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ECO No. 98/20

EUI WORKING PAPERS

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ECONOMICS DEPARTMENT

EUI Working Paper **ECO** No. 98/20

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BADIA FIESOLANA, SAN DOMENICO (FI)

Printed in Italy in July 1998
European University Institute
Badia Fiesolana
I – 50016 San Domenico (FI)
Italy

TESTING OF SEASONAL FRACTIONAL INTEGRATION IN U.K. AND JAPANESE CONSUMPTION AND INCOME*

L.A. Gil Alaña[1] and P.M. Robinson[2]

[1] European University Institute
Department of Economics
Badia Fiesolana
50016 SAN DOMENICO DI FIESOLE
ITALY

[2] London School of Economics
Department of Economics
Houghton Street
LONDON WC2A 2AE
ENGLAND

ABSTRACT

The seasonal structure of quarterly U.K. and Japanese consumption and income is examined by means of fractionally-based tests proposed by Robinson (1994). These series were analyzed from an autoregressive unit root viewpoint by Hylleberg, Engle, Granger and Yoo (HEGY, 1990) and Hylleberg, Engle, Granger and Lee (HEGL, 1993). We find that seasonal fractional integration, with amplitudes possibly varying across frequencies is an alternative plausible way of modelling these series.

Corresponding author: L.A. Gil-Alaña
Department of Economics
European University Institute
Badia Fiesolana
I-50016 San Domenico di Fiesole
ITALY

Phone: +39 (55) 4685 433/4
Fax: +39 (55) 4685 202

JEL Classification: C22

Keywords: Fractional integration; Nonstationarity; Seasonality.

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*: Research supported by ESRC Grant R000235892.

1. Introduction and summary

Many economic time series contain important seasonal components. A simple model for a time series y_t is a regression on dummy variables S_{it},

$$y_t = m_o + \sum_{i=1}^{s-1} m_i S_{it} + \epsilon_t, \qquad \epsilon_t \sim iid, \tag{1}$$

where s is the number of time periods in a year and the m_i are unknown coefficients. Stochastic processes have also been widely used in modelling seasonality, for example the stationary seasonal ARMA

$$\Phi_p(L^s) y_t = \Theta_q(L^s)\epsilon_t, \qquad \epsilon_t \sim iid, \tag{2}$$

where $\Phi_p(L^s)$ and $\Theta_q(L^s)$ are polynomials in L^s (the seasonal lag operator) of orders p and q respectively, with the zeros of $\Phi_p(L^s)$ outside the unit circle and the zeros of $\Theta_q(L^s)$ outside or on the unit circle. If moreover the zeros of $\Theta_q(L^s)$ are strictly outside the unit circle, (2) can be written as an infinite autoregression

$$\rho(L^s) y_t = \epsilon_t, \qquad \epsilon_t \sim iid, \tag{3}$$

with all roots of $\rho(L^s) = 0$ outside the unit circle, some of them in complex pairs with seasonal periodicities. As an alternative to (1) and (2), it may be appropriate to allow for stochastic seasonal nonstationarity, as is implicit in the practice of seasonal differencing (see eg. Box and Jenkins (1970)) whereby the operator $1-L^s$ produces a stationary weakly dependent sequence. For example, for quarterly data $\rho(L^s) = 1-L^4$ can be factored as $(1-L)(1+L)(1+L^2)$, containing four zeros of modulus unity: one at zero frequency; one at two cycles per year, corresponding to frequency π; and two complex pairs at one cycle per year, corresponding to frequencies $\pi/2$ and $3\pi/2$ (of a cycle 2π).

A good deal of empirical work has followed this approach: Hylleberg, Engle, Granger and Yoo (1990) (henceforth HEGY) found evidence for seasonal unit roots in quarterly U.K. nondurable consumption and disposable income, using a procedure that allows tests for unit roots at some seasonal frequencies without maintaining their presence at all such frequencies. This procedure allows inclusion of a constant, seasonal dummies and/or a time trend. Beaulieu and Miron (1993) extended the HEGY procedure to monthly data and examined twelve U.S. macroeconomic series in monthly and quarterly data. By contrast with previous studies, they concluded that evidence in favour of a seasonal unit root was weak. These findings have been seriously questioned by Hylleberg, Jorgensen and Sorensen (1993), who concluded that seasonality is in many cases variable, not fixed. Hylleberg, Engle, Granger and Lee (1993) (henceforth HEGL) performed the HEGY test on quarterly series of Japanese real consumption and real disposable income, suggesting that income is integrated of order 1 (I(1)) at 0 and all seasonal frequencies, $\pi/2$, π and $3\pi/2$, and consumption is I(1) at frequencies 0 and π, while some difficulty was found in separating unit roots at frequency $\pi/2$ (and $3\pi/2$) from a deterministic seasonal pattern. Osborn (1993) suggested that a nonstationary periodic AR(1) or a periodically integrated I(1) processes could better be more useful.

Seasonal unit roots can be viewed not only in an autoregressive framework but also as a particular case of seasonal fractionally integrated processes. Consider the process

$$(1 - L^s)^d \, y_t = u_t, \tag{4}$$

where $d > 0$ and u_t is an I(0) series, which is defined as a covariance stationary process with spectral density bounded and bounded away from zero at all frequencies. Clearly, y_t has s roots of modulus unity, all with the same integration order d. (4) can be extended to present different integration orders for each seasonal frequency, whereas y_t is stationary if all orders are smaller than 1/2. We say that y_t has seasonal long memory at a given frequency if the integration order at that frequency is greater than zero. A seasonal series might also display only a single root at a particular frequency. For example, an integrated process with a single root at two cycles per year is:

$$(1 + L)^d \, y_t = u_t, \tag{5}$$

and at one cycle per year:

$$(1 + L^2)^d \, y_t = u_t. \tag{6}$$

Thus, if u_t is I(0) and $0 < d < 1/2$, y_t will in both cases be covariance stationary with spectral density unbounded at frequency π in (5), and at frequencies $\pi/2$ and $3\pi/2$ (of a cycle 2π) in (6).

Few empirical studies have been carried out in relation to seasonal fractional models. The notion of fractional Gaussian noise with seasonality was suggested by Jonas (1981) and extended in a Bayesian framework by Carlin, Dempster and Jonas (1985) and Carlin and Dempster (1989). Porter-Hudak (1990) applied a seasonal fractionally integrated model to quarterly U.S. monetary aggregate with the conclusion that a fractional ARMA model could be more appropriate than standard ARIMAs. Advantages of seasonal fractionally differencing models for forecasting monthly data are illustrated in Sutcliffe (1994), and another empirical application is found in Ray (1993).

In the following section we briefly describe some common tests for seasonal integration, and compare them with Robinson's (1994) tests for nonstationary hypotheses which permit testing of seasonal fractional integration of any stationary or nonstationary degree. Section 3 describes models to be tested, using Robinson's (1994) approach, to macroeconomic data of United Kingdom (Section 4) and Japan (Section 5) analyzed in HEGY (1990) and HEGL (1993) respectively. Section 6 contains some concluding remarks.

2. Tests for seasonal integration

We first consider the Dickey, Hasza and Fuller (DHF) (1984) test of $\rho_s = 1$ in

$$(1 - \rho_s L^s) y_t = \epsilon_t, \qquad \epsilon_t \sim iid\ (0,\sigma^2). \tag{7}$$

The test is based on the auxiliary regression

$$(1 - L^s) y_t = \pi y_{t-s} + \epsilon_t, \tag{8}$$

the test statistic being the t-ratio corresponding to π in (8). Due to the nonstandard asymptotic distributional properties of the t-ratios under the null hypothesis, DHF (1984) provide the simulated critical values for testing against the alternative $\pi < 0$. In order to whiten the errors in (8), the auxiliary regression may be augmented by lagged $(1-L^s)y_t$, and with deterministic components, but unfortunately this changes the distribution of the test statistic. A limitation of DHF (1984) is that it jointly tests for roots at zero and seasonal frequencies, and therefore does not allow for unit roots at some but not all seasonal frequencies.

This defect is overcome by HEGY (1990) for the quarterly case. Their test is based on the auxiliary regression

$$(1 - L^4) y_t = \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \epsilon_t, \tag{9}$$

where $y_{1t} = (1+L+L^2+L^3)y_t$ removes the seasonal unit roots but leaves in the zero frequency unit root, $y_{2t} = -(1-L+L^2-L^3)y_t$ leaves the root at π and $y_{3t} = -(1-L^2)y_t$ leaves the roots at $\pi/2$ and $3\pi/2$. The existence of unit roots at 0, π, $\pi/2$ (and $3\pi/2$) implies that $\pi_1 = 0$, $\pi_2 = 0$, and $\pi_3 = \pi_4 = 0$ respectively. The t-ratio for π_1 and π_2 is shown by HEGY to have the familiar Dickey-Fuller distribution (see Fuller (1976)) under the null of $\pi_1 = 0$ and $\pi_2 = 0$ respectively, while the t-ratio for π_3, conditioned on $\pi_4 = 0$ has the distribution described by DHF (1984) for

$s=2$. Also a joint test of $\pi_3 = \pi_4 = 0$ is proposed based on the F-ratio, and the critical values of the distribution tabulated. A crucial fact in these tests is that the same limiting distributions are obtained when it is not known a priori that some of the π's are zero: if the π's other than the one to be tested are truly nonzero, then the process does not have unit roots at these frequencies and the corresponding y's are stationary. The regression is therefore equivalent to a standard augmented unit-root test. If however some of the other π's are zero, there are other unit roots in the regression, but the corresponding y's are now asymptotically uncorrelated and the null distribution of the test statistic will not be affected by the inclusion of a variable with a zero coefficient which is orthogonal to the included variables. As in DHF (1984), the auxiliary regression has to be augmented by lagged dependent variables in order to whiten the errors, and deterministic components can be introduced in the auxiliary regression (9), though again the distribution changes. An extension of this procedure to allow joint HEGY-type tests for the presence of unit roots at zero and all seasonal frequencies, and only for the seasonal frequencies, is given in Ghysels et al. (1994). It is shown that the test statistics will have the same limiting distribution as the sum of the corresponding squared t-ratios for π_i $(i=1,2,3,4)$ in the former, and π_i $(i=2,3,4)$ in the latter test.

All these procedures test for a unit root in the seasonal AR operator and have stochastic nonstationarity as the null hypothesis. Canova and Hansen (1995) seasonally extend the test of Kwiatkowski et al. (1992), and propose a Lagrange multiplier test (the CH test) based on the residuals from a regression extracting the seasonal and other deterministic components, for testing the null of stationarity about a deterministic seasonal pattern. Hylleberg (1995) compares small sample properties of the HEGY and CH tests for seasonal unit roots in quarterly series, concluding that both tests complement each other. More recently, Tam and Reinsel (1996) propose a test for a unit root in the seasonal MA operator, testing a deterministic seasonal null against a stochastic nonstationary alternative. They consider the (integrated) SMA(1) model,

$$y_t = \mu_t + \epsilon_t, \qquad t = 1-s, \ldots 0, \tag{10}$$

$$(1-L^s)\, y_t = (1-\alpha L^s)\, \epsilon_t, \qquad t = 1,2,\ldots, \tag{11}$$

where μ_t is a deterministic seasonal mean, so that $\mu_t - \mu_{t-s} = 0$, and ε_t is, initially, a white noise process. Thus, a test of $\alpha = 1$ in (11) can be interpreted as a test of deterministic seasonality against the alternative $\alpha < 1$ of stochastic integrated seasonality. The test can be extended to allow ε_t to be a stationary and invertible ARMA, and also to allow for a deterministic linear trend in y_t, leading to a different nonstandard null limit distribution.

The tests described above consider the possibility of only a single form of seasonal stochastic nonstationarity, in particular, unit roots. We now describe the tests of Robinson (1994), which can test any integer or fractional root of any order on the unit circle in the complex plane.

We observe $\{(y_t,z_t), t=1,2,...,n\}$ where

$$y_t = \beta' z_t + x_t, \qquad t = 1,2,..., \tag{12}$$

$$\rho(L;\theta)x_t = u_t, \qquad t = 1,2,..., \tag{13}$$

$$x_t = 0, \qquad t \leq 0, \tag{14}$$

where β is a (kx1) vector of unknown parameters and z_t is a (kx1) vector of deterministic variables that might include an intercept, a time trend and/or seasonal dummies; $\rho(L;\theta)$, a prescribed function of L and the unknown (px1) parameter vector θ, will depend on the model tested; u_t is an I(0) process with parametric spectral density

$$f(\lambda;\tau) = \frac{\sigma^2}{2\pi} g(\lambda;\tau), \qquad -\pi < \lambda \leq \pi,$$

where the positive scalar σ^2 and the (qx1) vector τ are unknown, but g is of known form.

