

Maladaptation to environmental degradation, and the interplay between negative and positive externalities

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Abstract

This paper investigates the possible dynamics that may emerge in an economy in which agents adapt to environmental degradation by increasing the produced output to repair the damages of environmental degradation. The analyzed economy is characterized by both positive and negative externalities. On the one hand, an increase in production-related environmental degradation lowers the net income left at disposal for consumption and investment; on the other hand, it induces an increase in labor and capital to repair environmental damages from production, which enhances the positive externalities occurring in the production process. From the analysis of the model we show that there can be two stationary states but only the one with lower capital level can be locally attractive. Both local and global indeterminacy may arise in the model, even with decreasing returns to scale. It follows that one cannot predict a priori which path the economy will follow when converging to an equilibrium, nor the equilibrium the dynamics will eventually converge to. In particular, the trajectories emerging from the model may eventually lead the economy to be trapped in a Pareto-dominated equilibrium with lower capital and higher environmental degradation levels. Moreover, the interplay between positive and negative externalities generates a rich set of possible trajectories that may lead to opposite extreme outcomes, namely, either infinite growth or the collapse of the economy.

Keywords: global indeterminacy, positive and negative externalities, growth, adaptation, climate change.

JEL classification: C61, C62, D62, O44.

1. Introduction

Environmental degradation phenomena caused by human activity are attracting large attention in the media and in the policy debate for their potentially catastrophic consequences that pose mankind and life on the earth at risk. Scientific evidence provides many examples of severe environmental problems with large-scale and possibly unknown consequences. The glaciers melting rate has increased almost everywhere and the ice mass loss rate has tripled in

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7 the Antarctic in the last decade (Shepard et al., 2018); the rise in the sea level has doubled its
8 rate in the last twenty years (Nerem, 2018); extreme weather events have increased in frequency
9 and intensity registering record numbers in terms of heavy rains and heat waves (USGCRP,
10 2017); oceans acidification has increased by 30 per cent due to the absorption of carbon dioxide
11 (NASA, 2018). The degradation of the ecosystems has large effects on mankind’s health and
12 economic activities. It is estimated that climate change causes about 250.000 deaths every
13 year (WHO, 2018). Moreover, the observed increase in floods and droughts causes crop failures
14 that encourage mass migrations, especially from agricultural countries (Cai et al., 2016; IPCC,
15 2018).

16 In facing these possibly catastrophic scenarios, mitigation and adaptation choices require
17 an unprecedented level of coordination. Mitigation actions by a single agent generate positive
18 (environmental) externalities on other agents and, therefore, agents tend to provide them at
19 a level lower than the social optimum (see, e.g., Shogren and Crocker, 1991). The opposite
20 holds for adaptation choices. They allow the single agent to protect herself from the damages
21 generated by environmental degradation (they generate “positive private effects”) but they
22 may contribute to a further increase of environmental degradation (“negative public effects”).
23 In such a case, according to Shogren and Crocker (1991), they tend to be adopted at a level
24 higher than the social optimum, and their adoption process exhibits a self-reinforcing nature:
25 an increase in adaptation effort determines an increase in environmental degradation, which in
26 turn determines a further increase in adaptation effort, and so on.¹

27 The aim of this paper is to propose an economic growth model in which adaptation choices
28 may generate indeterminacy of equilibrium selection, even in an economy in which economic
29 agents are endowed with perfect foresight. The economy we analyze is decentralized, that is,
30 the choices of economic agents are not coordinated by a policy maker. In our economy, agents
31 adapt to the negative effects of environmental degradation by sustaining adaptation costs that
32 allow to self-protect from it. To bear such costs, agents need to increase their labor input in
33 order to produce more output. The increase in output contributes to worsening the damages
34 deriving from environmental degradation. In this context, the economy may end up in a self-
35 reinforcing vicious circle that leads to an increase in environmental degradation and in the costs
36 incurred by agents to defend themselves against it, which may eventually reduce the agents’
37 welfare.

38 Following the terminology that has been recently proposed in the literature (Barnett and
39 O’Neill, 2010; IPCC, 2018; Antoci et al., 2019), the adaptive choices considered in our model
40 can be classified as “maladaptive”. The term “maladaptation” (originally introduced by IPCC,
41 2001), in fact, refers to the large set of situations in which adaptation ends up further increasing
42 environmental degradation. More precisely, maladaptation denotes self-protective strategies
43 (Shogren and Crocker, 1991; Antoci and Bartolini, 2004; Antoci and Borghesi, 2012) which
44 exacerbate environmental problems or shift negative impacts, risks, and exposure to other
45 individuals, population groups or countries (Antoci and Borghesi, 2010). This phenomenon is

¹In their seminal paper Shogren and Crocker (1991) distinguish two kinds of self-protection choices: those that filter (dilute) externalities versus those that transfer them to the others. The adaptation choices described in this paper enhance the environmental degradation suffered by all individuals, including the person who performs the adaptive choice. Think, for instance, of the numerous cases of self-protection from increasing temperatures that eventually contribute to global warming (see below). As such, the adaptation activity does not fully “transfer” the damage to the others (as in the terminology adopted by Shogren and Crocker, 1991), as it ends up being also a self-damaging activity.

46 increasingly observed in modern economies and it is regarded as one of the global emerging
47 environmental challenges (UNEP, 2019).

48 The empirical literature and many case studies report numerous instances of maladaptive
49 choices. Air conditioning is a paradigmatic example of maladaptation: global warming aug-
50 ments the demand and use of air conditioning systems that is rapidly growing all over the
51 world, particularly in middle-income countries (cf. Davis and Gertler, 2015).² This brings
52 about a dramatic increase in electricity consumption³ which produces additional emissions thus
53 worsening global warming.

54 Similarly, the rise in average temperatures increases the agricultural demand for irrigation,
55 the use of water pumps in more arid zones (Borghesi 2013, Beilin et al. 2012), of water transfer
56 schemes across basins (Barnett and O’Neill 2010), of desalination projects and of snow-making
57 machines (Abegg et al., 2007) which are all very energy-intensive activities contributing to
58 further enhance global warming.

59 The increasing use of fertilizers and pesticides is another example of maladaptation (Klein
60 et al., 2014). To counterbalance the observed land productivity loss, many agents use more and
61 more chemical products. The consumption of these products, however, ends up polluting land
62 and water (thus possibly further reducing the land productivity in the long run), while their
63 production produces additional greenhouse gases in the air which eventually contribute to the
64 desertification and crop yield loss in many areas, as well as to expected changes in land use in
65 the farming sector (Fezzi et al., 2015).

66 The pharmaceutical production of medicaments to cure environment-related diseases ac-
67 counts for another large area of maladaptive choices. As extensively shown in the literature
68 (IPCC, 2018), environmental problems are responsible for a large number of health problems.⁴
69 The need to address environment-related health issues pushes an increase in pharmaceutical
70 production which is itself polluting and, therefore, health-damaging.

71 Also the reconstruction of buildings and infrastructures can be seen as a maladaptive choice
72 if it simply re-establish the status quo. Global warming causes extreme weather events that
73 damage or destroy houses, roads, bridges etc. The need to repair these physical damages
74 supports production in the building sector which causes additional emissions (UNEP, 2020;
75 Rock et al, 2020).

76 As these few examples show, maladaptation choices represent a large category and a per-
77 vasive phenomenon. Indeed, the empirical literature has documented instances in many other
78 areas and sectors beyond those described above, such as the tourism sector, water manage-
79 ment, geoengineering, infrastructural development, disaster relief and resettlement, agriculture
80 practices, land use changes, migration choices, insurance schemes, and urban planning (Hamin
81 and Gurran, 2009; Barnett and O’Neill, 2010; Pouliotte et al., 2009; McEvoy and Wilder, 2012;
82 Klein at al., 2014; Fezzi et al., 2017; Wagner and Weitzman, 2015; Weitzman, 2015; Magnan

²In China, for instance, sales of air conditioners have nearly doubled over the last few years, becoming more than eight times as many as those sold in the United States (cf. Euromonitor International, 2014).

