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Competition Policy: Should Variable Cost Aid Be
Banned?

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State Aid to Attract FDI and the European Competition Policy: Should Variable Cost Aid Be Banned?

Mario Mariniello*

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Abstract

The purpose of this paper is to analyze the European Commission's approach to state aid for foreign direct investment in a competition policy framework. The Commission shows to consider variable cost aid (*VCA*) to be more distortive than start-up or fixed cost aid (*FCA*). This paper addresses that issue and checks whether allowing *FCA* while banning *VCA* is a first-best strategy for a rational Authority maximizing welfare.

The model shows that a rational forward-looking government maximizing domestic welfare always prefers *VCA* to *FCA* if both the incumbent and the entrant are foreign firms and if granting *VCA* does not cause to the incumbent firm to exit the market. On the other hand, a *VCA* which causes the incumbent firm to be crowded out by the entrant never occurs at the equilibrium.

The model shows that the Commission's approach may lead to sub-optimal equilibria where market competition and consumers' welfare are not maximized.

Key words: state aid, competition policy, start-up aid, variable cost aid

JEL: L11, L13, L40, L53

1 Introduction

The use of state aid is in principle banned by the Treaty of Rome. One of the main reasons for the ban lies in the fact that subsidies, altering the relative positions of competing firms, are usually linked to distortions in the market. The Treaty, however, allows for a number of exceptions to the general ban whenever the potential distortion of a subsidy is low enough to be overcome by its potential benefits, such as the support of a depressed area or the growth of a particular sector of a country's economy. The purpose of this paper is to analyze a well-established policy of the European Commission on the compatibility of state aid with the Treaty's rules according to which subsidies which lower firms' variable cost (*VCA*) are more distortive than subsidies which lower firms' fixed entry cost (*FCA*). To a certain extent, indeed, the definition of variable cost aid coincides with what the Commission calls *operating state aid*

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i.e. aid ordinarily associated with business' normal operations. As an illustration, consider a recent European competition policy case which is the *Ryanair - Charleroi* case¹. In short, the publicly controlled airport of Charleroi granted a certain number of benefits to the air carrier Ryanair in order to encourage the opening of new routes to Charleroi. These benefits have been considered state aid by the Commission, but only some of them have been found to be incompatible with EU rules, given the exceptional features of the depressed area of Charleroi. The Commission decided that those benefits which were sufficiently tied to the start-up of new routes and to the development of the airport could have been considered compatible with the Treaty of Rome under the provision of Article 87(3)(c). On the other hand, those aids which were intended to reduce Ryanair's variable cost had to be given back, since they did not meet the compatibility criteria established by the Commission. Examples of the first type of subsidies - those that were finally allowed - are: 160,000 euros per new route opened, up to a maximum amount of 1,920,000 euros; 200 square meters free of charge to be used for offices and as engineering store; a lump sum contribution to promotional activities. Examples of the second type of subsidies - those that were banned - are: a preferential rate for landing charges of 1 euro per boarding passenger, which is about one half of the official standard rate charged to airlines in Belgium; a rate of 1 euro per passenger for ground-handling services which is about ten times lower than the average rate charged to other airlines.

Rather than being an isolated case, the Ryanair - Charleroi case decision is a manifestation of a general approach which has become evident during past years in the Commission's official documents and decisions. The Commission's guidelines on national regional aid clearly states that, in the context of aid to stimulate the development of depressed areas, *aid to initial investment* is allowed while *aid aimed at reducing a firm's current expenses is normally prohibited*². Nevertheless, as a confirmation that this kind of approach is a general one and that it concerns aids which do not fall into the regional aid category as well, it is sufficient to have a look at the list of decisions taken in the past years and to notice that the likelihood of being considered illegal state aid is much higher for aids which use tax reduction instruments rather than a direct grant instruments. Think for example to cases such as the Italian tax breaks for companies listed for the first time on the EU stock exchanges, where the motivation for outlawing the aid scheme lies on the fact that the subsidy is proportionate to the revenues earned by the beneficiaries³. Or to the case of three aid schemes implemented by the Basque province, where the Commission states "*...as they [the aid schemes] also constitute operating aid, doubts exist about their compatibility with the common market*". In the words of the Commission, the aid schemes were indeed designed to relieve firms of cost tax charges they would normally have to bear as part of their everyday management of usual activities and are, as a consequence, illegal⁴.

This kind of approach does not seem to be based on a rigorous competition policy analysis. Although it is true that state subsidies may introduce distortions in the market, it is not generally true that banning variable cost subsidies and allowing start-up subsidies is a first best solution for a welfare maximizing Authority. Any optimal choice requires to consider the trade-

¹OJ 2004 L137/1, 12 February 2004

²Commission's guidelines on national regional aid, OJ C 74, 10/03/1998 0009-0031

³IP/05/304

⁴IP/00/1244

off between the possible gain in welfare due to an increase in competition (which may be brought by variable cost aids too) and the possible loss of welfare brought by the distortions introduced in the market. The approach taken by the European Commission on the Ryanair - Charleroi case and, in general, on state aid seems to lack of such consideration. The model presented in this paper addresses that concern focusing on the competition policy's aspect of state aid only and leaving aside alternative possible concerns such as lobbying or public choice issues. The focus is on a specific kind of aid which is aid to attract foreign direct investment (FDI) and in the basic setting the two firms playing the game are foreigners. Much have been said on FDI (Brander and Spencer [1985], Krugman [1987], Markusen et al. [1995], Markusen and Venables [1997], Barros and Cabral [2000], Fumagalli [2002]) but the models usually implemented have significant differences with the model presented in this paper. Usually the presence of more than one government competing in order to attract FDI is assumed, local firms are assumed to play a role in host markets and the analysis does not include a comparison among different possible aid instruments which could be implemented. In this paper, on the contrary, I abstract from those features to focus on the welfare effects of different ways of financing foreign direct investments. Therefore, in the model there is only one government, there are no local producers (the incumbent is also supposed to be foreign-owned) and externalities are not modelled for simplicity, that is: the focus is not on what is the reason why the government wants to subsidize entry - spillovers, labor market imperfections, and so on - (these two last assumptions are, however, relaxed in an extension to the basic model).

As we shall see, the model outlined in Section 2 suggests that *VCA* is always preferred to *FCA* whenever the government decides for intervention and both the two state aid instruments do not imply a crowding out effect in the market. At the same time, the model shows that a government which cares only about total domestic welfare never chooses to grant a *VCA* when the recipient firm would be able to force its competitor to exit the market. The conclusion is that a general ban that prevents governments from using variable cost aid may be welfare detrimental by reducing potential competition: there exists in fact an interval for the cost parameters in which, if the government cannot use variable cost aid, it does not intervene at all, thus leaving the potential entrant outside the market.

This paper proposes a specific point of view from which to assess the European state aid policy: the Competition Authority is assumed to adopt a consumers' welfare standard⁵. That is: the loss in incumbents' profits caused by entry is not included in the objective function of the Competition Authority. A reduction in potential competition in the domestic market is thus automatically linked to a loss in potential welfare given by the loss in the potential gain in terms of consumers' surplus. Under definite parameters conditions, banning *VCA* is then a sub-optimal policy.

Several reasons might justify this approach. From a policy making point of view, many experts report of an increasing weight attributed by Antitrust Authorities to consumers' welfare rather than to total welfare: for Schmalensee [2004] the benefits of entry are usually assessed by the U.S. Antitrust Authority solely on the basis of its impact on consumers' welfare; Derek Morris, former chairman of the Competition Commission in the U.K., stated that "*...in practice,*

⁵Or, in other words, it is assumed to have the same welfare objective of the domestic government. For an overview on welfare standards used in competition policy economics, see Motta [2004], pgg. 20-22.

competition policy effectively gives a very high weighting to consumer welfare and a very low weighting elsewhere"⁶. Not least, Neelie Kroes, the European Commissioner for Competition Policy, stated very recently: "*The consumer is at the heart of competition enforcement. [...] we are applying this 'consumer welfare standard' through better use of economic analysis in our work*"⁷. From a technical point of view, instead, many economists are still reluctant to suggest the use of a consumers' welfare standard in competition policy analysis. There is a growing literature on mergers, however, which tends to emphasize the benefits of a consumers' welfare approach with respect to a total welfare approach (see, for example, Lyons [2002] and Neven and Roller [2005]). My paper is one of the first academic papers trying to harmonize the welfare standards used in competition policy, merger control and state aid control and to move to common consumers' welfare approach. Nevertheless, a specific section of the paper is dedicated to the comparison between the two different welfare approaches and a discussion on how policy recommendation would change with the adoption of a total welfare standard is reported as well (see section 3.2.1).

Although the basic setting of the model is rather stylized, the results of the paper appear to be robust to generalization. Garcia and Neven [2005] show how the impact of state aid depends on market concentration: they find that the distortion induced by entry of a subsidized firm tend to be reduced when competition between domestic firms is increased. The results of my paper are shown to be robust to extension of the game to n playing firms and to the introduction of an externality function which links entry to the local economy⁸. The model is tested for the case in which the incumbent firm is domestic as well. In that case the government internalizes the negative effect on the incumbent's profits due to entry and state aid becomes less likely. Moreover, the unique kind of aid which is granted at the equilibrium is *FCA*, because the implicit further reduction in the incumbent's profits due to a reduction of the entrant's marginal cost makes *VCA* always inferior to *FCA* in terms of welfare yielded. This result suggests that the likelihood of a negative impact of a ban of variable cost aid by the European Commission is reduced whenever domestic firms play a significant role in the game.

The model's results eventually have a clear policy implication: an Antitrust Authority should assess the impact of an aid on competition and welfare independently of the way in which it is granted, *VCA* or *FCA*. A decision which depends largely on the kind of state aid instrument used might then require additional justification and should be carefully analyzed.

These results may be naturally compared with those few works in the literature where the authors focus more closely on subsidies and competition in the context of European Union's competition policy. Collie [2000, 2002] models a situation where governments subsidize, through distortionary taxation, their own firms in order to increase their competitiveness and to catch the increasing oligopolistic profits. The researcher then asks whether the prohibition of aid by the European Union would increase welfare, and concludes that there always exists a range of

⁶Lecture of the national consumer council, 30 April 2002.

⁷European Press Releases, <http://europa.eu.int/rapid/pressReleasesAction.do?reference=SPEECH/06/691&format=HTML&aged=0&language=EN&guiLanguage=en>

⁸The model with the externality function is not reported in the paper but can be provided by the author on request.

values for the shadow cost of subsidization where member states are willing to grant an aid and where the prohibition of doing so increases welfare of all member states. Contrary to the result of Collie, in the model outlined in this paper the shadow cost of the subsidy λ have a secondary impact on the general conclusions. An increase in λ obviously reduces state intervention's likelihood, so the impact of a ban of *VCA* on welfare would be lower. Nevertheless, even taking into consideration that *FCA* improves its relative advantage with respect to *VCA* when the entrant is inefficient, *FCA* would never arise for any value of λ in the basic setting i.e. when both the two competing firms are foreigners. Thus the general conclusions are unaltered.

Besley and Seabright [1999] analyze the role of subsidies in a static and dynamic framework and suggest a way of implementing the strategic trade literature for assessing the European Union's approach to state aid. Nicolaidis and Bilal [1999] check the validity of EU rules on state aid in promoting efficiency arguing that aid aimed to correct market failure should be allowed even if they may have cross-border effects. Compared to these researches, this paper proposes an analytical approach and a new setting on the basis of which to assess the European competition policy on state aid, from a consumers' welfare perspective.

The paper is organized as follows. In the following section I describe the basic setting and where the model. Extensions to the basic setting are proposed in section 3. Section 4 reports the conclusions and the policy implications. Appendix A contains the algebraic proofs while appendix B reports a description of the *non-crowding out* result i.e. the result by which the government would never let the entrant to force the incumbent firm to exit the market granting a state aid.

2 The Model

The players of the game are: the government G of a representative country, an entrant firm E and an incumbent firm I . At the moment the game starts, E is outside the market and I is inside the market producing $q_i^m > 0$. Firm j has constant marginal cost $0 \leq C_j \leq 1$. In addition, E has to pay a fixed cost of entry $K > 0$ in case it enters the market. Firm j 's net-of-costs profit is π_j . Consumers' demand is given by:

$$Q = 1 - P$$

where $Q = q_i + q_e$ is total output produced by the two firms and P is the associated market price. The government maximizes total domestic welfare $W(C_e, C_i, K, \lambda)$, where $\lambda \geq 1$ is the shadow cost of the subsidy⁹. In order to abstract from strategic trade policy (or rent-extraction) considerations, in this basic setting it is assumed that both I and E are foreign firms and that the government maximizes the following welfare function¹⁰:

$$W(C_e, C_i, K, \lambda) = CS(C_e, C_i) - \lambda S(C_e, C_i, K) \tag{1}$$

⁹ $\lambda = 1$ when lump-sum taxes are feasible (Collie [2002]). Moreover, empirical evidence shows that λ may vary from 1 (Kaplow [1996]) to 2.65 (Feldstein [1997]).

