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Abstract
If demand for human services is inelastic or manufactured goods are necessities, labour shifts from manufacturing to services and the budget share of services rises. Higher productivity growth in the market sector pushes up the tax rate and public employment if private goods and public services are poor substitutes, labour supply is inelastic and there are few dependants. Otherwise, private affluence and public squalor result. More dependants boost public employment if the market provides poor substitutes, but public services per dependent may fall due to tax base erosion. Extensions to market and public employment being imperfect substitutes, varying utility of money and public sector productivity depends on pay.

Keywords: Baumol’s cost disease, Wagner’s law, congestion, cost of public funds, dependency ratio

JEL code: E62, H0, J22, J31, J4, O40

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1. Introduction
The sustainability of the welfare state is top of the political agenda in many countries. This may be due to growing pressures of increasing globalisation, inflation in the price of public services, greying of the population, slowing down of labour productivity growth, rising unemployment and welfare state dependency, and disincentive effects and moral hazard inherent in tax and benefit systems (e.g., Lindbeck, 2006). Here we explore the implications of a greying population on the tax burden and provision of public services, but focus primarily on the inflation in the relative price of public services caused by productivity growth in care of the elderly, child care, education and other human services falling short of productivity growth in the market sector (Baumol, 1967; Baumol, Blackman and Wolff, 1985). Nurses and teachers cannot be replaced by machines. Consequently, there is less scope for productivity growth than in the market sector and relative prices of such human services increase steadily over time. Recent empirical evidence for US data for the period 1948-2001 suggests that stagnant industries indeed show higher growth in relative prices (Nordhaus, 2006). In fact, the higher growth in relative prices is almost exactly equal to the productivity growth differential. Although these findings are based on value added rather than output data and fraught with measurement problems especially in sectors like health, education and personal services where output measures are really measures of input, they support Baumol’s main hypothesis. Also, sectors with one percentage-point lower productivity growth are associated with three-quarters percentage-point lower growth in real output. The reason is that higher relative prices implied by the cost disease depress demand for those sectors. Interestingly, wages and profits are mainly determined by the aggregate economy rather than productivity experiences of individual sectors. Because the output composition has shifted away from sectors with rapid productivity growth like manufacturing to those with stagnant technologies like government, education and construction, Nordhaus concludes that aggregative productivity growth in the US has slowed down by more than one-half percentage-point over the last half century.

We provide more insight into Baumol’s cost disease and investigate the implications of stagnant human services being financed out of the public purse. We examine under which conditions the budget share of the stagnant sectors increases over time. With Cobb-Douglas preferences the budget share of human services and other stagnant sectors are constant. However, this budget share steadily increases over time if human services and other economic

1 Similarly, for the Netherlands the annual percentage growth rate in costs per unit of output of the education, care and police/justice were, respectively, 1.9, 1.1 and 4.5 percentage points above that in
products and services are poor substitutes. Also, if human services are luxuries, Wagner’s law kicks in and the budget share of human services increases over time. Human services such as care for children and the elderly release time for work and leisure, so their time cost falls less than their relative price. Hence, the national income share of human services and labour supply increase over time. It makes sense to reserve the term Baumol’s cost ‘disease’ when the budget share of human services increases over time. The term ‘disease’ is used but no normative implications are intended. In fact, as people grow wealthier due to technological advances, they are well able to afford the ever more expensive human services.

One way to avoid the cost disease is to allow the gap between public pay and market pay to steadily increase over time or to let the quality of human services deteriorate. Of course, it then becomes increasingly difficult to recruit, retain and motivate people to work in public services. Clearly, this is not feasible in the long run. Although nurses and teachers are dedicated professionals and may work for a relatively low wage, they will not accept an ever-increasing wage gap. There is also no empirical evidence for a growing gap (Nordhaus, 2006).

To make sure that demand and provision of these crucial public services do not fall, it is often argued that government subsidy is required. This is not really convincing, since the persistent productivity increases in the market sector give rise to steady increases in purchasing power. If people value human services, they will use their new riches to pay for it. Since these services typically have income elasticities greater than unity, the provision of these services may flourish as technical progress makes people wealthier. If demand for public services is inelastic, suppliers of these services can raise prices and revenues sufficiently to compensate for rising costs. Furthermore, Baumol's cost disease may induce offsetting trends. The rise in the relative price causes a shift towards less labour-intensive public services such as robots for the care of the elderly or the use of video and Internet in teaching. Technology may thus induce new economies of scale and substitution in consumption. In addition, both increased use of the Internet and lower transport costs induce a shift from small-scale to large-scale providers of public services. Still, the bulk of human services are in nursing, childcare and education. In those sectors the scope for productivity improvements is limited unless one sacrifices quality. There is then a danger that free and equal access of public services is contested as high-income people increasingly opt out of public services and make use of private supplements. It also threatens the quality of public services if the best teachers and medical specialists are drawn into the private sector. This is reinforced if mixing pupils of different abilities and backgrounds is important for the overall quality of education.

the market economy during 1990-2000 (Kuhry and van der Torre, 2002). The annual growth rate of the total of these public services was 1.6 percentage points higher than in the market economy.
Although politicians often believe that higher growth eases financial problems of the welfare state, growth may threaten viability of tax-financed public services (Anderson, 2006). Public services inherently have fewer possibilities for productivity growth and thus their cost will inexorably rise (Baumol’s law). In addition, the tax base erodes as people develop a taste for working less (holidays, early retirement, shorter working hours). There are also limits to the acceptability of raising tax rates to finance the growing demand for ever more expensive public services as distortions increase more than proportional with hikes in the tax rate. Also, growth pushes up the budget share of public services as people demand more and better public services (Wagner’s law\(^2\)). The key question is thus whether growth driven by technical progress and bigger demand really threatens the provision of tax-financed public services.

Our first objective is to analyse this ‘growth puzzle’ within a public finance framework and to examine whether rapid technical progress in the market sector really threatens viability of the welfare state. Is it possible to avoid public squalor and private affluence? Does growth lead to a decline in tax-financed public services? What happens to the tax rate and the cost of public funds as the relative price of public services inexorably rises over time?

Our second objective is to analyse the consequences of a growing demand for public services by a rising number of dependents and less people available for work on the sustainability of the welfare state. The provision of public services per dependant can have a rival nature (pensions or child allowances) or a partially non-rival nature (parks or care). In our setup dependants can be children, the sick or the elderly. Groezen, Meijdam and Verbon (2005) investigate more specifically the effects of an ageing population in a two-sector, two-period economy with endogenous growth in manufacturing and zero growth in human services. They make some strong assumptions: old people only consume services, young people consume only manufactured products, people either die when young or make it into retirement, and capital fully depreciates in one period. They show that ageing unambiguously reduces long-run growth in a small open economy, but this only occurs in a closed economy if the elasticity of substitution between labour and capital in manufacturing exceeds unity. Their result hinges on Cobb-Douglas preferences and a constant allocation across sectors despite a growing wage rate. The main novelty of our paper is that we highlight the reasons why labour may shift from manufacturing to services. To remain close to the framework of Baumol (1967), we abstract from saving and investment. In contrast to Groezen, Meijdam and Verbon (2005), we allow for elastic labour supply, an elasticity of substitution between commodities and services that is

\(^2\) In regressions explaining the log of the budget share of public spending one finds a significant positive coefficient on the log of national income (e.g., Ram, 1987). Even when carefully allowing for
different from unity, and non-homothetic preferences. Our focus is thus on the impact of differential productivity growth and a growing number of dependants on the dynamic allocation of labour to manufacturing and services, the tax rate and the marginal cost of public funds.

