Firm Heterogeneity and the Two Sources of Gains from Trade

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EUI Working Paper ECO No. 2006/38
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November 24, 2006

Abstract

Recent empirical work identifies two main channels through which consumers benefit from trade. Trade liberalization lowers prices, while it raises product variety. This paper develops the first model that connects both channels and interprets their interaction. It shows that heterogeneity in firm productivity is the source behind both. Upon liberalization efficient exporters enter, pushing out the least efficient domestic firms. Two countervailing forces emerge, both stylized facts. Liberalization leaves a more concentrated market. But exporters offer more variety than the firms that they replace. Remarkably, total variety unambiguously increases. Exploration of comparative statics leads to an intuitive explanation.

Keywords: Trade, Firm selection, Product Variety, Heterogeneous firms

JEL Classification: F12, F15, L11

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A recent, growing body of microeconometric evidence has come to indicate that the benefits of opening up to trade run mainly through two channels: firm selection and product variety. Firm selection refers to the pushing out of the least efficient domestic firms by relatively efficient foreign entrants. This channel implies gains for consumers through lower prices. The variety channel, on the other hand, means that consumers end up with a wider choice due to trade in differentiated varieties. If we want to thoroughly understand the way trade liberalization affects consumers, therefore, we need to comprehend both channels. Already since the seminal contributions of Krugman (1979, 1980) we have a framework that allows us to analyze the effect of trade on product variety. Moreover, with advent of models of trade with heterogeneous firms by Melitz (2003) we have obtained a mechanism to understand the forces that drive firm selection.\textsuperscript{1} Yet, no framework connects the two channels and interprets their interaction.

In the Melitz-model firms draw their productivities from an exogenous distribution. Production exhibits increasing returns to scale, and each firm produces a single horizontally differentiated variety. Exporting involves a sunk cost\textsuperscript{2}, which leads to a scale ranking: only the most productive firms export. Trade liberalization induces more firms into export. And entry by foreign exporters pushes out the least productive firms and raises the average productivity in the industry. This is the firm selection effect that, as documented in the surveys of Tybout (2003) and Greenaway and Kneller (2005), receives extensive support from microeconometric evidence. Moreover, as proven by Baldwin and Forslid (2006), in the Melitz-model each relatively efficient foreign entrant pushes out more than one domestic firm. Foreign entry raises market concentration. This is not only intuitive, but also empirically supported by Mirza (2006). Therefore, in itself, it is a desirable feature of the model.

However, since each firm produces only one variety, total variety available to consumers

\textsuperscript{1}See also Bernard et al. (2003) for an approach with heterogeneous firms and Bertrand competition, rather than S-D-S monopolistic competition as in Melitz (2003) and Krugman (1979, 1980).

\textsuperscript{2}Empirically, sunk costs are found to be very relevant for the decision to enter an export market. See Tybout (2003).
necessarily falls in the Melitz-model. The basic Krugman-model matches the link between trade and product variety, of course, but there we lose the element of firm heterogeneity, and the potential to understand how firm selection and product variety jointly arise and interact. Instead, we propose a simple solution: a Melitz-model where firms can optimally set their scope, instead of being restricted to producing a single variety. To achieve this, we combine the models of Melitz (2003) and Allanson and Montagna (2005). In the latter model, there is no trade and firms are homogeneous in productivity, but they produce differentiated varieties and optimally choose their scope. Allanson and Montagna apply the model to analyze industry shakeout: historically, industries have often moved from many firms each offering few varieties, to few firms offering many. This type of mechanism is, in fact, precisely what we require to bring together an increase in both market concentration and product variety upon trade liberalization.\(^3\)

In our setup firms make additional decisions compared to the Melitz-model: how many varieties to offer at home and abroad. From the increasing returns to scale technology, economies of scope come about naturally alongside economies of scale: the firm-wide fixed cost can be spread over the varieties. Moreover, the nested CES formulation of Allanson and Montagna ensures that there is also a diseconomy of scope: each additional variety cannibalizes on the demand for the firm’s existing line of varieties. The interplay between the economies and diseconomies of scope determines the optimal variety range. The scale ranking of the standard Melitz-model is now supplemented by a scope ranking: exporters offer more variety than non-exporters. In fact, our model is first in matching what is a stylized fact. Bernard et al. (2005 and 2006) and Manez et al. (2004) report that multiproduct firms are, on average, of a larger scale, more efficient and more likely to be exporters.

