Department of Economics

TRANSMISSION MECHANISMS OF SHOCKS IN OPEN ECONOMY AND NEW KEYNESIAN DSGE MODELS

Zeno Enders

Thesis submitted for assessment with a view to obtaining
the degree of Doctor of Economics of the European University Institute

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Part I

Introduction

Introduction

This thesis deals with the transmission of shocks, i.e. how economies adjust to unforseen changes in either exogenous circumstances or policy variables. It is divided in three parts, including this introduction. The second part, containing two chapters, is devoted to the transmission of monetary shocks in closed economies. Chapter 1 looks at frictions at the price-setters side and chapter 2 at frictions at the consumers' side. Both chapters are developing alternative, more micro-founded explanations to the nowadays standard model in New Keynesian Economics, in which price setters are exogenously forced to set prices only at random dates. A better understanding of the underlying dynamics of the slow price adjustments after monetary shocks should be helpful in predicting effects of unpreceded events and for drawing policy implications. The third part, consisting of chapter 3, considers the transmission of technology shocks in an open economy. Empirically assessing the transmission mechanism at work after a technology shock gives strong results on the structure of the economy and the resulting wealth effects of such a shock. In the following, I will shortly describe each chapter in more detail.

Chapter 1 microfounds the slow distribution of information about a change in aggregate demand throughout the economy. The basic ingredients are heterogeneous firms, a discrete money stock which is known to change only occasionally and firm-specific demand shocks. The firms combine private with public information, i.e. other firms' reactions, to form expectations on the money stock. No menu costs, staggered prices or ad-hoc noisy information channels are imposed; nevertheless plausible impulse-response functions after a monetary shock can be achieved, including a delayed impact on the maximal inflation response. The model does not generate disinflationary booms and the predictions about relative sales of large and small firms are empirically supported. Furthermore, different sizes of shocks give differently shaped impulse-response functions, a feature which cannot be replicated by linearized models.

In chapter 2 a model of limited participation in the asset market is developed, in which varieties of consumption bundles are purchased sequentially. By this, heterogeneity in money holdings and in the effective elasticity of substitution of consumers arises, which affects optimal markups chosen by oligopolistic firms. The model generates a short-term inflation-output tradeoff, although all firms can set their optimal price each period and no informational problems exist. The responses are persistent even after a one-time monetary shock due to an internal propagation mechanism that stems from the slow dissemination of newly injected money. Furthermore, a liquidity effect, countercyclical markups, procyclical profits and marginal costs after monetary shocks are obtained. The model is tractable, such that analytical results for the linearized model can be derived.

Chapter 3 is joint work with Gernot Müller from the Goethe University Frankfurt. Using vector autoregressions on U.S. time series, we find that technology shocks induce an 'S'-shaped cross-correlation function for the trade balance and the terms of trade (S-curve). In calibrating a prototypical international business cycle model to match the S-curve under complete and incomplete financial markets, we find two distinct sets of parameter values. While both model specifications deliver the S-curve, the underlying transmission mechanism of technology shocks is fundamentally different. Most importantly, only in the incomplete markets economy the terms of trade appreciate and thus amplify the relative wealth effects of technology shocks - as suggested by time series evidence.

Part II

Monetary Shocks

Chapter 1

Slow Information Diffusion

1.1 Introduction

After some time of neglect, there has been a recent resurrection of, as Mankiw (2001) calls it, 'The Inexorable and Mysterious Tradeoff Between Inflation and Unemployment'. Although few topics in economics have attracted so much research, there is still discontent with the existing theories. This dissatisfaction exists although a great variety of mechanism has been put forward on this topic since Phelps (1970) proposed his island economy.[1] In his model, firms increase production after observing rising demand due to lack of information. They cannot distinguish between a change in general demand and a change in demand for their product only, an idea which many later models build on, including the one used in the present chapter. Lucas (1972) showed that the short-term tradeoff based on Phelps' model does not contradict rational expectations.

With the growing focus on dynamics in economics, the static model of Lucas was unable to fulfill some of the empirical facts it should have explained; the real effects of monetary shocks in his model can only prevail one period. The disturbances become public information in the period after the (unanticipated) shock. This means that in $t+1$, people have full information and are not 'tricked' into producing more than they would have done if they had known the real money supply. The only way of getting around this is to assume a series of shocks, but the concept of rationality foresees that people would adjust expectations accordingly. This one-time effect of monetary policy stands in contrast to empirical findings (see Christiano, Eichenbaum and Evans (1999) for an extensive survey). Classic

[1] I will keep this historical survey brief, since most of it appears in introductory textbooks. For more extensive surveys see Mankiw (1990) and Ball and Mankiw (1994) among others.

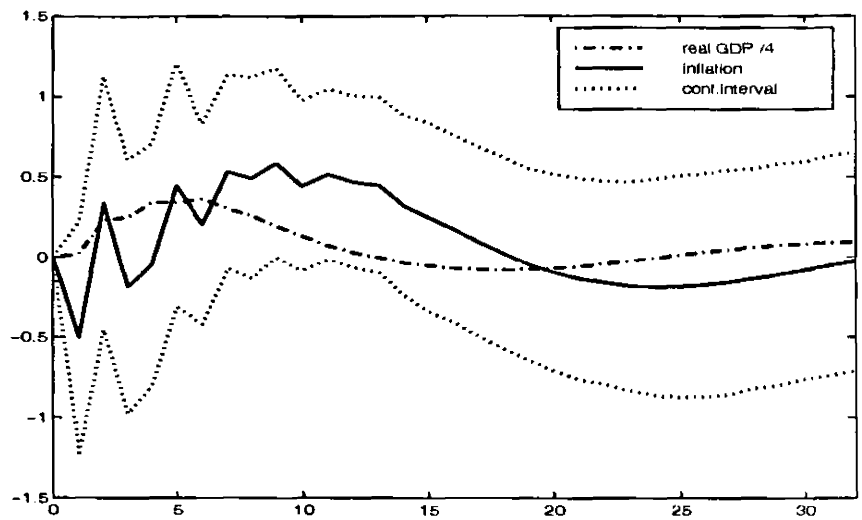

Figure 1.1: Estimated impulse-response function of real GDP and inflation after an unexpected interest-rate reduction by Christiano et al. (2001).

attempts to obtain more realistic dynamics include models of staggered price setting, such as Fisher (1977) and Taylor (1980). Calvo (1983) modified these models so that the possibility to adjust prices arrives randomly at the firms, a model which is still a benchmark in this area.

Nevertheless, besides the disadvantage of imposing nominal rigidities in a more or less ad-hoc way, these models also have troubles reconciling some empirical findings. As noted by, e.g., Mankiw (2001) and Karanassou, Sala and Snower (2002), the following problems arise: First, while persistence in the price level can be explained, the empirical observed *inflation* persistence is not achieved; inflation dies out rather quickly (see also Fuhrer and Moore (1995)). Second, the famous hump-shaped impulse-response functions of output after monetary shocks are not reproduced, either. Christiano, Eichenbaum and Evans (2001) estimate the impulse-response functions of GDP and inflation after an unexpected monetary loosening in period 0 using VAR techniques, depicted in picture 1.1.[2] Quarters are on horizontal, percentage points on the vertical axis. The dashed lines are the +/- 2 standard error confidence intervals. Third, as stated by Ball (1994), credible announcements of monetary contractions will lead to 'disinflationary booms' in staggered-price models, because prices set today that cannot adjust tomorrow are already lower in anticipation of the contraction. This is the opposite of empirical findings (see Ball (1997) and Ball (1999)). Finally, also criticized by Karanassou et al. (2002), the derived 'New Keynesian Phillips Curve' implies that an unemployment rate constantly

[2] The figure is taken from Woodford (2002).

above or below the NAIRU would lead to ever increasing rises or falls of the inflation rate. This does not fit observations, neither when unemployment rates did stay unusually high (e.g. in Europe in the 80's) nor low (e.g. in the US in the 90's) over longer periods.

Other, more recent, models concentrate on the informational problem of the Phelps-Lucas model. Woodford (2002) goes back to the original Lucas model and introduces two new features. Phelps' islands are now connected, so that the monopolistic competitors take the pricing decisions of others, which they can observe, into account. Not only does each producer's information matter now, but also the expectations about other producers' expectations and further higher order expectations. Additionally, price-setters could theoretically access all information in this model. Still, due to finite information processing capacity, as put forward by Sims (2003), their perception of the state of the world is noisy. By assuming this, Woodford achieves a sluggish adjustment of the higher-order expectations, such that prices also react this way. Thus, persistent and longer-lasting impulse-response functions of real variables in response to nominal shocks can be achieved. Nevertheless, some unrealistic assumptions have to be taken in order to achieve these results. While one might discuss the relevance of the limited information processing capacity for price setters, the assumptions that '[...] the person making the pricing decision does not actually observe (or does not pay attention to!) the quantity sold at that price'[3] is clearly less realistic.

Also building on Sims (2003), the model proposed by Maćkowiak and Wiederholt (2007) introduces a tradeoff between paying attention to aggregate and idiosyncratic conditions. Furthermore, observed variables used for information extraction are endogenous, i.e. they depend on the decisions of firms and therefore on the extraction problem itself. By this, the authors obtain a slow reaction of prices to monetary shocks, where the speed of adjustment depends on the attention paid to aggregate variables. Furthermore, real volatility increases if nominal shocks get larger. The idea that firms use aggregate and idiosyncratic endogenous variables to predict future states of the economy is also taken up in this chapter. However, no limited information processing capacity is imposed. One of the most influential recent papers on the output-inflation tradeoff is Mankiw and Reis (2002). Their model is related to the standard Calvo model, with the difference that not the possibility to readjust prices arrives randomly at the firms, but new information about the state of the economy. This is why they call their model a model of 'sticky information' as opposed to the 'sticky prices' of e.g. Taylor (1980). By this combi-

[3]Page 14

nation of elements of Calvo (1983) and Lucas (1972), the results are somewhat closer to the Fisher (1977) model. In this, some agents use outdated information, due to the nominal contracts that are fixed over some periods; in the Mankiw and Reis paper the random arrival of new information fulfills this role. This way the predictions are closer to the version of the backward-looking Philips Curve, which describes empirical findings better. While Mankiw and Reis get plausible results (e.g. impulse-response functions), they also write: 'Yet we must admit that information processing is more complex than the time-contingent adjustment assumed here.[...]Developing better models of how quickly people incorporate information about monetary policy into their plans, and why their response is faster at some times than at others, may prove a fruitful avenue for future research on inflation-output dynamics.' This is one aim of this chapter. As a result, the present model does not need ad-hoc nominal rigidities to obtain a short-term inflation-output tradeoff. Moreover, it does not exhibit real effects of merely announced changes in the money supply, in contrast to the Mankiw and Reis model.

Like Lucas and the other models based on the mechanism of imperfect information, I assume that producers do not observe the money stock (which can also be seen as a measure of aggregate demand, as stated by Romer (1996)) directly. To generate dynamic responses, I assume that this informational problem is not solved in the next period, just as in Woodford (2002) and Mankiw and Reis (2002). The main difference to the above models is, however, the heterogeneity of firms. By introducing this feature and combining it with informational imperfections, the 'imposed' nominal rigidities of Calvo contracts can be replaced by a rational behavior that leads nevertheless to a slow diffusion of new information throughout the economy. The model is also able to show where the uncertainty about the environment has its source, such that no noisy information channel has to be imposed, in contrast to Woodford (2002).

Preferences of the representative agent in the present model are such that some firms face a higher demand than other. This demand fluctuates around a mean, where the variance does not depend on this mean. According to the law of large numbers, larger firms get more accurate information about the mean by simply dividing observed demand by their size. This informational advantage of large firms plays a crucial role in the transmission of monetary shocks. If an exceptionally good year is attributed to unusually large realizations of the (serially uncorrelated) error term, the expected demand and the pricing decision for next period do not change. If, on the other hand, a good year is credited to a higher mean of demand, which indicates a higher money supply, also next period's demand is expected to

be higher; hence prices adjust upward. Another deviation from the mentioned models is the process of the money stock. In this model it is not assumed to change every period but known to do so only once a while to a considerable degree, which resembles in some way the adjustments of the central bank's interest rate. This alters the signal-extraction problem in such a way that price-setters do not change their prices after small changes in the observed demand, but only when they are convinced that a change in the money supply did really happen.[4]
The pricing decision of the firm is then based on its own information and the publicly observable price level. The latter comes in because there is a certain probability that a change in the price level is due to the recognition of some other firm, that the general demand has increased (but since they could be wrong this is not a perfect indicator, either). The resulting gradual increase of the price level results from the worse private information of the small firms. These firms do not change their prices based on their own inferior information until a certain number of other firms have increased the price level by raising their prices. Hence, the recognition about a general demand increase spreads slowly through the population.

Several problems can be solved with this mechanism. Of course, different distributions of firms over size can give different impulse-response functions, which include gradual responses and inflation persistence (high inflation today leads to many price adjustments tomorrow, i.e. again high inflation). Hump-shaped impulse-response functions for inflation can be achieved as a response to a one-time, permanent monetary shock, which is consistent with empirical findings. It is not necessary to introduce serial correlation of the growth rates of the money stock for this result, in contrast to most other mentioned papers.This implies, that empirical realistic results can be obtained for more general stochastic processes of the money stock.
Furthermore, the introduction of heterogeneous firms leads to the prediction, that small firms have a larger decline in sales after a negative monetary shock compared to large firms. This implication is empirically supported by Gertler and Gilchrist (1994) and shown in figure 1.2. They apply VARs on a data set constructed from the 'Quarterly Financial Report for Manufacturing Corporations' ranging from 1960 to 1991. They use a specification that includes the growth rate in GNP, Inflation, and the Federal Funds Rate for samples of small and large firms. While models with homogeneous firms cannot explain these differences in sales by construction, the informational disadvantages of the small

[4] Alternatively, this could also be justified by a continuous money supply in combination with small menu costs.

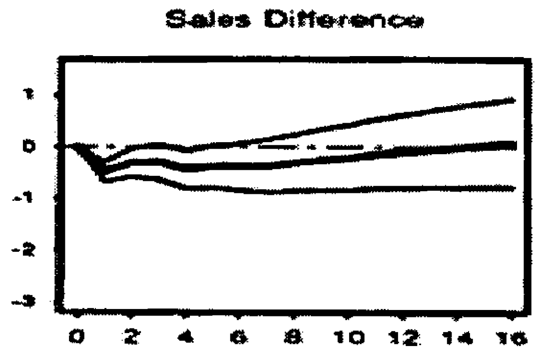

Figure 1.2: Cumulated response of the quarterly growth rate of small-firm sales versus large-firm sales to a negative one-standard deviation shock to the Funds Rate with one standard deviation error bands from Gertler and Gilchrist (1994). Time intervals are quarters.

firms in the model of this chapter gives a natural explanation for slower price adjustments of these price-setters after a negative shock, and therefore also for decreasing relative sales. Following the initial downturn, more and more firms realize that the money stock got increased, and change prices accordingly. The relative sales return to the old value, as also apparent in the data (see figure 1.2). Also in line with the predictions of the model of this chapter, Ehrmann (2000) finds that business conditions of German small firms worsen much more after a negative monetary shock then it is the case for large firms.

All of the mentioned features need certain distributions of firms over size, but not price staggering or menu costs. Also, credible monetary contractions do not lead to booms because firms are free to set their prices in the period of the shock, and do not have to adjust them beforehand in anticipation of the shock. Attempts of the monetary authorities to use the inflation-output tradeoff too often lead to adjustments of the expectations of the agents, such that the tradeoff gets more difficult to exploit. Furthermore, this mistrust against the authorities implies more frequent 'wrong' adjustments of prices due to price-setter's mistaking temporary demand shocks for changes in the money stock. Since this lowers welfare, monetary authorities should not generate surprise inflations too often.

In addition to the above mentioned features of the model, its main contributions are, first, the description of the way how information can spread through the economy without assuming the standard framework of Calvo contracts, an imposed noisy information channel or similar assumptions. By replacing these ad-hoc assumptions with a microfounded mechanism, it proposes a fundament to many papers based on Calvo contracts. Second, the non-linear structure of the model gives differently shaped impulse-response functions, depending on the size of the monetary shock. The above named, linearized, models generate differently scaled but equally shaped impulse-response functions. While more empirical work should be done on this, it is intuitive that a major shock to monetary policy has very different, and not just amplified, effects on the economy. This is especially true if one thinks of models with informational imperfections, since major shocks get much attention in the public, while minor ones might be observed only by experts.

The remainder of this chapter is organized as follows: Section 1.2 sets up the model, section 1.3 describes the simulation. Section 1.4 presents the results, while section 1.5 concludes.

1.2 A Model of Imperfect Information with Heterogeneous Firms

1.2.1 Households

The economy is populated by a representative agent who maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\int_0^1 L_t^{1+\phi}(j)}{1+\phi} \right], \tag{1.1}$$

where $L_t(j)$ are hours worked at firm j and C_t a composite consumption index defined by

$$C_t = \left[\int_0^1 N(j)^{\frac{1}{\gamma}} \left(C_t(j) + \varepsilon_t(j) \right)^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{\gamma}{\gamma-1}}. \qquad \gamma > 0 \tag{1.2}$$

$C(j)$ is a good produced by firm j. There is a continuum of firms indexed by $j\epsilon(0;1)$. $N(j)$ is a fixed taste parameter, which simply says that the agent demands some goods more than others. For consistency the following has to hold $\int_0^1 N(j) = 1$.

$\varepsilon(j) \sim N(0;\sigma_\varepsilon^2)$ is a taste shock, which can be interpreted in the following way: The representative agent sometimes feels like a bit more or a bit less of a certain good, i.e. she exhibits some variations in demand around the expected value. Today she gets the same utility from two coffees as she gets from three tomorrow. The variation of this fluctuations does not increase with the value of the good.

The agent underlies the following budget constraint

$$M_{t+1} + B_{t+1} + P_t C_t = M_t + (1+i_t)B_{i,t} + \Pi_{t+1} + \int_0^1 W_{t+1}(j) L_{t+1}(j) + S_{t+1}, \tag{1.3}$$

where i_t is the interest rate, B_t bonds that cost one unit of currency, P_t the price index defined in section 1.2.2, Π_t profits of the firms in the economy, and $W_t(j)$ the wage paid by firm j.[5] M_t is the money supply issued by the central bank, and S_t are monetary injections, given as transfers to the consumer. Additionally, the agent has to obey the following Cash-in-Advance constraint:

$$M_t \geq P_t C_t, \tag{1.4}$$

Note that M_t is beginning-of-period cash. Hence, visible from equation (1.3), the consumers can acquire cash needed for today's shopping with today's income (asset market opens first). The integral over all error terms is zero: $\int_0^1 \varepsilon(j)dj = 0$. The optimal allocation of expenditure between the goods results as [6]

$$C_t(j) = \left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} N(j)C_t + \varepsilon_t(j). \tag{1.5}$$

1.2.2 Aggregation

As Dixit and Stiglitz (1977), I follow Green (1964) (chapter 3) in constructing the price index. The basic relationship for consistency of the stated demand function (1.5) says that in equilibrium a change in one firm's price P_j is met by an equivalent increase in the demand for the other products, so that total expenditure remains equal to M.

$$\frac{\partial \int_0^1 P(j)C(j)dj}{\partial P(j)} = \frac{\partial P(j)C(j)}{\partial P(j)} + \frac{\partial \int_{l=0,l\neq j}^1 P(l)C(l)dl}{\partial P(j)} \equiv 0$$

This equation leads to the price index

$$P_t = \left[\int_0^1 P_{j,t}^{1-\gamma} N(j)dj\right]^{\frac{1}{1-\gamma}}. \tag{1.6}$$

It resembles the Dixit-Stiglitz index for homogeneous firms, except that bigger firms get weighted more. The case of identical firms ($N(j) = 1$) is a special case of the above, inserting the appropriate values leads to a confirmation of the results. Using formulae (1.4), (1.5) and (1.6), and the observation that it is not optimal to carry over cash between periods, one gets the following standard aggregate

[5] In equilibrium, the bond is in zero net supply; the interest rate adjusts in a way such that no trade in bonds takes place.

[6] There is a more plausible interpretation for this formula stating that each firm serves a different number of customers. However, modeling heterogeneity also on the consumers' side is not the focus of this chapter.

demand expression

$$C_t = \frac{M_t}{P_t}, \tag{1.7}$$

which is the same as in, among many others, Lucas (1972) and Mankiw and Reis (2002).

1.2.3 Firms

The monopolistic competitor j produces according to the following production function

$$Y_t(j) = A_t(j)L_t^{\rho}(j), \tag{1.8}$$

Setting production equal to demand gives the (real) profit of this firm as

$$\Pi_{j,t} = \left[\left(\frac{P_{j,t}}{P_t}\right)^{-\gamma} N(j)C_t + \varepsilon_t(j)\right]\frac{P_{j,t}}{P_t} - \frac{W_t(j)}{P_t}L_t(j). \tag{1.9}$$

Solving for the equilibrium wage rate of firm j, resulting from the household's and firm's problems:

$$W_t(j) = \left(\frac{C_t(j)}{A_t(j)}\right)^{\psi-\frac{1}{\rho}} C_t^{\sigma} P_t,$$

where $\psi = \frac{\phi+1}{\rho}$. The size of firms is governed by steady-state demand, which in turns depends on $N(j)$. There is a technology spillover from aggregate output to individual technology, where larger firms (i.e. a higher $N(j)$) are better able to capture this spillover: $A_j = [N(j)C]^{(\psi-1)/\psi}$. This ensures that each firm charges the same price in steady state.[7]

1.2.4 Price Setting

Most important for price setting is the process of the money stock M. In the time span covered by the model, M is known to change only once, in this case upward. The difference to the signal extraction problem of the Lucas model is therefore that M is not a continuous random variable. It resembles monetary policy that does not fluctuate every period, but only once a while. Additionally, firms do not know if monetary actions (which are undeniable at least partly publicly observable) translate into aggregate demand. As Mankiw and Reis (2002) write: 'For most people, it is easy to find out what the monetary authority is doing, but it is much harder to figure out what it means.' and Romer (1996) puts

[7] As alternative simulations show, this assumption is not crucial for the results, but it makes the model more intuitive and simple.

it in the following way: 'M should be thought of as a generic variable affecting aggregate demand rather than as money'. There are very few shop owners who take central bank decision into account when setting their prices; they rather look at the demand for their products and the general price level. I assume that M has only two known values, M_L (low) and M_H (high). The rational decision rule is therefore not to change prices according to a signal extraction method every period based on noisy information as in Lucas (1972), because this would lead to a wrong price in almost every period even if M did actually change. Hence, a firm changes its price only when it becomes convinced that an increase in M happened. 'Becoming convinced' could be interpreted as the arrival of new information in the sense of Mankiw and Reis.

One further assumption is taken in order to keep the solution simple and focus on the theoretical point of the chapter. I assume that firms can set their prices to only two values: namely P_L and P_H. The values of P_L and P_H are set to the prices in long-term, full-information equilibria with a low or high money stock. Using additionally the assumption that firms can set their prices to any value should strengthen the results of this model. Maximizing profits (1.9) for a known M yields the following result

$$P_X = \left(\frac{\gamma}{\gamma-1}\psi C^{\sigma+1-\psi} M_X^{\psi-1}\right)^{\frac{1}{\psi-1}}, \tag{1.10}$$

where X is either L or H. This, together with equation (1.7) and goods market clearing, also shows that in steady-state $Y = [(\gamma-1)/(\gamma\psi)]^{1/\sigma}$, independent of the nominal money supply.

The main problem of the firms is now to decide if the money stock has changed or not. They have two sources of information, private ($C_{t-1}(j)$) and public (P_{t-1}). Both stem from last period so that the firms use the latest available information.[8]

Private Information: Own Sales $C_{t-1}(j)$

It is most intuitive to look at 'core demand'[9]

$$\frac{C_{t-1}(j)}{N(j)} \equiv c_{j,t-1} = \left(\frac{P_{j,t-1}}{P_{t-1}}\right)^{-\gamma}\frac{M_{t-1}}{P_{t-1}} + \frac{\varepsilon_{t-1}(j)}{N(j)}. \tag{1.11}$$

$c_{j,t-1}$, given M_{t-1}, is normally distributed with mean $\left(\frac{P_{j,t-1}}{P_{t-1}}\right)^{-\gamma}\frac{M_{t-1}}{P_{t-1}}$ and variance $\frac{\sigma_\varepsilon^2}{N(j)^2}$, since the error term of equation (1.11) has a zero mean. Note that the only unknown variable in this equation

[8] I do not yet include the possibility of basing the decision on repeated observations of the demand, e.g. changing the price if the demand was above the expected value for a couple of periods. This will be done in a later paper.

[9] Equation (1.7) was used for the substitution of C_{t-1}.

is M_{t-1}, all other variables are observable in the last period. Figure 1.3 illustrates equation (1.11) for two different firms.[10] The lower panel corresponds to a small firm ($N(l)$ relatively small) with a corresponding higher variance, whereas the upper panel depicts a relatively large firm. Hence, the ratio of two normal distributions with different means, corresponding to M_H and M_L, and same variance gives the probability that last period's demand was due to either M_H or M_L (based on private information only). As visible, the variance of equation (1.11) plays a crucial role for the firm's recognition of a monetary shock. Since, by the law of large numbers, it depends negatively on the size of a firm, larger firms have an informational advantage over smaller ones.

