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Abstract

This paper analyzes the interplay of growth, distribution and public policies when the latter depend on economically important fundamentals. It is shown that not only pro-capital, but also pro-labour or income egalitarian policies lead to high growth. A wealth redistribution policy generally causes lower growth, but less so when there is technological progress. The model implies that high tax rates per se do not necessarily imply low growth. The paper argues that the long-run relationship between growth, post-tax factor incomes and public policies is more complicated in theory and especially when comparing countries as often suggested.

KEYWORDS: Growth, Distribution, Endogenous Policy

JEL Classification: O4, D3, H2

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1 Introduction

It is often shown that policies which are optimal for the accumulated factor of production maximize growth and that high (re-)distributive taxes slow down long-run growth. See, for instance, Perotti (1993), Alesina and Rodrik (1994), Bertola (1993) or Persson and Tabellini (1994).[1]

This paper argues that policies other than those optimal for the accumulated factor of production may also lead to high growth. The model analyzes endogenous policy and shows among other things that, when policy reacts to changes in economically important fundamentals, high (re-)distributive taxes may observationally go together with high long-run growth.

To make these points the structure of models is followed in which the optimal policy of the accumulated factor maximizes growth. As is common the accumulated factor of production is identified with capital and the non-accumulated factor of production with unskilled labour. The paper builds on Alesina and Rodrik and analyzes policies that imply different factor income distributions and long-run growth rates. In the model the governments are taken to be entirely pro-capital or entirely pro-labour. The qualitative results would not change if instead governments attached different social weights on the workers' or capital owners' welfare. As a benchmark policy for assessing income distributions a strictly '*income egalitarian*' policy is considered, which grants all agents an equal income.

By construction a pro-capital policy maximizes growth in the model. A pro-labour government sets higher taxes in order to redistribute wealth or secure high wages. Thus, even if the pro-labour government does not redistribute wealth it sets relatively high tax rates so that the paper distinguishes between redistributing and non-redistributing policies.

[1] Across countries these theoretical predictions do not appear to command strong empirical support, however. See, for example, Easterly and Rebelo (1993), Perotti (1994), Sala-i-Martin (1996) and discussions of those issues in Bénabou (1996), Bertola (1999), Temple (1999), Aghion, Caroli and García-Peñalosa (1999) or Jovanovic (2000).

In the model all policies depend on three fundamental variables: the rate of time preference, an index of the state of technology and the (pre-tax) share of capital (income in total income). When analyzing the consequences of *endogenous policy* the following results emerge:

For all policies considered, higher technological efficiency leads to higher growth and either higher taxes or no change in taxes, but lower redistribution. The reason for that lies in externalities that productive government expenditures exert on the private return on capital. See e.g. Aschauer (1989), Barro (1990) Barro and Sala–i–Martin (1992). An increase in efficiency for given taxes raises growth. For given efficiency an increase in taxes lowers growth. The combined effect of an increase in efficiency is to raise growth and the optimal tax rates. Thus, higher tax rates do not necessarily indicate that growth must be lower.

In the model an increase in efficiency raises the agents' and the governments' intertemporal welfare. Interestingly, the long-run welfare gains are relatively higher for a pro-labour than for a pro-capital government. Furthermore, an efficiency increase never benefits the workers less and often more than the capital owners.

The conditions for wealth redistribution and positive growth are shown to be restrictive. If there is redistribution, an increase in technological efficiency optimally leads to less resources being transferred to labour. This implies that if one compares two economies that are led by redistributing governments the one with a more efficient economy redistributes relatively less wealth, but has higher growth. That suggests an interesting trade-off between growth, wealth redistribution and technological efficiency.

A change in efficiency does not change the post-tax factor income distribution under pro-capital or income egalitarian policies and shifts relatively more post-tax factor income to the accumulated factor of production (capital) under all pro-labour policies. The result looks a bit odd and is explained by the fact that pro-labour governments are only concerned about the welfare of the workers and not about relative incomes as such. Thus, it may well be optimal for a pro-labour government to choose a policy that raises the workers' welfare and at the same time makes the capital owners get relatively more income.

Furthermore, it is shown that pro-labour or factor income egalitarian policies may be indistinguishable or even identical to a growth maximizing policy. Thus, income egalitarianism is not necessarily bad for growth. The result is interesting, because it shows - contrary to conventional wisdom - that other than entirely pro-capital objectives may lead to maximal growth.

Pro-labour and income egalitarian policies are generally different and induce different combinations of growth and post-tax factor income distributions. However, there exist instances where these policies coincide. In general it is ambiguous which of these policies induces higher or lower growth in comparison to the growth maximizing policy.

Finally, the effects of changes in the share of capital are investigated. In the model an increase in the share of capital raises growth under all, but the income egalitarian policies. Taxes increase under an income egalitarian policy, do not change under a redistributing policy and respond in an ambiguous way under all other policies considered.

The main insight to be gained from the paper's analysis is that the relationship between distributive policies and growth is more complicated - in theory and especially when comparing countries - as often suggested.

The paper is organized as follows: Section 2 sets up the model and presents the optimal policies of pro-capital and pro-labour governments. Section 2.1 provides a comparative steady state analysis of these policies and compares them to a strictly factor income egalitarian policy. Section 3 provides concluding remarks.

2 The Model

The economy is populated by two types of many, price-taking and infinitely lived individuals who are all equally patient. One group of agents, the capitalists, owns wealth equally and does not work. The other group is made up of workers who own (raw) labour equally, but no capital.[2] Population is stationary and consists of l workers and n capitalists of whom there are less, that is, $l > n$. Each individual derives logarithmic utility from the consumption of a homogeneous, malleable good. Aggregate output is produced according to

$$Y_t = A\, K_t^{\alpha}\, G_t^{1-\alpha}\, L_t^{1-\alpha} \quad , \quad 0 < \alpha < 1 \tag{1}$$

where Y_t denotes aggregate output, K_t is the real capital stock, L_t is labour supplied, and G_t are public inputs to production.[3] Capital is broadly defined and by assumption human capital is strictly complementary to physical capital.

Thus, in the model capitalists who, for instance, own computers know how to operate them as well. This eliminates a separate treatment of how human capital is accumulated and entails that the return on human capital services equals that of physical capital services in a perfectly competitive economy. For a justification of such an approach in a different context see Mankiw, Romer and Weil (1992).

The variable A is a constant efficiency index, which reflects the economy's state of technology. It depends on cultural, institutional and technological development and captures long-run, exogenous factors that play a role in the production process.

[2]The assumption uses a short-cut of a result in Bertola (1993). He has shown in an endogenous growth model that for utility maximizing, infinitely lived agents who do not own initial capital, it is not optimal to save/invest out of wage income along a *long-run*, i.e. steady state, balanced growth path. Similarly, it is not optimal to work for those who only own capital initially. Thus, the set-up is reminiscent of Kaldor (1956), where different proportions of profits and wages are saved. However, in Kaldorian models growth determines factor share incomes, whereas in endogenous growth models the direction is rather from factor shares to growth.

[3]Like Barro (1990) one may assume that the government owns no capital and that it buys a flow of output from the private sector and makes it available to the *individual* firm. Then public inputs to production would be rival. Alternatively one may assume that *total* government expenditure affects private production in a non-rival way. By assumption this empirically relevant distinction does not matter analytically in this model. Note that in the absence of a government, for instance, due to civil war or other forms of unrest, the economy would break down and the agents would starve.

Each worker inelastically supplies $\frac{1}{l}$ units of labour at each point in time. As there are l workers in the economy, $L_t = 1$ so that the total labour endowment equals unity. Furthermore, the model abstracts from problems arising from the depreciation of the capital stock so that output and factor returns are really defined in net terms. This has no consequences for the price-taking, market clearing logic of the model.

The Public Sector. The paper follows Alesina and Rodrik by analyzing a wealth tax scheme which is meant to serve as a metaphor capturing the *essential* features of many different sets of (re-)distributive policies.[4] The government taxes wealth at the constant rate τ and redistributes a constant share λ of its tax revenues to the workers. The tax on capital should be viewed as a tax on all resources that are accumulated, including human capital. Unskilled labour is not subject to taxation in this model.

Alesina and Rodrik (1994) call '*redistribution*' any policy that distributes *income* to the non-accumulated factor of production while reducing the incentive to accumulate. Thus, they assess income redistribution relative to growth maximizing policies. In terms of income distribution it is not entirely clear why those policies should serve as a benchmark. For example, it may well be the case that moving from a growth maximizing to some other policy may increase income inequality and decrease growth. Most people would assess such a redistributing policy shift with reference to a policy that grants equal incomes. Thus, here *redistribu-*

[4] As tax schemes differ widely across countries due to historical, institutional or political differences an answer to the question why a society chooses a particular scheme has to remain outside of this model. For similar arguments and example what redistributive mechanisms the wealth tax scheme may capture see Alesina and Rodrik's paper. Furthermore, in the same framework they show that the optimal policies are constant over time and, thus, time-consistent. For convenience constancy of policy is assumed from the beginning in this paper. In line with most of the literature on capital taxation the paper abstracts from taxation of raw labour. That allows one to focus on the distributional conflicts between accumulated and non-accumulated factors of production. By assumption expropriation of capital is ruled out for the governments. Although a command optimum in the model would involve expropriation of capital even for a government maximizing the welfare of the capital owners, it is ruled out since it is not very common in the real world.

tion is defined as taking real resources (wealth) from the accumulated factor of production by giving them to the non-accumulated factor of production.

The government faces the balanced budget constraint,

$$\tau K_t = G_t + \lambda \tau K_t. \tag{2}$$

Of the tax revenues τK_t the workers receive $\lambda\tau K_t$ as transfers and G_t is spent on public inputs to production. The parameter λ represents the degree of (capital) redistribution in the economy.[5]

The Private Sector There are many identical, profit-maximizing firms which operate in a perfectly competitive environment. They are owned by the capital owners who rent capital to and demand shares of the firms. The shares are collateralized one-to-one by capital. The markets for assets and capital are assumed to clear at each point in time. The firms take G_t as given, and rent capital and labour in spot markets in each period. The price of output serves as numéraire and is set equal to one. Profit maximization entails that firms pay each factor of production its marginal product

$$r = \alpha A[(1-\lambda)\tau]^{1-\alpha} \tag{3}$$

$$w_t \equiv \eta(\tau,\lambda)K_t = (1-\alpha)A[(1-\lambda)\tau]^{1-\alpha}K_t. \tag{4}$$

Because of the productive role of government services, policy has a bearing on the marginal products. The return on capital is constant over time while the wages grow with the capital stock. Notice that more redistribution lowers r and η, while higher taxes raise them.