\(^3\)The empirical literature on variety gains from trade consists of aggregate studies and case studies. The seminal study of the first type is Broda and Weinstein (2004). Applying the methodology developed by Feenstra (1994) for estimating an ideal price index, they find imported varieties have raised US welfare by 3% over the past 25 years. Other aggregate studies include Funke and Ruhlwedel (2003), Chen (2006) and Klenow and Rodriguez-Clare (1997). Case studies focus on the car market. Tovar (2004), for instance, considers the impact of trade liberalization on the Colombian car industry. He reports gains of 3000 US dollar per consumer, mostly due to increased variety. See also Ferstman and Gandal (1998) and Clerides (2005).
After deriving the basic equations that govern firms’ entry/exit, price setting and scope decisions, we parameterize the productivity distribution as a Pareto distribution, which is common in the literature. We are then able to derive a closed-form equilibrium solution. We observe that firm heterogeneity is the driving force behind both firm selection and variety gains. On the one hand, upon liberalization more domestic firms get pushed out than are replaced by foreign firms. Market concentration rises. But the increase in the market share of foreigners in itself implies a more than offsetting variety gain. The reason is that there is a scope gap between exporters and domestic firms. Remarkably, this gap unambiguously dominates the rise in market concentration. We show that for the extreme parameter values for which scope differences vanish, the rise in market concentration simultaneously disappears. Intuitively, imagine a closed economy in which economies of scope suddenly increase. Total variety must rise. But demand per variety drops, rendering the varieties of the least efficient firms unprofitable. Market concentration then rises due to firms’ greater economies of scope. Trade liberalization, by shifting market share to exporters who can spread their R&D costs over more markets, can be thought of in such terms.

A related question concerns the comparative statics of firm heterogeneity. When looking across industries with different productivity distributions, where will trade liberalization have the strongest impact through firm selection and variety? We prove that firm heterogeneity unambiguously strengthens both channels of gains from trade. This matters for two reasons. Firstly, much effort has been devoted to understand the effects of firm heterogeneity, but its impact on variety has never been considered. Secondly, given that greater heterogeneity affects both channels in the same direction, we have a policy implication: if due to political constraints policy makers are unable to liberalize all industries at once, our model suggests they should start from those industries in which firms’ productivities differ most. Finally, we consider an application of our model to a different issue. Anti-globalists often claim that globalization brings about a "standardization", meaning that firms with "local character"

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4We are able to mathematically identify the variety effect on consumer welfare, not just the number of varieties. The two can differ since in a nested CES formulation consumers also care about the number of firms from which they can purchase.
disappear and are replaced by international firms that are more similar to each other. We show that, surprisingly, such a "standardization" need not necessarily make consumers worse off.

The structure of the paper is as follows. The next section presents the model, while section 3 computes the equilibrium solution. In section 4 we derive and discuss our main results. Finally, section 5 concludes. The appendix contains all proofs.

2 Model

We first describe demand and then the decision problem that firms face. At the end of the section we will show precisely how the works of Melitz (2003) and Allanson and Montagna (2005) are contained in the model.

2.1 Demand

Preferences are given by a nested CES, as in Allanson and Montagna (2005). The representative consumer optimizes over three stages. In the first stage, the consumer optimally allocates expenditure, $E$, between the quantity index of a differentiated good $q$ (defined below), and an outside composite good, $z$, which is used as a numeraire:

$$U = z^{\eta} q^{1-\eta}$$  \hspace{1cm} (1)

with $\eta \in (0, 1)$. By optimization, the consumer spends $y = (1 - \eta) E$ on $q$, so that we can write the consumer’s budget constraint for the differentiated good as

$$y = pq$$  \hspace{1cm} (2)
where $p$ is the price index associated with the differentiated good. Second and third stage utility are given by

$$q = \left( \int_{i=0}^{n} \left( \frac{q_{i}^{\sigma}}{p_{i}} \right)^{\frac{\theta}{\sigma-1}} \right)^{\frac{\sigma}{\sigma-1}}$$  \hspace{1cm} \text{(3)}$$

and

$$q_{i} = \left( \int_{k=0}^{h_{i}} \left( \frac{q_{ik}}{p_{ik}} \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$  \hspace{1cm} \text{(4)}$$

where $q_{ik}$ is the demand for each variety $k$ of a given firm $i$, which produces a number (=mass) $h_{i}$ of varieties. Then, $q_{i}$ is the quantity index associated with the sales of a given firm, while $n$ is the number of firms. Importantly, $\sigma$ is the elasticity of substitution between different varieties of a given firm and $\theta$ is the inter-firm elasticity of substitution. We assume that $\sigma > \theta > 1$.

It is well-known that minimizing expenditure subject to the CES aggregator gives the solution for the welfare-based price indices (see Allanson and Montagna (2005) and Obstfeld and Rogoff (1996, pp. 227-228)):

$$p = \left( \int_{i=0}^{n} p_{i}^{1-\theta} \, di \right)^{\frac{1}{1-\theta}}$$  \hspace{1cm} \text{(5)}$$

and

$$p_{i} = \left( \int_{k=0}^{h_{i}} p_{ik}^{1-\sigma} \, dk \right)^{\frac{1}{1-\sigma}}$$  \hspace{1cm} \text{(6)}$$

and that final demand for varieties can then be expressed as

$$q_{ik} = \left( \frac{p_{i}}{p} \right)^{-\theta} \left( \frac{p_{ik}}{p_{i}} \right)^{-\sigma} q$$  \hspace{1cm} \text{(7)}$$