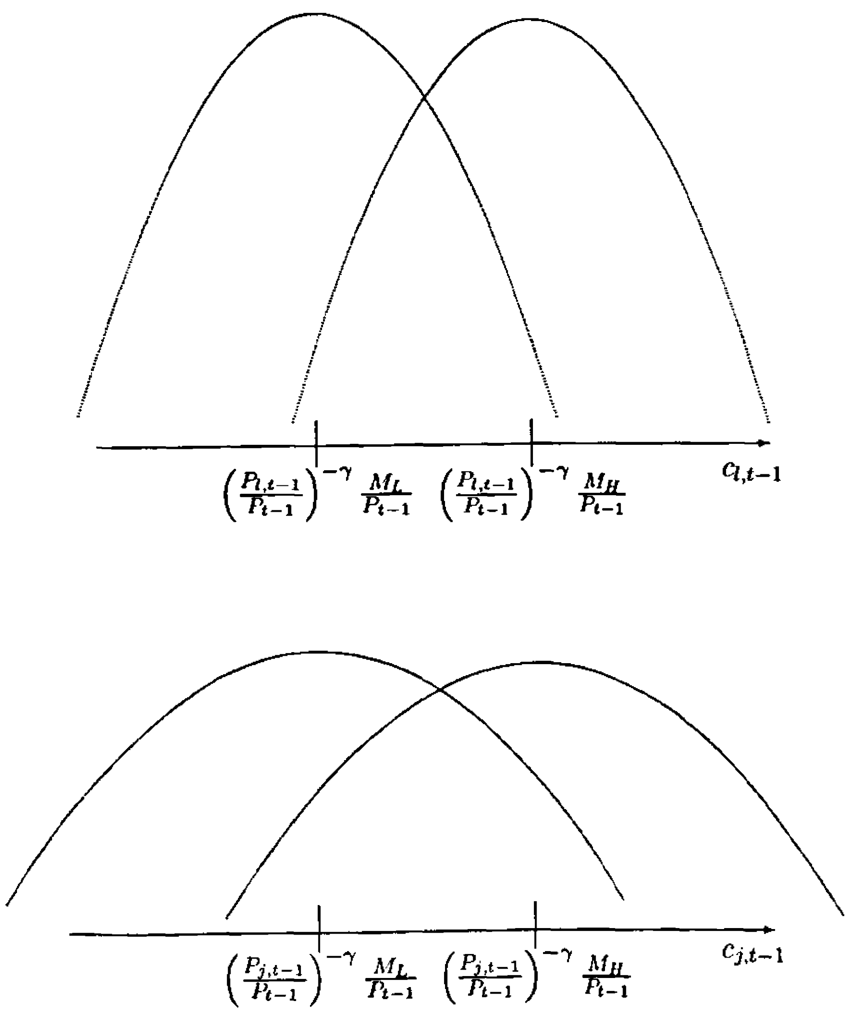

Figure 1.3: Distribution of $c_{j,t-1}$ of two differently sized firms

[10] The distributions are not exactly normal distributions but stylized.

Public Information: The Price Level P_{t-1}

Not only own observations provide information about the likelihood that a change in M happened, but also the aggregate price level. If it goes up, it shows that other firms changed their prices because they believe that the money stock has risen to M_H. There is a certain probability that they did this by mistake, which makes the price level an imperfect indicator about M, just as $c_{j,t-1}$.[11] Unfortunately, it is not easy to disclose the exact information about M embodied in P, the discussion about this is deferred to section 1.3.

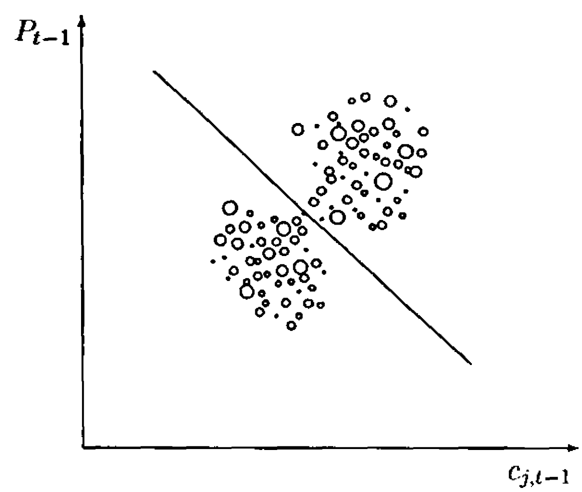

Figure 1.4: Discriminant Analysis

However, the firms need to somehow combine the public and private information from $c_{j,t-1}$ and P_{t-1}. Since there are only two possible states of the hidden state vector M, both sources of information can stem from either a low or a high money stock, which gives rise to the following technique.

[11] An earlier version of the model included also cost shocks, another reason why firms could raise their prices without M changing.

Discriminant Analysis

As said, the two variables can form two clusters, depending on the realization of M. To discriminate between these two clusters, which means to discriminate between M_L and M_H, the firm uses the statistical multivariate technique of ***Maximum Likelihood Discriminant Analysis***, as described in standard statistical textbooks, e.g. Härdle and Simar (2002). In the simplest case one wants to draw a line between the two clusters in the $(c_{j,t-1}, P_{t-1})$ space, as shown in figure 1.4.

In the present case this is not enough, since different kinds of misclassifications lead to different profits of the firms. That is why the probabilities of classification (correct and incorrect ones) have to be weighted by the corresponding profits (Π) in the following way:

Π(M_X guessed while M_Y is prevailing) * Prob(M_Y prevails) * Prob(classify observations as M_X) for all combinations of X and Y (which are H or L). Or, more formal:

$$\Pi(M_H|M_H)Pr(M_H)\int I(x\epsilon R_H)Pr(x|M_H)dx$$
$$+\Pi(M_H|M_L)Pr(M_L)\int I(x\epsilon R_H)Pr(x|M_L)dx$$
$$+\Pi(M_L|M_L)Pr(M_L)\int I(x\epsilon R_L)Pr(x|M_L)dx$$
$$+\,\Pi(M_L|M_H)Pr(M_H)\int I(x\epsilon R_L)Pr(x|M_H)dx, \qquad (1.12)$$

where $Pr(x|M_Y)$ is the probability that the observed vector x, consisting of $c_{j,t-1}$ and P_{t-1}, stems from a distribution which is valid if M_Y is the current money stock and $I(x\epsilon R_Y)$ indicates if x falls into the range, where it gets classified as stemming from M_Y. The integrals are taken over all possible realizations of x. Maximizing the above expression leads to the decision rule:

Coming from P_L (the initial state), change price to P_H if

$$\frac{Pr(x|M_H)}{Pr(x|M_L)} > \qquad (1.13)$$
$$\frac{\Pi(M_L|M_L)-\Pi(M_H|M_L)}{\Pi(M_H|M_H)-\Pi(M_L|M_H)}\frac{Pr(M_L)}{Pr(M_H)}.$$

The first part of the right hand side is in words: Expected gain from classifying *correctly* if M_L prevails / Expected gain from classifying *correctly* if M_H prevails. The second part gives the relative unconditional (prior) probability that M_L prevails. If the monetary authorities are trying to exploit possible inflation-output tradeoffs often, agents will adjust this term. Quicker reactions become more likely and the tradeoff more difficult to achieve.

The tricky part of solving the model is finding expressions for the elements of equation (1.13). First, I will describe the solution of the model, assuming that equation (1.13) can be solved, and delay the discussion of how to solve this equation to section 1.3.

1.2.5 Dynamics

With the specification so far, the dynamics can be described. The price level after a monetary shock can be calculated by

$$P_t = \left[\int_0^{s_{t-1}} P_L^{1-\gamma} N(j)dj + \int_{s_{t-1}}^1 P_H^{1-\gamma} N(l)dl\right]^{\frac{1}{1-\gamma}}. \tag{1.14}$$

s is the index of the marginal firm that is indifferent between changing its price or not. It is obtained by inserting an equality sign in equation (1.13) in time $t-1$. This means that all firms that were below this index in $t-1$ do not change their prices away from P_L in t, and all above it adjust to P_H.[12] Unfortunately, for general values of γ no analytical solution can be derived, so that numerical simulations have to be used. Once this new price level is calculated, one can insert it in equation (1.13) to find the marginal firm of period t. Now, a higher proportion of firms will adjust their prices in $t+1$, since it became more probable that the observed price level stems from a risen money stock. Repeating this exercise gives the dynamic path of the price level after a shift from M_L to M_H. Knowing the price level, it is easy to derve inflation ($\frac{P_t - P_{t-1}}{P_{t-1}}$) and Y through equation (1.7), so that impulse response functions for these variables can be drawn. How this is done in practice will be described next.

[12] In the simulations I check that there is not more than one marginal firm, which turns out to be never the case since equation (1.13) is strictly increasing in the size of the firm.

1.3 Numerical Simulation of the Model

In order to simulate the model, one needs to solve equation (1.13), inserting an equality sign. Four elements have to be found for this. First and easiest, I set the prior relative probabilities of both money supply levels equal, $\frac{Pr(M_L)}{Pr(M_H)} = 1$. Second, the profits in the different cases of false or correct classification have to be calculated. The profit when M_X is assumed while in reality M_Y prevails ($\Pi(M_X|M_Y)$) can be found by inserting the appropriate values in equation (1.9).

$$E_t(\Pi(M_X|M_Y)) = \left[\left(\frac{P_X}{E_t(P_t|M_Y)}\right)^{-\gamma} N(j)\frac{M_Y}{E_t(P_t|M_Y)}\right]\frac{P_X}{E_t(P_t|M_Y)} - E_t[W_t^r(j)L_t(j)], \tag{1.15}$$

where X and Y are again either L or H. P_X is therefore the either the high or the low price that a firm can set. $E_t[W_t^r(j)L_t(j)]$ are expected real costs (with W_t^r being the real wage). These depend on $E_t(P_t|M_Y)$ and higher moments of the distribution of ε, which are known to the price setter. Hence, the only unknown in the above equations is $E(P_t|M_Y)$. Here, I make another simplifying assumption, similar to the one about the only two possible prices. Because firms do not know the distribution of firms over size, they just approximate $E(P_t|M_Y)$ by the equilibrium values of the price level that correspond to the low or high money stock, i.e. $E(P_t|M_L) = P_L$. Third, the term $\frac{Pr(x|M_H)}{Pr(x|M_L)}$ has to be evaluated (remember that this is the relative probability that the observed variables $c_{j,t-1}$ and P_{t-1} stem from M_L or M_H). Since P_{t-1} consists of the prices set based on information of period $t-2$, it is independent of $c_{j,t-1}$ and the following is valid

$$\frac{Pr(x_{t-1}|M_H)}{Pr(x_{t-1}|M_L)} = \frac{Pr(c_{j,t-1}|M_H)}{Pr(c_{j,t-1}|M_L)}\frac{Pr(P_{t-1}|M_H)}{Pr(P_{t-1}|M_L)}. \tag{1.16}$$

As already described in section 1.2.4, the following ratio gives the probability that the observed $c_{j,t-1}$ stems from M_H relative to M_L

$$\frac{Pr(c_{j,t-1}|M_H)}{Pr(c_{j,t-1}|M_L)} = \frac{N\left(\left(\frac{P_{j,t-1}}{P_{t-1}}\right)^{-\gamma}\frac{M_H}{P_{t-1}}; \frac{\sigma_\varepsilon^2}{N(j)^2}\right)}{N\left(\left(\frac{P_{j,t-1}}{P_{t-1}}\right)^{-\gamma}\frac{M_L}{P_{t-1}}; \frac{\sigma_\varepsilon^2}{N(j)^2}\right)}. \tag{1.17}$$

Finally, and most difficult, the probabilities that the observed price level stems from M_L or M_H has to be calculated. In order to do this, the distributions of P after a change from M_L to M_H and in the case of no change have to be known.[13]

[13]The firms are assumed to know these distributions from experience. All other possible assumptions would be less plausible.

1.3.1 Distribution of the Price Level

Unfortunately, these distributions are only known after one has solved the model for P, i.e. the distributions are the result of a process that can only be solved if these distributions are known beforehand. Hence, I use an augmented Parameterized Expectations Algorithm in the following way:

- I draw shocks for the marginal firm
- I solve the model for each shock, assuming normal distributions for the price levels in the first period after an upward shift of the money stock and in the case of no change. I get a cloud of points for the resulting price level after a shift in M.
- Using a Kernel, I estimate a smooth, nonlinear function for this price level. Inserting a weighted average of the new and the old distribution in equation (1.16), I simulate the distribution of the price level if no change in M happened, again via a Kernel and a weighted average (which is needed for a slow convergence).
- I repeat this from the second step on, always using the simulated distributions of the step before, until both functions converge. Thus, I can evaluate the expression

$$\frac{Pr(P_{t-1}|M_H)}{Pr(P_{t-1}|M_L)} \tag{1.18}$$

from equation (1.16).

1.3.2 Simulation of the Dynamics

As said, because of equation (1.14) and the 0/1 decisions whether a firm changes its price or not, the model cannot be solved analytically and no easy Phillips-Curve can be derived. Hence, I simulate the model numerically as described in section 1.2.5, using the distributions for P that were found in the previous subsection. I used Matlab for this task.[14]

γ	ρ	ϕ	σ	σ_ε^2	M_L	M_H	$N(j)$	$\frac{Prob(M_H)}{Prob(M_L)}$	Tolerance	Shocks drawn
2	1	1	1	0.1	50	50.5	$4j^3$	1	0.01	100

Table 1.1: Values used in the baseline simulation

[14] The code is available from the author upon request.

Table 1.1 shows the values used in the simulation, which gave figures 1.5 and 1.6.[15] M_H and M_L are only interesting in their ratio (a 1% rise in the money stock, i.e. $S = 0.01M_L$). $\frac{Prob(M_H)}{Prob(M_L)}$ is set to 1 for the moment, since I did not yet consider any prior probabilities of a low or high money supply. Tolerance means that the algorithm for finding the distributions for P described in section 1.2.4 continued until both distributions did not change more than 0.01 at their maximal distances from the distributions found one iteration before. γ is set to a common value, $\phi = 1$ as in Galí (2003), $\rho = 1$ implies a linear production function, and σ is set to 1, i.e. the log case. Due to lack of computing power, I could only draw 100 shocks for which the distributions were calculated over and over again. σ_ε^2, the variance in the demand function, is set to a relatively small value, such that the agents are not too far from a standard neoclassical behavior (which is equivalent to $\sigma_\varepsilon^2 = 0$).

1.4 Results of the Simulation

For the simulations, I consider a permanent positive shock to the money supply from M_L to M_H in period 0, i.e. $S_t = 0 \; \forall \; t \neq 0$. For the example values given, the effects of a one percent change in the money stock on inflation and output are depicted in figures 1.5-1.6.[16] The mechanism of the model can be seen in these figures. The largest firms in the economy realize first the increase in the money stock and change their prices, which in turn triggers a price change by smaller firms one period later. So, additional firms adjust in the following periods and the price level increases only gradually. The change in the price level determines inflation, whereas output is influenced by the combination of the increased money stock and the slowly adjusting price level. Unfortunately, the shown results did not prove to be very robust to variations in the parameters.

Nevertheless, through this mechanism alone, empirical plausible hump-shaped impulse-response functions can be achieved, considering the stylized economy used. As mentioned in the introduction, these hump-shaped impulse-response functions can be obtained without assuming neither autocorrelation of the growth rates of the money stock nor random or noisy arrival of information, which differentiates the results to the ones obtained in Woodford (2002) and Mankiw and Reis (2002). The main requirement for this mechanism is the heterogeneity in firm sizes. If firms were of the same size, all firms would notice the change at the same time, i.e. either in the first period after the change in M or, if the

[15] Relevant is only the range between P_L and P_H, the initial downturn is to be ignored.

[16] The values at the Y axis for output are absolute values in order to depict the effect on the level of GDP. In these simulations the nominal values are of course only meaningful in relations.

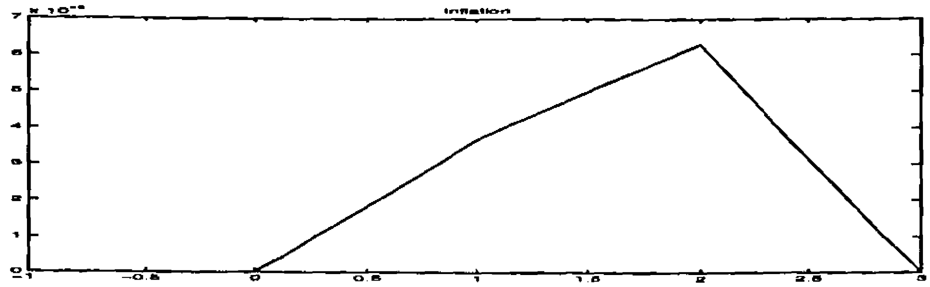

Figure 1.5: Response of Inflation...

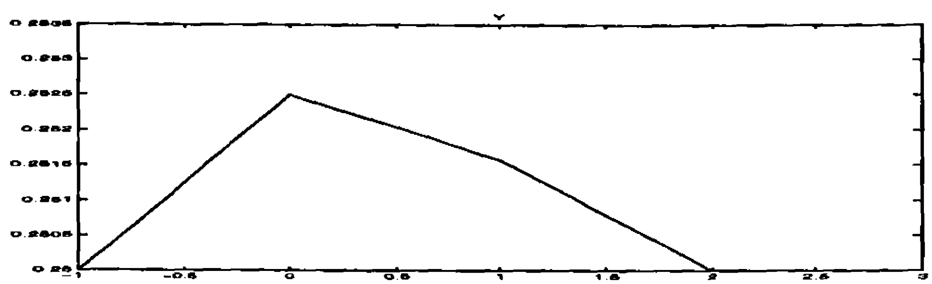

Figure 1.6: ..and Output to a 1% increase in the money stock.

change was very small, never. In the former case, the impulse-response functions would degenerate to one-time peaks in period 0 for GDP and in period 1 for inflation. In the latter, inflation would not occur at all, while output would be permanently slightly above the old steady state. For more realistic distributions of firms over size, however, inflation persistence can be generated. Note that the peak in inflation occurs after the peak in the response of output, an empirical fact observed by Christiano et al. (2001). The model is combinable with mechanisms used in other papers. Christiano et al. (2001) for example use habit formation in order to get a slow increase in demand after a monetary loosening. This delays the maximum impact of the change in monetary policy for some periods, which can also be motivated by the empirical fact that there is a substantial lag of the effects of monetary policy.[17] While the present model does exhibit a delay in the maximum impact on inflation (in contrast to sticky-price models), a slow increasing demand would also delay the effect on output. Besides this lag, incorporating the mentioned features in the present model would probably strengthen the general mechanism. Even if firms do not use the general price level at all for their estimation of the money stock, the slowly increasing demand would convince one firm after the other, hence again a gradual adjustment of the price level could be achieved.

[17]See, e.g., Bernanke and Gertler (1995).

1.5 Conclusion

The model does quite well in reproducing empirical plausible impulse-response functions to monetary policy. It does so by using ideas connected to the Lucas (1972) model in order to explain the slow dissemination of information in the spirit of Mankiw and Reis (2002), without using staggered contracts, menu costs or the like. Contrary to the stochastic process used in Mankiw and Reis, the arrival of new information is based on a modified signal extraction problem, in which the effects of temporary and permanent demand shocks are tried to be distinguished. The source of the uncertainty about demand is explained in the utility function, and not by an imposed noisy information channel as in Woodford (2002). Besides the mentioned hump-shaped impulse-response function, several other conclusions emerge.

First, there is a short-term tradeoff between inflation and output. The corresponding impulse-response functions change their shapes depending on the size of the shock to the money stock. If money supply is increased by 2%, one can already see a change in the shape of the impulse-response functions in figures 1.7-1.8. For simulation with a, e.g., 10% increase in the money stock, all firms react in the period after the shock. Hence, there is only a one-period deviation from the steady-state output. The larger the increase in the money stock, the faster its effects will vanish. Moreover, the monetary authorities cannot exploit the short-run tradeoffs as often as they want. If they do so, agents will adjust the prior probability of a high money stock in equation (1.13), which leads to quicker reactions and a shorter-lived tradeoff. Additionally, after an adjustment of these prior probabilities firms are more likely to switch to a higher price by mistaking a temporary demand shock with an increase in M, even if M stayed low. Since this introduces volatility, the increased mistrust in the authorities reduces welfare on average. Hence, monetary authorities are advised not to generate surprise increases in M too often.

Furthermore, the distribution of firms over size plays an important role for the inflation-output tradeoff. As an exercise in comparative statics, I report the impulse-response functions for a different distribution of firms over size, namely $N(j) = 5j^4$, in figures 1.9-1.10. All other parameters used are the same as in figures 1.5-1.6, i.e. the figures show responses after a 1% increase in M. Note that the new distribution reaches up to 5 at $j = 1$, whereas the previous distribution $N(j) = 4j^3$ only reached 4. This means that there are larger firms in absolute terms, and a larger proportion of relatively larger firms due to the increased steepness of the distribution. These firms on the upper end are better able to

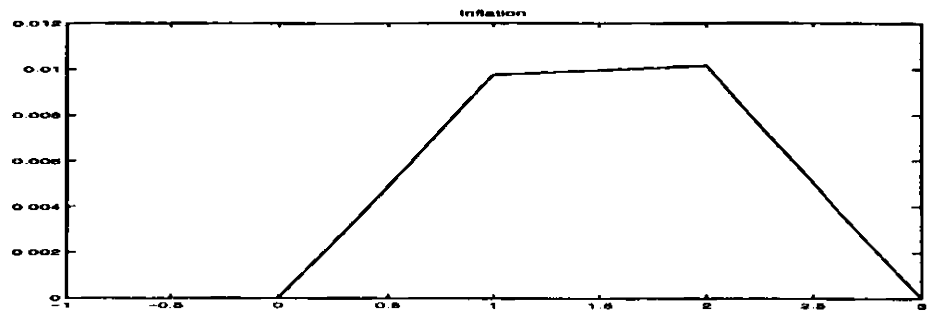

Figure 1.7: Response of Inflation...

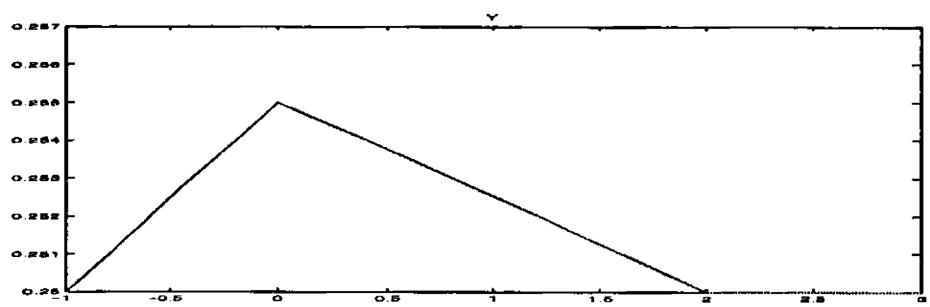

Figure 1.8: ..and Output to a 2% increase in the money stock.

detect changes in M, such that the initial response in inflation in figure 1.9 is larger than in the benchmark case reported in figures 1.5-1.6. A larger initial response of these 'leaders' triggers more price changes of smaller firms, the real effect of the change in money supply falls therefore quicker in the second period after the shock, visible in figure 1.10. Second, since there are no staggered contracts, a credible announcement of a future change in the money stock does not have any effects on real variables. If all firms believe this announcement, and are furthermore convinced that also all other firms believe it, each firm will adjust prices at the time of the money stock increase. Output will therefore remain constant in the absence of other shocks. This implies also that no disinflationary booms can arise, which was criticized by Ball (1994) for the staggered-contracts models.

This chapter predicts faster adjustments after a shock to the money stock from the larger firms, due to their informational advantage. This implies that the sales recover faster after a negative shock, since they adjust quicker to the optimal price. As mentioned in the introduction, Gertler and Gilchrist (1994) find exactly this result in the data. They observed that after a negative shock to the Funds Rate the difference in sales, starting from a normalization of 0, first swings in favor of larger firms, until returning back to the starting value, as seen in figure 1.2 (in the model this would be due to

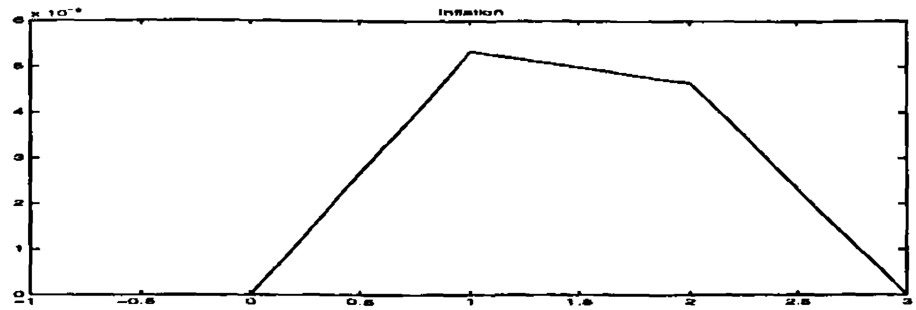

Figure 1.9: Response of Inflation...