Section 2 shows that more rapid technological progress in manufacturing pushes up the budget share of public services rises if manufactured goods are necessities or human services are luxury goods, manufactured goods and human services are poor substitutes, and human services release time for work or leisure. Sections 3 and 4 investigate the impact of rapid technical progress in the market economy and a growing number of dependants on provision of tax-financed public services and the cost of public funds paying attention to the adverse effects of taxes on labour supply and congestion in public services. Section 3 abstracts from income effects in labour supply while section 4 allows for a variable wage elasticity of labour supply and the negative impact of a higher dependency ratio on labour supply. Section 5 allows for immaterial joy of working in public services, imperfect substitution between public and private employment and the effect of public sector pay on public sector productivity. Section 6 concludes and offers some suggestions for further research.

2. Baumol’s cost disease: human services and manufacturing

We first consider a market economy with two sectors: human services and manufacturing. Human services include health care (not cure), education, etc. and cannot easily be standardised. Services S has a relatively low growth rate in labour productivity $\rho_S$ and the market sector M has a higher and persistent growth rate in labour productivity $\rho_M > \rho_S$. Following Baumol (1976) we refer to services as the stagnant sector and manufacturing as the progressive sector:

“The common element that characterizes all stagnant services is the handicraft attribute of their supply process. None of them has, at least so far, been fully automated and liberated from the requirement of a substantial residue of personal attention by their producers. That is, they have resisted reduction in the amount of labor expended per unit of output.”

Output levels are $Y_S = A_{S0}L_S \exp(\rho_ST)$ and $Y_M = A_{M0}L_M \exp(\rho_MT)$, where $L$ denotes employment, $t$ indicates time and $A_{S0}$ and $A_{M0}$ are constants. We abstract from other factors of production.\(^3\) With labour mobility between services and manufacturing, wages in services must equal those in manufacturing. Hence, wages in services grow at the same rate as in manufacturing, so that possibly non-stationary variables and correlated or heteroscedastic residuals and using cointegration and exogeneity tests, one finds evidence for Wagner’s law on USA data for 1929-1996 (Islam, 2001).\(^3\) However, if there is also capital and the interest rate is determined on world capital markets, the capital intensities and thus the constants $A_{M0}$ and $A_{S0}$ are also pinned down by the world interest rate. A companion paper explores Baumol’s cost disease in a general equilibrium setting (van der Ploeg, 2006).
$W = K \exp(\rho_M t)$ where $K$ is a constant. Unit labour costs in services rise at the labour productivity growth differential of manufacturing over services, but are constant in manufacturing:

\[ \frac{W_L}{Y_M} = K \exp\left[(\rho_M - \rho_S)t\right]/A \] and \[ \frac{W_L}{Y_M} = K/A. \]

If prices are a constant mark-up on unit labour costs, the price of services $P_S$ grows over time at the rate $\rho_M - \rho_S > 0$ while the price of manufacturing $P_M$ stays constant. Of course, there is no guilty party for the persistent increase in the real price of services. Greed or waste play no role.

With Cobb-Douglas preferences the budget shares of services and manufacturing are constant, so the allocation of labour across sectors $P_S Y_S / P_M Y_M = L_S / L_M$ is constant also. If total labour demand $L_S + L_M$ equals exogenous labour supply, $L_S$ and $L_M$ are constant. Hence, output of services grows at the rate $\rho_S$ and manufacturing at the rate $\rho_M$. Although the ratio of output of services to that of manufacturing dwindles away and prices of services rise at the rate $\rho_M - \rho_S > 0$, employment in services remains constant. Hence, Baumol’s cost disease does not destroy jobs or output of services. Technological progress in manufacturing boosts purchasing power of people sufficiently to keep up expenditures on services despite rising prices of services. Also, the cost of human services in terms of the hours one needs to work to afford them falls at the rate $\rho_S$. The hours work needed to buy a unit of manufacturing output falls at a higher rate, namely $\rho_M$.

As a result, the real consumption wage grows at the rate $\alpha \rho_M + (1 - \alpha) \rho_S$, where $0 < \alpha < 1$ indicates the budget share of manufacturing, even though the CPI grows at the rate $(1 - \alpha)(\rho_M - \rho_S) > 0$. In the public debate the steady growth in the price of human services is often confused with a steady growth in the real cost of human services, but the real cost of human services is at worst constant and more likely will fall at a modest rate each year.

2.1. Wagner’s law

One shortcoming of this discussion of Baumol’s cost disease is that Cobb-Douglas preferences and more generally homothetic preferences imply linear Engel curves. However, manufactured goods may be necessities and include basic needs such as food, drink, clothing and shelter while services such as care and education are luxury goods. Wagner’s law suggests that human services have an income elasticity greater than one. In that case, non-homothetic preferences are more appropriate. The budget share of services then rises over time, since people assign relatively more priority to basic needs if they are poor and more to luxury services as they grow richer. To illustrate, consider the Stone-Geary utility function $U = \alpha \log(Y_M - Y_M^*) + (1 - \alpha) \log(Y_S)$, where $0 < \alpha < 1$ and $Y_M^* > 0$ is the constant subsistence level of manufactured goods. The
household budget is \( P_MY + P_SY = Y \), where income \( Y \) grows at the rate \( \rho M \). If manufacturing is the numéraire with \( P_M = 1 \) and \( P_S = P \), the optimal budget shares of services (which equals the fraction of labour employed in human services) and manufactured goods are:

\[
(2) \quad P_Y Y = L_S (L_S + L_M) = (1-\alpha) (1-Y_M^*/Y) < 1-\alpha \quad \text{and} \quad Y_M^*/Y = \alpha + (1-\alpha) Y_M^*/Y > \alpha.
\]

Due to more rapid technical progress in manufacturing than in services, the budget share of services gradually rises over time as the demand for necessities is fulfilled. Since \( P_Y Y_M = L_S/L_M \) rises over time, labour moves from manufacturing to services.

### 2.2. Enjoying human services takes time

Human services take up time. Enjoying Wagner’s Ring requires many hours in the opera. Investing in education also requires hours of studying. Suppose thus that it takes \( \gamma \) units of time to consume human services and assume that labour supply is endogenous. Take a Cobb-Douglas utility function \( U(Y_M, Y_S, V) \) with leisure given by \( V = 1 - L - \gamma S \) and \( L \) indicates labour supply.\(^4\)

Maximising utility subject to the budget constraint \( Y_M + P Y_S = WL \) yields the budget share of human services, relative budget (and employment) share and labour supply:

\[
(3) \quad \frac{P S}{W L} = \frac{\beta P}{\alpha + \beta P \gamma W}, \quad \frac{P S}{C} = \frac{L_S}{L_M} = \frac{\beta P}{\alpha (P + \gamma W)} \quad \text{and} \quad L = \alpha + \beta \left( \frac{P}{P + \gamma W} \right),
\]

where \( P + \gamma W \) is the ‘time price’ of human services. If productivity growth in services is positive but less than in manufacturing, the cost of human services rises less fast than wages. The budget share of human services in wage income thus falls. Labour supply falls as more time is needed to consume human services. In equilibrium total labour supply matches total labour demand, so employment in manufacturing and human services are:

\[
(4) \quad L_M = \alpha \quad \text{and} \quad L_S = \beta P/(P + \gamma W) \quad \Rightarrow \quad \hat{L}_M = 0 \quad \text{and} \quad \hat{L}_S = -\left( \frac{\gamma W}{P + \gamma W} \right) \rho S \leq 0,
\]

\(^4\) A more general analysis requires disentangling income and substitution effects (e.g., Baumol, 1973).
where we used that the growth rate in \(W/P\) equals \(\rho_S\). We thus conclude that employment in manufacturing employment is unaffected by relatively rapid technical progress in manufacturing, whereas employment in human services gradually declines over time if \(\gamma > 0\).

Of course, other human services may free up time for leisure and work such as professional childcare or care of elderly parents. In that case, \(\gamma < 0\) and the results are reversed. In other words, with the passage of time, the national income share of care for children and pensioners increases and that of manufactured goods declines. The ‘time cost’ of these human services is relatively less strong than the relative price of human services. As a result, labour supply expands in order to satisfy the extra demand for labour in human services.