¹⁰Notice that the welfare function maximized by the government would be the same if the government adopts a consumers' surplus standard and one of the two or both firms are domestic.

where $CS(C_e, C_i)$ is the consumers' surplus and $S(C_e, C_i, K)$ is the subsidy. As it can be seen, the welfare function does not include any positive externalities to the local economy due to the entrance of a foreign firm in the market. It can be shown, however, that this assumption is not crucial and it does not influence the results reached by the model: it is sufficient to consider the consumers' surplus to achieve the main conclusions of the paper¹¹.

The focus of the analysis is on the effect of subsidies on the competition between E and I . For that reason, we can concentrate on the relative instead of the absolute efficiency of the two competing firms. So, for simplicity, in the rest of the paper it is assumed $C_i = \frac{1}{2}$. However, this assumption has to be removed in order to check whether the government would ever grant an aid such that the incumbent firm is crowded out from the market by the entrant firm (this can happen only if $C_i > \frac{1}{2}$). To that purpose, appendix B analyses what would be the choice of the government when C_i is allowed to vary and shows the non-crowding out result: if entry makes the incumbent to exit the market, the government does not grant any subsidy.

The game has four stages¹² (see Figure 1 for an illustration):

1. The government chooses between one of the following actions: financing a reduction in the size of K in order to allow the entrant to enter the market (*fixed cost aid, FCA*); financing a reduction in C_e (*variable cost aid, VCA*) for the same purpose; leaving the entrant's fixed and variable cost unchanged (*no intervention, NI*).
2. The government has to choose the amount of subsidy to be provided, if any. Define S_k as subsidy to fixed cost and S_c as subsidy to marginal cost. If the government has chosen *FCA* in the previous stage, it fixes $S_k > 0$ and $S_c = 0$. If it has chosen *VCA*, it fixes $S_c > 0$ and $S_k = 0$. If the government has chosen *NI* in the first stage, it sets $S_k = S_c = 0$.
3. After observing the government's decision, the entrant E chooses whether to enter the market or not
4. Firms in the market compete *à la Cournot* setting the output levels q_j

insert figure 1

To have a feeling of the kind of subsidies that can be granted by G we can refer to the Ryanair - Charleroi case: a lump sum contribution to the opening of new routes to Charleroi is an example of *FCA* while a discounted preferential rate for landing charges per passengers is an example of *VCA*.

The aim of the model is to outline situations where an entrant firm intends to enter the market but does not choose to do so because the presence of an entry barrier makes entry unprofitable. In other words the entry barrier is necessary and sufficient to prevent the entrant from entering the market if no subsidization occurs. To focus on the cases we are interested in, we hence need to impose some restrictions R on the cost parameters. In the following each of the three restrictions needed are described together with their formal expression:

¹¹The proof of this result can be provided by the author on request.

¹²Stage 1 and stage 2 could be brought together by saying that in stage 1 the government chooses the level of each type of subsidy which can also be zero. That would have no impact on the solution of the game, though.

In the paper stage 1 and 2 are separated because this structure of the game facilitates the analysis.

R 1 *The fixed cost barrier is sufficient to deter entry*

This restriction let us to focus on a situation where the intervention of the government with a sufficiently large subsidy is strictly needed by the entrant in order to enter the market and to make non-negative profits. In other words, the entrant should not be efficient enough to overcome by itself the entry barrier, otherwise there would be no need for subsidization.

R 2 *The fixed cost barrier is necessary to deter entry*

This restriction means that, if there were no entry barrier, both firms would be able to stay in the market producing non-negative quantities. Suppose that this was not the case i.e. that the market sustains only one firm. If the two competing firms cannot coexist in the market, then the comparison between *FCA* and *VCA* is pointless: both the two aid instruments would force the incumbent out if effective in making the entrant to enter the market. Then, regardless of the type of aid instrument implemented, competition would be distorted by an effective subsidy. This case is of marginal interest for the analysis proposed.

R 3 *VCA can be effective in triggering entry*

As the government cannot turn a cost into a benefit, the maximum amount of subsidy which can be granted to the entrant when *VCA* is implemented is $S_c = C_e$. In that case the post-subsidy marginal cost of the entrant would be $C_e^{S_c} = 0$. This restriction simply tells us that setting the subsidy with *VCA* at its maximum level is enough to let the entrant to enter the market and to produce a positive quantity i.e. *VCA* can be effective. If that was not the case, there would be no reason to compare *FCA* with *VCA*, because the only state aid instrument that could be implemented by the government would be *FCA*.

Restrictions R1 - R3 are formally summarized in the following table:

	<i>formulation</i>	<i>restriction</i>	<i>restriction with $C_i = \frac{1}{2}$</i>
R 1	$\pi_e < 0$	$\frac{1+C_i}{2} - \frac{3}{2}\sqrt{K} < C_e$	$\frac{3}{4} - \frac{3}{2}\sqrt{K} < C_e$
R 2	$q_e > 0$ if $K = 0$ $q_i > 0$ if $K = 0$	$C_e < \frac{1+C_i}{2}$ $C_i < \frac{1+C_e}{2}$	$C_e < \frac{3}{4}$ $C_e > 0$
R 3	$q_e > 0$ if $C_e = 0$	$K < \left(\frac{1+C_i}{3}\right)^2$	$K < \frac{1}{4}$

To understand how each restriction is formally expressed, it is sufficient to notice that in the last stage of the game firm j chooses $q_j = \frac{1+C_{l \neq j}-2C_j}{3}$ if $q_j > 0$ and, with $j = E$, if $\pi_e = \left(\frac{1+C_i-2C_e}{3}\right)^2 - K > 0$.

Let us solve the game by backward induction:

2.1 Stage 4: firms' output choice

In this stage firms in the market compete *à la Cournot* and thus decide how much to produce.

If the entrant has entered the market in the previous stage, equilibrium quantities and price are:

$$q_e = \frac{3-4(C_e-S_c^*)}{6}, \quad q_i = \frac{(C_e-S_c^*)}{3}, \quad Q = \frac{3-2(C_e-S_c^*)}{6}, \quad P = \frac{3+2(C_e-S_c^*)}{6} \quad (2)$$

If the entrant did not enter the market in the previous stage then quantities and price are:

$$q_e^o = 0, \quad q_i^o = Q^o = \frac{1}{4}, \quad P^o = \frac{3}{4} \quad (3)$$

Notice that $P^o > P$ and $Q^o < Q \forall C_e$.

2.2 Stage 3: the decision of the entrant

In the third stage the entrant decides whether to enter or not. To that purpose, the entrant compares the level of profits if she stays out ($\pi_e = 0$) with the profits given by the quantity produced if she enters the market. By assumption, the entrant enters whenever entry is weakly preferred i.e. when entering the market allow the entrant to make non negative profits. This means that the entrant enters if the following inequality holds:

$$\pi_e(C_e, K, S_c, S_k) = \frac{(1.5 - 2(C_e - S_c))^2}{9} - (K - S_k) \geq 0$$

2.3 Stage 2: the government sets the parameters

In stage two the government sets S_k and S_c . Depending on its choice in stage one, it sets the parameters in order to achieve the highest possible level of welfare given the method chosen (*FCA*, *VCA*, *NI*). First, let us define the threshold values $\bar{K}(C_e)$ and $\bar{C}_e(K)$ which are those values for K and C_e at which by entering the market, the entrant makes non-negative profits:

$$\bar{K}(C_e) := \frac{(1.5 - 2C_e)^2}{9} \quad (4)$$

$$\bar{C}_e(K) := \frac{3 - 6\sqrt{K}}{4} \quad (5)$$

If the government is adopting *FCA* then it has to set S_k s.t. the new entry barrier faced by the entrant $K - S_k$ is below or equal to \bar{K} . On the other hand, if the government is adopting *VCA*, then it has to set S_c s.t. the new marginal cost of the entrant $C_e - S_c$ is lower than or equal to \bar{C}_e . In the following the optimal choice of S_k and S_c made by the government is analyzed.

2.3.1 The optimal choice of S_k when FCA has been chosen

If the government has chosen FCA in the first stage, in the second stage it sets S_k such that domestic welfare is maximized. Let us call $W_k(C_e, S_k, \lambda)$ the welfare function when adopting FCA . The value of the welfare function is given by consumers' surplus CS_k minus the cost of the subsidy λS_k :

$$W_k(C_e, S_k, \lambda) := CS_k - \lambda S_k = \frac{(1.5 - C_e)^2}{18} - \lambda S_k$$

so the government chooses S_k such to:

$$\begin{aligned} & \max_{S_k} W_k(C_e, S_k, \lambda) \\ & s.t. \ K - S_k \leq \bar{K} \end{aligned}$$

the unique solution for the maximization problem is:

$$S_k^* = K - \frac{(1.5 - 2C_e)^2}{9} \quad (6)$$

Not surprisingly the government sets the new entry barrier $K - S_k^* = \bar{K}$. This is due to the absence of any additional effect of a FCA greater than the one needed to at least trigger entry. The government does not have any incentive to set $K - S_k < \bar{K}$. That would increase the cost of the subsidy keeping constant the gain in consumers' surplus associated with entry.

2.3.2 The optimal choice of S_c when VCA has been chosen

Let us define $W_c(C_e, S_c, \lambda)$ as the welfare function to be maximized when VCA has been chosen by the government in the first stage:

$$W_c(C_e, S_c, \lambda) := CS_c - q_e \cdot \lambda S_c = \frac{(1.5 - (C_e - S_c))^2}{18} - \frac{3 - 4(C_e - S_c)}{6} \cdot \lambda S_c$$

so the government chooses S_c such to:

$$\begin{aligned} & \max_{S_c} W_c(C_e, S_c, \lambda) \\ & s.t. \ C_e - S_c \leq \bar{C}_e \end{aligned}$$

The solutions are:

$$\begin{aligned} S_c^* &= C_e - \frac{3(3\lambda-1)+12\lambda C_e}{2(12\lambda-1)} & \text{if } C_e \leq \frac{6\lambda+1-2\sqrt{K}(12\lambda-1)}{8\lambda} = \widehat{C}_e(K, \lambda) \\ S_c^* &= C_e - \frac{3-6\sqrt{K}}{4} & \text{if } C_e > \widehat{C}_e(K, \lambda) \end{aligned} \quad (7)$$

From the solution of the optimization problem we can deduce that depending on the value of C_e with respect to K and λ two kinds of subsidy can be granted. A first type is an aid which is just sufficient to make the entrant to break even. That happens whenever the entrant is inefficient with respect to the entry barrier i.e. when subsidizing entry is relatively costly. A second type of aid consists, instead, in a subsidy which allows the entrant to enter the market

but goes beyond than that by reducing the marginal cost of the entrant even more than what is strictly needed to enter. This latter case arises whenever C_e is low with respect to K and to trigger entry is relatively cheap. In that case the gain in consumers' surplus brought by the entry of a more efficient firm offsets the relatively low additional cost represented by a greater subsidy. This leads us to the following lemma:

Lemma 1 *If the entrant is efficient enough (i.e. $C_e \leq \widehat{C}_e(K, \lambda)$), the amount of subsidy granted through VCA exceeds what is strictly necessary to make the entrant to enter the market.*

Notice that, by the defining a lower and upper bound for C_e , restrictions R1 and R2 implicitly identify those values for K for which just one of the two mentioned variable cost aid types may arise:

$$\begin{aligned} K < \frac{1}{4(12\lambda-1)^2} &\implies S_c^* > C_e - \overline{C}_e \\ \frac{1}{4(12\lambda-1)^2} \leq K \leq \frac{1}{4(6\lambda-1)^2} &\implies S_c^* \geq C_e - \overline{C}_e \\ K > \frac{1}{4(6\lambda-1)^2} &\implies S_c^* = C_e - \overline{C}_e \end{aligned}$$

so for very big (small) values of K the government chooses with certainty the first (second) type of variable cost aid.

Notice, moreover, that the shadow cost of subsidization λ is negatively related to $\widehat{C}_e(K, \lambda)$:

$$\frac{\partial \widehat{C}_e(K, \lambda)}{\partial \lambda} = -\frac{2\sqrt{K} + 1}{8\lambda^2}$$

the more costly is rising funds to finance state aid the less likely is that subsidization would be used by the government as an instrument to increase consumers' surplus through a reduction of the marginal costs of those firms that would anyhow enter the market.

