Economics Department

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# Sequential Methods for Detecting Structural Breaks in Cointegrated Systems*

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**Abstract**

We investigate the issue of breaks in systems of equations by analysing small systems where breaks in the deterministic components affect the equations at distinct time periods. A method is proposed of timing the breaks and a sensitivity-analysis undertaken to test the robustness of our findings to variations in the parameters characterising the data generation processes. Generalisations for dealing with empirically relevant models, via bootstrap methods, are proposed. An empirical example, based on data from the UK-trading sector concludes the paper.

**Keywords:** Systems, Structural Breaks, Cointegration

**JEL Classification:** C10, C12, C13, C15

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# 1 Introduction and Overview

In recent years, the econometrics literature on unit roots and cointegration has grappled with the problem of distinguishing deterministic breaks in trend or mean, of a time series, from genuine unit roots. Papers such as Perron (1989) have dealt with this issue in the framework of *a priori* imposed break dates, while others, such as Banerjee, Lumsdaine and Stock (1992) (henceforth called BLS (1992)) or Zivot and Andrews (1992), have used methods where the break date is endogenised.

It would be fair to say that there is no firm consensus in this literature on the best way of dealing with what have been called "structural breaks" (although this terminology itself may be regarded as misleading.) Except that is for noting that the presence of such breaks biases tests for unit roots in favour of acceptance and has important effects on tests for cointegration both in single equations and in systems (see for example Gregory and Hansen (1996)). Endogenised procedures usually require substantially higher critical values for the unit root hypothesis to be rejected, *i.e.* the inferential procedure is highly conservative, with the result that losses in power are evident in the use of such procedures. Procedures which impose break dates, on the other hand, are open to the charge of pre-test bias, although where break-dates are known with certainty these procedures are of necessity very powerful. In addition, the break dates are often imposed at dates which are "economically interpretable", with the 1929 Crash or the 1973 Oil Shock being the classical examples.

A major difficulty with this entire literature is the interpretation one puts on the finding of a structural break. It can be easily argued that a finding of the kind that suggests that the slope of the trend coefficient may have changed value or that the mean of the process may have changed is not a change in the fundamental "structure" of the process but rather that these changes are easily accommodated by the stochastic variability of a unit root process. It is in this sense that the endogenous break literature is more conservative, both statistically and in its economic interpretation.

Recent papers, due mainly to Bai and his co-authors (see, *inter alia*, Bai (1997), Bai and Perron (1998)) have made good progress in dealing with the more realistic case of multiple structural breaks in univariate models. Bai (1997), for example, provides tests for dating multiple breaks one at a time (conditioning on previously determined breaks). Our work investigates the problem of dating multiple breaks within the context of a small system of equations, an area in which progress has been more limited (Bai, Lumsdaine and Stock (1994)) but which can nevertheless be considered within the broad framework proposed by Bai for the univariate case.

The empirical relevance of this theoretical analysis is, in our view, substantial. Several recent empirical papers (Clements and Mizon (1991), Hendry and Mizon (1993), Doornik and Hendry (1994)) enforce stability of their system of equations by inserting break-dummies at different time-points. The issue that dogs our inquiry is whether *a priori* imposition of these dates can be avoided.

Our paper is an attempt to illustrate some of the challenges involved in dating multiple breaks in systems of equations by using simple data generation processes (DGPs) and models. The approach to the problem is *via* Monte Carlo simulations since the main findings are illustrated straightforwardly by this technique. The main issue we address is that of more than one structural break in the system when the breaks occur in individual equations in the system at different points in time. Depending upon the order of integration of the variables concerned (the size of the largest root) such a break can occur, upon cumulation, either in the drift of the process (stationary root) or its trend growth rate (unit root). The powers of the testing procedures are correspondingly affected. That the break dates can be different from each other is to be expected: some series, for example, may be more affected by an output shock than by a productivity shock and these shocks may occur at different points in time while the relationship between them is maintained in the long run.

In our testing methods, we operate within the framework of sequential tests, where the break date(s) is (are) not imposed *a priori*. This is of course not the only class of tests one might consider. In an earlier discussion paper (Banerjee and Urga, 1995) we have attempted to employ recursive Chow-type tests (of the kind discussed earlier in the context of Clements and Mizon (1991)) due to Anderson and Mizon (1984). The substantial difficulty we face with using these tests is in the appropriate choice of critical values. A comparison with fixed critical values (independent of the horizon over which we wish to test for structural stability, whether in-sample or out-of-sample) will lead asymptotically to rejections of a true null of structural stability with probability one. Any attempt to correct the critical values by adjusting them upwards leads to tests with disappointingly poor power properties.

We discuss a variety of testing procedures, using both the structural and reduced forms of models and single-equation and systems tests. In our simulations we concentrate on the reduced form because of its superior power properties. It should be noted that the flavour of the difficulties can be easily illustrated in the context of data generation processes which are entirely stable (in the sense of having roots outside the unit circle), *i.e.* there is nothing fundamentally important about the unit root aspects of the problem. Thus while we do not pursue our arguments here using stationary processes, the results which would arise from this case are qualitatively very similar to those derived from the study of integrated variables (see Banerjee and Urga (1995)).

The main choice we face is between estimating the potentially multiple breaks unconditionally as opposed to allowing some form of *a priori* imposition. Lumsdaine and Papell (1997) consider the issue of testing for unit roots in the presence of two structural breaks. When *a priori* imposition is not contemplated, in a sample of $T$ observations there are roughly $O(T^2)$ combinations of points to consider, even allowing for trimming (*i.e.* the break dates cannot occur too

near the end or the beginning of the sample) or imposing the restrictions that one break strictly precedes the other or that they cannot happen at the same time. Given the easy availability of computing power and highly efficient dynamic programming algorithms (see Bai and Perron (1998)), there is nothing inherently difficult about the unconditional problem since one could simply choose each of the $T^2$ points in turn. The issue is simply whether there is a more elegant way to proceed by allowing knowledge (derived from estimation or omniscience) of a break in one of our processes to inform or search for the break in the other process.

An interesting issue we face is that of non-standard critical values. For the class of problems considered, the critical values are non-standard and are not available in ready form. They are indeed specific to the data generation processes chosen and have to be recomputed for each different method of testing employed and each different specification of the DGP. While this introduces a degree of specificity to the exercise, our programs are sufficiently packaged for recomputations of this kind, both to recalculate the critical values following a redesign of the DGP and to work out the powers of the tests, to be accomplished at low cost. In related work we consider the virtues of bootstrapping empirically relevant models to detect structural breaks (see Banerjee, Lazarova and Urga (1998)).

In Section 2 we establish the main data generation process and model which will form the basis of the discussion for most of the paper. Here we consider a DGP where, under the null, the marginal process may be either weakly or strongly exogenous for the parameters of interest in the conditional model. We discuss the main and sequential methods used and illustrate their use for the simple special case where the breaks occur distinctly. We contemplate several ways of pinning down the break, either by methods which do not impose any *a priori* restrictions (UNCONDITIONAL PROCEDURE) or by more restrictive methods where we estimate the break in the marginal process first and then impose this knowledge in some form when searching for the break in the conditional process (CONDITIONAL PROCEDURE). Examples of such restrictive hypotheses might be that the break in the conditional model occurs within $k$ periods (before or after) the break in the marginal model (under the assumption that shocks propagate through the system (see Bai, Lumsdaine and Stock (1994)).