2.2 Firms

Next, we consider the decision problem of firms. In the first stage, firms are merely potential entrants. To start operating firms have to pay a one-time cost $F_{e}$, which entails, among
other things, plant setup, initial market research and setting up of a distribution network. Only after incurring this cost, firms discover their productivity. We think of firms as drawing their productivity, $\varphi$, from a time-invariant distribution, $g(\varphi)$. This is an essential building block of the Melitz (2003) model. It is empirically well documented that firms within a given industry differ widely in their productivities, and the relevance of this industrial feature for trade has been a topic of intense research.\(^5\)

Once firms know their productivity, they must decide whether to produce or exit. Stayers set their prices and variety offering. This is the second stage. But firms that choose to stay active face an exogenously probability, $\delta$, each period of being hit by a death shock. This feature of the Melitz-model draws on Hopenhayn’s (1992) work on industry dynamics. Being active on the domestic market brings about the following costs each period:

$$C_i(\varphi) = a + (b + f_h) h_i + \int_{k=0}^{h_i} \frac{w}{\varphi} q_{ik} dk$$

(8)

where $a$ and $b$ are firm-wide and variety-specific fixed costs, respectively. These represent, for instance, advertisement, management time and maintenance of the distribution networks. More generally, they are the fixed costs required to maintain activity on the domestic market, and are a standard part of the literature on monopolistic competition with increasing returns to scale (see Allanson and Montagna, 2005). Distinct from these is $f_h$, which is the cost of creating a new variety.\(^6\) Once we introduce exports, it will be clear why these must be kept distinct. In fact, it costs $F_h$ to set up a new variety. But firms are indifferent between paying this $F_h$ up front, or paying the amortized fixed cost $\delta F_h = f_h$ each period.\(^7\) The last term in the equation captures the variable costs of production. Marginal costs are inversely proportional to the productivity parameter, $\varphi$, and $w$ is the wage. We normalize $w = 1$, \(^5\)See the surveys of Greenaway and Kneller (2005) and Tybout (2003).

\(^6\)Notice that by the formulation in equation (8) we implicitly assume that R&D costs are linear in the number of varieties. We have also tried different specifications, however, such as convex costs (as proposed, for instance, by Brambilla (2006)). These make no difference for our final results, so that we stick to linearity for analytical convenience.

\(^7\)See the discussion in Melitz (2003, p.1708) on rewriting fixed costs to per-period notation.
without loss of generality.

At the beginning of the second stage a firm also chooses whether to become an exporter, how many of its varieties to export, and which prices to charge abroad. Yet, in order to export, firms face an additional hurdle. They must pay a so-called beachhead cost, \( f_x \), associated to setting up a new trade line (see footnote 2 for the empirical relevance of these costs). For simplicity, we let the fixed costs of maintaining activity on a market, \( a \) and \( b \), be the same in the domestic and foreign markets. To export a good, furthermore, a firm pays tariff and transport costs \( \tau > 1 \) per shipped unit. The firm’s per period profit function becomes:

\[
\pi_i(\varphi) = \int_{k=0}^{h_i} \left( p_{ik} - \frac{1}{\varphi} \right) q_{ik} dk - a - (b + f_h) h_i + \max \left\{ 0, \int_{k=0}^{h_i^X} \left( p_{ik}^X - \frac{\tau}{\varphi} \right) q_{ik}^X dk - (a + f_x) - bh_i^X \right\}
\]

where the max operator signifies the decision of entering the export market. The second term in the operator represents the profits from exporting, and the firm will only become an exporter if those profits are greater than zero. The terms \( h_i^X \), \( p_{ik}^X \) and \( q_{ik}^X \) stand for, respectively, the number of varieties exported, the price of variety \( k \) charged in the foreign market and the quantity of variety \( k \) sold abroad. We focus on a two-country model, though it would not make a difference to extend it to more countries. The main assumption we make is that all countries are identical and that the trading cost \( \tau \) is the same to each destination. Hence, if a firm would export to one country, it would export to all. The fact that the countries are identical also means that wages are equivalent (as in Melitz (2003)).

Before we can proceed to solving the model, there are a few important points to observe. Firstly, the first stage decision of the firm can be summarized by a free-entry condition:

\[
E \left[ \max \{ [\pi_i(\varphi)], 0 \} \right] \geq f_e
\]

where we have rewritten \( \delta F_e = f_e \). Firms will enter as long as the expected net present value of positive future profits covers the entry cost. Note that firms, if they pay \( f_e \), own a call
option. Their "underlying asset" is the productivity draw. If it is large enough to ensure positive per-period profits, they stay. Otherwise, they exit. Hence the max operator. We can therefore define a cutoff productivity level, $\tilde{\varphi}$, for which firms are precisely indifferent between continuing and ceasing production:

$$\pi_i (\tilde{\varphi}) = 0$$

(11)

Moreover, the model also contains a cutoff productivity for exporting, $\tilde{\varphi}^X$. This is the value that equates both sides in the max operator of equation (9). Hence, it is the productivity draw for which the firm is precisely indifferent between exporting and not exporting.

$$\int_{k=0}^{h_i^X} \left( p_{ik}^X - \frac{\tau}{\tilde{\varphi}^X} \right) q_{ik}^X dk - (a + f_x) - bh_i^X = 0$$

