EUI DEPARTMENT OF ECONOMICS

On the Role of Lenders in Sovereign Debt Crises

Adrien Wicht

Thesis submitted for assessment with a view to
obtaining the degree of Doctor of Economics
of the European University Institute

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Abstract

This thesis emphasizes the role of lenders in the analysis of sovereign debt crises, whereas the existing literature on sovereign debt has mostly focused on the borrower's default decision. In particular, it investigates two main dimensions of sovereign lending: coordination and heterogeneity among lenders. The first two chapters of the thesis apply the theory of optimal contracts and analyze the role of coordination among lenders. The third chapter builds on the standard approach of exogenous incomplete markets. Like the first chapter, it focuses on emerging economies. Like the second chapter, it analyzes the interaction between official and private lenders.

In the first chapter, I analyze the sovereign debt management of emerging economies. I consider a market economy in which a sovereign borrower trades non-contingent bonds of different maturities with two foreign lenders. The borrower is impatient and lacks commitment. I show that the market economy can implement the Planner's constrained efficient allocation through changes in maturity and costly debt buybacks. Defaults cannot substitute for such buybacks. Nevertheless, the market economy may fail to implement the Planner's allocation for the following reasons. First, market participants rely on Markov strategies; an assumption that I rationalize in the context of emerging economies. Second, there are multiple Markov equilibria owing to the strategic interaction of the lenders. I relate these equilibria to the experience of Argentina and Brazil since 1995. In particular, conducting buybacks and avoiding defaults, I find that Brazil has a more efficient sovereign debt management than Argentina.

In the second chapter, joined with Yan Liu and Ramon Marimon, we develop the optimal design of a Financial Stability Fund to overcome sovereign debt overhang issues. Similar to the previous chapter, we derive an optimal contract that we use as a normative benchmark with positive implications. We consider an environment in which a sovereign country can borrow long-term defaultable bonds on the private international market, while having access to a Fund, which provides insurance and credit. We interpret the Fund as the outcome of long-term contingent contract subject to limited enforcement constraints. Our main theoretical result is to characterize the Nash equilibrium

between a sovereign country, the Fund and private competitive lenders. The Fund is only required minimal absorption of the sovereign debt to achieve the constrained efficient allocation. The Fund and its lending policy are essential as private lenders alone may not attain such allocation as shown in Chapter 1. Quantitatively, we find that Italy would have had important welfare gains and a more efficient path of debt accumulation with the Fund.

In my final chapter, I study existing official multilateral lending institutions. This complements the previous chapter which derives the optimal design of an official lender. I first present new empirical facts on defaults involving the International Monetary Fund and the World Bank. In particular, defaults on such institutions are infrequent, last relatively longer and are associated with greater private creditors' losses. I subsequently build a theoretical model to rationalize those findings. I consider an incomplete markets model with heterogenous lenders and endogenous renegotiation. The key assumption is the greater enforcement power of multilateral lenders relative to private lenders emanating from a non-toleration of arrears combined with a greater output penalty upon default. This generates an important pecuniary spillover on private lending and rationalizes the aforementioned empirical facts.

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Contents

Chapter 1

Efficient Sovereign Debt Management in Emerging Economies

Abstract This paper assesses the efficiency of the sovereign debt management in emerging economies. I consider a market economy in which a sovereign borrower trades non-contingent bonds of different maturities with two foreign lenders. The borrower is impatient and lacks commitment. I show that the market economy cannot implement the Planner's constrained efficient allocation through defaults but instead by costly debt buybacks. Moreover, as the lenders must enforce those buybacks, the implementation generally requires history-dependent strategies. Nevertheless, interpreting the borrower's impatience as a form of bounded memory, small perturbations in the payoff of the market participants rule out any other strategies than Markov ones. In this case, the Planner's allocation can only be approximated by Markov strategies. I show that emerging economies such as Argentina and Brazil present evidence of such approximation albeit with different outcomes. The multiplicity in outcome comes from the strategic interaction between the two lenders. I find that Brazil has a more efficient sovereign debt management than Argentina.

1.1 Introduction

Argentina and Brazil experienced an economic crisis in the late 1990s. Maturity shortened in both countries during that period. Argentina defaulted in 2001 and got excluded from the international financial market until 2006. In opposition, Brazil did not default and started a sovereign debt buyback program in the early 2000s. Buybacks had the peculiarity that they entailed a premium and were therefore costly for the Brazilian government.[1] In light of this historical episode, the question that arises is twofold. What can explain these opposite debt management? And which of these two debt management is more efficient? I analyze the role of the lenders in the multiplicity of equilibria and the role of maturity, buyback and default in implementing or approximating the constrained (Pareto) efficient allocation.

The literature on sovereign debt has focused on the borrower's decision to default. Differences between countries come from differences in terms of preferences, default costs or productivity shocks. My view is that Argentina and Brazil were not dissimilar in terms of economic fundamentals at the end of the 1990s. What truly differentiates those two countries is the strategic interaction between the lenders.

In terms of maturity, the literature on fiscal policy with commitment suggests to trade non-contingent bonds of different maturities to replicate the return of Arrow securities.[2] The portfolio of bonds emanating from this approach is however empirically implausible.[3] To reconcile the model's prediction with the data, the literature has introduced different frictions such as limited commitment and trade constraints.[4] I provide an alternative explanation: market participants often lack the strategical sophistication required to implement the aforementioned maturity management. Focusing on emerging economies, one ought to consider Markov strategies under which market

[1] Argentina conducted buybacks at a discount (i.e. below par) which usually correspond to a default in the form of distressed debt exchange.

[2] See Kreps (1982), Angeletos (2002) and Barro (2003).

[3] Buera and Nicolini (2004) and Faraglia et al. (2010) show that the borrower ought to *sell* long-term bonds and *buy* short-term bonds in the magnitude of several multiples of GDP.

[4] Debortoli et al. (2017) introduce limited commitment in fiscal policy, Faraglia et al. (2019) limit the extent of debt repurchase and reissuance and Kiiashko (2022) adds limited commitment in repayment.

participants can usually only approximate the return of Arrow securities. Such an approximation is consistent with the data, though.

In terms of default and buyback, the literature on sovereign debt argues that it might be optimal to conduct the former as this provides a source of state contingency, while the latter is suboptimal as it only benefits the lenders.[5] I argue the opposite. A default generates deadweight losses which impact both the borrower and the lenders. Hence, it is Pareto improving to avoid such an event. In opposition, buybacks at a premium can generate state contingency without causing the aforementioned deadweight losses.[6] As the bond price incorporates such premium, it is possible to generate state-contingent capital losses and gains with the appropriate buyback policy. As a result, the optimal sovereign debt management consists of no default and occasional costly buybacks.

I consider a small open economy populated by two foreign lenders and one sovereign borrower. The lenders supply the capital input and buy bonds issued by the borrower. Conversely, the borrower takes decisions on behalf of the small open economy, runs the production technology and issues non-contingent defaultable bonds of different maturities. Domestic production is subject to persistent productivity shocks and the borrower is impatient. There is one friction: the borrower cannot commit to repay the lenders.

Relying on the concept of sustainable equilibria, I first show that the Second Welfare Theorem holds in the market economy. For this, I derive an optimal contract in which a Planner allocates resources between the borrower and the lenders. This defines a constrained efficient allocation which features state-contingent debt relief and production distortions. Particularly, the borrower records capital gains in low productivity states and capital losses otherwise. In addition, when the borrower receives sufficiently high utility in the contract, the contract can sustain the productivity-maximizing level of capital. Otherwise, the threat of autarky fades and the Planner reduces the level of capital to relax the participation constraint. The Planner never finds optimal to set capital

[5] See notably Grossman and Van Huyck (1988), Adam and Grill (2017), Roettger (2019) and Hatchondo et al. (2020a) for defaults as a source of risk sharing and Bulow and Rogoff (1988, 1991) and Cohen and Verdier (1995) for buybacks as suboptimal policy.

[6] I consider an exogenous premium on buybacks and provide foundations for such cost in Appendix 1.2.

to zero, though.

I then implement the optimal contract in the market economy. Given that the Planner never distorts capital to zero, defaults – which imply markets exclusion – cannot implement the constrained efficient allocation. Instead, the borrower conducts buybacks at a premium. Such buybacks implicitly introduce state contingency in the bond contract as the bond price incorporates the premium paid. They occur in high productivity states implying that the price of long-term bonds increases after the realization of high productivity shocks. This in turn increases the value of outstanding long-term debt resulting in capital losses for the borrower. The opposite happens after the realization of low productivity shocks. Thus, costly buybacks can generate the capital losses and gains necessary to mimic the state contingency in liabilities of the optimal contract.

The Second Welfare Theorem holds but the First Welfare Theorem generally fails even when I restrict the analysis to Markov equilibria. I first link the study of emerging economies with the concept of Markov equilibrium. Given that such economies suffer from important political instability, I interpret the borrower's impatience as a form of bounded memory. In addition, as emerging economies' fundamentals are difficult to assess for foreign creditors, I introduce small and independent perturbations in payoffs.[7] Under these two assumptions, I show that all sustainable equilibria are Markov. Hence, the study of emerging economies ought to be limited to Markov equilibria.

I then show that there are multiple Markov equilibria in the market economy. The multiplicity comes from strategic interactions between the lender holding legacy debt and the other lender. The legacy lender is willing to avoid default and supports costly buybacks while the other lender is indifferent. I find two Markov equilibria. The first one is a Markov equilibrium without default in which premium-bearing buybacks can occur on equilibrium path. The second one is a Markov equilibrium with default in which premium-bearing buybacks never occur as in Arellano and Ramanarayanan (2012). I relate the second Markov equilibrium to the experience of Argentina and

[7]See notably Bussière and Mulder (2000), Scholl (2017) and Andreasen et al. (2019) for the political instability and Tsyrennikov (2013) and Morelli and Moretti (2021) for the lack of financial transparency.

the first Markov equilibrium to the experience of Brazil. Hence, the difference between the two countries can be solely attributed to the lenders' interaction.

Beside suffering from multiplicity, Markov strategies generally fail to implement the constrained efficient allocation. The reason is that costly buybacks need to be enforced by the lenders. In a Markov equilibrium, such enforcement is possible only if the borrower does not issue assets and buybacks are not too costly. However, to replicate the Planner's allocation, the borrower needs to hold short-term assets unless the buyback premium is sufficiently large. Particularly, I find that Markov strategies fail to implement the Planner's allocation under empirically plausible buyback premia. Thus, being restricted to Markov strategies, emerging economies can only approximate the constrained efficient allocation.

To gauge the goodness of the Markov approximation, I calibrate the Markov equilibrium with default to match moments of the Argentine economy over the period 1995-2019. The calibrated model fits the data well and features default episodes in which indebtedness increases with respect to output and maturity shortens. Conversely, during restructurings, the level of debt remains substantial and the maturity lengthens. In addition, using the calibration for Argentina, I find that the Markov equilibrium without default is quantitatively close to Brazil. I therefore interpret Brazil as the counterfactual of Argentina with costly buybacks and without defaults. This supports my claim that the difference between Argentina and Brazil can be solely attributed to the lenders. Finally, in line with the literature on fiscal policy with commitment, I find that the implementation of the constrained efficient allocation generates unrealistic debt portfolios unlike Markov equilibria.

I then compare the Markov equilibria with the implementation of the optimal contract through various simulation exercises. Relying on costly buybacks instead of defaults implies important welfare gains for both the borrower and the lenders. In that logic, the Markov equilibrium without default is quantitatively the closest to the constrained efficient allocation. Nonetheless, it is far from the Pareto frontier indicating that the Markov approximation remains crude. Thus, the limitation to Markov strategies in emerging economies is very costly in terms of welfare.

The paper is organized as follows. I review the literature in Section 1.2. The economic environment is in Section 1.3 and the market economy in Section 1.4. I present the constrained efficient and the Markov debt management in Sections 1.5 and 1.6, respectively. The calibration and quantitative analyses are in Section 1.7. Finally, I conclude in Section 1.8.

1.2 Literature Review

The paper combines elements of the literature on sovereign defaults and buybacks with elements of the literature about optimal contracts and their implementation.

The literature on sovereign defaults assumes that markets are incomplete and agents follow Markov strategies (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008).[8] There, the borrower has access to only non-contingent claims and can obtain limited state contingency through defaults. My study is the closest to Arellano and Ramanarayanan (2012) and Niepelt (2014) given that I adopt two bonds with different maturities and to Mendoza and Yue (2012) given that the default cost is endogenous. Also, similar to Kovrijnykh and Szentes (2007), I consider strategic lenders. I contribute to this literature in two ways. First, I show that the reliance on defaults to obtain state contingency is inefficient. Second, I provide a foundation for the use of Markov strategies interpreting the assumption of impatience as evidence of bounded memory and then applying the result of Bhaskar et al. (2012) and Angeletos and Lian (2021). This second contribution relates to Krusell and Smith (1996) and Krusell et al. (2002) as it connects the equilibrium outcome with the sophistication of agents' strategies.[9]

On a similar note, this paper relies on costly buybacks as a way to implement the constrained efficient allocation. It therefore relates to the seminal contribution of Bulow and Rogoff (1988, 1991) who document that buybacks are suboptimal as they increase the recovery value per unit of bond and therefore fail to reduce indebtedness. In light of this, Cohen and Verdier (1995) show that buybacks are effective only if they remain secret. Similarly, Aguiar et al. (2019) find that buybacks reduce welfare as they shift the maturity structure and therefore affect the default risk. In opposition, Rotemberg (1991) shows that buybacks can be advantageous to all parties as they

[8] See also Aguiar and Amador (2014), Aguiar et al. (2016) and Aguiar and Amador (2021).

[9] See also the concept of robust prediction in Passadore and Xandri (2021).

lower the bargaining costs. Moreover, Acharya and Diwan (1993) find that buybacks provide a positive signal about the borrower's willingness to repay. Finally, Kovrijnykh and Szentes (2007) find that buybacks are socially efficient despite the fact that only lenders benefit from it *ex post*. My analysis goes in this direction as it emphasizes the efficiency of buybacks as a source of risk sharing between the borrower and the lenders.

Showing multiplicity in the Markov equilibrium, this study also relates to the work of Calvo (1988), Cole and Kehoe (2000), Conesa and Kehoe (2017), Lorenzoni and Werning (2019) and Aguiar et al. (2022).[10] Unlike Cordella and Powell (2021), I relate the enforcement of a no-default borrowing limit to strategic decisions of the lenders rather than commitment. This generates multiple equilibria different than in Alvarez and Jermann (2000) and Kirpalani (2017). Especially, multiplicity comes from the attitude of the lenders towards defaults and costly buybacks in the presence of long-term debt.

The paper derives an optimal contract between foreign lenders and a borrower and therefore relates to the seminal contributions of Kehoe and Levine (1993, 2001) and Thomas and Worrall (1994). My study accounts for limited commitment similar to Aguiar et al. (2009) and is close to Kehoe and Perri (2002) and Restrepo-Echavarria (2019) as it relies on the approach of Marcet and Marimon (2019) with the difference that I implement the contract in a market economy.[11]

The paper therefore addresses the literature on optimal contract's implementation. Note that I discuss the following studies in more details in Appendix 1.1. Unlike Aguiar et al. (2019) and Müller et al. (2019), my implementation is not generally Markov. On the one hand, Aguiar et al. (2019) account for multiple maturities but consider a Planner's problem which has no participation constraint, unlike my Planner problem. On the other hand, Müller et al. (2019) use preemptive restructurings and GDP-linked bonds, whereas I rely on the maturity structure. An alternative to this approach is Dovis (2019) who develop an implementation through partial defaults and an active debt maturity management. He builds on Angeletos (2002) and Buera and Nicolini (2004) who

[10] See also Stangebye (2020), Galli (2021), Corsetti and Maeng (2021) and Bloise and Vailakis (2022).

[11] The other difference is that I adopt a capital depreciation rate of 1 which simplifies the equilibrium computation and the proofs.

show that one can replicate the state-contingency of Arrow securities using non contingent bonds of different maturities.[12] My implementation is the closest to Dovis (2019) with the difference that I rely on debt buybacks without defaults and haircuts. Moreover, similar to Hatchondo et al. (2020a), I connect my implementation to the Markov allocation. Especially, as buybacks need to be enforced, I explain why and when history dependence matters.

1.3 Environment

Consider a small open economy over infinite discrete time $t = \{0, 1, \dots\}$ with a single homogenous good. The small open economy is populated by a benevolent government and a large number of homogenous households which own domestic firms, while two foreign lenders invest in the small open economy.[13]

Foreign lenders are risk neutral, strategic and break even in expectations. They discount the future at rate $\frac{1}{1+r}$ with r being the exogenous risk-free rate. The representative domestic household discounts the future at rate $\beta \leq \frac{1}{1+r}$. Preference over consumption is represented by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$ where c_t corresponds to the consumption at time t. The instantaneous utility function $u(\cdot)$ is continuous, increasing and concave.

Domestic households have access to a production technology, $F(k_t, l_t)$, where l_t is the labor input and k_t is the capital input at time t. They are endowed with one unit of labor in every t. The lenders supply the capital input, k_t, at price p_t in every t. I denote $f(k_t) \equiv F(k_t, 1)$ and assume it is continuous, increasing, concave, satisfies the Inada condition, $\lim_{k \to 0} f_k(k) = \infty$, and $f(0) > 0$. For tractability, capital depreciates at rate 1.

Domestic production is subject to a shock g_t which takes value on the discrete set $G \equiv \{g_L, g_H\}$ with $0 < g_L < g_H$ and follows a Markov chain of order one with $\pi(g_{t+1}|g_t)$ corresponding to the probability of drawing g_{t+1} at date $t+1$ conditional on drawing g_t at t. I further assume that shocks are persistent meaning that $\pi(g|g) > 0.5$ for all $g \in G$. The fact that $f(\cdot)$ is concave implies that

[12] See also the references in footnotes 2, 3 and 4.

[13] The present environment is similar to the one of Quadrini (2004), Aguiar et al. (2009) and Dovis (2019).

there exists a level $k^*(g_t)$ which maximizes the net production such that $g_t f_k(k^*(g_t)) = p_t$.

The government is benevolent and takes the decision on behalf of the small open economy.[14] There is a tax on the import of capital made by domestic firms at rate $\tau_t = 1 - \frac{1}{p_t}$ meaning that $p_t(1-\tau_t)k_t = k_t$.[15] Thus, the household's after-tax income is given by

$$y(g_t, k_t) \equiv g_t F(k_t, l_t) - k_t.$$

The government has access to bond contracts with two different maturities: short-term and long-term. The short-term bond $b_{st,t+1}$ is a one-period bond with unit price $q_{st,t}$ which pays a coupon of one next period. The long-term bond $b_{lt,t+1} \leq 0$ is a consol with unit price $q_{lt,t}$ which pays a coupon of one every period. I denote debt as a negative asset meaning that $b_j < 0$ is a debt, while $b_j > 0$ is an asset for all $j \in \{st, lt\}$. The government can hold short-term assets but not long-term assets.

A bond auction determines the new issuance of bond contracts. Similar to Kovrijnykh and Szentes (2007), the two lenders simultaneously offer a couple $(b_{st,t+1}, b_{lt,t+1})$ during the auction in t and the borrower chooses among the two offers. For the long-term bond, all outstanding bonds have to be repurchased before issuing new ones. This implies that only one of the two lenders is holder of legacy claim each period. I call this lender the *legacy* lender and the other the *new* lender.

As one will see, the *new* lender will have to decide which type of debt contracts to offer. For this, I introduce the indicator function $\varpi_0 \in \{0, 1\}$. The exogenous shock space is therefore given by $s_t \equiv (g_t, \varpi_0)$.

The bond contract specifies the conditions for default and buyback. A default corresponds to a missed coupon payment which triggers exclusion from both the capital and the bond markets. Nevertheless, the borrower can regain access to those markets with probability $\lambda \geq 0$. There is no recovery value. Conversely, a buyback is defined as any new long-term debt issuance below

[14] As in Aguiar et al. (2009) and Dovis (2019), I combine the income of households and government together. Households provide labor inelastically and receive lump sum transfers from the government.

[15] This technical assumption is necessary for the implementation of the constrained efficient allocation in the market economy. Having τ_t a choice variable would make taxation time inconsistent in Markov equilibria.

the initial amount outstanding, i.e. $b_{lt,t+1} \geq b_{lt,t}$. There are two types of buybacks: *official* and *unofficial*. In the latter, the government repurchases the outstanding debt at $q_{lt,t}$.[16] In the former, it repurchases the outstanding debt at $q^{bb}_{lt,t} = q_{lt,t} + \chi$ where $\chi > 0$ is the buyback premium. Only the *legacy* lender can make an offer entailing χ. Note that I provide foundations for $\chi > 0$ in Appendix 1.2.

The government cannot commit to pay the lenders. In particular, it cannot commit to pay the coupon due every period and to conduct *official* instead of *unofficial* buybacks.

The timing of actions is the following. At the beginning of each period $t \geq 0$, g_t realizes and the lenders jointly supply k_t. Subsequently, domestic production takes place, capital depreciates and is taxed. The government decides whether to default. Conditional on no default, the bond auction determines $b_{st,t+1}$ and $b_{lt,t+1}$.

1.4 The Market Economy

In this section, I define the set of sustainable equilibrium outcomes in the market economy following the approach of Abreu (1988) and Chari and Kehoe (1990). Keeping track of the entire history of play, it is possible to sustain multiple equilibrium outcomes.

1.4.1 The Government's problem

Define $D_t \in \{0, 1\}$ as the default policy at time t. If $D_t = 0$, the government repays, while if $D_t = 1$, it defaults. Correspondingly, as market re-access is stochastic, let the default status $\mathbb{I}_{D,t}$ be 1 if the government is in default at the beginning of t and 0 otherwise. Further define $M_t \in \{0, 1\}$ as the *official* buyback policy at time t. If $M_t = 1$, the government buys its debt back paying the premium χ, while if $M_t = 0$, it does not.

In addition, define the government's choice set as $\mathcal{C}_{b,t} = \{D_t, M_t, b_{st,t+1}, b_{lt,t+1}\}$ and the government's strategy as σ_b. Furthermore, let $h^t = (h^{t-1}, s_t, \mathbb{I}_{D,t}, p_t, k_t, \mathcal{C}_{b,t})$ be the history up to time t taking the initial debt $\{b_{j,0}\}_{j\in\{st,lt\}}$ as given. Due to the specific timing of actions, fur-

[16] *Unofficial* buybacks are a version of what Cohen and Verdier (1995) call a "secret buyback".

ther define the history of the two lenders and the government as $h_l^t = (h^{t-1}, s_t, \mathbb{I}_{D,t})$ and $h_b^t = (h^{t-1}, s_t, \mathbb{I}_{D,t}, p_t, k_t)$, respectively. I also define the history for the choice of capital as $h_k^t = (h^{t-1}, s_t, \mathbb{I}_{D,t}, p_t)$. Finally, denote the value of the two lenders and the government after any specific history by $W^l(h_l^t)$ and $W^b(h_b^t)$, respectively.

In the case in which the government decides to repay (i.e. $D_t = 0$), it determines its consumption and prospective borrowing given the realization of the history h_b^t. In the case of no *official* buyback (i.e. $M_t = 0$), the budget constraint reads

$$c_t + q_{st}(h_b^t, \mathcal{C}_{b,t})b_{st,t+1} + q_{lt}(h_b^t, \mathcal{C}_{b,t})(b_{lt,t+1} - b_{lt,t}) = y(g_t, k_t) + b_{st,t} + b_{lt,t}.$$

There is no restriction on the issue of long-term debt meaning that the government can potentially conduct *unofficial* buybacks. Conversely, in the case of an *official* buyback (i.e. $M_t = 1$), budget constraint is given by

$$c_t + q_{st}(h_b^t, \mathcal{C}_{b,t})b_{st,t+1} + q_{lt}(h_b^t, \mathcal{C}_{b,t})b_{lt,t+1} = y(g_t, k_t) + b_{st,t} + b_{lt,t}(1 + q_{lt}^{bb}(h_b^t, \mathcal{C}_{b,t})) \ \wedge \ b_{lt,t+1} \geq b_{lt,t}.$$

The government pays the premium χ and issues new long-term debt such that $b_{lt,t+1} \geq b_{lt,t}$. Conversely, if the government decides to default (i.e. $D_t = 1$), it gets excluded from the markets and consumes

$$c_t = g_t f(k_t).$$

Neither capital nor debt are repaid. Due to the specific timing of capital, the government enjoys $k_t \geq 0$ in the first period of autarky and then $k_t = 0$. The capital market exclusion therefore forms an endogenous cost of default. Upon markets re-access,

$$c_t + q_{st}(h_b^t, \mathcal{C}_{b,t})b_{st,t+1} + q_{lt}(h_b^t, \mathcal{C}_{b,t})b_{lt,t+1} = y(g_t, k_t).$$

There is no recovery value of debt. Thus, after any history h_b^t, the optimal strategy of the government, σ_b, is the solution of

$$W^b(h_b^t) = \max_{\{\mathcal{C}_{b,t}\}_{t=0}^{\infty}} u(c_t) + \beta \mathbb{E}\left[W^b(h_b^{t+1}) \middle| h_b^t, \mathcal{C}_{b,t}\right], \tag{1.1}$$

subject to the budget constraint.

1.4.2 Sustainable equilibria

Having derived the government's problem, I can define and characterize the set of sustainable equilibria. The lenders break even meaning that in expectations they make zero profit. The price of one unit of bond is given by,

$$q_{lt}(h^t) = \frac{1}{1+r}\mathbb{E}\left[(1 - D(h^{t+1}))\left\{1 + (1 - M(h^{t+1}))q_{lt}(h^{t+1}) + M(h^{t+1})q_{lt}^{bb}(h^{t+1})\right\}\middle| h^t\right], \tag{1.2}$$

$$q_{st}(h^t) = \frac{1}{1+r}\mathbb{E}\left[(1 - D(h^{t+1}))\middle| h^t\right].$$

As I rely on the entire history of play, it is sufficient to characterize the problem from the borrower's perspective. In Section 1.6, I develop the lenders' perspective.

Definition 1.1 (Sustainable Equilibrium)**.** *Given* $\{b_{j,0}\}_{j\in\{st,lt\}}$*, a sustainable equilibrium in this environment consists of strategy for the government,* σ_b*, policy for the firm's capital,* k *as well as price schedule for capital,* p*, and for bonds,* q_{st} *and* q_{lt} *such that*

1. *Taking* p*,* q_{st} *and* q_{lt} *as given,* σ_b *is the solution to* (1.1).
2. *Taking* p *as given, the choice of capital by domestic firms is such that*

$$\mathbb{E}\left[u_c(c(h_b^t))(g_t f_k(k(h^t)) - p(h^t))\middle| h_k^t\right] = 0. \tag{1.3}$$

3. *The price of capital is consistent with*

$$\max_{k_t} \mathbb{E}\left[p(1-\tau)k_t - k_t \middle| h_l^t\right]. \tag{1.4}$$

4. *The bond prices satisfy* (1.2).

Following the approach of Abreu (1988) and Chari and Kehoe (1990), I characterize the set of outcomes that can be sustained in equilibrium using reversion to the worst equilibrium. The following lemma shows that permanent autarky is the worst equilibrium outcome. All proofs are in Appendix 1.11.

Lemma 1.1 (Worst Equilibrium Outcome)**.** *In this environment, the worst possible outcome is permanent autarky which can be supported as an equilibrium.*

Keeping track of the entire history of play and relying on trigger strategies, I can sustain multiple equilibrium outcomes. The only two requirements are: the allocation and price satisfy the optimality conditions for all market participants and the borrower's value cannot be lower than the value of the worst equilibrium.

Lemma 1.2 (Sustainable Outcomes)**.** *Given* $\{b_{j,0}\}_{j\in st,lt}$*, an allocation* $\{\mathcal{C}_{b,t}, k_t\}_{t=0}^{\infty}$ *with prices* $\{q_{st,t}, q_{lt,t}, p_t\}_{t=0}^{\infty}$ *is the outcome of a sustainable equilibrium if and only if it satisfies* (1.1)*,* (1.2)*,* (1.3)*,* (1.4) *and for every* t, h_b^t*,* $W^b(h_b^t) \geq \sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(g_j f(0))$.

In what follows, I show that the Second Welfare Theorem holds in the market economy – Section 1.5 – but the First Welfare Theorem generally fails even when I restrict the analysis to Markov equilibria – Section 1.6.

1.5 Constrained Efficient Debt Management

This section presents the constrained efficient debt management. I first derive an optimal contract between the borrower and the two lenders and subsequently characterize the underlying constrained efficient allocation before implementing it the market economy.

1.5.1 The optimal contract

The optimal contract is the outcome of a problem in which a Planner allocates capital and consumption to maximize the lenders' and the borrower's weighted utility subject to a participation constraint. The participation constraint accounts for limited commitment in repayment (Thomas and Worrall, 1994). Denoting g^t as the history of realized value of g at time t, it must hold that for all t and g^t

$$\sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(c(g^j)) \geq V^D(g_t, 0, k_t). \tag{1.5}$$

If the borrower breaks the contract, it is sent to autarky for some time but can regain access to the market with probability λ and resumes the Markov equilibrium specified in Section 1.6 where $\varpi_0 = 1$.[17] I write $V^D(g_t, \mathbb{I}_{D,t}, k_t)$ to make explicit the dependence on k_t. As a result, the participation constraint ensures that the borrower's value of remaining in the contract is at least as large as the value of opting out.

Given the above constraint, the optimal contract between the borrower and the lenders in sequential form is the result of the following Planner's maximization problem

$$\max_{\{k(g^t), c(g^t)\}_{t=0}^{\infty}} \mu_{b,0} \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t|g_0) u(c(g^t)) + \mu_{l,0} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{g^t} \pi(g^t|g_0) T(g^t) \tag{1.6}$$

$$\text{s.t.} \quad (1.5),\ T(g^t) = g_t f(k(g^t)) - c(g^t) - k(g^t) \text{ for all } g^t, t \text{ with } (\mu_{b,0}, \mu_{l,0}) \geq 0 \text{ given.}$$

The given weights $\mu_{b,0}$ and $\mu_{l,0}$ are the initial non-negative Pareto weights assigned by the Planner to the borrower and the lenders, respectively. The above maximization problem combines the utility function $u(\cdot)$ with the production function $f(\cdot)$ and therefore might not be convex.

Assumption 1.1 (Convexity). *Define the optimal level of capital $k^*(g)$ such that $gf_k(k^*(g)) = 1$ and $h := gf(k) - k$ for $k \in [0, k^*(g)]$ with $h^*(g) = gf(k^*(g)) - k^*(g)$. Let $K(h)$ denote the*

[17] As we will see in Section 1.6, in this specific Markov equilibrium, the *new* lender always satiates the borrower's demand for bond. Fixing, $\varpi_0 = 1$, I can write g_t instead of s_t in the outside option.

inverse mapping from $[0, h^*(g)]$ *to* $[0, k^*(g)]$ *such that* $k = K(h)$. *For all* $g \in G$, $u(gf(k(h)))$ *is convex in* h *for* $h \in [0, h^*(g)]$.

Following, Aguiar et al. (2009), Assumption 1.1 ensures that there is no need for randomization. I now derive the recursive formulation of the above maximization problem. Following Marcet and Marimon (2019), I reformulate (1.6) as a saddle-point Lagrangian problem,

$$\begin{aligned}
\mathcal{SP} \min_{\{\gamma(g^t)\}_{t=0}^{\infty}} \max_{\{k(g^t),c(g^t)\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t|g_0)\mu_{b,t}(g^t)u(c(g^t)) + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{g^t} \pi(g^t|g_0)\mu_{l,t}(g^t)T(g^t) \\
& + \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi(g^t|g_0)\gamma(g^t)\Big[\sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t)u(c(g^j)) - V^D(g_t, 0, k_t)\Big] \\
\text{s.t.} \quad & T(g^t) = g_t f(k(g^t)) - c(g^t) - k(g^t), \\
& \mu_{b,t+1}(g^t) = \mu_{b,t}(g^t) + \gamma(g^t) \text{ and } \mu_{l,t+1}(g^t) = \mu_{l,t}(g^t) \text{ for all } g^t, t \\
& \text{with } \mu_{b,0}(g_0) \equiv \mu_{b,0} \text{ and } \mu_{l,0}(g_0) \equiv \mu_{l,0} \text{ given.}
\end{aligned}$$

In this formulation, $\gamma(g^t)$ denotes the Lagrange multiplier attached to the participation constraint at time t. As the value of the borrower appears in both the Planner's objective function and the participation constraint, there is a direct link between $\mu_{b,t}(g^t)$ and $\gamma(g^t)$. More precisely, the borrower's Pareto weight evolves according to $\mu_{b,t+1}(g^t) = \mu_{b,t}(g^t) + \gamma(g^t)$, while the lenders' Pareto weight, $\mu_{l,t}(g^t)$, remains constant.

Following Marcet and Marimon (2019), the saddle-point Lagrangian problem is homogenous of degree one in $(\mu_{b,t}(g^t), \mu_{l,t}(g^t))$. I can therefore redefine the contracting problem over $(x_t(g^t), 1)$ where $x_t(g^t) = \frac{\mu_{b,t}(g^t)}{\mu_{l,t}(g^t)}$ corresponds to the relative Pareto weight – i.e. the Pareto weight attributed to the borrower relative to the lenders. Given that $(\mu_{b,0}, \mu_{l,0}) \geq 0$ and $\gamma(g^t) \geq 0$ for all t, $x \in X \equiv [\underline{x}, \overline{x}]$ with $\underline{x} \geq 0$ and $\overline{x} \leq \infty$. Moreover,

$$x_{t+1}(g^t) = (1 + \nu(g^t))\eta x_t \quad \text{with} \quad x_0 = \frac{\mu_{b,0}}{\mu_{l,0}}, \tag{1.7}$$

where $\eta \equiv \beta(1 + r) \leq 1$ corresponds to the borrower's impatience relative to the lenders and

$\nu(g^t) \equiv \frac{\gamma(g^t)}{\mu_{b,t}(g^t)}$ represents the normalized multiplier attached to the participation constraint. Following Marcet and Marimon (2019), the state vector for the problem reduces to (g, x) and the Saddle-Point Functional Equation is given by

$$FV(g,x) = \mathcal{SP} \min_{\nu(g)} \max_{k(g),c(g)} x\Bigg[(1+\nu(g))u(c(g)) - \nu(g)V^D(g,0,k)\Bigg] \tag{1.8}$$

$$+ T(g) + \frac{1}{1+r}\sum_{g'} \pi(g'|g)FV(g',x')$$

$$\text{s.t.} \quad x'(g) = (1+\nu(g))\eta x,\ T(g) = gf(k(g)) - c(g) - k\ \forall g.$$

The value function takes the form of $FV(g,x) = xV^b(g,x) + V^l(g,x)$ with $V^b(g,x) = u\big(c(g)\big) + \beta\mathbb{E}_{g'|g}\Big[V^b\big(g',x'\big)\Big]$ being the value of the borrower and $V^l(g,x) = T(g) + \frac{1}{1+r}\mathbb{E}_{g'|g}\Big[V^l\big(g',x'\big)\Big]$ being the value of the two lenders together. I obtain the optimal consumption and capital policies by taking the first-order conditions in (1.8)

$$u_c(c(g)) = \frac{1}{x(1+\nu(g))} \quad \text{and} \quad gf_k(k(g)) - 1 = \nu(g)u_c(gf(k(g)))gf_k(k(g))x.$$

In terms of consumption, the binding participation constraint of the borrower (i.e. $\nu(g) > 0$) induces an increase in consumption. Regarding capital, the economy does not reach the production-maximizing level of capital $k^*(g)$ as long as the participation constraint binds in g. In the next subsection, I formalize this argument in Proposition 1.2.

1.5.2 Contract properties

I characterize the main properties of the contract in terms of Pareto frontier and risk sharing. Additional characterization can be found in Appendix 1.3.

I start with the definition of two threshold values for the relative Pareto weight: the one for which the borrower's participation constraint binds with $k = 0$ and with $k = k^*(g)$.

Definition 1.2 (Threshold)**.** *Define* $x_a(g)$ *such that* $V^b(g, x_a(g)) = V^D(g, 0, 0)$ *and* $x^*(g)$ *such that* $V^b(g, x^*(g)) = V^D(g, 0, k^*(g))$.

In words, $x^*(g)$ is the lowest relative Pareto weight that can sustain $k^*(g)$. Conversely, $x_a(g)$ is the weight associated with the autarkic allocation (i.e. $k = 0$) and is therefore the lowest possible weight in the contract. I show next that $x_a(g)$ is never attained.

Proposition 1.1 (Efficiency)**.** *Under Assumption 1.1, the autarkic allocation is not optimal meaning that $x \in \tilde{X} \equiv [\tilde{x}, \overline{x}]$ with $\tilde{x} > x_a(g_H)$. Moreover, $V^l(g,x)$ is strictly decreasing, while $V^b(g,x)$ is strictly increasing in $x \in \tilde{X}$ for all $g \in G$.*

The proposition is made of two parts. First, autarky (i.e. $k = 0$) is not optimal.[18] Due to the Inada condition on the production function, there are always strictly positive gains from trade between the borrower and the lenders when k is close to zero. This already means that defaults – which imply markets exclusion – cannot implement the Planner's constrained efficient allocation. Second, the proposition states that the optimal contract is constrained efficient which makes it the best achievable outcome in this environment.

The following proposition highlights the main properties of the constrained efficient allocation. The contract features production distortions, risk sharing across states and state-contingent debt relief.

Proposition 1.2 (Constrained Efficient Allocation)**.** *Under Assumption 1.1,*

I. (Production). $k(g,x) = k^*(g)$ *for* $x \geq x^*(g)$ *and* $x^*(g_H) > x^*(g_L)$. *Conversely, for all* $x, \ddot{x} \in \tilde{X}$ *with* $x^*(g) > x > \ddot{x}$, $0 < k(g,\ddot{x}) < k(g,x) < k^*(g)$.

II. (Risk-Sharing). $c(g_L,x) < c(g_H,x)$ *and* $x'(g_L,x) < x'(g_H,x)$ *for all* $x < x^*(g_H)$ *and* $c(g_L,x) = c(g_H,x)$ *and* $x'(g_L,x) = x'(g_H,x)$ *otherwise.*

III. (Liabilities). $V^l(g_L,x) < V^l(g_H,x)$ *for all* $x \in \tilde{X}$.

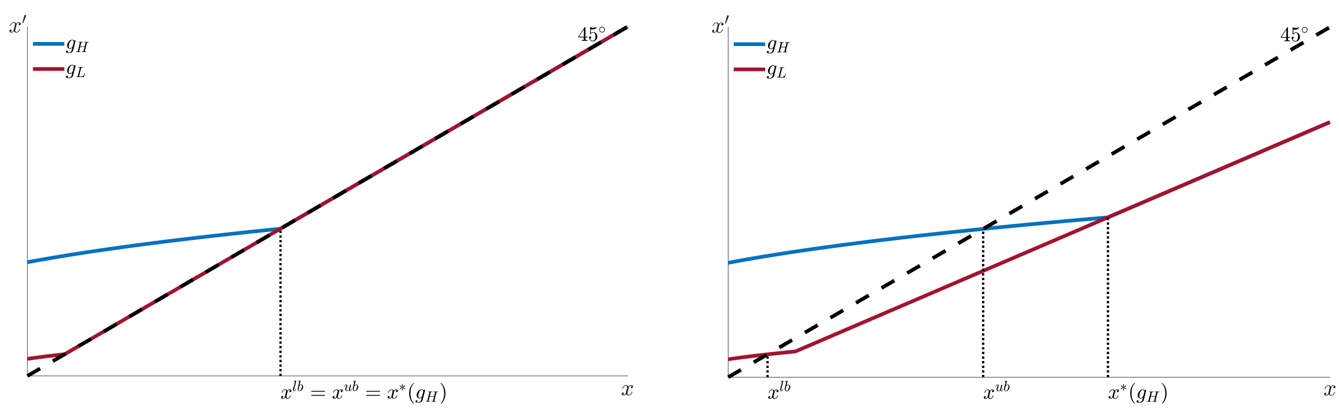
Part I of the above proposition states that the production-maximizing level of capital $k^*(g)$ such that $gf_k(k^*(g)) = 1$ is attained only if the relative Pareto weight, x, is above a certain threshold. Capital distortion is a consequence of the binding participation constraint (1.5). As the autarky value depends on the level of capital in the economy, the Planner finds optimal to reduce k to relax

[18]In Appendix 1.4, I show that if one introduces domestic capital, this result continues to hold. Note further that the depreciation rate of capital is irrelevant here.

the constraint. It continues to decrease k as long as x decreases but never finds optimal to set $k = 0$ following Proposition 1.1.

Part II states that the Planner always provides risk sharing to the extent possible. Equalization of consumption is possible whenever the borrower's participation constraint ceases to bind in all productivity states. Otherwise, the Planner provides more consumption and a greater continuation value when the high productivity state realizes.

Part III relates to the liabilities of the borrower. In this environment, $T(g)$ corresponds to the borrower's current account balance. Hence, the value of the two lenders represents the net foreign asset position in the contract. A positive value of $V^l(g, x)$ indicates the extent to which the borrower is indebted. The proposition states that the liabilities increase when g is high. This implies that the Planner adopts a state-contingent policy as it provides debt relief in low productivity states. This state contingency will be replicated through *official* buybacks in the market economy.



(a) Patience ($\eta = 1$) (b) Impatience ($\eta < 1$)

Note: The figure depicts the law of motion of the relative Pareto weight in the case of a patient (i.e. $\eta = 1$) and impatient (i.e. $\eta < 1$) borrower. The blue line corresponds to the law of motion in g_H and the red line to the law of motion in g_L. The black dotted line represents the $45°$ line. x^{lb} and x^{ub} correspond to the lower and upper bounds of the ergodic set, respectively. $x^*(g)$ corresponds to the weight at which the participation constraint ceases to bind in g.

Figure 1.1: Steady State Dynamic

Having determined the constrained efficient allocation, I now show that the long-term contract is characterized by an ergodic set of relative Pareto weights.[19]

Proposition 1.3 (Steady State)**.** *A steady state is defined by an ergodic set of relative Pareto weights*

[19]The term ergodic refers to the fact that the relative Pareto weights in this set are aperiodic and recurrent with non-zero probability.

$x \in [x^{lb}, x^{ub}] \subset \tilde{X}$. *Under Assumption 1.1, it holds that* $x'(g_H, x^{ub}) = x^{ub}$ *and* $x'(g_L, x^{lb}) = x^{lb}$ *and*

I. *If* $\eta = 1$, *then* $x^{lb} = x^{ub} = x^*(g_H)$.

II. *If* $\eta < 1$, *then* $x^{lb} < x^{ub} < x^*(g_H)$.

The proposition states that whenever the borrower is patient (i.e. $\eta = 1$), the steady state does not display any dynamic. Conversely, whenever the borrower is relatively impatient (i.e. $\eta < 1$), the steady state is dynamic. This dynamic is however bounded below by x^{lb} and above by x^{ub}. For instance, after a sufficiently long series of g_L (g_H), the contract hits x^{lb} (x^{ub}). It then stays there until g_H (g_L) realizes and that irrespective of the past realizations of the shock. The bounds of the ergodic set represent therefore regions of amnesia in the contract. Figure 1.1 illustrates each of the two types of steady states.

1.5.3 Contract implementation

Having derived and characterized the constrained efficient allocation, I construct a sustainable equilibrium that implements the constrained efficient allocation in the market economy. That is, I show that the Second Welfare Theorem holds in the market economy.

Proposition 1.4 (Implementation)**.** *Under Assumption 1.1, given a constrained efficient allocation, a sustainable equilibrium exists that implements it.*

The implementation works as follows. The government conducts *official* buybacks when the economy hits the upper bound of the ergodic set (i.e. $x = x^{ub}$). As this bound is reached after a sufficiently long series of high productivity shocks, this buyback policy generates a specific term structure in which high productivity shocks are related to relatively larger long-term bond prices than low productivity shocks, while the short-term bond price remains unchanged. Given this, I can equalize the value of debt in the contract, $V^l(g, x)$, with the value of the debt in the market economy, $b_{st} + b_{lt}[1 + q_{lt}]$, for each (g, x). As I have two productivity states and two bonds, this gives a system of two equations with two unknowns which has a unique solution for each x given the specified term structure.

Notice that the bounds of the ergodic set implicitly define endogenous borrowing and lending limits. On the one hand, when the borrower hits x^{lb}, borrowing remains the same every period as long as g_L realizes. Similarly, when the borrower hits x^{ub}, no additional lending takes place as long as g_H realizes. The borrowing limit attached to x^{lb} ensures that the borrower has no incentive to default. On the other hand, the lending limit attached to x^{ub} naturally arises due to discounting.

Lemma 1.3 (Official Buyback and No Default)**.** *Under Assumption 1.1, the implementation features official buybacks on equilibrium path and no default. However, official buybacks cannot occur when g_L realizes.*

Lemma 1.3 is made of two parts. First, the implementation does not rely on defaults. As shown in Proposition 1.1, the Planner never finds it optimal to distort capital to zero. This means that there is no proper markets exclusion in any region of the contract. It is therefore not possible to interpret the borrower's binding constraint as a default in my environment.[20] In Appendix 1.4, I show that ignoring domestic capital is without loss of generality here.

Second, *official* buybacks generate the capital losses and gains necessary to mimic the state contingency in liabilities of the optimal contract. In particular, as *official* buybacks involve a premium χ, they arise in the high productivity state. Unlike defaults, they are an efficient source of risk sharing. Defaults entail costs for both the lenders and the borrower, while *official* buybacks are solely costly for the latter. A default is therefore not renegotiation proof as both contracting parties would be strictly better off avoiding this event *ex post*.

A corollary of Lemma 1.3 is that the long-term bond spread is negative.[21] On the one hand, in the absence of default, there is no positive spread. On the other hand, *official* buybacks entail a premium χ implying that the long-term bond price exceeds the risk-free price.

Another corollary is that whether maturity shortens in the low productivity state depends on the

[20] Müller et al. (2019) and Restrepo-Echavarria (2019) interpret the borrower's binding constraint as a form of preemptive restructuring which does not trigger markets exclusion. Nonetheless, Asonuma and Trebesch (2016) show that even preemptive restructurings are followed by markets exclusion in the data.

[21] This is a feature that one finds in other implementations such as the ones of Ábrahám et al. (2019) and Liu et al. (2020). The mechanism is different though. The negative spread restricts the trade of state-contingent securities in a two-sided limited commitment problem when the participation constraint of the risk-neutral lenders binds.

exact parameters of the model. On the one hand, there is a substitution effect which pushes the maturity towards the long end in g_L. In particular, every successive realization of g_L makes q_{lt} less sensitive to changes in g. This is because the lenders anticipate that *official* buybacks are less likely to occur. More long-term debt is therefore required to replicate the state contingency in the contract. On the other hand, there is an income effect which increases the total indebtedness in g_L. In steady state $x'(g_L) \leq x$ as shown in Proposition 1.3. By Proposition 1.1, this implies that the value of the lenders increases as g_L realizes. Thus, more short-term and long-term debt are needed to replicate the liabilities of the contract.

In Appendix 1.5, I explore alternatives to *official* buybacks. Empirically, such alternatives do not exist or remain underdeveloped. Moreover, they raise similar enforcement issues as *official* buybacks. That is why I do not consider them in the main analysis.

1.6 Emerging Economies Debt Management

The previous section showed that the Second Welfare Theorem holds in the market economy. This section analyzes whether First Welfare Theorem holds. In particular, it restricts the analysis to Markov equilibria and shows that the First Welfare Theorem generally fails.

1.6.1 Foundation for Markov equilibrium

As highlighted in Section 1.4, the market economy features multiple equilibria. I therefore restrict my attention to Markov equilibria. In particular, I present two assumptions under which all sustainable equilibria are Markov and relate these assumptions to the characteristics of emerging economies.

Following Maskin and Tirole (2001), Markov equilibria rely on strategies conditioned the payoff-relevant state which corresponds to $\Omega \equiv (s, \mathbb{I}_D, b_{st}, b_{lt})$ in my environment. Formally,

Definition 1.3 (Markov Equilibrium)**.** *A sustainable equilibrium is Markov if for any* $(h_b^t, h_l^t) \neq (\tilde{h}_b^t, \tilde{h}_l^t)$ *ending with the same* $\Omega_t \equiv (s_t, \mathbb{I}_{D,t}, b_{st,t}, b_{lt,t})$*, strategies are the same such that* $W^b(h_b^t) = W^b(\tilde{h}_b^t) \ \wedge \ W^l(h_l^t) = W^l(\tilde{h}_l^t)$.

The definition relates to a strong-Markov equilibrium as it requires that strategies – and not only payoffs – be the same (Chari and Kehoe, 1993).[22] All Markov equilibria are sustainable equilibria as they restrict the information set to $\Omega_t \subset h^t$ for any $t > 0$.

I now present two assumptions under which all sustainable equilibria are Markov. The first one is that the government's memory is bounded. In other words, the government's memory goes back to a certain number of periods $\mathcal{T} = \frac{1-\psi}{\psi}$ with $\psi \in [0, 1]$.[23] This can be related to the government's relative impatience, $\beta(1 + r) < 1$. In the political economy literature, it is reduced form for the fact that governments are subject to re-elections and may lose office (Alesina and Tabellini, 1990).

The second assumption pertains to small utility shock perturbations. Following Bhaskar et al. (2012) and Angeletos and Lian (2021), in each period t, a utility shock $\epsilon\varrho_{b,t}$ and $\epsilon\varrho_{l,t}$ with $\epsilon \geq 0$ is drawn for the government and the lenders, respectively. It has compact support $P_i \subset \mathbb{R}^{|\mathcal{C}_i|}$ with absolutely continuous density $\varsigma_{P_i} > 0$ where $|\mathcal{C}_i|$ is the cardinality of the choice set of market participant $i \in \{b, l\}$. Moreover, it is independently distributed across market participants, histories and other shocks. If the market participant $i \in \{b, l\}$ chooses a particular action, say $a \in \mathcal{C}_i$, its utility is augmented by $\epsilon\varrho^a_{i,t}$.[24] Finally, the utility shock $\epsilon\varrho_{i,t}$ is privately observed by market participant $i \in \{b, l\}$.

Assumption 1.2 (Perturbation)**.** *The government's memory goes back to $\mathcal{T} = \frac{1-\psi}{\psi}$ periods in the past with $\psi \in [0, 1]$. In addition, in each t, a utility shock $\epsilon\varrho_{i,t}$ with $\epsilon \geq 0$ is drawn from the compact support $P_i \subset \mathbb{R}^{|\mathcal{C}_i|}$ with absolutely continuous and i.i.d. density $\varsigma_{P_i} > 0$ for each $i \in \{b, l\}$. The utility shock is additive and privately observed.*

Under Assumption 1.2, the market economy considered in Section 1.4 corresponds to $\psi = \epsilon = 0$. My equilibrium selection boils down to what happens when $(\psi, \epsilon) > 0$ but arbitrarily small. It means that (a) the government eventually forgets the history of play in the very distant past and (b) market participants have imperfect knowledge of the other participant's fundamentals.

[22] See also Definition A.2 in Appendix 1.11.

[23] I consider here that the lenders are long-run players. However this is without loss of generality. The result holds as long as at most one market participant has unbounded memory.

[24] For instance, the instantaneous utility of the government taking action a is given by $u(c_t) + \epsilon\varrho^a_{b,t}$.

These two assumptions particularly fit the case of emerging economies. On the one hand, such economies suffer from important political instability which can explain (a).[25] On the other hand, the fundamentals of these economies are difficult to assess from the perspective of foreign creditors. Particularly, governments of emerging economies often release data of poor quality or even distort some statistics which can explain (b).[26]

The presence of the privately observed shocks – albeit small and independent – coupled with the bounded memory of the government prevent both participants to rely on past history. This causes all non-Markov equilibria to unravel.

Proposition 1.5 (Foundation of Markov equilibria)**.** *Under Assumption 1.2, with* $(\psi, \epsilon) > 0$*, every sustainable equilibrium is a Markov equilibrium.*

The rationale behind that result follows Bhaskar (1998) and Bhaskar et al. (2012). Suppose the lenders condition their action at time t on a payoff-irrelevant past event, then the government must also condition on this past event given that moves are sequential. Nevertheless, as long as $\psi \neq 0$, the government eventually forgets everything that happened in an arbitrarily distant point in the past. This means that, asymptotically, the government – and consequently the lenders – cannot condition on past history. In addition, the utility shocks with $\epsilon \neq 0$ ensure that each decision node is a singleton and that the equilibrium outcome is strong instead of weak-Markov.

Both parts of Assumption 1.2 are necessary for Proposition 1.5 to hold. On the one hand, with $\psi = 0$ and $\epsilon \neq 0$, the markets participants could condition their actions on the entire history of play relying on the law of large numbers for the distribution of utility shocks. On the other hand, with $\psi \neq 0$ and $\epsilon = 0$, payoffs are independent of history but not necessarily strategies making the equilibrium weak-Markov as in Chari and Kehoe (1993).

1.6.2 Markov equilibrium

The Markov equilibrium I obtain in this environment is a version of Arellano and Ramanarayanan (2012) with strategic lenders and additional provisions on buybacks.

[25] See notably Bussière and Mulder (2000), Scholl (2017) and Andreasen et al. (2019).

[26] See notably Tsyrennikov (2013) and Morelli and Moretti (2021).

