EUI Working Papers
ECO 2007/20

Twin Deficits, Openness and the Business Cycle

Giancarlo Corsetti and Gernot J. Müller
Twin Deficits, Openness and the Business Cycle

Giancarlo Corsetti
and
Gernot J. Müller

Twin Deficits, Openness and the Business Cycle*

Giancarlo Corsetti  
European University Institute, University of Rome III and CEPR

Gernot J. Müller  
Goethe University Frankfurt

September 13, 2007

Abstract

In this paper, we study the co-movement of the government budget balance and the trade balance at business cycle frequencies. In a sample of 10 OECD countries we find that the correlation of the two time series is negative, but less so in more open economies. Moreover, for the US the cross-correlation function is S-shaped. We analyze these regularities taking the perspective of international business cycle theory. First, we show that a standard model delivers predictions broadly in line with the evidence. Second, we show that conditional on spending shocks the model predicts a perfect correlation of the budget balance and the trade balance. Yet, the effect of spending shocks on the trade balance is contained if an economy is not very open to trade.

Keywords: Fiscal Policy, Twin deficits, Openness, Business Cycle

JEL-Classification: F41, F42, E32

---

*We are grateful to Roel Beetsma, Massimo Giuliodori, Franc Klaassen, Tommaso Monacelli and Roberto Perotti as well as seminar participants at the EEA Meeting in Budapest, Goethe University Frankfurt and the Reserve Bank of Australia for helpful comments. The usual disclaimer applies. Please address correspondence to giancarlo.corsetti@eui.eu or muellerg@wiwi.uni-frankfurt.de
1 Introduction

The notion of ‘twin deficits’ emerged in the mid-1980s following the observation that the US trade balance moved into deficit at a time of increasing government budget deficits, suggesting that fiscal expansions caused the positive co-movement.\(^1\) On the other hand, in the debate on the need for fiscal consolidation to correct external imbalances, it has been observed that the correlation between the two time series is actually very small, or even negative in the data.\(^2\)

Any assessment of the co-movement of the budget and the trade balance, however, should take into account that both variables adjust endogenously not only to fiscal shocks but to the entire state of the economy. Therefore, we study the transmission of both fiscal and productivity shocks onto the government budget and net exports, taking the perspective of international business cycle theory.\(^3\)

We proceed in two steps. First, we document three regularities concerning the co-movement of the trade and the budget balance. Using quarterly time series for 10 OECD countries during the period 1973-2005, we show that: i) the contemporaneous correlation between the budget and the trade balance (both scaled by GDP) is typically negative at business cycle frequencies: budget surpluses are associated with trade deficits; ii) the correlation is less negative, the more open countries are to trade; iii) the cross-correlation for the budget balance and the trade balance in the US resembles a stretched ‘S’.

Second, we ask whether a standard international real business (IRBC) model can account for the above regularities. For the sake of transparency, we draw on the classical contribution by Backus, Kehoe, and Kydland [1994], henceforth BKK, assuming shocks to technology as well as government spending. To analyze the behavior of the government budget balance, we assume that government purchases are financed either through issuing debt or by taxing the income of domestic resi-

---

\(^1\)Recall that national accounting implies: current account deficit = budget deficit + private investment - private saving. Hence, unless fiscal shocks cause large swings in private net savings, policies that deteriorate the budget are bound to worsen the trade deficit.

\(^2\)See e.g. Backus, Henriksen, Lambert, and Telmer [2006] who dismiss the relevance of the twin deficits hypothesis on the ground of this observation.

\(^3\)By explicitly taking into account non-fiscal shocks for the co-movement of the budget and the trade balance, this paper complements a line of research which focuses on the transmission of fiscal shocks; see Erceg, Guerrieri, and Gust [2005] for an analysis using a general equilibrium model and Kim and Roubini [2003], Corsetti and Müller [2006], Monacelli and Perotti [2006] and Beetsma, Giuliodori, and Klaasen [2007] using VAR models.
We find that the model is able to replicate the empirical regularities, notably the negative correlation of the budget and the trade balance. However, simulating the model for each shock in isolation shows that the correlation is perfect conditional on domestic government spending shocks: consistent with the notion of twin deficits, fiscal expansions cause a joint deterioration of the budget and the trade balance. Yet, an almost perfect correlation does not translate into an economically significant effect: we find only a very small effect of fiscal shocks on the trade balance if an economy is relatively closed.

2 Properties of the data

In this section we characterize the business cycle properties of the primary budget balance and the trade balance. We consider quarterly time series for 10 OCED countries covering the post-Bretton Woods period 1973-2005. Table 1 displays several statistics of the HP-filtered series of net exports, \( nx \), the primary government budget balance, \( bb \), and real output, \( y \).

The first two panels of Table 1 show that standard deviations and autocorrelations display considerable variation across the 10 countries in our sample. However, the contemporaneous correlation of the trade balance and the budget balance, shown in the third panel of the table, is negative everywhere except in the Netherlands and Canada, where it is nonetheless close to zero. The correlation between the primary budget balance and output is positive in all countries, while the correlation between the trade balance and output is generally negative, as stressed by the early IRBC literature.