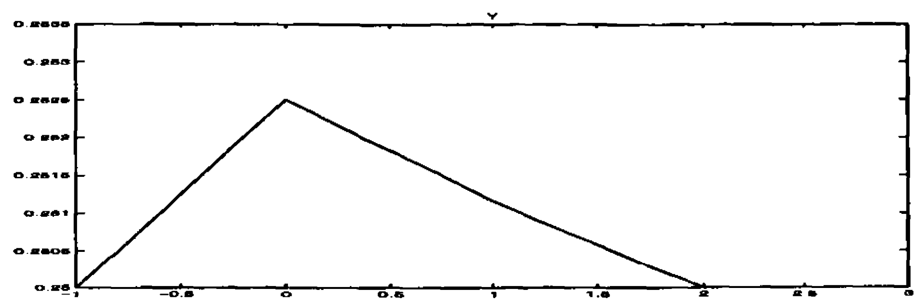

Figure 1.10: ..and Output for a steeper distribution of firms over size.

price adjustments of all firms after some time). Figure 1.11 plots the corresponding difference of the growth rate of sales of small firms (below the median) and large firms (above the median) after a 1%, permanent fall of the money supply in the model.[18] Also Ehrmann (2000) finds that 'The business conditions of all firms deteriorate after a monetary tightening, but those of small firms do so much more.'[19]

However, several critiques apply to the proposed approach. First, while it is an advantage to have nonlinear models to get more realistic impulse-response functions, it makes the model more complicated. As a result, the model is difficult to modify or to use as a benchmark for other models that start at a higher level of aggregation. Moreover, several simplifications were necessary in order to keep the model tractable. Especially the assumptions of only two levels of the money stock and two possible prices for each firm are quite stringent. It could prove fruitful to relax these assumptions, and substitute the discriminant analysis with a generalized Kalman Filter. Future work will incorporate extensions based on these features.

[18]To make the graph comparable to the one from Gertler and Gilchrist, only the dynamics after the shock are considered.
[19]Page 24

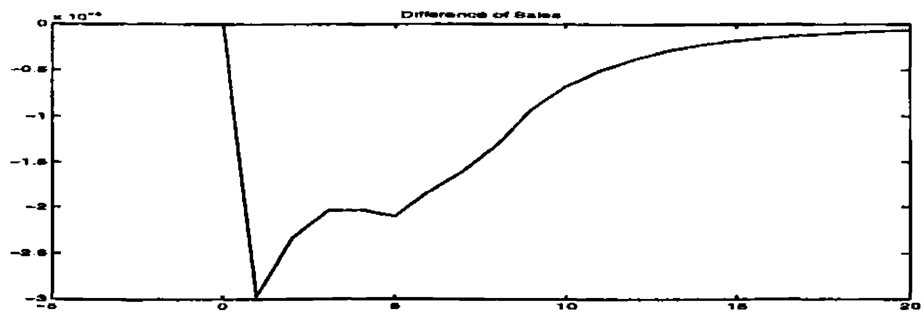

Figure 1.11: Difference of changes in sales of small vs. large firms after a negative 1% shock to M.

1.5.1 Two more general points of the chapter

In this model, the ad-hoc rigidities of models of homogeneous agents (e.g. Calvo contracts), needed to get satisfactory impulse response functions, could be replaced. However, the idea of the chapter may also be applicable to other situations, in which heterogeneity combined with observation of the decisions of others can explain spreading of information and hence slow adjustment to all sorts of shocks.

Another implication of the model is the fact that the response of the economy depends on the size of the monetary shock, since the nonlinear structure of the model has a strong impact on the shape of the impulse-response functions. As mentioned, if the money supply is increased a lot, all firms take notice and adjust immediately, such that the impulse response function does not simply get amplified but alters its shape. This is more realistic and part of a general point: Depending on the size of some shocks or movements in economic variables, different models should be used or, as in this case, a model which generates different reduced forms (e.g. through non-linearity). This is related to suggestions put forward in the context of perception of inflation (small amounts of inflation are not perceived, but large ones precisely) by, e.g., Khan (1977).

Chapter 2

Slow Money Dissemination

2.1 Introduction

Monetary economics has seen a considerable variety of models labeled limited participation models over the last two decades. Because of their common ability to replicate well some empirical features, '[t]hese models represent a serious alternative to the sticky-price and sticky-wage Keynesian models that have been popular in recent policy analysis.', according to Williamson (2005). Originally, Grossman and Weiss (1983) had in mind an infrequent adjustment of asset holdings by households, combined with overlapping shopping sequences. In their model, consumers are assumed to make staggered money withdrawals, i.e. in any given period only half the population visits the bank. The notion that households do not adjust their assets each period is theoretically shown as optimal by Jovanovic (1982) and empirically supported by Christiano, Eichenbaum and Evans (1996), who find that 'households do not adjust their financial assets and liabilities for several quarters after a monetary shock'. Households in the Grossman-Weiss model need money balances to finance their consumption, since they face a Cash-in-Advance constraint. A liquidity effect is obtained because falling interest rates are needed after an increase in the money supply to induce higher money holdings by the agents currently at the bank. Furthermore, prices rise slowly after a positive monetary shock. However, the path of adjustment is not satisfactory. The price level and the interest rate are reaching the new steady state in an oscillating manner. Because of the simplification of setting output exogenously and constant, no inflation-output tradeoff is modeled. Rotemberg (1984) uses the same timing structure, but introduces production and capital. He finds that after a monetary expansion, output increases and returns slowly to the steady state without oscillating. A Cobb-Douglas function for the combination of utility from both period's consumption is assumed. Therefore, households spend equal amounts of their cash holdings each period in between their visits to the asset market, such that one important aspect of intertemporal optimization of the households is not addressed. Also, because of perfect competition, the optimal markup is not considered.

The present chapter tries to shed some light on these and other questions by using a model with oligopolistic competition in the goods market with arbitrary values for the elasticities of substitution between varieties and periods. Note that oligopolistic competition here does not necessarily stem from few firms populating the economy, but from the fact that each consumer buys at a countable number of shops. Analytical results are derived, which are not available for the Grossman-Weiss and Rotemberg models. In these models, the distribution of wealth over time is difficult to track because of the heterogenous agents. Hence, the models are limited to study the effects of one-time monetary shocks in a deterministic setting. Maintaining the heterogenous money holdings and the sequential shopping sequence of the models mentioned, I assume an ownership structure of the shops that mimics the slow dissemination of newly injected money throughout the economy, and leads to a model that can be analyzed with the standard tools for dynamic stochastic general equilibrium models.

A different and widely used solution to the problem of tractability was found by Lucas (1990), who lets household members pool their trade receipts at the end of the period. By this, a degenerate money distribution, and therefore tractability is reached. However, the paper and many follow-ups deal mainly with the liquidity effect and asset pricing implications, but not with the inflation-output tradeoff. Because households undo the effects of monetary policy at the end of each period, only unanticipated monetary shocks have real effects, and last merely one period. Based on classical search models of money such as Kiyotaki and Wright (1989), papers like Shi (1997) use Lucas's solution to study the effects of monetary policy in this environment. Most search models discard the Walrasian auctioneer and decentralize trading activities. Typically, potential buyers and sellers meet with a certain probability and engage in trade if their wants coincide. In an alternative to Lucas's method, Lagos and Wright (2005) assume a periodic access for all agents to a centralized market in a search model, where they choose the same money balances because of a restriction on the utility function. While these solutions overcome the non-tractability and therefore the unsuitability of studying monetary policy, a non-degenerate wealth distribution is also likely to have considerable effects and would therefore be interesting to study.

Thus, some new models re-introduce the heterogenous money holdings into different settings. Like Lagos and Wright, Williamson (2006) assumes a search and a centralized location. A random, periodic re-allocation of agents between both allows for a slow spreading of new central bank money, which can only be injected at the centralized location. Closed form solutions for the stochastic version can be derived in the case of no monetary interventions. Monetary shocks, anticipated or not, lead to distributional and persistent effects. The model of the present chapter also features heterogenous money holdings, but goes back to the original setup of the Grossman-Weiss and Rotemberg models. As in their models, staggered money withdrawals are placed in a Walrasian environment. Shopping bundles of consumption goods takes time, in contrast to an implicit assumption taken in standard models of monopolistic competition. In the present model, consumers buy each variety of their bundles one after the other. If unexpected events occur during this sequence, the original plan is altered according to the new circumstances. Since it is unlikely that all consumers start and finish their sequences at the same points in time, these sequences overlap.

In the following section I will briefly discuss a benchmark model of oligopolistic competition with two households populating the economy. As it is usual in these standard models, no real effects of monetary interventions will arise. The following assumption has to be made: New money injected into the system by the central bank reaches all agents in an identical way. Heterogenous shocks to money holdings cannot be studied with this model because these shocks would lead to explosive behavior or a violation of the Euler equation. Since all agents are informed about the monetary shock, prices will immediately rise by the same percentage as the money stock.

In section 2.3, homogeneity is replaced by limited participation. One period is divided into two; in each of these new periods both consumers are shopping, but only one of them is visiting the asset market. Hence, monetary injections reaches only this consumer. It seems realistic to assume that not all agents are benefiting from central bank actions in the same way, or as Williamson (2005) puts it: 'For example, when the Fed conducts an open market operation, the economic agents on the receiving end of this transaction typically are large financial institutions that are not directly connected to all other economics agents in the economy through exchange. (...) This difference will be important for short-run movements in interest rates, aggregate output, and the distribution of wealth across the population.' Additionally, the assumption that each shop belongs to one consumer together with a sequential opening of these shops leads to a slow spreading of the newly injected money and keeps the model tractable. Because monetary injections take time to be distributed equally through second-round effects, unequal wealth levels arise which in turn affect the aggregate price elasticity firms are facing. The reason is the following: If consumers in the beginning of their shopping sequences have a larger weight due to monetary transfers to them via the asset market, aggregate elasticity is higher since these consumers spread the new income over the goods to follow in the sequence. This introduces strategic interaction in the price-setting behavior of firms and thereby lowers the optimal markup chosen. Hence, a short-term inflation-output tradeoff is reached. This approach is related to Bils (1989), where a monopolist faces a tradeoff between extracting profits from loyal customers and attracting new ones. In the model of this chapter the oligopolists face customers in the beginning of their shopping sequence, who can substitute with goods further down the sequence, and customers in a later stage, who will substitute less. This leads to a similar tradeoff. For monetary shocks, the model predicts countercyclical markups, as empirically found by Rotemberg and Woodford (1999). Specifically, markups are countercyclical at the firm level, coinciding with evidence in the supermarket industry presented by Chevalier and Scharfstein (1996). These authors also confirm that prices are strategic complements. The Phillips Curve derived in section 2.4 displays inflation inertia via the internal propagation mechanism, i.e. even one-time shocks lead to long lasting responses. The impulse response functions developed in section 2.5 are empirically plausible. Output and profits rise, marginal costs do so moderately while the interest rate falls (i.e. there is a liquidity effect) after a positive monetary shock; features that were found empirically by Christiano, Eichenbaum and Evans (1997). In appendix 2.A, the general formulae for any given number of agents are developed. This number can also be seen as the free parameter describing the lags in the monetary system suggested in the last paragraph of Lucas (1990). Section 2.6 concludes.

2.2 A Standard Two-Consumer Model

I will develop a two-consumer, two-firm model of oligopolistic competition as a reference model in order to make clear which assumptions in section 2.3 drive the results. This model is built on the standard monopolistic-competition Dixit and Stiglitz (1977) model, with two agents instead of a representative household, and two firms instead of a continuum. The stocks of each firm are owned by one of the two consumers, such that the households earn the profits of 'their' firm. Furthermore, consumers have to obey a Cash-in-Advance (CIA) constraint. Note that the oligopolistic structure stems mainly from the assumption that each consumer buys at two different firms, which does not have to translate literally into two firms populating the economy.

2.2.1 Setup

Households Each consumer maximizes the following standard utility function. The subscript $i = 1, 2$ denotes the consumer.

$$E_t(\overline{U}_i) = E_t \left(\sum_{t=0}^{\infty} \beta^t \frac{\overline{C}_{i,t}^{1-\sigma}}{1-\sigma} \right)$$

The consumption aggregate $\overline{C}_i$ combines the consumption goods of the two firms for each consumer:

$$\overline{C}_{i,t} = \left[C_{i,t}^{a\frac{\gamma-1}{\gamma}} + C_{i,t}^{b\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} \qquad \gamma > 1$$

where C_i^j is the current consumption of consumer $i = 1, 2$ at store $j = a, b$. The intertemporal budget constraint is

$$M_{i,t+1} + \overline{C}_{i,t} P_t + B_{i,t+1} = M_{i,t} + (1 + i_t) B_{i,t} + \Pi_t^i + S_{t+1}, \tag{2.1}$$

where Π_t^i is the nominal revenue $Y_t^j P_t^j$ of a firm, whose stocks are owned by household i. P_t^j is the current price of the good of firm j and P_t is the price index, to be defined below. Here and in what follows, I assume that consumers do not take into account their ownership of a firm while shopping at that particular firm. $B_{i,t}$ are bonds bought by household i, which cost one unit of the currency and earn the interest rate i_t between time t and $t + 1$. S_t are this period's nominal transfers, which are the same for each household, and $M_{i,t}$ are beginning of period cash holdings. Households are furthermore subject to a Cash-in-Advance constraint

$$M_{i,t} \geq P_t^a C_{i,t}^a + P_t^b C_{i,t}^b = \overline{C}_{i,t} P_t. \tag{2.2}$$

Households can acquire the cash needed for consumption in the same period (asset market opens first). This implies that at the beginning of each period, households have to decide how to divide available resources between money holdings needed for shopping and savings, i.e. bonds. Current income from business activity cannot be used for current consumption while the cash injections can be used contemporaneously, as in Lucas (1982).[1] As normal, the central bank can either set the money supply via S_t or the nominal interest rate i_t, where the respective other variable has to adjust accordingly. Furthermore, a transversality condition has to be obeyed

$$\lim_{T \to \infty} \Pi_{t=0}^{T} \beta^t \overline{C}_{i,t}^{-\sigma} B_{i,t-1} \geq 0, \tag{2.3}$$

where $\overline{C}_{i,t}^{-\sigma}$ is the marginal utility of period t consumption and $B_{i,t-1}$ are bond holdings at the beginning of the period.

Producers The two producers maximize the following profit function (the focus of the model is on the optimal markup, such that the cost side is not modeled in order to keep the model as simple as possible)

$$\Pi_t^j = Y_t^j P_t^j - CO_t(Y_t^j),$$

where the nominal costs are summarized in the function $CO_t(Y_t^j, P_t) = \frac{(Y_t^j/A_t)^{1+\rho}}{1+\rho} P_t$, which depends on the elasticity ρ of real marginal costs with respect to individual firm's output Y_t^j, common technology A_t, and the price index P_t, which is taken as given by the producers[2]

$$P_t = \left[P_t^{a\,1-\gamma} + P_t^{b\,1-\gamma} \right]^{\frac{1}{1-\gamma}}. \tag{2.4}$$

[1] In order to avoid the inflation-tax effects normally present in CIA models, the firms are 'de-personalized', i.e. their costs do not depend on the CIA constraint of the owner. Hence, no real effects of this restriction arises. See also footnote 2.

[2] Alternative specifications do not change the qualitative results, but do complicate the analysis. Forward looking costs would distort the pricing decision in the presence of expected inflation and would therefore give rise to real effects. While this could be undone by introducing a subsidy, it would be difficult to see where the differences between this and the next section arise.

2.2.2 First-Order Conditions

Optimizing the above stated problems leads to a set of first-order conditions for households and firms.

Households Division of demand between the two consumption goods is governed, as usual, by the elasticity of substitution γ and the relative price. The producers play a Cournot game. Since there are only two firms in the economy, I use the explicit demand schedule facing a producer who takes her influence on the price level into account. The different schedule comes about from the fact that purchases of one good reduce consumers' remaining resources. This income effect vanishes in a model with an continuum of firms, since consumers spend zero income on each variety. Demand for good a by consumer i is given as

$$C_{i,t}^{a} = \left[\left(\frac{P_t^a}{P_t^b}\right)^{\gamma} + \frac{P_t^a}{P_t^b}\right]^{-1} \frac{M_{i,t}}{P_t^b}, \tag{2.5}$$

and vice versa for good b.[3] The intertemporal consumption path is characterized by a normal Euler equation relating consumption today with consumption tomorrow

$$E_t\left(\frac{\overline{C}_{i,t+1}}{\overline{C}_{i,t}}\right)^{\sigma} = \beta(1+i_t)E_t\frac{P_t}{P_{t+1}}. \tag{2.6}$$

The budget constraint (2.1), the CIA constraint (2.2) and the transversality condition (2.3) complete the description of household's optimal behavior. Furthermore, it is not optimal to transfer money as cash between periods, such that the CIA constraint holds with equality.[4]

Producers Resulting from equation (2.5), producer a faces the following aggregate demand schedule

$$C_1^a + C_2^a = \left[\left(\frac{P_t^a}{P_t^b}\right)^{\gamma} + \frac{P_t^a}{P_t^b}\right]^{-1} \frac{M_{1,t}+M_{2,t}}{P_t^b}, \tag{2.7}$$

which is valid symmetrically for producer b. The optimal price setting for firm a and the real marginal costs MC_t^a are given by the formulae

$$P_t^a = \left[\frac{P_t^a(\gamma-1)}{MC_t^a P_t} - \gamma\right]^{\frac{1}{1-\gamma}} P_t^b, \qquad MC_t^a = \frac{Y_t^{a\rho}}{A_t^{1+\rho}}. \tag{2.8}$$

[3]One can rewrite this condition also in the standard form using the price index (2.4) as

$$C_{i,t}^{j} = \left(\frac{P_t^j}{P_t}\right)^{-\gamma} \frac{M_{i,t}}{P_t}.$$

In this case one has to make sure to account for the effect of P_t^j on P_t in firms' optimization processes.

[4]See section 2.3 for a longer discussion of this point. Here it is clearly not optimal to carry over cash in the case of positive nominal interest rates.

2.2.3 Equilibrium

Equilibrium requires market clearing, i.e. the complete amount of both varieties produced is sold to both agents

$$Y_t^a = C_1^a + C_2^a \qquad Y_t^b = C_1^b + C_2^b.$$

Because in this section households are identical, including the amount of transfers received from the government, there will be no borrowing nor lending between them in equilibrium. Combined with the fact that in this closed economy without investment net savings, i.e. bonds, have to be zero, private bond holding of each agent are zero, too. Hence, equation (2.6) merely determines the interest rate. Furthermore, both households consume the same amount of resources from each firm

$$C_1^a = C_2^a, \qquad C_1^b = C_2^b.$$

The resulting price is[5]

$$P_t^j = \frac{\gamma+1}{\gamma-1} MC_t^j P_t,$$

which by the symmetry assumption of $P_t = P_t^j$ leads to the standard result of

$$\frac{1}{MU} = MC_t \tag{2.9}$$

with MU being the constant markup $\frac{\gamma+1}{\gamma-1}$.

Results Equation (2.8) leads under symmetry assumptions to the following result:

$$Y_t = 2A_t^{\frac{1+\rho}{\sigma+\rho}} \left(\frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{\rho+\sigma}}.$$

From this equation it is obvious that output depends only on technology and deep parameters. Combined with the Euler equation (2.6), it becomes clear that the real interest rate stays constant after a change in the nominal interest rate, i.e. $\beta(1+i_t) = \frac{P_{t+1}}{P_t}$. Inflation jumps once, and is zero thereafter. Thus, the model exhibits neutrality and superneutrality with respect to monetary policy. As was already visible from equation (2.9), optimal (and realized) markups stay constant, a feature that has been empirically rejected.[6]

[5] Note that if the firms were not to take their influence on the general price level into account, the optimal markup would be $\frac{\gamma}{\gamma-1}$, which would not change the conclusions drawn in this section.

[6] Again, see Rotemberg and Woodford (1999) for a longer discussion of this point.

2.3 A Sequential-Purchases Two-Agent Model

In the above section, it is assumed that all actions are done simultaneously, although in discrete-time models one period is assumed to be of considerable length. Agents aggregate and consume the consumption goods they buy during one period and choose their consumption paths by relating these bundles across periods. It is crucial that it takes virtually no time to acquire these goods. Only with this assumption all prices of the goods in the consumption bundle are known at the time of the purchase. This implies also that all households receive their income at the same moment in time, such that they calculate equilibrium good prices while buying bonds. As a result of all this, any changes in income (including monetary shocks) lead to an instantaneous adjustment of prices, and therefore no real effects can be observed.

Only with the assumption of instantaneous purchases it can be justified that periods for all agents start and end at the same time. If one is to assume that it takes some time to acquire a consumption bundle, it is unlikely that consumers start and stop their shopping sequences and adjust their financial assets at the same dates. To account for this point, I am dividing one period into two and introduce overlapping shopping sequences. Instead of visiting both shops simultaneously, the two consumers now buy at one shop in the first period and at the other in the second period. Both goods still enter the same consumption bundle. After having completed the consumption bundle it is consumed at home, still in the second period. Consumers start their shopping sequences after having visited the bank, where they receive their income and adjust their financial positions. The sequence is therefore: 'Bank' (financial transactions), 'First Shop' (shopping), 'Second Shop' (shopping), 'Home' (consuming), where the first two places are visited in the first period, and the last two places in the following one. Because of the overlapping structure, I assume that one consumer receives income from business activity, bonds and transfers in even periods, while the other receives her income in odd periods. This aspect of the model is close to Grossman and Weiss (1983) and Rotemberg (1984). Each shop is therefore serving two customers, who are in different stages of their shopping sequence. Seen from the firm's perspective, this is equivalent to an economy with a representative consumer, but uncertainty about the current stage of the shopping sequence of this consumer. The assumption that consumers do not adjust their assets instantaneously is empirically supported by Christiano et al. (1996). Graphic 2.1 shall help explain the difference. In the upper half, people buy at both shops during one period, i.e. simultaneously. One could imagine that the shops open one after the other, but since all actions in the period are compressed to one single point in time, this does not play any role. In the lower half, shopping sequences overlap. Now it matters when which shop opens and who is buying there. Therefore, the length of the periods is cut in half with only one shop being open in each 'new' period. The arrows show the dates when the agents visit the bank and therefore between which points in time they can save by buying bonds. The limitation to a finite number of shops introduces strategic complementarity between the prices of the firms. Empirical evidence for this complementarity was found by Chevalier and Scharfstein (1996) in the sector of supermarkets.

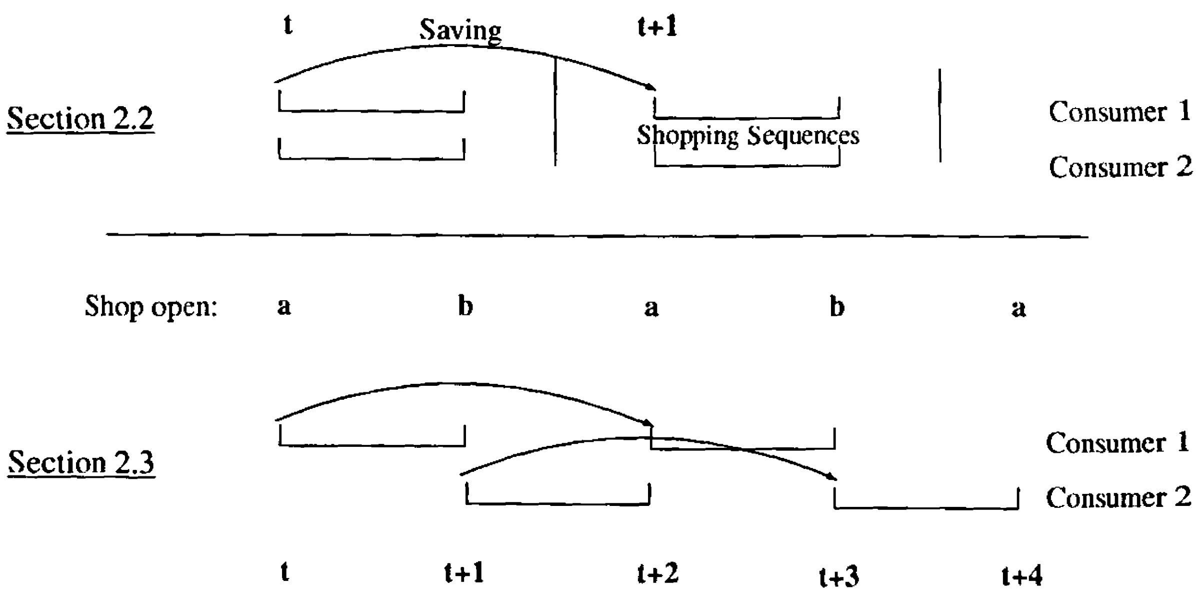

Figure 2.1: Difference in timing

2.3.1 Notation

Since now there is only one shop open each period, the time index is enough to distinguish between the two firms. The firm which is open in t transfers its profits to the bank account of its owner, the consumer who starts shopping in time $t+1$. Furthermore, I will use the subscripts $i = 1, 2$ to indicate consumers in the following way: 1 denotes the consumer that is at the beginning of its shopping sequence, while consumer 2 is at the end of this sequence. Hence, a person with the subscript 1 in t will have the subscript 2 in $t+1$ and again 1 in $t+2$. Decisions on bonds and the division of money spent on both goods are made at the beginning of the sequence, hence by the person with the index 1.

2.3.2 Setup

Households Most of the setup is similar to the one described in section 2.2. A CES-utility function is to be maximized

$$E_t(\overline{U}_{1,t}) = E_t\left(\sum_{l=0}^{\infty}\beta^{l+1}\frac{\overline{C}_{t+1+2l}^{1-\sigma}}{1-\sigma}\right),$$

where the consumption bundle $\overline{C}$ is acquired over the course of two periods and consumed in the second period, such that the counter for the first consumption seen from today is $t+1$ and increases in steps of two. Since only one agent consumes a consumption bundle in any given period, no index indicating the consumer is necessary for the bundle. This consumption aggregate $\overline{C}$ is defined as in section 2.2, but since the consumer buys sequentially, she does not know the price of the good to be purchased in the next period when starting her shopping sequence in this period. Hence, she maximizes the expected value of the bundle, given the information in period t

$$E_t(\overline{C}_{t+1}) = E_t\left[\left(C_{1,t}^{\frac{\gamma-1}{\gamma}} + C_{2,t+1}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}\right]. \qquad \gamma > 1 \tag{2.10}$$

Since the shopping is done over two periods, the consumer now faces alternating budget constraints. When entering the bank, the budget constraint is[7]

$$M_{1,t+1} + P_t C_{2,t} + B_{t+1} = M_{2,t} + (1+i_{t-1})B_{t-1} + \Pi_t + S_{t+1} \tag{2.11}$$

where Π_t is the revenue of the firm, whose stocks are owned by the household with index 1 in $t+1$. Since the asset market is visited only every second period, interest is earned on bonds bought two periods ago. One period later, i.e. after having adjusted financial positions, the budget constraint is simply

$$M_{2,t+1} + P_t C_{1,t} = M_{1,t}. \tag{2.12}$$

[7]Remember that the consumer-index is alternating between periods for the same agent, in order to indicate the stage of the shopping sequence.