In general we wish to test the null hypothesis

$$\mathbf{H_o: \theta = 0.} \tag{15}$$

Under (15), the residuals are

$$\tilde{u}_t = \rho(L)y_t - \tilde{\beta}' w_t, \qquad t = 1,2,...,$$

where

$$\rho(L) = \rho(L;0), \qquad \tilde{\beta} = \left(\sum_{t=1}^{n} w_t w_t'\right)\sum_{t=1}^{n} w_t\, \rho(L)y_t, \qquad w_t = \rho(L)z_t.$$

Unless g is completely known function (eg. $g \equiv 1$, as when u_t is white noise) we have to estimate the nuisance parameter vector τ, for example by

$$\hat{\tau} = \arg\min_{\tau\in T} \sigma^2(\tau), \tag{16}$$

where T is a suitable subset of R^q and

$$\sigma^2(\tau) = \frac{2\pi}{n}\sum_{j=1}^{n-1} g(\lambda_j;\tau)^{-1} I(\lambda_j),$$

where

$$I(\lambda) = |(2\pi n)^{-1/2}\sum_{t=1}^{n} \tilde{u}_t e^{it\lambda}|^2, \quad \lambda_j = \frac{2\pi j}{n}.$$

The test statistic, derived from the Lagrange multiplier (LM) principle is

$$\hat{R} = \frac{n}{\hat{\sigma}^4}\hat{a}'\hat{A}^{-1}\hat{a} = \hat{r}'\hat{r}, \tag{17}$$

where

$$\hat{r} = \frac{n^{1/2}}{\hat{\sigma}^2}\hat{A}^{-1/2}\hat{a}, \quad \hat{\sigma}^2 = \sigma^2(\hat{\tau}), \quad \hat{a} = \frac{-2\pi}{n}\sum_j^{*} \psi(\lambda_j) g(\lambda_j;\hat{\tau})^{-1} I(\lambda_j),$$

$$\hat{A} = \frac{2}{n}\left(\sum_j^{*}\psi(\lambda_j)\psi(\lambda_j)' - \sum_j^{*}\psi(\lambda_j)\hat{\epsilon}(\lambda_j)'\left(\sum_j^{*}\hat{\epsilon}(\lambda_j)\hat{\epsilon}(\lambda_j)'\right)^{-1}\sum_j^{*}\hat{\epsilon}(\lambda_j)\psi(\lambda_j)'\right),$$

$$\psi(\lambda_j) = Re\left(\frac{\partial}{\partial\theta}\log\rho(e^{i\lambda_j};0)\right), \quad \hat{\epsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau})$$

and $\sum_j^{*}$ is a sum over λ_j such that $-\pi < \lambda_j < \pi$. $\lambda_j \notin (\rho_l-\lambda_l, \rho_l+\lambda_l)$, $l = 1,2,...,s$, such that ρ_l $l = 1,2,...,s < \infty$ are the distinct poles of $\rho(L)$. Note that $\hat{R}$ is a function of the hypothesized differenced series which has short memory under (15) and thus, we must specify the frequencies and integration orders of any seasonal roots.

Robinson (1994) established under regularity conditions that

$$\hat{R} \to_d \chi_p^2 \qquad as \ n \to \infty, \tag{18}$$

and also the Pitman efficiency property of LM in standard problems. If p = 1, an approximate one-sided 100α% level test of (15) against alternatives

$$\mathbf{H_1:}\ \theta > 0 \tag{19}$$

rejects H_o if $\hat{r} > z_\alpha$, where the probability that a standard normal variate exceeds z_α is α, and conversely, a test of (15) against alternatives

$$\mathbf{H_1:\ \theta < 0} \qquad \mathbf{(20)}$$

rejects H_o if $\hat{r} < -z_\alpha$. A test against the two-sided alternative $\theta \neq 0$, for any p, rejects if $\hat{R}$ exceeds the upper critical value of the χ_p^2 distribution.

We can compare Robinson's (1994) tests with those in HEGY (1990). Extending (9) to allow augmentations of the dependent variable to render the errors white noise, and deterministic paths, the auxiliary regression in HEGY (1990) is

$$\phi(L)(1 - L^4)y_t = \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \eta_t + \epsilon_t, \qquad (21)$$

where $\phi(L)$ is a stationary lag polynomial and η_t is a deterministic process that might include an intercept, a time trend and/or seasonal dummies. If we cannot reject the null hypothesis $\pi_1 = 0$ against the alternative $\pi_1 < 0$ in (21), the process will have a unit root at zero frequency whether or not other (seasonal) roots are present in the model. In Robinson's (1994) tests, taking (13) with

$$\rho(L;\theta) = (1 - L)^{d+\theta} \qquad (22)$$

with d = 1, (15) implies a single unit root at zero frequency. However, we could have instead

$$\rho(L;\theta) = (1 - L^2)^{d+\theta}, \qquad (23)$$

or alternatively

$$\rho(L;\theta) = (1 - L + L^2 - L^3)^{d+\theta}, \qquad (24)$$

or

$$\rho(L;\theta) = (1 - L^4)^{d+\theta}. \qquad (25)$$

If again d = 1, under (15), x_t displays unit roots at frequencies zero and π in (23); zero and two complex ones corresponding to frequencies $\pi/2$ and $3\pi/2$ in (24), or all of them in (25). Using HEGY's (1990) tests, the non-rejection of the null $\pi_2 = 0$ in (21) will imply a unit root at frequency π independently of other possible roots, and this can be consistent with (12)-(14) jointly with (23) or (25) among other possibilities covered by Robinson's (1994) tests. Furthermore, testing sequentially, (or jointly as in Ghysels et al. (1994)), the different null hypotheses in (21), if we cannot reject that $\pi_i = 0$ for i = 1,2,3 and 4, the overall null hypothesized model in HEGY (1990) becomes:

$$\phi(L)(1 - L^4)y_t = \eta_t + \epsilon_t, \qquad t = 1,2,..., \tag{26}$$

and we can compare it with the set-up in Robinson (1994), using (12)-(14) and (25) with

$$\phi(L)\, u_t = \epsilon_t, \qquad t = 1,2,..., \tag{27}$$

which, with d = 1, under the null (15), becomes

$$\phi(L)(1 - L^4)y_t = \phi(L)\,\beta'(1 - L^4)z_t + \epsilon_t, \qquad t = 1,2,\ldots . \tag{28}$$

Clearly, if we do not include explanatory variables in (12) and (21), (i.e. $\eta_t = z_t \equiv 0$), (28) becomes (26), and including regressors, the difference between the two models will be due purely to deterministic components. Similarly, if we cannot reject $\pi_1 = \pi_2 = 0$ but reject $\pi_3 = \pi_4 = 0$ in (21), a plausible model in HEGY (1990) would be

$$\phi(L)(1 - L^2)y_t = \eta_t + \epsilon_t, \qquad t = 1,2,..., \tag{29}$$

and the corresponding setting in Robinson's (1994) tests would be (12)-(14) and (27) with

$$\rho(L;\theta) = (1 - L^2)^{1+\theta}.$$

Robinson's (1994) tests allow testing different integration orders for each of the seasonal frequencies. Thus, instead of (25) we could consider for instance,

$$\rho(L;\theta) = (1 - L)^{d_1+\theta_1}(1 + L)^{d_2+\theta_2}(1 + L^2)^{d_3+\theta_3} \tag{30}$$

and test the null $\theta = (\theta_1,\theta_2,\theta_3)' = 0$ for different values of d_1, d_2 and d_3. This possibility is also ruled out in HEGY (1990) and the other tests presented above, which just concentrate on the unit root situations.

We can also compare the tests of Robinson (1994) with those in Tam and Reinsel (1996), who considered

$$y_t = \mu_t + u_t, \qquad t = 1-s,\ldots 0, \tag{31}$$

$$(1 - L^s)y_t = (1 - \alpha L^s)u_t, \qquad t = 1,2,..., \tag{32}$$

where μ_t is as in (10), (i.e., $\mu_t - \mu_{t-s} = 0$), and u_t is a stationary and invertible ARMA process. They tested

$$\mathbf{H_o: \ \alpha = 1} \qquad \mathbf{(33)}$$

in (32) against the alternative $\alpha < 1$. The non-rejection of (33) in (31) and (32) would imply that y_t follows a deterministic seasonal pattern plus a stationary stochastic process, (i.e., like (31) with t = 1,2,..), while its rejection would be evidence of seasonal integration. We can take fractional operators instead of the AR and MA ones in (32):

$$(1 - L^s)^d \, y_t \;=\; (1 - L^s)^\gamma \, u_t, \qquad t \;=\; 1,2..., \qquad (34)$$

with d > 0, and given the common factors appearing in both sides in (34), calling $\delta = \gamma$ - d, the model can be rewritten as (31) with

$$(1 - L^s)^\delta \, y_t \;=\; u_t, \qquad t \;=\; 1,2,.., \qquad (35)$$

and we can test

$$\mathbf{H_o: \ \delta = 0,} \qquad \mathbf{(36)}$$

against the alternative $\delta > 0$. Thus (32) and (35) are identical under the null. The null and alternative versions of (35) are covered by Robinson's (1994) setting, with $\beta' z_t$ in (12) replaced by μ_t, and s = 4, d = 0 and $\theta = \delta$ in (25).

The null χ^2 limit distribution of Robinson's (1994) tests is constant across specifications of $\rho(L;\theta)$ and z_t and thus does not require case by case evaluation of a nonstandard distribution, unlike of the other tests described. Ooms (1997) proposes Wald tests based on Robinson's (1994) model in (12)-(14), which have the same limit behaviour as LM tests of Robinson (1994), but require efficient estimates of the fractional differencing parameters. He suggests a modified periodogram regression estimation procedure of Hassler (1994), whose distribution is evaluated under simulation. Robinson's (1994) tests are applied to non-seasonal data by Gil-Alaña and Robinson (1997), and given the vast amount of empirical work based on AR structures, an empirical study of fractional based tests for seasonal data seems overdue.

3. Empirical applications

The relationship between consumption and income is arguably one of the most important in macroeconomics. The most influential and perhaps most widely tested view of this relationship is the permanent income hypothesis (see Hall (1989)). We concentrate on the univariate treatment of these two variables, and apply different versions of Robinson's (1994) tests to some seasonally unadjusted, quarterly data for United Kingdom and Japan, using the same datasets as in HEGY (1990) and HEGL (1993) respectively.

For both countries we follow the same procedure. We test (15) in a version of (12),

$$y_t = \beta_1 + \beta_2 t + \beta_3 S_{1t} + \beta_4 S_{2t} + \beta_5 S_{3t} + x_t, \qquad t = 1,2,..., \qquad (37)$$

with (13) and (14), where S_{1t}, S_{2t} and S_{3t} are seasonal dummies. We test in a sequential fashion. Since the data are quarterly, we start by assuming that x_t in (37) has four roots and take $\rho(L;\theta)$ as in (25). Given that θ is scalar, we test H_0 (15) against the one-sided alternatives (19) and (20). In order to allow different integration orders at different frequencies we also consider

$$\rho(L;\theta) = (1 - L^2)^{d_1+\theta_1} (1 + L^2)^{d_2+\theta_2}, \qquad (38)$$

and more generally, (30). Therefore, $\theta = (\theta_1,\theta_2)'$ under (38) and $(\theta_1,\theta_2,\theta_3)'$ under (30) and we test here (15) against the two-sided alternative $\theta \neq 0$. Clearly, when departures are actually of the specialized form (25), a test of (15) directed against (25) will have greater power than ones directed against (38) or (30), but the tests have power against a wider range of alternatives.

Following this sequential way of testing we next assume x_t displays only three roots: two of them complex, corresponding to frequencies $\pi/2$ and $3\pi/2$, and one real that might be either at zero or at frequency π. Thus, we perform the tests in case of (24) and

$$\rho(L;\theta) = (1 + L + L^2 + L^3)^{d+\theta}, \qquad (39)$$

and extending now the tests to allow different integration orders at the complex and at the real roots, we also consider two-sided tests where

$$\rho(L;\theta) = (1 - L)^{d_1+\theta_1} (1 + L^2)^{d_2+\theta_2} \qquad (40)$$

and

$$\rho(L;\theta) = (1 + L)^{d_1+\theta_1} (1 + L^2)^{d_2+\theta_2}. \qquad (41)$$

In a further group of tests, we assume the hypothesized model contains only two roots, one at zero frequency and the other at π. Again we look first at one-sided tests against (23) and then at two-sided tests against

$$\rho(L;\theta) \quad = \quad (1 - L)^{d_1+\theta_1}\ (1 + L)^{d_2+\theta_2}. \tag{42}$$

Finally we consider the possibility of a single root (or perhaps two complex ones), and therefore look at (22) as well as

$$\rho(L;\theta) \quad = \quad (1 + L)^{d+\theta}, \tag{43}$$

and finally,

$$\rho(L;\theta) \quad = \quad (1 + L^2)^{d+\theta}. \tag{44}$$

The form of $\hat{A}$ for these various choices of ρ is derived in the appendix. It is found that $\hat{A}$, interestingly, does not vary with the null hypothesized integration order d or integration orders d_i, clearly facilitating the computations. In all these cases the tests will be performed for different model specifications in (37). First we assume that $\beta_i \equiv 0$ a priori; next $\beta_i = 0$, $i \geq 2$, (including an intercept); next $\beta_i = 0$, $i \geq 3$, (a time trend); next $\beta_2 = 0$, (an intercept and dummy variables); finally that all β_i are unknown. In all cases we consider a wide range of null hypothesized d (and d_i's when $p > 1$), from 0.50 through 2.25 with 0.25 increments, and white noise u_t, though in some cases of interest we extend to I(0) parametric autocorrelation in u_t, allowing seasonal or non-seasonal AR structure. Clearly, non-rejections of (15) when d (and the d_i's) equal 1 imply unit roots, and non-rejections with $d = 0$ will suggest deterministic models of form advocated by Tam and Reinsel (1996).