The total wage and transfer income is $\eta(\tau,\lambda)K_t + \lambda\tau K_t$. Each worker receives an equal share of it and derives utility from consuming

[5] As human and physical capital are strict complements by assumption this is a strong from of redistribution. It implies that if a capital good is given to the workers the corresponding services necessary to operate that good are also given to them. As a one good economy is contemplated, giving the capital good to the workers for consumption does not cause a problem.

his entire income. The representative worker's intertemporal welfare is given by

$$\int_0^\infty \ln c_t^W \, e^{-\rho t} dt \quad \text{where} \quad c_t^W = (\eta(\tau, \lambda) + \lambda\tau)\, \tilde{k}_t, \tag{5}$$

where $\tilde{k}_t \equiv \frac{K_t}{l}$. Thus, the owners of the non-accumulated factor of production do not invest and are not taxed by assumption.

The capitalists choose how much to consume or invest. They have perfect foresight about the price and tax rate paths, which they take as given. The representative capital owner maximizes his intertemporal utility according to

$$\max_{c_t^k} \int_0^\infty \ln c_t^k \, e^{-\rho t} dt \tag{6}$$

$$s.t. \qquad \dot{k}_t = (r - \tau)k_t - c_t^k \tag{7}$$

$$k(0) = \overline{k_0}, \quad k(\infty) = \text{free}, \tag{8}$$

where $k_t \equiv \frac{K_t}{n}$. Equation (7) is the dynamic budget constraint of the capitalist which depends on his after-tax income. The growth rate of consumption and wealth can be calculated in a standard way (see Appendix A) and is given by

$$\gamma \equiv \frac{\dot{c}_t^k}{c_t^k} = \frac{\dot{k}_t}{k_t} = (r - \tau) - \rho. \tag{9}$$

Growth is increasing in the after-tax return on capital and constant over time. Furthermore, from (9) and (7) one verifies that $c_t^k = \rho k_t$ is the capitalist's optimal level of consumption.

Market Equilibrium. Constant policies imply constant r and hence γ. The economy's overall resource constraint is

$$I_t = \dot{K}_t = (r - \tau)K_t + (\eta + \lambda\tau)K_t - C_t^k - C_t^W. \tag{10}$$

As the workers' consumption is $C_t^W = (\eta + \lambda\tau)K_t$ in the aggregate, this constraint is binding, simplifying (10) to $\dot{K}_t = (r - \tau)K_t - C_t^k$. The capitalists' consumption $C_t^k = nc_t^k$ and wealth $K_t = nk_t$ grow at the constant rate γ. Substitute $G_t = (1-\lambda)\tau K_t$ in (1). Recalling $L_t = 1$ and taking logarithms and time derivatives yields $\frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = \frac{\dot{G}_t}{G_t}$. Hence, the economy is characterized by *balanced growth* with $\gamma = \frac{\dot{c}_t^k}{c_t^k} = \frac{\dot{k}_t}{k_t} = \frac{\dot{c}_t^W}{c_t^W} = \frac{\dot{C}_t^k}{C_t^k} = \frac{\dot{K}_t}{K_t} = \frac{\dot{C}_t^W}{C_t^W} = \frac{\dot{G}_t}{G_t} = \frac{\dot{Y}_t}{Y_t}$.

As $\gamma = (r-\tau)-\rho$ and $r = \alpha A[(1-\lambda)\tau]^{1-\alpha}$, growth is first increasing and then decreasing, that is, concave in τ and maximized when $\lambda = 0$ and $\tau = [\alpha(1-\alpha)A]^{\frac{1}{\alpha}} \equiv \hat{\tau}$. Thus, if high taxes - for example for redistribution of wealth - are levied, growth is traded off against redistribution when $\tau \gtrless \hat{\tau}$.

Notice $r - \tau = \tau\left(\alpha A[(1-\lambda)\tau]^{-\alpha} - 1\right)$ so that for given policy an increase in A raises growth and implies an upward shift of the concave relationship between taxes and growth. Furthermore, the maximum after-tax return, $\hat{r} - \hat{\tau} = \hat{\tau}\left(\frac{\alpha}{1-\alpha}\right)$, is increasing in A since $\frac{d(\hat{r}-\hat{\tau})}{dA} = \left(\frac{\alpha}{1-\alpha}\right)\frac{d\hat{\tau}}{dA}$ where $\frac{d\hat{\tau}}{dA} = \hat{\tau}\,[\alpha A]^{-1} > 0$. Hence, $\frac{d\hat{\gamma}}{dA} > 0$ as well.

Lemma 1 *An increase in efficiency raises growth for given policy. Furthermore, it raises the maximum after-tax return, the growth maximizing tax rate and maximum growth.*

The result that a more efficient economy has higher growth corresponds to common economic intuition. Interestingly, however, for maximum growth taxes must also be higher in the model which is due to the externality of public inputs in production.

The Government. The governments represent the representative worker or capital owner. The intertemporal welfare of an entirely pro-capital, V^r, resp. entirely pro-labour government, V^l, is given by

$$V^r(c_t^k) = \frac{\ln(\rho k_0)}{\rho} + \frac{\gamma}{\rho^2} \quad \text{and} \quad V^l(c_t^W) = \frac{\ln\left[(\eta(\tau,\lambda) + \lambda\tau)\tilde{k_0}\right]}{\rho} + \frac{\gamma}{\rho^2}. \quad (11)$$

(See Appendix B.) The governments respect the right of private property and maximize the welfare of their clientele under the condition $\lambda \geq 0$. That restricts the governments in that even a pro-capital government does not tax workers, because a negative λ would effectively amount to a tax on wages.

The optimal *pro-labour* policy is derived in Appendix C and is given by

If $\rho \geq [(1-\alpha)A]^{\frac{1}{\alpha}}$ then:

$$\tau = \rho, \qquad \lambda = 1 - \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{\rho}. \tag{12}$$

If $\rho < [(1-\alpha)A]^{\frac{1}{\alpha}}$ then:

$$\tau[1 - \alpha(1-\alpha)A\tau^{-\alpha}] = \rho(1-\alpha), \qquad \lambda = 0. \tag{13}$$

Let $\check{\tau}$ solve these equations and notice that for a wide range of parameter values the pro-labour government chooses not to redistribute wealth. In contrast, the *pro-capital* government chooses $\tau = \hat{\tau}$, does not redistribute wealth and acts growth maximizing by granting the maximum after-tax return on capital.[6]

2.1 A Comparative Dynamic Analysis

Political preferences alone do not rule out the possibility of choosing a high growth policy in the model. An almost trivial, but important point in this context is that a pro-labour government mimics a growth maximizing policy if the workers are very patient. To see this let $\rho \to 0$ in equation (13).

Proposition 1 *A pro-labour government mimics a growth maximizing policy if the workers are very patient.*

[6] In Appendix C it is also shown that any government that attaches more social weight on the capitalists' welfare would choose taxes closer to $\hat{\tau}$ leading to higher growth. All the subsequent results would then hold in relative terms.

This is a special, but interesting case because it means that a government placing maximal weight on the non-accumulated factor of production, but at the same time putting almost equal weight on the welfare of future generations (low ρ) may act like a growth maximizer.[7] Furthermore, in this case the measured tax and growth rates under an optimal pro-labour, pro-capital or a growth maximizing policy would be *observationally* indistinguishable.

Suppose the government redistributes wealth in the optimum. Equation (12) implies $(1-\lambda)\check{\tau} = [(1-\alpha)A]^{\frac{1}{\alpha}}$. If $\Theta \equiv (1-\alpha)A$, then

$$r = \alpha A[(1-\lambda)\check{\tau}]^{1-\alpha} = \alpha A \Theta^{\frac{1-\alpha}{\alpha}} = \left(\frac{\alpha}{1-\alpha}\right)\Theta^{\frac{1}{\alpha}}.$$

But $\check{\tau} = \rho \geq \Theta^{\frac{1}{\alpha}}$ in (12), and $\gamma > 0$ requires $r - \check{\tau} - \rho > 0$. So $\check{\tau}$ has to satisfy

$$\check{\tau} > \Theta^{\frac{1}{\alpha}} \;\wedge\; \left(\frac{\alpha}{1-\alpha}\right)\Theta^{\frac{1}{\alpha}} > 2\check{\tau} \;\Leftrightarrow\; \check{\tau} \left(\frac{\alpha}{1-\alpha}\right)\Theta^{\frac{1}{\alpha}} > \Theta^{\frac{1}{\alpha}}\, 2\check{\tau} \;\Leftrightarrow \alpha > \frac{2}{3}.$$

Thus, the share of capital has to be sufficiently more important than that of public inputs or labour.[8] Furthermore, for an increase in A one finds $\frac{d\lambda}{dA} < 0$ so that λ would be lower in the new optimum.

Proposition 2 *If growth is positive under a redistributing policy then $\alpha > \frac{2}{3}$. Under its optimal policy an increase in efficiency makes the government redistribute less wealth.*

The proposition is empirically relevant and testable. It entails that an increase in efficiency causes the government to redistribute *less* wealth and place more weight on growth. Thus, there is an interesting trade-off

[7]The two policies coincide only if $\rho \to 0$ which causes problems for the convergence of the utility indices. For the observability argument it suffices that ρ is very low while the utility indices still converge.

[8]The Cobb-Douglas technology implies that the elasticity of output with respect to (broad) capital equals α and is constant over time. Coupled with the assumptions of perfect competition and profit maximization, α also denotes the (constant) *share of (broad) capital* (income) in total income $\frac{r_t K_t}{Y_t}$. Thus, α allows for two interpretations in the model, one referring to technology and the other one to distribution.

between growth, redistribution of wealth <u>and</u> technological efficiency in the model.

