

(III.3.c) $\overline{\theta}^{jR}<-\theta^i\leq\overline{\theta}^{jA}$, and therefore at least $\overline{\theta}^{jR}<\overline{\theta}^{jA}$. The decision rule becomes Accept if and only if θ^i satisfies

$$\left(\frac{\theta+h-\overline{\theta}^{jA}}{2(\theta+h)}\right)>\left(\frac{\theta+h-\overline{\theta}^{jR}}{2(\theta+h)}\right)\left(\left(\frac{\theta+h+\theta^i}{\theta+h-\overline{\theta}^{jR}}\right)-k\left(\frac{-\theta^i-\overline{\theta}^{jR}}{\theta+h-\overline{\theta}^{jR}}\right)\right)$$

which amounts to Accept if and only if

$$\theta^i<-\frac{1}{1+k}\left(\overline{\theta}^{jA}+k\overline{\theta}^{jR}\right)\in(-\overline{\theta}^{jA},-\overline{\theta}^{jR})$$

(III.3.d) And finally the case where $\overline{\theta}^{jR},\overline{\theta}^{jA}<-\theta^i$. This amounts to $\overline{\theta}^{jA}>\overline{\theta}^{jR}$. Hence, a project will be accepted if $\overline{\theta}^{jA}>\overline{\theta}^{jR}$, while it will be rejected if $\overline{\theta}^{jA}\leq\overline{\theta}^{jR}$. In summary, for $\overline{\theta}^{jR},\overline{\theta}^{jA}\in(-\theta-h,\theta+h)$, and $\overline{\theta}^{jR}>\overline{\theta}^{jA}$ a project will be accepted if and only if $\theta^i>-\frac{1}{k+1}\left(\overline{\theta}^{jR}+k\overline{\theta}^{jA}\right)$; if, however, $\overline{\theta}^{jR}<\overline{\theta}^{jA}$ a project will be accepted if and only if $\theta^i<-\frac{1}{k+1}\left(\overline{\theta}^{jA}+k\overline{\theta}^{jR}\right)$; and in case $\overline{\theta}^{jR}=\overline{\theta}^{jA}$ no project will be accepted at all, hence $\theta^i\geq\theta$.

Proof of Proposition 9 (I) As noted in the main body of the text, for $\overline{\theta}^{jR}\geq\theta+h$ an omniarchy turns into a hierarchy. The idea is then to check whether $\overline{\theta}^{jR}\geq\theta+h$ is a best reply to the consistent threshold value for Mr. i as given in proposition 3. This proposition suggests to split up the proof in four parts: (1) $-\theta<h\leq-\theta\frac{2k}{3k+1}$, (2) $h\in(-\theta\frac{2k}{3k+1},\theta\frac{2k}{k+1})$, (3) $h\geq\theta\frac{2k}{k+1}$, and (4) $\forall(\theta,h,k)$.

(I.1) For $-\theta<h\leq-\theta\frac{2k}{3k+1}$, $\overline{\theta}^i=(\theta+h)\frac{k-1}{k+1}\in(-\theta,\theta)$. Therefore $\overline{\theta}^{jR}(\overline{\theta}^i)=-\overline{\theta}^i\frac{1}{k+1}+\theta\frac{k}{k+1}=-h\frac{k-1}{(k+1)^2}+\theta\frac{k^2+1}{(k+1)^2}$, which is larger than or equal

to $\theta + h$ if and only if $h \leq -\theta\frac{2}{k+3}$. Combining this with $-\theta < h \leq -\theta\frac{2k}{3k+1}$, I obtain the condition $k \leq 1$ and $h \in (-\theta, -\theta\frac{2}{k+3})$, or $k > 1$ and $h \in \left(-\theta, -\theta\frac{2k}{3k+1}\right)$.

(I.2) For $h \in (-\theta\frac{2k}{3k+1}, \theta\frac{2k}{k+1})$, $\overline{\theta}^i = -\theta\frac{1}{2k+1} - h\frac{1+k}{2k+1} \in (-\theta, \theta)$, and therefore $\overline{\theta}^{jR}(\overline{\theta}^i) = -\overline{\theta}^i\frac{1}{k+1} + \theta\frac{k}{k+1} = \theta\frac{2k^2+k+1}{(2k+1)(k+1)} + h\frac{k+1}{(2k+1)(k+1)}$. This is bigger than or equal to $\theta + h$ if and only if $h \leq -\theta\frac{1}{k+1}$, which, given the initial assumption, amounts to $h \in (-\theta\frac{2k}{3k+1}, -\theta\frac{1}{k+1})$.

