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This Version: December 12, 2006

Abstract

This paper modifies the standard Mortensen-Pissarides job matching model in order to explain the cyclical behavior of vacancies and unemployment. The modifications include strategic wage bargaining [Hall and Milgrom, 2006] and convex labor adjustment costs. The results reveal that our model replicates the cyclical behavior of both variables remarkably well. First, we show that strategic wage bargaining increases the volatility of vacancies and unemployment enormously. Second, the introduction of convex labor adjustment costs makes both variables much more persistent. Third, our analysis indicates that both modifications are complementary in generating volatile and persistent labor market variables.

JEL Codes: E24, E32, J41

Keywords: Business Cycles, Matching, Strategic Bargaining

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* A previous version of this paper was entitled: 'Business Cycles, Strategic Bargaining, and the Beveridge Curve'. I would like to thank Morten Ravn and Salvador Ortigueira for their help and supervision. I am also very grateful to Marcus Hagedorn and seminar participants at the 2006 annual meeting of the Verein für Socialpolitik, and the 2006 doctoral workshop of the EBIM International Research Training Group for extensive comments and suggestions. All remaining errors are mine.

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1 Introduction

The Mortensen-Pissarides job search and matching model has become the standard theory of equilibrium unemployment. Moreover, starting with Merz (1995), Andolfatto (1996) and den Haan et al. (2000), several authors have introduced the Mortensen-Pissarides matching model in a DSGE setting. In the first place, this line of research focussed on the impact of labor market frictions on the propagation of shocks. In addition, beyond enhancing output propagation, this framework provides a workhorse for analyzing the behavior of the labor market over the business cycle. Recently, however, the job search and matching model has come under criticism. Following Shimer (2005a), a large literature has emerged which has shown that the Mortensen-Pissarides model cannot replicate the cyclical behavior of its two central elements – vacancies and unemployment.

First and foremost, Shimer (2005a) emphasizes that the standard job search and matching model cannot explain the observed high degree of volatility in vacancies and unemployment. In particular, Shimer (2005a) challenges not the job search and matching approach itself, but rather the commonly-used Nash (1953) bargaining assumption for wage determination. Accordingly, the households and the firms, respectively, receive period-by-period a constant share of the mutual surplus. As a result, the model generated wage bill per worker is almost as elastic as the underlying productivity shock. From the firms’ perspective, this means that the costs per worker are almost as elastic as the benefits. Thus, Nash bargaining gives firms only little incentive to adjust employment at the business cycle frequencies. In order to reconcile the job search and matching model with the data, Shimer (2005a) proposes to consider an alternative bargaining game which delivers real wage rigidity. In this context, Shimer (2004) gives evidence that real wage rigidity gives firms much stronger incentives to amplify their hiring activities.

Apart from that, the standard Mortensen-Pissarides model is challenged from a second perspective. As demonstrated by Fujita (2005), the model is incapable of generating sufficient vacancy persistence. This artifact follows from firms’ pattern of vacancy posting under linear vacancy posting costs. In response to a positive technology shock, forward-looking firms anticipate that the sharp and lasting rise in market tightness causes a surge in hiring costs. Hence, profit maximizing firms are likely to adjust employment instantaneously. For this reason, we observe that vacancies spike on impact, but fall back half way only one period later. Contrary to this pattern, several authors have found ample evidence that the impulse response function of vacancies displays a marked hump-shape, peaking with several quarters delay. This indicates that vacancies – like unemployment – react sluggishly to technology shocks. Fujita (2005) address this issue by considering frictions in the process of vacancy creation. These frictions change the first order condition for vacancy posting and, thus, prevent the immediate employment adjustment. As a result, the impulse response function of vacancies displays a marked hump-shape. Moreover, the persistence of model generated vacancies improves remarkably.

The main aim of our paper is to replicate the cyclical behavior of vacancies and unemployment along both dimensions – volatility and persistence. Therefore, we modify the standard job search and matching model in two ways. First, following Mortensen and Nagypál (2005), we
adopt strategic wage bargaining as introduced into the literature by Hall and Milgrom (2006). In contrast to (static) Nash bargaining, this approach assumes that wages are determined by a Rubinstein (1982) game of alternating offers. Hence, strategic wage bargaining accounts for the dynamic and interactive character of wage negotiations. The essential difference between these two bargaining concepts lies in the players’ threat points. Under Nash bargaining, both players’ threat points are determined by their respective outside alternative. Under strategic wage bargaining, however, the prospective mutual surplus gives both players strong incentives to hold-up the bargaining process until an agreement is reached. Thus, both players’ threat points are determined by their respective value of bargaining.

As argued by Hall and Milgrom (2006), the value of bargaining is much less sensitive to current labor market conditions than the outside alternative (i.e. the value of labor market search). Hence, strategic wage bargaining reduces the elasticity of the wage bill per worker by half. As a consequence, the elasticity of the net flow value of the marginal match rises enormously, providing firms much stronger incentives to hire new workers in economic upswings. Thus, strategic wage bargaining gives an endogenous explanation for the observed high degree of labor market volatility.

Second, we combine strategic wage bargaining with convex labor adjustment costs as used by Gertler and Trigari (2006). In contrast to linear vacancy posting costs, firms’ hiring costs now are determined by the number of vacancies that are filled, and not by the number of vacancies that are posted. Further, firms’ hiring costs now depend negatively on the current employment level. Consequently, marginal matching costs are no longer a function of market tightness, but of the gross hiring rate. In contrast to market tightness, the gross hiring rate is much less elastic and much less persistent in response to technology shocks. This altered behavior of marginal matching costs removes firms’ incentives to adjust employment instantaneously. Instead, the convex shape of the labor adjustment cost function gives firms strong incentives to smooth their hiring activities. For this reason, the impulse response function of vacancies shows a pronounced hump-shape, peaking several quarters after the shock. This result indicates that modifying firms’ first order condition might be an effective means to account for sluggish vacancy responses.

Moreover, we notice that strategic wage bargaining and convex labor adjustment costs are complementary in generating labor market volatility and persistence. This result is surprising given that, in the existing literature (Fujita and Ramey, 2005), the introduction of adjustment costs leads to more persistent labor market responses, but damps volatility at the same time. Unlike Fujita and Ramey (2005), however, we assume adjustment costs in matches, and not in vacancies. Moreover, we assume that hiring costs depend negatively on the employment level. The quantitative impact of this effect, however, is the more evident the larger the fluctuations in the labor market. Consequently, strategic wage bargaining amplifies the elasticity of labor market variables through two channels. On the one hand, strategic wage bargaining enhances employment volatility. On the other hand, the higher the stock of employment, the lower the costs of labor adjustment. Thus, the introduction of convex labor adjustment costs induces not only more persistence, but also more volatility in the labor market.

We conduct our analysis using a real model of the business cycle (Andolfatto, 1996). This framework seems advantageous, given that it allows for a proper calibration of the factor
income shares and small (accounting) profits (Hornstein et al., 2005). As demonstrated by Hagedorn and Manovskii (2006), profits have to be small in order to leverage a given productivity shock into large labor market fluctuations.

Furthermore, we find that our model gives rise to two distortionary effects. Given convex labor adjustment costs, social optimality requires that the wage bill per worker is equal to household’s outside alternative. In contrast, we assume that (i) the wage bill per worker is independent of the fluctuations in household’s outside alternative (ii) firms’ bargaining power \( \xi \) is smaller than unity. Hence, firms’ private gains from search effort are generally smaller than their social contribution. In this case, the dynamic behavior of the wage bill per worker is not socially optimal (Hosios, 1990). Hence, we compute the market solution to our modified job search and matching model, based on the paper of Cherom and Langot (2004).

The remainder of this paper is organized as follows. Section 2 presents our model. Section 3 calibrates the model and evaluates its quantitative performance against U.S. data. Section 4 concludes.

2 The Model

2.1 Labor Market Flows

The job matching model presumes that search on the labor market is frictional. These frictions are represented by a Cobb-Douglas matching function which relates aggregate job matches \( m_t \) to the number of vacancies that are posted \( v_t \) and the search effort of the unemployed \( e (1 - n_t) \):

\[
m_t(v_t, (1 - n_t)) = \chi v_t^\alpha (e (1 - n_t))^{1-\alpha} \leq \min[v_t, (1 - n_t)], \tag{1}
\]

where the searching time \( e > 0 \) per unemployed is taken to be constant. The ratio between vacancies and unemployed job searchers measures the tightness of the labor market. By linear homogeneity of the matching function, the vacancy filling rate \( q(\gamma_t) \) and the job finding rate \( q(\gamma_t) \gamma_t \) depend solely on the value of market tightness \( \gamma_t \):

\[
q(\gamma_t) &\equiv \frac{m_t}{v_t} = \chi \left( \frac{1 - n_t}{v_t} \right)^{1-\alpha} \tag{2a}, \\
q(\gamma_t) \gamma_t &\equiv \frac{m_t}{(1 - n_t)} = \chi \left( \frac{v_t}{(1 - n_t)} \right)^{\alpha}. \tag{2b}
\]

These ratios give the expected return on labor market search for firms and the unemployed, respectively. One can observe that the tighter the labor market, the longer the expected time to fill a vacancy, but the shorter the expected search for a job (and vice versa). However, firms do not internalize the congestion effect of their search activities on the aggregate return rates. Hence, the decision to post vacancies gives rise to two externalities.

We assume that new job matches \( m_t \) are formed at the end of each period. Simultaneously, a fraction of preexisting jobs is terminated. Consistent with the results of Shimer (2005b), we assume the job destruction rate \( \sigma \) to be constant. Hence, the law of motion for the aggregate employment level is given by:

\[
n_{t+1} = (1 - \sigma) n_t + m_t. \tag{3}
\]
2.2 The Problem of the Household

The representative household consists of a continuum of individuals who insure each other completely against idiosyncratic employment risk. The share of employed household members, \( n_t \), works \( l_t \) "hours" per period on the job while the share \( 1 - n_t \) (the unemployed) searches \( e \) "hours" on the labor market. Both activities affect utility negatively as they reduce the amount of leisure. We assume that total factor productivity \( a_t \) is subject to an exogenous shock specified by the following autoregressive process:

\[
\ln(a_t) = (1 - \rho) \ln(\bar{a}) + \rho \ln(a_{t-1}) + \epsilon_t, \quad \text{with} \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2_\epsilon) \quad \text{and} \quad iid.
\]


\[ u^N(c^N_t, 1 - l_t) = \ln(c^N_t) + \phi_1 \frac{(1 - l_t)^{1 - \eta}}{1 - \eta}, \quad u^U(c^U_t, 1 - e) = \ln(c^U_t) + \phi_2 \frac{(1 - e)^{1 - \eta}}{1 - \eta}. \]

The parameter \( \phi_i, i = 1, 2 \) captures the fact that the valuation of leisure depends on the employment status. Each employed household member earns the real wage rate \( w_t \) per hour \( l_t \). Hence, \( n_t w_t l_t \) constitutes the labor income of the representative household. In addition, households receive dividends remitted by firms \( \pi_t \) and rental income \( r_t k_t \) from perfectly competitive capital markets. The state space of the household is given by the set \( \Omega^H_t = \{k_t, n_t\} \). Thus, the maximization problem of the representative household can be formulated as:

\[
W(\Omega^H_t) = \max_{c^U_t, c^N_t, k_{t+1}} \{ n_t \, u^N(c^N_t, 1 - l_t) + (1 - n_t) \, u^U(c^U_t, 1 - e) + \beta E_t[H(\Omega^H_{t+1})] \}, \tag{4}
\]

s.t.

\[
k_{t+1} = (1 - \delta + r_t)k_t + \pi_t + n_t w_t l_t - n_t c^N_t - (1 - n_t)c^U_t, \tag{5}
\]

\[
n_{t+1} = (1 - \sigma)n_t + q(\gamma_t) \gamma_t (1 - n_t). \tag{6}
\]

Here, equation (3) is the budget constraint. Equation (6) is the law of motion for the household’s employment share. Provided stochastic processes for \( \{w_t, r_t, l_t, \pi_t, q(\gamma_t) \gamma_t | t \geq 0\} \) and a set of initial conditions \( \{k_0, n_0\} \), the representative household chooses contingency plans \( \{c^U_t, c^N_t, k_{t+1} | t \geq 0\} \) that maximize its expected discounted utility. These choices have to satisfy following first order conditions:

\[
c^N_t: \lambda_t = u^N_t(c^N_t, 1 - l_t), \tag{7}
\]

\[
c^U_t: \lambda_t = u^U_t(c^U_t, 1 - e), \tag{8}
\]

\[
k_{t+1}: \lambda_t = \beta E_t[\lambda_{t+1}(1 - \delta + r_{t+1})]. \tag{9}
\]

The first order conditions (7) and (8) show that perfect income insurance against idiosyncratic employment risk allocates the same consumption level to employed and unemployed workers. Equation (9) gives the familiar Euler equation for consumption.