Next, turn to a pro-labour government that does not redistribute. The effect of an increase in A on taxes in (13) is

$$\left(1-\alpha(1-\alpha)^2 A\tau^{-\alpha}\right) d\tau - \left(\alpha(1-\alpha)\tau^{1-\alpha}\right) dA = 0$$
$$\frac{d\tau}{dA} = \alpha(1-\alpha)\tau\left(\tau^{\alpha}-\alpha(1-\alpha)^2 A\right)^{-1}. \qquad (14)$$

As $\check{\tau} > \hat{\tau}$, the expression is positive.[9] Hence, an increase in efficiency makes a non-redistributing, pro-labour government increase its optimal tax rate. Next, $\frac{d\gamma}{dA} = r_A + (r_\tau - 1)\frac{d\tau}{dA} > 0$ if[10]

$$\alpha\tau^{1-\alpha} \;>\; \left(1-\alpha(1-\alpha)A\tau^{-\alpha}\right)\left[\alpha(1-\alpha)\tau\left(\tau^{\alpha}-\alpha(1-\alpha)^2 A\right)^{-1}\right]$$

$$\tau^{\alpha}-\alpha^2(1-\alpha)^2 A \;>\; (1-\alpha)\tau^{\alpha}-\alpha^2(1-\alpha)^2 A$$

which is equivalent to $1 > 1-\alpha$ and true. Thus, $\frac{d\check{\gamma}}{dA} > 0$ if $\lambda = 0$ in (13).

Suppose the government redistributes wealth. Then Proposition 2 and equation (12) imply $\frac{d\check{\tau}}{dA} = 0$ and $\frac{d\check{\lambda}}{dA} < 0$. Then $\frac{d\gamma}{dA} = r_A + r_\lambda \frac{d\lambda}{dA} > 0$ since $r_\lambda < 0$.

Proposition 3 *The optimal policies of a pro-capital or pro-labour government imply that higher efficiency leads them to choose either higher taxes when $\lambda = 0$ or the same taxes and lower redistribution. An increase in efficiency leads to higher growth under the optimal pro-capital or pro-labour policy.*

As a better technology raises long-run growth under both policies it is an interesting question what its welfare implications are. From (11) one verifies that $0 < \frac{dV^r}{dA}_{|\hat{\tau}} < \frac{dV^l}{dA}_{|\check{\tau}}$. See Appendix D. Thus, in the

[9]To see this notice that $\frac{d\tau}{dA} > 0$ requires $\tau^{\alpha} > \alpha(1-\alpha)^2 A$ which is equivalent to $\tau > \hat{\tau}(1-\alpha)^{\frac{1}{\alpha}}$ and always satisfied since $\check{\tau} > \hat{\tau}$ and $(1-\alpha)^{\frac{1}{\alpha}} < 1$.

[10]The partial derivative of function x w.r.t. a variable y (other than t) will be denoted by x_y.

model an advance in technology would benefit a pro-labour government relatively *more* in the long run than a pro-capital government.[11]

Proposition 4 *Governments that represent the non-accumulated factor of production only and that wish to redistribute resources from the accumulated to the non-accumulated factor of production have a relatively greater incentive in the long run to have an economy with a superior technology than governments representing the accumulated factor of production only.*

The result suggests interesting long-run consequences of the effects of e.g. institutional reform on growth and welfare. Of course, things may be different in the short run when workers might have to learn new technologies or there is resistance to reform. For models studying these issues see e.g. Fernandez and Rodrik (1991), Helpman and Rangel (1999) or Canton, de Groot and Nahuis (1999).

It is important to notice that Proposition 4 applies to governments. For a given policy the worker's or capital owner's welfare may react differently to changes in A. In this context Appendix D also shows the following

Proposition 5 *For given pro-capital or pro-labour policies the workers never benefit less from technical progress in the long run than the capital owners. Unless the pro-labour policy redistributes wealth, the workers benefit relatively more than the capital owners.*

That result allows for various interpretations. For instance, if the workers benefit relatively more from technical progress in the long run than the capital owners, they should be relatively more interested in innovations and should be willing to pay a higher (shadow) price for it. Such prices may, for instance, have to be paid for short-run (in the model

[11] The same would hold if the government represented the *group* of workers or of capitalists. Furthermore, the proposition provides a closed economy analogue to the argument in Rehme (1999) that redistributing *governments* in highly integrated economies may have a relatively stronger interest in technical progress in the *long* run than *governments* that do not redistribute towards the non-accumulated factor of production.

pre-t_0) phenomena such as the pain to learn new technologies, short-run unemployment or any adverse effects on the income distribution.

Distribution. *Pre-tax* total factor income inequality is denoted by $F^g = \frac{rK_t}{\eta K_t} = \frac{\alpha}{1-\alpha}$, which is independent of capital or policy, and increasing in the share of capital. Similarly, define *post-tax* total factor income inequality as

$$F(\tau, \lambda) = \frac{\text{total post-tax capital income}}{\text{total post-tax wage income}} = \frac{(r-\tau)K_t}{(\eta+\lambda\tau)K_t} = \frac{(r-\tau)}{(\eta+\lambda\tau)}, \quad (15)$$

which is also independent of capital, but depends on policy. By assumption there are more workers than capital owners and there is no inequality in intra-group incomes.

Obviously, these 'inequality measures' are extremely crude. They ignore intra-group inequality, the population composition and other things. The justification for employing them is the following: Any policy change in this model will affect the personal *and* the factor income distribution which may not always be the case when analyzing personal income distributions.

Suppose person i gets income 10 and person j gets income 20. If the government gives 10 to i and takes 10 from j, person i and j would swap places in the total personal income distribution. This is sometimes not recorded as a change in total personal income inequality, especially if i and j have the same utility functions. In this model, however, such a transfer would affect factor income inequality since j may be a capital owner and i may be a worker. The income transfer would make one worker better off and increase total wage income and make one capitalist worse off and reduce total capital income. Of course, if one used a personal income inequality measure that is decomposable so that one can group capital income and wage income recipients, where the groups are weighted, an income transfer from a capitalist to a worker would be recorded as a change in inequality, since intra and inter-group inequality would change. On the complexity of moving from a factor share to a personal income distribution analysis see, for example, Atkinson (1983) or Atkinson and Bourguignon (1999).

Thus, the paper concentrates on situations where policy changes have a direct impact on long-run income inequality among persons via changes in post-tax factor income shares. Below the policies considered so far are compared to an income egalitarian policy, which is strictly committed to granting equal after-tax incomes to each individual. The reason for introducing it is threefold.

Firstly, it allows one to compare policy induced after-tax factor income distributions to one where all agents get the same income. Thus, the income egalitarian policy provides a benchmark from which one may assess how much inequality other policies entail.[12]

Secondly, many people tend to associate pro-labour ('left-wing') with income egalitarian policies. The two clearly involve distinct objectives. A pro-labour government acts in the interest of one particular group. In this model it tries to maximize the welfare of the unskilled workers and is therefore concerned about their *level* of welfare. In contrast, the income egalitarian objective is *relative* in nature in that it compares a worker's and a capitalist's income. Thus, levels do not feature as an objective for an income egalitarian.

However, one has to be careful with this particular egalitarian objective. Many other egalitarian policies are possible and interesting to analyze. For example, a *utilitarian* will attempt to equalize marginal utilities of the agents. A strictly *utility egalitarian* government will try to make everybody equally happy in terms of total individual utility. Furthermore, a *Rawlsian* objective may involve comparing utilities relative to the least well-off. It also raises the complicated issue whether the objectives require equality at each point in time or equality of intertemporal welfare. These issues and other egalitarian objectives are discussed in more depth by, for instance, Sen (1982) or Atkinson and Stiglitz (1980),

[12]For consistency with the paper's definition of 'redistribution' it will be made explicit which benchmark is used for comparisons of alternative distributive policies. Furthermore, income egalitarianism is always meant to be 'strict' in this paper.

chpt. 11.[13] The reason for not considering these other egalitarian policies lies in the aim to analyze the factor income distribution.

Thirdly, as data on income are available, everybody getting equal factor incomes may be a natural reference point for assessing the effects of different distributional policies. In comparison, a factor income distribution that would make everybody equally happy, requiring knowledge about the exact form of welfare functions, appears far more difficult to determine - even in this simple model. This may justify restricting the analysis to strictly income egalitarian policies.[14]

To facilitate the analysis assume that a government with a strictly income egalitarian objective does not redistribute wealth to the workers. Then and for all non-redistributing governments F is given by

$$F(\tau) = \frac{r-\tau}{\eta} = \frac{\alpha}{1-\alpha} - \frac{\tau^{\alpha}}{A(1-\alpha)}. \tag{16}$$

Notice that an increase in taxes shifts income towards labour, reducing F.

The *strictly income egalitarian* policy grants each individual an equal after-tax income, which is achieved if $\frac{\eta K_t}{l} = \frac{(r-\tau)K_t}{n}$. Thus, it does not matter in the model whether the income egalitarian objective requires equality of income at each point in time or over the entire planning horizon. Furthermore, the objective is directly related to F and fixes it at $F^* = \frac{n}{l}$ where $n < l$. Thus, the income egalitarian objective is satisfied when setting taxes such that $F(\tau)$ equals its target F^*.[15] The

[13] Furthermore, these authors show that under some conditions the utility Rawlsian and the utility egalitarian solutions coincide. However, if the social welfare function takes individual utilities as its arguments but is no longer monotonically increasing in them, that is, if it is individualistic, but non-Paretian, the Rawlsian objective will no longer necessarily satisfy the egalitarian principle of equalizing utilities.

[14] This clarification is important since the results presented below apply *only* to the income egalitarian policy. Other egalitarian objectives may lead to different results. Furthermore, notice that in this model a strictly income egalitarian policy coincides with that of an income leximin policy, which may not be the case for total utility egalitarian and utility leximin policies.

[15] The reason for working with F^* rather than with l/n directly is to avoid confusion with the distribution of groups in the economy. Of course, they coincide in this model.

tax rate τ_e that satisfies this is

$$\tau_e = [A(\alpha - (1-\alpha)F^*)]^{\frac{1}{\alpha}} . \tag{17}$$

Note that F^* depends on the number of agents in each group. For consistency $F^* < min\{1, F^g\}$ which is easily met for $l << n$ and reasonable values of α. If there are only a few capital owners then the income egalitarian government chooses a very low F^*, and high τ_e. At the other extreme, assume that there are as many capitalists as workers. This would correspond to a representative agent economy where each household would derive equal income and under intra-group income equality would get equal wage and capital income. A (strictly) income egalitarian government would charge relatively lower taxes in that case. The income egalitarian policy implies

$$\frac{d\tau_e}{dA} = \frac{1}{\alpha}\left[A(\alpha-(1-\alpha)F^*)\right]^{\frac{1}{\alpha}-1}(\alpha-(1-\alpha)F^*) = \tau_e\,[\alpha A]^{-1} > 0$$

so that an increase in A leads to a higher choice of τ_e. For growth one finds

$$\gamma_e = \tau_e\left[\alpha A\tau_e^{-\alpha} - 1\right] = \tau_e\left[\frac{\alpha A}{A(\alpha-(1-\alpha)F^*)} - 1\right]$$

and $\frac{d\gamma_e}{dA} > 0$. Thus, an increase in A also increases growth under the income egalitarian policy.