The Cash-in-Advance constraint has to be satisfied in both periods

$$M_{1,t} \geq P_t C_{1,t} \tag{2.13}$$

$$M_{2,t+1} \geq P_{t+1} C_{2,t+1}. \tag{2.14}$$

The transversality condition is still valid.

$$\lim_{T\to\infty} \Pi_{t=0}^{T} \beta^t \overline{C}_t^{-\sigma} B_{t-1} \geq 0. \tag{2.15}$$

Producers The producer side is similar to section 2. The two producers maximize the same profit function

$$\Pi_t = Y_t P_t - CO_t(Y_t),$$

where the nominal costs are $CO_t(Y_t) = \frac{(Y_t/A_t)^{1+\rho}}{1+\rho} P_t$.

Monetary Authority The central bank controls the money supply. It does so by setting the monetary injections S_t according to a money growth rule

$$S_t = \eta_s S_{t-1} + \epsilon_t, \tag{2.16}$$

which is the same as specifying a movement of total money stock M_t according to

$$\Delta M_t = \eta_s \Delta M_{t-1} + \epsilon_t.$$

2.3.3 First-Order Conditions

Due to the different timing assumptions, some differences in the first order conditions arise. Notably, since shopping is now sequential, households plan their purchases at each store based on *expected* future relative prices of competitors. Due to this different consumption behavior, price setting of firms is also affected.

Households At time t, the household that is at the beginning of its shopping sequence has to decide how much cash to hold for this sequence, and how much to put into bonds, resulting in an Euler equation. This equation is quite standard, except that it is more convenient that the household maximizes directly over cash holdings, a certain variable, in contrast to consumption that is uncertain because the price of the second good purchased in $t+1$ is unknown as of time t. The results are the same.

$$M_{1,t}^{-\sigma} = \frac{E_t\left[M_{1,t+2}^{-\sigma} E_{t+2}\left(\overline{P}_{t+3}^{\sigma-1}\right)\right]}{E_t\left(\overline{P}_{t+1}^{\sigma-1}\right)} \beta^2 (1+i_t), \tag{2.17}$$

where i_t is the going interest rate, which is earned on bonds between t and $t+2$. In deriving this equation, I already made use of equations (2.12)-(2.14) that can be summarized as

$$M_{1,t} \geq P_t C_{1,t} + P_{t+1} C_{2,t+1} = \overline{P}_{t+1} \overline{C}_{t+1}, \tag{2.18}$$

where $\overline{P}_{t+1}$ is the price index, defined below. This equation holds with equality if no cash is carried over from period $t+1$ to $t+2$, i.e. resources are transferred to period $t+2$ only via the interest bearing bonds. In appendix 2.B, this is proven to be an optimal behavior for increases in the money supply that increase prices.[8] However, it turns out that this equality does not have to be the case for price decreases, which gives rise to an interesting possible extension of the model. The model as described from here on is mainly valid for increases in the money supply, whereas decreases would trigger an asymmetric, different response. This asymmetry is in line with conventional wisdom about different dynamics during times of inflations and deflations. After unexpected price decreases, consumers are delaying purchases, i.e. they carry cash over between periods. Extending the model also for this option could prove a fruitful exercise that highlights the asymmetric effects of monetary policy.

[8] Also Rotemberg (1984) argues that acting in such a way is optimal as long as a positive interest rate prevails, which is also the case here.

Turning back to the setup of the model, in order for the definition $P_t C_{1,t} + P_{t+1} C_{2,t+1} = \overline{C}_{t+1} \overline{P}_{t+1}$ in equation (2.18) to hold, the price index $\overline{P}_{t+1}$ has to be defined in a more complicated manner. This is the case because it involves P_{t+1}, which is unknown at the time of the decision on how much to consume in period t. Hence, to make the price index consistent with the optimal consumption chosen in t, based on $E_t\{P_{t+1}\}$, and the realized consumption in $t+1$, it has to involve the expected and realized P_{t+1}[9]

$$\overline{P}_{t+1} = \left[1 + \left(E_t\left\{P_{t+1}^{\frac{1-\gamma}{\gamma}}\right\} P_{t+1}^{\frac{\gamma-1}{\gamma}} \frac{P_t}{P_{t+1}}\right)^{\gamma-1}\right]^{\frac{\gamma}{1-\gamma}} \left[1 + \left(E_t\left\{P_{t+1}^{\frac{1-\gamma}{\gamma}}\right\}\right)^{\gamma} P_t^{\gamma-1}\right] P_t. \quad (2.19)$$

Having decided how much cash $M_{1,t}$ to hold at the beginning of period t for shopping in t and $t+1$, the consumer now has to decide how to divide the cash between the two goods. As argued above and shown in appendix 2.B, equation (2.18) will be binding in the analyzed cases. Hence, the household will spend all remaining cash in $t+1$ on the good purchased in that period

$$C_{2,t+1} = \frac{M_{2,t+1}}{P_{t+1}}. \quad (2.20)$$

Knowing this, one can also solve for the consumption of the good in period t by maximizing the expected value of the consumption bundle (2.10), resulting in the following demand schedule

$$C_{1,t} = (R_t + 1)^{-1} \frac{M_{1,t}}{P_t}, \quad (2.21)$$

with the expected transformed price ratio R_t equal to

$$R_t = P_t^{\gamma-1} \left[E_t\left(P_{t+1}^{\frac{1-\gamma}{\gamma}}\right)\right]^{\gamma}.$$

Again, the budget constraint (2.11) and the transversality condition (2.15) complete the description of households' optimal behavior.

Producers Since shopping periods overlap, i.e. at each point of time one consumer is at the beginning of her sequence (equation 2.21), and another one at the end (equation 2.20), total demand facing a producer at time t is

[9] See Green (1964), chapter 3, on a more general discussion on the requirements of a price index.

$$C_t = (R_t + 1)^{-1} \frac{M_{1,t}}{P_t} + \frac{M_{2,t}}{P_t}. \tag{2.22}$$

Note, that people at the beginning of their shopping sequence have a higher elasticity of substitution than the people further down the sequence (the consumer in the last period of the sequence just spends all her money). Hence, when setting its price, the firm faces a tradeoff between extracting more profits from the costumers with a low elasticity, and loosing profits from the customers at the beginning of the sequence, who might substitute to firms that come later in the row.

The optimal price is implicitly given by

$$\frac{\frac{M_{2,t}}{M_{1,t}}(R_t + 1)^2 + \gamma R_t + 1}{(\gamma - 1)R_t} = \frac{A_t^{1+\rho}}{Y_t^{\rho}}, \tag{2.23}$$

which corresponds the standard formula Markup=1/Marginal Costs. Steady-state markup is $MU = (\gamma + 3)/(\gamma - 1)$. Note that the firm is taking household expectations R_t as given, such that the game played between the producers does not change compared to section 2.2. Instead of turning into a Stackelberg leader, the individual firm does not assume that its price setting affects people's expectations of future prices, hence the shops still play a Cournot game.

2.3.4 Equilibrium

Equilibrium requires market clearing in each period

$$Y_t = C_{1,t} + C_{2,t}. \tag{2.24}$$

Like Grossman and Weiss (1983) and Rotemberg (1984), I make the simplifying assumption that inter-household borrowing and lending is not possible. This would contradict the structure of the model, in which consumers are not going to the bank during their shopping sequence. Hence, consumers currently at the bank do not engage in borrowing and lending with the consumers not at the bank. Together with the fact that aggregate savings in a closed economy without investment have to be zero, this leads to the same result as in section 2.2, namely that the interest rate adjusts in such a way that buying bonds is not optimal for households. In this respect, the Euler equation (2.17) only determines the interest rate. Due to these restrictions, it is possible to find a reduced form representation of the linearized system in the next section.

2.4 Linearized System

An advantage of this model is the possibility of finding an analytical solution. In a first step, money holdings are substituted out to find two equations summarizing the dynamics, plus one to determine the equilibrium interest rate. A monetary policy rule and a process for technology complete the description. Below, these equations will be derived for the model of two (kinds of) agents, as presented in the preceding section. An analytical solution, i.e. expressions for the variables as functions of the states, the exogenous variables and the shocks, is derived in section 2.4.2 and simulated in section 2.5. The formulae for any given number of agents are given in appendix 2.A.

2.4.1 Reduced System

Equation 1: Effects of Monetary Policy An equation for the effects of monetary policy can be found by combining the log-linearized versions of equations (2.11), (2.18), (2.20), (2.21), (2.23), and (2.24), substituting out consumption and money holdings. The following equation emerges[10]

$$s_t + y_{t-1} + \frac{\gamma+3}{4}(\rho+1)a_t = \pi_t + \frac{\gamma+7}{4}y_t, \tag{2.25}$$

where y_t is expressed in percentage deviations from its steady-state values, π_t is current inflation and $s_t = \frac{S_t}{M_1^{St.St.}}$.[11] Since firms can adjust prices at every period, the flex-price equilibrium is always reached. This implies, contrary to the standard Calvo-model, that the markup mu_t is the inverse of marginal costs, i.e. the negative of marginal costs in the linearized form, in each period. Due to this fact marginal costs are

$$mc_t = \rho y_t - (\rho+1)a_t = -mu_t. \tag{2.26}$$

Note that marginal costs here are, although not explicitly modeled, costs for labor only, since capital is missing. Furthermore, profits are $Y_t MU_t$, linearized $y_t + mu_t$. It is visible from equation (2.25) that monetary injections are likely to raise nominal GDP, where the division between inflation and real output depends on current and past expectations, see next equation.

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[10] Alternatively, one could also develop a formula describing the process of updating expectations: $E_t(\pi_{t+1}) = E_{t-1}(\pi_t) + \frac{\gamma+3}{\gamma-1}\rho \dot{y}_t + \frac{2}{\gamma-1}s_t$.

[11] The steady-state employed here is a stationary economy with no trending variables.

Equation 2: Phillips Curve In order to derive the Phillips Curve, the same mentioned equations can be used, resulting in

$$\pi_t = \frac{\gamma-1}{\gamma+1}E_t(\pi_{t+1}) + \frac{(\gamma+3)\rho/2-2}{\gamma+1}y_t + \frac{2}{\gamma+1}y_{t-1} - \frac{\gamma+3}{2(\gamma+1)}a_t + \frac{\gamma-1}{\gamma+3}\left[\pi_t - E_{t-1}(\pi_t)\right]. \tag{2.27}$$

All the coefficients have the expected sign, i.e. inflation today depends positively on expected inflation, current and past output, and the expectational error of inflation. It depends negatively on deviations of technology from its steady state.

Equation 3: Interest Rate In order to determine the interest rate one can log-linearize equation (2.17), using (2.11) and (2.19) to substitute out the money holdings to get

$$r_t = \frac{1-\sigma}{2}E_t(\pi_{t+3}) + (1-\sigma)E_t(\pi_{t+2}) + \frac{1+\sigma}{2}E_t(\pi_{t+1}) + \tag{2.28}$$

$$+\sigma\pi_t + \sigma[E_t(\Delta y_{t+1}) + \Delta y_t] + \sigma[E_t(\Delta s_{t+2} + \Delta s_{t+1})], \tag{2.29}$$

with r_t being the percentage deviation of the interest rate from its steady-state $\bar{i} = 1/\beta^2 - 1$.[12]

Equation 4: Exogenous Variables To close the model, a process for technology has to be assumed together with the money growth rule (2.16). Since I mainly want to present the internal propagation mechanism, I have kept these equations as simple as possible; namely AR(1) processes with coefficients $0 < \eta_s, \eta_a < 1$, the monetary policy shock ε and the shock to technology v[13]

$$\begin{aligned} s_t &= \eta_s s_{t-1} + \varepsilon_t \\ a_t &= \eta_a a_{t-1} + v_t. \end{aligned} \tag{2.30}$$

2.4.2 Solution

Once reduced to the two equations describing the dynamics of the model, (2.25) and (2.27), the endogenous variables π_t and y_t can be expressed as functions of the states and shocks only. However, since the intrinsic state variables M_1 and M_2 were substituted out because they are not directly observable, other variables have to fulfill the role of state variables. It turns out that yesterdays values

[12] Although the percentage deviation of a variable measured in percentage points is a bit awkward, by this we do not have to choose a value for β.

[13] The transformed monetary shock ε equals $\epsilon/M_1^{St.St.}$ from equation (2.16).

of output y_{t-1} and the exogenous variables s_{t-1} and a_{t-1} summarize all necessary information. The former is needed because M_1 depends on last periods revenue, while the latter variables are important for building expectations, required to determine M_2. Hence, the following guess can be postulated

$$\begin{aligned} y_t &= \lambda_{yy}y_{t-1} + \lambda_{ys}s_{t-1} + \lambda_{ya}a_{t-1} + \lambda_{y\varepsilon}\varepsilon_t + \lambda_{yv}v_t \\ \pi_t &= \lambda_{\pi y}y_{t-1} + \lambda_{\pi s}s_{t-1} + \lambda_{\pi a}a_{t-1} + \lambda_{\pi\varepsilon}\varepsilon_t + \lambda_{\pi v}v_t. \end{aligned} \tag{2.31}$$

This dynamic system is verified and solved using the method of undetermined coefficients. The stability of the system is governed by the value for λ_{yy}, for which a quadratic equation is obtained. For $\gamma > 1$ and $\rho > 0$ - as assumed in section 2.3 - both solutions to this equation are positive, with one solution being greater and the other lesser than one. The first root is therefore discarded, and the solution to the equation using the second root is given below together with the coefficients for the effects of past output and monetary policy

$$\begin{aligned}
\lambda_{yy} &= \frac{(\gamma+3)^2\rho + 8(\gamma-1) - \sqrt{(\gamma+3)\rho[(\gamma+3)^3\rho - 64(\gamma-1)]}}{2[(\gamma+3)\rho+4](\gamma-1)} && > 0; < 1 \\
\lambda_{\pi y} &= \frac{4 + \lambda_{yy}[(\gamma+3)\rho - 4]}{4 + 2(\gamma-1)(1-\lambda_{yy})} && > 0 \\
\lambda_{\pi s} &= 2\eta_s \frac{2(\gamma-1)\lambda_{\pi y} + (\gamma+3)\rho - 4}{4(\gamma-1)\lambda_{\pi y} + 4(\gamma+3)\rho + (\gamma-1)(1-\eta_s)[(\gamma+3)\rho+4]} && < \eta_s \\
\lambda_{y\varepsilon} &= \frac{4(1-\lambda_{\pi s})}{(\gamma+3)\rho+4} && > 0 \\
\lambda_{ys} &= \frac{4(\eta_s - \lambda_{\pi s})}{(\gamma+3)\rho+4} && > 0 \\
\lambda_{\pi\varepsilon} &= \frac{4(\gamma-1)\lambda_{\pi y} + (\gamma-1)\lambda_{\pi s}[(\gamma+3)\rho+4] + 2[(\gamma+3)\rho-4]}{4(\gamma-1)\lambda_{\pi y} + 4(\gamma+3)\rho}.
\end{aligned}$$

The effects of technology can be seen from the following coefficients

$$\begin{aligned}
\lambda_{\pi a} &= \eta_a \frac{(\gamma+3)(\rho+1)[(\gamma-1)\lambda_{\pi y} - 4]}{(\gamma-1)\{[(\gamma+3)\rho+4](1-\eta_a) + 4\lambda_{\pi y}\} + 4(\gamma+3)\rho} \\
\lambda_{\pi v} &= \frac{(\gamma-1)[(\gamma+3)\rho(\lambda_{\pi a} - \lambda_{\pi y}) + 4\lambda_{\pi a} - (\gamma+3)\lambda_{\pi y}] - (\gamma+3)(1+\rho)[(\gamma+3)\rho-4]}{(\gamma^2-9)\rho + 4(\gamma+1) - 2(\gamma-1)^2\lambda_{\pi y}} \\
\lambda_{ya} &= \frac{(\gamma+3)(\rho+1)\eta_a - 4\lambda_{\pi a}}{(\gamma+3)\rho+4} \\
\lambda_{yv} &= \frac{(\gamma+3)(\rho+1) - 4\lambda_{\pi v}}{(\gamma+3)\rho+4}.
\end{aligned}$$

Note that the effects $\lambda_{ys}, \lambda_{ya}, \lambda_{\pi s}, \lambda_{\pi a}$ of yesterdays exogenous variables a_{t-1} and s_{t-1} disappear if their AR(1) processes are reduced to white noise, i.e. $\eta_a = \eta_s = 0$. It can also be seen from $\lambda_{y\epsilon} > 0$ that expansionary monetary shocks unambiguously have a positive effect on current output, while $\lambda_{\pi y} > 0$ shows that past economic activity has a positive effect on inflation. Furthermore, positive monetary shocks have a stimulating effect on output also in the following period, see $\lambda_{ys} > 0$. The sign of the effect of monetary shocks on current inflation, $\lambda_{\pi\epsilon}$, depends on ρ and γ.

The New Keynesian Phillips Curve has problems in generating the empirically observed inflation persistence. In the present model, the state variable y_{t-1} can be replaced by π_{t-1} by reformulating the dynamic system (2.31). Having done so, it is evident that the effect of past inflation on current inflation is equal to λ_{yy}. This coefficient is smaller than one but larger than zero, such that inflation persistence also exists with the internal propagation mechanism only, no autocorrelated shocks or similar dynamics have to be assumed. One should keep in mind that this is a stylized model, which lacks any other mechanisms often employed for slow adjustment of real and nominal variables.

Finally, concerning the response of output, inflation, marginal costs, markup and the interest rate to shocks, I refer to section 2.5, where impulse response functions for the model in this chapter are plotted. While the responses of the variables are generally consistent with the empirical findings, the hardest point to fulfill is the slow and gradual effects of monetary shocks on inflation and output. Autocorrelated monetary shocks, as often used in the literature, low values for γ or increasing the number of agents generate hump-shaped impulse response functions.

2.5 Simulation

In this section I calculate the impulse-response functions to further check the dynamic implications of the model. In section 2.5.1 I simulate the model for $n = 2$ agents, whereas the model with three agents in section 2.5.2 allows for richer dynamics, i.e. agents react to shocks by adjusting their expectations and re-optimizing their plans in the middle of their shopping sequence. The general formulae for any given n are given in appendix 2.A.

2.5.1 Simulation for n=2

I use equations (2.25) to (2.30) to simulate the model with the parameter values $\gamma = 16$ (which implies a steady-state markup of 26%)[14], $\rho = 0.1$ (close to constant marginal costs) and $\sigma = 3$ (which merely determines the size of the interest rate reaction).[15] The steady-state markup and the value for σ are not as controversial as the value for ρ, for which values between zero and infinity are used in the literature. The value used here is rather low and implicitly implies a labor supply elasticity of 10%. Raising this value shortens the reactions by lowering the response of output while increasing the effect on inflation. Whereas otherwise the responses do not change qualitatively, this result seems to strengthen the point of Christiano et al. (1997) that labor market frictions (like predetermined wages that would dampen the initial impact on inflation) are probably needed if the model is to be completed with a labor market.[16]

In the left panel of figure 2.2, a one-time monetary injection of 1% to total money supply takes place (which corresponds to $s_1 = 3/2$, zero otherwise).[17] The horizontal axis shows periods, whilst the vertical axis depicts percentage deviations from the steady-state. Note that the interest rate got scaled down by the factor 10 to fit in the same picture; a 0.1 percent shift in the graph corresponds to a movement of one percent. In the right panel an autocorrelation of $\eta_s = 0.3$ in the monetary policy rule (2.30) with $\varepsilon_1 = 3/2$ is assumed. In the left panel of figure 2.3, the one-percent shock takes place twice, i.e. both agents receive the same positive monetary injection ($s_1 = s_2 = 3/4$, zero

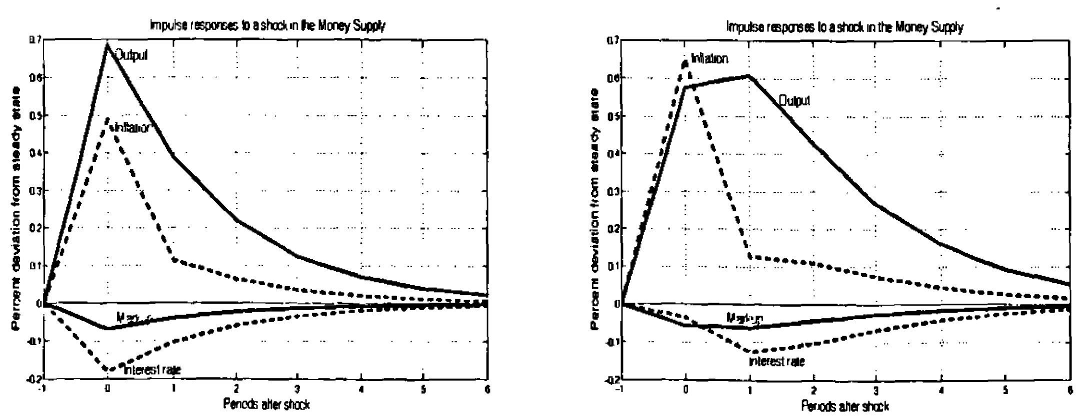

Figure 2.2: n=2. On the left no autocorrelated monetary policy rule, on the right $\eta_s = 0.3$

[14]Steady-state markup is $\frac{\gamma+3}{\gamma-1}$. Because of the oligopoly, the markup is generally relatively high for given values of γ.

[15]The function was calculated using the Matlab codes of Uhlig (1997).

[16]With a labor elasticity of 1 ($\rho = 1$), an initial effect on output around 0.18% is reached after the same shock. Lower values of γ increase this value.

[17]s is the percentage shock to M_1. For a 1% shock to total money supply $M_1 + M_2$, s has to be adjusted accordingly.

otherwise).[18] It is noteworthy that even for one-time monetary injections, the internal propagation mechanism already prolongs the responses. The model displays a liquidity effect, except on impact for the two subsequent shocks and very high autocorrelations of the monetary injections. Hump-shaped responses are obtained for autocorrelated monetary shocks and $n = 3$. Low values for γ lower the initial response of inflation, such that no initial reaction with a subsequent hump-shaped response can be reached for $\gamma = 7$, see the right panel of figure 2.3. Output would in this case rise initially to 1.2% because of strong substitution effects. However, in this case the steady-state markup is unrealistically high at 66%. Generally, the impulse-response functions are in line with qualitative empirical facts. The markup falls and moves countercyclically, both features are empirical observations which are difficult to reproduce in the standard new Keynesian framework. Real marginal costs in the linearized form are given by $mc_t = -mu_t$ (mu being the markup). Hence, they increase mildly after a positive monetary shock. Because of the lack of capital, procyclical marginal costs would equal wages after a possible introduction of a labor market. Linearized profits are $y_t + mu_t$ and procyclical as well, since output rises more than the markup falls. Thus, the model coincides with the observations of Christiano et al. (1997), who find that after a positive monetary shock output and profits rise, wages increase mildly and the interest rate falls.

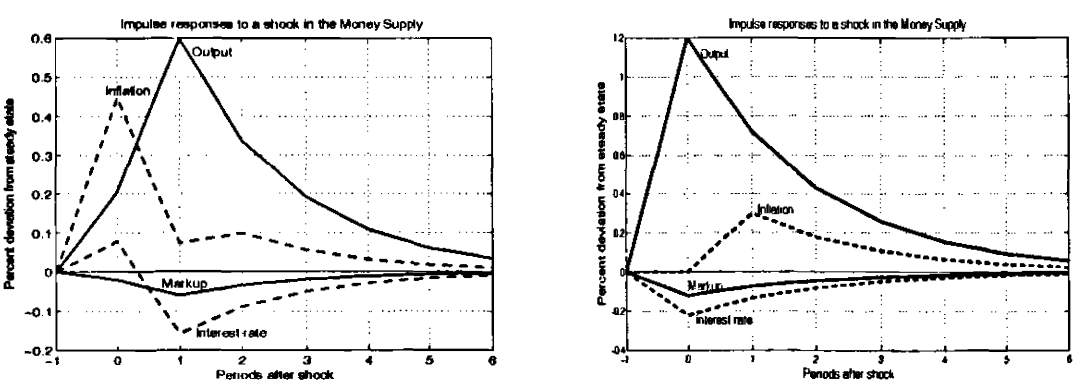

Figure 2.3: n=2. On the left two subsequent shocks; on the right $\gamma = 7$, one shock

[18] To stay with the picture of two agents with alternating trips to the bank, this would imply that both agents receive the same amount of money at the same time, but one agent 'picks up' the money one period later. The second shock is therefore anticipated one period ahead, which indirectly leads to the initial rise in the interest rate.