(12)

As in Melitz (2003), however, we require a condition on the parameters to make sure that $\tilde{\varphi}^X > \tilde{\varphi}$. That is, only the most productive fraction of active firms become exporters, as supported by microeconometric studies (see footnote 5). This is the driving force of firm selection. Moreover, we want ensure that $h_i > h_i^X$ and firms do not invent new varieties only for export. Empirical work indicates that firms indeed export only a part of their domestic varieties (Bernard et al. (2005)). We impose the following parameter restriction, which, as we will see later, ensures both:

$$b \left( \tau^{\theta-1} - 1 \right) > 1$$

(13)

Finally, it is worthwhile pointing out how our model nests the contributions of Melitz (2003) and Allanson and Montagna (2005). If we set $\sigma = \theta$, $f_h = 0$ and fix $h_i = h_i^X = 1$ we obtain a model with heterogeneous, single-variety firms that is equivalent to Melitz’s. A way to look at it is to say that the Melitz-model has no diseconomies of scope, as additional varieties do not reduce the demand for the firm’s current line ($\sigma = \theta$). But the presence of a firm-wide fixed cost then implies that firms would like to produce infinitely many varieties. To avoid this, the Melitz-model implicitly assumes a particular cost function of creating new varieties:
zero for the first variety, and infinity for any subsequent ones. Our model instead has a
continuous R&D cost function and bounds variety by $\sigma > \theta$. Conversely, if we cut out the
heterogeneity of firms by fixing $\varphi = \bar{\varphi}$ for all firms, and take away firms’ possibility to export,
we get to Allanson and Montagna’s model of homogeneous, multi-variety firms.

3 Equilibrium

In this section we work to obtain a closed-form equilibrium solution. We first solve for
the price setting of the firms. We replace $q_{ik}$ from equation (7) into equation (9) and set
$\frac{\partial \pi_i(\varphi)}{\partial p_{ik}} = 0$. Likewise, noting that $q_{ik}^X = \left( \frac{p_{ik}^X}{p} \right)^{-\theta} \left( \frac{p_{ik}^X}{p_i^X} \right)^{-\sigma} q$ - where $p_{ik}^X$ is the price index of
domestic consumers’ purchases from a foreign firm - we set $\frac{\partial \pi_i(\varphi)}{\partial p_{ik}} = 0$ to obtain prices charged
by exporters. Subsequently, $\frac{\partial \pi_i(\varphi)}{\partial h_{ik}} = 0$ and $\frac{\partial \pi_i(\varphi)}{\partial h_{ik}^X} = 0$ give us equations for $h_i$ and $h_{ik}^X$. Next,
we turn to the determination of $\bar{\varphi}$ and $\bar{\varphi}^X$, the cutoff productivity levels for activity on the
domestic and foreign markets. Taking the domestic sales part of equation (9) and setting it
equal to zero, we solve for $\bar{\varphi}$. Moreover, we know that for the firm that is indifferent between
exporting and not exporting, the two sides inside the max operator of equation (9) will equal.
This gives us a solution for $\varphi^X$.

The next step is to solve for the firms’ free-entry condition in equation (10). To do this,
we rewrite the max operators in the profit function to probabilistic terms. That is, with the
probability that $\varphi > \bar{\varphi}$ the firm will remain active in the domestic market after discovering its
productivity. This probability is simply $\int_{\varphi}^{\infty} g(\varphi) d\varphi$. Similarly, before entering the market,
the firm has a chance of $\int_{\bar{\varphi}}^{\infty} g(\varphi) d\varphi$ of becoming an exporter. Finally, we need to rewrite
the aggregate price level from equation (5) to

$$p = \frac{\sigma}{\sigma - 1} \left( n \int_{\bar{\varphi}}^{\infty} \varphi^{\theta-1} (h_{ik})^{\theta-1} \frac{\varphi}{\bar{\varphi}} g(\varphi \mid \varphi > \bar{\varphi}) \, d\varphi + (1-n) X \int_{\bar{\varphi}}^{\infty} \varphi^{\theta-1} (h_{ik}^X)^{\theta-1} g(\varphi \mid \varphi > \bar{\varphi}^X) \, d\varphi \right)^{\frac{1}{1-n}}$$

(14)

where $g(\varphi \mid \varphi > \bar{\varphi})$ is the conditional distribution of $\varphi$. That is, the distribution of produc-
tivities among only active firms. While \( g(\varphi \mid \varphi > \tilde{\varphi}^X) \) is that distribution among exporters. Furthermore, \( n^X \) is the number of foreign firms from which domestic consumers purchase. By the symmetry of countries this is equal to the number of domestic exporters. Formally,

\[
    n^X = n \frac{\int_{\tilde{\varphi}^X}^{\infty} g(\varphi) d\varphi}{\int_{\tilde{\varphi}}^{\infty} g(\varphi) d\varphi}
\]

(15)