Proposition 6 *Taxes and growth are higher under a strictly income egalitarian policy when A is larger,* $\frac{d\tau_e}{dA} > 0$*, and* $\frac{d\gamma_e}{dA} > 0$*.*

For a pro-capital policy one verifies that $\hat{F} = \frac{\alpha^2}{1-\alpha}$, which is independent of A. Suppose the pro-labour government chooses to redistribute. From equation (12) $\check{\tau}(1-\lambda) = [(1-\alpha)A]^{\frac{1}{\alpha}}$ and $\check{r} = \alpha A[(1-\alpha)A]^{\frac{1-\alpha}{\alpha}}$ so that $\check{r}$ and $\check{\eta}$ are independent of λ. Then $\eta + \lambda\check{\tau} = (1-\alpha)A[(1-\alpha)A]^{\frac{1-\alpha}{\alpha}} + \rho - [(1-\alpha)A]^{\frac{1}{\alpha}} = \rho$ so that $\check{F}(\lambda > 0) = \frac{\check{r}-\rho}{\rho} = \frac{\alpha A[(1-\alpha)A]^{\frac{1-\alpha}{\alpha}}-\rho}{\rho} = \frac{\alpha[(1-\alpha)A]^{\frac{1}{\alpha}}}{(1-\alpha)\ \rho} - 1$ where $\rho > [(1-\alpha)A]^{\frac{1}{\alpha}}$ and $\alpha > \frac{2}{3}$ if $\check{\gamma}(\lambda > 0) > 0$. Clearly, $\frac{d\check{F}(\lambda>0)}{dA} > 0$.

Next, suppose $\rho < [(1-\alpha)A]^{\frac{1}{\alpha}}$ so that $\lambda = 0$. Then $\check{\tau}$ solves equation (13), and from (16) such that $\check{F}(\lambda = 0) < \hat{F}$ as $\check{\tau} > \hat{\tau}$. Furthermore,[16]

$$\frac{d\check{F}(\lambda=0)}{dA} = -\frac{\alpha\check{\tau}^{\alpha-1}(1-\alpha)A\frac{d\check{\tau}}{dA} - (1-\alpha)\check{\tau}^{\alpha}}{(A(1-\alpha))^2} > 0.$$

Lemma 2 *Under the optimal* <u>*pro-capital*</u> *or the* <u>*income egalitarian*</u> *policy technological progress does not change the long-run post-tax factor income distribution.*

Under the optimal <u>*pro-labour*</u> *policy technological progress shifts the long-run post-tax factor income distribution towards capital!*

It is noteworthy that a higher A causes the pro-labour government to shift relatively more income towards the accumulated factor of production. This holds no matter whether the pro-labour government redistributes wealth or not.

Comparative Dynamics. Suppose one hypothetically compares the effects of different policies on the same economy. It is then an interesting question how the income egalitarian policy compares to that of a growth maximizing government. For $\gamma_e = \hat{\gamma}$ one needs $\tau_e = \hat{\tau}$ which is satisfied if $A[\alpha - (1-\alpha)F^*] = \alpha(1-\alpha)A$,

$$a^* = \frac{\sqrt{4F^* + F^{*2}} - F^*}{2}.$$

Thus, for a particular value of the share of capital $\gamma_e = \hat{\gamma}$ so that the income egalitarian policy would be equivalent to a growth maximizing one.

[16] Simplifying and substituting for $\frac{d\check{\tau}}{dA}$ from (14) yields

$$\frac{d\check{F}(\lambda=0)}{dA} = \frac{(1-\alpha)\check{\tau}^{\alpha}}{((1-\alpha)A)^2} - \frac{\alpha^2\check{\tau}^{\alpha}}{A}\left(\check{\tau}^{\alpha} - \alpha(1-\alpha)^2 A\right)^{-1}.$$

The expression is positive if $\check{\tau}^{\alpha} > \alpha(1-\alpha)^2 A + \alpha^2(1-\alpha)A$, that is, if $\check{\tau} > \hat{\tau}$ which the pro-labour government indeed chooses.

Proposition 7 *If* $\alpha^* = \frac{\sqrt{4F^* + F^{*2}} - F^*}{2}$, *then* $\tau_e = \hat{\tau}$, $\gamma_e = \hat{\gamma}$ *and* $F^* = \hat{F}$.

Hence, income egalitarianism is not necessarily bad for growth. The result crucially depends on the income egalitarian government's target F^*.[17] It also allows for another interpretation: If the share of capital equals α^*, then a growth maximizing policy would lead to minimal post-tax factor income inequality. Seen from this angle, the model provides an example that efficiency and equity orientated policies may lead to the same outcome.[18]

However, the model's income egalitarian policy does not maximize growth in general, because

$$\alpha \gtrless \alpha^* \quad \Leftrightarrow \quad \tau_e \gtrless \hat{\tau} \quad \Leftrightarrow \quad \gamma_e < \hat{\gamma}.$$

Suppose $\alpha < \alpha^*$. Then $\tau_e < \hat{\tau} < \check{\tau}$, as a pro-labour government chooses $\check{\tau} > \hat{\tau}$. As $F(\tau)$ is decreasing in τ it follows that $\check{F} < \hat{F} < F^*$, implying that the average capital owner would have a lower income than the average worker. It would also entail that the pro-capital policy would grant more income to a worker than to a capitalist. This is clearly consistent with the model's pro-capital government's objective, which is not concerned with relative income, but seems very implausible and unrealistic. Hence, this cases is not investigated any further.

Suppose $\alpha > \alpha^*$. Then $\tau_e > \hat{\tau}$ and the following holds:

[17] Interestingly, if there are as many capital owners as workers, then strict income egalitarianism calls for $F^* = 1$ in which case $\alpha^* = \frac{\sqrt{5}-1}{2}$, which equals the *golden ratio.*

[18] In general, the two interpretations are not equivalent, however. There may be instances where an income egalitarian policy leads to maximum growth. This may be so if that policy targets a particular personal income distribution, which implies a particular factor income distribution which *may* lead to maximum growth, as the proposition shows. The growth maximizer targets growth, implying a particular factor income distribution, implying many, possibly different personal income distributions. Thus, to get equivalence more assumptions are required. In this model the equivalence is due to the assumption of no intra-group inequality.

Proposition 8 *If $a > a^*$ the pro-capital policy grants more income to the capital owners in comparison to the income egalitarian one. The optimal polices imply*

1. $\rho > \rho_e: \quad \hat{\tau} < \tau_e < \check{\tau} \;\Leftrightarrow\; \hat{F} > F^* > \check{F} \quad \Leftrightarrow \quad \hat{\gamma} > \gamma_e > \check{\gamma}$

   or

2. $\rho < \rho_e: \quad \hat{\tau} < \check{\tau} < \tau_e \;\Leftrightarrow\; \hat{F} > \check{F} > F^* \quad \Leftrightarrow \quad \hat{\gamma} > \check{\gamma} > \gamma_e.$

Hence, the income egalitarian policy may also be close to a pro-labour one. In fact, the two policies coincide if there is a ρ, call it ρ_e, such that $\check{\tau} = \tau_e$. However, the exact relationship between the growth rates under these two policies is ambiguous. Also, it is not clear whether the pro-labour or the pro-capital policy is closer to the income egalitarian policy in terms of post-tax income inequality. In either case the pro-capital policy shifts more income towards capital in comparison to the income egalitarian government. This is what one would expect.

In contrast, the pro-labour government may shift relatively more income to capital or labour depending on how patient the workers are. If they are very impatient, $\tau_e < \check{\tau}$ and they will shift relatively more income to the workers, leading to lower growth than under the income egalitarian policy. If they are patient, the pro-labour government chooses to shift relatively more income to capital and there will be higher growth than under the income egalitarian policy. As $\frac{d\check{F}(\lambda \geq 0)}{dA} > 0$ the effect of technological progress under a pro-labour policy is also ambiguous. It is inequality reducing if $\check{F} < F^*$ and inequality enhancing if $\check{F} > F^*$.

Thus, the pro-labour government grants more or less income to the workers and it has lower or higher growth than the income egalitarian government. Technological progress may reduce or increase income inequality under the pro-labour policy compared to the income egalitarian policy. All these results depend on how patient the agents are. It may be noteworthy that for patient workers, the pro-labour policy leads to a post-tax factor income distribution that is more favourable to capital in comparison to the income egalitarian policy. The workers are, however, compensated for this by higher growth.

This highlights that the income egalitarian government is strictly concerned about relative incomes, whereas the pro-labour government cares about the level of the workers' welfare. Thus, $\check{F} > F^*$, which implies that the capitalists get relatively more income than the workers under a pro-labour government, is consistent with that government's objective, as it gives the workers the highest welfare.[19]

The workers would in general not prefer an income egalitarian policy, although it might give them more income at each date t. This is so, because growth is higher under the pro-labour policy, granting them higher consumption in the future which they prefer if they are patient. The model, thus, shows how misleading it may be to identify income egalitarian with pro-labour policies.

Hence, strictly income egalitarian and pro-labour policies are generally not the same in the model and lead to quite different post-tax factor income distributions and growth performances.

[19] Suppose a utility egalitarian chooses a policy somewhere between that of the income egalitarian and a Rawlsian. Furthermore, compare policies relative to F^*. If an outcome with some income inequality is better for the worse-off group than a strictly income egalitarian outcome then it is better according to a Rawlsian (utility leximin) policy. Thus, if $\check{F} > F^*$ and the workers are worse off than the capitalists under a pro-labour policy, then a Rawlsian would choose lower taxes and move closer to the workers' preferred, pro-labour policy. The reason is that such a move would be Paretian as all agents would prefer an F such that $F > F^*$. Thus, a (total) utility egalitarian objective satisfying the Pareto-principle would also imply a policy $F > F^*$. Thus, for patient agents a large class of egalitarian objectives satisfying the Pareto-principle would shift relatively more income to capital than to labour. The argument is different when $\hat{F} < F^* < \check{F}$, and the income egalitarian policy leads to higher growth than the pro-labour policy. If evaluated relative to F^*, it is not so clear what a utility leximin policy chooses, that is, whether it would be closer to $\hat{F}$ or $\check{F}$. A precise analysis would require whether that policy is concerned about the worst-off at any point in time or the worst-off intertemporal welfare of the agents. In either case any policy away from F^* would violate the Pareto-principle since it would make one group better off and another one worse off. In that case it is also unclear what a total utility egalitarian would choose relative to a Rawlsian.