2.4 Stage 1: the government chooses among different types of aid

In the first stage of the game the government compares each possible action and then chooses the one which is associated with the highest level of welfare. Given the optimal choices of the subsidy in stage two, the welfare yielded by *NI*, *FCA* and *VCA* are W_o^* , W_k^* and W_c^* respectively, as it is illustrated in the following table:

<i>NI</i>	$W_o^* = \frac{1}{32}$
<i>FCA</i>	$W_k^* = \frac{(1.5-C_e)^2}{18} - \lambda \left(K - \frac{(1.5-2C_e)^2}{9} \right)$
<i>VCA</i>	$W_c^* = \frac{\lambda^2(4C_e-9)^2}{8(12\lambda-1)^2} - \lambda \left(\frac{6\lambda+1-8\lambda C_e}{2(12\lambda-1)} \right) \left(C_e - \frac{3(3\lambda-1)+12\lambda C_e}{2(12\lambda-1)} \right)$ with $C_e \leq \widehat{C}_e(K, \lambda)$ $W_c^* = \frac{1}{2} \left(\frac{1+2\sqrt{K}}{4} \right)^2 - \lambda\sqrt{K} \left(C_e - \frac{3-6\sqrt{K}}{4} \right)$ with $C_e > \widehat{C}_e(K, \lambda)$

Let us compare the three options two-by-two:

- *FCA vs NI*

FCA is preferred to *NI* if:

$$W_k^*(C_e, K, \lambda) - W_o^*(C_e, K, \lambda) \geq 0$$

The welfare yielded by *NI* is simply the consumers' surplus when only the incumbent is in the market. The difference with the welfare yielded by *FCA* is then made up of two opposite effects: a gain in the consumers' surplus due to an increase in competition in the market and a loss in terms of the public resources needed to finance the aid. With the state's intervention the gain in consumers' surplus is always non-negative¹³. Solving for K we get that:

FCA \succ *NI* iff

$$K \leq \frac{16C_e^2(8\lambda + 1) - 48C_e(4\lambda + 1) + 9(8\lambda + 3)}{288\lambda} = \sigma(C_e, \lambda) \quad (8)$$

Whenever the entry barrier is lower than $\sigma(C_e, \lambda)$, the government prefers *FCA* to *NI*. This threshold is negatively related with λ and C_e in the interval defined by restrictions R2 i.e. whenever $0 < C_e < \frac{3}{4}$. Indeed, the less efficient the entrant, the more difficult for *FCA* to be preferred to *NI*: the gain in the consumers' surplus is higher and pulling down the entry barrier is cheaper if the entrant is relatively more efficient.

- *VCA vs NI*

Now let us compare the welfare yielded by *VCA* with the welfare yielded by *NI*:

$$\begin{aligned} W_{\widehat{C}}^*(C_e, K, \lambda) - W_o^*(C_e, K, \lambda) &\geq 0 \quad \text{with } C_e \leq \widehat{C}_e(K, \lambda) \\ W_{\widehat{C}}^*(C_e, K, \lambda) - W_o^*(C_e, K, \lambda) &\geq 0 \quad \text{with } C_e > \widehat{C}_e(K, \lambda) \end{aligned} \quad (9)$$

Concerning the first inequality, it is easy to see that the left hand side is convex in C_e and has a minimum in $C_e = \frac{6\lambda+1}{8\lambda}$ where it is equal to zero. So the government always prefers *VCA* to *NI* when the entrant is efficient enough, such that the government wishes to grant a subsidy higher than what is strictly necessary to make the entrant to break even.

Concerning the second inequality, solving for K we get:

VCA \succ *NI* iff

$$K \leq \left(\frac{6\lambda + 1 - 8\lambda C_e}{12\lambda - 1} \right)^2 = \tau(C_e, \lambda) \quad (10)$$

notice that $\tau(C_e, \lambda)$ is negatively related with C_e and with λ as in the case of $\sigma(C_e, \lambda)$.

In proof n. 1 in the appendix it is shown that threshold $\tau(C_e, \lambda)$ is always greater than threshold $\sigma(C_e, \lambda)$. That implies the following remark:

¹³Since $CS_k(C_e)$ is decreasing in C_e , to show that $CS_k(C_e) \geq CS_o$ it is sufficient to set C_e at its maximum value $C_e = \frac{3}{4}$ and notice that at that value $CS_k(\frac{3}{4}) = CS_o$

Remark 2 for any value of the parameters if *FCA* is preferred to *NI* then *VCA* is preferred to *NI*. On the other hand, if *NI* is preferred to *VCA*, then *NI* is preferred to *FCA* too.

- *VCA vs FCA*

Finally, let us compare *VCA* with *FCA*:

$$\begin{aligned} W_{\widehat{c}}^*(C_e, K, \lambda) - W_k^*(C_e, K, \lambda) &\geq 0 \quad \text{with } C_e \leq \widehat{C}_e(K, \lambda) \\ W_{\widehat{c}}^*(C_e, K, \lambda) - W_k^*(C_e, K, \lambda) &\geq 0 \quad \text{with } C_e > \widehat{C}_e(K, \lambda) \end{aligned}$$

As in the case of *VCA vs NI*, the first inequality (which refers to the case of an aid greater than what is strictly necessary to trigger entry) always holds [see proof n. 2 in the appendix]. That means that when the entrant is very efficient (but still not so efficient to overcome by herself the entry barrier) the first best choice of the government is *VCA*.

Lemma 3 If the entrant is very efficient compared to the entry barrier (i.e. $C_e \leq \widehat{C}_e(K, \lambda)$) the first best choice of the government in stage 1 is *VCA*.

Regarding the second inequality (*VCA* makes the entrant to break even), it might be useful to split the analysis in two parts to understand how the two possible choices of the government influence welfare through a gain in the consumers' surplus and a loss in public resources.

Concerning the first issue, we have that $CS_c > CS_k$ always:

Remark 4 the gain in consumer surplus yielded by *VCA* at the equilibrium is always superior to that yielded by *FCA*.

This can easily be seen:

$$\begin{aligned} CS_c = \frac{1}{2} \left(\frac{1.5 - (C_e - S_c^*)}{3} \right)^2 &> CS_k = \frac{1}{2} \left(\frac{1.5 - C_e}{3} \right)^2 \\ (C_e - S_c^*) &< C_e \end{aligned}$$

The reason is that while *FCA* raises consumers' surplus only through competition, *VCA* has the same effect plus an additional positive effect given by the increase in the efficiency of one of the two firms competing in the market.

For the second aspect - loss in public resources due to the subsidy - the analysis is less straightforward. As figure 2 shows, while the cost of providing *VCA* increases linearly with C_e , the cost of providing *FCA* is concave in C_e , so that the bigger is C_e the higher is the difference between the two cost functions. To understand why that is the case, notice that with *VCA* the equilibrium post-aid entrant's marginal cost is always the same and equal to $\widehat{C}_e(K)$, even if C_e

is increasing. On the other hand, with *FCA* the equilibrium post-aid entrant's marginal cost increases with C_e , because the aid leaves it unaltered, and thus the shape of the *FCA*'s cost function reflects (with negative sign) that of firm's profits (which are convex in C_e since firms compete à la Cournot).

insert figure 2

To see it algebraically, let $\Delta(C_e, K, \lambda)$ be a positive function of the difference between the total cost of subsidizing E with *VCA* minus the total cost of subsidizing E through *FCA*:

$$\Delta(C_e, K, \lambda) := \frac{q_e \cdot S_c^* - S_k^*}{\lambda} = \sqrt{K} \left(C_e - \frac{3 - 6\sqrt{K}}{4} \right) - \left(K - \frac{(1.5 - 2C_e)^2}{9} \right)$$

we can then differentiate Δ for C_e :

$$d\Delta = \sqrt{K} dC_e - \left(\frac{2}{3} - \frac{4}{9} C_e \right) dC_e$$

and notice that for any $C_e > \frac{3}{2} - \frac{9}{4}\sqrt{K}$, $dC_e > 0 \implies d\Delta > 0$.

So when the entrant is relatively inefficient *VCA* costs more than *FCA*. The total effect of the aid is however unclear because of the larger positive effect of *VCA* on consumers' surplus has to be taken into account. A corollary of that result is that, when the entrant is relatively inefficient, a greater K (i.e. an increase in the relative inefficiency of the entrant with respect to the entry barrier) makes the government more likely to prefer *FCA* with respect to *VCA*.

Consistently, *VCA* is preferred to *FCA* whenever K is *below* the following threshold:

$$K \leq \frac{(3(8\lambda + 3) - 4C_e(8\lambda + 1))^2}{36(4\lambda - 1)^2} = \psi(C_e, \lambda) \tag{11}$$

Notice that the relationship between $\psi(C_e, \lambda)$ and λ is negative. Indeed:

$$\frac{\partial \psi(C_e, \lambda)}{\partial \lambda} = \frac{2(4C_e - 5)(4C_e(8\lambda + 1) - 3(8\lambda + 3))}{3(1 - 4\lambda)^3} < 0$$

which means that an increase in the shadow cost of subsidy reduces *VCA*'s comparative advantage with respect to *FCA*. The explanation is rather straightforward: we have seen that when the entrant is inefficient, an increase in its marginal cost determines an increase in the (positive) difference between *VCA*'s and *FCA*'s granting costs. An increase in λ then magnifies the relative burden that choosing *VCA* entails with respect to choosing *FCA*. As we can infer from the following lemma, however, even if λ is bigger, *FCA* does not arise at the equilibrium because higher values of λ also mean that subsidy in whatsoever shape is not welfare enhancing (see figures 3 and 4 for an illustration when C_e and λ are hold constant). Indeed, proof n. 3 in the appendix shows that $\psi(C_e, \lambda)$ is always greater than $\tau(C_e, \lambda)$ and $\sigma(C_e, \lambda)$.

insert figures 3, 4

This leads to the following lemma:

Lemma 5 *whenever FCA is preferred to VCA, NI is preferred to FCA.*

We can thus conclude that *FCA* never arises at the equilibrium. The intuition behind this result is the following: for *FCA* to be preferred to *VCA* when the entrant is inefficient ($C_e > \widehat{C}_e(K, \lambda)$) it is necessary that the entry barrier is relatively big. That happens because the relative cost of the subsidy with *VCA* with respect to *FCA* increases with C_e when C_e is high enough. In that case, however, financing entry is too costly if compared to the benefits obtained in terms of surplus, even if a *FCA* is granted. Then the only two options which might arise at the equilibrium are *VCA*, if K or C_e are relatively small, or *NI* in the opposite case.

Eventually, the equilibrium choice of the government are:

$$\left\| \begin{array}{ll} \text{adopt FCA} & \text{never} \\ \text{adopt VCA} & \text{if } K \leq \tau(C_e, \lambda) \\ \text{not intervene} & \text{if } K > \tau(C_e, \lambda) \end{array} \right\|$$

Since at equilibrium the government never chooses to grant *FCA*, the model may appear to be in contrast with the Ryanair - Charleroi's facts, where both kinds of aids were chosen. This result however depends on the no mix-form aid assumption: were the government in the model be able to lower both K and C_e at the same time then both *FCA* and *VCA* could have arisen at the equilibrium. Nevertheless, even if we allow for mixed forms of aid, the welfare loss when *VCA* is banned would be even higher, thus strengthening the conclusions which are discussed below¹⁴. Another possible explanation for observing *FCA* is simply that the government might have anticipated that the likelihood of a negative decision from the European Commission is higher when *VCA* is implemented. Indeed, there exists an interval of values for the parameters where if the government cannot grant a *VCA* it grants a *FCA*, reaching a second best equilibrium. We are going to address that issue in the following paragraph.

2.5 The Equilibrium and the European Commission's Approach

insert figures 5,6

Appendix B shows that a rational forward-looking government never grants a state aid such that the incumbent firm is crowded out from the market. At the same time, as we have previously seen, for sufficiently low values of K and C_e , granting a *VCA* to an entrant firm is a first best solution for maximizing domestic welfare. It turns out that a specific competition policy which allows *FCA* but bans *VCA* may lead to sub-optimal equilibria where domestic welfare is not maximized.

¹⁴Suppose that for $S_c \geq \widehat{S}_c$ the incumbent firm is forced to exit the market. Appendix B shows that a pure form *VCA* is never granted when it implies a crowding out effect. So $S_c < \widehat{S}_c \forall C_i, C_e, K$. Now, if both S_c and S_k can be different from zero (i.e. mix-form aid is allowed) clearly S_c would still be lower than \widehat{S}_c because an increase in S_k does not alter the relative positions of firms. At the same time, as the pure form aid choices are a subset of the mix form aid choices and given the assumption of perfect rationality of the government, the welfare yielded by a mix form aid is always greater than the welfare yielded by pure form aid.

Figures 5 and 6 report the government's optimal choice for given variable cost parameters (the second-best choices when *VCA* cannot be chosen are in brackets). As expected, in both figures the greater are C_e and K , the wider is the area of no-intervention.

Figure 5 shows the equilibrium government's choices when *VCA* is allowed while figure 6 shows the government's choices when *VCA* is banned. As can be seen in the figures, if the European Commission allows *VCA*, the entrant enters the market through the government's help for the set of combinations for C_e and K which pick out points below $\tau(C_e, K)$. On the contrary, when *VCA* is not allowed, only those points lying below $\sigma(C_e, K)$ lead to equilibria where two firms compete in the market. Moreover, proof 1 actually shows that $\tau(C_e, \lambda) - \sigma(C_e, \lambda)$ is never empty. So a *VCA* banning-policy can never be optimal for every feasible size of the entry barrier in the interval identified by R1-R3.