2.5.2 Simulation for n=3

The linearized first-order equations for the case of $n = 3$ agents are developed from the general formulae in appendix 2.A, with optimal consumption for each agent already inserted. They are given as

Households

Euler Equation	$(\sigma - 1)E_t\{\overline{p}_{t+3} - \overline{p}_t\} = \sigma(E_t\{mr_{1,t+3}\} - mr_{1,t}) - r_t$
with expected Price Index	$3E_t\{\overline{p}_{t+2}\} = p_t + E_t\{p_{t+1} + p_{t+2}\}$
Money Holdings	$mr_{1,t} = p_{t-1} + y_{t-1} + s_t$
	$mr_{2,t} = mr_{1,t-1} - \frac{\gamma-1}{6}E_{t-1}\{\pi_{t+1} + 2\pi_t\}$
	$mr_{3,t} = mr_{2,t-1} - \frac{\gamma-1}{2}E_{t-1}\{\pi_t\}$

Pricing $30mr_{1,t} + 12mr_{2,t} - 42mr_{3,t} - 7(7\gamma + 11)mc_t = (\gamma - 1)E_t\{\pi_{t+2} + 25\pi_{t+1}\}$

Market Clearing $3y_t = mr_{1,t} + mr_{2,t} + mr_{3,t} + \dfrac{\gamma - 1}{3}E_t\{\pi_{t+2}\} + \dfrac{\gamma - 1}{2}E_t\{\pi_{t+1}\} - 3p_t$

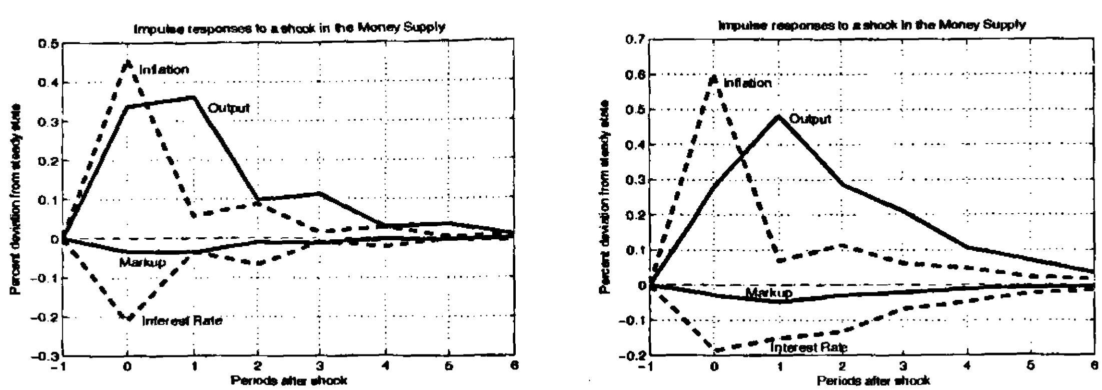

Figure 2.4: n=3. On the left no autocorrelated monetary policy rule, on the right $\eta_s = 0.3$

with marginal costs mc defined as in (2.26). The transversality condition also has to be obeyed. Steady-state markup is given by $\frac{\gamma+11/7}{\gamma-1}$. The equations for the money holdings can be used to substitute out these variables in the pricing and the market clearing equations, such that we get a system of two equations

$$(\gamma - 3)\Big[E_t\{25\pi_{t+1} + 2\pi_{t+2}\} + E_{t-1}\{2\pi_{t+1} - 17\pi_t\} - 7E_{t-2}\{\pi_t + 2\pi_{t-1}\}\Big]$$
$$= 30(\pi_{t-1} + \Delta y_{t-1} + \Delta s_t) + 42(\pi_{t-2} + \Delta y_{t-2} + \Delta s_{t-1}) + 7mc_t \qquad (2.32)$$

$$3(\pi_t + \Delta y_t) + 2(\pi_{t-1} + \Delta y_{t-1}) + \pi_{t-2} + \Delta y_{t-2}$$
$$= \frac{\gamma - 1}{6}\Big[E_t\{2\pi_{t+2} + 3\pi_{t+1}\} - E_{t-1}\{\pi_{t+1} + 5\pi_t\} - E_{t-2}\{\pi_t + 2\pi_{t-1}\}\Big]. \tag{2.33}$$

As before, one could label the first one 'Effects of monetary policy' and the second 'Phillips Curve', but because of their complicated structure this is less obvious. For the simulation, I use the following parameter values: Coefficient of relative risk aversion again $\sigma = 3$, elasticity of substitution $\gamma = 12$, coefficient of marginal disutility of labor again $\rho = 0.1$. As before, these values have been chosen to match standard values (σ) or to be close to standard values for resulting variables (the chosen γ implies a steady-state markup of 23%). The impulse response function for a shock of 1% the total money supply (i.e. $s_0 = 2$, zero otherwise) without a autocorrelated shock is depicted on the left panel of figure 2.4, while in the right panel $\eta_s = 0.3$ and $\varepsilon_1 = 2$. As can be seen, increasing the number of firms makes the responses longer lasting, but smaller in size. Output increases in a more or less hump-shaped manner before returning over time to the steady-state values. For illustrative purposes, an alternative value $\gamma = 4.5$ is chosen in figure 2.5. While this generates a very high steady state markup of 73%, the inflation response is lower on impact. On the left, all three agents receive the same shock one after another, such that the initial money stock is increased by 1%. On the right, only one agents receives the monetary transfer.

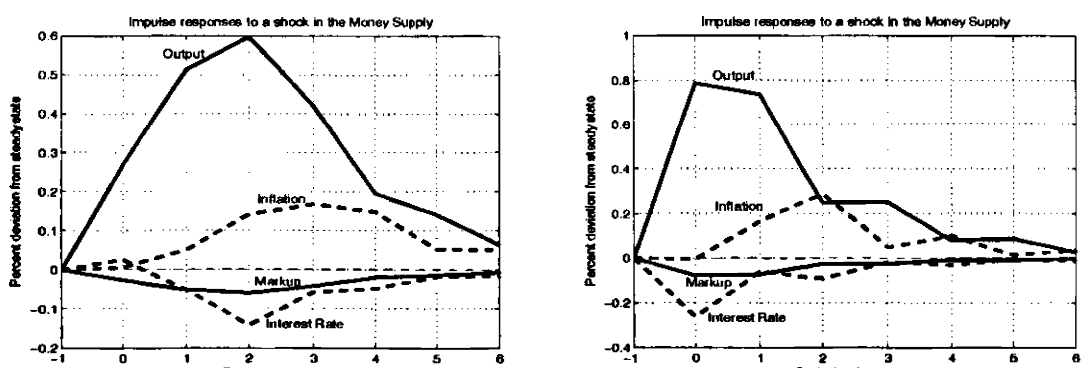

Figure 2.5: n=3, $\gamma = 4.5$. On the left three subsequent shocks; on the right one shock

2.6 Conclusion

With the present setup of the model, several empirical observations can be replicated: 1) a short-term inflation-output tradeoff after a monetary injection, 2) empirical plausible impulse-response functions for output and inflation after monetary injections 3) a liquidity effect, and 4) a countercyclical markup at the firm level after monetary shocks, which would also imply procyclical wages. The model generates a microfounded, internal propagation mechanism which does not rely on capital or sticky prices, but on the slow spreading of newly inserted money. This can be seen as a way of describing the effects of central bank actions in reality, where only parts of the population benefit through first-round effects, while others are affected indirectly and later. The underlying friction of limited participation of consumers is empirically supported by Christiano et al. (1996). A reduced form description of the linearized system including a Phillips Curve can be obtained, without assuming a degenerate money distribution.

As stated, after monetary shocks optimal markup falls. Strategic complementarity is important in this model. Each firm wants to maintain a higher markup, but would suffer too large a drop in sales if it raised prices first, because customers substitute away to other firms. As other firms slowly adjust their prices, each firm can raise prices itself only gradually, thereby limiting coming pricing increases of competitors and so on. This effect arises due to the sequential structure of the model. In discrete time models with symmetry assumptions on firms, this process of reacting to other firms' price adjustments is done instantaneously. Price setters calculate their own optimal price knowing that all firms are alike. Hence, other firms' price increases are completely anticipated before setting their own price, and the new steady state is reached instantaneously. This could be seen as an unrealistic feature, since most firms react to other firms' observed price setting behavior, and do not increase prices relying on the belief that all other firms will make identical price increases at the same time. Only if all firms adjust at the same time will costumers not have the possibility of substituting to a cheaper competitor, who did not yet adjust. Hence, the present model could be a different viewpoint on collusion incentives, adding to literature on implicit collusion as Rotemberg and Saloner (1986). Also in their paper, markup falls during times of higher economic activity, due to increased difficulties to maintain the collusion. These further insights into dynamic oligopolies and the role of coordination devices could be used to, e.g., study the mechanisms present during the introduction of the Euro.

Appendix

2.A General Case

In this section I explore the general case for any given number n of agents in the economy, which could also be seen as the free parameter determining the length of a whole period asked for in the last paragraph of Lucas (1990). The structure of the model is maintained, in particular the overlapping purchases and the assumptions about the timing of shop openings (in each period, one shop opens).

Households The utility function is the same as in the preceding sections

$$E_t(\overline{U}_{1,t}) = E_t\left(\sum_{l=0}^{\infty} \beta^{l+n} \frac{\overline{C}_{t+n(1+l)-1}^{1-\sigma}}{1-\sigma}\right).$$

The budget constraint for a consumer entering the bank is

$$M_{1,t+1} + B_{t+1} + P_t C_{n,t} = M_{n,t} + (1 + i_{t-n+1})B_{t-n+1} + \Pi_t + S_{t+1}. \qquad (2.A\ 1)$$

while in all other periods it is

$$M_{i+1,t+1} + P_t C_{i,t} = M_{i,t}. \qquad (2.A\ 2)$$

The Cash-in-Advance constraint has to be obeyed in all periods

$$M_{i,t} \geq P_t C_{i,t}. \qquad (2.A\ 3)$$

Again, the household receives last period's profits $\Pi_{t-1} = Y_{t-1}P_{t-1}$ from 'his' shop. Household i's expected value of the consumption bundle while buying the good of shop j is

$$E_t(\overline{C}_{t+n-i}) = E_t\left[\left(\sum_{l=1}^{i-1} C_{l,t-i+l}^{\frac{\gamma-1}{\gamma}} + \sum_{j=i}^{n} C_{j,t-i+j}^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}\right], \qquad \gamma > 1 \qquad (2.A\ 4)$$

where the goods 1 to $i-1$ already have been purchased and the consumption bundle consists of goods purchased between time $t-i+1$ and $t+n-i$. Maximizing (2.A 4) with respect to $C_{i,t}$ leads to

$$\frac{\partial E_t[\overline{C}_{t+n-i}]}{\partial C_{i,t}} = E_t\left[\overline{C}_{t+n-i}^{-\frac{1}{\gamma}}\left(C_{i,t}^{-\frac{1}{\gamma}} + \sum_{j=i+1}^{n} C_{j,t+j-i}^{-\frac{1}{\gamma}}\frac{\partial C_{j,t+j-i}}{\partial C_{i,t}}\right)\right] \equiv 0.$$

Solving this equation verifies homothetic preferences:

$$C_{i,t} \equiv \Psi_{i,t} M_{i,t}. \tag{2.A 5}$$

Recursive solution starts at the last shop n

$$C_{n,t} = \frac{M_{n,t}}{P_t}, \tag{2.A 6}$$

and then solves for optimal consumption until the currently open shop i, given the information set in the current period. Solving leads to

$$\Psi_{i,t} = \left[\Phi_{i,t}^{\gamma} P_t^{\gamma} + P_t\right]^{-1}, \tag{2.A 7}$$

with

$$\Phi_{i,t} = E_t\left[\sum_{j=i+1}^{n}\left(\Psi_{j,t+j-i}\prod_{k=i+1}^{j-1}\Psi_{k,t+k-i}\Phi_{k,t+k-i}^{\gamma}P_{k,t+k-i}^{\gamma}\right)^{\frac{\gamma-1}{\gamma}}\right], \tag{2.A 8}$$

and the definitions

$$\Phi_{n,t} \equiv 0, \qquad \prod_{k=i+1}^{i}\Psi_{k,t+k-i}\Phi_{k,t+k-i}^{\gamma}P_{k,t+k-i}^{\gamma} \equiv 1. \tag{2.A 9}$$

Producers Firm t, the only one selling at time t, maximizes profit

$$\Pi_t = P_t\sum_{j=1}^{n} C_{i,t} - CO_t^j(Y_t^j, P_t),$$

where $CO_t(Y_t^j, P_t) = \frac{(Y_t^j/A_t)^{1+\rho}}{1+\rho}P_t$ is the nominal cost function. Using equation (2.A 5), we get the following:

$$\Pi_t = P_t\sum_{i=1}^{n} M_{i,t}\Psi_{i,t} - \left(\frac{Y_t}{A_t}\right)^{1+\rho} P_t.$$

Plugging in the appropriate values leads to

$$\frac{\sum_{i=1}^{n}(\Phi_{i,t}^{\gamma}P_t^{\gamma-1}+1)^{-2}(\gamma\Phi_{i,t}^{\gamma}P_t^{\gamma-1}+1)M_{i,t}}{\sum_{i=1}^{n}(\Phi_{i,t}^{\gamma}P_t^{\gamma-1}+1)^{-2}\Phi_{i,t}^{\gamma}P_t^{\gamma-1}(\gamma-1)M_{i,t}} = \frac{A_t^{1+\rho}}{Y_t^{\rho}} \tag{2.A 10}$$

which equals $Markup = 1/MC$. This equation can be used together with equations (2.A 1), (2.A 3), (2.A 5)-(2.A 9), and a monetary policy rule to to simulate the economy or to derive a reduced form for the linearized version, as done in section 2.5 for $n = 3$.

2.B Carrying over Cash between Periods

This appendix shows when it is optimal to not carry over cash between the last and the first period of the shopping sequence, i.e. not holding money when visiting the bank. First note that it could only be optimal to carry over resources in the form of cash if unforseen shocks happen after the agent has visited the bond market, because with positive nominal interest rates it is clearly optimal to carry over resources with interest-bearing bonds. Hence, the shock has to take place after t. In the following I will derive the optimality condition for the two-shop case, which also gives intuition for the general case with n agents.

Carrying over cash in between periods is not optimal if the following condition holds

$$\frac{\partial U_t}{\partial M_{2,t+1}} > \beta E_{t+1}\left\{\frac{\partial U_{t+2}}{\partial M_{1,t+2}}\right\}, \tag{2.B 1}$$

where purchases of period t were already done because of the above argument, i.e. the decision considered takes place in $t+1$. $M_{2,t+1}$ is the money used for purchases in $t+1$.
In order for carrying cash over not to be optimal, the marginal utility of increasing these purchases has to be higher than the expected marginal utility of increasing expenditure in the coming periods. Since $\frac{M_{1,t+2}}{P_{t+3}} = \overline{C}_{t+3}$, the following derivative of the RHS of the above formula can be taken

$$E_{t+1}\left\{\frac{\partial U_{t+2}}{\partial M_{1,t+2}}\right\} = E_{t+1}\left\{\overline{P}^{\sigma}_{t+3}\right\}.$$

Considering the LHS of (2.B 1), it has to be taken into account that the decision to carry over money only affects consumption of good $C_{2,t+1}$, such that the consumption bundle
$\overline{C}_{t+1}$ looks like

$$\overline{C}_{t+1} = \left[C_{1,t}^{\frac{\gamma-1}{\gamma}} + \left(\frac{M_{2,t+1}}{P_{t+1}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}.$$

Hence, using the formulae from section 2.3 leads to the following marginal utility

$$\frac{\partial U_t}{\partial M_{2,t+1}} = \left(\frac{M_{1,t}}{\overline{P}_{t+1}}\right)^{\frac{1}{\gamma}-\sigma}\left(\frac{M_{2,t+1}}{P_{t+1}}\right)^{-\frac{1}{\gamma}}\frac{1}{P_{t+1}}.$$

Inserting all this into formula (2.B 1) leads to the following condition for carrying cash over to the next period not to be optimal

$$\left[\frac{M_{1,t}}{M_{2,t+1}}\frac{P_{t+1}}{\overline{P}_{t+1}}\right]^{\frac{1}{\gamma}} = \left[\frac{\overline{C}_{t+1}}{C_{2,t+1}}\right]^{\frac{1}{\gamma}} > \beta\frac{P_{t+1}}{\overline{P}_{t+3}}\left[\frac{M_{1,t}/\overline{P}_{t+1}}{M_{1,t+2}/\overline{P}_{t+3}}\right]^{\sigma}.$$

As stated above, I consider only increases in the money supply at $t+1$. $\overline{C}_{t+1} > C_{2,t+1}$ since $\overline{C}_{t+1}$ includes additional to $C_{2,t+1}$ also $C_{1,t}$. Remember that the discrete-number-of-agents indexes of section 2.3 are a sum, not an average. Hence, the LHS is larger than one. On the RHS, $\beta < 1$ and $P_{t+1} \leq \overline{P}_{t+3}$, since the prices increase monotonically after a positive monetary shock (even if prices jumped immediately to the new steady state, this would be true). Furhtermore, the consumer does not visit the asset market where he receives higher income until period $t+2$, but prices start rising already in $t+1$. Therefore $M_{1,t}/\overline{P}_{t+1} \leq M_{1,t+2}/\overline{P}_{t+3}$, i.e. the consumption of the agent who did not receive the monetary transfer first drops and picks up over time. Hence, the RHS is less than one and the condition is fulfilled.

While the necessary conditions are more complicated to derive for the general case with n shops, it should be intuitive that it is not optimal to carry over cash from one period to the next if prices are rising, as after a monetary expansion. Rather, it would be optimal to transfer money from coming periods to today, which is ruled out by the Cash-in-Advance constraint.

Part III

Technology Shocks

Chapter 3

S-Curve Redux

Jointly written with Gernot Müller

3.1 Introduction

Throughout the last 15 years international business cycle models have been used to analyze the international transmission of technology shocks. Irrespectively of specific assumptions on the structure of international asset markets and on firm's price setting behavior, these models generally provide a very similar account of how technology shocks impact the economy and are propagated over time and across countries. The *standard transmission mechanism* can be summarized as follows: in response to a country specific positive technology shock, domestic output expands and its relative price falls (i.e. the domestic terms of trade depreciate). At the same time, a surge of investment induces a trade deficit which turns into a surplus once the domestic capital stock has been built up. Note that the depreciation of the terms of trade increases the relative value of foreign output such that even in the absence of explicit risk-sharing foreign residents will generally reap some of the benefits of the domestic technology shock.

The empirical success of models based on this transmission mechanism has been mixed. In a seminal contribution Backus, Kehoe and Kydland (1994), hereafter BKK, show that the frictionless, complete markets variant of the model fails to replicate several key properties of the data, notably the volatility of relative prices.[1] At the same time BKK emphasize that - conditional on technology shocks - the

[1] Subsequent research has documented this failure as well as further anomalies and made various suggestions for their resolution. Examples for further evidence on anomalies include Backus, Kehoe and Kydland (1995), Baxter (1995), Ravn (1997), Ambler, Cardia and Zimmermann (2004); for partially successful resolutions see Stockman and Tesar (1995), Chari, Kehoe and McGrattan (2002), Heathcote and Perri (2002), Kehoe and Perri (2002) and, more recently, Corsetti, Dedola and

model delivers the *S-curve*, i.e. an S-shaped cross-correlation function for the trade balance and the terms of trade, which is 'one of the striking features of the data'.[2] The S-curve is a robust feature of the data and qualifies as a stylized fact characterizing international business cycles.

In the present chapter we rigorously assess the transmission mechanism of technology shocks implied by a prototypical international business cycle model - both under complete and incomplete financial markets. Given that the S-curve is one of the (few) dimensions where the prediction of the model squares well with the evidence, we focus on this statistic to pin down key parameters of the model. However, in contrast to earlier work, we compute the cross-correlation function for the trade balance and the terms of trade using counterfactual time-series obtained from purging the raw time series of the contributions of non-technology shocks. In other words, we compute a cross-correlation function conditional on technology shocks. In our view, it is important to focus on the conditional cross-correlation function given an emerging consensus according to which technology shocks are unlikely to be the only source of business cycle fluctuations.[3] In fact, while one-shock models that correctly describe the effects of technology shocks may or may not predict the unconditional S-curve which characterizes the data, it is a necessary condition for an empirical successful model to deliver the *conditional* cross-correlation function.

Of course, additional assumptions are required to obtain the counterfactual time-series. We therefore estimate a structural vector autoregression (VAR) model on U.S. time series data and identify technology shocks by assuming that only these shocks affect labor productivity in the long-run. The aim of the VAR analysis is twofold. First, we compute counterfactual time series that would have been obtained, had technology shocks been the sole source of fluctuations. These time series, in turn, are used to calculate the conditional cross-correlation function for the trade balance and the terms of trade. We find it to be S-shaped as well, but shifted to left relative to its unconditional counterpart. We use the conditional S-curve to calibrate our quantitative business cycle model where technology shocks are the only source of fluctuations.

Second, we use the VAR model to compute impulse response functions. We find that a positive technology shock induces a hump-shaped increase in output and investment and a lasting, hump-

Leduc (2006a).

[2]See BKK, page 94. If government shocks are considered in addition to technology shocks, the shape of the cross-correlation function barely changes. If government spending shocks were the sole impulse to the theoretical economy, the cross-correlation function would be tent-shaped.

[3]See Galí (1999), Altig, Christiano, Eichenbaum and Lindé (2005) or Chari, Kehoe and McGrattan (2005). While these papers disagree in various respects, they all suggest that the contribution of technology shocks to business cycle fluctuations is substantially lower than 70 percent as argued, for instance, in Kydland and Prescott (1991).

shaped decline in the trade balance. At the same time the relative price of domestic goods increases, i.e. we find a positive technology shock to induce an *appreciation* of the terms of trade and the real exchange rate. We treat these responses as an empirical characterization of the actual transmission mechanism and use them to assess the transmission mechanism implied by the prototypical business cycle model. In other words, the empirical impulse responses will serve as a sufficient condition for a successful theoretical account of the international transmission of technology shocks.

The business cycle model used to account for the evidence is a variant of the model originally proposed by BKK. In addition to complete financial markets, we also consider the possibility that only non-contingent bonds are traded across countries (incomplete financial markets). Moreover, we allow for investment adjustment costs and focus on exogenous differences in the level of technology across countries. We use the cross-correlation function conditional on technology shocks to calibrate the model: by matching the S-curve we pin down parameter values for the elasticity of substitution between domestic and foreign goods, investment adjustment costs and the persistence of relative technology. We consider both asset market structures. If financial markets are complete, we find a relatively high elasticity of substitution, while relative technology is moderately persistent. Investment costs are absent (calibration A). If financial markets are incomplete, we find a low elasticity of substitution and relative technology is very persistent. There is also evidence for mild investment adjustment costs under incomplete financial markets (calibration B).

Our assessment of the model starts with the observation that the S-curve is fairly well matched under both model specifications. We thus turn to the underlying transmission mechanism and compare the impulse responses of the theoretical economies with those obtained from the VAR model. Here we observe a striking difference across both specifications: under calibration A, the model predicts the terms of trade to depreciate and the trade balance to fall sharply on impact - in line with the standard transmission mechanism. In contrast, under calibration B, the model implies a transmission mechanism which turns the responses of the terms of trade and the trade balance upside down: it predicts an appreciation of the terms of trade and a hump-shaped decline in the trade balance - in line with the VAR evidence.

Regarding the role of asset markets in shaping the transmission process, it is important to stress that the difference across calibrations is not the result of different asset markets *per se*. In fact, for standard calibrations of the prototypical business cycle model, the transmission mechanism hardly differs across the two asset market structures. Hence, our results are not to be taken as evidence in favor of incomplete markets as such, but as evidence in favor of incomplete markets-*cum*-low-elasticity. Put differently, we provide evidence in favor of a particular transmission mechanism which is quite distinct from the standard transmission mechanism of technology shocks common to most international business cycle models. Corsetti, Dedola and Leduc (2004), henceforth CDL, were the first to observe such a possible alternative to the standard transmission mechanism.[4] Specifically, CDL show that if home bias is pervasive, the elasticity of substitution between domestic and foreign goods is low and financial markets are incomplete, technology shocks tend to appreciate the real exchange rate and the terms of trade. As a result, terms of trade movements amplify the effects of technology shocks on the distribution of wealth across countries. To assess this more formally, we follow King (1991) and compute a Hicksian decomposition in order to quantify the wealth effect of a technology shock, both at home and abroad. We find that under calibration A, both representative agents experience a positive wealth effect. In the incomplete markets-*cum*-low-elasticity case, however, the domestic agents faces a much larger wealth effect, while the foreign agent experiences a negative one, which is the result of a worsening of her terms of trade.

The remainder of the chapter is organized as follows. In the next section, we analyze U.S. time series computing the S-curve and empirical impulse response functions. We outline and calibrate the model under both asset market structures in section 3 and analyze the transmission mechanism and its implications in section 4. In section 5, we offer a brief conclusion. The appendix provides several robustness tests.

3.2 Time Series Evidence for the United States

In this section we use U.S. time series to establish evidence on the international transmission of technology shocks. First, we compute the unconditional cross-correlation function for the trade balance and the terms of trade - revisiting a key finding of BKK. Next, we estimate a VAR model and compute the cross-correlation function conditional on technology shocks as well as the contribution of technology shocks to the volatility of trade variables. Finally, we compute the impulse response functions to a technology shock.

Our analysis relies on U.S. data which are described in more detail in appendix 3.A. Regarding our

[4] In contrast to the present chapter, CDL do not investigate the cross-correlation function for the trade balance and the terms of trade. Instead, they focus on the relative consumption-real exchange rate anomaly identified by Backus and Smith (1993).

two key variables, we compute the terms of trade, p_t, as the log of the price index of non-commodity imports of goods and services divided by the price index of non-commodity exports of goods and services. The trade balance, nx_t, is measured as the ratio of nominal net exports to nominal GDP.[5] As macroeconomic volatility and, in particular, those of the terms of trade has been much higher in the 1970s relative to the post-1980 period, we limit our sample to data covering the period 1980:1-2005:4.