The share of capital. The importance of the share of capital for the relationship between distribution, politics and growth has, for instance, been emphasized by Saint-Paul (1992), Buiter (1993), Alesina and Rodrik (1994), ftn. 7, Stokey and Rebelo (1995), or Bertola (1999). Often a higher share of capital is shown to imply higher growth. That reflects the logic of models, in which growth is driven by the accumulated factor of production. In the present model and for given policy, that is, for given τ, an increase in α raises growth, and it increases the after-tax factor income share going to capital (higher F).[20]

However, treating policy as given may be misleading, as distributional changes often induce policy reactions which attempt to counteract any negative effects for a government's clientele. The following table summarizes the long-run effects of changes in the share of capital when policy reacts to these changes.

Table 1: Growth and Policy Effects

	PC			IE			PL, $\lambda = 0$			PL, $\lambda \geq 0$			
	$\hat{\tau}$	$\hat{\gamma}$	$\hat{F}$	τ_e	γ_e	F^*	$\check{\tau}$	$\check{\gamma}$	$\check{F}$	$\check{\tau}$	λ	$\check{\gamma}$	$\check{F}$
α	?	+	+	+	?	0	?	+	+	0	?	+	?

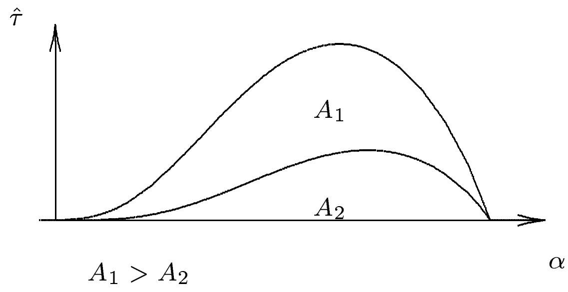
PC - pro-capital, IE - income egalitarian PL - pro-labour
Sign: (+) - positive, (−) - negative, (?) - ambiguous

Thus, there are no unambiguous responses of long-run growth, policy or the relative after-tax income shares to changes in α. For instance, a plot of the growth maximizing tax rate reveals a pattern as in Figure 1. Thus, it is possible for given A that two values of α lead to the same $\hat{\tau}$, but different long-run growth rates.

The pro-capital and non-redistributing, pro-labour policies react in the same way. Growth is higher as a result of a larger share of capital,

[20] The effects of α are derived in Appendix E.

Figure 1: The growth maximizing τ for different α and A



but the tax rate may go up or down. Under both policies it is optimal to shift relatively more after-tax factor income to capital. Thus, if capital gets relatively more income, taxes may be raised, but growth and the relative income of the capitalists may still increase as a result. Under a redistributing policy growth would also be higher, but taxes rates would not not change in the model. Redistribution and after-tax factor income inequality may go up or down.

The income egalitarian policy corrects for an increase in relative after-tax factor income inequality caused by a higher α. It raises the tax rate as a result which has an ambiguous effect on growth. The result is interesting, because it shows that policy plays a role for growth when the accumulated factor of production receives more pre-tax factor income. Depending on initial relative (pre-tax) factor shares an increase in inequality (higher α) may lead to higher or lower growth with higher tax rates.

Hence, distributional changes generally produce ambiguous effects on the long-run association between growth and policy.

3 Concluding Remarks

This paper investigates the long-run relationship between public policies and growth. Within a common theoretical framework it is shown that optimizing governments take account of fundamental economic variables when making their decisions. Thus, the paper focuses on endogenous policy.

In the model the optimal policies of pro-capital or pro-labour, and 'income egalitarian' governments are analyzed and changes of fundamental economic variables have interesting effects on policies and through the latter on growth and income distribution. Several findings of the paper are noteworthy.

First, under certain conditions growth maximizing policies may also be pursued if a government has welfare objectives other than those which are optimal for the the non-accumulated factor of production.

Second, an increase in technological efficiency generally raises taxes and growth but also the agents' welfare under the optimal policies considered. Interestingly, the relative welfare gains are found to be often higher for the workers and always higher for a pro-labour government.

Third, within a framework predicting that redistribution towards the non-accumulated factor of production slows down growth it is shown that redistribution is only optimal under restrictive conditions. In the optima considered an increase in technological efficiency reduces the incentive to redistribute so that a testable implication of the model is whether a more advanced country relies more or less on direct wealth transfers as a means to pursue redistributive objectives.

Obviously, economic growth is influenced by many things. This paper argues that analyzing the long-run interplay of fundamental economic variables and public policy may provide useful insights about differences in growth and distribution experiences within or across countries. But then further research on the effects of endogenous policy on long-run growth would seem to be desirable.

A The Capital Owners' Optimum

The necessary first order conditions for the maximization problem are given by equations (7), (8) and

$$\frac{1}{c_t^k} - \mu_t = 0 \tag{A1a}$$

$$\dot{\mu}_t = \mu_t \rho - \mu_t \left(r - \tau\right) \tag{A1b}$$

$$\lim_{t \to \infty} k_t \mu_t e^{-\rho t} = 0. \tag{A1c}$$

where μ_t is a positive co-state variable. From equation (A1a), (A1b) it follows that $\frac{\dot{c}_t^k}{c_t^k} = (r - \tau) - \rho$. Furthermore, for constant τ and from the transversality condition (A1c) and the budget constraint (7) it follows that $\frac{\dot{c}_t^k}{c_t^k} = \frac{\dot{k}_t}{k_t}$. Thus, $\gamma = \frac{\dot{c}_t^k}{c_t^k} = \frac{\dot{k}_t}{k_t}$.

B Welfare

The agents' welfare is $V^r = \int_0^t \ln c_t^k \; e^{-\rho t}$ and $V^l = \int_0^t \ln c_t^W \; e^{-\rho t}$. Let $t \to \infty$ and use integration by parts. For this define $v_2 = \ln c_t^j$, $dv_1 = e^{-\rho t} dt$ where $j = k, W$. Recall that $dv_2 = \frac{\dot{c}_t^j}{c_t^j} = \gamma$ for $j = k, W$ and constant in steady state. Then $v_1 = -\frac{1}{\rho} \; e^{-\rho t}$ so that

$$\begin{aligned} \int_0^\infty \ln c_t^j \; e^{-\rho t} \; dt &= \frac{1}{\rho}\Big[- \; \ln c_t^j \; e^{-\rho t}\Big]_0^\infty + \frac{1}{\rho}\int_0^\infty \gamma \; e^{-\rho t} \; dt \\ &= \frac{\ln c_0^j}{\rho} - \frac{1}{\rho^2}\Big[\gamma \; e^{-\rho t}\Big]_0^\infty \end{aligned}$$

where $c_0^k = \rho k_0$ and $c_0^W = (\eta + \lambda\tau)\tilde{k_0}$. Evaluation at the particular limits yields the expressions in (11).

C Optimal Policies

The government solves: $\max_{\tau,\lambda} \, (1-\beta)\, V^r + \beta\, V^l$ s.t. $\lambda \geq 0$ where β is the social weight attached to each group's welfare. The FOCs are

$$\beta \, \frac{\eta_\tau + \lambda}{(\eta + \lambda\tau)\rho} + \frac{\gamma_\tau}{\rho^2} = 0 \quad , \quad \lambda \left(\beta \, \frac{\eta_\lambda + \tau}{(\eta + \lambda\tau)\rho} + \frac{\gamma_\lambda}{\rho^2} \right) = 0.$$

Notice that γ_τ must be negative for the first equation to hold, so in the optimum $\tau > \hat{\tau}$. Concentrating on an interior solution for λ, simplifying, rearranging and division of the resulting two equations by one another yields

$$\frac{\eta_\tau + \lambda}{\eta_\lambda + \tau} = \frac{\gamma_\tau}{\gamma_\lambda}. \tag{C1}$$

Then $\gamma_\lambda = r_\lambda$ and $\gamma_\tau = r_\tau - 1$ imply $(\eta_\tau + \lambda) r_\lambda = (\eta_\lambda + \tau)(r_\tau - 1)$ which upon multiplying out becomes $\eta_\tau r_\lambda + \lambda r_\lambda = r_\tau \eta_\lambda + r_\tau \tau - \eta_\lambda - \tau$. Notice $r_\lambda \eta_\tau = r_\tau \eta_\lambda$ and $\eta = \frac{1-\alpha}{\alpha} r$. Then $\lambda r_\lambda = r_\tau \tau - \frac{1-\alpha}{\alpha} r_\lambda - \tau$ and so

$$\left(\lambda + \frac{1-\alpha}{\alpha}\right) r_\lambda = \tau r_\tau - \tau \quad \Leftrightarrow \quad \left(\lambda + \frac{1-\alpha}{\alpha}\right) = \frac{\tau r_\tau}{r_\lambda} - \frac{\tau}{r_\lambda}.$$

Recall $r_\tau = \alpha E(1-\lambda), r_\lambda = \alpha E(-\tau)$ where $E = (1-\alpha)A[(1-\lambda)\tau]^{-\alpha}$. Thus, $\frac{\tau r_\tau}{r_\lambda} = -\frac{\tau \alpha E(1-\lambda)}{\alpha E \tau} = -(1-\lambda)$ and $\lambda + (1-\lambda) + \frac{1-\alpha}{\alpha} = -\frac{\tau}{r_\lambda} \Leftrightarrow \frac{r_\lambda}{\alpha} = -\tau$ and so

$$\tau = \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{1-\lambda}. \tag{C2}$$

Notice that for this τ we have $E = 1$. For the first order condition for τ we note that $\eta = (1-\alpha)A[(1-\lambda)\tau]^{1-\alpha} = E[(1-\lambda)\tau] = [(1-\alpha)A]^{\frac{1}{\alpha}}$. Furthermore, $\eta_\tau = (1-\alpha)(1-\lambda),\; r_\tau = \alpha(1-\lambda)$. Eqn. (C2) implies $\lambda = 1 - \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{\tau}$ so that

$$\eta + \lambda\tau = [(1-\alpha)A]^{\frac{1}{\alpha}} + \tau \left(1 - \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{\tau}\right) = \tau.$$

Then the first order condition for τ becomes

$$\beta \frac{\eta_\tau + \lambda}{(\eta + \lambda\tau)} = -\frac{\gamma_\tau}{\rho} \quad \Leftrightarrow \quad \frac{\eta_\tau + \lambda}{\tau} = -\frac{\gamma_\tau}{\beta\rho} \quad \Leftrightarrow \quad \frac{\eta_\tau + \lambda}{\gamma_\tau} = -\frac{\tau}{\beta\rho}.$$

But from above $\frac{\eta_\tau+\lambda}{\gamma_\tau} = \frac{(1-\alpha)(1-\lambda)+\lambda}{\alpha(1-\lambda)-1} = -1$ so that $\tau = \beta\rho$. Thus,

$$\tau = \beta\rho \quad \text{and} \quad \lambda = 1 - \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{\beta\rho}. \tag{C3}$$

which is equation (12) when $\beta = 1$. Recall that these equations hold for $\lambda \geq 0$, thus for $\beta\rho \geq [(1-\alpha)A]^{\frac{1}{\alpha}}$.