Section 3 presents a sensitivity analysis of the power of the procedures. We do this by considering alternative timings of the break-dates (at the edges or the interior of the sample) or by the size of the break (calibrated by the standard deviation of the conditional innovation process) or the variance of the error processes.

Section 4 presents an empirical example on modelling the exports sector of the UK. Section 5 concludes.

# 2 Data Generation Process and Models

We consider, to begin with, a DGP which, under the null hypothesis, takes the form of a bivariate cointegrated system (when the variables are I(1)) while under the alternative the system is allowed to be structurally broken. This method allows us to disentangle the two hypotheses which are often tested together, *i.e.* no cointegration and the absence of a structural break, notably in Gregory and Hansen's recent work (1996). In the latter analysis it becomes difficult to attribute rejections of the null to the absence of cointegration or the presence of a structural break in the cointegrating relationship.

Thus, let us suppose that variable $\{y_t\}$ depends on $\{x_t\}$ while $\{x_t\}$ depends only on its own past. Thus consider:

$$\text{DGP A} \quad \begin{aligned} y_t &= \beta x_t + \gamma_1 D_t^1 + u_{1t}, \\ x_t &= \rho x_{t-1} + \gamma_2 D_t^2 + u_{2t} \end{aligned}$$

where:

$$\begin{aligned} y_0 &= x_0 = 0, \\ D_t^1 &= I\left(t \geq k_1\right), \\ D_t^2 &= I\left(t \geq k_2\right), \\ \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} &\sim IN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{pmatrix}\right), \\ k_1 &\neq k_2 \quad \text{(in general)}, \\ & I\left(\cdot\right) \quad \text{is the standard indicator function.} \end{aligned}$$

Data are generated according to DGP A for various configurations of parameter values for $\{\beta, \gamma_1, \gamma_2, \rho, k_1, k_2, \sigma_{11}, \sigma_{22}\}$. The values of the constant in the processes for $y_t$ and $x_t$ are set equal to zero possibly with loss of generality. In this paper we consider the case of where the covariance between the errors is taken to be zero so that here the marginal process is strongly exogenous for the parameter of interest. (In Banerjee and Urga (1995) we also experiment with cases where the covariance is non-zero). Later we relax this assumption on the strong exogeneity of the marginal process and present results for both the special and the general case.

Note of course that the null model has $\gamma_1 = \gamma_2 = 0$. For the majority of the experiments with DGP A, under the alternative, $\gamma_1$ and $\gamma_2$ are set to one. Similarly, the value of $\beta$ and the variances $\sigma_{11}$, $\sigma_{22}$ are also set equal to $\{0.5\}$. $\rho = 1.0$ and the system is therefore integrated of order 1. Finally, $k_1$ and $k_2$ are varied systematically as indicated in the tables below.

In Section 3, as part of the sensitivity analysis, we consider different sizes of the break (standardized by the magnitude of the variance). Note that, since $\rho$ is equal to 1, the dummy under the alternative represents a change in the trend growth rate of the process $x_t$, with the strength of the shift being determined by the value taken by $\gamma_2$. The effect of the value of $\gamma_2$ on the power of the test procedure is reported in Table 3 which provides other power calculations as part of a sensitivity analysis.

## 2.1 Tests

Critical values are computed for the following tests by looking at single equations:

**Model A (Structural form)**

The sequence of sequential $t$-tests for $\gamma_1$ in the model

$$y_t = \beta x_t + \gamma_1 D_t^1 + u_{1t}$$

where the test statistic used is

$$\tau_2 = t - \max = argmax \left\{ t_{\gamma_1 = 0}(k) \right\}, \quad k = 2, ..., T - 2.$$

Here, the regression is estimated over the full-sample each time but the location of the dummy variable is moved sequentially over the sample. (see BLS(1992) for further details.)

**Model B (Reduced form)**

The sequence of sequential $t$-tests for $\gamma_1$ and $\gamma_3$ (and the joint $F$-test) in the model

$$y_t = \delta_0 x_{t-1} + \delta_1 y_{t-1} + \gamma_1 D_t^1 + \gamma_3 D_t^2 + u_{1t},$$

where $\delta_0$,$\delta_1$ and $\gamma_3$ are simple functions of the parameters of the DGP.

Note that the difficulty with multiple (distinct) breaks arises immediately when dealing with the sequential test in the reduced form of the model in the case (where $k_1 \neq k_2$). The term of the form $\gamma_1 D_t^1 + \beta\gamma_2 D_t^2$ $(= \gamma_1 D_t^1 + \gamma_3 D_t^2)$, will not be proxied adequately by a single dummy variable in the conditional reduced-form model. In the case where $k_1 = k_2$, this is not likely to cause a problem.

Except for the next subsection, the discussion in this paper deals only with the reduced form of the model, not only because it highlights the central difficulty of break-dating better but also because we have some reason to believe that the dynamics help to provide better inference. The main arguments are entirely analogous for the structural form and simulation evidence for this case is available in an earlier (discussion paper) version of this paper (see Banerjee and Urga (1995)).

## 2.2 Unconditional Procedure

Here the issue boils down to finding an adequate way of handling the $k_1 \neq k_2$ case. Although this is not the main focus of our analysis, one way of dealing with the problem is to incorporate two structural break dummies in the conditional model. If one were to do this unrestrictedly, the dimensionality of the problem, for two structural breaks, is of $O(T^2)$. Although computationally intensive, this approach is eminently tractable.[1] However it is likely that such a procedure does not make full use of information available from the consideration of related series which may themselves have breaks in the neighbourhood of the breaks in, say, the $\{y_t\}$ series under consideration. In Banerjee and Urga (1995) we report a series of experiments (critical values and some power calculations) for the sequential procedure using the reduced form of the system only (since estimation is usually in reduced form). Recall that here the sequential procedure loops over roughly $T^2$ points for each replication, and is therefore extremely expensive in computing time. In terms of *detecting* the presence of breaks, the power properties are quite good. This is a surprising finding, given the conservativeness, implied by its use of maximal-order statistics, of the inferential procedure. However, as reported in the relevant histograms in Banerjee and Urga (1995), the procedure is much less competent at timing the breaks accurately. The problem described above could be generalized by allowing breaks of different kinds to occur at the different time points. Thus $D_t^1$ might be a break in the mean as before while $D_t^2$ may be generalized to allow a break in trend. The indicator function is modified for $D_t^2$ to $I(\cdot)(t-k)$, for $t > k$.[2]

We turn now to the introduction of an alternative procedure which we call the *conditional* procedure.

[1] Lumsdaine and Papell (1997) provide results for a similar problem but in the context of testing for unit roots with double structural breaks in a univariate series.

[2] Banerjee, Lazarova and Urga (1998) present some results for such generalizations.

## 2.3 Conditional Procedure

In order to reduce the dimensionality of the inferential problem, we consider a two-stage strategy. At the first stage, sequential-testing procedures are employed on the marginal model to date the break in this model. (The estimators and test statistics obey the density functions derived in BLS (1992).) Where the break is deemed to be significant, this knowledge is then used to search for the break in the conditional model. We call this the conditional procedure. A further restriction of the conditional procedure would involve restricting the break in the conditional model to occur within $[-k', k'']$ using the arguments that breaks in related series happen in the same neighbourhood of each other. We discuss the effects of imposing such restrictions, thereby moving from an *unconditional conditional* search to a *conditional conditional* search, on the critical values and powers of the test. Unsurprisingly, in a majority of cases, the move implies lower critical values and higher power where the breaks do occur within a neighbourhood of each other. This latter form of the test of course have no power if the true breaks are widely separated from each other.