This gives us enough to solve the free-entry condition and obtain an equation for \( n \). But to solve closed-form for \( p \) we need to parameterize the distribution of firm productivities, \( g(\varphi) \). As is standard in the trade literature with heterogeneous firms, we work with a Pareto distribution.\(^8\) The probability density function takes the form

\[
    g(\varphi) = cd^c\varphi^{-c-1}
\]

(16)

where \( d \) is the lower bound of the distribution, \( g(\varphi) \) is defined on \([d, \infty)\), and \( c \) is the parameter that measures heterogeneity. A smaller \( c \) implies a wider distribution and, thus, a more heterogeneous population of firms. As is common in the literature, we normalize \( d = 1 \). Moreover, as in Helpman et al. (2004) and Chaney (2006), we require a parameter restriction

\[
    c > \max \left\{ 2, \frac{(\sigma - 1)(\theta - 1)}{\sigma - \theta} \right\}
\]

(17)

to ensure finite variance of the distribution of productivity draws \( g(\varphi) \) and the conditional productivity distribution of active firms \( g(\varphi \mid \varphi > \tilde{\varphi}) \). Implementing the Pareto distribution

\(^8\)The Pareto distribution is both analytically convenient and empirically relevant. See Helpman et al. (2004) and Axtell (2001).
gives us, after quite a bit of algebra, the closed-form solution we sought:

\[ q = \frac{y}{p} \]

\[ p = \frac{1}{\phi} \frac{\sigma}{\sigma - 1} \left[ \frac{\sigma (\sigma - 1)}{y} \right]^{\frac{1}{\sigma - 1}} \left[ \frac{b + f_h}{\theta - 1} \right]^{\frac{1}{\sigma - 1}} \left[ \frac{a}{\sigma - \theta} \right]^{\frac{\sigma - \theta}{\sigma - 1}} \left( \frac{\sigma - 1}{\sigma - 1} \right)^{\frac{\sigma - \theta}{\sigma - 1}} \]

\[ \phi = \left[ \frac{a}{f_c (\sigma - \theta) c - (\sigma - 1) (\theta - 1)} \left( 1 + \frac{1 + f_x}{1 + \frac{f_x}{a} + \Psi (\tau)} \right) \right]^{\frac{1}{c}} \]

\[ \phi^X = \left[ \frac{a}{f_c (\sigma - \theta) c - (\sigma - 1) (\theta - 1)} \left( 1 + \frac{1 + f_x}{1 + \frac{f_x}{a} + \Psi (\tau)} \right) \right]^{\frac{1}{c}} \]

\[ n = \frac{y (\sigma - \theta) c - (\sigma - 1) (\theta - 1)}{a} \left( \frac{\sigma}{\sigma - 1} c \right) \left( \frac{1}{1 + \frac{f_x}{a} + \Psi (\tau)} \right) \]

\[ n^X = \frac{y (\sigma - \theta) c - (\sigma - 1) (\theta - 1)}{a} \left( \frac{\sigma}{\sigma - 1} c \right) \left( \frac{1}{1 + \frac{f_x}{a} + \Psi (\tau)} \right) \]

where we have defined

\[ \Psi (\tau) = \left( \tau \left[ \frac{b}{b + f_h} \right]^{\frac{1}{\sigma - 1}} \left[ \frac{a + f_x}{a} \right]^{\frac{\sigma - \theta}{\sigma - 1}} \right)^{\frac{c}{\sigma - 1}} \] (18)

to make things visually easier to absorb. Moreover, for a given firm with productivity draw \( \varphi \) we also have the following equations governing price setting and the optimal scope at home and abroad:

\[ p_{ik} = \frac{\sigma}{\sigma - 1} \frac{1}{\varphi} \] (19)

\[ P_{ik}^X = \frac{\sigma}{\sigma - 1} \frac{\tau}{\varphi} \] (20)

\[ h_i = \frac{\theta - 1}{\sigma - \theta b + f_h} \left( \frac{\varphi}{\varphi} \right)^{\frac{(\sigma - 1)(\theta - 1)}{\sigma - \theta}} \] (21)

\[ h_i^X = \left[ \frac{b + f_h}{b} \right]^{\frac{\sigma - 1}{\sigma - \theta}} \left( \frac{\theta - 1}{\sigma - \theta b + f_h} \left( \frac{\varphi}{\varphi} \right)^{\frac{(\sigma - 1)(\theta - 1)}{\sigma - \theta}} \right) \] (22)

It is interesting to observe here that the inter-firm elasticity of substitution, \( \theta \), does not affect price setting. Rather, firms set markups purely according to their intra-firm elasticity, \( \sigma \), and adjust for \( \theta \) completely along the variety margin. Note, moreover, that we indeed
have \( h_i > h_i^X \) and \( \widehat{\varphi}^X > \widehat{\varphi} \) by the condition in equation (13).

