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Abstract

In this paper we solve a monetary policy game where a government appoints a completely independent central banker whose preferences from a point of view of private agents are private information. We show that a bit of private information is sufficient to eliminate any incentive for the goverment to precommit monetary policy to a conservative agent: both in a separating equilibrium and in a pooling equilibrium the central banker's optimal degree of conservativeness is the same as the government's one.

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1 Introduction

The relationship between "society" and the agent in charge of monetary policy has been focused by recent literature as one of the main issues in the economic analysis of monetary policymaking, (see Persson and Tabellini, 1997, for a recent survey). The starting point for much of the recent literature has been a work by Kydland and Prescott (1977) who showed how, in a monetary policy game between a monetary authority and private agents the precommitment solution for the policymaker welfare dominates any discretionary solution. Rogoff (1985) interpreted their result as the possibility for a government prone to the temptation for inflation surprise to delegate monetary policy to an agent (central banker-CB henceforth) who does not take into account the benefits that unexpected inflation may have on government's target variables (unemployment, service on outstanding nominal public debt). Rogoff (1985) also showed that, in a model with nominal rigidity and scope for stabilization policy, the optimal CB's preferences are not such that the agent in charge for monetary policy sets an infinite weight on the inflation rate, though his preferences penalize inflation more than the median voter does[1]. This is the celebrated Rogoff's result about the convenience for a Government to commit monetary policy to an independent and "conservative" CB though not an infinitely conservative one, by trading off a certain amount of flexibility in the policy response for credibility.

The crucial hypothesis for this result is that the preferences of the

[1] See Alesina and Grilli (1992) for an explicit reinterpretation of the Rogoff's model in terms of a political game where a population of citizens, differing only with respect to the relative weight which they assign to inflation and stabilisation, votes upon the preferences of the "governor" to appoint. For an alternative approach on the analysis of the relationship between "society" and the agent in charge of monetary policy see Walsh (1995) and Persson and Tabellini (1993). Persson and Tabellini (1997), pp. 37 ss. offer a critical analysis of the main differences and analogies between the precommitment and the contracting approach to the normative analysis of monetary policy making.

appointed CB are common knowledge among the players of the game. Different authors[2] have emphasised that the environment in which the policymaker makes decisions is characterised by the presence of asymmetric information by the policymaker on its own preferences about alternative objectives to which policy is targeted. In the case of private information the action of the policymaker is constrained by private agents beliefs, but also has scope for policy actions that would be ineffective in the presence of complete information.

In this paper we ask whether the "conservative CB" result still holds in the presence of CB's private information on its preferences. To answer this question we consider a simple model as in Barro and Gordon (1983) and as in Vickers (1986)[3]. The model is easily summarised: at time t=0 (delegation stage) a government is elected endowed with preferences over inflation and unexpected inflation and delegates monetary policy to a CB in charge for two periods whose preferences are private information. At time t=1 (monetary policy stage), given agents expectations, the CB will set the inflation rate taking into account the fact that future expectations (at time t=2) by private agents will be set conditional on the observation of the CB's current choice. [4]. We consider both pooling and separating

[2]The issue of private information in monetary policy games has been analysed, for example, by Backus and Driffil (1985), Canzonery (1985), Rogoff (1987), Vickers (1986).

[3]The question we address in this paper naturally arises in Rogoff (1985) who states: "We have assumed that the preferences of the agent appointed to head the central bank is well known. Clearly many strategic problems arise when this assumption is relaxed". However our analysis is cast in the original set up used by Barro and Gordon (1983) and extended by Vickers (1986) to the case of private information. This is mainly due to computational difficulties originating by the fact that the first order condition to the Government's problem, defined by eq. 22a in Rogoff (1985) does not have closed form solution. Differently from Rogoff (1985), in the original set up used here there is no trade off between flexibility and credibility and, as it will be seen in a moment, the choice of the government trades off the possibility to exploit private information and credibility.

[4]Vickers (1986) shows that the sub game played by the CB at time 1 is a signalling game with both a separating and a pooling equilibrium strategies. He also shows that only the separating equilibrium survives the application of some refinement criteria. Differently from Vickers we analyse the signalling sub-game under the hypothesis of a

equilibrium and show how, in either case, the incentive to exploit the CB's degree of conservativeness as a commitment device is modified by the presence of private information.