Next, we ask whether the correlation of the budget and the trade balance vary with

\[ ^4 \text{Note that we adopt a parsimonious model setup in order to convey our main argument in a transparent and efficient way; namely that non-fiscal shocks are important for the co-movement of the trade and the budget balance. Clearly, quantitative aspects of our analysis might be refined using richer specifications.} \]

\[ ^5 \text{We use a smoothing parameter of 1600. All data are obtained from the OECD economic outlook database (Economic Outlook 81, Annual and Quarterly data, Vol. 2007 release 1). The primary budget balance in percent of GDP is available at quarterly frequency for the following OECD 10 countries: Australia, Canada, Finland, Ireland, Japan, Republic of Korea, Netherlands, Sweden, United Kingdom, United States. The trade balance is computed as the difference of exports and imports scaled by GDP, at current prices. Data for Korea and the Netherlands are only available from 1975 and 1980, respectively. In the working paper version of this paper we also compute statistics using annual time series for 16 countries, see Corsetti and Müller [2007].} \]
Table 1: Properties of net exports, output and the budget balance

<table>
<thead>
<tr>
<th></th>
<th>nx</th>
<th>y</th>
<th>bb</th>
<th>nx</th>
<th>y</th>
<th>bb</th>
<th>(nx,bb)</th>
<th>(nx,y)</th>
<th>(bb,y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>1.06</td>
<td>1.38</td>
<td>1.04</td>
<td>0.76</td>
<td>0.74</td>
<td>0.87</td>
<td>-0.23</td>
<td>-0.21</td>
<td>0.62</td>
</tr>
<tr>
<td>CAN</td>
<td>0.94</td>
<td>1.46</td>
<td>1.29</td>
<td>0.72</td>
<td>0.89</td>
<td>0.78</td>
<td>0.02</td>
<td>0.05</td>
<td>0.63</td>
</tr>
<tr>
<td>FIN</td>
<td>1.67</td>
<td>2.14</td>
<td>1.67</td>
<td>0.44</td>
<td>0.87</td>
<td>0.94</td>
<td>-0.05</td>
<td>-0.30</td>
<td>0.67</td>
</tr>
<tr>
<td>GBR</td>
<td>0.99</td>
<td>1.49</td>
<td>1.32</td>
<td>0.66</td>
<td>0.85</td>
<td>0.71</td>
<td>-0.15</td>
<td>-0.34</td>
<td>0.31</td>
</tr>
<tr>
<td>IRL</td>
<td>1.95</td>
<td>1.66</td>
<td>1.27</td>
<td>0.79</td>
<td>0.77</td>
<td>0.92</td>
<td>-0.03</td>
<td>-0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>JPN</td>
<td>0.75</td>
<td>1.38</td>
<td>0.71</td>
<td>0.85</td>
<td>0.81</td>
<td>0.92</td>
<td>-0.30</td>
<td>-0.42</td>
<td>0.45</td>
</tr>
<tr>
<td>KOR</td>
<td>2.90</td>
<td>2.55</td>
<td>0.97</td>
<td>0.73</td>
<td>0.84</td>
<td>0.90</td>
<td>-0.28</td>
<td>-0.43</td>
<td>0.52</td>
</tr>
<tr>
<td>NLD</td>
<td>0.90</td>
<td>1.15</td>
<td>0.99</td>
<td>0.45</td>
<td>0.76</td>
<td>0.89</td>
<td>0.02</td>
<td>-0.03</td>
<td>0.35</td>
</tr>
<tr>
<td>SWE</td>
<td>1.10</td>
<td>1.38</td>
<td>2.08</td>
<td>0.45</td>
<td>0.69</td>
<td>0.92</td>
<td>-0.00</td>
<td>-0.09</td>
<td>0.57</td>
</tr>
<tr>
<td>USA</td>
<td>0.45</td>
<td>1.59</td>
<td>1.12</td>
<td>0.78</td>
<td>0.88</td>
<td>0.81</td>
<td>-0.34</td>
<td>-0.45</td>
<td>0.74</td>
</tr>
</tbody>
</table>


Figure 1: Correlation of trade balance, nx, and budget balance, bb; left panel: contemporaneous correlation vs. average import share for 10 OECD countries and model (dashed line); right panel: ccf for US data (solid line and shaded area for 95 percent confidence bounds) and model (dashed line, baseline calibration), vertical axis: \( \rho(b_t, nx_{t+k}) \), horizontal axis: \( k \).

the degree of openness of a country, as measured by the import share in GDP (openness). The left panel of Figure 1 plots these two variables against each other for the countries in our sample. As our second finding, we note that, by and large, the correlation is less negative, the more open an economy. Finally, we focus on the dynamic relationship between the budget balance and the trade balance in the US, plotting the cross-correlation function (ccf) of \( bb_t \) and \( nx_{t+k} \) for \( k = -8 \ldots 8 \) in the right panel of Figure 1. As our third finding, we note that for the US the ccf resembles a stretched ‘S’.
3 The model

Can the empirical regularities established above be accounted for by a standard international business cycle model? Are these facts inconsistent with the twin deficit hypothesis? In the rest of this paper we address these questions by adopting a parsimonious specification of the BKK model.\textsuperscript{6} The main features of the model are as follows. Letting $c_{it}$ denote consumption and $n_{it}$ the amount of labor supplied, the preferences of the representative household in country $i$ ($i = 1, 2$) are given by the following expression

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} [c_{it}^{\mu} (1 - n_{it})^{1-\mu}]^{1-\gamma}. \quad (1)$$

Households supply labor and rent capital to a representative firm which produces a country-specific intermediate good, denoted by $a$ and $b$, in country 1 and 2, respectively. Labor and capital are internationally immobile; households in each country own the capital stock $k_{it}$ of that country. Investment, $x_{it}$, increases the existing capital stock in the following way,

$$k_{it+1} = (1 - \delta)k_{it} + x_{it}, \quad (2)$$

where $\delta$ is the depreciation rate. Households’ labor and capital income are both taxed at the same rate, $\tau_{it}$. Households maximize (1) subject to (2), a no-Ponzi-game condition and a budget constraint, where we allow for international trade in a complete set of state-contingent securities.