Let $\Omega_P \equiv (s, 0, b_{st,t}, b_{lt,t})$ be the payoff-relevant space of the borrower in repayment. Given this, the government's overall beginning of the period value is

$$W^b(\Omega_P) = \max_{D\in\{0,1\}} \Big\{(1-D)V^P(\Omega_P) + DV^D(s,0,k)\Big\}, \tag{1.9}$$

where V^P and V^D correspond to the value of repayment and default, respectively. Under repayment, the government chooses whether to conduct *official* buybacks. Thus

$$V^P(\Omega_P) = \max_{M\in\{0,1\}} \Big\{(1-M)V^{NB}(\Omega_P) + MV^B(\Omega_P)\Big\}, \tag{1.10}$$

where V^B and V^{NB} are the values under *official* buyback and no *official* buyback, respectively. If the government decides to officially repurchase its long-term debt,

$$\begin{aligned} V^B(\Omega_P) = &\max_{b'_{st}, b'_{lt}} u(c) + \beta \mathbb{E}_{s'|s}\Big[W^b(\Omega'_P)\Big] \\ &\text{s.t.} \quad c + q_{st}(s, b'_{st}, b'_{lt})b'_{st} + q_{lt}(s, b'_{st}, b'_{lt})b'_{lt} = y(g,k) + b_{st} + b_{lt}(1 + q^{bb}_{lt}(s, b'_{st}, b'_{lt})), \\ &\qquad b'_{lt} \geq b_{lt}. \end{aligned}$$

Conversely, under no *official* buyback,

$$\begin{aligned} V^{NB}(\Omega_P) = &\max_{b'_{st}, b'_{lt}} u(c) + \beta \mathbb{E}_{s'|s}\Big[W^b(\Omega'_P)\Big] \\ &\text{s.t.} \quad c + q_{st}(s, b'_{st}, b'_{lt})b'_{st} + q_{lt}(s, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}) = y(g,k) + b_{st} + b_{lt}. \end{aligned}$$

The value under default is given by

$$V^D(s,0,k) = u(gf(k)) + \beta \mathbb{E}_{s'|s}\Big[(1-\lambda)V^D(s',1,0) + \lambda W^b(s',0,0,0)\Big]. \tag{1.11}$$

This represents the optimal contract's outside option used in Section 1.5. After the choice of capital,[27] the value of the *legacy* lender is given by

$$W^l_{\text{legacy}}(\Omega) = \max_{b'_{st}, b'_{lt}} -\Big[b_{st} + b_{lt}(1 + q_{lt}(s, b'_{st}, b'_{lt}) + M(\Omega)\chi)\Big](1 - D(\Omega)). \tag{1.12}$$

The continuation value is equal to zero given the break-even assumption. The value of the *new* lender is then $W^l_{\text{new}}(\Omega) = 0$. The value of the two lenders together is then $W^l(\Omega) = W^l_{\text{legacy}}(\Omega) + W^l_{\text{new}}(\Omega)$. To avoid redundancy with Section 1.4, the pricing equations are presented in Appendix 1.6.

1.6.3 Equilibrium multiplicity

I now characterize the Markov equilibrium and show that there are two equilibria: one without default where *official* buybacks can occur and one with default where *official* buybacks never occur on equilibrium path.

The multiplicity of equilibria originates from the interaction between the *legacy* and the *new* lender. On the one hand, the former lender is unwilling to let the borrower dilute legacy debt as this directly reduces its payoff. Similarly, it is supportive of *official* buybacks as the payment of the premium χ increases its payoff. On the other hand, the *new* lender is indifferent to both dilution and *official* buybacks owing to the break-even assumption.[28]

Proposition 1.6 (Legacy Debt)**.** *The legacy lender is unwilling to dilute legacy long-term debt claim and is willing to have official buybacks. The new lender is always indifferent.*

Being indifferent, the *new* lender can offer debt contracts that either satisfy the borrower's problem in (1.10) or that satisfy the *legacy* lender's problem in (1.12). The indicator function, ϖ_0, takes value one in the former case and zero in the latter. This has implications on whether borrowing is risky and on whether buybacks entail a premium.

Regarding risky borrowing, define the endogenous borrowing limit as $\mathcal{B} \geq \min_{s'}\{(b'_{st}, b'_{lt})$:

[27] The value before the capital choice should include $p(1-\tau)k - k$ with $p = \infty$ if $D = 1$ implying $k = 0$.

[28] Note that the break even assumption with zero recovery value rules out any monopoly power of the *legacy* lender close to the default decision. My environment therefore does not feature the default cycles characterized by Kovrijnykh and Szentes (2007).

$V^P(\Omega'_P) = V^D(s', 0, k')\}$.[29] The borrower has no incentive to default if it does not accumulate debt beyond $\mathcal{B}$. Moreover, the *legacy* lender never offers $(b'_{st}, b'_{lt}) < \mathcal{B}$ given Proposition 1.6. Thus, if the *new* lender decides to satisfy the *legacy* lender's problem, the borrower never enters the risky borrowing region.[30] In opposition, if it decides to satisfy the borrower's problem, the borrower can enter the risky borrowing region if willing to do so. As a result, whether an equilibrium entails defaults on equilibrium path depends on the behavior of the *new* lender.

Regarding *official* buybacks, the behavior of the *new* lender is also crucial. An *official* buyback is a reverse default as it corresponds to an overpayment, whereas a default is an underpayment of liabilities. While underpayments are sanctioned by markets exclusion, there is no direct reward after overpayments. Especially, the government can avoid the payment of χ through *unofficial* buybacks. As a result,

Proposition 1.7 (Official Buyback Aversion)**.** *In any state Ω_P, the borrower cannot commit to conduct official buybacks as it always strictly prefers unofficial buybacks for the same (b'_{st}, b'_{lt}).*

This means that for two debt contracts offering the same (b'_{st}, b'_{lt}), the borrower would never choose the one in which it has to pay χ. Thus, if the *new* lender decides to satisfy the borrower's problem, *official* buybacks never occur on equilibrium path.

In opposition, if the *new* lender decides to satisfy the *legacy* lender's problem, *official* buybacks may occur on equilibrium path. The key point is that only the *legacy* lender can offer debt contracts entailing the payment χ. This means that an *official* buyback occurs only if the borrower chooses the offer of the *legacy* lender. Thus, the *new* lender has to offer an unfavorable debt contract to force the borrower to pick the offer of the *legacy* lender. In particular, it can offer a sudden stop debt contract consisting of $b'_{st} \geq 0$ and $b'_{lt} \geq b_{lt}$. This does not necessarily mean that an *official* buyback occurs every period.

Lemma 1.4 (Official Buyback Enforcement)**.** *When the new lender offers $b'_{st} \geq 0$ and $b'_{lt} \geq b_{lt}$, the borrower accepts the legacy lender's official buyback offer if it does not want to save (i.e. $b'_{st} < 0$)*

[29]This is what Zhang (1997) defines as a no-default borrowing constraint.

[30]This corresponds to the "saving equilibrium" defined by Aguiar and Amador (2020).

and either $-b_{st} > 0$ is sufficiently large or $-b_{lt}q_{lt}^{bb}$ is not too large.

The lemma states that *legacy* lender's offer is preferable when the borrower does not possess any assets and *official* buybacks are not too costly. I further develop this argument in Appendix 1.2 when I endogenize χ. A direct corollary of Lemma 1.4 is that the state space Ω_P can be separated in two zones: the enforcement and the impunity zone.

1. The enforcement zone:

   In this zone, the borrower is worse off with a sudden stop rather than paying the premium χ. Hence, the borrower picks the *legacy* lender's offer as of Lemma 1.4. In other words, the *official* buyback takes place.

2. The impunity zone:

   In this zone, the borrower is always worse off paying the premium χ. The *official* buybacks therefore does not happen as of Proposition 1.7.

In the impunity zone, $V^{NB}(\Omega_P) > V^B(\Omega_P)$ even if new bond issuance is restricted to $b'_{lt} \geq b_{lt}$ and $b'_{st} \geq 0$ when $M(\Omega_P) = 0$. Hence, the enforcement zone is the only zone in which an *official* buyback occurs.[31]

Given the above multiplicities, there are two Markov equilibria. The first one is a Markov equilibrium without default in which *official* buybacks can occur on equilibrium path. It happens when the *new* lender offers debt contracts that satisfy the *legacy* lender's problem in (1.12) (i.e. $\varpi_0 = 0$). The second one is a Markov equilibrium with default in which *official* buybacks never occur. It happens when the *new* lender offers debt contracts that satisfy borrower's problem in (1.10) (i.e. $\varpi_0 = 1$).

The Markov equilibrium with default is the one characterized by Arellano and Ramanarayanan (2012). On the one hand, defaults arise on equilibrium path and especially when productivity is low. On the other hand, maturity shortens during debt crises. The repayment of long-term debt is laddered through multiple periods which implies that the claim of the *legacy* lender can be

[31] In the enforcement zone, the *new* lender offers a sudden stop contract, while the *legacy* offers a contract satisfying (1.10) subject to $M = 1$. The borrower picks the latter offer. In the impunity zone, no *official* buybacks can be enforced. The two lenders offer the same contract satisfying (1.10) subject to $M = 0$ and $(b'_{st}, b'_{lt}) \geq \mathcal{B}$.

diluted. The long-term bond therefore admits a greater default premium than the short-term bond. As a result, close to default, the long-term debt price drastically drops which encourages shorter maturity.[32] Finally, *official* buybacks never occur. In terms of sovereign debt management, this equilibrium is the closest to what is observed in Argentina as one will see in the next section.

Furthermore, the Markov equilibrium without default predicts the opposite of what the previous Markov equilibrium does. Defaults do not arise on equilibrium path, while *official* buybacks can. Moreover whether maturity shortens in the low productivity state depends on the exact parameters of the model. The predictions of the model are therefore the closest to what is observed in Brazil as one will see in the next section.

The Markov equilibrium without default is very close to the implementation presented in Section 1.5. However, under Markov strategies, *official* buybacks can only arise in the enforcement zone. The following lemma shows that *official* buybacks that implement the constrained efficient allocation do not generally arise in this zone.

Lemma 1.5 (Non-Markov Implementation)**.** *Under Assumption 1.1, for a given implementation of the constrained efficient allocation, the point of official buyback is not necessarily located in the enforcement zone.*

Lemma 1.4 states that *official* buybacks are enforceable in a Markov equilibrium when there is no short-term assets and *official* buybacks are not too costly. However, Lemma 1.5 shows that, in the implementation of the constrained efficient allocation, at the point of *official* buyback, the borrower needs to hold short-term assets unless the buyback premium is sufficiently large. Hence, *official* buybacks are not automatically enforceable through Markov strategies. Especially in the next section, I show quantitatively that that Markov strategies fail to implement the constrained efficient allocation under empirically plausible buyback premia. Trigger strategies are therefore more often than not necessary, which puts the implementation at the mercy of Proposition 1.5.

[32] As shown by Niepelt (2014), this result is also a consequence of the fact that a default implicates the entire long-term and short-term debt. It would not arise if the government would only default on the maturing portion of the long-term debt. See also Perez (2017).

1.7 Quantitative Analysis

This section starts with a comparison of Argentina and Brazil since 1995. I then calibrate the Markov equilibrium with default to Argentina and assess the fit of the model to the data. I show that the Markov equilibrium without default is quantitatively close to Brazil and contrast the two Markov equilibria with the constrained efficient allocation.

1.7.1 Argentina vs. Brazil

I compare the experience of Argentina and Brazil starting in 1995 as Brazil defaulted last in the 1980s and regained access to the international market after the implementation of the Brady Plan in 1994.[33] Additional results can be found in Appendix 1.8.

Figure 1.2 presents the main statistics of interests for both countries. The blue (green) line represents Argentina (Brazil). As one can see, the two countries recorded a sudden drop in output at the end of the 1990s. Brazil experienced a major currency crisis following a speculative attack on the real, while Argentina suffered from a banking crisis. Consumption and investment drastically reduced, while indebtedness and the spread largely increased. In addition, the average maturity shortened for both countries in the years preceding the crisis.[34] Most importantly, Argentina eventually defaulted in 2001 (represented by the grey area), whereas Brazil did not. This triggered markets exclusion for Argentina which could re-access the market in 2006 (Cruces and Trebesch, 2013).

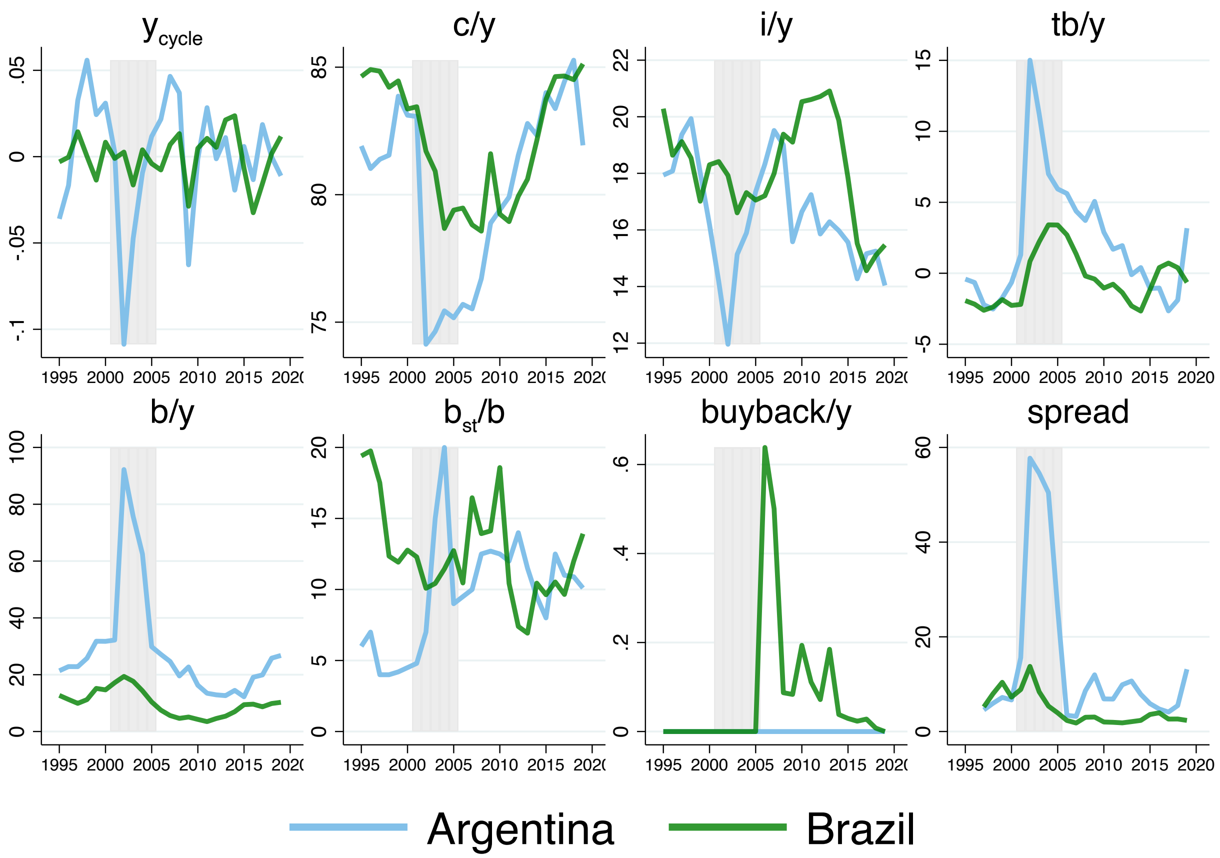
Moreover, Brazil conducted *official* buybacks while Argentina did not under the period considered.[35] Figure 1.3 depicts the *official* buybacks conducted by Brazil. In 2006, the country started the Early Redemption Program which aimed at correcting the average maturity of the debt and reducing the potential refinancing risk.[36] Repurchases were conducted by the Brazilian National

[33] The Brady Plan is an extensive debt restructuring program aimed at resolving the numerous of sovereign debt defaults in 1980s especially in Latin America. In general, see Buera and Nicolini (2021) for for the economic history of Argentina and Ayres et al. (2021) for Brazil.

[34] See also the maturity regression analysis in Appendix 1.8.

[35] Note that Argentina repurchased external debt at a discount (i.e. below par) on the secondary market in various occasions. However, this usually corresponds to a default in the form of distressed debt exchange.

[36] See https://www.gov.br/tesouronacional/en/federal-public-debt/external-market/buyback-program.



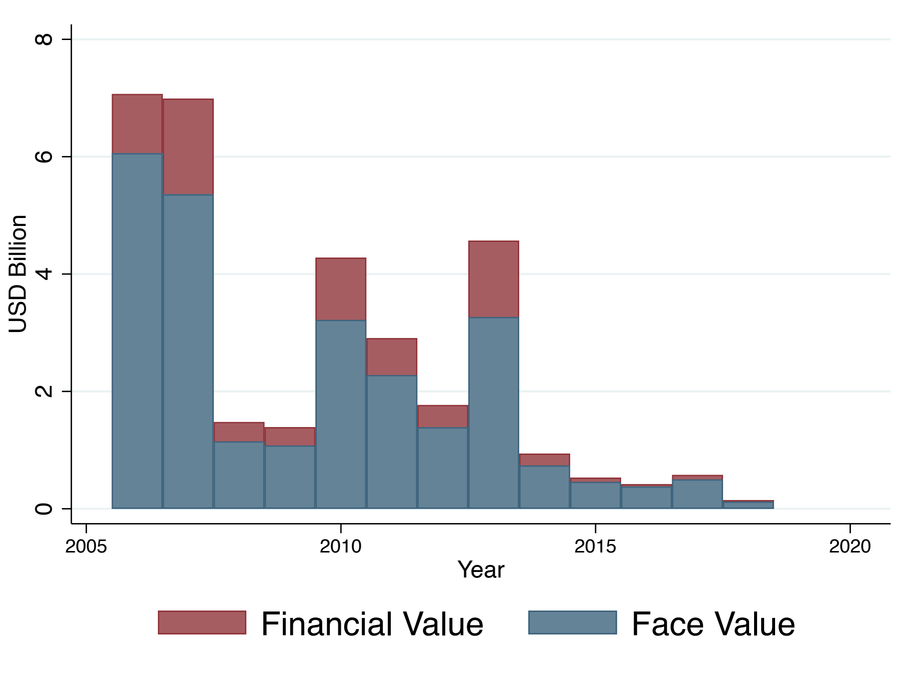
Note: y_{cycle} corresponds to output detrended using the Hodrick–Prescott filter with a smoothing parameter of 6.25, spread is the EMBI spread, i is the investment, tb/y is the trade balance over output and b is the public sector external debt stock with $b = b_{st} + b_{lt}$. Series for Argentina are depicted in blue and in green for Brazil. The grey area represents the periods in which Argentina is in default.

Source: Author's calculation, Buera and Nicolini (2021), Tesouro Nacional, Global Financial Data and World Bank.

Figure 1.2: Argentina vs. Brazil

Treasury either directly on the secondary market or indirectly through call options and special auctions. I identify two features in the Brazilian *official* buybacks. First, such buybacks are costly. The financial value (i.e. the red bar) is systematically above the face value (i.e. the blue bar) meaning that the Brazilian government always paid a premium to extract its debt out of the market.[37] This premium is primarily explained by the fact that the repurchased Brazilian bonds entailed high coupon rates relative to the market interest rate. On average, the financial value is 24.5% above the face value. This figure provides the basis of calibration of $\chi > 0$ in the next subsection. Second, those buybacks were the largest when the output of the Brazilian economy was on or above trend

[37]The Financial value corresponds to the amount required for payment of the securities redeemed, while the face value in blue corresponds to the value of debt in the national statistics.



Note: The figure depicts the amount of external debt bought back by the Brazilian government in USD billion. The Financial value in red corresponds to the amount required for payment of the securities redeemed. The face value in blue corresponds to the value of debt in the national statistics.

Source: Author's calculation and Tesouro Nacional.

Figure 1.3: Official Buyback in Brazil

consistent with the predictions in Section 1.5. I provide more details on that in Appendix 1.8.

As a result, the experience of Argentina and Brazil qualitatively relate to the predictions of the Markov equilibrium with and without default, respectively. In what follows, I gauge the quantitative fit.

1.7.2 Calibration

I calibrate the Markov equilibrium with default as it corresponds to the workhorse model in the literature on sovereign defaults. The calibration aims at matching some specific moments of the Argentine economy over the period 1995-2019. Table 1.1 summarizes each parameter.

The instantaneous utility function takes the CRRA form with a coefficient of relative risk aversion of ϑ, i.e. $u(c) = \frac{c^{1-\vartheta}}{1-\vartheta}$. I adopt $\vartheta = 2$ as it is standard in the real business cycle literature and set the discount factor to $\beta = 0.8$ to match the average public sector external debt-to-GDP ratio of 28.71%. This corresponds to a quarterly discounting of 0.945 which is standard in studies on

Table 1.1: Calibration

Parameter	Value	Description	Targeted Moment
A. Based on Literature			
ϑ	2.00	Relative risk aversion	
r^f	0.01	Risk-free rate	
B. Direct Measure from the Data			
$\pi(g_H \mid g_H)$	0.93	Probability staying high state	
$\pi(g_L \mid g_L)$	0.68	Probability staying low state	Real total factor productivity
g_L	0.44	Productivity in low state	
$1-\alpha$	0.70	Labor share	Labor income share
χ	4.59	*Official* buyback premium	Financial over face value of debt
r^e	0.04	Excess return	US excess return on debt
C. Based on Model solution			
β	0.80	Discount factor	Debt-to-GDP ratio
g_H	1.12	Productivity in high state	Correlation consumption and output
ϕ	1.50	CES production	Investment-to-GDP ratio
λ	0.281	Probability re-accessing market	Average spread

emerging economies. In addition, the production function has the CES form

$$F(k,l) = \left[\alpha k^{\frac{\phi-1}{\phi}} + (1-\alpha) l^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}},$$

where α represents the capital share and ϕ the CES parameter. The value of $1-\alpha$ is set to the standard labor share in GDP adopted in the literature on emerging economies (Mendoza and Yue, 2012). The CES parameter is $\phi = 1.5$ to match the share of investment in GDP and is within the range of admissible values in the business cycle literature. In addition, I estimate the Markov transition matrix by means of a Markov-switching AR(1) process with two states. For this, I use data on the real total factor productivity of Argentina from 1990 to 2019 from the Penn World Table 10.0 (Feenstra et al., 2015). I then select g_H to match the correlation between consumption and output and accordingly set g_L to obtain the average real TFP of Argentina given the estimated transition matrix.

Regarding the exogenous rate r, I relax the assumption of zero-profit lending and set $r = r^f + r^e$ where r^f represents the risk-free rate and r^e corresponds to the lenders' excess return. This means that the lenders borrow at r^f and lend at $r > r^f$. This has two purposes. First, it better captures the potential risk premium US investors demand on emerging market bonds. I therefore set $r^e =$

0.0434 consistent with the US excess return on debt instruments estimated by Gourinchas et al. (2017) and $r^f = 0.01$ as it is standard in the literature. Second, as the spread is calculated with respect to the risk-free rate, modelling an excess return enables to correct the negative spread which has little empirical support for the countries under study.

I choose $\lambda = 0.281$ to match the average (EMBI) spread of 14.17%. The value selected implies an expected default length of roughly 3.5 years. This is below the value of 5.1 years Cruces and Trebesch (2013) find in the data. Finally, for the *official* buyback premium, I set $\chi = 4.59$ to match the wedge between the financial and the face value recorded on the Brazilian Early Redemption program highlighted in the previous subsection.

In light of Section 1.6, the difference between Argentina and Brazil lies in the lenders' strategic interaction. I therefore consider that the two countries are identical in terms of economic fundamentals and use the same calibration to solve the two Markov equilibria and the constrained efficient allocation. The only difference lies in the specification the *new* lender's offer (i.e. whether ϖ_0 is equal to one or zero).

1.7.3 Numerical results

This subsection presents the result of the calibration. It gauges the fit of the model with respect to the data for both targeted and non targeted moments. It also compares the outcome of the Markov allocation with default (MA), without default (MAND) and the constrained efficient allocation (CEA) together.

The upper part of Table 1.2 presents the fit of the MA with respect to the Argentine economy in terms of targeted moments. It also reports the result of the CEA and the MAND. As one can see, the MA replicates relatively well the main features of the Argentine economy in terms of consumption, investment, spreads and indebtedness.

The lower part of Table 1.2 presents the fit of the MA in terms of non-targeted business-cycle moments. In general, the fit is poor. This is because I only consider 2 productivity states meaning that I rule out tail events. The MA generates too low volatilities for most variables. Moreover, the

Table 1.2: Targeted and Non-Targeted Moments

Variable	Argentina	MA	Brazil	MAND	CEA
A. Targeted Moments					
Variable	Argentina	MA	Brazil	MAND	CEA
i/y	14.26	14.22	17.98	16.22	14.61
$-b/y$	28.71	28.15	10.12	7.18	-353.20
Spread	14.17	12.88	4.97	3.85	3.95
corr(c, y)	0.96	0.94	0.88	0.95	0.68
B. Non-Targeted Moments					
Variable	Argentina	MA	Brazil	MAND	CEA
c/y	78.90	85.80	81.43	82.90	82.52
$\sigma(c)/\sigma(y)$	1.04	0.84	0.83	0.71	0.21
$\sigma(i)/\sigma(y)$	3.31	0.08	3.21	0.08	1.25
$\sigma(tb/y)/\sigma(y)$	1.22	0.40	1.38	0.40	0.96
$\sigma(spread)/\sigma(y)$	4.53	0.77	2.47	0.00	0.01
corr(i, y)	0.97	0.91	0.93	1.00	0.99
corr$(tb/y, y)$	-0.60	0.53	-0.34	0.79	0.97
corr$(spread, y)$	-0.63	-0.24	-0.20	-0.71	-0.67

Note: The variable $\sigma(\cdot)$ denotes the volatility, tb/y denotes the trade balance over output and i the investment which corresponds to k in the model given the depreciation of 1. For the volatilities and correlation statistics, I filter the simulated data – except the spread – through the HP filter with a smoothness parameter of 6.25. MA refers to the Markov equilibrium with default, MAND to the Markov equilibrium without default and CEA to the constrained efficient allocation.

trade balance is pro-cyclical unlike the data. The model however produces empirically plausible correlations for the spread and investment relative to output.

Having said that, the MA generates a realistic debt dynamic. Table C.7 depicts the underlying debt structure of the Markov equilibria and the CEA. Two points deserve to be noted. First, the MA replicates well the data as maturity shortens during debt crises, while indebtedness relative to GDP increases. Second, during a restructuring, the maturity lengthens and the level of debt remains substantial.[38]

Turning to the MAND, Table 1.2 presents the similarities with Brazil. As discussed at the beginning of the section, Brazil has not defaulted since the end of the 1980s, whereas Argentina defaulted 3 times since 1995 with the most recent episode being in 2023. Second, Brazil conducted an official buyback program from 2006 to 2018. Third, maturity shortens in the low productivity state. Fourth, in terms of economic fundamentals, Brazil records a lower average debt ratio, a

[38] See Benjamin and Wright (2013), Mihalache (2020) and Dvorkin et al. (2021) for related results.

Table 1.3: Debt Structure

	Mean $-b/y$ (percent)	Mean $-b/y$ in g_H (percent)	Mean $-b/y$ in default (percent)	Mean $-b/y$ in restructuring (percent)
Argentina	28.7	22.0	65.7	29.9
MA	28.2	24.1	216.4	17.5
Brazil	10.1	9.0	-	-
MAND	7.2	2.3	-	-
CEA	-353.2	-353.3	-	-
	Mean b_{st}/b (percent)	Mean b_{st}/b in g_H (percent)	Mean b_{st}/b in default (percent)	Mean b_{st}/b in restructuring (percent)
Argentina	9.7	8.3	11.7	9.0
MA	44.0	43.6	84.1	64.5
Brazil	12.6	12.4	-	-
MAND	21.7	11.3	-	-
CEA	112.5	112.0	-	-

Note: In the CEA, $b_{st}/b > 1$ as $b_{st} > 0$ in some states while $b_{lt} \leq 0$. MA refers to the Markov equilibrium with default, MAND to the Markov equilibrium without default and CEA to the constrained efficient allocation.

greater average investment ratio and a lower average spread than Argentina for the period 1995 to 2019. The MAND is capable of matching most of the main moments of the Brazilian economy despite the fact that none of them where directly targeted.[39] This suggests that Brazil can be interpreted as the counterfactual of Argentina with buybacks and without default in the period 1995-2019.

Looking at the CEA in the last column of Table 1.2, one directly observes that it predicts an empirically implausible average indebtedness. In fact, the borrower holds a net asset position. Such prediction is well known in the literature on fiscal policy under commitment as highlighted by notably Buera and Nicolini (2004) and Faraglia et al. (2010). Even though I consider an alternative environment without commitment, the bond portfolio implementing the CEA remains at odds with the data.

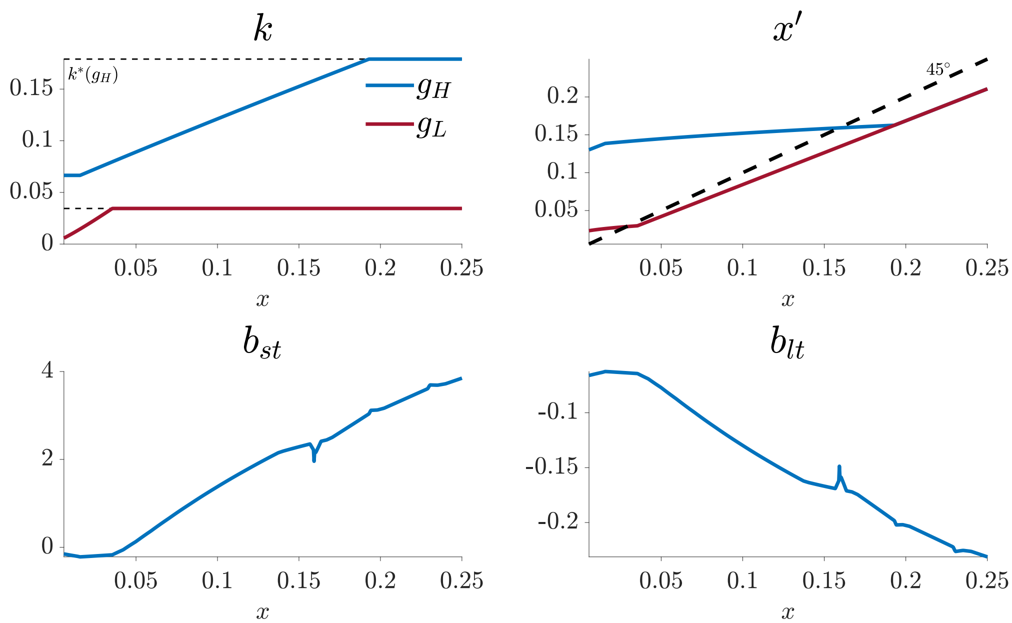
The MAND and the CEA achieve better risk sharing than the MA. In the CEA, consumption corresponds to a lower share of output, correlates less with output and is less volatile. Investment corresponds to a larger share of output, correlates more with output and is more volatile. Finally, the bond spread is lower than in the MA given that defaults do not arise on equilibrium path and

[39]Note that the excess return corrects the negative spread in the model.

official buybacks exceed the risk-free price. The same holds true for the MAND with the exception of a slightly larger consumption correlation than in the MA.

1.7.4 Implementation and buyback cost

In this subsection, I discuss the implementation of the CEA in the market economy. Under empirically plausible *official* buyback premia, the point of *official* buyback is located outside the enforcement zone.

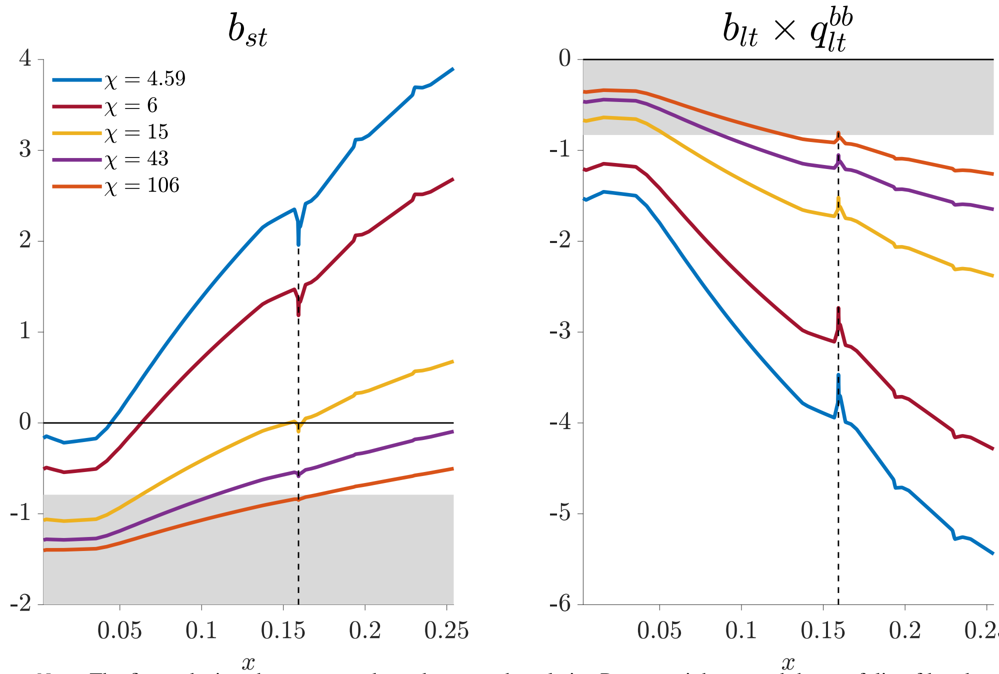


Note: The figure depicts the policy functions for capital, k, prospective relative Pareto weight, x', short-term debt, b_{st}, and long-term debt, b_{lt}.

Figure 1.4: Main Policy Functions of the CEA

Figure 1.4 depicts the main policy functions related to the optimal contract. The law of motion of the relative Pareto weight is consistent with the fact that the borrower is impatient (i.e. $\eta < 1$). Similarly, capital remains distorted in steady state in line with Propositions 1.2 and 1.3. Regarding borrowing, when x is low, the government accumulates more short-term debt and less long-term debt. In opposition, when x gets larger, the opposite is true. Furthermore, the borrower holds short-term assets – especially when *official* buybacks occur. This means that the point of *official* buyback is in the impunity zone which explains the reliance on trigger strategies.

To obtain short-term debt holdings when *official* buybacks occur, the premium χ should be larger



Note: The figure depicts the correspondence between the relative Pareto weight, x, and the portfolio of bonds, (b_{st}, b_{lt}), for different *official* buyback premia, χ. The grey area depicts the enforcement zone in the Markov equilibrium.

Figure 1.5: Implementation and χ

than the calibrated one. The rationale behind this is that with a larger χ, the long-term bond price is more sensitive to the realization of g. As a result, more short-term debt and less long-term debt are required to replicate the state-contingent liabilities of the optimal contract. Figure 1.5 depicts the portfolio of bonds necessary to implement the CEA for different values of χ. The black dashed line represents the relative Pareto weight at which the *official* buyback occurs – i.e. $x = x^{ub}$. As one can see, it is possible that the borrower holds short-term debt – and not asset – by more than tripling χ relative to the calibration benchmark. This means that Markov strategies fail to implement the Planner's allocation under empirically plausible *official* buyback premia in emerging economies.

Thus, with respect to the literature on fiscal policy under commitment and the findings of Buera and Nicolini (2004) and Faraglia et al. (2010), I reconcile the model's prediction with the data by arguing that the borrower lacks the strategical sophistication to implement the CEA. Under an empirically plausible cost of *official* buyback, the borrower can only approximate – as opposed to replicate – the returns of Arrow securities with non-contingent bonds of multiple maturities. Nevertheless, such approximation is consistent with the sovereign debt management of emerging economies as shown previously.

1.7.5 Equilibria comparison

I explore in more details the differences between the Markov equilibria and the CEA. For this purpose, I conduct two main exercises. First, I compute welfare gains with respect to the MA. Second, I measure the distance of each equilibria from the Pareto frontier. Additional results can be found in Appendix 1.9.

Table 1.4: Steady State Welfare Analysis

State	Borrower welfare gains (percent)		Lenders welfare gains (percent)		$\mathscr{F}(g)$ (percent)		
	MAND	CEA	MAND	CEA	MA	MAND	CEA
g_H	0.01	0.07	0.5	1.7	23.6	26.3	100.0
g_L	0.84	0.85	1.9	3.4	18.7	21.2	100.0
average	0.17	0.22	0.8	2.0	22.6	25.3	100.0

Note: The table presents the welfare gains in consumption equivalent relative to the MA. See Appendix 1.10 for details on the computation. MA refers to the Markov equilibrium with default, MAND to the Markov equilibrium without default and CEA to the constrained efficient allocation.

Table 1.4 depicts the welfare gains of the CEA and the MAND in consumption equivalent terms with respect to the MA for both the borrower and the lenders. Welfare gains are computed through the simulation of 5,000 independent shock histories starting with initial debt holdings and relative Pareto weights drawn from the ergodic set. The details of the welfare computation are presented in Appendix 1.10.

As one can see, the CEA and the MAND imply substantial welfare gains compared to the MA, on average for both the lenders and the borrower. The CEA leads to the largest welfare gains in all states for all market participants. Those are more pronounced when g_L realizes. Hatchondo et al. (2020a) find similar results when comparing the MA with a Ramsey plan. Note that in the MAND, the borrower's welfare gains become negligible compared to the MA when g_H realizes. This is due to the fact that *official* buybacks occur whenever the borrower enters the enforcement zone, unlike the implementation of the CEA in which such buybacks occur conditional on a certain portfolio holding in addition to the realization of g_H. It therefore seems that the borrower in the MAND conducts *official* buybacks too frequently.

Table 1.5: Borrower Welfare Decomposition

State	MAND		CEA	
	State contingency (percent)	Cost of default (percent)	State contingency (percent)	Cost of default (percent)
g_H	1.90	98.10	15.67	84.33
g_L	99.13	0.87	98.70	1.30
average	20.38	79.62	31.45	68.55

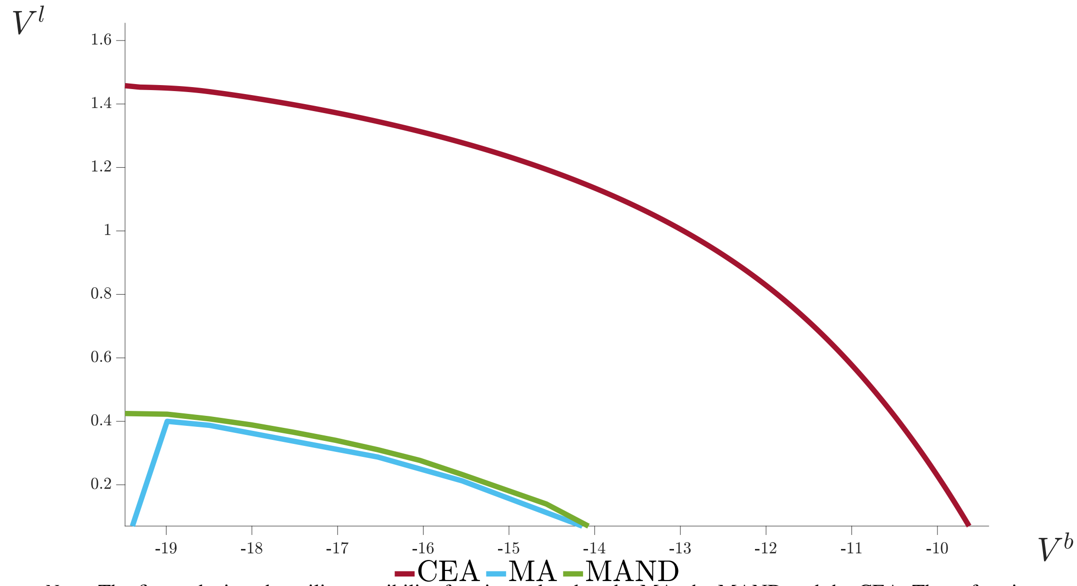
Note: The table presents the decomposition of the borrower's welfare gains in two factors: cost of default and state-contingency. The gains related to state contingency come from the computation of the MAND without *official* buybacks. The residual welfare gains is attributed to the cost of default. MA refers to the Markov equilibrium with default, MAND to the Markov equilibrium without default and CEA to the constrained efficient allocation.

In Table 1.5, I decompose the borrower's welfare gains of the MAND and the CEA by calculating the percentage of gains that can be attributed to the following two factors: cost of default and state-contingency. I isolate those two factors in the following way. To compute the gains related to state contingency, I compute the MAND without *official* buybacks. The residual welfare gains can then be attributed to the cost of default. Doing so I find that, in the MAND, 20% of the welfare gains come from the state contingency on average and the remaining part can be attributed to the cost of default. In the CEA, we find that 31% of the gains come from state contingency and the rest comes from the cost of default. In both cases, the share of gains related to state contingency is the highest in g_L.

Besides the welfare gains, I can compute the distance with respect to the Pareto frontier. For this purpose, I derive a metric measuring the distance between the constrained efficient allocation and any alternative allocation. From Proposition 1.4, I obtain a direct correspondence between x and (b_{st}, b_{lt}). I can therefore express the value of the lenders in any Markov equilibrium as a function of x instead of (b_{st}, b_{lt}), i.e. $\ddot{V}^l : G \times X \to \mathbb{R}$. I then define

$$\mathscr{F}(g) = \frac{\int_{\underline{x}}^{\overline{x}} \ddot{V}^l(g,x)dx}{\int_{\underline{x}}^{\overline{x}} V^l(g,x)dx}.$$

The metric $\mathscr{F}(g)$ measures the distance between the Markov allocation and the CEA. Given Proposition 1.1, it is bounded between 0 and 1. A value of $\mathscr{F}$ near 1 indicates that an allocation is close



Note: The figure depicts the utility possibility frontiers related to the MA, the MAND and the CEA. Those frontiers express the value of the lenders V^l as a function of the value of the borrower V^b. The CEA is in red, the MA in blue and the MAND in green. MA refers to the Markov equilibrium with default, MAND to the Markov equilibrium without default and CEA to the constrained efficient allocation.

Figure 1.6: Distance to Pareto Frontier in g_L

to the constrained efficient benchmark, whereas a value close to 0 indicates the opposite.[40]

Figure 1.6 depicts the different frontiers: in red the Pareto frontier and in blue and green the utility possibility frontier related to the MA and the MAND, respectively. Defaults in the MA produce an upward sloping part of the frontier in which both the borrower and the lenders can be made better off. Neither the CEA nor the MAND display such upward slope. This shows the inefficiency of default (see Fudenberg et al. 1990).

Looking at the metric $\mathscr{F}(g)$ in the last column of Table 1.4, the MAND is superior to the MA but not to the CEA. More precisely, the MA is relatively far from the CEA and the MAND can get the economy closer to it. The MAND therefore provides a better approximation of the CEA than the MA. Nevertheless, the MAND remains far from the CEA meaning that the Pareto improvement is small relative to what can be achieved with trigger startegies. The metric $\mathscr{F}(g)$ is important as it relates to the entire value of the debt contract (i.e. the combined value for the borrower and the lenders) and not only on the steady state unlike the welfare gains computed above.

[40]The metric $\mathscr{F}(g)$ is based on the same concept as the Gini coefficient which measures the distance between the Lorenz curve and the equity line.

1.8 Conclusion

This paper derives the constrained efficient allocation emanating from an optimal contract to deduce optimal sovereign debt management policies. The bottom line is that the reliance on defaults on equilibrium path is inefficient. Instead, changes in maturity and costly debt buybacks can implement the constrained efficient allocation. Nevertheless, the implementation often requires highly sophisticated agents capable of building on past history. Less sophisticated agents – in the spirit of emerging economies – would in fact rely on Markov strategies. Given this, I derive history-invariant debt management policies inspired by the optimal contract and assess their efficiency. I show that there are multiple Markov equilibria depending on the behavior of the lender which does not hold legacy claims. These equilibria can be Pareto ranked and can rationalize the experience of Argentina and Brazil since 1995.

This paper stresses two points. The first one is that incomplete markets might not be the reason why a market economy fails to attain constrained efficiency. Rather it can be linked to the incapacity of market participants to build on past history. I show that this restriction in the strategies followed by the market participants makes sense in the context of emerging economies. In that logic, Markov equilibria as (time-invariant) approximation of the constrained efficient allocation are not only the empirically-relevant but also the policy-relevant equilibrium concept for such economies.

The second point this paper highlights is that the strategic interaction of the lenders is key. The literature on sovereign debt and default has focused on the borrower's side. However, it is possible to explain a variety of alternative dynamics in equilibrium by looking at the lenders and the way they interact.

Chapter 2

Making Sovereign Debt Safe with a Financial Stability Fund

Joint with Yan Liu and Ramon Marimon

Abstract We develop an optimal design of a Financial Stability Fund that coexists with the international debt market. The sovereign can borrow defaultable bonds on the private international market, while having with the Fund a long-term contingent contract subject to limited enforcement constraints. The Fund contract does not have *ex ante* conditionality, but requires an accurate country-specific risk-assessment (DSA), accounting for the Fund contract. The Fund periodically announces the level of liabilities the country can sustain to achieve the constrained efficient allocation. The Fund is only required *minimal absorption* of the sovereign debt, but it must provide insurance (Arrow-securities) to the country. Furthermore, with the Fund *all sovereign debt is safe independently of the seniority structure*; however, for the Fund, seniority may require a greater *minimal absorption* than a *pari passu* regime. We calibrate our model to the Italian economy and show it would have had a more efficient path of debt accumulation with the Fund.

2.1 Introduction

In the last few years, the public debt-to-GDP ratio has reached historic levels in the European Union (EU).[1] This is the result of three consecutive crises — the global financial crisis of 2007–2009, the European sovereign debt crisis of 2010–2012 and the COVID-19 crisis. In response to these crises, important institutional and policy changes took place, making the Euro area and the EU more resilient but, for the time being, more indebted.[2] As a result of these changes, at the end of 2021, Euro area institutions were playing a leading role in their sovereign debt market, holding more than 30% of the sovereign debt of all Euro area countries.[3] Nevertheless, the question of how to efficiently stabilise the sovereign debt — for example, with complementary official lending programmes — remains open.

To address this question, we design a Financial Stability Fund (Fund) as a *constrained efficient mechanism*, in line with Ábrahám et al. (2022).[4] While the latter assumes that the Fund absorbs all the sovereign debt of a country and focuses on the borrower's perspective, we emphasize the lender's side of the contract and derive the optimal relationship between the private competitive lenders and the Fund. More precisely, we assume that sovereign countries can raise debt in the private international market and in the Fund.[5] While private international lenders solely offer credit (i.e. long-term non-contingent defaultable bonds), the Fund provides both credit and insurance (i.e. Arrow securities) in the form of long-term state-contingent securities. The Fund's intervention is constrained to prevent default and, therefore, it also takes into account the country's indebtedness

[1] According to AMECO, General Government Gross Debt in 2022: Euro area 94%, Italy 145%, Portugal 115%, Spain 114% and Greece 171%.

[2] In particular, starting the European Banking Union, founding the European Stability Mechanism, implementing asset purchasing programmes by the ECB, some including purchases of Euro area sovereign debt, and the COVID-19 Next Generation EU (NGEU) programme of the EU making, *de facto*, the European Commission the world's largest official lender, with unprecedented emissions of EU debt.

[3] Particularly, the sovereign debt holdings by Euro area institutions represents for Cyprus, Italy, Portugal and Spain more than 40% of their GDP and for Greece more than 120%.

[4] The main difference with respect to Ábrahám et al. (2022) is threefold. First, we do not consider an exclusive contract between the Fund and the contracting countries. Second, we use growth shocks to better analysze the interest rate-growth differential (i.e. $r - g$). Third, we abstract from moral hazard as we focus on the lending side of the contract.

[5] The adjective 'private' is used to distinguish lenders on the international market relative to the Fund.

(i.e. commitments) with private lenders, which brings the issue of whether the Fund possesses seniority. In line with the official lending practice, we consider two regimes: *pari passu* (i.e. no seniority) and *seniority* of Fund's liabilities over private liabilities. The Fund is also constrained to satisfy a strict *debt sustainability analysis* (DSA), which requires that the expected present value of the sovereign's future surpluses (net savings) can *always* cover the country's debt liabilities with the Fund and the private lenders. This constraint has three simultaneous consequences: *i*) it prevents permanent transfers from the Fund to the country which, in the context of a union (as share holders of the Fund), means that there are 'no undesired transfers' across countries, nor debt mutualization, *ii*) it prevents excessive lending when debt is safe (i.e. it is a *non-excessive-lending* contraint), and *iii*) it provides more recursive structure to the Fund contract since a no point in time the Fund would have a negative overhang if it had to restart.[6]

The Fund does not impose *ex ante* conditions, provided there can be feasible contracts with the country, given its existing sovereign debt.[7] This requires upfront a detailed risk-assessment of the country and a calibration of the economy which allow the Fund to compute the optimal borrowing policy the sovereign should adopt. This policy defines the *total* debt holdings and insurance necessary to reach the *constrained efficient allocation*. Then, in any given period, the Fund plays a dual role with respect to the country with a long-term contract: first, it announces the total liabilities that the country can sustain for next period, provided they maintain the contract with the Fund; second, after the country has contracted some, or all, of its debt liabilities with private lenders, the Fund implements its contract, for the period, with its insurance and, if needed, its additional lending. Our characterization of the Fund is a Nash *Recursive Competitive Equilibrium* (RCE). The Fund does not play the role of a Ramsey planner, since it lacks the authority to fully control the market transactions between the private lenders and the sovereign borrower. In particular, it takes the decisions of the private lenders as given and *vice versa*.

[6]We keep this *strict* feature through our analysis, however, it can easily be extended to allow for 'desired transfers' in particular states — say, a pandemic — not properly accounted in the risk-sharing component of the Fund contract.

[7]There may be very high levels of debt that may require restructuring to make the Fund contract feasible or the country may prefer to implement some *ex ante* reforms to improve its risk-profile; that is, the Fund can, and should, have a *menu of Fund contracts* depending on different risk profiles.

The characterization of this RCE implementation is remarkable. First, the Fund stabilizes the entire indebtedness of the sovereign. In other words, *the entire sovereign debt becomes safe*, without default risk. Second, as we assume that there is sufficient private demand for safe assets, there is only need for a *minimal intervention policy* (MIP) of the Fund in the sovereign debt market. Such intervention consists of an insurance component with an additional guarantee on long-term debt holdings by private lenders when the DSA binds. Third, all sovereign debt is safe independently of the seniority structure. However, seniority of the Fund may require a greater debt absorption by the Fund than a *pari passu* regime. Fourth, the Fund — as capacity announcer and provider of insurance and, when needed, debt — implements a unique *constrained efficient* allocation which features no default, therefore, no debt-dilution, and no excess lending. In sum, the literature on sovereign debt has mostly focused on the borrower's default decision, we contribute by characterising the lenders' optimal policy and its impact on the sovereign debt market.

The first three elements contain novel aspects that deserve explanation.[8] First, *the entire sovereign debt* is split between the competitive private lenders and the Fund. The Fund contract makes the privately held debt safe, while the Fund's debt holdings become a safe asset in its balance sheet allowing the Fund to issue safe debt (say, eurobonds) to finance its absorption, therefore *the entire sovereign debt becomes safe assets*.

Second, the depreciation of the value of the debt can take different forms: when debt is nominal, with inflation; when debt is real and defaultable, with default and dilution, and when the debt is real and perceived safe, with excessive lending. As we have already mentioned, the DSA constraint is, in fact, a *non-excessive-lending* constraint, when it is binding results in a negative spread, a price signal that lenders should not purchase new debt, but also that, if they can, they should sell their holdings of long-term debt in exchange for riskless assets with a better return. Expectations of these *sudden stop* turbulences can harm the value of long-term bonds. That is, the Fund's MIP can be seen as a prudential policy: if the DSA binds, the Fund is willing to absorb "whatever it takes" of the existing stock of long-term debt, while keeping its commitment to provide insurance,

[8]We postpone the explanation of the 'fourth element' to the discussion of the literature.

in order to repel the turmoil.

Third, seniority is usually rationalized based on the fact that official lenders are, ultimately, backed by public resources and therefore should have priority in default proceedings. As a result, seniority introduces a partial default risk (default to private lenders but not the Fund), which increases with the fraction of privately held sovereign debt. The Fund contract is designed to make debt safe independently of the seniority structure. Thus, from the perspective of the borrowing country and the private lenders, as long as debt is safe, the seniority structure is irrelevant. However, to avoid partial default, the Fund must be able to commit to absorb enough private debt as to make the sovereign debt country indifferent between partial default and the repayment of private lenders. This commitment — the MIP with seniority — may be substantially larger than the MIP with *pari passu*; therefore, seniority may be a burden for the Fund.

As we said, our analysis enables a comparison with existing lending institutions such as the European Stability Mechanism (ESM) and the International Monetary Fund (IMF). We show that the Fund without seniority might need to absorb less debt in our environment, while the ESM and the IMF usually require seniority in their lending programs.[9] Moreover, while it is true that official lending institutions conduct DSAs as a necessary condition to guarantee credits, it is not the case that their resulting debt contracts provide insurance against future DSAs, as the Fund does. In other words, international lending institutions base their lending policy on one of several scenarios — e.g. the 'most likely,' the 'politically preferred,' or the 'worst case' scenario. In contrast, the Fund contract risk-shares among these different scenarios or paths. That is, it provides additional transfers in the worst scenario in exchange for higher payments in the best scenario.[10]

[9]The IMF together with the World Bank have a *de facto* seniority, but it is not a formal contractual feature (see Schlegl et al., 2019). In opposition, the ESM has a *de jure* seniority with respect to the market. The only exception to this is Spain. The Spanish program was initially agreed with the EFSF with a standard *pari passu* clause and managed to extend this feature into the ESM loan.

[10]Recently, the IMF DSA analysis takes the form of a *Stochastic Debts Sustainability Analysis*, (SDSA), where risk paths are 'statistically calibrated.' There are two differences with our analysis. First, we calibrate the parameters of a stochastic dynamic model to the macro-history of the country, in order to generate an exogenous stochastic structure, which provides a risk assessment without the Fund's contract. Second, we compute the constrained efficient contract design, given our calibration. Furthermore, as it is also done with standard DSA or SDSA, we obtain our 'counterfactual' DSA accounting with the Fund contract.

We conduct a quantitative analysis in which we calibrate the outside option of the Fund — an incomplete market economy with defaults — to Italy for the period 1992Q1–2019Q4. Unlike Greece, Portugal and Spain, Italy did not participate to any official lending support during the European sovereign debt crisis. It therefore offers the possibility to conduct counterfactual analyses.

The main results of our quantitative inquiry are twofold. First, with the Fund, the Italian debt would have been free of default risk. This is due to the Fund state-contingent credit line being designed to support a countercyclical fiscal policy with respect to exogenous shocks, but also contingent to the states that endogenous enforcement constraints become binding: reassessing the value of primary surpluses to avoid default, and risk-sharing across states when the DSA would be binding in some state. Importantly, we show that the sovereign benefits from a greater debt absorption capacity compared to the standard incomplete market economy with defaults. Particularly, receiving state-contingent transfers from the Fund, the sovereign can accumulate debt in states in which defaults would usually happen. Quantitatively, we find in the steady-growth economy with the Fund substantial welfare gains.

Second, we argue that by accessing the Fund, Italy would have had a more stable evolution of its indebtedness. Using the decomposition of Cochrane (2020, 2022), we show that, in the last two decades, Italy largely increased its public indebtedness despite large primary surpluses. This is due to a strongly positive interest rate-growth differential $(r - g)$ dominating the debt accumulation process. The positive differential is a combination of a relatively low, and unstable, growth of the Italian economy with an important risk premium on the Italian sovereign debt. We show that, by accessing the Fund, the Italian government would have reduced these perverse effects and therefore would have ended up with a lower indebtedness. The model predicts that the Italian indebtedness by the end of 2019 would have been around 80% of GDP rather than 135% if Italy could have joined the Fund in 2000.