The remaining of the paper is organized as follows: in section 2 we outline the model and define the separating equilibrium and the pooling equilibrium in the monetary policy sub-game; in section 3 we derive the optimal CB's degree of conservativeness set by the government in the delegation stage; in section 4 we conclude.

2 Equilibria in the monetary policy signalling game

The model is described as follows: at time zero a Government characterised by the preferences of his median voter is elected and appoints a CB in charge for two periods whose preferences about the trade off between the costs of inflation versus the benefits from unexpected inflation[5] are private information from the point of view of private agents. In each of the two periods when it is in charge, the CB plays a two stage subgame vis à vis private agents such that: in the first stage agents set their expectations about the inflation rate to be played by the CB and, given this expected level of inflation, they set nominal contracts. In the second stage, after observing contracts set by private agents, the CB sets out the optimal inflation rate. The incentive to reveal private information for the CB arises since private agents, setting contracts in the first stage of the second period will condition their expectations on what they observed in

continuum of types that the CB may incarnate. Then we solve for the optimal CB's type that the government may want to select under either equilibrium.

[5] See Cukiermann (1993) for a detailed exposition of the main issues and interpretations involved in the use of this utility function as a charcterisation of the government objectives.

the second stage of the first period. The pay off to the CB is given by:

$$W_t = -\frac{1}{2}\pi_t^2 - \alpha\left(\pi_t^e - \pi_t\right) \tag{1}$$

where $\alpha > 0$ represents the preference parameter that describes CB's relative weight set on unexpected inflation, $\alpha \in [a, A]$ is private information to the CB i.e., from the point of view of private agents, it is distributed according to $F(\alpha)$,[6] t indicates the period of the game. π_t is the inflation rate which, for the sake of simplicity is assumed to be a perfectly controlled instrument, π_t^e represents expected inflation[7].

Private agents seek to minimize forecast error on inflation rate according to the following per period payoff function[8]:

$$u_t = -\left(\pi_t - \pi_t^e\right)^2 \tag{2}$$

We solve the monetary policy game for the Bayes Nash equilibrium strategies and we obtain a couple of inflation rates played by CB: $s = \{\pi_1(\alpha), \pi_2(\alpha)\}$, and a couple of expected inflation rates played by private agents: $e = \{\pi_1^e, \pi_2^e\}$, where $\pi_1^e = E(\pi_1)$ represents the first period expected inflation rate based on the prior beliefs and $\pi_2^e = E(\pi_2 \mid \pi_1)$ represents the second period expected inflation rate contingent on the actual inflation rate observed in the first period and it is set according to the Bayes rule.

Lemma 1

For any arbitrary support $[a, A]$ of $F(\alpha)$, a separating equilibrium exists in the range $[\alpha^s, A]$. Equilibrium strategies are the following ones: $s^s = \{\pi_1(\alpha), \pi_2(\alpha)\}$, $e^s = \{\pi_1^e, \pi_2^e\}$,

[6] Both the support and the distribution function are common knowledge and can be arbitrarily defined. However, it can be shown that, conditions for the existence of a separating equilibrium may restrict the support, cfr. Mailath (1987).

[7] See Cukierman (1993), pp. 34 e ss., for a critical analysis of the welfare function and the macroeconomic structure used in monetary policy games.

[8] For a thorough discussion of (2) see Rogoff (1987, p.146).

where $\pi_1^s(\alpha) = \phi(\alpha)$, $\pi_2^*(\alpha) = \alpha$, $\pi_1^e(\alpha) = E[\phi(\alpha)]$, $\pi_2^e(\alpha) = E(\pi_2 \mid \pi_1) = \alpha$.

$\pi_1^s(\alpha) = \phi(\alpha)$ satisfies the following first order non linear differential equation:

$\frac{d\phi}{d\alpha} = \frac{\alpha}{\alpha - \phi}$ and is such that $0 < \phi(\alpha) < \pi_1^*(\alpha) = \alpha$.