2.3 The Problem of the Firm

Output is produced by firms that use capital \( k_t \) and labor hours \( (n_t l_t) \) as input factors. The production function is taken to be Cobb-Douglas. This implies that the model has a representative firm. We assume that total factor productivity \( a_t \) is subject to an exogenous shock specified by the following autoregressive process:

\[
\ln(a_t) = (1 - \rho) \ln(\bar{a}) + \rho \ln(a_{t-1}) + \epsilon_t, \quad \text{with} \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2_\epsilon) \quad \text{and} \quad iid. \tag{10}
\]
The specification of the firm’s cost function follows Gertler and Trigari (2006). The firm incurs rental costs of capital $r_t k_t$, aggregate wage payments $n_t w_t l_t$, and labor adjustment costs $\psi_t$:

$$\psi(m_t, n_t) = \frac{\kappa n_t^2}{2}$$  \hspace{1cm} (11)

In contrast to the standard specification, labor adjustment costs are determined by the squared number of new job matches $m_t^2$, the employment level $n_t$, and a constant scale parameter $\kappa/2$. Consequently, firms’ labor adjustment costs are determined by the number of vacancies that are filled, and not by the number of vacancies that are posted. In addition, notice that firms take the aggregate vacancy filling rate $q(\gamma_t)$ as given. Hence, from the firm’s perspective, the number of new job matches $m_t = q(\gamma_t)v_t$ is linear in vacancies.

Moreover, we assume that the representative firm is large in the sense that it has many workers, and that it is large enough to eliminate all uncertainty about $n_{t+1}$. Thus, we ensure that all firms in the model remain of the same size. However, the representative firm in our model is small in the sense that it is competitive. Hence, it takes not only the aggregate vacancy filling rate, but also the wage bill per worker, $w_t l_t$, as given. The state space of the firm is given by the set $\Omega_t^F = \{ n_t \}$. Thus, the representative firm’s problem can be formulated as:

$$\mathcal{V}(\Omega_t^F) = \max_{k_t, v_t} \left\{ y_t - n_t w_t l_t - r_t k_t - \frac{\kappa n_t^2}{2} + \beta E_t \left[ (\frac{\lambda_{t+1}}{\lambda_t}) \mathcal{V}(\Omega_{t+1}^F) \right] \right\},$$  \hspace{1cm} (12)

s.t.

$$y_t = a_t k_t^\theta (n_t l_t)^{(1-\theta)},$$  \hspace{1cm} (13)

$$n_{t+1} = (1 - \sigma) n_t + q(\gamma_t)v_t,$$  \hspace{1cm} (14)

Given stochastic processes for $\{ a_t, w_t, r_t, l_t, q(\gamma_t) \geq 0 \}$ and an initial condition for $n_0$, the representative firm chooses contingency plans $\{ k_t, v_t, n_{t+1} \geq 0 \}$ that maximize the expected present value of the dividend flow. The first order conditions are given as:

$$k_t : \quad r_t = \theta \frac{y_t}{k_t^\gamma},$$  \hspace{1cm} (15)

$$n_{t+1} : \quad \kappa x_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} (1 - \theta) \frac{y_{t+1}}{n_{t+1}} - w_{t+1} l_{t+1} + \frac{\kappa n_t^2}{2} + (1 - \sigma) \kappa x_{t+1} \right],$$  \hspace{1cm} (16)

where the gross hiring rate $m_t/n_t$ is denoted by $x_t$. Equation (15) shows the familiar relation between the real interest rate and the marginal product of capital under perfectly competitive capital markets. The hiring condition (16) states that the representative firm posts the optimal number of job vacancies $v_t$ that equalizes expected marginal hiring costs $\kappa x_t$ (the left hand side) with the expected present value of the marginal match in the future (the right hand side). The expected present value of the marginal match depends on the marginal product per worker $(1 - \theta)(y_{t+1}/n_{t+1})$, the expected wage bill per worker $w_{t+1} l_{t+1}$, expected savings on adjustment costs.

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3See Appendix A.1 for the firm’s problem with linear vacancy posting costs $\psi(v_t) = \kappa v_t$.

4Several recent studies (Nilsen and Schiantarelli 2003; King and Thomas 2005; Merz and Yashiv 2005) provide evidence for the empirical relevance of convex adjustment cost functions at the macro level.

5In other words, firms do not internalize the impact of their hiring activities on the expected wage bill per worker $(\partial w_{t+1} l_{t+1})/(\partial n_{t+1}) = 0$. Nevertheless, firms anticipate the future wage bill per worker $w_{t+1} l_{t+1}$ correctly. See Section 2.5.1 for more information.
costs \((\kappa/2) x^2_{t+1}\), and expected savings on hiring costs \((1 - \sigma) \kappa x_{t+1}\). Savings on adjustment costs capture the fact that each marginal match increases the stock of employment in the next period, irrespective of when the match is terminated. On the contrary, savings on hiring costs are only realized if the match survives the following period.

Furthermore, the hiring condition captures one central aspect of our model. Since firms incur costs only for successful job matches, it is irrelevant how many vacancies that are posted are required to obtain the optimal gross hiring rate. In other words, vacancy posting \textit{per se} is costless.

### 2.4 The Resource Constraint

The following equation gives the resource constraint of our economy. The resource constraint postulates that output is divided into consumption, gross investment and labor adjustment costs:

\[
y_t = c_t + k_{t+1} - (1 - \delta) k_t + \frac{\kappa m^2_t}{2 n_t}. \tag{17}
\]

### 2.5 Wage Determination

#### 2.5.1 The Bargaining Set

The matching frictions in the labor market create a prospective mutual surplus between firm-worker matches. This surplus equals the value added of the match compared to the payoff of both parties on the labor market. Following Pissarides (2000, chapter 3), we assume that the wage bill per worker \(w_t l_t\) is determined for each match separately while wages in all other matches are taken as given. Hence, the relevant surplus share of the household and the firm, respectively, is determined by the marginal job match:

\[
W_2(\Omega^H_t) = \left\{ \lambda_t(w_t l_t + c^U_t - c^N_t) + (1 - \sigma) \beta E_t [W_2(\Omega^H_{t+1})] \right\} - \\
- \left\{ u^U(c^U_t, 1 - e) - w^N(c^N_t, 1 - l_t) + q(\gamma_t) \gamma_t \beta E_t [W_2(\Omega^H_{t+1})] \right\}, \tag{18}
\]

\[
V_1(\Omega^F_t) = (1 - \theta) F_2 l_t - w_t l_t + \frac{\kappa}{2} x^2_t + (1 - \sigma) \beta E_t [(\lambda_{t+1}/\lambda_t) V_1(\Omega^F_{t+1})]. \tag{19}
\]

The surplus share of the household \(W_2(\Omega^H_t)\) equals the difference between the value of employment and the value of unemployment. The value of employment is made up of the sum of the wage bill per worker and household’s expected present value of the match in the future. The value of unemployment, i.e. household’s outside alternative, consists of the current utility gain from leisure and household’s continuation payoff from labor market search. The surplus of the firm \(V_1(\Omega^F_t)\) is composed of \(i\) the contribution of the marginal worker to output \(ii\) the wage bill per worker \(iii\) savings on adjustment costs and \(iv\) the expected present value of the match in the future. Provided a non-arbitrage condition, the outside alternative of the firm (i.e. the ex-ante value of an unfilled vacancy) is zero. The sum of the marginal product per worker \(i\) and savings on adjustment costs \(iii\) is defined as gross flow value of the marginal match. Given that the weight of the marginal match is small, the firm takes the gross flow value of the marginal match as given during the bargaining process.

\(^6\)With perfect insurance against unemployment, the level of consumption is independent of the employment status \((c^N_t = c^U_t)\).
Thus, the mutual surplus \( S_t \) of the marginal firm-worker match (in units of the consumption good) is given as the sum of the two shares:

\[
S_t = \left( W_2(\Omega^H_t)/\lambda_t \right) + V_1(\Omega^F_t).
\]

The allocation of the mutual surplus between the household and the firm determines the wage bill per worker \( w_t l_t \). In order to satisfy individual rationality, the equilibrium wage bill per worker has to make each party at least indifferent between accepting the contract and the forgone outside alternative of continued labor market search. We obtain the reservation value of the household and the firm, respectively, by setting the surplus share equal to zero. Equation (18) shows that the reservation value of the household \( (wl)_{min} \) is given by the value of unemployment less household’s expected value of the match in the future:

\[
(wl)_{min} = \frac{1}{\lambda_t} \left\{ u(c_t^U, 1 - \epsilon) - u(c_t^N, 1 - l_t) + q(\gamma_t)\gamma_t \beta E_t \left[ W_2(\Omega^H_{t+1}) \right] \right. \\
- (1 - \sigma) \beta E_t \left[ W_2(\Omega^F_{t+1}) \right] \right\}. 
\] (20)

Analogously, the reservation value of the firm \( (wl)_{max} \) is defined as the gross flow value of the marginal match plus firm’s expected present value of the marginal match in the future:

\[
(wl)_{max} = (1 - \theta) F_{2,t} l_t + \frac{\kappa}{2} x_t^2 + (1 - \sigma) \beta E_t \left[ (\lambda_{t+1}/\lambda_t) V_1(\Omega^F_{t+1}) \right]. 
\] (21)

These two reservation values constitute the lower and the upper bound of the bargaining set which contains all feasible wage bills \(^{[19]}\)Malcomson, 1999\). In other words, the equilibrium value of the wage bill per worker is indeterminate. Therefore, we assume that wages are determined by an ex-post bargaining game between the household and the firm. In particular, we consider two alternative approaches – standard Nash (1953) bargaining and a Rubinstein (1982) game of alternating offers. In addition, the wage bill per worker is subject to continuous renegotiation whenever new information arrives. In our discrete-time model, this implies that new matches are formed at the end of each period. However, bargaining does not start until the beginning of the next period when the new state of technology can be observed.

### 2.5.2 The Optimal Wage Contract

For the standard job search and matching model (with linear vacancy posting costs), Hosios (1990) has established a necessary and sufficient condition under which both congestion externalities just offset one another. As mentioned above, the congestion externalities arise from the fact that firms take the aggregate vacancy filling rate \( q(\gamma_t) \) as given when deciding upon the optimal number of vacancies \( v_t \). Thus, from the firm’s perspective, the number of new job

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7Note that firms treat marginal hiring costs \( \kappa x_t \) as sunk costs. Hence, a firm would generate negative profits if it accepted a wage bill per worker close to \( (wl)_{max} \) \(^{[20]}\)Hall and Milgrom, 2006\). However, this possibility is ruled out by our calibration.

matches \( m_t = q(\gamma_t) v_t \) is linear in vacancies. Accordingly, the firm’s private gain of the marginal vacancy is given as (see Appendix A.1):

\[
\kappa = q(\gamma_t) \nu_1(\Omega_t^F).
\]  

\( (22) \)

In contrast, the social planner solution accounts for the fact that new job matches are a concave function of vacancies (see equation (1)). Hence, the social planner internalizes that the vacancy filling rate decreases in the number of vacancies that are posted. Therefore, the marginal vacancy yields following social benefit (see Appendix A.2):

\[
\kappa = \alpha q(\gamma_t) S_t.
\]  

\( (23) \)

Social optimality requires that firms’ private gains from search effort equals their social benefit. Consequently, firms’ incentives to post vacancies are efficient if and only if

\[
\nu_1(\Omega_t^F) = \alpha S_t
\]  

holds. In words, the Hosios (1990) condition postulates that the private gain per match equals the share \( \alpha \) of the mutual surplus \( S_t \) per match.