4. The U.K. case

We analyze the quarterly United Kingdom dataset used in HEGY (1990). c_t is log consumption expenditure on non-durables and y_t is log personal disposable income, from 1955.1 through 1984.4. The conclusions of HEGY (1990) were that c_t could be I(1) at each of the frequencies 0, $\pi/2$ (and $3\pi/2$) and π; y_t may contain only two roots, at zero and π; c_t-y_t can have four unit roots if dummies are not introduced, but two unit roots of the same form as in c_t if they are.

Table 1 reports results for the one-sided statistic $\hat{r}$, when $\rho(L;\theta)$ in (13) is (25). First, in Table 1(i), we take u_t as a white noise process, and we observe that for the two individual series (c_t and y_t), the null is never rejected when d = 0.75 and d = 1, and also that d = 1.25 is not rejected when we include as regressors an intercept and dummies. For the differenced series (c_t-y_t), the values of d where H_0 is not rejected are slightly smaller (d = 0.50 and d = 0.75), and we see that the unit root null is clearly rejected in all cases, in favour of less nonstationary alternatives, suggesting that if the two individual series were in fact I(1), a degree of fractional cointegration may exist for a given cointegration vector (1,-1), using a simplistic version of the permanent income hypothesis theory as discussed by Davidson et al. (1978) for instance. The fact that the unit root null is never rejected for c_t is consistent with HEGY (1990), but this hypothesis is not rejected for y_t while HEGY (1990) found evidence of only two unit roots (at frequencies 0 and π) in this series. Various tests of this hypothesis will be performed later in a further group of tests. Also, HEGL (1990) allowed augmentations incorporating significant lagged values of the series. Thus, we also performed the tests with AR u_t. In Tables 1(ii) and (iii)

(Table 1 about here)

we give results for AR(1) and AR(2) u_t respectively. Tests allowing higher order AR u_t were also performed obtaining similar results. When allowing seasonal AR structures of form $\Phi(L^s)u_t = \varepsilon_t$, or mixed seasonal/non-seasonal ARs we observed a lack of monotonic decrease in $\hat{r}$ with respect to d in many cases. Such monotonicity is to be expected given correct specification and adequate sample size. In Tables 1(ii) and (iii) monotonicity is achieved in nearly all cases and the unit root null is always rejected. The non-rejection values are d = 0.50 and d = 0.75, and in those cases where the former is rejected, always it is in favour of stationary alternatives. The lower integration orders observed in these two tables compared with Table 1(i) can in large part be due to the fact that the AR estimates are Yule-Walker ones, entailing roots that cannot exceed one in absolute value but can be arbitrarily close to it, so they pick up part of the nonstationary component.

Table 2 reports results of the two-sided tests $\hat{R}$ in (17) when θ is (2x1). $\rho(L;\theta)$ is now given in (38) and therefore we allow different integration orders for the real and complex roots. We concentrate on the cases of no regressors, an intercept and a time trend. If there are no regressors, H_0 is rejected in all cases for the individual series and the lowest test statistics are achieved when $d_1 = 1$ and $d_2 = 0.5$, indicating perhaps the importance of real roots over complex ones. For c_t-y_t, all non-rejections correspond to values of d_2 (i.e. the integration order of the complex roots) smaller than d_1 (i.e. the integration order for the two real roots), and the lowest value is now at $d_1 = 0.75$ and $d_2 = 0.50$. Including a constant or a time trend, results are similar in both cases: for c_t, all non-rejections occur when d_1 = 1.00, 1.25 or 1.50 and when d_2 = 0.50 and 0.75, with the lowest statistic at $d_1 = 1$ and $d_2 = 0.5$. For y_t, we observe only three non-rejection cases corresponding to $d_1 = 1.00$, 1.25 and 1.50, with $d_2 = 0.50$, which might indicate that complex roots are not required when modelling this series, as pointed out in HEGY (1990). For c_t-y_t, there are some more non-rejections, with the lowest value at $d_1 = 0.75$ and $d_2 = 0.5$. Thus, we observe in all these tables a greater degree of integration for real roots than complex ones, and also smaller integration orders for c_t-y_t than for c_t and y_t.

(Tables 2 and 3 about here)

In Table 3 we extend these tests to allow different integration orders at zero and π and thus $\rho(L;\theta)$ is in (30). The results are consistent with the previous ones: in fact, when there are no regressors, the null hypothesis is always rejected for c_t and y_t while for c_t-y_t there are some non-rejections, with the lowest value achieved at $d_1 = 1$ and $d_2 = d_3 = 0.50$ (i.e. the same alternative as in Table 2). Including a constant or time trend, the lowest value of the statistics occurs when $d_1 = 1$ and $d_2 = d_3 = 0.50$ for c_t and c_t-y_t, and when $d_1 = 1.50$, $d_2 = 1.00$ and $d_3 = 0.50$ for y_t. All these results seem to emphasize the importance of the root at zero frequency over the others, given its greater integration order.

Following this sequential way of testing we next assume x_t can be modelled with three roots and thus, remove from (25) the root, at zero frequency (in which case $\rho(L;\theta)$ adopts the forms (39) or (41)), or at π (i.e., $\rho(L;\theta)$ as in (24) or (40)). Though we do not present the results, they show that H_0 is rejected in all series and across all cases, indicating the importance of these two roots, as was suggested in HEGY (1990).

In the next group of tables we suppose x_t has only two roots, at zero and π. First we take $\rho(L;\theta)$ as in (23), so the same integration order is assumed at both frequencies. This way of specifying the model is interesting in view of results in HEGY (1990), who suggested that only two unit roots at these frequencies were present in y_t, and in some cases for c_t-y_t. Results for white noise u_t are given in Table 4(i). Monotonicity is now always achieved and the non-rejection values occur when $d = 0.75$ and 1 for c_t and y_t, and when $d = 0.50$ for c_t-y_t, suggesting again the possibility of a fractional cointegration relationship at these two frequencies for the cointegrating vector (1,-1). The hypothesis of two unit roots (d=1) is always rejected for c_t if we include regressors. These rejections are in line with HEGY (1990),

(Table 4 about here)

who indicated that complex unit roots should be included. For y_t we observe that d=1 is not rejected in 3 of the 5 possible specifications in (37), which is also consistent with HEGY (1990). If u_t follows a seasonal AR (Tables 4(ii) and (iii)), the non-rejections occur for d between 0.50 and 1 for the individual series, but only when d = 0.50 there are non-rejections for c_t-y_t. We observe in these tables more non-rejection cases for y_t than for the other two series when testing the unit root null, as is once more consistent with HEGY (1990).

In Table 5 we allow integration orders to differ between zero and π frequencies and thus, $\rho(L;\theta)$ is as in (42). If there are no regressors, H_0 is always rejected and the lowest statistics are obtained at d_1 = 1.25 and d_2 = 0.50 for c_t and y_t, and at d_1 = 0.50 and d_2 = 1.50 for c_t-y_t, so if there are no regressors but x_t displays two real roots, the root at zero appears more important than the seasonal one for the individual series but the one at π is most important when modelling c_t-y_t. Including a constant or a time trend, the results are consistent with those in Table 4(i), where the only non-rejection case with an intercept or a time trend was d = 0.75 for y_t. In Table 5 this alternative is narrowly rejected but not d_1 = 0.75 and d_2 = 0.50, and in all the other situations, H_0 is rejected as in Table 4(i).

(Tables 5 and 6 about here)

Finally we assume x_t has only two complex roots, at $\pi/2$ and $3\pi/2$, or a single one either at π or zero. Thus $\rho(L;\theta)$ takes the form given in (44), (43) and (22) respectively. As expected, H_0 is always rejected in the first two cases, given the importance of the root at zero frequency to describe trending behaviour. Table 6 gives results of $\hat{r}$ for white noise u_t and $\rho(L;\theta)$ as in (22), and we observe here that if there are no regressors, the I(1) null is not rejected for c_t and y_t, but is strongly rejected for c_t-y_t with stationary alternatives (d < 0.5) being more plausible. There are few non-rejections in this table and they correspond to values of d ranging between 0.50 and 1 for the individual series. For c_t-y_t, the only two non-rejection cases occur at d = 0.50 if dummies are included, but for the remaining specifications, this null is strongly rejected in favour of stationary alternatives. The fact that the unit root is rejected in this table for all series when some regressors are included in (37) is consistent with HEGY (1990), who suggest the need of at least one seasonal unit root.

Summarizing now the main results obtained in the U.K. case, we can say that if x_t in (37) is I(d) with four roots of the same order and u_t is white noise, the values of d where the null is not rejected range between 0.75 and 1 for the individual series and are slightly smaller for the difference c_t-y_t. If u_t is AR, d ranges between 0.50 and 0.75 for the three series considered. Allowing different integration orders at each frequency, we observe that the root at zero frequency seems more important than the seasonal ones, and the seasonal root at π appears also more important than the two complex ones at $\pi/2$ and $3\pi/2$. Modelling x_t as an integrated process with three roots, the null is strongly rejected when the excluded root is at zero. If the excluded root is the real seasonal π, the null is also rejected in practically all cases, suggesting the importance of these two roots. If we take x_t as I(d) with two real roots, the model seems more appropriate for y_t than for c_t or c_t-y_t, which is in line with results in HEGY (1990). Finally, modelling x_t as fractionally integrated with a single root at zero frequency, the range of d where H_0 is not rejected goes from 0.50 to 1 for the individual series but close to stationarity for c_t-y_t, but using a single seasonal root at frequency π or a pair of complex ones at frequencies $\pi/2$ and $3\pi/2$ seems inappropriate in view of the great proportion of rejections.

5. The Japanese case

We analyze here the log of total real consumption (c_t), the log of real disposable income (y_t), and the difference between them (c_t-y_t) in Japan from 1961.1 to 1987.4 in 1980 prices. These series have been analyzed in HEGL (1993) to test the presence of seasonal integration and cointegration. In this work (and in an earlier version (HEGL (1991)), they apply the HEGY (1990) tests to these data and their conclusions can be summarized as follows: for c_t, integration is obtained at all frequencies 0, $\pi/2$, $3\pi/2$ and π if there are no regressors in the model or if only a time trend is included; however, if dummies are also included, only two unit roots are observed, one at zero frequency and one at frequency π. For y_t, unit roots are not rejected at any frequency when there are no regressors or when a time trend and/or dummies are introduced, but if only an intercept is included the unit root at zero frequency is rejected. Finally, for c_t-y_t, unit root nulls are not rejected at any frequency, independently of the regressors used.

Table 7 is analogous to Table 1, showing the one-sided test statistic $\hat{r}$ when $\rho(L;\theta)$ in (13) takes the form (25). Table 7(i) reports results for white noise u_t, and the first thing that we observe is that if $\beta_i \equiv 0$ in (37), we cannot reject (15) for d = 0.75 and d = 1 in either c_t or y_t, while in c_t-y_t, these two cases are also not rejected, along with d = 0.50. When regressors such as an intercept, a trend or seasonal dummies are included, the unit root hypothesis is rejected in both series in favour of more nonstationary alternatives (d > 1), but in some cases we observe a lack of monotonicity with respect to d, in particular when we include an intercept, and an intercept and dummies for c_t, and an intercept and dummies for y_t. Looking at c_t-y_t, monotonicity is now always achieved and the nulls of d = 0.75 and d = 1 are never rejected. We could conclude from this table that if $\rho(L;\theta) = 1-L^4$, and u_t is in fact white noise, the two individual series are clearly nonstationary with d greater than 1 in most cases; however their difference seems less nonstationary (with $d \leq 1$), suggesting that some fractional cointegration could exist between both series, for the cointegrating vector (1,-1). The fact that d = 1 is not rejected for c_t and y_t when there are no regressors, and for c_t-y_t independently of the regressors used in (37), is consistent with the results in HEGL (1993) though they allow AR structure in the differenced series. Therefore in Tables 7(ii) and (iii) we suppose that u_t in (13) is an AR(q) with q = 1 and 2. Monotonicity is now observed in many cases, especially for c_t-y_t. The range of non-rejection values of d goes from 0.50 through 1 for c_t and c_t-y_t, and from 0.50 through 1.25 for y_t. When d > 1.25, H_0 is rejected in all cases where monotonicity is achieved. As we explained before for the U.K. case, this smaller degree in the integration order

(Table 7 about here)

of the series (compared with Table 7(i)), could be in large part due to competition between integration order and AR parameters in describing the nonstationary component. If we concentrate on the AR(1), we see that the unit root is not rejected for y_t but is for c_t when dummy variables are included in the model, again in line with HEGL (1993).