Proof see Appendix

The separating strategy is such that, in the first period any CB in the range $[\alpha^s, A)$, will select an inflation rate lower than under complete information (reducing the inflationary bias). Private agents will anticipate this and will lower the expected level of inflation in the first period.[9]

In a signalling game such as this one there will also exist many pooling equilibria. A natural candidate for a pooling is the one in which the CB in the first period plays a pooling strategy that validates agents prior beliefs. This result is described in the following lemma.

Lemma 2

A candidate for a pooling equilibrium is described by the following strategies $s^p\{\pi_1^p(\alpha), \pi_2^p(\alpha)\}$, $e^p = \{\pi_1^e, \pi_2^e\}$ where, $\pi_1^p(\alpha) = \bar{\alpha}$, $\pi_2^p(\alpha) = \alpha$, $\pi_1^e(\alpha) = \bar{\alpha}$, $\pi_2^e(\alpha) = \bar{\alpha}$. When out of equilibrium beliefs are such that after observing any deviation from $\pi_1^p(\alpha) = \bar{\alpha}$, agents will set $\pi_2^e(\alpha) = A$, this pooling equilibrium exists for $\alpha \in \left[A - \sqrt{A^2 - \bar{\alpha}^2}, A\right]$. When out of equilibrium beliefs are such that agents do not update beliefs after observing any deviation (passive conjectures), deviation is always worthwhile and the pooling equilibrium does not exist.

Proof See Appendix

[9] A graphic characterisation of the separating equilibrium is in D' Amato and Pistoresi (1996). The analytical solution for the separating schedule is provided in the appendix.

3 The optimal CB's degree of conservativeness when his preferences are private information

In this section we analyse the optimal strategy of a government that is willing to appoint a CB but is not able to release to private agents credible information about the CB's preferences about the trade off between the cost of inflation and the benefits of unexpected inflation[10].

We finally come to the main point of the paper: in the economic framework outlined in the previous section, is it still the case that a government (i.e. a median voter) with given preferences has the incentive to appoint a conservative CB?[11] To answer this question consider that at the outset of the monetary policy game played by CB a government is elected and is willing to appoint a completely independent[12] CB whose preferences are not common knowledge.

As a benchmark, consider the case when the CB' s type is complete information, then the monetary policy outcome will be the usual inflationary equilibrium played in the two periods. Under complete information on the preferences of the CB it is immediate to show that the optimal solution would be such that $\alpha = 0$ (precommitment solution).

[10] As it is well known, a less extreme solution than "precommitment" (Kydland and Prescott, 1977) is provided in Rogoff (1985) where it is assumed that the CB's preferences are complete information, in his setting, rather than an infinite penalty on inflation, the median voter, ("society") will be willing to appoint a "conservative" CB, weighting inflation more than the median voter itself.

[11] A problem similar to the one analysed in this section is analysed and solved by Ziv (1993) in the context of information sharing among oligopolists.

[12] Complete independence of the CB in the monetary policy game makes the information about the Goverment's type irrelevant for the private agents. In this case the contract between the CB and the government only has internal relevance and there is no point for the government to reveal its own preferences. On the same ground there is no point for the government in revealing the CB's preferences any announcement would not be credible.

When the CB holds private information about its preferences we have two possible cases: either a separating or a pooling equilibrium strategies will be played by the CB[13]. Assuming that the monetary policy game equilibrium is a separating equilibrium, the government will take into account that the appointed CB must satisfy the incentive compatibility constraint for truthful revelation described in Lemma 1 and given by (A2) in the Appendix.[14] When the equilibrium outcome of the monetary policy game is assumed to be in pooling strategies, the government will take into account that the CB's behaviour will be described by strategies and constrained by beliefs as in Lemma 2. Let us solve the program for the government in either case.

The payoff function for the government is given by:

$$W^G = -\pi_1^2/2 - \alpha^G\left[\pi_1^e - \pi_1(\alpha)\right] - \pi_2^2/2 - \alpha^G\left[\pi_2^e - \pi_2(\alpha)\right] \qquad (3)$$

where α^G represents the government's preference parameter for surprise inflation, and α represents the CB's preferences in charge of monetary policy for the next two periods and represents the government's choice variable.