In contrast, if firms internalized the congestion effect on the aggregate vacancy filling rate correctly, social optimality would require firms to gain the entire mutual surplus per match, i.e.

\[
\nu_1(\Omega_t^F) = S_t.
\]  

\( (25) \)

However, if firms gained the entire mutual surplus, even though they did not internalize the congestion effects, the private gain from the marginal vacancy would be larger than the social benefit. Thus, firms would be likely to overhire. For this reason, in order to avoid the overhiring effect, the Hosios (1990) condition requires that firms gain only the share \( \alpha \) of the mutual surplus per match.

Under convex labor adjustment costs, on the contrary, firms’ hiring costs depend on the number of vacancies that are filled \( q(\gamma_t) v_t \), and not on the number of vacancies that are posted \( v_t \). Consequently, the congestion externalities bias not only firms’ private gains, but also – to the same extent – firms’ hiring costs. This removes firms’ incentives to overhire, even if they gained the entire mutual surplus per match. Under these circumstances, the congestion externalities exactly offset each other if and only if the entire mutual surplus per match accrues to the firms (see Appendix A.3), i.e. if equation (25) holds.

2.5.3 Nash Bargaining

Nash bargaining has become the standard method for wage determination in job matching models. This approach postulates a number of axioms and derives a unique equilibrium sharing rule for the mutual surplus. In addition, Nash (1953) proves that exactly the same solution can be generated by a simultaneous one-shot game. This bargaining game presumes that both parties threaten each other to terminate the bargain unilaterally rather than to conclude an agreement. Subsequently, both parties simultaneously reveal their demands. If these demands
are not compatible, the match is broken up and both players gain only their respective outside alternative, i.e. they return to labor market search. However, given perfect information and rational players, Nash (1953) shows that both parties agree on following unique sharing rule:

$$w_t \ell_t = \arg \max_{w_t \ell_t} \left\{ \left( \frac{W_2(\Omega^H_t)}{\lambda_t} \right)^{1-\xi} \left( \frac{V_1(\Omega^F_t)}{\xi} \right)^{\xi} \right\} , \quad (26)$$

where the original version assumes symmetric bargaining power ($\xi = 1/2$). The generalized version, however, allows any value for $\xi$ in the interval $(0,1]$. Hence, the solution to our model is given by the wage bill per worker which maximizes the weighted product of both parties’ surplus shares. This sharing rule allocates period-by-period a constant share of the mutual surplus to each of the two parties:

$$\xi \left( \frac{W_2(\Omega^H_t)}{\lambda_t} \right) = (1 - \xi)\frac{V_1(\Omega^F_t)}{\xi} . \quad (27)$$

Hence, in the case of linear vacancy posting costs, the Nash solution generates the socially optimal bargaining outcome if and only if firms’ bargaining power $\xi$ coincides with the matching elasticity of vacancies $\alpha$. With convex labor adjustment costs, however, social optimality requires that the entire match surplus $S_t$ accrues to the firms (i.e. $\xi = 1$). In words, this implies that the wage bill per worker $w_t \ell_t$ is equal to household’s outside alternative. Nevertheless, we consider the general case $\xi \in (0,1]$ throughout our analysis.

The resulting wage bill per worker equals the weighted average of the gross flow value of the marginal match and household’s outside alternative:

$$w_t \ell_t = (1 - \xi) \left[ (1 - \theta) \frac{y_t}{n_t} + \frac{m_t \kappa x_t}{1 - n_t} \right] + \xi \left[ \frac{u^U_t - u^N_t}{\lambda_t} + \frac{(1 - \xi)}{\xi} \frac{m_t \kappa x_t}{1 - n_t} \right] . \quad (28)$$

Household’s outside alternative depends on the flow value of unemployment (i.e. the current utility gain in leisure $(u^U_t - u^N_t)/\lambda_t$) and the continuation payoff from labor market search. The latter, in turn, depends on the current job finding rate $m_t/(1 - n_t)$ times her adjusted share $(1 - \xi)/\xi$ of the expected present value of a prospective future match $\kappa x_t$. Consequently, household’s outside alternative is very sensitive to current labor market conditions.

Notably, the expected present value of the current match (see equations (18) and (19)) does not enter equation (28). This is due to the fact that the mutual surplus is always allocated according to the same sharing rule (26). Hence, both expressions widen the bargaining set proportionally, but have no impact on the bargaining outcome. We define the replacement rate $b$ as the ratio between the flow value of unemployment and the gross flow value of the marginal match.

### 2.5.4 Strategic Wage Bargaining

Hall and Milgrom (2006) highlight that Nash bargaining abstracts from the dynamic and interactive character of wage negotiations. For that reason, they argue that wages in the job matching model should be determined by a Rubinstein (1982) game of alternating offers. In particular, Hall and Milgrom (2006) emphasize the crucial importance of the prospective mu-

Note that equations (25) and (26) are not defined for $\xi = 0$. 

9
tual surplus. The mutual surplus gives both players strong incentives to conclude the bargaining successfully. Hence, neither party seriously considers to break up the bargaining process completely. Given perfect information, this implies that threatening the opponent to terminate the bargaining process is not an credible option (Schelling, 1960). Instead, both parties threaten each other to reject unfavorable demands. Since both parties are impatient, this strategy causes costly delays and gives them the incentive always to make acceptable demands. Consequently, once a firm-worker match has successfully been formed, it is the value of bargaining – and not the outside alternative – that determines the relevant surplus.

In their analysis, Hall and Milgrom (2006) focus on the limiting case in which the time interval between successive offers decreases to zero. Under these circumstances, they show that both parties agree on the equilibrium wage bill per worker instantaneously. This allows us to approximate the solution to the dynamic bargaining game by a corresponding static game (Binmore et al., 1986). The solution to this new game can be found by maximizing the weighted product of the two surplus shares – like in the standard Nash solution. However, the solution to this dynamic bargaining problem is inherently different from the Nash solution as the surplus of each party is no longer determined by the respective outside alternative, but by the losses associated with delays.

Following Hall and Milgrom (2006), we calibrate the dynamic bargaining model to the same steady state as the standard bargaining model. This simplifying assumption implies that the steady state value of bargaining coincides with the outside alternative. Furthermore, Hall and Milgrom (2006) emphasize that the value of bargaining might depend less sensitive on current labor market conditions than the outside alternative. Thus, they take the value of bargaining to be time-invariant. For this reason, we replace all variables in equation (28) that derive from the outside alternative with their steady state values (denoted by an overline):

\[ w_t l_t = (1 - \xi) \left[ (1 - \theta) \frac{y_t}{n_t} + \frac{\kappa}{2} x_t^2 \right] + \xi \left[ \frac{\bar{u}^U - \bar{u}^N}{\lambda} + \frac{(1 - \xi)}{\xi} \frac{\bar{m} \bar{\kappa} \bar{x}}{(1 - \bar{n})} \right]. \tag{29} \]

This sharing rule is equivalent to Nash bargaining with a constant outside alternative. Given that the outside alternative is typically procyclical, the dynamic bargaining game generates a less elastic wage bill per worker than Nash bargaining. Hence, the households’ share of the surplus falls below \((1 - \xi)\) in economic upswings (and vice versa). Note that the wage bill per worker satisfies individual rationality as long as it remains within the bargaining set.

In conclusion, strategic wage bargaining gives rise to two distortionary effects. As discussed above, social optimality under convex labor adjustment costs requires that the wage bill per worker \(w_t l_t\) is equal to household’s outside alternative. In contrast, we assume that (i) the wage bill per worker is independent of the fluctuations in household’s outside alternative (ii) firms’ bargaining power \(\xi\) is generally smaller than unity, i.e. \(\xi \in (0, 1]\). Hence, firms’ private gains from search effort are generally smaller than their social contribution. In this case, the dynamic behavior of the wage bill per worker is not socially optimal (Hosios, 1990).
2.6 Optimal Labor Effort

The model is closed with the condition for optimal labor effort $l_t$ ('hours'). We assume that both parties have a joint interest to maximize the value of the mutual surplus $S_t$. Provided that the marginal product of labor $F_{2,t}$ is taken as given by both parties, the maximization problem of $S_t$ with respect to $l_t$ yields following condition:

$$
(1 - \theta) \frac{y_t}{n_t l_t} = \frac{1}{\lambda_t (1 - l_t)^y}.
$$

(30)

This condition determines how the wage bill per worker is split up into the real wage rate per 'hour' $w_t$ and 'hours' per worker $l_t$.

2.7 Competitive Equilibrium

The competitive equilibrium is a set of allocations $\{c_t, k_{t+1}, v_t, n_{t+1}\}$ and prices $\{r_t, w_t\}$, such that:

(i) employment relationships are governed by the matching function and the law of motion of employment

(ii) $\{c_t, k_{t+1}\}$ solves the household’s problem subject to the budget constraint and the law of motion for its employment share

(iii) total factor productivity follows the exogenous stochastic process

(iv) $\{k_t, v_t\}$ solves the firm’s problem subject to the production technology and the law of motion for its stock of employment

(v) the resource constraint holds and the perfectly competitive capital market clears

(vi) the wage bill per worker is determined either by Nash bargaining or by strategic wage bargaining

(vii) hours per worker maximize the mutual surplus

(viii) an initial condition for the state space $(k_0, n_0, z_0)$ is given

Consequently, the competitive equilibrium is defined by following conditions: and the law of motion for its stock of employment, either or , and (30).

3 Model Evaluation

3.1 Calibration

We calibrate the model so that one period corresponds to a month. This seems advantageous given that, in the U.S., the job finding rate is very high. When we simulate the model, we time-aggregate the artificial data to quarterly data in order to make it comparable to the U.S. aggregate time series. Table summarizes the parameter values of our model.

Using data on aggregate income shares, Cooley and Prescott (1995) calibrate the production elasticities of capital ($\theta = 0.40$) and labor ($1 - \theta = 0.60$). We adopt their conventional values, this follows from the fact that the weight of the marginal match is small.
even though the production elasticity of labor is slightly larger than the average labor share in job matching models (0.58, see Table 2). In addition, we set the monthly depreciation rate $\delta$ to match an annual rate of 10% (Kydland and Prescott, 1982).

$\beta$ is chosen to be consistent with a quarterly real interest rate of 1 percent. Following Juster and Stafford (1991), we set the steady state working time of employed household members to $\bar{l} = 1/3$ of their discretionary time endowment. Moreover, Barron and Gilley (1981) estimate that the typical unemployed primarily engaged in random job search (approximately one half of the sample) spends between 8 and 9 hours per week to contact potential employers. This corresponds to about 25% of the average working time $\bar{l}$. For the given specification of preferences (Andolfatto, 1996), the elasticity of intertemporal substitution in labor supply is given as: $\nu = \eta^{-1}[(1/\bar{l}) - 1]$. Blundell and Macurdy (1999) provide robust evidence that the value of $\nu$ for annual hours of employed men is between 0.1 and 0.3. For employed women, Blundell et al. (1988) and Triest (1990) estimate values in the same range. However, Browning et al. (1999) observe that leisure is more substitutable over shorter intervals than longer ones. Using monthly data on employed men, MaCurdy (1983) finds significantly higher elasticities (0.3 – 0.7). Hence, we choose $\nu$ equal to 0.5, which implies setting $\eta = 4$.

