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Abstract

We build a dynamic model where the economy is vulnerable to belief-driven slowmoving debt crises at intermediate debt level, and rollover crises at both low and high debt levels. Vis-à-vis the threat of slow-moving crises, countercyclical deficits generally welfare-dominate debt reduction policies. In a recession, optimizing governments only deleverage if debt is close to the threshold below which belief-driven slow-moving crises can no longer occur. The welfare benefits from deleveraging instead dominate if governments are concerned with losing market access even at low debt levels. Long bond maturities may fully eliminate belief-driven rollover crises but not slow-moving ones.

Keywords

Sovereign default; Self-fulfilling crises; Expectations; Debt sustainability

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1 Introduction

After the global financial crisis, the average public debt to GDP ratio in advanced countries rose from below 80 percent to well above 100 percent at the end of 2008. After 2020, the global distress of the COVID-19 pandemic has sparkled a further hike in this ratio, raising a host of issues in financial and macroeconomic stability. The academic and policy literature has long reflected on the possibility that countries with relatively high debt face disruptive belief-driven turmoil in the sovereign bond market. This may take the form of hikes in the borrowing costs that raise deficits and feed unsustainable debt dynamics—with a (slow) build-up of liabilities eventually leading to default. It may also coincide with a sudden stop of market financing, where default is immediate in the face of a (fast) rollover crisis. The exercise pursued by this literature is far from a theoretical *curiosum*. The turmoil in the euro area after 2010 provides a vivid and striking example of the widespread disruption caused by this type of crises even among advanced countries.[1]

In this paper, we reconsider the logic of debt crises, specifying a stylised model suitable to address two key open issues in the literature. First, sovereign risk and rollover crises appear to be pervasive in the data: under what conditions sovereigns may face hikes in borrowing costs, as opposed to losing market access, due to market beliefs coordinating on a "bad equilibrium"? In particular, is lengthening the debt maturity an effective way to shield countries from these adverse scenarios? Second, and most crucially, is the threat of belief-driven crises enough to motivate optimal deleveraging even when the economy is in a downturn—as opposed to borrowing more, "gambling" on a future economic recovery?

We address these questions by specifying a model with the same setting of Conesa and Kehoe (2017), except that we follow Calvo (1988) in letting the government act as a price taker in the bond market. Hence our economy features a feedback loop between equilibrium bond prices and debt issuance not modelled in the literature after Cole and Kehoe (2000). Using this framework, we reconsider the mechanisms through which market beliefs may cause a variety of sovereign crises, eliciting different policy responses by optimizing governments.

As in Calvo (1988), lenders set the bond price based on their expectations of current deficits and future default. Conditional on this price, a discretionary government optimally adjusts its fiscal surplus and takes debt repayment/issuance decisions. Market beliefs about future default thus drive the equilibrium fiscal policy and debt dynamics—beliefs determine the debt tolerance thresholds based on which lenders price bonds, and the government takes its default decisions.[2]

[1] In the words of the ECB president Mario Draghi: "The assessment of the Governing Council is that we are in [...] a 'bad equilibrium', namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios." ECB Press Conference, Transcript from the Q&A, September 6 2012.

[2] See the discussion in Lorenzoni and Werning (2019), Corsetti and Dedola (2016) and Ayres et al. (2018).

Our main results are as follows. First, it is well understood that, in a dynamic setting after Calvo (1988), a switch from a regime of "optimistic" to one of "pessimistic" beliefs may drive sovereign risk hikes that cause debt crises to be "slow moving", as characterized by Lorenzoni and Werning (2019). As a contribution to the debate in the literature, we underscore that rollover crises are also possible in the same setting. Conditional on a switch of lenders' expectations from optimistic to pessimistic, sudden stops occur if lenders realize that the outstanding stock of debt is too high to be sustained at the risky interest rate. When this is the case, the government loses market access. For comparison with the literature, we also study the model allowing for beliefs that we dub 'Cole-Kehoe', whereby lenders are willing to finance the government only at the risk-free rate, provided the government is able and willing to service its debt in the current and future periods, independently of market financing. We show that, conditional on lenders coordinating on Cole-Kehoe beliefs, our model comes close to reproduce the dynamics of rollover crises in Cole and Kehoe (2000).

Rollover crises under pessimistic and Cole-Kehoe beliefs have different quantitative implications. In a numerical example, we show that, under pessimistic beliefs, rollover crises are possible at high levels of debt—between 122% and 206% of GDP—, as opposed to slow-moving crises, which are possible at intermediate levels of initial debt—between 72% and 122% of GDP—. Under Cole-Kehoe beliefs, the debt threshold at which rollover crises can occur is instead very low—in our numerical example, well below 40% of GDP. The main message is that, driven by self-validating expectations of a crisis, the market for government bonds (prior to the crisis traded at the riskless price) may suddenly disappear at either low or high levels of debt. At intermediate levels, shifts in expectations may result in a hike in borrowing costs and accelerate debt accumulation.

Second, lengthening the maturity of government debt *per se* does not rule out equilibrium multiplicity. Slow-moving debt crises remain pervasive for *all debt maturities*. However, longer maturities may rule out fast rollover crises. We study the conditions under which this is the case using our model relative to a variant of it, replicating the debt-limit framework adopted by Lorenzoni and Werning (2019). In either specification of the models, ruling out fast crises requires, in addition to a long debt maturity, a sizable probability of a recovery. In our model, however, the parameter restrictions are much more stringent.

Third, focusing on sunspot equilibria where either pessimistic or Cole-Kehoe beliefs are assigned a positive, arbitrarily small probability, we show that the threat of welfare-damaging beliefs-driven crises may not motivate optimizing governments to undertake debt reduction policies. Consider a high-debt economy in a persistent recession: the optimal fiscal policy weighs the benefits from running deficits to support consumption, against the benefits from reducing debt over time, up to rule out belief-driven crises altogether. We show that the consumption support motive tends to dominate policy choices when policymakers are only concerned with slow-moving debt crises scenarios. In this case, policymakers generally find it optimal to run anti-cyclical deficits, adding to the stock of debt. In a recession they run pro-cyclical surpluses only if the outstanding stock of liabilities is sufficiently close to the safe debt threshold (in our numerical example, close to 72% of GDP)—implying that the consumption costs of ruling out vulnerability to belief-driven crises is quite contained. The incentive to reduce debt is instead much stronger when lenders and policymakers are concerned with rollover crises driven by Cole-Kehoe beliefs. In this case, the benefits from eliminating vulnerability to beliefs-driven crises remain high also for debt levels away from the safe debt threshold. Even in recessions, optimizing governments generally opt for precautionary austerity, and run pro-cyclical fiscal policies. The circumstances under which the government optimally runs deficits, 'gambling for the redemption', are analyzed by Conesa and Kehoe (2017). We elaborate on the difference between this paper and ours in Section 5.

From a policy perspective, our analysis has key implications for debt sustainability analysis and the design of policies to enhance sustainability. First, our result that a long debt maturity is not a cure-all solution to the problem of multiple equilibria warns against betting solely on debt management strategies rebalancing the debt maturity structure. Second, our results show that pervasive crisis risk may not provide enough of a welfare incentive for implementing (optimally smoothed) debt reduction strategies. Indeed, our model provides a benchmark against which to assess political economy factors, e.g., the role of short-sighted or self-interested policymaking. In our framework, even a forward-looking benevolent government generally finds it optimal to raise debt in a recession, smoothing consumption at the cost of keeping the country in a state of vulnerability to self-fullling crises.

Last but not least, estimates of debt tolerance thresholds are a crucial input in assessing the extent to which a country can steer away from default. Our results reiterate that these thresholds are not only contingent on the current and future states of the economy and/or preferences of the policymakers. They also change with the lenders' beliefs. This consideration is a challenge to debt sustainability analysis, motivating an investment in sharpening the analytical toolkit employed in the assessment exercises.

The literature. This paper draws on the seminal contributions by Calvo (1988) and Cole and Kehoe (2000), in turn related to Conesa and Kehoe (2017). Calvo (1988) focuses on a two-period model, where the government financing need is taken as given, and the price and quantity of bonds are jointly determined in equilibrium. Self-fulfilling expectations of default generate market "runs" that manifest themselves in a surge in the interest rate charged by lenders to the government—but no rollover crisis is modelled in the same context. Conversely, Cole and Kehoe (2000) focus on liquidity crises whereby the market may suddenly become unwilling to roll over government debt in anticipation of a default. In our paper we aim to reconsider the nature and dynamics of belief-driven crises—we do so by specifying a model in the style of Calvo (1988) model, but adopting a dynamic setting using the same environment as Conesa and Kehoe (2017) except for the specification of auctions underlying their view of rollover crises.

It is virtually impossible to provide a fair account of the rich literature on debt crises that has contributed to these two paradigms, directly and indirectly.[3] Lorenzoni and Werning (2019) reconsider Calvo (1988) in a dynamic setting, stressing that the increase in the sovereign's borrowing costs driven by self-fulfilling expectations of default leads a country to accumulate debt slowly but relentlessly over time. As the debt stock rises, at some point default occurs unless the conditions of the economy improve sufficiently. Ayres et al. (2018) adopt a framework similar to Arellano (2008) but for the timing assumption, to investigate the likelihood that a country becomes vulnerable to belief-driven crises. Also drawing on Calvo (1988), Corsetti and Dedola (2016) and Bacchetta et al. (2018) write monetary models and discuss how the central bank can backstop government debt, i.e., eliminate self-fulfilling crises by using, respectively, either unconventional (balance sheet) policy, or conventional (inflation) policy.[4]

Several papers have developed the model with rollover crises of Cole and Kehoe (2000) in new directions. By way of example, Bocola and Dovis (2019) characterize how the maturity of sovereign debt can be structured to respond to rollover risk and fundamental risk. The importance of liquidity lending for a currency union is emphasized by Aguiar et al. (2015), suggesting that the co-existence of high debt and low debt countries in a currency area may create incentive for liquidity provisions that benefit also relatively virtuous countries—and the sustainability of the area overall. Rollover crises are also modelled and discussed in early work by Giavazzi and Pagano (1989), Alesina et al. (1992) and Cole and Kehoe (1996).

The goal of our paper is similar to the goal pursued by Aguiar et al. (2022), who also address the need to develop a unified framework to account for the variety of crises that we observe in the data. Aguiar et al. (2022) enrich Cole and Kehoe (2000) allowing for uncertainty about the default decision by the government once the debt auction is closed. Their model specification creates the possibility of belief-driven hikes in borrowing rates and debt accumulation as in Calvo (1988), but generated by a totally different economic mechanism. We defer a discussion of the differences with our work to Section 3.3.

[3]A partial list includes Alesina et al. (1989), Aguiar and Gopinath (2006), Chamon (2007), Yue (2010), Grauwe (2011), Chatterjee and Eyigungor (2012), Mendoza and Yue (2012), Arellano and Ramanarayanan (2012), Araujo et al. (2013), Cruces and Trebesch (2013), Ghosh et al. (2013), Lizarazo (2013), the Handbook chapter by Aguiar and Amador (2014) and Aguiar et al. (2016), Aguiar et al. (2015), Collard et al. (2015), Hatchondo et al. (2016), Tirole (2015), Chatterjee et al. (2016), Park (2017), Bianchi et al. (2018), Ayres et al. (2019), Bocola et al. (2019), Chatterjee and Eyigungor (2019), Stangebye (2020), Galli (2021), Ayres and Paluszynski (2022), Paluszynski (2023).

[4]See also Aguiar et al. (2013).

This paper is organized as follows. Section 2 lays out the model. Sections 3 analyzes equilibrium multiplicity with optimistic and pessimistic beliefs in an economy with short-term debt. Section 4 carries out numerical exercises in a calibrated version of the model with long-term debt. Section 5 focuses on the question of whether the perceived threat of belief-driven crises can motivate an optimizing government to pursue debt reduction policies, up to running pro-cyclical surpluses during recessions. Section 6 analyses whether longer debt maturities may rule out self-validating expectations of debt crises. Section 7 concludes.

2 Model

We specify a model of debt sustainability and default following Conesa and Kehoe (2017), except that we do not assume that the government commits to debt issuance as in Eaton and Gersovitz (1981) and Cole and Kehoe (2000).[5] Rather, we posit that in each period the government sets its primary surplus and thus how much debt to issue taking the bond price offered by lenders as given, as in Calvo (1988).

2.1 Environment

The model features a small open economy populated by a continuum of identical households, a government, and a continuum of risk-neutral competitive lenders with measure one. Time is discrete and indexed by $t = 0, 1, 2, \cdots$. To focus attention to the sovereign's behaviour, every period the representative household is assumed to consume all its income after paying tax.

The country's output is exogenous and random, given by $y(a, z) = A^{1-a} Z^{1-z} \bar{y}$, with $A < 1$ and $Z < 1$. The parameter a indicates whether the economy is in a recession ($a = 0$) or not ($a = 1$); z denotes the government decision to default ($z = 0$) or repay ($z = 1$). If the government defaults, $z = 0$ forever, and productivity permanently drops by the factor Z. The economy starts out with $a_0 = 0$ and $z_{-1} = 1$. From period 1, the economy recovers with probability p and once recovered, never falls back into a recession again.[6] In Figure 1, we depict possible endowment paths together with the recession indicator a, conditional on full repayment of debt.

[5] Specifically, we draw extensively on previous work on debt bailout, especially on Corsetti et al. (2017), which introduces official lending in a Conesa and Kehoe (2017) framework, as well as on Corsetti et al. (2020), Conesa and Kehoe (2014), Roch and Uhlig (2018) and Marin (2017).

[6] The model can be extended to adopt the bimodal income process used by Ayres et al. (2018), Chatterjee and Eyigungor (2019), Ayres et al. (2019) and Paluszynski (2023). In such a setting, there are equilibria in which a small increase in the bond price creates extra demand of bonds. For the sake of clarity and simplicity, we adopt the discrete distribution such that the debt Laffer curve is locally increasing at all points to filter unstable equilibria without the need to impose an equilibrium selection criterion. Equilibria presented in our model are in line with stable equilibria from a bimodal income process.

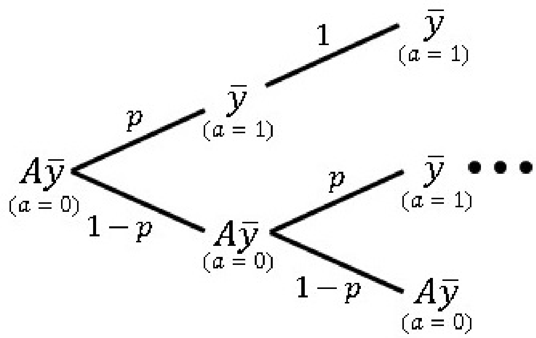

Figure 1: Paths of possible output realization

The government issues non-contingent bonds to a continuum of risk-neutral lenders. As is customary after Hatchondo and Martinez (2009), we model the maturity of government bonds assuming geometrically decreasing coupons: a bond issued at t pays the sequence of coupons

$$\kappa, (1-\delta)\kappa, (1-\delta)^2\kappa...$$

where $\delta \in [0, 1]$. Hence, under risk neutrality of lenders with discount factor β, the price of a default-risk-free bond is

$$q = \frac{\beta\kappa}{1-\beta(1-\delta)}$$

To normalize bond prices, it is convenient to set $\kappa = 1-\beta+\beta\delta$ so that the price of a default-free bond is β. The parameter δ indexes the maturity of debt, where $\delta = 0$ corresponds to the case of "consols" (or perpetuities) and $\delta = 1$ corresponds to the case of short-term bonds. Since a bond issued at $t - m$ is equivalent to $(1-\delta)^m$ bonds issued at t, the stock of outstanding bonds can be summarized by a single state variable B.

Crucial to equilibrium multiplicity is the assumption that lenders can coordinate on different regimes of beliefs and these in turn may impinge on the bond price market participants are willing to offer the government. Following the literature, coordination is driven by an exogenous state ρ. The aggregate state variable of the economy is then summarized by $s = \{(B, z_{-1}, a, \rho)\}$.

2.2 Timeline of Fiscal and Lenders' Decisions

The sequence of fiscal and lenders' decisions is summarized in Figure 2. Each period starts out with the realization of the business cycle shock a and the beliefs regime ρ. The aggregate state $s = \{(B, z_{-1}, a, \rho)\}$ is known to all agents at the beginning of the period. Upon observing the aggregate state s, lenders offer to buy sovereign bonds at price q (subject to a bound on aggregate issuance discussed below). The sovereign takes its fiscal decisions taking this price as given—it issues bonds B' to cover its financing need, and then chooses whether to default or to repay. Note that, the government is able to decide whether to default or to make good within the period, after the market for new bonds clears. If it defaults, under standard simplifying assumptions in the literature, the government receives the value $\mathcal{V}^D(a)$ that depends only on the business cycle shock a.[7]

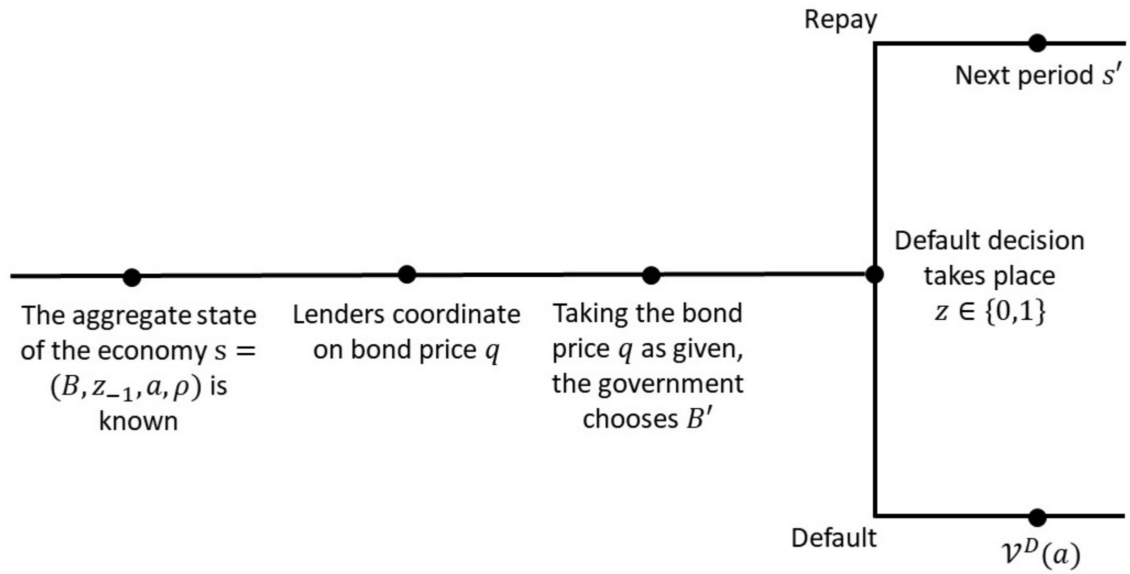

Figure 2: Timeline

Two comments are in order concerning the sequence of decisions detailed above. To start with, we model a discretionary government that is unable to commit to repay within the period (deviating from Eaton and Gersovitz (1981)). Cole and Kehoe (2000) shows that multiple equilibria are possible under this specification, even when, different from our timing specification, the government could credibly commit to its preferred issuance B' without taking the market bond price q as given. This is because lenders could coordinate their expectations on an end-of-period default and refuse to finance the government altogether. Under rational expectations, of course, they would do so in anticipation that, upon losing market access, an optimizing government with enough legacy debt would choose to default, rather than sustaining the high costs of large adjustment in current spending and primary surplus, required to repay outstanding liabilities.

Following Calvo (1988), however, we deviate from Cole and Kehoe (2000) in recognizing that, under discretionary policymaking, B' may respond to current market conditions. Specifically, we posit that lenders coordinate their beliefs on an equilibrium and set the equi-

[7]For simplicity, different from Cole and Kehoe (2000), we assume that the revenue raised from new issuance cannot be used by the government if default takes place within the period. This assumption has been made by other authors (see, for instance, Aguiar et al. (2019) and Bocola and Dovis (2019)), as a way to simplify the government problem without affecting qualitatively the key features of the model.

librium bond price; based on this price, the sovereign takes its decisions on how much to spend and borrow, or default. So, the amount of debt the government owes in future periods B' varies with its optimal (discretionary) fiscal response to the interest rate on government bonds set by markets.[8] Since, in turn, the lenders' valuation of debt depends on their anticipation of the fiscal decisions by policymakers in the current and future periods, the model features a two-way endogeneity of the entire future path of government debt accumulation and current borrowing costs. It is precisely this two-way endogeneity that (in a Calvo setting) creates multiple equilibrium paths. Belief-driven high or low borrowing costs can alter the pace of debt accumulation.[9]

It is straightforward to see that the two-way endogeneity just described would not arise under the assumption that, while acting under discretion, the government could nonetheless commit to auction the optimally chosen B', and guide the equilibrium offer price by lenders.[10] To put it differently, the fact that the government credibly sets B' at the beginning of the period plays the role of an equilibrium selection device, that rules out the type of crises highlighted by Calvo (1988).