The main purpose of this section is to presents the critical values and power for the sequential procedure when applied to a more general DGP, called DGP B, of which DGP A is a special case. We demonstrate our conditional sequential procedure when applied to a more general DGP, but still under the maintained restriction of zero covariance between the error processes in the marginal and conditional equations of the DGP.

$$\begin{array}{ll} \text{DGP B} & y_t = \beta x_t + \gamma_1 D_t^1 + u_{1t} \\ & x_t = \rho_1 x_{t-1} + \rho_2 \Delta y_{t-1} + \gamma_2 D_t^2 + u_{2t} \\ \text{Model} & y_t = const + \delta_0 x_{t-1} + \delta_1 y_{t-1} + \gamma_1 D_t^1 + \gamma_3 D_t^2 + u_{1t} \\ & x_t = \rho_1 x_{t-1} + \rho_2 \Delta y_{t-1} + \gamma_2 D_t^2 + u_{2t} \end{array}$$

This allows for the possibility that $x_t$ is either weakly or strongly exogenous with respect to $y_t$. $x_t$ is strongly exogenous for the parameter of interest in the conditional model if $\rho_2 = 0$. Otherwise $x_t$ is only weakly exogenous, the correlation between $u_{1t}$ and $u_{2t}$ being equal to zero by assumption. For a range of parameter values, the critical values are computed by Monte-Carlo simulation.[3] The simulations proceed in three stages. The first stage determines the critical value for the marginal dummy, $\gamma_2$. In the second step, the critical values for the sequential dummy, $\gamma_1$, is determined using the critical value from the first step to determine when the estimation of the system should include a common dummy.[4] In the final stage the power of the sequential procedure is determined using the critical values from the previous stages. In estimating the marginal process the variance is estimated whereas both the variances are assumed know in the estimation of the system.

The following parameters were chosen as a reference case: $\beta = 1$, $\rho_1 = 1$, and $\rho_2 = 0$ or $0.05$ and with variance $\sigma_{11} = \sigma_{22} = 0.5$, $\sigma_{12} = 0$. The results for a range of experiments are reported in Table 2. Let us consider the case of $T = 60$ and searching between 10 and 20. The power of the procedure under the alternative of unit breaks ($\gamma_1 = \gamma_3 = 1$) is calculated using break dates at $\tau_1 = 18$ in the conditional process and at $\tau_2 = 15$ in the marginal process.

The critical value for $\gamma_2$ and $\gamma_1$ at 5% level was found to be 3.58 and 4.11 respectively for the weakly exogenous case and 3.56 and 4.16 for the strongly exogenous case. The power of the test is 98.00% and 97.85% for the two cases. If the search instead is made unconditional from 3 to 57, i.e., without assuming knowledge of approximately where the breaks occur, the critical values are 4.03 for $\gamma_2$ and 4.97 for $\gamma_1$ for the weakly exogenous case (the power is 94.20%). For $\rho_2 = 0$ the critical values for $\gamma_2$ is 4.02 and for $\gamma_1$ 5.02 (the power is 95.20%).[5]

Table 2 presents the further results for longer time series ($T$ = 100) and/or different search window (5–15, 20–30 and 8–18) and break

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[3]In all the simulations the number of iterations used was 2,000. Running 5,000 iterations were tried in a few cases but our findings did not change.

[4]If included, the coefficient for this dummy will be re-estimated rather than restricted to the value found from the first step estimation of the dummy in the marginal process. The new coefficient is call $\gamma_3$.

[5]It is worth noticing that in Table 2 we report the estimation results that $\tau_1$ and $\tau_2$ are estimated jointly. In Banerjee and Urga (1995) all simulations assume $\tau_2$ as *known*.

position $((\tau_1, \tau_2) = (10, 6), (28, 25)$, and $(15, 10))$. In the next section we also investigate the impact on the results of changing most of the important features of the DGP.

TABLE 2: CRITICAL VALUES AND POWER

DGP B $\quad y_t = \beta x_t + \gamma_1 D_t^1 + u_{1t}$

$x_t = \rho_1 x_{t-1} + \rho_2 \Delta y_{t-1} + \gamma_2 D_t^2 + u_{2t}$

Model $\quad y_t = const + \delta_0 x_{t-1} + \delta_1 y_{t-1} + \gamma_1 D_t^1 + \gamma_3 D_t^2 + u_{1t}$

$x_t = \rho_1 x_{t-1} + \rho_2 \Delta y_{t-1} + \gamma_2 D_t^2 + u_{2t}$

| $T$ | Search window | $(\tau_1, \tau_2)$ | $\rho_2 = 0.05$ | | | $\rho_2 = 0$ (DGP A) | | |
|---|---|---|---|---|---|---|---|---|
| | | | $CV_{\gamma_2}$ | $CV_{\gamma_1}$ | $Power_{\gamma_1}$ | $CV_{\gamma_2}$ | $CV_{\gamma_1}$ | $Power_{\gamma_1}$ |
| 60 | 10–20 | (18, 15) | 3.58 | 4.11 | 98.00 | 3.56 | 4.16 | 97.85 |
| 60 | 5–15 | (10, 6) | 3.62 | 3.99 | 87.75 | 3.58 | 4.02 | 91.95 |
| 60 | 3–57 | (18, 15) | 4.03 | 4.97 | 94.20 | 4.02 | 5.02 | 95.20 |
| 60 | 3–57 | (10, 6) | 4.03 | 4.97 | 76.00 | 4.02 | 5.02 | 82.50 |
| 100 | 20–30 | (28, 25) | 3.31 | 3.86 | 100.00 | 3.29 | 3.90 | 100.00 |
| 100 | 8-18 | (15, 10) | 3.39 | 3.94 | 99.40 | 3.36 | 3.98 | 99.55 |
| 100 | 5–95 | (28, 25) | 3.99 | 5.14 | 99.60 | 3.97 | 5.20 | 99.80 |
| 100 | 5–95 | (15, 10) | 3.99 | 5.15 | 96.45 | 3.97 | 5.20 | 97.45 |

The critical values are calculated at 5% and the power reports the frequency that $\gamma_1$ was significant. $\sigma_{11} = \sigma_{22} = 0.5$.

# 3 Sensitivity Analysis

Our next step was to undertake a sensitivity analysis in order to investigate the robustness of the methods proposed to changes in the parameters generating the data and checking the effect this had on the size and power calculations.

Sensitivities of three kinds were investigated, some of which are reported in Table 3 below:

i. to the location of the break points. Here we were particularly interested in the power properties of our testing procedures if the breaks occurred at extremal points of the samples. More generally, we investigated the impact on the tests of increasing distance between the breaks. We also studied the impact of the symmetry or order in which the breaks occur, *i.e.* $(D_t^1 = k_2, D_t^2 = k_1)$ instead of $(D_t^1 = k_1, D_t^2 = k_2)$.

ii. to the value of $\rho_2$, the so-called feedback or ECM or weak-exogeneity parameter;

iii. to $\gamma_1$ and $\gamma_2$, *i.e* the magnitudes of the breaks, relative to the variances of the processes;

Our main conclusion can be summarized as follows. Under (i.) the test is robust to the order of the breaks while power falls marginally the closer to the end-points the break occurs. For example, in a sample of T = 60, when the breaks occur at (45, 48) instead of at (15, 18) power of the test falls from 98.40 to 97.70. When the breaks occur at (6, 10) the power is 94.35 while when they occur at (50, 54) the power is 95.25.