2.3. Services and manufacturing are poor substitutes

We also for an elasticity of substitution between services and manufacturing less than unity, since manufactured goods are poor substitutes for care or education. A CES utility function with an elasticity of substitution less than unity generates a budget share of public services that rises over time. For example, the homothetic utility function \(U(Y_M, Y_S)\) yields the first-order optimality condition \(U_s/U_M = \theta\). Loglinearisation gives \(\frac{\bar{Y}_M - \bar{Y}_S}{\bar{Y}} = \theta\), where \(\theta > 0\) is the elasticity of substitution between manufactured goods and services. Together with \(\bar{P} = \rho_M - \rho_S\) and the budget constraint, \(\phi \bar{Y}_M + (1-\phi)(\bar{P} + \bar{Y}_S) = \bar{Y}\) where \(0 < \phi = Y_M/Y < 1\), we obtain:

\[
\frac{\bar{Y}_M}{\bar{Y}} = -(1-\theta)(1-\phi)(\rho_M - \rho_S) < 0, \quad \frac{\bar{P}Y_S}{\bar{Y}} = (1-\theta)\phi(\rho_M - \rho_S) > 0
\]

and \(\bar{L}_M - \bar{L}_S = (1-\theta)(\rho_M - \rho_S) > 0\) if \(\theta < 1\).

If manufactured goods and services are poor substitutes (\(\theta < 1\)) and manufacturing has relatively high productivity growth (\(\rho_M > \rho_S\)), the budget share of manufactured goods declines and that of services rises over time. Again, workers gradually move from manufacturing towards services.

2.3. Endogenous growth in manufacturing

It may be realistic to have endogenous growth in manufacturing (Romer, 1990) and zero growth in services (cf., Groezen, Meijdam and Verbon, 2005). One could allow for learning by doing and knowledge spill-over effects and postulate that growth in manufacturing increases with the number employed in manufacturing, e.g., \(\rho_M = \rho_M^* + \Phi(L_M)\) with \(\Phi' \geq 0\), where \(\rho_M^*\) is autonomous productivity growth in manufacturing. We then see that autonomous productivity growth is offset by less learning by doing as the manufacturing sector diminishes and labour
shifts to services. With Wagner’s law in operation, services taking up time or services and manufacturing being poor substitutes, we thus see that over time some of labour thus moves back from services to manufacturing. Still, steady-state employment in manufacturing is lower than before the shock to autonomous productivity growth in manufacturing.

2.4. Summing up

**Proposition 1:** *In a two-sector market economy, the relative price of human services increases at the excess of the rate of productivity growth in manufacturing over that in services. With Cobb-Douglas preferences the allocation of employment across the sectors remains constant. However, the budget share of services and the fraction of labour employed in services rise over time if services are luxury goods or poor substitutes for manufactured goods. If human services replace household services and release time for work and leisure, the national income share of human services and the level of employment in human services increase over time as well.*

There are good reasons to believe that the budget share and employment share of human services gradually increase over time. People become sufficiently rich due to technological advances to be able to afford rising relative prices of human services that and welfare benefits from more rapid technical progress in manufacturing. Furthermore, the stagnant sectors may not be that stagnant after all and there may be a shift towards less labour-intensive human services which are due to the Internet and falling labour costs supplied on a much larger scale. Indeed, Cowen (1996) in response to Baumol (1996) and Triplett and Bosworth (2003) argue that Baumol’s cost disease has been ‘cured’. Still, the term ‘disease’ is used by lobbies in care, education and the arts to obtain more government subsidies. They rightly argue that these public services suffer from ever-increasing costs, but this does not imply that they should be compensated for these rising costs by the government. If the government has access to non-distorting taxes, there is no problem succumbing to these lobbies as rapid technical progress in the market sectors is making the country wealthier. However, this rationale is less clear if the extra subsidies are financed by distorting taxes. The term ‘disease’ could then refer to the adverse incentive effects on labour supply and saving of a growing demand for taxation. This way of looking at Baumol’s cost disease is of utmost policy relevance. The key question is whether Baumol’s cost disease inevitably leads to public squalor and private affluence. In the next sections we thus modify our discussion of Baumol’s cost disease and assume that services are the responsibility of the government and financed by taxes on labour income.
3. Public finance aspects of Baumol’s cost disease

We investigate the implications of Baumol’s cost disease for labour supply and the tax burden within the context of a two-sector model of the economy. Sector M is the market and sector P the public sector. We abstract from saving and investment. We thus address sustainability of social spending and the welfare state in the light of (i) stagnant public services and rapid technological advances in manufacturing, and, (ii) a rise in the dependency ratio arising from, say, greying of the population or a rise in the number of sick people.

To simplify and set the scene for the next section, we abstract from income effects in labour supply and assume that market and public employment are perfect substitutes for households. We focus on provision of public services and abstract from a public pension system for dependants; households take care of their own pensions. The main effect of greying is thus on the rise in demand for public services especially if they are rival goods. Households therefore maximise utility

$$U = C - \left( \frac{\varepsilon}{1+\varepsilon} \right) \left( L_M + L_P \right)^{\frac{1+\varepsilon}{\varepsilon}}$$

subject to the budget constraint

$$C = (1-T) \left( W_M L_M + W_P L_P \right)$$

where $C$, $L_M$, $L_P$, $W_M$ and $W_P$ denote private consumption (including dependants), market employment, public employment, the market wage and the public wage, respectively. This gives labour supply

$$L = L_M + L_P = \left[ (1-T)W_M \right]^\varepsilon$$

with $\varepsilon \geq 0$ the constant Frisch wage elasticity of labour supply. Labour mobility ensures that public pay matches that in the market, i.e., $W_M = W_P = A_M$ where $A_M$ stands for productivity of market employment. The provision of public services per dependant is

$$S = \frac{A_P L_P}{D^\nu}$$

where $A_P$ is public sector productivity and $D$ denotes the number of dependants. If public services are rival goods, more public employment is needed to take care of a larger group of dependants and thus $\nu=1$. If public services are non-rival goods, $\nu=0$. In general, $\nu$ indicates the degree of rivalry of public services. The optimal tax rate $T$ and provision of public services per dependant $S$ follow from the following optimisation problem for the government:

$$\begin{align*}
\max_{T,L_P} \Omega &= \left( \frac{1}{1+\varepsilon} \right) \left[ (1-T)A_M \right]^{\frac{\varepsilon}{1+\varepsilon}} + V \left( A_P L_P / D^\nu \right) \quad \text{subject to} \quad L_P = T \left[ (1-T)A_M \right]^\varepsilon
\end{align*}$$

where the first term stands for indirect private utility and the second term $V(S)$, $V'>0$, $V''<0$ captures the value of public services in social welfare $\Omega$. The marginal value of the provision of public services should equal its cost times the marginal cost of public funds $\eta$: 


If public services are non-rival the cost is simply the price \( P \), but if public services are rival the cost corresponds to the price times the number of dependants, i.e., \( PD \). Congestion effects in public services (\( \nu < 0 \)) imply that more dependants requires more public employment to maintain the same quality of public services and thus pushes up the cost of public services. The cost of public funds is large if the wage elasticity of labour supply \( \varepsilon \) and the tax rate on labour are large. If labour supply is inelastic, taxes are not distorting, the cost of public funds is fixed at unity and thus (7) reduces to the Samuelson rule \( V'(S) = PD \). In general, the cost of public funds simply increases with the tax rate (provided \( \varepsilon > 0 \) and \( \eta > 1 \)):

\[
\hat{\eta} = \left( \frac{\eta - 1}{T} \right)^{\hat{T}},
\]

where a hat indicates a relative change (e.g., \( \hat{\eta} \equiv d\eta / \eta \) except for \( \hat{T} \equiv dT / (1 - T) \)). With the aid of (8), we loglinearise (7) and obtain the change in the demand for public services:

\[
\bar{L}_p = (1 - \psi) \left( \nu D - \bar{P}_p \right) - \psi \left[ \bar{A}_M + \left( \frac{\eta - 1}{T} \right)^{\hat{T}} \right], \quad \text{and}
\]

\[
\bar{S} = -\psi \left[ \nu D + \bar{P} + \left( \frac{\eta - 1}{T} \right)^{\hat{T}} \right], \quad \text{where} \ \psi \equiv -V' / SV \geq 0.
\]

The provision of public services per dependent \( S \) thus decreases if the cost of funds, the relative price of public services or the number of dependants increases. Public employment also decreases if the cost of funds or the productivity in the market sector rises. Furthermore, provided the output effect dominates the substitution effect (i.e., \( \psi < 1 \)), greying of the population or a fall in public sector productivity boosts public employment.