Our work is related to the sovereign debt literature pioneered by Eaton and Gersovitz (1981) and subsequently extended by Aguiar and Gopinath (2006) and Arellano (2008).[11] As in Ábrahám et al.

[11] See also Aguiar and Amador (2014), Aguiar et al. (2016) and Aguiar and Amador (2021).

(2022), our benchmark economy with defaultable debt builds on Chatterjee and Eyigungor (2012) who introduce long-term bonds. Within this literature, our work is closely related to Hatchondo et al. (2017), who consider the case of adding a non-defaultable bond into the otherwise standard defaultable bond economy, and show that there are welfare gains by swapping defaultable bonds into non-defaultable bonds. Our work also relates more closely to Roch and Uhlig (2018) who model a bailout agency with a *minimal intervention policy* but focus on self-fulfilling debt crises.

Besides this, our study addresses the literature on optimal contracts with limited enforcement constraints such as Kehoe and Levine (2001), Kocherlakota (1996) and, in particular, Kehoe and Perri (2002) and Restrepo-Echavarria (2019) who already applied the Lagrangian-recursive approach developed by Marcet and Marimon (2019). Unlike Aguiar et al. (2019) and Aguiar and Amador (2020), our planner's problem integrates two-sided limited enforcement constraints. Our focus is close to Thomas and Worrall (1994) who already studied international lending contracts, with one-sided limited commitment. We decentralize the Fund contract using state-contingent securities and endogenous debt constraints as in Alvarez and Jermann (2000) and show the First and Second Welfare Theorem hold. As a result, in the economy with the Fund, the competitive equilibrium implements the *unique* constrained efficient allocation. Callegari et al. (2023) extend our framework in the environment of Cole and Kehoe (2000) and show that the Fund continues to implement the unique constrained efficient allocation by eliminating self-fulfilling debt crises.

A more recent literature merges these last two strands of literature and it is the most closely related one to our work. In particular, Dovis (2019) decentralises optimal contracts through partial default and an active debt maturity management, and Müller et al. (2019) through *ex post* state-conditionality given by default and renegotiation procedures. Our approach is not to 'rationalise' *ex post* observed behaviour, but to account for existing constraints. In view of this, we adopt a Nash specification in which the Fund takes the decision in the private bond market as given. We then characterise the constraint efficient allocation and assess it quantitatively in relation to a calibrated version of the benchmark defaultable debt economy.

Finally, as a theoretical foundation for the design of a — effectively running — fiscal fund, able to stabilise sovereign debt and expand the supply of safe assets, our work is related to a large literature regarding the IMF and other international institutions lending practices, and to the debate on the need to develop the Fiscal Union within the European Economic and Monetary Union (EMU) and expand its supply of eurobonds (as it has been done with the Next Generation EU (NGEU) program) as safe assets.[12]

The paper is organised as follows. We lay down the environment in Section 2.2 and present the Fund contract in Section 2.3. We expose the decentralized economy in Section 2.4, which includes the sovereign's, the private lenders' and the Fund's problems. Section 2.5 develops the Fund's intervention with seniority. After this, we calibrate our model to Italy in Section 2.6 and present the underlying results in Section 2.7. Finally, we conclude in Section 2.8.

2.2 Environment

We assume an infinite-horizon small open economy with a single homogenous consumption good in discrete time. There is a sovereign borrower acting as a representative agent and taking decisions on behalf of the small open economy, a Fund acting as official lender and a continuum of competitive private lenders.

2.2.1 The Sovereign Borrower

The sovereign's preference is represented by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$, where $\beta \in (0,1)$ is the discount factor, n_t is the labor, $1 - n_t$ the leisure and c_t the consumption at time t. The sovereign is relatively impatient as $\beta < 1/(1+r)$. We adopt a specific form of utility function so as to obtain a (stochastic) balanced growth path and to simplify the detrended formulation of the problem: $U(c,n) = u(c) + h(1-n) = \log(c) + \xi \frac{(1-n)^{1-\zeta}}{1-\zeta}$.

The sovereign has access to a labor technology $y = \theta f(n)$ subject to decreasing returns to scale, where $f_n(n) > 0$, $f_{nn}(n) < 0$. Moreover, $\theta \in \Theta$ represents a trend shock to the productivity. It is the only source of uncertainty in the economy. The law of motion of the shock is given by

[12] See Marimon and Wicht (2021) for a discussion on how our Fund proposal relates to this literature and it can be implemented within EMU.

$\theta_t = \gamma_t \theta_{t-1}$, where $\gamma_t \in \Gamma$ represents the growth rate at time t. We denote the history of θ up to time t by θ^t. The exact form of the shock is detailed in Section 2.6.[13]

Finally, the sovereign has access to a long-term state-contingent contract with the Fund — a credit-insurance line that we specify below — and long-term debt contracts with a continuum of competitive private lenders. However, it cannot commit to honour the terms of any contract. Given that contracts are all long term, this gives rise not only to *default risk* — i.e. non repayment — but also to *dilution risk* — i.e. devaluation of legacy debt.

In the first part of our analysis, we assume that the Fund contract has no seniority with respect to the private debt contracts. That is, every default is a *full* default as the sovereign reneges its entire debt position. Under such default, the sovereign receives a penalty in the form of a reduced output, $\theta^d \leq \theta$, and loses access to both the private bond market and the Fund. Later, it can reintegrate the private bond market with some probability, λ, but cannot obtain the assistance of the Fund anymore. In the second part of our analysis, we consider the case in which the Fund possesses seniority with respect to the private bond market.[14]

2.2.2 The Private Lenders

There is a continuum of competitive private lenders which have access to international financial markets. They are risk neutral and discount the future at $\frac{1}{1+r}$ where r is the risk-free rate. Private lenders' contracts are a continuum of simple long-term debt contracts.[15] We denote by $b_{l,t}$ the debt held by the private lenders, while we denote by b_t the debt issued by the sovereign at time t. By market clearing, $b_t = -b_{l,t}$ for all t. Furthermore, $b_t > 0$ denotes an asset and $b_t < 0$ denotes a debt from the point of view of the sovereign.

[13] We present in the main text the model with the stochastic trend and keep track of θ in the state space. The detrended version is presented in the Appendix 2.2. There we only keep track of γ in the state space.

[14] We do not consider the case in which the Fund is junior relative to the private lenders as official multilateral lending institutions generally enjoy a preferred creditor status (see Schlegl et al., 2019).

[15] We assume that private lenders do not offer Arrow securities, basically for two reasons. One is factual: Arrow securities are more complex than, for example, insurance contracts against natural disasters, for which private insurance companies provide contracts to households and firms, but — not surprisingly — not to countries; the other is redundancy (and convenience): we expect the same results would hold but would require to always keep track of the fraction of Arrow securities in the private sector, which is not just a problem of more baroque notation, but also of properly restating some results.

2.2.3 The Financial Stability Fund

Similar to the private lenders, the Fund has access to international financial markets, is risk neutral, discounts the future at $\frac{1}{1+r}$ and breaks even in expectation.

While private lenders are competitive, the design of the Fund contract is based on a risk assessment of the country which, as it is common practice in *debt sustainability analysis* (DSA), also accounts for the effect of the same Fund contract in enhancing the sustainability of the country's sovereign debt. In our Nash specification, the Fund takes the decisions in the private bond market as given. At the same time, the private lenders take the lending decisions of the Fund as given.

In addition, the Fund provides a state-contingent contract, whereas private lenders offer non-contingent debt contracts.[16] Particularly, the Fund contract is a state-contingent asset, $a_{l,t}$, which can be decomposed into a debt, $\bar{a}_{l,t}$, and an insurance components, $\hat{a}_{l,t}(\theta^t)$. As before, by market clearing, $a_t = -a_{l,t}$ for all t where $a_t > 0$ denotes an asset and $a_t < 0$ denotes a debt from the point of view of the sovereign. Importantly, liabilities in the Fund contract cannot be arbitrary. There is a limit on the extent of losses the Fund can make given by $\theta_{t-1}Z \leq 0$. The reason is that any contract with permanent losses has to be compensated with other contracts having permanent gains. For instance, in a union of sovereign countries, expected losses must be mutualized if the Fund is only backed with the union's primary surplus. Thus assuming that $Z = 0$ means that there is no permament transfer across the different Fund's contracts.[17]

Finally, the Fund's withdrawal in the case of a *full* default is permanent, whereas the private bond market's exclusion is temporary.

2.2.4 Timing of Actions

The timing of actions within the period is:

1. Given $(\theta_{t-1}Z, a_{l,t}, b_{l,t})$, after the realization of the growth shock θ_t, the Fund announces what

[16] See notably Hatchondo and Martinez (2012), Mateos-Planas and Seccia (2014) and Kirpalani (2017) for models with private state-contingent contracts.

[17] As already noted in footnote 6, we could also assume more generally that $Z(\theta^t)$ meaning that there can be bounded solidarity transfers among union countries depending on the realization of θ^t. For example, the 'grant component' of the NGEU recovery plan mentioned in footnote 2.

is the (state-contingent) sustainable debt capacity of the sovereign country for next period: $\{\omega_l'(\theta^{t+1})\}_{\theta^{t+1}|\theta^t}$ which can be decomposed into a debt component, $\bar{\omega}_l'(\theta^t) = b_{l,t} + \bar{a}_{l,t}$, to be allocated between the private lenders and the Fund, and insurance components, $\omega_l'(\theta^{t+1}) - \bar{\omega}_l'(\theta^t) = \hat{a}_{l,t}(\theta^{t+1})$, which must be part of the Fund contract.

2. The sovereign decides whether to default or not and, in the latter case, the sovereign then determines its borrowing with the private bond market before going to the Fund.
3. Conditional on no default, the Fund and the sovereign implement the corresponding debt and insurance part of their contract.[18]

2.3 The Financial Stability Fund

We specify the Fund contract in a Nash specification where the actions in the private bond market are taken as given.

2.3.1 Debt and Sustainability

The private lenders' and Fund's contracts establish that at time t and state-history θ^t the country must transfer $\tau_f(\theta^t)$ for its state-contingent liabilities with the Fund and $\tau_p(\theta^t)$ for its non-contingent debt liabilities with the private lenders. We denote $\tau(\theta^t) \equiv \tau_f(\theta^t) + \tau_p(\theta^t)$ as the total transfer the country pays. That is, given a consumption and employment plan $\{c(\theta^t), n(\theta^t)\}_{t=0}^{\infty}$, in period-state (t, θ^t) feasibility implies that

$$\tau(\theta^t) = \theta_t f(n(\theta^t)) - c(\theta^t); \tag{2.1}$$

that is, $\tau(\theta^t)$ is the *primary surplus* in state-history θ^t. Therefore, if the country's debt with private lenders is $-b_{l,t}$ and its asset position with the Fund is $-a_{l,t}$ for a total amount of $-\omega_{l,t} = -(b_{l,t} + a_{l,t})$, *debt sustainability* requires that the expected present value of future transfers discounted with

[18]This timing rules out self-fulfilling debt crises (Ayres et al., 2018). See Callegari et al. (2023) for a version of the model with self-fulfilling debt crises.

the risk free rate r should cover the outstanding amount of debt:

$$\mathbb{E}_t \sum_{j=t}^{\infty} \left(\frac{1}{1+r}\right)^{j-t} \tau(\theta^j) \geq \omega_{l,t}.$$

In particular, there is a decomposition of total transfers such that:

$$\mathbb{E}_t \sum_{j=t}^{\infty} \left(\frac{1}{1+r}\right)^{j-t} \tau_p(\theta^j) \geq b_{l,t} \quad \text{and} \quad \mathbb{E}_t \sum_{j=t}^{\infty} \left(\frac{1}{1+r}\right)^{j-t} \tau_f(\theta^j) \geq a_{l,t}. \tag{2.2}$$

Without loss of generality, we assume that $a_{l,0} = 0$, therefore the initial state is given by $(\theta_0, b_{l,0})$. In contrast with private lenders which only issue non-contingent debt contracts, the Fund provides a state-contingent contract, i.e. $a_{l,t} = \bar{a}_{l,t} + \hat{a}_{l,t}(\theta^t)$. More precisely, it defines contingent transfers for $(t+1, \theta^{t+1})$ at (t, θ^t); i.e. $\tau_f'(\theta^{t+1}) = \tau_f(\theta^t) + \hat{\tau}_f'(\theta^{t+1})$, with $\sum_{\theta^{t+1}|\theta^t} \hat{\tau}_f'(\theta^{t+1}) = 0$ and

$$\tau_f(\theta^t) = \sum_{\theta^{t+1}|\theta^t} \tau_f'(\theta^{t+1}). \tag{2.3}$$

We later specify the form that these transfers have in a decentralized economy.

However, for the debt to be sustainable two other factors must be taken into account. First, a *sovereign country can default* on its liabilities. Therefore, if in state θ_t the value of the outside default option is $V^{af}(\theta_t)$, to prevent *full* default the Fund contract must satisfy:

$$\mathbb{E}\left[\sum_{j=t}^{\infty} \beta^{j-t} U(c(\theta^j), n(\theta^j)) \middle| \theta^t \right] \geq V^{af}(\theta_t). \tag{2.4}$$

Second, the Fund contract must account that the *liabilities with the Fund cannot be arbitrary*. Therefore, since the Fund takes into account the private debt liabilities $b_{l,t}$, and both debt liabilities are treated *at par* (a feature we analyze in detail in Section 2.5) the Fund contract must satisfy:[19]

$$\mathbb{E}\left[\sum_{j=t}^{\infty}\left(\frac{1}{1+r}\right)^{j-t}\tau(\theta^j)\middle|\theta^t\right] \geq \theta_{t-1}Z + b_{l,t}. \tag{2.5}$$

The above constraint depends on $Z \leq 0$ and b_l. The former variable indicates the level of redistribution of the Fund. In order to prevent that the Fund provides permanent transfers to the sovereign we will assume that $Z = 0$, i.e. that in no state the Fund contract has expected losses. Similarly, b_l indicates the level of outstanding private debt the sovereign needs to repay. Larger b_l tightens the constraint. We therefore interpret (2.5) as a DSA as it corresponds to an evaluation of the present value of the sovereign's future surpluses.

The literature has mainly considered one-sided limited enforcement contracts in which (2.4) is the standard constraint. We focus on a two-sided limited enforcement contract in which we introduce (2.5) alongside (2.4). Without (2.5), the Fund prevents defaults on equilibrium path and is unconcerned by the extent of losses in the contract. In opposition, with (2.5), the Fund actively monitors the sovereign's capacity to generate surpluses — i.e. τ. In particular, in states where the sovereign's future surpluses might not appropriately cover additional amount of debt — say, when (2.5) is binding at θ^t — there is a lending 'sudden stop' to avoid losses that would go beyond the contract's terms. Thus, with (2.5), the Fund internalizes the fact that marginal lending can be excessive.

[19] To obtain equation (2.5), observe that, conditional on θ^t,

$$\mathbb{E}\left[\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}\tau(\theta^{t+j})\middle|\theta^t\right] = \mathbb{E}\left[\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}\left(\tau_f(\theta^{t+j})+\tau_p(\theta^{t+j})\right)\middle|\theta^t\right].$$

Using the valuation formula in (2.2), the previous equation simplifies into

$$\mathbb{E}\left[\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}\tau(\theta^{t+j})\middle|\theta^t\right] \geq \mathbb{E}\left[\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}\tau_f(\theta^{t+j})\middle|\theta^t\right] + b_l(\theta^t).$$

The present value constraint on Fund's lending is $\mathbb{E}\left[\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}\tau_f(\theta^{t+j})\middle|\theta^t\right] \geq \theta_{t-1}Z$, thus the overall participation constraint of the Fund reduces to (2.5). Note that we cannot consider $\mathbb{E}\left[\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}\tau_f(\theta^{t+j})\middle|\theta^t\right] \geq \theta_{t-1}Z$ and $\mathbb{E}\left[\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^{j}\tau_p(\theta^{t+j})\middle|\theta^t\right] \geq b_l(\theta^t)$ as two separate constraints because the Fund takes the actions in the private bond market as given.

The design of the Fund contract has two distinct features. First, it establishes the levels of debt which are sustainable next period, $\{\omega_l'(\theta^{t+1})\}_{\theta^{t+1}|\theta^t}$, according to a DSA. The announcement of $\{\omega_l'(\theta^{t+1})\}_{\theta^{t+1}|\theta^t}$ by the Fund makes such levels of debt common knowledge at the beginning of the period and therefore coordinates the private lenders' beliefs. Second, the Fund defines the long-term contract between the Fund and the sovereign, which here takes the form of financial transfers. In particular, it commits to a debt and insurance level $\{a_{l,t+1}(\theta^{t+1}, b_{l,t+1})\}_{\theta^{t+1}|\theta^t}$, where $a_{l,t+1}(\theta^{t+1}, b_{l,t+1}) = \omega_{l,t+1}(\theta^{t+1}) - b_{l,t+1}$.[20]

2.3.2 The Fund Contract Problem

We now turn to the specific design of the Fund's announcement and contract. Once the corresponding country's risk assessment regarding $\{\theta_t\}_{t=0}^{\infty}$ has been done, the Fund solves a planner's problem with two agents — the sovereign and the Fund itself — taking into account the participation of a continuum of private lenders in absorbing credit needs. This defines an allocation, of consumption and employment, which the Fund takes as the benchmark policy the sovereign will follow, and the corresponding transfers of the sovereign to the lenders.

We say that $\{c(\theta^t), n(\theta^t)\}_{t=0}^{\infty}$ is a Fund's *constrained efficient allocation* in *sequential form*, given $b_{l,0}$, if there exist sequences of transfers $\{\tau_p(\theta^t), \tau_f'(\theta^{t+1})\}_{t=0}^{\infty}$, with associate $\{b_{l,t}\}_{t=0}^{\infty}$ satisfying (2.2), such that:

$$\max_{\{c(\theta^t),n(\theta^t)\}_{t=0}^{\infty}} \mathbb{E}\left[\mu_{b,0} \sum_{t=0}^{\infty} \beta^t U(c(\theta^t), n(\theta^t)) + \mu_{l,0} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \tau(\theta^t) \middle| \theta_0\right] \tag{2.6}$$

$$\text{s.t. } (2.5), (2.4), (2.3) \text{ and } (2.1), \text{ for all } (t, \theta^t), t \geq 0.$$

The *constrained efficient allocation* prescribes that, in period t, the sovereign consumes $c(\theta^t)$ and provides labor $n(\theta^t)$.[21] Furthermore, the Fund's break-even assumption determines the initial weights $(\mu_{b,0}, \mu_{l,0})$. If without private debt there is an interior solution to the Fund's contract-

[20] Note that the Fund contract is state contingent with respect to the productivity shocks θ_{t+1} but also with respect to $V^{af}(\theta_{t+1})$ and $\theta_{t-1}Z + b_{l,t+1}$ being binding.

[21] Our contract accounts for all the lenders on equal footing in the maximization. While the Fund *directly* specifies contingent transfers $\tau_f'(\theta^{t+1})$ taking as given $\tau_p(\theta^t)$, effectively the contract accounts for the total surplus, $\tau(\theta^t)$.

ing problem, then an optimal solution exists and there are feasible paths of private debt, starting at $b_{l,0}$, subject to an upper bound on how large the initial debt $b_{l,0}$ can be. We come back to this later.

Using the recursive contracts approach of Marcet and Marimon (2019), we can formulate the Fund's problem (2.6) in *recursive form*. Defining $s \equiv \{\theta^-, \gamma\}$ and $\eta \equiv \beta(1+r) < 1$,

$$\begin{aligned} FV(s,x,b_l) = \mathcal{SP} \min_{\{\nu_b,\nu_l\}} \max_{\{c,n\}} \; & x\Big[(1+\nu_b)U(c,n) - \nu_b V^{af}(\theta)\Big] \\ & + \Big[(1+\nu_l)\tau - \nu_l(\theta^- Z + b_l)\Big] + \tfrac{1+\nu_l}{1+r}\mathbb{E}\Big[FV(s',x',b_l')\Big|\theta\Big] \end{aligned} \tag{2.7}$$

$$\begin{aligned} \text{s.t. } & \tau = \theta f(n) - c, \\ & x' = \frac{1+\nu_b}{1+\nu_l}\eta x \quad \text{with } x_0 \text{ given.} \end{aligned} \tag{2.8}$$

The Appendix 2.1 presents all the details of such exposition. We denote by x' the prospective Pareto weight of the sovereign relative to the Fund where $\nu_b \geq 0$ and $\nu_l \geq 0$ are the normalized multipliers attached to the sovereign's and the Fund's participation constraints, respectively. The value function of the contracting problem satisfies:

$$FV(s,x,b_l) = xV^b(\theta,x,b_l) + V^l(s,x,b_l), \text{ with}$$
$$V^b(\theta,x,b_l) = U(c,n) + \beta\mathbb{E}[V^b(\theta',x',b_l')|\theta] \quad \text{and} \quad V^l(s,x,b_l) = \tau + \frac{1}{1+r}\mathbb{E}[V^l(s',x',b_l')|\theta].$$

It should be underlined that we take a specific planner's perspective in solving for the Fund contract. The Fund is designing a constrained efficient contract with the sovereign borrower while taking as given the lending policies of the private lenders in the market, and at the same time, the private lenders are aware of the lending decisions of the Fund in the contract. However, the Fund does not play the role of a Ramsey planner in our framework, since it lacks the authority to fully control the market transactions between the private lenders and the sovereign borrower, neither directly through planned allocations nor indirectly via policy instruments. In other words, as we will see more explicitly when, in the next section, we characterize the decentralized economy, the equilibrium between the Fund, the sovereign and the continuum of private lenders has a

Nash-competitive equilibrium characterization, not a Ramsey policy implementation.

Our present formulation is close to the current rules of international lending institutions such as the IMF or the ESM. The Fund takes into account all the sovereign's debt liabilities — within and outside the Fund — that satisfy the DSA in every possible state. The difference with current practices is that the DSA is usually only conducted at the beginning of the contract, or at certain time intervals, while in our characterisation of the Fund contract, DSA, i.e. (2.5), is contingent to all states that the contract specifies, including those where participation constraints are binding. This means that our DSA has a different definition of 'sustainability' than existing official multilateral lending institutions. Particularly, sovereign liabilities have to remain sustainable *ex ante* and *ex post* in all considered paths.

Another difference is that in this framework, the Fund provides state-contingent transfers, key component that averts default on equilibrium path as we will see. Finally, the Fund has no seniority over privately owned debt. This is in general not the case when official multilateral lenders intervene (cf. footnote 9). In Section 2.5, we consider an alternative formulation where the Fund liabilities has seniority over privately held sovereign debt. We show that the seniority structure of the Fund might affect the Fund's intervention.

2.3.3 The Sovereign's Outside Option

The autarky value of the standard incomplete market model with default represents the sovereign's outside option (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008). Since the Fund has no seniority with respect to the privately held sovereign debt, the sovereign reneges its entire debt position if it decides to default. This is what we call a *full* default. The Bellman equation in such situation reads

$$V^{af}(\theta) = \max_{n} \Big\{ U(\theta^d f(n), n) \Big\} + \beta \mathbb{E}\Big[(1-\lambda) V^{af}(\theta') + \lambda J(\theta', 0) \Big| \theta \Big], \tag{2.9}$$

where $\theta^d \leq \theta$ contains the penalty for defaulting. Furthermore, V^{af} corresponds to the value under financial autarky and J to the value of reintegrating the private bond market without the

Fund. More precisely, $J(\theta, b) = \max\{V^o(\theta, b), V^{af}(\theta)\}$, with

$$V^o(\theta, b) = \max_{\{c,n,b'\}} U(c, n) + \beta \mathbb{E}\Big[J(\theta', b')\Big|\theta\Big] \tag{2.10}$$
$$\text{s.t. } c + \tau_p(b') \leq \theta f(n).$$

Given equation (2.2), the sequence of private transfers $\{\tau_p(\theta^t)\}_{t=0}^{\infty}$ directly relates to a sequence of private debt $\{b(\theta^t)\}_{t=0}^{\infty}$. Hence, for a given b, by picking b', the borrower directly chooses a certain level of transfer τ_p. By a slight abuse of notation, we write τ_p as a function of b'.

2.3.4 Properties of the Fund Contract

This subsection demonstrates the main properties of the Fund contract. Other properties such as the inverse Euler equation and the steady state are presented in the Appendix 2.3. Proofs are in the Appendix 2.4.

We start with the existence of the Fund contract and, for this, we need the following interiority assumption (Marcet and Marimon, 2019).

Assumption 2.1 (Interiority). *There is an $\epsilon > 0$, such that, for all $\theta^t \in \Theta^t$ with associate $\{b_{l,t}\}_{t=0}^{\infty}$ satisfying* (2.2)*, there is a sequence $\{\ddot{c}(\theta^t), \ddot{n}(\theta^t)\}$ satisfying for all $t \geq 0$,*

$$\mathbb{E}\Big[\textstyle\sum_{j=t}^{\infty} \beta^{j-t} U(\ddot{c}(\theta^j), \ddot{n}(\theta^j))\Big|\theta^t\Big] \geq V^{af}(\theta_t) + \epsilon,$$
$$\mathbb{E}\Big[\textstyle\sum_{j=t}^{\infty} \left(\tfrac{1}{1+r}\right)^{j-t} \Big(\theta_j f(\ddot{n}(\theta^j)\Big) - \ddot{c}(\theta^j))\Big|\theta^t\Big] \geq \theta_{t-1} Z + b_{l,t} + \epsilon.$$

This assumption ensures the uniform boundedness of the Lagrange multipliers. For equations (2.4) and (2.5), it requires that, in spite of the enforcement constraints, there are strictly positive rents to be shared among the contracting parties. In our environment, since rents to be shared are positively correlated with productivity shocks, this assumption is easily satisfied given that default is costly. Otherwise, there may not exist a constrained efficient risk-sharing agreement.

Proposition 2.1 (Existence and uniqueness)**.** *In the specified environment,*[22] *if Assumption 2.1 is satisfied, for every* θ *there is a* $\underline{b}_l(\theta) > 0$ *such that if* $b_{l,0}(\theta) \leq \underline{b}_l(\theta)$*, then there exists a unique Fund's allocation with initial condition* $(\theta, b_{l,0}(\theta))$*. Furthermore, there is a* $\underline{t}(\theta, \underline{b}_l(\theta))$ *such that for* $t > \underline{t}(\theta, \underline{b}_l(\theta))$ *the detrended Fund contracts are at the steady state.*

Proposition 2.1 is made of three parts. First, a Fund contract exists if — among other requirements — the initial level of private indebtedness is not too high, as to Assumption 2.1 to be satisfied. However, if an economy is in an initial state $(\theta, b_{l,0}(\theta))$ but $b_{l,0}(\theta) > \underline{b}_l(\theta)$ then the private debt will need to be restructured — i.e. to a $\ddot{b}_{l,0}(\theta) \leq \underline{b}_l(\theta)$ — for a Fund contract to exist. In other words, there is a strict risk-assessment of the sovereign and, provided that the existing level of private liabilities is sustainable, if there is a Fund contract then no other *ex ante* conditionality is needed. Second, the Fund contract allocation in terms of consumption and employment is unique and, third, it is characterised by an ergodic distribution which we detail in the Appendix 2.3.

Corollary 2.1 (No Full Default)**.** *In a Fund contract, there is no full default.*

The sovereign's participation constraint (2.4) implies no (*full*) default on equilibrium path. The Fund always provides state-contingent transfers to the sovereign. This sustains the chosen sequence of private liabilities, $\{\tau_p(\theta^t)\}_{t=0}^{\infty}$, and ensures that the sovereign finds optimal not to default. This shows the importance of the state contingency of the Fund's transfer. Without this feature, the Fund would not be capable of accounting for the possibility of default in each state $\theta' \in \Theta$ specifically.

2.4 The Decentralized Economy

The previous section derived the Fund contract from the perspective of a mixed centralized-private economy. It had the advantage that it allowed a full characterization of the Fund contract, but the disadvantage of having the sovereign in the shadow, with its actions being decided by the Fund contract. We now consider the decentralized version of the economy in which the Fund and

[22] We define a specific functional form for $U(c, n)$, but it is enough to assume that U is continuous, strictly monotone and concave in $\mathbb{R}^+ \times [0, 1]$.

the private lenders trade securities with the sovereign.

The financial market is composed of private lenders and the Fund. The sovereign has therefore two funding opportunities. On the one hand, it can borrow long-term defaultable bonds, b', on the private bond market at a unit price of $q_p(\theta, \bar{\omega}')$, where $\bar{\omega}$ is defined momentarily. On the other hand, it can trade $|\Theta|$ state contingent securities $a'(\theta')$ at a unit price of $q_f(\theta', \omega'(\theta')|\theta)$. A fraction $1-\delta$ of each financial asset matures today and the remaining fraction δ is rolled-over and pays a coupon κ. Given this, the transfer to the Fund and the private lenders are, respectively

$$\tau_f(\theta) = \sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)(a'(\theta') - \delta a(\theta)) - (1 - \delta + \delta\kappa)a(\theta), \tag{2.11}$$

$$\tau_p(\theta) = q_p(\theta, \bar{\omega}')(b'(\theta) - \delta b(\theta)) - (1 - \delta + \delta\kappa)b(\theta). \tag{2.12}$$

The assets provided by the Fund are state contingent, while private bonds are not. More precisely, the portfolio $a'(\theta')$ can be decomposed into a common bond $\bar{a}'$ that is independent of the next period state, traded at the implicit bond price $q_f(\theta, \bar{\omega}') \equiv \sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)$, and an insurance portfolio of $|\Theta|$ Arrow securities $\hat{a}'(\theta')$. Thus we have that $a'(\theta') = \bar{a}' + \hat{a}(\theta')$ with $\bar{a}' = \frac{\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)a'(\theta')}{q_f(\theta, \bar{\omega}')}$ and $\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)\hat{a}'(\theta') = 0$ which represents the market clearing condition of Arrow securities.[23]

Given that the Fund has no seniority with respect to the private lenders, the bond prices are a function of the total liabilities next period. We denote the entire position — including insurance and debt — by $\omega = a + b$ and a total *debt* position by $\bar{\omega} = \bar{a} + b$.

The Fund takes the decisions in the private bond market as given and *vice versa*. In addition, as long as there are no spreads — positive or negative — on the debt, private lenders are willing to provide all the debt the sovereign asks for.

The timing of actions is the one presented in Section 2.2. Given $(\theta^{-}Z, a_l, b_l)$, after the realization of the growth shock θ, the Fund announces what is the (state-contingent) sustainable debt capacity

[23]Note that $\tau_f'(\theta') = \tau_f(\theta) + \hat{\tau}_f(\theta')$, where $\tau_f(\theta) = q_f(\theta, \bar{\omega}')(\bar{a}'(\theta) - \delta a(\theta)) - (1 - \delta + \delta\kappa)a(\theta)$ and $\hat{\tau}_f(\theta') = q_f(\theta', \omega'(\theta')|\theta)\hat{a}'(\theta')$.

of the sovereign country for next period: $\{\omega_l'(\theta')\}_{\theta'|\theta}$. The sovereign decides whether to default or not and, in the latter case, the sovereign then determines its borrowing with the private bond market (i.e. b_l') before going to the Fund (i.e. $\{a_l'(\theta')\}_{\theta'|\theta}$). Conditional on no default, the Fund and the sovereign implement the corresponding debt and insurance part of their contract.

2.4.1 The Sovereign's and Private Lender's Problems

The economy is decentralised as a competitive equilibrium with endogenous borrowing and lending constraints following Alvarez and Jermann (2000) and Krueger et al. (2008). Under the above market structure, the sovereign's problem reads

$$W^b(\theta, a, b) = \max_{\{c,n,b',\{a'(\theta',b')\}_{\theta'\in\Theta}\}} U(c,n) + \beta\mathbb{E}\Big[W^b(\theta', a'(\theta', b'), b')\Big|\theta\Big] \tag{2.13}$$

$$\text{s.t. } c + \sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)(a'(\theta', b') - \delta a) + q_p(\theta, \bar{\omega}')(b' - \delta b) \tag{2.14}$$

$$\leq \theta f(n) + (1 - \delta + \delta\kappa)(a + b), \text{ and}$$

$$\omega'(\theta') = a'(\theta', b') + b' \geq \mathcal{A}_b(\theta'). \tag{2.15}$$

Equation (2.15) is the equivalent to the participation constraint (2.4), which prevents defaults. The endogenous borrowing limit $\mathcal{A}_b(\theta')$ is such that

$$W^b(\theta', \ddot{a}'(\theta', \ddot{b}'), \ddot{b}') = V^{af}(\theta') \text{ for all } \ddot{a}'(\theta', \ddot{b}') + \ddot{b}' = \mathcal{A}_b(\theta'). \tag{2.16}$$

In words, the endogenous borrowing limit is such that the sovereign's expected lifetime utility from repaying its debts is at least as high as that of defaulting. It is therefore a no-default borrowing constraint (Zhang, 1997). Particularly, it is tight enough in the sense of Alvarez and Jermann (2000) to prevent default but allows as much risk sharing as possible. We explain the dependence of $a'(\theta', b')$ on b' when we derive the decentralized Fund's problem.

Private lenders solve a static problem. However, we express it in recursive form to later formulate the DSA of the Fund. We have

$$W^p(\theta, a_l, \bar{a}_p, b_l) = \max_{\{c_p, b_l', \bar{a}_p'\}} c_p + \frac{1}{1+r}\mathbb{E}\Big[W^p(\theta', a_l', \bar{a}_p', b_l')\Big|\theta\Big] \tag{2.17}$$
$$\text{s.t. } c_p + q_p(\theta, \bar{\omega}')(b_l' - \delta b_l) + q_f(\theta, \bar{a}_p')(\bar{a}_p' - \delta \bar{a}_p) \leq (1 - \delta + \delta\kappa)(b_p + \bar{a}_p).$$

An important object which emanates from this problem is the private lending policy, $b_l' = B_l(\theta, a_l, b_l)$ which is taken as given by the Fund.

The private lenders also have access to the bonds issued by the Fund. This enables that the bond price in the Fund and in the private bond market coincide through arbitrage. We will consider equilibria where, without loss of generality, $\bar{a}_p = 0$, therefore we simplify notation by eliminating $\bar{a}_p$ if not necessary. Notably, we write $W^p(\theta, a_l, 0, b_l) \equiv W^p(\theta, a_l, b_l)$. Besides this, the trade of private bonds satisfies the following transversality condition:

$$\lim_{n\to\infty} \mathbb{E}\Big\{\Big[\prod_{j=0}^{n} Q_p\Big(\theta^{t+j}, \bar{\omega}\Big(\theta^{t+j}\Big)\Big)\Big] b_l\Big(\theta^{t+j}\Big)\Big|\theta^t\Big\} = 0, \quad \text{with} \tag{2.18}$$

$$Q_p\Big(\theta^{t+j}, \bar{\omega}\Big(\theta^{t+j}\Big)\Big) = \frac{q_p\Big(\theta^{t+j}, \bar{\omega}\Big(\theta^{t+j}\Big)\Big)}{1 - \delta + \delta\kappa + \delta q_p\Big(\theta^{t+j+1}, \bar{\omega}\Big(\theta^{t+j+1}\Big)\Big)}. \tag{2.19}$$

The implicit interest rate in the private bond market is $r_p(\theta, \bar{\omega}') \equiv \frac{1}{Q_p(\theta, \bar{\omega}')} - 1$. As we will see, it is possible that $r_p(\theta, \bar{\omega}') < r$ generating a wedge between the lenders' discount factor and the pricing kernel. That is why the valuation equation (2.2) holds with inequality.

2.4.2 The Decentralised Fund Contract

We can further decentralise the Fund contract. We show that, given the realization of the state, the Fund formulates an announcement stating the level of indebtedness that remains sustainable in all future states. The maximization problem of the Fund is given by

$$W^f(s, a_l, b_l) = \max_{\{c_f, \{a_l'(\theta', b_l')\}_{\theta' \in \Theta}\}} c_f + \tfrac{1}{1+r}\mathbb{E}\Big[W^f(s', a_l'(\theta', b_l'), b_l')\Big|\theta\Big] \tag{2.20}$$

$$\text{s.t. } c_f + \textstyle\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)(a_l'(\theta', b_l') - \delta a_l) \leq (1 - \delta + \delta\kappa)a_l, \tag{2.21}$$

$$\omega_l'(\theta') = a_l'(\theta', b_l') + b_l' \geq \mathcal{A}_f(\theta', b_l'), \tag{2.22}$$

$$\text{with } b_l' = B_l(\theta, a_l, b_l) \text{ given,}$$

where $B_l(\theta, a, b)$ is the lending policy of the private lenders and $s \equiv \{\theta^-, \gamma\}$. Again, we remove $\bar{a}_p$ to simplify notation as $\bar{a}_p = 0$ in equilibrium.

Note that in (2.22), $\omega_l'(\theta')$ and $a_l'(\theta', b_l')$ are simultaneously determined for a given b_l'.[24] That is, the Fund, as a security trader choosing $a_l'(\theta', b_l')$, determines $\omega_l'(\theta')$ by (2.22); alternatively, the Fund, as capacity announcer, could have chosen $\omega_l'(\theta')$ and use (2.22) to determine $a_l'(\theta', b_l')$. The variable $\mathcal{A}_f(\theta', b_l)$ represents an endogenous limit defined as

$$W^f(s', \mathcal{A}_f(\theta', b_l') - b_l', b_l') = \theta Z. \tag{2.23}$$

This condition restricts the extent of losses. Particularly, it ensures that the present discounted value of the Fund's assets are at least equal to $\theta^- Z \leq 0$. Specifically, when $Z = 0$, $\mathcal{A}_f(\theta', b_l')$ ensures that the sovereign's liabilities can be absorbed by the Fund without incurring permanent losses. Adding equations (2.23) to the value of the lender (2.17) and applying the transversality condition (2.18), we obtain

$$W^f(s', \mathcal{A}_f(\theta', b_l') - b_l', b_l') + W^p(\theta', \mathcal{A}_f(\theta', b_l') - b_l', b_l') = \theta Z + b_l'.$$

This gives the decentralised counterpart of the Fund's participation constraint in (2.5),

$$W^l(s', a_l'(\theta', b_l'), b_l') \equiv W^f(s', a_l'(\theta', b_l'), b_l') + W^p(\theta', a_l'(\theta', b_l'), b_l') \geq \theta Z + b_l', \tag{2.24}$$

[24]Even if it will not happen in equilibrium, the Fund must have a policy for the case that the interaction between the sovereign and private lenders ends with an over-lending which makes the continuation of the contract unfeasible. Then, its policy is to do as it does at the beginning of the Fund contract: discontinue the contract unless there is a debt restructuring that makes its intervention possible and credible.

We interpret condition (2.24) as a proper DSA since it links the value of the current lending with its prospective stream of transfers. This DSA takes into account the sovereign's entire debt position — within and outside the Fund — in every possible state. Moreover, owing to the trade of Arrow securities, it is contingent on all the states that the contract specifies, including those states where participation constraints are binding.

Note that, with $\bar{a}_p(\theta) = 0$, the market clearing condition in the Fund is given by $a(\theta, b) + a_l(\theta, b) = 0$ for all (θ, b). In addition, the initial asset holdings of the sovereign in the Fund, $a(\theta_0, b_0) = -a_l(\theta_0, b_0) = 0$, are given.

2.4.3 Properties of the Competitive Equilibrium

We first define a (recursive) competitive equilibrium in this environment and then characterize the price dynamic and the optimal holdings of assets.

Definition 2.1 (Recursive Competitive Equilibrium (RCE))**.** *Given the outside options of the sovereign, $V^{af}(\theta')$, and of the lenders, $\theta^{-}Z + b_l$, a* Recursive Competitive Equilibrium *(RCE) consists of: prices $q_f(\theta', \omega'(\theta')|\theta)$ and $q_p(\theta, \bar{\omega}')$; value functions $W^b(\theta, a, b)$, $W^f(s, a_l, b_l)$, and $W^p(\theta, a_l, b_l)$; endogenous limits, $\mathcal{A}_b(\theta')$ and $\mathcal{A}_f(\theta', b_l')$; and policy functions $c(\theta, a, b)$, $c_f(\theta, a_l, b_l)$, $c_p(\theta, a_l, b_l)$, $n(\theta, a, b)$, $a'(\theta', b') = A(\theta', \theta, a, b, b')$, $a_l'(\theta', b_l') = A_l(\theta', \theta, a_l, b_l, b_l')$, $b' = B(\theta, a, b)$ and $b_l' = B_l(\theta, a_l, b_l)$, which are solutions to the problems of the sovereign, the private lenders and the Fund, and all markets clear. Particularly, the announcement $\omega_l'(\theta')$ is equal to its equilibrium value, i.e. $\omega_l'(\theta') = a_l'(\theta', b_l') + b_l' = -\omega'(\theta')$.*

The definition of the RCE is made of two parts. The first part follows Alvarez and Jermann (2000) requiring optimality and markets clearing with the endogenous limits $\mathcal{A}_b(\theta')$ and $\mathcal{A}_f(\theta', b_l')$ defined as equilibrium objects. The second part of the definition makes it clear that the RCE has a Nash specification. On the one hand, the Fund takes the private lending policy as given in (2.20). On the other hand, the Fund's announcement $\{\omega_l'(\theta')\}_{\theta'|\theta}$ is not a constraint in either (2.13) or (2.17), while, as we said, it is part of (2.20).

We now characterize the price dynamic and the optimal holdings of assets in the decentralised environment. Using the fact that the borrowing constraints of the sovereign and the Fund do not bind at the same time, the price is determined by the agent whose constraint is not binding (Krueger et al., 2008).[25] Defining $\eta \equiv \beta(1+r)$, it follows that

$$q_f(\theta',\omega'(\theta')|\theta) = \frac{\pi(\theta'|\theta)}{1+r}\left[(1-\delta+\delta\kappa)+\delta\sum_{\theta''|\theta'} q_f(\theta'',\omega''(\theta'')|\theta')\right]\max\left\{\frac{u_c(c(\theta',a',b'))}{u_c(c(\theta,a,b))}\eta, 1\right\}. \tag{2.25}$$

Given the above price schedule, the intertemporal discount factor is defined by

$$Q_f\Big(\theta',\omega'(\theta')|\theta\Big) \equiv \frac{q_f\Big(\theta',\omega'(\theta')|\theta\Big)}{1-\delta+\delta\kappa+\delta\sum_{\theta''|\theta'} q_f\Big(\theta'',\omega''(\theta'')|\theta'\Big)}. \tag{2.26}$$

The implicit interest rate in the Fund is then defined by $r_f(\theta,\bar{\omega}') \equiv \frac{1}{Q_f(\theta,\bar{\omega}')} - 1$ with $Q_f(\theta,\bar{\omega}') \equiv \sum_{\theta'|\theta} Q_f\Big(\theta',\omega'(\theta')|\theta\Big)$.

Provided that the private lenders have access to the Fund's securities, no arbitrage is possible between the Fund and the private bond market for the borrower. Hence, the bond prices in the Fund and the private bond market are alike.

Proposition 2.2 (Bond Price)**.** *In an RCE, for all* $(\theta,\omega'(\theta'))$*,*

$$\sum_{\theta'|\theta} q_f\left(\theta',\omega'(\theta')|\theta\right) = q_p(\theta,\bar{\omega}').$$

Moreover, whenever (2.24) *binds,* $\sum_{\theta'|\theta} q_f\left(\theta',\omega'(\theta')|\theta\right) > \frac{1-\delta+\delta\kappa}{1+r-\delta}$.

Given the definition of the price in (2.25), if the Fund's DSA is binding, the price of a bond reads $q_f(\theta,\bar{\omega}') > \frac{1}{1+r}\sum_{\theta'|\theta}\pi(\theta'|\theta)\Big[(1-\delta+\delta\kappa)+\delta q_f(\theta',\bar{\omega}'')\Big]$, or equivalently $Q_f(\theta,\bar{\omega}') > \frac{1}{1+r}$ implying that $r_f(\theta,\bar{\omega}') < r$. In words, when the Fund's DSA binds, a *negative spread* appears. The Fund's binding DSA has therefore two opposite effects. On the one hand, accumulating debt, $\bar{a}_l' > 0$, is cheaper owing to the fact that $q_f(\theta,\bar{\omega}')$ is above the risk-free price. On the other hand, buying insurance, $\hat{a}_l'(\theta') < 0$, becomes more expensive.

[25] If both constraints would bind at the same time, Assumption 2.1 would be violated.

The negative spread is a strong signal that the Fund refrains from further lending and causes private lenders to stop lending to the sovereign as the rate of return settles below r. At this rate, the private lenders are willing to *borrow* from the Fund in terms of a portfolio of securities which constitutes risk free asset a_p, and investing the funds to earn a risk free rate r. Nevertheless, the binding DSA of the Fund also prevents such trading activities. As a result, a binding DSA in (2.24) not only restricts the provision of the Fund's insurance to the sovereign, it also sustains a no-trade equilibrium in the private bond market: there is a private lending 'sudden stop'.

From Proposition 2.2, without (2.24), there is no negative spread and the private lenders are willing to provide all the debt the sovereign asks for. Thus, without negative spread, the private lenders fail to realize that the present value of future transfers does not cover additional lending. As a result, they would lend "too much" in the sense that the sovereign becomes a permanent net debtor to the rest of the world. The negative spread prevents such excessive lending. In particular, it ensures any amount lent is appropriately covered by future transfers and therefore guarantees no permanent positive transfers to the sovereign when $Z = 0$. Thus, (2.24) internalizes *the pecuniary externality of a negative spread* that competitive private lenders do not: the fact that marginal lending can be excessive.

When (2.24) binds, the maximal amount of debt the Fund may have to absorb is $\theta^- Z + \delta b_l - \min\left\{\hat{a}_l'(\theta') \,:\, \text{(2.24) binds} \;\wedge\; \pi(\theta'|\theta) > 0\right\}$. On the one hand, the Fund provides the transfer component $\theta^- Z \leq 0$ and the complement to the maximal amount of insurance the sovereign may receive with positive probability. On the other hand, from the perspective of the private lenders, the Fund has to guarantee a maximal absorption of δb_l. In other words, the Fund must stand ready to guarantee just enough lending for the sovereign to honour its long-term liabilities. This is because the private lending sudden stop endangers the ability of the sovereign to maintain the value of its long-term debt, either directly — under the counterfactual interpretation that each period the sovereign buys and sells the long-term debt — since it may not be able to borrow from the private lenders to cover it; or, indirectly since private lenders may want to sell their holdings of over-priced, low-return, long-term debt in exchange for safe assets. The Fund's guarantee is therefore a form of prudential policy which is active as long as debt is long term (i.e. $\delta > 0$).

Proposition 2.3 (Private Debt)**.** *In a RCE, in the states in which* (2.24) *binds,* $b_l' \leq \delta b_l$*. Conversely, in the states in which* (2.24) *does not bind, the division of* $\bar{\omega}_l'$ *between* b_l' *and* $\bar{a}_l'$ *is indeterminate.*

However, when the Fund's DSA in (2.24) does not bind, the sovereign can equally access the private bond market and the Fund. In this case, given Proposition 2.2, debt is as expensive in the Fund as in the private bond market and the sovereign can accumulate debt in both locations. Therefore, the sovereign is indifferent between holding debt in the private bond market or in the Fund. It is then without loss of generality that we can set $\bar{a}_l' = 0$ whenever (2.24) does not bind. As we have said, our underlying assumption is that as long as there are no spreads (positive or negative) on the debt's interest rates, private lenders are willing to buy all the debt being offered by the sovereign. We can therefore define the Fund's *minimal intervention policy* (MIP) in the following terms.

Definition 2.2 (The Fund's Minimal Intervention Policy (MIP))**.** *For a given state* (θ, b_l)*, we say that the the Fund implements a Minimal Intervention Policy (MIP) if* $\bar{a}_l' = \underline{a}(\theta, b_l)$ *where, if* (2.24) *binds* $\underline{a}(\theta, b_l) \in [\check{a}_l, \check{a}_l + \delta b_l]$ *with* $\check{a}_l \equiv \theta^{-} Z - \min \left\{ \hat{a}_l'(\theta') \, : \, (2.24) \textit{ binds } \wedge \; \pi(\theta'|\theta) > 0 \right\}$ *and* $\underline{a}(\theta, b_l) = 0$ *otherwise.*

Having characterized the bond price and the Fund's MIP, we first show that the Second Welfare Theorem (SWT) holds before turning to the First Welfare Theorem (FWT).

Proposition 2.4 (Second Welfare Theorem (SWT))**.** *Given initial conditions* $\{\theta_0, b_0, x_0\}$*, the Fund's constrained efficient allocation can be decentralised as a RCE with endogenous borrowing and lending limits.*

In solving the Fund contract, the Fund takes the strategy of the private lenders and the sovereign as given. In particular, it solves for consumption and leisure and the corresponding transfers, which, given the lending strategy of the private agents, split between the private lenders and the Fund, with the latter also providing insurance. The proof of the SWT requires to map the structure of the Fund's contract, accounting for its lending from competitive private lenders, into the more decentalized market structure of the RCE. A key step is to map the state of the Fund (θ, x) to the state of the RCE (θ, a_l, b_l), given the lending strategy of the lenders; that is, to map from (θ, x) to (θ, ω_l) and, giving b_l, a_l is determined. This map is given by the identification of the consumption policies and the Fund's consumption first-order condition

$$u_c(c(\theta, a, b)) = u_c(c(\theta, x, b_l)) = \frac{1 + \nu_l(\theta, x, b_l)}{1 + \nu_b(\theta, x, b_l)} \frac{1}{x},$$

and, since by (2.8) the right hand side is equal to η/x', the law of motion of the co-state variable x maps into the borrower's Euler equation. Using this, we define the Fund contract as a long-term state-contingent asset and derive the corresponding asset prices. Then, we map policies and value functions and show that they satisfy the RCE conditions of Definition 2.1. Furthermore, by Proposition 2.1, the constrained efficient allocation is unique (when Assumption 2.1 is satisfied), therefore the RCE of Proposition 2.4 can take different forms (e.g. different asset structures), but the corresponding RCE allocation is also unique.

The SWT is satisfied in many environments. This is not the case for the FWT since multiplicity of equilibria usually prevails; in particular, inefficient equilibria, such as autarky. We first introduce an assumption that, similar to Assumption 2.1, ensures the uniform boundedness of the Lagrange multipliers in the decentralized economy.

Assumption 2.2 (Decentralized interiority)**.** *There is an* $\epsilon > 0$*, such that, for all equilibrium states* (θ, a, b) *the sovereign, lenders and Fund problems* — (2.13)*,* (2.17) *and,* (2.20) — *have a solution when the right hand sides of constraints* (2.15) *and* (2.22) *are replaced by* $\mathcal{A}_b(\theta') + \epsilon$ *and* $\mathcal{A}_f(\theta', b_l') + \epsilon$*, respectively.*

In general, equilibrium boundedness follows from standard monotonicity of preferences and an interiority, *free disposal*, assumption. Assumption 2.2 introduces the equivalent to *free disposal* when there are endogenous limit constraints. In particular, it dismisses autarky as the Lagrange multiplier of the budget constraint is unbounded in the autarkic allocation.[26]

[26]With Assumption 2.2 we restrict our attention to allocations enabling risk sharing between the contracting parties, which rules out an autarky equilibrium and, by Proposition 4.10 in Alvarez and Jermann (2000), their *high implied interest rates* condition is satisfied in our constrained efficient equilibrium; i.e. our Assumption 2.2 implies their condition.

Proposition 2.5 (First Welfare Theorem (FWT))**.** *Given initial conditions $\{\theta_0, b_0, a_0\}$, a RCE with endogenous borrowing and lending limits, satisfying Assumption 2.2 implements the constrained efficient allocation of the Fund.*

In the decentralized economy it is even more explicit that the Fund takes the strategy of the private lenders and the sovereign as given, as well as asset prices contingent on the sovereign's liabilities. The proof of the FWT requires the (inverse) map from the market structure — given by problems (2.13), (2.17) and (2.20), and the corresponding equilibrium conditions — to the structure of the Fund contract problem. The starting point is the first-order condition of the sovereign's problem (2.13): $u_c(c(\theta, a, b)) = \varkappa(\theta, a, b)$, where $\varkappa(\theta, a, b)$ is the Lagrange multiplier of the budget constraint (2.14). From the sovereign's and Fund's Euler equations we obtain the following intertemporal relation between these multipliers:

$$\varkappa(\theta, a, b) = \eta \frac{1 + \mathring{\nu}_b(\theta', a', b')}{1 + \mathring{\nu}_l(\theta', a', b')} \varkappa'(\theta', a', b'),$$

where $\mathring{\nu}_b(\theta', a', b')$ and $\mathring{\nu}_l(\theta', a', b')$ are normalized Lagrange multipliers of the endogenous limit constraints (2.15) and (2.22). As it can be seen, this intertemporal relation mirrors the law of motion of the co-state variable (2.8), which is at the core of Fund's problem. With the (inverse) map of value and policy functions it follows that the RCE allocation is a solution to the Fund's problem. Furthermore since, again by Proposition 2.1, the solution is unique the RCE allocation (when Assumption 2.2 is satisfied) must also be unique.

2.5 The Seniority Structure of the Fund

So far, we assume that the Fund has no seniority with respect to the privately held sovereign debt. We therefore consider that a default always implicates both the Fund and the private lenders. We now relax this assumption allowing for a *partial* default in which the sovereign defaults only on its private liabilities while remaining in the Fund.

2.5.1 The Sovereign and the Private Lenders under Seniority

Compared to the case without seniority, the sovereign possesses two outside options. On the one hand, it can default on both the private lenders and the Fund. This represents the case of *full* default considered previously. On the other hand, the sovereign can repudiate its private debt while remaining in the Fund. We refer to this situation as a *partial* default because the sovereign solely defaults on the private lenders. That is, if the sovereign has an outstanding debt of $\omega = a + b$, it defaults on b and repays a. We assume that the default penalty and the re-access probability are the same in *partial* and *full* defaults.