The critical values and power of the test are not sensitive to $\rho_2$ as may be seen in Table 3.

They are however sensitive to respecifications under (iii.), so that increasing noise relative to the magnitude of the break increases the critical values and reduces power. This may be seen in rows 5-7 for T=60 and rows 12-14 for T=100 of Table 3.

We also implemented other simulations not reported here where we kept the values of the variances fixed to one and systematically moved the values of $\gamma_1$ and $\gamma_2$ away from unity. This also resulted in the test becoming less powerful. For example, when $\gamma_1$ was changed from 0.9 to 0.5, keeping $\gamma_2$ fixed at 1, the power moved from 63.65 to 49.30. Keeping $\gamma_1$ equal to 1 and changing $\gamma_2$ from 0.9 to 0.5 the power moves from 80 to 60. The powers and sizes of more borderline to unity cases for $\gamma_1$ and $\gamma_2$ correspond closely to values reported in Table 3.

Details of the joint distribution of the estimation of the location of the breaks is shown in Tables 4A and 4B and in Figure 1A and 1B. In roughly 30% of the cases, the breaks are determined to within one of their true value and about 80% of the estimates lie within three of the true values. Table 5 shows the marginal distribution of the estimated location of the breaks for the actual breaks being located at different position.

It can be seen that the procedure is in general good at locating the break in the marginal process by being within one of the true value in approximately 70% of the estimations.

However, estimates of the break in the conditional model are biased towards the true location of the break in the marginal process. When the true breaks are three periods apart, the procedure is about 75% as likely to conclude that the break in the conditional model is at the same position as the break in the marginal process. When the distance between the true breaks is greater this bias falls—if the true breaks are at 18 and 12 the procedure is 'only' 32% as likely to incorrectly find a 'common break' at the position of the break in the marginal process. This is in accord with the theoretical results reported in Bai (1997).

In Figures 1A and 1B the estimates of the sizes of the breaks is plotted conditional on the estimated location of the break in the conditional process being: (a) at the true location ($\widehat{\tau}_1 = \tau_1$), or (b) at the location of the 'common break' ($\widehat{\tau}_1 = \tau_2$). The bias does not seem to depend on where the breaks were found nor does there appear to be any dependence between the bias in the two estimates.

An interesting exercise would be to compare the performance of the sequential procedure to an exhaustive search procedure of all combinations of break dates. The results reported here cover the cases where the breaks in the marginal and conditional models are fairly close to each other, and thus provide a conservative lower bound for the power of our procedure.

TABLE 3: SENSITIVITY TO CHANGE IN VARIANCE AND BREAK SIZE

| $T$ | $\sigma_{11}$ | $\sigma_{22}$ | Break sizes | $\rho_2 = 0.05$ | | | $\rho_2 = 0$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | $CV_{\gamma_2}$ | $CV_{\gamma_1}$ | $Power_{\gamma_1}$ | $CV_{\gamma_2}$ | $CV_{\gamma_1}$ | $Power_{\gamma_1}$ |
| 60 | 0.50 | 0.50 | 1.00 | 3.58 | 4.11 | 98.00 | 3.56 | 4.16 | 97.85 |
| 60 | 1.00 | 0.50 | 1.00 | 3.55 | 3.40 | 81.30 | 3.56 | 3.47 | 81.20 |
| 60 | 0.50 | 1.00 | 1.00 | 3.59 | 5.20 | 80.80 | 3.56 | 5.23 | 81.35 |
| 60 | 1.00 | 1.00 | 1.00 | 3.58 | 4.02 | 65.40 | 3.56 | 4.10 | 64.35 |
| 60 | 0.50 | 0.50 | 0.50 | 3.58 | 4.11 | 72.25 | 3.56 | 4.16 | 73.95 |
| 60 | 0.13 | 0.13 | 0.50 | 3.58 | 4.66 | 99.90 | 3.56 | 4.67 | 99.95 |
| 60 | 0.03 | 0.03 | 0.25 | 3.58 | 5.70 | 98.85 | 3.56 | 5.67 | 99.45 |
| 100 | 0.50 | 0.50 | 1.00 | 3.31 | 3.86 | 100.00 | 3.29 | 3.90 | 100.00 |
| 100 | 1.00 | 0.50 | 1.00 | 3.29 | 3.25 | 99.05 | 3.29 | 3.30 | 98.90 |
| 100 | 0.50 | 1.00 | 1.00 | 3.31 | 4.90 | 93.80 | 3.29 | 4.91 | 94.95 |
| 100 | 1.00 | 1.00 | 1.00 | 3.31 | 3.82 | 85.85 | 3.29 | 3.87 | 86.80 |
| 100 | 0.50 | 0.50 | 0.50 | 3.31 | 3.86 | 89.60 | 3.29 | 3.90 | 90.20 |
| 100 | 0.13 | 0.13 | 0.50 | 3.31 | 4.41 | 100.00 | 3.29 | 4.40 | 100.00 |
| 100 | 0.03 | 0.03 | 0.25 | 3.31 | 5.33 | 100.00 | 3.29 | 5.23 | 100.00 |

The critical values are calculated at 5% and the power reports the frequency that $\gamma_2$ was significant. For $T = 60$ the search window is 10–20 and the breaks are at $(18, 15)$. For $T = 100$ the search is over 20–30 with the breaks located at $(28, 25)$.

TABLE 4A: JOINT DISTRIBUTION OF ESTIMATES OF BREAK LOCATIONS

| | | $\tau_2$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | |
| $\tau_1$ | 10 | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.2% | 0.0% | 0.2% | 0.1% | 0.0% | 0.0% | 0.5% |
| | 11 | 0.0% | 0.0% | 0.0% | 0.0% | 0.1% | 0.3% | 0.3% | 0.1% | 0.0% | 0.0% | 0.0% | 0.8% |
| | 12 | 0.1% | 0.0% | 0.0% | 0.0% | 0.1% | 0.5% | 0.1% | 0.0% | 0.2% | 0.2% | 0.0% | 1.2% |
| | 13 | 0.0% | 0.0% | 0.0% | 0.1% | 0.2% | 0.9% | 0.3% | 0.0% | 0.0% | 0.0% | 0.2% | 1.7% |
| | 14 | 0.2% | 0.1% | 0.1% | 0.0% | 1.1% | 1.7% | 0.4% | 0.1% | 0.2% | 0.2% | 0.2% | 4.3% |
| | 15 | 0.9% | 0.8% | 0.8% | 2.0% | 5.6% | 13.8% | 4.8% | 2.5% | 1.8% | 1.5% | 2.1% | 36.6% |
| | 16 | 0.3% | 0.2% | 0.5% | 1.3% | 2.3% | 5.0% | 2.0% | 0.8% | 0.7% | 0.9% | 0.4% | 14.4% |
| | 17 | 0.4% | 0.2% | 0.4% | 1.1% | 0.3% | 3.7% | 1.6% | 0.9% | 0.4% | 0.1% | 0.4% | 9.5% |
| | 18 | 0.5% | 0.7% | 1.5% | 1.4% | 2.9% | 9.7% | 2.7% | 1.0% | 0.2% | 0.3% | 0.1% | 21.0% |
| | 19 | 0.0% | 0.1% | 0.0% | 0.4% | 0.9% | 3.3% | 1.0% | 0.2% | 0.0% | 0.0% | 0.0% | 5.9% |
| | 20 | 0.1% | 0.0% | 0.1% | 0.2% | 0.3% | 2.1% | 0.5% | 0.1% | 0.2% | 0.0% | 0.0% | 3.6% |
| | | 2.5% | 2.1% | 3.4% | 6.5% | 13.8% | 41.2% | 13.7% | 5.9% | 3.8% | 3.2% | 3.4% | |

The distribution is for the estimates where both breaks were significant.
The model is the reference case where $\rho_2 \neq 0$.