(I.3) For $h \geq \theta\frac{2k}{k+1}$, $\overline{\theta}^i \leq -\theta$, and therefore $\overline{\theta}^{jR}(\overline{\theta}^i) \in (-\infty, \infty)$, and therefore $\overline{\theta}^{jR} \geq \theta + h$ is a best response.

(I.4) $\forall(\theta, h, k)$, $\overline{\theta}^i \geq \theta$ is a consistent cut-off value. $\overline{\theta}^{jR}(\overline{\theta}^i) = \theta\frac{k-1}{k+1}$, which is bigger than or equal to $\theta + h$ if and only if $h \leq -\theta\frac{2}{k+1}$.

(II) Similarily, the case where $\overline{\theta}^{jA} \leq -\theta - h$, amounts to a polyarchy. The proof proceeds in the same way as under (I), by checking whether $\overline{\theta}^{jA} \leq -\theta - h$ is a best response to the threshold value $\overline{\theta}^i$ as determined in proposition 6. Three subcases will be discussed in turn: (1) $-\theta < h \leq -\theta\frac{2}{k+3}$, (2) $h \in (-\theta\frac{2}{k+3}, \theta\frac{2}{k+1})$, and (3) $h \geq \theta\frac{2}{k+1}$.

(II.1) $-\theta < h \leq -\theta\frac{2}{k+3}$ and therefore $\overline{\theta}^i = (\theta + h)\frac{k-1}{k+1}$. Therefore, $h \leq -\theta\frac{2k}{3k+1}$ to ensure that $\overline{\theta}^{jA}(\overline{\theta}^i) \leq -\infty$.

(II.2) If $h \in (-\theta\frac{2}{k+3}, \theta\frac{2}{k+1})$, then $\overline{\theta}^i = \theta\frac{k}{k+2} + h\frac{k+1}{k+2} \in (-\theta, \theta)$, and therefore $\overline{\theta}^{jA}(\overline{\theta}^i) = -\overline{\theta}^i\frac{k}{k+1} - \theta\frac{1}{k+1} = -\theta\frac{k^2+k+2}{(k+2)(k+1)} - h\frac{k^2+k}{(k+2)(k+1)}$, which is smaller than or equal to $-\theta - h$ if and only if $h \leq -\theta\frac{k}{k+1}$. This combined with the initial assumption leads to $h \in (-\theta\frac{2}{k+3}, -\theta\frac{k}{k+1})$.

(II.3) $h \geq \theta\frac{2}{k+1}$, and therefore $\overline{\theta}^i \geq \theta$. Hence, $\overline{\theta}^{jA}(\overline{\theta}^i) \in (-\infty, \infty)$, which allows for $\overline{\theta}^{jA} < -\theta - h$.

(III) For $\overline{\theta}^{jR} = \overline{\theta}^{jA} \in (-\theta - h, \theta + h)$ Mr. i 's best response equals $\overline{\theta}^i \geq \theta$, to which Ms. j 's best response is $\overline{\theta}^{jR} = \theta\frac{k-1}{k+1}$, and $\overline{\theta}^{jA} \in (-\infty, \infty)$. Hence, if $\overline{\theta}^{jR} = \overline{\theta}^{jA} = \theta\frac{k-1}{k+1} \in (-\theta - h, \theta + h)$, a consistent set of threshold levels exists. $\theta\frac{k-1}{k+1} \in (-\theta - h, \theta + h)$ if and only if $k \leq 1$ and $h > -\theta\frac{2k}{k+1}$, or $k \geq 1$ and $h > -\theta\frac{2}{k+1}$.

(IV) Finally, suppose $\overline{\theta}^{jR}, \overline{\theta}^{jA} \in (-\theta - h, \theta + h)$ and $\overline{\theta}^{jA} < \overline{\theta}^{jR}$. Therefore $\overline{\theta}^{i} = -\frac{1}{k+1}\left(\overline{\theta}^{jR} + k\overline{\theta}^{jA}\right)$. I split this case up in three subcases: (1) $\overline{\theta}^{i} \in (-\theta, \theta)$, (2) $\overline{\theta}^{i} \geq \theta$, and (3) $\overline{\theta}^{i} \leq -\theta$.

(IV.1) For $\overline{\theta}^{i} \in (-\theta, \theta)$, $\overline{\theta}^{i} = 0$, $\overline{\theta}^{jR} = \theta\frac{k}{1+k}$, and $\overline{\theta}^{jA} = -\theta\frac{1}{1+k}$ is the unique solution if the condition $\overline{\theta}^{jR}, \overline{\theta}^{jA} \in (-\theta - h, \theta + h)$ is satisfied. These constraints are satisfied if and only if $k < 1$ and $h > -\theta\frac{k}{k+1}$, or $k \geq 1$ and $h > -\theta\frac{1}{k+1}$.