There is a clear tradeoff when deciding whether to enter *partial* default. On the one hand, in *partial* default, the sovereign is less productive — i.e. $\theta^d \leq \theta$ — for some time. On the other hand, the sovereign repudiates is private liabilities — i.e. $b = 0$ — and continues to receive support from the Fund. That is, unlike in *full* default, it can still trade bonds and insurance with the Fund. Given this, the state space in the decentralized economy is now (θ, a, b, d_p) where $d_p = 1$ if the sovereign is in *partial* default and $d_p = 0$ otherwise. Hence, in a given state (θ', a', b'), the sovereign does not enter in *partial* default if

$$W^b(\theta', a'(\theta', b', 0), b', 0) \geq W^b(\theta', a'(\theta', 0, 1), 0, 1), \tag{2.27}$$

where the value upon *partial* default reads

$$\begin{aligned}
W^b(\theta, a, 0, 1) = \max_{\{c, n, \{a'(\theta', 0, d_p')\}_{\theta', d_p'}\}} \; & U(c, n) + \beta \mathbb{E}\Big[(1 - \lambda) W^b(\theta', a'(\theta', 0, 1), 0, 1) \\
& + \lambda W^b(\theta', a'(\theta', 0, 0), 0, 0) \Big| \theta\Big] \\
\text{s.t.} \quad & c + \sum_{\theta' | \theta, d_p'} q_f(\theta', a'(\theta', 0, d_p'), 0 | \theta)(a'(\theta', 0, d_p') - \delta a) \leq \theta^d f(n) + (1 - \delta + \delta\kappa) a, \\
& a'(\theta', 0, d_p') = \bar{a}'(0) + \hat{a}'(\theta', d_p') \geq \mathcal{A}_b(\theta', d_p').
\end{aligned}$$

We then define $V^{ap}(\theta,a) \equiv W^b(\theta,a,0,1)$.[27] In the case of *partial* default, the endogenous borrowing limit is defined as $W^b(\theta',\mathcal{A}_b(\theta',1),0,1) = V^{af}(\theta')$, while in the case of repayment $W^b(\theta',\ddot{a}'(\theta',\ddot{b}',0),\ddot{b}',0) = V^{af}(\theta')$ for all $\ddot{a}'(\theta',\ddot{b}',0)+\ddot{b}' = \mathcal{A}_b(\theta',0)$.

Compared to the case without seniority, a' is now a function of the *partial* default status next period, d_p'. As the bond component $\bar{a}'$ is not contingent, it is the Arrow component, $\hat{a}'$, that depends on d_p'. This is because the sovereign is less productive in *partial* default — i.e. $\theta^d \le \theta$ — and repudiates its liabilities towards private lenders — i.e. $b = 0$. The Fund's insurance component must therefore discriminate whether the sovereign is in default as the sovereign's risk profile changes.

The private lenders' problem remains static as in Section 2.4. Thus, when the DSA does not bind, the private bond price is given by

$$q_p(\theta,\bar{a}',b') = \frac{\mathbb{E}\{(1-D(\theta',a',b'))[1-\delta+\delta\kappa+\delta q_p(\theta',\bar{a}'',b'')]|\theta\}}{1+r}, \tag{2.28}$$

where $D(\theta,a,b) = D_p(\theta,a,b)+D_f(\theta,a,b)$ with $D_p(\theta,a,b) = 1$ if $V^{ap}(\theta,a) > W^b(\theta,a,b,0)$ and $V^{ap}(\theta,a) \ge V^{af}(\theta)$ and $D_p(\theta,a,b) = 0$ otherwise, while $D_f(\theta,a,b) = 1$ if $V^{af}(\theta,a) > W^b(\theta,a,b,0)$ and $V^{af}(\theta,a) > V^{ap}(\theta)$ and $D_f(\theta,a,b) = 0$ otherwise. The value under *full* default might coincide with the value under *partial* default. Hence, if the sovereign is indifferent between the two types of default, we assume it selects the *partial* default.

However, the price may not depend on the total level of debt $\bar{\omega}'$ anymore but on $\bar{a}'$ and b' separately. As we will see, the split of $\bar{\omega}'$ between $\bar{a}'$ and b' becomes relevant as in *partial* default the sovereign defaults on b' but repays $\bar{a}'$.

2.5.2 The Fund under Seniority

The Fund still aims at making the sovereign's debt safe. Thus, even though it possesses seniority, its announcement continues to relate to the sovereign's entire indebtedness as in the case without seniority. However, in addition to (θ,a,b), the announcement now includes the default status d_p. That is, depending on the *partial* default decision, the sovereign does not necessarily receive

[27]The value under repayment is a simple extension of (2.13) with the additional state variable $d_p = 0$.

the same amount of resource from the Fund. Again, this is because a *partial* default affects the sovereign's risk profile.

As we have seen previously, the sovereign's participation constraint continues to relate to the case of *full* default. The rationale is that, in the contract with seniority, the sovereign defaults on the Fund only in the case of a *full* default. A *partial* default solely affects private lenders. This means that if the value under *partial* default is greater than the value of *full* default in some states, *partial* defaults can occur. In other words, the sovereign's participation constraint alone is not sufficient to prevent *partial* defaults.

The Funds's participation constraint may change in the case of seniority. Particularly, the transfer to the private lenders is now given by $\tau_{p,t} = q_p(\theta_t, \bar{a}_{t+1}, b_{t+1})b_{t+1} - (1 - \delta + \delta\kappa + \delta q_p(\theta_t, \bar{a}_{t+1}, b_{t+1}))b_t(1 - D_t)$. Hence, depending on whether there are *partial* defaults, given (2.2), b_t might not be the same as in the Fund's participation constraint without seniority. The difference comes from the private bond market exclusion and the haircut following a default. Furthermore, given that $\theta^d \leq \theta$, a *partial* default impacts the sovereign's output which also affects the transfer to the Fund, τ_f. Hence, only without *partial* default does the Fund's participation constraint with and without seniority coincide.

2.5.3 The Fund's Minimal Intervention Policy under Seniority

To evaluate the importance of the seniority assumption, we need to check whether the sovereign is willing to follow the Fund's announcement when we impose seniority. For this purpose, we define the Fund's MIP under seniority such that a *partial* default is never optimal.

First, observe that a *partial* default can occur only when the sovereign holds private debt. In other words, if $\bar{a}'_l = \bar{\omega}'_l$ and $b' = 0$, there is no *partial* default. In opposition, if *partial* default is optimal next period, the sovereign would like to set $b' = \bar{\omega}'$ and $\bar{a}' = 0$. Moreover, if $\theta^d = \theta$ for all θ, there is no penalty upon *partial* default meaning that it is not possible to sustain debt in the private bond market. This follows from the standard result in Bulow and Rogoff (1989). In the same logic, if $\theta^d < \theta$ for at least one θ, then the sovereign can hold some level of private debt without being willing to enter *partial* default. Thus, the MIP is the minimal level of debt the Fund should absorb such that (2.27) holds for all θ' for which $\pi(\theta'|\theta) > 0$.

Definition 2.3 (The Fund's Minimal Intervention Policy (MIP) under Seniority)**.** *For a given state θ, we say that the the Fund implements a Minimal Intervention Policy (MIP) under seniority if $\bar{a}_l' = \max\{\underline{a}(\theta, b_l), \underline{\underline{a}}(\theta)\}$ where $\underline{a}(\theta, b_l)$ is given in Definition 2.2 and $\underline{\underline{a}}(\theta) = \max\Big\{\{-\bar{a}' > 0 : (2.27)\text{ binds } \wedge\ \pi(\theta'|\theta) > 0\} \cup \{0\}\Big\}$.*

The sovereign does not have any incentive to enter *partial* default if Definition 2.3 is satisfied. Thus we come up with the following proposition.

Proposition 2.6 (MIP and Partial Default)**.** *In equilibrium, if $\bar{a}_l' \geq \underline{\underline{a}}(\theta)$, the sovereign never enters in partial default. Conversely, if $0 < \bar{a}_l' < \underline{\underline{a}}(\theta)$, the sovereign is willing to enter in partial default in at least one θ'.*

The first part of the proposition directly follows from Definition 2.3: the MIP under seniority is such that there is no *partial* default. Note that depending on the severity of the output penalty and the duration of the private bond market exclusion, (2.27) may hold in all states with $\underline{\underline{a}}(\theta) = 0$. In other words, under the Fund's seniority, the Fund's MIP can be identical to the one without seniority given in Definition 2.2. This is the case in our calibration below. The second part of the proposition states that if the MIP is violated there is a strictly positive probability of a *partial* default next period.

Nevertheless, the next proposition shows that the sovereign cannot deviate from the Fund's MIP. If the DSA binds with strictly positive probability next period, as we have seen in Section 2.4, a negative spread arises and the private lenders prefer borrowing from the Fund rather than lending to the sovereign. In other words, private lenders would like that the Fund absorbs all the sovereign debt. In this case, there is no possibility for the sovereign to deviate — say by accumulating more private debt — from the Fund's MIP.

In contrast, if the DSA does not bind next period, without the MIP (Definition 2.3), there is no private lending sudden stop and the sovereign can freely accumulate debt in the private bond market. In addition, the Fund adapts the insurance it provides to the sovereign according to the *partial* default status. Arrow securities therefore aim at equating wealth not only across productivity states but also across repayment states. Thus, for a given level of debt in the Fund, the repayment decision is not state contingent when the DSA does not bind. However, with the MIP (Definition 2.3), the private lenders anticipate that, giving the Fund debt holdings, partial default will occur with probability one, and, therefore, set the bond price to zero consistent with (2.28). In this case, there is a *lending sudden stop*, not because the private lenders are trying to borrow from the Fund but to escape from a *partial* default. Again, the sovereign cannot deviate from the Fund's MIP.

Proposition 2.7 (No Partial Default)**.** *For a given Fund's announcement $\bar{\omega}_l'$, the sovereign cannot deviate from the Fund's MIP given in Definition 2.3:*

I. If the DSA binds in at least one θ', the private lenders do not lend as of Proposition 2.3.

II. If the DSA does not bind, then for all $b' < -(\bar{\omega}_l' - \underline{a}(\theta))$ we have that $\mathbb{E}[D_p(\theta', -\bar{\omega}_l' - b', b')|\theta] = 1$ implying that $q_p(\theta, -\bar{\omega}_l' - b', b') = 0$.

All in all, depending on the output penalty upon default and the re-access probability, the Fund's MIP might differ in the case with and without seniority. In the former, the Fund may need to absorb relatively more debt. However, in equilibrium, the sovereign cannot profitably deviate from the Fund's MIP. Thus, the entire debt position remains safe as no default — either *partial* or *full* — arise on equilibrium path.[28] Thus, the seniority only affects the Fund and, in that view, a *pari passu* clause is preferable to seniority.

2.6 Calibration

We calibrate the parameters of the model economy by fitting the sovereign debt model (2.9)–(2.10), i.e. the one without the Fund, to quarterly data of Italy over the period 1992Q1 to 2019Q4.[29] Table 2.1 summarizes the value of each parameters.

We calibrate the productivity growth rate shock γ_t with a Markov regime switching AR(1) pro-

[28] Wicht (2023) shows that in an environment without state-contingent contracts, seniority is actually preferable to a *pari passu* regime.

[29] The calibration starts in 1992 due to data availability and ends in 2019 owing to the pandemic. The Appendix 2.5 contains detailed explanations on data sources, measurement, and additonal information on shock process estimation.

Table 2.1: Parameter Values

Parameter	Value	Definition	Targeted Moment
A. Direct measures from data			
α	0.5295	labor share	labor share
λ	0.032	return probability	average exclusion period
r	0.0132	risk-free rate	annual real short-term rate
δ	0.9297	bond maturity	bond maturity
κ	0.0543	bond coupon rate	bond coupon rate
B. Based on model solution			
β	0.96	discount factor	average b'/y
ψ	0.746	productivity penalty	corr(spread, y)
ζ	0.29	labor elasticity	average n, $\sigma(c)/\sigma(y)$ and corr(n, y)
ξ	1.265	labor utility weight	
C. By assumption			
Z	0	Fund's outside option	

Note: The variable $\sigma(\cdot)$ denotes the volatility.

cess to the sample productivity series of Italy. We choose a specification of 2 regimes that we denote by $\varsigma \in \{1, 2\}$, with the first regime capturing the crisis period (i.e. the Great Financial Crisis) observed in the data. Specifically, we estimate the following model for the (net) growth rate $\gamma_t - 1$ with the expectation maximization (EM) algorithm of Hamilton (1990):

$$\gamma_t - 1 = (1 - \rho(\varsigma_t))\mu(\varsigma_t) + \rho(\varsigma_t)(\gamma_{t-1} - 1) + \sigma(\varsigma_t)\epsilon_t, \tag{2.29}$$

where ς_t denotes the regime at t, $\rho(\varsigma_t)$, $\mu(\varsigma_t)$, $\sigma(\varsigma_t)$ are the regime-specific autocorrelation, mean and variance of the process, respectively, and ϵ_t follows an i.i.d. standard normal distribution. As shown in the Appendix 2.5, such a regime switching process can capture the sudden drop in productivity dynamics around crisis periods. In the computation, we further discretize the shock process using the method of Liu (2017) with 15 grid points for each regime. Aguiar and Gopinath (2006) show that given a CRRA utility in consumption $\frac{c^{1-\sigma}}{1-\sigma}$, one requires that $\lim_{t\to\infty} \mathbb{E}_0 \beta^t (\theta_{t-1}^{1-\sigma} - 1)/(1-\sigma) = 0$, so that the discount utility can be well defined with stochastic trend. For the case of log utility, this amounts to $\lim_{t\to\infty} \mathbb{E}_0 \beta^t \log \theta_{t-1} = 0$, which holds automatically in our setup. We subsequently detrend an 'allocation' variable x_t by θ_{t-1}: $\tilde{x}_t = x_t/\theta_{t-1}$.

The preference parameters for labor supply are set to $\zeta = 0.29$ and $\xi = 1.265$. These are used to match the average fraction of working hours and its correlation with GDP, together with the volatility of consumption relative to GDP. The risk free interest rate is fixed to $r = 1.32\%$, the average real short-term interest rates of the Euro area. We further set $\delta = 0.9297$ and $\kappa = 0.0543$ to match the average Italian bond maturity and coupon rate (coupon payment to debt ratio), respectively. Finally, we fix $\beta = 0.96$ to match the average indebtedness relative to annual output. The production function is Cobb-Douglas $f(n) = n^{\alpha}$, and we set $\alpha = 0.5295$ to match the average labor share in Italy.

The default penalty is asymmetric as in Arellano (2008). To ensure that we can properly detrend the penalty, we consider

$$\theta_t^d = \theta_{t-1}\psi\mathbb{E}\gamma_t \text{ if } \theta_t \geq \theta_{t-1}\psi\mathbb{E}\gamma_t \quad \text{and} \quad \theta_t^d = \theta_t \text{ if } \theta_t < \theta_{t-1}\psi\mathbb{E}\gamma_t.$$

One sets $\psi = 0.746$ to match the correlation of spread with respect to output. Furthermore, we fix $\lambda = 0.032$ which corresponds to an average default duration between 7 and 8 years. This is consistent with the average default length Italy recorded during its defaults on external debt in the 1930s and the 1940s (Reinhart and Rogoff, 2011). Note that under such parameter values, the MIP is the same with and without seniority.

2.7 Quantitative Analysis

In this section, we first assess the fit of the model to the data. We then compare the economy with and without the Fund through various exercises.

2.7.1 Model Fit and Comparison

The fit of the model with respect to the data is depicted in Table 2.2. As we calibrate the model to Italy, the relevant benchmark is the economy without the Fund. To compute the moments we run 5,000 simulations of the model with 600 periods each, and we discard the first 200. For the volatilities and correlation statistics, we filter the simulated data — except the spread — through the HP filter with a smoothness parameter of 1600.

As one can see, the model replicates well the average indebtedness of Italy owing to the long-

Table 2.2: Data and Models

Targeted Moments				Non-Targeted Moments			
Variable	Data	Without Fund	With Fund	Variable	Data	Without Fund	With Fund
			A. First Moments				
$b'/y\%$	117.64	116.20	221.00	$\tau/y\%$	2.09	6.49	9.54
$n\%$	38.64	38.23	39.93	spread%	2.50	0.43	0.00
			B. Second Moments				
$\sigma(c)/\sigma(y)$	1.27	1.25	0.28	σ(spread)	0.96	0.11	0.00
corr(n, y)	0.68	0.63	0.99	$\sigma(n)/\sigma(y)$	0.75	1.42	0.62
corr(spread, y)	-0.16	-0.25	0.00	corr(c, y)	0.53	0.04	0.95
				$\sigma(\tau/y)/\sigma(y)$	1.09	2.32	0.72
				corr$(\tau/y, y)$	0.29	0.71	0.98

Note: The variable $\sigma(\cdot)$ denotes the volatility and τ/y denotes the primary surplus (i.e $\theta f(n) - c$) over output. We simulate 5,000 economies with 600 periods each, and we discard the first 200. For the volatilities and correlation statistics, we filter the simulated data — except the spread — through the HP filter with a smoothness parameter of 1600.

term debt structure (Chatterjee and Eyigungor, 2012). We are also matching the share of hours worked and its correlation with output given the specification of the shocks. The same holds true for the volatility of consumption. In addition, the model replicates well the correlation of the spread with output. However, it cannot match the average spread observed in the data.[30] In terms of other non-targeted moments, the model also exaggerates the volatility and the correlation of the primary surplus.

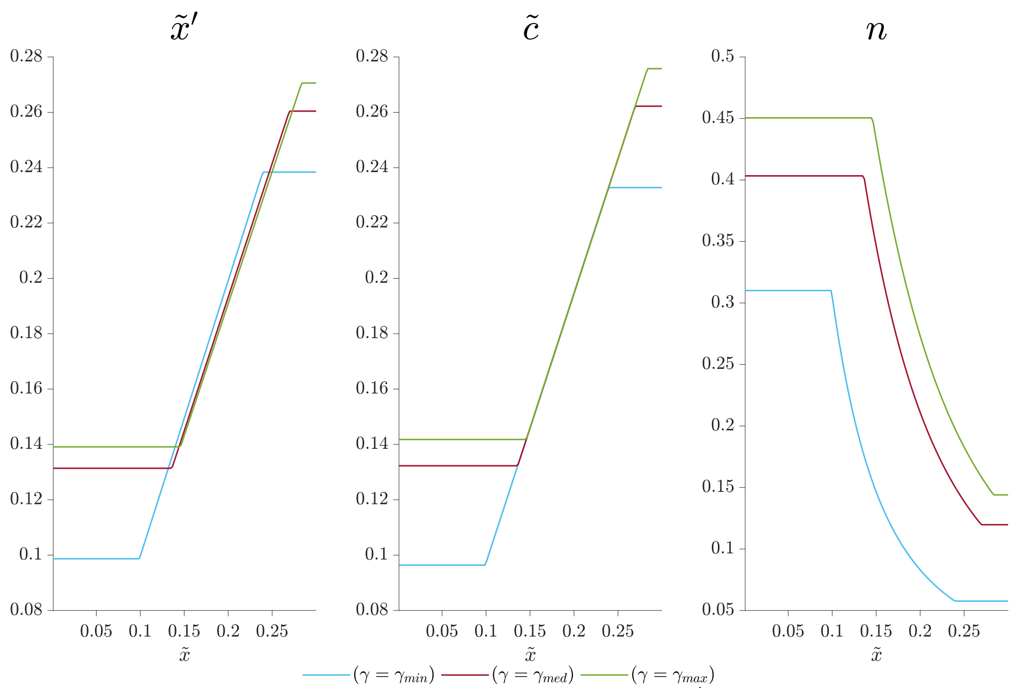
In addition, Table 2.2 compares the economy with and without the Fund. The difference between the two is important. First, the Fund enables a greater accumulation of debt in total. Particularly, the Fund almost doubles the debt capacity of the economy. Nevertheless, with the MIP, the Fund's debt holdings is nil given that the Fund's DSA never binds in steady state as we will see. Second, there is no spread with the Fund, while the spread attains 0.43% without the Fund. Hence, the Fund achieves the goal of making sovereign debt safe — i.e. without default risk. Third, consumption is much less volatile in the presence of the Fund. This means that there is a greater risk sharing across states. This comes from the highly pro-cyclical surplus. In other words, in periods of distress, the Fund provides resources to sustain consumption. Such mechanism is less marked in the economy

[30] Models of sovereign defaults following Aguiar and Gopinath (2006) and Arellano (2008), with stochastic growth shocks and risk-neutral lenders, have similar difficulty to match the average spreads typical for emerging economies. Chatterjee and Eyigungor (2012) manage to match an average spread of 8% by means of long-term debt and quadratic output penalty but do not use growth shocks. Bocola and Dovis (2019) also match the average spread using multiple maturities but target an average spread of 0.61%.

without the Fund owing to the risk premium attached on the debt and the lack of state contingency.

2.7.2 Policy Functions and Financial Variables

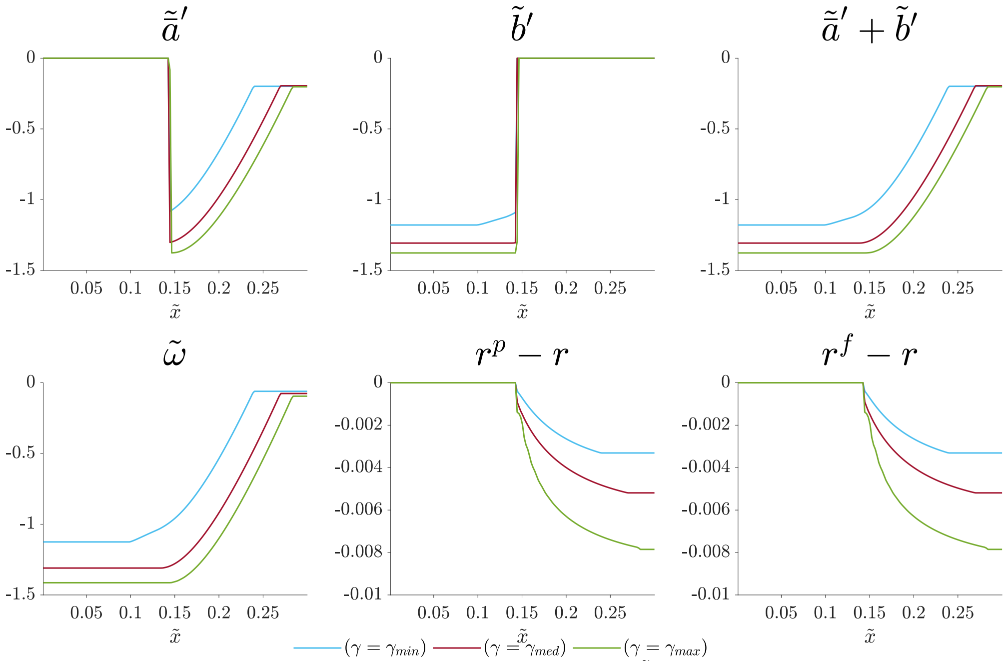
To gain better understanding of the working of the Fund, we first present the numerical solutions of the policy functions of the Fund under our calibration. Figure 2.1 depicts the the different policy functions for zero private debt as a function of $(\gamma, \tilde{x})$, while Figure 2.2 depicts the main financial variables. All figures relate to the detrended version of the model presented in the Appendix 2.2. We focus on three main values of the growth rate: the smallest one, γ_{min}, the median one, γ_{med}, and the highest one, γ_{max}. We denote the annual output by $\tilde{y}$.



Note: The Figure depicts the optimal policies for the relative Pareto weight, $\tilde{x}'$, consumption, $\tilde{c}$, and labor, $\tilde{n}$ as a function of $(\gamma, \tilde{x})$. We fix $\tilde{b} = 0$ and consider three main values of the growth rate: the smallest one, γ_{min}, the median one, γ_{med}, and the highest one, γ_{max}.

Figure 2.1: Optimal Policies with Zero Private Debt as Function of $(\gamma, \tilde{x})$

Figure 2.1 presents the optimal policies with respect to the future relative Pareto weight, consumption and labor as function of $(\gamma, \tilde{x})$. As explained in Section 2.3 and in the Appendix 2.1, the recursive formulation of the Fund relies on the relative Pareto weight $\tilde{x}$ which keeps track of the binding constraints. With a logarithmic utility, one has that $\tilde{c} = \tilde{x}'\frac{\gamma}{\eta}$. Both $\tilde{c}$ and $\tilde{x}'$ are increasing, while n is decreasing in the current relative Pareto weight $\tilde{x}$. In each panel, the horizontal line



Note: The Figure depicts the optimal policies for the debt in the Fund, $\tilde{\tilde{a}}'$, the debt in the private bond market, $\tilde{b}'$, together with the outstanding total liabilities, $\tilde{\omega}$, the spread in the Fund, $r^f - r$, and the spread in the private bond market, $r^p - r$. We fix $\tilde{b} = 0$ and consider three main values of the growth rate: the smallest one, γ_{min}, the median one, γ_{med}, and the highest one, γ_{max}.

Figure 2.2: Financial Variables with Zero Private Debt as Function of $(\gamma, \tilde{x})$

on the left hand side is determined by the sovereign's binding participation constraint, while the horizontal line on the right hand side is determined by the Fund's binding participation constraint. The line rejoining both horizontal lines is determined by the first best allocation and has a slope of $\eta < 1$.

We now turn to the financial variables depicted in Figure 2.2. The first row of the figure represents the prospective debt holdings of the sovereign. Consistent with the definition of MIP, when the Fund's DSA does not bind, the credit line of the Fund is nil. Conversely, when the Fund's DSA binds, there is a private lending sudden stop. With zero initial private debt this translates into a complete stop of private lending activities. In this case, the debt accumulation is largely reduced.

The second row of Figure 2.2 depicts the current asset holdings and the interest spreads. One sees that when the Fund's DSA is binding, $\tilde{\omega}$ is very close to zero because of Definition 2.2 and the fact that $Z = 0$ and $\tilde{b} = 0$. As $\tilde{\omega} = \tilde{\tilde{\omega}} + \tilde{\tilde{a}}(\gamma)$, this tells us that if the Fund's DSA is binding today then the value of the sovereign's debt is in great part offset by the value of the realized Arrow security. Hence, when the Fund's DSA binds, the sovereign is limited in the trade of both Arrow securities and bonds.

Regarding interest rates, the Fund's and private bonds market's spreads are nil when the Fund's DSA is not binding consistent with Corollary 2.1. In contrast, spreads are negative when the Fund's DSA is binding consistent with Proposition 2.2. As one can see, the negative spread remains relatively modest in terms of magnitude.

2.7.3 Steady State Analysis

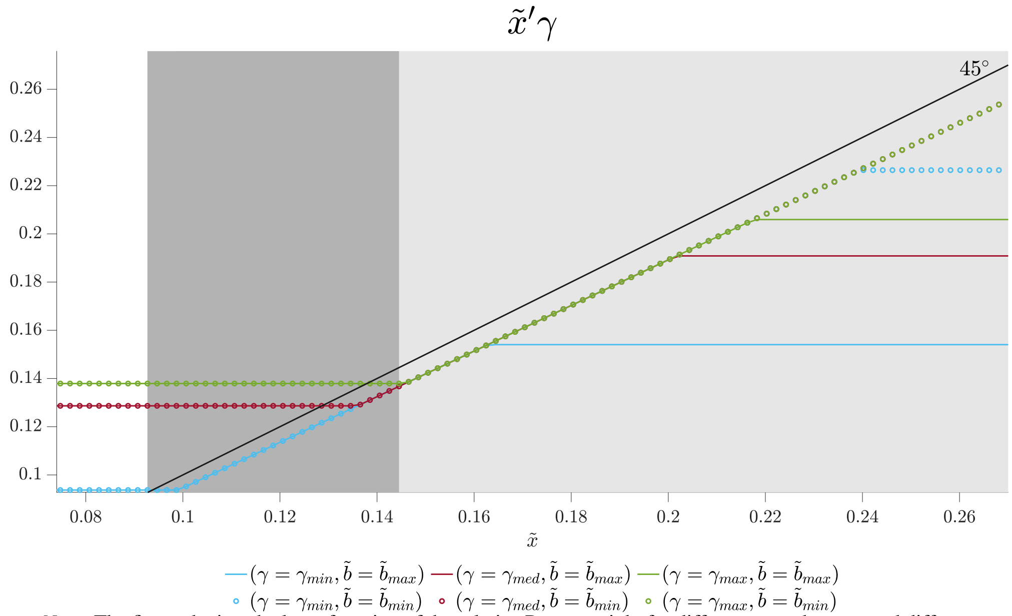
As detailed in Appendices 2.1 and 2.3, the relative Pareto weight, $\tilde{x}$, is key to the dynamics of the model economy as it represents a sufficient statistic of the contract's binding constraints. We first explain the dynamic of the relative Pareto weight before simulating the economy with and without the Fund in steady state.

Figure 2.3 displays the law of motion of the relative Pareto weight. The dark grey region represents the ergodic set given in Definition B.1. The light grey region represents the basin of attraction of the ergodic set. As one can see, the convergence path to the steady state depends on the level of privately held debt. Especially, the larger is the level of private debt, the closer the economy gets to the ergodic set. This is different than in Ábrahám et al. (2022) where the convergence path solely depends on $\tilde{x}$ ad γ.

Most importantly, we see that the Fund's DSA does not bind in steady state. This has two main consequences. First, the private lending sudden stop exposed in Proposition 2.3 does not arise in the long run. Second, in line with Definition 2.2, the Fund's holding of sovereign debt is nil — i.e. $\bar{a}' = 0$. In other words, the Fund solely provides insurance.[31]

We simulate the economy within the ergodic set of relative Pareto weights. For this purpose, we generate one history of shocks for 500 periods in steady state starting with the lowest Pareto weight in the ergodic set. To avoid that the initial conditions blur the results, the first 200 periods are discarded. To gauge the impact of the Fund's intervention in this exercise, we simulate both

[31] Also, consistent with Corollary B.1, the average bond maturity is irrelevant.

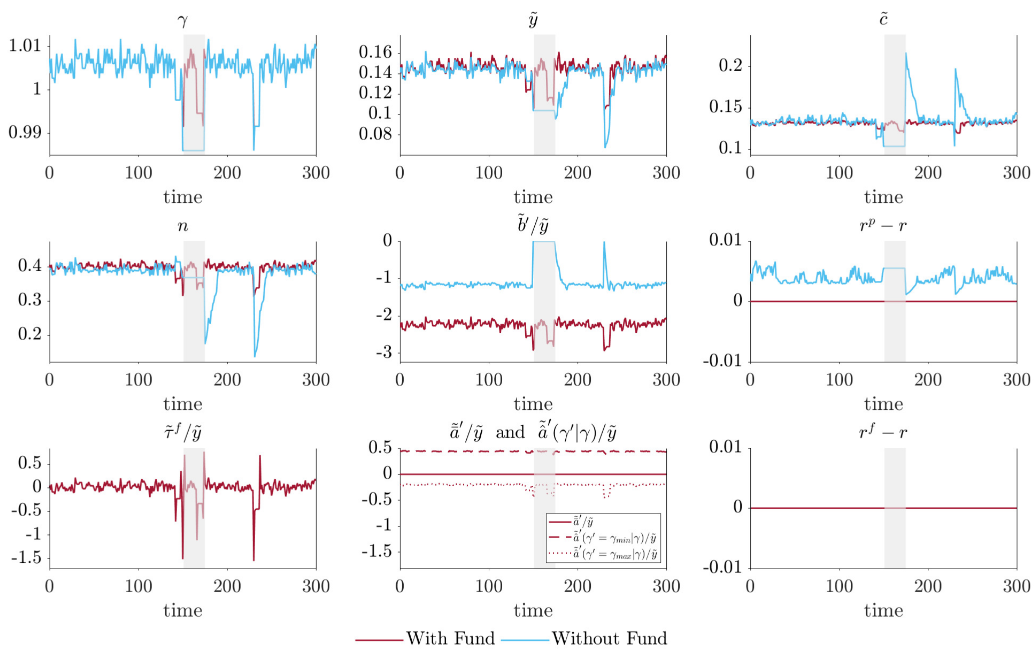


Note: The figure depicts the law of motion of the relative Pareto weight for different growth states and different private debt levels. The dark grey x-axis region is the ergodic set which defines the steady state of the economy. The light grey region is the basin of attraction of the ergodic set. We take $\tilde{b} = \tilde{b}_{min} = 0$ and $\tilde{b} = \tilde{b}_{max}$ and consider three main values of the growth rate: the smallest one, γ_{min}, the median one, γ_{med}, and the highest one, γ_{max}.

Figure 2.3: Evolution of the Relative Pareto Weight in Steady State as a Function of $(\gamma, \tilde{b}, \tilde{x})$

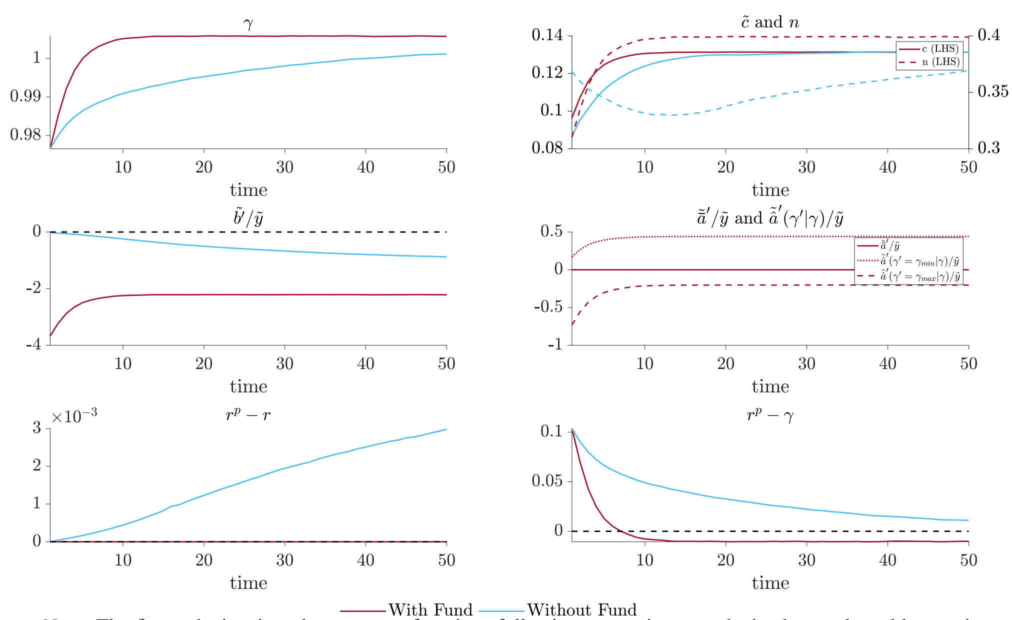
the economy with and without the Fund in parallel.

Figure 2.4 depicts the simulation result with the grey region representing the periods in which the economy without the Fund is in default. With the Fund's intervention, the economy has a more stable consumption path over time. The sovereign avoids the major fluctuations of consumption that characterise the standard incomplete market economy with defaults. Moreover, the sovereign is able to accumulate private debt at the risk-free rate in regions where it would normally default without the Fund. This is entirely due to the fact that the entire debt position is hedged by Arrow securities. To get a sense of the insurance component, we display the Arrow securities purchased today for the highest and the lowest states tomorrow. Two points deserve to be noted. First, the portfolio of Arrow securities is procyclical as it closely follows the shock process. Second, the positions taken in Arrow securities are substantial. If one focuses on $\tilde{a}'(\gamma'|\gamma)$ for $\gamma' = \gamma_{min}$, we see that it amounts on average to roughly 50% of annual GDP. Instead of looking at the Arrow securities one can observe the Fund's primary surplus, $\tilde{\tau}_f$, which also moves procyclically and largely oscillates around zero since $Z = 0$.



Note: The figure depicts the simulation of a specific steady state path on selected key variables. The economy without (with) the Fund is in blue (red). The grey area represents the region in which the economy without the Fund is in default. We simulate one history of shocks for 500 periods in steady state starting with the lowest Pareto weight in the ergodic set. To avoid that the initial conditions blur the results, the first 200 periods are discarded.

Figure 2.4: Simulation of a Specific Steady State Path



Note: The figure depicts impulse response functions following a negative growth shock on selected key variables. The economy without (with) the Fund is in blue (red). Impulse response functions are obtained by averaging the simulation of 5,000 independent shock histories for 50 periods starting with $\gamma = \gamma_{min}$ and initial debt holding and relative Pareto weight drawn from the ergodic set.

Figure 2.5: Impulse Response Functions to a Negative γ Shock

Figure 2.5 depicts the impulse response functions resulting from a stark negative growth shock on selected key variables.[32] The responses are computed as the mean of 5,000 independent shock histories starting with the lowest growth shock as well as initial debt holdings and relative Pareto weights drawn from the ergodic set. In the very first periods following the negative shock's realization, the Fund provides additional insurance to the sovereign. This sustains the existing level of debt and prevents a large decrease in consumption and a large increase in labor supply. Hence, without the Fund's intervention, the sovereign repudiates its debt and is obliged to provide more labor to avoid a massive reduction in consumption. Thus, the immediate impact of a sudden low growth shock is more severe in the absence of the Fund. In the long run, the sovereign without the Fund is likely to repudiate debt again and therefore reaches a lower level of steady state indebtedness. Besides this, the economy with the Fund avoids the positive spread in the private bond market. It can therefore reach more quickly a low level of $r^p - \gamma$ easing debt management.

2.7.4 Welfare Analysis

Sharp difference in the dynamics of the economy with and without Fund translates into superior welfare implications of the Fund. The first column of Table 2.3 represents the welfare gains of the Fund's intervention in consumption equivalent terms at zero initial debt holdings. Recall that the sovereign which has access to the Fund can hold debt in the Fund or in the private bond market. Thus, to adequately compare the two economies, we compare them for the same *total* debt holdings. That is, the welfare comparisons are computed at the points where $\tilde{\omega} = 0$ for the economy in the Fund and at $\tilde{b} = 0$ for the economy outside the Fund. The welfare computation is presented in the Appendix 2.6.

Welfare gains are significant with the Fund's intervention. With zero initial debt, the consumption-equivalent welfare gains are on average 14%. Moreover, the largest welfare gains are recorded in low growth states. Thus, the Fund's intervention is mostly valued when the sovereign is in a difficult economic situation. As mentioned above, welfare gains are the consequence of two main

[32]Figures B.2 and B.3 in the Appendix 2.7 present the impulse response functions to a negative and to a positive shock of all relevant variables in the model, respectively.

Table 2.3: Welfare Comparison at Zero Initial Debt

State	Welfare Gains (%)	Maximal Debt Absoption (% of GDP)	
		With Fund	Without Fund
$\gamma = \gamma_{min}$	15.27	496	224
$\gamma = \gamma_{med}$	14.01	244	125
$\gamma = \gamma_{max}$	13.82	204	104
Average	14.05		

Note: The table reports welfare gains of the Fund's intervention at zero initial debt in consumption equivalent terms. The welfare computation is presented in the Appendix 2.6.

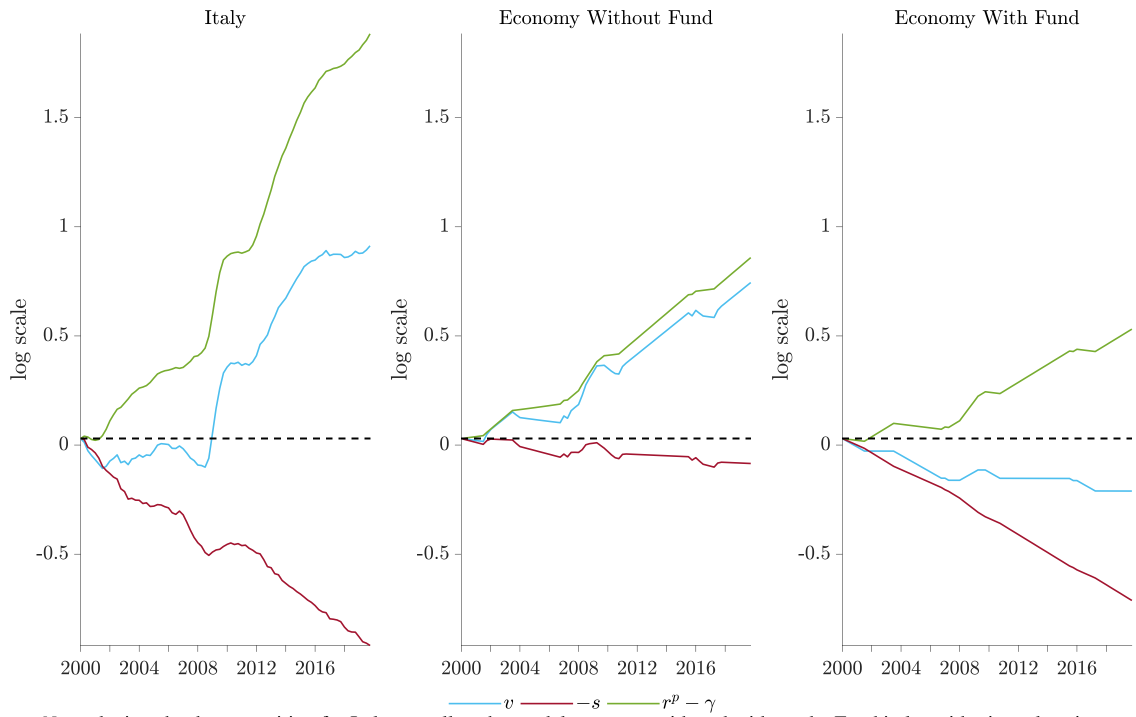
features of the Fund's intervention. First, the Fund provides state-contingent transfers and therefore enhances consumption smoothing. Second, it enables a greater accumulation of debt in general. As one can see in the last two columns of Table 2.3, the the maximal debt absorption of the economy is almost always twice larger with the Fund than without.

To be more precise on the source of the aforementioned welfare gains, in the Appendix 2.6, we provide a decomposition of the welfare gains. We show that they are mostly due (i.e. above 90%) to the greater debt capacity and the insurance component; among these two factors debt capacity represents the largest share of total gains (i.e. circa 85%).

2.7.5 Debt Dynamic Decomposition

We further decompose the evolution of the debt according to Cochrane (2020, 2022): sovereign debt at the end of the year, v_{t+1}, is equal to its value at the beginning of the year, v_t, plus the net cost of keeping debt, $r_t^p - \gamma_t$, and the year's primary deficit (excluding interest payment), $-s_t$, so that $v_{t+1} = v_t + r_t^p - \gamma_t - s_t$, assuming no discounting for simplification. In our environment, the primary surplus without interest payment corresponds to $\tilde{b}_{t+1} - \tilde{b}_t$ for the economy without the Fund and $\tilde{\tilde{\omega}}_{t+1} - \tilde{\omega}_t$ for the economy with the Fund.

Figure 2.6 depicts the decomposition for Italy as well as the model economy with and without the Fund in logarithmic scale. We generate the two panels for the model economy by feeding the smoothed growth path of Italy over 2000Q1–2019Q4 into the model and start with the same



Note: depicts the decomposition for Italy as well as the model economy with and without the Fund in logarithmic scale using the decomposition proposed by Cochrane (2020, 2022). The variable v corresponds to the value of debt, $-s$ is the primary deficit and $r^p - \gamma$ is the interest-growth differential. A line above zero contributes positively to debt accumulation, while the opposite holds for a line below zero.

Figure 2.6: Cochrane Decomposition

level of debt of Italy in 2000Q1.[33] We then obtain the path of debt and interest rate through the optimal policy functions. The blue line represents the evolution of the value of debt which is the combination of the green line (i.e. $r^p - \gamma$) and the red line (i.e. $-s$). In view of this, had the accumulation of debt been costless (i.e. $r^p - \gamma = 0$), then the blue line would coincide with the red line.

We observe that the evolution of Italy's debt is the result of two conflicting forces: a remarkable history of increasing accumulated primary surpluses and two decades of growth decline resulting in accumulated costs $r^p - \gamma$. The model without the Fund replicates well the dynamic of the Italian public indebtedness. It nonetheless minimises the positive impact of primary surpluses and the negative impact of the interest rate-growth differential.

Turning to the economy with the Fund, we see that the evolution of debt is flatter than in the economy without. This comes from two components. On the one hand, the rate at which the

[33] We consider a smoothed version of the growth path to avoid defaults in the economy without the Fund.

sovereign issues debt is at most risk free. This therefore largely reduces the $r^p - \gamma$ cost compared to the economy without the Fund. On the other hand, the Fund provides insurance through Arrow securities. This eases debt management by making fiscal policy countercyclical as shown previously. As a result, the debt path is more smooth. Particularly, the model predicts that the Italian indebtedness by the end of 2019 would have been around 80% of GDP rather than 135% if Italy could have joined the Fund in 2000.[34]

This shows that the path followed by the Italian economy in the last two decades was highly inefficient. The Italian government's perseverance in maintaining positive primary surpluses, in spite of growth reversals, can be seen as a commitment to debt sustainability, in line with the European Union's fiscal policy. Indeed, the accumulation of large primary surpluses dampened the increase in Italian indebtedness, but this was a highly inefficient path to have been followed compared with the path that could have been followed with the Fund.

2.8 Conclusion

A starting point of this research has been the recognition that in a monetary union, such as the Euro area and as the result of the 21st Century crises, not only sovereign debt is very high, but also that a large fraction of the union-countries' sovereign debt is being held in Euro area institutions. This has helped 'stressed countries', reducing sovereign debt spreads — for example, in the Euro crisis in 2012. However, a simple maturity transformation or a long-term holding of sovereign debts may not be the most efficient debt management policy for the union. In fact, Ábrahám et al. (2022) has already shown that there can be high efficiency and welfare gains from having a *Financial Stability Fund*, with the proviso that the Fund absorbs all the sovereign debt of a country. We remove this proviso and show that the gains are still very high. Particularly, we show that Fund's intervention needs only to be minimal. Such *minimal intervention policy* (MIP) consists of an insurance component with an additional guarantee on long-term debt holdings by private lenders when the DSA binds, as *prudential policy* to prevent 'excessive lending' when

[34] We obtain this figure by computing the model implied debt-to-GDP ratio at the end of the sample period using the decomposition of Cochrane (2020, 2022).

sovereign debt is safe, we call it *the pecuniary externality of a negative spread*.

In sum, there are many interesting features to our results but we want to emphasize the two key elements that give the Fund a leading role in 'making sovereign debt safe' even with a MIP. The two elements also require innovation with respect of existing official lender's practices. First, the existence of a proper country risk-assessment, accounting for the effect of the constrained efficient Fund contract. Second, the role of the Fund state-contingent contract in defining a thick contingent (and contention) wall between the level of liabilities which is sustainable and the level which is not. And, linking the two, its role in coordinating lenders' and sovereign's beliefs with its announcements. As we said, most of the sovereign literature has focused on default problems, but in a mature union, outright default or exit may be rare events.[35] However, with the uncertainty and challenges that even advanced economies face, debt sustainability can remain a persistent concern for years to come and, even if sovereign debt is perceived to be safe, excessive lending can be a problem that private lenders may not internalize. We hope our work will not only contribute to the existing literature but also to face these challenges.

Finally, we show in our calibration to the Italian economy and subsequent simulations and computations, how important welfare gains can be achieved by improving existing official lending practices offering long-term state-contingent Fund contracts, even when there is debt accumulation or $r - g$ uncertainty, as most countries nowadays face.

[35] The recent Brexit shows that exit can happen or, alternatively, that the union was still immature.

Chapter 3

Seniority and Sovereign Default: The Role of Official Multilateral Lenders

Abstract Sovereign countries do not necessarily repay all its creditors. There is a clear pecking order in which official multilateral lenders – i.e. mainly the International Monetary Fund and the World Bank – are given priority in repayment. Yet, this preferred status is a market practice and is not legally binding. This paper documents the source and the consequences of the *de facto* seniority of official multilateral lenders. Empirically, I present evidence that defaults involving such lenders are infrequent, last relatively longer and are associated with greater private creditors' losses. To rationalize those findings, I build a model of endogenous defaults and renegotiations with heterogenous lenders. The key component behind the *de facto* seniority is the typical policy of non-toleration of arrears adopted by official multilateral lenders. Combined with the default penalty, this policy rationalizes the aforementioned empirical facts and generates important spillovers on other creditors. The borrower values the use of official multilateral debt and would not necessarily prefer other seniority regimes.

3.1 Introduction

Excluding advanced economies, debt from (official) multilateral lenders – i.e. mainly the International Monetary Fund (IMF) and the World Bank (WB) – represents more than 35% of the total sovereign debt and is beside bonds the second largest category of sovereign borrowing.[1] It has the peculiarity that multilateral lenders are paid ahead of other creditors and, when payments are deferred, are usually repaid in full (Schlegl et al., 2019). Yet, legally speaking, nothing enforces this pecking order. In other words, market participants give a special rank to multilateral lenders even though they have no legal obligations to do so. This suggests the existence of a *de facto* – as opposed to *de jure* – seniority structure. The literature on sovereign debt and default has generally overlooked this implicit seniority structure modelling defaults as symmetric across creditors. The present study seeks to fill this gap. It investigates the source and the consequences of the preferred creditor status of multilateral lenders.

I begin this inquiry by establishing new empirical facts on multilateral creditors based on 187 episodes of external debt's default from 1970 to 2014.[2] First and foremost, defaults with multilateral creditors are infrequent. In the sample at hand, such events represent around 18% of all reported episodes. Second, they usually last longer than other defaults taking roughly 9 years to be resolved. In opposition, defaults on other types of creditors last on average 3 years. Third, I find that for default episodes with multilateral creditors, the average haircut on private creditors raises to 59%, while it falls to 33% otherwise. Finally, multilateral lenders always lend at preferential rates. All these facts hold after controlling for the default's and the country's characteristics.

Having identified the main empirical facts linking defaults with multilateral creditors, I build a model capable of rationalizing them. For this purpose, I augment the standard model of Eaton and Gersovitz (1981) with heterogenous lenders and endogenous restructurings. I assume the existence of a continuum of competitive private lenders and one multilateral lender. There is

[1] See Appendix 3.1 for a breakdown of the world sovereign debt between 1970 and 2020.

[2] Default's dates come from Asonuma and Trebesch (2016) and haircuts from Cruces and Trebesch (2013). I then identify the creditors involved in each default using the database of Beers et al. (2022).

limited enforcement in repayment and the sovereign can decide to default either on private creditors – *partial* default – or on both creditors – *full* default. Each default is followed by a complete market exclusion and an output penalty. In a *partial* default, the sovereign continues to service the multilateral debt while being in autarky.

A central idea in my theory is that the multilateral lender has a greater enforcement power than the private lenders. On the one hand, defaulting on the multilateral debt entails a greater output penalty than defaulting on the private debt. On the other hand, the multilateral lender does not lend when the sovereign is in arrears and the outstanding multilateral debt has not been repaid in full.[3] This second element corresponds to a policy of non-toleration of arrears characteristic of the IMF's and the WB's practice. The greater enforcement power implies that the multilateral debt is *de facto* senior to private debt.[4] The policy of non-toleration of arrears safeguards the preferential rate of multilateral lending, whereas the output penalty controls the frequency of multilateral debt's defaults.[5]

The key feature in the model is that the *de facto* seniority of multilateral debt generates important spillovers on private lenders. On the one hand, the private debt is subordinated meaning that private creditors receive what is left after the full repayment of multilateral debt. Hence, in a *full* default, the level of multilateral debt directly affects the private debt's recovery value. On the other hand, a larger stock of multilateral debt reduces the value of a *partial* default as it raises the multilateral debt servicing costs in autarky. Thus, while the multilateral debt raises the subordination risk of private liabilities, it can reduce the default risk up to a certain point. The net effect depends on the size of the multilateral debt.

Given this, the *de facto* seniority impacts the sovereign borrowing, the default's decision and the restructuring's process. In terms of borrowing, the multilateral debt is less sensitive to the default risk but at the cost of subordinating private debt. Thus, more multilateral debt may or may

[3]In a default, there is no accumulation of arrears. Missed coupon payments are forgone.

[4]We say that seniority is *de jure* when enforced by *ex ante* contractual requirements. In my analysis, seniority is *de facto* as it emerges from *ex post* sanctions.

[5]Notice that I abstract from conditionality in my analysis which is another feature of multilateral lending.

not increase the marginal benefit of debt issuance. In the former case there is a seniority benefit, while in the latter a seniority cost to the repayment incentive. I find that the seniority benefit usually manifests when the stock of multilateral debt is relatively small. In addition, the reduced sensitivity to the default risk renders the multilateral debt less prone to dilution than the private debt. The possibility to dilute private debt reduces the marginal cost of debt issuance as it reduces the future debt burden. This is what I call the subordination benefit. There is therefore a tradeoff between insurance and incentive shaping the seniority structure.

Regarding the default's decision, there are two effects. On the one hand, more multilateral debt reduces the value of a *partial* default owing to larger multilateral debt servicing costs in autarky. As a result, such type of default becomes less attractive. On the other hand, a larger portion of multilateral debt increases the probability of *full* default. The default's decision is therefore shaped by the level of punishment on the different types of defaults and the level of multilateral indebtedness relative to the stock of private debt.

Finally, regarding the debt restructuring's process, the model predicts larger haircuts and longer default's durations when the multilateral lender is involved. This is a consequence of the non-toleration of arrears which renders restructurings more costly. In particular, the sovereign can issue new multilateral debt only after clearing arrears (i.e. after the restructuring). This combined with the full repayment of outstanding multilateral debt reduces the sovereign's value of restructuring which in turns increases both the private creditors' loss and the default's length. Thus, what is at the source of the *de facto* seniority also explains the greater default's lengths and haircuts observed in the data.

The model is calibrated to match moments related to Argentina. Except for the share of *full* defaults, none of the aforementioned empirical facts is directly targeted. I find that the model fits the data particularly well. It is capable of generating the observed default's lengths and haircuts depending on the different types of creditors involved. Nevertheless, it somewhat underestimates the average duration in a *full* default. The larger haircut can be directly attributed to the two building blocks of the policy of non-toleration of arrears: the full repayment of the multilateral debt and the inability to issue new multilateral debt when clearing arrears. The longer duration mainly comes from a greater private debt accumulation triggered by the aforementioned policy.

I subsequently conduct a series of counterfactual analyses. For instance, I study what happens when one introduces a *de jure* seniority or a *pari passu* clause. Under a *de jure* seniority, the sovereign adopts a more reckless debt management with a larger debt ratio and a larger default rate than in the benchmark model. This is because the multilateral debt trades at the risk-free rate irrespective of the default rate. In opposition, under a *pari passu* clause, debt accumulation is more disciplined than in the benchmark model. This is because the default rate largely impacts the multilateral debt price. Hence, reinforcing the seniority of the multilateral debt seems detrimental to fiscal discipline.

Moreover, for those two exercises, I find welfare losses for the sovereign especially in regions of debt crises. A *de jure* seniority is too strict and does not allow for full debt repudiation, while a *pari passu* clause drastically limits the last-resort aspect of the multilateral debt. Hence, the sovereign values the *de facto* seniority of multilateral debt and would not necessarily prefer other seniority regimes.

The paper is organized as follows. Section 3.2 reviews the existing literature. Section 3.3 introduces the conventions on sovereign debt seniority. Section 3.4 presents the empirical analysis. Section 3.5 describes the economic environment of the model. Sections 3.6 and 3.7 present the decisions regarding repayment and renegotiation, respectively. Section 3.8 characterizes the optimal seniority structure. Sections 3.9 and 3.10 present the calibration and the result of the quantitative analysis, respectively. Finally, Section 3.11 concludes.