**Table 4B: Joint Distribution of Estimates of Break Location**

| | | $\tau_2$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | |
| $\tau_1$ | 10 | 0.0% | 0.0% | 0.0% | 0.1% | 0.0% | 0.3% | 0.2% | 0.2% | 0.0% | 0.0% | 0.1% | 0.9% |
| | 11 | 0.0% | 0.0% | 0.0% | 0.0% | 0.0% | 0.4% | 0.2% | 0.1% | 0.1% | 0.0% | 0.0% | 0.8% |
| | 12 | 0.1% | 0.0% | 0.0% | 0.0% | 0.1% | 0.4% | 0.1% | 0.1% | 0.2% | 0.1% | 0.1% | 1.2% |
| | 13 | 0.1% | 0.0% | 0.0% | 0.1% | 0.8% | 1.7% | 0.7% | 0.3% | 0.6% | 0.1% | 0.2% | 2.0% |
| | 14 | 0.1% | 0.1% | 0.1% | 0.2% | 0.8% | 1.7% | 0.7% | 0.3% | 0.6% | 0.1% | 0.2% | 4.9% |
| | 15 | 1.5% | 0.6% | 1.0% | 2.4% | 4.1% | 14.7% | 4.8% | 2.1% | 1.5% | 1.2% | 1.6% | 35.5% |
| | 16 | 0.3% | 0.5% | 0.4% | 1.4% | 2.2% | 4.7% | 1.9% | 0.7% | 0.7% | 0.6% | 0.4% | 13.8% |
| | 17 | 0.3% | 0.4% | 0.4% | 0.8% | 1.5% | 4.1% | 1.6% | 0.8% | 0.5% | 0.2% | 0.3% | 10.9% |
| | 18 | 0.6% | 1.0% | 1.2% | 1.3% | 2.9% | 10.2% | 2.5% | 1.2% | 0.3% | 0.3% | 0.2% | 21.7% |
| | 19 | 0.2% | 0.2% | 0.3% | 0.3% | 0.8% | 2.7% | 0.9% | 0.3% | 0.1% | 0.0% | 0.1% | 5.9% |
| | 20 | 0.0% | 0.0% | 0.1% | 0.3% | 0.4% | 2.1% | 0.5% | 0.0% | 0.1% | 0.0% | 0.0% | 3.5% |
| | | 3.2% | 2.8% | 3.5% | 6.9% | 12.9% | 42.2% | 13.6% | 5.9% | 4.4% | 2.5% | 3.2% | |

The distribution is for the estimates where both breaks were significant.

The model is the reference case where $\rho_2 = 0$.

TABLE 5: MARGINAL DISTRIBUTION OF ESTIMATED BREAK POSITION

| | $\rho_2 \neq 0$ | | | | $\rho_2 = 0$ | | | |
|---|---|---|---|---|---|---|---|---|
| $(\tau_1, \tau_2)$ | $\widehat{\tau}_1 = \tau_1$ | $\widehat{\tau}_1 = \tau_2$ | $\widehat{\tau}_2 = \tau_2$ | $\widehat{\tau}_2 = \tau_1$ | $\widehat{\tau}_1 = \tau_1$ | $\widehat{\tau}_1 = \tau_2$ | $\widehat{\tau}_2 = \tau_2$ | $\widehat{\tau}_2 = \tau_1$ |
| $(18, 15)$ | 21.0% | 36.6% | 41.2% | 3.8% | 21.7% | 35.5% | 42.2% | 4.4% |
| $(15, 18)$ | 20.3% | 34.3% | 45.9% | 3.3% | 20.8% | 33.7% | 45.8% | 3.1% |
| $(12, 15)$ | 23.7% | 33.9% | 43.2% | 3.6% | 21.7% | 35.7% | 42.1% | 3.6% |
| $(15, 12)$ | 19.9% | 36.9% | 46.2% | 3.6% | 20.5% | 36.3% | 46.4% | 3.6% |
| $(18, 12)$ | 20.3% | 27.4% | 46.4% | 0.9% | 20.6% | 26.9% | 46.8% | 0.9% |

$\widehat{\tau}_i = \tau_i$ refer to the estimates where the break $i$ was found at the true location and $\widehat{\tau}_i = \tau_j$, $i \neq j$ to the estimates where the break $i$ was incorrectly found at the true location of the other break. The model is the reference case.

# 4 Empirical Application

The usefulness of our approach is illustrated by applying our analysis to estimate equations for the export sector employed by the London Business School model (LBS, 1995). In order to do this, while keeping the empirical analysis within the theoretical framework of one break each in the conditional and marginal models, we need to take two further steps.

First, we have to devise a way of judging the significance of the breaks in the sample, since the critical values developed in previous sections of this paper are not exactly applicable to our particular data set. A simple extension of our method however helps to overcome this difficulty. While the full details of this extension are described in a companion paper (Banerjee, Lazarova and Urga (1998)), in essence it involves resampling from the estimated equation and using the resampled or bootstrapped coefficient values to estimate confidence bands for the parameters of interest.

The second simplification is to truncate out the full sample. which extends from 1970:I to 1994:III, to include only the period 1977:I-1991:IV. This has the important effect of eliminating the instabilities of the early seventies and the early nineties, and makes the search for breaks here a relatively straightforward exercise. We would however emphasize that the methods in Banerjee *et al.* (1998) allow us to estimate our equations over the full sample. This full sample result is thus also reported, while our analysis of the truncated sample provides an intuitive illustration of the more general method.

The export equation makes use of the conventional approach of modelling the volume of trade by emphasizing only the "demand-side" of the trade volumes. The equation also typically focuses on two main explanatory variables, the output to income ratio and relative prices, and models exports of goods and services as a function of world income (as a proxy for world demand) and relative competitiveness. The imposition of a unit coefficient on world income is supported by the data. The inclusion of a competitiveness term means that an appreciation of the exchange rate makes world export price lower, and hence UK exports less competitive, unless the price of UK exports falls proportionately.

The exports equation in the long run can then be expressed as follows[6]

$$lx - ly = f\left(lpx - lpw\right),$$

where

$$\begin{aligned}
lx &= \log\left(\text{exports of goods and services, sterling 1990}\right), \\
ly &= \log\left(\text{GNP, major 6}\right), \\
lpx &= \log\left(\text{price of exports of goods} + \text{services, } 1990 = 1\right), \\
lpw &= \log\left(\text{price of world exports of manufactures, } 1990 = 1\right)
\end{aligned}$$

[6]A cointegration analysis of the system given by $lx$, $ly$, $lpx$ and $lpw$ confirms the existence of one cointegrating vector with the homogeneity restrictions implied by the parameterisation below being accepted.

Graphs of the series (levels and first differences) are reported in Figures 2-5. These show that the series are very likely to be integrated of order 1 with breaks in mean and (or) trend. We next estimate sequential tests for the marginal model where we allow for a break in the mean of the series.