³See Deschênes and Greenstone (2011) and Auffhammer and Aroonruengsawat (2011) for an analysis of the relationship between electricity consumption and temperature shocks in the US residential sector.

⁴Climate change is estimated to cause about 250’000 additional deaths each year and health losses that amount to 2-4 billions USD/year from 2030 (WHO, 2018). According to Takakura et al. (2017), the economic costs of preventing heat-related illnesses in the workplace may casue GDP losses ranging between 0.5 and 4 per cent of world GDP in 2100 depending on the emissions scenario. Ng and Zhao (2011) estimate that an increase in 1 °C is associated with a fall by 3 per cent in global GDP.

83 et al., 2016; UNEP, 2019).

84 The present paper intends to contribute to the literature on this issue focusing attention
85 on the dynamics that might emerge in the economy in the presence of maladaptive choices.
86 To provide a more exhaustive analytical framework, we will account not only for the negative
87 externalities deriving from environmental degradation but also for the positive ones that may
88 characterize the production process.

89 As pointed out above, we assume no government action and no mitigation activities by the
90 agents. We are fully aware that mitigation policies are gaining increasing importance and that
91 governments can play a key role in spurring/implementing such policies. However, here we want
92 to focus on the dynamic effects of maladaptation and show that multiple equilibria may exist
93 in a decentralized system in which agents do not care for environmental degradation (at least,
94 not directly for the sake of it but only for its production consequences) and take no mitigation
95 action but try only to defend from it, possibly provoking further environmental damages. In
96 other words, we want to look at the consequences of two (possibly simultaneous) undesirable
97 behaviors: on the one hand, inaction in terms of mitigation, on the other hand, wrong action
98 in terms of adaptation.⁵

99 The analysis of our model shows that, in the scenario described above, the interplay between
100 positive and negative externalities may generate a rich set of possible dynamic regimes, and
101 very complex global indeterminacy scenarios may emerge (see the seminal paper by Matsuyama,
102 1991). More specifically, starting from the same initial values of the state variables (“history”,
103 in the terminology of Krugman, 1991), different initial values of the jumping variable (whose
104 choice is conditioned by agents’ “expectations”) may collocate the economy along equilibrium
105 trajectories approaching very different outcomes.⁶ Along such trajectories, the economy may
106 eventually converge to one or more steady states, to an infinite growth path or to the opposite
107 extreme case in which the amount of capital in the economy goes to zero. Our results suggest,
108 therefore, that the degree of uncertainty surrounding the costs generated by environmental
109 degradation may be extremely high and that opposite outcomes can happen once the domino
110 effects generated by adaptive choices are at work.

111 Global indeterminacy is the object of many contributions in economic growth theory (see
112 Mino, 2017, for a review of the literature). Other growth models with environmental assets ex-
113 hibit global indeterminacy scenarios (see, among the others, Antoci et al., 2011; Yanase, 2011;
114 Fernández et al., 2012; Carboni and Russu, 2013, 2014; Bretschger and Schaefer, 2017; Bella and
115 Mattana, 2018; Caravaggio and Sodini, 2018; Russu, 2021). However, this literature neglects
116 the possible role of maladaptive choices in generating global indeterminacy. This paper aims
117 to contribute to that literature in three main respects: (i) it extends the scope of application
118 of global indeterminacy to maladaptation problems, (ii) it focuses on the case in which agents
119 try to adapt to environmental degradation rather than coordinating their activities to mitigate
120 environmental problems, (iii) differently from previous contributions in this research line (e.g.

⁵While these assumptions may look too pessimistic as compared to the increasing commitments and call for environmental actions by many individuals and governments worldwide, they may capture the relative inaction in the fight against many environmental problems that has prevailed in past years and the lack of coordination among agents in their adaptation choices. See, for instance, Bird (2014) for a discussion of the relative unbalance between adaptation and mitigation actions against global warming.

⁶On the relationship between history and expectations see the interesting contribution by Bretschger and Schaefer (2017) who study the impact of energy policy on the relevance of expectations compared to history in driving the economy towards different steady states.

121 Antoci et al, 2021), we deliberately assume that people do not care for environmental degrada-
 122 tion per se, but only for its production and consumption consequences. While environmental
 123 awareness is certainly increasing all over the world, most people seem to care or even realize
 124 about environmental degradation only because this impacts their consumption and life habits.
 125 To provide an example, people probably do not care for (or hardly perceive) an increase in tem-
 126 perature by 1°C, but they care about the impact this may have on agricultural production, on
 127 the probability of suffering adverse consequences from extreme events, on the insurance costs to
 128 protect against the environmental risks and so on. Assuming a “non-environmentalist” utility
 129 function allows to enrich and extend the indeterminacy results obtained in previous studies,
 130 showing that indeterminacy can occur also if agents have just an “instrumental” view of the
 131 environment and do not care for the environment per se.

132 Finally, differently from previous contributions in the global indeterminacy literature based
 133 on bifurcation techniques (e.g., Mattana and Nishimura, 2009; Bella et al., 2017), it derives re-
 134 sults through an analytical characterization of the invariant surfaces separating different regimes
 135 of the trajectories (e.g. Antoci et al., 2011, 2014).

136 The present paper is structured as follows. Sections 2 and 3 define the set-up of the model
 137 and the associated dynamic system. Section 4 deals with the existence and local stability
 138 of steady states. Section 5 is devoted to the global analysis of dynamics, while Section 6
 139 summarizes and discusses the main results emerging from the paper.

140 2. Set-up of the model

141 The economy we analyze is constituted by a continuum of identical economic agents; the
 142 size of the population of agents is normalized to unity. At each instant of time $t \in [0, \infty)$, the
 143 representative agent produces an output Y by the following Cobb-Douglas technology

$$Y = AK^\alpha L^\beta, \text{ with } \alpha + \beta < 1 \quad (1)$$

144 where K is the stock of physical capital accumulated by the representative agent, L is the
 145 agent’s labor input, and A represents the positive externality

$$A = \bar{K}^a \bar{L}^b, \text{ with } a, b > 0 \quad (2)$$

146 \bar{K} and \bar{L} denoting the economy-wide average values of K and L , respectively.⁷

147 We assume that production activities cause environmental degradation and the latter deter-
 148 mines a reduction in output via a damage function (see, among the others, Hackett and Moxnes,
 149 2015; Golub and Toman, 2016; Bretschger and Pattakou, 2019). We denote with $\Omega \in (0, 1]$ the

⁷Following the seminal papers by Lucas (1988) and Romer (1994), by positive externalities we mean that a higher level of L and K generates improved/increased knowledge that becomes common knowledge (i.e. it is transferred to the rest of society) through a learning-by-doing mechanism. To provide just a few examples in the context of the adaptive choices discussed here, producing medicaments against the numerous health problems provoked by pollution may increase knowledge on how to deal also with other diseases. Analogously, producing air conditioners to defend from increasingly frequent heat waves may improve knowledge on cooling systems used for other purposes (e.g. to refrigerate industrial engines and computer servers). A similar reasoning applies to the reconstruction of buildings or infrastructures that have been damaged/destroyed by extreme weather events, which may bring about knowledge spillovers (e.g. in terms of construction materials and techniques) increasing the productivity in the building sector as a whole.

150 share of the output Y that can be used either for consumption or investment in physical capital,
 151 which is determined by the following damage function

$$\Omega(P) = \frac{1}{1 + P^\gamma}, \text{ with } \gamma > 0 \quad (3)$$

152 where the variable P represents an index of environmental degradation caused by the pro-
 153 duction activity. So, $\Omega(P) \cdot Y$ represents the net output, while $[1 - \Omega(P)] \cdot Y$ represents the
 154 output required to repair the damages generated by P (i.e. the cost of adaptation to environ-
 155 mental degradation).