Given a separating equilibrium, the reduced form of equation (3), is given by:

$$W^{Gs} = -\frac{1}{2}[\phi(\alpha)]^2 - \alpha^G\left[E(\phi(\alpha)) - \phi(\alpha)\right] - \frac{1}{2}\alpha^2 \qquad (4)$$

Given a pooling equilibrium, the reduced form of equation 3 is given by:

$$W^{Gp} = -\frac{1}{2}\overline{\alpha}^2 - \frac{1}{2}\alpha^2 - \alpha^G(\overline{\alpha} - \alpha) \qquad (5)$$

[13]Differently from Vickers (1986), we do not apply refinement criteria in order to check if some of the equilibria can be eliminated. The main reason being that, under the hypothesis of continuity of types no refinement criterium can be safely applied. See Mailath (1987), (1992).

[14] Notice that, since the action of the CB is observable in the present model, the same solution can be obtained if the government, instead of appointing a CB with suitable preferences sets an inflation target in the two periods.

The optimal CB's degree of conservativeness can be obtained by maximising (4) and (5) with respect to α. We get the following proposition

Proposition 1

In the political stage of the game given either (i) a separating or (ii) a pooling equilibrium in the monetary policy game, the optimal CB's degree of conservativeness is the same as that of the government, i.e. $\alpha^G = \alpha$.

Proof

As for (i), the first order condition will be given by: $W_\alpha^{Gs} = -\phi\phi' + \alpha^G\phi' - \alpha = 0$. Using equation (A.2) for ϕ', derived in the Appendix and described in Lemma 1, we get $(\alpha^G - \alpha)/(\alpha - \phi) = 0$, which is satisfied for $\alpha^G = \alpha$. In the Appendix we will show that this is a sufficient for a global maximum.

As for (ii), the first order condition will be given by $W_\alpha^{Gp} = -\alpha + \alpha^G = 0$. The second order condition is satisfied since $W_{\alpha\alpha}^{Gp} = -1$.

Let us discuss now the reasons why, under private information of the CB on its preferences parameter, any incentive for the government to appoint a "conservative CB" disappears. Consider first the case of a separating equilibrium. In such an equilibrium, the appointment of a conservative CB will imply delegation costs in the first period and benefits in the second period. The first period cost is due to the fact that by appointing a CB with $\alpha < \alpha^G$ implies that the CB will set a level of inflation lower than the one that would be selected by the government in the absence of delegation. This will prevent a weak government to exploit reduction in the expected inflation by private agents and will inflict "strong" types of governments separating costs that are higher than in the absence of delegation. In the second period, when the equilibrium outcome is just the inflationary equilibrium, the appointment of a conservative CB would benefit any type of government. For the selected specification of the payoff functions, the cost of appointing a conservative CB will exactly counterbalance the benefits, this explains point (i) in proposition 1. In the pooling equilibrium considered here things are slightly different, since the first period inflation rate is constrained by prior beliefs, the choice of the banker is completely irrelevant for the first period outcome. Hence a rational government will trade-off only benefits and costs in the second period of the game. In this case a weak government will not waste the opportunity to artificially booming the economy by surprise inflation and hence will not select a conservative CB. A strong type of Government, on the other hand, will not like appointing a conservative CB since this latter will trigger a deflation in the second period which is larger than the deflation that the government itself is willing to pay in order to keep inflation low.

4 Conclusions

In this paper we have shown that when the government appoints a CB in charge for two periods, whose preferences are private information, the optimal degree of conservativeness is the same as the government's one. That is, private information, in the form considered in this paper, eliminates any incentive for society to appointing a CB whose preferences penalise inflation more than the median voter. The intuition for the result is that the introduction of private information on CB' s preferences increases the cost for government's commitment up to the level that in the optimum, it will not be used to constrain future behaviour of the government. The general implication of our result, for the institutional design indicate that in the case of uncertainty of the public about the true CB preferences a simple commitment argument can not be considered as a sound foundation for the set up of strategic setting of goals and legislative features shielding central banks from political pressures.