We calibrate the monthly job separation rate to 3.5% (Shimer, 2005b). This implies that the average job duration is $2 \frac{1}{2}$ years. Furthermore, the steady state unemployment rate is set to 10 percent (Hall, 2005b). This measure includes the officially unemployed job searchers and the pool of marginally attached non-participants (Jones and Riddell, 1999). Thus, our calibration implies that the average job finding rate, $q(\gamma)$, is equal to 0.32 (see Table 2, which is consistent with the results of Hall (2005b). The monthly vacancy rate is set to match the quarterly value $q(\gamma) = 0.71$ estimated by van Ours and Ridder (1992). Based on the fact that

\begin{table}
\centering
\begin{tabular}{|l|c|c|}
\hline
Description & Variable & Value & Source \\
\hline
Technology & production elasticity of capital & $\theta$ & 0.40 & Cooley and Prescott (1995) \\
& depreciation rate & $\delta$ & 0.0083 & Kydland and Prescott (1982) \\
\hline
Preferences & discount factor & $\beta$ & 0.9967 & Kydland and Prescott (1982) \\
& working time per worker & $\bar{l}$ & 1/3 & Juster and Stafford (1991) \\
& effort per job seeker & $e$ & 1/12 & Barron and Gilley (1981) \\
& individual labor supply & $\nu$ & 0.5 & MaCurdy (1983) \\
\hline
Labor Market & job destruction rate & $\sigma$ & 0.035 & Shimer (2005b) \\
& unemployment rate & $1 - \pi$ & 0.10 & Hall (2005b) \\
& vacancy filling rate & $q(\gamma)$ & 0.3381 & van Ours and Ridder (1992) \\
& adjustment costs/output ratio & $\psi$ & 0.01 & Hamermesh and Pfann (1996) \\
& matching elasticity of vacancies & $\alpha$ & 0.5 & Petrongolo and Pissarides (2001) \\
& firm's bargaining power & $\xi$ & 0.5 & Svejnar (1986) \\
\hline
Technology Shock & 1st order autocorrelation & $\rho$ & 0.9830 & Cooley and Prescott (1995) \\
& standard deviation & $\sigma_\epsilon$ & 0.004395 & Cooley and Prescott (1995) \\
\hline
\end{tabular}
\caption{The Monthly Parametrization of the Model}
\end{table}
per-period labor adjustment costs "are not much more than one percent of per-period payroll cost" (Hamermesh and Pfann, 1996), we calibrate average labor adjustment costs $\bar{\psi}$ equal to 0.01. This implies that the average replacement ratio $\bar{\psi}$ is equal to 63% (see Appendix A.2). This value is somewhat larger than the upper bound ($\bar{b} = 40\%$) estimated by Shimer (2005a). However, Shimer (2005a) interprets $b$ entirely as an unemployment benefit. In our model $b$ includes also utility costs of working, e.g. leisure value of unemployment or the value of home production (Hagedorn and Manovskii, 2006). Unfortunately, empirical evidence on the size of hiring costs is scarce (Holmlund, 1998). According to Costain and Reiter (2005), the utility costs of working might sum up to $\bar{b} = 75\%$. Hence, our value seems reasonable.

We calibrate the matching elasticity of unemployment to $\alpha = 0.5$. This value is within the plausible range ($0.5 - 0.7$) proposed by Petrongolo and Pissarides (2001). In addition, we assume symmetrically distributed bargaining power, i.e. $\xi = 0.5$ (Svejnar, 1986). Hence, as mentioned above, our model gives rise to two distortionary effects. On the one hand, we assume that the wage bill per worker is independent of the fluctuations in household’s outside alternative. On the other hand, firms’ bargaining power $\xi$ is strictly smaller than unity. Hence, their private gains from search effort are generally smaller than their social contribution (Hosios, 1990). Nevertheless, setting $\alpha = \xi$ makes it easier to compare our results with the existing literature.

We calibrate the law of motion for the technology shock by setting the monthly autocorrelation coefficient $\rho$ equal to 0.98 and the standard deviation $\sigma_\epsilon$ equal to 0.0044. The autocorrelation coefficient is chosen to match the conventionally used quarterly value of 0.95 (Cooley and Prescott, 1995). Furthermore, we set the standard deviation of the monthly process so that the volatility of the time-aggregated Solow-residual is in accordance with a standard-calibrated quarterly real business cycle model (Cooley and Prescott, 1995).

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>job finding rate</td>
<td>$q(\gamma)$</td>
<td>0.3150</td>
<td>vacancies</td>
<td>$\bar{v}$</td>
<td>0.0932</td>
</tr>
<tr>
<td>matches</td>
<td>$\bar{m}$</td>
<td>0.0315</td>
<td>gross hiring rate</td>
<td>$\bar{x}$</td>
<td>0.0350</td>
</tr>
<tr>
<td>hiring costs parameter</td>
<td>$\kappa$</td>
<td>18.1406</td>
<td>matching function parameter</td>
<td>$\chi$</td>
<td>1.1305</td>
</tr>
<tr>
<td>real interest rate</td>
<td>$\bar{r}$</td>
<td>0.0034</td>
<td>capital</td>
<td>$\bar{k}$</td>
<td>34.2200</td>
</tr>
<tr>
<td>investment</td>
<td>$\delta k$</td>
<td>0.2852</td>
<td>consumption</td>
<td>$\bar{c}$</td>
<td>0.7048</td>
</tr>
<tr>
<td>aggregate wage bill/labor share</td>
<td>$\bar{w}l$</td>
<td>0.5881</td>
<td>wage bill per worker</td>
<td>$\bar{w}$</td>
<td>0.6534</td>
</tr>
<tr>
<td>production function parameter</td>
<td>$\zeta$</td>
<td>0.5012</td>
<td>real wage</td>
<td>$\bar{l}$</td>
<td>1.9603</td>
</tr>
<tr>
<td>household’s reservation value</td>
<td>$(\bar{wl})_{min}$</td>
<td>0.0164</td>
<td>firm’s reservation value</td>
<td>$(\bar{wl})_{max}$</td>
<td>1.2905</td>
</tr>
<tr>
<td>leisure parameter employed</td>
<td>$\phi_1$</td>
<td>0.5605</td>
<td>leisure parameter unemployed</td>
<td>$\phi_2$</td>
<td>0.0504</td>
</tr>
<tr>
<td>leisure exponent</td>
<td>$\eta$</td>
<td>4</td>
<td>replacement ratio</td>
<td>$\bar{b}$</td>
<td>0.6331</td>
</tr>
</tbody>
</table>

Table 2: Implied Steady State Values

Notice that the calibration implies that the reservation value of the firm $(\bar{wl})_{max}$ is larger than the reservation value of the household $(\bar{wl})_{min}$.  

13In the model with linear vacancy posting costs, $\bar{\psi} = 0.01$ implies that the average replacement ratio is equal to 81%. This value is slightly larger than the one $(\bar{b} = 75\%$) suggested by Costain and Reiter (2005), but still below the estimate $\bar{b} = 94\%$ of Hagedorn and Manovskii (2000). Nevertheless, the choice for $b$ is crucial, because a larger $b$ decreases the surplus. Consequently, the higher the value of $\bar{b}$, the easier it is to leverage a given shock into labor market fluctuations.

14The social planner’s problem is documented in Appendix A.2.

15Cooley and Prescott (1995) estimate the quarterly parameters ($\rho = 0.95$, $\sigma_\epsilon = 0.007$) assuming that the labor income share equals $1 - \theta$. In labor search models, this assumptions holds only as an approximation.
3.2 Results

This section examines the quantitative performance of the modified job matching model. We analyze how the chosen wage determination mechanism and the costs of labor adjustment, respectively, affect the dynamics of the model in response to technology shocks. In particular, we focus on the interactions between both modifications.

We evaluate the model generated time series against quarterly U.S. data from 1964:1 to 1999:4. Most of the time series are from the Federal Reserve Bank of St. Louis (FRED®). In addition, we use the expanded unemployment series from Hall (2005b). From this data we construct a set of time series which corresponds to the variables in our model (see Appendix A.6). We log and detrend all series using the Hodrick and Prescott (1997) filter assuming a smoothing parameter 1600.

Table 3 reports the well-known business cycle statistics of the U.S. labor market. In the first place, we focus on the cyclical behavior of vacancies $v$ and unemployment $1 - n$. The data reveal that both variables are highly volatile and very persistent. In addition, vacancies are strongly procyclical whereas unemployment is strongly anticyclical. Consequently, vacancies and unemployment are almost perfectly negatively correlated ($\rho_{vu} = -0.95$). Due to the strong persistence of both variables, we observe that the negative correlation between vacancies and unemployment remains also at leads and lags (Fujita, 2005). Hence, the dynamic correlation structure between vacancies and unemployment follows a pronounced U-shape (see Figure 3). This pattern is known as the "dynamic Beveridge Curve". Furthermore, we observe that the wage bill per worker $w_l$ is significantly less volatile and less procyclical than output per worker $y/n$.

To begin with, we log-linearize the model around the steady state (see Appendix A.5) and solve for the recursive law of motion using the 'Toolkit' from Uhlig (1998). Corresponding to the U.S. data sample period, we simulate the model to 432 "monthly" data points. Subsequently, we transform the artificial data as described above and compute the statistics over 10,000 simulations.

3.2.1 Comparative Impulse Response Analysis

We now inspect the model’s impulse responses to an one percent shock in total factor productivity. In particular, we explore the role of the chosen wage determination mechanism and the costs of labor adjustment, as well as the interactions between them. Therefore, Figure 1 compares the model’s impulse responses (solid line) with the corresponding Nash solution (dashed line) and the strategic wage bargaining model under linear vacancy posting costs (dotted line). The graphs depict the evolution of the relevant variables over 96 months (32 quarters).

**Strategic Wage Bargaining under Convex Labor Adjustment Costs**

The hiring condition (16) reveals that the amplitude and pattern of cyclical employment adjustment is governed by two main determinants: First, the net flow value of the marginal match captures cyclical variations in the return to additional employment. Second, the structure...
of labor adjustment costs determines how fast and at what cost firms adjust employment over the business cycle. Hence, in the following, we focus on the impact of both factors.

In response to the technology shock, we note that the gross flow value of the marginal match rises by about one percent. The elasticity of the wage bill per worker, in contrast, is significantly lower. This follows directly from strategic wage bargaining (see equation (29)). Accordingly, the elasticity of the wage bill per worker is given by the elasticity of the gross flow value of the marginal match times household’s bargaining power \((1 - \xi)\). As a result, the costs per worker increase much less than the benefits. Hence, we observe that the net flow value of the marginal match increases by about 17 percent, giving firms strong incentives to adjust employment over the business cycle.

In the case of convex labor adjustment costs, firms choose the optimal number of new job matches \(m_t\) that equalizes expected marginal matching costs \(\kappa x_t\) and the expected present value of the marginal match. However, new job matches seem to be much less elastic than the net flow value of the marginal match. This is due to the convex shape of \(\psi_t\) which induces firms to smooth hiring over several periods. Indeed, new job matches rise on impact by somewhat more than 4 percent and then remain well above their steady state value for the entire observation period. This continuous inflow of new job matches leads to a pronounced hump-shape in the impulse response function of employment, which peaks about 2 1/2 years after the shock. Accordingly, the impulse response function of unemployment follows a distinct U-shape.

Moreover, the strong reaction in employment feeds back to the expected marginal matching costs. Given that employment \(n_t\) is a state variable, the impulse response function of expected marginal hiring costs increases on impact by exactly the same amount as new job matches. In the following period, however, the long-lasting increase in employment has a dampening effect on the expected marginal hiring costs. Hence, we observe that their impulse response converge relatively quickly to their steady state value. This pattern gives firms incentives to smooth their hiring activities and thus, might explain the remarkable slow convergence of new job matches.

Finally, we analyze the impulse response function of vacancies. As mentioned above, we assume that firms’ hiring costs depend on the number of vacancies that are filled, and not on the number of vacancies that are posted. Therefore, firms always post the number of vacancies that is necessary to obtain the optimal number of new job matches. According to the aggregate matching function (see equation (14)), the number of new job matches is given by the current level of unemployment and the number of vacancies that are posted. In response to a positive technology shock, firms face following scenario: On the one hand, firms have to maintain a continuous inflow of new job matches. On the other hand, the impulse response function of unemployment decreases sharply over more than 2 1/2 years. This leads to a strong fall in the vacancy filling rate. The lower the vacancy filling rate, the more vacancies have to be posted in order to obtain the optimal number of new matches. Thus, we observe that the impulse response function of vacancies increases on impact by about 10 percent. In the following periods, the vacancies continue to rise and reach a maximum 18 percent increase with 2 1/2 years delay. Hence, the impulse response of vacancies follows a marked hump-shape. This pattern is found to be consistent with the data.\(^{18}\)

\(^{18}\)See, amongst others, Blanchard et al. (1989), Fujita (2005), Braun et al. (2006), and Ravn and Simonelli (2006).
The Impact of Strategic Wage Bargaining

In order to illustrate the impact of strategic wage bargaining on the cyclical behavior of vacancies and unemployment, we now discuss the model’s impulse responses if wages are determined by the corresponding Nash solution. Under Nash bargaining, the elasticity of the wage bill per worker is not only determined by the gross flow value of the marginal match, but also by household’s outside alternative. Given that household’s outside alternative is clearly procyclical, we note that the elasticity of the wage bill per worker increases substantially. As a result, the wage bill per worker is nearly as elastic as the gross flow value of the marginal match. In other words, the costs per worker increase almost as much as the benefits. Thus, the elasticity of the net flow value of the marginal match decreases enormously. Additionally, due to the hump-shape in the household’s outside alternative, the net flow value of the marginal match is also less persistent than under strategic wage bargaining. In summary, Nash bargaining gives firms only little incentive to hire new workers.