How much detrimental for welfare can the European Commission's policy be? As a limit case, assume that lump sum taxes are feasible ($\lambda = 1$), that K is such that $W_k^* = W_o^* = \frac{1}{32}$ and that C_e is very close to its lowest feasible value given the previous conditions¹⁵ (this in turn means that K is very close to its upper bound defined by R3).

We thus have the following values for the parameters:

$$\begin{aligned} K &= 0.2499 \\ C_e &= 0.1214 \end{aligned}$$

and welfare is:

$$\begin{aligned} W_k^* &= W_o^* = 0.03125 \\ W_c^* &= 0.06432 \approx 2 \times W_k^* \end{aligned}$$

with those values for the parameters, allowing the government to grant *VCA* implies to double domestic welfare. If the European Competition Authority has the same objective function of the government (i.e. it adopts a consumers' surplus standard) and bans *VCA*, it might generate a loss of potential gain in welfare of up to 100% with respect to a more permissive policy. These results thus cast doubts on the opportunity of an a priori general ban of *VCA*.

In the following section, two extensions to the basic setting are considered: in the first one I allow for n firms to play the game; in the second one the same game is played by a domestic incumbent firm and a foreign entrant firm.

3 Extensions

3.1 More than one incumbent firm

As the marginal gain in consumers' surplus associated with entry is expected to be lower when more than two firms are playing the game, the results that have been shown above might change if we increase the number of playing firms. In this subsection I show that this is not the case.

¹⁵It is easy to see that at these conditions the the gain in welfare yielded by *VCA* with respect to the other two options is maximum.

Let us suppose that in the domestic market $(n - 1) > 1$ symmetric firms are operating producing $q_i > 0$ and making positive profits. Their marginal cost is $c = \frac{1}{2}$. An entrant firm E with marginal cost C_e may enter the market paying K .

Restrictions R1-R3 are then modified accordingly:

	<i>formulation</i>	<i>restriction</i>
R 1	$\pi_e < 0$	$\frac{1+n}{2n} - \frac{1+n}{n}\sqrt{K} < C_e$
R 2	$q_e > 0$ if $K = 0$ $q_i > 0$ if $K = 0$	$C_e < \frac{1+n}{2n}$ $C_e > 0$
R 3	$q_e > 0$ if $C_e = 0$	$K < \frac{1}{4}$

Let us playing the game. If entry does not occur, in the last stage we have:

$$\begin{aligned} q_i &= \frac{1}{2n} \forall i \\ Q &= \frac{(n-1)}{2n} \\ P &= \frac{n+1}{2n} \\ CS &= \frac{(n-1)^2}{8n^2} \end{aligned}$$

If entry occurs, instead, we have:

$$\begin{aligned} q_i &= \frac{(C_e - S_c)}{1+n} \forall i \\ q_e &= \frac{1 - 2n(C_e - S_c) + n}{2(1+n)} \\ Q &= \frac{1+n - 2(C_e - S_c)}{2(1+n)} \\ P &= \frac{1+n + 2(C_e - S_c)}{2(1+n)} \\ CS &= \frac{(1+n - 2(C_e - S_c))^2}{8(1+n)^2} \end{aligned}$$

In stage 3, entry decision depends on E 's profits. Entry then occurs whenever:

$$\pi_e = \frac{1 - 2n(C_e - S_c) + n}{2(1+n)} \left(\frac{1+n + 2(C_e - S_c)}{2(1+n)} - (C_e - S_c) \right) - (K - S_k) \geq 0$$

which leads to the following threshold levels for entry:

$$\begin{aligned} \overline{K}^n &:= \frac{(1 - 2nC_e + n)^2}{4(1+n)^2} \\ \overline{C}_e^n &:= \frac{(n+1)(1 - 2\sqrt{K})}{2n} \end{aligned}$$

When in stage 2, the government faces an optimization problem depending on the choice made in stage 1. If *FCA* has been chosen, the optimal choice of S_k is, as usual, the one that makes the entrant firm to break even:

$$S_k^{n*} = K - \frac{(1 - 2nC_e + n)^2}{4(1+n)^2}$$

If *VCA* has been chosen, then the government faces the following maximization problem:

$$\max_{S_c} W_c^n(C_e, S_c, \lambda, n) = \frac{(1+n-2(C_e-S_c^n))^2}{8(1+n)^2} - \frac{1-2n(C_e-S_c^n)+n}{2(1+n)} \lambda S_c^n$$

$$s.t. C_e - S_c^n \leq \overline{C_e^n}$$

the optimal choice of the government is then:

$$S_c^{n*} = C_e - \frac{(n+1)(2n\lambda C_e + n\lambda + \lambda - 1)}{2(2n^2\lambda + 2n\lambda - 1)} \quad \text{if } C_e \leq \frac{n^2\lambda + n(\lambda+1) - 1 - 2\sqrt{K}(2n^2\lambda + 2n\lambda - 1)}{2n^2\lambda} = \widehat{C_e^n}(K, \lambda, n)$$

$$S_c^{n*} = C_e - \frac{(n+1)(1-2\sqrt{K})}{2n} \quad \text{if } C_e > \widehat{C_e^n}(K, \lambda, n)$$

the three welfare levels which can be reached with each of the three options, *NI*, *FCA*, *VCA*, are:

<i>NI</i>	$W_o^{n*} = \frac{(n-1)^2}{8n^2}$
<i>FCA</i>	$W_k^{n*} = \frac{(1+n-2C_e)^2}{8(1+n)^2} - \lambda \left(K - \frac{(1-2nC_e+n)^2}{4(1+n)^2} \right)$
<i>VCA</i>	$W_c^{n*} = \frac{\lambda(4C_e^2n^2\lambda - 4C_en^2\lambda + n(\lambda+1) - 1) + n^2(\lambda+2) + 2n\lambda + \lambda - 2}{8(2n^2\lambda + 2n\lambda - 1)} \quad \text{with } C_e \leq \widehat{C_e^n}(K, \lambda, n)$ $W_c^{n*} = \frac{4\sqrt{K}(n^2\lambda + n(\lambda+1) - 1) + n^2 - 2n + 1 - 8C_e\sqrt{K}n^2\lambda - 4K(2n^2\lambda + 2n\lambda - 1)}{8n^2} \quad \text{with } C_e > \widehat{C_e^n}(K, \lambda, n)$

Finally, in stage 1, the government chooses one of the three options, and the following thresholds can be shown to hold when comparing the different levels of welfare:

- *FCA vs NI*

FCA \succ *NI* iff

$$K \leq \frac{4C_e^2n^2(2n^2\lambda + 1) - 4C_en^2(n+1)(2n\lambda + 1) + (n+1)^2(2n^2\lambda + 2n - 1)}{8n^2\lambda(n+1)^2} = \sigma^n(C_e, \lambda, n)$$

- *VCA vs NI*

once again, if S_c^{n*} is greater than what is strictly needed to make the entrant firm to break even, then *VCA* is always better than *NI* [see proof n. 4 in the appendix]

in the case in which $C_e - S_c^{n*} = \overline{C_e^n}$ (i.e. the government grants a *VCA* such that the entrant breaks even), *VCA* \succ *NI* if the following threshold holds:

$$K \leq \left(\frac{n^2\lambda + n(\lambda+1) - 1 - 2C_en^2\lambda}{2n^2\lambda + 2n\lambda - 1} \right)^2 = \tau^n(C_e, \lambda, n)$$

Consistently with the $n = 2$ case, proof n. 5 shows that $\tau^n(C_e, \lambda, n) > \sigma^n(C_e, \lambda, n)$.

- *VCA vs FCA*

if S_e^{n*} is greater than what is strictly needed to make the entrant firm to break even, *VCA* is preferred to *FCA* if:

$$K > \frac{4C_e^2(3n^4\lambda^2 + 2n^3\lambda^2 - n^2\lambda^2 + 2n\lambda - 1) - 4C_e(n+1)(3n^3\lambda^2 + n^2\lambda(2\lambda + 1) - n\lambda^2 + \lambda - 1)}{8\lambda(n+1)^2(2n^2\lambda + 2n\lambda - 1)} +$$

$$+ \frac{(n+1)^2(3n^2\lambda^2 + 2n\lambda(\lambda + 1) - \lambda^2 - 1)}{8\lambda(n+1)^2(2n^2\lambda + 2n\lambda - 1)} = \phi^n(C_e, \lambda, n)$$

as in the case where $n = 2$, restriction R1 implies $K > \phi^n \forall C_e, n, \lambda$ which means that in that case *VCA* is always preferred to *FCA* [see proof n. 6 in the appendix].

If, instead, $C_e - S_e^{n*} = \overline{C_e}^n$, *VCA* \succ *FCA* if the following threshold holds:

$$K > \left(\frac{2n^3\lambda + 2n^2(\lambda + 1) + n - 1 - 2C_en(2n^2\lambda + 1)}{2(n+1)(2n\lambda - 1)} \right)^2 = \psi^n(C_e, \lambda, n)$$

as in the $n = 2$ case, it is possible to show that $\psi^n(C_e, \lambda, n) > \tau^n(C_e, \lambda, n) > \sigma^n(C_e, \lambda, n)$ [see proof n.7 in the appendix].

The equilibrium results are then the following:

$$\left\| \begin{array}{ll} \text{adopt FCA} & \text{never} \\ \text{adopt VCA} & \text{if } C_e < \tau^n(C_e, \lambda, n) \\ \text{not intervene} & \text{if } C_e > \tau^n(C_e, \lambda, n) \end{array} \right\|$$

The result of the basic setting are then robust to the generalization to n firms. Proof n. 8 in the appendix moreover shows that in the interval defined by R2 the following inequality holds:

$$\frac{\partial \tau^n(C_e, \lambda, n)}{\partial n} < 0$$

so when the number of incumbents increases, the condition for subsidization becomes stricter (i.e. state aid is less likely). This happens because the marginal positive effect of entry on consumers' surplus is reduced as n becomes larger. Nevertheless, restrictions R1 and R2 are 'elastic' with respect to n : they become less strict when n becomes larger. As a consequence, there is always an interval of values for K such that *VCA* is preferred to *NI* and in which a general ban of *VCA* would lead to sub-optimal equilibria (although that interval becomes shorter the bigger is n : at the limit case, with $n \rightarrow \infty$, the interval is empty).

3.2 The incumbent is domestic

Coming back to the $n = 2$ setting, one might wonder whether including the incumbent's profits in the objective function of the government would change the results of the basic model. Indeed, if the incumbent firm is domestic, the welfare function maximized by the government is given by:

$$W(C_e, K, \lambda) = CS(C_e) + \pi_i(C_e) - \lambda S(C_e, K)$$

where π_i are the incumbent's profits.

As the players' equilibrium choices in stage 4 and 3 do not change with respect to the case of two foreign firms, let us focus on stage 1 and 2.

According to the entry decision welfare is:

$$\begin{array}{ll} E \text{ did not enter the market} & W = \frac{3}{32} \\ E \text{ entered the market} & W = \frac{(1.5 - (C_e - S_c))^2}{18} + \frac{(C_e - S_c)^2}{9} - \frac{3 - 4(C_e - S_c)}{6} \cdot \lambda S_c - \lambda S_k \end{array}$$

When in stage 2, the fact that incumbent's profits are now included in the welfare function maximized by the government distorts its choice concerning the amount of *VCA* to be granted. Indeed, it is sufficient to notice that when $\lambda = 1$ (i.e. when subsidy is least expensive), maximizing over S_c leads to the following result:

$$S_c = \frac{C_e - 1}{3} < 0$$

that means that if the incumbent's profits are included in the welfare function, the optimal level of variable subsidy chosen by the government is the one that makes the entrant firm to break even. Indeed, the government internalizes the negative effect that S_c has on the incumbent's profits by reducing her competitor's marginal cost. It turns out that whatever is the level of C_e , the optimal choice of S_c when *VCA* is chosen is given by the following expression (as in the case of foreign incumbent when $C_e > \widehat{C}_e(K, \lambda)$):

$$S_c^* = C_e - \frac{3 - 6\sqrt{K}}{4}$$

The following table represents the government's options in stage 1 with the associated welfare levels:

<i>NI</i>	$W_o^{d*} = \frac{1}{32} + \frac{1}{16}$
<i>FCA</i>	$W_k^{d*} = \frac{(1.5 - C_e)^2}{18} - \lambda \left(K - \frac{(1.5 - 2C_e)^2}{9} \right) + \frac{C_e^2}{9}$
<i>VCA</i>	$W_c^{d*} = \frac{1}{2} \left(\frac{1 + 2\sqrt{K}}{4} \right)^2 - \lambda \sqrt{K} \left(C_e - \frac{3 - 6\sqrt{K}}{4} \right) + \left(\frac{3 - 6\sqrt{K}}{12} \right)^2$

where the last term in each equation are the incumbent's profits¹⁶.