(IV.2) $\overline{\theta}^{i} \geq \theta$, and therefore $\overline{\theta}^{jR}(\overline{\theta}^{i}) = \theta\frac{k-1}{k+1}$ which should be within the interval $(-\theta - h, \theta + h)$. Hence, $k \leq 1$ and $h > -\theta\frac{2k}{k+1}$, or $k > 1$ and $h > -\theta\frac{2}{k+1}$. Moreover, $\overline{\theta}^{jA} < \overline{\theta}^{jR}$ and $\overline{\theta}^{jA} \in (-\theta - h, \theta + h)$. $\overline{\theta}^{i}(\overline{\theta}^{jR}, \overline{\theta}^{jA}) \geq \theta$ and $\overline{\theta}^{jR} = \theta\frac{k-1}{k+1}$ is equivalent to $\overline{\theta}^{jA} \leq -\theta\frac{k+3}{k+1}$ which is strictly smaller than $-\theta < \theta + h$. Hence, for $-\theta\frac{k+3}{k+1}$ to be within $(-\theta - h, \theta + h)$, $-\theta - h < -\theta\frac{k+3}{k+1}$ should hold, which is the case if and only $h > \theta\frac{2}{k+1}$. Consequently, there are consistent threshold values if and only if $h > \theta\frac{2}{k+1}$.

(IV.3) $\overline{\theta}^{i} \leq -\theta$ and therefore $\overline{\theta}^{jA}(\overline{\theta}^{i}) = \theta\frac{k-1}{k+1}$ which should be within the interval $(-\theta - h, \theta + h)$. Hence, $k \leq 1$ and $h > -\theta\frac{2k}{k+1}$, or $k > 1$ and $h > -\theta\frac{2}{k+1}$. Moreover, $\overline{\theta}^{jA} < \overline{\theta}^{jR}$ and $\overline{\theta}^{jR} \in (-\theta - h, \theta + h)$. $\overline{\theta}^{i}(\overline{\theta}^{jR}, \overline{\theta}^{jA}) \leq -\theta$ and $\overline{\theta}^{jA} = \theta\frac{k-1}{k+1}$ is equivalent to $\overline{\theta}^{jR} \geq \theta\frac{3k+3}{k+1}$ which is strictly smaller than $\theta > -\theta - h$. Hence, for $-\theta\frac{k+3}{k+1}$ to be within $(-\theta - h, \theta + h)$, $\theta + h > \theta\frac{3k+3}{k+1}$ should hold, which is the case if and only $h > \theta\frac{2k}{k+1}$. Consequently, there are consistent threshold values if and only if $h > \theta\frac{2k}{k+1}$.

Proof of Proposition 10 For $h = 0$, there are two vectors of consistent threshold values: $(\overline{\theta}^{i}, \overline{\theta}^{jR}, \overline{\theta}^{jA}) = (\theta, \theta\frac{k-1}{k+1}, \theta\frac{k-1}{k+1})$ and $(\overline{\theta}^{i}, \overline{\theta}^{jR}, \overline{\theta}^{jA}) = (0, \theta\frac{k}{k+1}, -\theta\frac{1}{k+1})$. In the first case, expected profits equal

$$\frac{X_1}{2}\frac{2\theta^2 - \frac{1}{2}(\overline{\theta}^{jR} + \theta)^2 - \frac{k}{2}(\overline{\theta}^{jR} + \theta)^2}{2\theta^2}$$

which equals $X_1/(2(k+1))$. In the second case, the expected pay-off amounts to

$$\frac{X_1}{2}\frac{2\theta^2 - \frac{1}{2}(\overline{\theta}^{jR} - \theta)^2 - \frac{1}{2}(\overline{\theta}^{jA} + \theta + h)^2 - k\left(\frac{1}{2}(\theta + h - \overline{\theta}^{jR})^2 - \frac{1}{2}(-\overline{\theta}^{jA})^2\right)}{2\theta^2}$$

which is equal to $X_1(k+2)/(4(k+1))$.

Proof of Proposition 11 First note that if the pair (θ, h) is such that $h \geq \theta$, then the pair (θ', h'), with $\theta' = \theta + h$, and $h' = -h$, satisfies $h' \leq -\frac{1}{2}\theta'$. I first determine (I) which of the possible vectors of threshold values maximizes the expected pay–off for a given pair (θ, h) and for a given ordering of agents (first i, then j), and then (II), taking the optimal vectors for every pair, I determine the optimal ordering of agents.