2.3 The Lenders' Problem

The model features risk-neutral lenders who are "deep-pocketed", an assumption that rules out corner solutions in each lender's problem. Optimal pricing thus must satisfy the break-even condition below, equating the expected return on sovereign debt to the risk-free rate:

$$q(s) = z\beta\mathbb{E}\left[z'\Big(\kappa + (1-\delta)q(s')\Big)\Big|s\right] \tag{1}$$

Two features of this pricing equation are noteworthy. First, optimal pricing depends on the exogenous state variable ρ, as this drives lenders' coordination on a particular regime of beliefs, possibly impinging on the (self-validating) equilibrium price $q(s)$ at which bonds are traded in the period. Second, the inclusion of z on the right hand side of (1) indicates that lenders are aware of inability of the government to make any within-period commitment to repay. Deviating from Eaton and Gersovitz (1981), holders of previously and newly issued debt suffer a haircut on their bond portfolio (for simplicity, set to 100% in our model) in the

[8]Precisely, with discount bonds, it would vary with the price that lenders are willing to offer to buy government bonds.

[9]See Lorenzoni and Werning (2019) for a discussion of the logic and microfoundations of Calvo-style models. These authors underscore that discretionary governments cannot credibly commit not to re-open a debt auction, if lenders offer a low bond price so that their planned bond issuance fails to cover their planned gross financing need.

[10]This assumption ensures a unique equilibrium in Eaton and Gersovitz (1981) with short-term debt. Auclert and Rognlie (2016) expand on Eaton and Gersovitz (1981) and discuss conditions such that a unique equilibrium exists.

event of a within-period default.

2.4 The Government's Problem

The government takes its fiscal decisions taking the equilibrium bond price offered by lenders (reflecting the belief state ρ) as given. Formally, the value for the government $\mathcal{V}(s)$ and the level of debt issuance $B'(s)$ solve the Bellman equation below:[11]

$$
\begin{aligned}
\mathcal{V}(s) = \max_{B'} \ & \mathcal{U}(c,g) + \beta \mathbb{E}\big[\mathcal{V}(s')|s\big] \\
\text{subject to } & c = (1-\tau) y(a, z(s,B')) \\
g + z(s,B')\kappa B = \ & \tau y(a, z(s,B')) + z(s,B') q(B' - (1-\delta)B)) \\
& g \geq \bar{g}
\end{aligned}
\tag{2}
$$

where c denotes consumption—the representative household spends all its income after paying tax every period—; g denotes endogenous government spending—we stipulate that spending cannot fall below some critical expenditure level $g \geq \bar{g}$. Both the private consumption of the representative household and government spending yield utility, so that we model $\mathcal{U}(c,g)$ as a concave function in both arguments.[12]

The optimal fiscal decision on default at the end of the period solves the following problem:

$$
\begin{aligned}
\max_{z \in \{0,1\}} \ & \mathcal{U}(c,g) + \beta \mathbb{E}\big[\mathcal{V}(s')|s\big] \\
\text{subject to } & c = (1-\tau) y(a,z) \\
g + z\kappa B = \ & \tau y(a,z) + zq(B' - (1-\delta)B)) \\
z = \ & 0 \text{ if } z_{-1} = 0
\end{aligned}
\tag{3}
$$

z is binary variable that indicates whether the government decides to default ($z = 0$) or to repay ($z = 1$). Default takes place if and only if the value of repaying is lower than that of defaulting.

As in Conesa and Kehoe (2017), we assume that, for any feasible B such that $\tau A\bar{y} - B$

[11] To make sure the government's problem is well-defined, we postulate that the total debt issuance is subject to a bound. Else, the government would be able arbitrage by issuing large amount of debt at the current market price, violating rational expectations. A bound on aggregate issuance is detailed in Section 3.1.

[12] Consistent with the assumption spelled out in footnote 7, the bond revenue term $q(B' - (1-\delta)B)$ is multiplied by $z(B', s)$—the government is unable to use the revenue if it defaults.

is an element of the feasible set of government spending g, the following condition holds

$$\mathcal{U}_g((1-\tau)A\bar{y}, \tau A\bar{y} - B) > \mathcal{U}_g((1-\tau)\bar{y}, \tau\bar{y} - B)$$

This ensures that the marginal benefit of spending in a recession is high enough that the government always has an incentive to raise debt.

2.5 Equilibrium

An equilibrium is a value function for the government $\mathcal{V}(s)$, the policy functions $z(s, B')$ and $B'(s)$, and an equilibrium bond price $q(s)$ such that

1. Given $\mathcal{V}(s)$ and the policy functions $B'(s)$ and $z(s, B')$, the bond price $q(s)$ determined under the (exogenous) belief state ρ satisfies the break-even condition (1), and lenders make zero expected profits.

2. Given the bond price $q(s)$ consistent with the belief state ρ and the policy function $z(s, B')$, $\mathcal{V}(s)$ and $B'(s)$ solve the government's problem in (2).

3. Given $\mathcal{V}(s)$ and $B'(s)$, $z(s, B')$ solves (3) at the end of the period.

For tractability, the notion of equilibrium we consider follows a simple Markov structure.

2.6 Beliefs

The root of equilibrium multiplicity in Calvo-type crises lies in the fact that the government, facing the bond price q, optimally adjusts its issuance policy and default decision, which creates the possibility that multiple triples of (q, B', z) satisfy (1), (2) and (3). The (exogenous sunspot) state variable ρ coordinates market expectations, that selects among alternative equilibrium.

As in Calvo (1988), in our analysis we let lenders to hold "optimistic" (*opt*) and "pessimistic" (*pes*) beliefs, writing $\rho = opt$ and $\rho = pes$, defined as follows.

- In a regime of optimism, lenders are willing to finance the government at the best possible equilibrium price—implying that they coordinate on the bond price which maximizes the government (societal) welfare. This price may/may not be the riskless price.

- In a regime of pessimistic beliefs, lenders approach the bond market entertaining systematically the possibility that debt is at the risk of default. Hence, in an equilibrium with debt rollover, they naturally coordinate their expectations on the default-risky price, provided debt is high enough for this price to be self-validating in equilibrium.

Since the government is unable to commit to repayment within a period, the model also features the type of rollover crises analyzed by Cole and Kehoe (2000). Lenders may coordinate their expectations on default at the end of the period and refuse to lend, i.e., $q = 0$.[13] Since this type of beliefs and crises has been amply discussed by the literature, we will focus most of our analysis on Calvo-type crises, carrying out comparative discussion in sections 3.3 and 5.2.

3 Lenders' Beliefs and Equilibrium Selection with Short-Term Debt

In this section, we illustrate analytically and graphically the logic of belief-driven crises in the model. To do so, we specialize the model assuming that, first, debt is short-term only. Second, all agents in the economy consider the current regime of lenders' beliefs as 'constant', i.e., agents do not expect to switch across regimes. A switch, if it occurs, is completely unanticipated. Also, for ease of exposition, we focus on economies in which the government is always better off when lenders offer the riskless price, relative to the risky price. This implies that, under optimism, in an equilibrium with debt rollover lenders invariably offer the riskless bond price.[14] We relax all these assumptions later on in the text.

The section is structured as follows. First, we derive sovereign's debt "tolerance thresholds" conditional on output and optimistic and pessimistic beliefs. Next, we provide insight on the economics of self-fulfilling crises using a simple graphical apparatus. Finally, we will compare our model with the canonical rollover crises in Cole and Kehoe (2000) and slow-moving crises in Aguiar et al. (2022).

3.1 The Debt Tolerance Thresholds

The structure of the model is such that the equilibrium can be characterized by critical levels of debt such that the equilibrium bond prices change discretely, from riskless to risky or to zero. These thresholds are contingent on both the state of the economy a and most crucially, on the regime of beliefs ρ (in this section assumed to be constant in the eyes of lenders). Thresholds will be denoted by $\bar{B}(a)_\rho$ with $\rho = opt$ or pes (e.g., in a recession ($a = 0$) with pessimistic beliefs ($\rho = pes$), the maximum sustainable debt level will be denoted by $\bar{B}(0)_{pes}$).

[13] Provided that the outstanding debt is large enough, given the high costs of adjusting the primary surplus upon losing market access, i.e., upon losing the ability to refinance the outstanding debt, the government then validates the lenders' expectations of default within the period and repudiates its liabilities.

[14] Without this restriction on the parameters of the model, it might be possible that, as debt rises, optimistic lenders first start to price default risk, then at some point stop lending altogether. Under our restriction, the intermediate stage is not an equilibrium. Optimistic lenders either lend risk free, or do not lend at all.

3.1.1 The Threshold in Normal Times $\bar{B}(1)$.

By assumption, once out of a recession, our economy remains in the high output state permanently. Accordingly, the default-risky price βp can no longer be an equilibrium—since pessimistic views anticipating default in the low-output recessionary state are no longer rational. Given our simplifying assumption, we derive the debt sustainability threshold positing that, after a recovery, lenders hold the view that a government with a sustainable level of debt will honor its liabilities regardless of whether beliefs are optimistic or pessimistic. Hence lenders will be lending at the risk-free rate with debt rollover,[15] and the government's optimization problem conditional on $a = 1$ is deterministic. Conditional on deciding to honor its debt, the government pays $(1 - \beta)B$ in each period to satisfy the no-Ponzi condition. The maximum sustainable debt level in normal times will simply be $\bar{B}(1)_{opt} = \bar{B}(1)_{pes} = \bar{B}(1)$.

Let $\mathcal{V}^R_\rho(B, a)$ denote the government utility of repaying debt, with no history of defaulting, starting with beliefs state ρ:

$$\mathcal{V}^R_{opt}(B, 1) = \mathcal{V}^R_{pes}(B, 1) = \frac{\mathcal{U}((1-\tau)\bar{y}, \tau\bar{y} - (1-\beta)B)}{1-\beta}$$

The utility of defaulting $\mathcal{V}^D(a)$[16] when the economy is not in a recession $(a = 1)$ is

$$\mathcal{V}^D(1) = \frac{\mathcal{U}((1-\tau)Z\bar{y}, \tau Z\bar{y})}{1-\beta}$$

It follows that the debt threshold at which the government is indifferent between defaulting and repaid $\bar{B}(1)$ solves:

$$\mathcal{V}^R_{opt}(\bar{B}(1), 1) = \mathcal{V}^R_{pes}(\bar{B}(1), 1) = \mathcal{V}^D(1)$$

and that the utility of the government $\mathcal{V}(B, z_{-1}, a, \rho)$ with no history of default $(z_{-1} = 1)$ in normal times $(a = 1)$ is summarized by

$$\mathcal{V}(B, 1, 1, \rho = opt \text{ or } pes) = \max\left\{\mathcal{V}^R_{opt}(B, 1), \mathcal{V}^D(1)\right\}$$

3.1.2 The Threshold in a Recession under Optimistic Beliefs $\bar{B}(0)_{opt}$

To derive the debt thresholds in a recession, we move from the observation that, in an optimistic regime, lenders always choose the price that maximizes the welfare of the government

[15] In the model, the recovery state is an absorbing state, in which no default takes place.

[16] $\mathcal{V}^D(a)$ is pinned down by the simplifying assumption that, in case of default, the country loses market access *and* experiences a discrete but permanent contraction in output by Z—output stays at either $AZ\bar{y}$ or $Z\bar{y}$ forever.

(i.e., social welfare). Recall that we restrict parameters of the model such that, under optimism, an equilibrium with debt rollover is always characterized by a riskless price (i.e., the value of repaying is the highest when bonds are traded at the riskless price). Thus, contingent on optimistic beliefs, the utility of repaying debt $\mathcal{V}^R_{opt}(B,0)$ solves the following optimization problem:

$$\begin{aligned}\mathcal{V}^R_{opt}(B,0) = \max_{0\leq B'\leq \bar{B}(0)_{opt}} \quad & \mathcal{U}(c,g)+\beta\Big(p\mathcal{V}(B',1,1,\rho=opt)+(1-p)\mathcal{V}(B',1,0,\rho=opt)\Big)\\ \text{s.t.} \quad & g+B=\tau A\bar{y}+\beta B',\\ & c=(1-\tau)A\bar{y}\end{aligned} \tag{4}$$

where debt issuance is bounded above at $\bar{B}(0)_{opt}$ to eliminate the possibility of arbitrage. Given the utility of defaulting in a recession, which is independent of beliefs

$$\mathcal{V}^D(0)=\frac{\mathcal{U}((1-\tau)AZ\bar{y},\tau AZ\bar{y})}{1-\beta(1-p)}+\beta\frac{p\mathcal{U}((1-\tau)Z\bar{y},\tau Z\bar{y})}{(1-\beta)(1-\beta+\beta p)}$$

$\bar{B}(0)_{opt}$ is derived by solving the equation below:

$$\mathcal{V}^R_{opt}(\bar{B}(0)_{opt},0)=\mathcal{V}^D(0)$$

Let $\mathbb{B}_{opt}$ denote the set of debt levels such that repayment is optimal conditional on optimistic beliefs:

$$\mathbb{B}_{opt}\equiv\{B \mid \mathcal{V}^R_{opt}(B,0)\geq\mathcal{V}^D(0)\}$$

On this domain, $\mathcal{V}(B,1,0,\rho=opt)=\mathcal{V}^R_{opt}(B,0)$. For $B>\bar{B}(0)_{opt}=\sup\mathbb{B}_{opt}$, lenders refuse to lend and the government defaults.

3.1.3 The Threshold in a Recession under Pessimistic Beliefs $\bar{B}(0)_{pes}$

Lenders who hold pessimistic beliefs are concerned with the possibility of future default if the current recession persists. The utility of repaying consistent with these beliefs, $\mathcal{V}^R_{pes}(B,0)$, is analogous to (4)—except that we now need to allow for the fact that, at the risky bond price, B' may or may not grow above the debt tolerance threshold in the current state of recession, $\bar{B}(0)_{pes}$.

The utility of repaying under pessimistic beliefs therefore needs to allow for these two cases, indexed by “nd” (no default) and “d” (default), respectively,

$$\mathcal{V}^R_{pes}(B,0)=\max\{\mathcal{V}^R_{pes,nd}(B,0),\mathcal{V}^R_{pes,d}(B,0)\}, \tag{5}$$

where:

$$\begin{aligned}
\mathcal{V}^R_{pes,nd}(B,0) = \max_{0 \le B' \le \bar{B}(0)_{pes}} \quad & \mathcal{U}(c,g) + \beta\Big(p\mathcal{V}(B',1,1,\rho = pes) + (1-p)\mathcal{V}(B',1,0,\rho = pes)\Big) \\
\text{s.t.} \quad & g + B = \tau A\bar{y} + \beta p B', \\
& c = (1-\tau)A\bar{y}
\end{aligned} \tag{6}$$

$$\begin{aligned}
\mathcal{V}^R_{pes,d}(B,0) = \max_{\bar{B}(0)_{pes} < B' \le \bar{B}(1)} \quad & \mathcal{U}(c,g) + \beta\Big(p\mathcal{V}(B',1,1,\rho = pes) + (1-p)\mathcal{V}^D(0)\Big) \\
\text{s.t.} \quad & g + B = \tau A\bar{y} + \beta p B', \\
& c = (1-\tau)A\bar{y}.
\end{aligned} \tag{7}$$

Since issuing debt above $\bar{B}(1)$ leads to default with certainty in the next period, bond issuance is bounded by $\bar{B}(1)$.

The debt threshold $\bar{B}(0)_{pes}$, the maximum sustainable debt under pessimistic beliefs regime, solves the equation below:

$$\mathcal{V}^R_{pes}(\bar{B}(0)_{pes}, 0) = \mathcal{V}^D(0)$$

The necessary condition for pessimistic beliefs to be self-validating is $B' > \bar{B}(0)_{pes}$ in $\mathcal{V}^R_{pes}(B,0)$. Together with the upper bound $\bar{B}(1)$, this condition identifies the range of outstanding debt causing the government to default if the recession persists in the future.

Let $\mathbb{B}_{pes}$ denote the set of initial debt levels such that "pessimistic beliefs" are validated in equilibrium, defined as follows:

$$\mathbb{B}_{pes} \equiv \{B \mid B' \text{ that solves } \mathcal{V}^R_{pes}(B,0) \text{ satisfies } B' > \bar{B}(0)_{pes} \text{ and } \mathcal{V}^R_{pes}(B,0) \ge \mathcal{V}^D(0)\}$$

The following proposition establishes a sufficient condition for the set $\mathbb{B}_{pes}$ to be non-empty, which provides insight on the important role played by the lower bound on government spending in the economics of self-fulfilling crises.

Proposition 1. *For a strictly concave utility function $\mathcal{U}$, a sufficient condition for $\mathbb{B}_{pes} \neq \varnothing$ is a high enough critical expenditure $\bar{g}$, such that under pessimistic beliefs regime, there exists some debt level at which the government is unable to sustain $\bar{g}$ unless it issues debt above $\bar{B}(0)_{pes}$.*

Proof. See Appendix H.1. □

For future reference, we should note that, for $B \in \mathbb{B}_{pes}$, the utility of the government in a recession is $\mathcal{V}(B,1,0,\rho = pes) = \mathcal{V}^R_{pes}(B,0)$—rolling over debt at the high borrowing costs foreshadows a crisis if the recession persists, which validates the lenders' pessimism. Conversely, if the initial debt level is to the left of the set, $B < \inf \mathbb{B}_{pes}$, even if lenders hold pessimistic beliefs, the risky price cannot be an equilibrium price.

We conclude our analysis by stressing an important implication of our restriction for equilibrium pricing under optimism, stated in the proposition below.

Proposition 2. *Provided lenders, under optimism, always lend at the risk-free rate in an equilibrium with debt rollover, the economy features distinct debt tolerance thresholds, with* $\bar{B}(0)_{opt} > \bar{B}(0)_{pes}$.

Our restriction ensures that the debt tolerance derived as shown above will always be higher in the optimistic relative to the pessimistic regime.

3.2 An Intuitive Graphical Analysis

In this section, we provide economic insight on equilibrium multiplicity using a graphical apparatus—the algorithm for computing each equilibrium is detailed analytically in Appendix B. Our goal is to clarify how and why, depending on the initial debt, the equilibrium in a given period may not be unique.

Figures 3-5 are drawn for three different levels of initial debt—low, intermediate and high. To highlight the role of beliefs, each figure includes two panels, the left panel referring to optimistic beliefs, the right panel to pessimistic beliefs. The graph plots the resources that the government can obtain by issuing debt at the market price, qB', on the y axis, against the amount of discount bonds the government issues during the period, B', on the x axis. Vertical dashed lines denote the debt tolerance thresholds derived above. In the left panel, lenders offer the risk-free price, $q = \beta$.[17] Issuance at this price is bounded by the threshold $\bar{B}(0)_{opt}$. In the right panel, drawn under pessimistic beliefs, lenders offer the risky price βp, but this price may/may not be an equilibrium price. Issuance at this price is also bounded above, by $\bar{B}(1)$, beyond which default would be unavoidable regardless of the economic conditions, one period ahead.

In each panel, horizontal lines show the endogenous Government Financing Need (GFN) $GFN^R_\rho(B, a)$, calculated conditional on full repayment of the inherited debt in a recession $(a = 0)$ using $\mathcal{V}^R_\rho(B, 0)$ in (4) and (5):

$$GFN^R_{opt}(B, 0) \equiv g + B - \tau A\bar{y} = \beta B' \text{ and } GFN^R_{pes}(B, 0) \equiv g + B - \tau A\bar{y} = \beta p B'$$

For a given initial debt, a lower bond price (higher borrowing costs) causes the government to optimally trim its spending/deficit: in each figure, given initial debt, the GFN is lower

[17] This is by virtue of our assumption that riskless pricing always exists in the optimistic beliefs regime.

under risky pricing (pessimism) than under riskless pricing (optimism). Conversely, for any given bond price, a higher stock of liabilities inherited by the government, B, raises the $GFN^R_\rho(B, 0)$, i.e., it shifts the GFN line upwards.

3.2.1 Riskless Pricing for a Low Outstanding Stock of Debt

Figure 3 illustrates a case in which, for a relatively low initial level of liabilities B_{low}, government bonds are traded at the default-free price regardless of whether lenders hold optimistic or pessimistic beliefs.