It will also prove useful in our analysis to have expressions for the total number of varieties available to domestic consumers, \( H \), and the total number of domestic plus foreign firms from which they buy, \( N \).

\[
H = n \int_{\widehat{\varphi}}^{\infty} h_i g(\varphi | \varphi > \widehat{\varphi}) d\varphi + n^X \int_{\widehat{\varphi}^X}^{\infty} h_i^X g(\varphi | \varphi > \widehat{\varphi}^X) d\varphi = \frac{y (\theta - 1) 1 + \frac{\ell_x}{a} + \left[ \frac{b}{b + f_h} \right] \Psi (\tau)}{\sigma (\sigma - 1) b} \frac{1 + \frac{\ell_x}{a} + \Psi (\tau)}{1} \tag{23}
\]

\[
N = n + n^X = \frac{y (\sigma - \theta) c - (\sigma - 1) (\theta - 1) 1}{\sigma (\sigma - 1) c} \left[ \frac{\Psi (\tau) + 1}{\Psi (\tau) + 1 + \frac{\ell_x}{a}} \right] \tag{24}
\]

### 4 Analysis

As stated at the outset, our aim is to come to a unified framework of the two sources of gains from trade, firm selection and product variety. Using our closed-form solution, we are now ready to analyze how these channels arise together, and how they interact. We proceed as follows. First, we mathematically identify the selection and variety effects. Then, we prove that both channels have the sign found in empirics, and analyze what makes them come about in our model. In particular, the variety effect is driven in opposite directions by, on the one hand, the impact of firm selection and, on the other, the scope differences between exporters and domestic firms. We consider why despite the opposing effects the sign nonetheless comes out unambiguous. We then turn to the comparative statics of firm heterogeneity: how are the two channels affected by the degree of productivity differences between firms? Finally, we briefly consider an extension of our model, which allows us to analyze anti-globalists' "standardization" critique.

#### 4.1 Firm selection and product variety

Since our purpose is to further our understanding of the sources of gains from trade, we start out by the mathematical identification of those sources. Looking at our closed-form
solution, it is immediately apparent that trade liberalization always improves welfare. The welfare-based price index, $p$, decreases when tariffs, $\tau$, decrease: $\frac{\partial p}{\partial \tau} > 0$. This does not tell us what causes the improvement in welfare, however. Luckily, we have a way to precisely identify the two channels.

**Lemma 1** *The firm selection effect can be found as*

$$\frac{\partial p}{\partial \phi} \frac{\partial \phi}{\partial \tau} \bigg|_{h_i=1} = \frac{c}{c+1-\theta} \frac{\partial \phi}{\partial \tau}$$  \hspace{1cm} (25)

*whereas the variety effect can be identified as*

$$\frac{\partial p}{\partial \phi} \frac{\partial \phi}{\partial \tau} - \frac{\partial p}{\partial \phi} \frac{\partial \phi}{\partial \tau} \bigg|_{h_i=1} = \frac{(\sigma-\theta) c - (\theta - 1) (\sigma - \theta)}{(\sigma-\theta) c - (\theta - 1) (\sigma - 1)} \left( \frac{\theta - 1}{\sigma - \theta b + f_h} \right)^{\frac{\theta-1}{\sigma-1}} \frac{\partial \phi}{\partial \tau}$$  \hspace{1cm} (26)

*where*

$$\left| \frac{\partial \phi}{\partial \tau} \right| = \frac{1 + \frac{L_x}{a}}{\tau \Psi (\tau) + 1 + \frac{L_x}{a}}$$  \hspace{1cm} (27)

The expressions in equations (25) and (26) are exactly what we were in need of. The first thing we can use them for, is understanding the signs of the two effects.

**Proposition 1** *Upon liberalization, welfare rises through both a positive firm selection effect and a positive variety effect. The signs of both effects are unambiguous. That is, $\frac{\partial p}{\partial \phi} \frac{\partial \phi}{\partial \tau} \bigg|_{h_i=1} > 0$ and $\frac{\partial p}{\partial \phi} \frac{\partial \phi}{\partial \tau} - \frac{\partial p}{\partial \phi} \frac{\partial \phi}{\partial \tau} \bigg|_{h_i=1} > 0$.*

Now, there are two questions to ask. How come that, contrary to Melitz (2003), the variety effect can be positive at all? And why is the sign of that effect unambiguous? As we shall see, these are two distinct questions. To answer the first, we need to identify the forces that drive the variety effect in our model. Take a look at equation (24), which represents the total number of firms that consumers purchase from. Keeping in mind that $\Psi' (\tau) > 0$, it is easy to see that $\frac{\partial N}{\partial \tau} > 0$. That is, the total number of firms increases in protection, and likewise, decreases in liberalization. In this sense our model is precisely as Melitz’s. When trade is liberalized firm selection leaves a market that is more concentrated. This feature
of deepening trade linkages is empirically supported by Mirza (2006). Hence, the impact of firm selection on the variety effect is negative.