So far we have assumed that the four roots in x_t must have the same integration order. In the following tables we allow integration orders to differ between complex roots and real ones. Table 8 corresponds to two-sided tests when $\rho(L;\theta)$ in (13) takes the form given in (38) and we present results for $\beta_i \equiv 0$, $\beta_i = 0$, $i \geq 2$, and finally $\beta_i = 0$, $i \geq 3$. When there are no regressors, the null is rejected in all cases for both c_t and y_t with the lowest value of the statistics achieved when $d_1 = 1$ and $d_2 = 0.50$, suggesting that perhaps the complex roots are not required and only two roots (at frequencies zero and π) are needed. Looking at c_t-y_t, we observe some non-rejection cases: if $d_1 = d_2$, the null is not rejected when the integration order is 0.50, 0.75 and 1. These three possibilities were not rejected in Table 7(i) when we considered the one-sided tests, but the lowest test statistics are now achieved when $d_1 = 0.75$ and $d_2 = 0.50$. Including an intercept or a time trend, we observe now some non-rejections for c_t and y_t. Starting with c_t, H_0 is not rejected when $d_1 = 1.25$ or 1.50 and $d_2 = 0.50$, 0.75 or 1, observing therefore a greater degree of integration at zero and π frequencies than at $\pi/2$ and $3\pi/2$. Similarly, for y_t, all non-rejections occur when d_1 is slightly greater than d_2, and for c_t-y_t, the lowest test statistics are obtained at $d_1 = d_2 = 0.75$. The null hypothesis of a unit root at all frequencies ($d_1 = d_2 = 1$) is not rejected in this series which is again consistent with Table 7(i) and with results given in HEGL (1993).

(Tables 8 and 9 about here)

In Table 9 we are slightly more general in the specification of $\rho(L;\theta)$ in (13), and a different integration order is allowed at each frequency. Therefore $\rho(L;\theta)$ takes the form (30) and again in this table, we present results for cases of no regressors, an intercept, and a time trend, with white noise u_t. Similarly to Table 8, when there are no regressors the null is always rejected for the individual series with the lowest value obtained at $d_1 = 1.50$ and $d_2 = d_3 = 0.50$, indicating therefore the importance of the root at zero frequency. For c_t-y_t there are non-rejections at some alternatives with the lowest value obtained at $d_1 = 1.50$, $d_2 = 0.50$ and $d_3 = 1$, but the case of $d_1 = d_2 = d_3 = 1$ is rejected. Finally, including an intercept or a time trend, the results are similar in both cases. For c_t, the lowest test statistic is obtained when $d_1 = 1.50$, $d_2 = 1.00$ and $d_3 = 0.50$; for y_t, when $d_1 = 1.50$, and $d_2 = d_3 = 1.00$, and for c_t-y_t, when $d_1 = 1.00$, $d_2 = 0.50$ and $d_3 = 1.00$. All these results corroborate the importance of the root at zero frequency over the others for the three series.

Performing the tests under the assumption that $\rho(L;\theta)$ is of forms (24) or (39)-(41), we always rejected. Thus, following this sequential way of performing the tests, we next assume that x_t has only two roots, one at zero frequency and the other at π. First we take $\rho(L;\theta)$ as in (23), so θ consists of a single parameter. Tables 10(i)-(iii) give results for one-sided tests with white noise and seasonal AR u_t. In Table 10(i) we observe that monotonicity is always achieved, though the results are quite variable across the different specifications of (37). Starting with c_t, if there are no regressors, the non-rejection values of d range between 0.75 and 1.25; when a time trend is considered, the only non-rejection case occurs at $d = 0.50$, and including dummies the values of d where the null is not rejected are 1 and 1.25. For y_t, if there are no regressors, the null is not rejected when $d = 0.75$ and 1; including an intercept, the only non-rejection value occurs at $d = 0.5$, and with seasonal dummies, the only non-rejection value of d is 0.75. For c_t-y_t, the null is rejected in favour of stationary alternatives for the first three cases, however, including dummies, it is not rejected when $d = 0.50$. For the unit root null, our results are consistent with those of HEGL (1993). In fact,

(Table 10 about here)

the unit root null is not rejected for c_t when dummies are included, but is nearly always rejected for y_t and c_t-y_t, due perhaps to exclusion of unit roots at frequencies $\pi/2$ and $3\pi/2$, as was suggested by these authors. Modelling u_t with seasonal AR, in Tables 10(ii) and (iii), we observe that for c_t, the values of d range between 0.5 and 1.25, and the unit root null is now never rejected. However, looking at y_t, the unit root null is rejected in favour of less nonstationary alternatives in all cases except when there are no regressors where the unit root is not rejected. Since this null hypothesis is not rejected for c_t, but it is for y_t and c_t-y_t, again results in this case with seasonal AR u_t support the evidence found in HEGL (1993) that only two unit roots (at frequencies zero and π) were present in c_t. For c_t-y_t, only when there are no regressors and d = 0.50 is the null not rejected, and in all other cases, stationary alternatives seem more plausible, so again here, a certain degree of fractional cointegration seems to exist at these two frequencies, according to the permanent income hypothesis.

Table 11 reports results extending the tests to allow different integration orders at the same two frequencies. We observe across this table just a single case where the null is not rejected and it corresponds to c_t when there are no regressors and d_1 = 1.25 and d_2 = 0.50. Results here are consistent with those given in Table 10(i) when we tested a scalar θ, especially for cases of an intercept and a time trend: with an intercept, we saw in Table 10(i) that the only non-rejection case was for y_t with d = 0.50. In Table 11 this hypothesis is rejected but it corresponds to the lowest value of the test statistics obtained across the table. Similarly for the case of a time trend, the only non-rejection in Table 10(i) corresponded to c_t with d = 0.50 and again this hypothesis produces the lowest statistic in Table 11.

(Tables 11 and 12 about here)

Finally, we examine the case of x_t containing a single root, and concentrate on the case when this root is at zero, i.e. (22). Table 12 shows results merely for white noise u_t, and we observe that the unit root null is not rejected for c_t and y_t when there are no regressors, but strongly rejected for c_t-y_t, in favour of stationary alternatives (with d < 0.5). There are few non-rejections in this table (only 5 of the 120 cases presented), and apart from the two cases of a unit root, the other three non-rejection cases correspond to d = 0.5 with a time trend for c_t, and d = 0.75 with seasonal dummies for y_t. In case of c_t-y_t, the null is rejected in favour of stationary alternatives for the whole variety of specifications in (37), suggesting that at this zero frequency, a certain degree of fractional cointegration might also occur and referring again to the permanent income hypothesis. We also performed the tests allowing AR u_t, but we observed here very few cases where monotonicity was achieved across the different values of d. This can be explained because seasonality is not captured now by first differences, and the deterministic components do not seem sufficient to pick up this effect. As a complement to this, on including a seasonal AR, monotonicity was achieved in practically all cases, with results very similar to those for white noise u_t in Table 12. Modelling x_t with a single root at frequency π (i.e., (43)) or as an I(d) process with two complex roots corresponding to frequencies $\pi/2$ and $3\pi/2$ (i.e., (44)), produced rejections for all cases and across all series.

As a conclusion we can summarize the main results obtained for the Japanese case by saying that if x_t is I(d) with four seasonal roots of the same order d, and u_t is white noise, the values of d where the null is not rejected are at least one for c_t and y_t, and less than or equal to one for c_t-y_t. If u_t is AR, d ranges in most cases from 0.50 to 1 for the three series, and allowing different integration orders for the different frequencies, the most noticeable fact is the relative importance of the root at zero frequency over the others. Excluding one of the real roots (either at zero or at frequency π), H_0 is rejected in practically all situations, indicating the importance of these roots. Taking x_t as I(d) with two roots, at zero and at frequency π, if u_t is white noise, the null is not rejected for c_t when d ranges between 0.75 and 1.25 while for y_t and c_t-y_t the non-rejection cases correspond to $d < 1$. Modelling here u_t as seasonal AR, the unit root null is not rejected for c_t but is for the other two series, and if we permit different integration orders at these two frequencies, the only non-rejection case occurs for c_t, with the integration order at zero frequency slightly greater than at π. Finally, if we assume that x_t has a single root at zero or at frequency π (or two complex ones corresponding to frequencies $\pi/2$ and $3\pi/2$), the unit root hypothesis will be rejected in practically all cases in favour of less nonstationary alternatives.

6. Concluding remarks

We have presented a variety of model specifications for quarterly consumption and income data in Japan and U.K.. Given the number of possibilities covered by Robinson's (1994) tests, one cannot expect to draw unambiguous conclusions about the very best way of modelling these series. In fact, using these tests, the null hypothesized model will permit different deterministic paths; different lagged structures allowing roots at some or all seasonal frequencies (as well as at zero frequency), each of them with a possibly different integration order; and different ways of modelling the I(0) disturbances

u_t. Looking at the results presented above as a whole, some common features are observed for all series in both countries, however, and they can be summarized as follows:

First, modelling x_t as a quarterly I(d) process (so $\rho(L) = 1-L^4$) seems to be appropriate when u_t is white noise or a non-seasonal AR, but not if u_t is a seasonal AR. This can be explained because seasonality can be captured in this case either by quarterly integration or by seasonal dummies in (37). We also observe that integration orders seem slightly smaller if u_t is AR rather than white noise, due perhaps to the AR picking up part of the nonstationary component. The results emphasize the importance of real roots over complex ones, given the greater integration order observed for the former, and this is even clearer when we allow different integration orders for each frequency. Excluding one real root results in rejecting the null in practically all situations. If $\rho(L;\theta)$ is given by (23), we observe some non-rejections if u_t is white noise, and allowing I(0) parametric autocorrelation, the results are now better for the case of seasonal AR than for non-seasonal AR processes. This can be explained because the lagged function $\rho(L)$ does not now seem to capture seasonality at all and therefore the seasonal AR component may play an important role in this situation. Separating here the roots at zero and at π, the results emphasize the importance of the root at zero, but modelling the series as a simple I(d) process with a single root does not seem appropriate in most of the cases.

Another common feature observed across all these tables is the fact that integration orders for the individual series seem to range between 0.50 (or 0.75) and 1.25, independently of the lagged function used when modelling x_t in (13) and the inclusion or not of deterministic parts in (37), indicating clearly the nonstationary nature of these series. (In fact, though it was not shown in the tables, the null was practically always rejected when d ranged between 0 and 0.50 and therefore, we found conclusive evidence against deterministic patterns of the form proposed in Tam and Reinsel (1996)); however, c_t-y_t seems less integrated in practically all situations. Therefore, if we consider that the series are well modelled by a given function $\rho(L)$, a certain degree of fractional cointegration would exist between consumption and income for a given cointegration vector (1,-1), using a very simplistic version of the permanent income hypothesis.

We can finally compare these results with those obtained in HEGL (1993) and HEGY (1990) for unit root situations. Results in HEGL (1993) for Japanese data indicated the presence of unit roots at all frequencies for y_t and c_t-y_t, and the same conclusions hold for c_t if dummies were not included in the model but only the two real unit roots would be present if these dummy variables were included. If we look now at our tables we observe that the unit root null is not rejected for y_t in any specification in (37) when $\rho(L;\theta)$ adopts the form in (25) with AR u_t. Similarly for c_t-y_t, we cannot reject the unit root null for the same $\rho(L;\theta)$ and white noise u_t. For c_t, the null of four unit roots is not rejected when there are no dummies, but if they are included non-rejections will occur when $\rho(L;\theta)$ takes the form of (23) with white noise or seasonal AR u_t. For the U.K. case, results in HEGY (1990) suggested that four unit roots could be present for c_t, and for c_t-y_t if dummies were not included, and two real unit roots for y_t, and for c_t-y_t if they were included. Our results again show a certain consistency with theirs, given that the unit root null is not rejected for consumption if $\rho(L;\theta)$ is (25) with white noise u_t, and for income this hypothesis is not rejected if $\rho(L;\theta)$ takes the form of (23) and u_t is white noise or a seasonal AR.