156 We assume that the representative agent's instantaneous utility function depends on leisure
 157 $1 - L$ and consumption C of the net output $\Omega(P) \cdot Y$. More precisely, we consider a *constant*
 158 *intertemporal elasticity of substitution* (CIES) utility function (a function of this type is used,
 159 among the others, by Mino, 1999; Bennet and Farmer, 2000; Itaya, 2008; Antoci et al., 2011)

$$U(C, L) = \frac{[C(1 - L)^\theta]^{1-\eta} - 1}{1 - \eta} \quad (4)$$

160 where $\theta, \eta > 0$ and $\eta \neq 1$. This function is concave in C and in $1 - L$ if $\eta > \frac{\theta}{1+\theta}$. The
 161 parameter η denotes the inverse of the inter-temporal elasticity of substitution in consumption
 162 and leisure. Our function possesses the property that income and substitution effects exactly
 163 balance each other in the labor supply equation.

164 Notice that environmental quality does not enter the CIES utility function adopted here.
 165 It follows that environmental degradation affects the agents' utility only indirectly, through its
 166 impact on consumption and leisure. This is equivalent to assuming (somehow provocatively)
 167 that agents do not care for environmental quality per se, but for the consequences of environ-
 168 mental degradation which -in the present context- induce them to work harder to repair the
 169 environmental damages.⁸

170 The time evolution of K (assuming, for simplicity, that the depreciation rate of K is equal
 171 to zero) is represented by the differential equation

$$\dot{K} = \Omega A K^\alpha L^\beta - C \quad (5)$$

172 where \dot{K} is the time derivative of K .

173 The time evolution of P is determined by

$$\dot{P} = \delta \bar{Y} - \varepsilon P e^{-\zeta P} \quad (6)$$

174 where \dot{P} is the time derivative of P , \bar{Y} represents the economy-wide average output, and
 175 the parameters satisfy the conditions $\varepsilon, \delta > 0, \zeta \geq 0$.

176 The parameter δ measures the positive impact of \bar{Y} on P . According to equation (6),
 177 environmental degradation P depletes at the rate $-\varepsilon e^{-\zeta P}$, which is a decreasing function of
 178 P : when environmental degradation increases, the environment's self-regeneration capacity

⁸Although the CIES utility function is sufficiently generic and widely used, the results of the model obviously hinge upon the specific utility function adopted here. Under alternative functional specifications, labour supply might shrink rather than grow in response to a negative productivity shock, thus preventing the self-reinforcing mechanism described here. We thank an anonymous reviewer for pointing this out.

179 deteriorates and may eventually become zero (see Xepapadeas, 2005).⁹

180 Equations (3), (5), and (6) represent a context in which the increase in adaptation cost
 181 $[1 - \Omega(P)] \cdot Y$ due to environmental degradation is financed via an increase in gross output Y ,
 182 which in its turn determines an increase in P . So, the adaptation choices we consider can be
 183 classified as maladaptation (IPCC, 2018; Barnett and O'Neill, 2010).

184 The representative agent solves the optimization problem

$$MAX_{C,L} \int_0^{\infty} \frac{[C(1-L)^{\theta}]^{1-\eta} - 1}{1-\eta} e^{-rt} dt \quad (7)$$

185 subject to (5) and (6), with $K(0)$ and $P(0)$ given, $K(t)$, $P(t)$, $C(t) \geq 0$ and $1 \geq L(t) \geq 0$
 186 for every $t \in [0, +\infty)$; $r > 0$ is the discount rate.

187 We assume that, in solving problem (7), the representative agent considers as exogenously
 188 determined the total productivity factor A (positive externalities) and the impact on P gener-
 189 ated by \bar{Y} (negative externalities), since, being economic agents a continuum, the impact on A
 190 and \bar{Y} of each individual is null. However, since agents are identical, ex post $\bar{Y} = Y$, $\bar{K} = K$
 191 and $\bar{L} = L$ hold. Therefore, from (1) and (2), we get $Y = AK^{\alpha}L^{\beta} = K^{\alpha+a}L^{\beta+b}$. This implies
 192 that the private marginal productivity of L and K (obtained taking A as exogenously given) is
 193 lower than the social marginal productivity of these factors (obtained by replacing A with its
 194 correspondent value) and that trajectories resulting from our model are not optimal (i.e. they
 195 do not describe the social optimum). However, they represent Nash equilibria in the sense that,
 196 along them, no agent has an incentive to modify her choices if the others don't modify theirs.

197 3. Dynamics

198 Following Wirl (1997), the current value Hamiltonian function associated to problem (7) is

$$H = \frac{[C(1-L)^{\theta}]^{1-\eta} - 1}{1-\eta} + \lambda (\Omega AK^{\alpha}L^{\beta} - C)$$

199 where λ is the co-state variable associated to K . By applying the Maximum Principle, the
 200 dynamics of the economy are described by the equations

$$\begin{aligned} \dot{K} &= \frac{\partial H}{\partial \lambda} = \Omega AK^{\alpha}L^{\beta} - C \\ \dot{\lambda} &= \theta \lambda - \frac{\partial H}{\partial K} = \lambda (r - \alpha \Omega AK^{\alpha-1}L^{\beta}) \end{aligned}$$

201 with the constraint

$$\dot{P} = \delta \bar{Y} - \varepsilon P e^{-\zeta P} \quad (8)$$

⁹Notice that posing $\zeta = 0$, we get the equation $\dot{P} = \delta \bar{Y} - \varepsilon P$, which exhibits a constant decay rate of P , which implies an exponential decay function. If $\zeta > 0$, instead, the decay function is not exponential but takes an inverted-U shape.

202 where C and L satisfy the following conditions¹⁰

$$\frac{\partial H}{\partial C} = C^{-\eta}(1-L)^{\theta(1-\eta)} - \lambda = 0 \quad (9)$$

$$\frac{\partial H}{\partial L} = 0, \text{ i.e. } \theta C^{1-\eta}(1-L)^{\theta(1-\eta)-1}(1+P^\gamma) - \beta AK^\alpha L^{\beta-1}\lambda = 0 \quad (10)$$

203 Since our system meets the Mangasarian hypotheses, the above conditions plus the transversality condition
204

$$\lim_{t \rightarrow +\infty} \lambda(t)K(t)e^{-rt} = 0 \quad (11)$$

205 are sufficient for solving problem (7). This is the case also if $\alpha + \beta + a + b > 1$ (remember
206 we assumed $\alpha + \beta < 1$), because \bar{Y} and $A = \bar{K}^a \bar{L}^b$ are considered as exogenously given in the
207 decision problem of the representative agent.

208 By replacing $\bar{Y} = Y$ and $AK^\alpha L^\beta = K^{\alpha+a} L^{\beta+b}$, the Maximum Principle conditions yield a
209 dynamic system with two state variables, K and P , and one jumping variable, λ .

210 Equations (9) and (10) allow us, after straightforward computations, to get the following
211 system, defined in $R = \{K > 0, P > 0, 0 < L < 1\}$, equivalent to (12)

$$\begin{aligned} \dot{K} &= \frac{1}{\vartheta(1+P^\gamma)} K^{a+\alpha} L^{b+\beta-1} [(\vartheta + \beta)L - \beta] \\ \dot{P} &= \delta K^{a+\alpha} L^{b+\beta} - \varepsilon P e^{-\zeta P} \\ \dot{L} &= \frac{L(1-L)}{(1-b-\beta)(1-L) + \left(1 - \frac{\vartheta(1-\eta)}{\eta}\right)L} \left[(a+\alpha) \frac{\dot{K}}{K} - \gamma \frac{P^{\gamma-1}}{1+P^\gamma} \dot{P} + \frac{1}{\eta} \left(r - \alpha \frac{1}{1+P^\gamma} K^{a+\alpha-1} L^{b+\beta} \right) \right] \end{aligned} \quad (12)$$

212 where $1 - \vartheta(1-\eta)/\eta > 0$, according to the concavity of the utility function (4), which requires
213 $\eta > \theta/(1+\theta)$.

214 In such a context, the jumping variable is L , instead of λ . As a consequence, given the
215 initial values of the state variables, K_0 and P_0 , the representative agent has to choose the initial
216 value L_0 of L so as to solve the maximization problem (7).