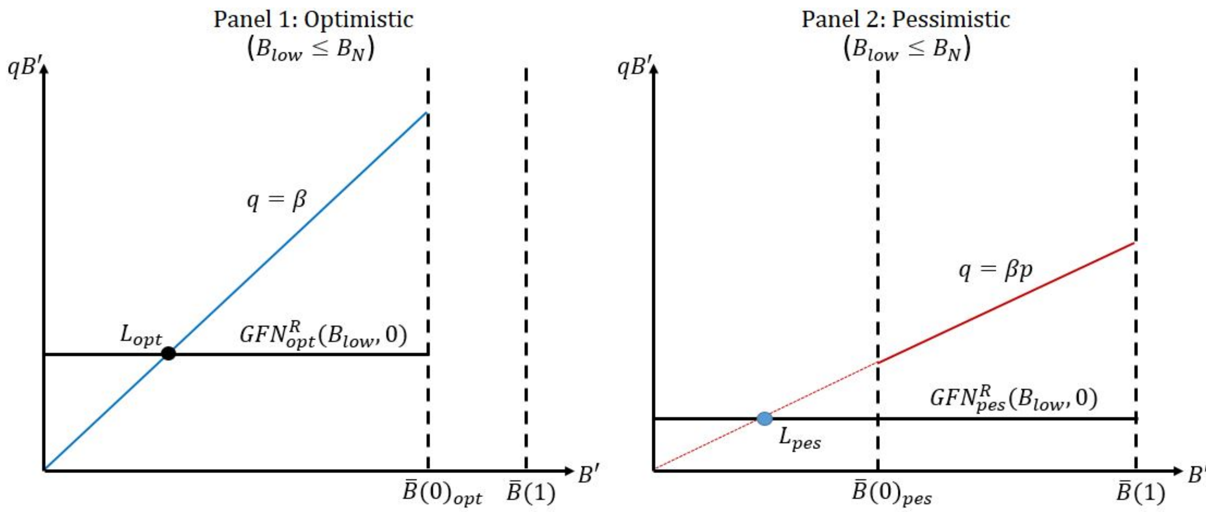

Figure 3: Riskless bond price at a low initial debt B_{low}

In the left panel, given the risk-free bond price, the financing need GFN^R_{opt} is satisfied with issuance up to L_{opt}, well below the debt tolerance threshold $\bar{B}(0)_{opt}$. In the right panel, the GFN^R_{pes} valued at the risky bond price is lower, so is the threshold $\bar{B}(0)_{pes}$, derived conditional on the default-risky price. It is apparent that GFN^R_{pes} is also satisfied with issuance—up to L_{pes}—well below the threshold $\bar{B}(0)_{pes}$. It follows that only the risk-free price is an equilibrium price. Indeed, suppose lenders holding pessimistic beliefs offered the low risky bond price: even at this price, the government moderate issuance would keep the outstanding stock of public liabilities at safe, low levels, proving pessimism wrong (i.e., $q = \beta p$ is not the solution to lenders' first order condition (1)). Note that this would be so, in spite of the fact that under pessimism lenders' debt tolerance is significantly lower than under optimism ($\bar{B}(0)_{pes} < \bar{B}(0)_{opt}$).

For analytical purpose, it is useful to keep track of the level of outstanding debt (determining the position of the GFN function in the graphs) at which the economy becomes vulnerable to crises driven by pessimistic beliefs. To do so, we introduce a new debt threshold, labelled B_N, defined as the maximum amount of the initial debt level such that, in a recession, there is no equilibrium in which bonds are traded at the risky price (i.e., the country is immune to pessimism), $B_N \equiv \inf \mathbb{B}_{pes}$. This threshold will play an important role in our analysis in Section 5.

3.2.2 Multiplicity with Slow-Moving Debt Crises

The GFN lines in Figure 4 are drawn for a level of initial debt larger than B_N but smaller than $\bar{B}(0)_{pes}$—in the graph, this initial debt level is referred to as B_{mid}. In the figure, the level of debt issuance required to satisfy the GFN is below the threshold $\bar{B}(0)_{opt}$ in the left panel, but above the threshold $\bar{B}(0)_{pes}$ in the right panel. It follows that, in equilibrium, debt issuance may either remain below or exceed the relevant threshold, depending on whether lenders offer the riskless or the risky bond price. Either price can be a self-validating equilibrium price.

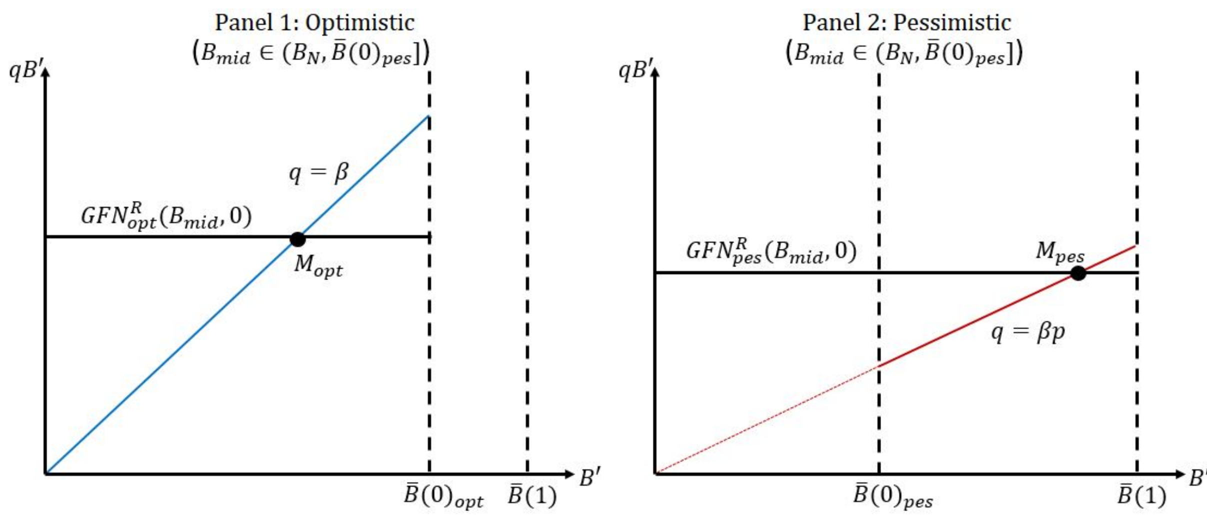

Figure 4: Multiple equilibria when $B_{mid} \in (B_N, \bar{B}(0)_{pes}]$

To expand on the economics of multiplicity: in the panel to the left, lenders have optimistic beliefs and offer to buy newly issued sovereign debt at the riskless price β; borrowing is moderate, and the stock of debt remains below the threshold $\bar{B}(0)_{opt}$. This validates ex post the lenders' optimistic beliefs. In the panel to the right, lenders have pessimistic beliefs and offer to buy bonds at the lower risky price. The government optimally adjusts its primary surplus to the higher borrowing costs. The cut in GFN is however insufficient to keep the new issuance debt debt below the tolerance threshold $\bar{B}(0)_{pes}$ (which is lower than $\bar{B}(0)_{opt}$). Since the large issuance foreshadows a default in the future if the economy fails to recover, it validates ex post pessimism among lenders.

The graph provides economic insight on the main results in Calvo (1988). For intermediate levels of debt, once lenders coordinate their expectations on the equilibrium in which sovereign bonds trade at the default-risky price βp, trade occurs at this price. Even if the government optimally responds by cutting spending and the deficit, under discretion this response is insufficient to prevent an increase in B' above the tolerance threshold. While the risk-free price β is also a (much preferred) equilibrium price, a discretionary government has no credible way/instrument to coordinate the regime of expectations prevailing in the market away from pessimism.[18]

The type of equilibrium with belief-driven default shown in Figure 4 corresponds to a scenario that Lorenzoni and Werning (2019) dubs a 'slow-moving' debt crisis. Interest rates are high because lenders expect the government to default if a recession persists. Because of high borrowing costs, the stock of government debt rises markedly prior to default. But default only occurs if and only if the country remains in a recessionary state in the future.

3.2.3 Multiplicity with Fast Debt Crises

By the logic of the model, however, there is also another type of equilibrium with belief-driven default. Figure 5 is drawn assuming a relatively high initial debt level, B_{high}, higher than $\bar{B}(0)_{pes}$. The left panel depicts the equilibrium where, despite the high debt, under optimistic beliefs a relatively large GFN is financed at the risk-free rate. The right panel shows what happens when lenders turn pessimistic. At the risky price, the adjustment required to reduce the GFN and roll over the large outstanding debt becomes exceedingly costly in terms of social welfare. Default welfare-dominates repaying. Since lenders anticipate this, a "fast" debt crisis occurs: lenders set $q = 0$, the government loses market access and default occurs the moment markets become pessimistic.

One can look at this fast debt crisis from two angles. From the vantage point of the government, even if, counterfactually, lenders were willing to finance the deficit at the default-risky bond price βp, default would be the government's preferred option at the end of the period. Knowing this, from the lenders' vantage point, it is rational not to finance the government at all: the country experiences a rollover crisis.

3.3 Comparison with Cole and Kehoe (2000) and Aguiar et al. (2022)

The rollover crisis discussed in relation to Figure 5 differs from the canonical rollover crisis modelled by Cole and Kehoe (2000). In Cole and Kehoe (2000), it is the sudden loss of market access that makes the surplus adjustment (required to repay existing obligations)

[18] We should note here that, different from the original analysis in Calvo (1988), the equilibrium when $\rho = pes$ in our model is "stable" in the sense discussed by Lorenzoni and Werning (2019). In Figure 4, a small deviation from the equilibrium does not create the kind of instability discussed by these authors, whereby the government can raise more revenue by reducing issuance at the margin

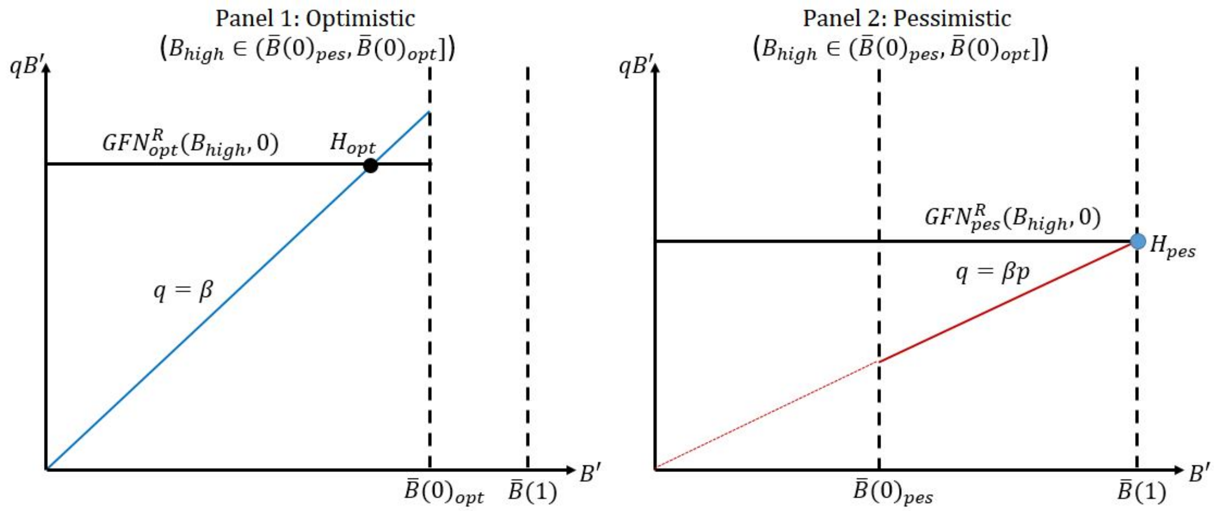

Figure 5: Fast crisis when $B_{high} \in (\bar{B}(0)_{pes}, \bar{B}(0)_{opt}]$

too large and harsh, for the choice of servicing the outstanding debt to welfare-dominate default. The crisis is generated by a within-period two-way feedback loop between a 'failed auction' and the default decision. This model highlights a type of beliefs that may make countries vulnerable to rollover crises already at relatively low levels of debt. In the model underlying Figure 5, instead, rollover crises occur only at a relatively high level of debt. The government loses market access precisely because lenders understand that debt would be too high to be sustainable, even if they were willing to finance the government at the (candidate) equilibrium default-risk premium. The two way feedback loop that generates the crisis is inherently *intertemporal*. Relative to the literature, our model highlights a new dimension of vulnerability to self-validating crises. Countries that are able to finance a high debt at the risk-free rate may suddenly be cut off from market financing.

In recent work, Aguiar et al. (2022) reconsiders the Cole and Kehoe (2000) framework, showing how multiplicity generated by a two-way within-period loop between market beliefs and debt issuance may drive risk premia and debt accumulation over time. In other words, a 'slow-moving debt crisis' can materialize independently of the intertemporal feedback loop modelled in Calvo-style analyses. These authors amend the canonical model by adding a state variable affecting the costs and benefits of default, whose realization is unknown when new debt is issued, but precedes the decision to repay or default at the end of the period. This new state variable, intervening between the debt auction and the decision to repay vs. default, introduces in the model intra-period risk—say, after the auction, the government may choose to repudiate debt because, ex post, it becomes less tolerant of the cuts in social spending planned at the beginning of the period, or because an international institution may discontinue an ongoing financial assistance program. Even if such developments may occur with a very small probability, lenders' concerns with them profoundly affect the optimal issuance strategy by the government as well as the equilibrium debt pricing by lenders, generating a rich set of equilibria.

Aguiar et al. (2022) first shows that, holding B' fixed, intra-period risk of default would give rise to equilibria with 'depressed' bond price, validating lenders' "concerns" (i.e., beliefs) about within-period repayment. They further show that the government can (optimally) eliminate this risk by over-issuing debt, so to obtain extra resources that can sustain its spending for any realization of the new state variable.[19] But over-issuing debt does not eliminate multiplicity. When lenders becomes "concerned" about within-period default risk, the optimal B' required to rule out such risk correspondingly hikes. The optimal issuance strategy thus comes at a cost: it prevents risk premia from arising as a reflection of lenders' concerns about end-of-period default, by letting debt accumulation deteriorate the credit prospects of the country over time. In equilibrium, intra-temporal risk results in intertemporal risk: as stated in Aguiar et al. (2022), "multiplicity is fundamentally static in nature". In our Calvo-style model, instead, the nature of multiplicity is fundamentally intertemporal, also when it results in intra-period default risk of rollover crisis.

4 Numerical Analysis

A key conclusion from our short-term debt analysis is that a switch from optimistic to pessimistic beliefs might result in either "slow" or "fast" debt crises, depending on debt levels. In the rest of the paper, we revisit this result and discuss some of its key implications using numerical examples that generalize the economies depicted in Figure 3-5. We allow for long-term debt and calibrate the model with standard parameter values from the literature.[20] In this section, we keep the assumption that agents hold 'constant beliefs', i.e., they assign a zero probability to a switch in the regime of expectations. In our experiment, agents are initially optimistic; we study an unanticipated permanent switch to pessimistic beliefs. In the next section, we relax this assumption, introducing sunspots, where agents trade bonds assigning a small positive probability to a switch between beliefs regimes.

[19] A key assumption in Aguiar et al. (2022) is that the government can neither re-open the auction after the realization of intra-period shock, nor accumulate the cash buffers—the resources raised from markets are used to repay existing liabilities and finance current spending.

[20] In Appendix C, we work out a numerical example assuming one-period bonds ($\delta = 1.0$), using the same calibration as in this section.

4.1 Calibration

In solving the model with long-term debt, we adopt the following functional form for the utility function:

$$\mathcal{U}(c,g) = \log(c) + \gamma \log(g - \bar{g}); \tag{8}$$

In our calibration, we set benchmark parameters following Conesa and Kehoe (2017). The parameter values are shown in Table 1.

Table 1: Parameter values

$\bar{y}$	Output	100
Z	Cost of Default	0.95
β	Discount factor	0.98
γ	Relative weight of c and g in the utility function	0.20
τ	Government revenue as a share of output	0.36
$\bar{g}$	Level of the critial government expenditure	25
A	Fraction of output during recession	0.9
p	Probability of leaving the recession	0.2
δ	Amortization rate of market debt	0.2

As shown in the table, we normalize output $\bar{y}$ to 100 so that the units in the model can be interpreted as percentage of GDP: e.g. $B = 50$ means that debt to GDP ratio is 50% in normal times. We set cost of default as $5\% = 1 - Z$. Our default cost is lower relative to the literature (e.g. Alesina et al. (1992)), on the grounds that we assume this cost to be permanent.[21] We assume the relative weight of government utility is 0.2; sensitivity analysis shows that this parameter is unimportant for our result.

The severity of recession A is set to 0.9 so that a recession results in a decrease in output by 10% for the benchmark scenario. This parameter is crucial to generating gambling on a recovery in an optimistic world. A more severe recession leads to a stronger smoothing motive for the government—we report results for different A in Appendix G.

We set the critical government expenditure $\bar{g}$ at 25% of GDP in normal times: the higher this value, the smaller the room for discretionary spending. Government revenue as a fraction of output is determined by the constant tax rate τ. In normal times, the government income is 36, but in a recession, it drops to 32. We set $\delta = 0.2$ to match average maturity from 2000-2009 for Greece, Italy and Spain, which is about 5 years. We set $p = 0.2$ so that the expected waiting time for recovery is 5 years.

[21] Upon a default, in our benchmark scenario Z cannot be too small in that the government spending cannot fall below $\bar{g}$. In other words, the conditions $\tau A Z \bar{y} > \bar{g}$ and $\tau Z \bar{y} > \bar{g}$ must be satisfied.

4.2 Slow-Moving and Rollover Crises with Long-Term Debt

Figure 6 plots the policy functions conditional on a recession, together with the debt tolerance thresholds (in a recession and in normal times), in the optimistic world (left panel) and the pessimistic world (right panel). Recall that in this numerical example agents are initially optimistic and view a regime switch as the zero probability event.

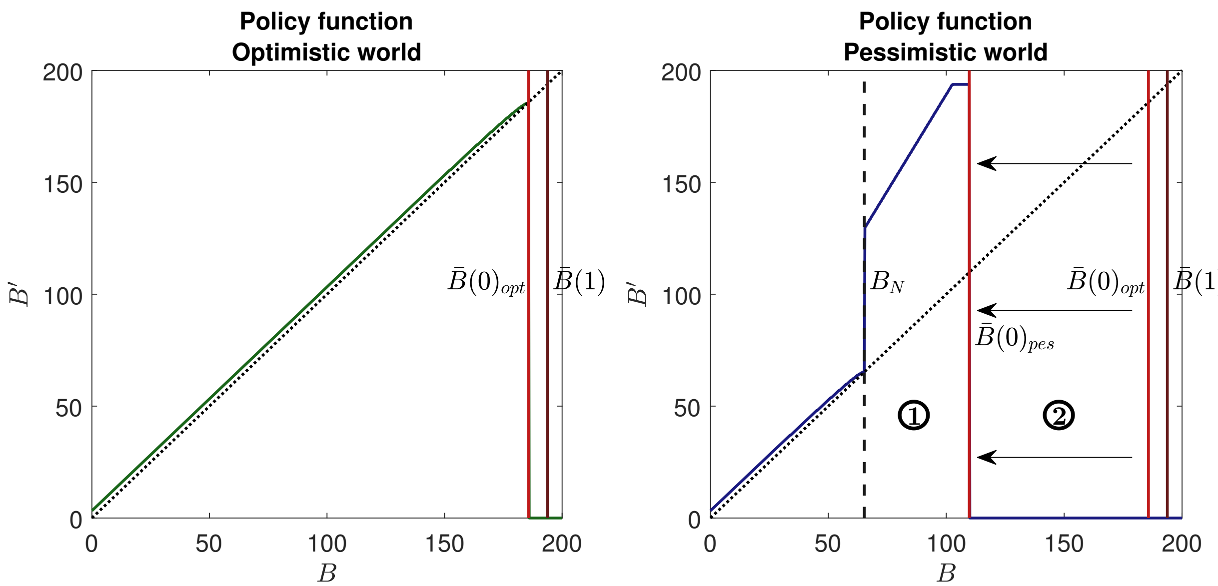

Figure 6: Policy functions in the optimistic world and the pessimistic world

In the optimistic equilibrium depicted on the left panel of Figure 6, the debt tolerance threshold in a recession is strikingly high, about 206% of GDP (or 186% as a ratio of GDP in normal times). As further discussed in Section 6, in our exercises we find that $\bar{B}(0)_{opt}$ is generally quite insensitive to the probability of recovery p or debt maturity δ. In a recession, the government smooths consumption by borrowing at the risk-free rate until the stock of outstanding debt reaches $\bar{B}(0)_{opt}$. As shown in the figure, the dynamics of debt are mildly increasing, until the stock of debt reaches the upper threshold, at which point it stabilizes.

The right panel of Figure 6 depicts what happens when lenders unexpectedly change their view on government solvency, from optimistic to pessimistic. While $\bar{B}(1)$ is not affected (reflecting our assumption that, after recovering, the economy never falls back into a recession again), the switch in beliefs regimes leads to a sharp fall in the threshold from $\bar{B}(0)_{opt}$ to $\bar{B}(0)_{pes}$, and the consequences for debt dynamics are stark.

If the outstanding debt is in the crisis region between B_N and $\bar{B}(0)_{pes}$, i.e., in the region labelled ① in our figure, the economy is vulnerable to slow-moving debt crisis. As shown in the right-hand panel, debt accumulates rapidly, driven by the beliefs-driven hike in interest rates. Observe that a lower debt tolerance threshold makes pessimistic views 'easier to validate' in equilibrium. Under our calibration, a crisis can arise for a debt to GDP ratio as low as 72% (65% if GDP is measured in normal times).[22]

If debt is in the crisis region between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$—the region labelled ② in the figure—the crisis is "fast": a default occurs simultaneously with the (unanticipated) shift in beliefs. Once more, when debt is in the fast crisis region, as long as lenders are optimistic, the government can actually issue debt at the risk-free rate. But once lenders change their view, the bond market dries out altogether: there is no "slow-moving" accumulation of debt. In our calibration, fast crises can occur with a debt to GDP ratio between about 122% and 206% (at the GDP measured in the recessionary state). Facing a collapse of the bond market, the government defaults immediately.