Intermediate goods are produced using the following production function

$$y_{it} = e^{z_{it} k_{it}^{\theta} n_{it}^{1-\theta}}, \quad (3)$$

where $z_{it}$ is an exogenous technology shock. Defining $z_{t+1} = [z_{1t} \ z_{2t}]'$, we assume $z_{t+1} = A z_t + \varepsilon_{t+1}$, where $\varepsilon_{t+1}$ is a bivariate vector of innovations to technology. The law of one price holds for intermediate goods $a$ and $b$. Final goods,

\textsuperscript{6}The model differs from BKK in two respects: First, we assume that government spending falls entirely on domestic goods, because of the evidence discussed in Corsetti and Müller [2006] suggesting that the import content in government spending is generally less than half the import content in private spending. As a first approximation it is thus reasonable to assume zero import content in government spending. Second, we assume that governments have no access to lump-sum taxes but instead levy a flat tax rate on income, which adjusts to the level of government debt.
are assembled on the basis of the following technology

\[
f_{it} = \begin{cases} 
\left[ \frac{\omega^i}{\sigma} a_{it}^{(\sigma-1)/\sigma} + (1 - \omega)^i b_{it}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, & \text{for } i = 1 \\
\left[ (1 - \omega)^i a_{it}^{(\sigma-1)/\sigma} + \omega^i b_{it}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, & \text{for } i = 2 
\end{cases}
\]

where \(\sigma\) is the elasticity of substitution between goods \(a\) and \(b\) and \(\omega\) measures the home bias in final goods. Firms are operating under perfect competition both at the intermediate and final good level. Domestic households thus earn the entire domestic intermediate output as income.

Government purchases, \(g_{it}\), are purely dissipate and financed by taxing income or by issuing risk-free debt, \(d_{it}\). Letting \(R_t\) denote the risk-free interest rate, the period budget constraint of the government is given by

\[
d_{it+1} R_t^{-1} - d_{it} = g_{it} - \tau_{it} y_{it}.
\]

Government spending is determined exogenously as follows

\[
g_{it} = (1 - \rho_g) g_t + \rho_g g_{it-1} + \varepsilon^g_{it},
\]

where \(g_t\) denotes government spending in steady state and \(\rho_g\) captures the persistence of deviations from steady state. \(\varepsilon^g_{it}\) is an exogenous innovation to government spending. The tax rate adjusts to the level of debt scaled by steady state output, \(y_i\):

\[
\tau_{it} = \tau_i + \phi \frac{d_{it}}{y_i},
\]

where \(\phi\) measures the debt-elasticity of the tax rate. In our analysis below, taking the perspective of country 1, we focus on the co-movement of the primary budget balance scaled by GDP, \((\tau_{it} y_{1t} - g_{1t})/y_{1t}\) and the trade balance \((a_{2t} - p_t b_{1t})/y_{1t}\), where \(p_t\) denotes the terms of trade measured as the price of good \(b\) relative to the price of good \(a\).

4 Properties of theoretical economies

We study the business cycle properties of the model using log-linear approximation of the equilibrium conditions near a symmetric zero-debt steady state.\(^7\) To
calibrate the model we follow BKK, as regards both the parameters governing preferences and technology, and the forcing processes in technology and government spending. Note that as government spending is assumed to fall entirely on domestically produced goods, assuming an import share of 15 percent in final goods \((\omega = 0.85)\) implies an import share of 12 percent of GDP, the average value in US time series.\(^8\) To pin down \(\phi\), we aim at matching the degree of autocorrelation of the budget balance in US data, which is equal to 0.81, subject to the constraint that the path of government debt is non explosive. We find that the constraint is binding at \(\phi = 0.0143\), implying that the tax rate adjusts very slowly to government debt. As a result, fluctuations in government spending and output induce persistent movements in the government budget balance.

In a first step, we assess the ability of the calibrated model to account for the key features of the data regarding twin deficits, openness and the business cycle. In Table 2 we compare second moments of US time series (first line) with those generated by the model under our baseline calibration (second line). The contemporaneous correlation of the ‘twins’ is negative. The budget and trade balance show a stronger correlation with output than in the data, but of the right sign. The theoretical standard deviation of the trade balance is somewhat below those characterizing in US time series; the model does slightly better in matching the volatility of output, but not as well as regards the budget balance. By the same token, the three variables show less persistence in the model than in the data.

In Figure 1 we assess the performance of the model in two additional dimensions. In the left panel the dashed line plots the contemporaneous correlation of the trade and the budget balance against openness. The model is able to replicate a key feature characterizing the cross-section of the data, namely the positive association between openness and the correlation of the budget and the trade balance. In the right panel, the dashed line displays the ccf implied by the baseline calibration of the model, which is close to the empirical cross-correlation function for the US. In light of our numerical results, we find that, overall, the model is able to provide a satisfactory account of the empirical regularities characterizing the co-movement of the budget and trade balance.