Indeed, we observe that firms’ hiring activities decline dramatically. On impact, new job matches rise only by less than one percent and then fall back quickly to their steady state value. Consequently, the impulse response of employment is substantially smaller. For the same reason, the U-shaped response of unemployment is much weaker. Moreover, due to the mild response of employment, the feedback effect from employment on lower expected marginal matching costs is almost not present.

The modest increase in matches, in conjunction with the weak reduction in unemployment, implies that the vacancy filling rate reduces only slightly. Thus, vacancies rise on impact only by somewhat more than one percent, continue to increase slightly for about 3 quarters, and then return slow and monotone to their steady state value. Hence, vacancies are much less elastic than under strategic wage bargaining. In addition, we observe that the hump-shaped dynamics of the impulse response functions are less distinct.

Therefore, we conclude that strategic wage bargaining amplifies the elasticity of employment, unemployment and vacancies enormously. In addition, the hump-shaped (U-shaped) responses of labor market variables are more distinct than under Nash bargaining.

The Impact of Convex Labor Adjustment Costs

In the following, we examine the impact of convex labor adjustment costs on the dynamic behavior of the labor market. Therefore, we compare the model’s impulse response functions with the impulse responses under linear vacancy posting costs. In both cases under consideration, the wage bill per worker is determined by strategic wage bargaining. Hence, we observe that the instantaneous elasticities of the gross flow value of the marginal match, household’s outside alternative, and the wage bill per worker, respectively, are very similar. However, the relative response of the net flow value of the marginal match under linear vacancy posting costs is significantly larger than under convex labor adjustment costs. Consequently, one should expect that employment fluctuations under linear vacancy posting costs are more amplified than under convex labor adjustment costs.

\[ \text{Note that the net flow value of the marginal match under convex labor adjustment costs is about twice as large as under linear vacancy posting costs (see Footnote 13). Thus, the absolute response is about equally large under both specifications.} \]
In fact, we note that the instantaneous elasticity of new job matches under linear vacancy posting costs is about three times as large as under convex labor adjustment costs. In the following periods, however, firms’ hiring activities decrease sharply. Thus, the impulse response function of employment peaks already after about 9 months. This is due to the modified hiring mechanism. Given linear vacancy posting costs, firms post vacancies in order to equalize expected marginal hiring costs $\kappa/q(\gamma_t)$ and the expected present value of the marginal match. In contrast to the case of convex labor adjustment cost, expected marginal hiring costs now depend on the inverse vacancy filling rate $1/q(\gamma_t)$ and not on the gross hiring rate $x_t$. In response to the technology shock, we observe that the inverse vacancy filling rate increases by about 13 percent and then remains persistently well above its steady state value over the whole observation period. This behavior differs substantially from the rather moderate and temporary increase of the gross hiring rate under convex labor adjustment costs.

Since firms are forward looking, they anticipate the future fall in unemployment when deciding upon the optimal number of vacancies. The future fall in unemployment tightens the labor market and, thus, raises the expected marginal matching costs in the future. For this reason, firms post vacancies instantaneously as long as the number of unemployed job searchers is still high. This pattern makes it impossible for the model to generate a hump-shaped impulse response function of vacancies. Instead, vacancies spike on impact and fall back half way only one period later. This behavior is in sharp contrast to the empirical evidence.

Under convex labor adjustment costs, however, the mechanism works into the other direction. In this case, firms’s expected marginal hiring costs depend on the gross hiring rate $x_t$. Thus, a high level of employment (i.e. a low level of unemployment) lowers expected marginal costs. In comparison to the inverse vacancy filling rate, the gross hiring rate is much less elastic and much less persistent. This removes firms’ incentive to adjust employment instantaneously. On the contrary, it gives firms strong incentives to smooth hiring over a long period. Hence, we observe that the overall employment impact under linear vacancy costs is substantially lower than under convex labor adjustment costs.

The Interactions between Strategic Wage Bargaining and Convex Labor Adjustment Costs

So far, we have seen that (i) strategic wage bargaining amplifies labor market fluctuations (ii) convex labor adjustment costs account for hump-shaped impulse response functions. Indeed, beyond understanding how both modifications work in isolation, it is important to explore their interactions.

As discussed above, the impact of current labor market conditions on expected marginal matching costs depends crucially on the specification of firms’ hiring costs. Under linear vacancy posting cost, a tight labor market raises expected marginal matching costs. Instead, under convex labor adjustment costs, a high level of employment lowers expected marginal matching costs. Importantly, the quantitative impact of this effect is the more evident the larger the fluctuations in the labor market. Consequently, strategic wage bargaining amplifies the elasticity of labor market variables through two channels. On the one hand, strategic wage bargaining dampens the cyclical fluctuations in the wage bill per worker. This stimulates hiring of new

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20See Appendix A.1 for the firm’s problem with linear vacancy posting costs.
workers. On the other hand, the higher the stock of employment, the lower the costs of labor adjustment. Thus, the introduction of convex labor adjustment costs induces not only more persistence, but increases also the elasticity of labor market variables.

However, if we add strategic wage bargaining to convex labor adjustment costs, we observe that labor market variables not only more elastic, but also more persistent. This is due to the fact that strategic wage bargaining removes the impact of the hump-shaped outside alternative. Hence, the net flow value of the marginal match is more persistent, translating into more persistent labor market fluctuations. In summary, we note that strategic wage bargaining and convex labor adjustment costs are complementary in generating elastic and persistent labor market variables.

**Robustness of the Hump-Shaped Vacancy Dynamics**

In the following, we analyze the robustness of the hump-shaped vacancy dynamics. In particular, we focus on the impact of the two distortionary effects. As shown in a previous paragraph, vacancies under Nash bargaining are much less elastic than under strategic wage bargaining. In addition, the hump-shape is flattened considerably.

Next, we set firms’ bargaining power in our model equal to unity. On impact, we note that vacancies rise by about 15 percent. This increase is about one and a half times higher than in the case of symmetrically distributed bargaining power. Moreover, vacancies display a marked hump-shape, albeit the hump is slightly weaker than in our model.

Indeed, the solution to the model with convex labor adjustment costs is socially optimal if and only if the wage bill per worker equals household’s outside alternative. This condition is only satisfied if we assume Nash bargaining and set firms’ bargaining power equal to unity. We now observe that vacancies increase on impact by about 7 percent, reach a maximum with about 3 quarters delay, and then return relatively quickly to their steady state value. These results indicate that the combination of both distortionary effects dampens the hump to some degree. Nevertheless, we note that hump-shaped vacancy dynamics are a robust result if convex labor adjustment costs are assumed.

**3.2.2 Simulation Results**

This section evaluates our model against U.S. data. Thereby, our analysis focuses on the cyclical behavior of vacancies and unemployment (Table 3). In particular, we examine the model in terms of its capability to generate sufficient volatility and persistence in both variables.

**Strategic Wage Bargaining under Convex Labor Adjustment Costs**

Strategic wage bargaining makes the wage bill per worker independent of fluctuations in household’s outside alternative. Hence, the wage bill per worker \((w)\) is significantly less volatile than output per worker \((y)\), giving firms strong incentives to expand their hiring activities in economic upswings. Thus, we observe that the model resembles the cyclical volatility of vacancies \((v)\), unemployment \((1 - n)\) and market tightness \((\gamma)\) closely. This result is in line with the insight in [Hall and Milgrom (2006)](Hall and Milgrom (2006)): Strategic wage bargaining generates endogenous real wage rigidity. This increases the volatility of the net flow value of the marginal match. As a result, labor market variables become more volatile.

18
Furthermore, we note that vacancies, unemployment and market tightness are highly persistent. This can be ascribed to the modified hiring condition which alters the qualitative pattern of firms’ hiring behavior decisively. As a result, the model generates hump-shaped responses in unemployment and vacancies. For the same reason, we observe that the model is capable to replicate the U-shaped pattern of the dynamic Beveridge Curve (see Table 9 and Figure 3 in the Appendix). Consistent with the data, the negative relation between model generated vacancies and unemployment remains for more than 4 quarters.

Apart from that, the model accounts for the fact that unemployment and market tightness lag the cycle by one quarter. This indicates that the combination of strategic wage bargaining and convex labor adjustment costs enhances the model’s ability to propagate technology shocks in the labor market. However, the model cannot match the cyclical comovement of two other variables – output per worker and the wage bill per worker. In the data, the contemporaneous correlation between output and output per worker is close to unity. The wage bill per worker, on the other hand, shows a much weaker contemporaneous correlation with output. Consequently, both variables are only moderately positively correlated (see Table 4). In the model, on the contrary, we observe that output per worker and the wage bill per worker are perfectly correlated.

<table>
<thead>
<tr>
<th></th>
<th>U.S. Data</th>
<th>SB Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.12</td>
<td>-0.17</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>-0.07</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>0.09</td>
</tr>
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<td></td>
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</tr>
<tr>
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<td>1.00</td>
</tr>
<tr>
<td></td>
<td>0.53</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>0.46</td>
<td>0.33</td>
</tr>
<tr>
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<td>0.09</td>
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<td></td>
<td>0.27</td>
<td>-0.06</td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

Table 4: The Dynamic Cross-Correlation Pattern. The table shows the correlation coefficients between the wage bill per worker \((w_t l_t)\) and output per worker \(y_{t+k}/n_{t+k}\), lagged respectively leaded by \(k\) quarters.

Indeed, note that the almost perfect correlation between output per worker and the wage bill per worker is generated essentially by construction. Equation (29) shows that variations in the wage bill per worker are closely related to changes in output per worker. Since this paper is motivated by the cyclical behavior of vacancies and unemployment, we allow only for total

Table 3: Simulation Results. This table shows the results of the model simulations. For each variable, we report the relative standard deviation \((\sigma_X/\sigma_Y)\), the first order autocorrelation \((\rho_{X,T,X_{T+1}})\), the phase shift relative to output \((\rho_{X,Y})\), and the contemporaneous correlation with output \((\rho_{XY})\).
factor productivity shocks in our model. We conjecture, though, that adding a shock to the value
of bargaining may help to bring the comovement of labor market variables closer to the data.
Alternatively, (Knabe 2005) addresses the determinants of the value of bargaining explicitly.
This analysis provides an endogenous explanation for fluctuations in the value of bargaining.
However, the study of such questions is beyond the scope of our analysis.

The Impact of Strategic Wage Bargaining

If the wage bill per worker is determined by Nash bargaining, both parties receive period-by-period a constant share of the mutual surplus. Hence, the wage bill per worker is almost as volatile as output per worker, giving firms little incentive to adjust employment over the business cycle. This contrasts sharply with the data. Consequently, the cyclical fluctuations of vacancies and unemployment are insufficiently small. The same applies to market tightness, confirming the conclusion reached by Shimer (2005a).

Indeed, the Nash bargaining assumption does not alter the qualitative pattern of employment adjustment. Hence, the model generated time series of vacancies and unemployment remain almost as persistent as under strategic wage bargaining. As a result, the model generated dynamic Beverdige Curve maintains the U-shaped pattern. However, we note that the negative relation between vacancies and unemployment remains now only for somewhat more than 3 quarters (instead of more than 4 quarters in the data). This might be due to the fact that Nash bargaining reduces not only the volatility, but also the persistence of the net flow value of the marginal match.

The Impact of Convex Labor Adjustment Costs

Due to strategic wage bargaining, we observe that the wage bill per worker is about half as volatile as output per worker, giving firms strong incentives to amplify hiring activities. This result holds independently of the hiring cost function. Under linear vacancy costs, however, we observe that vacancies spike on impact and fall back very quickly. Thus, the cumulative inflow of new job matches is much weaker, inducing less volatility in employment, unemployment and market tightness than under convex labor adjustment costs.

By the same token, labor market variables under linear vacancy posting costs are much less persistent than under convex labor adjustment costs. This pattern can be ascribed to the modified hiring condition. Given linear vacancy posting costs, firms anticipate the fall in the vacancy filling ratio and, hence, adjust employment instantaneously. On the contrary, convex labor adjustment costs give firms strong incentives to smooth their hiring activities over several periods. This leads to a continuous inflow of new job matches, generating highly persistent labor market variables.