If debt is in the safe debt region between 0 and B_N (the maximum initial debt level below which the country is "immune to pessimism"), the dynamics of debt are comparable to the case of optimism. In this region, the government bond price is unaffected by the switch in the regime of beliefs: when debt is low enough, by the logic of the model, even if lenders were to offer the depressed bond price, the dynamics of debt accumulation at this price would not validate their pessimism (see Figure 3). Consequently, lenders would still lend at the risk-free rate despite the switch to the pessimistic beliefs regime.

There is nonetheless a notable difference relative to the case of optimistic beliefs. The bound on safe debt issuance under pessimism B_N is much lower than $\bar{B}(0)_{opt}$, around 70%. This means that a switch to pessimism indeed imposes tighter borrowing constraint on the government—debt must be kept low enough to fully eliminate the risk of a fall in the value of bonds that are not fully amortized each period. Under optimism, the government does not face such a tight safe debt limit, as lenders perceive no risk of suffering a capital loss on their bond holding driven by a deterioration of market expectations. Indeed, in the left panel, around levels of debt corresponding to B_N (around 70%), the government keeps borrowing, letting debt gradually grows up to $\bar{B}(0)_{opt}$. These considerations together provide a useful theoretical insight: conditional on the same (risk-free) bond price, the optimal deficit may be different, depending on the anticipated state of beliefs in the future—the regime of pessimistic beliefs induces endogenous austerity, as highlighted in the figure.

Overall, the numerical exercise in this section emphasizes two significant novel results from our analysis. First, the threat of "slow-moving" and "fast" debt crises remain pervasive with long-term debt. Second, even if the market bond price is risk-free, optimal fiscal policies can vary depending on the anticipated state of beliefs in the future—market pessimism makes the government more cautious in its budget and debt accumulation policies.

[22] In our calibration, a slow-moving crisis actually gets resolved rather quickly, as the probability of a default one period ahead is 0.8, and the risk of default disappears if the economy recovers.

5 Costs and Benefits of Debt Reduction Policies in a Recession

So far, we have imposed the simplifying assumption that lenders and the government attribute zero probability to a switch to pessimism. In this section, we model sunspot equilibria where all agents in the economy anticipate the possibility of a change in beliefs regime. This brings our specification more closely in line with Conesa and Kehoe (2017), and allows us to reconsider the main issue these authors address in their paper.

In our baseline exercise, we posit that lenders are initially optimistic, but understand that, in each period, the beliefs state ρ transits between optimistic and pessimistic ($\rho \in \{opt, pes\}$) with probability $\pi = 0.04$. Conditional on the realization of the sunspot, a switch in equilibrium pricing and fiscal policy occurs if and only if the GFN is such that pessimistic beliefs are self-validating. This is the case if the government, when a switch occurs, either borrows more than $\bar{B}(0)_{pes}$ (slow-moving crises), or defaults ("fast" crises). For simplicity, when pessimistic beliefs are self-fulfilling, we assume that lenders stick to this expectations regime *forever* afterwards. To save space, in the following we focus on policy functions and the equilibrium bond price in a recession. The debt tolerance threshold in a recession is now denoted with $\bar{B}(0)_{\pi}$.

A natural question that arises in this framework is if and under what conditions, facing the possibility of beliefs-driven crises, a government would optimally choose to decrease debt to safe levels—running surpluses even if the economy is in a recession. The motivation for doing so lies in avoiding the large drop in expected utility the economy would suffer when it becomes vulnerable to belief-driven crises, and/or benefiting from lower borrowing costs associated to reduced vulnerability. We refer to the former as the welfare '*cliff effect*' (see Appendix D for details), as opposed to the latter—the '*price effect*'.

5.1 Baseline with Pessimistic Beliefs

Figure 7 displays the policy function (left) and the equilibrium bond price (right) in our baseline with long-term debt, where lenders assign a small probability to a switch to pessimistic beliefs. For debt levels in the region between 0 and B_N, the debt dynamics are the same as in the right panel of Figure 6, and the government is able to issue safe debt.

In the region between B_N and $\bar{B}(0)_{\pi}$, where the economy is vulnerable to sunspot crises, the debt dynamics are different from what we have seen so far—it is no longer uniform. This region can be split into two sub-regions. For an initial debt level exceeding but close to B_N, the government chooses to run surpluses and reduce its borrowing. However, for large enough initial debt, the government prefers to keep borrowing. It will do so until its debt

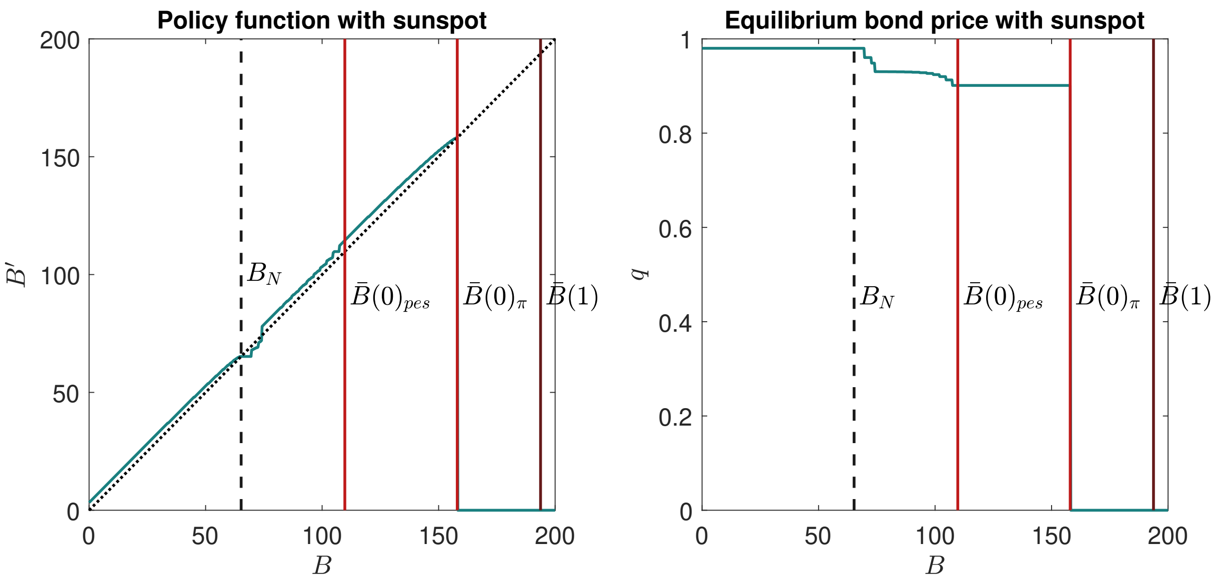

Figure 7: Baseline ($\rho \in \{opt, pes\}$), $\delta = 0.2$

level reaches $\bar{B}(0)_{\pi}$, even for debt levels above (but close to) $\bar{B}(0)_{pes}$, where self-fulfilling crises, if they occur, are fast.

Why is deleveraging optimal for debt levels just above B_N, but not so for debt levels just above $\bar{B}(0)_{pes}$? The key insight is that keeping debt below $\bar{B}(0)_{pes}$ shields the country from fast crises, but not from slow-moving ones. Hence, while the government may still have some advantage not to let debt trespass $\bar{B}(0)_{pes}$, this advantage is exclusively in terms of lower borrowing costs (the 'price effect'), not in terms of eliminating the possibility of crises 'tout-court' (the welfare 'cliff effect' is less relevant here). When debt is long term, the borrowing costs advantage alone (about 1.4 percentage point) is not enough to offset the benefits from smoothing consumption in a recession via borrowing.[23]

This is a distinctive result of our analysis. In our baseline, deleveraging is preferred over smoothing with debt accumulation only for a small range of debt above the threshold at which slow-moving crises become a possibility (associated with the 'cliff effect' in welfare). For a very wide range of debt levels, the government prefers to accumulate liabilities and smooth consumption, "gambling" on the prospective recovery.

[23]Crucial to this result is the low pass-through of higher interest rates into borrowing costs when the outstanding stock of debt has long maturity—an observation that resonates with the analysis of debt dilution in Aguiar and Amador (2020), also in Aguiar et al. (2019). The gains in terms of lower borrowing cost from deleveraging are shared between the lenders (as a capital gain) and the government (the 'price effect') for long debt maturity. In Appendix C, we present a simulation result of our baseline with short-term debt. In this case, the 'price effect' is stronger, so is the incentive to deleverage for the government.

5.2 Comparison with Conesa and Kehoe (2017)

The results above are best appreciated in relation to the analysis by Conesa and Kehoe (2017). A comparison with their work highlights the value added of our analysis, in showing how the economic forces underlying the optimal choice between smoothing and deleveraging play out very differently, theoretically and quantitatively, depending on the specification of the sunspot, hence on beliefs.

To compare our analysis with Conesa and Kehoe (2017), we specify a sunspot allowing for "Cole-Kehoe (CK) beliefs". In a regime of CK beliefs, lenders are only willing to finance the government if this remains able and willing to honour its outstanding liabilities in the *current* and *future* period *independent of market financing*. While leaving the full characterization of the model to Appendix E, hereafter we state our main results. First, we show that, by assigning a small probability to a switch from optimism to CK beliefs, i.e., $\rho \in \{opt, CK\}$, our model is able to fully replicate debt dynamics in Conesa and Kehoe (2017). Second, in sunspot equilibria with CK beliefs, the incentive to reduce debt generally dominates the government's debt policy—it deleverages over a much wider range of debt relative to our baseline with $\rho \in \{opt, pes\}$. Third, yield rates on long-maturity debt are much higher. Since bond prices respond strongly to the sunspot probability, the price effect as a motivation for optimal deleveraging is substantial.

In Table 2, we compare sunspot equilibria with pessimistic and CK beliefs, i.e., $\rho \in \{opt, pes\}$ vs. $\rho \in \{opt, CK\}$, contrasting economies with long- vs. short-term debt. The first three columns of the table report the debt tolerance thresholds B_N and $\bar{B}(0)_\pi$, together with the range of debt in the crisis region over which the government finds it optimal to deleverage (expressed in percentage of the total width of the crisis region). The last three columns report the equilibrium interest spread over the risk-free rate, conditional on an initial debt to GDP ratio equal to, respectively, 33%, 66% and 99%.

Table 2: Relevant thresholds, debt dynamics, and interest spread

Model	The maximum debt to GDP ratio immune to debt crises (%)	$B(0)_\pi/(A\bar{y})$ (%)	Proportion of deleveraging (%)	$r-r^*$ when $\frac{B}{A\bar{y}}=33\%$	$r-r^*$ when $\frac{B}{A\bar{y}}=66\%$	$r-r^*$ when $\frac{B}{A\bar{y}}=99\%$
			Long-term bonds ($\delta = 0.2$)			
Baseline, $\rho \in \{opt, pes\}$	73	176	9.38	0%	0%	5.52%
CK beliefs, $\rho \in \{opt, CK\}$	38	112	83.66	0%	11.07%	17.78%
			One-period bonds ($\delta = 1.0$)			
Baseline, $\rho \in \{opt, pes\}$	13	141	13.83	0%	3.38%	3.38%
CK beliefs, $\rho \in \{opt, CK\}$	8	83	84.66	4.25%	4.25%	Default

The first row of Table 2 refers to our baseline with long-term debt. The debt levels in the region where crises are possible ranges from 73 to 176 percent of GDP. The government deleverages over around 1/10 of this region, when B is close to B_N. Replacing pessimistic with CK beliefs (see second row of the table), the government is exposed to the risk of debt crises, in the form of rollover crises, at much lower levels of debt—between 38 to 112 percent of GDP. Deleveraging is optimal over much of this region—about 84 percent of it![24] As shown in the third and fourth row of the table, sustainability substantially deteriorates with short-term debt. With one-period bonds, the economy becomes vulnerable to crisis at much lower level of debt, 13 and 8 percent of GDP, respectively, for $\rho \in \{opt, pes\}$ and $\rho \in \{opt, CK\}$. Relative to the case of long-term debt, the range over which deleveraging is optimal widens somewhat (especially for $\rho \in \{opt, pes\}$).

The last three columns of Table 2 show that, for any given initial debt, the government pays higher borrowing costs when lenders price the possibility of a rollover crisis $\rho \in \{opt, CK\}$, relative to the threat of slow-moving debt crises $\rho \in \{opt, pes\}$. Moreover, with long-term debt, the borrowing costs rise more steeply with the level of debt. By way of example, for $\rho \in \{opt, CK\}$, the spread of long-term bonds increases from 0 to 17.78% when debt to GDP ratio rises from 33% to 99%; for $\rho \in \{opt, pes\}$, the spread increases from 0 to only 5.52%. A stronger 'price effect' contributes to explain why deleveraging is preferred for a wider range of debt when $\rho \in \{opt, CK\}$.

6 Does a Long Debt Maturity Rule out Self-Fulfilling Crises?

For any given stock of debt, longer maturities may help sustainability by reducing the exposure to rollover risk and the pass-through of hikes on interest rates onto the total cost of servicing the outstanding debt. Our model is particularly suitable to revisit the questions of whether and under what circumstances maturity can rule out multiplicity, as it distinguishes between slow-moving and fast debt crises.

We therefore conclude our study with a close up analysis of a country's vulnerability to self-fulfilling debt crises for a different maturity of public debt, varying the expected persistence of a downturn.[25] We focus our analysis on the cases of optimistic and pessimistic beliefs.[26] Our specific goal is to highlight conditions under which self-fulfilling debt crises may/may not occur.

In Figure 8 we plot debt tolerance thresholds in a recession against variable debt maturity (left panel) and probability of recovery (right panel). Focus first on the left panel of Figure

[24] As the policy function when $\rho \in \{opt, CK\}$ is similar to Conesa and Kehoe (2017), to save space, we show it in Appendix E.

[25] In Appendix G, we also vary the depth of the recession.

[26] To save space, we do not include an analysis of CK beliefs. Note that, under these beliefs, a longer maturity and/or a higher probability of recovery have the strongest effect on the safe debt threshold. Reducing δ and increasing p, everything else equal, raises this threshold much more (in percentage terms) than $\bar{B}(0)_{opt}$ and $\bar{B}(0)_{pes}$.

8. A first observation is that $\bar{B}(0)_{opt}$ (solid line) is largely insensitive to debt maturity, but for extremely small values of δ ($\delta \to 0$), corresponding to very long-term debt. Why? Recall that optimistic lenders offer the riskless price as long as such a price maximizes the welfare of the government. This is the case when debt maturity is not extremely long—then, under optimism, longer-term and shorter-term bonds are basically equivalent in the eye of the lenders, and $\bar{B}(0)_{opt}$ is invariant to the maturity structure of bonds. However, when $\delta \to 0$ (debt maturity is very long), the bond price that maximizes societal welfare can be lower than the risk-free price. Intuitively, a low bond price raises borrowing costs at the margin (i.e., the interest rate on new issuance), but also deteriorates the value of outstanding bonds (the government benefits from diluting existing debt). When the amount of outstanding debt is large enough, the benefit from debt dilution outweighs the higher borrowing costs: welfare is higher under default-risky pricing than under riskless pricing. Consequently, optimistic lenders offer the default-risky price—same as pessimistic lenders. In the figure, in the limit case of consols, the threshold $\bar{B}(0)_{opt}$ is higher and coincides with $\bar{B}(0)_{pes}$.

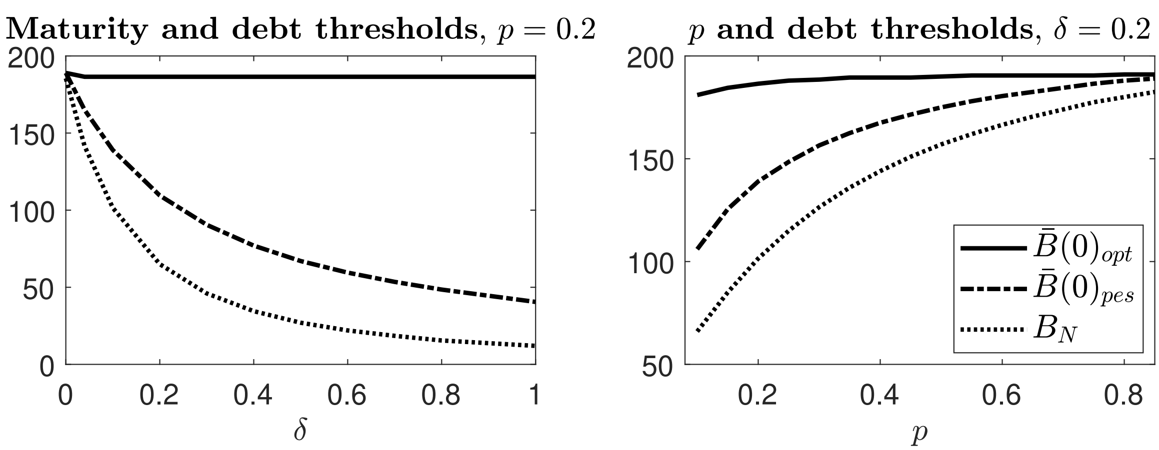

Figure 8: Debt thresholds given $A = 0.9$

In contrast to $\bar{B}(0)_{opt}$, both B_N and $\bar{B}(0)_{pes}$ (the dotted and dashed lines in the figure) decrease sharply with δ; that is, they increase with a longer maturity of debt.[27] To see why, consider the net bond revenue in a pessimistic world, $\beta p(B' - (1-\delta)B) - \kappa B$, where $\beta p B'$, $\beta p(1-\delta)B$ and κB denote, respectively, revenue from newly issued bonds, the value of the outstanding stock of bonds, and interest payment to lenders. Maturity has opposite effects on these terms. As the maturity of bonds becomes longer ($\delta \downarrow$), the value of the outstanding stock of bonds $\beta p(1-\delta)B$ rises but the interest payments due in the period κB fall. The first effect decreases, while the second effect increases the net bond revenue. Rearranging

[27] Note that, the market value of bonds around the threshold under pessimistic beliefs regime falls with lower δ (longer debt maturity), due to debt dilution. By contrast, under optimistic beliefs regime, the market value of bonds generally does not vary with δ, unless fast debt crises is fully eliminated.

the net revenue equation as follows $\beta p B' - [1 - \beta(1-p)(1-\delta)]B$ makes it clear that the second effect always dominates the first one: a fall in δ unambiguously increases net debt revenue—explaining why $\bar{B}(0)_{pes}$ and B_N are larger as the debt maturity becomes longer.

To gauge whether longer debt maturities can eliminate multiplicity, we let δ decrease from 1 to 0 (from one-period debt to consols) and track the changes in debt tolerance thresholds. In the left-hand side panel, as debt maturity grows, the distance between $\bar{B}(0)_{opt}$ and $\bar{B}(0)_{pes}$—the fast crisis region—becomes progressively narrower, up to disappearing in the limit case of consols. On the contrary, the distance between B_N and $\bar{B}(0)_{pes}$—the "slow-moving" crisis zone—remains approximately unchanged at short to moderately long maturities, and only starts to narrow significantly when debt maturity becomes very long. While fast crises disappear with very long bond maturities, slow-moving crises are always a possibility, as B_N never coincides with $\bar{B}(0)_{opt}$. Not even maturities approaching "consols" can rule out the multiplicity—slow-moving crises remain pervasive.

The right panel of Figure 8 shows that the probability of recovery p does not have much of an effect on $\bar{B}(0)_{opt}$, while it has a significant impact on both B_N and $\bar{B}(0)_{pes}$. The net bond revenue in an optimistic world, $\beta(B' - (1-\delta)B) - \kappa B$, does not vary with p, while the net bond revenue in a pessimistic world, $\beta p(B' - (1-\delta)B) - \kappa B$, is unambiguously increasing in p. A higher probability of recovery p significantly narrows the "fast" crisis zone. It also narrows, but to a lesser extent, the "slow-moving" crisis zone. In the left panel of Figure 8, this means that the dotted and dashed lines get both closer to the solid line when p rises.

In the appendix, we extend our analysis specifying a "debt-limit" (as opposed to "strategic default") version of our model (see Appendix F). We show that also in this framework belief-driven crises (fast and slow) remain a pervasive possibility. However, the conditions under which rollover crises can be eliminated are relatively mild: in addition to a sufficiently long debt maturity, the probability of a recovery cannot be too small. The parameter restrictions that rule out rollover crises in our model are somewhat more stringent—see Appendix G for details.

A takeaway from our exercises is that fast (rollover) crises may be eliminated, but multiplicity remains pervasive for *all debt maturities* in both our model and its debt-limit variant. A long debt maturity may reduce vulnerability by eliminating the possibility of fast crises (especially in the debt-limit model) and by narrowing the "slow-moving" crises zone. Yet, slow-moving ones never disappear—even with consols.[28]

[28] We should note here that the differences between our results and Lorenzoni and Werning (2019) originate from a different assumption concerning output uncertainty. In Lorenzoni and Werning (2019), output is drawn from normal distribution with a single peak—which in their debt-limit framework rules out multiple equilibria for one-period bonds. Our model, instead, features a bimodal distribution for which, as explained in footnote 6, stable equilibria exist at high interest rates for all debt maturities.

7 Conclusion

The literature has long emphasized that, once a country debt is sufficiently high, the equilibrium is no longer unique and the country is vulnerable to disruptive self-fulfilling crises. As the COVID-19 pandemic is causing widespread economic crises across the globe, it is unavoidable that debt stocks rise virtually everywhere, potentially undermining stability in the bond markets in advanced countries and raising issues in which instruments are available to keep these markets in a "good equilibrium".