APPENDIX

In this appendix we analyze the matrix $\hat{A}$ in $\hat{R}$ in (17) when $\rho(L;\theta)$ in (13) adopts the form in (30), and u_t is white noise so that

$$\hat{A} = \frac{2}{n}\sum_{j}^{*} \psi(\lambda_j)\,\psi(\lambda_j)',$$

where $\psi(\lambda) = (\psi_1(\lambda), \psi_2(\lambda), \psi_3(\lambda))'$ for $|\lambda| \leq \pi$, with

$$\psi_1(\lambda) = Re[\log(1 - e^{i\lambda})] = \log\left|2\sin\frac{\lambda}{2}\right| = -\sum_{r=1}^{\infty}\frac{\cos r\lambda}{r},$$

$$\psi_2(\lambda) = Re[\log(1 + e^{i\lambda})] = \log\left(2\cos\frac{\lambda}{2}\right) = -\sum_{r=1}^{\infty}(-1)^r\frac{\cos r\lambda}{r},$$

$$\psi_3(\lambda) = Re[\log(1 + e^{2i\lambda})] = \log|2\cos\lambda| = -\sum_{r=1}^{\infty}(-1)^r\frac{\cos 2r\lambda}{r}.$$

Then $\hat{A}$ can be approximated in large samples by

$$\tilde{A} = \frac{1}{\pi}\int_{-\pi}^{\pi}\psi(\lambda)\,\psi(\lambda)'d\lambda = (\tilde{A}_{ij}),$$

where

$$\tilde{A}_{11} = \tilde{A}_{22} = \tilde{A}_{33} = \sum_{r=1}^{\infty} r^{-2} \simeq \frac{\pi^2}{6} = 1.644,$$

$$\tilde{A}_{13} = \tilde{A}_{31} = \tilde{A}_{23} = \tilde{A}_{32} = \frac{1}{2}\sum_{r=1}^{\infty}(-1)^r r^{-2} \simeq -0.411,$$

$$\tilde{A}_{12} = \tilde{A}_{21} = \sum_{r=1}^{\infty}(-1)^r r^{-2} \simeq -0.822.$$

$\hat{A}$ in (17) approximates n times the expected value of the second derivative matrix of the log-likelihood with respect to the (px1) parameter vector θ. (See Robinson (1994), page 1433). Thus, given the non-diagonality of $\hat{A}$, we rule out the possibility of testing, as in HEGY (1990), for the presence of roots independently of the existence of other roots at any other frequencies in the process.

For the remaining specifications of $\rho(L;\theta)$, $\tilde{A}$ can be easily obtained from the above expressions. Thus, if $\rho(L;\theta)$ is given by (25), $\psi(\lambda) = \psi_1(\lambda)+\psi_2(\lambda)+\psi_3(\lambda)$ and $\tilde{A} = 1.64$; under (38), $\psi(\lambda) = [\psi_1(\lambda)+\psi_2(\lambda),\psi_3(\lambda)]'$ and the (2x2) matrix $\tilde{A} = [(1.64,-0.82)'; (-0.82, 1.64)']$; under (24), $\psi(\lambda) = \psi_1(\lambda)+\psi_3(\lambda)$ and $\tilde{A} = 2.46$; under (39), $\psi(\lambda) = \psi_2(\lambda)+\psi_3(\lambda)$ and $\tilde{A} = 2.46$; under (40), $\psi(\lambda) = [\psi_1(\lambda),\psi_3(\lambda)]'$ and $\tilde{A} = [(1.64,-0.41)'; (-0.41, 1.64)']$; under (41), $\psi(\lambda) = [\psi_2(\lambda),\psi_3(\lambda)]'$ and $\tilde{A} = [(1.64,-0.41)'; (-0.41, 1.64)']$; under (23), $\psi(\lambda) = \psi_1(\lambda)+\psi_2(\lambda)$ and $\tilde{A} = 1.64$; under (42), $\psi(\lambda)=[\psi_1(\lambda),\psi_2(\lambda)]'$ and $\tilde{A} = [(1.64,-0.82)'; (-0.82, 1.64)']$; under (22), (43) or (44), $\psi(\lambda) = \psi_1(\lambda)$, $\psi_2(\lambda)$ or $\psi_3(\lambda)$ respectively, with $\tilde{A} = 1.64$ in each case.

Allowing AR (q) u_t, $g(\lambda;\tau)$ below (14) takes the form

$$|1 - \sum_{j=1}^{q} \tau_j e^{ij\lambda}|^{-2}$$

and $\hat{A}$ will be given by the expression below (17), with the l^{th} element of $\hat{\epsilon}(\lambda)$ given by

$$\hat{\epsilon}_l(\lambda) = 2\left(\cos l\lambda - \sum_{j=1}^{q} \hat{\tau}_j \cos(l-j)\right) g(\lambda;\hat{\tau}).$$

A diskette with the FORTRAN code for the tests is available from the authors on request.

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TABLE 1

$\hat{r}$ in (17) with $\rho(L;\theta) = (1-L^4)^{d+\theta}$ (U.K. data)

(i) With white noise u_t

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
	--	3.31	1.02'	-1.00'	-2.43	-3.32	-3.88	-4.25	-4.51
	I	5.09	1.31'	-1.11'	-2.00	-2.79	-3.42	-3.86	-4.18
c_t	I,T	2.65	0.41'	-1.26'	-2.33	-3.02	-3.46	-3.75	-3.99
	I,S	5.17	1.32'	-1.09'	-1.87'	-2.62	-3.24	-3.70	-4.04
	I,T,S	2.70	0.31'	-1.25'	-2.23	-2.87	-3.34	-3.72	-4.04
	--	3.29	1.01'	-1.00'	-2.42	-3.31	-3.87	-4.24	-4.50
	I	5.16	1.25'	-0.96'	-1.81'	-2.61	-3.25	-3.72	-4.08
y_t	I,T	2.50	0.45'	-1.06'	-2.11	-2.84	-3.37	-3.76	-4.07
	I,S	5.16	1.21'	-0.97'	-1.76'	-2.53	-3.16	-3.64	-4.00
	I,T,S	2.41	0.39'	-1.06'	-2.06	-2.76	-3.28	-3.69	-4.02
	--	-0.66'	-1.48'	-2.21	-2.84	-3.32	-3.69	-3.99	-4.24
	I	1.09'	-1.37'	-2.39	-3.05	-3.53	-3.88	-4.15	-4.37
c_t - y_t	I,T	-0.20'	-1.44'	-2.39	-3.06	-3.53	-3.86	-4.11	-4.32
	I,S	1.34'	-1.19'	-2.21	-2.89	-3.41	-3.79	-4.08	-4.32
	I,T,S	-0.01'	-1.26'	-2.21	-2.92	-3.43	-3.82	-4.11	-4.35

(ii) With AR(1) u_t

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
	--	-3.26	-3.62	-3.96	-4.27	-4.52	-4.72	-4.87	-4.98
	I	-0.84'	-0.78'	-2.10	-3.13	-3.76	-4.17	-4.44	-4.63
c_t	I,T	1.07'	-0.82'	-2.32	-3.25	-3.81	-4.16	-4.39	-4.55
	I,S	-2.27	-2.65	-3.34	-3.75	-4.05	-4.29	-4.49	-4.65
	I,T,S	-1.08'	-2.64	-3.38	-3.81	-4.10	-4.32	-4.50	-4.65
	--	-3.26	-3.62	-3.96	-4.27	-4.52	-4.71	-4.86	-4.98
	I	-1.81'	-1.77'	-2.59	-3.32	-3.85	-4.23	-4.49	-4.69
y_t	I,T	-0.24'	-1.69'	-2.69	-3.40	-3.90	-4.25	-4.50	-4.68
	I,S	-2.43	-2.52	-3.01	-3.47	-3.87	-4.18	-4.43	-4.62
	I,T,S	-1.23'	-2.32	-2.99	-3.51	-3.90	-4.21	-4.44	-4.63
	--	-0.86'	-1.85'	-2.60	-3.17	-3.59	-3.91	-4.17	-4.38
	I	-0.30'	-1.79'	-2.66	-3.25	-3.69	-4.01	-4.25	-4.45
c_t - y_t	I,T	-0.62'	-1.80'	-2.66	-3.26	-3.69	-3.99	-4.22	-4.41
	I,S	-0.29'	-1.67'	-2.52	-3.13	-3.58	-3.93	-4.20	-4.41
	I,T,S	-0.57'	-1.69'	-2.52	-3.14	-3.60	-3.94	-4.21	-4.43

(iii) With AR(2) u_t

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
	--	-3.30	-3.62	-3.91	-4.21	-4.48	-4.69	-4.85	-4.98
	I	-1.11'	-1.10'	-2.25	-3.18	-3.77	-4.16	-4.42	-4.61
c_t	I,T	0.45'	-1.17'	-2.47	-3.32	-3.85	-4.18	-4.39	-4.54
	I,S	-2.35	-2.80	-3.49	-3.88	-4.15	-4.36	-4.54	-4.68
	I,T,S	-1.29'	-2.81	-3.53	-3.93	-4.20	-4.39	-4.55	-4.68
	--	-3.29	-3.61	-3.91	-4.21	-4.47	-4.68	-4.85	-4.98
	I	-2.13	-2.27	-2.89	-3.47	-3.92	-4.26	-4.51	-4.69
y_t	I,T	-1.10'	-2.19	-2.96	-3.54	-3.97	-4.29	-4.51	-4.69
	I,S	-2.62	-2.81	-3.20	-3.59	-3.92	-4.20	-4.43	-4.61
	I,T,S	-1.79'	-2.64	-3.18	-3.61	-3.95	-4.23	-4.45	-4.62
	--	-0.90'	-2.02	-2.79	-3.31	-3.69	-3.97	-4.20	-4.40
	I	-0.68'	-1.99	-2.83	-3.39	-3.78	-4.07	-4.29	-4.47
c_t - y_t	I,T	-0.71'	-1.96	-2.82	-3.39	-3.78	-4.06	-4.27	-4.44
	I,S	-0.69'	-1.90'	-2.72	-3.29	-3.70	-4.02	-4.26	-4.46
	I,T,S	-0.67'	-1.88'	-2.71	-3.29	-3.71	-4.03	-4.28	-4.47

': Non-rejection values for the null hypothesis (15) at 95% significance level; --: No intercept, no time trend and no seasonal dummies; I: Intercept; I,T: Intercept and time trend; I,S: Intercept and seasonal dummies; I,T,S: Intercept, time trend and seasonal dummies.

TABLE 2: $\hat{R}$ in (17) with $\rho(L;\theta) = (1\text{-}L^2)^{d1+\theta1}\,(1+L^2)^{d2+\theta2}$ and white noise u_t (U.K. data)

		No intercept and no trend			Intercept			Intercept and time trend		
d_1	d_2	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t
0.50	0.50	52.45	52.15	3.42'	79.34	83.17	11.36	33.55	40.66	3.65'
0.50	0.75	60.69	60.37	9.92	88.99	91.84	22.06	46.54	48.22	10.31
0.50	1.00	67.35	66.99	14.87	96.04	99.10	31.11	54.20	53.60	15.80
0.50	1.25	72.87	2.47	18.35	102.02	105.50	38.64	59.62	57.75	19.94
0.50	1.50	77.53	77.09	20.95	107.41	111.28	45.04	63.87	61.09	23.15
0.75	0.50	19.80	19.76	1.05'	12.96	18.85	0.86'	7.51	14.80	0.86'
0.75	0.75	25.89	25.85	5.65'	23.48	26.73	4.90'	16.69	21.37	4.82'
0.75	1.00	31.25	31.19	10.25	31.01	33.24	9.40	23.30	26.30	9.24
0.75	1.25	36.06	35.98	13.73	36.87	38.92	13.17	28.11	30.26	12.80
0.75	1.50	40.45	40.34	16.43	41.85	44.05	16.36	31.94	33.59	15.69
1.00	0.50	8.31	8.29	2.03'	0.86'	5.43'	2.76'	1.03'	5.61'	2.75'
1.00	0.75	11.56	11.57	4.20'	6.07	10.23	4.48'	6.47	10.40	4.46'
1.00	1.00	14.42	14.44	7.73	11.13	14.03	7.61	11.48	14.06	7.59
1.00	1.25	17.08	17.10	10.62	14.86	17.17	10.23	15.03	17.03	10.22
1.00	1.50	19.61	19.62	12.90	17.78	19.92	12.30	17.74	19.60	12.30
1.25	0.50	8.60	8.55	4.99'	0.98'	3.89'	5.88'	1.36'	4.47'	5.91'
1.25	0.75	10.58	10.56	5.34'	4.14'	7.44	6.20	4.78'	7.98	6.26
1.25	1.00	12.05	12.04	7.84	8.23	10.23	8.52	8.93	10.61	8.58
1.25	1.25	13.24	13.24	10.04	11.18	12.46	10.57	11.71	12.59	10.63
1.25	1.50	14.30	14.31	11.73	13.34	14.42	12.07	13.60	14.27	12.13
1.50	0.50	11.09	11.01	8.22	2.96'	5.40'	8.93	3.22'	6.04	8.89
1.50	0.75	12.97	12.92	7.49	5.14'	8.19	8.41	5.57'	8.93	8.37
1.50	1.00	14.16	14.12	9.30	8.68	10.28	10.22	9.35	11.04	10.20
1.50	1.25	14.90	14.87	11.08	11.10	11.67	12.03	11.89	12.38	12.04
1.50	1.50	15.39	15.36	12.34	12.54	12.76	13.28	13.35	13.35	13.31