However this implication can not be taken too far since, as stated above, we analysed a monetary policy game within a model where the trade off between credibility and flexibility is not present. However the simple mode analysed in this paper can be safely considered relevant in circumstances where the surprise inflation is a strong temptation for the government regardless of the shocks that may hit the economy (stabilisation motive). An interesting task left for future work will be to set the same question addressed in the present paper within a model where the trade-off between flexibility and credibility is present. The point made here, however, is very likely to carry over: signalling costs increase with the degree of conservativeness of the CB and tend to countervail the benefits of commitment.

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A Appendix

Proof of Lemma 1

By backward induction let us solve first the second period equilibrium outcome. This is obtained by maximising $W_2 = -\pi_2^2/2 - \alpha\left(\pi_2^e - \pi_2\right)$ so that the CB will select: $\pi_2 = \alpha$. Private agents, to minimize error forecasts, will set expectations given by $\pi_2^e = E(\alpha \mid \pi_1 = \phi) = \hat{\alpha}$.

Given the equilibrium outcome of the second period, the reduced form pay-off function to the CB in the first period is given by:

$$\tilde{W} = -\frac{1}{2}\pi_1^2 - \alpha\left(E(\pi_1) - \pi_1\right) - \alpha^2/2 - \alpha\left(\hat{\alpha} - \alpha\right) \tag{A1}$$

The separating strategy will be obtained by maximising (A1) with respect to π_1 subject to $\pi_1 = \phi(\alpha)$, and $\pi_2^e = \hat{\alpha} = \phi^{-1}(\alpha)$. However, before solving this program, we need to check that (A1) satisfies regularity conditions for a separating equilibrium as stated in Mailath (1987). Belief monotonicity condition: $\tilde{W}_{\widehat{\alpha}} = -\alpha < 0$, (the CB has incentive to be believed an inflation fighter, i.e. low); Type monotonicity condition: $\tilde{W}_{\pi_1 \alpha} = 1 > 0$, (that is the larger the weight that CB sets on unexpected inflation the larger the welfare effect of increasing π_1 for any given belief of private agents); Single crossing condition is given by: $\partial(\tilde{W}_{\pi_1}/\tilde{W}_{\hat{\alpha}})/\partial\alpha = -\pi_1/(-\alpha)^2$, which is satisfied since it does not change sign for a positive inflation rate.

The first order condition for maximising (A1) will yield

$$\frac{d\phi}{d\alpha} = \frac{\alpha}{\alpha - \phi} \tag{A2}$$

A sufficient condition for (A2) to be a local maximiser for (A1) is the following one (see Mailath, 1987,p.1355): $\phi'\tilde{W}_{\pi_1\alpha} + \tilde{W}_{\widehat{\alpha}\alpha} \geq 0$ and hence, since $\pi_1 \in \phi$, and $\tilde{W}_{\widehat{\alpha}\alpha} = -1$ and $\tilde{W}_{\pi_1\alpha} = 1$, we get the following restriction on ϕ: $\phi' > 1$ the is, from (A2), $\phi > 0$.

A sufficient condition for (A2) to be a global maximiser for (A1), is the following one $\phi'\left\{\tilde{W}_{\pi_1\alpha} - (\tilde{W}_{\pi_1}/\tilde{W}_{\widehat{\alpha}})\tilde{W}_{\alpha\widehat{\alpha}}\right\} \geq 0$, which is satisfied for $\phi'\left\{1 - (\alpha - \phi)/\alpha\right\} \geq 0$, i.e. for $0 \leq \phi \leq \alpha$. This completes the proof of Lemma 1.

Equation (A2) can be solved by separating the variables and integrating to obtain the implicit function describing the separating strategy. A characterisation theorem in Mailath (1987), p.1353, plus an initial condition will allow us to select the relevant branch of the implicit function as the unique separating equilibrium strategy. In particular, set $\phi = \alpha \cdot u$, and after some algebra write equation (A2) in the following separable form: $\frac{1-u}{1-(1-u)u}du = \frac{1}{\alpha}d\alpha$. By integrating both members and by eliminating the auxiliary variable u, we get the following implicit solution: $\frac{1}{\sqrt{3}}Arc\tan\left(\frac{2u-1}{\sqrt{3}}\right) - \frac{1}{2}\log\left(1 - u + u^2\right) = \log\alpha + k$, where k represents the constant of integration which can be obtained by the initial value condition, $\phi(A) = A$. This solution is such that the relevant branch of the separating strategy is an increasing convex function such that, $0 \leq \phi \leq \alpha$, $\phi(A) = A$, and $1 \leq \phi' < \infty$. Numerical simulations showing contourplots of the implicit function above by using Mathematica confirm the characterisation.