However, compared to the Melitz-model, there is a new, counteracting force at work in our model. Entering foreign exporters offer more variety than the domestic firms they push out. The fact that exporters are relatively more efficient implies not only lower prices, but also a positive adjustment along the scope margin. As an empirical matter, Bernard et al. (2005) and Manez et al. (2004) report that exporters are indeed of a larger scope than the average domestic firm. In our model, this arises because exporters’ larger efficiency dominates the fact that they sell only a subset of their varieties abroad. Formally we have that

$$
[ h_i^X \mid \varphi = \hat{\varphi}^X ] - [ h_i \mid \varphi = \hat{\varphi} ] = \frac{\theta - 1}{\sigma - \theta} \left[ \frac{a + f_X}{b} - \frac{a}{b + f_h} \right] > 0 \tag{28}
$$

so that foreign entrants always bring with them more variety than the firms that they push out.

But now the question becomes: how can it still be unambiguous that trade liberalization increases variety? Looking at equation (28), can we not just take $f_h \to 0$ and $f_x \to 0$, or $\sigma \to \infty$ and $\theta \to 1$, thereby cancelling out the scope difference between exporters and domestic firms? The answer is: indeed we could, but that would also cancel out the rise in market concentration caused by firm selection. For $f_h \to 0$ and $f_x \to 0$ we can see from equation (24) that $\frac{\partial N}{\partial \tau} \to 0$, so that the total number of firms stays the same upon liberalization. The reason for this is that $[ p_{ik} \mid \varphi = \hat{\varphi} ] \to [ p_{ik}^X \mid \varphi = \hat{\varphi}^X ]$: the marginal exporter that enters upon liberalization, sets the same price as the least efficient domestic firm. The aggregate price index does not change when we replace the one firm with the other, hence the "space" for firms in the market is not reduced through liberalization. When $\sigma \to \infty$ and $\theta \to 1$, moreover, we have that $\hat{\varphi} \to 0$ and firm selection as represented by equation (25) vanishes. The number of firms becomes insensitive to liberalization.

Yet, the question remains what the intuition is for the outcome that the variety effect is unambiguously positive. We can give the intuition most easily by means of an extreme
example. Imagine a closed economy. First, all firms have a per-variety fixed cost of \((b + f_h)\).
Suddenly, their cost per variety changes to \(b\). Clearly, economies of scope rise. And diseconomies of scope, through the cannibalization represented by \((\sigma - \theta)\), remain constant. The same holds true for a change from \(a\) to \((a + f_x)\) as the firm-wide fixed costs. With greater economies of scope, and constant diseconomies, among the firms that fill the domestic market, total variety offered must be larger. As there are more varieties, demand per variety diminishes. Hence, the varieties of the least efficient firms become unprofitable. Consequently, the number of firms will end up smaller. This, in fact, is a way to look at the process following trade liberalization, as a larger part of the market share shifts to foreign firms, who spread their R&D costs over more markets.

### 4.2 Comparative statics of firm heterogeneity

Thus, we have a framework to understand how firm selection and variety effects jointly arise. But what precisely is the role of firm heterogeneity? It is quite apparent that wider firm productivity differences strengthen firm selection. This could already be seen form the models of Melitz (2003) and Bernard et al. (2003). But, as concerns variety, the comparative statics of firm heterogeneity have never been considered.

**Proposition 2** Greater firm heterogeneity implies both a stronger selection effect and a stronger variety effect. That is, \(\frac{\partial}{\partial c} \left[ \frac{\partial p}{\partial c} \right]_{h_i=1} < 0\) and \(\frac{\partial}{\partial c} \left[ \frac{\partial p}{\partial c} - \frac{\partial p}{\partial \tau} \right]_{h_i=1} < 0\), since a higher \(c\) implies less heterogeneity.

Why does greater heterogeneity strengthen the variety effect? As before, there are two countervailing effects. On the one hand, firm selection is strengthened, which means that more domestic varieties get pushed out upon liberalization. On the other hand, the scope difference between exporters and non-exporters widens. The latter effect again dominates. Hence, both channels of gains from trade are more potent in industries where productivity differences are more pronounced.
Finally, we would like to point out that the results we have derived in sections 4.1 and 4.2 do not simply extend to non-tariff barriers such as, for instance, regulatory barriers to trade. Such barriers are captured by the term \( f_x \). But, as we can observe from our equilibrium solution, it is not even clear that a reduction in such barriers improves welfare: \( \frac{\partial p}{\partial f_x} \) is of ambiguous sign. Though lower costs to start exporting clearly increase the number of exporters, they also flatten the productivity difference between exporters and domestic firms. As we saw before, \( f_x \rightarrow 0 \implies [p_{ik} | \phi = \hat{\phi}] \rightarrow [p_{ik}^X | \phi = \hat{\phi}^X] \) and the gains to consumers from lower prices vanish.

4.3 An extension

Although we have constructed our model for the purpose of analyzing the gains from trade, it is quite a rich framework and can be applied to other questions as well. For instance, anti-globalists have long claimed that globalization brings about a "standardization" of products, which flattens the choice of consumers. According to this view, local firms have their own "character", whereas large, international firms are more similar to each other. Globalization forces local firms to either adjust or exit. Yet, we can use our model to show that things may be a notch more subtle. If globalization makes firms more similar, consumers end up with more variety.