\(^8\)See Corsetti and Müller [2007] for a list of the parameter values used in the baseline specification and sensitivity analyses showing the robustness of our results with respect to alternative specifications.
We thus turn to counterfactual experiments and simulate the model drawing from the distribution of each shock in isolation. Results are shown in rows 2 to 5 of Table 2, which report the second moments predicted by the model for the main variables of interest, conditional on specific shocks.

Three observations are in order. First, the contemporaneous correlation of the trade and the budget balance conditional on domestic government spending shocks is perfect (third row). This squares well with the notion of twin deficits whereby fiscal shocks induce co-movement of the budget and the trade balance. Second, the correlation is strongly negative conditional on technology shocks (fourth row). Third, technology shocks seem to dominate the unconditional correlation which is close to the correlation conditional on technology shocks. Put differently, government spending shocks and foreign technology shocks have only a limited effect on the unconditional moments of the simulated data.

The strong positive correlation of the trade and the budget balance that the model predicts conditional on government spending shocks, however, does not necessarily imply a strong economic effect of fiscal shocks on the trade balance. To clarify this issue, we display in the columns of Figure 2 the impulse responses to each of the four shocks, both for the baseline economy (solid line) and a model economy which is identical to the baseline case except for a higher import share of 30 percent (line with diamonds).

---

**Table 2: Properties of Key Variables in Theoretical Economies**

<table>
<thead>
<tr>
<th></th>
<th>Standard deviation (percent)</th>
<th>Autocorrelation</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>nx</td>
<td>y</td>
<td>bb</td>
</tr>
<tr>
<td>US data</td>
<td>0.45</td>
<td>1.59</td>
<td>1.12</td>
</tr>
<tr>
<td>Benchmark</td>
<td>0.30</td>
<td>1.33</td>
<td>0.28</td>
</tr>
<tr>
<td>Only $g_1$</td>
<td>0.01</td>
<td>0.04</td>
<td>0.09</td>
</tr>
<tr>
<td>Only $z_1$</td>
<td>0.21</td>
<td>1.27</td>
<td>0.26</td>
</tr>
<tr>
<td>Only $g_1$, $z_1$</td>
<td>0.21</td>
<td>1.27</td>
<td>0.27</td>
</tr>
</tbody>
</table>

First row reports data moments for US, see Table 1; consecutive rows contain theoretical counterparts for different assumptions on forcing process; for theoretical moments standard deviations are in parentheses.

---

9The last row of Table 2 reports the moments conditional on both domestic shocks. In Corsetti and Müller [2007] we also report the conditional ccf illustrating how domestic technology shocks dominate the unconditional correlation.
Figure 2: Shock transmission in theoretical economies. Notes: Columns 1-4 show, in turn, effect of shocks to domestic and foreign government spending and domestic and foreign technology; solid lines display responses of baseline economy (12 percent import share: $\omega = 0.85$), line with diamonds corresponds to an economy with import share of 30 percent ($\omega = 0.625$). Vertical axis: percent of GDP, horizontal axis: quarters.

In the first column we show the responses to an increase of government spending by one percent of GDP: it decreases consumption and investment, and raises output by about 0.5 on impact (baseline economy). The trade balance falls, but its movement is quite contained (about 0.1 percent), while the budget balance moves into a significant deficit (about 0.85 percent). So, while the conditional correlation of the trade and the budget balance is virtually perfect, only a small fraction of the fiscal expansion is reflected in the trade balance. The picture changes considerably in economies which are more open to trade. In this case, the effect of fiscal shocks on the trade balance increases significantly, a result which is analyzed in detail by Corsetti and Müller [2006] and Corsetti, Meier, and Müller [2007]. We observe that the response of output is virtually
unaltered, but the responses of investment and consumption increase relative to the baseline scenario. Hence, the trade balance falls significantly.

Figure 2 also reports the effect of an increase in foreign government spending, displayed in the second column: domestic consumption and investment fall; yet the economy experiences mild trade and budget surpluses. To complete our analysis, columns three and four show the effects of technology shocks in the domestic country and abroad. As in BKK, a domestic technology shock worsens the trade balance, because investment and consumption rise more than output in the short run. Symmetrically, the trade balance improves if the technology shock originates in the foreign country. The budget balance improves persistently in response to a domestic technology shock: as government spending is constant and the tax rate responds slowly to government debt, tax revenues move proportional to domestic output. Domestic technology shocks thus induce a negative correlation of budget and trade balance, but less so, the more open the economy.\(^\text{10}\)

5 Conclusion

In this paper we reconsider the notion of twin deficits in light of empirical evidence from a sample of 10 OECD countries, and quantitative results from a standard international business cycle model.

Our analysis highlights two points which are potentially relevant for the policy debate on twin deficits. First, the negative correlation found in the data is not inconsistent with the twin deficit hypothesis: our results suggest that conditional on fiscal shocks, the budget and the trade balance co-move strongly, although their overall correlation is determined by other shocks driving the business cycle. Second, even if conditional on fiscal shocks the correlation between the two deficits is positive and strong, the quantitative response of the trade balance may still be quite contained, especially in economies with a low import share in GDP.

\(^\text{10}\)The correlation becomes less negative in more open economies, because the terms of trade depreciation following the technology shock alters the intertemporal margin governing investment decisions, see Corsetti and Müller [2006] for a discussion of the underlying mechanism in the context of fiscal shocks. Corsetti and Müller [2007] consider alternative values for \(\phi\) finding some effect on the response of \(n_x\) to fiscal shocks. As a result, the correlation between the trade and the budget balance conditional on spending shocks falls for higher values of \(\phi\), but remains positive.
A Data

In the main text we computed statistics using quarterly time series, thereby limiting the sample to 10 OCED countries. Annual time series data are available for 16 countries, plotted in Figure A1.