The full details of the marginal model estimated over the sample 1977:I-1991:IV are reported as Equation (1) below. The break in the marginal model is dated at 1985:IV (*i.e.* where the $t$-test on the step-dummy $D_t^1$ achieves its maximum), a period characterised by strong and sustained appreciation of the dollar, while the estimated coefficient on the lagged $(lpx - lpw)$indicates that the series may be integrated, although this finding is sensitive to the extent to which the sample is trimmed, and to the inclusion of additional break dummies.

**Equation (1)**

Modelling $(lpx - lpw)_t$ by OLS (sequential dummy),

Sample period 1977:I–1991:III

$$(lpx - lpw)_t = -\underset{(2.8)}{0.08}\, const + \underset{(12.9)}{0.74}\,(lpx - lpw)_{t-1} + \underset{(2.4)}{0.001}\, trend$$

$$Seq.dummy = -\underset{(3.5)}{0.065} \text{ occurring at } t = 36 \text{ (1985:IV)}.$$

$$R^2 = 0.88 \quad F(3, 55) = 136.76\ [0.0000] \qquad \sigma = 0.028 \quad DW = 1.76$$

| | | |
|---|---|---|
| AR 1–5 | $F(5, 50) = 0.303$ | [0.91] |
| ARCH 4 | $F(4, 47) = 0.165$ | [0.95] |
| Normality | $\chi^2(2) = 2.235$ | [0.33] |
| $\mathrm{Xi}^2$ | $F(5, 49 = 1.29$ | [0.28] |
| RESET | $F(1, 54) = 0.316$ | [0.58] |

Equation (2) below presents the results of bootstrapping a more general version of Equation (1). The headings give the regressors in the model and, reading from the left, the abbreviations refer to the constant, trend, the autoregressive parameter (AR), the break in constant (br con), the break in trend (br tre) and the break date (t0) respectively. The output provided includes the central estimates alongwith the confidence interval for these estimates.

In order to interpret the results consider for example the estimated constant. Its value is given as -0.043 and with 95% probability it may be deduced that the constant is significant and negative with -0.043 representing an unbiased estimate of the true coefficient. Similarly, the break in constant (br con) coefficient has a coefficient of -0.065 and it is again deemed to be significantly negative (at 95% confidence, since the confidence interval excludes 0). The histogram of the break dates, each one computed on a bootstrap replication of the sample (10,000 replications), is presented in Figure 6 and shows a strong central tendency in the neighbourhood of 1985:IV.

Thus, from the bootstrapping, we find unequivocal and confirmatory evidence for significance of the estimated coefficient for the break in constant and we take this as strong reason for including it in the marginal model. The diagnostics in Equation (1) indicate a well behaved equation, with no evidence of instability, and we do not proceed with any further searches for breaks.

**Equation (2) (Bootstrapping (1))**

Export equation - reduced form for $(lpx - lpw)_t$

Number of replications: 10000

Number of observations: 59

Sample: 1977.1 1991.3

Trimming: lower bound (%) 5

upper bound (%) 95

| constant | trend | AR | br con | br tre | t0 | percentiles |
|---|---|---|---|---|---|---|
| -0.043 | 0.001 | 0.765 | -0.065 | 0.000 | 1985.IV | |
| | | | | | | |
| -0.131 | -0.000 | 0.425 | -0.128 | -0.005 | 1984.I | 2.5% |
| -0.022 | 0.004 | 0.823 | -0.029 | 0.003 | 1986.III | 97.5% |

An interesting point to note is the following. Say we had estimated Equation (1), having imposed 1985:IV and the model had still displayed structural instability as exemplified by the failure of Chow-test or residual non-normality. The logical step, as part of this exercise, would have been to look at further occurrences of breaks in the sample. Our method, described in detail in Banerjee *et al.* (1998) is (a) to impose the break already found in the equation as a regressor; (b) to re-bootstrap the resulting augmented (with break) model over either the full sample or over the two separate subsamples 1977:I to 1985:IV and 1986:I to 1994:III, with symmetric trimming at the end-points; and (c) iterate over further sub-samples if necessary[7]

While there is no need to adopt such a procedure in this case - either for the marginal or for the conditional model- for the sample period investigated, in order to estimate the models over the full sample of available data (1970:1 to 1994:III), steps (b) and (c) above were found

[7]Our bootstrapping algorithm which is supplemented on to the estimation exericise allows us to use the method on any data set.

to be necessary. The consequent selection of dates, reported as part of Equations (5) (marginal) and (6) (conditional) below stabilise and adequately specify the two equations.

Next, substituting the value of the break in the marginal process in the conditional model and bootstrapping this equation, we find a break in the 10th period of the trimmed sample, corresponding to 1979:II. This is reported as Equation (3) and the headings, read from left to right again, are the same as above plus three additional columns to include the estimated values (and their confidence intervals), respectively, for the lagged marginal variable (AR(M)), and the imposed dummies from the marginal process for break in constant and break in trend (br con(M), br tre(M)).

### Equation (3) (Bootstrapping (conditional model))

Export equation - reduced form for $(lx - ly)_t$

Number of replications: 10000

Number of observations: 59

Sample: 1977.1 1991.3

Trimming: lower bounds (%) 5

upper bounds (%) 95

| constant | trend | AR | br con | br tre | t0 | AR(M) | br con(M) | br tre(M) | percentile |
|---|---|---|---|---|---|---|---|---|---|
| 2.166 | -0.012 | 0.092 | 0.085 | 0.012 | 1979.II | -0.295 | 0.005 | -0.000 | |
| 1.834 | -0.020 | -0.225 | 0.058 | 0.006 | 1979.II | -0.445 | -0.022 | -0.002 | 2.5% |
| 2.923 | -0.006 | 0.234 | 0.121 | 0.020 | 1979.III | -0.144 | 0.037 | 0.003 | 97.5% |

Equation (4) provides a parsimonious version of the conditional model bootstrapped above. Note that here we have allowed for mean-changes over three successive quarters in order to model more precisely the gradual adjustment in the structure of the export series. This adds additional weights against the discrete change interpretation, attributable to a dock strike. There is no evidence of misspecification, and the model is structurally stable in the presence of the dummies. The histogram of the break dates from the bootstrapping of the conditional model having imposed 1985:IV is given in Figure 7 and shows strong central tendency around 1979:II. The 95% confidence interval of this dummy (0.085) taken from the bootstrap confirms significance.

**Equation (4)**

Modelling $(lx - ly)_t$ by OLS (sequential dummy),

Sample period 1977:1–1991:3

$$
\begin{aligned}
(lx - ly)_t \quad = \quad & \underset{(5.4)}{1.54}\, const \\
& - \underset{(2.4)}{0.16}\, (lpx - lpw)_{t-1} + \underset{(2.8)}{0.34}\, (lx - ly)_{t-1} + \\
& \underset{(1.7)}{0.012}\, s1985qIV - \underset{(4.9)}{0.093}\, s1979qI \\
& + \underset{(7.1)}{0.18}\, s1979qII - \underset{(3.3)}{0.081}\, s1979qIII
\end{aligned}
$$

$R^2 = 0.75 \quad F(6, 52) = 25.53 \ [0.0000] \qquad \sigma = 0.018 \quad DW = 2.01$

| | | |
|---|---|---|
| AR 1–5 | $F(5, 47) = 1.133$ | [0.36] |
| ARCH 4 | $F(4, 44) = 0.587$ | [0.67] |
| Normality | $\chi^2(2) = 0.283$ | [0.87] |
| $\mathrm{Xi}^2$ | $F(13, 38) = 0.416$ | [0.95] |
| RESET | $F(1, 51) = 0.975$ | [0.33] |

We conclude then there is evidence of a structural shift in 1979, after allowing for changes in the mean of the components of relative prices. The fact that a dock strike occurred at the same time raises some uncertainty about this conclusion of a shift, but, and as demonstrated in details in Hall, Urga and Whitley (1996), we can gain comfort (and evidence against such a simplistic interpretation) in that estimates of the break point do not always coincide with the dock strike period. In their paper the authors show that even allowing for the dock strike there is sufficient support for a supply-side interpretation of our results, in so far as there appear to have been changes in the behaviour of prices, which do not appear to be closely related to the changes in exports behaviour and a shift in the underlying demand for UK export.[8] Figures 2 and 4 reinforce this interpretation of a permanent shift (evidenced by a shift in the intercept of the trend growth rate in Figure 2 and a spike in Figure 4) of the export series around 1979.[9] [10]

Equations (5) and (6) provide the estimates from the full sample of observations. Note that, for example, there are additional step dummies imposed at 1992:IV, 1993:I and 1972:III,IV, 1973:I, in both the marginal and conditional models respectively, in order to take account of changes related to the ERM re-adjustments and the first oil shock. The details of the full exercise are available on request from the authors.