3.2 Literature Review

The paper combines elements of the empirical literature about sovereign debt with elements of the theoretical literature about sovereign debt, seniority and official lending.

In the empirical literature on sovereign debt, Benjamin and Wright (2013) are one of the first to document the main statistics on sovereign debt renegotiations.[6] Building a more comprehensive dataset, Cruces and Trebesch (2013) refine the previous analysis and present evidence that haircuts impact the bond spreads and the market exclusion's length. Similarly, Asonuma and Trebesch (2016) show that preemptive restructurings are associated with shorter durations and lower creditors' losses relative to post-default restructurings. In addition, Asonuma and Joo (2020) present evidence that the economic conditions on the side of foreign creditors largely influence the length and the terms of a restructuring. Closer to my analysis, Asonuma et al. (2023) document that haircuts are greater on short-term than on long-term bondholders. I contribute to this literature by showing that haircuts and default duration also depend on the type of creditors involved in the default episode.

The starting point of the theoretical literature on sovereign debt is the study of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008).[7] To replicate the characteristics of emerging economies, the original model has been expanded in five main dimensions. First, Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) introduced long-term debt. Subsequently, Arellano and Ramanarayanan (2012) and Niepelt (2014) included mixed maturities. Third, Mendoza and Yue (2012) endogenized the income process and the default cost. Fourth, Arellano et al. (2023) introduced partial defaults to account for arrears accumulation. Finally, Bi (2008), Yue (2010) and Benjamin and Wright (2013) endogenized the renegotiation process assuming either a cooperative or a non-cooperative game between the lenders and the borrower.[8] All aforementioned studies assume that haircuts and default durations are symmetric across creditors. This paper first documents that this assumption is not supported by the data. It then introduces two creditors with different enforcement power to address this matter.

The paper is further related to the literature on the seniority structure of sovereign debt. Erce and Mallucci (2018) present evidence that countries discriminate between domestic and foreign

[6] See also the earlier work of Lindert and Morton (1989), Rieffel (2003), Finger and Mecagni (2007), Díaz-Cassou et al. (2008), Sturzenegger and Zettelmeyer (2008) and Trebesch (2011).

[7] See also Aguiar and Amador (2014) and Aguiar et al. (2016) and Aguiar and Amador (2021).

[8] Their framework has been recently extended by Dvorkin et al. (2021) and Mihalache (2020) to account for mix maturities, by Asonuma and Trebesch (2016) to distinguish between preemptive and post-default restructurings, by Asonuma and Joo (2020) to introduce risk-averse creditors and by Fourakis (2021) to account for reputation.

creditors when defaulting. Among foreign creditors, Schlegl et al. (2019) show that the seniority is mostly *de facto* and that multilateral lenders enjoy the highest seniority. Theoretical models only partially addressed this issue, though. While many studies take the seniority structure as given,[9] few explicitly model the mechanism leading to a seniority structure of sovereign debt. Chatterjee and Eyigungor (2015) define senior tranches as the tranches which were issued first. Dellas and Niepelt (2016) and Ari et al. (2018) generate an implicit seniority structure by means of the default penalty. Conversely, Bolton and Jeanne (2009) develop a model in which the *de facto* seniority emerges from the renegotiation process. Finally, Cordella and Powell (2021) generate a preferred creditor status through commitment in lending. I focus on the ability of the lenders to enforce repayment through the output penalty and the renegotiation process. This generates a tradeoff between multilateral and private debts similar to the one between short-term and long-term debts in Arellano and Ramanarayanan (2012) and Niepelt (2014). The main difference is that more multilateral debt – unlike short-term debt – does not always increase the incentive to repay.

Finally, the paper also connects to the literature on official lending. Building on Ábrahám et al. (2019), Liu et al. (2020) find that the seniority of a multilateral lending institution is not necessarily preferable to a *pari passu* regime.[10] In opposition, I show that the seniority structure of sovereign debt is necessary to sustain the last-resort function of multilateral lending. Such function is important as it often relates to the catalytic effect of multilateral lending. Corsetti et al. (2006), Morris and Shin (2006) and Rochet and Vives (2010) show theoretically that the provision of multilateral debt can bolster the inflow of private funds. However, empirical analyses remain inconclusive and present at most mixed evidence. Focusing on the IMF, the most recent studies have therefore sought to explain this ambivalence.[11] For instance, extending the framework of Corsetti et al. (2006), Krahnke (2020) shows that the *de facto* seniority of the IMF can lead to a crowding-out

[9]For example, Hatchondo et al. (2017) consider the case of adding a non-defaultable bonds beside traditional defaultable bonds. Similarly, Gonçalves and Guimaraes (2014) analyze the link between fiscal policy and sovereign default taking the seniority structure as given. Analysing the interaction between default, private and multilateral debt, Boz (2011) and Fink and Scholl (2016) adopt the same modelling strategy.

[10]Relatedly, focusing on self-fulfilling debt crises, Galli (2021) argues that the seniority of the IMF can give rise to more coordination failures among private lenders than a *pari passu* clause.

[11]See notably Saravia (2013), Erce and Riera-Crichton (2015), Gehring and Lang (2018) and Krahnke (2020) for empirical analyses and Zwart (2007) and Krahnke (2020) for theoretical ones.

of private funds if the IMF support is sufficiently large. I find a similar effect. In addition, similar to Cordella and Powell (2021), I stress the importance of the policy of non-toleration of arrears in shaping the *de facto* seniority. Finally, I show that different seniority regimes imply different behaviors of debt accumulation and default.

3.3 Multilateral Lenders and Seniority

This section reviews the existing conventions on sovereign debt seniority. Having supreme and unrestricted power as a sovereign state, a government can always choose to breach the terms of its debt obligations. Despite major improvements in the 1990s, international law remains limited in enforcing reimbursements of sovereign debt and offers little guidance on the repayment priority of creditors.[12] Furthermore, there exists no supranational entity capable of prosecuting defaults or supervising restructurings of sovereign debt.[13] Thus, the seniority structure of sovereign debt is mostly implicit (Gelpern, 2004). That is why one refers to a *de facto* seniority, as a matter of *ex post* conduct, in contrast to a *de jure* seniority, as a matter of *ex ante* legal requirement.

More precisely, a *de jure* seniority structure relates to *ex ante* enforceable legal clauses that give priority to some creditors. The European Stability Mechanism (ESM), for example, has a *de jure* seniority with respect to the market, meaning that countries obtaining financial support from that institution are legally compelled to prioritize the ESM's repayment.[14] In opposition, a *de facto* seniority structure does not originate from initial contracting clauses or laws. Rather it is a feature that is the result of some *ex post* practice or convention.

Yet, it is the multilateral lending institutions such as the IMF and the WB which enjoy *de facto*

[12]Even though there exist eminent litigation cases in which creditors successfully enforced repayments (e.g. Bank and Trust Company against the Central Bank of Brazil or Elliott Associates against the Republic of Panama and Banco de la Nación in Peru), few cases managed to obtain full repayment. The existing legal framework therefore remains relatively limited in enforcing debt repayments (Panizza et al., 2009). Plus, it provides no explicit priority system for creditors involved in restructurings (Martha, 1990; Gelpern, 2004). Nevertheless, it has gained in importance since the 1990s with notably the development of specialized distressed debt funds and the use of *pari passu* clauses (Schumacher et al., 2021).

[13]See Krueger (2001) for one of the most influential proposals on that matter.

[14]The only exception relates to the program with Spain which was not senior only because of a transitional agreement with the European Financial Stability Facility.

seniority.[15] Neither the IMF's nor the WB's Articles of Agreement mention any seniority or preferred creditor status (Raffer, 2009). However, the market participants acknowledge and respect this implicit seniority structure (Standard & Poor's, 2000). That is, those lending institutions are paid ahead of other creditors and, when payments are deferred, are usually repaid in full (Schlegl et al., 2019). As one can see in Figure C.3 in Appendix 3.1, the IMF and the WB never represented more than 4% of the total amount of debt in default over the years. Similarly, from Figure C.4, the two institutions combined never accounted for more than 11% of the countries in default. None of the other reported creditors such as the Paris Club and other official creditors has a better record.

Interestingly, the aforementioned international financial institutions did not initially endorse their *de facto* seniority status (Martha, 1990; Raffer, 2009). Regarding the IMF, many of its loans were restructured jointly with other types of debts in the 1960s (Beers and Mavalwalla, 2018). Subsequently, in the 1970s and until the late 1980s, multiple countries started to accumulate substantial arrears with respect to crisis loans the IMF provided (Reinhart and Trebesch, 2016; Schlegl et al., 2019). This resulted to the official endorsement of the preferred creditor status at the end of the 1980s (IMF, 1988). Regarding the WB, the International Bank for Reconstruction and Development's (IBRD) and the International Development Association's (IDA) loans were initially meant to be subordinated to private claims (Raffer, 2009). Moreover, the major credit agencies waited more than a decade after the WB's creation to attribute it the highest rating.

This implicit seniority structure provides an, albeit imperfect, shelter to multilateral institutions, allowing them to provide loans to countries with major economic difficulties at preferential rates (Fischer, 1999). To maintain this preferred status, multilateral lenders have developed a set of policies. For example, the IMF has established a clear policy of non-toleration of arrears consisting of two main lines of conduct.[16] First, it does not tolerate defaults on official creditors and forbids the use of funds to member states with arrears to the IMF (IMF, 1989; IMF, 2015). More precisely,

[15]This is a well established fact documented by numerous studies, explicitly supported by the Paris Club and repeatedly acknowledged by the main rating agencies. See notably Jeanne and Zettelmeyer (2001), Roubini and Setser (2003), Gelpern (2004), Raffer (2009), Schadler (2014) and Schlegl et al. (2019).

[16]The IMF's policy of non-toleration of arrears has evolved over time. Moreover, as noted by Reinhart and Trebesch (2016), the IMF applies this policy with some degrees of freedom. See Buchheit and Lastra (2007) for the history of the policy and Erce (2014) for a critical appraisal.

countries need to clear arrears to regain access to IMF lending. Second, if a sovereign receives support from an IMF program and defaults on its private creditors, the program should, absent immediate corrective actions by the authorities, be suspended (IMF, 1999). The WB follows a similar scheme as it does not lend into arrears and reserves the right to withdraw its funds in case of lacking reforms (IDA, 2007; IBRD, 2021).[17]

When building the model, I will assume that the multilateral lenders adopt a (simplified) version of the aforementioned policy of non-toleration of arrears. In particular, the multilateral lender refrains from lending into arrears as in Cordella and Powell (2021) and requests full repayment. As one will see, this safeguards the preferential rates of multilateral lending. It is also at the source of a longer defaults and greater private creditors' losses.

3.4 Empirical Facts

In this section, I introduce the main empirical regularities linking defaults with multilateral creditors.[18] My analysis relies on 187 default episodes from 1970 to 2014, which all involve external debt and private creditors.

Data on default durations and haircuts come from Asonuma and Trebesch (2016) and Cruces and Trebesch (2013), respectively. I then identify the different creditors involved in each default episode using the database of Beers et al. (2022). In particular, I focus on multilateral lenders which consist of the IMF, the IBRD and the IDA. A default episode *with* multilateral lenders consists of an episode in which a country defaults on at least one of the these three lending institutions.[19] The alternative case corresponds to a default *without* multilateral lenders. Appendix 3.2 gives a detailed overview of the data used in this section. In particular, Table C.1 presents the sample used and Table C.2 specifies the source.

[17] See for example the case of Somalia in March 2020 and Sudan in March 2021 which both could re-access the WB after successfully clearing their arrears and conducting requested reforms.

[18] The analysis in this section and in Appendix 3.3 is not necessarily causal.

[19] As noted by Cordella and Powell (2021), multilateral lenders do not identify these episodes as defaults but simply as arrears because they eventually expect full repayment. I nevertheless use the term default as it corresponds to a missed payment consistent with the definition of Cruces and Trebesch (2013).

Table 3.1: Duration and Haircut Statistics

	Mean	Median	Min	Max	Std. Dev.	Obs.
Default Duration (year)						
Overall	3.6	1.6	-0.2	27.4	4.71	187
With multilateral creditors	8.5	7.6	0.3	27.4	6.98	33
Without multilateral creditors	2.6	1.3	-0.2	18.2	3.25	154
SZ Haircut on Private Lenders (%)						
Overall	37.5	32.5	-9.8	97.0	27.93	187
With multilateral creditors	59.0	55.2	12.3	97.0	27.68	33
Without multilateral creditors	32.9	29.0	-9.8	97.0	25.83	154

Note: The table depicts the default duration in years and the haircut on private lenders in percent for all the defaults in the sample (overall) and separately for defaults with and without multilateral lenders. SZ haircuts are computed according to Sturzenegger and Zettelmeyer (2008). The negative default duration corresponds to the preemptive restructuring of Nicaragua in 1982.

Source: Default dates are from Asonuma and Trebesch (2016) and the haircuts are from Cruces and Trebesch (2013). See Appendix 3.2 for more details.

Table 3.1 presents the main figures related to the default's duration and private creditors' haircut. For each statistic, I distinguish between defaults with and without multilateral lenders. Overall, I identify four main empirical facts. The first one states that defaults with multilateral creditors are infrequent. Out of the 187 default episodes presented here only 33 are with multilateral creditors.[20]

Fact I. *A default with multilateral lenders is infrequent.*

In addition, I find that sovereign defaults take between 3 and 4 years to be resolved. More importantly, if one conditions the length on the type of creditors involved, a default with multilateral creditors takes roughly 9 years to be resolved. In opposition, a default without such lenders takes on average 3 years to be resolved. Looking at the median the wedge between the two statistics is even larger. Hence, defaults with multilateral creditors are associated with a tripling of the length of default on average. The second fact is thus:

Fact II. *A default with multilateral lenders takes longer to be resolved.*

To go beyond the analysis of simple stylised facts, I conduct a more comprehensive econometric analysis. However, for the continuity of the argument, I only highlight here the main findings. The detailed regression analysis is presented in Appendix 3.3. To gauge the robustness of Fact II, I conduct two main exercises: ordinary least squares (OLS) regressions and Cox proportional

[20] See also the discussion in Section 3.3 and the related Figures C.3 and C.4 in Appendix 3.1.

hazard (Cox) duration regressions. There I control for the specificity of each default (i.e. amount restructured, presence of a Brady deal and private creditor's losses) but also for the economic and political stands of the countries in default.

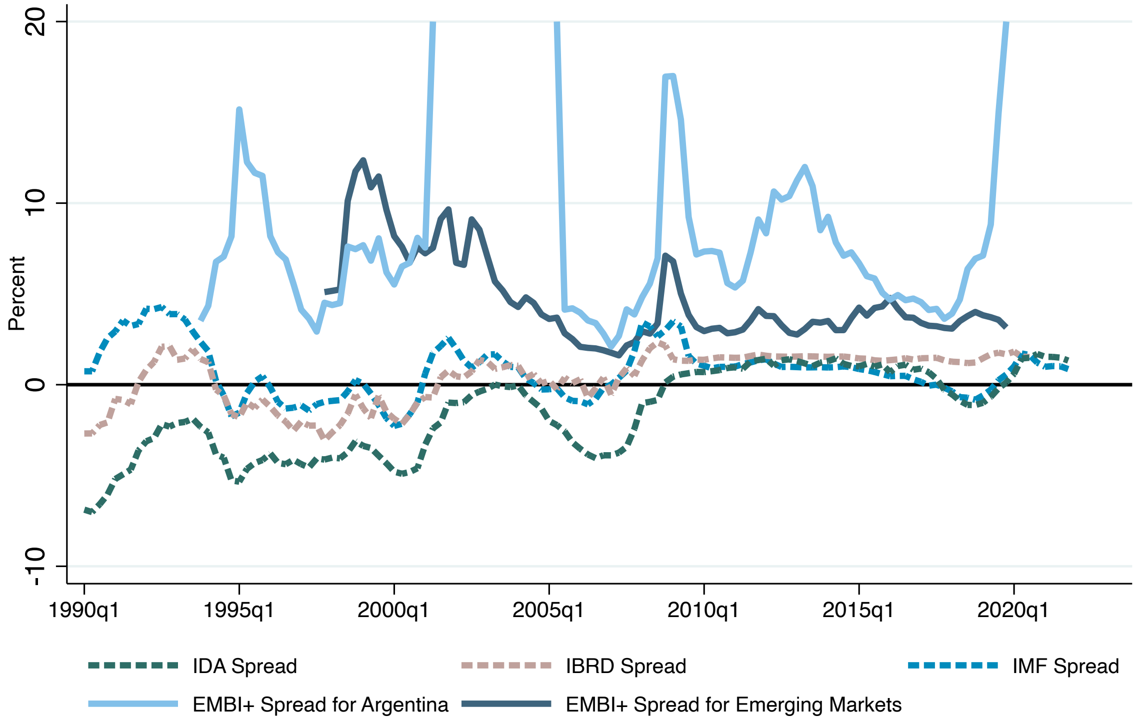
The outcome of the OLS duration regressions is depicted in Table C.3. There is a strong and positive association between defaults with multilateral creditors and the length of the default duration. Particularly, a default with multilateral debt is associated with a default's duration between 3 and 7 additional years depending on the model's specification. I draw similar conclusions from the outcome of the Cox model presented in Table C.4. Notably, a default with multilateral creditors is associated with a reduction of the probability of exiting default between 36% and 69% depending on the model's specification. In view of those results, it seems that this newly established fact is relatively robust. Controlling for the specificity of each default episode and the countries' characteristics does not undermine the association between the default's length and multilateral creditors.

The second part of Table 3.1 presents the private creditors' haircut computed according to Sturzenegger and Zettelmeyer (2008) (henceforth SZ). Private creditors' haircuts are 38% on average. However, for default episodes with multilateral creditors, the average haircut raises to 59%, while it falls to 33% otherwise. Looking at the median the wedge between the two statistics is of similar magnitude. This leads to the third empirical fact:

Fact III. *A default with multilateral lenders is associated with larger private creditors' losses.*

However, the association between large haircuts and multilateral creditors might simply be a by-product of other factors not necessarily related to the creditor's identity. Thus, I conduct an econometric analysis to disentangle the forces at play. For this purpose, I run OLS regressions controlling for the specificity of each default episode (i.e. amount restructured, presence of a Brady deal and the duration) as well as the economic and political situations of each country in default, like I did for Fact II.

Table C.5 in Appendix 3.3 presents the results of the haircut regressions. The coefficient related



Note: The figure depicts the spread for different types of sovereign debt. The EMBI+ spread series track the spread on emerging market fixed and floating-rate sovereign debt instruments. The EMBI+ series for Argentina has been truncated to 20% for expositional reasons. The IMF spread corresponds to the adjusted rate of charge minus the yield on 1-year US government bonds. The IBRD spread corresponds to the lending rate minus the yield on 1-year US government bonds. The IDA spread corresponds to the service charge minus the yield on 1-year US government bonds.

Source: See Appendix 3.2.

Figure 3.1: Spreads on Sovereign Debt

to multilateral lenders is economically important although the statistical significance is less pronounced than for Fact II. Defaulting on such creditors is associated with an increase of the private creditors' haircut between 8 and 15 percentage points for the SZ haircut depending on the model's specification. I therefore conclude that the third empirical fact is relatively robust as well. Controlling for specific components such as the default duration, IMF programs, WB loans or the HIPC initiative does not undermine the association between the private creditors' loss and multilateral creditors.[21]

Finally, the last empirical fact relates to the lending conditions. While private creditors can request substantial risk premia, multilateral creditors always provide funds at preferential rates. The fourth and last empirical fact is thus

[21]In 1996, the IMF and the WB started the Heavily Indebted Poor Countries (HIPC) initiative which aims at providing immediate debt relief to low-income countries.

Fact IV. *Multilateral lenders lend at rates close to the risk-free rate.*

Figure 3.1 depicts the spread of the IMF adjusted rate of charge as well as the IBRD lending rate and the IDA service charge with respect to the yield on 1-year US government bonds. It also presents the EMBI+ spread for Argentina and emerging economies to have a sense of the premium charged by the market in general. As it is clear, multilateral lenders always charge a rate close to the risk-free one.[22] Boz (2011) already highlighted this particularity for the IMF lending.

3.5 Environment

Having established new empirical facts, the following sections aim at building a model capable of rationalizing them. I consider a small open economy in infinite discrete time $t = \{0, 1, \dots\}$ with a single homogenous good. The small open economy is populated by a benevolent government a continuum of competitive private lenders and one (official) multilateral lender.

The sovereign acts as a representative agent and takes the decision on behalf of the small open economy. Preference over consumption is given by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $\beta \in (0, 1)$ is the discount factor and c_t denotes the consumption at time t. The instantaneous utility function $u(\cdot)$ is differentiable, strictly increasing and strictly concave. Moreover, I assume that the sovereign is relatively impatient meaning that $\beta < 1/(1 + r)$ where r is the exogenous risk-free rate. Each period the sovereign receives an exogenous endowment, $y(z)$, which follows a first-order Markov process with a compact support $Z = \{z_1, z_2, ..., z_n\}$.

The sovereign faces two funding opportunities. On the one hand, it can issue private debt, b'_p, at the unit price $q_p(z, b'_m, b'_p)$. On the other hand, it can issue multilateral debt, b'_m, at the unit price $q_m(z, b'_m, b'_p)$. I consider that $b_i < 0$ denotes a debt, while $b_i > 0$ denotes an asset for all $i \in \{m, p\}$. Both types of debt are long-term and follow the structure of Chatterjee and Eyigungor (2012). More precisely, a fraction $1 - \delta$ of the bond portfolio matures every period and the remaining fraction δ is rolled over and pays a coupon κ. Both types of bonds have the same

[22]The average IMF, IBRD and IDA spreads are 0.76%, 0.30% and -1.78%, respectively. In opposition, the EMBI+ spread for Argentina and emerging economies amount to and 13.51% and 4.72%, respectively.

(κ, δ) and the risk-free return is given by $\bar{q} \equiv \frac{1-\delta+\delta\kappa}{1+r-\delta}$. The financial market is incomplete as bonds do not discriminate the returns across z.

There is limited enforcement in repayment. The sovereign has two default options: *partial* or *full*. In the former case, the sovereign solely defaults on its private debt, whereas in the latter case it defaults on its entire debt position.[23] Both types of default are followed by a complete bond market exclusion and an output penalty. I denote $y^{DP}(z)$ and $y^{DF}(z)$ as the endowment upon a *partial* and *full* default, respectively.

The private lenders are risk-neutral and competitive. Similarly, the multilateral lender is risk-neutral and breaks even in expectation. Nevertheless, the multilateral lender has a greater enforcement power than the private lenders. First, defaulting on the multilateral debt entails greater output cost – i.e. $y \geq y^{DP} > y^{DF}$. Second, following the discussion in Section 3.3, the multilateral lender follows a stringent policy of non-toleration of arrears which consists of two main components. First, repayment of outstanding multilateral debt is always in full. Second, the multilateral lender does not provides new debt until arrears have been completely cleared. For tractability, missed coupon payments are nevertheless forgone.

Following Dvorkin et al. (2021), I introduce additive utility shocks for computational reasons. I assume that debt takes values in a discrete support $B_p = \{b_{p,1}, \ldots, b_{p,\mathcal{P}}\}$ with $|B_p| = \mathcal{P}$ for the private debt and $B_m = \{b_{m,1}, \ldots, b_{m,\mathcal{M}}\}$ with $|B_m| = \mathcal{M}$ for the multilateral debt. Define the vectors $\boldsymbol{b}_p$ and $\boldsymbol{b}_m$, where (b_p^i, b_m^i) are the ith elements of each vector.

$$\boldsymbol{b}_p = [\underbrace{B_p, \ldots, B_p}_{\mathcal{M} \text{ times}}] \quad \text{and} \quad \boldsymbol{b}_m = [\underbrace{b_{m,1}, \ldots, b_{m,1}}_{\mathcal{P} \text{ times}}, \underbrace{b_{m,2}, \ldots, b_{m,2}}_{\mathcal{P} \text{ times}}, \ldots, \underbrace{b_{m,\mathcal{M}}, \ldots, b_{m,\mathcal{M}}}_{\mathcal{P} \text{ times}}]$$

There is a utility shock vector $\boldsymbol{\epsilon}$ of size $\mathcal{P} \times \mathcal{M} + 2 \equiv \mathcal{J} + 2$, which corresponds to the number of all possible combinations of the entries in B_p and B_m plus two additional elements that accounts for the choices of *partial* and *full* defaults. The random vector is drawn from a multivariate distribution with joint cumulative distribution function $F(\boldsymbol{\epsilon}) = F(\epsilon_1, \epsilon_2, \ldots, \epsilon_{\mathcal{J}}, \epsilon_{\mathcal{J}+1}, \epsilon_{\mathcal{J}+2})$ and joint density

[23] The sovereign does not have the possibility of defaulting only on the multilateral debt. This is consistent with the fact that, in the empirical analysis, default episodes always involve private creditors.

function $f(\epsilon)$.

In this environment, the sovereign faces two problems. On the one hand, it decides whether to repay or not. This is the repayment problem. On the other hand, under default, the sovereign has to renegotiate its debt. This is the renegotiation problem.

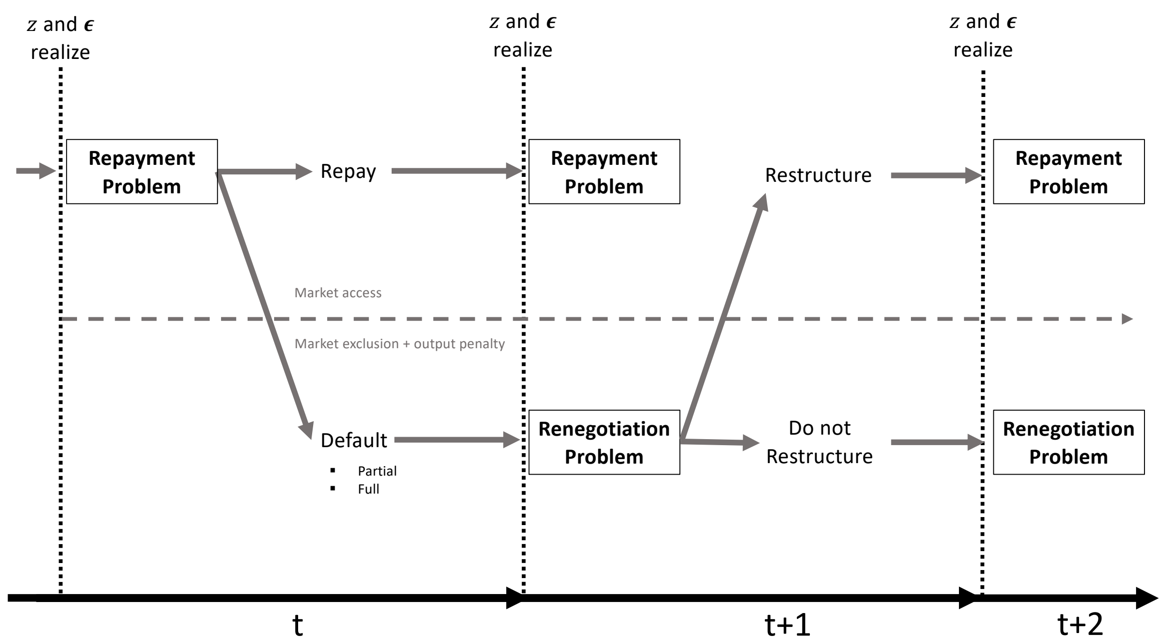


Figure 3.2: Timing and Problems

The timing of the model is depicted in Figure 3.2. In the repayment problem, given the realization of (z, ϵ), the sovereign decides whether to repay or not. If it repays, it maintains its market access, determines its prospective borrowing and faces the repayment problem again in the next period. Upon default, the sovereign receives the output penalty, is excluded from the bond market and faces the renegotiation problem in the next period. In this problem, it has to directly bargain with the lenders to restructure its debt. The creditors and the sovereign propose stochastically over multiple rounds. If the negotiating parties agree on a restructuring, the sovereign regains access to the market and faces the repayment problem in the next period. Otherwise, the sovereign remains in autarky and the renegotiation problem repeats next period.

3.6 The Repayment Problem

This section develops the repayment problem. Given the prices, the outcome of the renegotiation problem, the realisation of $(z, \boldsymbol{\epsilon})$ and the current stock of debt, the sovereign decides whether to repay. When defaulting, the sovereign can choose to enter into *partial* or *full* default. The overall beginning of the period value function is then given by

$$V(z, \boldsymbol{\epsilon}, b_m^i, b_p^i) = \max\{V^P(z, \boldsymbol{\epsilon}, b_m^i, b_p^i), V^{DP}(z, \epsilon_{\mathcal{J}+1}, b_m^i, b_p^i), V^{DF}(z, \epsilon_{\mathcal{J}+2}, b_m^i, b_p^i)\}, \tag{3.1}$$

where $V^P(\cdot)$ is the value function under repayment, $V^{DP}(\cdot)$ under *partial* default and $V^{DF}(\cdot)$ under *full* default.

In the case in which the sovereign decides to honor the terms of all its debt contracts, the Bellman equation reads

$$\begin{aligned} V^P(z, \boldsymbol{\epsilon}, b_m^i, b_p^i) = \max_{j \in \{1,2,\ldots,\mathcal{J}\}} \quad & u(c) + \epsilon_j + \beta \mathbb{E}_{z'|z} \mathbb{E}_{\boldsymbol{\epsilon}'} V(z', \boldsymbol{\epsilon}', b_m^j, b_p^j) \\ \text{s.t.} \quad & c + q_m(z, b_m^j, b_p^j)(b_m^j - \delta b_m^i) + q_p(z, b_m^j, b_p^j)(b_p^j - \delta b_p^i) \\ & = y(z) + [1 - \delta + \delta\kappa]\,(b_m^i + b_p^i). \end{aligned} \tag{3.2}$$

If the sovereign decides to enter into *partial* default, it receives an output penalty and is excluded from the bond market. Moreover, it continues to service the multilateral debt. The Bellman equation for the case of *partial* default is given by

$$\begin{aligned} V^{DP}(z, \epsilon_{\mathcal{J}+1}, b_m^i, b_p^i) = & u(c) + \epsilon_{\mathcal{J}+1} + \beta \mathbb{E}_{z'|z} \mathbb{E}_{\boldsymbol{\epsilon}'} V^{RP}(z', \boldsymbol{\epsilon}', b_m^j, b_p^i) \\ \text{s.t.} \quad & c = y^{DP}(z) + [1 - \delta + \delta\kappa]\, b_m^i, \\ & b_m^j = \delta b_m^i. \end{aligned} \tag{3.3}$$

The continuation value $V^{RP}(\cdot)$ is the expected payoff from the renegotiation process with the private creditors and is specified in the next section. Consistent with the policy of non-toleration of arrears, there is no multilateral lending, while the sovereign continues to service its multilateral debt which decays at the rate δ.[24] Hence, the larger is $-b_m^i$, the less attractive is this type of default. Moreover, in the case of one-period debt (i.e. $\delta = 0$), the debt is repaid in one instalment. This further renders the *partial* default unappealing. In opposition, the longer is the average maturity (i.e. $\delta \to 1$), the lower is the debt service incurred every period.

Finally, the Bellman equation in the case of *full* default reads as follows

$$\begin{aligned} V^{DF}(z, \epsilon_{\mathcal{J}+2}, b_m^i, b_p^i) =& u(c) + \epsilon_{\mathcal{J}+2} + \beta \mathbb{E}_{z'|z} \mathbb{E}_{\boldsymbol{\epsilon}'} V^{RF}(z', \boldsymbol{\epsilon}', b_m^i, b_p^i) \\ \text{s.t.} \quad & c = y^{DF}(z). \end{aligned} \tag{3.4}$$

The continuation value $V^{RF}(\cdot)$ is the expected payoff derived from the debt renegotiation process with the two creditors. Unlike a *partial* default, the sovereign does not need to service any debt while being in autarky. It is therefore crucial that $y^{DF} < y^{DP}$. I show in Section 3.10, that when $y^{DP} = y^{DF}$, a *partial* default becomes almost completely unattractive.

3.7 The Renegotiation Problem

The previous section developed the repayment problem taking as given the outcome of the renegotiation problem. This section does the opposite. To endogenize the renegotiation process, I mainly draw from the framework developed by Bi (2008) and Benjamin and Wright (2013) as it is capable of generating endogenous delays and haircuts. The exact form of the renegotiation process follows Dvorkin et al. (2021).

3.7.1 Partial default

The renegotiation is a multi-round non-cooperative game in which the private lenders and the sovereign propose stochastically. Figure 3.3 depicts the sequence of actions in the renegotiation with the sovereign's payoffs.

[24]The absence of multilateral lending is for tractability. There are occurrence in the data in which multilateral lenders continue to lend when the sovereign is in arrears with respect to the private creditors. In light of this, my model adopts a stringent interpretation of the policy of non-toleration of arrears.

With probability ϕ the private lenders have the opportunity to propose and if so the sovereign decides whether to accept. Conversely, with probability $1-\phi$, the sovereign can propose and if so the private lenders decide whether to accept. The probability ϕ directly reflects the private lenders' bargaining power as it represents the probability of having the first-mover advantage (Merlo and Wilson, 1995).

An offer states the value of the restructured private debt, W. If the proposer does not propose or the recipient does not accept the offer, the renegotiation is delayed, the sovereign stays in autarky and the game repeats next period. Otherwise, the negotiating parties settle, the game ends and the sovereign can return to the repayment problem. Formally,

$$V^{RP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i) = \phi\Omega^{RP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W_l^{RP}) + (1-\phi)\Omega^{RP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W_b^{RP}). \tag{3.5}$$

$\Omega^{RP}(\cdot)$ is the value derived from a specific offer and W_l^{RP} and W_b^{RP} represent the offer made by the private lenders and the sovereign, respectively.

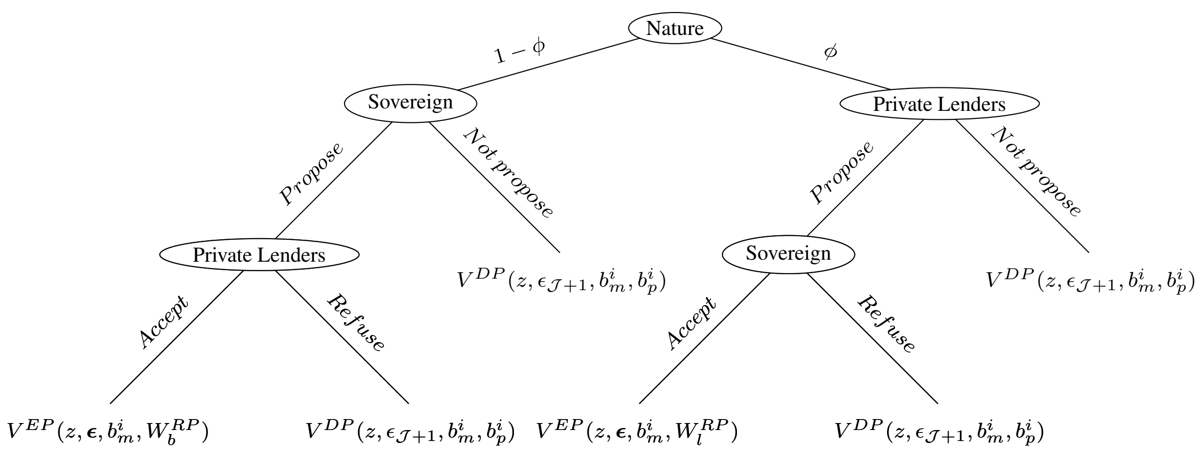


Figure 3.3: Renegotiation Game Tree in Partial Default

In each round, the sovereign compares the value of remaining in autarky with the value of paying W and re-accessing the market. Hence, one has that

$$\Omega^{RP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W) = \max\left\{V^{DP}(z, \epsilon_{\mathcal{J}+1}, b_m^i, b_p^i), V^{EP}(z, \boldsymbol{\epsilon}, b_m^i, W)\right\}, \tag{3.6}$$

where $V^{DP}(\cdot)$ is the value of remaining in autarky and $V^{EP}(\cdot, W)$ is the value of exiting the negotiation with a restructured private debt of value W. This defines a stopping function $A^{RP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W)$ which takes value one if the restructuring is preferred and zero otherwise. The value upon restructuring is given by

$$V^{EP}(z, \boldsymbol{\epsilon}, b_m^i, W) = \max_j u(c) + \epsilon_j + \beta \mathbb{E}_{z'|z} \mathbb{E}_{\boldsymbol{\epsilon}'} V(z', \boldsymbol{\epsilon}', b_m^j, b_p^j) \tag{3.7}$$

$$\begin{aligned} \text{s.t.} \quad & c = y(z) + \tau + [1 - \delta + \delta\kappa] b_m^i, \\ & \tau = q_p(z, b_m^j, b_p^j)(-b_p^j) - W, \\ & \tau \geq 0, \\ & b_m^j = \delta b_m^i. \end{aligned}$$

During the restructuring, the sovereign repays the value of the restructured debt, W, and gets rid of the output penalty. As in Dvorkin et al. (2021), the value of restructured debt has to be financed by new debt issuance (i.e. $\tau \geq 0$). Due to the policy of non-toleration of arrears, the sovereign can issue new multilateral debt only after clearing private arrears.

The haircut corresponds to $1 - \frac{W}{-b_p^i \bar{q}}$. The numerator in the fraction corresponds to the present value of restructured debt, whereas the denominator is the present value of the defaulted debt.

Let's now determine W_b^{RP}. Given the risk neutrality of the private lenders, the sovereign cannot offer less than the current market value of defaulted debt if it wants to settle. There is also no reason to offer more. Thus,

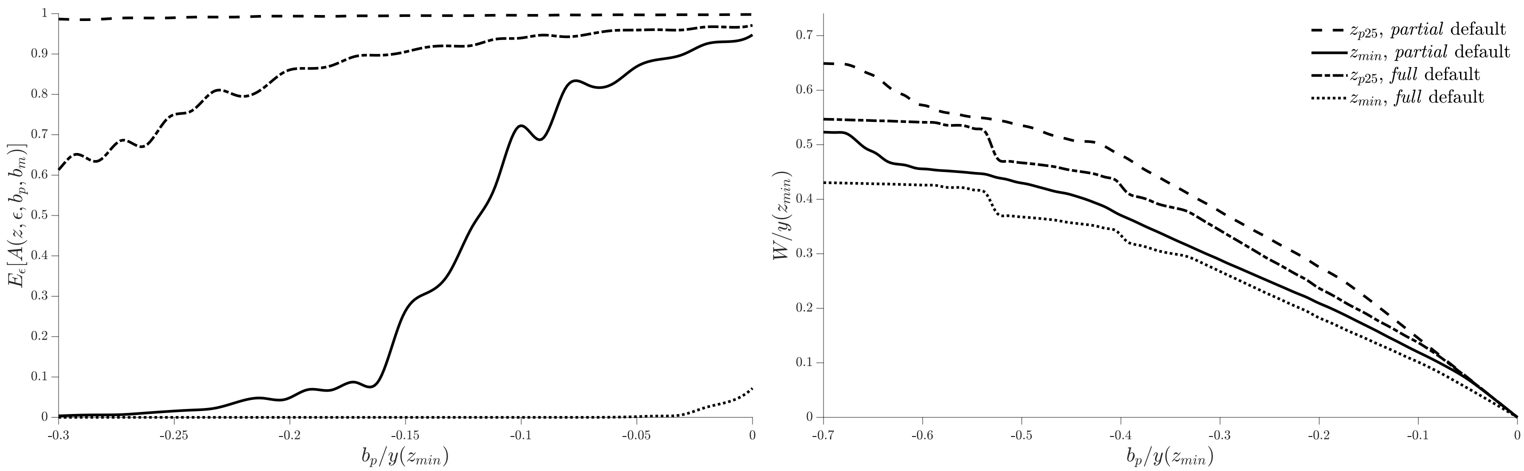
$$W_b^{RP}(z, b_m^i, b_p^i) = -b_p^i q_p^{DP}(z, b_m^i, b_p^i),$$

where $q_p^{DP}(\cdot)$ is specified in the next section. The sovereign's offer corresponds the private lenders' reservation value. The private lenders will therefore always accept the sovereign's offer. Nevertheless, the sovereign might decide not to propose (i.e. $A^{RP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W) = 0$) if it is better off staying in autarky.

When the private lenders propose, they consider two main aspects. On the one hand, they should come up with a settlement that the sovereign is likely to accept. On the other hand, they have to make sure to maximize the recovery value. The private lenders' offer is therefore the result of

$$W_l^{RP}(z, b_m^i, b_p^i) = \arg\max \left[\mathbb{E}_{\boldsymbol{\epsilon}} A^{RP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W)W + (1 - \mathbb{E}_{\boldsymbol{\epsilon}} A^{RP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W))W_b^{RP}(z, b_m^i, b_p^i)\right]$$
$$\text{s.t.} \quad W \leq -b_p^i \bar{q}.$$

In words, the private lenders seek to maximize the recovery value the sovereign is willing to accept under the constraint that the proposed restructuring does not exceed the present value of the defaulted debt.



(a) $\mathbb{E}_\epsilon[A(z, \epsilon, b_p, b_m)]$ (b) W/y_{min}

Note: Figure 3.4a depicts the sovereign's acceptance probability and Figure 3.4b the optimal offer from the private lenders in *partial* and *full* defaults as a function of b_p fixing (z, b_m). The different lines correspond to different levels of endowment and b_m is set to the largest level of debt in the grid. z_{pX} corresponds to the X^{th} percentile of the endowment.

Figure 3.4: Acceptance Probability and Private Lender's Offer

Figure 3.4a depicts the sovereign's acceptance probability and Figure 3.4b the private lenders' offer. As one can see, W decreases in the level of private debt and in the level of endowment. Regarding the acceptance probability, delays are more likely in low endowment states and with larger levels of debt.

What is the source of delays in this set-up? The sovereign usually defaults in low endowment states with a relatively high level of debt. If the sovereign desires to settle at the lowest cost, the least it could pay is $W_b^{RP} = -q_p^{DP}(z, b_m^i, b_p^i)b_p^i$. To get out of default, it would need to issue new private debt. The problem is that in low endowment states, $q_p(z, b_m^i, b_p^i)$ is very close to $q_p^{DP}(z, b_m^i, b_p^i)$ due to the persistence of the shocks. Owing to the constraint $\tau \geq 0$, the sovereign should accumulate a prospective level of debt similar to the one it just defaulted on if it wants to settle. As a result, it runs the risk of falling into default once again next period lowering $V^{EP}(\cdot)$. It is then optimal for the sovereign to wait that the endowment state improves and $q_p(z, b_m^i, b_p^i)$ recovers in order to settle its debt. Note that it is also optimal for the private lenders to wait. When the default risk is high, the recovery value of debt is very low. However, as the default risk diminishes, the private lenders can recover more from the sovereign.

As in Benjamin and Wright (2013), delays therefore originate from the limited enforcement in repayment. In other words, delays in the renegotiation emanate from the same force that generates the default itself.

3.7.2 Full default

To simplify the renegotiation under *full* default, I assume that there are neither coordination nor cooperation problems between the private and the multilateral lenders. The former acknowledge the full repayment of the latter. Hence, the two types of lender jointly propose a common offer with probability ϕ.[25] The value under renegotiation is given by

$$V^{RF}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i) = \phi \Omega^{RF}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W_l^{RF} - b_m^i \bar{q}) + (1-\phi)\Omega^{RF}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W_b^{RF} - b_m^i \bar{q}).$$

where W_b^{RF} and W_l^{RF} represent the offer for the private debt made by the sovereign and the two types of lenders, respectively. Consistent with the policy of non-toleration of arrears, irrespective of the proposer, the multilateral debt is always repaid in full – i.e. $-b_m^i \bar{q}$.[26] Nevertheless, the

[25] It is similar to assume that the sovereign renegotiates with each lender separately but the multilateral lender has a bargaining power of 1.

[26] Note that it is enough to keep track of W instead of $W - b_m^i \bar{q}$ in the state space. However, this makes the dependence on the full repayment of multilateral debt clear.

multilateral lender forgoes the missed coupon payments. In other words, there is no accumulation of arrears.

Following the same logic as before, the stopping function $A^{RF}(z, \boldsymbol{\epsilon}, b^i_m, b^i_p, W - b^i_m\bar{q})$ is the result of

$$\Omega^{RF}(z, \boldsymbol{\epsilon}, b^i_m, b^i_p, W - b^i_m\bar{q}) = \max\left\{V^{DF}(z, \epsilon_{\mathcal{J}+2}, b^i_m, b^i_p), V^{EF}(z, \boldsymbol{\epsilon}, b^i_m, W - b^i_m\bar{q})\right\}. \quad (3.8)$$

where $V^{DF}(\cdot)$ is the value of remaining in full default and $V^{EF}(\cdot, W - b^i_m\bar{q})$ is the value of exiting the renegotiation with a restructured private debt of value W and multilateral debt of value $-b^i_m\bar{q}$. The value under restructuring is given by

$$\begin{aligned} V^{EF}(z, \boldsymbol{\epsilon}, b^i_m, W - b^i_m\bar{q}) = \max_j\, & u(c) + \epsilon_j + \beta\mathbb{E}_{z'|z}\mathbb{E}_{\boldsymbol{\epsilon}'}V(z', \boldsymbol{\epsilon}', b^j_m, b^j_p) \\ \text{s.t.}\quad & c = y(z) + \tau, \\ & \tau = q_p(z, b^j_m, b^j_p)(-b^j_p) - (W - b^i_m\bar{q}), \\ & \tau \geq 0, \\ & b^j_m = 0. \end{aligned} \quad (3.9)$$

Upon restructuring, the sovereign repays the value of the restructured debt, $W - b^i_m\bar{q}$, and gets rid of the output penalty. In addition, due to the policy of non-toleration of arrears, the sovereign cannot access multilateral funds as it is clearing its arrears in the current period. The sovereign's offer for the private debt is given by

$$W^{RF}_b(z, b^i_m, b^i_p) = -b^i_p q^{DF}_p(z, b^i_m, b^i_p).$$

Conversely, the lenders' offer for the private debt is the result of

$$W^{RF}_l(z, b^i_m, b^i_p) = \arg\max\Big[\mathbb{E}_{\boldsymbol{\epsilon}}A^{RF}(z, \boldsymbol{\epsilon}, b^i_m, b^i_p, W - b^i_m\bar{q})W$$

$$+ (1 - \mathbb{E}_{\boldsymbol{\epsilon}} A^{RF}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W - b_m^i \bar{q})) W_b^{RF}(z, b_m^i, b_p^i) \Big]$$

$$\text{s.t.} \quad W \leq -b_p^i \bar{q}.$$

The full repayment of multilateral lenders affects the stopping function $A^{RF}(\cdot, W - b_m^i \bar{q})$. The level of multilateral debt therefore directly impacts W_l^{RF}.

How is this setting supposed to generate additional delay? If the multilateral lender was not requesting full repayment, the sovereign could offer $q_m^{DF}(z, b_m^i, b_p^i)(-b_m^i)$ with $q_m^{DF}(z, b_m^i, b_p^i) \leq \bar{q}$ instead of $\bar{q}(-b_m^i)$. In words, the complete repayment of the multilateral debt renders the debt restructuring more costly which depreses the value of restructuring. In addition, compared to a *partial* default, the value of staying in default may be larger in a *full* default as the sovereign does not need to service the multilateral debt in autarky. Hence, the sovereign prefers to stay in default for a longer period of time. Figure 3.4a depicts the acceptance probability in *partial* and *full* defaults. A successful restructuring is always less likely in *full* defaults for a given state.

Given additional delays, the absence of multilateral debt issuance is necessary to generate haircuts in line with Fact III. In this bargaining game, the haircut is shaped by two opposing forces as shown in Figure 3.4b. On the one hand, for a given level of endowment, the larger is the level of debt, the larger is the haircut. On the other hand, for a given level of debt, the higher is the endowment, the lower is the haircut due to the lower default risk. As mentioned above, in the case of a *full* default, delays in the renegotiation process are more pronounced which mechanically lead to lower haircuts. The restriction on multilateral debt issuance is thus necessary to counterbalance this effect. As one can see in Figure 3.4b, for a given state, the private lenders' offer in *full* default is always strictly lower than in *partial* default.

3.8 Prices and Seniority Structure

The previous two sections exposed the repayment problem and, subsequently, the renegotiation problem the sovereign faces. The present section aims at defining the prices and characterising the optimal seniority structure.

3.8.1 Bond prices

Define $D^{DP}(z, \boldsymbol{\epsilon}, b^i_m, b^i_p)$ as the *partial* default policy which takes value one in case of such default and zero otherwise. Similarly, define $D^{DF}(z, \boldsymbol{\epsilon}, b^i_m, b^i_p)$ as the *full* default policy. Regarding borrowing, $b_p(z, \boldsymbol{\epsilon}, b^i_m, b^i_p)$ and $b_m(z, \boldsymbol{\epsilon}, b^i_m, b^i_p)$ correspond to the private and multilateral bond policies, respectively.

Private lenders are competitive meaning that in expectations they make zero profit. The price of one unit of bond can therefore be separated into two parts: the return when the sovereign decides to repay and the recovery value when the sovereign defaults.

$$\begin{aligned} q_p(z, b^j_m, b^j_p) = \frac{1}{1+r}\mathbb{E}_{z'|z}\mathbb{E}_{\boldsymbol{\epsilon}'}\Bigg[&\Big(1 - D^{DP}(z', \boldsymbol{\epsilon}', b^j_m, b^j_p) - D^{DF}(z', \boldsymbol{\epsilon}', b^j_m, b^j_p)\Big)\times \\ &\Big(1 - \delta + \delta\kappa + \delta q_p(z', b_m(z', \boldsymbol{\epsilon}', b^j_m, b^j_p), b_p(z', \boldsymbol{\epsilon}', b^j_m, b^j_p))\Big) + \\ &D^{DP}(z', \boldsymbol{\epsilon}', b^j_m, b^j_p) q^{DP}_p(z', b^j_m, b^j_p) + \\ &D^{DF}(z', \boldsymbol{\epsilon}', b^j_m, b^j_p) q^{DF}_p(z', b^j_m, b^j_p)\Bigg]. \end{aligned} \tag{3.10}$$

If the sovereign decides to repay, the private lenders receive the fraction of bond maturing, $1-\delta$, the coupon for the share of debt that is rolled-over, $\delta\kappa$, and the value of the outstanding debt in the next period, $\delta q'_p$. If the sovereign decides to renege the debt contract, the private lenders receive the recovery value which depends on the acceptance probability, the bargaining power and the proposed offer. In the case of *partial* default,

$$\begin{aligned} q^{DP}_p(z, b^i_m, b^i_p) = \frac{1}{1+r}\mathbb{E}_{z'|z}\mathbb{E}_{\boldsymbol{\epsilon}'}\Big[&(1 - \phi A^{RP}(z', \boldsymbol{\epsilon}', \delta b^i_m, b^i_p, W^{RP}_l)) q^{DP}_p(z', \delta b^i_m, b^i_p) + \\ &\phi A^{RP}(z', \boldsymbol{\epsilon}', \delta b^i_m, b^i_p, W^{RP}_l)\frac{W^{RP}_l(z', \delta b^i_m, b^i_p)}{-b^i_p}\Big]. \end{aligned}$$

The price is again shaped by the break-even condition. If the private lenders propose and the sovereign accepts the deal, then the recovery value per unit of bond is $\frac{1}{-b^i_p}W^{RP}_l(z', \delta b^i_m, b^i_p)$. Conversely, if the sovereign proposes, the private lenders receive their outside option, $q^{DP}_p(z', \delta b^i_m, b^i_p)$.

Finally, if the sovereign refuses to settle or does not propose, it does not disburse anything now, but in present value it pays $q_p^{DP}(z', \delta b_m^i, b_p^i)$. Similarly, in the case of *full* default,

$$q_p^{DF}(z, b_m^i, b_p^i) = \frac{1}{1+r}\mathbb{E}_{z'|z}\mathbb{E}_{\boldsymbol{\epsilon}'}\Big[(1 - \phi A^{RF}(z', \boldsymbol{\epsilon}', b_m^i, b_p^i, W_l^{RF}))q_p^{DF}(z', b_m^i, b_p^i) + \phi A^{RF}(z', \boldsymbol{\epsilon}', b_m^i, b_p^i, W_l^{RF})\frac{W_l^{RF}(z', b_m^i, b_p^i)}{-b_p^i}\Big].$$

I can now pass to the price of multilateral debt. Given the risk neutrality and the break-even assumption, the price formula is similar to the one of private debt,

$$q_m(z, b_m^j, b_p^j) = \frac{1}{1+r}\mathbb{E}_{z'|z}\mathbb{E}_{\boldsymbol{\epsilon}'}\Bigg[\Big(1 - D^{DF}(z', \boldsymbol{\epsilon}', b_m^j, b_p^j)\Big)\times \Big(1 - \delta + \delta\kappa + \delta q_m(z', b_m(z', \boldsymbol{\epsilon}', b_m^j, b_p^j), b_p(z', \boldsymbol{\epsilon}', b_m^j, b_p^j))\Big) + D^{DF}(z', \boldsymbol{\epsilon}', b_m^j, b_p^j)q_m^{DF}(z', b_m^j, b_p^j)\Bigg]. \tag{3.11}$$

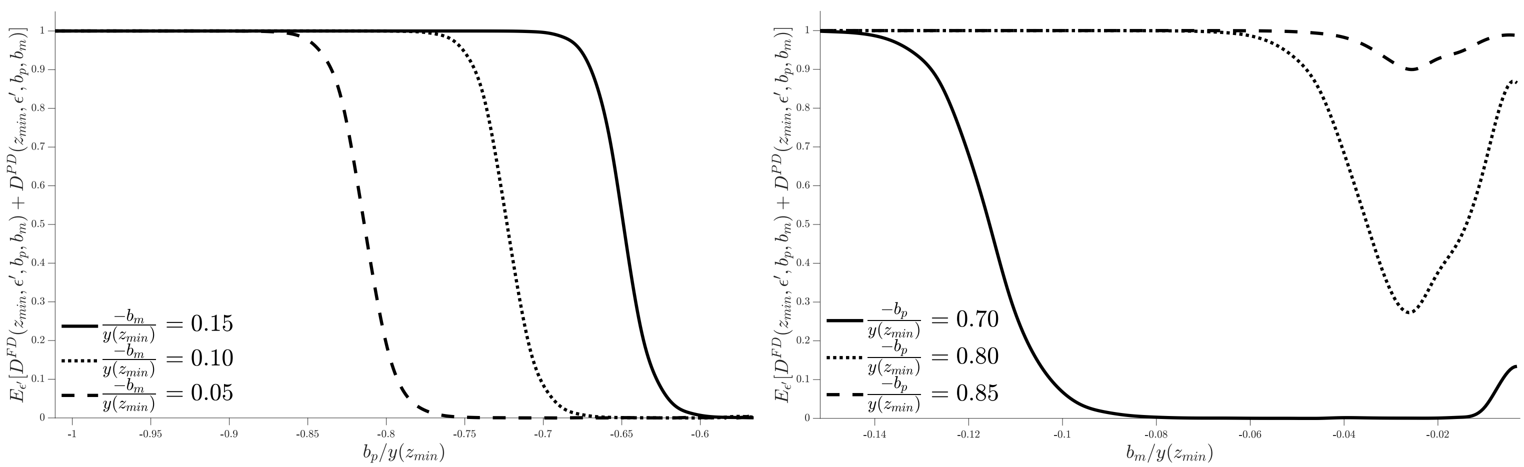
As the multilateral lender is always repaid in full, the recovery value upon *full* default is

$$q_m^{DF}(z, b_m^i, b_p^i) = \frac{1}{1+r}\mathbb{E}_{z'|z}\mathbb{E}_{\boldsymbol{\epsilon}'}\Big[(1 - A^{RF}(z', \boldsymbol{\epsilon}', b_m^i, b_p^i, W_l^{RF}))q_m^{DF}(z', b_m^i, b_p^i) + A^{RF}(z', \boldsymbol{\epsilon}', b_m^i, b_p^i, W_l^{RF})\bar{q}\Big].$$

The potential wedge between q_m^{DF} and $\bar{q}$ is solely due to the fact that the multilateral lender forgoes the missed coupon payments. Thus, unlike in Cordella and Powell (2021), the multilateral lender does not necessarily lend at the risk-free rate.