We use the labour supply curve to loglinearise the government budget \( L_p = T L \):

\[
\bar{L}_p = \varepsilon \bar{A}_M + \left( \frac{1 - T}{T} - \varepsilon \right) \hat{T} = \varepsilon \bar{A}_M + \left( \frac{1 - T}{\eta T} \right)^{\hat{T}}.
\]
A higher tax rate leads to more revenues and permits more public spending. Part of the extra revenues is choked off by the disincentive effect on labour supply, especially if the wage elasticity of labour supply \( \varepsilon \) and thus the cost of public funds \( \eta \) are large. Furthermore, a higher productivity in the market sector boosts the wage and labour supply, expands the tax base and thus allows for more spending on public services (if \( \varepsilon > 0 \)).

We solve for the optimal tax rate and provision of public services from (9) and (10):

\[
\hat{T} = \left[ (1-\psi)(\nu \bar{D} - \bar{A}_p) - (\psi + \varepsilon)\bar{A}_M \right] / \Delta,
\]

\[
\hat{L}_p = \Delta \left\{ (1-\psi)\left(\nu \bar{D} - \bar{A}_p\right) - \psi \left[ 1 - \varepsilon T \left( \frac{\eta}{1-T} \right) \right]^2 \bar{A}_M \right\}, \text{ and}
\]

\[
\hat{S} = -\Delta \eta \psi \left[ 1 + \varepsilon T \left( \frac{\eta}{1-T} \right) \right] \left( \nu \bar{D} - \bar{A}_p \right) - \left[ 1 - \varepsilon^2 T \left( \frac{\eta}{1-T} \right) \right]^2 \bar{A}_M,
\]

where \( 0 < \Delta = \left( \frac{1-T}{\eta T} \right)/\left[ \psi \left( \frac{\eta - 1}{T} \right) + \left( \frac{1-T}{\eta T} \right) \right] < 1 \). We now interpret the solution (11)-(12).

Providing a better quality of public services and, assuming that public services are to some extent rival public goods (\( \nu > 0 \)), greying of the population (i.e., lower \( A_p \) or higher \( D \)) induce, if the output effect dominates the substitution effect (i.e., \( \psi < 1 \)), an expansion of public employment. To balance the budget, this necessitates a higher tax rate. After-tax wages in both the public sector and market economy fall and thus labour supply falls, which drives up the cost of public funds. These effects are particularly strong if \( \psi \) is small, the wage elasticity of labour supply \( \varepsilon \) is small, and the initial tax rate is not too high already. In contrast, greying of the population and a better quality (i.e., lower productivity) of public services reduces the provision of public services per dependant, \( S \), despite the increase in public employment.

Rapid technical progress in the market sector (i.e., higher \( A_M \)) unambiguously drives up wages and broadens the tax base. This permits cuts in the tax rate and the cost of public funds. In fact, due to the falling tax rate, after-tax wages rise faster than the rate of technical progress in the market economy. Consequently, labour supply and the tax base rise even more, especially if the wage elasticity of labour \( \varepsilon \) is large, thus permitting a substantial fall in the tax rate or a boost to the level of public employment. Rapid technical progress also means that it
is attractive to substitute from public to private utility and thus public employment will fall, especially if $\nu$ is large. Some of this fall in public employment will be off-set by the increase in labour supply and tax revenues, especially if the pre-existing tax rate and wage elasticity of labour supply (i.e., $T$ and $\varepsilon$) are large. Rapid productivity growth in the market cuts public employment only if $\varepsilon T [\eta(1-T)]^2 < 1$ holds. Consensus estimates of $\varepsilon$ for males and females are, respectively, 0.1 and 0.5 (Evers, et. al., 2005). If the wage elasticity of labour and the tax rate are as high as 0.5 and 0.4, respectively, the left hand side equals 1.25 and thus the inequality is violated. Hence, public employment only falls as productivity growth in the market economy takes off if the wage elasticity of labour supply and the tax rate are not too large. However, if the wage elasticity of labour supply and the tax rate are large, it is a real possibility that rapid productivity growth in the market economy boosts public employment. The elasticity of labour supply on the extensive margin is rather larger than that on the intensive margin, which makes it even harder to rule out a boost to public employment (e.g., Blundell and MaCurdy, 1999; Evers, et. al., 2005).

**Proposition 2:**

(A) A growing number of dependants in the population and a lower productivity of public services increase public employment and push up the tax rate and the cost of public funds provided that substitution possibilities between public services provision and private utility are small. Even so, the provision of public services per dependant will fall.

(B) Rapid technical progress in the market economy expands labour supply and the tax base and drives down the tax rate and the cost of funds. It also encourages substitution from public services provision towards material welfare and thus leads to a cut in public employment. However, if the tax rate is high and labour supply is fairly elastic, the expansion of labour supply may boost public employment. In that case, Baumol’s cost disease does not lead to public squalor and does not threaten the viability of the welfare state.

The main impact of greying is on demand for public services. However, an unsatisfactory feature of the above analysis is that labour supply does not fall as a consequence of a rising number of dependants. To remedy this, we could have utility increasing in leisure rather than falling in hours worked. If we use the utility function $U = C - \left( \frac{\varepsilon}{1+\varepsilon} \right) (L_m + L_p + D)^{\frac{\varepsilon}{1+\varepsilon}}$ where the time constraint is $L+D=1-V$ and $V$ indicates leisure, we obtain the labour supply schedule
We still obtain the optimality condition (7), but the cost of public funds now increases as the ratio of dependants to workers (i.e., \( D/L \)) increases:

\[
V'(S) = PD^\varepsilon \eta \quad \text{with} \quad \eta \equiv \frac{1}{1 - \varepsilon \left( \frac{T}{1-T} \right) \left( \frac{L+D}{L} \right)} \geq 1.
\]

Not surprisingly, a growing dependency ratio makes it harder to raise public funds and thus the cost of public funds rises and demand for public services falls. To save space, we do not analyse this extension.

4. Sustainable social spending with variable elasticity of labour supply

We do extend the analysis of section 3 to allow for: (i) a varying marginal utility of consumption and both income and substitution effects in labour supply, and (ii) the effect of more dependants on the time available to potentially active households. This implies a variable elasticity of labour supply, so labour supply is more elastic if people enjoy a lot of leisure and inelastic if people have hardly any spare time left. It also implies that an increasing dependency ratio has an adverse effect on labour supply.