This paper shows that different types of self-fulfilling crises, one emphasized by Calvo (1988), the other by Cole and Kehoe (2000), may occur in the same dynamic Calvo (1988) setting. In particular, both slow-moving debt crises and rollover crises are possible when lenders coordinate on what we dub "pessimistic" beliefs, while rollover crises are a form of self-fulfilling debt crises specific to "Cole-Kehoe" beliefs.

We revisit debt dynamics and the incentive to deleverage when governments operate under the threat of self-fulfilling debt crisis. This is an important issue, that may dominate debates on fiscal policy in the post-COVID-19 high-debt regime. Under the threat of rollover crises driven by lenders coordinating on "Cole-Kehoe" beliefs, in line with the literature, we find that a forward-looking benevolent government would generally reduce its debt even in a recession. As a contribution to the literature, however, we also show that, if crises are anticipated to be slow-moving—driven by "pessimistic" beliefs—, deleveraging is optimal only over a relatively small range of debt. We stress that this result is obtained independently of political economy considerations, with policymakers modelled as short-sighted or self-interested. Even for forward-looking benevolent governments, the threat of slow-moving debt crises is generally not enough to motivate precautionary fiscal policy of risk reduction. This suggests that, after a crisis causing an abrupt rise in borrowing, debt may remain at high levels for a long time, even if governments are aware that their failure to deleverage keeps their country exposed to the threat of belief-driven crises.

As a direction for future research, theory can be brought to bear on debt sustainability when the government can choose the maturity of its debt and rely on external bailouts or liquidity assistance. The logic of our model suggests that the way debt management and/or official support can impinge on the dynamics of debt and vulnerability to crisis crucially rests on how these policies are able to affect the incentives of a government to gamble on prospective recovery—which, as we have shown, depends on the regime of beliefs. A key question is under what conditions maturity swaps and bailouts may help and speed up deleveraging—as opposed to give an extra incentive to smooth adjustment by borrowing. Bailouts may create a trade-off between resilience to rollover crises and vulnerability to default at high levels of debt. Understanding this trade-off is crucial in the design of an efficient governance of official lending institutions.

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Appendices

A The Model: Two-Period Version

In this section, we specify a two-period economy, with the goal of providing a transparent analytical discussion of the driving forces of debt accumulation and crises. The section is comprehensive and self-contained—it can be used independently of the main text of the paper. Tractability allows us to provide analytical insight on different types of debt crises and the role of lenders' beliefs as driver of instability. We also offer a graphical representation of the equilibria complementary to the one in the text.

A.1 The Model Setup

The economy is populated by a continuum of identical households, a continuum of risk-neutral competitive lenders with measure one, and a welfare-maximizing government. In period 0, the economy starts out in a recession ($a_0 = 0$, assuming no output penalization that in an intertemporal model may result from past default, i.e., $z_{-1} = 1$). In period 1, the economy recovers with the probability p. Hence, the output distribution at $t = 1$, conditional on full repayment of debt in both periods, is

$$y_1 = \begin{cases} \bar{y} & \text{with probability } p \\ A\bar{y} & \text{with probability } 1 - p \end{cases}$$

To focus on the sovereign's behaviour, we assume that in each period the representative household passively consumes all its (exogenous) income after paying taxes, charged at a constant rate τ by assumption. As a further simplifying assumption, without loss of generality in this section we let $\tau \to 1$, so to abstract from the (passive) consumption level altogether.

When financing the government, lenders coordinate on a regime of beliefs ρ_t that shape their expectations about the fiscal outlook. The aggregate state variable of the economy is then summarized by $s_t = (B_t, z_{t-1}, a_t, \rho_t)$.

A.1.1 Timing

The sequence of fiscal and lenders' decisions is summarized in Figure 9. The first period is subdivided in 3 sub-periods, $t = 0^1, t = 0^2, t = 0^3$. At $t = 0^1$, the economy starts out in a recession ($a_0 = 0$); the exogenous shock determining the beliefs regime ρ_0 is realized. The aggregate state $s_0 = (B_0, z_{-1}, a_0, \rho_0)$ is known at the beginning of the period. Conditional on the realized regime of beliefs ρ_0 and the outstanding debt stock B_0, the continuum of measure one of lenders form rational expectations of fiscal decisions of the government, and coordinate their expectations on the bond price q; lenders offer to buy bonds at that price (subject to a bound on aggregate issuance discussed below).

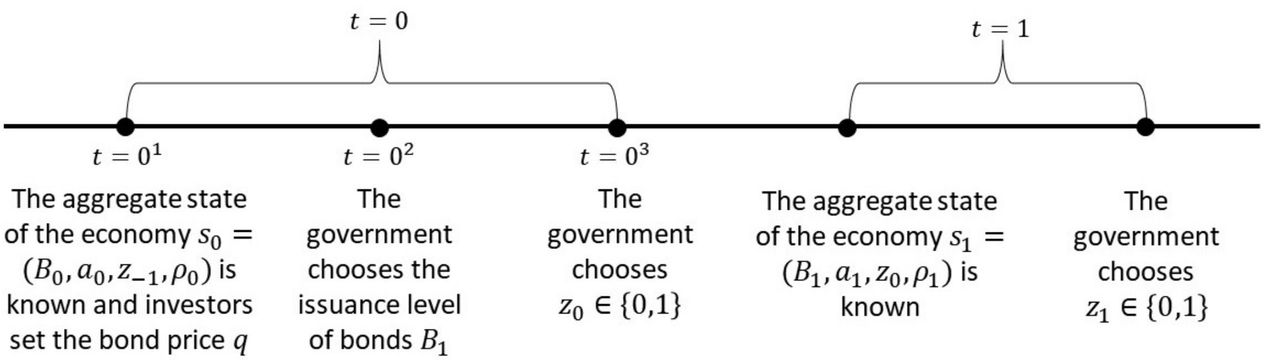

Figure 9: Timeline in two-period economy

At $t = 0^2$ and 0^3, fiscal decision of the government takes place; the government sets how much new debt to issue B_1 at $t = 0^2$, and then decides whether to default on inherited debt z_0 at $t = 0^3$. Due to inability of the government to commit to repay at the end of the period, our model admits the possibility of the canonical rollover crises Cole and Kehoe (2000). However, as in the main text, we leave aside the belief regime that provokes rollover crises of Cole and Kehoe (2000) in two-period economy.

In period $t = 1$, depending on the amount of maturing debt B_1, and the exogenous realization of the output shock a_1, the government decides whether to repay or default—if a default has already occurred in $t = 0$, the agent simply consumes a share of the penalized endowment.[29]

A.1.2 The Government's Problem

Given the two-period horizon, the government problem in the last period is straightforward. If the government has already defaulted in period 0, spending will be set equal to current output scaled down by Z, the penalty for defaulting. Hence in normal times $Z\bar{y}$ while in a recession $AZ\bar{y}$. Otherwise, for a given amount of maturing debt, B_1, the government will honor its liabilities as long as government spending under repayment will be higher than under default. The decision will in turn depend on whether the economy is in normal times

[29] In the two-period model, the realization of the beliefs regime shock ρ_1 is irrelevant, since no new debt is issued.

(g_1^H), or in a recession (g_1^L), that is:

$$g_1^H = \max\{\bar{y} - B_1, Z\bar{y}\} \tag{9}$$
$$g_1^L = \max\{A\bar{y} - B_1, AZ\bar{y}\} \tag{10}$$

where we have used our assumption $\tau \to 1$. In terms of tractability, the key advantage of working with the two-period model is that, in the last period, spending and welfare are independent of lenders' beliefs in the period ρ_1—obviously, there is no further debt issuance in the terminal date. Lenders' beliefs only matter in period 0.

At $t = 0^2$ and 0^3, the government takes its fiscal decision subject to the bond price q offered by lenders and an upper bound on aggregate issuance. The government's problem is reduced to:[30]

$$V_0(s_0) = \max_{B_1, z_0 \in \{0,1\}} u(g_0) + \beta \mathbb{E}_0[u(g_1)] \tag{11}$$
$$\text{subject to } g_0 = AZ^{1-z_0}\bar{y} - z_0 B_0 + z_0 q B_1 \text{ and } g_0 > \bar{g}$$
$$g_1^H = z_0 \times \max\{\bar{y} - B_1, Z\bar{y}\} + (1 - z_0)Z\bar{y}$$
$$g_1^L = z_0 \times \max\{A\bar{y} - B_1, AZ\bar{y}\} + (1 - z_0)AZ\bar{y}$$

where, as already mentioned, we stipulate that spending cannot fall below the critical expenditure level $\bar{g}$.[31]

Note that, from g_1^L and g_1^H in (9) and (10), it is also straightforward to derive two "debt tolerance thresholds" for B_1, $\underline{B}$ and $\bar{B}$, defining debt levels above which the government optimally defaults, respectively, in a recession and in normal times at $t = 1$.

$$\underline{B} = (1 - Z)A\bar{y}$$
$$\bar{B} = (1 - Z)\bar{y}$$

When the government issues B_1 below $\underline{B}$ in $t = 0^2$, always repays in period 1, whether output is low or high; when B_1 lies in between $\underline{B}$ and $\bar{B}$ instead, the government defaults in period 1 if the economy fails to recover. Note that $\bar{B}$ is the maximum debt level at which the government can be expected to honor its liabilities in period 1 when the economy recovers—beyond this threshold, default occurs with probability one.

[30] As in infinite-period version of our model, the total debt issuance is subject to a bound that depends on the bond price offered, to eliminate the possibility of violating rational expectations.

[31] We implicitly assume that sovereign's tax revenue after default, $Z\bar{y}$ and $AZ\bar{y}$, are larger than $\bar{g}$.

A.1.3 The Lenders' Problem

We assume that the risk-neutral lenders are "deep-pocketed", such that corner solutions in each lender's problem are ruled out in equilibrium. Hence, the lenders' break-even condition requires that:

$$q(s_0) = z_0 \beta \mathbb{E}[z_1] \tag{12}$$

There may exist multiple triples (q, B_1, z_0) that simultaneously solve (11) and (12). The equilibrium price is a function of the regime (ρ_0) on which lenders coordinate their beliefs. In our analysis, we will focus only on "optimistic" (*opt*) vs "pessimistic" (*pes*), writing $\rho_0 = opt$ and $\rho_0 = pes$.

A.2 Debt Levels and Self-Fulfilling Debt Crises

We now use the model to show how progressively higher levels of initial debt give rise to the possibility of multiple equilibria—where a 'good' equilibrium in which government bonds are priced at the riskless rate, coexists with different types of bad equilibrium. For intermediate level of debt, in the bad equilibrium government bonds are traded at the default-risky; at higher level of debt, the government may lose market access altogether.

To show this result, in what follows, we will first study the value of repaying at different levels of initial debt, conditional on either the risk-free or the default-risky bond price. Second, given the value of repaying at these bond prices, we will characterize three regions of the initial debt defining, respectively, a 'safe region' with no default; a region where the economy is at the risk of prospective debt crises in period 1 conditional on a recession; and a region at the risk of rollover crises and default already in period 0.

For ease of exposition, as in the main text, we restrict the focus of our analysis to economies in which an equilibrium with risk-free pricing of government bonds always exists under a regime of optimistic expectations.

A.2.1 The Value of Repaying

We start our analysis by studying the value of repaying debt when bonds are traded at the riskless price β, as opposed to the default-risky price βp, at different levels of initial debt. We plot the results in Figure 10. This figure has three panels, corresponding to three arbitrary levels of debt, respectively, in the safe and the two default-risky regions (to be formally defined below). In each panel, the utility of repaying debt conditional on the riskless bond price is a solid blue line, dubbed V_{β}^{R}; the utility conditional on the default-risky price is a solid red line, dubbed $V_{\beta p}^{R}$. In the figure, we also plot a horizontal line corresponding to the utility of defaulting in period 0—which, in our two-period framework, is a constant. Vertical dashed lines mark $\underline{B}$ and $\bar{B}$ defined above.

The utility of repaying debt when bonds are issued at the riskless price is

$$V_{\beta}^{R}(B_0, B_1) = u(A\bar{y} - B_0 + \beta B_1) + \beta \underbrace{(pu(\bar{y} - B_1) + (1-p)u(A\bar{y} - B_1))}_{\equiv \mathbb{E}_0[u(g_1)]} \tag{13}$$

where we can restrict the analysis to issuance below the upper bound $\underline{B}$.[32]

As apparent from the figure, $V_{\beta}^{R}(B_0, B_1)$ is hump-shaped. Since $\mathbb{E}_0[u(g_1)]$ is strictly decreasing in B_1 in the region $B_1 \in [0, \underline{B}]$, the hump shape originates from the fact that increasing B_1 boosts current consumption but at the same time reduces the continuation value. The peak of $V_{\beta}^{R}(B_0, B_1)$ indicates the optimal issuance policy when lenders offer the riskless price. Facing the risk-free bond price in the market, the government optimally chooses spending—i.e., its gross financing need—issuing B_{β}^{*}.

The utility of repaying when bonds are issued at the default-risky price, $V_{\beta p}^{R}$, is analogous to (13)—except that with risky bond pricing, there exists a possibility that B_1 grows above $\underline{B}$—implying that contingent default may take place depending on the output at period 1. We therefore consider two cases, indexed by "nd" (no default) and "d" (default), respectively, and bond issuance is now bounded by $\bar{B}$. Write

$$V_{\beta p,nd}^{R}(B_0, B_1) = u(A\bar{y} - B_0 + \beta p B_1) + \beta(pu(\bar{y} - B_1) + (1-p)u(A\bar{y} - B_1)) \tag{14}$$

$$V_{\beta p,d}^{R}(B_0, B_1) = u(A\bar{y} - B_0 + \beta p B_1) + \beta(pu(\bar{y} - B_1) + (1-p)u(AZ\bar{y})), \tag{15}$$

which can be combined synthetically as follows

$$V_{\beta p}^{R}(B_0, B_1) = \mathbb{1}_{B_1 \in [0, \underline{B}]} V_{\beta p,nd}^{R}(B_0, B_1) + \mathbb{1}_{B_1 \in (\underline{B}, \overline{B}]} V_{\beta p,d}^{R}(B_0, B_1). \tag{16}$$

In this expression, $\mathbb{1}_{B_1 \in [0, \underline{B}]}$ is an indicator function equal to 1 if bond issuance is below $\underline{B}$, equal to 0 if bond issuance is comprised between $\underline{B}$ and $\bar{B}$. We define $\mathbb{1}_{B_1 \in (\underline{B}, \overline{B}]}$ in an analogous way. The figure highlights a key property of $V_{\beta p}^{R}$, that is, the function is piecewise-defined and continuous—the red line depicting the utility of repaying when bonds are priced at the risky rate has a kink at $\underline{B}$. The kink corresponds to the discrete change in the continuation value when issuance is high enough to foreshadow a prospective default in period 1.[33] Below and above $\underline{B}$, this utility is hump-shaped, for the same reason explained

[32] Without such a bound, the government could boost utility by borrowing a large amount at t=0 at the riskless price, to finance a large hike in current government spending, up to overcompensate the cost of default, either conditionally or unconditionally, in period 1.

[33] When the new issuance $B_1 \to \underline{B}$, the government is indifferent between defaulting and repaying in a recession at $t = 1$. This can be easily seen from $\lim_{B_1 \to \underline{B}^-} V_{\beta p,nd}^{R} = \lim_{B_1 \to \underline{B}^+} V_{\beta p,d}^{R}$. Hence, $V_{\beta p}^{R}$ is continuous. But the marginal effect of issuing extra unit of debt is different between $B_1 \to \underline{B}^-$ and $B_1 \to \underline{B}^+$. When $B_1 \to \underline{B}^-$, issuing a marginal unit of debt would marginally decrease the utility of consumption in

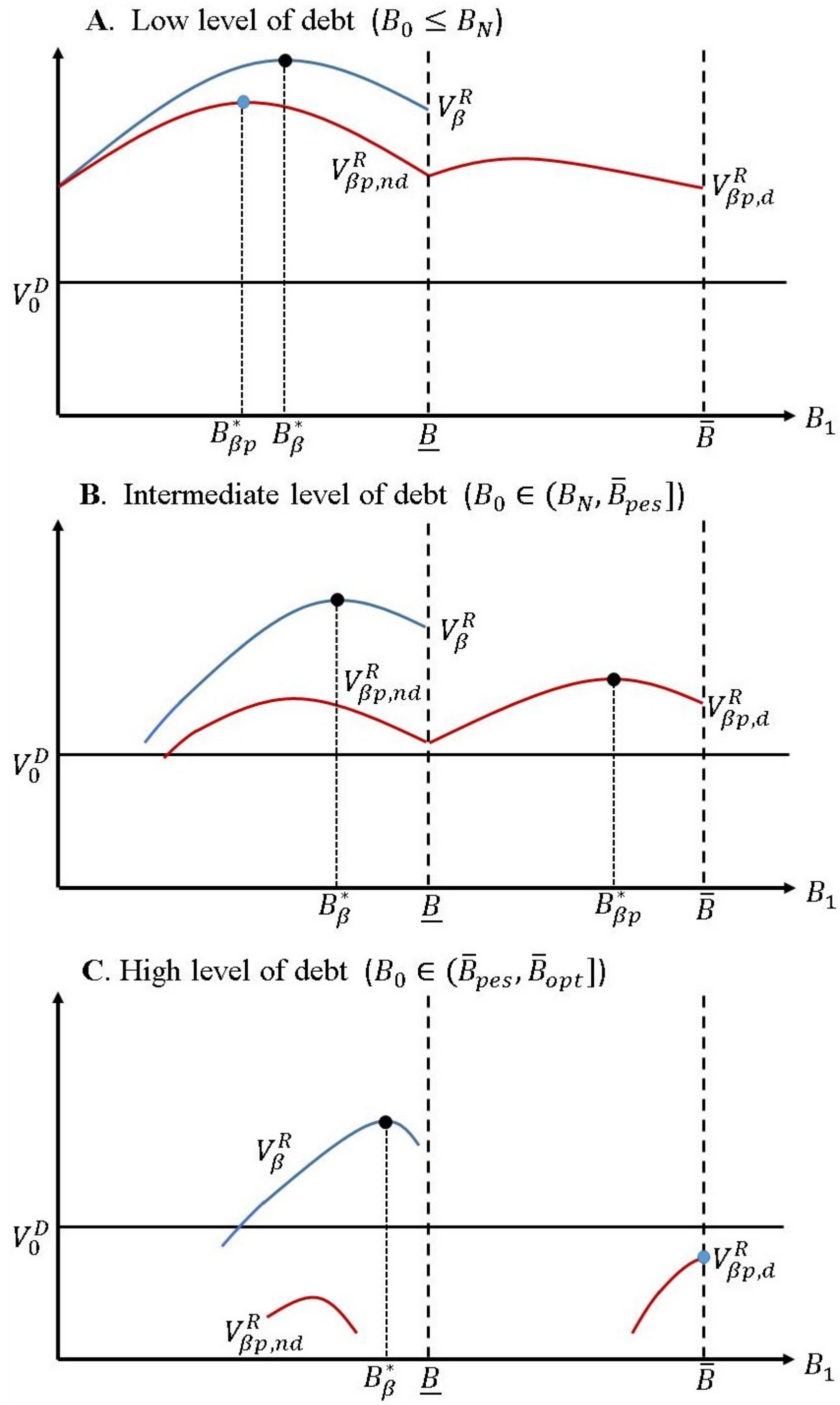

Figure 10: Value functions. The figure depicts V_q^R, $q \in \{\beta, \beta p\}$, as a function of B_1 for different levels of initial outstanding debt. **A** depicts a case of low initial debt which features unique equilibrium; In **B**, multiple equilibria is observed for intermediate levels of debt; In **C**, fast crises occur under pessimistic beliefs regime, when debt level is high.

above when discussing V^R_β.

As for $V^R_{\beta p}$, conditional on the lenders offering the risky price, the government optimally adjusts its primary surplus and issues $B^*_{\beta p}$ corresponding to the (global) peak in the value of this utility. In the upper plot in the figure, for a moderate initial debt level, the government would respond to the default-risky price by issuing $B^*_{\beta p}$ below $\underline{B}$, hence this price cannot be an equilibrium price.[34] In the middle plot, for intermediate debt levels, the optimal issuance $B^*_{\beta p}$ is above $\underline{B}$. Hence there is an equilibrium with default-risky pricing of bonds, high issuance and prospective default. In the bottom plot, for higher debt levels, $B^*_{\beta p}$ lies at $\bar{B}$, i.e., the maximum debt level at which the government can be expected to honor its liabilities if only with probability p. At the largest possible issuance $B_1 = \bar{B}$, however, the value of repaying would be below the value of default. Here is the key implication: rolling over public debt at the risky price would require a too harsh adjustment in current and future spending, up to reducing welfare below the level associated with immediate default, i.e., $V^R_{\beta p}(B_0, \bar{B}) < V^D_0$. Hence, there is no equilibrium in which lenders offer to buy government bonds at the default-risky price.