TABLE 3: $\hat{R}$ in (17) with $\rho(L;\theta) = (1\text{-}L)^{d1+\theta1}\,(1+L)^{d2+\theta2}\,(1+L^2)^{d3+\theta3}$ and white noise u_t (U.K. data)

			No intercept and no trend			Intercept			Intercept and time trend		
d_1	d_2	d_3	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t
0.50	0.50	0.50	127.05	126.62	10.53	164.90	171.34	28.14	76.44	95.29	11.08
0.50	0.50	1.00	152.82	152.31	26.92	193.94	198.38	59.74	112.61	117.81	28.76
0.50	0.50	1.50	169.81	169.18	35.71	212.63	218.33	81.57	127.96	130.52	39.38
0.50	1.00	0.50	142.22	141.65	26.75	184.11	191.31	59.23	104.44	118.39	29.54
0.50	1.00	1.00	165.31	164.67	53.77	209.65	215.12	105.01	142.48	139.65	59.23
0.50	1.00	1.50	180.43	179.68	67.65	226.66	232.99	133.04	158.39	151.31	75.56
0.50	1.50	0.50	150.03	149.37	37.56	196.00	203.51	80.41	117.98	128.19	42.68
0.50	1.50	1.00	170.47	169.75	65.60	218.48	224.71	126.01	150.78	146.37	73.77
0.50	1.50	1.50	184.05	183.24	78.90	234.23	241.05	151.84	164.90	156.38	89.06
1.00	0.50	0.50	21.14	21.23	2.00'	2.11'	7.68'	3.10'	2.15'	7.91	3.05'
1.00	0.50	1.00	32.90	33.08	11.08	13.72	18.10	12.88	13.78	18.22	12.76
1.00	0.50	1.50	42.95	43.14	17.44	21.12	25.66	19.76	20.99	25.47	19.62
1.00	1.00	0.50	34.51	34.56	4.70'	11.11	23.34	4.20'	11.61	24.20	4.21'
1.00	1.00	1.00	50.50	50.61	14.55	35.02	42.05	11.58	35.77	42.70	11.60
1.00	1.00	1.50	63.55	63.64	23.00	49.17	55.45	18.64	49.29	55.41	18.68
1.00	1.50	0.50	43.38	43.39	9.64	19.96	35.22	8.32	20.30	35.77	8.33
1.00	1.50	1.00	59.88	59.92	27.72	49.19	56.68	23.42	49.65	56.71	23.42
1.00	1.50	1.50	72.94	72.96	41.97	64.53	70.88	37.01	64.43	70.11	36.92
1.50	0.50	0.50	11.07	10.99	9.41	8.22	12.24	10.38	8.67	12.65	10.37
1.50	0.50	1.00	14.13	14.11	26.61	28.72	28.64	27.95	29.62	29.17	27.95
1.50	0.50	1.50	15.38	15.37	38.31	41.79	42.13	39.74	42.71	42.74	39.74
1.50	1.00	0.50	15.57	15.54	6.04'	2.54'	6.03'	6.53'	2.62'	6.32'	6.50'
1.50	1.00	1.00	21.47	21.53	9.41	8.79	11.69	10.53	8.87	11.84	10.52
1.50	1.00	1.50	25.43	25.52	13.63	13.54	15.15	15.15	13.47	14.94	15.17
1.50	1.50	0.50	20.77	20.74	8.93	6.09'	12.07	9.28	6.03'	12.23	9.24
1.50	1.50	1.00	29.37	29.42	11.63	19.63	23.26	11.43	19.50	23.13	11.42
1.50	1.50	1.50	35.65	35.72	16.43	29.30	31.77	15.58	28.84	31.08	15.60

': Non-rejection values for the null hypothesis (15) at 95% significance level.

TABLE 4

$\hat{r}$ in (17) with $\rho(L;\theta) = (1\text{-}L^2)^{d+\theta}$ (U.K. data)

(i) With white noise u_t

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
	--	5.23	2.04	-0.47’	-2.00	-2.87	-3.38	-3.72	-3.95
	I	2.06	-4.26	-4.74	-4.86	-4.95	-5.01	-5.04	-5.06
c_t	I,T	-3.21	-4.30	-4.71	-4.89	-4.98	-5.03	-5.06	-5.09
	I,S	7.14	0.17’	-2.49	-3.40	-3.98	-4.33	-4.53	-4.66
	I,T,S	2.60	-0.66’	-2.50	-3.48	-4.03	-4.34	-4.54	-4.66
	--	5.18	2.00	-0.51’	-2.03	-2.89	-3.40	-3.74	-3.97
	I	6.47	-0.69’	-2.81	-3.64	-4.16	-4.47	-4.65	-4.76
y_t	I,T	1.99	-1.05’	-2.80	-3.72	-4.23	-4.49	-4.65	-4.76
	I,S	7.52	1.52’	-1.16’	-2.38	-3.23	-3.75	-4.07	-4.28
	I,T,S	4.09	0.96’	-1.18’	-2.50	-3.29	-3.78	-4.08	-4.28
	--	-3.97	-4.47	-4.77	-4.93	-5.01	-5.05	-5.07	-5.08
	I	-3.11	-4.35	-4.70	-4.86	-4.94	-4.98	-5.01	-5.03
c_t - y_t	I,T	-3.76	-4.40	-4.70	-4.86	-4.94	-4.99	-5.02	-5.04
	I,S	-0.54’	-3.03	-3.84	-4.27	-4.51	-4.66	-4.75	-4.82
	I,T,S	-1.64’	-3.06	-3.85	-4.27	-4.51	-4.66	-4.75	-4.81

(ii) With seasonal AR(1) u_t

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
	--	1.93’	1.17’	-0.49’	-2.09	-3.10	-3.66	-3.99	-4.19
	I	0.62’	-1.40’	-2.27	-2.66	-2.92	-3.12	-3.27	-3.40
c_t	I,T	-0.77’	-1.68’	-2.28	-2.67	-2.94	-3.15	-3.34	-3.51
	I,S	1.87’	-0.54’	-2.40	-3.13	-3.59	-3.87	-4.05	-4.17
	I,T,S	0.80’	-1.12’	-2.41	-3.17	-3.60	-3.88	-4.05	-4.18
	--	1.91’	1.13’	-0.54’	-2.12	-3.12	-3.69	-4.01	-4.20
	I	1.65’	-1.06’	-2.45	-3.09	-3.50	-3.76	-3.92	-4.03
y_t	I,T	0.56’	-1.26’	-2.44	-3.13	-3.53	-3.77	-3.94	-4.06
	I,S	2.05	0.55’	-1.46’	-2.59	-3.39	-3.88	-4.16	-4.35
	I,T,S	1.95’	0.17’	-1.49’	-2.69	-3.45	-3.90	-4.17	-4.34
	--	-2.14	-2.76	-3.25	-3.58	-3.80	-3.93	-4.02	-4.08
	I	-2.06	-2.80	-3.19	-3.42	-3.58	-3.70	-3.79	-3.88
c_t - y_t	I,T	-2.19	-2.81	-3.19	-3.42	-3.58	-3.71	-3.81	-3.91
	I,S	-1.35’	-2.92	-3.54	-3.91	-4.14	-4.29	-4.40	-4.48
	I,T,S	-1.86’	-2.92	-3.54	-3.91	-4.14	-4.29	-4.40	-4.49

(iii) With seasonal AR(2) u_t

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
	--	1.94’	1.08’	-0.49’	-2.11	-3.14	-3.71	-4.03	-4.22
	I	1.27’	-0.85’	-2.00	-2.65	-3.11	-3.42	-3.64	-3.80
c_t	I,T	-0.19’	-1.15’	-2.00	-2.66	-3.13	-3.46	-3.71	-3.92
	I,S	2.08	-0.45’	-2.14	-2.90	-3.47	-3.85	-4.09	-4.26
	I,T,S	0.74’	-0.99’	-2.15	-2.93	-3.47	-3.84	-4.09	-4.26
	--	1.92’	1.05’	-0.54’	-2.15	-3.17	-3.73	-4.04	-4.23
	I	1.94’	-0.70’	-1.95’	-2.69	-3.26	-3.67	-3.94	-4.11
y_t	I,T	0.54’	-0.91’	-1.95’	-2.72	-3.28	-3.67	-3.93	-4.12
	I,S	2.24	0.54’	-1.50’	-2.59	-3.38	-3.87	-4.16	-4.34
	I,T,S	1.86’	0.10’	-1.53’	-2.70	-3.43	-3.89	-4.17	-4.35
	--	-1.91’	-2.65	-3.30	-3.75	-4.02	-4.19	-4.29	-4.34
	I	-1.77’	-2.73	-3.28	-3.62	-3.83	-3.97	-4.07	-4.15
c_t - y_t	I,T	-1.95’	-2.74	-3.28	-3.62	-3.84	-3.98	-4.10	-4.19
	I,S	-1.34’	-2.91	-3.54	-3.92	-4.16	-4.31	-4.42	-4.49
	I,T,S	-1.92’	-2.91	-3.54	-3.92	-4.16	-4.31	-4.41	-4.48

’: Non-rejections values for the null hypothesis (15) at 95% significance level; --: No intercept, no time trend and no seasonal dummies; I: Intercept; I,T: Intercept and time trend; I,S: Intercept and seasonal dummies; I,T,S: Intercept, time trend and seasonal dummies.

TABLE 5

$\hat{R}$ in (17) with $\rho(L;\theta) = (1-L)^{d1+\theta1}(1+L)^{d2+\theta2}$ and white noise u_t (U.K. data)

		No intercept and no trend			Intercept			Intercept and a time trend		
d_1	d_2	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t
0.50	0.50	81.29	80.38	16.06	26.81	101.13	10.85	11.79	25.48	14.58
0.50	0.75	91.34	90.37	16.37	36.50	115.60	10.93	11.41	34.07	14.88
0.50	1.00	99.46	98.47	15.89	47.24	128.02	10.49	10.60	41.85	14.42
0.50	1.25	106.20	105.24	15.08	59.01	139.06	10.00	9.86	49.40	13.68
0.50	1.50	111.95	111.01	14.15	71.59	149.02	9.68	9.40	56.85	12.84
0.75	0.50	25.29	24.99	19.09	18.45	4.73'	18.05	18.45	5.34'	18.37
0.75	0.75	32.42	32.03	20.13	18.45	8.54	19.12	18.89	8.81	19.50
0.75	1.00	38.66	38.21	20.12	17.44	11.96	19.18	18.42	11.25	19.61
0.75	1.25	44.23	43.76	19.63	16.05	15.71	18.78	17.62	13.57	19.26
0.75	1.50	49.27	48.81	18.88	14.48	20.06	18.17	16.65	16.15	18.68
1.00	0.50	7.24	7.25	20.56	21.21	6.40	19.78	21.08	6.44	19.80
1.00	0.75	10.61	10.54	22.31	22.44	9.50	21.59	22.28	9.60	21.61
1.00	1.00	13.63	13.50	22.84	22.56	10.58	22.20	22.37	10.70	22.22
1.00	1.25	16.43	16.26	22.80	22.32	10.84	22.27	22.13	10.98	22.30
1.00	1.50	19.09	18.90	22.46	21.90	10.88	22.08	21.71	11.03	22.11
1.25	0.50	6.36	6.50	20.82	21.75	8.43	20.02	21.94	8.80	20.02
1.25	0.75	8.21	8.30	23.13	23.41	12.62	22.36	23.62	13.09	22.36
1.25	1.00	9.65	9.70	24.05	23.77	14.25	23.32	23.99	14.76	23.32
1.25	1.25	10.86	10.87	24.35	23.76	14.59	23.67	24.01	15.12	23.67
1.25	1.50	11.97	11.94	24.33	23.62	14.39	23.73	23.88	14.92	23.73
1.50	0.50	8.26	8.43	20.47	21.94	9.54	19.68	22.14	9.82	19.69
1.50	0.75	9.86	10.02	23.22	24.01	14.73	22.45	24.23	15.11	22.46
1.50	1.00	10.93	11.06	24.43	24.53	17.12	23.70	24.77	17.55	23.72
1.50	1.25	11.67	11.77	24.95	24.64	17.94	24.24	24.89	18.40	24.26
1.50	1.50	12.21	12.28	25.13	24.62	18.04	24.45	24.88	18.51	24.47

': Non-rejection values for the null hypothesis (15) at 95% significance level.