4.1. Labour supply and demand for market goods

The representative household consists of potentially active members who work or enjoy leisure and inactive members such as children, disabled and pensioners who depend on the active members for support. Total time available to all members of the representative household is one unit. Households face the utility function \( \Omega(U,S) \), where \( U=U(C,V) \) indicates a homothetic sub-utility function in consumption of market goods \( C \) and leisure \( V \), and \( S \) denotes provision of public services per dependant. Households maximise utility subject to the budget constraint \( C=W(1-T)L \) and the time constraint \( L+V=1-D \). Over time the hours available for work of leisure decreases as the fraction of dependants \( D \) increases with greying of the population. Utility maximisation by households takes provision of public services \( S \) as exogenous and requires that the marginal rate of substitution between leisure and consumption equals the net wage, \( U_r/U_c=(1-T)W \). The relative changes in labour supply, leisure and private consumption follow readily:
\[ \dot{L} = \varepsilon (\hat{W} - \hat{T}) - \delta \hat{D}, \quad \dot{V} = -\varepsilon \left( \frac{L}{V} \right) (\hat{W} - \hat{T}) - \delta \hat{D}, \quad \hat{C} = (1 + \varepsilon)(\hat{W} - \hat{T}) - \delta \hat{D}, \]

where \( \varepsilon = (\sigma - 1) \left( \frac{V}{1 - D} \right), \delta = \left( \frac{D}{1 - D} \right), \delta \hat{D} = D \delta \)

and \( \delta \) denotes the dependency ratio (i.e., number of inactive people divided by number of active people in population). The uncompensated wage elasticity of labour supply \( \varepsilon \) is positive if the substitution effect exceeds the income effect, that is if the elasticity of substitution between consumption and leisure \( \sigma \) exceeds one. Labour supply becomes inelastic as the number of hours households work approaches the maximum, so a very high wage is needed to encourage them to work a bit more. Labour supply is most elastic if households work few hours. Labour supply, leisure and private consumption fall as the number of dependants rises. Private part of utility is lower when there are more dependants and a higher after-tax wage:

\[ \hat{U} = \left( \frac{L}{1 - D} \right) (\hat{W} - \hat{T}) - \delta \hat{D}. \]

### 4.2. Trend increase in the relative cost of public services

Output in the market sector is produced by private employment \( L_M \) with a linear technology and an exogenous productivity growth rate of \( \rho_M > 0 \) and is sold for consumption to households:

\[ C = A_M L_M \quad \text{where} \quad A_M = A_{M0} \exp(\rho_M t). \]

Clearly, with this technology the market wage is given by \( \hat{W} = A_M \) and grows at the rate of exogenous productivity growth in the market sector. Output of public services \( G \) results from public employment and productivity growth at the rate \( 0 < \rho_P < \rho_G \):

\[ G = A_P L_P \quad \text{and} \quad PG = W_P L_P \quad \text{where} \quad A_P = A_{P0} \exp(\rho_P t) \]

and \( P \) indicates the relative price of public services. Since public employment and private employment are perfect substitutes in private utility, workers in both sectors are paid the same wage. It follows that the price of public services must rise at a rate equal to the excess of the rate of productivity growth in the private over that in the public sector:
(17) \[ P = A_M / A_P = (A_{MW} / A_{W}) \exp[(\rho_M - \rho_P) t] \] as \[ W = PA_p = A_M. \]

The delivery of social services offers less scope for productivity improvements. With a stagnant public sector and fast-growing market sector, prices of public services must inexorably rise, \( P / P = \rho_M - \rho_P > 0 \) (Baumol, 1967; Baumol, Blackman and Wolff, 1985). The key question is whether this leads to an ever-increasing tax base and a continuous fall in demand for public services or not. To answer these questions, we turn to the government.

4.3. Demand for public services

The government values the provision of public services per dependant and maximises:

(18) \[ \Omega(U(C,V), S) \] where \( S = A_pL_p / D^\nu, \quad 0 < \nu < 1. \)

If public services are a rival good, \( \nu=1 \). If they are a non-rival good, \( \nu=0 \). With greying of the population and \( \nu>0 \), the government needs to supply more public services to prevent social welfare from falling. The government faces the constraint that public services must be financed by tax revenues, that is \( PG = W L_p = T W L \). Labour market equilibrium requires that \( L = L_M + L_P \). It follows that the government solves:

(19) \[ \begin{array}{l}
\max \quad \Omega^*(U((1-T)A_M, D), A_pL_p / D^\nu) \\
\text{subject to} \quad L_p = TL^*(1-T)A_M, D),
\end{array} \]

where \( L^*(.) \) is the labour supply schedule and \( U^*(.) \) the indirect private utility function defined by (13) and (14), respectively. The optimal provision of public services follows from:

(20) \[ \frac{\Omega^*(U(S), C(1-D))}{\Omega^*_U(U, S)} \frac{C(1-D)}{LU} = PD^\nu \eta \quad \text{with} \quad \eta = \frac{1}{1 - \epsilon \left( \frac{T}{1-T} \right)} \geq 1. \]

It says that the marginal rate of substitution between public services and private consumption must equal the relative cost of public services corrected for the dependency ratio, i.e., \( PD^\nu \), times the marginal cost of public funds \( \eta \). If public services suffer from congestion and are to some degree rival goods (e.g., nursing care, teaching), \( \nu>0 \) and a higher dependency ratio raises the effective cost of providing human services. The cost of public funds is again high if
the wage elasticity of labour supply is large and the tax rate is large. Substituting the household budget constraint into the optimality condition (20) and loglinearising gives:

\[
U - (\hat{A}_p + \hat{L}_p - \nu \hat{D}) = \theta \left[ \hat{P} + \hat{\eta} + \hat{U} + \hat{T} - \hat{A}_u + (\nu + \delta) \hat{D} \right]
\]

where \( \theta \) is the elasticity of substitution between \( U \) and \( S \) in social welfare \( \Omega \). Using (17) and substituting \( \hat{U} \) from (14) into (21) gives the demand for employment in public services:

\[
\hat{L}_p = (1 - \theta) \left[ \left( \frac{1}{1 - D} \right) (\hat{A}_u - \hat{T}) - \hat{A}_p + \nu \hat{D} \right] - \theta (\hat{\eta} + \hat{T}) - \delta \hat{D} \quad \text{and}
\]

\[
\hat{S} = (1 - \theta) \left[ \left( \frac{1}{1 - D} \right) (\hat{A}_u - \hat{T}) - \theta (\hat{\eta} + \hat{T} - \hat{A}_p) - (\delta + \theta \nu) \hat{D} \right].
\]

The final term on the right-hand side of (22) indicates that a growing number of dependants curbs labour supply and thus reduces public employment. The middle term states that demand for public services is less if the marginal cost of public funds is high, especially if substitution between public services and private utility is easy. The middle term also indicates that demand for public employment falls if the tax rate is high, since this implies a low after-tax wage (i.e., the opportunity cost of leisure) and a low level of private utility. Hence, it is optimal to substitute away from public services towards private utility. The first-term on the right-hand side of (22) indicates that, if the private part of utility and public services are poor substitutes (i.e., \( \theta < 1 \)), technical progress in the market sector, productivity slowdown in public services and a lower tax rate boost the demand for employment in public services. To the extent that public services are rival goods, an increasing dependency ratio also boosts demand for public employment. Conversely, the first term indicates that a higher price of public services driven by faster productivity growth in the market sector than in public services, a lower tax rate, lower productivity of public services and an increasing dependency ratio depresses public employment if the substitution effect dominates the income effect in social welfare (i.e., if \( \theta > 1 \)).