Allanson and Montagna (2005) have, in fact, applied their model to study standardization in the context of the product life-cycle. The emergence of a dominant design in an industry, pushing out the designs of various small firms, is caught by an increase in the inter-firm elasticity of substitution, \( \theta \). This increase is both in absolute terms and relative to the intra-firm elasticity of substitution, \( \sigma \). In our context, we can simply use the expression in equation (23) to take the derivative \( \frac{\partial H}{\partial \theta} \). Since \( \Psi (\tau) \) is decreasing in \( \theta \), it follows that \( \frac{\partial H}{\partial \theta} > 0 \). When \( \theta \) is higher, the cannibalization effect is weaker, and each active firm offers more variety. Therefore, if it is true that \( \frac{\partial \theta}{\partial \tau} < 0 \), as anti-globalists claim, then consumers indeed end up buying from more similar firms. But they also have more varieties to choose from. It is not
clear that "standardization" makes consumers worse off, therefore. In fact, a glance at our closed-form solution reveals that the derivative $\frac{\partial \nu}{\partial \varphi}$ is of ambiguous sign.

5 Conclusions

In this paper, we presented what we believe to be the next step in understanding the gains from trade: a model that brings together firm selection and variety gains, showing that it is firm heterogeneity that drives both. Matching a stylized fact, exporters offer more variety than non-exporters do. This is counteracted by the other empirically supported element: the rise in market concentration, which leaves consumers with fewer firms to buy from. But the scope difference between exporters and non-exporters unambiguously dominates, so that the variety effect is always positive. This can be understood intuitively by seeing trade liberalization as an event that raises average economies of scope in the economy, by shifting market share to firms that can spread their R&D costs over more markets. That rise in economies of scope both increases variety and drives out the least efficient firms.

Our work gives rise to two policy implications. Firstly, both sources of gains from trade are stronger in industries where productivity differences between firms are larger. Hence, if policy makers are constrained to make a second best choice about which industries to liberalize, our model suggests they should choose those with the largest firm heterogeneity. Secondly, policy makers need not be overly concerned about the product "standardization" effect of globalization, as suggested by anti-globalists. The disappearance of small firms that differ more from each other in "character" may be more than compensated for by the increase in product variety.
A Appendix

Proof of Lemma 1. From our closed-form solution we know that the entire welfare gain from trade liberalization is summarized by the term $\frac{\partial p}{\partial \hat{\varphi}} \frac{\partial \hat{\varphi}^2}{\partial r}$. We can now turn back to equation (14) and observe that $\frac{\partial p}{\partial \hat{\varphi}} = \left[ \frac{\partial}{\partial \hat{\varphi}} \int_0^{\infty} \varphi^{\theta-1} (h_i)^{\theta-1} \varphi g (\varphi > \hat{\varphi}) d\varphi \right] \frac{\partial \hat{\varphi}^2}{\partial r}$. But, within this term, the variety effect on welfare runs through $h_i$, whereas firm selection runs through $\varphi^{\theta-1}$. Hence, we can restrict the model to $h_i = 1$, to cut out endogenous scope and observe the part of firm selection in the entire derivative $\frac{\partial p}{\partial \hat{\varphi}} \frac{\partial \hat{\varphi}^2}{\partial r}$. That is, firm selection can be found as $\frac{\partial p}{\partial \hat{\varphi}} \frac{\partial \hat{\varphi}^2}{\partial r} \bigg|_{h_i=1}$. Then, the remainder of the derivative is explained by the variety effect: $\frac{\partial p}{\partial \hat{\varphi}} \frac{\partial \hat{\varphi}^2}{\partial r} - \frac{\partial p}{\partial \hat{\varphi}} \frac{\partial \hat{\varphi}^2}{\partial r} \bigg|_{h_i=1}$. ■

Proof of Proposition 1. This follows directly from substituting equation (27) into equations (25) and (26). All terms are positive. ■

Proof of Proposition 2. The proof follows directly from equations (25), (26) and (27) and the expressions for $\tilde{\varphi}$ and $\Psi (\tau)$ from our closed-form solution. First, by $\frac{\partial \Psi (\tau)}{\partial c} > 0$ and $\frac{\partial \tilde{\varphi}}{\partial c} < 0$ we have $\frac{\partial}{\partial c} \left[ \frac{\partial \tilde{\varphi}}{\partial \tau} \right] < 0$. Together with $\frac{\partial}{\partial c} \frac{c}{c+1-\theta} < 0$ this implies $\frac{\partial}{\partial c} \left[ \frac{\partial p}{\partial \hat{\varphi}} \frac{\partial \hat{\varphi}^2}{\partial r} \bigg|_{h_i=1} \right] < 0$. And together with $\frac{\partial}{\partial c} \frac{c}{(\sigma-\theta)(\theta-1)(\sigma-1)} < 0$ we have $\frac{\partial}{\partial c} \left[ \frac{\partial p}{\partial \hat{\varphi}} \frac{\partial \hat{\varphi}^2}{\partial r} - \frac{\partial p}{\partial \hat{\varphi}} \frac{\partial \hat{\varphi}^2}{\partial r} \bigg|_{h_i=1} \right] < 0$. ■
References