3.2.1 The unconditional S-curve

Before turning to our VAR model, we follow BKK and compute the unconditional cross-correlation function for the trade balance and the terms of trade. In order to separate short-run fluctuations from long-run movements in both time series we employ the HP-filter using a smoothing parameter of 1600. The dashed line in figure 3.1 displays the cross-correlation function for the terms of trade (t) and the trade balance (t+k) for k ranging from -8 to 8 quarters, i.e. for leads and lags up to two years. As

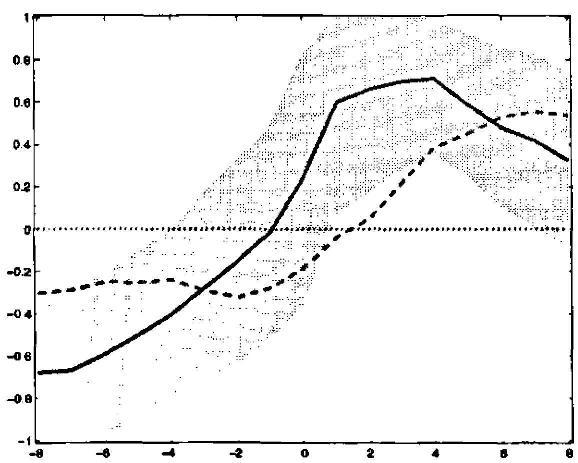

Figure 3.1: CROSS-CORRELATION FUNCTION FOR THE TRADE BALANCE (T+K) AND THE TERMS OF TRADE (T); SAMPLE: U.S. DATA 1980:1-2005:4; DASHED LINE: UNCONDITIONAL, COMPUTED AFTER APPLYING HP-FILTER TO RAW TIME SERIES; SOLID LINE: CONDITIONAL, COMPUTED AFTER APPLYING HP-FILTER TO COUNTERFACTUAL TIME SERIES OBTAINED FROM THE VAR MODEL; SHADED AREA: BOOTSTRAPPED 90 PERCENT CONFIDENCE INTERVALS

described by BKK, the shape of the cross-correlation function resembles an horizontal 'S'. Note that for our time series, the S-shape of the cross-correlation function is more pronounced and resembles

[5]We use price indices for non-commodity imports and exports in order to limit the impact of oil price fluctuations on the terms of trade. We follow BKK and consider net exports in current prices and thus allow valuation effects to play an important role in the dynamics of the trade balance. Note that this is quite distinct from analyzing the dynamics of the trade balance in constant prices, see Raffo (2006).

more closely what BKK report for the non-U.S. countries in their sample.[6] The function is negative at $k = 0$ and crosses the axis to the right of this point: the correlation between p_t and nx_{t+k} becomes increasingly positive for $k > 0$ such that future trade balance realizations are positively associated with current terms of trade.

BKK rationalize the S-curve by appealing to a specific transmission mechanism of technology shocks that, partly as a result of their work, may be considered the standard transmission mechanism.[7] After a one-time positive shock to technology domestic output increases and its relative price falls (p_t increases). Investment increases strongly and induces a fall in net exports (nx_t falls). After the surge in investment dissipates, the trade balance moves into a surplus. The contemporaneous correlation of both variables is therefore negative, while p_t and nx_{t+k} are positively correlated for $k > 1$.

3.2.2 The conditional S-curve

To obtain a cross-correlation function conditional on technology shocks we estimate a structural VAR model. In order to identify technology shocks we follow Galí (1999) and others and assume that these shocks are the only shocks which affect the long-run level of average labor productivity. The implementation follows Christiano, Eichenbaum and Vigfusson (2003) and is discussed in more detail in appendix 3.B.1. Our VAR model contains the change in the log of labor productivity (output/hour), the log of per capita hours worked, the log of the terms of trade and net exports (scaled by GDP). In addition to these four variables we also consider the real exchange rate and investment. To economize on the degrees of freedom, we re-estimate the VAR model by replacing, in turn, the terms of trade with the log of the real effective exchange rate, rx_t, and the trade balance with the log of investment over GDP. We include a constant and four lags of each endogenous variable.

Given the estimated model and the identified technology shocks, we compute counterfactual time series that would have been the result, had technology shocks been the only source of business cycle fluctuations. We then calculate the cross-correlation function for the trade balance and the terms of trade after HP-filtering the simulated series. Figure 3.1 displays the result. The solid line gives the point estimate, while the shaded area displays 90 percent confidence intervals computed by bootstrap based on 1000 replications.

The conditional cross-correlation function displays a pattern which is similar to the the unconditional

[6] Differences with respect to BKK are mostly due to considering a different sample period; only very small changes result from considering price indices for non-commodity imports and exports to compute the terms of trade.

[7] The cross-correlation pattern is also consistent with the notion of a J-curve, whereby a depreciation of the terms of trade (i.e. a rise in p_t) - through sluggish expenditure switching effects - leads to an increase in net exports only with a delay. This consideration provides the starting point for the analysis of BKK.

cross-correlation function (dashed line); in fact, it also resembles an horizontal 'S'. However, relative to its unconditional counterpart, the conditional cross-correlation function shifts to the left, i.e. while the unconditional contemporaneous correlation is negative, the conditional contemporaneous correlation is positive. This suggests that actual business cycle fluctuations of the trade balance and the terms of trade are to some extent driven by non-technology shocks. Hence, in order to understand the transmission of technology shocks it seems important to focus on those fluctuations of the data that can be attributed to technology shocks.

3.2.3 Business cycle variance decomposition

To assess quantitatively the contribution of technology shocks to fluctuations of the trade balance and the terms of trade, we perform a business cycle variance decomposition following Altig et al. (2005). Again, we rely on the counterfactual series that would have been the result if only technology shocks had occurred. We then compute the variance of these series relative to the variance of the series that result from all shocks occurring. Table 3.1 displays the results. The numbers give the fraction of the variance that can be attributed to technology shocks (standard errors computed by bootstrap based on 1000 replications are given in parentheses). Of course, the importance of technology shocks in accounting for business cycle fluctuations has been a topic of considerable debate in macroeconomics since the early 1980s and is clearly beyond the scope of this chapter. Here we are only interested

Table 3.1: BUSINESS CYCLE VARIANCE DECOMPOSITION

Output	Hours	p	nx	rx	Investment
0.33	0.21	0.46	0.29	0.55	0.22
(0.23)	(0.22)	(0.23)	(0.15)	(0.21)	(0.18)

Notes: Fraction of variance accounted for by technology shocks after application of HP filter (standard errors in parentheses).

in the importance of technology shocks in accounting for the shape of the S-curve: we find that technology shocks account for 46 and 29 percent of fluctuations of the terms of trade and the trade balance, respectively. These numbers are relatively high; in particular, if compared to the contribution of technology shocks to the fluctuation of the other variables included in the VAR. Technology shocks thus appear to be an important source of the short-run fluctuations of the trade balance and the terms of trade.[8]

[8]We also estimated a larger VAR model and identified monetary and fiscal policy shocks in addition to technology shocks. Technology shocks always accounted for the bulk of the business cycle fluctuations of the trade balance and the terms of trade.

3.2.4 Impulse response functions

In order to gain further insights into *how* technology shocks impact on the trade balance and the terms of trade we compute the impulse response functions of the estimated VAR model. Below, we will treat these responses as an empirical characterization of the actual transmission mechanism. Figure 3.2 displays the responses to a positive, one percent increase in technology. All variables are measured in

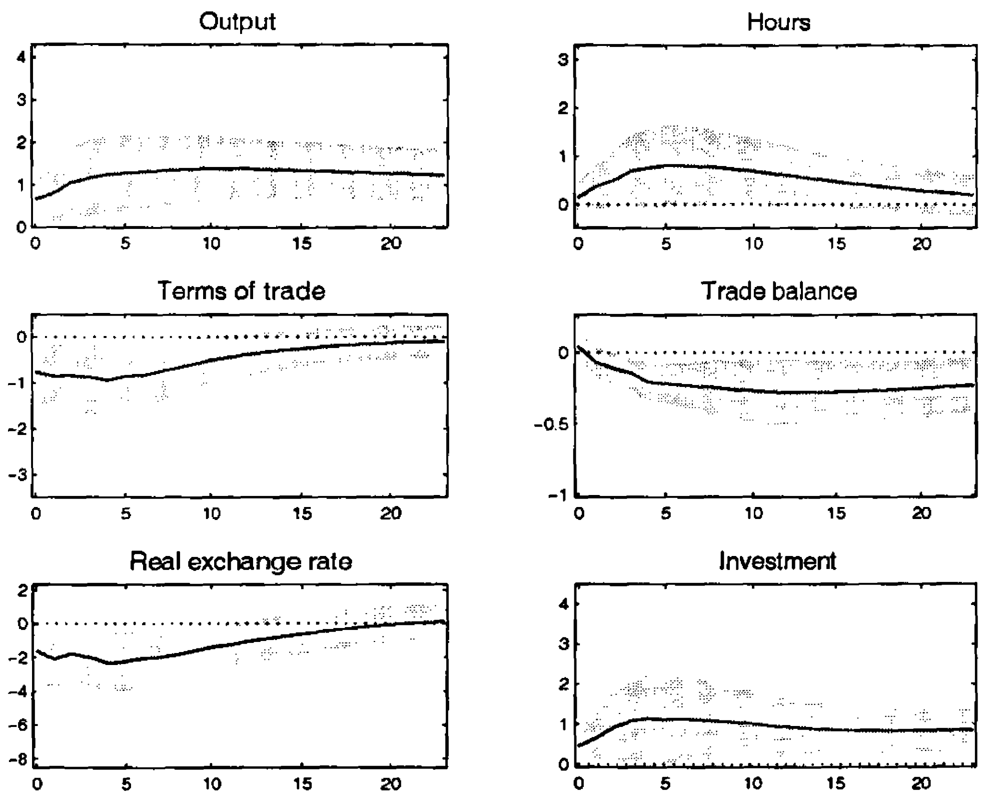

Figure 3.2: TRANSMISSION OF A ONE PERCENT INCREASE IN TECHNOLOGY; SAMPLE: U.S. DATA 1980:1-2005:4; SHADED AREAS INDICATE BOOTSTRAPPED 90 PERCENT CONFIDENCE INTERVALS; HORIZONTAL AXES: QUARTERS; VERTICAL AXES: PERCENT, EXCEPT FOR TRADE BALANCE (PERCENTAGE POINTS OF OUTPUT)

percentage deviations from trend, except for the trade balance which is measured in percentage points of GDP. The shaded area displays 90 percent confidence intervals, computed by bootstrap based on 1000 replications.

The panels in the first row give the responses of output and hours: both increase in response to a positive shock to technology.[9] The second row gives the responses for the terms of trade and the trade

[9]The response of hours to technology shocks has been the topic of a considerable debate. Some authors, notably Galí (1999), have argued that hours are difference stationary only. If first differences of hours instead of levels are used in the

balance. The terms of trade fall (appreciate) significantly on impact, i.e. the price of U.S. imports falls relative to the price of U.S. exports. The trade balance displays a lasting, hump-shaped and significant decline. We find these responses to be robust with respect to variations of the sample period and to inclusions of additional variables in the VAR model. They will therefore play a key role in our assessment of the transmission mechanism implied by different specifications of the standard two country business cycle model.

In the last row we display the responses of the real exchange rate and investment. As the real exchange rate also appreciates, pricing behavior is unlikely to play a key role in accounting for the appreciation of the terms of trade, see Obstfeld and Rogoff (2000). As investment displays a hump-shaped increase, we will allow for investment adjustment costs in our theoretical model below.

Before turning to the international business cycle model, we note that our identification scheme is meant to capture technology shocks by assuming that only these shocks affect U.S. labor productivity in the long-run.[10] These shocks are likely to consist of both a country-specific (idiosyncratic) and a global (common) component. However, to the extent that the terms of trade and the trade balance respond to technology shocks, we are likely to pick up the idiosyncratic component, because the global component will affect all countries similarly and is therefore unlikely to have a substantial effect on the terms of trade and the trade balance.[11] The conditional S-curve can therefore be interpreted as resulting from the country-specific component of technology shocks, i.e. exogenous changes in the level of U.S. technology *relative* to its trading partners.

A more formal assessment of our VAR model is provided in appendix 3.B.3, where we use the calibrated business cycle model outlined below to carry out a Monte Carlo experiment. Overall, we find that the VAR performs quite well. In particular, the S-curve obtained from the VAR is almost identical to the true cross-correlation function and the impulse responses have the correct sign.

VAR model, hours tend fall in response to a technology shock. In appendix 3.B.2 we consider this specification, finding a similar result. However, the responses of trade variables are strikingly robust with respect to this modification.

[10]Corsetti, Dedola and Leduc (2006b) make an alternative identification assumption in a differently specified VAR model. Specifically, they consider relative variables and assume that technology shocks are the only shocks which have a long-run effect on *relative* labor productivity across countries. Interestingly, their results are broadly in line with ours, notably regarding the behavior of international relative prices.

[11]Glick and Rogoff (1995) test a small open economy version of the international business cycle model by comparing the effect of country-specific and global technology shocks on the current account. They find no effect of the global component. Similarly, Gregory and Head (1999) estimate a dynamic factor model to identify common and country-specific factors driving productivity, investment and the current account. A key finding is that common factors account for almost none of the variations in current accounts. Finally, Normandin and Fosso (2006) decompose technology into a country-specific and a global component using a (one-good) international business cycle model. They find no role for global technology shocks in accounting for current account movements.

3.3 The Model

In this section we analyze the international transmission of technology shocks in a standard two country business cycle model. The model is a variant of the model originally proposed by BKK. In the next subsection, we closely follow the exposition of Heathcote and Perri (2002) and then discuss our strategy to solve the model numerically around a deterministic steady state. In a third subsection, we calibrate the model by matching the S-curve conditional on technology shocks.

3.3.1 Setup

The world economy consists of two countries, each of which produces a distinct good and is populated by a representative household. Regarding internationally traded assets, we consider the possibility of complete and incomplete financial markets, where only non-contingent bonds are traded across countries.[12] In the following, s^t denotes the history of events before and including time t, consisting of all events $s_\tau \in S, \tau \leq t$, where S is the set of possible events. The probability of history s^t at time 0 is given by $\pi(s^t)$.

Households allocate consumption expenditures on final goods, $c_i(s^t)$, and supply labor, $n_i(s^t)$, to intermediate good firms. The representative household in country i maximizes

$$\sum_{t=0}^{\infty}\sum_{s^t}\pi(s^t)\beta(\{c_i(s^\tau)\}_{\tau=0}^{\tau=t-1},\{n_i(s^\tau)\}_{\tau=0}^{\tau=t-1})U(c_i(s^t),n_i(s^t)), \tag{3.1}$$

subject to a budget constraint which depends on the structure of international asset markets. As also further detailed below, the discount factor β may depend on the sequence of consumption and labor. Instantaneous utility is non-separable in consumption and leisure, $1-n_i(s^t)$:

$$U(c_i(s^t),n_i(s^t)) = \frac{1}{1-\gamma}[c_i(s^t)^\mu(1-n_i(s^t))^{1-\mu}]^{1-\gamma}. \tag{3.2}$$

The representative household in each country owns the capital stock, $k_i(s^t)$, and rents it to intermediate good firms. Capital and labor are internationally immobile. As in Christiano, Eichenbaum and Evans (2005), we assume that it is costly to adjust the level of investment, $x_i(s^t)$. Specifically, the law

[12] While BKK consider only complete financial markets, Heathcote and Perri (2002) also investigate a third case: financial autarky. In fact, they find that the model performs relatively well under this assumption. However, by definition trade is always balanced in this case, which is thus not suited for our analysis. Note that we depart from the model in Heathcote and Perri (2002) by i) introducing an endogenous discount factor under incomplete financial markets to ensure the stationarity of bond holdings; ii) introducing investment adjustment costs to account for the hump-shaped investment response observed in the data; and iii) assuming that technology is non-stationary and labor augmenting.

of motion for capital is given by

$$k_i(s^{t+1}) = (1-\delta)k_i(s^t) + H(x_i(s^t), x_i(s^{t-1})), \text{ with } H = [1 - G(x_i(s^t)/x_i(s^{t-1}))]x_i(s^t). \quad (3.3)$$

Restricting $G(1) = G'(1) = 0$ ensures that the steady state level of capital is independent of investment adjustment costs captured by the parameter $\chi = G''(1) > 0$.

Intermediate good firms specialize in the production of a single intermediate good, $y_i(s^t)$. It is produced by combining capital and labor according to a standard Cobb-Douglas production function:

$$y_i(s^t) = k_i(s^t)^\theta [z_i(s^t) n_i(s^t)]^{1-\theta}, \quad (3.4)$$

where $z_i(s^t)$ denotes technology. Letting $w_i(s^t)$ and $r_i(s^t)$ denote the wage rate and the rental rate of capital in terms of the local intermediate good, the problem of intermediate good firms is given by

$$\begin{aligned} &\max_{n_i(s^t), k_i(s^t)} y_i(s^t) - w_i(s^t) n_i(s^t) - r_i(s^t) k_i(s^t), \\ &\text{subject to} \quad k_i(s^t), n_i(s^t) \geq 0. \end{aligned} \quad (3.5)$$

Intermediate goods are sold on to final good producers in both countries while the law of one price is assumed to hold throughout.

Final good firms assemble intermediate goods produced both domestically and abroad. Let $a_i(s^t)$ and $b_i(s^t)$ denote the uses of the two intermediate goods in country i, originally produced in country 1 and 2, respectively. Then final goods are produced on the basis of the following constant returns to scale technology

$$F_i(a_i(s^t), b_i(s^t)) = \begin{cases} \left[\omega^{1/\sigma} a_i(s^t)^{(\sigma-1)/\sigma} + (1-\omega)^{1/\sigma} b_i(s^t)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, & \text{for } i = 1 \\ \left[(1-\omega)^{1/\sigma} a_i(s^t)^{(\sigma-1)/\sigma} + \omega^{1/\sigma} b_i(s^t)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, & \text{for } i = 2 \end{cases} \quad (3.6)$$

where σ measures the elasticity of substitution between foreign and domestic goods and $\omega > 0.5$ the extent to which the composition of final goods is biased towards domestically produced intermediate goods. Final good firms solve the following problem

$$\max_{a_i(s^t),b_i(s^t)} F_i(s^t) - q_i^a(s^t)a_i(s^t) - q_i^b(s^t)b_i(s^t),$$
$$\text{subject to} \quad a_i(s^t), b_i(s^t) \geq 0, \tag{3.7}$$

where q_i^a and q_i^b denote the prices of intermediate goods a and b in terms of the final good F_i, respectively. The budget constraint of the representative household depends on the asset market structure. We consider both incomplete and complete international financial markets.

Incomplete financial markets

In this case only a non-contingent bond is traded across countries. It pays one unit of the intermediate good a in period $t+1$ in each state of the world. Letting $B_i(s^t)$ and $Q(s^t)$ denote the quantity and the price of this bond bought by the representative household in country i at the end of period t, then the budget constraint of household 1 reads as follows

$$c_1(s^t)+x_1(s^t)+q_1^a(s^t)Q(s^t)B_1(s^t) = q_1^a(s^t)[w_1(s^t)n_1(s^t)+r_1(s^t)k_1(s^t)]+q_1^a(s^t)B_1(s^{t-1}). \tag{3.8}$$

The budget constraint for the representative household in country 2 is analogously defined in terms of final good 2.

To ensure stationarity of bond holdings, we follow Mendoza (1991) by assuming that the time discount factor depends on the sequence of consumption and leisure. Therefore we assume

$$\beta(\{c_i(s^\tau)\}_{\tau=0}^{\tau=t-1}, \{n_i(s^\tau)\}_{\tau=0}^{\tau=t-1}) = \exp\left[\sum_{\tau=0}^{t-1} -\nu(c_i(s^\tau), n_i(s^\tau))\right], \tag{3.9}$$

where

$$\nu(c_i(s^t), n_i(s^t)) = \ln(1 + \psi[c_i(s^t)^\mu(1-n_i(s^t))^{1-\mu}]), \tag{3.10}$$

with $\psi > 0$ set to determine the discount factor in steady state.[13]

[13]As discussed in Bodenstein (2006) the assumption of an endogenous discount factor also ensures the uniqueness of the steady state - in contrast to other assumptions which induce stationarity of bond holdings. While Bodenstein warns against excluding a priori the multiplicity of steady states, note that an endogenous discount factor will generally pick the symmetric one. Regarding impulse responses functions to technology shocks, Bodenstein also points out the possibility of multiplicities, which are ignored if a linearized version of the model is used. We will rely on such a version of the model in our simulations below. This seems sensible, because we thereby ignore time paths, which induce implausibly large jumps in consumption and output in response to technology shocks.

Complete markets

Alternatively, we consider the case in which a complete set of state-contingent securities is traded on international financial markets. Letting $B_i(s^t, s_{t+1})$ denote the quantity of bonds bought by household i in period t that pay one unit of the intermediate good a in $t+1$ if the state of the economy is s_{t+1}, then the budget constraint of the household 1 reads as

$$\begin{aligned} c_1(s^t) + x_1(s^t) + q_1^a(s^t) \sum_{s_{t+1}} Q(s^t, s_{t+1}) B(s^t, s_{t+1}) \\ = q_1^a(s^t)[w_1(s^t)n_1(s^t) + r_1(s^t)k_1(s^t)] + q_1^a(s^t)B(s^{t-1}, s_t). \end{aligned} \tag{3.11}$$

The budget constraint for the representative household in country 2 is analogously defined in terms of final good 2. For convenience, we assume that the time discount factor is constant in this case, i.e. $\beta(\{c_i(s^\tau)\}_{\tau=0}^{\tau=t-1}, \{n_i(s^\tau)\}_{\tau=0}^{\tau=t-1}) = \beta^t$.

Equilibrium is a set of prices for all s^t and all $t \geq 0$ such that when intermediate and final good firms, as well as households take these prices as given, households solve (3.1) subject to the capital accumulation equation (3.3) and to either budget constraint (3.8) or (3.11); firms solve their static problems (3.5) and (3.7) subject to the production functions (3.4) and (3.6); furthermore all markets clear, i.e. for intermediate goods we have

$$a_1(s^t) + a_2(s^t) = y_1(s^t), \tag{3.12}$$

$$b_1(s^t) + b_2(s^t) = y_2(s^t); \tag{3.13}$$

for final goods

$$c_i(s^t) + x_i(s^t) = F_i(s^t), \qquad i = 1, 2;$$

holds, and - under incomplete financial markets - we have

$$B_1(s^t) + B_2(s^t) = 0,$$

or - under complete financial markets

$$B_1(s^t, s_{t+1}) + B_2(s^t, s_{t+1}) = 0, \;\; \forall \; s_{t+1} \in S.$$

Additional variables of interest are the terms of trade, $p(s^t)$, the trade balance, $nx(s^t)$, and the real exchange rate, $rx(s^t)$. For the terms of trade and the real exchange rate in country 1, we have

$$p(s^t) = q_1^b(s^t)/q_1^a(s^t) \quad \text{and} \quad rx(s^t) = q_1^a(s^t)/q_2^a(s^t),$$

respectively. Its trade balance is defined as the ratio of net exports to output

$$nx(s^t) = \frac{a_2(s^t) - p(s^t)b_1(s^t)}{y(s^t)}.$$

3.3.2 Model solution

We linearize the model around a symmetric steady state and consider the deviations of a variable from its steady state value. More precisely, we focus on relative variables, i.e. the deviation from steady state of a variable in the home country (country 1) minus the deviation from steady state of the corresponding variable in the foreign country (country 2). Note that the terms of trade and net exports in one country are just the negative of the value of the variable in the other country. We assume that domestic and foreign technologies, written in logs using 'hats', follow the joint process

$$\begin{bmatrix} \hat{z}_1(s^t) \\ \hat{z}_2(s^t) \end{bmatrix} = \begin{bmatrix} \rho_1 & \rho_2 \\ \rho_2 & \rho_1 \end{bmatrix} \begin{bmatrix} \hat{z}_1(s^{t-1}) \\ \hat{z}_2(s^{t-1}) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(s^t) \\ \varepsilon_2(s^t) \end{bmatrix}, \tag{3.14}$$
$$\text{with} \quad \begin{bmatrix} \varepsilon_1(s^t) \\ \varepsilon_2(s^t) \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{\varepsilon_1}^2 & \sigma_{\varepsilon_1\varepsilon_2} \\ \sigma_{\varepsilon_1\varepsilon_2} & \sigma_{\varepsilon_2}^2 \end{bmatrix} \right).$$

Note that, as in the calibrated models of BKK and Heathcote and Perri (2002), technology spillovers are assumed to be symmetric. In addition, to be consistent with our identification strategy used in the VAR model, we assume that $\rho_1 + \rho_2 = 1$ such that innovations to technology have permanent effects on the level of technology. In addition, we assume that $\rho_1, \rho_2 > 0$. As a result there is a cointegration relation between $\hat{z}_1(s^t)$ and $\hat{z}_2(s^t)$, with the cointegrating vector $[\ 1 \quad -1\ ]$.
This allows us to focus on relative technology $\tilde{z}(s^t) = \hat{z}_1(s^t) - \hat{z}_2(s^t)$, which is stationary and follows the AR(1) process

$$\tilde{z}(s^t) = \rho\tilde{z}(s^{t-1}) + \varepsilon(s^t), \quad \varepsilon(s^t) \sim N(0, \sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 - 2\sigma_{\varepsilon_1\varepsilon_2}) \tag{3.15}$$

with $\rho = \rho_1 - \rho_2$. As stressed in Kollmann (1998), in the standard two country business cycle model only relative technology matters for the dynamics of relative variables as well as for the dynamics of the terms of trade and the trade balance. Given that we are primarily interested in the joint dynamics of these two variables, we focus on the parameter ρ, i.e. on the persistence of relative technology, without having to take a stand on the relative size of ρ_1 to ρ_2. We thus rely on the process (3.15) in calibrating the model.