217 4. Stationary states and local indeterminacy

218 This section deals with the steady states of system (12).

219 Previous studies (Benhabib and Farmer, 1994; Boldrin and Rustichini, 1994) found indeter-
220 minacy assuming increasing or constant social returns to scale ($a+b+\alpha+\beta \geq 1$). In the present
221 context, instead, we will not constraint the value of the sum of the exponents $a+b+\alpha+\beta$
222 to allow for any possible social returns to scale. For this purpose, we will exclude the case in
223 which $b+\beta \geq 1$ as this would automatically imply constant or increasing returns to scale, and
224 will limit our analysis to the case $b+\beta < 1$ which allows the overall sum of the exponents

¹⁰Notice that the utility function we adopted implies $C > 0$ and $0 < L < 1$.

225 $a + b + \alpha + \beta$ to be greater, equal or less than 1. We will describe and summarise results based
 226 on the possible values of $a + \alpha$.

227 The following theorem holds.

228 **Theorem 1.** *Assume $\zeta > 0$. Then*

- 229 1. *If $a + \alpha = 1$, there exists at most one steady state E^* in the region $R = \{K, P > 0, 0 < L < 1\}$.
 230 It exists iff $\alpha \left(\frac{\beta}{\vartheta + \beta}\right)^{b + \beta} > r$, and it is either a repeller or a saddle point with a two-
 231 dimensional stable manifold (the Jacobian determinant in E^* is positive).*
- 232 2. *If $a + \alpha < 1$, there exist, generically, zero or two steady states in R . In the latter case,
 233 call them $E_A = (K_A, P_A, L^*)$ and $E_B = (K_B, P_B, L^*)$, with $K_B > K_A$, $P_B < P_A$, and
 234 $L^* = \frac{\beta}{\vartheta + \beta}$. The steady state E_A is either an attractor or a saddle point with a one-
 235 dimensional stable manifold, while the steady state E_B is either a repeller or a saddle
 236 point with a two-dimensional stable manifold (the Jacobian determinant is negative in
 237 E_A , and positive in E_B).*
- 238 3. *If $a + \alpha > 1$, there exist, generically, zero or two steady states in R , $E_A = (K_A, P_A, L^*)$
 239 and $E_B = (K_B, P_B, L^*)$, with $K_B > K_A$, $P_B > P_A$. The stability properties of E_A and E_B
 240 are as in case $a + \alpha < 1$.*

241 **Proof.** Let $E^* = (K^*, P^*, L^*)$ be a steady state of system (12). Then the Jacobian matrix is
 242 easily checked to have the form

$$J(E^*) = \begin{pmatrix} 0 & 0 & p \\ q & s & t \\ u & v & w \end{pmatrix} \quad (13)$$

243 while $\det J(E^*)$ is easily computed to have the sign of $\left(\frac{\partial \dot{P}}{\partial K} \frac{\partial \dot{\lambda}}{\partial P} - \frac{\partial \dot{P}}{\partial P} \frac{\partial \dot{\lambda}}{\partial K}\right)(E^*)$. When the
 244 Jacobian determinant is negative, the matrix (13) has either one or three eigenvalues with
 245 negative real part; when it is positive, the matrix (13) has either zero or two eigenvalues with
 246 negative real part.

247 In case $a + \alpha = 1$, being $\frac{\partial \dot{\lambda}}{\partial K} = 0$, such a determinant turns out to be positive, which proves
 248 claim 1. Suppose now $a + \alpha \neq 1$. Then the two possible steady states, $E_A = (K_A, P_A, L^*)$
 249 and $E_B = (K_B, P_B, L^*)$, with $K_B > K_A$ and $L^* = \frac{\beta}{\vartheta + \beta}$, are the intersections of two curves in
 250 the positive quadrant of the (K, P) plane: namely, the graphics of the functions $K = f(P) =$
 251 $\left[\frac{\varepsilon}{\delta(L^*)^{b + \beta}} P e^{-\zeta P}\right]^{\frac{1}{a + \alpha}}$ and $K = g(P) = \left[\frac{\vartheta}{\alpha(L^*)^{b + \beta}} (1 + P^\gamma)\right]^{\frac{1}{a + \alpha - 1}}$. It is easily checked that the
 252 former graphic is *bell-shaped* (with a maximum at $P = \zeta^{-1}$) if $\zeta > 0$, while the latter one is
 253 the graphic of a function decreasing or increasing if, respectively, $a + \alpha < 1$ or $a + \alpha > 1$.
 254 Now, let $E^* = (K^*, P^*, L^*)$ be, as above, a steady state of system (12). Hence, it follows from
 255 straightforward computations that, when $a + \alpha < 1$, $\text{sign}(\det J(E^*)) = \text{sign}(f'(P^*) - g'(P^*))$,
 256 whereas, when $a + \alpha > 1$, $\text{sign}(\det J(E^*)) = \text{sign}(g'(P^*) - f'(P^*))$: which easily implies
 257 claims 2 and 3 of the theorem.¹¹ ■

¹¹In the limit case $\zeta = 0$ there exists exactly one steady state if $a + \alpha < 1$ or $a + \alpha - 1 > \gamma(a + \alpha)$
 (the Jacobian determinant being positive in the first case and negative in the second one). Vice-versa, if
 $0 < a + \alpha - 1 < \gamma(a + \alpha)$ there exist, generically, zero or two steady states and, in the latter case, the Jacobian
 determinant is negative in E_A , and positive in E_B .

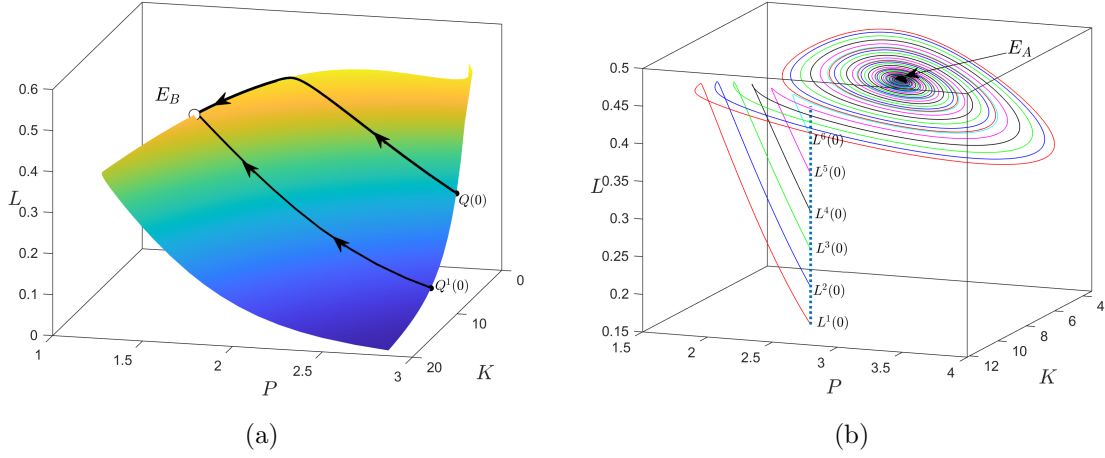


Figure 1: Parameter values: $\alpha = 0.08$, $\beta = 0.8$, $\gamma = 0.9$, $\delta = 1$, $\epsilon = 0.85$, $\eta = 0.6$, $\theta = 1$, $\zeta = 0.5$, $a = 0.02$, $b = 0.1$, $r = 0.002$.

258 According to Theorem 1, the value of $a + \alpha$ affects the number and properties of the steady
 259 states. Remember that the output Y is produced according to the production function $Y =$
 260 $AK^\alpha L^\beta$ (with $\alpha + \beta < 1$), where $A = \bar{K}^a \bar{L}^b$ (with $a, b > 0$) is a positive externality. So we can
 261 interpret the condition $\alpha + a > 1$ (respectively, $\alpha + a < 1$) as representing a scenario in which the
 262 positive externalities generated by the economy-wide average value \bar{K} of K are “high” (“low”).
 263 According to Theorem 1, if positive externalities are high ($\alpha + a > 1$), then the values of K and
 264 P at the steady states $E_A = (K_A, P_A, L^*)$ and $E_B = (K_B, P_B, L^*)$ are positively correlated, so
 265 that the steady state E_A has both lower capital and lower pollution (i.e. $K_B > K_A$, $P_B > P_A$).
 266 Vice-versa, if positive externalities are low ($\alpha + a < 1$), the values of K and P at the steady
 267 states are negatively correlated. In this case, the steady state E_A has lower capital but higher
 268 pollution than the steady state E_B (i.e. $K_B > K_A$, $P_B < P_A$).