TABLE 6

$\hat{r}$ in (17) with $\rho(L;\theta) = (1-L)^{d+\theta}$ and white noise u_t (U.K. data)

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
	--	9.89	3.91	-0.30'	-2.55	-3.73	-4.43	-4.87	-5.18
	I	1.57'	-4.49	-4.76	-5.01	-5.23	-5.42	-5.59	-5.74
c_t	I,T	-3.32	-4.31	-4.74	-5.02	-5.25	-5.44	-5.61	-5.76
	I,S	11.91	-0.91'	-3.37	-4.28	-4.83	-5.18	-5.42	-5.61
	I,T,S	3.84	-1.13'	-3.34	-4.34	-4.87	-5.21	-5.45	-5.64
	--	9.83	3.87	-0.31'	-2.55	-3.73	-4.42	-4.86	-5.17
	I	8.65	-3.00	-4.31	-4.95	-5.37	-5.65	-5.85	-6.00
y_t	I,T	1.13'	-2.69	-4.27	-4.99	-5.41	-5.67	-5.87	-6.02
	I,S	11.76	-0.86'	-3.49	-4.60	-5.24	-5.61	-5.85	-6.02
	I,T,S	4.76	-0.77'	-3.44	-4.66	-5.28	-5.64	-5.87	-6.04
	--	-3.66	-4.26	-4.63	-4.87	-5.06	-5.22	-5.38	-5.52
	I	-3.00	-4.20	-4.61	-4.87	-5.07	-5.24	-5.40	-5.54
c_t - y_t	I,T	-3.50	-4.23	-4.61	-4.87	-5.07	-5.24	-5.39	-5.54
	I,S	-1.09'	-3.67	-4.42	-4.85	-5.13	-5.34	-5.51	-5.65
	I,T,S	-1.95'	-3.63	-4.42	-4.85	-5.13	-5.34	-5.50	-5.65

': Non-rejection values for the null hypothesis (15) at 95% significance level; --: No intercept,no time trend and no seasonal dummies; I: Intercept; I,T: Intercept and time trend; I,S: Intercept and seasonal dummies; I,T,S: Intercept, time trend and seasonal dummies.

TABLE 7

$\hat{r}$ in (17) with $\rho(L;\theta) = (1-L^4)^{d+\theta}$ (Japanese data)

(i) With white noise u_t

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
	--	2.61	0.77'	-1.02'	-2.36	-3.22	-3.76	-4.12	-4.37
	I	4.36	2.64	3.05	1.36'	-0.89'	-2.54	-3.50	-4.04
c_t	I,T	9.12	7.28	3.83	0.00'	-2.72	-3.76	-4.01	-4.17
	I,S	4.41	2.80	4.39	2.95	0.34'	-1.78'	-3.06	-3.76
	I,T,S	10.02	8.34	5.14	1.04'	-2.11	-3.51	-3.99	-4.24
	--	2.54	0.72'	-1.05'	-2.38	-3.23	-3.77	-4.13	-4.38
	I	4.70	3.34	2.21	-0.08'	-2.10	-3.37	-4.06	-4.44
y_t	I,T	7.80	6.04	2.54	-0.91'	-3.11	-3.76	-3.77	-3.86
	I,S	4.95	4.12	4.78	2.33	-0.57'	-2.63	-3.72	-4.25
	I,T,S	10.28	8.48	5.10	0.84'	-2.30	-3.69	-4.19	-4.44
	--	1.53'	-0.08'	-1.77'	-2.93	-3.63	-4.05	-4.33	-4.52
	I	2.41	0.46'	-1.54'	-2.84	-3.60	-4.05	-4.34	-4.54
c_t - y_t	I,T	2.34	0.45'	-1.54'	-2.86	-3.58	-3.82	-3.89	-4.02
	I,S	3.42	0.35'	-1.79'	-3.06	-3.76	-4.15	-4.39	-4.55
	I,T,S	3.31	0.34'	-1.79'	-3.06	-3.76	-4.15	-4.39	-4.55

(ii) With AR(1) u_t

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
	--	-3.13	-3.50	-3.83	-4.13	-4.38	-4.57	-4.71	-4.83
	I	-1.59'	-0.67'	-0.51'	-1.79'	-2.78	-3.50	-3.99	-4.30
c_t	I,T	2.57	1.01'	-0.65'	-2.01	-3.19	-3.82	-4.09	-4.27
	I,S	-2.87	-3.21	-3.31	-3.51	-3.73	-4.05	-4.35	-4.56
	I,T,S	-1.05'	-2.67	-3.30	-3.63	-4.12	-4.48	-4.64	-4.74
	--	-3.01	-3.47	-3.82	-4.12	-4.37	-4.57	-4.71	-4.83
	I	-0.03'	0.87'	0.23'	-1.38'	-2.67	-3.52	-4.03	-4.34
y_t	I,T	3.09	2.07	0.24'	-1.64'	-3.09	-3.67	-3.80	-3.96
	I,S	-2.51	-2.37	-1.71'	-1.88'	-2.50	-3.34	-3.99	-4.36
	I,T,S	0.29'	-1.41'	-1.61'	-1.98	-3.08	-3.91	-4.28	-4.49
	--	0.87'	-0.84'	-2.29	-3.21	-3.77	-4.13	-4.37	-4.54
	I	1.94'	-0.01'	-1.78'	-2.91	-3.59	-4.01	-4.28	-4.48
c_t - y_t	I,T	1.89'	-0.02'	-1.78'	-2.93	-3.58	-3.86	-4.00	-4.16
	I,S	1.34'	-1.29'	-2.66	-3.46	-3.95	-4.25	-4.44	-4.58
	I,T,S	1.29'	-1.29'	-2.66	-3.46	-3.95	-4.25	-4.45	-4.58

(iii) With AR(2) u_t

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
	--	-3.19	-3.53	-3.81	-4.09	-4.34	-4.54	-4.70	-4.82
	I	-1.53'	-0.51'	-0.85'	-2.14	-2.96	-3.56	-4.01	-4.36
c_t	I,T	1.77'	0.16'	-1.35'	-2.37	-3.30	-3.88	-4.15	-4.34
	I,S	-2.90	-3.26	-3.56	-3.82	-3.99	-4.24	-4.48	-4.66
	I,T,S	-1.23'	-2.84	-3.60	-3.92	-4.29	-4.60	-4.74	-4.83
	--	-3.08	-3.50	-3.80	-4.09	-4.34	-4.54	-4.70	-4.82
	I	-0.29'	0.75'	0.20'	-1.31'	-2.54	-3.41	-3.96	-4.30
y_t	I,T	2.69	1.61'	0.04'	-1.55'	-3.04	-3.66	-3.77	-3.93
	I,S	-2.54	-2.57	-2.53	-2.78	-3.05	-3.57	-4.07	-4.39
	I,T,S	0.11'	-1.99	-2.59	-2.72	-3.33	-3.97	-4.31	-4.51
	--	0.80'	-0.88'	-2.27	-3.18	-3.75	-4.11	-4.36	-4.53
	I	1.85'	0.03'	-1.72'	-2.89	-3.60	-4.02	-4.30	-4.49
c_t - y_t	I,T	1.81'	-0.01'	-1.72'	-2.91	-3.59	-3.85	-3.97	-4.12
	I,S	0.45'	-1.67'	-2.77	-3.47	-3.94	-4.24	-4.44	-4.58
	I,T,S	0.40'	-1.68'	-2.77	-3.47	-3.94	-4.24	-4.44	-4.58

': Non-rejection values for the null hypothesis (15) at 95% significance level; --: No intercept, no time trend and no seasonal dummies; I: Intercept; I,T: Intercept and time trend; I,S: Intercept and seasonal dummies; I,T,S: Intercept, time trend and seasonal dummies.

TABLE 8: $\hat{R}$ in (17) with $\rho(L;\theta) = (1-L^2)^{d1+\theta1}\,(1+L^2)^{d2+\theta2}$ and white noise u_t (Japanese data)

d_1	d_2	No intercept and no trend c_t	y_t	c_t-y_t	Intercept c_t	y_t	c_t-y_t	Intercept and time trend c_t	y_t	c_t-y_t
0.50	0.50	41.03	39.76	5.25'	64.79	63.91	6.83	167.85	107.69	6.49
0.50	0.75	47.92	46.58	12.86	72.81	75.60	15.02	192.19	150.17	14.47
0.50	1.00	53.35	51.97	19.48	79.24	83.19	23.76	201.74	168.45	23.03
0.50	1.25	57.72	56.32	24.32	84.92	89.31	30.32	207.12	178.07	29.41
0.50	1.50	61.28	59.88	28.05	90.11	94.67	35.22	210.65	183.90	34.14
0.75	0.50	17.12	16.72	0.42'	22.81	13.95	4.30'	77.49	29.81	4.23'
0.75	0.75	22.42	22.01	2.95'	34.46	30.38	0.50'	117.38	68.97	0.52'
0.75	1.00	27.06	26.61	8.97	42.28	43.89	5.08'	137.50	100.85	5.16'
0.75	1.25	31.19	30.72	14.18	48.55	53.78	11.17	150.13	123.02	11.27
0.75	1.50	34.94	34.45	18.52	54.07	61.56	16.86	159.27	138.97	16.96
1.00	0.50	7.76	7.64	3.58'	8.74	8.21	10.28	11.04	8.56	10.27
1.00	0.75	10.73	10.62	1.45'	22.43	5.76'	2.66'	29.89	6.90	2.67'
1.00	1.00	13.33	13.22	4.71'	35.55	14.50	2.39'	48.00	18.11	2.41'
1.00	1.25	15.72	15.59	7.98	45.91	26.86	4.86'	62.37	33.81	4.89'
1.00	1.50	17.98	17.84	10.53	54.34	38.69	7.33	74.08	49.01	7.35
1.25	0.50	8.07	7.98	8.32	1.82'	11.98	15.19	1.96'	14.05	15.31
1.25	0.75	9.93	9.91	4.61'	3.85'	2.95'	7.92	0.36'	5.22'	8.04
1.25	1.00	11.30	11.30	6.64	11.73	0.30'	6.31	5.01'	0.43'	6.41
1.25	1.25	12.40	12.40	9.30	20.03	4.29'	8.09	10.88	2.47'	8.20
1.25	1.50	13.37	13.37	11.08	27.56	9.91	9.77	16.30	6.18	9.88
1.50	0.50	10.37	10.25	12.16	3.37'	16.22	18.62	6.01	19.15	19.08
1.50	0.75	12.16	12.13	7.72	0.37'	9.18	11.92	3.78'	14.25	12.22
1.50	1.00	13.30	13.31	8.85	2.37'	3.32'	9.31	5.14'	7.65	9.29
1.50	1.25	13.99	14.02	11.58	6.04	3.81'	11.01	7.96	8.00	10.92
1.50	1.50	14.45	14.48	13.53	9.44	5.56'	13.00	9.94	9.71	12.92

TABLE 9: $\hat{R}$ in (17) with $\rho(L;\theta) = (1-L)^{d1+\theta1}\,(1+L)^{d2+\theta2}\,(1+L^2)^{d3+\theta3}$ and white noise u_t (Japanese data)

d_1	d_2	d_3	No intercept and no trend c_t	y_t	c_t-y_t	Intercept c_t	y_t	c_t-y_t	Intercept and time trend c_t	y_t	c_t-y_t
0.50	0.50	0.50	103.66	101.27	21.28	141.71	136.00	18.03	281.38	181.08	17.54
0.50	0.50	1.00	125.45	122.99	49.44	166.31	169.76	49.62	334.06	276.40	48.41
0.50	0.50	1.50	138.97	136.53	63.60	183.92	188.44	66.73	346.47	298.25	64.88
0.50	1.00	0.50	117.27	114.99	43.61	154.99	157.32	44.85	320.99	259.76	43.82
0.50	1.00	1.00	136.62	134.40	94.74	177.59	188.17	129.43	366.44	370.88	127.32
0.50	1.00	1.50	148.39	146.29	120.92	194.11	205.87	176.67	377.28	395.97	173.92
0.50	1.50	0.50	123.50	121.33	57.72	164.44	169.23	63.62	335.74	292.35	62.24
0.50	1.50	1.00	140.31	138.24	107.66	185.10	196.22	152.38	371.11	383.19	150.03
0.50	1.50	1.50	150.64	148.71	131.71	200.60	212.57	196.08	379.89	403.28	193.24
1.00	0.50	0.50	18.90	18.50	2.03'	9.87	3.73'	4.01'	10.73	3.66'	4.01'
1.00	0.50	1.00	29.47	28.91	2.04'	32.10	4.74'	0.53'	36.42	4.94'	0.54'
1.00	0.50	1.50	38.39	37.60	3.03'	45.26	8.71	1.04'	50.98	8.85	1.04'
1.00	1.00	0.50	31.34	30.89	6.50'	24.98	12.03	11.13	28.27	12.33	11.12
1.00	1.00	1.00	45.88	45.45	16.30	81.47	39.02	7.86	100.22	44.08	7.87
1.00	1.00	1.50	57.62	57.14	29.12	113.61	79.39	17.82	142.03	92.80	17.82
1.00	1.50	0.50	39.66	39.20	8.21	40.61	16.16	11.31	47.66	17.03	11.30
1.00	1.50	1.00	54.65	54.24	26.23	106.91	65.41	15.61	135.75	77.02	15.62
1.00	1.50	1.50	66.40	65.97	43.02	138.79	115.71	32.58	179.06	142.14	32.60
1.50	0.50	0.50	10.33	10.11	2.94'	9.57	3.89'	3.94'	11.15	4.33'	3.99'
1.50	0.50	1.00	13.25	13.06	1.78'	31.88	3.95'	1.19'	35.86	4.40'	1.20'
1.50	0.50	1.50	14.41	14.16	2.00'	44.10	5.65'	1.34'	48.52	6.02'	1.35'
1.50	1.00	0.50	14.23	14.01	11.25	3.24'	14.26	16.43	4.79'	16.22	16.73
1.50	1.00	1.00	19.69	19.56	7.84	3.62'	1.58'	7.17'	3.57'	3.72'	7.13'
1.50	1.00	1.50	23.28	23.11	11.94	11.54	5.77'	10.30	9.18	8.36	10.26
1.50	1.50	0.50	19.04	18.81	12.79	5.20'	16.52	18.79	6.49'	19.23	19.22
1.50	1.50	1.00	27.05	26.95	12.38	14.48	6.84'	10.62	9.75	8.65	10.47
1.50	1.50	1.50	32.83	32.72	20.29	30.45	13.79	16.00	18.91	12.28	15.69