In particular, the first order autocorrelation of vacancies under linear vacancy posting costs is much weaker than under convex labor adjustment costs. This follows directly from the counterfactual shape of the model generated impulse response function. For the same reason, the shape of the model generated dynamic Beverdige Curve is biased. Despite the strong negative contemporaneous correlation, the cross-correlation between unemployment and leaded vacancies is close to zero beyond 2 quarters. Hence, the model predicts rather a J-shaped relationship, echoing the findings of Fujita (2005).
We summarize that (i) strategic wage bargaining amplifies the volatility of the labor market (ii) convex labor adjustment costs improve labor market persistence. Accordingly, we conclude that only the combination of both features generates sufficient volatility and persistence in the labor market.

Moreover, the results presented above indicate that strategic wage bargaining and convex labor adjustment costs are complementary in generating volatility and persistence. This result is surprising given that, in the existing literature (Fujita and Ramey, 2005), the introduction of adjustment costs leads to more persistent responses in vacancies, but dampens the volatility of all labor market variables at the same time. Unlike Fujita and Ramey (2005), however, we assume adjustment costs in matches, and not in vacancies. Moreover, we assume that hiring costs depend negatively on the level of employment. Thus, marginal matching costs under convex labor adjustment costs are much less volatile and persistent than under linear vacancy posting costs. Consequently, the introduction of convex labor adjustment costs enhances the volatility of unemployment and market tightness. Apart from that, the volatility of vacancies remains virtually unchanged.

In addition, strategic wage bargaining removes the impact of the hump-shaped outside alternative on the wage bill per worker. Hence, strategic wage bargaining induces not only more volatility, but also more persistence in all labor market variables.

The complementarity between strategic wage bargaining and convex labor adjustment costs is also illustrated by the dynamic Beveridge Curve. Only if we combine both features simultaneously, the negative relation between vacancies and unemployment remains for more than 4 quarters. Indeed, the impact of convex labor adjustment costs seems to be more important in this respect.

**Business Cycle Analysis**

The last section has shown that our model resembles the cyclical behavior of the labor market quite well. Hence, in the following, we conduct a more comprehensive business cycle analysis. The main features of business cycles are well-known (Cooley and Prescott, 1995): The fluctuations of output $y$ and total hours $n_l$ are nearly equal, while consumption $c$ fluctuates less and investment $i$ fluctuates more. Employment $n$ is almost as volatile as output, indicating that fluctuations in total hours are generated for the most part by the extensive margin. This conjecture is confirmed by the relatively tiny fluctuations in hours per worker $l$. In addition, also labor productivity $y/(nl)$ and the real wage rate $w$ fluctuate considerably less than output. All these variables are procyclical, albeit labor productivity and real wages show clearly less procyclicality than the other variables.

Table 5 compares the model’s business cycle statistics with the data. In total, the model economy captures properly the cyclical behavior of consumption, investment and employment. In particular, the model works well along the extensive margin of employment adjustment. Beyond matching the standard business cycle facts, the model accounts additionally for the low positive correlation between employment and the wage bill per worker found in U.S. data (see Table 6).
The data reveals that the empirical correlation between total hours and the real wage rate is essentially zero. This pattern is often referred to as 'Dunlop-Tarshis Observation'. On that score, however, the model’s performance drops relative to the data. Even though, the combination of both modifications improves the performance of the model significantly. This indicates that the model cannot replicate the cyclical comovement of hours per worker. In fact, while hours per worker are procyclical in the data, the model generated contemporaneous correlation between output and hours per worker is close to zero. This artifact follows from the interactions between strategic wage bargaining and convex labor adjustment costs. As described above, the combination of both modifications induces larger employment fluctuations than all other scenarios. This leads to larger output fluctuations, implying a strong income effect. On the other hand, that the combination of strategic wage bargaining and convex labor adjustment leads to a fast decline in labor productivity. Hence, the intertemporal substitution effect is relatively weak. Consequently, workers grant more value to leisure and, thus, put less (additional) effort in economic upswings.

Unfortunately, the counterfactual behavior of hours per worker biases also the cyclical properties of some other variables, like the real wage rate. Given that the real wage rate is defined as the wage bill per worker over hours per worker, the real wage rate has to account for almost the whole procyclicality of the individual wage bill. Thus, the model generated real wage rate is highly procyclical – in stark contrast to the data. Furthermore, labor productivity is too procyclical and total hours are too less volatile.

By the same token, we observe that the model cannot fully account for the relatively high volatility of the aggregate wage bill. In addition, the aggregate wage bill is too procyclical. Hence, the model generates too much volatility in the labor share and underestimates its lead. As

---

21 See, inter alia, Christiano and Eichenbaum (1992) and the references therein.

22 Note that we observe hours per worker in the data. In the model, however, \( l_t \) might capture rather (unobservable) labor effort.

---

Table 5: Business Cycle Statistics. For each variable, the table reports the relative standard deviation (\( \sigma_X/\sigma_Y \)), the first order autocorrelation (\( \rho_{X,T,X_{T+1}} \)), the phase shift relative to output (in parenthesis), and the contemporaneous correlation with output (\( \rho_{XY} \)).

<table>
<thead>
<tr>
<th>( \sigma_X/\sigma_Y )</th>
<th>( \rho_{X,T,X_{T+1}} )</th>
<th>( \rho_{XY} )</th>
</tr>
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<tbody>
<tr>
<td>0.42</td>
<td>0.75</td>
<td>(0) 0.81</td>
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<tr>
<td>2.37</td>
<td>0.91</td>
<td>(0) 0.97</td>
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<tr>
<td>0.68</td>
<td>0.79</td>
<td>(+1) 0.76</td>
</tr>
<tr>
<td>0.79</td>
<td>0.90</td>
<td>(+1) 0.82</td>
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<td>(0) 0.77</td>
</tr>
<tr>
<td>0.17</td>
<td>0.74</td>
<td>(-1) 0.27</td>
</tr>
<tr>
<td>0.26</td>
<td>0.84</td>
<td>(-4) 0.27</td>
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<tr>
<td>0.63</td>
<td>0.84</td>
<td>(+1) 0.79</td>
</tr>
<tr>
<td>0.30</td>
<td>0.84</td>
<td>(-5) -0.07</td>
</tr>
</tbody>
</table>

Table 6: The Dunlop-Tarshis Observation. The table reports the correlation coefficients between employment and the wage bill per worker and the correlation coefficients between total hours and the real wage rate, respectively.

<table>
<thead>
<tr>
<th>( \rho(n_t, w_t/l_t) )</th>
<th>( \rho(n_t, l_t, w_t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.34</td>
<td>0.03</td>
</tr>
<tr>
<td>0.22</td>
<td>0.55</td>
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<tr>
<td>0.87</td>
<td>0.98</td>
</tr>
<tr>
<td>0.87</td>
<td>0.88</td>
</tr>
</tbody>
</table>
a result, our model improves the dynamic behavior of the labor share only slightly (Andolfatto, 1996).

Moreover, we note that output volatility is slightly lower than in the data. This may be partly due to the somewhat understated volatility in total hours and investment. However, it is likely to increase output volatility by allowing for variable capital utilization (Burnside and Eichenbaum, 1996).

Finally, we analyze the cyclical behavior of the wage bill per worker, relative to the cyclical behavior of the bargaining set. As explained above, the wage bill per worker satisfies individual rationality only if it lies in the bargaining set. For this purpose, Figure 4 reports the evolution of the reservation value of the firm (upper graph), the wage bill per worker (middle graph) and the reservation value of the household (lower graph) over 12000 simulated periods. We highlight the steady state value of household’s reservation value as well as its 95% confidence interval. The graphs show that the wage bill per worker is always in the bargaining set during any period in this long simulation. Moreover, the upper confidence bound of household’s reservation value is far below the graph of the wage bill per worker. Consequently, all employment formations are efficient (Hall, 2005a). In other words, the critique of Barro (1977) does not apply here.

4 Conclusion

This paper modifies the standard Mortensen-Pissarides job matching model in order to explain the cyclical behavior of vacancies and unemployment. The modifications include convex labor adjustment costs and strategic wage bargaining as introduced into the literature by (Hall and Milgrom, 2006). The main contribution of our paper is to improve the cyclical behavior of vacancies and unemployment along two dimensions – volatility and persistence.

First, we show that strategic wage bargaining increases the volatility of both variable enormously. This is due to the fact that strategic wage bargaining makes the wage bill per worker independent of the fluctuations in household’s outside alternative. This reduces the elasticity of firms’ costs per worker by half. As a consequence, firms have much stronger incentives to hire new workers in economic upswings.

Second, the introduction of convex labor adjustment costs leads to more persistent labor market responses. In particular, the impulse response function of vacancies shows a marked hump-shape, peaking with several quarters delay. This can be attributed to firms’ altered optimization problem. In contrast to the case of linear vacancy posting costs, firms’ hiring costs now depend on the number of vacancies that are filled, and not on the number of vacancies that are posted. Thus, marginal hiring costs under convex labor adjustment costs are less volatile and less persistent than under linear vacancy posting costs, giving firms strong incentives to smooth hiring activities.

Moreover, we observe that strategic wage bargaining and convex labor adjustment costs are complementary in generating labor market volatility and persistence. This surprising result follows from the assumption that firms’ hiring costs depend negatively on the stock of employment. The quantitative impact of this effect, however, is the more evident the larger the fluctuations in the labor market. Hence, strategic wage bargaining amplifies the elasticity of labor market variables through two channels. Consequently, we note that the introduction of convex labor adjustment costs induces not only more persistence, but also more volatility in the labor market.
Apart from that, we find that our model gives rise to two distortionary effects. Given convex labor adjustment costs, social optimality requires that the wage bill per worker is equal to household’s outside alternative. In contrast, we assume that (i) the wage bill per worker is independent of the fluctuations in household’s outside alternative (ii) firms’ bargaining power $\xi$ is strictly smaller than unity. Hence, firms’ private gains from search effort are generally smaller than their social contribution. In this case, the dynamic behavior of the wage bill per worker is not socially optimal (Hosios, 1990).

It would be interesting to extent our analysis towards endogenizing the value of bargaining. As mentioned above, Knabe (2005) addresses the determinants of the value of bargaining explicitly. Thus, this analysis provides an endogenous explanation for fluctuations in the value of bargaining.
References


A Appendix

A.1 Firm’s Maximization Problem with Linear Vacancy Posting Costs

Corresponding equations (12) - (14), the optimization problem of the representative firm is:

$$\mathcal{V}(\Omega_t^F) = \max_{k_t, v_t} \left\{ y_t - w_t n_t l_t - r_t k_t - \kappa v_t + \beta E_t \left[ (\lambda_{t+1}/\lambda_t)\mathcal{V}(\Omega_{t+1}^F) \right] \right\}, \tag{31}$$

subject to

$$y_t = a_t (n_t l_t)^{(1-\theta)}, \tag{32}$$

$$n_{t+1} = (1 - \sigma) n_t + q(\gamma_t) v_t. \tag{33}$$

Corresponding equations (15) - (16), the first order conditions are given as:

$$k_t : \quad r_t = \theta \frac{y_t}{k_t}, \tag{34}$$

$$n_{t+1} : \quad \frac{\kappa}{q(\gamma_t)} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( 1 - \theta \frac{y_{t+1}}{n_{t+1}} - w_{t+1} l_{t+1} + (1 - \sigma) \frac{\kappa}{q(\gamma_{t+1})} \right) \right]. \tag{35}$$

Corresponding equation (17), the resource constraint of the economy is:

$$y_t = c_t + k_{t+1} - (1 - \delta) k_t + \kappa v_t. \tag{36}$$

Corresponding equation (28) the wage bill per worker under Nash bargaining is:

$$w_t l_t = (1 - \xi) \left[ (1 - \theta) \frac{y_t}{n_t} + \xi \left[ \frac{u^U - u^N}{\lambda_t} + \frac{(1 - \xi)}{\xi} \frac{v_t}{(1 - n_t)} \right] \right]. \tag{37}$$

Corresponding equation (29) the wage bill per worker under strategic wage bargaining is:

$$w_t l_t = (1 - \xi) \left[ (1 - \theta) \frac{y_t}{n_t} + \xi \left[ \frac{u^U - u^N}{\lambda_t} + \frac{(1 - \xi)}{\xi} \frac{\bar{v}}{(1 - \bar{n})} \right] \right]. \tag{38}$$

Additionally, we replace condition (16) by (35), condition (17) by (36), condition (28) by (37), and condition (29) by (38) in order to obtain a competitive equilibrium.