The net effect of an increasing dependency ratio is to boost public employment if substitution possibilities between private utility and public services are small, public services are a rival good and the dependency ratio is not too high (i.e., if \( (1 - \theta) \nu > \delta \)). Otherwise, an increasing dependency ratio reduces public employment. The net effect on the provision of public services per dependant is negative even if public employment rises.
4.4. The cost of public funds and the government budget constraint

Although public services and private utility are likely to be poor substitutes, briefly consider the special case of a Cobb-Douglas social welfare function $\Omega = \alpha \log(U) + (1-\alpha) \log(S)$ with $\theta = 1$. The demand for public employment is then unaffected by relative prices and productivities:

\[
L_p = \left(\frac{(1-\alpha)(1-T)(1-D)}{\alpha \eta}\right), \quad \text{so that} \quad \hat{L}_p = -\hat{\eta} - \hat{T} - \delta \hat{D}.
\]

Demand for public services simply declines with the cost of public funds, the tax rate and the dependency ratio. However, even in this special case, the productivity in the market sector affects the demand for public employment through the cost of public funds. To see this, we loglinearise the cost of public funds defined in (13) whereby we recognise that the elasticity of labour supply $\varepsilon = (\sigma - 1)V/(1-D)$ is variable:

\[
\hat{\eta} = (\eta - 1)\left(\hat{\varepsilon} + \frac{1}{T}\hat{T}\right) = (\eta - 1)\left[\left(1 + \hat{\varepsilon} \frac{L}{V}\hat{T} - \hat{\varepsilon} \frac{L}{V} \hat{A}_m\right)\right] \quad \text{with} \quad \eta - 1 = \left\{\frac{\varepsilon \frac{T}{1-T}}{1 - \varepsilon \frac{T}{1-T}}\right\}.
\]

The cost of public funds exceeds one, especially if the tax rate is high and the elasticity of labour supply is large. If labour supply is inelastic ($\sigma = 1$) or becomes inelastic as people work so much that they run out of time, the cost of public funds drops to one. The two terms in the square brackets associated with $\varepsilon$ result from the endogenous nature of labour supply. They imply that a higher after-tax wage, induced by a cut in the tax rate or an increase in the productivity in the market sector, boosts labour supply and the tax base (if $\varepsilon > 1$), and consequently lowers the cost of public funds. Note that the cost of funds does not depend on the dependency ratio $\delta$. The government budget constraint must now take account of the adverse effect of greying on labour supply (see equation (13)) and is thus given by:

\[
\hat{L}_p = \varepsilon \hat{A}_m + \left(\frac{1-T}{\eta T}\right)\hat{T} - \delta \hat{D}.
\]
4.5. Equilibrium

The demand curve (22), the cost curve (23) and the budget constraint \((10')\) can be solved simultaneously for the level of public services, the cost of public funds and the tax rate. To see this, we solve (22) and \((10')\) for the demand for public revenue:

\[
(24) \quad \hat{T} = \frac{(1 - \theta) \left( \frac{L}{1 - D} \right) - \varepsilon}{\left( \frac{1 - T}{\eta T} \right) + \theta + (1 - \theta) \left( \frac{L}{1 - D} \right)} \hat{A} + \delta \hat{D} + (1 - \theta)(\nu \hat{D} - \hat{A}) - \theta \hat{\nu}.
\]

Substituting (23) into (24) and solving finally gives the optimal tax rate:

\[
(25) \quad \hat{T} = \frac{(1 - \theta) \left( \frac{L}{1 - D} \right) - \varepsilon + \theta(\eta - 1) \varepsilon \left( \frac{L}{V} \right) \hat{A} + \delta \hat{D} + (1 - \theta)(\nu \hat{D} - \hat{A})}{\left( \frac{1 - T}{\eta T} \right) + \theta + (1 - \theta) \left( \frac{L}{1 - D} \right) + \theta(\eta - 1) \left( \frac{1 + \varepsilon \frac{L}{V}}{T} \right)}.
\]

Substitution of (25) into (23) and \((10')\) gives the changes in the cost of public funds and public employment. It helps to illustrate this solution diagrammatically. The demand curve (24) and the upward-sloping cost curve (23) are drawn in the right panel of Figure 1. The demand curve shows that the tax rate demanded for the finance of public services declines with the cost of public funds. The cost curve slopes upwards, since the marginal cost of public funds increases as the tax rate increases. The government budget constraint is given in the left panel of Figure 1. It indicates that a high tax rate allows more public employment.

Increasing dependency ratio (rise in \(D\)) implies that less people work, so the tax base shrinks, the tax rate must rise and the government budget constraint \((10')\) shifts back and the demand curve for public services shifts up. A higher dependency ratio does not affect the cost of funds schedule (23). The main effect of a higher dependency ratio is thus upward pressure on the tax rate and the cost of funds and a declining level of employment in public services. However, there is also a secondary effect of a higher dependency ratio. If public services are partly rival goods \((\nu > 0)\) and poor substitutes for private goods \((\theta < 1)\), a higher dependency ratio implies a further boost to the higher tax rate to finance a higher level of public services for a given cost of funds so that the demand curve (24) shifts out. Effectively, society demands more public services as it is difficult to substitute for material welfare. A higher
dependency ratio thus shifts the equilibrium from E to E’ and induces both a higher tax rate and a higher cost of public funds. This boosts employment in public services, but some or all of this may be offset by the shrinking of the tax base due to greying of the population. However, in the less likely case that public services are very good substitutes for private goods, the dependency ratio is small and rivalry in the provision of public services is substantial \((\theta>1+\delta'\nu)\), a higher dependency ratio shifts down the demand curve and moves the equilibrium from E to E”. The result is a lower tax rate, a smaller cost of funds and unambiguously less employment in public services.

**Figure 1: Increasing dependency ratio and better quality of public services**

*Key:* Increasing dependency ratio shifts equilibrium from E to E’ if \(\theta<1\) and to E” if \(\theta>1\). Better quality of public services yields the same effects in the right-hand panel, but shifts the equilibrium in the left-hand panel to A’ if \(\theta<1\) and to A” if \(\theta>1\).

Better quality in provision of public services goes together with more teachers in the classroom and more nurses on the ward and thus with a fall in productivity in public services \((A_p\) lower). The cost curve (16) and the government budget constraint (17) are unaffected. However, if private goods and public services are poor substitutes \((\theta<1)\), the demand for public services increases and thus the demand curve shifts out and the equilibrium moves from E to A’. Clearly, both the tax rate and the cost of public funds increase to make possible more employment in public services. If private goods and public services are very good
substitutes ($\theta > 1$), the equilibrium moves from $E$ to $A''$. The private component of utility increases and thus public employment falls. Consequently, the demand curve shifts back and both the tax rate and the cost of public funds fall.

Technical progress in the market sector boosts wages in the market sector and in public services. Figure 2 traces out the consequences for the tax rate, cost of funds and public sector employment. The tax base expands and thus, for a given tax rate, the government can afford more public services. The budget curve (17) thus shifts out. The cost of public funds must fall, since the higher wage boosts labour, reduces leisure and thus lowers the elasticity of labour supply. Consequently, the cost curve (16) shifts back. The demand curve (18) shifts out if private goods and public services are poor substitutes, the wage elasticity of labour supply is small and the ratio of active to inactive people is large, i.e., $(1 - \theta L/(1-D) > \varepsilon$.

**Figure 2: More rapid technological progress in the market economy**

Key: Technical progress in the market economy shifts the equilibrium from $E$ to $E'$ if $(1 - \theta L/(1-D) > \varepsilon$ and to $E''$ otherwise.

Technical progress in the market sector thus shifts the equilibrium outcome from $E$ to $E'$, so that the tax rate rises and demand for public employment expands. The cost of public funds increases (decreases) if the positive effect on the increases demand for public services on the cost of funds outweighs (is less than) the negative effect on leisure and the cost of funds. However, if labour supply becomes very elastic, the number of active to inactive people falls...
and private goods and public services become better substitutes, rapid productivity improvements in the market economy will shift back the demand curve. In that case, the equilibrium may shift from E to E″. Since the demand effect and the cost effect now reinforce each other, the net result is an unambiguous drop in the cost of funds. The tax rate rises if the cost effect outweighs the demand effect. In that case, public employment expands. However, if the demand effect outweighs the cost effect, the tax rate falls and this tends to offset or even reverse the expansion of public services.

Proposition 3:

(A) If public services suffer from congestion effects and are poor substitutes for private goods, a higher dependency ratio boosts demand for public employment and thus pushes up the tax rate and the cost of funds. Provision of public services per dependent may, however, fall due to the growing number of dependants and the consequent shrinking of the tax base. A better quality of public services also boosts the tax rate and public employment. However, if public services and private goods are good substitutes, a higher dependency ratio or a better quality of public services lowers the tax rate and always reduces provision of public services.