Suppose $\lambda = 0$, then the first order condition becomes

$$\frac{\eta_\tau}{\eta} = -\frac{r_\tau - 1}{\beta\rho} \quad \Leftrightarrow \quad \frac{(1-\alpha)E}{\tau E} = -\frac{\alpha E - 1}{\beta\rho} \quad \Leftrightarrow \quad (1-\alpha)\beta\rho = \tau - \alpha\tau E$$

so that the solution with $\lambda = 0$ is given by

$$(1-\alpha)\beta\rho = \tau\left[1 - \alpha(1-\alpha)A\tau^{-\alpha}\right] \tag{C4}$$

which holds only if $\beta\rho < [(1-\alpha)A]^{\frac{1}{\alpha}}$. For $\beta = 1$ this is equation (13) in the text.

If $\beta = 0$ it follows that $\lambda = 0$, $\gamma_\tau = r_\tau - 1 = 0$ and $\tau = \hat{\tau}$. Thus, the pro-capital government acts growth maximizing in the model.

Lemma $\gamma(\tau)$ is inversely related to β.

Proof: $\gamma_\tau < 0$ for $\check{\tau} > \hat{\tau}$ in (12) and (13). Also $\gamma(\tau) = \alpha A\left((1-\lambda)\tau\right)^{1-\alpha} - \tau - \rho$. Clearly, if $\lambda > 0$, then $\frac{d\check{\tau}}{d\beta} > 0$ in (C3), and $(1-\lambda)\tau = [(1-\alpha)A]^{\frac{1}{\alpha}}$. Thus, $\frac{d\gamma}{d\beta} < 0$.

Suppose $\beta > 0$ and $\lambda = 0$. Then $\check{\tau}$ is given as in (C4) so that by the implicit function theorem $\frac{d\check{\tau}}{d\beta} > 0$. Thus, $\frac{d\gamma}{d\beta} = \gamma_\tau \frac{d\check{\tau}}{d\beta} < 0$ which proves the lemma.

D Technology Effects on Welfare

Under the optimal policies the welfare in (11) is given by $V^i(A, \tau(A), \lambda(A))$ where $i = l, r$. An increase in A changes welfare by

$$dV^i = \frac{\partial V^i}{\partial A}\, dA + \frac{\partial V^i}{\partial \tau}\, \frac{\partial \tau}{\partial A}\, dA + \frac{\partial V^i}{\partial \lambda}\, \frac{\partial \lambda}{\partial A}\, dA.$$

By the *envelope theorem* $\frac{\partial V^r}{\partial \tau} = 0$ under the optimal pro-capital policy and $\frac{\partial V^l}{\partial \tau} = \frac{\partial V^l}{\partial \lambda} = 0$ under the optimal pro-labour policy. Thus,

$$\frac{dV^r}{dA}_{|\hat{\tau}} = \frac{\partial \hat{\gamma}}{\partial A}_{|\hat{\tau}} \left(\frac{1}{\rho^2}\right),\ \frac{dV^l}{dA}_{|\check{\tau},\lambda} = \frac{\partial(\eta + \lambda\check{\tau})}{\partial A}_{|\check{\tau},\lambda} \left(\frac{1}{(\eta + \lambda\check{\tau})\rho}\right) + \frac{\partial \check{\gamma}}{\partial A}_{|\check{\tau},\lambda} \left(\frac{1}{\rho^2}\right).$$

Notice that under the optimal pro-labour policy $(\eta + \lambda\check{\tau}) = \rho$ when $\lambda > 0$ and $\frac{\partial \eta}{\partial A}_{|\check{\tau},\lambda=0} \left(\frac{1}{\eta\rho}\right) > 0$ when $\lambda = 0$. But $\frac{\partial \hat{\gamma}}{\partial A}_{|\hat{\tau}} = \alpha\hat{\tau}^{1-\alpha} < \frac{\partial \check{\gamma}}{\partial A}_{|\check{\tau},\lambda} = \alpha\check{\tau}^{1-\alpha}$ because $\check{\tau} > \hat{\tau}$. Hence, $0 < \frac{dV^r}{dA}_{|\hat{\tau}} < \frac{dV^l}{dA}_{|\check{\tau},\lambda}$ so that a *government* representing the average worker benefits relatively more than a *government* representing the average capital owner.

Quite another question is how each *individual*'s welfare is affected by a change in A *given* policy. For instance, under a *pro-capital* policy an increase in A implies $\frac{dV^r}{dA}_{|\hat{\tau}} = \frac{\partial \gamma}{\partial A}\frac{1}{\rho^2}$ for a capital owner. For the worker it implies $\frac{dV^l}{dA}_{|\hat{\tau}} = \frac{\partial V^l}{\partial A} + \frac{\partial V^l}{\partial \tau}\frac{\partial \tau}{\partial A} = \left(\frac{\partial \eta}{\partial A} + \frac{\partial \eta}{\partial \tau}\frac{\partial \tau}{\partial A}\right)\frac{1}{\eta\rho} + \left(\frac{\partial \gamma}{\partial A} + \frac{\partial \gamma}{\partial \tau}\frac{\partial \tau}{\partial A}\right)\frac{1}{\rho^2}$. When $\tau = \hat{\tau}$ none of these derivatives is negative so that $\frac{dV^r}{dA}_{|\hat{\tau}} < \frac{dV^l}{dA}_{|\hat{\tau}}$ and a worker benefits more from technical progress than a capital owner under a pro-capital policy.

Under a $\lambda = 0$, pro-labour policy and using the envelope theorem the welfare changes are $\frac{dV^r}{dA}_{|\check{\tau}} = \left(\frac{\partial \gamma}{\partial A} + \frac{\partial \gamma}{\partial \tau}\frac{\partial \tau}{\partial A}\right)\frac{1}{\rho^2}$ and $\frac{dV^l}{dA}_{|\check{\tau}} = \frac{\partial \eta}{\partial A}\frac{1}{\eta\rho} + \frac{\partial \gamma}{\partial A}\frac{1}{\rho^2}$. As $\frac{\partial \gamma}{\partial \tau} < 0$ and $\frac{\partial \tau}{\partial A} > 0$ when $\tau = \check{\tau}$ it follows that $\frac{dV^r}{dA}_{|\check{\tau}} < \frac{dV^l}{dA}_{|\check{\tau}}$.

Under a $\lambda > 0$, pro-capital policy $(\eta + \lambda\check{\tau}) = \rho$ so that changes in A only affect γ in V^i in (11). But then $\frac{dV^r}{dA}_{|\check{\tau},\lambda} = \frac{dV^l}{dA}_{|\check{\tau},\lambda}$ and the workers and capital owners would benefit equally.

E The Share of Capital

Exogenous Policy. For given τ and positive after-tax returns on capital

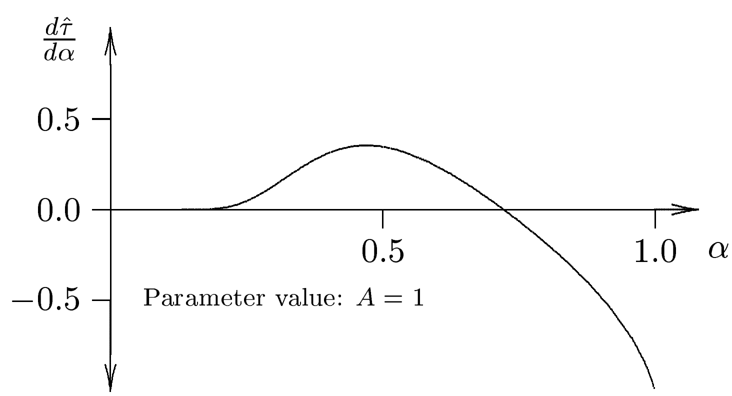
$$\frac{d\gamma}{d\alpha}_{|\tau\ given} = A\tau^{1-\alpha} - \alpha A\tau^{1-\alpha}\ln\tau > 0$$
$$\frac{dF}{d\alpha}_{|\tau\ given} = \frac{1}{1-\alpha} + \frac{\alpha}{(1-\alpha)^2} - \frac{\tau^{\alpha}}{A(1-\alpha)^2} - \frac{\tau^{\alpha}\ln\tau}{A(1-\alpha)} > 0$$

because $\frac{\alpha}{(1-\alpha)^2} - \frac{\tau^{\alpha}}{A(1-\alpha)^2} = \frac{\alpha A\tau^{1-\alpha}-\tau}{A(1-\alpha)^2\tau^{1-\alpha}} > 0$. Hence, for given policy F and γ are increasing in α.