3.3.3 Calibration

The model outlined in the previous subsections is meant to provide a structural interpretation of the time series evidence established in section 3.2. A subset of the results of the VAR analysis will therefore play a key role in calibrating the model. In a first step, we use the conditional S-curve to calibrate the model, given that its unconditional counterpart is one of the dimensions where the prediction of the model has been shown to square well with the evidence. Simple experimentation shows that the shape of the cross-correlation function for the trade balance and the terms of trade implied by the model is mostly governed by the values of three parameters: the elasticity of substitution between domestic and foreign goods, σ, investment adjustment costs, χ, and the persistence of the process of relative technology, ρ.

Our calibration strategy is to pin down values for these model parameters in order to match the conditional S-curve obtained from the VAR model. This strategy is particularly suitable, given that values for all three parameters are not identified by first moments of the data and are at the focus of the debate on the international transmission process.[14] Other parameters have little bearing on the cross-correlation function for the trade balance and the terms of trade and are less controversial in the literature. We therefore simply follow BKK's choice of parameter values.

More formally, our calibration strategy can be stated as follows. Let m_d denote a 17×1 vector containing the empirical cross-correlation function for 8 lags and leads and let $m(\lambda)$ denote the corresponding cross-correlation function obtained from a simulation of the model (averages over 20 simulations of 104 observations, corresponding to the number of observations used in the VAR). As the theoretical moments depend on $\lambda = \{ \sigma \quad \chi \quad \rho \}$, we find values for these parameters by solving the following problem

$$\min_{\lambda} \left(m(\lambda) - m_d\right)' W \left(m(\lambda) - m_d\right), \tag{3.16}$$

[14]This is particularly true for σ, see CDL. Regarding the process for technology, the traditional approach is to estimate an AR(1) process on Solow residuals for the U.S. and the rest of the world. Our approach allows us to avoid the construction of these series which are likely to be contaminated by measurement error.

where W is the efficient weighting matrix, i.e. the inverse of the (bootstrapped) variance-covariance matrix of m_d. We solve (3.16) for both asset market structures - complete and incomplete international financial markets. In a recent contribution, Canova and Sala (2005) stress identification issues that may arise in the calibration and estimation of richly specified DSGE models. While the present model is relatively parsimonious, we nevertheless assess whether the structural parameters σ, ρ and χ are jointly identified by our criterion function (3.16). Figure 3.12 in appendix 3.C.1 shows that our criterion function is well suited to pin down the parameters of interest.

Table 3.2: PARAMETER VALUES OF THEORETICAL ECONOMIES

Standard values:		
Discount factor (steady state)	$\beta = 0.99$	
Consumption share	$\mu = 0.34$	
Risk aversion	$\gamma = 2$	
Capital share	$\theta = 0.36$	
Depreciation rate	$\delta = 0.025$	
Import share (steady state)	$1 - \omega = 0.12$	
	Financial markets	
	Complete	Incomplete
Matching the S-curve:	(Calibration A)	(Calibration B)
Elasticity of substitution between intermediate goods	$\sigma = 2.182$	0.250
Investment adjustment costs	$\chi = 0.000$	0.025
Autoregressive coefficient of technology	$\rho = 0.871$	0.987
Loss function:	8.911	6.116

Notes: Standard parameter values are taken from Backus et al. (1994). Values for parameters in the second part of table are obtained by solving the objective (3.16); the last line gives its value in the optimum.

Table 3.2 displays the results. The upper part of the table reports parameter values which are assumed independently of the asset market structure. All values are taken from BKK, except for the import share which we assume to be 0.12, the average in our sample. The lower part of table 3.2 reports the values for the elasticity of substitution between domestic and foreign goods, σ, investment adjustment costs, χ, and the persistence of cross-country technology differential, ρ, obtained by solving (3.16). The set of parameter values obtained under the assumption that financial markets are complete defines calibration A (left column). The elasticity of substitution between intermediate goods, σ, takes a value of about 2.2. This is relatively close to 1.5, the value used in the benchmark economy of BKK. Investment adjustment costs are also absent and the value for the persistence of technology differentials is $\rho = 0.87$.

The set of parameter values obtained by solving (3.16) under the assumption that financial markets are incomplete defines calibration B (right column). In this case, the elasticity of substitution between

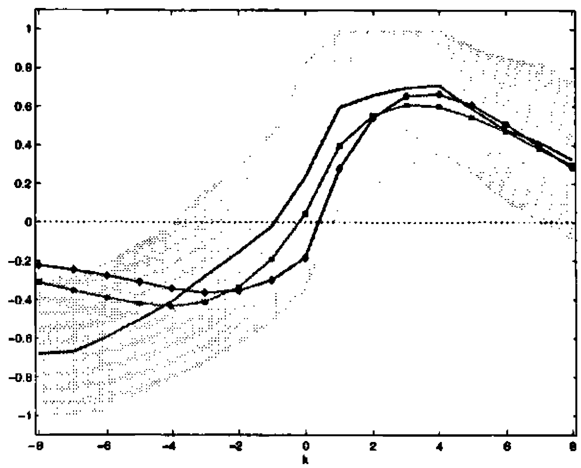

Figure 3.3: CONDITIONAL CROSS-CORRELATION FUNCTION FOR THE TRADE BALANCE AND THE TERMS OF TRADE; SOLID LINE: COMPUTED AFTER APPLYING HP-FILTER TO COUNTERFACTUAL TIME SERIES OBTAINED FROM THE VAR MODEL; SHADED AREA: BOOTSTRAPPED 90 PERCENT CONFIDENCE INTERVALS; MODEL SIMULATIONS: ◇ CALIBRATION A, □ CALIBRATION B

intermediate goods, σ, takes a value of 0.25.[15] Under calibration B the model is also characterized by very mild investment adjustment costs as $\chi = 0.025$. Christiano et al. (2005), using the same specification in a different context, report an estimate of approximately 2.5. Finally, note that calibration B is also characterized by very persistent technology differentials.[16] Regarding the calibration of the model under incomplete financial markets, it is interesting to observe that there is also a local minimum which is characterized by parameter values close to those defining calibration A.[17]

In figure 3.3 we plot the cross-correlation function for the trade balance and the terms of trade. Both economies deliver a cross-correlation function quite close to the conditional S-curve obtained from the VAR. Moreover, the theoretical S-curves are generally well within the 90 percent confidence interval. This is noteworthy, given that we match 8 leads and lags as well as the contemporaneous

[15]This number is lower than the values often used or found in the literature. Recent estimates in a similar order of magnitude, however, are reported by Lubik and Schorfheide (2006). Other recent papers which suggest a relatively low elasticity of substitution between intermediate goods include Kollmann (2006) and de Walque, Smets and Wouters (2005). Note, moreover, that such a low effective elasticity may be the result of a higher elasticity in an economy with a distribution sector as in CDL.

[16]Note, that Kollmann (1998) cannot reject the null hypothesis of no cointegration for the process of U.S. total factor productivity and total factor productivity in the G6 countries estimated on the basis of Solow residuals.

[17]More generally, the economy defined by this local minimum has properties similar to calibration A. However, the global optimum defines an economy (calibration B) which is characterized by a particularly low elasticity of substitution and this - as we show below - will fundamentally alter the international transmission mechanism of technology shocks.

moment of the cross-correlation function when pinning down 3 structural parameters. In other words, we impose 14 overidentifying restrictions. On the other hand it is also interesting to note that the contemporaneous correlation is positive only in calibration B.[18]

3.4 The international transmission of technology shocks

Given the calibrated model, we now turn to the international transmission of a domestic technology shock. First, we consider the responses of those variables which are included in the VAR model in order to assess the empirical performance of both model calibrations. Second, we discuss the implications of both calibrations for risk sharing across countries. Third, we compute the wealth effect triggered by the shock in order to gain further insights into the underlying transmission mechanism.

3.4.1 Comparing DSGE and VAR impulse responses

We assess the ability of the model under calibration A and B to account for the time series evidence as represented by the impulse response functions displayed in figure 3.2. Given the joint process for domestic and foreign technologies (3.14), we consider a one percent, permanent increase in the level of domestic technology.[19] To compare the transmission process of the theoretical economy to the data, we focus on the responses of those variables included in the VAR.

Figure 3.4 displays the results. The upper left panel shows the response of domestic output, which increases by about 0.8 percent on impact in both economies. Hours also increase in line with the VAR evidence (level specification). Both, the response of output and hours are quantitatively similar to the responses obtained from the VAR model. Under calibration B, as a result of mild investment adjustment costs, the model also predicts small humps in the responses of output and hours, corresponding to the VAR evidence.

The responses of the terms of trade and the trade balance are displayed in the second row. Here one observes a striking difference between calibrations A and B. Under calibration A the terms of trade depreciate, i.e. the price of imports increases relative to the price of exports. Under calibration B,

[18] Relative to the cross-correlation function reported by BKK, the S-curve which characterizes calibration A is shifted to the left. The analysis in BKK shows that such a shift is likely to result from an increase in the elasticity of substitution between intermediate goods. BKK's benchmark case is defined by a value of 1.5.

[19] Recall that in calibrating the model we have relied on relative variables only. Now we are also interested in the value of domestic variables *per se*. Therefore we have to specify the parameters governing (3.14). From the assumption $\rho_1 + \rho_2 = 1$ (see section 3.3.2) and the value obtained for the persistence of relative technology $\rho = \rho_1 - \rho_2$ in the calibration of the model (see table 3.2), we obtain $\rho_1 = (1 + \rho)/2$ and $\rho_2 = 1 - \rho_1$.

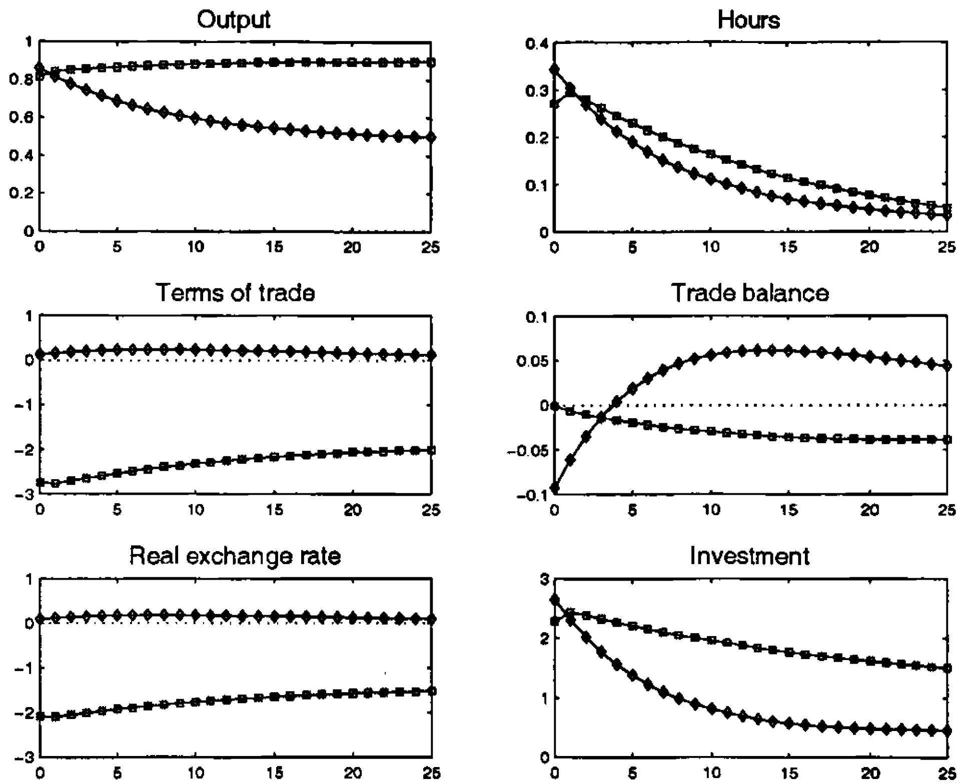

Figure 3.4: TRANSMISSION OF A ONE PERCENT INCREASE IN TECHNOLOGY; HORIZONTAL AXES: QUARTERS; VERTICAL AXES: PERCENT, EXCEPT FOR TRADE BALANCE (PERCENTAGE POINTS OF OUTPUT); ◇ CALIBRATION A, □ CALIBRATION B

in contrast, the terms of trade appreciate - in line with the VAR evidence. The response of the trade balance is also markedly different across calibrations. It displays a lasting, hump-shaped decline under calibration B - a pattern very similar to the response obtained from the VAR model. In contrast, under calibration A the trade balance falls sharply on impact and moves into surplus after about four quarters. Similarly, only under calibration B the model correctly predicts the sign of the response of the real exchange rate.

To sum up, we note that while both economies deliver the S-curve (figure 3.3) the underlying transmission process is quite distinct (figure 3.4). In fact, as far as the terms of trade and the trade balance are concerned, the transmission mechanism under calibration B turns the process under calibration A upside down! Put differently, we find that only under calibration B, the model correctly predicts a terms of trade appreciation and a lasting, hump-shaped decline of the trade balance. As a caveat it should be noted that the quantitative performance of the model under calibration B is not fully satisfactory. While the terms of trade respond too much relative to the VAR evidence, the trade balance responds too little to the technology shock.[20]

3.4.2 Implications for implicit risk-sharing under incomplete financial markets

Under calibrations A and B the model predicts fundamentally different responses of the terms of trade and the trade balance. These responses reflect deep differences with respect to the extent to which country-specific risk is shared internationally. To see this, it is important to recall that the difference in the transmission of technology shocks is not the result of different asset market structures *per se*. This follows from results established by earlier literature, showing that - all else equal - moving from complete to incomplete financial markets does generally not affect the equilibrium allocation very much. In fact, if there is no trade in a complete set of state contingent securities across countries, there are two important mechanism through with implicit risk-sharing may be achieved under incomplete markets. First, Baxter and Crucini (1995) show in a one-good model that intertemporal trade in a single non-contingent bond allows to achieve allocations close to the one obtained under complete markets. A condition for this result to go through is that technology shocks are not too persistent.

Second, Cole and Obstfeld (1991) consider risk-sharing in a two-good world and find that terms of trade movements can also provide implicit risk sharing under incomplete markets. Specifically, if the elasticity of substitution between domestic and foreign goods is unity, the real allocation under incomplete markets is identical to the allocation obtained under complete markets within their model. To see how this works, consider the standard transmission mechanism in an economy with incomplete financial markets, where the home country faces a favorable technology shock. As a result, output expands relative to foreign. At the same time the *terms of trade depreciate*, i.e. the price of domestically produced goods falls relative to foreign intermediate goods. This change in relative prices implies a wealth transfer from home to foreign, such that the wealth effect of the domestic technology shock is spread equally across countries.

[20]To see this more formally, we computed the standard deviations of the variables included in the VAR relative to output: for the U.S. time series, for the counterfactual time series conditional on technology shocks, and for calibrations A and B. Table 3.4 in appendix 3.C.2 shows the results. It illustrates that under calibration A, the model clearly fails in delivering the volatility of relative prices which characterizes the data, while under calibration B, the model fails to deliver the volatility of the trade balance. This tradeoff is related to the elasticity of substitution between intermediate goods, as noted by Backus et al. (1994, 1995).

Our calibration B, however, is characterized by a low elasticity of substitution and very persistent shocks to relative technology. While these features make it likely that the allocations under complete and incomplete markets differ, it is *a priori* not clear how they affect the response of the terms of trade. The appreciation is particularly striking, because thereby terms of trade movements not only fail to provide implicit risk sharing, but will instead amplify the wealth effect of technology shocks. The terms of trade appreciation raises the value of domestic output relative to foreign output, despite the fact that domestic output is now produced under a more favorable technology.

CDL analyze the possibility of such a 'negative' international transmission of technology shocks. They find that the domestic terms of trade appreciate in response to a positive technology shock if i) financial markets are incomplete, ii) home bias is substantial and iii) the elasticity of substitution between domestic and foreign goods is low. To see how these features induce a terms of trade appreciation, consider an increase in domestic technology. Ceteris paribus, this increases domestic wealth relative to foreign if financial markets are incomplete. As a result domestic absorption increases relative to foreign. If, in addition, home bias is pervasive and substitution elasticities are low, this induces a more than proportional increase in the demand for domestically produced goods. In equilibrium this leads the price of domestically produced goods to rise relative to the price of foreign goods. The rise in the price of domestic goods, in turn, supports the initial rise in domestic absorption as it transfers wealth from foreign to domestic residents.

3.4.3 Wealth Effects

We now turn to a more formal assessment of the explicit or implicit risk-sharing arrangements operating under the transmission mechanism implied by calibration A and B, respectively. Following King (1991), we compute the dynamic Hicksian decomposition of the consumption response to a domestic technology shock into a wealth and a substitution effect. We proceed as follows. First, we compute the change in lifetime utility triggered by the technology shock. Second, we compute a permanent lump-sum transfer that would induce the same change in lifetime utility assuming counterfactually that no shock occurs.[21] Finally, we compute the percentage change in steady state consumption induced by the transfer payment. This change provides a measure for the wealth effect of the technology shock. Figure 3.5 displays the response of consumption to a permanent domestic technology shock. The

[21] We assume that both the domestic and the foreign household obtain the same transfer payment such that equilibrium prices are not affected by the counterfactual experiment.

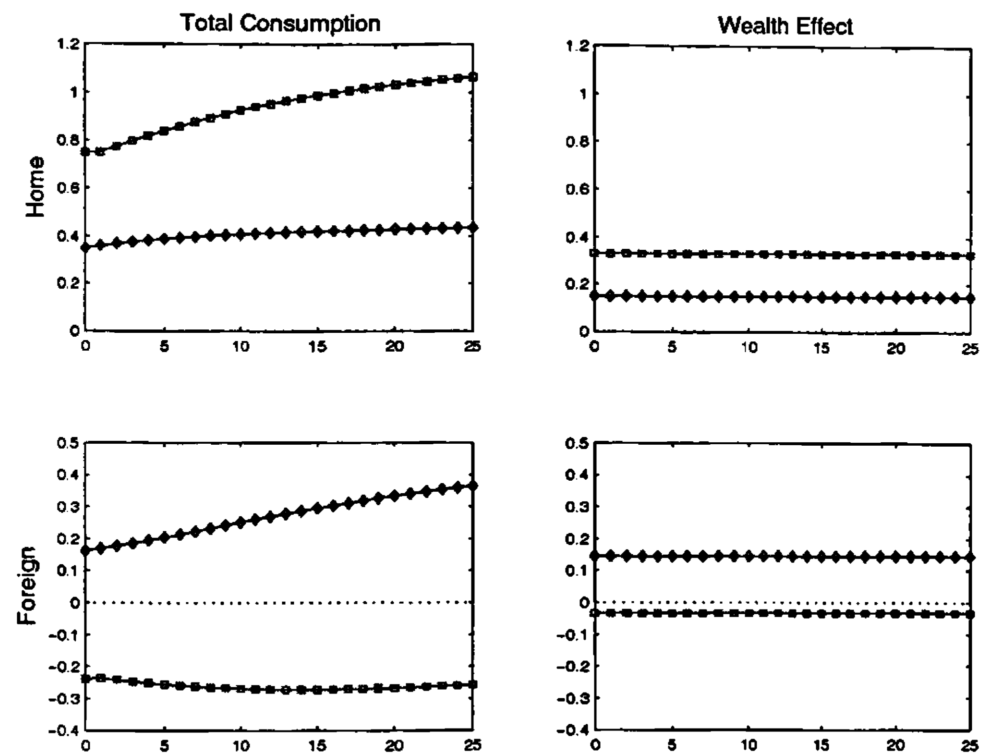

Figure 3.5: HICKSIAN DECOMPOSITION OF CONSUMPTION RESPONSE TO A ONE PERCENT INCREASE IN TECHNOLOGY (PERCENT OF STEADY STATE CONSUMPTION); ◇ CALIBRATION A, □ CALIBRATION B

panels in the left column show the total consumption response in the domestic economy (upper panel) and the foreign economy (lower panel) for calibrations A and B. Under calibration A, both domestic and foreign consumption increases in response to the domestic technology shock. Under calibration B, domestic consumption increases considerably more relative to calibration A, while foreign consumption falls in response to the shock. The panels in the right column display the component of the consumption response which can be attributed to the wealth effect of the domestic technology shock. They illustrate the fundamental differences in the transmission mechanism of the model under both calibrations. The wealth effect of the domestic technology shock on domestic consumption is positive under both calibrations. Under calibration A, however, it is about half the size as in calibration B. Given full risk sharing under complete markets, there is a positive wealth effect of the domestic technology shock on foreign consumption. Under calibration B, in contrast, there are no signs of implicit risk sharing. In fact, the opposite happens: the terms of trade appreciation lowers the value of foreign output which, in turn, is reflected in a negative wealth effect manifest in the consumption response.

In order to obtain a comprehensive measure of the overall wealth effect, we use the following strategy. We calculate the above mentioned permanent transfer that would make the agents indifferent between the transfer and a permanent 1% shock to technology at home. This transfer, expressed in percentage of steady-state consumption, is reported in table 3.3. The left column shows that one would have to pay the domestic agent 0.48% of her steady-state consumption in order to make her as well off as after a 1% positive technology shock in her country. Similarly, the foreign agent would have to receive 0.47% of her steady-state consumption to be indifferent between the transfer and the shock in the home country. In contrast, under calibration B one would have to take away some of the foreign agents' resources, indicating that she is harmed by the domestic technology shock.

We also conduct a counterfactual experiment disentangling the effects of the asset market structure and the parameter values on the wealth effects within calibration B. In table 3.3, we report the wealth effects of a technology shock in a counterfactual economy with the parameter values of calibration B, but complete markets (third column). Relative to calibration B, moving to complete markets would require positive transfers to the both agents to make them indifferent, showing in this scenario foreigners are able to reap some of the benefits of the technology shock.

Table 3.3: DYNAMIC WEALTH EFFECTS

	Calibration A	Calibration B	Counterfactual
Home	0.48%	1.06%	0.69%
Foreign	0.47%	-0.11%	0.26%

Notes: Transfers as percentage of steady-state consumption that would make the agents indifferent between the transfer and a permanent 1% shock to technology at home. Counterfactual: Parameter values of Economy B, but complete markets.

3.5 Conclusion

In this chapter we have analyzed the international transmission of technology shocks by confronting the transmission mechanism of a standard international business cycle model under complete and incomplete financial markets with evidence from U.S. time series for 1980-2005.

We have estimated a VAR model and identified technology shocks assuming that these are the only shocks affecting labor productivity in the long run. In a next step, we have computed counterfactual time series assuming that technology shocks had been the sole source of business cycle fluctuations. We have calculated the cross-correlation function for the trade balance and the terms of trade on the basis of these counterfactual time series (conditional S-curve). Relative to its unconditional counterpart, the conditional S-curve shifts to the left, i.e. while the unconditional contemporaneous correlation is negative, the conditional contemporaneous correlation is positive. A second result from our VAR analysis is that a positive technology shock appreciates the terms of trade and induces a lasting, hump-shaped decline in the trade balance.

We have then calibrated a prototypical international business cycle model to match the S-curve conditional on technology shocks, both under complete and incomplete financial markets. Assuming complete financial markets, the model matches the S-curve for parameter values close to those used in the baseline calibration of BKK. The elasticity of substitution between domestic and foreign goods is about 2.2 and investment adjustment costs are absent (calibration A). Assuming incomplete financial markets, the elasticity of substitution is substantially lower and there is evidence for mild investment adjustment costs. Technology differentials appear to be much more persistent (calibration B).

To assess the ability of the model under both calibrations to account for the transmission of technology shocks apparent from the data, we have computed the impulse responses functions to a one percent increase in technology. It turns out that the transmission process is quite distinct. Under calibration A, the model predicts a depreciation of the terms of trade and a sharp decline the trade balance (standard transmission mechanism). Under calibration B, in contrast, the model predicts the terms of trade to appreciate and a lasting, hump-shaped decline in the trade balance. Hence, the model can can account for the time series evidence only under calibration B.

In our view, the main result of our analysis may be summarized as follows: while both theoretical economies deliver the S-curve conditional on technology shocks, the underlying transmission process is fundamentally different. In fact, as far as the terms of trade and the trade balance are concerned, the transmission mechanism implied by calibration B turns the responses implied by calibration A upside down. Its predictions are qualitatively in line with time series evidence for the U.S.

We have also stressed that this result is not evidence against the assumption of complete markets *per se*. More generally, it is evidence against the standard transmission mechanism of technology shocks. The standard transmission mechanism may - depending on the parameterization - be obtained under complete and incomplete financial markets. In fact, the transmission mechanism of technology shocks implied by the prototypical business cycle model is the result of assumptions on the asset market structure *and* parameter values.

Analyzing the wealth effects triggered by a technology shock under both calibrations highlights that much is at stake regarding the international transmission mechanism. If the terms of trade appreciate in response to a positive technology shock, terms of trade movements fail to provide implicit insurance against country-specific risks; instead they amplify the relative wealth effect of technology shocks.

Against this background, we conclude that further research into the international transmission of technology shocks is necessary. Specifically, the role of relative prices in the transmission of technology shocks is of particular interest. It therefore seems worthwhile to allow for richer dynamics in that respect, for instance, by considering a non-tradable sector or consumer durables, as in earlier work by Stockman and Tesar (1995) and Burda and Gerlach (1992).