¹⁶Notice that, given R1 and R2, $\left(\frac{3 - 6\sqrt{K}}{12} \right)^2 \leq \frac{C_e^2}{9} \leq \frac{1}{16}$ i.e. the incumbent has the following order of preferences concerning a subsidy to be granted to her competitor: $NI \succ FCA \succ VCA$.

Notice that in this setting, even if omit to consider subsidy's cost, *VCA* can be inferior to *NI* if the marginal cost of the entrant is sufficiently high. Indeed, it is easy checking that for $C_e > \frac{1}{4}$ domestic welfare is reduced with entry even if the government does not sustain any cost in order to trigger entry (i.e. $K = 0$). The loss in terms of incumbent profits is indeed higher than the gain in consumers' surplus.

If we compare the three choices two-by-two, we get the following results:

- *FCA vs NI*

$$FCA \succ NI \text{ iff:}$$

$$K < \frac{16C_e^2(8\lambda + 3) - 48C_e(4\lambda + 1) + 9(8\lambda + 1)}{288\lambda} = \sigma^d(C_e, \lambda)$$

- *VCA vs NI*

$$VCA \succ NI \text{ iff:}$$

$$K < \left(\frac{6\lambda - 1 - 8\lambda C_e}{3(4\lambda - 1)} \right)^2 = \tau^d(C_e, \lambda) \cap C_e < \frac{6\lambda - 1}{8\lambda}$$

- *VCA vs FCA*

$$VCA \succ FCA \text{ iff:}$$

$$K < \frac{(4C_e(8\lambda + 3) - 3(8\lambda + 1))^2}{36(4\lambda - 3)^2} = \psi^d(C_e, \lambda) \cap C_e < \frac{3(8\lambda + 1)}{4(8\lambda + 3)}$$

For *VCA* to be preferred, both K and C_e must be low: the bigger is λ the more likely is that *VCA* is worst with respect to *NI* or *FCA* whatever is the value of the entry barrier. Proof n. 9 in the appendix moreover shows that:

$$C_e > \frac{3(8\lambda + 1)}{4(8\lambda + 3)} \implies K > \sigma^d(C_e, \lambda)$$

that means that if C_e is sufficiently high, neither of the two *FCA* and *VCA* can arise at the equilibrium, independently of the entry barrier.

With respect to the foreign incumbent case, however, *FCA* can now arise at the equilibrium, whenever $\psi^d(C_e, \lambda) < K < \sigma^d(C_e, \lambda)$. A necessary condition for that to be possible is that $\psi^d(C_e, \lambda) < \sigma^d(C_e, \lambda)$, which happens whenever:

$$\frac{48\lambda^2 + 32\lambda - 9}{4(16\lambda^2 + 16\lambda - 3)} < C_e < \frac{3(8\lambda + 1)}{4(8\lambda + 3)}$$

Although this interval for C_e is very small¹⁷, we can conclude that *FCA* may arise at the equilibrium when the incumbent firm is domestic.

On the other hand, contrary to the case of two competing foreign firms, the following lemma holds:

¹⁷think for example at the case in which $\lambda = 1$. In that case the interval defined by that condition is:

$$0.61207 < C_e < 0.61364$$

Lemma 6 *when the incumbent is domestic, VCA is never chosen by the government.*

To see why, notice that a necessary condition for *VCA* to be preferred is that it contemporaneously yields an higher level of welfare with respect to both *NI* and *FCA*. That implies the following necessary (but not sufficient) conditions, according to the thresholds defined above:

$$C_e < \frac{6\lambda - 1}{8\lambda}$$

$$K < \psi^d(C_e, \lambda)$$

R1 implies however $K > \psi^d(C_e, \lambda)$ whenever $C_e < \frac{6\lambda - 1}{8\lambda}$ [proof n. 10 in the appendix].

The conclusion is that when the incumbent is domestic, state intervention is less likely and that if eventually the government decides to subsidize the entrant firm, it will use *FCA* rather than *VCA*. Indeed, the following strategy is implemented by the government at the equilibrium:

$$\left\| \begin{array}{ll} \text{adopt } FCA & \text{if } K < \sigma^d(C_e, \lambda) \\ \text{adopt } VCA & \text{never} \\ \text{not intervene} & \text{if } K > \sigma^d(C_e, \lambda) \end{array} \right\|$$

The overall effect of considering domestic firm's profits in the welfare function maximized by the government is a downward shift of the threshold for non intervention. To see that, it is sufficient to compare the two equilibria in the two different settings:

	<i>I is foreign</i>	<i>I is domestic</i>
<i>subsidize E</i>	$K < \tau(C_e, \lambda)$	$K < \sigma^d(C_e, \lambda)$
<i>not subsidize E</i>	$K > \tau(C_e, \lambda)$	$K > \sigma^d(C_e, \lambda)$

suppose, for simplicity¹⁸, that $\lambda = 1$. We thus have:

$$\tau(C_e, 1) - \sigma^d(C_e, 1) = \frac{4311 - 2864C_e^2 - 3216C_e}{34848} > 0$$

where the above inequality holds because by R2 $C_e < \frac{3}{4}$. Not surprisingly, the lower is C_e the higher is the difference between $\tau(C_e, 1)$ and $\sigma^d(C_e, 1)$: the more efficient is the entrant the more reluctant would the government be to let her entering in the market if the incumbent is domestic instead of foreigner.

In conclusion, we can state that a general ban of *VCA* when the incumbent is domestic does not lead to second best equilibria, since it does not impact the government's choice.

¹⁸It is nevertheless possible to show that as λ increases, $\sigma^d(C_e, \lambda)$ becomes even smaller with respect to $\tau(C_e, \lambda)$, in the interval of possible values for C_e .

3.2.1 Consumers' vs Total Welfare Approach

The results obtained when the incumbent is domestic have an interesting consequence on the policy implications that the basic model has if we assume the Competition Commission to adopt a total welfare standard instead of a consumers' welfare standard. In that case, in fact, even if both E and I are foreigners with respect to the domestic government, their profit functions must be included in the objective function maximized by the Antitrust Authority (assuming that both I and E belong to the same integrated market i.e. the EU). However, when the entrant is inefficient enough ($C_e > \widehat{C}_e(K, \lambda)$), we have seen that the government sets both FCA and VCA in order to trigger entry but making the entrant to break even. Indeed, if $C_e > \widehat{C}_e(K, \lambda)$, the entrant's profits are always 0, whatever is the option chosen by the government in stage 1. It turns out that the objective function maximized by an Antitrust Authority which adopts a total welfare standard coincides with that maximized by the government when the incumbent is domestic. Since in that case VCA is never superior to both FCA and NI , we can conclude that a general ban of VCA has no impact on total welfare and would be consistent with the objectives of an Antitrust Authority maximizing total welfare. The same conclusion is reached when the entrant is relatively efficient ($C_e \leq \widehat{C}_e(K, \lambda)$), as proof. 11 shows. If the two firms are foreigners, moreover, the government and the Antitrust Authority (which now is assumed to adopt a total welfare standard) maximize different objective functions. In that case a ban of VCA is desirable from the point of view of the Antitrust Authority because it prevents the government from choosing an aid instrument which would increase domestic welfare but decrease total welfare (which includes incumbent's profits as well).

Proposition 7 *If the Antitrust Authority adopts a total welfare standard, then a general ban of VCA is desirable whenever the government and the Antitrust Authority maximize different objective functions.*

4 Conclusions

This paper addresses the economic grounds of the European Commission's approach to state aid to attract foreign investment. In particular, it sheds light on a well-established policy of the Commission according to which state aid aimed to reduce variable cost of production (VCA or *operating* aid, in the terminology used by the Commission) is more distortive than state aid aimed to reduce fixed cost of entry (FCA or *start-up* aid).

In the basic setting of the model, two foreign firms are playing the game: one incumbent firm I already present in the domestic market and one entrant firm E which is unable to enter the market without the help of the domestic government.

Solving the game led us to the following conclusions: if the minimum VCA necessary to make the entrant to break even and entering the market does not cause to the incumbent firm to exit the market, then the only type of aid which can occur at the equilibrium is VCA . That conclusion holds independently of the shadow cost of subsidy λ , although an increase in it reduces the comparative advantage of VCA with respect to FCA (when the entrant is

inefficient enough). The reason why a positive change in λ does not impact that result is that while *VCA* loses its appeal with respect to *FCA*, at the same time government's intervention likelihood is lowered as well, so that neither *FCA* or *VCA* can arise at the equilibrium. This result thus differs with the recent literature on state aid in which the shadow cost of subsidy plays a determinant role (see, for example, Besley and Seabright [1999] or Collie [2000, 2002]).

On the other hand, if the minimum amount of *VCA* necessary to let the entrant to enter the market is sufficient to force the incumbent out, then *VCA* is never granted by the government. In other words, a rational, forward-looking government never grants an aid such to give to the more efficient aid-endowed entrant firm the ability to crowd her competitor out from the market.

The same results are obtained when the basic setting is expanded in order to allow for positive externalities in the domestic economy given by FDI and for a number n of firms playing the game. In the latter case, an increase in the number of incumbents decreases state aid's likelihood, since the marginal contribution of entry to consumers' surplus is reduced. Whatever is the number of playing firms, however, *VCA* can always occur at the equilibrium.

The results mentioned above are not robust to the case of domestic (instead of foreign) incumbent firm, though. If the objective function of the government includes incumbent's profits, then two results are obtained: first, the government is much less likely to subsidize entry. Second, *VCA* never occurs and the only type of aid which can occur at the equilibrium is *FCA*. The reason why this happens is rather obvious: by including the incumbent's profits in its objective function, the government internalizes the negative effect of entry on the domestic competitor. The model shows that this negative effect is not offset by the potential gain in consumers' surplus of a more efficient entrant, thus identifying *FCA* as the unique type of aid instrument which can be chosen by the government, when definite conditions for the parameters hold.

Given these findings, the model shows that a general ban of *VCA* can be harmful for welfare from a consumers' surplus perspective. Indeed, if we stick to the basic setting, in the model is shown that there exists an interval of values for K and C_e in which two firms are producing positive quantities in the market if *VCA* is allowed while just one if *VCA* is banned. This is the interval of values where *VCA* is preferred to *NI* that is itself preferred to *FCA*. So banning *VCA* may lead to second best equilibria where both welfare and competition are lower with respect to what they could be if there were no ban.

The primary message of the model is thus that an Antitrust Authority should not apply a general a priori rule that discriminates between operative aid and start-up aid. More precisely, the model casts doubts about the validity of the 'state aid instrument' argument supporting a Commission's decision. There might be several reasons why a *VCA* should not be allowed: bounded rationality of the government or lobbying are examples. But the mere fact of using one state aid instrument instead of another should not be a discriminant for accepting or rejecting the state aid programme: further economic analysis is needed in order for such Commission's decisions to be fully legitimated.

Finally, this paper belongs to a stream of literature, competition economics and state aid, which is still at its infancy. The model adopts a consumers' welfare approach and addresses the validity of the European Commission's arguments concerning the use of different state

aid instruments. As this approach is almost new to the literature, further research should be undertaken in order to have a full understanding of the topic. It could be of interest, for example, to address the same issue from a political economy point of view and try to account for lobbying issues which might be one of the main reasons underlying the European Commission's worries for *VCA*. I plan to address that issue in a new research project.

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A Proofs

Proof n. 1. We need to show that $\tau(C_e, \lambda) > \sigma(C_e, \lambda)$.

Let

$$F^{\tau-\sigma}(C_e, \lambda) := (\tau(C_e, \lambda) - \sigma(C_e, \lambda))$$

notice that, given $\lambda \geq 1$, $F^{\tau-\sigma}(C_e, \lambda)$ is convex in C_e :

$$\frac{\partial^2 F^{\tau-\sigma}(C_e, \lambda)}{\partial C_e^2} = \frac{48\lambda^2 + 16\lambda - 1}{9\lambda(12\lambda - 1)^2} > 0$$

moreover:

$$\frac{\partial F^{\tau-\sigma}(C_e, \lambda)}{\partial C_e} = 0 \iff C_e = \underline{C_e^{\tau-\sigma}}$$

$$\underline{C_e^{\tau-\sigma}} = \frac{3(48\lambda^2 + 20\lambda - 1)}{2(48\lambda^2 + 16\lambda - 1)}$$

$$\frac{\partial \underline{C_e^{\tau-\sigma}}}{\partial \lambda} < 0$$

$$\lim_{\lambda \rightarrow \infty} \underline{C_e^{\tau-\sigma}} = \frac{3}{2} > \frac{3}{4}$$

Hence, we know that $F^{\tau-\sigma}(\underline{C_e^{\tau-\sigma}}, \lambda)$ is a global minimum and that $\underline{C_e^{\tau-\sigma}}$ lies to the right of the maximum possible value of C_e , $\frac{3}{4}$ as implied by R2.