269 As to the dynamic properties of the two steady states $E_A = (K_A, P_A, L^*)$ and $E_B =$
 270 (K_B, P_B, L^*) , the theorem proves that only the steady state E_A (characterized by a lower
 271 accumulation of physical capital, in both the cases $\alpha + a \geq 1$) can be locally attractive, while
 272 only the steady state E_B can have a two-dimensional stable manifold.

273 When the steady state E_A is attractive, then local indeterminacy occurs: if the economy
 274 starts from initial values $K(0)$ and $P(0)$ sufficiently close to K_A and P_A , respectively, then
 275 there exist a continuum of initial values $L(0)$ such that the trajectory from $(K(0), P(0), L(0))$
 276 approaches E_A . If E_A is a saddle with a one-dimensional stable manifold, it cannot (generically)
 277 be reached by the economy. When the steady state E_B is a saddle point with a two-dimensional
 278 stable manifold, then it possesses saddle-point stability: if the economy starts from initial values
 279 $K(0)$ and $P(0)$ sufficiently close to K_B and P_B , respectively, then there (generically) exists a
 280 unique initial value $L(0)$ of the jumping variable L such that the trajectory starting from
 281 $(K(0), P(0), L(0))$ approaches E_B . If E_B is a repeller, it cannot (generically) be reached by the
 282 economy.

283 Figure 1(a) illustrates the dynamics around the steady state E_B , when it possesses saddle-
 284 point stability. The black trajectories belong to the two-dimensional stable manifold (the
 285 colored surface), and so they approach E_B starting from different initial values of the state
 286 variables, K and P . Figure 1(b) illustrates the dynamics around the steady state E_A , when it
 287 is locally attractive. All the trajectories approaching E_A start from the same initial values of

288 the state variables, K and P .

289 What is the welfare level at the two steady states?¹² It is easy to check that, in both the
290 cases $\alpha + a \geq 1$, the value in E_A of the objective function of problem (7) is lower than in E_B . So
291 E_A is always a poverty trap, when it is attractive. This result is obvious under the assumption
292 $\alpha + a < 1$ since, in such a context, the economy is poorer and more polluted at E_A than at E_B
293 ($K_B > K_A$ and $P_B < P_A$ hold). In the context $\alpha + a > 1$, instead, the economy is poorer but less
294 polluted at E_A than at E_B ($K_B > K_A$ and $P_B > P_A$). However, even in this case the economy
295 is better-off at E_B than at E_A . Indeed, the higher positive externalities in E_B – generated by
296 a higher equilibrium value of K – overcome the higher adaptation costs generated by a higher
297 equilibrium value of P . It follows that the net welfare effect of the interplay between positive
298 and negative externalities is always higher at E_B than at E_A .

299 Figure 2 illustrates numerical examples concerning the values of the state variables K and
300 P at the steady states $E_A = (K_A, P_A, L^*)$ and $E_B = (K_B, P_B, L^*)$, corresponding to different
301 values of parameters γ and δ (panel (a)), and of parameters a and b (panel (b)). The color scale
302 is set in such a way that the steady state values are increasing from blue to yellow. The red
303 curve in the diagram denotes the Hopf bifurcation curve H . The latter separates the parameter
304 space in two regions such that a limit cycle arises when crossing the curve H . The black curve
305 LP in the diagram represents the limit point curve. The system admits one steady state along
306 this bifurcation curve, two steady states to its left and no steady state to its right. In the white
307 region, therefore, no steady state exists, being to the right of the curve LP . In these examples,
308 the steady state E_B is always saddle-point stable. In panel (a), the steady state E_A is locally
309 attractive on the right of the Hopf bifurcation curve H , while it has a one-dimensional stable
310 manifold on the left of H . The opposite holds in panel (b).

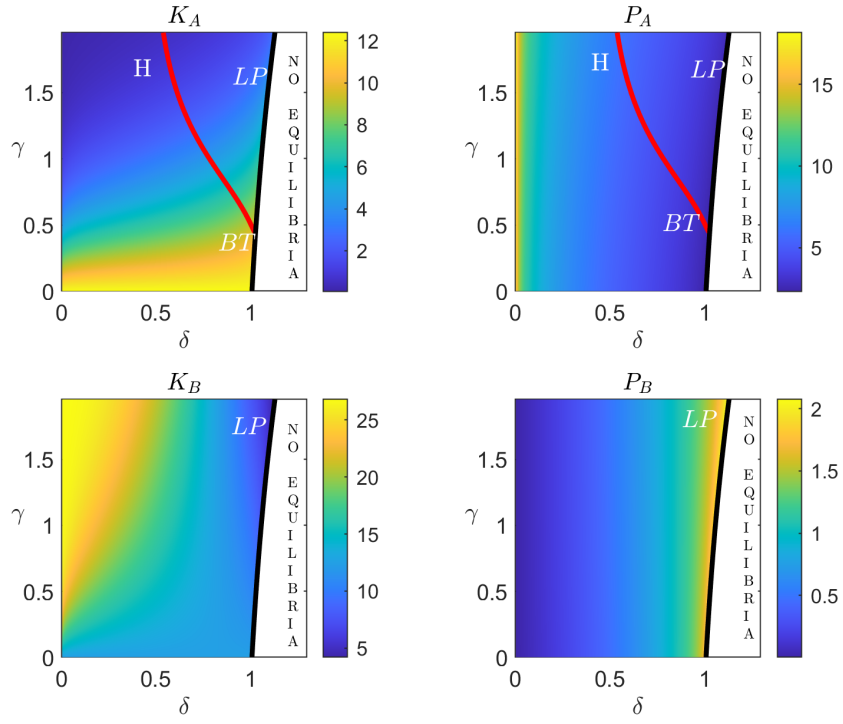
311 Notice that local indeterminacy can be observed even if positive externalities are very low
312 (i.e., a and b are very low, see panel (b)), and in the context in which the sum $\alpha + \beta + a + b$ is
313 lower than 1. In fact, consider panel (b). In this case, as stated above, E_A is locally attractive
314 (therefore, there is local indeterminacy) on the left of the Hopf bifurcation curve H . As the
315 figure shows, this area exists even at extremely low values of both a and b (i.e. very low positive
316 externalities) and low values of α and β ($\alpha = 0.08$ and $\beta = 0.5$ as indicated in the caption), so
317 that the sum $\alpha + \beta + a + b$ is much lower than 1.

318 This result enriches the literature by showing that indeterminacy may occur even when
319 social returns to scale are very low. Indeed, early studies (see, among the others, Benhabib and
320 Farmer, 1994; Boldrin and Rustichini, 1994) found indeterminacy assuming high social returns
321 to scale (much larger than 1). Subsequent studies proved that indeterminacy may emerge also
322 with lower but still increasing returns to scale (slightly above 1).¹³ In the present case, instead,
323 very low social returns to scale (much below 1) turn out to be sufficient for indeterminacy to
324 occur.