Appendix

3.A Data

The data used to calculate the S-curve and estimate the VAR model are obtained from the Bureau of Economic Analysis (National Income and Product Accounts, NIPA), the Bureau of Labor Statistics (BLS) and the OECD. Specifically, we compute six time series, displayed in figure 3.A:

- Labor productivity growth: first difference of log output per hour in the non-farm business sector (BLS: PRS85006093)
- Average hours: log of hours in non-farm business sector (BLS: PRS85006033) divided by population (NIPA: B230RC0)
- Terms of trade: log of relative price of imports to exports, calculated on the basis of the price index of non-commodity imports of goods and services (OECD, Economic Outlook: USAP-MGSX) and exports of non-commodity goods and services (OECD, Economic Outlook: US-APXGSX)
- Trade balance: nominal net exports (NIPA: A019RC1) divided by nominal GDP (NIPA: A191RC1)
- Investment to output ratio: gross private domestic investment (NIPA: A006RC1) divided by nominal GDP (NIPA: A191RC1)
- Real exchange rate: log of inverted real effective exchange rate as provided by OECD (Main Economic Indicators)

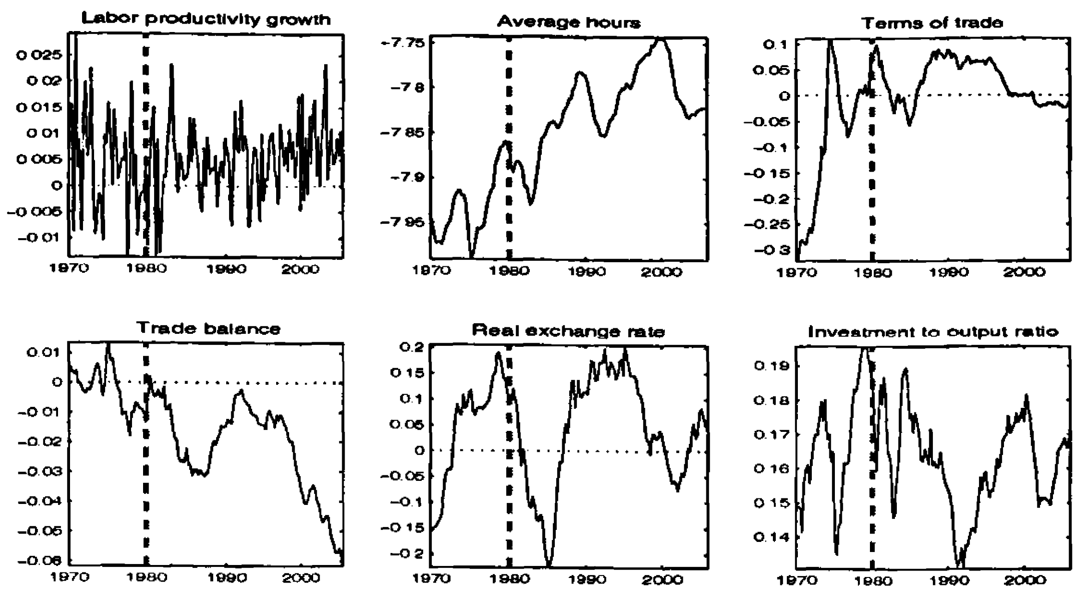

Figure 3.6: U.S. TIME SERIES 1970-2005, VERTICAL DASHED LINES INDICATE START OF SAMPLE (DEPENDENT VARIABLE)

3.B The VAR model

3.B.1 Identification

Our identification strategy is based on Christiano et al. (2003) or Altig et al. (2005). As described in the main text, our VAR model contains the following variables

$$Y_t = \begin{pmatrix} \Delta \ln(\text{GDP}_t/\text{Hours}_t) \\ \ln \text{Hours}_t \\ \ln(\text{Terms of Trade}_t) \\ \text{Net Exports}_t/\text{GDP}_t \end{pmatrix} = \begin{bmatrix} \Delta a_t \\ h_t \\ p_t \\ nx_t \end{bmatrix}. \tag{3.B 1}$$

The structural VAR model of the economy is given by

$$A(L)Y_t = \varepsilon_t, \tag{3.B 2}$$

where a constant is omitted to simplify the exposition and $A(L)$ denotes a p^{th}-ordered polynomial in the lag operator L. Specifically, we consider four lags, i.e.

$$A(L) = A_0 + A_1 L + A_2 L^2 + A_3 L^3 + A_4 L^4,$$

such that A_0 allows for contemporaneous interaction of the variables contained in Y_t. The fundamental economic shocks are contained in the 4×1 vector ε_t. We assume that these fundamental shocks are mutually uncorrelated such that

$$E\left(\varepsilon_t \varepsilon_t'\right) = D,$$

is a diagonal matrix and the diagonal elements of A_0 are normalized to one.

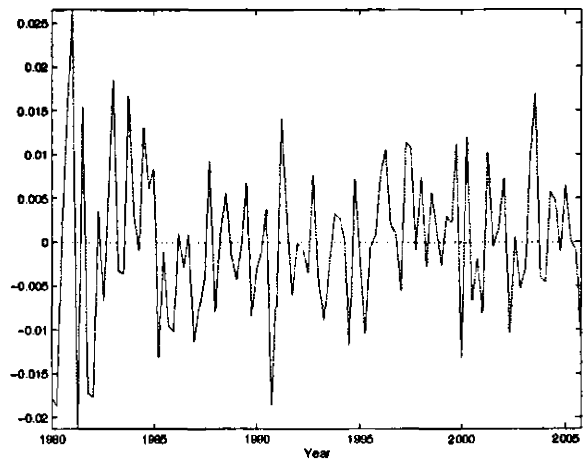

Figure 3.7: TECHNOLOGY SHOCKS IDENTIFIED IN U.S. DATA USING BASELINE VAR MODEL

We want to estimate the coefficients of the structural VAR model (3.B 2). To ensure identification further assumptions have to be made. To simplify the discussion define $Z_t' = \begin{bmatrix} h_t & p_t & nx_t \end{bmatrix}$. Also define an element in $A_i = \alpha_{i,kl}$, where k denotes the row and l the column of A_i. Note that evaluating (3.B 2) in the long-run with $p = 4$ gives

$$\underbrace{\begin{pmatrix} \underset{1\times 1}{\sum_{i=0}^{4} \alpha_{i,11}} & \underset{1\times 3}{\sum_{i=0}^{4} \alpha_{i,12}} \\ \underset{3\times 1}{\sum_{i=0}^{4} \alpha_{i,21}} & \underset{3\times 3}{\sum_{i=0}^{4} \alpha_{i,22}} \end{pmatrix}}_{\equiv A(1)} \begin{bmatrix} \underset{1\times 1}{\Delta a_t} \\ \underset{3\times 1}{Z_t} \end{bmatrix} = \begin{bmatrix} \underset{1\times 1}{\varepsilon_t^a} \\ \underset{3\times 1}{\varepsilon_t^Z} \end{bmatrix}.$$

Technology shocks are identified through the assumption that only technology shocks have a long run effect on labor productivity. This imposes the following restriction on the long-run multiplier $A(1)$:

$$\sum_{i=0}^{4} \alpha_{i,12} = 0. \tag{3.B 3}$$

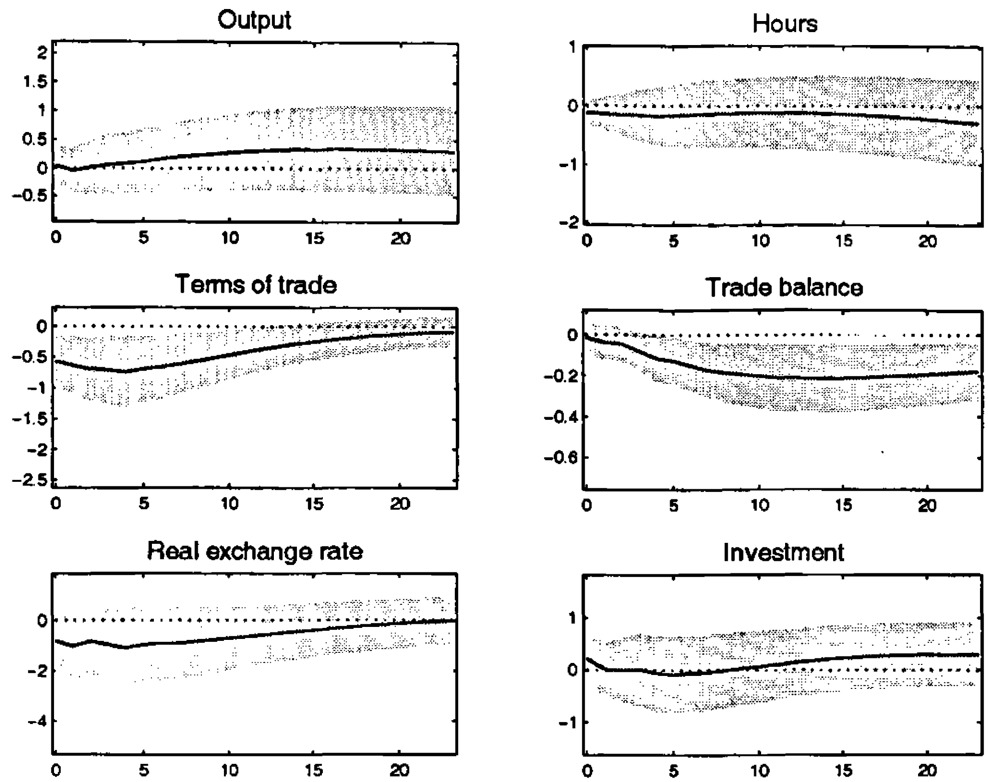

Figure 3.8: TRANSMISSION OF ONE PERCENT INCREASE IN TECHNOLOGY, SEE FIGURE 3.2; HERE: VAR ESTIMATED WITH HOURS IN DIFFERENCES INSTEAD OF LEVELS

To see this, assume to the contrary that this sum was not zero. Then, given that other shocks induce Z_t to be different from zero in the long run, also labor productivity may be affected by these shocks in the long run, which is ruled out by assumption. Christiano et al. (2003) provide a more detailed discussion. In practice we impose this restriction on the first equation of (3.B 2), given by

$$\Delta a_t = -\sum_{i=1}^{4} \alpha_{i,11} L^i \Delta a_t - \sum_{i=0}^{4} \alpha_{i,12} L^i Z_t + \varepsilon_t^a, \qquad (3.B\ 4)$$

which, after imposing (3.B 3), reads as[22]

$$\Delta a_t = -\sum_{i=1}^{4} \alpha_{i,11} L^i \Delta a_t - \sum_{i=0}^{3} \alpha'_{i,12} L^i \Delta Z_t + \varepsilon_t^a.$$

[22]Here we use the fact that $\alpha(L) = \alpha(1) + \alpha'(1-L)$ together with $\alpha(1) = 0$ implies

$$\alpha_{i,1k} = \alpha'_{i,1k} - \alpha'_{i-1,1k}.$$

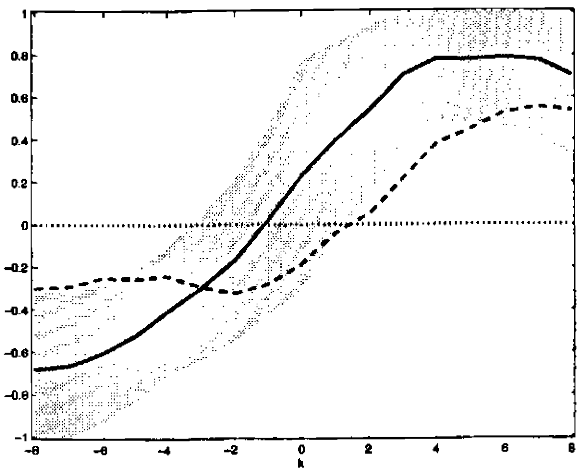

Figure 3.9: CROSS-CORRELATION FUNCTION FOR THE TRADE BALANCE (T+K) AND THE TERMS OF TRADE (T), SEE FIGURE 3.1; HERE: VAR ESTIMATED WITH HOURS IN DIFFERENCES INSTEAD OF LEVELS

Note, however, that since $\alpha_{0,21} \neq 0$, this equation cannot be estimated by OLS. Instead, as originally proposed by Shapiro and Watson (1988), we use $Y_{t-1}, \ldots Y_{t-4}$ as instruments in a two-stage least squares regression.

Finally, the structural shocks related to Z_t cannot be identified, as the mapping from the reduced form to the structural form is not unique, see the discussion in the technical appendix to Altig et al. (2005). In order to estimate the structural model - leaving ϵ_t^Z unidentified (we do not give a structural interpretation to these estimated shocks) - we assume that $\alpha_{0,22}$ is lower triangular. Given these restrictions we are in a position to estimate the structural VAR model (3.B 2) and identify technology shocks. Figure 3.7 displays the identified technology shocks. In the main text we report several statistics computed on the basis of the estimated VAR model and these shocks.

3.B.2 Sensitivity of results with respect to hours specification

To explore the robustness of the VAR results, we also consider the specification where hours enter the VAR in first differences. Figure 3.8 displays the impulse response functions, which are different for output, investment and, in particular, for hours relative to the baseline specification. This has been the topic of a considerable debate, see, for instance, Galí (1999) and Christiano et al. (2003). However, for the present analysis it is important to note that the responses of the trade balance, the terms of trade, and the real exchange rate are robust with respect to the trend specification of hours in the VAR model. This is also reflected in the counterfactual S-curve displayed in figure 3.9.

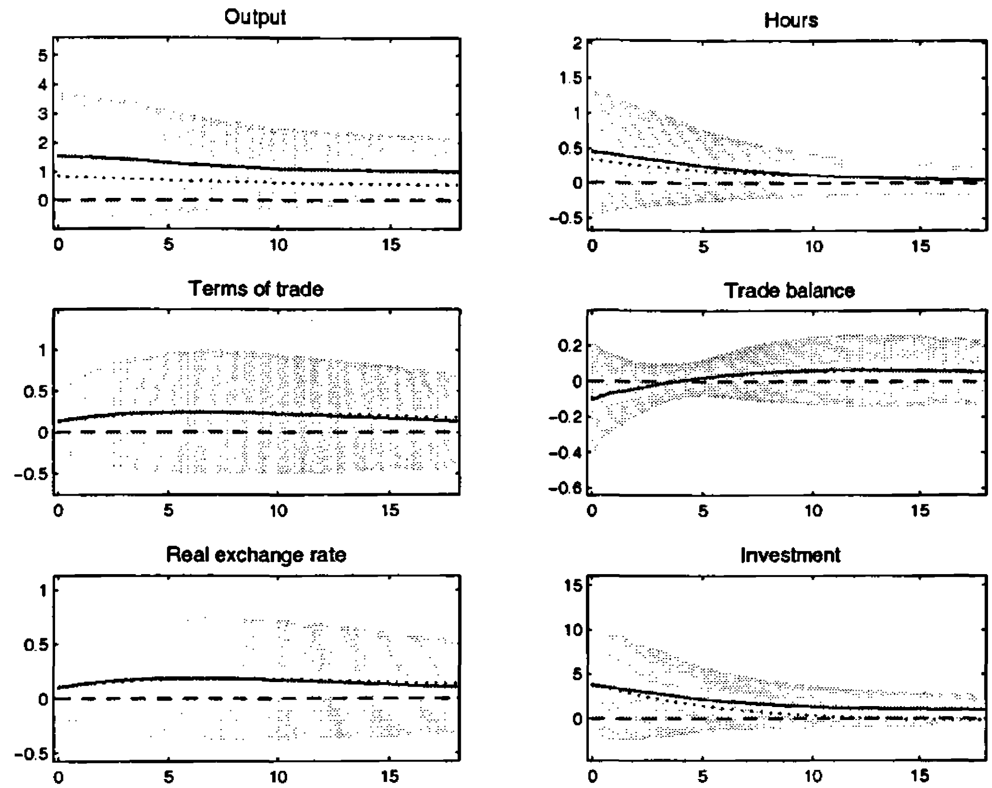

Figure 3.10: TRUE (DOTTED LINE) VS. ESTIMATED (STRAIGHT LINE) RESPONSES TO ONE PERCENT INCREASE IN TECHNOLOGY WITH BOOTSTRAPPED 90 PERCENT CONFIDENCE INTERVALS (SHADED AREA)

3.B.3 VAR Assessment

The use of VAR models to identify technology shocks on the basis of long-run restrictions has been criticized by, among others, Cooley and Dwyer (1998) and Chari et al. (2005). We therefore perform a Monte Carlo experiment similar to Christiano, Eichenbaum and Vigfusson (2006). Note, however, that the scope of our analysis is limited to a specific case: we assess whether the VAR model used in section 3.2 is able to uncover the true impulse responses and the true cross-correlation function for the trade balance and the terms of trade if our calibrated business cycle model is used as the data generating process. Since we estimate the VAR on four time series, while there are only two fundamental shocks in the model, we add measurement error to labor productivity growth and the trade balance to avoid stochastic singularity, as, for instance, in Ireland (2004). We set the standard deviation of the measurement errors to 3%. The standard deviations of the innovations to technology are assumed to be uncorrelated and are taken from the estimates reported in Heathcote and Perri (2002). Another issue is related to the existence of a VAR representation of the DSGE model. Fernández-Villaverde, Rubio-Ramírez, Sargent and Watson (2006) show that DSGE models only have (infinite-order) VAR

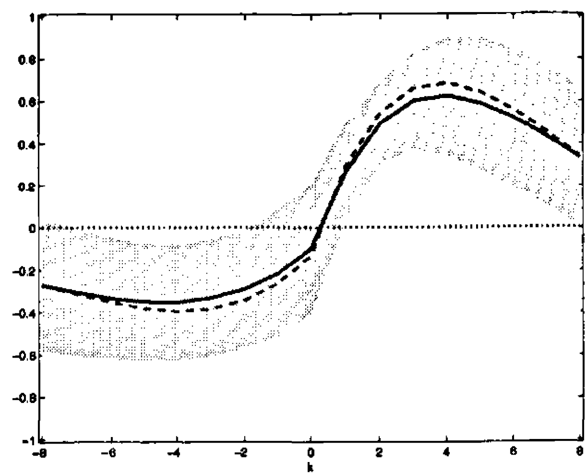

Figure 3.11: TRUE (DASHED LINE) VS. ESTIMATED (SOLID LINE) CROSS-CORRELATION FUNCTION WITH BOOTSTRAPPED 90 PERCENT CONFIDENCE INTERVALS (SHADED AREA)

representations if certain conditions are satisfied. We find that these conditions hold for our calibration and the VAR model.[23] We then generate data using the calibrated DSGE model (calibration A). Specifically, we simulate the model for 104 periods (corresponding to the number of observations used to estimate the VAR model), estimate the VAR model, and repeat this exercise a 1000 times. Figure 3.10 displays the results. The dotted lines display the true impulse response functions of the model, the solid lines display the mean of the estimated response functions and the shaded areas indicate 90 percent confidence intervals. In our view, the VAR performs quite well. Except for output it is difficult to detect any bias.

Figure 3.11 displays the cross-correlation function for the trade balance and the terms of trade calculated on the basis of the simulated data using the same procedure as in section 3.2. The shaded areas indicate 90 percent confidence intervals. The dashed line is the S-curve stemming from the simulation of the model with technology shocks only. Again, the VAR performs quite well in detecting the true S-curve.[24]

[23]The theoretical model can be written using the following representation

$$\begin{aligned} x_{t+1} &= Ax_t + Bw_{t+1} \\ y_{t+1} &= Cx_t + Dw_{t+1}, \end{aligned}$$

where x_t is a $n \times 1$ vector of state variables, y_t is a $k \times 1$ vector of the variables which are observed in the empirical VAR model, and w_t is a $m \times 1$ vector of shocks to the states and the observables. The condition for invertibility then reads as follows: The eigenvalues of $A - BD^{-1}C$ have to be strictly less than one in modulus. We find that the highest eigenvalue for economies A and B is 0.95 and 0.978, respectively, such that both economies have a (infinite-order) VAR representation.

[24]The VAR performs similarly well calibration B is used in the simulation of the model. In particular, the shape of the cross-correlation function and the signs of the impulse responses are correctly estimated. However, due to highly persistent technology differentials, the estimated impulse responses functions are somewhat biased.

3.C Empirical and theoretical moments

3.C.1 Identification of DSGE model parameters

In a recent contribution, Canova and Sala (2005) take up identification issues that typically arise in the estimation and calibration of richly specified DSGE models. In principle, different combinations of values for the structural model parameters may induce identical values for the criterion function. To assess whether our approach to calibration is prone to identification problems, we compute the value for our criterion function (3.16) for the relevant range of parameter values. Figure 3.12 displays the results for each combination of χ, ρ and σ keeping, in turn, the third parameter constant at the value obtained in the global minimum and for both asset market structures. We find that the objective function is well behaved and reasonably curved. In our view, this experiment lends support to our calibration strategy.

3.C.2 Unconditional and conditional volatilities

In the main text we focus on the cross-correlation function for the trade balance and the terms of trade considering, in turn, the raw time series, the counterfactual time series and the different calibrations of the model. For completeness, we also compute the relative volatilities of the variables included in our VAR model. Table 3.4 shows the results.

Table 3.4: STANDARD DEVIATIONS RELATIVE TO GDP

	U.S. Data		Theoretical	
Statistics	Unconditional	Conditional	Calibration A	Calibration B
Investment	4.72	2.38	3.15	3.01
Trade balance	0.29	0.20	0.15	0.02
Terms of Trade	1.16	1.03	0.27	3.91
Real Exchange Rate	2.83	2.81	0.21	2.97
Hours	1.20	0.33	0.42	0.38

Notes: HP-filtered series have been used to calculate the standard deviations; Conditional refers to calculations based on counterfactual time series obtained from the VAR model assuming that only technology shocks had occurred.

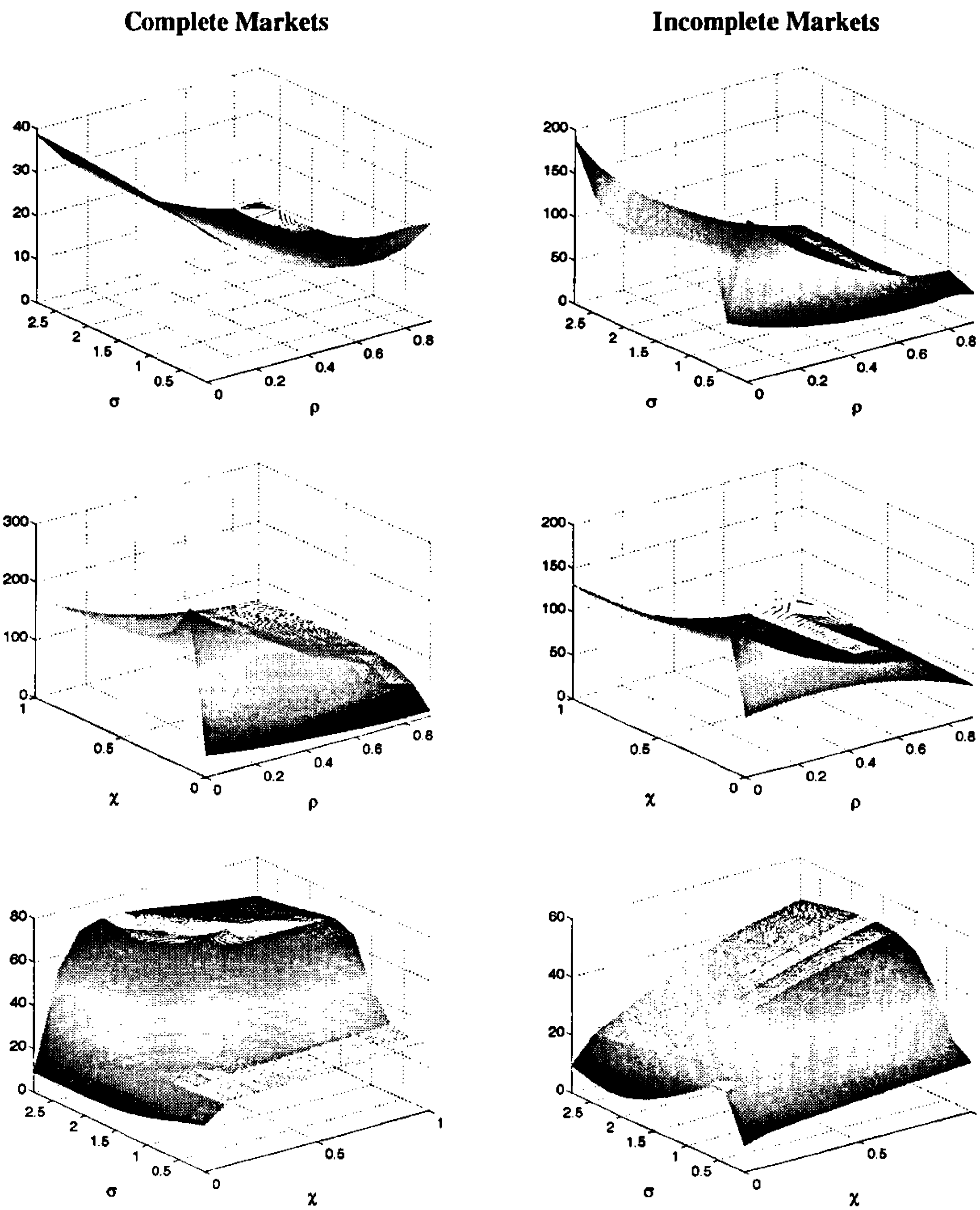

Figure 3.12: CRITERION FUNCTION (VERTICAL AXIS) EVALUATED FOR DIFFERENT VALUES OF STRUCTURAL MODEL PARAMETERS (HORIZONTAL AXES)

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