Proof of lemma 2

Consider first out of equilibrium conjectures such that, observing any deviation from equilibrium strategy, private agents will set expectations for the second period to the inflation rate that would be selected by the worst possible type that the CB may incarnate in the support, i.e. $\pi_2^e = A$. In this case benefits from deviation will be given by $W^d = \alpha^2 - \alpha(\bar{\alpha} + A)$. Welfare from a pooling strategy will be given by $W^p = (\alpha^2 - 2\alpha\bar{\alpha} - \bar{\alpha}^2)/2$. Hence, deviation will be worthwhile for any type in the support if $W^d > W^p$, i.e. $\alpha^2 - \alpha(\bar{\alpha} + A) > (\alpha^2 - 2\alpha\bar{\alpha} - \bar{\alpha}^2)/2$ which is satisfied if $\alpha^2 - 2A\alpha + \bar{\alpha}^2 > 0$, that is for values of outside the values of the two roots $\alpha_i = A \mp \sqrt{A^2 - \bar{\alpha}^2}$. This implies that deviation is worthwhile for $\alpha \in [a, A - \sqrt{A^2 - \bar{\alpha}^2}]$, on the other hand, pooling equilibrium will exist in the range $\alpha \in [A - \sqrt{A^2 - \bar{\alpha}^2}, A]$.

Consider now out of equilibrium beliefs defined by passive conjectures (no updating after deviation), to obtain welfare from deviation substitute $\pi_1 = \alpha$ and $\pi_2^e = \bar{\alpha}$ and obtain $W^d = \alpha^2 - 2\alpha\bar{\alpha}$, so that deviation is worthwhile if $\alpha^2 - 2\alpha\bar{\alpha} > (\alpha^2 - 2\alpha\bar{\alpha} - \bar{\alpha}^2)/2$, that is $(\alpha - \bar{\alpha})^2 > 0$, which is always satisfied.

Proof of proposition 1

Given a pooling equilibrium, the optimality of $\alpha^G = \alpha$ has been proved in the text.

Given a separating equilibrium a sufficient condition for $\alpha^G = \alpha$ to be the optimal CB's type would be that (4) is a concave function in α . However we will show that, though (4) is not globally concave in α, $\alpha^G = \alpha$ is a global maximiser.

To show that (4) is not globally concave we compute $W^G_{\alpha\alpha} = -(\phi')^2 + (\alpha^G - \phi)\phi'' - 1$ and use $\phi' = \alpha/(\alpha - \phi)$, and hence $\phi'' = (\alpha\phi' - \phi)/(\alpha - \phi)^2$. From the characterisation of ϕ in Lemma 1,we know that it is a convex increasing function in α, i.e. $\phi''(\alpha) > 0$.

Though (4) is not globally concave, we can show that for any $\alpha^G < \alpha$ ($\alpha^G > \alpha$), W^G is always increasing (decreasing). To this aim consider the inequality

$$W^G_\alpha = \phi\phi' + \alpha^G\phi' - \alpha > 0 \tag{A3}$$

substitute the definition of ϕ' from (A2) and get: $[-\alpha\phi + \alpha^G\alpha - \alpha(\alpha - \phi)]/(\alpha - \phi) > 0$. In a separating equilibrium $(\alpha - \phi) < 0$, so condition (A3) will be equivalent to $-\alpha\phi+\alpha^G\alpha-\alpha(\alpha-\phi)<0$, i.e. (A3) holds for $\alpha<\alpha^G$. By reverting inequality (A3) we immediately obtain that W^G is always decreasing for $\alpha>\alpha^G$. This proves the result.