In what follows, I highlight three main features of the bond prices. First, the multilateral debt price is higher than the private bond price and that for two reasons. On the one hand, the default probability on the multilateral debt is lower than on the private debt owing to the greater output penalty. On the other hand, the recovery value for the multilateral debt is higher than the recovery value of private debt due to the policy of non-toleration of arrears.

Second, with monotonic bond policy functions, the private (multilateral) debt price decreases in the amount of private (multilateral) debt. This follows the standard argument of Chatterjee and Eyigungor (2012).



(a) b_p (b) b_m

Note: The figure depicts the overall default probability as a function of the debt level. Figure 3.5a plots the overall default probability as a function of b_p where I fix z at z_{min} and b_m at respectively 15%, 10% and 5% of output. Figure 3.5b plots the overall default probability as a function of b_m where I fix z at z_{min} and b_p at respectively 70%, 80% and 85% of output.

Figure 3.5: Overall Default Probability

Third, the effect of the multilateral debt on the private debt price is ambiguous. The reason is that the multilateral debt has two opposite effects on the default risk. On the one hand, more multilateral debt reduces the probability of a *partial* default given the additional multilateral debt servicing costs in autarky. On the other hand, more multilateral debt increases the probability of a *full* default. Thus, if additional multilateral debt decreases the probability of a *partial* default without a one-to-one increase in the probability of *full* default, the overall default risk effectively decreases.[27] Figure 3.5b depicts the default probability as a function of b_m and shows it is indeed U-shaped in some states. In opposition, Figure 3.5a depicts the same statistic as a function of b_p and shows no such pattern.

In particular, the effect of the multilateral debt on the private bond price depends on the interaction between the default and the subordination risks. The multilateral debt increases the subordination risk of private debt. As seen in Figure 3.4b, in a *full* default, absolute priority is given to the

[27]This is possible given that $y \geq y^{DP} > y^{DF}$.

repayment of the multilateral debt and the private lenders receive what is left. Hence, if additional multilateral debt increases the default probability, then the private bond price unambiguously decreases in the amount of multilateral debt as the default and the subordination risks go in the same direction. In opposition, if additional multilateral debt decreases the default probability, the private debt price may not necessarily decrease. The reduction in the default probability might be sufficiently large to compensate the reduction in the recovery value of private debt and potential future dilutions. Figure 3.5b suggests that this can only happen when the level of multilateral debt is not too large.

3.8.2 Optimal seniority structure

Having determined the prices, I can now characterize the optimal seniority structure. The definition of the competitive equilibrium can be found in Appendix 3.4.

To understand the tradeoff involved in the borrowing decision, I analyze the optimality conditions for the sovereign. I adopt a heuristic approach relying on three main assumptions. First, the bond choices have a continuous and compact support $B_p = [b_p^1, b_p^{\mathcal{P}}]$ and $B_m = [b_m^1, b_m^{\mathcal{M}}]$ without utility shocks. Second, I disregard buybacks and consider the case in which $b_j' - \delta b_j < 0$ for all $j \in \{m, p\}$. Third, as Arellano and Ramanarayanan (2012) and Arellano et al. (2023), I assume that the bond price functions $q_m(\cdot)$ and $q_p(\cdot)$ and the value of repayment $V^P(\cdot)$ are differentiable everywhere.[28] I then derive the first-order necessary conditions of the sovereign's problem given in (3.2) with respect to b_m',

$$u_c(c)\left[\frac{\partial q_m}{\partial b_m'}(b_m' - \delta b_m) + q_m + \frac{\partial q_p}{\partial b_m'}(b_p' - \delta b_p)\right] = \beta \mathbb{E}^R_{z'|z}\left[u_c(c')(1 - \delta + \delta\kappa + \delta q_m')\right], \quad (3.12)$$

and with respect to b_p',

$$u_c(c)\left[\frac{\partial q_m}{\partial b_p'}(b_m' - \delta b_m) + \frac{\partial q_p}{\partial b_p'}(b_p' - \delta b_p) + q_p\right] = \beta \mathbb{E}^R_{z'|z}\left[u_c(c')(1 - \delta + \delta\kappa + \delta q_p')\right], \quad (3.13)$$

[28] Mateos-Planas et al. (2022) show that this is generally not the case.

where $\mathbb{E}^R$ is the expectation in repayment, $u_c(\cdot)$ represents the first derivative of $u(\cdot)$ with respect to c and $q'_j = q_j(z', b''_m, b''_p)$ for $j \in \{m, p\}$ is the bond price next period. The left-hand side of each first-order condition represents the marginal benefits of issuing one additional unit of the type of debt concerned, whereas the right-hand side represents the marginal costs of this additional issuance. Following the argument in the previous subsection, one has

$$\frac{q_m}{q_p} \geq 1, \quad \frac{\partial q_p}{\partial b'_p} \geq 0, \quad \frac{\partial q_m}{\partial b'_m} \geq 0 \quad \text{and} \quad \frac{\partial q_p}{\partial b'_p} \lesseqgtr \frac{\partial q_p}{\partial b'_m}.$$

This together with the first-order conditions unveil two effects that shape the optimal holdings of debt in the model: the seniority benefit or cost and the subordination benefit. The former relates to the multilateral debt and is given by the ratio of the left-hand side of (3.12) and (3.13) each divided by the private debt price,

$$\text{Seniority benefit or cost} = \frac{\frac{q_m}{q_p} + \frac{\partial q_m}{\partial b'_m}\frac{(b'_m - \delta b_m)}{q_p} + \frac{\partial q_p}{\partial b'_m}\frac{(b'_p - \delta b_p)}{q_p}}{1 + \frac{\partial q_m}{\partial b'_p}\frac{(b'_m - \delta b_m)}{q_p} + \frac{\partial q_p}{\partial b'_p}\frac{(b'_p - \delta b_p)}{q_p}}.$$

The numerator (denominator) corresponds to the marginal impact of issuing multilateral (private) debt on the incentive to repay. Whether there is a seniority benefit or a seniority cost depends on the relative sensitivity of the private bond price with respect to the two types of debt. Given that the multilateral debt is eventually repaid in full, one would expect that $\frac{\partial q_m}{\partial b'_m} \approx \frac{\partial q_m}{\partial b'_p}$. Thus, if $\frac{\partial q_p}{\partial b'_p} > \frac{\partial q_p}{\partial b'_m}$, the sovereign has a greater incentive to repay when it issues multilateral debt. As argued before, this typically happens when $-b'_m$ is relatively small. In this case there is a seniority benefit to the repayment incentive. Conversely, when $\frac{\partial q_p}{\partial b'_p} < \frac{\partial q_p}{\partial b'_m}$, there are two cases. If $\frac{q_m}{q_p}$ is sufficiently large, there is still a seniority benefit. Otherwise, the sovereign has a lower incentive to repay when it issues multilateral debt and there is therefore a seniority cost.

The subordination benefit relates to the private debt and corresponds to the ratio of the right-hand side of (3.12) and (3.13),

$$\text{Subordination benefit} = \frac{\mathbb{E}^R_{z'|z}\left[u_c(c')(1-\delta+\delta\kappa+\delta q'_m)\right]}{\mathbb{E}^R_{z'|z}\left[u_c(c')(1-\delta+\delta\kappa+\delta q'_p)\right]},$$

which one can reformulate as

$$\frac{\mathbb{E}^R_{z'|z}\left[u_c(c')\right]\mathbb{E}^R_{z'|z}\left[1-\delta+\delta\kappa+\delta q'_m\right]+\operatorname{cov}(u_c(c'),\delta q'_m)}{\mathbb{E}^R_{z'|z}\left[u_c(c')\right]\mathbb{E}^R_{z'|z}\left[1-\delta+\delta\kappa+\delta q'_p\right]+\operatorname{cov}(u_c(c'),\delta q'_p)}.$$

Owing to the *de facto* seniority, it is difficult to dilute the multilateral debt. The sovereign is less likely to renege multilateral debt and when it does it has to repay in full what it defaulted on. Hence, q'_m remains relatively close to $\bar{q}$ due to the high recovery value, while q'_p can get closer to 0. This means that in low endowment states, the price of private debt tomorrow, q'_p, can decrease relatively more when the prospective consumption is low. If this is the case, then the above ratio is greater than one as $\operatorname{cov}(u_c(c'), q'_p) \leq \operatorname{cov}(u_c(c'), q'_m) < 0$ and $\mathbb{E}^R_{z'|z}\left[q'_m\right] \geq \mathbb{E}^R_{z'|z}\left[q'_p\right]$. The private debt becomes therefore more attractive to the sovereign than the multilateral debt. The possibility to dilute private debt reduces the marginal cost of debt issuance as it reduces the future debt burden.

All in all, the multilateral debt has a dual effect. In small amount, it can generate a large value at the issuance, whereas, in large amount, it overly depresses the value of private debt issuance owing to subordination. In addition, it is less prone to dilution than private debt making it more costly to repay at the maturity. This shapes the borrowing choice of the sovereign. In particular, the optimal seniority structure is determined such that the seniority benefit or cost equates the subordination benefit.[29]

This tradeoff closely relates to the one in Arellano and Ramanarayanan (2012) and Niepelt (2014) in which the sovereign has to choose between short-term and long-term debts. The former debt has to be repaid in the next period, while only a fraction of the latter matures. The price of

[29]Note that with one-period debt (i.e. $\delta = 0$), there is no subordination benefit. In this case, the sovereign issues private and multilateral debt such that the seniority benefit or cost equates the ratio of expected marginal utilities in repayment.

long-term bonds therefore includes the prospective value of debt rendering it more sensitive to the default risk. Given this, the short-term debt has beneficial effects on the incentive to repay, whereas the long-term debt provides an hedge against future low endowments. In my model, the tradeoff is similar with the main exception that more multilateral debt – unlike short-term debt – does not always increase the incentive to repay.

The spillover effect of multilateral debt on private debt addresses the catalytic function of official multilateral lending. As noted by Krahnke (2020), the seniority can crowd out private capital flows if the amount of senior debt becomes too large. A similar effect arises in this model. While some level of multilateral debt encourages the sovereign to repay its debt, large amounts of multilateral debt considerably dilute the private debt.

3.9 Calibration and Model Evaluation

This section presents the calibration of the model and evaluates the goodness of fit with respect to targeted moments, the empirical facts of Section 3.4 and other non-targeted moments.

3.9.1 Calibration and targeted moments

The model is solved using numerical methods presented in Appendix 3.5 and is calibrated in the following way. Some parameters are borrowed from the literature, some are estimated directly from the data and the remainders are selected to match some specific moments.

I calibrate the model to Argentina for the period 1970 to 2018 with a yearly frequency. Table 3.2 summarizes the main parameters of the model. The utility function takes the constant relative risk aversion (CRRA) form,

$$u(c) = \frac{c^{1-\varrho}}{1-\varrho},$$

where the risk aversion parameter, ϱ, is set to the standard value of 2 adopted in the real business cycle literature. The risk-free rate is 4.2% to match the average real 10-year US Treasury bonds yield (Dvorkin et al., 2021).[30] Finally, the stochastic process follows a log-normal AR(1) process $\log y_t = \rho \log y_{t-1} + \varepsilon_t$ with $\varepsilon \sim N\left(0, \sigma_\varepsilon^2\right)$. Following the estimation of Aguiar and Gopinath (2006) for Argentina, the persistence of the endowment shock ρ is set to 0.9 and the standard deviation σ_ε to 0.034. The stochastic process is discretized into a 8-state Markov chain following the approach of Tauchen (1986).

Table 3.2: Parameters

Parameter	Value	Description	Targeted Moment	Target	Model
A. Based on Literature					
ϱ	2	Risk aversion			
B. Direct Measure from the Data					
r	0.042	Risk-free rate	Average 10-year US real Treasury yield		
δ	0.9	Reciprocal of average maturity	Average maturity structure		
κ	0.12	Coupon payments	Average coupon rate		
ρ	0.9	Output persistence	Argentina GDP		
σ_ϵ	0.034	Standard deviation			
C. Based on Model solution					
β	0.9415	Discount factor	Debt-to-GDP ratio (%)	41.58	40.77
$\mathcal{A}$	-0.12	Multilateral borrowing limit	Multilateral-debt-to-GDP ratio (%)	5.89	6.27
ϕ	0.4915	Bargaining power	Average SZ haircut (%)	37.50	35.46
ψ	0.882	General default cost	Average default duration (year)	3.60	3.11
$\varkappa^{DP}$	0.89	Initial *partial* default cost	Default rate (%)	2.50	2.46
$\varkappa^{DF}$	0.85465	Initial *full* default cost	Share *full* default (%)	17.65	18.52
α_1	10^{-8}	Debt issuance cost (intercept)	Average issuance costs (%)	0.20	0.11
α_2	19.5	Debt issuance cost (slope)	Debt increase prior to default (percentage point)	22.00	26.84
ω	0.0075	Utility shock variance parameter	Standard deviation debt-to-GDP ratio	8.00	9.84
υ	0.2	Utility shock correlation parameter	Standard deviation duration	4.67	3.37

Following Chatterjee and Eyigungor (2012), I set $\kappa = 0.12$ to directly match the average coupon rate of Argentina. I choose $\delta = 0.9$ to match the average maturity which I estimate as the ratio of the external debt over the external debt service.[31] I subsequently select the value of the discount factor to match the average external debt-to-GDP ratio of Argentina. I obtain $\beta = 0.9415$ which is within the bounds admitted in the real business cycle and sovereign debt literature. In addition, the bargaining power is set so as to match the overall average SZ haircut. The value of 0.4915 gives a minor advantage to the borrower, whereas Benjamin and Wright (2013) and Dvorkin et al. (2021) set ϕ slightly above 0.5 giving a minor advantage to the lenders. Note that the average haircut for defaults with multilateral creditors (i.e. *full* defaults) is not targeted.

I introduce an exogenous limit $\mathcal{A} \leq 0$ to multilateral debt and that for two reasons. First, without

[30] Average 10-year US Treasury bond rate minus PCE inflation between 1980 and 2010.

[31] Based on Broner et al. (2013), many studies target an average maturity of 5 years for Argentina. My target of 10 years diverges from this benchmark as Broner et al. (2013) estimate the maturity using private bonds, while my estimate takes into account the total external debt.

this, the sovereign would accumulate mostly multilateral debt and very little private debt. Second, a borrowing limit is consistent with the fact that the IMF and the WB impose lending quotas. I calibrate $\mathcal{A}$ to match the multilateral debt-to-GDP ratio of Argentina.

Similar to Dvorkin et al. (2021), I differentiate the output cost when entering and staying in default. When the sovereign enters a *partial* default, its endowment is given by $y^{DP}(z) = \varkappa^{DP} y^D(z)$, while if it enters a *full* default, it receives $y^{DF}(z) = \varkappa^{DF} y^D(z)$. Conversely, if the sovereign stays in default its endowment is given by $y^{DP}(z) = y^{DF}(z) = y^D(z)$, where

$$y^D(z) = \begin{cases} \bar{y}, & \text{if } y(z) \geq \bar{y} \\ y(z) & \text{if } y(z) < \bar{y} \end{cases} \quad \text{with } \bar{y} = \psi \mathbb{E}[y(z)]$$

The output cost is made of two components: ψ and $(\varkappa^{DF}, \varkappa^{DP})$. The former relates to the standard asymmetric cost of Arellano (2008). It directly impacts the length of default and is therefore not specific to the type of default as I target the overall average default duration. The initial default cost impacts the default rate. Hence, I calibrate $\varkappa^{DP}$ to match a 2.5% default rate (Tomz and Wright, 2007, 2013) and $\varkappa^{DF}$ to match the share of defaults on multilateral debt reported in Fact I.

Owing to the positive recovery value of debt, this model is subject to large increases in indebtedness and consumption booms prior to default. This problem is further reinforced by the fact that additional multilateral debt can largely dilute private debt. There are different ways of dealing with this problem. Hatchondo et al. (2016) impose a limit on the private bond spread, Dvorkin et al. (2021) set a transaction cost on portfolio adjustments and Fourakis (2021) adds a premium related to the default risk. To avoid to distort too much the tradeoff between private and multilateral debt and the choice between *partial* and *full* default, I adopt an issuance cost of the following form

$$\varpi(b_p^j, b_p^j) = \alpha_1 \exp\left(\alpha_2 |b_p^j + b_m^j|\right) - \alpha_1.$$

The parameter α_1 commands the intercept, while α_2 gives the slope of the issuance cost. The former is calibrated to match an issuance cost of 0.2% and the latter to match the increase of the debt ratio prior to default.

Finally, I calibrate the variance and the correlation parameters of the utility shocks to match the standard deviation of the debt-to-GDP ratio and the standard deviation of the duration, respectively. In Appendix 3.6, I show that with this calibration, the utility shocks do not significantly impact the default rate, the haircuts, the duration or the debt choices.

3.9.2 Facts and other non-targeted moments

In terms of non-targeted moments, I first assess how the model matches the empirical facts of Section 3.4. As one can see in Table 3.3, without directly targeting such moments, the model generates haircuts and durations that are in line with Facts II and III and that for defaults with and without multilateral creditors. The model nonetheless underestimates the average length and haircut of defaults implicating multilateral creditors. It can get closer to the median values but cannot exactly match the duration. Finally, the multilateral lender lends at a rate very close to the risk-free rate consistent with Fact IV.

I also gauge how the model matches non-targeted business cycles moments presented in Table 3.4. I find that it replicates moments related to consumption in an accurate way except perhaps for the relative volatility which is below 1. For the trade balance, the model indicates a surplus as in the data. However, it fails to generate countercyclical trade balance. In addition, the model can reproduce neither the mean nor the volatility of the private debt spread. Nevertheless, the maximum private debt spread amounts 31% which is in-between the maximum observed in Argentina and emerging economies. The spread on multilateral debt is 0.44% which is close to the IMF's and the IBRD's spread. Besides this, it is roughly three times smaller than the private spread. The same holds true for the relative volatility which is around 0.40 for the private debt, but only amounts 0.12 for the multilateral debt.

In comparison to previous studies, my model matches relatively well moments related to emerging economies. Dvorkin et al. (2021) report an average spread of 1.01% overall and 1.37% in bad

Table 3.3: Empirical Facts

	Data		Model	
	Mean	Median	Mean	Median
Default length (year) (overall)	3.60	1.58	3.11	1.69
Default length (year) (with multilateral lenders)	8.50	7.58	4.95	4.15
Default length (year) (without multilateral lenders)	2.60	1.33	2.68	1.58
Private creditors' haircut (%) (overall)	37.50	32.50	35.46	36.08
Private creditors' haircut (%) (with multilateral lenders)	59.00	55.20	49.34	49.71
Private creditors' haircut (%) (without multilateral lenders)	32.90	29.00	32.14	34.25
Share *full* default (%)	17.65	-	18.52	-
Multilateral debt spread (%)	0.54	0.89	0.44	0.26

Note: Data moments come from Table 3.1. The multilateral debt spread corresponds to the quarterly average spread removing the concessional lending of the IDA.

time. They also cannot generate a countercyclical trade balance. Calibrating a model with long-term bonds and exogenous restructuring to Argentina, Chatterjee and Eyigungor (2012) report an average spread of 8.15% with a standard deviation of 0.04.[32] Moreover, they obtain a correlation between consumption and output of 0.99 and a correlation between the trade balance (over output) and output of -0.44. Finally, they report a volatility of consumption relative to output of 1.11 and a volatility of the trade balance (over output) relative to output of 0.2. Hence, except for the countercyclical trade balance and the spread moments, my model generates statistics very close the aforementioned ones. Especially, Chatterjee and Eyigungor (2012) obtain a better fit for the spread for two reasons. First, they assume a recovery value of zero, while the present model generates strictly positive recovery values. Second, they use a quadratic default penalty function to match the volatility of spread, while I adopt the standard asymmetric penalty.

[32] See Asonuma and Joo (2020) for a comparison of models with endogenous and exogenous restructurings under short-term debt.

Table 3.4: Selected Business-Cycle Moments

x	Mean(x)	Max(x)	Min(x)	Std(x)	Std(x)/Std(y)	Corr(x, y)
c						
Model	98.05	120.08	86.94	0.03	0.96	0.92
Argentina	75.27	86.08	63.79	0.04	1.17	0.98
$(y-c)/y$						
Model	1.95	13.06	-20.08	0.01	0.39	0.29
Argentina	1.83	9.11	-2.90	0.01	0.31	-0.90
$r_p - r$						
Model	1.17	31.40	0.00	0.01	0.40	-0.45
EMBI+ Spread for Emerging Markets	4.72	12.36	1.62	0.02	0.74	-
EMBI+ Spread for Argentina	13.51	57.23	3.20	0.16	4.77	-0.63
$r_m - r$						
Model	0.44	7.89	0.00	0.00	0.12	-0.51
IMF Spread	0.76	4.13	-1.76	0.01	0.43	-0.21
IBRD Spread	0.30	1.96	-2.41	0.01	0.41	-0.20
IDA Spread	-1.78	1.31	-6.63	0.02	0.70	-0.31

Note: The sample runs from 1970 to 2018. Consumption mean, min and max are with respect to output. Output, consumption and the trade balance are detrended with the Hodrick-Prescott filter with a smoothing parameter of 6.25. The IMF, the IBRD and the IDA spreads correspond respectively to the IMF adjusted rate of charge, the IBRD lending rate and the IDA service charge from which is deducted the yield on 1-year US government bonds. See Appendix 3.2 for more details.

3.10 Quantitative Analysis

In this section, I first study the dynamic of default through an event analysis. I then conduct counterfactual analyses regarding the seniority assumption and assess the changes in welfare. In Appendix 3.7, I relax the assumption of equal maturity for the two types of debt.

3.10.1 Default dynamic

This subsection aims at explaining the dynamic of defaults in the model. For this purpose, I first compute the statistics of endowment and indebtedness close to default episodes. I subsequently conduct an event analysis in a window of five years before and after a default.

Table 3.5 depicts the main statistics of the model around defaults. The general dynamic is consistent with the empirical findings of Benjamin and Wright (2013). First, default's settlements usually arise when the sovereign's economic situation recovers. Most notably, defaults tend to start when the sovereign's GDP is below trend, whereas it usually ends when the sovereign's GDP settles back on the trend. Second, default's resolutions are not associated with a substantial reduction

Table 3.5: Endowment and Debt Around Default

		Endowment (percent of $\overline{y}$)	Private debt (percent of y)	Multilateral debt (percent of y)	Total debt (percent of y)
Partial default	Before	87.3	39.5	7.5	47.1
	At	76.9	68.2	0.5	68.8
	After	91.4	44.5	0.4	44.9
Full default	Before	90.5	59.6	8.0	67.6
	At	75.2	98.0	15.8	113.8
	After	97.1	64.2	0.0	64.2

Note: The table depicts the average endowment and debt around *partial* and *full* defaults. The averages come from simulation over 2000 economies for 600 periods where the initial 200 periods are discarded. The variable $\bar{y}$ corresponds to the average output.

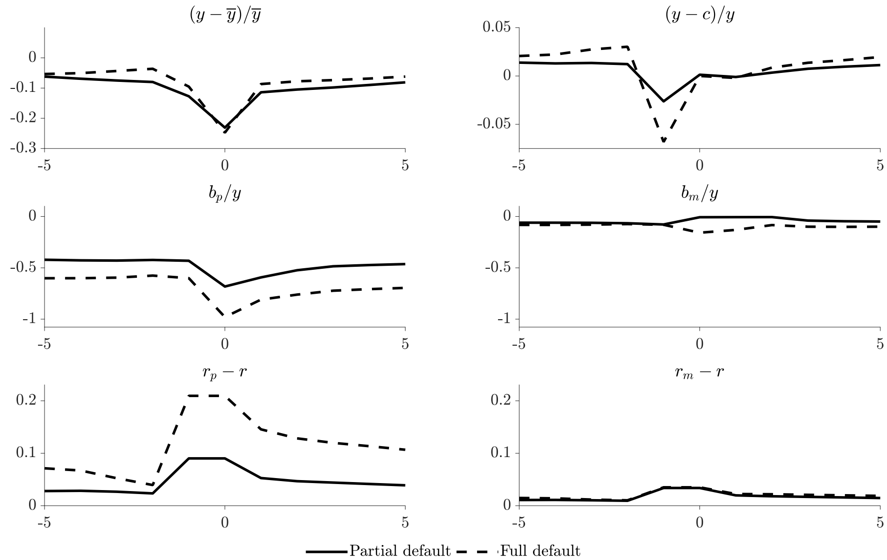
of indebtedness.[33] Third, the sovereign does not necessarily accumulate the same level of debt depending on the type of default. Particularly, *partial* defaults are associated with lower levels of debt than *full* defaults.

To construct the event analysis, I simulate 2000 economies for 600 periods. To make sure that the initial conditions do not matter, I discard the first 200 periods. I then identify the five periods preceding and succeeding a default and take the average over the simulated panel. I discriminate between *partial* and *full* defaults both in the model and in the data.

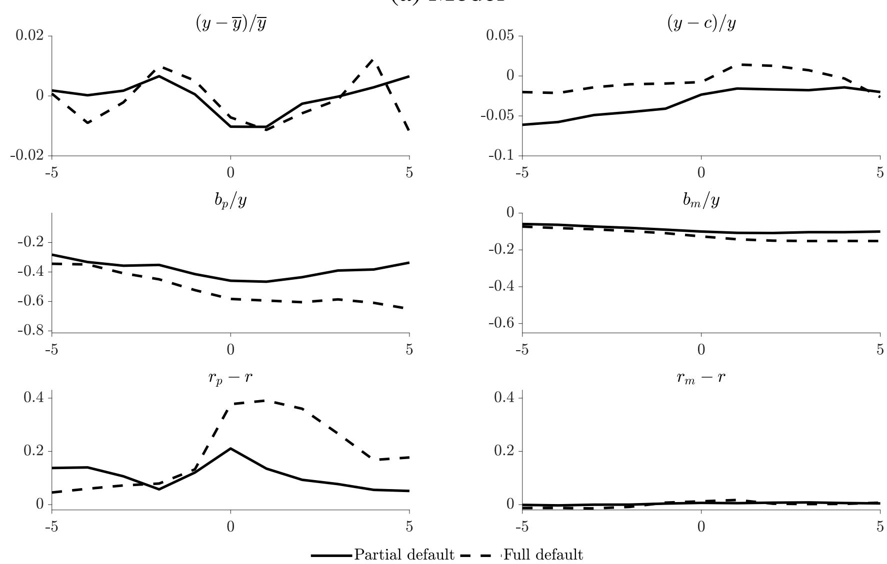
Figure 3.6a depicts the event analysis for some selected variables in the model.[34] Period 0 corresponds to the occurrence of default. The solid line relates to a *partial* default, while the dashed line corresponds to a *full* default. As one can clearly see, *full* defaults are precedented by a rapid output contraction and a greater accumulation of debt. Furthermore, the sovereign country records a trade balance reversal at time 0. In the year before a *full* default, the trade balance becomes negative and suddenly reverts with the default. Finally, the private bond spread experiences a sudden and very large increase shortly before the occurrence of default. The multilateral debt spread also reacts but in a negligible manner.

[33]Nevertheless, as noted by Arellano et al. (2023), the model does not generate a larger indebtedness at the end of default compared to the beginning of default.

[34]See Figure C.5 for the median instead of the average over the simulated panel.



(a) Model



(b) Data

Note: The figure depicts the evolution of endowment, trade balance, debt and spreads around *partial* and *full* defaults. Period 0 corresponds to the occurrence of default. In the model, the five-year window averages come from simulation over 2000 economies for 600 periods where the initial 200 periods are discarded. The variable $\bar{y}$ corresponds to the average output. In the data, averages come from the entire sample used in Section 3.4. The variable $(y-\overline{y})/\overline{y}$ corresponds to the deviation from the GDP trend using the HP filter with a smoothing parameter of 6.25. See Appendix 3.2 for more details.

Figure 3.6: Event Analysis

In opposition, *partial* defaults are related to a slower output contraction. Despite different output costs, the average endowment at time 0 is similar to the one in *full* defaults. Moreover, there is a relatively limited trade balance reversal in the vicinity of default. Besides this, one observes a reduction of multilateral indebtedness on the default path. Most of the indebtedness comes from the private sector. The private bond spread increases as the economy approaches default but far less than in the case of *full* defaults. Consistent with what has been said before, the multilateral debt spread remains relatively modest in comparison to the private debt spread.

Figure 3.6b depicts the event analysis for some selected variables in the data. As in the model, a *full* default arises after sudden and sharp reduction in output, when the level of indebtedness is large. The average output at time 0 is similar to the one in *full* defaults. In addition, the private debt spread reacts more than in a *partial* default. The trade balance reversal is nonetheless less marked in the data. Thus, except for the trade balance, the models replicates very closely the movements in the data in the vicinity of a default.

3.10.2 Towards a de jure seniority

This subsection analyzes the role of the output penalty. In the calibration, I assumed that the output penalty differs between *partial* and *full* defaults only when entering default. I therefore consider two extreme cases. First, I equalize $\varkappa^{DF}$ and $\varkappa^{DP}$ to show that this mainly affects the share of *full* default. Second, I set $\varkappa^{DF} = 0$ which, given the form of the utility function, implies a *de jure* seniority on multilateral debt.

Table 3.6 depicts the moments related to the model with different output penalties alongside the benchmark model. In the case of equal output penalty – i.e. $\varkappa^{DF} = \varkappa^{DP} = 0.85465$ – the share of *full* default becomes almost 100%. The default's length and the haircut reduce compared to the benchmark prediction but the wedge between *partial* and *full* defaults remains. As seen in the previous subsection, *full* defaults are associated with larger indebtedness. This explains the greater average debt ratio. Moreover, despite a lower default rate, the private bond spread is larger as the private debt is subordinated in *full* defaults. Finally, even though *full* defaults are more frequent, the multilateral debt spread remains stable.

The model with equal output penalty exaggerates the occurrence of *full* defaults compared to the data but does not significantly affect the wedge in duration and haircut between *partial* and *full*

Table 3.6: Alternative Settings

	Benchmark	$\varkappa^{DF} = \varkappa^{DP}$	*de jure*	$\mathcal{A} = 0$	*pro rata*	$b_m^j \leq 0$	*pari passu*
Default length (year) (with multilateral lenders)	4.95	3.51	-	-	3.01	5.58	2.63
Default length (year) (without multilateral lenders)	2.68	1.87	5.73	5.59	2.82	5.88	2.96
Private creditors' haircut (%) (with multilateral lenders)	49.34	44.52	-	-	29.93	25.47	30.54
Private creditors' haircut (%) (without multilateral lenders)	32.14	37.95	21.94	22.68	28.98	24.29	29.59
Share *full* default (%)	18.52	97.80	0.00	-	10.07	65.72	24.51
Default rate (%)	2.46	2.15	3.43	3.78	2.34	3.61	2.34
Total debt increase (percentage point) (prior to default)	26.84	39.49	25.25	23.89	20.69	30.05	21.83
Total debt to GDP (%)	40.77	44.59	58.61	54.39	40.71	59.55	40.24
Multilateral debt to GDP (%)	6.27	6.62	6.46	-	6.28	6.99	6.22
Private debt spread (%)	1.17	1.27	1.46	1.56	0.90	1.61	0.94
Multilateral debt spread (%)	0.44	0.29	0.00	-	0.66	0.69	0.73

defaults. It therefore shows the role of such channel in generating the *de facto* seniority.

Turning to the case in which $\varkappa^{DF} = 0$, the multilateral debt becomes *de jure* senior. The model is similar to the one of Hatchondo et al. (2017) with the difference that the default's length and the haircut are endogenous. As one can see in Table 3.6, the default's duration becomes more pronounced, while the average haircut reduces relative to the benchmark case. Furthermore, the default rate, the private bond spread and the debt ratio increase. I come back later on why the default's duration and the total indebtedness increase so much.

To complement the above argument, I also consider the case in which the sovereign has only access to private debt – i.e. $\mathcal{A} = 0$. This brings my analysis closer to the one of Benjamin and Wright (2013). As one can see in Table 3.6, the model without the multilateral lender generates predictions very close to the case of *de jure* seniority in terms of haircut, default length, default rate and spread. However, it produces unrealistic default durations compared to the ones reported in Table 3.1. Hence, even though multilateral debt represents a small portion of the total debt and *full* defaults are infrequent, the presence of multilateral debt directly affects the average haircut and duration. The coexistence of multilateral and private debt seems therefore a key element explaining the dynamic of emerging economies.

3.10.3 Towards a pari-passu clause

I introduce a *pari passu* clause between the multilateral and the private lender which consists of two components. On the one hand, there is the possibility of a net multilateral debt issuance (i.e. $b^j_m \leq 0$) upon restructuring. On the other hand, the multilateral and private lenders make a common offer W which is split *pro rata*. That is the multilateral lender is not anymore repaid in full. To identify the impact of each of these two components, I consider them separately before joining them together.

To analyze the *pro rata* split, consider that the two types of lenders make a joint offer X for the entire debt. Formally, the value of a restructuring upon *full* default reads

$$V^{RF}(z,\boldsymbol{\epsilon},b^i_m,b^i_p) = \phi\Omega^{RF}(z,\boldsymbol{\epsilon},b^i_m,b^i_p,X^{RF}_l) + (1-\phi)\Omega^{RF}(z,\boldsymbol{\epsilon},b^i_m,b^i_p,X^{RF}_b).$$

where the sovereign's offer is given by

$$X^{RF}_b(z,b^i_m,b^i_p) = -b^i_p q^{DF}_p(z,b^i_m,b^i_p) - b^i_m q^{DF}_m(z,b^i_m,b^i_p).$$

Conversely, the joint offer of the private and multilateral lender is

$$\begin{aligned} X^{RF}_l(z,b^i_m,b^i_p) = \arg\max & \Big[\mathbb{E}_{\boldsymbol{\epsilon}} A^{RF}(z,\boldsymbol{\epsilon},b^i_m,b^i_p,X)X \\ & + (1-\mathbb{E}_{\boldsymbol{\epsilon}} A^{RF}(z,\boldsymbol{\epsilon},b^i_m,b^i_p,X))X^{RF}_b(z,b^i_m,b^i_p)\Big] \\ \text{s.t.} \quad & X \leq -(b^i_p + b^i_m)\bar{q}. \end{aligned}$$

Finally, the transfer upon restructuring is given by

$$\tau = q_m(z, b^j_m, b^j_p)(-b^j_m) - X \geq 0,$$

where the private lenders get a share $\frac{b^j_p}{b^j_p + b^j_m}$ of X upon restructuring and the multilateral lender the remaining part. There is no multilateral debt issuance upon restructuring yet.

Table 3.6 presents the result of the *pro rata* split. *Partial* and *full* defaults have now similar average duration and haircut. Moreover, despite similar debt ratio and default rate, one observes a larger multilateral debt spread compared to the benchmark case. Thus, the *pro rata* split does weaken the *de facto* seniority of multilateral lenders. More precisely, the full repayment of multilateral lending institutions is a prerequisite to safeguard lending at preferential rates. However, this comes at the cost of private debt subordination implying a larger spread and haircut on the private debt.

Having shown the impact of relaxing the full repayment of multilateral lenders, I now consider that, upon restructuring, there is a net multilateral debt issuance (i.e. $b^j_m \leq 0$). Formally, one has that

$$\begin{aligned} V^{EF}(z, \boldsymbol{\epsilon}, b^i_m, W - b^i_m \bar{q}) = \max_j\, & u(c) + \epsilon_j + \beta \mathbb{E}_{z'|z} \mathbb{E}_{\boldsymbol{\epsilon}'} V(z', \boldsymbol{\epsilon}', b^j_m, b^j_p) \\ \text{s.t.} \quad & c = y(z) + \tau, \\ & \tau = q_m(z, b^j_m, b^j_p)(-b^j_m) + q_p(z, b^j_m, b^j_p)(-b^j_p) - (W - b^i_m \bar{q}), \\ & \tau \geq 0. \end{aligned}$$

Thus, b^j_m is not anymore restricted to be zero. This means that the sovereign's value of restructuring in a *full* default is larger than in the benchmark case. Nevertheless, the multilateral lender is still repaid in full – i.e. $-b^i_m \bar{q}$.

Table 3.6 presents the result of introducing multilateral debt issuance upon restructuring. As before, *partial* and *full* defaults have similar average duration and haircut. The multilateral debt spread increases but to a lesser extent than in the case of a *pro rata* split. Unlike the *pro rata* split, this comes from the larger default rate combined with the greater share of *full* default compared to the benchmark case. Thus, the the inability to issue new multilateral debt at the restructuring largely impacts the private creditors' haircut and the default duration.

Joining the multilateral debt issuance together with the *pro rata* split, I obtain a *pari passu* clause between the two types of lenders.[35] The renegotiation process under *full* default is now isomorphic to the one under *partial* default.[36] The exact outcome is in-between the above two cases, albeit closer to the *pro rata* split. As shown in Table 3.6, the two types of default have analogous average haircut and duration. The default rate and debt ratio are lower than in the benchmark case. Moreover, the multilateral debt spread largely increases and comes closer to the private one. Private and multilateral debt become therefore closer substitutes. As a result, the sovereign accumulates more private debt and less multilateral debt than in the benchmark case.[37] This exercise therefore shows the role of the policy of non-toleration of arrears in generating the *de facto* seniority.

3.10.4 Debt, duration and haircut

The comparison between the *de facto*, the *de jure* and the *pari passu* regimes is instructive on what is the source of longer durations and larger haircuts in the benchmark model.

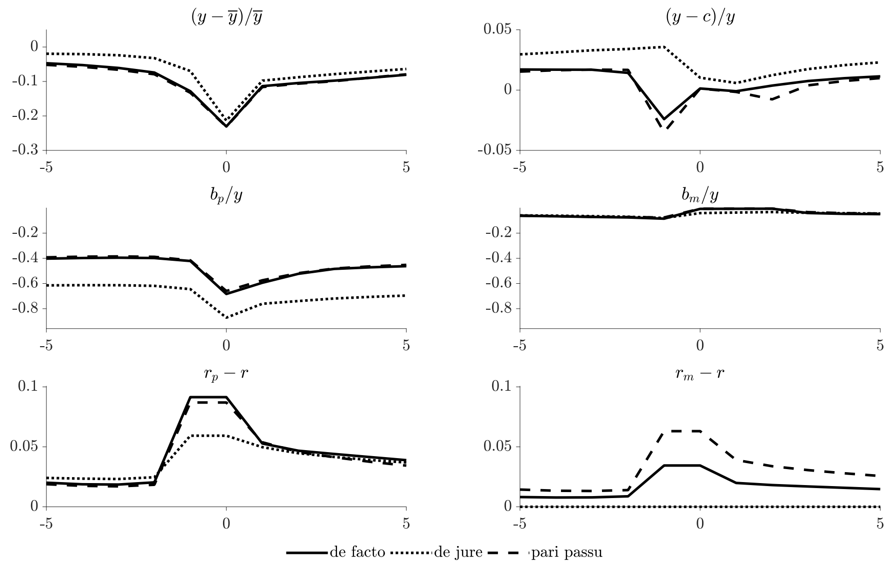
Regarding the haircut, it is clear from Table 3.6 that the policy of non-toleration of arrears is the key component behind larger private creditors' losses. Once one removes either the full repayment of multilateral debt (i.e. the *pro rata* split) or the non-access of multilateral borrowing upon restructuring (i.e. $b_m^j \leq 0$), the haircut in a *full* default drastically decreases and becomes similar to the one in a *partial* default. In particular, Table 3.7 shows that the *de facto* seniority

[35]In this case, the transfer upon restructuring is given by $\tau = q_m(z, b_m^j, b_p^j)(-b_m^j) + q_p(z, b_m^j, b_p^j)(-b_p^j) - W \geq 0$, where W is split *pro rata*.

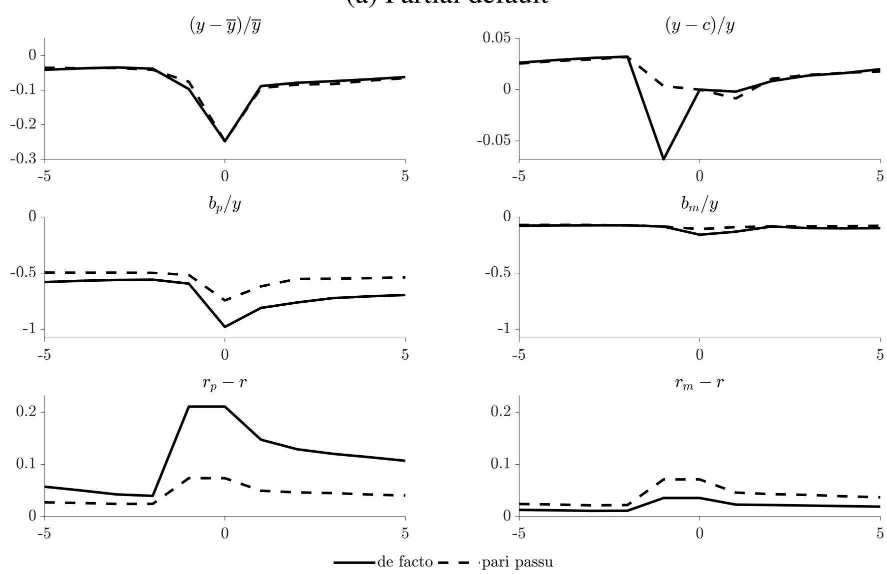
[36]The only difference is the value of autarky. Under *partial* default, the sovereign continues to service the multilateral debt in autarky.

[37]Note that if in addition to the *pari passu* clause, I equalize the output penalty between the *partial* and the *full* default, the private and multilateral debt become perfect substitutes. In this case, the share of *full* default goes to 100% and the spread equalizes between the two types of debt.

always generates the largest haircuts in a *full* default.



(a) Partial default



(b) Full default

Note: The figure depicts the evolution of endowment, trade balance, debt and spreads around defaults for the different seniority regimes. The five-year window averages come from simulation over 2000 economies for 600 periods where the initial 200 periods are discarded. Period 0 corresponds to the occurrence of default. The variable $\bar{y}$ corresponds to the average output.

Figure 3.7: Event Analysis in Partial and Full Defaults

Regarding the duration, what mainly explains the differential between a *full* and a *partial* default in the benchmark model is the private debt accumulation. Notably, larger total indebtedness mechanically produces longer defaults as the sovereign waits that the default risk reduces before restructuring. In addition, as I consider an exogenous borrowing limit $\mathcal{A}$, the multilateral debt has a limited impact on the total indebtedness – and therefore on the duration – when the stock of private debt is relatively large. As one can see in Figure 3.7a, in a *partial* default, the sovereign accumulates less private debt under the *de facto* seniority than under the *de jure* seniority. This translates into a lower *partial* default duration in the benchmark model. In opposition, as shown in Figure 3.7b, the sovereign accumulates a higher amount of private debt in a *full* default under the *de facto* seniority. As a result, the length of a *full* default is longer than in a *pari passu* regime.

To disentangle the effect of the larger private debt accumulation relative to the multilateral debt stock, I compute the default duration for private debt levels around $\mathcal{A}$ at z_{min} for the different seniority regimes. In the benchmark model, a *full* default is always related to a longer average duration than a *partial* default. Moreover, the wedge is more pronounced when the multilateral debt is high. Besides this, under the *pari passu* regime, durations are never higher than in the *de facto* seniority.[38]

Table 3.7: Duration and Haircut in Partial and Full Defaults

	Private debt	Multilateral debt	Average duration (year)		Average haircut (%)	
			Partial default	*Full* default	*Partial* default	*Full* default
de facto	$\mathcal{A}$	$\mathcal{A}$	1.7	6.3	6.2	9.2
	$\mathcal{A}$	$\mathcal{A}/3$	1.2	3.8	6.1	7.7
	$\mathcal{A}/3$	$\mathcal{A}$	1.1	5.1	5.0	6.4
	$\mathcal{A}/3$	$\mathcal{A}/3$	1.0	1.4	5.0	5.8
de jure	$\mathcal{A}$	$\mathcal{A}$	1.1	-	6.1	-
	$\mathcal{A}$	$\mathcal{A}/3$	1.1	-	5.9	-
	$\mathcal{A}/3$	$\mathcal{A}$	1.0	-	4.9	-
	$\mathcal{A}/3$	$\mathcal{A}/3$	1.0	-	4.9	-
pari passu	$\mathcal{A}$	$\mathcal{A}$	1.4	3.7	6.1	8.0
	$\mathcal{A}$	$\mathcal{A}/3$	1.4	1.9	6.1	7.6
	$\mathcal{A}/3$	$\mathcal{A}$	1.0	1.5	5.0	5.9
	$\mathcal{A}/3$	$\mathcal{A}/3$	1.0	1.2	5.0	5.4

Note: The table depicts the average duration and haircut in *partial* defaults for the different seniority regimes. The averages come from simulation over 2000 economies for 600 periods where the initial 200 periods are discarded. I fix the endowment at z_{min}. $\mathcal{A}$ corresponds to the multilateral borrowing limit.

Thus, in my specific calibration, the longer duration of a *full* default in the benchmark model

[38] Notice that the larger duration in a *full* compared to a *partial* default in the *pari passu* regime can be solely attributed to the repayment of multilateral debt in autarky during a *partial* default.

comes from the larger private debt accumulation. The borrowing limit $\mathcal{A}$ is too tight relative to the total stock of private debt for the multilateral debt to affect the default duration directly.

Having said that, the larger private debt accumulation in a *full* default relative to a *partial* default is a consequence of the *de facto* seniority. Under the *de jure* seniority, the sovereign accumulates more debt than in the benchmark case. In opposition, under the *pari passu* clause, the sovereign accumulates less debt than in the benchmark case. Thus, the effect of multilateral debt on the *full* default duration is indirect.

This shows that different seniority regimes are associated with different behaviors in terms of debt accumulation and default. Under the *de jure* seniority, the sovereign is more reckless as both the debt ratio and the default rate are high. This is because the multilateral debt trades at the risk-free rate irrespective of the default rate. Conversely, under the *pari passu* clause, the sovereign adopts a more rigorous debt management. The reason is that the default rate directly affects the price of multilateral debt. In particular, the multilateral debt has the same repayment priority as the private debt rendering the multilateral debt price very sensitive to the default risk. The *de facto* seniority is in-between these two cases.

3.10.5 Welfare Analysis

I calculate the consumption-equivalent welfare gains with respect to the benchmark model for the sovereign. The computation of the welfare is exposed in Appendix 3.8. I consider each of the above exercises one by one.

As shown in Table 3.8, an equal output penalty is associated with mostly welfare gains. The only welfare loss is recorded when there is no multilateral debt as the sovereign does not have access to a "cheap" *partial* default in which it would get a lower output penalty. However, one observes mainly welfare losses in the case of a *de jure* seniority. In regions in which debt crises occur – i.e. low endowment states with a large level of debt – losses come from the incapacity of the sovereign to repudiate its entire debt. The sovereign can only enter in *partial* default in which it continues to service the multilateral debt which adds to the default cost.

Table 3.8: Welfare Gains Relative to Benchmark

Endowment state	Private debt	Multilateral debt	Welfare gains (%)					
			$\varkappa^{DF} = \varkappa^{DP}$	*de jure*	$\mathcal{A} = 0$	*pro rata*	$b_m^j \leq 0$	*pari passu*
z_{min}	High	High	0.02	-1.27	-	-0.09	-0.46	-0.21
	High	Zero	-0.27	-0.14	-0.10	-0.15	-0.15	-0.22
	Zero	High	0.06	0.10	-	0.06	-0.07	0.02
	Zero	Zero	0.06	0.10	-0.20	-0.02	-0.06	-0.04
z_{max}	High	High	0.03	0.03	-	0.02	0.01	0.00
	High	Zero	0.03	0.03	-0.09	0.01	0.00	-0.00
	Zero	High	0.01	-0.02	-	0.01	-0.05	0.00
	Zero	Zero	0.01	-0.02	-0.02	0.01	-0.05	-0.00
average	High	High	0.04	-0.79	-	-0.10	-0.58	-0.29
	High	Zero	-0.07	-0.12	-0.22	-0.16	-0.14	-0.22
	Zero	High	0.02	-0.03	-	0.03	-0.09	0.01
	Zero	Zero	0.02	-0.01	-0.12	0.02	-0.07	-0.00

Note: The computation of the welfare is exposed in Appendix 3.8. A high level of debt corresponds to the largest level of debt in the grid for both the private and the multilateral debt.

Regarding the *pari passu* clause, I find welfare losses in most states. Hence, as already argued by Hatchondo et al. (2017), the sovereign highly values the use of last-resort funds at a near risk-free rate. The *pari passu* clause weakens the *de facto* seniority of multilateral debt which is particularly valued in bad times when the default risk is high. The same holds true when I completely remove the multilateral lenders – i.e. $\mathcal{A} = 0$.

All in all, the *de facto* seniority seems to be beneficial for the sovereign. Except in a few states, the sovereign is better off than in a *de jure* or a *pari passu* regime. The former is certainly too strict and does not allow for full debt default, while the latter limits the multilateral debt's capacity of being a last-resort source of funding.

3.11 Conclusion

This paper uncovers the source and the consequences of the *de facto* seniority of (official) multilateral lenders – i.e. mainly the International Monetary Fund (IMF) and the World Bank (WB). I first present evidence that defaults involving such lenders are infrequent, last relatively longer and are associated with greater private haircuts.

To rationalize these findings, I augment the standard model of Eaton and Gersovitz (1981) with heterogenous creditors and endogenous restructurings. The key assumption is that the multilateral lender has a greater enforcement power than the private lenders. This greater power emanates from the larger output penalty upon default and a stringent policy of non-toleration of arrears in the spirit of the one adopted by the IMF and the WB.

Given this, the multilateral debt has an important impact on the price of private debt. On the one hand, the multilateral debt drastically reduces the private debt's recovery value owing to its repayment priority upon default. On the other hand, it can increase the sovereign's willingness to repay by rendering a *partial* default on private debt more costly. Thus, while the multilateral debt raises the subordination risk of private liabilities, it can reduce the default risk up to a certain point.

The model quantitatively matches the empirical regularities relating to the default durations, the multilateral lending rate and private creditors' haircuts. The policy of non-toleration of arrears is behind most of the model's dynamic. Such policy ensures that multilateral creditors can lend at preferential rates. Coupled with the larger output penalty, it generates important spillovers on private creditors. I find that the borrower values the use of official multilateral debt and would not necessarily prefer other seniority regimes.

My analysis focuses on multilateral lending institutions and abstracts from the Paris Club, which is a major actor in sovereign debt renegotiations. Very few studies analyze this entity which does not properly enjoy a preferred creditor status but largely impacts the private creditors' haircuts and imposes a comparability of treatment among creditors. I leave this inquiry for future work.

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Appendix A

Appendix to Chapter 1

1.1 Discussion on Alternative Implementations

This section discusses the relationship between the implementation presented in Section 1.5 and the main alternatives that exist in the literature.

Dovis (2019) considers an environment similar to the one presented in Section 1.3 with the only difference that g is privately observed by the borrower. He derives an optimal contract subject to a participation and an incentive compatibility constraint to account for limited commitment and adverse selection, respectively. He subsequently decentralizes the aforementioned contract through partial defaults and an active debt maturity management. The main difference with my study is that he explicitly uses defaults – instead of costly buybacks – to implement the constrained efficient allocation. This is because the combination of limited commitment and adverse selection generates a region of *ex post* inefficiencies in which the Planner sets $k = 0$.[1] As I only consider limited commitment, this region does not exist in my analysis – as shown in Proposition 1.1. Nevertheless, my implementation works in the environment of Dovis (2019), while the opposite is not true. In general, his implementation does not apply to renegotiation-proof contracts, while mine applies to contracts with or without *ex post* inefficiencies.

[1] Using different environments, Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007) and Yared (2010) also characterize a region of *ex-post* inefficiencies in optimal contracts.

Besides this, Alvarez and Jermann (2000) propose a way to implement the allocation derived in Kehoe and Levine (1993) through Arrow securities and endogenous borrowing limits. I apply their approach in my environment in Appendix 1.7. The main difference with my analysis is that the two authors assume a greater financial sophistication as securities are state contingent, while I generally need higher sophistication in the strategy of the market participants – unless the implementation works under Markov strategies.

The study of Müller et al. (2019) considers a small open economy with a stochastic default cost and two productivity states: recession and normal time. The authors assume a financial market formed by two securities: a one-period non-contingent defaultable bond and a state-contingent bond which pays out only in normal time (i.e. GDP-linked bond). The authors additionally assume that the borrower lacks commitment only in recession and renegotiation upon default is endogenous. This coupled with the aforementioned market structure, enables the two bonds to act as Arrow securities. In other words, the defaultable bond is recession contingent and spans the different stochastic default costs through renegotiation, while the contingent bond spans the good state which is free from default risk. Hence, as the bonds act as proper Arrow securities, there is no need to rely on past history.