(I) I distinguish the cases (1) $h \in (-\theta, -\frac{1}{2}\theta)$, (2) $h = -\frac{1}{2}\theta$, (3) $h \in (-\frac{1}{2}\theta, \theta)$, and (4) $h \geq \theta$.

(I.1) For $h \in (-\theta, -\frac{1}{2}\theta)$, there are two vectors of consistent threshold values: $(\theta, 0, 0)$, and $(0, \theta + h, -\theta - h)$. The first gives rise to an expected pay–off of $X_1 \dfrac{\theta + h}{4\theta}$, whereas the second yields $X_1 \dfrac{\theta - h}{4\theta}$. The latter is clearly the largest of the two. Hence,

$$E(\Pi | h \in (-\theta, -\frac{1}{2}\theta)) = X_1 \frac{\theta - h}{4\theta}$$

(I.2) For $h = -\frac{1}{2}\theta$, only $(\theta, 0, 0)$ is a consistent vector, giving rise to a pay–off equal to

$$E(\Pi | h = -\frac{1}{2}\theta) = X_1 \frac{1}{8}$$

(I.3) For $h \in (-\frac{1}{2}\theta, \theta)$, both $(0, \frac{1}{2}\theta, -\frac{1}{2}\theta)$ and $(\theta, 0, 0)$ are consisent vectors. I split this case up in (a) $h \in (-\frac{1}{2}\theta, 0)$ and (b) $h \in (0, \theta)$. The case of $h = 0$ can be left unstudied as it implies identical agents.

(I.3.a) That the vector $(0, -\frac{1}{2}\theta, \frac{1}{2}\theta)$ leads to the highest profit can easily be seen. Hence

$$E(\Pi | h \in (-\frac{1}{2}\theta, 0)) = X_1 \frac{\theta^2 + 4\theta h + 2h^2}{8\theta(\theta + h)}$$

(I.3.b) $(0, \frac{1}{2}\theta, -\frac{1}{2}\theta)$ maximizes the expected pay–off. Therefore

$$E(\Pi | h \in (0, \theta)) = X_1 \frac{3\theta + 4h}{4\theta + h}$$

(I.4) For $h \geq \theta$, there are four different vectors of consistent threshold values, three of which lead to the same expected pay–off:

$$(\theta, 0, 0), (\theta, 0, -\theta - h), \text{ and } (-\theta, \theta + h, 0)$$

all lead to an expected pay–off of $X_1 \dfrac{\theta + 2h}{4(\theta + h)}$; the vector $(0, \frac{1}{2}\theta, -\frac{1}{2}\theta)$ leads to an expected profit equal to $X_1 \dfrac{3\theta + 4h}{8(\theta + h)}$, which is the larger of the two profit expressions. Therefore

$$E(\Pi | h \geq \theta) = X_1 \frac{3\theta + 4h}{8(\theta + h)}$$

(II) I follow the same division as in (I).

(II.1) Suppose $h \in (-\theta, -\frac{1}{2}\theta)$, and therefore $h' \geq \theta'$. $E(\Pi | O(i,j)) = X_1 \dfrac{\theta - h}{4\theta}$, while

$$E(\Pi | O(j,i)) = X_1 \frac{3\theta' + 4h'}{8(\theta' + h')} = X_1 \frac{3(\theta + h) - 4h}{8\theta} = X_1 \frac{3\theta - h}{8\theta}$$

Clearly, for the interval under consideration, $E(\Pi | O(j,i)) > E(\Pi | O(i,j))$; the person with the smallest variance should be the first.

(II.2) Suppose $h = -\frac{1}{2}\theta$, and so $h' = \theta'$. Therefore $X_1 \frac{7}{16} = E(\Pi | O(j,i)) > E(\Pi | O(i,j)) = X_1 \frac{1}{4}$, and thus the person with the smallest variance should be the first.

(II.3.a) For $h \in (-\frac{1}{2}\theta, 0)$, and hence $h' \in (0, \theta')$,

$$X_1 \frac{3\theta - h}{8\theta} = E(\Pi | O(j,i)) > E(\Pi | O(i,j)) = X_1 \frac{\theta^2 + 4\theta h + 2h^2}{8\theta(\theta + h)}$$

which implies that the agent with the smallest variance should be the first.

(II.3.b) For $h \in (0, \theta)$, and hence $h' \in (-\frac{1}{2}\theta', 0)$,

$$X_1 \frac{3\theta + 4h}{8(\theta + h)} = E(\Pi | O(i,j)) > E(\Pi | O(j,i)) = X_1 \frac{\theta}{4(\theta + h)}$$

so once again, the agent with the smallest variance should be the first.