![Graphs of annual time series for primary government budget balance and trade balance for 16 OECD countries: USA, Sweden, Japan, Italy, Ireland, Iceland, Greece, UK, Spain, France, Finland, Denmark, Canada, Belgium, Austria, Australia.](image)

Figure A1: Annual time series for the primary government budget balance (solid line) and the trade balance (circles) for 16 OECD countries: 1973-2005

We also compute the statistics reported in Table 1 for all 16 countries after HP-filtering - using a smoothing parameter of 100. The results, displayed in Table A1, show that the correlation between the budget and the trade balance is mostly negative - as with quarterly data. In Figure A2 we plot the correlation of the budget and the trade balance both for quarterly data (left panel) and for annual data (right panel) against openness. In both cases the relationship between openness and the correlation of the budget and the trade balance is positive, significantly so only in the case of quarterly data.
Table A 1: Properties of net exports, output and the budget balance

<table>
<thead>
<tr>
<th>Standard deviation</th>
<th>Autocorrelation</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(percent)</td>
<td>nx</td>
<td>y</td>
</tr>
<tr>
<td>Australia</td>
<td>0.95</td>
<td>1.66</td>
</tr>
<tr>
<td>Austria</td>
<td>0.84</td>
<td>1.32</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.81</td>
<td>1.41</td>
</tr>
<tr>
<td>Canada</td>
<td>1.15</td>
<td>2.15</td>
</tr>
<tr>
<td>Denmark</td>
<td>1.29</td>
<td>1.90</td>
</tr>
<tr>
<td>Spain</td>
<td>1.49</td>
<td>2.12</td>
</tr>
<tr>
<td>Finland</td>
<td>1.81</td>
<td>3.52</td>
</tr>
<tr>
<td>France</td>
<td>0.85</td>
<td>1.37</td>
</tr>
<tr>
<td>UK</td>
<td>1.14</td>
<td>2.07</td>
</tr>
<tr>
<td>Greece</td>
<td>1.08</td>
<td>2.61</td>
</tr>
<tr>
<td>Ireland</td>
<td>2.57</td>
<td>2.95</td>
</tr>
<tr>
<td>Iceland</td>
<td>3.02</td>
<td>3.03</td>
</tr>
<tr>
<td>Italy</td>
<td>1.24</td>
<td>1.57</td>
</tr>
<tr>
<td>Japan</td>
<td>0.81</td>
<td>1.92</td>
</tr>
<tr>
<td>Sweden</td>
<td>1.23</td>
<td>1.99</td>
</tr>
<tr>
<td>USA</td>
<td>0.59</td>
<td>1.99</td>
</tr>
</tbody>
</table>


Figure A 2: Correlation of trade balance, nx, and budget balance, bb, vs. openness; left panel: quarterly data slope coefficient of regression line is 0.60 (s.e. 0.21); right panel: annual data slope coefficient is 0.54 (s.e. 0.40)

B Model solution

To solve the model, we first complete the problem of households and firms, discussed in the main text. As in Heathcote and Perri [2002], let $q_{it}^a$ and $q_{it}^b$ denote the prices of goods $a$ and $b$ in country $i$ in units of the final good produced in country $i$. Further, let $w_{it}$ and $r_{it}$ denote the real wage rate and the real return on capital, both in terms of domestic intermediate goods. Let $Q_{t,t+1}$ denote the stochastic discount
factor used to price the portfolio of securities in period $t$, $A_{t+1}$. Then the risk free interest rate is defined by $R_t = E_t Q_{t,t+1}$ and the representative household’s budget constraint is given by

$$c_{it} + x_{it}$$

$$= \begin{cases} 
q^a_{it}[(1 - \tau_{it})(w_{it}n_{it} + r_{it}k_{it})] + q^a_{it}A_{it} - q^a_{it}E_t Q_{t,t+1}A_{it+1}, & \text{for } i = 1 \\
q^b_{it}[(1 - \tau_{it})(w_{it}n_{it} + r_{it}k_{it})] + q^a_{it}A_{it} - q^a_{it}E_t Q_{t,t+1}A_{it+1}, & \text{for } i = 2
\end{cases}$$

The final good firm’s objective is given by

$$\max_{a_{it},b_{it}} \left(f_{it} - q^a_{it}a_{it} - q^b_{it}b_{it}\right).$$

subject to (4). The intermediate goods objective is given by

$$\max_{k_{it},n_{it}} \left(y_{it} - w_{it}n_{it} - r_{it}k_{it}\right),$$

subject to (3).

Given initial values for household wealth and a specification of the tax rule (7) which is consistent with government solvency, an equilibrium is given by a set of prices for all $t \geq 0$ such that when firms and households take prices as given, households maximize (1) subject to (2), (B1) and a no-Ponzi game condition, firms solve their static problems (B3) and (B2) and all markets clear for any particular realization of the shock processes for technology and for government spending, both given by BKK.

Market clearing for intermediate goods requires that

$$y_{1t} = a_{1t} + a_{2t} + g_{1t},$$

$$y_{2t} = b_{1t} + b_{2t} + g_{2t};$$

while market clearing for final goods requires that

$$f_{it} = c_{it} + x_{it}, \quad i = 1, 2.$$

The asset market clears by Walras’ law.