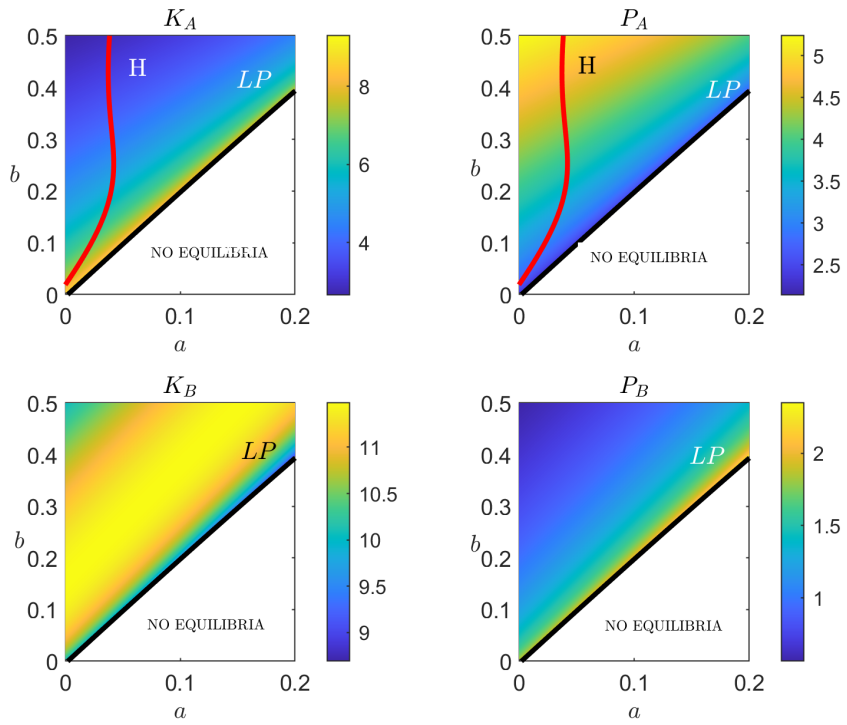
325 Finally, figure 3 digs deeper into the results shown in figure 2 providing further insights on the
326 possible outcomes of the model from a different perspective. The figure helps the reader visualize
327 the existence of a bifurcation which separates the stationary states E_A and E_B at different

¹²Notice that, since utility depends on consumption and leisure and labor is constant at the steady state ($L=L^*$, see Theorem 1), differences in welfare reflect differences in consumption levels at the steady states.

¹³See, among the others, Perli (1998), Benhabib and Nishimura (1998), Bennet and Farmer (2000), Nishimura et al. (2009), Brito and Venditti (2010). For a review of the literature on this issue see Benhabib and Farmer (1999) and Mino (2017).



(a) $a = 0.02, b = 0.1, \beta = 0.8$



(b) $\delta = 0.9, \gamma = 0.9, \beta = 0.5$

Figure 2: The steady state values of the state variables (K and P) as function of the parameters: δ and γ (panel (a)); a and b (panel (b)). The color scale is set such that the steady state values are increasing from blue to yellow. H= Hopf curve, LP= limit Point curve, BT=Bogdanov Takens point. Parameter values: $\alpha = 0.08, \eta = 0.6, \epsilon = 0.85, \theta = 1, \zeta = 0.5, r = 0.002$.

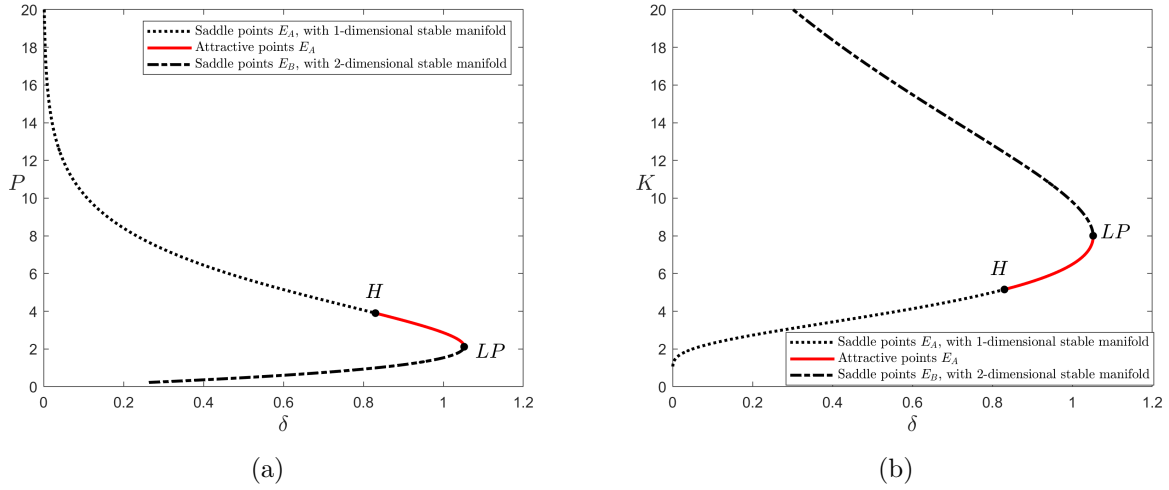


Figure 3: Parameter values: $a = 0.02$, $b = 0.1$, $\alpha = 0.08$, $\beta = 0.8$, $\eta = 0.6$, $\epsilon = 0.85$, $\theta = 1$, $\gamma = 0.9$, $\zeta = 0.5$, $r = 0.002$.

328 values of environmental degradation. Panel 3a (3b) reports on the vertical axis the value of
329 pollution (capital) at the stationary state, and on the horizontal axis the production-related
330 environmental degradation level. The shape and color of the curves illustrate the properties of
331 the stationary states that can be attractive (along the red portion of the curves), or saddle-point
332 stable (with 1 or 2-stable manifolds, along the dotted portions of the curves). To get a better
333 understanding of the diagram, it is convenient to interpret the diagram moving leftwards along
334 the horizontal axis (i.e. from high to low values of δ). As the figure shows, at very high values
335 of δ (to the right of LP in the diagram) no stationary state exists as there is no portion of
336 the curve corresponding to such values. In simple words, this suggests that if environmental
337 degradation is too high (i.e. if production is very polluting) the system does not converge to
338 any stationary state. If δ decreases to $\delta_{LP} = 1.052078439$, then there exists a unique stationary
339 state (indicated as LP in the figure). If δ keeps falling and gets lower than δ_{LP} (moving further
340 to the left of LP along the horizontal axis), then the stationary state splits into two alternative
341 equilibria, corresponding to the two branches of the curve (one for E_A and the other for E_B).
342 As the figure shows, only the poverty trap E_A can be attractive, as the red portion occurs only
343 along the branch corresponding to E_A . Notice that the latter is first attractive (red portion)
344 and then gets saddle point stable (dotted portion). Point H ($\delta_H = 0.8300851851$) indicates the
345 Hopf bifurcation showing where this transformation occurs, and thus also where a limit cycle
346 may possibly arise around E_A .

347 5. Taxonomy of dynamic regimes and global indeterminacy

348 This section deals with global analysis of the dynamic system (12), in order to highlight the
349 dynamic regimes that can be observed, and the role played by agents' expectations in deter-
350 mining the future evolution of the economy. As we will show, in our model, the economy may
351 face global indeterminacy scenarios: given the initial values of the state variables, K and P ,
352 different initial values of L may collocate the economy along equilibrium trajectories approach-
353 ing different ω -limit sets (for example, different steady states). In such a context, performing
354 local stability analysis alone may be misleading, since it refers only to a neighborhood of a

355 steady state, whereas the initial values of the jumping variable L may not belong to such a
 356 neighborhood.

357 The mathematical results about global dynamics are in the Supplementary material. Ac-
 358 cording to such results, there always exist trajectories along which the variables K , P and L
 359 all go to zero, thus leading to a clean environment but at the cost of having no production at
 360 all. Moreover, there always exist other trajectories along which both K and P tend to $+\infty$, as
 361 $t \rightarrow +\infty$, and L tends to 1, if and only if the following conditions are satisfied:

- 362 1. $\gamma \leq 1$: that is, the negative impact of P on net output ΩY is low enough;
- 363 2. $(a + \alpha)(1 - \gamma) < 1$: that is, the positive externality due to the economy-wide average
 364 value of K (measured by the parameter a) is low enough, given the value of γ . Notice
 365 that this condition is always satisfied if $a + \alpha \leq 1$.