(II.4a) For $h = \theta$, $h' = -\frac{1}{2}\theta'$. Therefore,

$$X_1 \frac{7}{16} = E(\Pi|O(i,j)) > E(\Pi|O(j,i)) = X_1 \frac{1}{4}$$

(II.4.b) $h > \theta$ is equivalent to $h' < -\frac{1}{2}\theta'$.

$$X_1 \frac{3\theta + 4h}{8(\theta + h)} = E(\Pi|O(i,j)) > E(\Pi|O(j,i)) = X_1 \frac{\theta}{4(\theta + h)}$$

and therefore, the agent with the smallest variance should be the first to analyze the project.

Proof of Proposition 12 This is based on a comparison of equations (37), (38), (39), and (40) for a particular vector (θ, h, k), with equations (47), (48), (49), and (50).

Proof of Proposition 13 Use the expressions $E(\Pi|S) = X_1 \frac{1}{2(k+1)}$, $E(\Pi|H) = X_1 \frac{1+k}{4k+2}$, $E(\Pi|P) = X_1 \frac{2}{2k+4}$, and $E(\Pi|O) = X_1 \frac{k+2}{4k+4}$.

Proof of Proposition 14 Use proposition 1, 2, 4, 5, 7, 8, and the profit expressions obtained in the proof of 11 to derive the result.

Proof of Proposition 15 The indices take on the following values: $D(S) = -X_1 \frac{k^2-k-1}{2(k+1)^2}$, $D(H) = X_1 \frac{3k+1}{2(2k+1)^2}$, $D(P) = -X_1 \frac{k^2-2k-4}{2(k+2)^2}$, and $D(O) = X_1 \frac{3k+2}{4(k+1)^2}$

Proof of Proposition 16 The proof is based on a comparison of the following expressions. For $h \in (-\theta, -\frac{1}{2}\theta)$, $D(S) = D(H) = D(P) = X_1 \dfrac{\theta - 3h}{8\theta}$, and $D(O) = X_1 \dfrac{5\theta - 3h}{16\theta}$.

For $h = -\frac{1}{2}\theta$, $D(S) = D(H) = D(P) = X_1 \frac{5}{16}$, while $D(O) = X_1 \frac{13}{32}$.

For $h \in (-\frac{1}{2}\theta, 0)$ the indices amount to $D(S) = X_1 \dfrac{2\theta - 6h}{16\theta}$, $D(H) = X_1 \dfrac{13h^2 + 16h\theta + 16\theta^2}{72\theta}$, $D(P) = X_1 \dfrac{5\left(h^2 + 4h\theta + 4\theta^2\right)}{72\theta}$, and $D(O) = X_1 \dfrac{5\theta - 3h}{16\theta}$.

For $h \in (0, \theta)$, $D(S) = X_1 \frac{\theta + 4h}{8(\theta + h)}$, $D(H) = X_1 \frac{13h^2 + 16h\theta + 16\theta^2}{72\theta}$,

$D(P) = X_1 \frac{5\left(h^2 + 4h\theta + 4\theta^2\right)}{72\theta}$, and $D(O) = X_1 \frac{5\theta + 8h}{16\theta}$.

For $h \geq \theta$, the indices become $D(S) = D(H) = D(P) = X_1 \frac{\theta + 4h}{8(\theta + h)}$, while $D(O)$ equals $X_1 \frac{5\theta + 8h}{16(\theta + h)}$.

Proof of Proposition 17 The proof is based on the following expressions for the expected profit of an omniarchy.

For $h \in (-\theta, -\frac{1}{2}\theta)$, $X_1 \frac{1}{2}(\frac{\theta - h}{4\theta} + \frac{3\theta - h}{8\theta}) = X_1 \frac{1}{16}(\frac{5\theta - 3h}{\theta})$.

For $h = -\frac{1}{2}\theta$, the expression for the omniarchy becomes $X_1 \frac{11}{32}$.

For $h \in (-\frac{1}{2}\theta, 0)$, the expression amounts to $X_1 \frac{1}{16} \frac{4\theta^2 + 6\theta h + h^2}{\theta(\theta + h)}$.

For $h \in (0, \frac{1}{2}\theta)$, it boils down to $X_1 \frac{1}{16} \frac{4\theta^2 + 2\theta h - h^2}{\theta(\theta + h)}$.

For $h = \theta$, performance equals $X_1 \frac{11}{32}$.

And for $h \geq \theta$ it becomes $X_1 \frac{1}{16} \frac{5\theta + 4h}{\theta + h}$.

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