Under our assumptions, the three plots in the figure highlight that optimistic lenders will invariably coordinate on the equilibrium where bonds are traded at the riskless price, independently of the initial debt level—optimistic lenders always coordinate on the equilibrium that maximizes welfare. Conversely, when lenders coordinate on pessimistic beliefs, the equilibrium bond price depends on the debt level. A switch from optimism to pessimism is not consequential at low debt level, in the upper plot; it causes government bonds to be traded at the risky price for intermediate level of debt, the middle plot in the figure; it causes the government to lose market access altogether at higher level of debt, in the bottom plot.

A.2.2 Regions of Multiplicity by Initial Debt

Using the value of repaying at the riskless and the risky bond prices, one can derive debt thresholds for the *initial* level of debt beyond which the economy is at the risk of default, either prospective or immediate. To start with, consider the case of a stock of initial liabilities low enough that the equilibrium is unique, and bonds are traded at the default-free price. This is the case corresponding to Figure 10A: whether beliefs are optimistic or pessimistic,

both the recessionary and the recovery state at $t = 1$; when $B_1 \to \underline{B}^+$, instead, issuing an extra unit of debt would not decrease the consumption in a recessionary state at $t = 1$. The effect of issuing marginal debt varies depending on the direction the economy approaches $\underline{B}$, which is apparent from the inequality $\lim_{B_1 \to \underline{B}^-} \frac{\partial V^R_{\beta p,nd}}{\partial B_1} < \lim_{B_1 \to \underline{B}^+} \frac{\partial V^R_{\beta p,d}}{\partial B_1}$, i.e., the left derivative is not equal to the right derivative at $B_1 = \underline{B}$. Hence, $V^R_{\beta p}$ is not differentiable at $B_1 = \underline{B}$.

[34] Note that, or $B_1 < \underline{B}$, the gap between V^R_β and $V^R_{\beta p,nd}$ increases in the debt carried forward. Since the continuation value is identical for each value of B_1, the difference between these two utility functions depends only on the difference of current consumption, which is rising in B_1—reflecting the fact the default-risky price is strictly lower than the risk-free price.

the government issuance will remain below $\underline{B}$ and B_1 will be repaid with probability one in period 1. Indeed, if lenders offered the default-risky bond price βp under the pessimistic beliefs regime, their beliefs would not be validated in equilibrium. Facing the lower, risky price, the government would issue $B^*_{\beta p}$, lower than $\underline{B}$, hence too low for the government to find it optimal to default in a recession in period 1. The low-risky bond price is not supported in equilibrium. In contrast, at the optimal issuance $B^*_{\beta} < \underline{B}$, the risk-free price is validated.

Define B_N as the maximum amount of outstanding debt in period $t = 0$ at which the equilibrium is unique with bonds priced at the risk-free rate, in two-period version of our model. B_N can be found by solving:

$$B_N = \sup_{B_0} \left\{ B^*_{\beta p} < \underline{B} \right\}$$

This threshold defines the upper bound of the safe region for the initial debt. For an initial stock of debt larger than B_N, we can then characterize crisis regions using V^R_{β} and $V^R_{\beta p}$ in conjunction with the constant value of default. In particular, we can derive two thresholds for the level of initial debt B_0, above which (i) rational lenders would not find it optimal to offer the riskless price, $\bar{B}_{\beta}$, and (ii) rational lenders would not find it optimal to offer the risky price, $\bar{B}_{\beta p}$. To do so, we solve

$$\begin{aligned} \max_{B_1 \in [0, \underline{B}]} V^R_{\beta}(\bar{B}_{\beta}, B_1) &= V^D_0 \\ \max_{B_1 \in [0, \overline{B}]} V^R_{\beta p}(\bar{B}_{\beta p}, B_1) &= V^D_0 \end{aligned}$$

Under our assumption that the riskless pricing of sovereign bonds always exists for $\rho_0 = opt$, social welfare is always higher when bonds are traded at the riskless price, for any level of initial debt. It follows that $\bar{B}_{\beta}$ is unambiguously larger than $\bar{B}_{\beta p}$. Let $\bar{B}_{\rho_0}$ ($\rho_0 = opt$ or pes) denote the debt tolerance thresholds conditional on the beliefs regime ρ_0. We state our main result in the proposition below.

Proposition A.1. *Provided $\bar{B}_{\beta}$ is strictly larger than $\bar{B}_{\beta p}$, the economy features distinct debt tolerance thresholds, with $\bar{B}_{opt} = \bar{B}_{\beta}$ and $\bar{B}_{pes} = \bar{B}_{\beta p}$.*

Intuitively, optimistic lenders are comparably more confident on debt sustainability—hence the government can sustain more debt (at a better price) in a regime of optimism. With $\bar{B}_{opt} > \bar{B}_{pes}$, the two debt thresholds split the multiplicity region into two sub-regions. When the initial debt is comprised between B_N and $\bar{B}_{pes}$, belief-driven crises take the form of a sudden switch in borrowing costs—the case depicted in Figure 10B; when debt is above $\bar{B}_{pes}$, the equilibrium price is either the risk-free price or zero—i.e., the economy is at the risk of rollover crises. This type of multiplicity is illustrated in 10C.

A.3 Slow and Fast Crises Dissected

We are now ready to discuss in detail the different types of crises predicted by the model, as represented by the second and the third panel of Figure 10.

A.3.1 "Slow-Moving" Debt Crises

Belief-driven crisis taking the form of a sudden change in borrowing costs, illustrated in Figure 10B, may arise for any outstanding initial debt level larger than B_N but smaller than $\bar{B}_{pes}$. To reiterate, the equilibrium is not unique: if lenders offer the risk-free price, the government optimally borrows B^*_{β}, below $\underline{B}$, validating ex-post the lenders' optimistic beliefs; if, instead, lenders offer the lower default-risky price, the optimal borrowing $B^*_{\beta p}$ is above $\underline{B}$—one period ahead, unless the output recovers to $\bar{y}$, the probability of defaulting is one, again validating lenders' pessimistic beliefs.

An important question is whether, facing a belief-driven surge in borrowing costs, a *welfare-maximizing* discretionary government would optimally cut spending, to moderate the pace of debt accumulation. It turns out that this is not necessarily the case—as the consumption smoothing motive dominates fiscal choices. The next proposition identifies instructive sufficient conditions for optimal fiscal austerity, when multiplicity takes the form of a surge in borrowing costs:

Proposition A.2 (A sufficient condition for fiscal austerity)**.** *For an initial stock of debt in the crisis region comprised between B_N and $\bar{B}_{pes}$, a sufficient condition for the equilibrium Gross Financing Need (GFN) to fall in the switch from a regime of optimistic to a regime of pessimistic beliefs among lenders is that $p < A$ and the value of B_N satisfies*

$$\frac{p}{1-p}(1+\beta)(1-A)\bar{y} < B_N < (A - Z + \beta p(1-Z))\bar{y}.$$

Proof. See H.2. □

Intuitively, $p < A$ ensures that the maximum bond revenue the government is able to raise given the risk-free price (i.e., $\beta\underline{B}$), is strictly larger than the maximum revenue conditional on the risky price (i.e., $\beta p\bar{B}$).[35] Given $p < A$, the left inequality $\frac{p}{1-p}(1+\beta)(1-A)\bar{y} < B_N$ guarantees that the government faces lower marginal utility cost of raising extra unit of bond revenue under optimistic beliefs regime—implying that it is optimal to run a larger GFN in equilibrium. The right inequality $B_N < (A - Z + \beta p(1-Z))\bar{y}$ ensures that there is some initial debt level in the region $(B_N, \bar{B}_{pes}]$, such that the optimal choice of B_1 by the sovereign does not bind the borrowing limit $\bar{B}$ conditional on the default-risky bond price.

[35] Else, when the borrowing limit binds under either beliefs regime for a certain level of debt, GFN under pessimistic beliefs regime is larger. We eliminate this possibility by imposing $p < A$.

A.3.2 Fast Debt Crises

At a relatively high initial debt level, higher than $\bar{B}_{pes}$ but below $\bar{B}_{opt}$, a switch from optimism to pessimism causes a loss of market access and immediate default—the case shown in Figure 10C. Under optimism, lenders buy government bonds at the riskless price β. Despite the high stock of initial liabilities, the optimal issuance B^*_β remains below $\underline{B}$. With a regime switch to pessimism, however, lenders anticipate that the government would not be willing to cut its primary needs to repay current liabilities at the default-risky bond price at the end of the period—the adjustment required to service debt at the higher borrowing rates is welfare-dominated by a default. The debt crisis is "fast", in that default occurs the moment markets become pessimistic.

B The Algorithm for Computing Value Functions

In this section, we present the algorithm that computes two debt thresholds in an optimistic world, and three debt thresholds in a pessimistic world with short-term debt under the assumption that the risk-free bond pricing always exists in a regime of optimistic beliefs.

B.1 Optimistic Beliefs

1. Compute the debt tolerance threshold in normal times $\bar{B}(1)$ by solving:

$$\frac{\mathcal{U}((1-\tau)\bar{y}, \tau\bar{y} - (1-\beta)\bar{B}(1))}{1-\beta} = \mathcal{V}^D(1)$$

   After the economy recovers, the government's optimization problem is deterministic. Thus, the value function in normal times can be characterized by

$$\mathcal{V}(B,1,1,\rho = opt) = \mathcal{V}(B,1,1,\rho = pes) = \begin{cases} \frac{\mathcal{U}((1-\tau)\bar{y},\tau\bar{y}-(1-\beta)B)}{1-\beta} & \text{if } 0 \leq B \leq \bar{B}(1) \\ \mathcal{V}^D(1) & \text{if } \bar{B}(1) < B \end{cases}$$

2. Derive $\bar{B}(0)_{opt}$ by solving the following equation.

$$\frac{\mathcal{U}((1-\tau)A\bar{y}, \tau A\bar{y} - (1-\beta)\bar{B}(0)_{opt})}{1-\beta(1-p)} + \frac{\beta p \mathcal{U}((1-\tau)\bar{y}, \tau\bar{y} - (1-\beta)\bar{B}(0)_{opt})}{(1-\beta(1-p))(1-\beta)} = \mathcal{V}^D(0)$$

3. Given $\bar{B}(0)_{opt}$, guess the value function $\tilde{\mathcal{V}}(B,1,0,\rho = opt)$ in an optimistic world. Perform value function iteration until it satisfies convergence criterion $\max_B \big|\mathcal{V}(B,1,0,\rho = opt) - \tilde{\mathcal{V}}(B,1,0,\rho = opt)\big| < \epsilon$.

where $\mathcal{V}(B,1,0,\rho=opt)=\max\{\mathcal{V}^R_{opt}(B,0),\mathcal{V}^D(0)\}$ (see (4) for details).

B.2 Pessimistic Beliefs

1. Repeat B.1 step 1.

2. Guess initial values for the threshold $\bar{B}(0)_{pes}$ and derive value function $\mathcal{V}^R_{pes,d}(B,0)$ (see (7) for details).

3. According to the proof of Proposition 1 in Appendix H, we have $\mathcal{V}^R_{pes,d}(\bar{B}(0)_{pes},0) > \mathcal{V}^R_{pes,nd}(\bar{B}(0)_{pes},0)$. Hence, derive $\bar{B}(0)_{pes}$ by solving:
$$\mathcal{V}^R_{pes,d}(\bar{B}(0)_{pes},0)=\mathcal{V}^D(0)$$

4. Guess the value function in a pessimistic world $\tilde{\mathcal{V}}(B,1,0,\rho=pes)$ and the threshold B_N.

5. Perform value function iteration and update initial guess until it satisfies convergence criterion $\max_B \big|\mathcal{V}(B,1,0,\rho=pes)-\tilde{\mathcal{V}}(B,1,0,\rho=pes)\big|<\epsilon$.
$$\mathcal{V}(B,1,0,\rho=pes)=\begin{cases}\mathcal{V}^R_{safe}(B,0) & \text{if } 0\le B\le B_N\\ \mathcal{V}^R_{pes,d}(B,0) & \text{if } B_N<B\le\bar{B}(0)_{pes}\\ \mathcal{V}^D(0) & \text{if } \bar{B}(0)_{pes}<B\end{cases}$$
where
$$\begin{aligned}\mathcal{V}^R_{safe}(B,0)=\max_{0\le B'\le\bar{B}(0)_{pes}}\quad & \mathcal{U}(c,g)+\beta\Big(p\mathcal{V}(B',1,1,\rho=pes)+(1-p)\tilde{\mathcal{V}}(B',1,0,\rho=pes)\Big)\\ \text{s.t.}\quad & g+B=\tau A\bar{y}+\beta B',\\ & c=(1-\tau)A\bar{y}\end{aligned}$$

6. Compute the government utility and policy function given the bond price $q=\beta p$, denoted as $\mathcal{V}^R_{pes}(B,0)=\max\{\mathcal{V}^R_{pes,nd}(B,0),\mathcal{V}^R_{pes,d}(B,0)\}$ and $B'_{pes}(B,0)$, respectively (see (6) and (7) for details).

7. Derive a new value of B_N by solving equation below.
$$B_{N,new}=\sup_B\{B'_{pes}(B,0)\le\bar{B}(0)_{pes}\}$$

8. If $\left|B_N - B_{N,new}\right| > \epsilon$, then update values: $B_N = B_{N,new}$, and go back to 5. Else, exit.

C Numerical Analysis with Short-Term Bonds

In this section, we show simulation results for an economy with short-term bonds. Below, we first characterize the economy under 'constant beliefs'. Then, we show results for our baseline with $\rho \in \{opt, pes\}$.

C.1 Short-Term Bonds with 'Constant Beliefs'

In Figure 11, we show the policy functions conditional on a recession for the case of one-period bonds ($\delta = 1.0$), taking the beliefs regime as 'constant' over time. Comparing this with Figure 6, it is apparent that, as δ converges to unity, $\bar{B}(0)_{pes}$ is much lower, while $\bar{B}(0)_{opt}$ is not affected. As discussed in Section 6, the result that $\bar{B}(0)_{opt}$ is not sensitive to debt maturity follows from the fact that, when lenders hold an optimistic view on government solvency, they lend the government at the risk-free rate: thus there is little scope for maturity to make a difference. Indeed, the left panel in Figure 11 features exactly the same dynamics as the left panel of Figure 6.

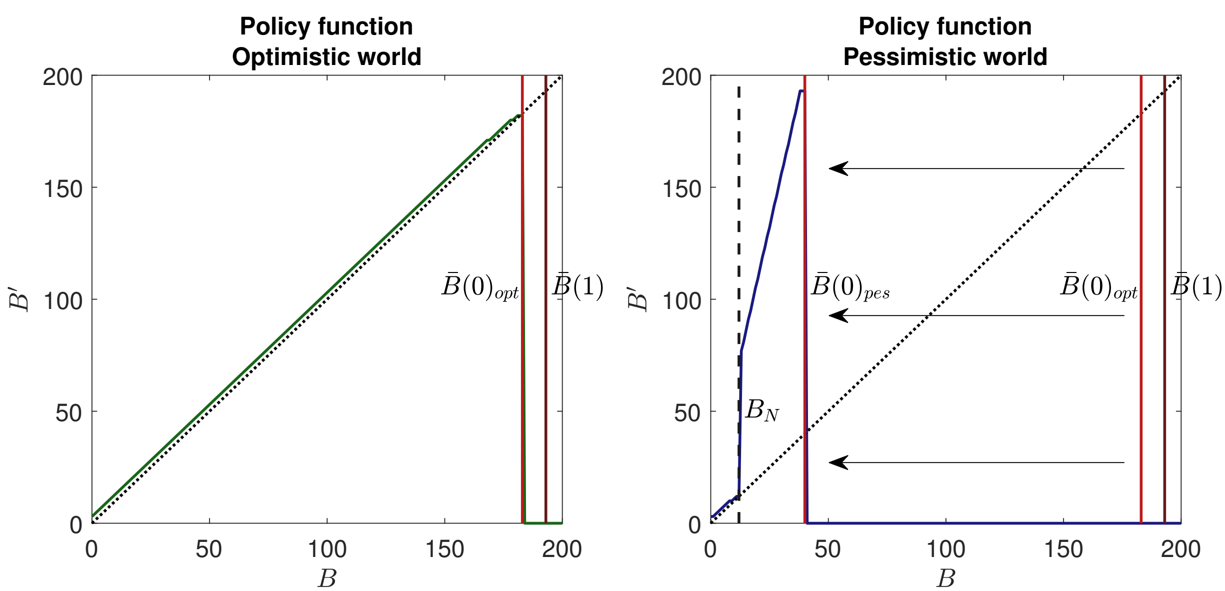

Figure 11: Policy functions for one-period bonds, $\delta = 1.0$

Maturity instead makes a difference for the region of debt in which fast and slow crises are possible. The region between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$ is much wider with short-term debt. The rollover crises might occur for low levels of debt (above 41% of GDP in normal times). Moreover, a larger region of fast crises is not fully compensated by a narrowing of the region of slow crises, as both B_N and $\bar{B}(0)_{pes}$ shrink when maturity is shorter. B_N falls from 53 to 12! When government bonds are all short-term, a country in a recession might suffer a slow-moving crisis even when its outstanding debt is relatively low.

C.2 Baseline with Short-Term Bonds

The welfare advantage from reducing borrowing costs through deleveraging, with one-period bonds, is higher. In Figure 12, we show a version of our baseline with one-period bonds ($\delta = 1$). Relative to the long-term maturity baseline in Figure 7, three differences are apparent. First, the debt thresholds are substantially lower. Second, the equilibrium bond price remains risk-free in the region between B_N and $\bar{B}(0)_{pes}$. Recall that debt is fully rolled over in each period: when B' lies at $(B_N, \bar{B}(0)_{pes}]$, the risk of a prospective slow-moving crisis one period ahead does not have any impact on the current equilibrium bond prices.

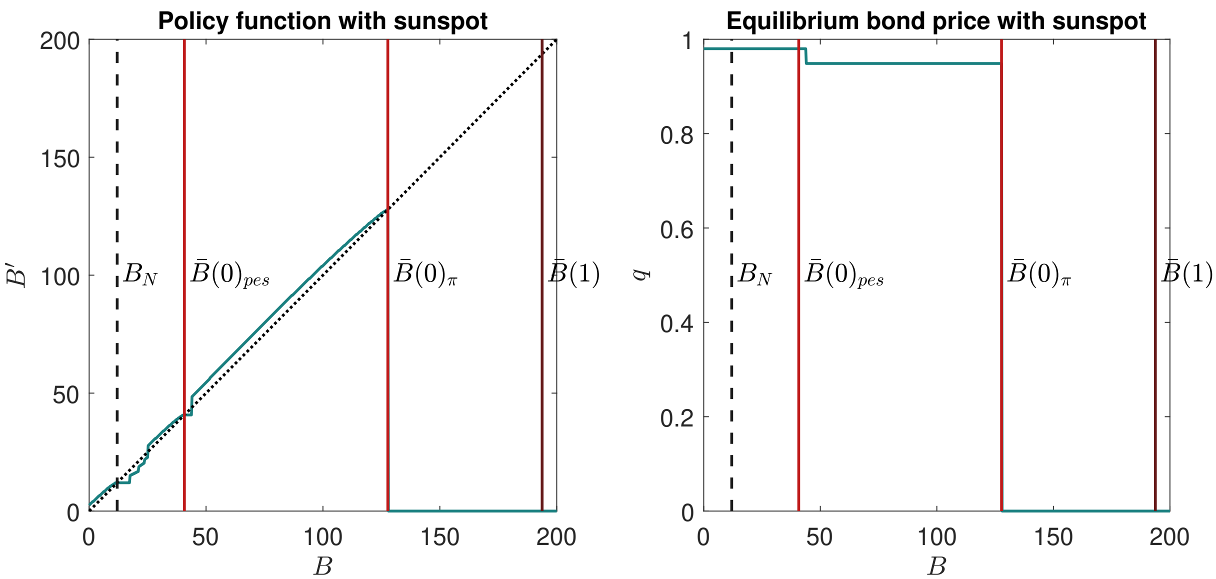

Figure 12: Baseline ($\rho \in \{opt, pes\}$), $\delta = 1.0$

Last but not least, debt reduction polices are optimal around both B_N and $\bar{B}(0)_{pes}$. Similar to the economy with long-term debt, deleveraging is optimal for a limited range of debt just above B_N. With short-term debt, deleveraging around this threshold is driven *exclusively* by the 'cliff effect'. Distinct from the case of long-term debt, however, the government deleverages also over a small range of debt just above $\bar{B}(0)_{pes}$. Since there is no cliff effect around $\bar{B}(0)_{pes}$, deleveraging here is motivated by the fact that the price effect at this threshold is much higher—the pass-through of the change in market interest rates onto government borrowing costs is full in one period, about 3.4 percentage points.[36] Note that

[36] When the maturity of the outstanding debt is short, the government internalizes the gains from reducing

the range of debt over which deleveraging is optimal is much wider around B_N—driven by the 'cliff effect'. The 'price effect' around $\bar{B}(0)_{pes}$ is less strong, still active—for comparison, this effect is the main driver of debt reduction in Aguiar and Amador (2020).[37]

D The 'Cliff Effect' in Welfare due to Self-Fulfilling Crises

In Figure 13, we show the government value function in a pessimistic world in our baseline with $\delta = 0.2$. We refer to a discontinuity in the value function at the debt threshold as the 'welfare cliff.' A cliff is apparent at B_N. In a sunspot equilibrium, the large loss of utility the economy suffers when it becomes vulnerable to belief-driven crises motivates the government to engage in policies of debt reduction and keep debt at safe levels, below B_N. Observe, however, that there is no cliff around the other, higher threshold—the welfare incentive for the government to deleverage is significant only around B_N.