We solve the model numerically by approximating the first order conditions around a symmetric zero-debt steady state. To derive the first order conditions, let $\lambda_t$ denote the multiplier on the budget constraint. Optimality of the household program
then requires (the index ‘i’ is dropped to simplify the exposition) the following to hold

\[ u_c(c_t, n_t) = \lambda_t \]  

(B 7)

\[ u_n(c_t, n_t) = -\lambda_t q_t^a(1 - \tau_t)w_t \]  

(B 8)

\[ \lambda_t q_t^a Q_t = \beta E_{t+1} \left[ \lambda_{t+1} q_{t+1}^a \right] \]  

(B 9)

\[ \lambda_t = \beta E_{t+1} \left[ \lambda_{t+1} q_{t+1}^a (1 - \tau_{t+1})r_{t+1} + \lambda_{t+1}(1 - \delta) \right] n \]  

(B 10)

Combining the first order condition for state-contingent securities in country 1 and 2 and iterating backwards gives the risk sharing condition (see, for instance, Chari, Kehoe, and McGrattan [2002]):

\[ u_c(c_{1,t}, n_{1,t}) q_{1,t}^a = u_c(c_{2,t}, n_{2,t}) q_{2,t}^a. \]  

(B 11)

An optimal intratemporal allocation of expenditure requires

\[ a_1 : \frac{\partial f_1}{\partial a_1} = q_1^a \implies a_1 = (q_1^a)^{-\sigma} \omega f_1 \]  

(B 12)

\[ b_1 : \frac{\partial f_1}{\partial b_1} = q_1^b \implies b_1 = (q_1^b)^{-\sigma}(1 - \omega) f_1 \]  

(B 13)

\[ b_2 : \frac{\partial f_2}{\partial b_2} = q_2^b \implies b_2 = (q_2^b)^{-\sigma} \omega f_2 \]  

(B 14)

\[ a_2 : \frac{\partial f_2}{\partial a_2} = q_2^a \implies a_2 = (q_2^a)^{-\sigma}(1 - \omega) f_2. \]  

(B 15)

The first order conditions to \( B3 \) define the wage and the rental rate of capital (in terms of intermediate goods)

\[ w_{it} = (1 - \theta) \frac{y_{it}}{n_{it}} \]  

(B 16)

\[ r_{it} = \theta \frac{y_{it}}{k_{it}}. \]  

(B 17)

We consider a symmetric steady state with balanced trade such that \( a_2 = b_1 \) and zero government debt. For simplicity we focus our analysis on country 1 (symmetric expressions hold for country 2). First, we relate the home bias parameter \( \omega \) to openness, i.e. the share of imports in GDP. Divide the FOC for \( a_1 \), equation (B12), by the FOC for \( b_1 \), equation (B13). Noting that because of symmetry the prices for intermediate goods \( a \) and \( b \) are equal in steady state (such that \( q_1^a = q_1^b \)), we thus obtain

\[ \frac{a_1}{b_1} = \frac{\omega}{1 - \omega}. \]  

(B 18)
Letting \( w_d \) denote the share of output net of government spending \( (y' = y - g) \) which is not exported (=not imported) in steady state we have

\[
\begin{align*}
a_1 &= w_d y'_1 \\
b_1 &= (1 - w_d) y'_1.
\end{align*}
\] (B 19)

Substituting into \((B 18)\) gives \( \omega = w_d \). Hence, the home bias parameter \( \omega \) measures the share of net output which is not exported, and \( 1 - \omega \) is a measure for openness, as it measures imports (=exports) as a share of net output in steady state.

Let \( g \) denote the steady state share of government spending in GDP and assume that government spending falls on domestic goods only. Such that

\[
y' = y - g = (1 - g)y
\]

We can then pin down \( \omega \) on the basis of \((B 19)\) using the share of imports in total output (which is observable):

\[
\omega = 1 - \frac{b_1}{y'_1} \iff \omega = 1 - \frac{1}{1 - g} \frac{b_1}{y'_1}.
\] (B 20)

Total final goods \( f \) equal net output in steady state \( y' \), which can be seen by substituting \((B19)\) into the production function for final goods, i.e. the Armington aggregator given by (4). Relative to the specification of the weights in the Armington aggregator in BKK, we thus impose a priori that \( y' = f \). \(^{11}\) This in turn implies

\[
c_1 + x_1 = f_1 = y'_1 = y_1 - g_1 \iff y_1 = c_1 + x_1 + g_1,
\]

i.e. in steady state, consumption, investment and government spending add up to total GDP.

Next, we consider relative prices in steady state. Applying the Euler theorem to the Armington aggregator allows to write

\[
f_1 = \left( \omega^{1/\sigma} a_{1t}^{(\sigma-1)/\sigma} + (1 - \omega)^{1/\sigma} b_{1t}^{(\sigma-1)/\sigma} \right) a_1^{1/\sigma} a_{1t}^{-1/\sigma} a_1
\]

\[
+ \left( \omega^{1/\sigma} a_{1t}^{(\sigma-1)/\sigma} + (1 - \omega)^{1/\sigma} b_{1t}^{(\sigma-1)/\sigma} \right) b_1^{1/\sigma} b_{1t}^{-1/\sigma} b_1
\]

\[
= q_1^a a_1 + q_2^b b_1.
\]