Now, if we substitute for $C_e = \frac{3}{4}$ we get:

$$F^{\tau-\sigma}\left(\frac{3}{4}, \lambda\right) = \frac{1}{(12\lambda - 1)^2} > 0$$

we can conclude that $F^{\tau-\sigma}(C_e, \lambda)$ is always positive in the interval of values for C_e defined by R1. So $\tau > \sigma$. ■

Proof n. 2. To see that $W_c^* \geq W_k^*$ whenever $S_c^* > \overline{S_c}$, we just need to solve for K the difference between the two levels of welfare and get:

$$W_c^* \geq W_k^* \text{ if}$$

$$K \geq \frac{4C_e^2(60\lambda^2 + 4\lambda - 1) - 12C_e(30\lambda^2 + 5\lambda - 1) + 9(15\lambda^2 + 4\lambda - 1)}{72\lambda(12\lambda - 1)} = \phi(C_e, \lambda)$$

However restriction R1 implies K to be always bigger than $\phi(C_e, \lambda)$. Notice, in fact, that the difference:

$$\frac{(1.5 - 2C_e)^2}{9} - \phi(C_e, \lambda) = \frac{(2C_e(6\lambda - 1) + 3(1 - 3\lambda))^2}{72\lambda(12\lambda - 1)} > 0$$

is quadratic and positive, given $\lambda \geq 1$. It turns out that

$$K > \frac{(1.5 - 2C_e)^2}{9} \geq \phi(C_e, \lambda)$$

and $W_c^* \geq W_k^*$ i.e. when the government grants a subsidy greater than what is strictly necessary, VCA is always preferred to FCA . ■

Proof n. 3. We need to show that $\psi(C_e, \lambda) > \tau(C_e, \lambda)$.

By R2:

$$\begin{aligned} (3(8\lambda + 3) - 4C_e(8\lambda + 1)) &> 0 \\ (6\lambda + 1 - 8C_e) &> 0 \end{aligned}$$

so

$$\sqrt{\psi(C_e, \lambda)} > \sqrt{\tau(C_e, \lambda)} \Rightarrow \psi(C_e, \lambda) > \tau(C_e, \lambda)$$

let

$$\begin{aligned} F^{\psi-\tau}(C_e, \lambda) &:= \sqrt{\psi(C_e, \lambda)} - \sqrt{\tau(C_e, \lambda)} \\ F^{\psi-\tau}(C_e, \lambda) &= \frac{4C_e(48\lambda^2 + 16\lambda - 1) - 3(48\lambda^2 + 32\lambda - 1)}{6(1 - 4\lambda)(12\lambda - 1)} \end{aligned}$$

notice that $F^{\psi-\tau}(C_e, \lambda)$ is decreasing in C_e :

$$\frac{\partial(F^{\psi-\tau}(C_e, \lambda))}{\partial C_e} = \frac{2(48\lambda^2 + 16\lambda - 1)}{3(1 - 4\lambda)(12\lambda - 1)} < 0$$

if we substitute for the maximum value which can be assumed for C_e according to R2, $C_e = \frac{3}{4}$ we get:

$$F^{\psi-\tau}\left(\frac{3}{4}, \lambda\right) = \frac{8\lambda}{(4\lambda - 1)(12\lambda - 1)} > 0$$

where the positive sign is given by $\forall \lambda > 1$. Hence $\sqrt{\psi(C_e, \lambda)} > \sqrt{\tau(C_e, \lambda)} \Rightarrow \psi(C_e, \lambda) > \tau(C_e, \lambda)$. ■

Proof n. 4. We need to show that $W_{\tilde{c}}^{n*}(C_e, n, \lambda) \geq W_o^{n*}(C_e, n, \lambda) \forall n$.

To see it, let:

$$\begin{aligned} F_n^{\tilde{c}-o}(C_e, n, \lambda) &= W_{\tilde{c}}^{n*}(C_e, n, \lambda) - W_o^{n*}(C_e, n, \lambda) \\ \frac{\partial F_n^{\tilde{c}-o}(C_e, n, \lambda)}{\partial C_e} &= 0 \iff C_e = \underline{C_e} \end{aligned}$$

$$\frac{\partial^2 F_n^{\tilde{c}-o}(C_e, n, \lambda)}{\partial C_e^2} = \frac{n^2 \lambda^2}{2n^2 \lambda + 2n\lambda - 1} > 0$$

$$F_n^{\tilde{c}-o}(C_e, n, \lambda) = 0$$

hence $F_n^{\tilde{c}-o}(C_e, n, \lambda)$ reaches a minimum and is equal to zero whenever $C_e = \underline{C}_e$. That in turns means that $W_{\tilde{c}}^{n*}(C_e, n, \lambda) \geq W_o^{n*}(C_e, n, \lambda)$. ■

Proof n. 5. This is a generalization to n number of firms of proof n. 1. We need to show that $\tau^n(C_e, \lambda, n) > \sigma^n(C_e, \lambda, n)$ and we proceed in the same way of proof n. 1.

Let

$$F_n^{\tau-\sigma}(C_e, \lambda, n) := (\tau^n(C_e, \lambda, n) - \sigma^n(C_e, \lambda, n))$$

notice that, given $\lambda \geq 1$ and $n \geq 2$, $F_n^{\tau-\sigma}(C_e, \lambda, n)$ is convex in C_e :

$$\frac{\partial^2 F_n^{\tau-\sigma}(C_e, \lambda, n)}{\partial C_e^2} = \frac{4n^4 \lambda^2 + 2n^2 \lambda(1 - 2\lambda) + 4n\lambda - 1}{\lambda(n+1)^2(2n^2 \lambda + 2n\lambda - 1)^2} > 0$$

moreover:

$$\frac{\partial F_n^{\tau-\sigma}(C_e, \lambda, n)}{\partial C_e} = 0 \iff C_e = \underline{C}_e^{\tau-\sigma}$$

we know then that $F_n^{\tau-\sigma}(\underline{C}_e^{\tau-\sigma}, \lambda, n)$ is a global minimum. It is easy to notice, moreover, that $\underline{C}_e^{\tau-\sigma}$ lies at the right hand side with respect to $\frac{1+n}{2n}$ which is the maximum value that can be assumed for C_e according to R2:

$$\underline{C}_e^{\tau-\sigma} - \frac{1+n}{2n} = \frac{n\lambda(n^2 - 1)}{4n^4 \lambda^2 + 2n^2 \lambda(1 - 2\lambda) + 4n\lambda - 1} + \frac{n^2 - 1}{2n} > 0$$

Now, if we substitute for $C_e = \frac{1+n}{2n}$ we get:

$$F_n^{\tau-\sigma}\left(\frac{1+n}{2n}, \lambda, n\right) = \frac{(n-1)^2}{(2n^2 \lambda + 2n\lambda - 1)^2} > 0$$

we can conclude that $F_n^{\tau-\sigma}(C_e, \lambda, n)$ is always positive in the interval of values for C_e defined by R1. So $\tau^n > \sigma^n$. ■

Proof n. 6. We need to show that $K > \phi^n \forall C_e, n, \lambda$.

R1 implies

$$K > \left(\frac{1 - 2nC_e + n}{2(1+n)}\right)^2 > \phi^n(C_e, \lambda, n)$$

to see that, let:

$$F^\phi(C_e, \lambda, n) := \left(\frac{1 - 2nC_e + n}{2(1+n)}\right)^2 - \phi^n(C_e, \lambda, n)$$

$$\frac{\partial (F^\phi(C_e, \lambda, n))}{\partial C_e} = 0 \iff C_e = \underline{C}_e^\phi$$

since $n \geq 2$ and $\lambda \geq 1$ we moreover have:

$$\frac{\partial^2 (F^\phi(C_e, \lambda, n))}{\partial C_e^2} = \frac{(n^2\lambda + n\lambda - 1)^2}{\lambda(n+1)^2(2n^2\lambda + 2n\lambda - 1)} > 0$$

finally:

$$F^\phi(\underline{C}_e^\phi, \lambda, n) = 0$$

so $F^\phi(C_e, \lambda, n)$ is convex in C_e and $F^\phi(\underline{C}_e^\phi, \lambda, n) = 0$ is a global minimum. ■

Proof n. 7. This is a generalization to n number of firms of proof n. 3. We need to show that $\psi^n(C_e, \lambda, n) > \tau^n(C_e, \lambda, n)$.

By R2:

$$\begin{aligned} n^2\lambda + n(\lambda + 1) - 1 - 2C_e n^2\lambda &> 0 \\ 2n^3\lambda + 2n^2(\lambda + 1) + n - 1 - 2C_e n(2n^2\lambda + 1) &> 0 \end{aligned}$$

so

$$\sqrt{\psi^n(C_e, \lambda, n)} > \sqrt{\tau^n(C_e, \lambda, n)} \Rightarrow \psi(C_e, \lambda) > \tau(C_e, \lambda)$$

let

$$F_n^{\psi-\tau}(C_e, \lambda, n) := \sqrt{\psi^n(C_e, \lambda, n)} - \sqrt{\tau^n(C_e, \lambda, n)}$$

notice that $F_n^{\psi-\tau}(C_e, \lambda, n)$ is decreasing in C_e :

$$\frac{\partial(F_n^{\psi-\tau}(C_e, \lambda, n))}{\partial C_e} = \frac{n(n^4\lambda^2 + 2n^2\lambda(1 - 2\lambda) + 4n\lambda - 1)}{(n+1)(1 - 2n\lambda)(2n^2\lambda + 2n\lambda - 1)} < 0$$

if we substitute for the maximum value which can be assumed for C_e according to R2, $C_e = \frac{1+n}{2n}$ we get:

$$F_n^{\psi-\tau}\left(\frac{1+n}{2n}, \lambda, n\right) = \frac{2n^2\lambda(n-1)}{(2n\lambda-1)(2n^2\lambda+2n\lambda-1)} > 0$$

where the positive sign is given by $\lambda \geq 1$ and $n \geq 2$. Hence $\sqrt{\psi^n(C_e, \lambda, n)} > \sqrt{\tau^n(C_e, \lambda, n)} \Rightarrow \psi^n(C_e, \lambda, n) > \tau^n(C_e, \lambda, n)$. ■

Proof n. 8. To show that $\frac{\partial\tau^n(C_e, \lambda, n)}{\partial n} < 0$, first of all, notice that the numerator of $\tau^n(C_e, \lambda, n)$ is positive:

$$n^2\lambda + n(\lambda + 1) - 1 - 2C_e n^2\lambda > 0$$

since by R2

$$C_e < \frac{n+1}{2n}$$

and substituting $C_e = \frac{n+1}{2n}$ into the previous expression gets:

$$\begin{aligned} n^2\lambda + n(\lambda + 1) - 1 - 2\left(\frac{n+1}{2n}\right)n^2\lambda &= \\ n - 1 &> 0 \end{aligned}$$

(the denominator of $\tau^n(C_e, \lambda, n)$ is positive as well, as can be easily noticed)
then, let

$$F^{\sqrt{\tau}}(C_e, \lambda, n) := \sqrt{\tau^n(C_e, \lambda, n)} = \frac{n^2\lambda + n(\lambda + 1) - 1 - 2C_e n^2\lambda}{2n^2\lambda + 2n\lambda - 1}$$

as $F^{\sqrt{\tau}}(C_e, \lambda, n) > 0$, $\frac{\partial F^{\sqrt{\tau}}(C_e, \lambda, n)}{\partial n} < 0 \Rightarrow \frac{\partial \tau^n(C_e, \lambda, n)}{\partial n} < 0$.

Let us take the first order partial derivative of $F^{\sqrt{\tau}}(C_e, \lambda, n)$ with respect to n :

$$\frac{\partial F^{\sqrt{\tau}}(C_e, \lambda, n)}{\partial n} = - \left(\frac{4C_e n \lambda (n\lambda - 1) + 2n\lambda(n-1) - \lambda + 1}{(2n^2\lambda + 2n\lambda - 1)^2} \right)$$

the denominator is obviously positive. The numerator is positive as well; notice in fact that

$$2n\lambda(n-1) - \lambda > 0$$

as $n \geq 2$. Given the negative sign, that means that $\frac{\partial \tau^n(C_e, \lambda, n)}{\partial n} < 0$. This in turn means that $\frac{\partial \tau^n(C_e, \lambda, n)}{\partial n} < 0$ in the interval defined by R2. ■

Proof n. 9. we need to show that $C_e > \frac{3(8\lambda+1)}{4(8\lambda+3)} \implies K > \sigma^d(C_e, \lambda)$. If we solve $\sigma^d(C_e, \lambda)$ as a function of K , we get:

$$FCA \succ NI$$

iff

$$C_e < \frac{3(2(4\lambda + 1) - \sqrt{32K\lambda(8\lambda + 3) + 1})}{4(8\lambda + 3)} = \Sigma^d(K, \lambda)$$

let

$$F^{\Sigma^d}(K, \lambda) := \frac{3(8\lambda + 1)}{4(8\lambda + 3)} - \Sigma^d(K, \lambda)$$

$$F^{\Sigma^d}(K, \lambda) = \frac{3(\sqrt{32K\lambda(8\lambda + 3) + 1} - 1)}{4(8\lambda + 3)} > 0$$

hence $\frac{3(8\lambda+1)}{4(8\lambda+3)} > \Sigma^d(K, \lambda)$. That implies $K > \sigma^d(C_e, \lambda)$ whenever $C_e > \frac{3(8\lambda+1)}{4(8\lambda+3)}$. ■

Proof n. 10. We need to show that $K > \psi^d(C_e, \lambda)$ whenever $C_e < \frac{6\lambda-1}{8\lambda}$.