Growth Maximizing Policies. $\hat{F} = \frac{\alpha^2}{1-\alpha}$, $\hat{\gamma} = \frac{\alpha\ \hat{\tau}}{1-\alpha} - \rho$, and $\hat{\tau} = [\alpha(1-\alpha)A]^{\frac{1}{\alpha}}$. Then $\frac{d\hat{F}}{d\alpha} = \frac{2\alpha(1-\alpha)+\alpha^2}{(1-\alpha)^2} > 0$ for all $\alpha \in (0,1)$. Furthermore,

$$\frac{d\hat{\tau}}{d\alpha} = \frac{[\alpha(1-\alpha)A]^{\frac{1}{\alpha}-1}\ (1-2\alpha)A}{\alpha} - \frac{[\alpha(1-\alpha)A]^{\frac{1}{\alpha}}\ \ln[\alpha(1-\alpha)A]}{\alpha^2}$$
$$= \frac{\hat{\tau}\ (1-2\alpha)}{\alpha^2(1-\alpha)} - \frac{\hat{\tau}\ \ln\hat{\tau}}{\alpha} \qquad \text{(E1)}$$

which is not easy to evaluate. Clearly, $\ln\hat{\tau} < 0$ so that $-\frac{\hat{\tau}\ \ln\hat{\tau}}{\alpha} > 0$. But for $\alpha > \frac{1}{2}$ the first expression is negative so that the sign of $\frac{d\hat{\tau}}{d\alpha}$ depends on α. The following plot establishes that $\frac{d\hat{\tau}}{d\alpha} \gtreqless 0$ for a particular level of A.



Thus, for two different values of the share of capital one may have the same $\hat{\tau}$. As $\frac{d\hat{\tau}}{d\alpha} \gtreqless 0$ the sign of $\frac{d\hat{\gamma}}{d\alpha}$ is not clear. For the calculation of $\frac{d\hat{\gamma}}{d\alpha}$

rearrange to get $(\hat{\gamma}+\rho) = \frac{\alpha\,\hat{\tau}}{1-\alpha}$. Then

$$\begin{aligned}
\ln(\hat{\gamma}+\rho) &= \ln\alpha - \ln(1-\alpha) + \ln\hat{\tau} \\
&= \ln\alpha - \ln(1-\alpha) + \frac{\ln(\alpha(1-\alpha)A)}{\alpha} \\
&= \left(\frac{\alpha+1}{\alpha}\right)\ln\alpha + \left(\frac{1-\alpha}{\alpha}\right)\ln(1-\alpha) + \left(\frac{1}{\alpha}\right)\ln A.
\end{aligned}$$

For the effect of a change in α on this expression one gets

$$\begin{aligned}
\frac{d\ln(\hat{\gamma}+\rho)}{d\alpha} &= -\left(\frac{1}{\alpha^2}\right)\ln\alpha + \left(\frac{\alpha+1}{\alpha^2}\right) - \left(\frac{1}{\alpha^2}\right)\ln(1-\alpha) - \frac{1}{\alpha} - \left(\frac{1}{\alpha^2}\right)\ln A \\
&= \frac{1}{\alpha}\left[\frac{1}{\alpha} - \left(\frac{1}{\alpha}\right)\ln(\alpha(1-\alpha)A)\right] = \frac{1}{\alpha}\left[\frac{1}{\alpha} - \ln\hat{\tau}\right].
\end{aligned}$$

As $\hat{\tau} < 1$ the expression is positive. Then $\frac{d\ln(\hat{\gamma}+\rho)}{d\alpha} > 0$ which implies $\frac{d\hat{\gamma}}{d\alpha} > 0$.

Income Egalitarian Policies. Under that policy $\frac{dF^*}{d\alpha} = 0$. Rearranging one obtains $\gamma_e + \rho = \tau_e(\alpha A \tau_e^{-\alpha} - 1)$ where $\tau_e = [A(\alpha - (1-\alpha)F^*)]^{\frac{1}{\alpha}}$. Then

$$\frac{d\tau_e}{d\alpha} = -\frac{1}{\alpha^2}\tau_e\ln\tau_e + \frac{1}{\alpha}\left[A(\alpha-(1-\alpha)F^*)\right]^{\frac{1}{\alpha}-1} A(1+F^*) > 0, \qquad \text{(E2)}$$

that is, $\frac{d\tau_e}{d\alpha}$ is positive. Substitution and simplification imply

$$\begin{aligned}
\ln(\gamma_e+\rho) &= \ln\tau_e + \ln(1-\alpha) + \ln F^* - \ln(\alpha-(1-\alpha)F^*) \\
&= \frac{1}{\alpha}\ln A + \frac{1-\alpha}{\alpha}\ln(\alpha-(1-\alpha)F^*) + \ln(1-\alpha) + \ln F^*.
\end{aligned}$$

Taking the derivative with respect to α yields

$$\begin{aligned}
\frac{d(\gamma_e+\rho)}{d\alpha} &= -\frac{1}{\alpha^2}\ln A + -\frac{1}{\alpha^2}\ln(\alpha-(1-\alpha)F^*) \\
&\quad + \frac{1-\alpha}{\alpha}\left[\frac{1+F^*}{\alpha-(1-\alpha)F^*}\right] - \frac{1}{1-\alpha} \\
&= -\frac{1}{\alpha}\ln\tau_e + \frac{1}{\alpha(1-\alpha)}\left[\frac{(1-\alpha)^2(1+F^*)}{\alpha-(1-\alpha)F^*} - \alpha\right].
\end{aligned}$$

The expression is positive if $(1-\alpha)^2(1+F^*) > \alpha^2 - \alpha(1-\alpha)F^*$, that is, if $F^* > \frac{2\alpha-1}{1-\alpha}$ and $\alpha \leq \frac{1}{2}$. For $F^* < \frac{2\alpha-1}{1-\alpha}$ it may be negative, if α is sufficiently large. So if $\alpha \leq \frac{1}{2}$, then definitely $\frac{d\gamma_e}{d\alpha} > 0$. Thus, $\frac{d(\gamma_e+\rho)}{d\alpha}$ is positive for $\alpha \leq \frac{1}{2}$ and may be ambiguous if $\alpha > \frac{1}{2}$.

Redistributing, Pro-Labour Policies. Equation (12) implies $\frac{d\check{\tau}}{d\alpha} = 0$ since $\check{\tau} = \rho$. For $\lambda = 1 - \frac{[(1-\alpha)A]^{\frac{1}{\alpha}}}{\rho}$ let $c \equiv (1-\alpha)A$, then

$$\frac{d\lambda}{d\alpha} = -\frac{1}{\rho}\left[-\frac{Ac^{\frac{1-\alpha}{\alpha}}}{\alpha} - \frac{c^{\frac{1}{\alpha}}\ln c}{\alpha^2}\right] = \frac{c^{\frac{1}{\alpha}}}{\alpha\rho}\left[\frac{1}{1-\alpha} + \frac{\ln(1-\alpha)+\ln A}{\alpha}\right].$$

Suppose A is low (e.g. $A < e^{-\frac{\alpha}{1-\alpha}}$), then $\frac{d\lambda}{d\alpha} < 0$. Next, suppose $c \approx 1$, then $\frac{d\lambda}{d\alpha} > 0$. Thus, the sign of $\frac{d\lambda}{d\alpha}$ is ambiguous. Since $\check{F} = \frac{\alpha}{1-\alpha} - \frac{\rho^\alpha}{(1-\alpha)A}$ one gets

$$\frac{d\check{F}}{d\alpha} = \frac{1}{(1-\alpha)^2} - \frac{A(1-\alpha)\rho^\alpha \ln\rho + A\rho^\alpha}{A^2(1-\alpha)^2}.$$

For $A \to 0$ the expression becomes negative. For $\rho < e^{-\frac{1+x}{1-\alpha}}$ with some small, positive x, the expression becomes positive. Hence, the sign of $\frac{d\check{F}}{d\alpha}$ is ambiguous.

Under the $\lambda > 0$ policy (see (12)) $r = \alpha A[(1-\lambda)\tau]^{1-\alpha} = \frac{\alpha}{1-\alpha}[(1-\alpha)A]^{\frac{1}{\alpha}}$. The growth rate can be rearranged as follows

$$\ln(\gamma + 2\rho) = \ln\alpha + \frac{1-\alpha}{\alpha}\ln(1-\alpha) + \frac{1}{\alpha}\ln A.$$

Taking the derivative yields

$$\frac{d\ln(\gamma+2\rho)}{d\alpha} = \frac{1}{\alpha} - \frac{1}{\alpha^2}\ln[(1-\alpha)A] - \frac{(1-\alpha)}{\alpha(1-\alpha)}$$

which is positive since $(1-\alpha)A = (1-\lambda)\tau < 1$ in (12). Hence, $\frac{d\check{\gamma}}{d\alpha} > 0$.

Non-Redistributing, Pro-Labour Policies. For $\lambda = 0$ the optimal tax rate $\check{\tau}$ solves equation (13), that is,

$$z = \frac{\tau}{1-\alpha} - \alpha A \tau^{1-\alpha} - \rho = 0$$

The partial derivatives of z are given by

$$z_\tau = \frac{1}{1-\alpha} - (1-\alpha)\alpha A \tau^{-\alpha} \quad \text{and} \quad z_\alpha = \frac{\tau}{(1-\alpha)^2} - A\tau^{1-\alpha} + \alpha A \tau^{1-\alpha} \ln \tau$$

with z_τ being positive for all $\tau > \hat{\tau}$. Then

$$\begin{aligned} \frac{d\tau}{d\alpha} = -\frac{z_\alpha}{z_\tau} &= -\frac{\frac{\tau}{(1-\alpha)} - (1-\alpha)A\tau^{1-\alpha} + \alpha(1-\alpha)A\tau^{1-\alpha}\ln\tau}{1-(1-\alpha)^2\alpha A\tau^{-\alpha}} \\ &= \frac{\tau\left[-\frac{1}{(1-\alpha)} + (1-\alpha)A\tau^{-\alpha} - \alpha(1-\alpha)A\tau^{-\alpha}\ln\tau\right]}{1-(1-\alpha)^2\alpha A\tau^{-\alpha}} \end{aligned}$$

where the first term in z_α is positive, but the sum of the other two terms is negative. However, $\check{\tau} \in \left((\alpha(1-\alpha)A)^{\frac{1}{\alpha}}, ((1-\alpha)A)^{\frac{1}{\alpha}}\right)$. Suppose $\check{\tau} \rightarrow (\alpha(1-\alpha)A)^{\frac{1}{\alpha}}$ and $A = 1$. Then the $\frac{d\tau}{d\alpha}$ reduces to

$$\frac{\hat{\tau}\ (1-2\alpha)}{\alpha^2(1-\alpha)} - \frac{\hat{\tau}\ \ln\hat{\tau}}{\alpha}$$

which is the same expression as that for $\frac{d\hat{\tau}}{d\alpha}$ in (E1). For $\alpha = \frac{1}{2}$ the expression is positive. For $\alpha \rightarrow 1$ a plot of the expression is similar to the one under a growth maximizing policy and reveals that the expression becomes negative. Hence, there exist A, ρ, α such that $\frac{d\check{\tau}}{d\alpha} \gtreqless 0$.