A.2 Social Planner Solution

The set $\mathcal{U}((\Omega_t) = \{k_t, n_t\}$ denotes the state space of the social planner.

$$\mathcal{U}(\Omega_t) = \max_{c_t, l_t, k_{t+1}, n_{t+1}, v_t} \left\{ \ln(c_t) + n_t \phi_1 \frac{(1 - l_t)^{1-\eta}}{1 - \eta} + (1 - n_t) \phi_2 \frac{(1 - e)^{1-\eta}}{1 - \eta} + \beta E_t \left[ \mathcal{U}(\Omega_{t+1}) \right] \right\}, \tag{39}$$

subject to

$$k_{t+1} = F(k_t, n_t l_t) + (1 - \delta) k_t - \frac{\kappa}{2} x_t^2 n_t - c_t,$$

$$n_{t+1} = (1 - \sigma) n_t + m_t.$$ 

The first order conditions are given as:

$$c_t : \quad \lambda_t = 1/c_t,$$

$$l_t : \quad \lambda_t F_2(k_t, n_t l_t) = \phi_1 (1 - l_t)^{-\eta},$$

$$k_{t+1} : \quad \lambda_t = \beta E_t \left[ \mathcal{U}(\Omega_{t+1}) \right],$$

$$n_{t+1} : \quad \mu_t = \beta E_t \left[ \mathcal{U}(\Omega_{t+1}) \right],$$

$$v_t : \quad \mu_t = \lambda_t \kappa x_t. \tag{40}$$
The envelope conditions are given as:

\[
U_1(\Omega_t) = \lambda_t[F_1(k_t, n_t l_t) + 1 - \delta],
\]

\[
U_2(\Omega_t) = \phi_1(1 - l_t)^{1-\eta} - \phi_2(1 - e)^{1-\eta} + \lambda_t F_2(k_t, n_t l_t) l_t + \mu_t \left[1 - \sigma - (1 - \alpha) \frac{m_t}{1 - n_t}\right].
\]

Consequently, the social planner solution is defined by following conditions:

\[
\lambda_t = \beta E_t \lambda_{t+1}[F_1(k_{t+1}, n_{t+1} l_{t+1}) + 1 - \delta],
\]

\[
\lambda_t F_2(k_t, n_t l_t) = \phi_1(1 - l_t)^{-\eta},
\]

\[
\mu_t = \lambda_t \kappa x_t,
\]

\[
\mu_t = \beta E_t \left[\phi_1 \frac{(1 - l_{t+1})^{1-\eta}}{1 - \eta} - \phi_2 \frac{(1 - e)^{1-\eta}}{1 - \eta}\right]
+ \lambda_{t+1} F_2(k_{t+1}, n_{t+1} l_{t+1}) l_{t+1} + \mu_{t+1} \left[1 - \sigma - (1 - \alpha) \frac{m_{t+1}}{1 - n_{t+1}}\right],
\]

\[
k_{t+1} = F(k_t, n_t l_t) + (1 - \delta) k_t - \frac{\kappa}{2} \frac{m_t^2}{n_t} - c_t,
\]

\[
n_{t+1} = (1 - \sigma)n_t + m_t.
\]

A.3 The Market Solution is generally not Pareto-Optimal

The surplus \(S_t\) (in utility units) equals the social benefit the marginal match:

\[\lambda_t S_t = U_2(\Omega_t).\]

We substitute this result into the first order condition (39):

\[\mu_t = \beta E_t [\lambda_{t+1} S_{t+1}].\]

The Nash sharing rule (26) implies that the firm gains the share \(\xi\) of the surplus:

\[V_1(\Omega_t^F) = \xi S_t.\]

Hence:

\[\mu_t = \beta \xi^{-1} E_t [\lambda_{t+1} V_1(\Omega_t^F)].\] (41)

Recall the first order condition of the firm (16):

\[\kappa x_t = \beta E_t \left[(\lambda_{t+1}/\lambda_t) V_1(\Omega_t^F)\right].\] (42)

Substituting (41) into (42) yields:

\[\kappa x_t = \beta \mu_t \xi E_t \left[(\lambda_{t+1}/\lambda_t) \beta^{-1} \lambda_{t+1}^{-1}\right]\]

Hence:

\[\kappa x_t \lambda_t = \mu_t \xi\]

Instead, the first order condition of the social planner (10) postulates:

\[\lambda_t \kappa x_t = \mu_t.\]

Hence, the market solution is pareto-optimal, if and only if \(\xi = 1\) holds.
A.4 Steady State Calculation

A.4.1 With Convex Adjustment Costs

The real interest rate ($R_t = 1 - \delta + r_t$):

$$\tilde{R} = 1/\beta.$$  

The number of new matches:

$$\tilde{m} = \sigma\tilde{n}.$$  

The number of vacancies:

$$\tilde{v} = \tilde{m}/\tilde{q}.$$  

The gross hiring rate:

$$\tilde{x} = \tilde{m}/\tilde{n}.$$  

The hiring cost parameter:

$$\kappa = (2\tilde{\psi})/((\tilde{x}^2\tilde{n})).$$  

The matching function parameter:

$$\chi = \tilde{m}/((\tilde{v}^\alpha)((1 - \tilde{n})e)^{1-\alpha}).$$  

The aggregate wage bill, respectively, the income share of labor (as $\bar{y} = 1$):

$$\bar{wnl} = (1 - \theta)\bar{y} - \left[\frac{\kappa\tilde{m}}{\beta} \left(1 - \beta \left(1 - \frac{\sigma}{2}\right)\right)\right].$$  

The capital stock:

$$\tilde{k} = (\theta\bar{y})/((\tilde{R} - 1 + \delta)).$$  

The production function parameter:

$$\zeta = \bar{y}/((\tilde{k}\theta)(\tilde{n}\tilde{l})^{-\theta}).$$  

The consumption level:

$$\tilde{c} = \bar{y} - \delta\tilde{k} - \tilde{\psi}.$$  

The Lagrange multiplier:

$$\tilde{\lambda} = 1/\tilde{c}.$$  

The leisure curvature parameter:

$$\eta = ((1/\bar{I}) - 1)/\nu.$$  

It is convenient to define $u_t = 1 - n_t$ and $f_t = 1 - l_t$:

$$\bar{u} = 1 - \bar{n},$$  

$$\bar{f} = 1 - \bar{l}.$$  

The level of investment:

$$\bar{i} = \delta\bar{k}.$$  

The real wage per "hour":

$$\bar{w} = \bar{wnl}/(\bar{n}\bar{l})$$  

The leisure parameter of the employed:

$$\phi_1 = (1 - \bar{I})^\eta(1 - \theta)\bar{y}/(\bar{n}\bar{c})$$
The flow value of unemployment:
\[
\frac{\bar{u}^U - \bar{u}^N}{\lambda} = \frac{(\bar{w}l) - [(1 - \alpha)((1 - \theta)\bar{y} + \frac{\kappa}{2} \bar{x}^2 + \kappa \bar{v}m)]}{\alpha}
\]

The leisure parameter of the unemployed:
\[
\phi_2 = \left[ \frac{\phi_1(1 - \bar{\bar{\lambda}})^{1 - \eta}}{1 - \eta} + [\bar{u}^U - \bar{u}^N] \right] \left( \frac{1 - \eta}{(1 - e)^{1 - \eta}} \right)
\]

The minimum and the maximum wage bill per worker:
\[
(wl)_{\text{min}} = \left( \frac{\bar{u}^U - \bar{u}^N}{\lambda} \right) + \left[ \frac{1 - \alpha}{\alpha} \kappa \left( \frac{\bar{m}^2}{\bar{n} \bar{u}} \right) \right] - \left[ \frac{1 - \alpha}{\alpha} \right] (1 - \sigma) \kappa \bar{x} \]
\[
(wl)_{\text{max}} = (1 - \theta) \frac{\bar{y}}{n} + \kappa \frac{\bar{x}^2}{2} + (1 - \sigma) \kappa \bar{x}
\]

The replacement ratio:
\[
\bar{b} = \left[ \left( \frac{\bar{u}^U - \bar{u}^N}{\lambda} \right) \right] / \left[ (1 - \theta) \frac{\bar{y}}{n} + \kappa \frac{\bar{x}^2}{2} \right]
\]

The net flow value per match:
\[
\bar{p} = \left[ (1 - \theta) \frac{\bar{y}}{n} + \kappa \frac{\bar{x}^2}{2} \right] - \bar{w}l
\]

A.4.2 With Linear Vacancy Posting Costs

The vacancy cost parameter
\[
\kappa = \bar{\psi} / \bar{v}
\]

The aggregate wage bill, respectively, the income share of labor:
\[
\bar{w}nl = (1 - \theta) \bar{y} - \left[ \frac{\kappa \bar{v}}{\sigma \beta} \right] (1 - \beta(1 - \sigma))
\]

The flow value of unemployment:
\[
\frac{\bar{u}^U - \bar{u}^N}{\lambda} = \frac{(\bar{w}l) - [(1 - \alpha)((1 - \theta)\bar{y} + \frac{\kappa}{2} \bar{x}^2 + \kappa \bar{v}m)]}{\alpha}
\]

The minimum and the maximum wage bill per worker:
\[
(wl)_{\text{min}} = \left( \frac{\bar{u}^U - \bar{u}^N}{\lambda} \right) + \left[ \frac{1 - \alpha}{\alpha} \kappa \left( \frac{\bar{v}}{\bar{u}} \right) \right] - \left[ \frac{1 - \alpha}{\alpha} \right] (1 - \sigma) \left( \frac{\kappa \bar{v}}{\bar{m}} \right)
\]
\[
(wl)_{\text{max}} = (1 - \theta) \frac{\bar{y}}{n} + (1 - \sigma) \left( \frac{\kappa \bar{v}}{\bar{m}} \right)
\]

The replacement ratio:
\[
\bar{b} = \left[ \left( \frac{\bar{u}^U - \bar{u}^N}{\lambda} \right) \right] / \left[ (1 - \theta) \frac{\bar{y}}{n} \right]
\]

The net flow value per match:
\[
\bar{p} = \left[ (1 - \theta) \frac{\bar{y}}{n} \right] - \bar{w}l
\]
A.5 The Log-Linearized Model

A.5.1 Strategic Wage Bargaining with Convex Labor Adjustment Costs

The auxiliary variables:

\[
\begin{align*}
0 &= \bar{n}\hat{n} + \bar{u}\hat{u} \\
0 &= \bar{l}\hat{l} + \bar{f}\hat{f}
\end{align*}
\]

The matching technology, corresponding equation (1): \[ 0 = -\hat{m} + \alpha\hat{v} + (1 - \alpha)\hat{u} - 1 \]

The law of motion for employment, corresponding equation (3): \[ 0 = -\bar{n}\hat{n} + (1 - \sigma)\bar{n} + \bar{m}\hat{m} \]

The first order conditions of the household (equations (7) - (9)), collapsed into one line:

\[ 0 = \hat{R} + \hat{c} - \hat{c} + 1 \]

The production technology, corresponding equation (13):

\[ 0 = -\hat{y} + \hat{a} + \theta\hat{k} - 1 + (1 - \theta)\hat{n} - (1 - \theta)\hat{l} \]

For convenience, we compute the gross hiring rate:

\[ 0 = -\hat{x} + \bar{m} - \bar{n} \]

The marginal product of capital, corresponding equation (15):

\[ 0 = -\bar{R} + \left[ \frac{\theta}{k} \right] \hat{y} - \left[ \frac{\theta}{k} \hat{k} \right] \hat{k} - 1 \]