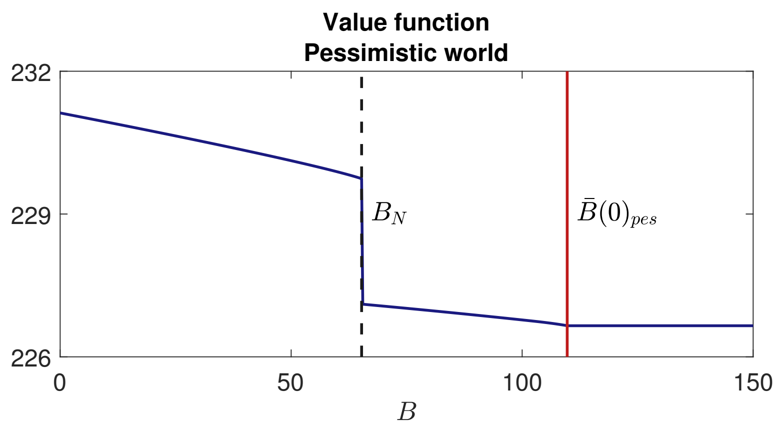

Figure 13: Cliff effect ($\delta = 0.2$)

E Cole-Kehoe Beliefs

This sections offers analytical details on crises under CK beliefs. For clarity, we start, as in Section 3, by studying an economy with short-term debt and constant beliefs regime. We next move to an economy with long-term debt and sunspots with $\rho \in \{opt, CK\}$.

the borrowing cost fully, and optimally chooses to deleverage for debt levels close to $\bar{B}(0)_{pes}$. Longer debt maturity would not provide enough of an incentive to do so as the gains are shared with lenders. See again Aguiar and Amador (2020).

[37] The role of short maturities in motivating deleveraging is also discussed by Conesa and Kehoe (2017).

E.1 Constant Beliefs, Short-Term Debt

Under constant CK beliefs, the utility of repaying short-term debt $\mathcal{V}^R_{CK}(B,a)$ is

$$\mathcal{V}^R_{CK}(B,a) = \max_{0\leq B'} \mathcal{U}((1-\tau)y(a,1), \tau y(a,1) - B + 0 \times B') + \beta\mathbb{E}[\mathcal{V}(s')|s] \tag{17}$$

When $\mathcal{V}^R_{CK}(B,a) \geq \mathcal{V}^D(a)$, the government does not default *today* and it will not default *next period*. As a result, $q = 0$ is not an equilibrium price and "CK beliefs" are not validated. In contrast, when $\mathcal{V}^R_{CK}(B,a) < \mathcal{V}^D(a)$, the government defaults at the end of the period and the loss of market access implied by the lenders' CK beliefs is validated in equilibrium. $\bar{B}(a)_{CK}$ is derived by solving the equation below:

$$\mathcal{V}^R_{CK}(\bar{B}(a)_{CK}, a) = \mathcal{V}^D(a)$$

The following proposition shows that $\bar{B}(a)_{CK}$ is always positive:

Proposition E.1. *A strictly concave utility function with the property* $\lim_{g\to\bar{g}}\mathcal{U}(c,g) = -\infty$ *ensures* $\bar{B}(a)_{CK}$ *is positive.*

Proof. See H.3. □

Note that, the debt threshold under CK beliefs in a recession, i.e., $\bar{B}(0)_{CK}$, is always lower than the threshold above which fast crisis can materialize under pessimistic beliefs:

Proposition E.2. *Strict concavity of the utility function implies* $\bar{B}(0)_{pes} > \bar{B}(0)_{CK}$

Proof. See H.4. □

E.2 Sunspot Equilibria with Optimistic and CK Beliefs

Figure 14 shows the policy function and the equilibrium bond price for our calibration with long-term bonds. To study the sunspot equilibrium with CK beliefs, we define a new debt threshold conditional on recovery, denoted $\bar{B}(1)_\pi$. The reason to introduce this new threshold is that, with CK beliefs, the government remains exposed to the risk of rollover crises even after the economy exits the recession, if its outstanding debt level is large enough—indeed larger than $\bar{B}(1)_{CK}$.[38]

[38]In accordance with (17), $\bar{B}(1)_{CK}$ is pinned down by solving the following equation: $\mathcal{U}((1-\tau)\bar{y}, \tau\bar{y} - \kappa\bar{B}(1)_{CK}) + \beta\frac{\mathcal{U}((1-\tau)\bar{y},\tau\bar{y}-(1-\beta)(1-\delta)\bar{B}(1)_{CK})}{1-\beta} = \mathcal{V}^D(1)$, unambiguously smaller than $\bar{B}(1)$ where $\rho = opt$ or $\rho = pes$. Even after the recovery, a switch to CK beliefs may lead to default—these beliefs are self-validating when a collapse of bond market induces an immediate default, regardless of the output states. This added vulnerability deteriorates the utility of repaying conditional on the recovery, hence $\bar{B}(1)_\pi$ in the figure is lower than $\bar{B}(1)$ in Figure 7.

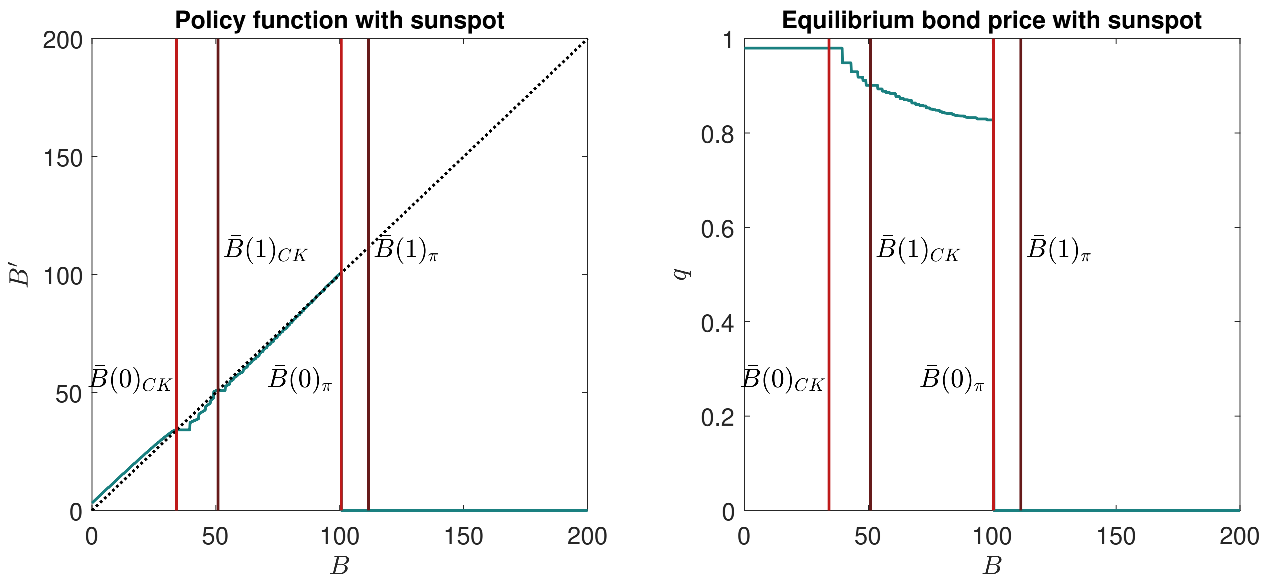

Figure 14: $\delta = 0.2$ and $\rho \in \{opt, CK\}$

As shown in the figure, when debt is in the region $[0, \bar{B}(0)_{CK}]$, the government borrows in a recession, until the outstanding debt stock reaches $\bar{B}(0)_{CK}$, at which debt is kept stationary. When debt is in the region $(\bar{B}(0)_{CK}, \bar{B}(0)_{\pi}]$, the borrowing dynamics are quite complex. The government mostly prefers to deleverage (running surpluses) unless debt is lower than, but close enough to either $\bar{B}(1)_{CK}$ or $\bar{B}(0)_{\pi}$ (see Table 2 in the text).

The right panel of Figure 14 displays the bond price. Compare this figure with Figure 7 for our baseline with pessimistic beliefs. With CK beliefs, bond prices are much lower. For instance, in terms of bond yields, when B is 70 (77.8% of GDP in the recessionary state), yields surge to 16.5%, relative to 5.4% in Figure 7. Although it may not be apparent from the Figure 14, the bond price falls (the bond yield increases) smoothly as debt rises from $\bar{B}(0)_{CK}$ to $\bar{B}(0)_{\pi}$—at a pace that slows down as B approaches $\bar{B}(0)_{\pi}$. The incentive to deleverage is strong when the initial debt level is low enough (far enough from $\bar{B}(0)_{\pi}$), so that by cutting deficit, the government can not only reduce borrowing costs to a sizable degree, but also steer away from belief-driven crises altogether.

F A Debt-Limit Version of the Model

In this appendix, we reconsider our main results using a debt-limit version of our model. The default condition is assessed at period-by-period maximum adjustment in the primary surplus, denoted with $\mathcal{PS}^{Max}$, as follows:

$$\mathcal{PS}^{Max} + \max_{B'}\{q(B' - (1-\delta)B)\} < \kappa B \tag{18}$$

Below, we first characterize the debt tolerance thresholds based on this condition, focusing on the case of one-period bonds. Then, we show how to derive the debt-limit model as a variant of our "strategic-default" model, and carry out numerical analysis for a generic bond maturity.

F.1 The Debt Tolerance Threshold with Short-Term Debt

In the debt-limit framework, the debt tolerance thresholds are pinned down by the maximum adjustment in primary surpluses the government is willing/able to generate. As for our "strategic-default" model, these thresholds may be shifting in response to the regime of lenders' expectations. We derive the thresholds conditional on short debt maturity in the following.

F.1.1 The Debt Tolerance Threshold in Normal Times $\bar{B}(1)$

In normal times, the government budget constraint is

$$B = \tau\bar{y} - g + qB'$$

Since by assumption, once the economy recovers, it never falls back into a recession again, there is no reason to borrow or lend for consumption smoothing purposes. The government's optimization problem is deterministic independently of whether the regime of beliefs is optimistic or pessimistic ($\rho = opt$ or pes). If no default has occurred in the past, the government will simply service its existing debt at the risk-free rate, paying $(1-\beta)B$ to lenders each period, to satisfy the no-Ponzi condition.

Given τ, the government does not default if and only if

$$B \leq \frac{\tau\bar{y} - \bar{g}}{1-\beta} = \bar{B}(1)$$

where $\bar{g}$ is the critical expenditure level.[39]

F.1.2 The Debt Tolerance Threshold(s) in a Recession $\bar{B}(0)_\rho$

In a recession, the debt thresholds depend on the regime of beliefs. As in Section 3 and 4, we posit that belief state stays "constant" over time. Below we consider pessimistic and optimistic beliefs respectively.

[39]In normal times, the debt threshold may still depend on CK beliefs. In the debt limit model, this threshold is derived by solving B in the following equation $B = \tau\bar{y} - \bar{g}$.

Pessimistic Beliefs

In a recession, the government budget constraint reflects the decline in tax revenue due to the downturn in activity ($A < 1$):

$$B = \tau A\bar{y} - g + qB'$$

In a pessimistic world, lenders are only willing to buy bonds at the low risky price. Given the definition of the debt tolerance threshold, the maximum the government can borrow is capped by the stock of debt that the government can service if the economy recovers, that is, $\max\{qB'\} = \beta p\bar{B}(1)$. Hence, to rule out immediate default, the current debt level must be low enough to satisfy:

$$B \leq \tau A\bar{y} - \bar{g} + \beta p\bar{B}(1) = \bar{B}(0)_{pes}, \tag{19}$$

an expression that gives us the current debt tolerance threshold $\bar{B}(0)_{pes}$. Analogously, we can also derive the threshold B_N below which the government is immune to pessimism:

$$B \leq \tau A\bar{y} - \bar{g} + \beta p\bar{B}(0)_{pes} = B_N.$$

If this condition is satisfied, the government would keep the debt level below $\bar{B}(0)_{pes}$, even if bonds were traded at the default-risky price.

Optimistic Beliefs

In an optimistic world, we need to examine two possible issuance strategies for the government. One consists of issuing a lot of debt, at a low, risky price—essentially this is the same strategy as described above, and is therefore associated to the same debt threshold in (19). The other one consists of keeping new issuance in check, so to ensure that debt remains safe. This can be dubbed as a "low-risk low-debt" issuance strategy. By using the same steps above, we can derive the maximum sustainable debt conditional on the safe-debt strategy as:

$$B \leq \frac{\tau A\bar{y} - \bar{g}}{1-\beta}$$

Since optimistic lenders coordinate on the price that maximize the sovereign's welfare, $\bar{B}(0)_{opt}$ can be characterized as follows:

$$\bar{B}(0)_{opt} = \max\left\{\frac{\tau A\bar{y} - \bar{g}}{1-\beta}, \bar{B}(0)_{pes}\right\}$$

Which strategy gives the government higher revenue in an optimistic world depends on parameters. If all government debt is short-term, we find that $\bar{B}(0)_{opt} > \bar{B}(0)_{pes}$, and thus a safe-debt strategy makes the government better off.[40]

F.2 The Government Welfare Function

To study the debt-limit model, we specify a variant of our "strategic-default" model building on the idea that the government suffers a utility cost Γ if it cuts spending below $\bar{g}$. Specifically, we replace (8) with a new objective function:

$$\mathcal{U}(c, g) = \mathbb{1}_{g>\bar{g}}\big(\log(c) + \gamma \log(g - \bar{g} + \epsilon)\big) - (1 - \mathbb{1}_{g>\bar{g}}) \times \Gamma,$$

where $\mathbb{1}_{g>\bar{g}}$ is an indicator function equal to 0 if spending falls below the critical value. We assume an arbitrary small positive ϵ to ensure that $\mathcal{U}(c, g)$ is bounded below when $g \to \bar{g}$. This is the key implication: if defaulting brings spending below the critical level $\bar{g}$ and a utility penalty Γ is cruel enough, the value of repaying will never be below that of defaulting—the government never defaults strategically, and thus (18) holds. Yet, as shown below, crises are still possible, depending on the initial conditions, the persistence of recessions and lenders' expectations.

Using this new framework, we now set $Z = 0.8$, $\tau = 0.35$, $\bar{g} = 30$ such that government spending falls below the critical level $\bar{g}$ upon a default. For the other parameters, we adopt the same values as in Table 1. We discuss the case of "constant" beliefs in Appendix F.3, a more general analysis of sunspots in Appendix F.4, and the resilience to the crises in Appendix F.5. We will show that, in this debt-limit framework, long-term debt tends to rule out "fast" debt crises more easily, but remains ineffective in ruling out "slow-moving" debt crises.

F.3 Policy Functions

The main results from our exercise are shown in the two panels of Figure 15, which depicts the policy functions with long-term bonds (left panel) and one-period bonds (right panel). Each panel illustrates both the optimistic and the pessimistic world.

With one-period bonds—the case shown in the right panel of Figure 15—the debt dynamics are very similar to the ones in our numerical example with long-term debt in Section 4.[41] In an optimistic world, the government accumulates debt over time to smooth consumption till it reaches $\bar{B}(0)_{opt}$. In a pessimistic world, the government issues safe debt at a slow

[40] In the case of CK beliefs, lenders gauge the current debt sustainable if it satisfies the following condition: $B \leq \tau A\underline{y} - \bar{g} = \bar{B}(0)_{CK}$. This means that debt will be repaid even if the government loses market access in a recession.

[41] Relative to Figure 6, the thresholds in Figure 15 are much lower, reflecting the difference in debt maturity in the two figures.

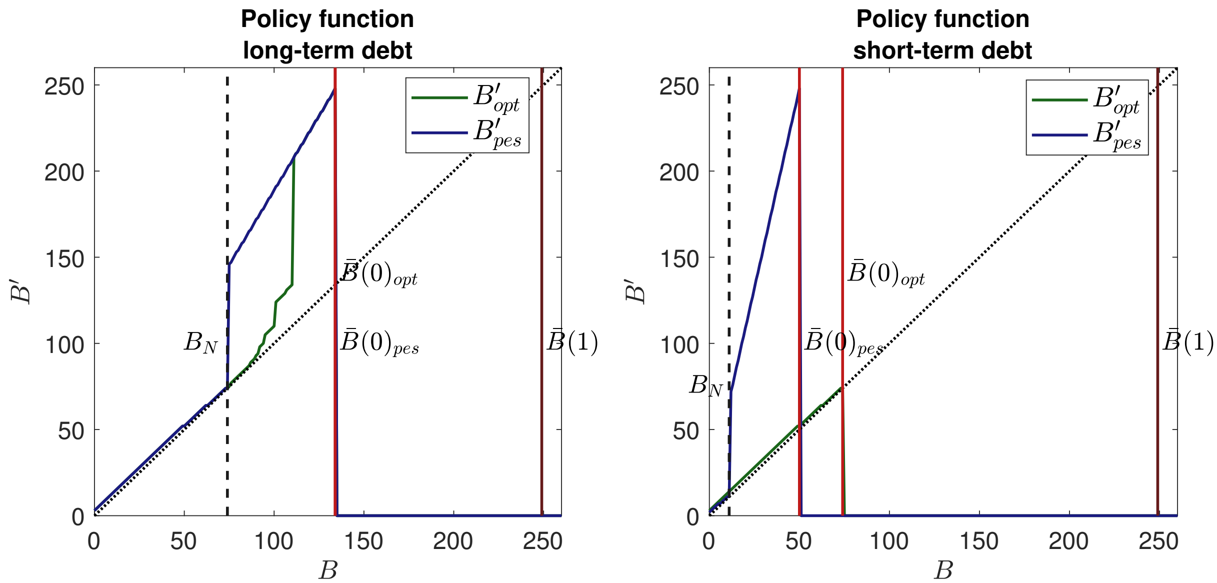

Figure 15: Policy functions where $\delta = 0.2$ (left) and $\delta = 1.0$ (right)

pace in the region between 0 and B_N; it starts to accumulate risky debt at a fast pace in the region between B_N and $\bar{B}(0)_{pes}$; fast, rollover crises can nonetheless occur for debt levels between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$.

The debt dynamics with long-term bonds shown by the left panel in Figure 15 are instead different from our numerical example in Figure 6. Under optimism, lenders lend at the risk-free rate up to $\frac{\tau A\bar{y}-\bar{g}}{1-\beta}$, since, in this region, the riskless pricing is able to guarantee that g is above $\bar{g}$.[42] The difference narrows under pessimism. Similar to our baseline, pessimistic lenders lend at the risk-free rate when debt lies in the region $[0, B_N]$; they lend at the default-risky price when debt is above B_N but below $\bar{B}_{pes}$. In the left panel of the figure, at the low debt level, the sovereign's policy function looks visually identical regardless of beliefs regime, due to the fact that the value of $\frac{\tau A\bar{y}-\bar{g}}{1-\beta}$ is almost the same as the value of B_N in our long-term debt simulation.

Looking at the left panel (with long-term bonds), focus first on the intermediate debt region. As lenders price the risk of a surge in the outstanding stock of debt in the future regardless of beliefs regimes, the bond price is decreasing in the level of debt. This drives a fast pace of debt accumulation under either pessimistic or optimistic beliefs. The beliefs regime nonetheless matters: from the figure, it is apparent that (with a persistent recession) debt accumulates faster under pessimistic beliefs.

An important difference relative to Figure 6, is that the thresholds $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$ coincide, and the region at the risk of faster debt accumulation under pessimistic beliefs

[42]Borrowing at the risky rate is not optimal for the government in this region as it would expose the government to the prospective default next period, which may lead to a contingent fall in g below $\bar{g}$.

is narrower than the region between the thresholds B_N and $\bar{B}(0)_{pes} = \bar{B}(0)_{opt}$. In our simulations for the debt-limit framework, debt accumulates at a faster pace for a debt-to-GDP ratio comprised between 82 and 122 percent (74 and 110 percent if GDP is measured in normal times); the overlapping debt thresholds are close to 150 percent.

Notably, when debt is long term (again, in the left panel), the economy features a unique equilibrium at a high level of debt. The reason is as follows. For a high enough stock of outstanding debt, keeping issuance below the safe-debt threshold may not yield enough income to avoid immediate default, even if lenders hold optimistic beliefs. When this is the case, lenders are only willing to lend at the default-risky price. The government has no alternative but issuing risky debt (i.e., issuing debt above the threshold), to avoid immediate default. Hence, the equilibrium is unique, with the country entering a slow-moving crisis mode regardless of beliefs regime. The unique equilibrium region at high debt levels widens with a longer bond maturity, since the net bond revenue from issuing risky bonds rises with maturity. Remarkably, this rules out the possibility of "fast" debt crises.

F.4 Sunspot Equilibria in a Debt-Limit Framework

We study how sunspots can affect government behavior in the debt-limit framework. To keep things simple and for the sake of comparison with Lorenzoni and Werning (2019), in the following we restrict our attention to a scenario with optimistic and pessimistic beliefs.