366 In other words, conditions 1 and 2 suggest that if environmental degradation has relatively
 367 low negative effects on net income, and capital has sufficiently low positive externalities then K
 368 and P will keep growing for ever while economic agents keep working more and more (eventually
 369 all the time) to repair the damages produced from environmental degradation. If condition 1
 370 does not hold, then K does not tend to $+\infty$ when (P, L) tend to $(+\infty, 1)$. In this case, in
 371 fact, environmental degradation has high negative effects on net income which prevent capital
 372 from keep growing even if agents work all the time. If condition 1 holds, but condition 2 is
 373 violated (i.e. $(a + \alpha)(1 - \gamma) > 1$), then K tends to $+\infty$ in finite time, and consequently the
 374 transversality condition (11) cannot be satisfied.

375 The above results can be summarized by saying that, for all sets of admissible parame-
 376 ters, there exist trajectories (filling open regions) showing extreme opposite behaviors: that is,
 377 tending either to the boundary plane $P = 0$ or to the boundary plane $P = +\infty$.

378 5.1. The context with “low” positive externalities ($a + \alpha < 1$)

379 The analysis in Supplementary material focuses on the context in which two steady states
 380 exist, a saddle $E_B = (K_B, P_B, L^*)$ and an attractor $E_A = (K_A, P_A, L^*)$. In such a context,
 381 beyond the trajectories described above, there exist the trajectories converging to the steady
 382 states E_A and E_B . Furthermore, either the sink E_A (the poverty trap) or the saddle E_B can be
 383 reached starting from the same initial values (K_0, P_0) of the state variables (K, P) . Precisely,
 384 starting from a sufficiently small neighborhood U of E_B , the economy can follow three different
 385 development paths:

- 386 a) It will converge to the saddle E_B if economic agents choose an initial value L_0 of the
 387 jumping variable L equal to a suitable \tilde{L} .
- 388 b) If $L_0 > \tilde{L}$ it will converge to the poverty trap E_A .
- 389 c) If $L_0 < \tilde{L}$ it will converge to $(K, P, L) = (0, 0, 0)$.

390 To provide an heuristic explanation of our results, consider the initial value $L_0 = \tilde{L}$ as
 391 a benchmark value: starting from (K_0, P_0, \tilde{L}) , the economy will approach the steady state
 392 E_B , which always Pareto dominates the locally attractive steady state E_A . If economic agents
 393 choose an initial value L_0 higher than \tilde{L} (i.e. they work
 394 “harder”) then they produce a higher gross output Y . This generates an increase in
 395 environmental degradation P and, consequently, an increase in the adaptation cost $[1 - \Omega(P)] \cdot Y$
 396 to environmental degradation. Such a process of adaptation is self-reinforcing and drives the

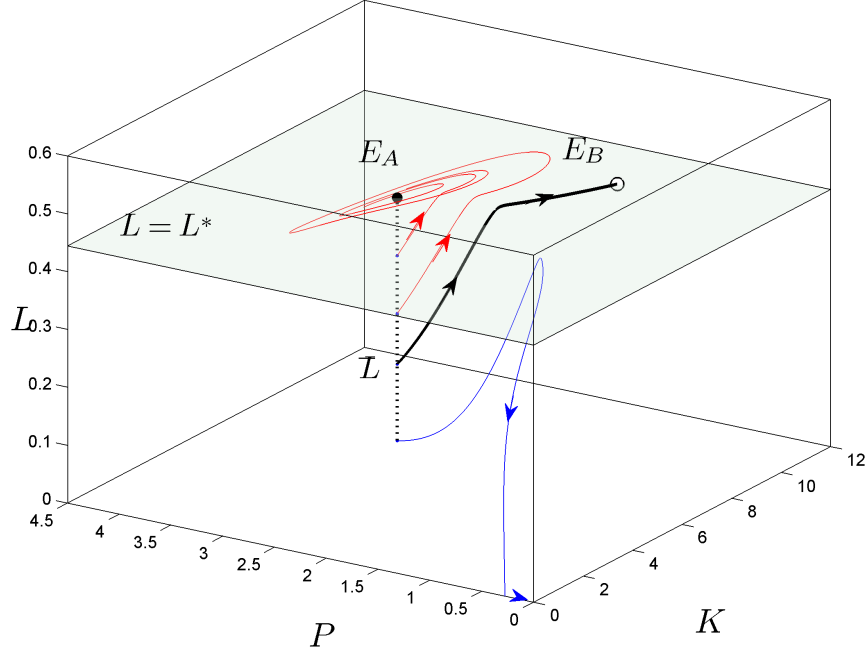


Figure 4: Global indeterminacy in the space (K, P, L) : case with $a + \alpha < 1$. Parameter values: $\alpha = 0.08$, $\beta = 0.8$, $\delta = 1$, $\epsilon = 0.85$, $\eta = 0.6$, $\theta = 1$, $\zeta = 0.5$, $a = 0.02$, $b = 0.15$, $r = 0.002$.

economy towards the poverty trap E_A , where the accumulation of physical capital is lower and environmental degradation is higher, with respect to the steady state E_B .

What does it happen in the opposite case, that is, if the initial choice L_0 is lower than \tilde{L} (i.e. agents work “little”)? In such a case, the trajectory starting from (K_0, P_0, L_0) will converge to $(K, P, L) = (0, 0, 0)$. In other words, if agents work little and have low positive externalities at the beginning, the economy eventually leads to an equilibrium without environmental degradation but also without capital.

Figure 3 illustrates the above global indeterminacy results, to help the reader visualize them. As the figure shows, starting from a given level of P and K but different levels of L , the economy can converge to totally different final outcomes. Consider, for instance, the vertical dashed line in the 3D-space (K, P, L) . All points along the vertical line correspond to equal values of P and K but different levels of L . If $L_0 < \tilde{L}$, as the arrows show, the economy moves along the trajectory converging to the lower right vertex of the cube in which $P = K = L = 0$. In this case, at the end of the day people will enjoy a clean environment without pollution ($P = 0$), but the economic system eventually collapses ($K = L = 0$). If $L_0 = \tilde{L}$ the economy moves along the solid bold line and converges to the steady state E_B . If $L_0 > \tilde{L}$ the system converges to the Pareto-dominated steady state E_A in which pollution is higher and capital lower than in E_B . Stated differently, agents basically work too much as a result of a coordination failure, leading the economy to end up in a poverty trap.

5.2. The context with “high” positive externalities ($a + \alpha > 1$)

When positive externalities are “high”, then starting from the same initial values (K_0, P_0) belonging to the set U , the economy can follow again three different development paths:

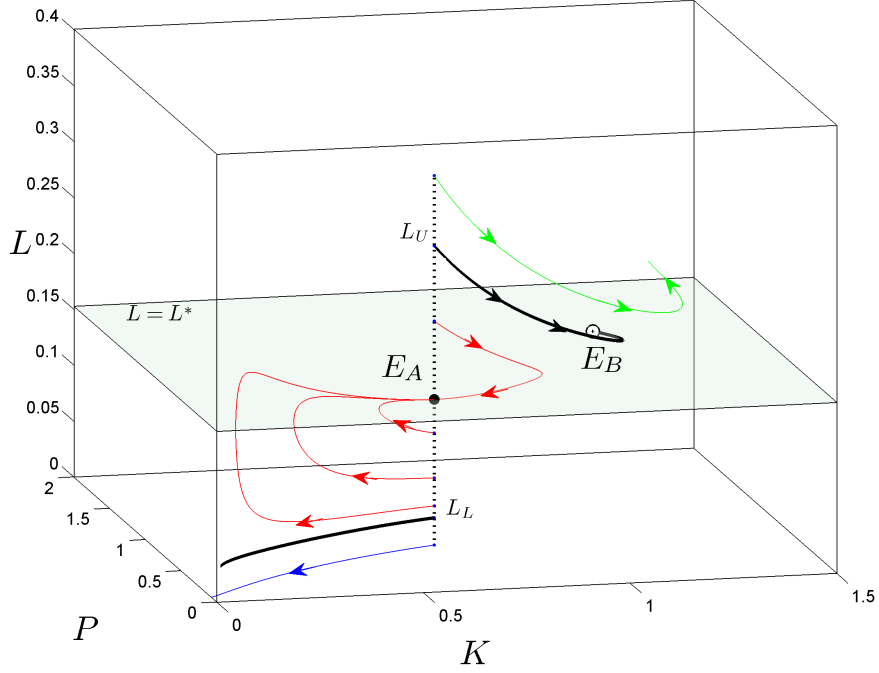


Figure 5: Global indeterminacy in the space (K, P, L) : case with $a + \alpha > 1$. Parameter values: $\alpha = 0.94$, $\beta = 0.018$, $\gamma = 0.04$, $\delta = 1$, $\epsilon = 1.15$, $\eta = 0.185$, $\theta = 0.1$, $\zeta = 0.7$, $a = 0.1$, $b = 0.32$, $r = 0.249$.