**Equation (5)**

Modelling $(lpx - lpw)_t$ by OLS (sequential dummy),

Sample period: 1970:I-1993:III

$$\begin{aligned}(lpx - lpw)_t \quad = \quad & \underset{(1.2)}{-0.01}\, const \\ & + \underset{(26.2)}{0.94}\, (lpx - lpw)_{t-1} \\ & \underset{(1.9)}{-0.2}\, s1985qIV \underset{(1.4)}{+0.001}\, s1985qIV * trend \\ & - \underset{(3.3)}{0.11}\, s1992qIV + \underset{(3.1)}{0.10}\, s1993qI\end{aligned}$$

[8]Hall, Urga and Whitley (1996) also found that variables which proxy changes in the quality of UK exports, or *other* supply-side influences, do not properly account for observed structural break in UK export performance.

[9]A simple dock-strike interpretation would not allow for such permanent shifts in export behaviour, but would instead imply a spike in the *level* series.

[10]The sample period is the same as that used for the marginal model.

$R^2 = 0.89 \quad F(5,92) = 144.16 \ [0.0000] \qquad \sigma = 0.029 \quad DW = 1.85$

| | | |
|---|---|---|
| AR 1–5 | $F(5,87) = 0.552$ | [0.74] |
| ARCH 4 | $F(4,84) = 0.436$ | [0.78] |
| Normality | $\chi^2(2) = 3.060$ | [0.22] |
| $Xi^2$ | $F(7,84) = 0.891$ | [0.52] |
| RESET | $F(1,91) = 0.000$ | [0.99] |

**Equation (6)**

Modelling $(lx - ly)_t$ by OLS (sequential dummy),

Sample period: 1970:I-1993:III

$$
\begin{aligned}
(lx - ly)_t \;=\; & + \underset{(3.98)}{0.64}\, const \\
& + \underset{(5.49)}{0.52}\, (lx - ly)_{t-1} - \underset{(2.6)}{0.11}\, (lpx - lpw)_{t-1} \\
& + \underset{(3.7)}{0.26}\, (lx - ly)_{t-2} + \underset{(1.00)}{0.07}\, (lx - ly)_{t-3} - \underset{(2.1)}{0.14}\, (lx - ly)_{t-4} \\
& - \underset{(4.5)}{0.10}\, s1972qIII + \underset{(7.3)}{0.21}\, s1972qIV + \underset{(3.7)}{0.09}\, s1973qI \\
& - \underset{(4.0)}{0.08}\, s1979qI + \underset{(7.4)}{0.21}\, s1979qII - \underset{(4.6)}{0.10}\, s1979qIII \\
& + \underset{(2.1)}{0.02}\, s1992qIV
\end{aligned}
$$

$R^2 = 0.90 \quad F(12,82) = 63.03 \ [0.0000] \qquad \sigma = 0.019 \quad DW = 1.80$

| | | |
|---|---|---|
| AR 1–5 | $F(5,77) = 0.872$ | [0.50] |
| ARCH 4 | $F(4,74) = 0.194$ | [0.94] |
| Normality | $\chi^2(2) = 0.351$ | [0.84] |
| $Xi^2$ | $F(17,64) = 0.577$ | [0.90] |
| RESET | $F(1,81) = 0.069$ | [0.79] |

# 5 Conclusions

Our methods provide a way of dating multiple structural breaks in the absence of any of *a priori* imposition in systems of equations. We make use of sequential break dating procedures (Banerjee, Lumsdaine and Stock (1992), Zivot and Andrews (1992)), procedures for estimating breaks one at a time (Bai (1997), Bai and Perron (1998)) and bootstrapping (Banerjee, Lazarova and Urga (1998)). Our innovation is to consider the whole problem within the framework of multivariate systems of equations and to use bootstrapping techniques as a way of establishing the statistical significance or otherwise of the breaks in the deterministic components of the time series considered. Empirical modelling of the export sector of the UK demonstrates the power and ease of applicability of our techniques.

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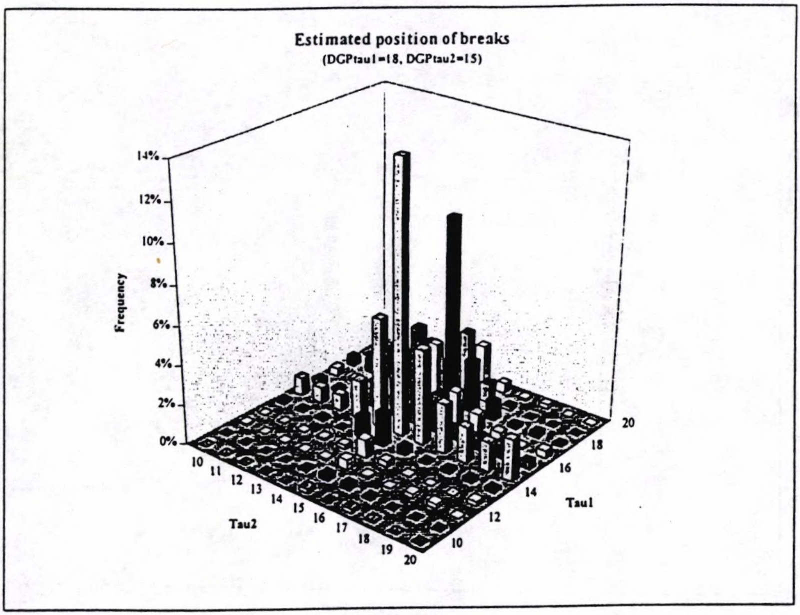
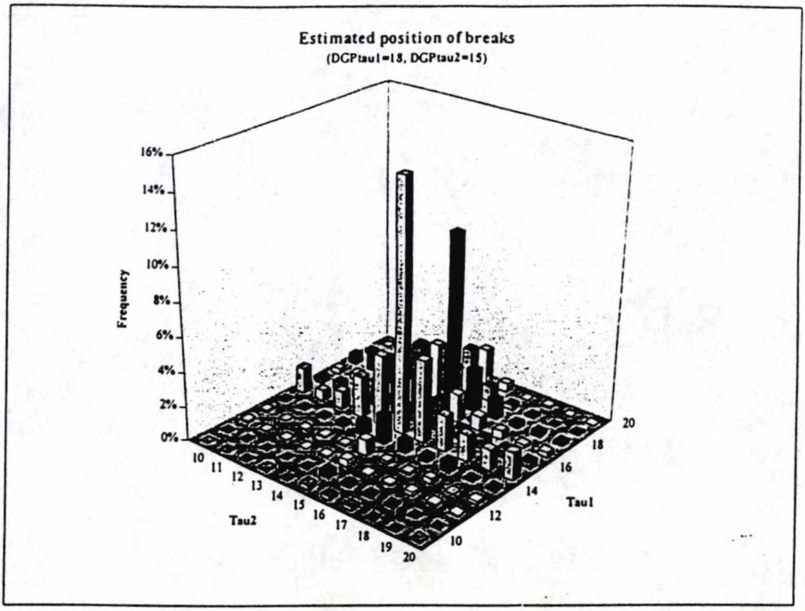
FIGURE 1A: PLOT OF THE JOINT DISTRIBUTION OF ESTIMATES OF BREAK LOCATIONS




For $\rho_2 \neq 0$.