(B) Rapid technical progress in the market economy pushes up the tax rate and boosts public employment if private services and market goods are poor substitutes, the wage elasticity of labour supply is small and there are not too many dependants. Public employment falls, however, if labour supply is very elastic, the number of pensioners and children is relatively large and the market provides good substitutes for public sector services.

There is a striking difference with Proposition 2, because a higher dependency ratio now diminishes labour supply. Comparing equations (11) and (25), we see that a higher dependency ratio now erodes the tax base and thus has an additional upward effect on the tax rate (indicated by the term \( \delta \hat{D} \) in the numerator) and the cost of funds. The negative effect of more dependants on labour supply and public employment is now thus partially offset by an increase in the tax rate and the resulting increase in the demand for public employment. If substitution possibilities between public services and private welfare are small (\( \psi<1 \) or \( \theta<1 \)), equation (12) indicates that a higher dependency ratio always increases public employment provided that there are congestion effects in public services (i.e., \( \nu>0 \)). Now Proposition 3 indicates that with adverse effects of greying on labour supply, the demand for public employment may well fall (especially if \( \delta \) is large).
Proposition 3 also entails an important difference with Proposition 2 for the effect of more rapid technical progress in the market economy. The main reason is that the ensuing growth in wages leads to less growth in labour supply as the wage elasticity of labour supply diminishes. As a result, the tax rate needs to rise more than in section 3. This is indicated by the third term in the square bracket in the numerator of expression (25) for the tax rate, since there is no equivalent of this term in expression (11).

4.6. Some illustrative calculations

To better understand the order of magnitudes involved, assume that the wage elasticity of labour supply equals 0.2, the share of leisure taken by workers is 0.4, the elasticity of substitution between provision of public services and private utility is 0.1, provision of public services is a rival good, technical progress in the stagnant sectors is zero, and the initial tax rate is 40 per cent. Consider a doubling of the dependency ratio from 20 per cent to 40 per cent in the next four decades. With average productivity growth in the market of 3 per cent per annum, productivity growth will increase by 226 per cent over the next forty years. We thus have \( \varepsilon = 0.2, \frac{L}{(L+V)} = 0.6, \theta = 0.1, \nu = 1 \) and \( A_p = 0, A_m = 226\%, \hat{\delta} = 100\%, \hat{D} = 83.3\% \).

We start off with \( \hat{\delta} = 0.2 \) and \( T = 0.4 \), and thus \( \eta = 1.1538 \). Expressions (25) and (10') then give:

\[
\hat{d}T = 0.1100 \hat{A}_m - 0.2723 \hat{A}_p + 0.3328 \hat{D} = (24.87 + 27.73)\%-point = 52.6\%-point, \\
\hat{L}_p = 0.4374 \hat{A}_m - 0.5877 \hat{A}_p + 0.5183 \hat{D} = (98.86 + 43.19)\% = 142.05\% \text{ and } \hat{S} = 58.74\%.
\]

Equations (13) and (14) give the corresponding effects on labour supply and private utility:

\[
\hat{L} = (28.5 - 16.2)\% = 12.3\% \text{ and } \hat{U} = (110.7 - 61.1)\% = 49.7\%.
\]

We thus see that the combination of a higher dependency ratio and Baumol’s cost disease (resulting from stagnant public services) more than doubles the tax rate. Labour supply increases due to the ongoing increase in the wage, even though it is offset to some extent by the hike in the tax rate. More dependants leads to shrinking of labour supply and thus to a smaller tax base and lower level of private utility, but the net effect on labour supply is positive. The net effect on private utility is also positive. Demand for public employment increases by 142\% both due to the greater wealth arising from ongoing technical progress in the market economy and from the growing number of dependants. The provision of public services per dependent grows much less, since a growing number of dependants makes it less easy to afford public services.

If there are more substitution possibilities between public services and market goods (e.g., \( \theta = 0.4 \)), the increases in the tax rate and public employment due to a bigger dependency ratio and Baumol’s cost disease are less severe:
\[ dT = 0.0481 \hat{A}_m - 0.1613 \hat{A}_p + 0.2150 \hat{D} = (10.87 + 17.92) \text{-point} = 28.79\% \text{-point}, \]

\[ \hat{L}_p = 0.3038 \hat{A}_m - 0.3481 \hat{A}_p + 0.2640 \hat{D} = (68.66 + 22.00) \% = 90.66\% , \quad \hat{S} = 7.33\% , \]

\[ \hat{L} = (35.6 - 16.2)\% = 19.4\% \quad \text{and} \quad \hat{U} = (146.5 - 67.9)\% = 78.6\% . \]

We now see that congestion of public services arising from a population with more dependants almost completely wipes out the increase in demand for public services arising from the higher wealth ensuing from ongoing technical progress in the market economy. Since market goods substitute for provision of public services, we see that the increase in labour supply and private utility are bigger than before.

These illustrative calculations suggest that both a higher dependency ratio and Baumol’s cost disease boost the tax burden, but neither lead to public squalor nor threaten the welfare state. If public services manage to show some productivity growth the rise in the tax burden will be less severe while public employment need not rise as much. For example, if public services manage a productivity growth of one per cent per year, we have \( \hat{A}_p = 48.89\% \). As a result, the tax rate will be 13.31 percentage-points if \( \theta = 0.1 \) and 7.89 percentage-points lower if \( \theta = 0.4 \). The growth of employment in public services will then be only 113.32\% and 73.64\%, respectively.

If substitution between market goods and public services is easy enough, it is possible that the tax rate falls. For example, if \( \theta = 1 \), productivity improvements in the public sector do not affect the tax rate or public employment. However, the combined effects of Baumol’s cost disease and a greying population lead to a fall in the tax rate of -0.2 percentage points. Public employment increases then by 9.8\% while the provision of public services falls by 73.6\%.

5. ‘If you pay peanuts, you get monkeys’

Public sector employees may accept lower pay than in the market if they value the good they do for others in society. Allow for this immaterial value by adding the term \( \phi L_p \) to private utility. Public pay is now lower than market pay due to the immaterial utility from working in the public sector, \( W_p = W_M - \phi (1-T) < W_M \). Adding a gap between public and market pay reduces the cost of public employment, so the government needs to levy less taxes and can afford more public employment. As technical progress in the market economy advances, this effect vanishes. Since the joy from working in the public sector is untaxed, a higher tax rate resulting from greying of the population drives a bigger wedge between public and market sector pay and makes it easier to afford public employment.

More interesting is to allow private and public employment to be imperfect substitutes for households. Households can work in the market sector at wage \( W_M = A_M \) or
work in public services at wage $W_p$. Extending the analysis of section 3 in this way and allowing for non-constant marginal utility of money, households maximise the utility function:

\[
U = \log(C) - \left( \frac{e}{1 + e} \right) L^{\frac{1 + e}{\epsilon}} = \left( \frac{e}{1 + e} \right) L^{\frac{1 + e}{\epsilon}}, \text{ where } \epsilon \geq 0, 
\]

subject to the budget constraint $C = (1 - T)(W_M L_M + W_P L_P)$. This yields the supply of labour to public services and to the market sector and the level of private consumption:

\[
L_p = \left[ 1 + \left( \frac{A_M}{W_p} \right)^{1 + e} \right]^{-\frac{1}{1 + e}}, \quad L_M = \left[ 1 + \left( \frac{W_p}{A_M} \right)^{1 + e} \right]^{-\frac{1}{1 + e}}
\]

and $C = (1 - T)\left[ W_p^{1 + e} + A_M^{1 + e} \right]^{-\frac{1}{1 + e}}$,

where private sector employment is paid its marginal productivity. Mobility of labour between the public and private sector is thus not perfect. We see that higher public sector pay, higher market sector pay and a lower tax rate boosts private consumption. Since the elasticity of substitution for private consumption is exactly unity, income and substitution effects cancel out exactly and thus the after-tax wage does not affect labour supply to the public sector and to the market. We thus focus on substitution from private to public jobs, which occurs if pay in the public sector rises relative to that in the market economy. Substitution of labour supplies into the government budget constraint $G = W_p L_P = T(W_p L_P + A_M L_M)$ yields:

\[
\frac{T}{1 - T} = \left( \frac{W_p}{A_M} \right)^{1 + e} \quad \text{or} \quad L_p = T^{\frac{e}{1 + e}}.
\]