- 419 a) It will converge to the saddle E_B if economic agents choose an initial value L_0 of the
420 jumping variable L equal to a suitable \tilde{L} .
421 b) If $L_0 < \tilde{L}$ it will converge to the poverty trap E_A .
422 c) If $L_0 > \tilde{L}$ it will converge to $(\hat{K}, +\infty, 1)$, where $\hat{K} = +\infty$ if $\gamma \leq 1$; so the economy will
423 follow an unlimited growth path $((K, P) \rightarrow (+\infty, +\infty) L_0 > \tilde{L}$ and $\gamma \leq 1)$.

424 According to b), when agents work “little” ($L_0 < \tilde{L}$), the trajectory starting from (K_0, P_0, L_0)
425 will approach the poverty trap E_A . Working less, economic agents are not able to benefit from
426 the positive externalities, and the economy converges to the poverty trap E_A with lower capital
427 accumulation and lower environmental degradation than in E_B .

428 According to c), when agents work “hard” ($L_0 > \tilde{L}$), then the economy will follow a growth
429 path characterized by an unbounded growth of environmental degradation and of the stock of
430 physical capital (remember that the latter occurs provided the negative impact of environmental
431 degradation is not too high, that is, $\gamma \leq 1$).

432 Figure 4 illustrates the global indeterminacy scenario described above. Observe that, if
433 $L_0 < \tilde{L}$ along the vertical dashed line, the dynamics of the system lead to the lower left vertex
434 of the cube (in which $P = K = L = 0$) and the economy eventually collapses (see the blue
435 line in the figure). If $L_L < L_0 < L_U$, trajectories lead to the poverty trap E_A along the red
436 lines. If $L_0 = L_U$, the economy moves along the solid bold line converging to the saddle E_B
437 in which both pollution and capital are higher than in E_A . Finally, if $L_0 > L_U$ trajectories tend
438 to infinity along the green line. This suggests that pollution and capital can keep growing for
439 ever when positive externalities are high.

440 Table 1 reports the results of a numerical exercise that were performed to compute welfare

441 levels along the possible trajectories. Starting from similar initial values of K and P (columns
 442 2 and 3), a higher level of labour may lead to a never-ending growth of capital and pollution
 443 (cf. row 2 in the table). Along this trajectory, however, welfare will be lower than along the
 444 path leading to the sink E_A (see column 5). This confirms that an unlimited growth process
 can be undesirable since it turns out to be welfare-reducing.

convergence	K(0)	P(0)	L(0)	\mathcal{J}
$(K(t), P(t)) \rightarrow (+\infty, +\infty)$	0.583	0.3315	0.3525	-11.043
$(K(t), P(t)) \rightarrow (K_A, P_A)$	0.583	0.3315	0.0575	-7.5173

Table 1: Numerical Simulation of welfare levels.

445

446 6. Conclusions

447 Environmental degradation requires mitigation and adaptation choices. Adaptation activ-
 448 ities, however, may sometimes exacerbate environmental problems or shift negative impacts,
 449 risks, and exposure to other individuals, population groups or countries, what is known as
 450 “maladaptation“ (IPCC, 2001, 2018; Barnett and O’Neill, 2010).

451 The present paper tries to contribute to the debate on this issue, which is considered as one
 452 of the global emerging environmental challenges (UNEP, 2019), and to enrich the analytical
 453 framework by taking both negative and positive externalities into account. To this purpose, we
 454 investigated an intertemporal optimization problem characterized by negative environmental
 455 externalities that reduce the net output at disposal of the agents and positive externalities in
 456 production that increase the productivity of labor and capital. The co-existence of positive
 457 and negative externalities generates two counteracting mechanisms which are simultaneously
 458 at work. On the one hand, an increase in production-related environmental degradation lowers
 459 the net income left at disposal (for consumption and investment) of the individuals; on the
 460 other hand, it generates a push effect inducing people to work harder and accumulate more
 461 capital to repair the higher environmental damages deriving from production. The consequent
 462 increase in labor and capital enhances the positive externalities occurring in the production
 463 process. As it emerges from the model, the co-existence of these two mechanisms may lead to
 464 a welfare-increasing or welfare-reducing outcome depending on the relative size of the (positive
 465 versus negative) externalities and thus on which one of the two opposite forces will eventually
 466 prevail.

467 The analysis of the model shows that even with optimizing agents both local and global
 468 indeterminacy arise in the context described above, so that one cannot predict a priori which
 469 path the economy will follow when converging to an equilibrium, nor the equilibrium the dy-
 470 namics will eventually converge to. This suggests that the degree of uncertainty surrounding
 471 the effects of maladaptive behaviors is extremely high and that the trajectories may eventu-
 472 ally lead the economy to be trapped in a Pareto-dominated equilibrium. This result - which
 473 derives from lack of coordination among rational, self-interested, maximizing agents - calls for
 474 policy intervention and coordinated mitigation activities which were here deliberately ignored
 475 to focus on the dynamic effects of self-adaptation only. As shown in previous contributions
 476 (e.g. Bretschger and Schaefer, 2017; Antoci et al., 2021), suitable policy interventions may pre-
 477 vent coordination failures leading the economy towards a Pareto-dominant steady state. For
 478 instance, using a two-sector model with clean and dirty capital, Bretschger and Schaefer (2017)

479 show that government intervention may lead agents to select the trajectories converging to the
480 Pareto-dominant steady state in which clean capital prevails even if the economy is originally in
481 the neighborhood of the Pareto-dominated steady state. In the context examined by Antoci et
482 al. (2021), when a sufficiently high output tax is introduced in the model the Pareto-dominated
483 steady state can no longer be reached and the economy converges towards a unique saddle-point
484 stable steady state, so that global indeterminacy eventually disappears. A similar outcome may
485 be obtained also in the present context introducing an output tax and using the correspondent
486 revenues to abate pollution.¹⁴

487 Much research remains to be done to further enrich the present analysis in the future.
488 While this paper concentrates on the possible perverse effects of adaptation, the model could
489 be extended to account for mitigation. In particular, it would be interesting to see how results
490 change if the revenues accruing from mitigation policies (e.g. a pollution tax) are used to
491 finance adaptation activities. The present analytical framework, moreover, could be extended
492 to a 2-sector model in which each sector uses a different stock of capital: one for adaptation,
493 the other for mitigation. Indeed, one could argue that adaptation requires a specific capital
494 which differs from the one used for mitigation. The presence of two capital stocks would likely
495 affect the dynamics of the model: based on their expectations, agents might coordinate actions
496 leading the economy to converge to either a poverty trap in which adaptation actions prevail
497 or to a virtuous equilibrium in which mitigation actions prevail.

498 Finally, it is important to stress that the global indeterminacy and complex dynamics
499 pointed out above were obtained from a very simple environmental dynamics. Complexity
500 is obviously bound to increase even further if one accounts for the complex dynamics character-
501 izing many forms of environmental degradation (such as, for instance, climate change). Future
502 research should, therefore, be devoted to investigate more complex environmental dynamics
503 that can better approximate the complex (and partially still uncertain) relationship between
504 economic activity and environmental degradation.

505

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¹⁴Numerical simulations confirm our a priori expectations: allowing the output taxation rate to range between 0 and 1, we find that the Pareto-dominated steady state can no longer be reached if the taxation rate gets above a given threshold level. The correspondent results, omitted here for space reasons, are available from the authors upon request.

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