FIGURE 1B: PLOT OF THE JOINT DISTRIBUTION OF ESTIMATES OF BREAK LOCATIONS




For $\rho_2 = 0$.

Figure 2

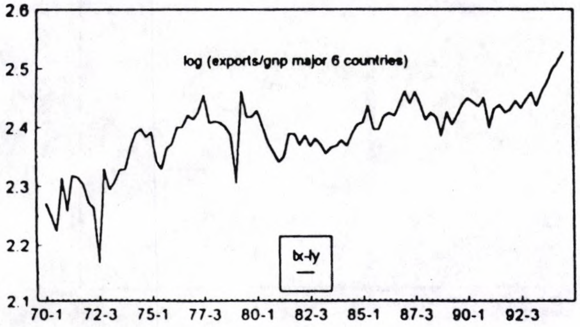



Figure 3

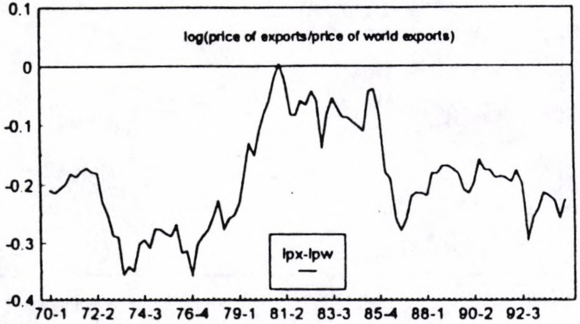



Figure 4

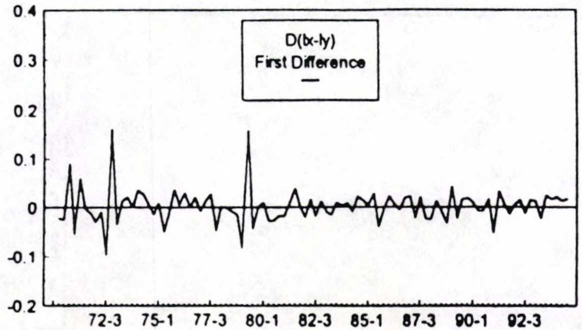
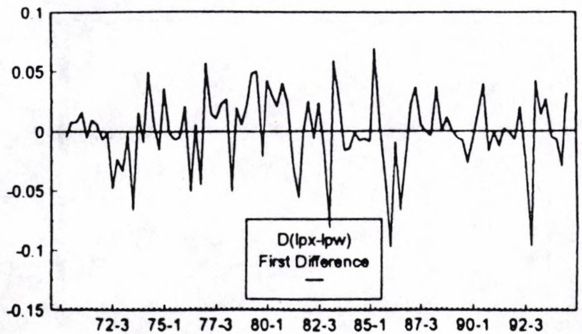



Figure 5



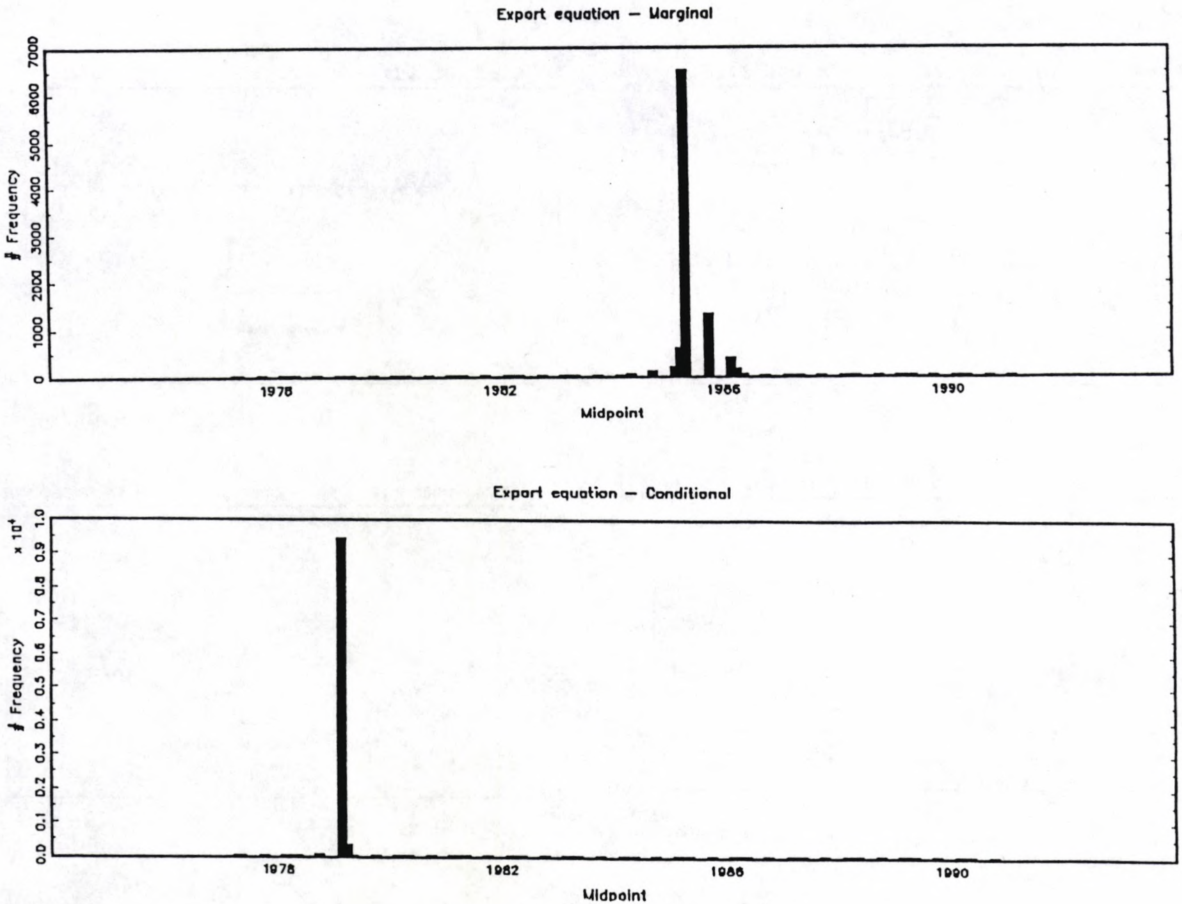

Export equation — Marginal
# Frequency
0
1000
2000
3000
4000
5000
6000
7000
1978
1982
1986
1990
Midpoint
Export equation — Conditional
# Frequency
× 10⁴
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1978
1982
1986
1990
Midpoint

FIGURE 7

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