\(^{11}\)In BKK the weights in the Armington aggregator are set such as intermediate output is equal to final goods in steady state, see also Ravn [1997].
Note again that in steady state \( q^a_1 = q^b_1 \) and exploiting symmetry \( a_2 = b_1 \) yields
\[
f_1 = q^a_1 (a_1 + a_2) = q^a_1 y'_1 = > q^a_1 = q^b_1 = 1. \tag{B 21}
\]
The above allows us to evaluate the FOC for \( k_{t+1} \) in steady state and to derive the capital-GDP-ratio in steady state:
\[
k_y = \frac{\theta \beta (1 - \tau)}{1 - \beta (1 - \delta)}
\]
The law of motion for capital implies that \( x_y = \delta k_y \). For consumption this implies
\[
c_y + x_y = f_y = \frac{y'_1}{y} = (1 - g_y) = > c_y = 1 - g_y - \delta k_y.
\]
Combining the FOC for hours and consumption in steady state implies:
\[
n = \frac{\mu (1 - \tau) (1 - \theta) y_c}{1 + \mu ((1 - \tau) (1 - \theta) y_c - 1)}
\]
such that, given \( c_y \), the parameters \( \theta, \tau \) and \( \mu \), pin down hours in steady state.

Assuming zero debt in steady state we have from equation (5): \( \tau = g_y \).

In the following, we list the linearized equilibrium conditions that we use in the actual simulation of the model, denoting their sequence with an 'X'. Unless noted otherwise all variables denote percentage deviations from steady state. Market clearing for intermediate goods (B 4) is approximated as
\[
y_{1t} = \omega (1 - g_y) a_{1t} + (1 - \omega) (1 - g_y) a_{2t} + \hat{g}_{1t} \tag{X1}
y_{2t} = \omega (1 - g_y) b_{2t} + (1 - \omega) (1 - g_y) b_{1t} + \hat{g}_{2t}, \tag{X2}
\]
where \( \hat{g}_t = (g_t - g)/y \). Market clearing for final goods (B 6) is approximated as
\[
(1 - g_y) f_{1t} = c_y c_{1t} + x_y x_{1t} \tag{X3}
(1 - g_y) f_{2t} = c_y c_{2t} + x_y x_{2t} \tag{X4}
\]
Production function of intermediate goods (3) is approximated as
\[
y_{1t} = z_{1t} + \theta k_{1t} + (1 - \theta) n_{1t} \tag{X5}
y_{2t} = z_{2t} + \theta k_{2t} + (1 - \theta) n_{2t} \tag{X6}
\]
Production function of final goods (4) is approximated as
\[
f_{1t} = \omega a_{1t} + (1 - \omega) b_{1t} \tag{X7}
f_{2t} = \omega b_{2t} + (1 - \omega) a_{2t} \tag{X8}
\]
16
Demand for intermediate goods (B 12)-(B 15) is approximated as

\[ a_{1t} = -\sigma q_{1t}^a + f_{1t} \] (X9)
\[ b_{1t} = -\sigma q_{1t}^b + f_{1t} \] (X10)
\[ b_{2t} = -\sigma q_{2t}^b + f_{2t} \] (X11)
\[ a_{2t} = -\sigma q_{2t}^a + f_{2t} \] (X12)

The first order conditions (FOC) for consumption are

\[ \lambda_{1t} = (\mu(1 - \gamma) - 1)c_{1t} - \frac{n}{1 - n}(1 - \mu)(1 - \gamma)n_{1t} \] (X13)
\[ \lambda_{2t} = (\mu(1 - \gamma) - 1)c_{2t} - \frac{n}{1 - n}(1 - \mu)(1 - \gamma)n_{2t} \] (X14)

The FOCs for labor are

\[ \lambda_{1t} + q_{1t}^a - \frac{\tau}{1 - \tau} \tau_{1t} + w_{1t} = \mu(1 - \gamma)c_{1t} + \left\{ \frac{n}{1 - n}[\gamma(1 - \mu) + \mu]\right\} n_{1t} \] (X15)
\[ \lambda_{2t} + q_{2t}^b - \frac{\tau}{1 - \tau} \tau_{2t} + w_{2t} = \mu(1 - \gamma)c_{2t} + \left\{ \frac{n}{1 - n}[\gamma(1 - \mu) + \mu]\right\} n_{2t} \] (X16)

The FOCs for capital are

\[ \lambda_{1t} = (1 - \beta(1 - \delta))E_t(q_{1t+1}^a - \frac{\tau}{1 - \tau} \tau_{1t+1} + r_{1t+1}) + E_t\lambda_{1t+1} \] (X17)
\[ \lambda_{2t} = (1 - \beta(1 - \delta))E_t(q_{2t+1}^a - \frac{\tau}{1 - \tau} \tau_{2t+1} + r_{2t+1}) + E_t\lambda_{2t+1} \] (X18)

The FOCs for intermediate good firms (B16) and (B17) are

\[ r_{1t} = y_{1t} - k_{1t} \] (X19)
\[ r_{2t} = y_{2t} - k_{2t} \] (X20)
\[ w_{1t} = y_{1t} - n_{1t} \] (X21)
\[ w_{2t} = y_{2t} - n_{2t} \] (X22)

The law of motion of capital in the two countries is

\[ k_{1t+1} = (1 - \delta)k_{1t} + \delta x_{1t} \] (X23)
\[ k_{2t+1} = (1 - \delta)k_{2t} + \delta x_{2t} \] (X24)
By linearizing the risk sharing condition \((B11)\) we obtain
\[
\lambda_{1t} + q^a_{1t} = \lambda_{2t} + q^a_{2t}
\] (X25)