The change in the growth rate is given by

$$\begin{aligned} \frac{d\check{\gamma}}{d\alpha} &= A\tau^{1-\alpha} - \frac{d\tau}{d\alpha} - \alpha A\left[\tau^{1-\alpha}\ln\tau\right] + \alpha(1-\alpha)A\tau^{-\alpha}\ \frac{d\tau}{d\alpha} \\ &= A\tau^{1-\alpha} - \alpha A\left[\tau^{1-\alpha}\ln\tau\right] - \left[1-\alpha(1-\alpha)A\tau^{-\alpha}\right]\ \frac{d\tau}{d\alpha}. \end{aligned}$$

I want to show that $\frac{d\check{\gamma}}{d\alpha} > 0$ for any $\check{\tau} \in \left((\alpha(1-\alpha)A)^{\frac{1}{\alpha}}, ((1-\alpha)A)^{\frac{1}{\alpha}}\right)$. For that it suffices to show that $\frac{d\tau}{d\alpha} < A\tau^{1-\alpha}$, since $-\alpha A\left[\tau^{1-\alpha}\ln\tau\right]$ is non-negative.

For the rest of the proof it is convenient to represent the solution space $\check{\tau}$ in the form

$$\check{\tau} = x\left((1-\alpha)A\right)^{\frac{1}{\alpha}} \quad \text{where} \quad x \in \left(\alpha^{\frac{1}{\alpha}}, 1\right) \quad \Leftrightarrow \quad \check{\tau} \in \left((\alpha(1-\alpha)A)^{\frac{1}{\alpha}}, ((1-\alpha)A)^{\frac{1}{\alpha}}\right).$$

Higher x implies a higher optimal $\check{\tau}$. I want to show that $\frac{d\tau}{d\alpha} < A\tau^{1-\alpha}$, that is,

$$\frac{\frac{\tau}{(1-\alpha)} - (1-\alpha)A\tau^{1-\alpha} + \alpha(1-\alpha)A\tau^{1-\alpha}\ln\tau}{1-(1-\alpha)^2\alpha A\tau^{-\alpha}} < A\tau^{1-\alpha}$$

$$\frac{\tau^{\alpha}}{A(1-\alpha)^2} - 1 + \alpha\ln\tau < \frac{1}{1-\alpha} - \alpha(1-\alpha)A\tau^{-\alpha}.$$

Substituting $\check{\tau}$ for τ yields

$$\begin{aligned} \frac{x^{\alpha}}{1-\alpha} - 1 + \alpha\ln\check{\tau} &< \frac{1}{1-\alpha} - \frac{\alpha}{x^{\alpha}} \\ \frac{\alpha}{x^{\alpha}} - 1 + \alpha\ln\check{\tau} &< \frac{1-x^{\alpha}}{1-\alpha} \end{aligned}$$

and holds since $\alpha\ln\check{\tau}$ is unambiguously negative, $\frac{\alpha}{x^{\alpha}} < 1$ and $x^{\alpha} > 1$ for all $x \in \left(\alpha^{\frac{1}{\alpha}}, 1\right)$. Hence, $\frac{d\check{\gamma}}{d\alpha} > 0$.

For $\check{F}$ with $\lambda = 0$ I obtain

$$\begin{aligned} \frac{dF}{d\alpha} &= \frac{1}{(1-\alpha)^2} - \frac{\tau^{\alpha}}{(1-\alpha)^2 A} - \frac{\tau^{\alpha}\ln\tau + \alpha\tau^{\alpha-1}\frac{d\tau}{d\alpha}}{(1-\alpha)A} \\ &= \frac{A - \tau^{\alpha} - (1-\alpha)\tau^{\alpha}\left[\ln\tau + \alpha\tau^{-1}\frac{d\tau}{d\alpha}\right]}{(1-\alpha)^2 A}. \end{aligned}$$

I want to show that this expression is positive. Its denominator is positive. Thus, for checking the sign of $\frac{dF}{d\alpha}$ it suffices to check the sign of the numerator. For simplicity

$$\begin{aligned} \frac{d\tau}{d\alpha} &= \frac{\tau\ H}{1-\alpha(1-\alpha)^2 A\tau^{-\alpha}} \quad \text{where} \\ H &= \left[-\frac{1}{(1-\alpha)} + (1-\alpha)A\tau^{-\alpha} - \alpha(1-\alpha)A\tau^{-\alpha}\ln\tau\right]. \end{aligned}$$

Then the numerator becomes

$$A - \tau^\alpha - \frac{(1-\alpha)\tau^\alpha}{1-\alpha(1-\alpha)^2 A\tau^{-\alpha}}\left[(1-\alpha(1-\alpha)^2 A\tau^{-\alpha})\ln\tau + \alpha H\right] \qquad \text{(E3)}$$

The expression in the square brackets is given by

$$(1-\alpha(1-\alpha)^2 A\tau^{-\alpha})\ln\tau + \alpha\left[-\frac{1}{(1-\alpha)} + (1-\alpha)A\tau^{-\alpha} - \alpha(1-\alpha)A\tau^{-\alpha}\ln\tau\right]$$

and simplifies to $(1-\alpha(1-\alpha)A\tau^{-\alpha})\ln\tau - \frac{\alpha}{(1-\alpha)} + \alpha(1-\alpha)A\tau^{-\alpha}$ which upon substituting back into (E3) and simplification yields

$$A - \tau^\alpha - \frac{[(1-\alpha)\tau^\alpha - \alpha(1-\alpha)^2 A]\ln\tau - \alpha\tau^\alpha + \alpha(1-\alpha)^2 A}{1-\alpha(1-\alpha)^2 A\tau^{-\alpha}}.$$

Expressing this as a fraction of $(1-\alpha(1-\alpha)^2 A\tau^{-\alpha})$ amounts to

$$\begin{aligned}(1-\alpha(1-\alpha)^2 A\tau^{-\alpha})(A-\tau^\alpha) \quad &- \quad \left[(1-\alpha)\tau^\alpha - \alpha(1-\alpha)^2 A\right]\ln\tau \\ &+ \quad \alpha\tau^\alpha - \alpha(1-\alpha)^2 A \qquad \text{(E4)}\end{aligned}$$

as the corresponding numerator. Now evaluate at $\check{\tau}$ and use $\check{\tau} = x((1-\alpha)A)^{\frac{1}{\alpha}}$. Clearly, $1 > \alpha(1-\alpha)^2 A\check{\tau}^{-\alpha} = \alpha(1-\alpha)x^{-\alpha}$ since the lowest value x could assume is $\alpha^{\frac{1}{\alpha}}$. Thus, the denominator of the fraction is positive. For the numerator in (E4) I find

$$\begin{aligned}(1-\alpha(1-\alpha)x^{-\alpha})A(1-(1-\alpha)x^\alpha) \quad &- \quad \left[(1-\alpha)^2(x^\alpha - \alpha)\right] A\ln\check{\tau} \\ &+ \quad \alpha(1-\alpha)A(x^\alpha - (1-\alpha))\end{aligned}$$

The term $-\left[(1-\alpha)^2(x^\alpha-\alpha)\right] A\ln\check{\tau}$ is non-negative since $x^\alpha > \alpha$. I wish to show that the sum of the other two terms is positive. Multiplying out and collecting terms I get

$$\begin{gathered}A - \alpha(1-\alpha)x^{-\alpha} - (1-\alpha)Ax^\alpha + \alpha(1-\alpha)^2 A + \alpha(1-\alpha)Ax^\alpha - \alpha(1-\alpha)^2 A, \\ A\ (1-\alpha(1-\alpha)x^{-\alpha} - (1-\alpha)^2 x^\alpha) \equiv M(x).\end{gathered}$$

It is not difficult to verify that if $x \to \alpha^{\frac{1}{\alpha}}$, then $M \to A\alpha(1-(1-\alpha)^2) > 0$ and if $x \to 1$, then $M \to \alpha A > 0$. Thus, at the boundaries of x the numerator is positive. For showing that it is positive for all values in x, I look for extrema of $M(x)$. The function M is differentiable in x and so continuous. I look for either maxima or minima of M. If one finds a unique x in $(a^{\frac{1}{\alpha}}, 1)$ that maximizes M, then by the sign found for the endpoints in that interval, it follows that M is positive. I will now show that all extrema in the relevant range maximize M so that it cannot be negative. Taking the derivative yields

$$\frac{dM}{dx} = A\left(\alpha^2(1-\alpha)x^{-\alpha-1} - \alpha(1-\alpha)^2 x^{\alpha-1}\right)$$

and setting it equal to zero establishes $x^* = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2\alpha}}$ as the value of x that yields a unique extremum of M for given α. Suppose the extremum were a minimum and $\alpha \geq \frac{1}{2}$. Then $x^* \geq 1$ and by the boundary argument M would be positive. Thus, I concentrate on $\alpha < \frac{1}{2}$ for which it is possible that $\alpha^{\frac{1}{\alpha}} < x^* < 1$. For showing that x^* maximizes M I calculate

$$\begin{aligned}\frac{d^2M}{dx^2} &= A\left(-\alpha^2(1-\alpha)(1+\alpha)x^{-\alpha-2} + \alpha(1-\alpha)^2 x^{\alpha-2}\right)\\ &= Ax^{-2}\left(\alpha(1-\alpha)^3 x^{\alpha} - \alpha^2(1+\alpha)(1-\alpha)x^{-\alpha}\right).\end{aligned}$$

Substituting in x^* one obtains

$$\begin{aligned}\frac{d^2M}{dx^2} &= Ax^{-2}\left(\alpha(1-\alpha)^3\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{2}} - \alpha^2(1+\alpha)(1-\alpha)\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{2}}\right)\\ &= Ax^{-2}\alpha^{\frac{3}{2}}(1-\alpha)^{\frac{3}{2}}\left((1-\alpha)-(1+\alpha)\right) < 0.\end{aligned}$$

Hence, $x^* \in \left(\alpha^{\frac{1}{\alpha}}, 1\right)$ maximizes M. Thus, the infimum of M is at $\alpha^{\frac{1}{\alpha}}$ which establishes that M is positive. As all other terms of $\frac{d\check{F}}{d\alpha}$ are positive it follows that $\frac{d\check{F}}{d\alpha} > 0$.

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