The hiring condition, corresponding equation (16):

\[ 0 = [\beta \left( (1 - \theta) \frac{\bar{y}}{\bar{n}} - \bar{w} + (1 - \sigma)\bar{w} + \frac{\kappa}{2}\hat{x} \right) ] \hat{c} + [\beta \left( (1 - \theta) \frac{\bar{y}}{\bar{n}} - \bar{w} + (1 - \sigma)\bar{w} + \frac{\kappa}{2}\hat{x} \right) ] \hat{c} + 1 + [\beta (1 - \sigma)\hat{k} + [\beta (1 - \sigma)\bar{k}] \hat{n} + \left[ \frac{\beta \kappa \hat{x}^2}{2} \right] 2\hat{x} + 1 - [\kappa \hat{x} \bar{m} + \kappa \hat{x} \hat{n} - 1 \]

The resource constraint, corresponding equation (12) and (17):

\[ 0 = -\bar{y} + \bar{c} + \bar{c} + \bar{c} + \left[ \frac{\kappa}{2}\hat{x}^2 \right] \hat{n} + \left[ \frac{\kappa}{2}\hat{x}^2 \hat{n} \right] \hat{n} - 1 \]

Capital dynamics:

\[ 0 = -\bar{k} + \left[ (1 - \delta)\bar{k} \right] \hat{k} + \hat{i} + \hat{i} \]
The real wage per worker, determined by strategic wage bargaining \([29]\):

\[
0 = -[\bar{w} \bar{t}] \hat{w}_t - [\bar{w} \bar{t}] \hat{l}_t + [(1 - \alpha)(1 - \theta) \bar{y} \bar{n}] \hat{y}_t - [(1 - \alpha)(1 - \theta) \bar{y} \bar{n}] \hat{n}_{t-1} + [(1 - \alpha) \frac{\kappa^2}{2}] 2 \hat{x}_t
\]

The optimal labor effort, corresponding equation \([30]\):

\[
0 = \hat{y}_t - \hat{n}_{t-1} - \hat{\bar{c}}_t + \eta \hat{\bar{f}}_t
\]

### A.5.2 Nash Bargaining with Convex Labor Adjustment Costs

The real wage per worker, determined by Nash bargaining \([28]\):

\[
0 = -[\bar{w} \bar{t}] \hat{w}_t - [\bar{w} \bar{t}] \hat{l}_t + [(1 - \alpha)(1 - \theta) \bar{y} \bar{n}] \hat{y}_t - [(1 - \alpha)(1 - \theta) \bar{y} \bar{n}] \hat{n}_{t-1} + \left[(1 - \alpha) \frac{\kappa^2}{2}\right] 2 \hat{x}_t
\]

\[
+ \left[(1 - \alpha) \frac{\kappa^2}{2}\right] 2 \hat{x}_t + \left[(1 - \alpha) \frac{\kappa m^2}{u n}\right] 2 \hat{m}_t - \left[(1 - \alpha) \frac{\kappa m^2}{u n}\right] \hat{n}_{t-1} - \left[(1 - \alpha) \frac{\kappa m^2}{u n}\right] \hat{t}_{t-1}
\]

\[
+ \left[\alpha \left(\bar{u} - \bar{u}^N\right) \right] \hat{c}_t - \left[\alpha \frac{\bar{u}^N}{\lambda}\right] (1 - \eta) \hat{f}_t
\]

### A.5.3 Strategic Wage Bargaining with Linear Vacancy Posting Costs

The vacancy posting condition, corresponding equation \([35]\):

\[
0 = \left[\beta \left((1 - \theta) \frac{\bar{y}}{\bar{n}} - \bar{w} \bar{l} + (1 - \sigma) \frac{\bar{k} \bar{v}}{\bar{m}}\right)\right] \hat{c}_t - \left[\beta \left((1 - \theta) \frac{\bar{y}}{\bar{n}} - \bar{w} \bar{l} + (1 - \sigma) \frac{\bar{k} \bar{v}}{\bar{m}}\right)\right] \hat{c}_{t+1}
\]

\[
+ \left[\beta(1 - \theta) \frac{\bar{y}}{\bar{n}}\right] \hat{y}_{t+1} - \left[\beta(1 - \theta) \frac{\bar{y}}{\bar{n}}\right] \hat{n}_t - \left[\beta \bar{w} \bar{l}\right] \hat{w}_{t+1} - \left[\beta \bar{w} \bar{l}\right] \hat{l}_{t+1}
\]

\[
+ \left[\beta(1 - \sigma) \frac{\bar{k} \bar{v}}{\bar{m}}\right] \hat{v}_{t+1} - \left[\beta(1 - \sigma) \frac{\bar{k} \bar{v}}{\bar{m}}\right] \hat{m}_{t+1} - \left[\frac{\bar{k} \bar{v}}{\bar{m}}\right] \hat{v}_t + \left[\frac{\bar{k} \bar{v}}{\bar{m}}\right] \hat{m}_t
\]

The real wage per worker, determined by strategic wage bargaining \([37]\):

\[
0 = -[\bar{w} \bar{t}] \hat{w}_t - [\bar{w} \bar{t}] \hat{l}_t + [(1 - \alpha)(1 - \theta) \bar{y} \bar{n}] \hat{y}_t - [(1 - \alpha)(1 - \theta) \bar{y} \bar{n}] \hat{n}_{t-1}
\]

Corresponding equations \([17]\) and \([31]\), the aggregate resource constraint is given as:

\[
0 = -\bar{y} \hat{y}_t + \bar{c} \hat{c}_t + \bar{\bar{c}}_t + [\kappa \bar{v}] \hat{v}_t
\]

### A.5.4 Nash Bargaining with Linear Vacancy Posting Costs

The real wage per worker, corresponding equation \([37]\):

\[
0 = -[\bar{w} \bar{t}] \hat{w}_t - [\bar{w} \bar{t}] \hat{l}_t + [(1 - \alpha)(1 - \theta) \bar{y} \bar{n}] \hat{y}_t - [(1 - \alpha)(1 - \theta) \bar{y} \bar{n}] \hat{n}_{t-1}
\]

\[
+ \left[(1 - \alpha) \frac{\bar{v} \bar{l}}{\bar{u}}\right] \hat{v}_t - \left[(1 - \alpha) \kappa \frac{\bar{v}}{\bar{u}}\right] \hat{t}_{t-1} + \left[\alpha \frac{(\bar{u} \bar{v} - \bar{u} \bar{v}^N)}{\lambda}\right] \hat{c}_t - \left[\alpha \frac{\bar{u} \bar{v}^N}{\lambda}\right] (1 - \eta) \hat{f}_t
\]

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A.6 Information on the Data Series

<table>
<thead>
<tr>
<th>Key</th>
<th>Raw Series</th>
<th>Frequency</th>
<th>Database</th>
<th>Series ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>[4]</td>
<td>Hours per Worker</td>
<td>monthly</td>
<td>St. Louis Fed: FRED®</td>
<td>AWHNONAG</td>
</tr>
<tr>
<td>[5]</td>
<td>Total Hours</td>
<td>monthly</td>
<td>St. Louis Fed: FRED®</td>
<td>AWHI</td>
</tr>
<tr>
<td>[9]</td>
<td>Services</td>
<td>quarterly</td>
<td>St. Louis Fed: FRED®</td>
<td>PCESVC96</td>
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<tr>
<td>[10]</td>
<td>Investment</td>
<td>quarterly</td>
<td>St. Louis Fed: FRED®</td>
<td>GPDIC96</td>
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</tbody>
</table>

Table 7: Raw Data Series

<table>
<thead>
<tr>
<th>Key</th>
<th>Constructed Series</th>
<th>Variable</th>
<th>Construction</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>Consumption</td>
<td>c</td>
<td>= ([8] + [9])/[1]</td>
</tr>
<tr>
<td>(2)</td>
<td>Investment</td>
<td>i</td>
<td>= ([7] + [10])/[1]</td>
</tr>
<tr>
<td>(3)</td>
<td>Output</td>
<td>y</td>
<td>= (1) + (2)</td>
</tr>
<tr>
<td>(4)</td>
<td>Employment</td>
<td>n</td>
<td>= ([1] − [2])/[1]</td>
</tr>
<tr>
<td>(5)</td>
<td>Unemployment</td>
<td>1 − n</td>
<td>= [2]/[1]</td>
</tr>
<tr>
<td>(6)</td>
<td>Vacancies</td>
<td>v</td>
<td>= [3]/[1]</td>
</tr>
<tr>
<td>(7)</td>
<td>Market Tightness</td>
<td>v/(1 − n)</td>
<td>= (6)/(5)</td>
</tr>
<tr>
<td>(8)</td>
<td>Hours per Worker</td>
<td>l</td>
<td>= [4]</td>
</tr>
<tr>
<td>(9)</td>
<td>Total Hours</td>
<td>n · l</td>
<td>= [5]</td>
</tr>
<tr>
<td>(10)</td>
<td>Real Wage</td>
<td>w</td>
<td>= [6]</td>
</tr>
<tr>
<td>(11)</td>
<td>Aggregate Wage Bill</td>
<td>w · n · l</td>
<td>= (9) · (10)</td>
</tr>
<tr>
<td>(12)</td>
<td>Labor’s Share</td>
<td>(w · n · l)/y</td>
<td>= (11)/(3)</td>
</tr>
<tr>
<td>(13)</td>
<td>Labor Productivity</td>
<td>y/(n · l)</td>
<td>= (3)/(9)</td>
</tr>
<tr>
<td>(14)</td>
<td>Output per Worker</td>
<td>y/n</td>
<td>= (3)/(4)</td>
</tr>
<tr>
<td>(15)</td>
<td>Individual Wage Bill</td>
<td>w · l</td>
<td>= (10) · (8)</td>
</tr>
</tbody>
</table>

Table 8: Constructed Data Series

A.7 The Dynamic Beveridge Curve

<table>
<thead>
<tr>
<th>U.S. Data</th>
<th>-0.13</th>
<th>-0.34</th>
<th>-0.56</th>
<th>-0.75</th>
<th>-0.90</th>
<th>-0.95</th>
<th>-0.86</th>
<th>-0.67</th>
<th>-0.45</th>
<th>-0.20</th>
<th>0.03</th>
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<tbody>
<tr>
<td>SB/CC Model</td>
<td>-0.30</td>
<td>-0.50</td>
<td>-0.69</td>
<td>-0.86</td>
<td>-0.96</td>
<td>-0.94</td>
<td>-0.76</td>
<td>-0.54</td>
<td>-0.32</td>
<td>-0.13</td>
<td>0.04</td>
</tr>
<tr>
<td>NB/CC Model</td>
<td>-0.22</td>
<td>-0.41</td>
<td>-0.62</td>
<td>-0.82</td>
<td>-0.95</td>
<td>-0.93</td>
<td>-0.71</td>
<td>-0.46</td>
<td>-0.23</td>
<td>-0.04</td>
<td>0.11</td>
</tr>
<tr>
<td>SB/LC Model</td>
<td>-0.07</td>
<td>-0.22</td>
<td>-0.40</td>
<td>-0.62</td>
<td>-0.84</td>
<td>-0.90</td>
<td>-0.51</td>
<td>-0.22</td>
<td>-0.05</td>
<td>0.06</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 9: The Dynamic Beveridge Curve. The table show the correlation coefficients between unemployment ut and vacancies v_t+k, lagged respectively leaded by k quarters
A.8 Impulse Response Functions

Figure 1: Impulse Response Functions. The graphs depict the evolution of the strategic wage bargaining model with convex labor adjustment costs (solid line), the Nash bargaining model with convex labor adjustment costs (dashed line), and the strategic wage bargaining model with linear vacancy posting costs (dotted line) over 96 months (32 quarters).
A.9 Plots

Figure 2: Robust Hump-Shaped Vacancy Dynamics. The solid line represents our model. The dashed line represents the model with Nash bargaining. The dot-dashed line represents the model with firms’ bargaining power equal to unity. The dot-dot-dashed line represents the model with Nash bargaining and firms’ bargaining power equal to unity.

Figure 3: The Dynamic Beveridge Curve (graphical representation of Table 1). The solid line with square shaped markers represents U.S. data. The solid line with triangle-shaped markers represents the strategic wage bargaining model with convex labor adjustment costs. The dashed line represents the Nash bargaining model with convex labor adjustment costs. The dotted line represents the strategic wage bargaining model with linear vacancy posting costs.

Figure 4: The Bargaining Set. The graphs depict the evolution of the reservation value of the firm (upper graph), the wage bill per worker (middle graph) and the reservation value of the worker (lower graph) over 12000 simulated periods. In addition, we highlight the steady state value of the worker’s reservation value as well as its 95% confidence interval.