Hence, a higher tax rate makes it possible to afford a higher level of public employment and a higher ratio of public sector pay to the market wage. By making use of (22) and (23), we obtain an expression for indirect utility of households and for social welfare:

\[
\Omega = \left( \frac{e}{1 + e} \right) \left[ \log(1 - T) - 1 \right] + \log(A_M) + V \left( \frac{A_M T^{\frac{e}{1 + e}}}{D'} \right).
\]

Maximising (24) with respect to the tax rate we obtain:
Since income and substitution effects in labour supply cancel out, the cost of funds is unity. The only thing that changes is that marginal utility of public services is converted from utility units to resource units by dividing $V'(S)$ by the marginal utility of private consumption (i.e., $1/C$). We see from the demand curve (25) and the cost curve (23) that, provided substitution possibilities between public services and market goods are small ($\psi < 1$), a higher dependency ratio or slower productivity growth in public services (perhaps to raise quality of public services) pushes up the tax rate and the level of public employment. To make people move from private sector jobs to public sector jobs, the ratio of public sector pay to market wages must rise. Interestingly, ongoing rapid technical progress in the market economy does not affect the tax rate or the level of public employment. Neither does it affect public sector pay as a fraction of the market wage. Both public sector pay and the market wage increase at the same pace as productivity growth in the market economy. Prices of public services steadily increase at the excess of productivity growth in the market economy over that in the public sector. Of course, if we replace $\log(C)$ by $C^{1/\varphi}/(1 - 1/\varphi)$ in the household utility function, we find that after-tax wages have a positive impact on labour supplies if the substitution effect dominates the income effect (i.e., if $\varphi > 1$). In that case, more rapid technical progress in the market economy boosts labour supply and thus expands the tax base. This makes possible gradual cuts in the tax rate and bigger provision of public services.

Another extension allows the government to encourage public employees to provide better public services. Efficiency-wage arguments suggest that higher public pay relative to market pay makes it easier to recruit, retain and motivate public employees (Summers, 1988; Layard, Nickell and Jackman, 1991). It avoids the trap of ‘if you pay peanuts, you get monkeys’. The government chooses public pay and public employment to ensure a decent quality of public services and takes account of the higher tax rate that is needed to do this. If public sector pay lags behind market pay, this cuts the public sector wage bill. However, this undermines public sector morale and productivity. Public sector productivity and pay are low if labour supply is relatively inelastic and the tax rate on labour is low. In the limit as the tax rate goes to zero, the government cannot afford to pay any public sector wages whatsoever. A low wage elasticity of labour supply implies that the government can pay its employees less
without risking them withdrawing their labour too much. More rapid ongoing technical progress in the market economy than in public services expands labour supply to the market more than that to the public sector, since the latter is more inelastic. Consequently, the government can afford to gradually push down the tax rate and increase the relative size of the market economy at the expense of the size of the public sector. The government reacts by trailing public sector pay behind productivity and pay in the market sector and as a result public sector morale and public sector productivity gradually fall. Since the tax rate falls with time, after-tax market wages rise by more than productivity growth in the market economy and thus labour supply to the market rises over time by more than it would otherwise.

6. Concluding remarks

If human services are provided by the market, more rapid technical progress in manufacturing than in services drives up their cost. Many worry that this strains budgets of families, municipalities and central governments. The inevitable rise in the cost of human services above the rate of inflation implies that supply of these services will fall both in quantity and quality. However, the increased wealth of the nation permits a growing demand for human services. Each year less hours of work are needed to purchase human services. Education, care and other human services become steadily cheaper, even though they appear more unaffordable. In fact, if human services are luxury goods and poor substitutes for manufactured goods or if commercial human services substitute for household services within the household and thus release time for work and leisure, the budget share of human services and employment in human services will inexorably rise over time. Even so, in a market economy where households voluntarily decide on consumption of human services and other goods, they happily spend a steadily increasing share of their budget on human services.

Problems may arise, however, if services such as education and health care are publicly provided and paid for by taxation. The big worry then is whether one can avoid public squalor and private affluence. We must thus allow for the adverse incentive effects on labour supply. More rapid technical progress in the market economy drives up the relative cost of public services (Baumol’s law). Also, growing incomes may boost demand for public services if they are luxury goods (Wagner’s law). A higher dependency ratio also increases the demand for public services, especially if they are rival public goods. Each of these three factors drives up the required tax revenue needed to finance public services. At the same time, as people become richer, they may choose to enjoy more leisure and work less. This together with the growing number of dependants narrows the tax base and necessitates a higher tax rate to finance the same provision of public services. These two factors also reduce the
viability of the welfare state. There are at least three offsetting factors. First, the higher cost of public services reduces demand for public sector jobs. Second, growing incomes reduce the preference for work and thus increase the elasticity of labour supply. This raises the cost of public funds and thus depresses demand for public sector jobs as well. Third, technical progress fuels growth in wages and thus boosts the tax base. This brings in extra tax revenues.

The key question is thus whether these offsetting factors are strong enough to contain the aforementioned threats to sustainability of the welfare state. We have demonstrated that, if the wage elasticity of labour supply and the initial tax rate are not too high, private goods and public services are poor substitutes and the number of dependants is not too large, relatively rapid progress in the market economy drives up the tax rate and thus permits more employment in public services. As the market provides better substitutes for public services and greying boosts the number of dependants, technical progress can eventually reduce provision of public services. We also showed that a better quality of provision of public services and, if public services are rival goods, greying of the population induce more demand for public sector employment financed by a higher tax rate. Still, the provision of public services per dependent will fall as a result of a greying population.

If working in public services yields immaterial utility, public sector employees are paid below market pay. More dependants drive up the tax rate and thus increase the wedge between pay in the public and market sector. This makes public employment more affordable. If public sector and market jobs are imperfect substitutes, a higher dependency ratio and a lower productivity growth of public services lead to a gradual increase in the tax rate, the ratio of public sector pay to the market wage and the level of public employment. If the substitution effects dominate income effects in labour supply, ongoing rapid technical progress in the market economy set of a process in the other direction. The increase in the tax base allows a gradual reduction in the tax rate and increase in the provision of public services despite the gradual increase in the relative price of public services. If the government realises that paying public sector workers below the market wage reduces productivity, the extent to which public sector employees are underpaid is greater as the tax rate is lower and the labour supply is less elastic. Relatively rapid technical progress in the market economy then induces a downward trend in the tax rate and a gradual expansion of the market economy and decline of public services. Public sector pay trails more and more behind market pay.

There at least four interesting avenues for further research. First, one can allow for household production of human services as growing incomes make it less attractive to provide household services oneself. Second, one can allow for public services and market services and allow for
income inequality. Higher income groups may value choice and quality, so opt out of public services and demand market services such as private schools or private health clinics. An interesting question is whether the shift from public to market services induced by rapid technological progress undermines solidarity, access or quality of services. Third, one can allow for the effects of Baumol’s cost disease on saving and investment. Finally, some public services such as curative health care display rapid technical progress (e.g., Jones, 2002). If these services are financed by the public purse, there will be a strong demand for new possibilities for life extension to be covered by health insurance. Obviously, this drives up the tax burden and the cost of public funds, and stimulates calls for privatisation of curative care.

References


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