R1 implies

$$K > \frac{(1.5 - 2C_e)^2}{9}$$

taking the difference

$$\frac{(1.5 - 2C_e)^2}{9} - \psi^d(C_e, \lambda) = \frac{6\lambda - 1 - 8C_e\lambda}{4\lambda - 3} > 0$$

it is easy noticing that the above condition always holds whenever $C_e < \frac{6\lambda-1}{8\lambda}$. ■

Proof n. 11. When the entrant is efficient, i.e. $C_e < \widehat{C}_e(K, \lambda)$, and the government has chosen VCA in stage 1, the amount of subsidy granted exceeds that strictly necessary to trigger entry. That means that E 's are positive. From a total welfare perspective, however, VCA is a suboptimal policy. To see that assume that $C_e < \widehat{C}_e(K, \lambda)$. Total welfare is then:

NI	$W_o^{tw*} = W_o^{d*} = \frac{1}{32} + \frac{1}{16}$
FCA	$W_k^{tw*} = W_k^{d*} = \frac{(1.5-C_e)^2}{18} - \lambda \left(K - \frac{(1.5-2C_e)^2}{9} \right) + \frac{C_e^2}{9}$
VCA	$W_{\bar{c}}^{tw*} = \frac{\lambda^2(4C_e-9)^2}{8(12\lambda-1)^2} - \lambda \left(\frac{6\lambda+1-8\lambda C_e}{2(12\lambda-1)} \right) \left(C_e - \frac{3(3\lambda-1)+12\lambda C_e}{2(12\lambda-1)} \right) +$ $+ \frac{(3(3\lambda-1)+12\lambda C_e)^2}{9} + \left[\frac{(1.5-2(\frac{3(3\lambda-1)+12\lambda C_e}{2(12\lambda-1)}))^2}{9} - K \right]$

where the term in square brackets represents E 's profits. VCA is superior to NI iff:

$$K < \frac{192C_e^2\lambda^2(4\lambda+3) - 16C_e\lambda(72\lambda^2+42\lambda+11) + 432\lambda^3 + 180\lambda^2 + 96\lambda + 13}{32(12\lambda-1)^2} = \tau^{tw}(C_e, \lambda)$$

since the comparative advantage of VCA with respect to NI is surely decreasing in λ in the interval defined by $C_e < \widehat{C}_e(K, \lambda)$, let us impose $\lambda = 1$, so that VCA yields the maximum possible level of welfare. We thus have:

$$\tau^{tw}(C_e, 1) = \frac{1344C_e^2 - 2000C_e + 721}{3872}$$

By R1, K has to be bigger than $\frac{(1.5-2C_e)^2}{9}$ it turns out that it can never be below $\tau^{tw}(C_e, 1)$. Indeed, let

$$F^{tw}(C_e) := \frac{(1.5-2C_e)^2}{9} - \tau^{tw}(C_e, 1) = \frac{3392C_e^2 - 5232C_e + 2223}{34848}$$

It is easy to see that $F^{tw}(C_e)$ is strictly convex in C_e and that

$$\arg \min F^{tw}(C_e) = \frac{5}{848} > 0$$

hence $F^{tw}(C_e)$ is always greater than zero and K cannot be lower than $\tau^{tw}(C_e, \lambda)$, so VCA is never a first best strategy for an Authority which maximizes total welfare, even if the entrant is very efficient. ■

B The *non-crowding out* choice of the government

If we remove the assumption that $C_i = \frac{1}{2}$, then it becomes possible that choosing *VCA* would causes to the incumbent to exit the market (crowding out effect). In what follows, it is shown that this never happens. Notice that for a firm to be considered forced out from the market in the definition used here, strictly negative profits must be linked to non negative quantities produced.

Three cases have to be considered:

$$\begin{aligned} \text{case 1} & C_i \leq \frac{1}{2} \\ \text{case 2} & \frac{1}{2} < C_i \leq 1 - \sqrt{K} \\ \text{case 3} & \max \left\{ 1 - \sqrt{K}; \frac{1}{2} \right\} < C_i \end{aligned}$$

For the analysis of all the three cases let us assume that lump sum taxes are feasible¹⁹ i.e. $\lambda = 1$.

B.1 case 1

It is straightforward to see that whenever $C_i \leq \frac{1}{2}$ the incumbent can never be crowded out since consumers' demand is given by $Q = 1 - P$ and at equilibrium $q_i = \frac{1+(C_e-S_c)-2C_i}{3} \geq 0 \forall C_i \leq \frac{1}{2}$.

B.2 case 2

Now the incumbent can be crowded out depending on the amount of *VCA* granted by the government to the entrant. At the same time, the government *can* choose a level for S_c which allows the entrant to enter the market but does not force the incumbent to exit. Let us define \widehat{C}_e the threshold for the marginal cost of the entrant below which the incumbent is crowded out:

$$\widehat{C}_e := 2C_i - 1$$

welfare without and with crowding out are $W_{in}(C_i, C_e, S_c)$ and $W_{out}(C_i, C_e, S_c)$ respectively:

$$\begin{aligned} W_{in}(C_i, C_e, S_c) &= \frac{1}{2} \left(\frac{2-C_i-(C_e-S_c)}{3} \right)^2 - \frac{1+C_i-2(C_e-S_c)}{3} \cdot S_c \quad \text{if } \overline{C}_e \geq C_e - S_c \geq \widehat{C}_e \\ W_{out}(C_i, C_e, S_c) &= \frac{1}{2} \left(\frac{1-(C_e-S_c)}{2} \right)^2 - \frac{1-(C_e-S_c)}{2} \cdot S_c \quad \text{if } \widehat{C}_e > C_e - S_c \end{aligned}$$

In stage 2, the government chooses S_c as to

$$\begin{aligned} \max_{S_c} W_{in}(C_i, C_e, S_c) \quad & \text{s.t. } \overline{C}_e \geq C_e - S_c \geq \widehat{C}_e \\ \max_{S_c} W_{out}(C_i, C_e, S_c) \quad & \text{s.t. } \widehat{C}_e > C_e - S_c \end{aligned} \tag{12}$$

¹⁹In the following it is shown that the government never chooses to grant an aid which crowds out the incumbent firm when the shadow cost of the subsidy is equal to 1. Setting $\lambda > 1$ would increase the total cost of the subsidy and decrease *VCA* likelihood, thus strengthening the non-crowding out result. It is then possible to simplify the analysis by setting $\lambda = 1$ and still get a general result.

The solutions are:

$$\begin{aligned} S_c^* &= C_e - \frac{1+4C_i+6C_e}{11} & \text{if } C_i &\geq 11\sqrt{K} - 3 + 4C_e \\ S_c^* &= C_e - \frac{1+C_i-3\sqrt{K}}{2} & \text{if } C_i &< 11\sqrt{K} - 3 + 4C_e \end{aligned}$$

As $C_i \leq 1 - \sqrt{K}$, $S_c > C_e - \widehat{C}_e$ is always sub-optimal. Hence the government never chooses a subsidy level which gives the entrant the ability to force the incumbent out from the market.

B.3 case 3

In this case, the level of subsidy strictly necessary to let the entrant to enter the market is sufficient to cause the crowding out effect. In other words, if the government chooses *VCA*, it has to reduce the marginal cost of the entrant at a level such that the incumbent cannot sustain competition if it wishes to let the entrant enter the market. In that case, the optimal choice of S_c in stage 2 would be:

$$\begin{aligned} \max_{S_c} W_{out}(C_i, C_e, S_c) \\ \text{s.t. } C_e - S_c \leq \overline{C}_e \end{aligned}$$

the unique solution of the maximization problem is:

$$S_c^* = C_e - \frac{1 + C_i - 3\sqrt{K}}{2}$$

Now to prove that *VCA* is never chosen in stage 1, it is sufficient to prove that *VCA* is never preferred to *FCA* in the interval $C_i \in [1 - \sqrt{K}; 1)$. To do so, let us compare the two levels of welfare. *VCA* would be weakly preferred if:

$$W_{out}(C_i, C_e, S_c) - W_k(C_i, C_e, S_c) \geq 0$$

which implies:

$$C_e \leq -\frac{3\sqrt{2}\sqrt{15(C_e - S_c^*)^2 - 10(C_e - S_c^*)(C_i + 2) + 4C_i^2 + 2C_i + 28K + 9} + 9(C_e - S_c^*) - 10C_i - 13}{14}$$

as $C_e \leq 1$. By R2 we have that $C_e > 2C_i - 1$. So, for the above inequality to hold it must be that:

$$2C_i - 1 + \frac{3\sqrt{2}\sqrt{15(C_e - S_c^*)^2 - 10(C_e - S_c^*)(C_i + 2) + 4C_i^2 + 2C_i + 28K + 9} + 9(C_e - S_c^*) - 10C_i - 13}{14} < 0$$

Let us call α the left hand side of the above inequality. Substituting for S_c^* and simplifying we get:

$$\alpha = \frac{3(\sqrt{2}\sqrt{11C_i^2 - 2C_i(15\sqrt{K} + 11) + 247K + 30\sqrt{K} + 11} + 15C_i - 9\sqrt{K} - 15)}{28}$$

Taking the first order derivative with respect to C_i and rearranging we get:

$$\frac{\partial \alpha}{\partial C_i} = \frac{3(15\sqrt{11(1-C_i)^2 + (1-C_i)30\sqrt{K} + 247K} - \sqrt{2}(11(1-C_i) + 15\sqrt{K}))}{28\sqrt{11(1-C_i)^2 + (1-C_i)30\sqrt{K} + 247K}}$$

Clearly the denominator is positive. To show that $\frac{\partial \alpha}{\partial C_i} \geq 0$ is then sufficient to prove that the numerator is also positive. Notice that:

$$11(1-C_i) + 15\sqrt{K} > 0$$

since $C_i \leq 1$ and $K > 0$; so it is possible to take the square of the two terms in the numerator and then compare them:

$$55125K + 6090\sqrt{K}(1-C_i) + 2233(1-C_i)^2 > 0$$

which is true by $C_i \leq 1$ and $K > 0$. Now, since $\frac{\partial \alpha}{\partial C_i} > 0$, we can substitute inside α the lower value attainable by C_i in the interval we're studying: $C_i = 1 - \sqrt{K}$ and find $\alpha = 0$. So α can never be lower than zero. This means that whenever $C_i \in [1 - \sqrt{K}; 1)$, $W_{out}(C_i, C_e, S_c) < W_k(C_i, C_e, S_c)$ and VCA can never be preferred to FCA .

As supposed to be shown, in none of the three cases the incumbent is crowded out by the entrant through the use of VCA .