In a debt-limit framework, a sunspot equilibrium differs from our model. First, when government bonds are long-term, there is a sharp acceleration of debt accumulation at intermediate levels of debt. Second, when debt is short-term, debt thresholds become sensitive to the probability attributed to pessimism—they shift at low values of these probabilities.

In Figure 16, we display the equilibrium bond price at different level of debt (left panel), and debt accumulation in the time domain for two different levels of debt (center and right panel). Here bonds are long term. Each panel contrasts the optimistic equilibrium and the sunspot equilibrium. We do omit the policy function from the graph because this is visually very similar to the optimistic world in Figure 15.

At intermediate levels of debt (center panel of Figure 16), the two economies are quite different. In the sunspot equilibrium, the government has to pay a higher spread than under optimism. This accelerates debt accumulation. Since debt crises arrive earlier, the spread rises even further, larger than $\pi = 4\%$ as B approches B_N (see the left panel of Figure 16).

When debt level is already sufficiently high, however, the difference narrows. In the right panel of Figure 16, the debt paths are identical in the sunspot and the optimistic equilibrium. Intuitively, the government always chooses the risky-debt high-debt issuance strategy at $T = 1$, regardless of the lenders' beliefs (there is no multiplicity at high debt levels, see Figure 15). The sunspot is immaterial for the equilibrium.

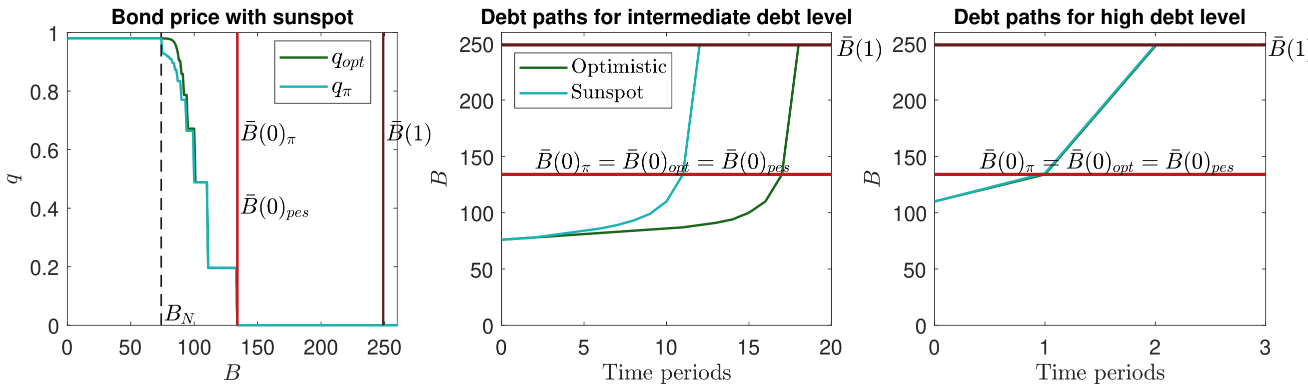

Figure 16: Bond price schedules and debt paths ($\delta = 0.2$)

F.5 Bond Maturities and Resilience to Crises

We report the results of sensitivity analysis varying debt maturity and probability of a recovery. Results are shown in Figure 17.

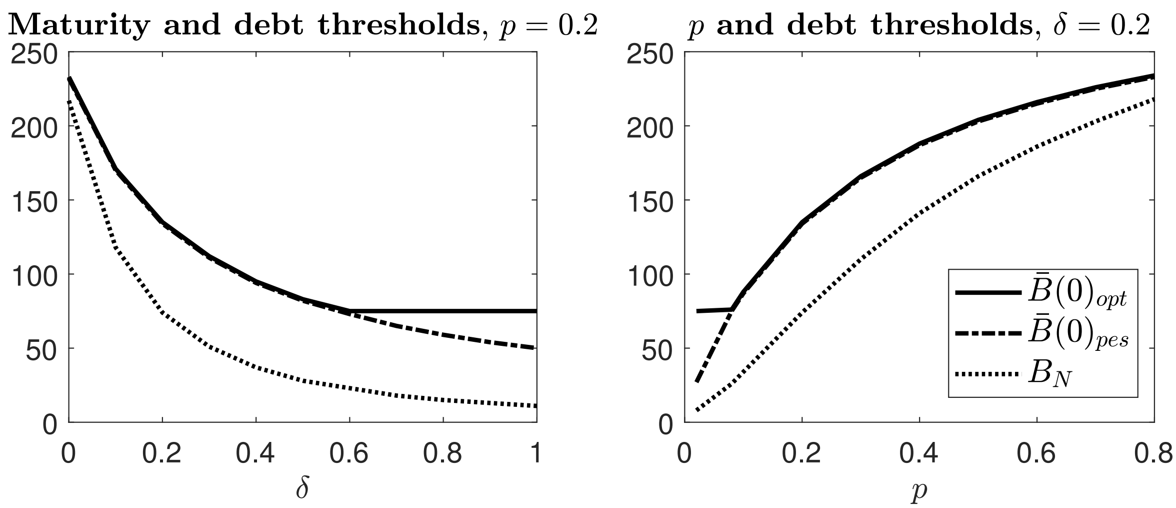

Figure 17: Debt thresholds in the debt-limit framework given $A = 0.9$

Looking at the left-hand panel of Figure 17, $\bar{B}(0)_{opt}$ remains insensitive to debt maturity only for relatively short maturities—for a δ higher than 0.57. As δ falls below this value, i.e., for longer maturities, all thresholds are increasing and, most crucially, $\bar{B}(0)_{opt}$ coincides with $\bar{B}(0)_{pes}$—different from the left panel of Figure 8, these two thresholds overlap away from the limit case of consols.

With longer maturities, issuing more debt at the risky prices imparts a capital loss on outstanding lenders, while the pass-through of higher borrowing costs onto the interest bill is slow. Dilution of existing bond holders strengthens the government incentives to borrow. Indeed, with long bond maturities and a high stock of outstanding liabilities, issuing (more) debt at the risky price may yield more revenue than issuing (less) debt at the risk-free price, even if beliefs are optimistic. It is therefore possible that, independently of the regime of beliefs prevailing in the market, a government may avoid immediate default only by issuing a large amount of risky debt—when the alternative of keeping issuance below the safe-debt threshold would not generate sufficient revenue to satisfy its gross financing need. In this case, of course, avoiding an immediate crisis comes at the cost of putting debt on a trajectory that foreshadows a crisis in the future if the recession persists. Similar considerations apply in relation to the probability of recovery; sensitivity to this parameter is depicted in the right panel of Figure 17. The two thresholds overlap and the government starts issuing risky debt regardless of beliefs regime at a relatively low level of p.

Overall, Figure 17 shows that, in the debt-limit framework, $\bar{B}(0)_{opt}$ coincides with $\bar{B}(0)_{pes}$ over a large region of parameters—both thresholds increase as debt maturity lengthens (δ falls) and the probability of recovery rises. As discussed above (see Section F.3), when the two thresholds coincide, multiple equilibria are still possible and the beliefs regime makes a difference at intermediate debt levels (bond prices are lower and debt accumulates faster when lenders are pessimistic). However, as debt approaches the thresholds, the government eventually pursues the same risky-debt issuance strategy regardless of the beliefs regime prevailing in the market. The equilibrium is unique and fast crises are no longer a possibility. With overlapping thresholds, therefore, the multiplicity region is narrower and, most crucially, belief-driven crises can only occur in the form of faster debt accumulation. We should stress that this result is not specific to the debt-limit model. As shown in the next section (Appendix G), $\bar{B}(0)_{opt}$ may coincide with $\bar{B}(0)_{pes}$ also in our baseline specification, but the conditions are more stringent than the debt-limit framework à la Lorenzoni and Werning (2019).

To conclude, we report that “fast” debt crises are ruled out in the debt-limit version of our model for any δ below 0.57, corresponding to a debt maturity of seven quarters. For longer debt maturities, “none” and “slow” are the only possible outcomes.

G Ruling out “Fast” Debt Crises

In the previous subsection of this appendix, we have seen that long debt maturities are effective in eliminating equilibria with fast (rollover) crises in the debt-limit version of our model. We now extend our analysis and characterize three conditions under which the equilibria with fast crises disappear also in our original baseline model. These are: (i) the country is in a very deep recession (A is very low), (ii) the probability of recovery is quite high, and (iii) debt maturity is sufficiently long—as to mute the pass-through of high interest rates on the total cost of debt servicing. Under these conditions, the government has a strong incentive to pursue high-debt risky-debt issuance strategy even when lenders' expectations are optimistic.

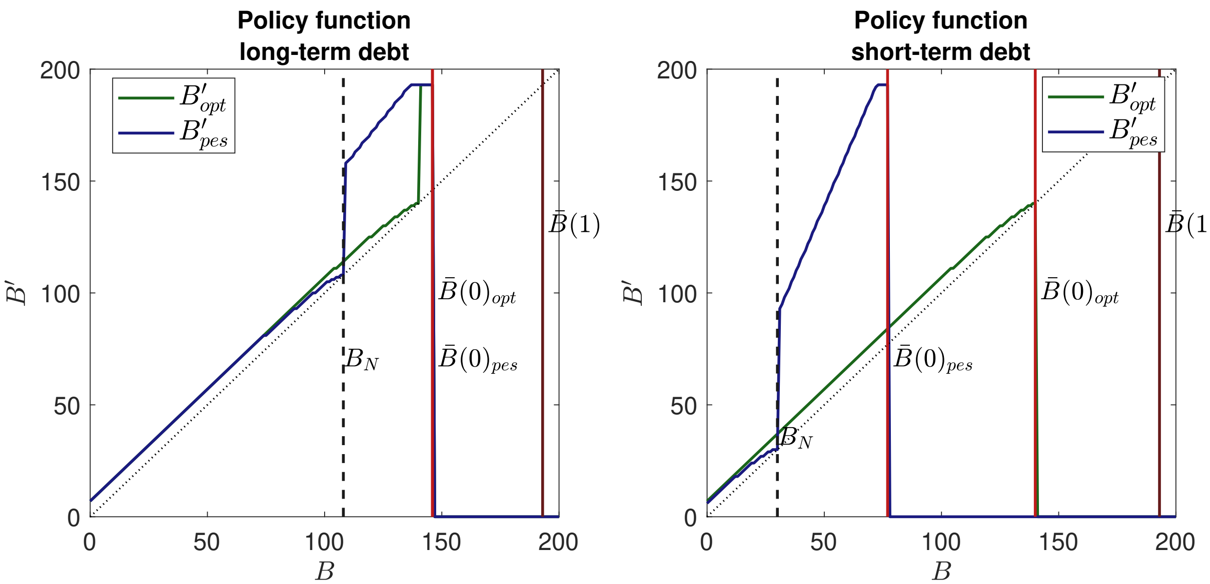

Figure 18: Policy functions in a very severe recession and high probability of recovery, $A = 0.8$ and $p = 0.4$, where $\delta = 0.2$ (left) and $\delta = 1.0$ (right)

In Figure 18 we set $A = 0.8$ and $p = 0.4$: the current recession is exceptionally deep (with a loss of output equal to 20%), but the probability of a recovery one period ahead is substantially high—40%. Figure 18 shows the policy functions when debt is long-term ($\delta = 0.2$) in the left panel, and when debt is short-term ($\delta = 1.0$) in the right panel.

When government bonds are short-term—the right panel of Figure 18—the two thresholds $\bar{B}(0)_{opt}$ and $\bar{B}(0)_{pes}$ are distinct, and thus fast debt crises are possible. With short debt maturity (high δ), the pass-through of interest rate on borrowing costs is very rapid. In an optimistic world, the government smooths consumption through the recession sticking to a safe-debt low-issuance strategy, and keeps debt stationary at the threshold $\bar{B}(0)_{opt}$. In a pessimistic world, a slow-moving debt crisis materializes at intermediate levels of debt; a fast one occurs at high levels of debt.

In contrast, as shown in the left panel of Figure 18, when debt maturity is sufficiently long, the two threshold coincide, and the equilibrium is unique before the outstanding level of debt reaches $\bar{B}(0)_{opt} = \bar{B}(0)_{pes}$, implying that fast debt crises are no longer possible. When debt is long-term, facing a deep recession and a high probability of recovery, the government has a stronger incentive to borrow. For high enough of outstanding stock of debt, the government switches to the risky-debt issuance strategy and keeps accumulating debt also in an optimistic world. Borrowing against the future (uncertain) recovery is the preferred option regardless of the prevailing regime of beliefs.

Overall, these results suggest that debt maturity is much more consequential in the debt-limit version of our model, than in our baseline.

H Proofs

H.1 Proof of Proposition 1

Proof. To prove that $\mathbb{B}_{pes}$ is non-empty, we only need to show that there exists some debt level that simultaneously satisfies $\mathcal{V}^R_{pes} = \max\{\mathcal{V}^R_{pes,nd}, \mathcal{V}^R_{pes,d}\} = \mathcal{V}^R_{pes,d}$ and $\mathcal{V}^R_{pes,d} \geq \mathcal{V}^D(0)$, i.e., the government chooses to issue debt above the threshold $\bar{B}(0)_{pes}$ and repay the existing debt at the default-risky rate.

Posit that, for some $\tilde{B} \leq \bar{B}(0)_{pes}$, the critical expenditure $\bar{g}$ is large enough to exceed $\tau A\bar{y} - \tilde{B} + \beta p\bar{B}(0)_{pes}$. $\bar{B}(0)_{pes}$ is derived by solving the following equation (see $\mathcal{V}^R_{pes,d}$ in (7)):

$$\mathcal{V}^R_{pes,d}(\bar{B}(0)_{pes}, 0) = \mathcal{V}^D(0)$$

For $B \in [\tilde{B}, \bar{B}(0)_{pes}]$, since $\bar{g} > \tau A\bar{y} - \tilde{B} + \beta p\bar{B}(0)_{pes}$, if the government issued bonds below $\bar{B}(0)_{pes}$ at the risky rate (see $\mathcal{V}^R_{pes,nd}$ in (6)), it would have to cut spending below $\bar{g}$.

Hence, it must be the case that $\mathcal{V}^R_{pes}(B, 0) = \mathcal{V}^R_{pes,d}(B, 0)$ for all $B \in [\tilde{B}, \bar{B}(0)_{pes}]$. The amount of newly issued bonds B' in this range must be unambiguously larger than $\bar{B}(0)_{pes}$, validating the pessimistic beliefs.

It follows that, when bonds trade at the default-risky price, a sufficient condition for a non-empty set $\mathbb{B}_{pes}$ is a large enough $\bar{g}$. Note that the proof above also implies that $\bar{B}(0)_{pes}$ is pinned down by solving the equation $\mathcal{V}^R_{pes,d}(\bar{B}(0)_{pes}, 0) = \mathcal{V}^D(0)$, as referred to the computation algorithm. See Appendix B for details. □

H.2 Proof of Proposition A.2

Proof. We proceed with the first condition $p < A$, which ensures larger maximum bond revenue under optimistic beliefs regime.

$$\beta\underline{B} - \beta p\bar{B} = \beta(1-Z)A\bar{y} - \beta p(1-Z)\bar{y} = \beta(1-Z)\bar{y}(A-p)$$

Hence, as long as $A > p$, $\beta\underline{B}$ is unambiguously larger than $\beta p\bar{B}$. Next, we show that the left inequality in the proposition ensures fiscal austerity when $q = \beta p$, given that a bound on borrowing does not bind for $B_0 \to B_N$. Since $B_0 \in (B_N, \bar{B}_{pes}]$, $\max_{B_1} V^R_{\beta p}(B_0, B_1) = \max_{B_1} V^R_{\beta p,d}(B_0, B_1)$. We take the first order condition with respect to B_1, without taking into account the boundary constraint ($B_1 \leq \bar{B}$):

$$u'(A\bar{y} - B_0 + \beta p B_1) = u'(\bar{y} - B_1) \tag{20}$$

We take an analogous step for V_β^R in the following:

$$u'(A\bar{y} - B_0 + \beta B_1) = pu'(\bar{y} - B_1) + (1-p)u'(A\bar{y} - B_1) \tag{21}$$

Substitute $\beta p B_1$ and βB_1, respectively, with $GFN_{\beta p}$ and GFN_β in (20) and (21), which gives us:

$$u'(A\bar{y} - B_0 + GFN_{\beta p}) = u'(\bar{y} - \frac{GFN_{\beta p}}{\beta p}) \tag{22}$$

$$u'(A\bar{y} - B_0 + GFN_\beta) = pu'(\bar{y} - \frac{GFN_\beta}{\beta}) + (1-p)u'(A\bar{y} - \frac{GFN_\beta}{\beta}) \tag{23}$$

The left hand side in (22) and (23) refers to the marginal benefit of raising extra unit of bond revenue under risky and riskless pricing, respectively. The right hand side refers to the marginal cost of raising extra unit of bond revenue.

From (22), the optimal value of $GFN^*_{\beta p}$ can be solved analytically:

$$GFN^*_{\beta p} = \min\left[\frac{\beta p((1-A)\bar{y} + B_0)}{1+\beta p}, \beta p \bar{B}\right]$$

Hence, to ensure the equilibrium GFN is larger under optimistic beliefs regime, we only need to ensure the marginal cost of raising extra unit of revenue is smaller in (23) relative to (22), when $GFN_\beta = GFN^*_{\beta p}$:

$$pu'(\bar{y} - \frac{GFN^*_{\beta p}}{\beta}) + (1-p)u'(A\bar{y} - \frac{GFN^*_{\beta p}}{\beta}) < u'(\bar{y} - \frac{GFN^*_{\beta p}}{\beta p})$$

which can be characterized by a stronger condition in the following:

$$A\bar{y} - \frac{GFN^*_{\beta p}}{\beta} > \bar{y} - \frac{GFN^*_{\beta p}}{\beta p} \Rightarrow GFN^*_{\beta p} > \frac{\beta p(1-A)\bar{y}}{1-p} \tag{24}$$

The value of $GFN^*_{\beta p}$ is the lowest when $B_0 \to B_N$:

$$\inf\left\{GFN^*_{\beta p} = \min\left[\frac{\beta p((1-A)\bar{y} + B_0)}{1+\beta p}, \beta p\bar{B}\right] : B_0 \in (B_N, \bar{B}_{pes}]\right\} = \min\left[\frac{\beta p((1-A)\bar{y} + B_N)}{1+\beta p}, \beta p\bar{B}\right]$$

The condition $\frac{(1-A)\bar{y}+B_N}{1+\beta p} < \bar{B}$ guarantees that the borrowing limit does not bind for $B_0 \to$

B_N, which can be reduced to:

$$B_N < (A - Z + \beta p(1 - Z))\bar{y} \tag{25}$$

Given that (25) holds, we can rewrite the condition (24) as:

$$B_N > \frac{p}{1-p}(1+\beta)(1-A)\bar{y}$$

□

H.3 Proof of Proposition E.1

Proof. Define $\mathcal{E}(B) \equiv \mathcal{V}^R_{CK}(B, a) - \mathcal{V}^D(a)$. Since $\mathcal{V}^D(a)$ is a constant, by the properties of the value of repaying debt, $\mathcal{E}(B)$ is a continuous and monotonically decreasing function. Given that $\lim_{g \to \bar{g}} \mathcal{U}(c, g) = -\infty$, by the intermediate value theorem the following inequalities imply existence and the uniqueness of a debt threshold $\bar{B}(a)_{CK}$ in the region $(0, \tau y(a, 1) - \bar{g})$:

$$\begin{aligned} \mathcal{E}(0) &> 0 \\ \lim_{B \to \tau y(a,1) - \bar{g}} \mathcal{E}(B) &= -\infty \end{aligned}$$

□

H.4 Proof of Proposition E.2

Proof. Rewrite $\mathcal{V}^R_{pes,nd}(B, 0)$ when $B = \bar{B}(0)_{CK}$:

$$\begin{aligned} \mathcal{V}^R_{pes,nd}(\bar{B}(0)_{CK}, 0) = \max_{0 \leq B' \leq \bar{B}(0)_{pes}} \quad & \mathcal{U}(c, g) + \beta\Big(p\mathcal{V}(B', 1, 1, \rho = pes) + (1-p)\mathcal{V}(B', 1, 0, \rho = pes)\Big) \\ \text{s.t.} \quad & g + B = \tau A\bar{y} + \beta p B', \\ & c = (1 - \tau)A\bar{y} \end{aligned}$$

If we set the choice variable B' to zero, the value of $\mathcal{V}^R_{pes,nd}(\bar{B}(0)_{CK}, 0)$ is equal to $\mathcal{V}^R_{CK}(\bar{B}(0)_{CK}, 0)$ (see $\mathcal{V}^R_{CK}$ in (17)). However, the optimal choice of B' is unambiguously positive in $\mathcal{V}^R_{pes,nd}(\bar{B}(0)_{CK}, 0)$. By strict concavity of $\mathcal{U}$, this implies that $\mathcal{V}^R_{pes,nd}(\bar{B}(0)_{CK}, 0)$ is larger than $\mathcal{V}^R_{CK}(\bar{B}(0)_{CK}, 0)$. Hence, $\bar{B}(0)_{pes} > \bar{B}(0)_{CK}$. □

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