The last study that I would like to discuss is the one of Aguiar et al. (2019) who consider a small open economy with a stochastic default cost and two productivity states as in Müller et al. (2019). The authors assume a continuum of maturities. They show the equivalence between the Markov equilibrium and the constrained efficient equilibrium. The Planner's problem is nonetheless peculiar as it does not take into consideration the legacy creditors in the surplus maximization. In other words, the Planner problem is sequential and only accounts for the current creditors, taking as given the inherited debt level. Furthermore, there is no participation constraint of the borrower. That is, the Planner cannot prevent the occurrence of defaults on equilibrium path. Hence, in the absence of a participation constraint – i.e. a forward-looking constraint – the Planner needs not build on past history. This combined with the disregard of legacy creditors directly leads to the Markov equilibrium in the spirit of Eaton and Gersovitz (1981).

1.2 Foundations for Costly Debt Buybacks

In what follows, I endogenize the cost of *official* buybacks in two ways. First, I develop a standard Nash bargaining in the Markov equilibrium. Second, I present a signalling game in which costly *official* buybacks enable the borrower to signal its productivity.

Before that, I present the mechanism of Bulow and Rogoff (1988, 1991) and highlight why this does not suit my framework. The two authors show that a buyback increases the value of debt as the recovery value is divided among fewer creditors.

To see this, consider a Markov equilibrium in which $\mathbb{E}_{s'|s}D(s', 0, b'_{st}, b'_{lt}) > 0$ for all (s, b'_{st}, b'_{lt}). In addition, suppose that there is a fixed recovery value of w after default. The bond price can therefore be separated into two parts: the return when the government decides to repay and the recovery value when the government defaults.

$$q_{lt}(s, b'_{st}, b'_{lt}) = \mathbb{E}_{s'|s}\Bigg[(1 - D(\Omega'_P))q^P_j(s', b'_{st}, b'_{lt}) + D(\Omega'_P)q^D_j(g', b'_{st}, b'_{lt})\Bigg],$$

where the recovery value is given by

$$q^D_{lt}(s', b'_{st}, b'_{lt}) = \frac{1}{1+r}\Bigg[(1-\lambda)q^D_{lt}(s', b'_{st}, b'_{lt}) + \lambda\frac{w}{b'_{lt}}\Bigg],$$

and the repayment value reads

$$q^P_{lt}(s', b'_{st}, b'_{lt}) = \frac{1}{1+r}\Bigg[1 + (1 - M(\Omega'_P))q_{lt}(s', b''_{st}, b''_{lt}) + M(\Omega'_P)q^{bb}_{lt}\Bigg].$$

The buyback premium naturally emerges from the bond price as w is constant. When $-b'_{lt}$ increases, q^D_{lt} decreases which implies that q_{lt} decreases given the strictly positive default probability. This is what the literature calls dilution. With a buyback the opposite happens as $-b'_{lt}$ decreases. There is a *reverse* dilution which increases q^D_{lt} and therefore q_{lt}.

This mechanism however works as long as there is a strictly positive default probability. If defaults never arise on equilibrium path, the long-term bond price remains constant. Moreover, this mechanism can only rationalize buybacks at a discount (i.e. below par) on the secondary market. For instance, it cannot explain the case of Brazil which bought back its debt at a premium (i.e. when the financial value is above the face value) as shown in Section 1.7.

1.2.1 Nash Bargaining in Markov Equilibrium

In this section, I introduce a Nash bargaining game in the Markov equilibrium. This first shows how to endogenize the *official* buyback premium. It also reinforces the argument made in Lemma 1.4 about the enforcement of *official* buybacks in Markov equilibria.

I modify the debt auction as follows. The borrower can choose either a take-it-or-leave-it offer from the *new* lender or a Nash bargaining with the *legacy* lender. I assume below that $\varpi_0 = 0$ meaning that the *new* lender makes an offer that satisfies the *legacy* lender's problem. When $\varpi_0 = 1$, the outcome is the same as the one presented in the main text.

Given that $\varpi_0 = 0$, the threat point of the game is that the borrower is not able to roll over its debt in the current period if the *official* buyback does not take place. In such circumstance, the borrower's value is given by

$$
\begin{aligned}
\bar{V}^{NB}(\Omega_P) = & \max_{b'_{st}, b'_{lt}} u(c) + \beta \mathbb{E}_{s'|s}\Big[W^b(\Omega'_P)\Big] \\
& \text{s.t.} \quad c + q_{st}(s, b'_{st}, b'_{lt})b'_{st} + q_{lt}(s, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}) = y(g,k) + b_{st} + b_{lt}, \\
& \qquad\quad b'_{lt} \geq b_{lt}, \\
& \qquad\quad b'_{st} \geq 0.
\end{aligned}
$$

Notice that the borrower can issue short-term assets. For any *official* buyback premium χ, I define the surplus of the borrower as

$$
\Delta^b(\Omega_P; \chi) = V^B(\Omega_P; \chi) - \max\Big\{\bar{V}^{NB}(\Omega_P), V^D(s, 0, k)\Big\}.
$$

The borrower's surplus corresponds to the difference between the value of conducting the *official* buyback and the value of rejecting it and suffering the underlying sudden stop.

To define the surplus of the *legacy* lender, I first need to derive the *legacy* lender's value under *official* buyback, under no *official* buyback and under default. The former reads

$$V^l_{\text{legacy},B}(\Omega_P) = \max_{b'_{st}, b'_{lt}} \; c_l$$
$$\text{s.t.} \quad c_l + b_{st} + b_{lt}(1 + q^{bb}_{lt}(s, b'_{st}, b'_{lt})) = 0,$$

while under no *official* buyback

$$\bar{V}^l_{\text{legacy},NB}(\Omega_P) = c_l$$
$$\text{s.t.} \quad c_l + b_{st} + b_{lt}(1 + q_{lt}(s, b'_{st}, b'_{lt})) = 0.$$

and finally, under default

$$V^l_{\text{legacy},D}(s, 0, k) = -\,k.$$

The continuation value is always zero given the break even assumption and the zero recovery value upon default. The surplus of the lenders corresponds to the difference in the value under *official* buyback and no *official* buyback

$$\Delta^l(\Omega_P;\chi) = V^l_{\text{legacy},B}(\Omega_P;\chi) - \Bigg[(1 - D(\Omega_P))\bar{V}^l_{\text{legacy},NB}(\Omega_P) + D(\Omega_P)V^l_{\text{legacy},D}(s, 0, k)\Bigg].$$

If the *legacy* lender has all the bargaining power, then it could extract a large *official* buyback premium (i.e. $\chi \to \infty$). In opposition, if the borrower has all the bargaining power, it can conduct *official* buybacks at low cost (i.e. $\chi \to 0$). To consider the case in between those two extremes, I assume that the lenders have a bargaining power of $\zeta \in [0, 1]$ and the borrower of $1 - \zeta$. In Ω_P, the *official* buyback premium $\chi(\Omega_P)$ is the solution to

$$\chi(\Omega_P) = \arg\max_{\tilde{\chi}\in(0,1)} \left[\Delta^l(\Omega_P;\tilde{\chi})^\zeta + \Delta^b(\Omega_P;\tilde{\chi})^{1-\zeta}\right]$$
$$\text{s.t.}\quad \Delta^l(\Omega_P;\tilde{\chi}) \geq 0,$$
$$\Delta^b(\Omega_P;\tilde{\chi}) \geq 0.$$

In light of Lemma 1.4, the above bargaining problem has a solution only if the threat of the sudden stop is credible. If the threat is not credible in a given state Ω_P, $\Delta^b(\Omega_P;\chi) < 0$ for all $\chi > 0$ meaning that there is no premium for which the borrower is willing to conduct *official* buybacks instead of being punished. In other words, there is no solution to the Nash bargaining program meaning that *official* buybacks are not enforceable.

1.2.2 Signalling in Markov Equilibrium

Besides Nash Bargaining, I can rationalize costly *official* buybacks with a signalling game. For this purpose, consider that g is privately observed by the borrower. The lenders must therefore form beliefs on g – i.e. the borrower's type.

To be an equilibrium, beliefs have to be consistent with the market participants' strategies and, given the beliefs, each market participant's strategy must be optimal. A belief system for the lenders, $\Gamma(b_{st}, b_{lt})$, specifies a probability distribution over G,

$$\Gamma(b_{st}, b_{lt}) = Pr(g = g_H | b_{st}, b_{lt}).$$

The lenders rely on the debt repayment, say S, as signal for the borrower's type. I therefore construct a separating equilibrium in which the borrower signals its type through debt repayment as in Cole et al. (1995) and Phan (2017a,b). The timing of actions is the following. First, g realizes and is privately observed by the borrower which then decides how much debt to repay, S. Conditional on the repayment, the lenders offer capital $k(S)$ and a bond price schedule $q_{st}(S, b'_{st}, b'_{lt,H})$ and

$q_{lt}(S, b'_{st}, b'_{lt})$ for the short-term bond, b'_{st}, and the long-term bond, b'_{lt}, respectively.[2]

The repayment signal works as follows. If the repayment is sufficiently large, then the lenders believe that g_H realized. In opposition, a low repayment signals that g_L realized. The signal therefore fully reveals the shock. However, to be an equilibrium, the low type should not be willing to choose a high repayment and *vice versa*.

I assume the following. If the borrower draws g_H, it chooses to conduct an *official* buyback. In opposition, if it draws g_L, there is neither *official* buyback nor default. Hence, the repayment of the high type for a given (b_{st}, b_{lt}) is

$$S_H(b_{st}, b_{lt}) = b_{st} + b_{lt}(1 + q^{bb}_{lt}),$$

and for the low type,

$$S_L(b_{st}, b_{lt}) = b_{st} + b_{lt}(1 + q_{lt}).$$

With $\chi > 0$, it directly follows that $S_L(b_{st}, b_{lt}) > S_H(b_{st}, b_{lt})$ for all (b_{st}, b_{lt}). Thus, costly *official* buybacks are necessary to signal types in the absence of defaults. Whenever the lenders receive a repayment lower than $-S_H(b_{st}, b_{lt})$, it believes that g_L realized. Obviously, those beliefs are consistent only if the high type has no incentive to repay according to the low type and *vice versa*. Thus, it must hold that for any (b_{st}, b_{lt}),

$$\begin{aligned} &u(y(g_H, k(S_H)) + S_H(b_{st}, b_{lt}) - q_{lt}(S_H, b'_{st,H}, b'_{lt,H})b'_{lt,H} - q_{st}(S_H, b'_{st,H}, b'_{lt,H})b'_{st,H}) = \\ &u(y(g_H, k(S_L)) + S_L(k, b_{st}, b_{lt}) - q_{lt}(S_L, b'_{st,L}, b'_{lt,L})b'_{lt,L} - q_{st}(S_L, b'_{st,L}, b'_{lt,L})b'_{st,L}), \end{aligned} \tag{A.1}$$

where $b'_{j,i}$ denotes the bond choice of maturity $j \in \{st, lt\}$ of reported borrower's type $i \in \{L, H\}$. Equation (A.1) makes the high type indifferent between paying S_H or S_L. Moreover, given that

[2] Note that similar to the timing in Eaton and Gersovitz (1981), it is implicitly assumed that the borrower can commit to repayment decision made before the debt auction. See Cole and Kehoe (2000) and Ayres et al. (2018) for more details.

$y(g_H, k(S_H)) > y(g_H, k(S_L))$ as $g_H > g_L$, (A.1) implies by concavity of the utility function that

$$u(y(g_L, k(S_H)) + S_H(b_{st}, b_{lt}) - q_{lt}(S_H, b'_{st,H}, b'_{lt,H})b'_{lt,H} - q_{st}(S_H, b'_{st,H}, b'_{lt,H})b'_{st,H}) \leq$$
$$u(y(g_L, k(S_L)) + S_L(k, b_{st}, b_{lt}) - q_{lt}(S_L, b'_{st,L}, b'_{lt,L})b'_{lt,L} - q_{st}(S_L, b'_{st,L}, b'_{lt,L})b'_{st,L}).$$

As a result, if the the high type indifferent between paying S_H or S_L, the low type has no incentive to pay S_H instead of S_L. Thus, if (A.1) holds, the beliefs are updated according to

$$\Gamma(b_{st}, b_{lt}) = \begin{cases} 1 & \text{if} \quad S \leq S_H(b_{st}, b_{lt}) \\ 0 & \text{else} \end{cases}$$

By Proposition 3 in Phan (2017b), the set of strategies and beliefs presented in this section constitutes a separating Markov equilibrium.

We see that from the definition of S_H and S_L, the *official* buyback premium χ ought to be strictly larger than zero for the signal to be informative.[3] On the other hand, as $\chi \to \infty$, $S_H(b_{st}, b_{lt}) \to \infty$. By (A.1) this would imply that the low type has to accumulate an infinite amount of assets. There is therefore a cap on how large χ can be and – similar to Lemma 1.4 – the lower is the *official* buyback premium, the easier is (A.1) satisfied in a given state Ω.

1.3 Further Theory Developments

I start this section with the existence and uniqueness of the optimal contract. For this, following Marcet and Marimon (2019), I need the following interiority assumption.

Assumption A.1 (Interiority)**.** *There is an* $\epsilon > 0$*, such that, for all* $g^t, t \geq 0$*, there is a sequence* $\{\tilde{c}(g^t), \tilde{k}(g^t)\}$ *satisfying,*

$$\sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(\tilde{c}(g^j)) \geq V^D(g_t, 0, \tilde{k}_t) + \epsilon.$$

[3]Otherwise, instead of *official* buybacks, the borrower would need to rely on defaults to signal its type as in Cole et al. (1995) and Phan (2017a,b).

Assumption A.1 ensures the uniform boundedness of the Lagrange multipliers. It states that there are strictly positive rents to be shared among the contracting parties. In my environment, this assumption is satisfied given the Inada condition on the production function.[4]

Proposition A.1 (Existence and Uniqueness)**.** *Under Assumptions 1.1 and A.1, there exists a unique contract allocation with initial condition* (g_0, x_0).

Having shown existence and uniqueness of the contract allocation, the following lemma derives the inverse Euler Equation which gives the consumption dynamic in the contract.

Lemma A.1 (Inverse Euler Equation)**.** *Under Assumption 1.1, the inverse Euler equation for a given* $g \in G$ *reads*

$$\mathbb{E}_{s'|s}\left[\frac{1}{u_c(c(g'))(1+\nu(g'))}\right] = \eta \frac{1}{u_c(c(g))},$$

If the participation constraint never binds, I obtain that for all (g, x),

$$\frac{1}{u_c(c(g))} \geq \mathbb{E}_{s'|s}\left[\frac{1}{u_c(c(g'))}\right]$$

with strict inequality when $\eta < 1$. In this case, the inverse Euler Equation is a positive super-martingale. Immiseration is a consequence of Doob's theorem stating that such super-martingales converge almost surely. With $\eta < 1$, the inverse of the marginal utility of consumption converges to 0. Under limited commitment of the borrower (i.e. $\nu(g) \geq 0$), one obtains a left bounded positive submartingale. The borrower's participation constraints therefore sets an upper bound on the supermartingale and prevents immiseration. Alternatively, when $\eta = 1$ consumption remains constant.

[4]See Proposition 1.1.

1.4 Domestic and Foreign Capital

In this section I show that the absence of domestic capital in my environment is without loss of generality for the implementation of the constrained efficient allocation.

Suppose that k is made of the aggregation of domestic capital, say k_d, and foreign capital, say k_f. More precisely, I consider a CES aggregator such that

$$k = \left[\alpha_k k_d^{\frac{\phi_k - 1}{\phi_k}} + (1-\alpha_k) k_f^{\frac{\phi_k - 1}{\phi_k}}\right]^{\frac{\phi_k}{\phi_k - 1}},$$

where $\alpha_k \in (0,1)$ is the share of domestic capital and ϕ_k is the elasticity of substitution between domestic and foreign capital. During a default, the borrower loses access to the international capital market (i.e. $k_f = 0$) but now can rely on the domestic capital market (i.e. $k_d > 0$). This dampens the cost of default for the borrower. Given this, I consider two cases: $\frac{\phi_k - 1}{\phi_k} = 1$ and $\frac{\phi_k - 1}{\phi_k} \neq 1$.

When $\frac{\phi_k - 1}{\phi_k} = 1$, aggregation becomes linear. Particularly, with $\alpha_k = 0.5$ domestic and foreign capital are perfect substitutes. This means that during a default, the borrower can substitute k_f by k_d one to one. In other words, the borrower completely averts the output cost of default.

However, this does not mean that the economy can rely on defaults to implement the constrained efficient allocation. A default generates (international) markets exclusion with a fixed re-access probability λ. This prevents the implementation of the constrained efficient allocation. If the default arises at the lower bound of the ergodic set (i.e. $x = x^{lb}$), then markets re-access should occur as soon as g_H realizes. This is not guaranteed in my environment as market re-access follows a poisson process independent of g.[5] Besides this, markets exclusion means that $b_{lt} = b_{st} = 0$ which is not necessarily what the Planner would choose.

When $\frac{\phi_k - 1}{\phi_k} \neq 1$, domestic and foreign capital are not perfect substitutes. This means that it is not possible for the borrower to completely avert the output cost of default. In other words, production

[5] Endogenizing the post-default renegotiation would not necessarily solve the problem. With a Nash bargaining, there is no delay meaning that a default would be settled on the spot (Yue, 2010). With a Rubinstein bargaining, there is the possibility to generate delays (Bi, 2008; Benjamin and Wright, 2013; Dvorkin et al., 2021). Depending on the parameters, the model can predict re-access when g_H realizes.

without foreign capital is not as efficient in terms of output maximization as production with both types of capital together. This means that, by averting default, a strict Pareto improvement is possible. It is therefore not possible to implement the constrained efficient allocation through defaults.

1.5 Alternatives to Official Buybacks

In this section, I provide alternatives to *official* buybacks: "excusable" defaults, variable-coupon bonds and variable-maturity bonds.

First, Grossman and Van Huyck (1988) develop the concept of "excusable" defaults.[6] The idea is that defaults which are on the path of play agreed by all market participants are not punished. In other words, the debt contract specifies *ex ante* the circumstances in which the borrower is allowed to repudiate its debt without suffering from markets exclusion. Given this, if defaults were "excusable", then the borrower's binding constraint – i.e. $x = x^{lb}$ – could be interpreted as a default. The issue is that the borrower might be willing to repudiate debt more often than what the debt contract specifies. To deal with this, one can either use trigger strategies or introduce an endogenous borrowing limit. Nevertheless, the concept of "excusable" defaults has little empirical relevance. The closest policy that has been implemented to this date is a sovereign debt standstill analyzed by Hatchondo et al. (2020b) with the only difference that there is no arrears accumulation in "excusable" defaults. In addition, Mateos-Planas et al. (2023) show that if the borrower were to choose the conditions for "excusable" defaults, such events would be extremely rare if not inexistent.

Second, the long-term debt can have variable coupon as in Faraglia et al. (2019) and Aguiar et al. (2021). Particularly, assume that the coupon payment is a choice variable, say $\kappa \in [0, 1]$, for the borrower. Obviously, the variability of the coupon is a covenant in the debt contract. In other words, changes in coupon are agreed by the contracting parties *ex ante* and do not pertain to a contract renegotiation – e.g. an outright default in case of reduced coupon payment. With

[6] See also the recent proposal on contingent convertible bonds by Hatchondo et al. (2022).

such debt contract, it is possible to implement the constrained efficient allocation in two ways: the borrower sets a standard coupon payment $\tilde{\kappa}$ and either increases it to $\bar{\kappa} > \tilde{\kappa}$ when $x = x^{ub}$ or decreases it to $\underline{\kappa} < \tilde{\kappa}$ when $x = x^{lb}$. In the former case, a variant of Proposition 1.7 applies as the borrower is not willing to pay a larger coupon payment. Hence, the same enforcement issue arises as with *official* buybacks and trigger strategies remain necessary in general. In opposition, in the case of reduced coupon payment, the borrower might be tempted to reduce the coupon payment more frequently than the Planner would. Thus, the lenders would also need to supervise the coupon policy.

Lastly, bonds can have variable maturities. That is, the maturity of outstanding short-term (long-term) debt can be lengthened (shortened). Similar to variable-coupon bonds, this is a feature which should be explicitly mentioned in the debt contract. To implement the constrained efficient allocation, the borrower ought to either lengthen the maturity of short-term debt when $x = x^{ub}$ or shorten the maturity of long-term debt when $x = x^{lb}$. Implicitly, by shortening the maturity, the borrower pays less coupons than it initially promised. In other words, the claim of legacy creditors is reduced. The opposite happens in the case of maturity lengthening. Thus, similar to variable-coupon bonds, maturity lengthening would need to be enforced, while maturity shortening should be closely supervised to avoid lowering legacy creditors' claim too frequently.

1.6 Price in Markov Equilibrium

The definition of price and equilibrium directly follow from Definition 1.3 stating that Markov equilibria are sustainable equilibria restricted to the payoff-relevant space. Thus, the price of one unit of bond is given by

$$q_{lt}(s, b'_{st}, b'_{lt}) = \frac{1}{1+r}\mathbb{E}_{s'|s}\left[(1 - D(\Omega'_P))\left\{1 + (1 - M(\Omega'_P))q_{lt}(s', b''_{st}, b''_{lt}) + M(\Omega'_P)q^{bb}_{lt}(s', b''_{st}, b''_{lt})\right\}\right],$$
$$q_{st}(s, b'_{st}, b'_{lt}) = \frac{1}{1+r}\mathbb{E}_{s'|s}\left[(1 - D(\Omega'_P))\right],$$

The equilibrium definition follows directly from Definitions 1.1 and 1.3.

1.7 Alternative Implementation

In what follows, I propose an alternative implementation as the one derived in Section 1.5. More precisely, I rely on the approach of Alvarez and Jermann (2000) using trade in state-contingent securities and an endogenous borrowing limit.

The structure of the financial market is the following. At the start of a period, the government holds a perpetual security a.[7] The government can trade $|G|$ state contingent securities $a'(g')$ with a unit price of $q(g', a'(g')|g)$. The portfolio $a'(g')$ can be decomposed into a common bond $\bar{a}'$ that is independent of the next period state, traded at the implicit bond price $q(g, a') \equiv \sum_{g'|g} q(g', a'(g')|g)$, and an insurance portfolio of $|G|$ Arrow securities $\hat{a}'(g')$. Thus we have that $a'(g') = \bar{a}' + \hat{a}(g')$ with

$$\bar{a}' = \frac{\sum_{g'|g} q(g', a'(g')|g) a'(g')}{q(g, a')} \quad \text{and} \quad \sum_{g'|g} q(g', a'(g')|g)\hat{a}'(g') = 0.$$

The last equation represents the market clearing condition of the Arrow securities. Given that in equilibrium, there is going to be no default, I omit $\mathbb{I}_D$ when not necessary.

As I am only interested in applying the Second Welfare Theorem, I disregard ϖ_0 in the state space. Nevertheless, it is true that such variable is important as the First Welfare Theorem does not generally hold in the environment of Alvarez and Jermann (2000).

The capital market is the same as in the main text: the lenders supply k at price p which is taxed at rate $\tau = 1 - \frac{1}{p}$. The government's problem therefore reads

$$W^b(g, a) = \max_{c, \{a'(g')\}_{g' \in g}} u(c) + \beta \mathbb{E}_{g'|g}\Big[W^b(g', a'(g'))\Big] \tag{A.2}$$

$$\text{s.t. } c + \sum_{g'|g} q(g', a'(g')|g)(a'(g') - a) \leq y(g, k) + a$$

$$\bar{a}' + \hat{a}(g') \geq \mathcal{A}(g', k'), \tag{A.3}$$

[7] The maturity is unimportant in this implementation. The security a could also be a one-period security.

where $\mathcal{A}(g', k')$ represents the endogenous borrowing limit and is defined such that

$$W^b(g', \mathcal{A}(g', k')) = V^D(g', 0, k'). \tag{A.4}$$

One can see here the similarity with the borrowing limit defined in Section 1.6. The lenders' problem is static. I nonetheless express it in recursive form.

$$\begin{aligned} W^l(g, a_l) = &\max_{c_l, k_l, \{a_l'(g')\}_{g' \in g}} c_l + \frac{1}{1+r}\mathbb{E}_{g'|g}\Big[W^l(g', a_l'(g'))\Big] \\ &\text{s.t. } c_l + \sum_{g'|g} q(g', a_l'(g')|g)(a_l'(g') - a_l) \leq p(1-\tau)k_l - k_l + a_l. \end{aligned} \tag{A.5}$$

Given this environment, I can determine a recursive competitive equilibrium in the following terms.

Definition A.1 (Recursive Competitive Equilibrium (RCE))**.** *A recursive competitive equilibrium is a sequence of prices $q(g', a'(g')|g)$ and $p(g, a)$, value functions, $W^b(g, a)$ and $W^l(g, a)$, an endogenous borrowing limit, $\mathcal{A}(g', k')$, as well as policy functions for (i) consumption, $c(g, a)$ and $c_l(g, a)$, (ii) capital, $k = k(g, a)$ and $k_l = k_l(g, a)$, (iii) asset holdings $a'(g') = A(g', g, a)$ and $a_l'(g') = A_l(g', g, a)$ such that,*

1. *Given the value function for the outside option of the government, $V^D(g', 0, k')$ as well as the prices $q(g', a'(g')|g)$ and $p(g, a)$,*
   - (a) *the policy functions $c(g, a)$ and $A(g', g, a)$, together with the value function $W^b(g, a)$, solve the government problem* (A.2) *with the endogenous limit, $\mathcal{A}(g', k')$.*
   - (b) *the policy functions $c_l(g, a_l)$, $k_l(g, a)$, and $A_l(g', g, a_l)$, together with the value function $W^l(g, a_l)$, solve the lenders' problem* (A.5) *and*
2. *Taking p as given, $k(g, a)$ is such that $u_c(c)(gf_k(k) - p) = 0$.*
3. *The price of capital is consistent with $\max_k\{p(1-\tau)k - k\}$.*
4. *The asset market clears, $a'(g') + a_l'(g') = 0$ for all $g' \in G$.*
5. *The product and capital markets clear, $c(g, a) + c_l(g, a_l) = gf(k(g, a))$ and $k(g, a) =$*

$k_l(g, a)$.

For the government's problem, taking the first-order conditions with respect to consumption and assets, one obtains

$$
\begin{aligned}
u_c(c) &= \mu_{BC}(g, a), \\
q(g', a(g')|g) &= \beta\pi(g'|g)\frac{u_c(c')}{u_c(c)}[1 + \sum_{g''|g} q(g'', a''(g'')|g')] + \frac{\mu_{EBL}(g', a'(g'))}{u_c(c)},
\end{aligned}
$$

where μ_{BC} and μ_{EBL} are the Lagrange multipliers attached to the budget constraint and the endogenous borrowing limit, respectively. Especially, $\mu_{EBL}(g', a'(g')) \geq 0$ with $\mu_{EBL}(g', a'(g')) = 0$ if $a'(g') > \mathcal{A}(g', k')$.

Conversely, taking the first-order conditions with respect to consumption and assets of the lenders' problem

$$
q(g', a(g')|g) = \frac{1}{1 + r}\pi(g'|g)(1 + \sum_{g''|g} q(g'', a''(g'')|g')).
$$

Following Krueger et al. (2008), the price is determined by the agent whose constraint is not binding. Therefore the price is determined by

$$
q(g', a(g')|g) = \pi(g'|g)(1 + \sum_{g''|g} q(g'', a''(g'')|g')) \max\left\{\beta\frac{u_c(c(g', a'(g'))}{u_c(c(g, a))}, \frac{1}{1 + r}\right\}. \tag{A.6}
$$

The following lemma states that the constrained efficient allocation can be implemented as a RCE with state-contingent securities and an endogenous borrowing limit.

Proposition A.2 (Alternative Implementation)**.** *Given initial conditions* (g_0, x_0)*, a constrained efficient allocation can be implemented as a RCE with state-contingent securities and an endogenous borrowing limit.*

The benchmark implementation presented in Section 1.5 relies on changes in the term premium to mimic the state-contingency in the optimal contract, while this alternative implementation relies

Table A.1: Alternative Implementation

	Benchmark	Alternative
$-b/y$	-353.20	15.87
Spread	3.95	4.34
$\sigma(b/y)/\sigma(y)$	8.34	0.31
$\sigma(spread)/\sigma(y)$	0.00	0.00
corr$(b/y, y)$	-0.72	0.67
corr$(spread, y)$	-0.67	0.00

Note: The variable $\sigma(\cdot)$ denotes the volatility. In the alternative implementation, $\bar{a} = b$. For the volatilities and correlation statistics, I filter the simulated data – except the spread – through the HP filter with a smoothness parameter of 6.25.

on changes in security holdings provided that securities are state-contingent. More importantly, given that securities are state contingent, the assumption that the borrower and the lenders keep track of the entire history of play is not anymore necessary. The implementation of the constrained efficient allocation now lies on the assumption of a greater financial sophistication.

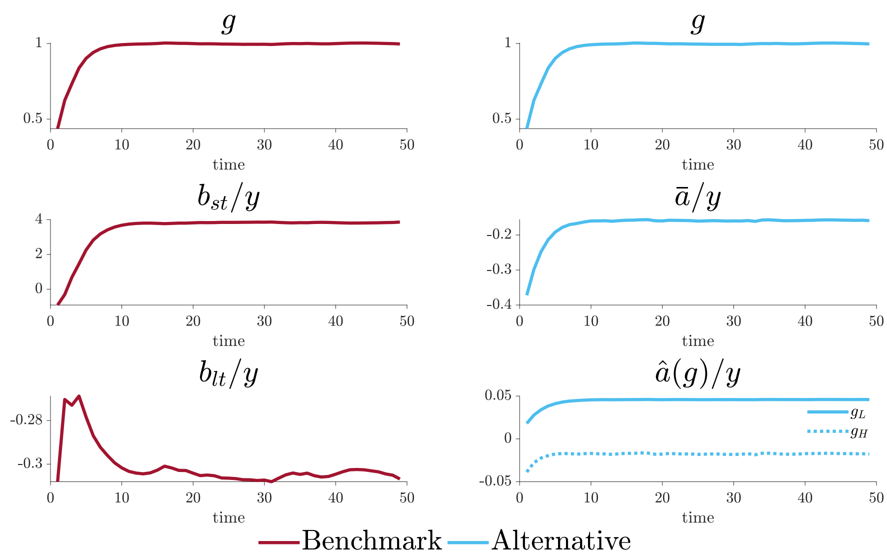


Figure A.1: Impulse Response Functions to a Negative g Shock

Having properly defined the alternative implementation, I now compare it quantitatively to the one presented in Section 1.5 using the calibration in Section 1.7.

Table A.1 presents the main difference between the two implementations. The benchmark case is related to a net asset position and a larger volatility of the debt ratio. This comes from the fact that bonds are non-contingent and the borrower alternates between short-term assets and long-

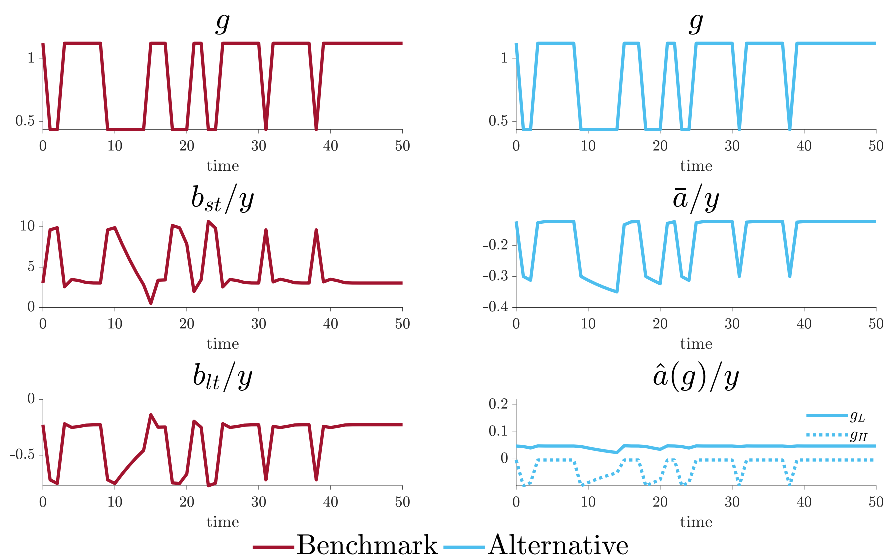


Figure A.2: Simulation of a Steady State Path

term debt. Thus, large movements in debt holdings are necessary to replicate the state contingency in the contract as shown by Buera and Nicolini (2004) and Faraglia et al. (2010). Particularly, we see that the volatility of debt-to-GDP ratio is 26 times larger in the benchmark than in the alternative implementation. Besides this, the benchmark implementation displays a lower spread owing to *official* buybacks. As explained before, the reason behind this is that the alternative implementation does not rely on changes in prices to mimic the state-contingency of the contract given that securities are state-contingent by definition.

Similar to the previous section, I construct impulse responses to see how the two implementations work. Figure A.1 depicts the responses in red for the benchmark implementation and in blue for the alternative one. The Arrow securities complement the accumulation of bonds in the alternative case, while the benchmark implementation needs to adapt both the long-term and the short-term bonds in opposite directions.

Turning to the simulation in Figure A.2, one can see that the level of long-term bond in the benchmark case closely follows the pattern of bonds in the alternative case. The magnitude of change in the former is nonetheless larger than in the latter. In terms of Arrow securities, $\hat{a}(g_H)$ closely follows the evolution of $\bar{a}$, while $\hat{a}(g_L)$ has the opposite sign. The evolution of $\hat{a}(g_H)$ is therefore closely mimicking the evolution of b_{lt}. Given that $\hat{a}(g)$ is state contingent, the alternative implementation needs to change the debt portfolio with lower magnitude.

1.8 Empirical Analysis on Maturity and Buyback in Brazil

In this section, I establish four main facts related to the prediction of the model for Brazil. The first fact relates to the sovereign debt maturity management. As already noted by Arellano and Ramanarayanan (2012), Broner et al. (2013), Perez (2017) and Bai et al. (2017), maturity shortens during debt crises and lengthens otherwise.

Fact V. *Average maturity shortens during debt crises and lengthens otherwise.*

Table A.2 presents the result of a typical maturity regression analysis. The dependent variable corresponds to the average maturity (in years) on new external debt retrieved from the World Bank's International Debt Statistics from 1995 to 2019. The explanatory variable is the EMBI spread which I obtain from the Global Financial Data. As one can see, when the spread increases, the average maturity of new issuance shortens.

The second fact relates to *official* buybacks. As already highlighted in Section 1.7, the Brazilian government always paid a premium to extract its debt out of the market. More precisely, on average, the financial value is 24.5% above the face value.

Fact VI. *Official buybacks are costly.*

While the second fact states that *official* buybacks are costly, the third fact explains when *official* buybacks occur. Consistent with the predictions of the model, *official* buybacks tend to arise in good times. Particularly, there is a positive association between the amount repurchased and the economic situation of the country.

Fact VII. *Substantial official buybacks are more likely to arise in good economic times.*

Table A.2 presents the result of the regression analysis on the Brazilian buyback. The dependent variable corresponds to the amount of external debt bought back in USD billion. I use the same

Table A.2: Regression Analysis

	(1) Maturity	(2) Buyback	(3) Buyback
EMBI Spread	-0.52**	-0.33***	-1.95***
	[0.20]	[0.10]	[0.59]
Foreign Share of External Debt			9.05*
			[4.55]
Observations	25	25	15
R^2 adjusted	0.17	0.24	0.45

Note: *** $p < .01$, ** $p < .05$, * $p < .10$. Robust standard errors in brackets.

Source: Author's calculation, Global Financial Data, Tesouro Nacional and World Bank.

explanatory variables as before. As one can see, when the EMBI spread increases, the amount bought back diminishes.

Using the database of Onen et al. (2023), I include as an additional explanatory variable the share of foreign holdings of long-term external government bonds. This variable only starts in 2005, though. Given this, I come up with the last fact

Fact VIII. *Substantial official buybacks are more likely with a low share of domestic holdings of external debt.*

As shown in Table A.2, a larger share of foreign holdings is associated to higher buybacks. However, this fact does not directly relate to the predictions of the model given that I do not consider domestic investors.

1.9 Additional Quantitative Results

This section provides additional quantitative results. First, I construct impulse response functions following a stark negative shock in both the Markov equilibria and the CEA. Second, I look at the dynamic of a specific shock path in steady state.

Figure A.3 depicts the impulse response functions resulting from a stark negative shock on selected key variables. The responses are computed as the mean of 2,000 independent shock histories starting with the lowest shock as well as initial debt holdings and relative Pareto weight drawn from the ergodic set. The blue line represents the Markov equilibrium with default (MA), the green line the Markov equilibrium without default (MAND) and the red line the constrained efficient allocation (CEA).

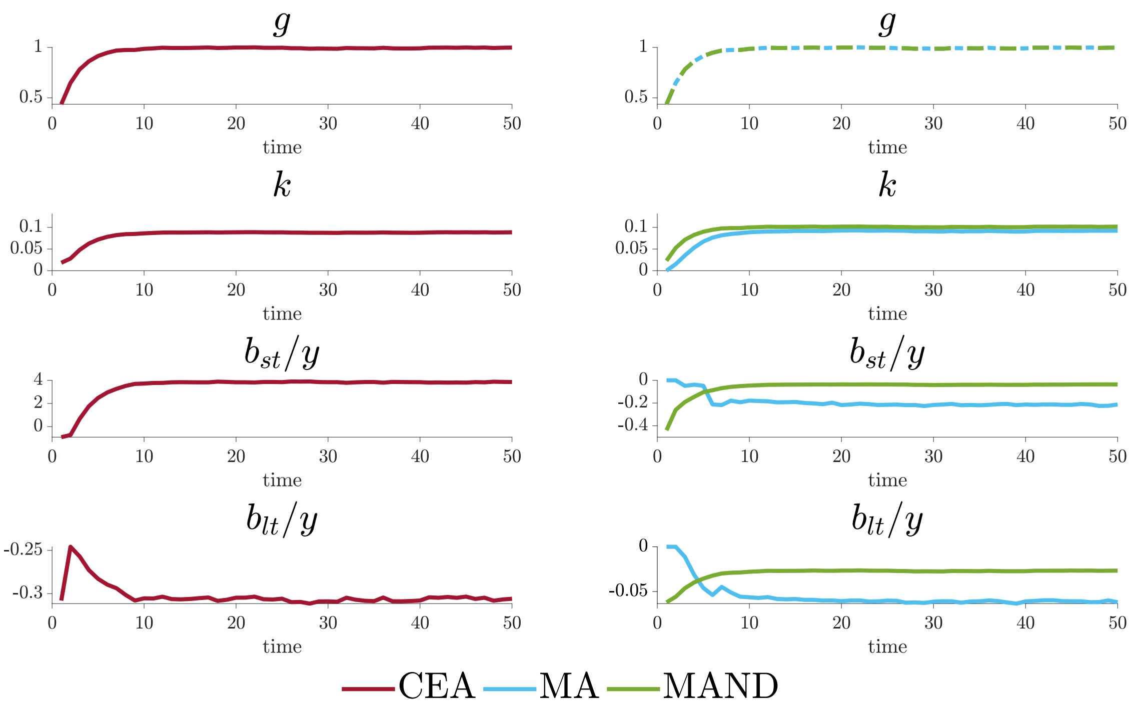


Figure A.3: Impulse Response Functions to a Negative g Shock

We see that at the outbreak of the shock's realization, capital decreases in the MA, the MAND and the CEA – albeit to a lesser extent in the latter two cases. Capital and debt go to zero as economies in the MA fall into default. In opposition, defaults do not arise in the CEA and the MAND which can both increase the indebtedness on impact. Maturity shortens at the outbreak of the bad shock's realization in both the MAND and the CEA. The Markov equilibria rely mostly on short-term debt, while the CEA uses both in opposite directions. Note further that the CEA switches from short-term debt to asset holdings.

The impulse response functions give an idea of the long-run dynamic of the economy. However, it does not tell how the economy reacts in the short run especially when there is a transition between two values of g. Thus, I simulate the economy and generate one history of shocks for 200 periods. To avoid that the initial conditions blur the results, the first 150 periods are discarded. Again, the blue line represents the MA, the green line the MAND and the red line the CEA. In addition, the

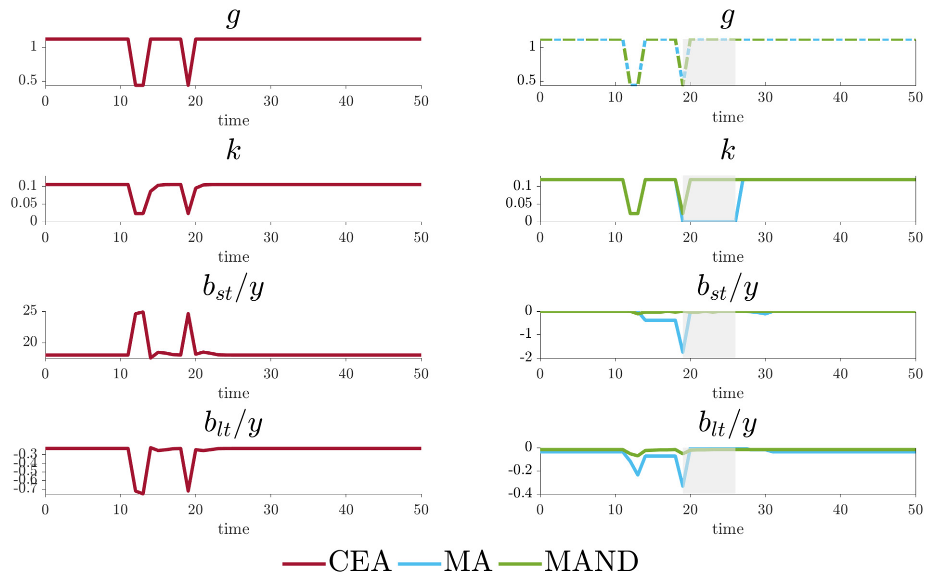


Figure A.4: Simulation of a Typical Path

grey area represents the period in which the MA is in default.

Figure A.4 depicts the simulation results. One observes that, in the MA, the economy defaults in the transition from g_H to g_L. This causes market exclusion and therefore $b_{st} = b_{lt} = k = 0$. In opposition, there are no defaults in the CEA and the MAND. In the transition from g_H to g_L, the government adapts the maturity of the debt and increases its indebtedness. Especially, one sees that the level of short-term bonds have opposite movements in the MAND and the CEA. The magnitude of the changes is also very different. Consistent with the findings of Buera and Nicolini (2004) and Faraglia et al. (2010), the movements in debt holdings are the most pronounced for the CEA. Particularly, holdings of short-term debt oscillate between 2000% and 2500% of output.

1.10 Welfare Analysis

To compute the borrower's welfare, first define the borrower's value for a sequence of consumption $\{c(s^t, \mathbb{I}_D^t)\}$ starting from an initial state at $t = 0$ as

$$W^b(\{c(s^t, \mathbb{I}_D^t)\}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c(s^t, \mathbb{I}_D^t)) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c(s^t, \mathbb{I}_D^t)^{1-\vartheta}}{1-\vartheta},$$

I denote the borrower's consumption allocation in the benchmark model by $\{c^b(s^t, \mathbb{I}_D^t)\}$ and the consumption allocation in the alternative model by $\{c^a(s^t, \mathbb{I}_D^t)\}$. In addition, I define the consumption-equivalent welfare gain of the alternative model with respect to the benchmark model by ι such that

$$W^b(\{(1+\iota)c^b(s^t, \mathbb{I}_D^t)\}) = W^b(\{c^a(s^t, \mathbb{I}_D^t)\}).$$

Given the functional form of the instantaneous utility one obtains

$$(1+\iota)^{1-\vartheta}\Big[W^b(c^b(s^t, \mathbb{I}_D^t)\})\Big] = W^b(\{c^a(s^t, \mathbb{I}_D^t)\}).$$

The borrower's welfare gain therefore boils down to

$$\iota = \left[\frac{W^b(\{c^a(s^t, \mathbb{I}_D^t)\})}{W^b(\{c^b(s^t, \mathbb{I}_D^t)\})}\right]^{\frac{1}{1-\vartheta}} - 1.$$

The lenders' welfare gains can be computed in the same way by setting $\vartheta = 0$ owing to the risk neutrality.

1.11 Proofs

Proof of Lemma 1.1

The value of permanent autarky is given by

$$v_a(g_t) = \sum_{j=t}^{\infty} \beta^{j-t} \sum_{g^j} \pi(g^j|g_t) u(g_j f(0)), \tag{A.7}$$

as the lenders set $k = 0$ in case of default. Permanent autarky is the worst equilibrium outcome as the government could always be better off with $k = \epsilon$ for small $\epsilon > 0$ given the Inada conditions on the production function. I show this in Proposition 1.1.

Permanent autarky is an equilibrium of the market economy. Suppose that the lenders believe that $D_t = 1$ for all t. Then, they set $p_t = \infty$ and $q_{j,t} = 0$ for all $j \in \{st, lt\}$. Given this, the government finds optimal to choose $D_t = 1$ for all t confirming the lenders' beliefs. □

Proof of Lemma 1.2

Necessity:

Conditions (1.1), (1.2), (1.3) and (1.4) follow directly from the equilibrium's definition. The budget constraints in the repayment (including *official* buybacks) and default states is required by feasibility. Finally, the fact that the value of the borrower is larger than the value under the worst equilibrium – i.e. (A.7) – ensures that the allocation can be sustained by trigger strategies.

Sufficiency:

Let's rely on trigger strategy (Abreu, 1988). That is, each player is punished by the worst outcome of the game (i.e. permanent autarky which is an equilibrium as shown in Lemma 1.1) if it decides to deviate. Since the outcome satisfies (1.1), (1.2), (1.3) and (1.4), it is optimal. Also as it satisfies the different budget constraints it is feasible. Finally, no deviations from play is profitable given that the value of the borrower is larger than the value under the worst equilibrium.

□

Proof of Proposition 1.1

I first show that the autarkic allocation is not optimal. The proof follows Aguiar et al. (2009). Consider a version of the optimal contract in which the outside option corresponds to the value of permanent autarky is given by (A.7). In autarky, $k = 0$ and from Definition 1.2 there is an $x_a(g)$ such that $u(c(g, x_a(g))) = u(gf(0))$ at all histories. Using the definition of h in Assumption 1.1, consider that one increases h by Δh and $u(c(g, x_a(g)))$ by $\theta u_c(gf(0))\Delta h$ where

$$\theta = \frac{u_c(g_L f(0))}{u_c(g_L f(0)) + \frac{\beta}{1-\beta}\mathbb{E}_{g'|g_L} u_c(g' f(0))} < 1.$$

I defined θ such that the borrower's participation constraint holds. To see this, note that the increase of h increases the borrower's outside option by $u_c(gf(0))\Delta h$ as it can benefit from the additional level of capital before going to autarky forever. However, if the borrower does not choose autarky, its value increases by $\theta(u_c(gf(0)) + \frac{\beta}{1-\beta}\mathbb{E}_{g'|g}u_c(g'f(0)))\Delta h \geq u_c(gf(0))\Delta h$ by definition of θ. Hence the borrower's participation constraint is satisfied. Furthermore, the value of the lenders changes by

$$\Delta h \frac{1}{1+r}\left(1 - \mathbb{E}_{g'|g_L}\left[\frac{u_c(g'f(0))}{u_c(c(g', x_a(g)))}\right]\theta\right) = \Delta h \frac{1}{1+r}(1-\theta) > 0,$$

where the equality comes from the fact that we consider the autarkic allocation (i.e. $c(g, x_a(g)) = gf(0)$) and the inequality from the fact that $\theta < 1$. As a result, the autarkic allocation is not optimal. The contract is therefore restricted to relative Pareto weights $x > \max_{g \in G}\{x_a(g)\} = x_a(g_H)$.

Consider the interval $[\tilde{x}, \overline{x}]$ with $\tilde{x} > x_a(g_H)$. From the law of motion of the relative Pareto weight, x' is strictly increasing in x. From the first-order conditions on consumption, c is strictly increasing in x. Hence, so does the value of the borrower. In opposition, with a greater c or x', the value of the lenders decreases. That is the lenders' value is strictly decreasing in x. Moreover, note that the relative Pareto weight, $x_{t+1} = \frac{\mu_{b,t}+\gamma(g_t)}{\mu_{l,t}}$, cannot be negative as $(\mu_{b,0}, \mu_{l,0}) \geq 0$ and $\gamma(g^t) \geq 0$ for all t. Hence, any continuation of an efficient allocation is itself efficient. $\square$

Proof of Proposition 1.2

– Part I

The optimal level of capital is given by

$$gf_k(k(g)) - 1 = \nu(g)u_c(gf(k(g)))gf_k(k(g))x.$$

As one can see, as soon as the participation constraint does not bind (i.e. $\nu(g) = 0$), the contract can attain the production-maximizing level of capital $k^*(g)$ such that $gf_k(k^*(g)) =$

1. When this condition is not met, $k < k^*(g)$. Thus, define $x^*(g)$ such that

$$V^b(g, x^*(g)) = V^D(g, 0, k^*(g)).$$

By the above definition, if $x < x^*(g)$, capital is distorted in state g, while if $x \geq x^*(g)$, capital is at the production-maximizing level. Moreover as $V^D(g_L, 0, k^*(g_L)) < V^D(g_H, 0, k^*(g_H))$, $x^*(g_H) > x^*(g_L)$.

Assume by contradiction that for $x_1 < x_2 < x^*(g)$ one has that $k(g, x_1) \geq k(g, x_2)$ for all $g \in G$. From Proposition 1.1, we have that $V^b(g, x_1) < V^b(g, x_2)$. Moreover, as $k^*(g) > k(g, x_1) \geq k(g, x_2)$, the borrower's participation constraint binds which gives

$$\begin{aligned} V^b(g, x_1) &= u(gf(k(g, x_1))) + \beta \mathbb{E}_{g'|g} V^D(g', 0, k') \\ &< V^b(g, x_2) \\ &= u(gf(k(g, x_2))) + \beta \mathbb{E}_{g'|g} V^D(g', 0, k'), \end{aligned}$$

where the two equalities come from the binding borrower's participation constraint and the inequality for the fact that $x_1 < x_2$ for a given g. This is contradiction. Hence, it must be that $k(g, x_1) < k(g, x_2)$ for any $x_1 < x_2$.

The fact that that $k(g, x) > 0$ for all (g, x) follows directly from Proposition 1.1 which shows that the autarkic allocation is not optimal.□

– Part II

The law of motion of the relative Pareto weight is given by $x'(g) = (1 + \nu(g))\eta x$, while the first-order condition on consumption reads $u_c(c(g)) = \frac{1}{1+\nu(g)}$.

Given the first-order condition, $c(g_L, x) \leq c(g_H, x)$ only when $\nu(g_L) \leq \nu(g_H)$. Assume by contradiction that $\nu(g_L) > \nu(g_H)$. This implies that $c(g_L, x) > c(g_H, x)$ and $x'(g_L, x) > x'(g_H, x)$. Especially, consider that $\nu(g_L) > \nu(g_H) \geq 0$. In this case,

$$u(c(g_H, x)) + \beta \mathbb{E}_{g'|g_H} V^b(g', x'(g_H, x)) \geq u(g_H f(k(g_H))) + \beta \mathbb{E}_{g'|g_H} V^D(g', 0, k'),$$

$$u(c(g_L,x)) + \beta \mathbb{E}_{g'|g_L} V^b(g', x'(g_L,x)) = u(g_L f(k(g_L))) + \beta \mathbb{E}_{g'|g_L} V^D(g', 0, k').$$

Given that $g_H > g_L$ and $\pi(g|g) > 0.5$ for all $g \in G$, $u(g_H f(k(g_H))) > u(g_L f(k(g_L)))$ and $V^D(g_H, 0, k) > V^D(g_L, 0, k)$. This implies that

$$u(c(g_H,x)) + \beta V^b(g', x'(g_H,x)) > u(c(g_L)) + \beta V^b(g', g_L, x'(g_L,x)).$$

which contradicts the fact that $c(g_L,x) > c(g_H,x)$ and $x'(g_L,x) > x'(g_H,x)$. Hence, $\nu(g_L) \leq \nu(g_H)$ which gives $c(g_L,x) \leq c(g_H,x)$ and $x(g_L,x) \leq x(g_H,x)$ as desired.
Especially, by definition, when $x \geq x^*(g_H)$, then $\nu(g) = 0$ for all g implying that $c(g_L,x) = c(g_H,x)$ and $x(g_L,x) = x(g_H,x)$. Otherwise, $c(g_L,x) < c(g_H,x)$ and $x(g_L,x) < x(g_H,x)$.
□

– Part III

This proofs is a modified version of Thomas and Worrall (1990, Lemma 4). The value of liabilities in the optimal contract is given by

$$V^l(g,x) \equiv g f(k(g)) - k(g) - c(g,x) + \frac{1}{1+r} \mathbb{E}_{g'|g} V^l(g', x'(g,x)).$$

Assume by contradiction that for a given x it holds that $V^l(g_H,x) \leq V^l(g_L,x)$. For $x \geq x^*(g_H)$, one directly reaches a contradiction as $c(g_L,x) = c(g_H,x)$ and $x(g_L,x) = x(g_H,x)$ which implies that $V^l(g_H,x) > V^l(g_L,x)$.
For $x < x^*(g_H)$, consider the pooling allocation in which $u(\ddot{c}(g_H,x)) = u(\ddot{c}(g_L,x)) = u(c(g_H,x))$ and $\ddot{V}^b(g_H,x) = \ddot{V}^b(g_L,x) = V^b(g_H,x)$. Under this allocation, the participation constraint is trivially satisfied. This leads to

$$\ddot{V}^l(g_H,x) > \ddot{V}^l(g_L,x)$$

which is a direct contradiction. Hence, $V^l(g_H,x) \geq V^l(g_L,x)$. However, $V^l(g_H,x) =$

$V^l(g_L, x)$ is ruled out by fact that there is no perfect risk sharing when $x < x^*(g_H)$. $\square$

Proof of Proposition 1.3

Recall the law of motion of the relative Pareto weight

$$x'(g) = (1 + \nu(g))\eta x.$$

The motion of the relative Pareto weight is dictated by the relative impatience, η, and the binding participation constraint, ν. I consider two cases. On the one hand, if $\eta < 1$, the relative Pareto weight increases only if $\nu(g) > 0$ is sufficiently large to overcome impatience. As we know, when $x \geq x^*(g)$, $\nu(g, x) = 0$ meaning that impatience eventually dominates the limited commitment issue. On the other hand, if $\eta = 1$ immiseration due to impatience does not exist and the relative Pareto weight remains constant.

When $\eta = 1$, the upper bound of the ergodic set coincides with the lower bound. As shown in Proposition 1.2, $x'(g_L, x) \leq x'(g_H, x)$. Moreover, by definition of $x^*(g_H)$ in Proposition 1.2, $x^{ub} = x^{lb} = x^*(g_H)$. Conversely, when $\eta < 1$, impatience prevents the contract to reach $x^*(g_H)$ as $\nu(g_H, x^*(g_H)) = 0$. Hence, $x^*(g_H) > x^{ub}$. Moreover, $x'(g_L, x) < x'(g_H, x)$ when $x < x^*(g_H)$ implying that $x^{ub} > x^{lb}$. In other words, impatience immiserates the relative Pareto in the low productivity state implying that $x^*(g_H) > x^{ub} > x^{lb}$.