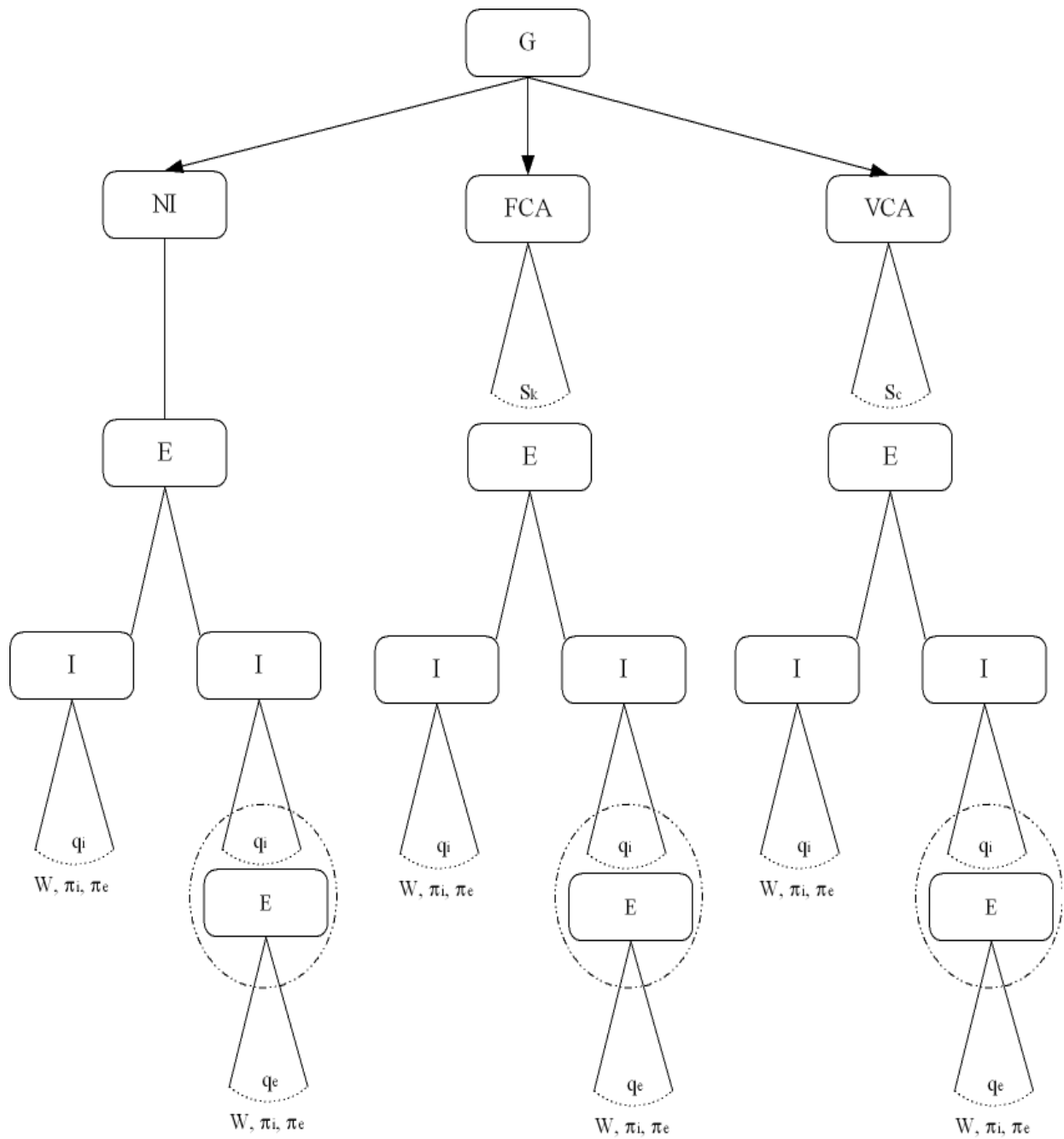


Figure1 – The game in extensive form

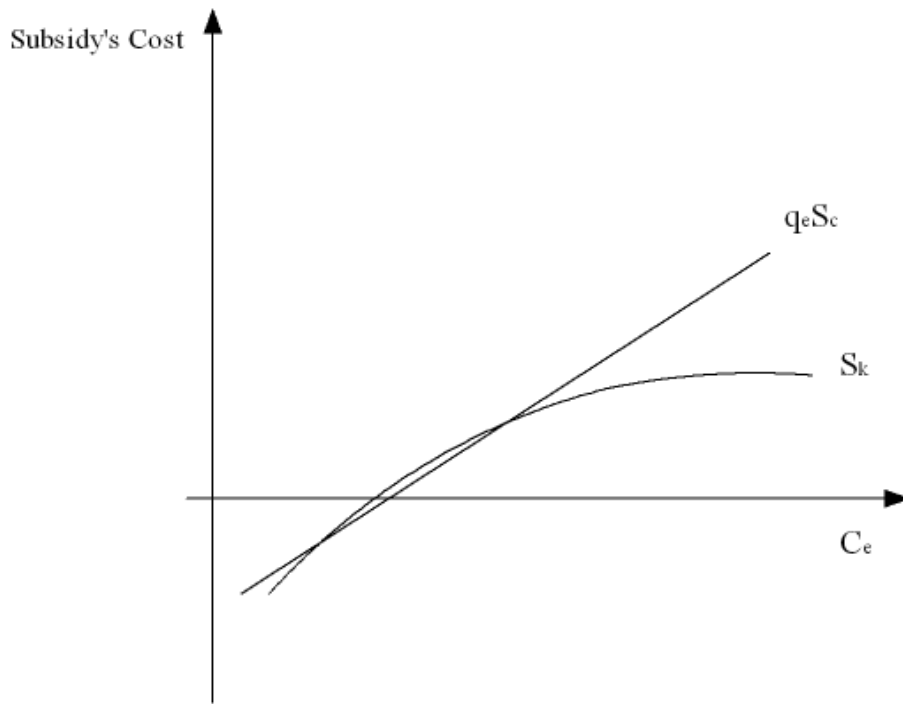


Figure 2 – State aid instruments' costs

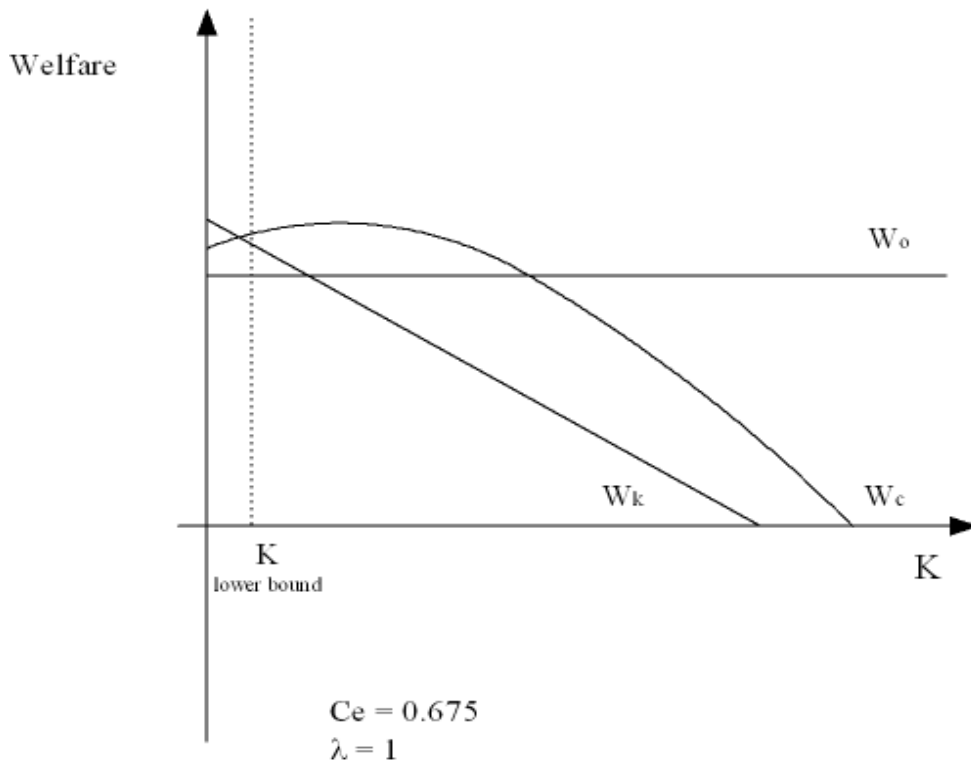


Figure 3 – Welfare levels for given C_e

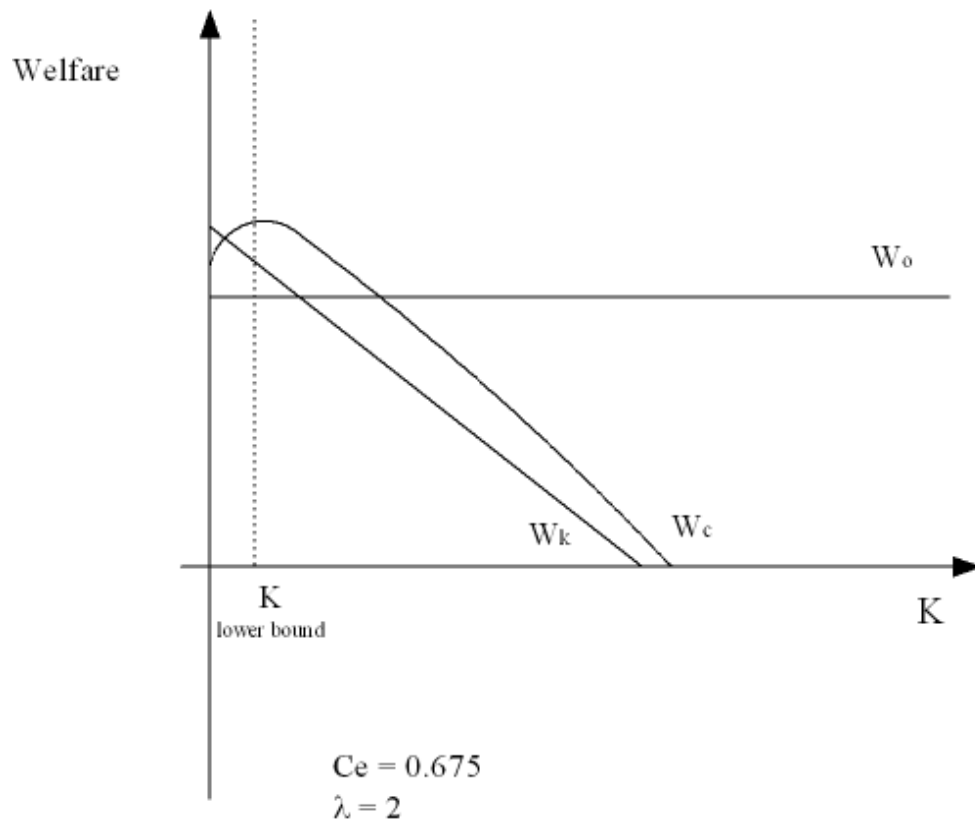


Figure 4 – Welfare levels for given C_e

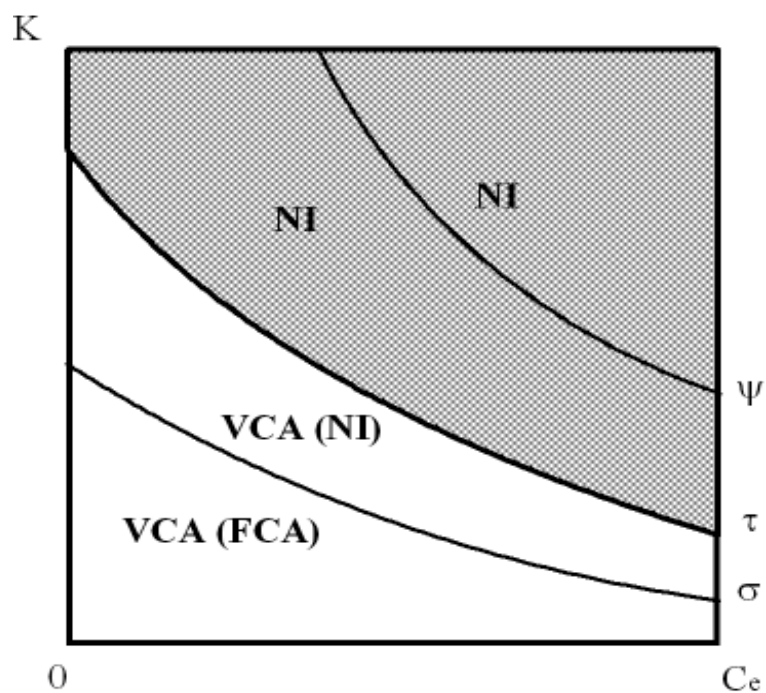


Figure 5 – The government's choice when VCA is allowed

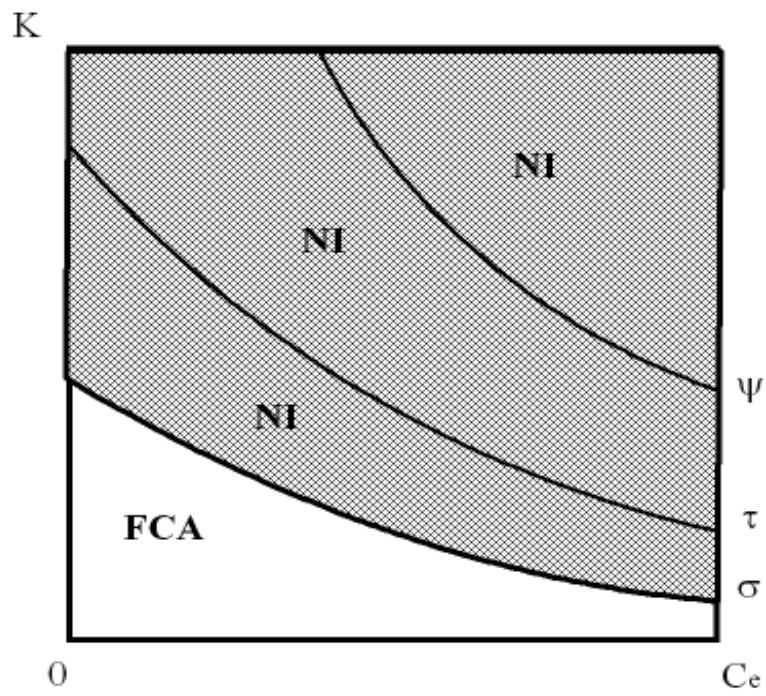


Figure 6 – The government's choice when VCA is not allowed