To close the model, we require that the law of one price holds, i.e. the relative price of each good is identical across countries. Therefore we define the real exchange rate, \(rx_t\), and impose two conditions:
\[
rx_t = q^b_{1t} - q^b_{2t}
\] (X26)
\[
rx_t = q^a_{1t} - q^a_{2t}
\] (X27)

As regard the government, approximating (5) gives
\[
\beta \dot{d}_{1t+1} - \dot{d}_{1t} = g_{1t} - \tau \tau^1_t - \tau y_{1t}
\] (X28)
\[
\beta \dot{d}_{2t+1} - \dot{d}_{2t} = g_{2t} - \tau \tau^2_t - \tau y_{2t},
\] (X29)

where \(\dot{d}_t = d_t/y\). The tax rule (7) is already linear. Note however that below we use \(\tau_t\) to denote percentage deviations from steady state, i.e. \(\frac{\tau_t - \tau_s}{\tau_s}\):
\[
\tau \tau^1_t = \phi \dot{d}_{1t}
\] (X30)
\[
\tau \tau^2_t = \phi \dot{d}_{2t}.
\] (X31)

These equations characterize the equilibrium in the neighborhood of the steady state, given the processes for the shocks. In addition, we define the primary budget balance
\[
bb_{1t} = \tau \tau^1_t - g_{1t} + g_y y_{1t}
\] (X32)
\[
bb_{2t} = \tau \tau^2_t - g_{2t} + g_y y_{1t};
\] (X33)

the terms of trade
\[
p_t = q^b_{1t} - q^a_{1t}
\] (X34)

and, finally, the trade balance
\[
nx_t = (1 - \omega)(1 - g_y)(a_{2t} - b_{1t} - p_t)
\] (X35)

Rewriting (6) gives
\[
\ddot{g}_{1t+1} = \rho_y \dot{g}_{1t} + g_y \epsilon_{1t}
\] (X36)
\[
\ddot{g}_{2t+1} = \rho_y \dot{g}_{2t} + g_y \epsilon_{2t}
\] (X37)

For technology we have two additional equations, given the exogenous process for technology specified by BKK.
The simulations of the model discussed in the main text are based on assigning parameter values for the structural model parameters, listed in Table C1.

Table C 1: Parameter values of theoretical economies

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor (steady state)</td>
<td>$\beta = 0.99$</td>
</tr>
<tr>
<td>Consumption share</td>
<td>$\mu = 0.34$</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma = 2$</td>
</tr>
<tr>
<td>Capital share</td>
<td>$\theta = 0.36$</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta = 0.025$</td>
</tr>
<tr>
<td>Home bias (steady state)</td>
<td>$\omega = 0.85$</td>
</tr>
<tr>
<td>Fraction of government spending (steady state)</td>
<td>$g_y = 0.2$</td>
</tr>
<tr>
<td>Trade elasticity</td>
<td>$\sigma = 1.5$</td>
</tr>
<tr>
<td>Debt stabilization</td>
<td>$\phi = 0.0143$</td>
</tr>
</tbody>
</table>

Notes: deep parameter values are taken from BKK; $\phi$ is obtained by matching the autocorrelation of $bb$.

Figure C 1: Cross-correlation function for the budget-output-ratio and the net export-output ratio in theoretical economies. Notes: ccf computed conditional on specific shocks indicated by letter on top of panel; vertical axis $\rho(bb_t, nx_{t+k})$, horizontal axis: $k$. Solid line: baseline specification; diamonds: low trade price elasticity; squares: high trade price elasticity; circles: import content in government spending.

In the following we perform a sensitivity analysis to explore the robustness of our results with respect to alternative model specifications. A first set of variations concerns the value of the trade price elasticity $\sigma$ and the extent of home
bias in government spending. Specifically, we consider a low value for the trade price elasticity of 0.2 and a high value, 3. Regarding government spending we assume in a third experiment that government spending has the same import con-
tent as private spending (12 percent). In table C2 we report the unconditional and conditional moments for all specifications, while figure C1 displays both the unconditional (lower right panel) and the conditional ccf. Figure C2 displays the corresponding impulse responses. Generally, the results are similar to those obtained under the baseline specification of the model. An interesting exception is the case of a low trade price elasticity. In this case, the correlation of the trade and the budget balance is negative. As shown by Figure C2, the reason is that the trade balance improves in response to a government spending shock: with a low trade price elasticity and complete financial markets, valuation effects dominate substitution effects, a possibility discussed in detail in Müller [2006].
A second experiment concerns the fiscal rule. Instead of determining the parameter value for the debt elasticity of the tax rate, $\phi$, by matching the autocorrelation of the budget balance, we consider a range of values. Results are reported in table C 3 and figure C3. It turns out that the value $\phi$ has some bearing on the response
of the trade balance notably to fiscal shocks: the stronger the response of taxes to debt, the stronger the fall in investment. Hence, one may actually observe an increase in the trade balance in the early period after the shock. Overall, however, the conditional correlations are qualitatively similar to those obtained under the baseline specification: conditional on spending shocks the contemporaneous correlation of the budget and the trade balance is positive, conditional on technology shocks it is negative - as is the unconditional correlation.
References


Tommaso Monacelli and Roberto Perotti. Fiscal policy, the trade balance and the real exchange rate: Implications for international risk sharing. mimeo, 2006.