': Non-rejection values for the null hypothesis (15) at 95% significance level.

TABLE 10

$\hat{r}$ in (17) with $\rho(L;\theta) = (1-L^2)^{d+\theta}$ (Japanese data)

(i) With white noise u_t

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
c_t	--	4.42	1.78’	-0.47’	-1.91’	-2.75	-3.25	-3.58	-3.80
	I	2.75	-4.04	-4.61	-4.75	-4.84	-4.88	-4.91	-4.92
	I,T	-0.96’	-3.71	-4.58	-4.82	-4.89	-4.92	-4.94	-4.95
	I,S	6.87	3.85	1.94’	-0.84’	-2.80	-3.73	-4.14	-4.34
	I,T,S	12.14	7.99	2.04	-1.93’	-3.49	-3.99	-4.21	-4.35
y_t	--	4.13	1.55’	-0.66’	-2.06	-2.87	-3.35	-3.67	-3.89
	I	-1.23’	-4.72	-4.83	-4.87	-4.90	-4.92	-4.93	-4.94
	I,T	-3.38	-4.51	-4.81	-4.89	-4.92	-4.94	-4.95	-4.96
	I,S	6.57	0.44’	-2.84	-4.05	-4.55	-4.73	-4.79	-4.81
	I,T,S	7.78	1.25’	-2.86	-4.28	-4.66	-4.72	-4.72	-4.74
$c_t - y_t$	--	-4.25	-4.63	-4.80	-4.87	-4.91	-4.93	-4.94	-4.95
	I	-4.55	-4.81	-4.87	-4.89	-4.91	-4.92	-4.92	-4.93
	I,T	-4.51	-4.79	-4.86	-4.89	-4.91	-4.92	-4.93	-4.94
	I,S	-1.11’	-3.40	-4.20	-4.50	-4.63	-4.69	-4.73	-4.76
	I,T,S	-1.14’	-3.39	-4.20	-4.50	-4.62	-4.66	-4.67	-4.69

(ii) With seasonal AR(1) u_t

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
c_t	--	1.67’	0.96’	-0.52’	-2.00	-2.95	-3.50	-3.81	-4.01
	I	0.50’	-1.25’	-1.74’	-2.10	-2.42	-2.67	-2.89	-3.09
	I,T	0.95’	-0.84’	-1.73’	-2.18	-2.50	-2.76	-3.00	-3.21
	I,S	1.91’	1.42’	-0.31’	-1.68’	-2.85	-3.48	-3.78	-3.95
	I,T,S	3.15	1.32’	-0.29’	-2.23	-3.24	-3.61	-3.79	-3.93
y_t	--	1.41’	0.63’	-0.85’	-2.21	-3.05	-3.52	-3.80	-3.98
	I	-0.18’	-2.44	-2.46	-2.58	-2.76	-2.94	-3.11	-3.26
	I,T	-0.91’	-2.00	-2.39	-2.62	-2.83	-3.03	-3.22	-3.41
	I,S	1.53’	-1.56’	-3.19	-3.66	-3.94	-4.11	-4.22	-4.30
	I,T,S	0.09’	-2.01	-3.20	-3.76	-4.00	-4.10	-4.13	-4.14
$c_t - y_t$	--	-1.86’	-2.34	-2.69	-2.94	-3.13	-3.29	-3.42	-3.53
	I	-2.56	-2.79	-2.90	-3.01	-3.13	-3.25	-3.37	-3.48
	I,T	-2.42	-2.72	-2.88	-3.02	-3.16	-3.29	-3.41	-3.55
	I,S	-2.28	-3.26	-3.70	-3.94	-4.10	-4.20	-4.28	-4.34
	I,T,S	-2.28	-3.25	-3.70	-3.94	-4.10	-4.18	-4.22	-4.23

(iii) With seasonal AR(2) u_t

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
c_t	--	1.69’	0.88’	-0.53’	-2.02	-2.99	-3.53	-3.84	-4.03
	I	1.23’	-0.64’	-1.65’	-2.14	-2.57	-2.90	-3.16	-3.38
	I,T	1.84’	-0.38’	-1.62’	-2.27	-2.70	-3.01	-3.28	-3.51
	I,S	2.11	1.31’	-0.52’	-1.79’	-2.85	-3.47	-3.81	-4.02
	I,T,S	3.64	1.45’	-0.51’	-2.26	-3.19	-3.58	-3.81	-3.98
y_t	--	1.49’	0.58’	-0.87’	-2.21	-3.05	-3.52	-3.81	-4.01
	I	0.80’	-2.57	-2.65	-2.79	-3.02	-3.22	-3.41	-3.58
	I,T	-0.65’	-2.06	-2.56	-2.85	-3.10	-3.31	-3.51	-3.71
	I,S	1.78’	-1.53’	-3.37	-3.85	-4.13	-4.29	-4.40	-4.46
	I,T,S	0.33’	-2.11	-3.39	-3.94	-4.18	-4.27	-4.29	-4.30
$c_t - y_t$	--	-1.71’	-2.39	-2.89	-3.23	-3.45	-3.61	-3.74	-3.85
	I	-2.68	-3.01	-3.16	-3.31	-3.45	-3.58	-3.71	-3.82
	I,T	-2.51	-2.93	-3.15	-3.32	-3.48	-3.61	-3.74	-3.89
	I,S	-2.33	-3.33	-3.81	-4.06	-4.22	-4.33	-4.40	-4.45
	I,T,S	-2.33	-3.33	-3.81	-4.06	-4.22	-4.30	-4.33	-4.34

’: Non-rejection values for the null hypothesis (15) at 95% significance level; --: No intercept, no trend and no seasonal dummies; I: Intercept; I,T: Intercept and trend; I,D: Intercept and seasonal dummies; I,T,D: Intercept, trend and seasonal dummies.

TABLE 11

$\hat{R}$ in (17) with $\rho(L;\theta) = (1-L)^{d1+\theta 1}\ (1+L)^{d2+\theta 2}$ with white noise u_t (Japanese data)

		No intercept and no time trend			Intercept			Intercept and time trend		
d_1	d_2	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t	c_t	y_t	c_t-y_t
0.50	0.50	63.67	58.38	18.26	34.93	7.74	20.83	7.15	12.41	20.43
0.50	0.75	72.47	67.13	17.99	44.65	9.18	20.85	8.78	11.44	20.46
0.50	1.00	79.47	74.24	17.20	54.82	11.70	20.29	11.30	10.22	19.93
0.50	1.25	85.18	80.15	16.25	65.42	15.58	19.54	15.21	9.05	19.23
0.50	1.50	89.94	85.14	15.22	76.26	20.99	18.66	20.81	8.10	18.40
0.75	0.50	21.45	19.22	21.17	17.19	22.08	22.54	14.80	20.34	22.36
0.75	0.75	27.81	25.23	21.55	16.67	22.32	23.18	14.41	20.46	22.98
0.75	1.00	33.37	30.59	21.26	15.33	21.87	23.15	13.36	20.00	22.94
0.75	1.25	38.30	35.44	20.80	13.70	21.14	23.00	12.12	19.35	22.78
0.75	1.50	42.74	39.86	20.26	11.99	20.19	22.77	10.85	18.57	22.56
1.00	0.50	6.44	6.18	22.31	20.65	22.58	22.75	20.50	22.47	22.73
1.00	0.75	9.51	8.92	23.11	21.44	23.32	23.62	21.24	23.17	23.60
1.00	1.00	12.28	11.41	23.09	21.33	23.38	23.72	21.09	23.20	23.70
1.00	1.25	14.85	13.76	22.92	20.94	23.29	23.71	20.67	23.08	23.68
1.00	1.50	17.28	16.02	22.70	20.41	23.14	23.67	20.12	22.91	23.64
1.25	0.50	5.75'	6.22	22.60	21.39	22.70	22.72	21.80	22.84	22.73
1.25	0.75	7.42	7.73	23.69	22.60	23.65	23.79	23.08	23.81	23.81
1.25	1.00	8.73	8.86	23.82	22.76	23.79	23.95	23.31	23.98	23.97
1.25	1.25	9.84	9.80	23.77	22.65	23.77	23.96	23.28	23.99	23.99
1.25	1.50	10.86	10.64	23.68	22.45	23.72	23.95	23.16	23.96	23.98
1.50	0.50	7.58	8.19	22.55	21.63	22.67	22.56	21.92	22.80	22.58
1.50	0.75	9.01	9.59	23.91	23.20	23.84	23.86	23.55	24.00	23.89
1.50	1.00	9.97	10.48	24.13	23.53	24.06	24.08	23.93	24.24	24.11
1.50	1.25	10.63	11.03	24.15	23.55	24.08	24.11	24.00	24.29	24.16
1.50	1.50	11.11	11.39	24.11	23.49	24.05	24.11	23.99	24.28	24.16

': Non-rejection values for the null hypothesis (15) at 95% significance level.

TABLE 12

$\hat{r}$ in (17) with $\rho(L;\theta) = (1-L)^{d+\theta}$ and white noise u_t (Japanese data)

Series	z_t \ d	0.5	0.75	1.00	1.25	1.50	1.75	2.00	2.25
	--	8.47	3.43	-0.37'	-2.49	-3.61	-4.27	-4.70	-4.99
	I	3.17	-4.31	-4.61	-4.83	-5.02	-5.18	-5.33	-5.46
c_t	I,T	-1.51'	-3.93	-4.59	-4.85	-5.04	-5.19	-5.33	-5.46
	I,S	12.74	3.01	-2.47	-4.54	-5.37	-5.68	-5.83	-5.93
	I,T,S	16.98	5.30	-2.52	-4.86	-5.47	-5.69	-5.82	-5.91
	--	7.35	2.47	-1.07'	-2.98	-3.98	-4.57	-4.95	-5.21
	I	-2.71	-4.98	-5.11	-5.27	-5.42	-5.55	-5.67	-5.78
y_t	I,T	-4.03	-4.82	-5.10	-5.28	-5.43	-5.56	-5.68	-5.78
	I,S	11.76	-0.13'	-3.38	-4.26	-4.62	-4.81	-4.96	-5.08
	I,T,S	10.31	0.31'	-3.42	-4.35	-4.64	-4.79	-4.90	-5.00
	--	-4.74	-5.09	-5.31	-5.47	-5.60	-5.72	-5.82	-5.91
	I	-4.95	-5.16	-5.32	-5.47	-5.60	-5.71	-5.82	-5.91
c_t - y_t	I,T	-4.89	-5.14	-5.32	-5.47	-5.60	-5.72	-5.83	-5.91
	I,S	-2.88	-4.56	-5.10	-5.35	-5.51	-5.63	-5.74	-5.82
	I,T,S	-2.91	-4.56	-5.10	-5.35	-5.50	-5.60	-5.67	-5.73

': Non-rejection values for the null hypothesis (15) at 95% significance level; --: No intercept, no trend and no seasonal dummies; I: Intercept; I,T: Intercept and trend; I,S: Intercept and seasonal dummies; I,T,S: Intercept, trend and seasonal dummies.

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