To show the existence of the ergodic set, one shows that the dynamic of the contract satisfies the conditions given by Stokey et al. (1989, Theorem 12.12). Set $\ddot{x}$ as the midpoint of $[x^{lb}, x^{ub}]$ and define the transition function $Q : [x^{lb}, x^{ub}] \times \mathcal{X}([x^{lb}, x^{ub}]) \to \mathbb{R}$ as

$$Q(x, G) = \sum_{g'|g} \pi(g'|g)\mathbb{I}\{x' \in G\}$$

One wants to show is that $\ddot{x}$ is a mixing point such that for $M \geq 1$ and $\epsilon > 0$ one has that $Q(x^{lb}, [x, x^{ub}])^M \geq \epsilon$ and $Q(x^{ub}, [x^{lb}, x])^M \geq \epsilon$. Starting at x^{ub}, for a sufficiently long but finite series of g_L, the relative Pareto weight transit to x^{lb} (either through impatience or because $x^{lb} =$

x^{ub}). Hence for some $M < \infty$, $Q(x^{ub}, [x^{lb}, \ddot{x}])^M \geq \pi(g_L|g_L)^M > 0$. Moreover, starting at x^{lb}, after drawing $M < \infty$ g_H, the relative Pareto weight transit to x^{ub} (either through the binding constraint or because $x^{lb} = x^{ub}$) meaning that $Q(x^{lb}, [\ddot{x}, x^{ub}])^M \geq \pi(g_H|g_H)^M > 0$. Setting $\epsilon = \min\{\pi(g_L)^M, \pi(g_H)^M\}$ makes $\ddot{x}$ a mixing point and the above theorem applies. $\square$

Proof of Proposition 1.4

The proof of this proposition is by construction. Similar to Dovis (2019), I express the policy functions of the implemented contract as a function of the relative Pareto weight, x, and the productivity state, g. Formally, define

$$\bar{D}, \bar{M} : G \times X \to \{0, 1\},$$
$$\bar{k}, \bar{p}, \bar{q}_{st}, \bar{q}_{tl}, \bar{b}_{st}, \bar{b}_{lt} : G \times X \to \mathbb{R}.$$

Given the timing of actions, the price schedules and bond policies depend on the prospective relative Pareto weights after the productivity shock realizes. Those objects can therefore be rewritten as

$$\bar{b}_j(g, x) = \bar{b}_j(x'(g, x)),$$
$$\bar{q}_j(g, x) = \bar{q}_j(x'(g, x)) \quad \text{for all } j \in \{st, lt\}.$$

I first determine the default and *official* buyback policies. Subsequently, I compute the underlying prices. I then define the portfolio of bonds to match the total value of debt $V^l(g, x)$ implied by the constrained efficient allocation. Finally, I determine the optimal capital pricing from the optimality conditions of the lenders and the domestic firms.

Given Proposition 1.1, autarky is never optimal in the contract. Hence, the government never enters into default. That is $\bar{D}(g, x) = 0$ for all (g, x). The government will therefore rely on changes in the maturity structure and *official* buybacks as in the Markov equilibrium without default. I assume that *official* buybacks arise only if the economy hits the upper bound of the ergodic set,

$$\bar{M}(g,x) = \begin{cases} 1 & \text{if} \quad g = g_H \text{ and } x = x^{ub} \\ 0 & \text{else} \end{cases}$$

Given the above policies, the short-term bond price equates the risk-free price,

$$\bar{q}_{st}(x) = \frac{1}{1+r},$$

while the long-term bond price,

$$\bar{q}_{lt}(x) = \frac{1}{1+r}\mathbb{E}_{g'|g}[1 + \bar{q}_{lt}(x')].$$

Note further that, the long-term bond price has the following properties.

Lemma A.2 (Bond Price). *With $q_{lt}^{bb} = q_{lt} + \chi$ and $\chi > 0$, the long-term bond price is the unique fixed point of $\bar{q}_{lt}$, is decreasing and is such that*

$$\frac{1}{r} + \chi > \bar{q}_{lt}(x'(g_H, x)) \geq \bar{q}_{lt}(x'(g_L, x)) > \frac{1}{r},$$

with strict inequality when $\eta < 1$.

Proof. Recall that the long-term bond price is given by

$$\bar{q}_{lt}(g,x) = \frac{1}{1+r}\mathbb{E}_{g'|g}\left[(1 - \bar{D}(g',x'))\left\{1 + (1 - \bar{M}(g',x'))\bar{q}_{lt}(g',x') + \bar{M}(g',x')q_{lt}^{bb}\right\}\right],$$

I consider that $\bar{D}(g',x') = 0$ for all (g',x') and $\bar{M}(g',x') = 1$ if $g' = g_H$ as well as $x' = x^{ub}$ and $\bar{M}(g',x') = 0$ otherwise. From Proposition 1.3, g_H and $x = x^{ub}$ arises with strictly positive probability for any (g, x),

$$\frac{1}{r} + \chi > \bar{q}_{lt}(g, x) > \frac{1}{r}.$$

Define Q_{lt} as the space of bounded functions $\bar{q}_{lt} : [\underline{x}, \bar{x}] \to [0, \frac{1}{r} + \chi]$ and $\mathbb{T} : Q_{lt} \to Q_{lt}$ as

$$\mathbb{T}\bar{q}_{lt}(g, x) = \frac{1}{1+r} \sum_{g'} \pi(g'|g)[1 + \bar{q}_{lt}(g', x')].$$

By the Blackwell sufficient conditions $\mathbb{T}$ is a contraction mapping. As a result, there exists a unique fixed point to $\mathbb{T}$, $\bar{q}_{lt}$ which is increasing as $\mathbb{T}$ maps increasing functions into increasing functions. This implies that $\bar{q}_{lt}(x'(g_H, x)) \geq \bar{q}_{lt}(x'(g_L, x))$ as $x'(g_H, x) > x'(g_L, x)$ for all x in the above specified domain. Assume now that there exists a x such that $\bar{q}_{lt}(x'(g_H, x)) = \bar{q}_{lt}(x'(g_L, x))$. This requires that $x'(g_H, x)$ and $x'(g_L, x)$ belongs to a subset $[x_t, x_{t+1}]$ where $\bar{q}_{lt}$ stays constant. Hence, for any $\ddot{x} \in [x_t, x_{t+1}]$, it must be that $x'(g_H, \ddot{x}), x'(g_L, \ddot{x}) \in [x_t, x_{t+1}]$ which is a contradiction as $x'(g_H, x_{t+1}) > x_{t+1}$ when $\eta < 1$. Therefore it must be that $\bar{q}_{lt}(x'(g_H, x)) > \bar{q}_{lt}(x'(g_L, x))$ when $\eta < 1$ and $\bar{q}_{lt}(x'(g_H, x)) \geq \bar{q}_{lt}(x'(g_L, x))$ otherwise. $\square$

Having properly determined the different price schedules, I can now determine the bond holdings and the maturity in order to match the total value of the debt implied by the constrained efficient allocation. Particularly, it must hold that when $x = x^{ub}$,

$$-V^l(g_H, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + q^{bb}_{lt}], \tag{A.8}$$

$$-V^l(g_L, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \bar{q}_{lt}(x'(g_L, x))]. \tag{A.9}$$

Otherwise, the relationship is given by

$$-V^l(g_H, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \bar{q}_{lt}(x'(g_H, x))],$$

$$-V^l(g_L, x) = \bar{b}_{st}(x) + \bar{b}_{lt}(x)[1 + \bar{q}_{lt}(x'(g_L, x))].$$

This is a system of 2 equations with 2 unknowns for which Lemma A.2 ensures that there exists a unique solution. The maturity structure of the bond portfolio is therefore properly determined.

To complete the proof, I determine the optimal capital price. Given that $\tau = 1 - \frac{1}{\bar{p}(g,x)}$, from the optimality conditions of the domestic firms and the lenders I get

$$g f_k(k) = \bar{p}(g,x). \tag{A.10}$$

Hence, the constrained efficient allocation can be replicated with the above policies for default, *official* buyback, and bond holdings. The optimality conditions of the lenders, the government and the domestic firms are satisfied as well as the price schedules.

This concludes the proof as the market allocation satisfies the necessary and sufficient conditions provided in Lemma 1.2. Especially, I used the budget constraints to determine the optimal bond holdings given the prices computed according to (1.2). The capital pricing is set to match the conditions (1.3) and (1.4). Finally, the resource constraint and the condition for reversion to the worst equilibrium outcome are satisfied as the constrained efficient allocation meet those requirements. Especially, note that $V^D(g_t, 1, 0) \geq v_a(g_t)$ as defined in (A.7) given that $\lambda \geq 0$. Thus, the participation constraint (1.5) ensures that the outcome can be sustained by trigger strategies. □

Proof of Lemma 1.3

The fact that default never occurs is a direct corollary of Proposition 1.1. Regarding buybacks, I consider that the government conducts *official* buybacks when it hits the upper bound of the ergodic set – i.e. $x = x^{ub}$ and $g = g_H$. I therefore need to consider two alternatives.

First, could *official* buybacks occur in the lower bound of the ergodic set – i.e. $x = x^{lb}$? The answer is negative. To reach the lower point of amnesia, the relative Pareto weight needs to decrease. More precisely, in steady state $x'(g_L) \leq x$ as shown in Proposition 1.3. This implies that the value of the lenders increases as g_L realizes. This goes against the idea of a debt repurchase which (weakly) reduces indebtedness. Furthermore, from Part III of Proposition 1.2, it holds that

$V^l(g_H, x) > V^l(g_L, x)$ which implies debt relief in the low productivity state. However, the price of long-term debt would increases as g_L realizes (i.e. the reverse of Lemma A.2) which goes against the idea of a debt relief. Hence, it is not possible to have an *official* buyback at any point related to the realization of g_L.

Second, could *official* buybacks occur before the contract hits the upper bound of the ergodic set – i.e. $x < x^{ub}$ and $g = g_H$? The answer is positive as the realization of g_H is associated with a debt decrease. However, one has to be careful that each *official* buyback should be such that $b'_{lt} \geq b_{lt}$. Moreover, note that if *official* buybacks happen at say $\tilde{x} < x^{ub}$ and $g = g_H$, then for all $x \in (\tilde{x}, x^{ub}]$ $\bar{q}_{lt}(x'(g_H, x)) \leq \bar{q}_{lt}(x'(g_L, x))$. $\square$

Proof of Proposition 1.5

I prove the proposition following Bhaskar et al. (2012). I first show that every equilibrium under Assumption 1.2 with $(\psi, \epsilon) > 0$ is essentially sequentially strict. I then prove that every essentially sequentially strict equilibrium is a Markov (perfect) equilibrium.

I start the proof with some definitions. Given the information structure, I split the histories into two categories: public and private. Public histories are the ones defined in Section 1.4 – that is h_b^t and h_l^t. Private histories of the borrower and the lenders at time t are the ones tracking the utility shocks – that is $p_b^t = (p_b^{t-1}, \varrho_{b,t})$ and $p_l^t = (p_l^{t-1}, \varrho_{l,t})$, respectively. Finally, I denote he entire history of the play including the privately observed utility shocks by $\hat{h}^t$.

In addition, I denote σ_b and σ_l as the strategy profile of the borrower and the lenders, respectively. Besides this, $\mathcal{C}_i$ corresponds to the countable set of actions with typical element a_i for market participant $i \in \{b, l\}$. For instance, actions taken by the borrower relate to borrowing, defaults and buybacks, while the lenders choose capital and its price. Moreover, $W^b(\sigma_b, \sigma_l | h_b^t, p_b^t)$ and $W^l(\sigma_b, \sigma_l | h_l^t, p_l^t)$ represent respectively the value of the borrower and the lenders from the strategy profile (σ_b, σ_l) at the relevant histories.

Given that each market participant has some private information regarding their payoff, they need to form beliefs about the unobserved utility shock of the other participants. Denote the belief of agent $i \in \{b, l\}$ over the entire history $\hat{h}^t$ as $\omega_i^{(h_i^t, p_i^t)}$. I follow Bhaskar et al. (2012) and put the least structure possible on such beliefs. They simply need to be independent of the private payoff shocks and put zero weight to events that history $\hat{h}^t$ is inconsistent with h_i^t. With this, I define a Markov equilibrium as

Definition A.2 (Markov Equilibrium)**.** *A strategy σ_i for $i \in \{b, l\}$ is Markov if for any two histories $(h_i^t, p_i^t) \neq (\tilde{h}_i^t, \tilde{p}_i^t)$ ending with the same state Ω_t,*

$$\sigma_i(h_i^t, p_i^t) = \sigma_i(\tilde{h}_i^t, \tilde{p}_i^t).$$

A strategy profile (σ_b, σ_l) is a Markov equilibrium if (σ_b, σ_l) is Markov and for any alternative strategy $(\tilde{\sigma}_b, \tilde{\sigma}_l)$,

$$W^b(\sigma_b, \sigma_l) \geq W^b(\tilde{\sigma}_b, \tilde{\sigma}_l) \ \wedge \ W^l(\sigma_b, \sigma_l) \geq W^l(\tilde{\sigma}_b, \tilde{\sigma}_l).$$

Note that this definition is equivalent to Definition 1.3 in the main text accounting for Assumption 1.2. Furthermore, I define a sequential best response as

Definition A.3 (Sequential Best Response)**.** *A strategy σ_i is a sequential best response to (σ_{-i}, ω_i), if for each history (h_i^t, p_i^t) and each alternative strategy $\tilde{\sigma}_i$*

$$\int W^i(\sigma_i, \sigma_{-i} | \hat{h}^t) d\omega_i^{(h_i^t)}(\hat{h}^t) \geq \int W^i(\tilde{\sigma}_i, \sigma_{-i} | \hat{h}^t) d\omega_i^{(h_i^t)}(\hat{h}^t).$$

Strategy σ_i is a sequential best response to σ_{-i} if strategy σ_i is a sequential best response (σ_{-i}, ω_i) for some ω_i.

Given the information structure, there is no general solution concept which can be used here. That is why, Bhaskar et al. (2012) appeal to the very weak concept of sequential optimality. Nonetheless, a profile of mutual sequential best response for the borrower and the lenders represents a perfect Bayesian equilibrium.

The other concept defined by the aforementioned authors is the current shock strategy which relies at most on the current value of the private shock. Formally

Definition A.4 (Current Shock Strategy)**.** *A strategy σ_i is a current shock strategy, if for any public history (h_i^t, p_i^t) and for any two histories, p_i^t and $\tilde{p}_i^t$, both finishing with the same ϱ_i, then for almost all ϱ_i*

$$\sigma_i(h_i^t, p_i^t) = \sigma_i(h_i^t, \tilde{p}_i^t).$$

The next lemma links Definitions A.3 and A.4. In words, any sequential response relies at most on the current value of the private shock. As a result the history of past private shocks becomes irrelevant.

Lemma A.3 (Sequential Strictness and Current Shock Strategy)**.** *If σ_i is a sequential best response to σ_{-i}, then σ_i is a current shock strategy.*

Proof. Consider a market participant i with history (h_i^t, p_i^t). The expected continuation value from choosing a certain action a_i under the strategy profile σ is given by

$$W^i(a_i, \sigma_{-i}, \omega_i | h_i^t, p_i^t) = \mathbb{E}_{g'|g} \int \int \max_{\sigma_i} W^i(\sigma_i, \sigma_{-i} | a_i, g', \varrho_i', \hat{h}^t) d\varsigma_{P_i}(\varrho_i') d\omega_i^{(h_i^t)}(\hat{h}^t).$$

Since σ_{-i} and $\omega_i^{(h_i^t, p_i^t)}$ do not depend on the private history, the value $W^i(a_i, \sigma_{-i}, \omega_i | h_i^t, p_i^t)$ is also independent of private history. Furthermore, since the density of ϱ_i is absolutely continuous, the market participant i can only be indifferent between two actions on a zero measure of the support. For different values of ϱ_i, the action is unique and independent of the past values of the shock. □

Given that beliefs on the history of past private shock do not matter, I can suppress the dependence on the beliefs and the private shock realization in the value function. Thus, the expected continuation value from choosing a certain action a_i under the strategy profile σ is given by

$$W^i(a_i, \sigma_{-i} | h_i^t) = \int \mathbb{E}_{g'|g} \max_{\sigma_i} W^i(\sigma_i, \sigma_{-i} | a_i, g', \varrho_i', h_i^t) d\varsigma_{P_i}(\varrho_i').$$

I then arrive to the first step of the proof. Given that beliefs over private histories are irrelevant for optimality, every perfect Bayesian equilibrium (i.e. a profile of mutual sequential best responses) satisfying Assumption 1.2 with $(\psi, \epsilon) > 0$ are essentially sequentially strict.

Lemma A.4 (Sequential Best Response and Perfect Bayesian Equilibrium)**.** *Every perfect Bayesian equilibrium satisfying Assumption 1.2 with* $(\psi, \epsilon) > 0$ *is essentially sequentially strict.*

Proof. I need to show that for any period, history and for almost all values of the private shock, the optimal action is unique. I consider the case of the borrower first. The borrower's value from action a_b after the realization of ϱ_b is given by

$$W^b(a_b, \varrho_b, \sigma_b | h_b^t) = u(a_b, g) + \epsilon \varrho_b^a + \beta \mathbb{E}_{g'|g} W^i(\sigma_b | a_b, g', h_b^t).$$

Suppose two actions a_b and $\tilde{a}_b$, the equality $W^b(a_b, \varrho_b, \sigma_b | h_b^t) = W^b(\tilde{a}_b, \varrho_b, \sigma_b | h_b^t)$ implies that

$$\epsilon(\varrho_b^{a_b} - \varrho_b^{\tilde{a}-b}) = u(a_b, g) - u(\tilde{a}_b, g) + \beta \mathbb{E}_{g'|g} \left[W^b(\sigma_b | a_b, g', h_b^t) - W^b(\sigma_b | \tilde{a}_b, g', h_b^t) \right].$$

The set of actions is countable, whereas the set of values of private shocks for which a market participant can be indifferent has measure zero. Hence, for almost all values of ϱ_i, the set of maximizing actions must be a singleton, and the profile is essentially sequentially strict. The proof naturally extends to the case of the lenders and is therefore omitted. □

Now that we have that all equilibria satisfying Assumption 1.2 with $(\psi, \epsilon) > 0$ are essentially sequentially strict, I simply need to show that sequentially strict equilibria are Markov equilibria.

Lemma A.5 (Sequential Strictness and Markov Equilibrium)**.** *Every essentially sequentially strict perfect Bayesian equilibrium is a Markov perfect equilibrium.*

Proof. Consider a t period history h^t. As shown previously, the private history matters, so the focus is on public history. Under Assumption 1.2 with $(\psi, \epsilon) > 0$, the borrower's behavior will not depend on h^t anymore from $t + \mathcal{T} + 1$ periods onward given that its memory is bounded to $\mathcal{T}$ periods back. This means that the lenders' value will not depend on h^t from $t + \mathcal{T} + 1$ periods for sure. As a result, if the lenders' strategy is sequentially strict, then h^t becomes irrelevant from $t + \mathcal{T} + 1$ periods.

What happens in period $t + \mathcal{T}$? This represents the last period in which strategies could be conditioned on h^t. However, at that time, the borrower's maximization problem is independent of h^t as no conditioning is possible next period. In addition, sequential strictness implies that the maximizing action is a singleton. Applying this argument recursively completes the proof. □

I have therefore shown that, under the assumption of bounded memory of the borrower, small perturbations in the payoff of the market participants suppress all equilibria except Markov ones. □

Proof of Proposition 1.7

Assume by contradiction that in a given state Ω_P, the borrower wants to conduct an *official* buy-back. That is, the borrower picks a pair (b'_{st}, b'_{lt}) such that

$$V^{NB}(\Omega_P) < V^B(\Omega_P).$$

The consumption under *official* buyback is given by

$$c^B(\Omega_P) = y(g,k) + b_{st} + b_{lt}(1 + q^{bb}_{lt}) - q_{st}(g, b'_{st}, b'_{lt})b'_{st} - q_{lt}(g, b'_{st}, b'_{lt})b'_{lt},$$

and the expected continuation value by

$$\mathbb{E}_{g'|g}\Bigg[W^b(g', b'_{st}, b'_{lt})\Bigg].$$

Now consider the alternative strategy of picking the same pair (b'_{st}, b'_{lt}) but conducting an *unofficial* buyback. In such circumstance, consumption is given by

$$c^{NB}(\Omega_P) = y(g,k) + b_{st} + b_{lt} - q_{st}(g, b'_{st}, b'_{lt})b'_{st} - q_{lt}(g, b'_{st}, b'_{lt})(b'_{lt} - b_{lt}).$$

It is clear from the *official* buyback premium that $c^{NB}(\Omega_P) > c^B(\Omega_P)$. Moreover, as the borrower chooses the same (b'_{st}, b'_{lt}), the continuation value is the same as before. Hence,

$$V^{NB}(\Omega_P) = u(c^{NB}(\Omega_P)) + \beta \mathbb{E}_{g'|g}\Big[W^b(g', b'_{st}, b'_{lt})\Big] > u(c^B(\Omega_P)) + \beta \mathbb{E}_{g'|g}\Big[W^b(g', b'_{st}, b'_{lt})\Big] = V^B(\Omega_P),$$

which contradicts the fact that an *official* buyback is ever optimal. $\square$

Proof of Lemma 1.4

– If $b'_{st} \geq 0$

The proof follows the same logic as the one of Proposition 1.7. Suppose by contradiction that the lenders can enforce *official* buybacks in a state Ω_P such that $B_{st}(\Omega_P) \geq 0$. Formally, in the case of an *official* buyback, the borrower chooses $b'_{st} = B_{st}(\Omega_P) \geq 0$ and $b'_{lt} = B_{lt}(\Omega_P) \geq b_{lt}$ to maximize its utility.

Now consider the case in which the borrower does not conduct the *official* buyback but mimics the debt choice in the case of *official* buyback. This is possible as the *new* lenders offer $b'_{st} \geq 0$.

The contradiction is immediate as the continuation value is the same in the two cases and $\bar{c}^{NB} < c^B$. Thus, *official* buybacks are not enforceable in case of short-term asset issuance.

– If $b'_{st} < 0$

I consider a state Ω_P in which, $B_{st}(\Omega_P) < 0$ and $B_{lt}(\Omega_P) \geq b_{lt}$. Moreover, I assume without loss of generality that the choice of private debt is the same in the case with and without *official* buyback. Given this, we have that for all s', $W^b(s', B_{st}(\Omega_P), B_{lt}(\Omega_P)) \leq W^b(s', 0, B_{lt}(\Omega_P))$. In words, the continuation value under the no-roll-over punishment is weakly larger than the continuation value under an *official* buyback.

I consider two cases. First, for a given g, if the level of short-term liabilities is large, then the incapacity to issue new short-term debt can lead to a default. In such case it is preferable for the borrower to pay the premium χ instead of entering default.

Second, given that the continuation value under punishment is weakly larger, to obtain that

$c^B > \bar{c}^{NB}$, it must be that

$$q_{st}(g, b'_{st}, b'_{lt})b'_{st} < b_{lt}(q_{lt}^{bb} - q_{lt}(g, b'_{st}, b'_{lt})).$$

Hence, provided that $q_{lt}^{bb} = \frac{1}{(1-\chi)r}$, if $\chi \to 0$ and $b_{lt} \to 0$, it is possible to have $V^B(\Omega_P) > \bar{V}^{NB}(\Omega_P)$ ensuring the enforcement of *official* buybacks.

□

Proof of Proposition 1.6

Consider the value of the *legacy* lender when there is no dilution. In that case

$$W^l_{\text{legacy},ND}(\Omega) = -\left[b_{st} + b_{lt}(1 + \frac{1}{r})\right].$$

In opposition, when there is dilution

$$W^l_{\text{legacy},D}(\Omega) = -\left[b_{st} + b_{lt}(1 + q_{lt}(y, B_{st}(\Omega), B_{lt}(\Omega)))\right] < W^l_{\text{legacy},ND}(\Omega),$$

where the inequality comes from the fact that $q_{lt}(y, B_{st}(\Omega), B_{lt}(\Omega)) < \frac{1}{r}$. As a result, the *legacy* lender is never willing to dilute. Similarly, the value of the *legacy* lender when there is an *official* buyback is

$$W^l_{\text{legacy},B}(\Omega) = -\left[b_{st} + b_{lt}(1 + q_{lt}(y, B_{st}(\Omega), B_{lt}(\Omega))) + \chi)\right],$$

while when there is no *official* buyback

$$W^l_{\text{legacy},NB}(\Omega) = -\left[b_{st} + b_{lt}(1 + q_{lt}(y, B_{st}(\Omega), B_{lt}(\Omega)))\right] < W^l_{\text{legacy},B}(\Omega),$$

As a result, the *legacy* lender is always willing to have an *official* buyback. In opposition, the value of the *new* lender is

$$W^l_{\text{new}}(\Omega) = W^l_{\text{new},ND}(\Omega) = W^l_{\text{new},D}(\Omega) = W^l_{\text{new},B}(\Omega) = W^l_{\text{new},NB}(\Omega) = 0.$$

As a result, the *new* lender is indifferent to dilution and *official* buyback. □

Proof of Lemma 1.5

From (A.8) and (A.9), the short and long-term holdings at the *official* buyback are respectively

$$\begin{aligned}
\bar{b}_{st}(x) &= \frac{V^l(g_H, x)[1 + \bar{q}_{lt}(x'(g_L, x))] - V^l(g_L, x)[1 + q^{bb}_{lt}]}{q^{bb}_{lt} - \bar{q}_{lt}(x'(g_L, x))} < 0, \\
\bar{b}_{lt}(x) &= -\frac{V^l(g_H, x) - V^l(g_L, x)}{q^{bb}_{lt} - \bar{q}_{lt}(x'(g_L, x))} \lesseqgtr 0.
\end{aligned}$$

From Part III of Proposition 1.2, it holds that $V^l(g_H, x) > V^l(g_L, x)$ meaning that $b_{lt} < 0$. However, it is not guaranteed that $b_{st} < 0$. Particularly, b_{st} can be negative only if q^{bb}_{lt} is very large with respect to q_{lt}. Moreover, recall that the *official* buyback takes place in the point of amnesia meaning that $b'_{st} = b_{st}$ and $b'_{lt} = b_{lt}$. As a result, Lemma 1.4 does not generally apply as b'_{st} is negative when χ is sufficiently large. □

Proof of Lemma A.1

The law of motion of the relative Pareto weight is given by

$$x'(g) = (1 + \nu(g))\eta x.$$

and the level of consumption by

$$u_c(c(g)) = \frac{1}{x(1 + \nu(g))}.$$

Isolating x leads to

$$x = \frac{1}{u_c(c(g))(1+\nu(g))}. \tag{A.11}$$

Plugging this back into the law of motion gives

$$x'(g) = (1+\nu(g))\eta\frac{1}{u_c(c(g))(1+\nu(g))}.$$

Replacing $x'(g)$ by with the forward equivalent of (A.11) gives

$$\frac{1}{u_c(c(g'))(1+\nu(g'))} = \eta\frac{1}{u_c(c(g))}.$$

Taking expectations on both sides,

$$\mathbb{E}_{g'|g}\left[\frac{1}{u_c(c(g'))(1+\nu(g'))}\right] = \eta\frac{1}{u_c(c(g))},$$

which gives the inverse Euler equation. □

Proof of Proposition A.1

Existence and uniqueness follow from Theorem 3 in Marcet and Marimon (2019). The two authors make the following assumptions: A1 a well defined Markov chain process for g, A2 continuity in $\{c, k\}$ and measurability in g, A3 non-empty feasible sets, A4 uniform boundedness, A5 convex technologies, A6 concavity for the lenders and strict concavity for the borrower, and a strict interiority condition. Assumption A1, A2, A5 and A6 are trivially met given my environment and Assumption 1.1. Since feasible c and k are bounded, payoffs functions are bounded as well. This combined with the fact that the outside options are also bounded ensure that A4 is met. Whether A3 is satisfied depends on the initial condition (g_0, x_0). Assumption A.1 ensures feasibility and that the strict interiority condition is satisfied.

It should be noted that Theorem 3 in Marcet and Marimon (2019) is the recursive, saddle-point, representation corresponding to the original contract problem (1.6). To obtain the recursive formulation of the contract, I have normalized the co-state variable. I relied on the the homogeneity of degree one in (μ_b, μ_l) to redefine the contracting problem using x – i.e. effectively $(x, 1)$ – as a co-state variable. Given this and the fact that multipliers are uniformly bounded, the theorem applies. That is, if I define the set of of feasible Lagrange multipliers by $L = \{(\mu_b, \mu_l) \in \mathbb{R}^2_+\}$ and the set of feasible consumption and capital by $A = \{(c, k) \in \mathbb{R}^2_+\}$, the correspondence SP : $A \times L \to A \times L$ mapping non-empty, convex, and compact sets to themselves, is non-empty, convex-valued, and upper hemicontinuous. I can therefore apply Kakutani's fixed point theorem and existence immediately follows.

Marcet and Marimon (2019) additionally show that the the saddle point functional equation (1.8) is a contraction mapping. Thus, given the concavity assumptions of $u(\cdot)$ and $f(\cdot)$, the allocation is unique. □

Proof of Proposition A.2

Following Alvarez and Jermann (2000) we prove the proposition by construction. First, define the asset price as

$$q(g', x'|g) = \frac{\pi(g'|g)}{1+r}\left[1 + \sum_{g''|g'} q(g'', x''|g')\right] \max\left\{\frac{u'(c(g', x'))}{u'(c(g, x))}\eta, 1\right\}.$$

Second, iterating over the budget constraint of the government and applying the transversality condition gives

$$a(g^t) = \mathbb{E}_t \sum_{j=0}^{\infty} Q(g^{t+j}, x(g^{t+j})|g^t)[c(g^{t+j}, x(g^{t+j})) - Y(g^{t+j}, x(g^{t+j}))], \tag{A.12}$$

where, $Y(g^t, x(g^t)) = g_t f(k(g^t, x(g^t))) - k(g^t, x(g^t))$ for all t and g^t. Similarly, iterating over the budget constraint of the lenders leads to

$$a_l(g^t) = \mathbb{E}_t \sum_{j=0}^{\infty} Q(g^{t+j}, x(g^{t+j})|g^t) c_l(g^{t+j}, x(g^{t+j})) \tag{A.13}$$

$$
\begin{aligned}
&= \mathbb{E}_t \sum_{j=0}^{\infty} Q(g^{t+j}, x(g^{t+j})|g^t)[Y(g^{t+j}, x(g^{t+j})) - c(g^{t+j}, x(g^{t+j}))] \\
&= \ - a(g^t).
\end{aligned}
$$

The market clearing condition implies that $a_l(g^t) + a(g^t) = 0$ for all t and g^t.

To ensure that the capital level is the same as in the constrained efficient allocation, I set the capital price according to

$$
gf_k(k) = p(g, a).
$$

I now need to establish the correspondence between the initial conditions, x_0, in the contract and the initial conditions in the RCE, a_0. Given (A.12) and (A.13) evaluated at $t = 0$, one can determine $\bar{a}'$ using the budget constraint

$$
c(g_0, x_0) + q(g_0, a_1)(\bar{a}' - a_0) + \sum_{g_1|g_0} q(g_1, a_1(g_1)|g_0)\hat{a}'(g_1) \leq y(g_0, k) + a_0.
$$

and the fact that $\sum_{g_1|g_0} q(g_1, a_1(g_1)|g_0)\hat{a}'(g_1) = 0$. Once, $\bar{a}'$ is determined, one can find the holdings of Arrow securities $\hat{a}'(g', g_0, a_0)$ for all $g' \in G$. We can then retrieve the entire portfolio recursively for $t > 0$.

Third, define the endogenous borrowing limits such that

$$
\mathcal{A}(g, k) = a(g, \tilde{x}(g, k)),
$$

where $\tilde{x}(g, k)$ is the relative Pareto weight when the participation constraint binds at (g, k). This definition implies that $a'(g', g, a) \geq \mathcal{A}(g', k')$. Hence, the constructed asset holdings satisfy the competitive equilibrium constraints.

Fourth, to ensure optimality of the policy functions by setting

$$\mu_{BC}(g,a) = \frac{1}{x(1+\nu(g))}$$

Hence, since $c(g,x)$ satisfies the optimality conditions in the Planner's problem, it is also optimally determined in the RCE. For the lenders, $c_l(g,x)$ is optimal if the asset portfolio is optimally determined. For this observe that

$$\begin{aligned}
q(g',a'(g')|g) &= \frac{1}{1+r}\pi(g'|g)\left[1+\sum_{g''|g'} q(g'',a''(g'')|g')\right] \\
&> \frac{1}{1+r}\pi(g'|g)\frac{u'(c(g',a'(g')))}{u'(c(g,a))}\eta\left[1+\sum_{g''|g'} q(g'',a''(g'')|g')\right] \\
&\quad \text{if } a'(g',g,a) = \mathcal{A}(g',k').
\end{aligned}$$

Hence the portfolio is optimally determined. We therefore obtain a one-to-one map between x and a for a given g. More precisely, $c(g,a) = c(g,x)$, $c_l(g,a) = T(g,x)$ and $k(g,a) = k(g,x)$. Thus, $W^b(g,a) = W^b(g,x)$ and $W^l(g,a) = W^l(g,x)$. Furthermore, the endogenous limits binds uniquely and exclusively when the participation constraints of the government binds. $\square$

Appendix B

Appendix to Chapter 2

2.1 The Fund Contract in Recursive Form

Using the recursive contracts approach, defining $s \equiv \{\theta^-, \gamma\}$, we say that $c(\theta, x, b_l)$, $n(\theta, x, b_l)$, $\nu_b(\theta, x, b_l)$ and $\nu_l(\theta, x, b_l)$ are a saddle-point solution to the Fund's contacting problem in *recursive form*, given $b_{l,0}$, if there exist a Fund's value function $FV(s, x, b_l)$, transfer policies $\tau_p(\theta, x, b_l)$ and $\tau_f'(\theta', x, b_l)$, with associate private lending policy $b_l' = B_l(\theta, x, b)$ satisfying (2.2), i.e., $\mathbb{E}_{t+1} \sum_{j=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{j-t-1} \tau_p($
$b_{l,t+1}$, such that:

$$FV(s, x, b_l) = \mathcal{SP} \min_{\{\nu_b, \nu_l\}} \max_{\{c,n\}} x\Big[(1+\nu_b)U(c,n) - \nu_b V^{af}(\theta)\Big] \tag{B.1}$$

$$+ \Big[(1+\nu_l)\tau - \nu_l(\theta^- Z + b_l)\Big] + \tfrac{1+\nu_l}{1+r}\mathbb{E}\Big[FV(s', x', b_l')\Big|\theta\Big]$$

$$\text{s.t. } \tau = \theta f(n) - c,$$

$$x' = \frac{1+\nu_b}{1+\nu_l}\eta x, \tag{B.2}$$

with x_0 given.

The value function of the contracting problem satisfies:

$$FV(s, x, b_l) = xV^b(\theta, x, b_l) + V^l(s, x, b_l), \text{ with} \tag{B.3}$$

$$V^b(\theta, x, b_l) = U(c, n) + \beta \mathbb{E}[V^b(\theta', x', b_l')|\theta], \tag{B.4}$$

$$V^l(s, x, b_l) = \tau + \frac{1}{1+r}\mathbb{E}[V^l(s', x', b_l')|\theta]. \tag{B.5}$$

We denote by x' the prospective Pareto weight of the sovereign relative to the Fund where $\eta \equiv \beta(1+r) < 1$ and $\nu_b \geq 0$ and $\nu_l \geq 0$ are the normalized multipliers attached to the sovereign's and the Fund's participation constraints, respectively.[1] That is, the constraint qualification constraints are

$$\nu_b \left[V^b(\theta, x, b_l) - V^{af}(\theta) \right] = 0, \tag{B.6}$$

$$\nu_l \left[V^l(s, x, b_l) - (\theta^- Z + b_l) \right] = 0. \tag{B.7}$$

The contracting problem in recursive form takes into account the existence a private lending policy, $b_l' = B_l(\theta, x, b)$.[2] The sequence of private transfers $\{\tau_p(\theta^t)\}_{t=0}^{\infty}$ directly relates to a sequence of private debt $\{b_l(\theta^t)\}_{t=0}^{\infty}$. Hence, for a given b_l, by picking b_l', the private lenders directly choose a certain level of transfer τ_p. The exact relationship between τ_p and b_l' is detailed in Section 2.4.

We obtain the optimal consumption and leisure policies, $c(\theta, x, b_l)$ and $n(\theta, x, b_l)$ by taking the first-order conditions of problem (B.1),[3]

$$u_c(c) = \frac{1+\nu_l}{1+\nu_b}\frac{1}{x}, \tag{B.8}$$

$$\theta f_n(n) = \frac{h_n(1-n)}{u_c(c)}. \tag{B.9}$$

This results in a total transfer policy $\tau(\theta, x, b_l)$ which corresponds to the lending capacity the Fund computes and announces every period. The lending capacity enables the economy to reach the constrained efficient allocation.

[1] The normalization of the Pareto weights is the same as the one in Ábrahám et al. (2022).

[2] In this Nash specification of the Fund contract, the effect of τ_f on $B(\theta, x, b)$ is not taken into account.

[3] The first-order condition with respect to consumption tells us that the sovereign can infer x_t from $u'(c_{t-1})$ given $(x_0, b_{l,0})$.

The relative Pareto weight, x, evolves according to the binding participation constraints. Particularly, it increases when the sovereign's constraint binds (i.e. $\nu_b > 0$) and decreases when the Fund's constraint binds (i.e. $\nu_l > 0$). In the former case, the sovereign's consumption increases not to generate default incentives, while in the latter case, the sovereign's consumption decreases to avoid expected losses from the lenders' perspective.

2.2 Detrended Model

As in Aguiar and Gopinath (2006), we consider a growth shock to the productivity of the following form $\theta_t = \gamma_t \theta_{t-1}$, where γ_t represents the growth rate and θ_t the trend at time t. We detrend the variables for allocations (except for labor n_t where we normalize the time endowment to 1) of the model by dividing them by θ_{t-1}. We normalize $\theta_{-1} = 1$, then the initial states satisfy $\theta_0 = \gamma_0$. We then denote by $\tilde{c}_t$ the detrended form of c_t such that $\tilde{c}_t = \frac{c_t}{\theta_{t-1}}$ represents the deviation from the trend. It follows that $U(c_t, n_t) = \ln(\theta_{t-1}) + U(\tilde{c}_t, n_t)$, and clearly, $\ln(\theta_{t-1})$ does not affect optimal choice. By the homogeneity of the sovereign's recursive problem, we have the detrended formulation as

$$\widetilde{W}^b(\gamma, \tilde{a}, \tilde{b}) = \max_{\left\{\tilde{c}, n, \tilde{b}', \{\tilde{a}'(\gamma')\}_{\gamma' \in \Gamma}\right\}} U(\tilde{c}, n) + \beta \mathbb{E}\left[\widetilde{W}^b(\gamma', \tilde{a}'(\gamma', \tilde{b}'), \tilde{b}') \middle| \gamma\right] \tag{B.10}$$

$$\begin{aligned} \text{s.t. } & \tilde{c} + \sum_{\gamma'|\gamma} q_f(\gamma', \tilde{\omega}'(\gamma')|\gamma)(\gamma \tilde{a}'(\gamma', \tilde{b}') - \delta \tilde{a}) + q_p(\gamma, \tilde{\bar{\omega}}')(\gamma \tilde{b}' - \delta \tilde{b}) \\ & \leq \gamma f(n) + (1 - \delta + \delta\kappa)(\tilde{a} + \tilde{b}), \text{ and} \\ & \tilde{\omega}'(\gamma') = \tilde{a}'(\gamma', \tilde{b}') + \tilde{b}' \geq \tilde{\mathcal{A}}_b(\gamma'). \end{aligned}$$

The sovereign's outside option in detrended form takes the following form

$$\widetilde{V}^{af}(\gamma) = \max_n \left\{U(\gamma^d f(n), n)\right\} + \beta \mathbb{E}\left[(1-\lambda)\widetilde{V}^{af}(\gamma') + \lambda \tilde{J}(\gamma', 0) \middle| \gamma\right],$$

The detrended Fund's problem in sequential form is given by

$$\max_{\{\tilde{c}(\gamma^t),n(\gamma^t)\}_{t=0}^{\infty}} \mathbb{E}\left[\mu_{b,0}\sum_{t=0}^{\infty}\beta^t U(\tilde{c}(\gamma^t),n(\gamma^t)) + \mu_{l,0}\sum_{t=0}^{\infty}\left(\frac{1}{1+r}\right)^t\left(\prod_{i=0}^{t-1}\gamma_i\right)\tilde{\tau}(\gamma^t)\middle|\theta_{-1}\right] \tag{B.11}$$

$$\text{s.t.}\quad \mathbb{E}\left[\sum_{j=t}^{\infty}\beta^{j-t}U(\tilde{c}(\gamma^j),n(\gamma^j))\middle|\gamma^t\right] \geq \widetilde{V}^{af}(\gamma_t), \tag{B.12}$$

$$\mathbb{E}\left[\sum_{j=t}^{\infty}\left(\frac{1}{1+r}\right)^{j-t}\left(\prod_{i=t}^{j-1}\gamma_i\right)\tilde{\tau}(\gamma^j)\middle|\gamma^t\right] \geq Z - \tilde{b}(\gamma^t), \tag{B.13}$$

$$\tilde{\tau}(\gamma^t) = \gamma_t f(n(\gamma^t)) - \tilde{c}(\gamma^t), \quad \forall\gamma^t, t \geq 0,$$

with $\mu_{b,0},\ \mu_{l,0},\ \{\tilde{b}(\gamma^t)\}_{t=0}^{\infty}$ given.

And in recursive form

$$\widetilde{FV}(\gamma,(\tilde{x},1),\tilde{b}_l) = \mathcal{SP}\min_{\{\nu_b,\nu_l\}}\max_{\{\tilde{c},n\}} \tilde{x}\Big[(1+\nu_b)U(\tilde{c},n) - \nu_b\widetilde{V}^{af}(\gamma)\Big] \tag{B.14}$$

$$+\Big[(1+\nu_l)\tilde{\tau} - \nu_l(Z+\tilde{b}_l)\Big] + \frac{1+\nu_l}{1+r}\gamma\mathbb{E}\Big[\widetilde{FV}(\gamma',(\tilde{x}',1),\tilde{b}_l')\Big|\gamma\Big]$$

$$\text{s.t.}\quad \tilde{\tau} = \gamma f(n) - \tilde{c},$$

$$\tilde{x}' = \frac{1+\nu_b}{1+\nu_l}\frac{\eta}{\gamma}\tilde{x}, \tag{B.15}$$

Note that we have also enlarged the co-state $\tilde{x}$ to $(\tilde{x},1)$, which contains the same information, but now $\widetilde{FV}$ is homogeneous of degree one in $(x,1)$, which is convenient in the proof of existence of a Fund contract. Nevertheless, we will only make this explicit extension when it is necessary. The value function takes the form of

$$\widetilde{FV}(\gamma,(\tilde{x},1),\tilde{b}_l) = \tilde{x}\widetilde{V}^b(\gamma,(\tilde{x},1),\tilde{b}_l) + \widetilde{V}^l(\gamma,(\tilde{x},1),\tilde{b}_l), \text{ with} \tag{B.16}$$

$$\widetilde{V}^b(\gamma,(\tilde{x},1),\tilde{b}_l) = U(\tilde{c},n) + \beta\mathbb{E}[\widetilde{V}^b(\gamma',(\tilde{x}',1),\tilde{b}_l')|\gamma], \text{ and}$$

$$\widetilde{V}^l(\gamma,(\tilde{x},1),\tilde{b}_l) = \tilde{\tau} + \frac{1}{1+r}\gamma\mathbb{E}[\widetilde{V}^l(\gamma',(\tilde{x}',1),\tilde{b}_l')|\gamma].$$

Taking the first-order conditions with respect to $\tilde{c}$ and n leads to

$$u_c(\tilde{c}) = \frac{1+\nu_l}{1+\nu_b}\frac{1}{\tilde{x}} \quad \text{and} \quad \gamma f_n(n) = \frac{h_n(1-n)}{u_c(\tilde{c})}.$$

The consumption is therefore equal to $\tilde{c} = \tilde{x}'\frac{\gamma}{\eta} \equiv \tilde{z}'\gamma$. From this, we see that whenever the growth rate of the economy settles below one, the relative Pareto weight increases. However, the consumption does not react to changes in γ.

Finally, for completeness, the decentralised Fund problem in detrended form is given by

$$\widetilde{W}^f(\gamma, \tilde{a}_l, \tilde{b}_l) = \max_{\{\tilde{c}_f, \{\tilde{a}_l'(\gamma', \tilde{b}_l')\}_{\gamma'\in\Gamma}\}} \tilde{c}_f + \frac{1}{1+r}\gamma\mathbb{E}\left[\widetilde{W}^f(\gamma', \tilde{a}_l'(\gamma', \tilde{b}_l'), \tilde{b}_l')\middle|\gamma\right] \tag{B.17}$$

$$\text{s.t. } \tilde{c}_f + \sum_{\gamma'|\gamma} q_f(\gamma', \omega'(\gamma')|\gamma)(\gamma\tilde{a}_l'(\gamma', \tilde{b}_l') - \delta\tilde{a}_l) \leq (1-\delta+\delta\kappa)\tilde{a}_l,$$

$$\tilde{a}_l'(\gamma', \tilde{b}_l') + \tilde{b}_l' \geq \tilde{\mathcal{A}}_f(\gamma', \tilde{b}_l'). \tag{B.18}$$

with $\tilde{b}_l' = \tilde{B}_l(\gamma, \tilde{a}_l, \tilde{b}_l)$ given

2.3 Further Theory Development

In this section we present other properties of the Fund contract. We start with the inverse Euler equation which is a key concept determining the dynamic of consumption in the contract.

Proposition B.1 (Inverse Euler Equation)**.** *In the Fund contract, the inverse Euler equation is given by*

$$\mathbb{E}\left[\frac{1}{u_c(c(\theta', x', b_l'))}\frac{1+\nu_l(\theta', x', b_l')}{1+\nu_b(\theta', x', b_l')}\middle|\theta\right] = \eta\frac{1}{u_c(c(\theta, x, b_l))},$$

and risk sharing is imperfect.

We obtain the inverse Euler equation by means of the first-order condition on consumption and the law of motion of the relative Pareto weight. This equation gives the intertemporal dynamic of consumption. If none of the constraints are ever binding (i.e. $\nu_b = \nu_l = 0$), it becomes

$$\mathbb{E}\left[\frac{1}{u_c(c(\theta', x', b'))}\middle|\theta\right] \leq \frac{1}{u_c(c(\theta, x, b))},$$

with strict inequality if $\eta < 1$, in our case. We therefore obtain a positive martingale, which by the supermartingale theorem, converges almost surely to 0. This is what the literature has called immiseration.

Thus, with $\eta < 1$, when none of the constraints are binding, consumption decreases. However, this reduction cannot go on indefinitely given the sovereign's participation constraint. This constraint puts a lower bound to the supermartingale and therefore acts as a stopper for immiseration. Conversely, the Fund's constraint puts an upper bound to the supermartingale which prevents consumption to increase indefinitely. As a result, in a contract with two-sided limited enforcement constraints and impatient borrower, risk sharing is only partial. The contract cannot converge to the first-best allocation characterised by constant consumption over time.

The long-run property of the Fund contract is related to the definition of an ergodic set of relative Pareto weights, x. The term ergodic refers to the fact that the relative Pareto weights in this set are aperiodic and recurrent with non-zero probability. In other words, the economy will move around the same set of relative Pareto weights over time and over histories. The following definition relies on the model in detrended form presented in the Appendix 2.2.

Definition B.1 (Steady State)**.** *Given a Markov chain of γ with a unique ergodic set in Γ, a* Steady State Equilibrium *is defined by an ergodic set with a lower bound $\underline{x} = \min_{\gamma \in \Gamma}\{x : \widetilde{V}^b(\gamma, x, \tilde{b}_l) = \widetilde{V}^{af}(\gamma)\}$ and an upper bound $\overline{x} = \max_{\gamma \in \Gamma}\{x : \widetilde{V}^b(\gamma, x, \tilde{b}_l) = \widetilde{V}^{af}(\gamma)\}$, satisfying $\underline{x} < \overline{x}$, for the relative Pareto weights.*[4]

The lower bound of the ergodic set is determined by the lowest achievable relative Pareto weight in the contract. It represents the lowest value that the sovereign accepts in the contract, which keeps it away from immiseration. The upper bound represents the highest relative Pareto weight

[4]The value functions marked with $\widetilde{V}$ are the detrended value functions presented in the Appendix 2.2. In Section 2.6, Figure 2.3 shows (in gray) the ergodic set of our calibrated economy.

that makes the sovereign's constraint bind; therefore it is the highest weight that the lender may need to accept. We can further characterise the bounds of the ergodic set with the following lemma, validating their independence on b_l.[5]

Lemma B.1 (Bounds of the Ergodic set)**.** *The bounds of the ergodic set solely depend on the current growth state, θ, thus for the detrended form, solely depend on γ.*

This lemma states that the bounds of the ergodic set are independent of b_l. In other words, the sovereign's participation constraint is solely determined by the realised growth state. From Definition B.1, the bounds of the ergodic set depend on the binding borrower's constraint. Thus, as the value of default is independent of b_l, so does the constraint.

Besides this, in the decentralised economy, we relate the level of debt with the present value of the budget constraint. This leads to the following lemma.

Lemma B.2 (Debt and Budget Constraint)**.** *At any period t for $\bar{a}_{p,t} = 0$,*

$$\begin{aligned} a_t(\theta^t) + b_t = \mathbb{E}_t \sum_{j=0}^{\infty} & Q_f(\theta^{t+j}, \omega(\theta^{t+j})|\theta^t) \\ & \times [c(\theta^{t+j}, a(\theta^{t+j}), b(\theta^{t+j})) - Y(\theta^{t+j}, a(\theta^{t+j}), b(\theta^{t+j}))], \\ a_{l,t}(\theta^t) + b_{l,t} = \mathbb{E}_t \sum_{j=0}^{\infty} & Q_f(\theta^{n+j}, \omega(\theta^{n+j})|\theta^n) \\ & \times [c_f(\theta^{n+j}, a(\theta^{n+j}), b(\theta^{n+j})) + c_p(\theta^{n+j}, a(\theta^{n+j}), b(\theta^{n+j}))]), \end{aligned}$$

with $Y(\theta^t, x(\theta^t), b(\theta^t)) \equiv \theta(\theta_t) f(n(\theta^t, x(\theta^t), b(\theta^t)))$ for all t and θ^t.

We end this section with a result regarding the maturity. We have taken the maturity structure of the debt as given. However,

Corollary B.1 (Debt maturity)**.** *If the sovereign takes into account the Fund contract in deciding its maturity structure, it prefers to choosing an alternative $\hat{\delta} = 0$ whenever* (2.24) *binds in steady state with $\delta > 0$. If for no $\delta > 0$* (2.24) *binds, the choice of δ is irrelevant.*

[5]It should be noted that if the sovereign and the Fund are equally patient (i.e. $\eta = 1$), then the upper bound would be determined by $\min_{\gamma \in \Gamma}\{x : \widetilde{V}^l(\gamma, x, \tilde{b}_l) = Z + \tilde{b}_l\}$, which depends on the endogenous b_l.

This result follows from the fact that with long-term debt, in particular with the MIP, the Fund's guarantee towards private lenders can be up to δb_l. Hence, the closer is δ to 0, the lower is the amount of private debt the Fund may need to absorb every period when (2.24) binds. In other words, the choice of maturity in the presence of the Fund solely matters when (2.24) binds for some $\theta' \in \Theta$ with $\delta > 0$ in steady state.

2.4 Proofs

Proof of Proposition 2.1

We need to prove that (B.14) has a unique solution which in finite time reaches a 'golden path' with a stationary growth distribution. We show first that existence and uniqueness follows from Theorem 3 in Marcet and Marimon (2019) . They make the following assumptions: A1 a well defined Markov chain process for γ,[6] A2 continuity in $\{c, n\}$ and measurability in γ, A3 non-empty feasible sets, A4 uniform boundedness, A5 convex technologies, A6 concavity for the lender and strict concavity for the sovereign, and a *Strict Interiority Condition* **SIC**. Assumption A1, A2, A5 and A6 are trivially met in the economies described in Sections 2.2. Since feasible c and n are bounded, payoffs functions are bounded as well. This combined with the fact that the outside options are also bounded ensure that A4 is met. Whether a feasible contract exists (i.e. A3) as the statement of Proposition 2.1, it amounts to show that for every θ there is a $\underline{b}_l(\theta) > 0$ for which a feasible contract exists, then by monotonicity, it also exists if $b_{l,0}(\theta) \leq \underline{b}_l(\theta)$. However, given that the borrower is more impatient and risk averse than the Fund, and its participation constraint is the value of being in the IMD economy with $b(\theta^t) = 0$, while $Z \leq 0$, there is a $\underline{b}_l(\theta_0) = \tilde{\underline{b}}_l(\theta_0) > 0$ for which a feasible contract exists. Similarly, the same argument shows that assumption 2.1 ensures that the *Strict Interiority Condition* **SIC** is also met.

It should be noted that Theorem 3 in Marcet and Marimon (2019) is the recursive, saddle-point, representation corresponding, in our framework, to the original contract problem (2.6). While we have renormalized the co-state variables — to $(x, 1)$ — and detrended allocations, multipliers and

[6] Being a discrete process A1b, stated in their theorem, is redundant.

states and co-states. Nevertheless, given that $\theta_t = \gamma_t\theta_{t-1}$, we normalize $\gamma{-1} = 1$ and given that multipliers are uniformly bounded, the theorem also applies to our normalized and detrended version.

Regarding uniqueness, since the *contraction mapping theorem* also applies and there is a unique value function (B.16) and, furthermore, given the strict concavity assumptions of U and f, the allocation is unique.

Regarding the steady state, as defined in Definition B.1, the lower bound of the ergodic set is determined by the lowest achievable relative Pareto weight in the contract. It represents the lowest value that the sovereign accepts in the contract. The upper bound represents the highest relative Pareto weight that makes the sovereign's constraint bind; therefore it is the highest weight that the lender may need to accept. This means that every time the highest productivity shock hits (i.e. γ_{max}), the sovereign climbs to the top of the ergodic set. In opposition, for a sufficiently long string of lowest productivity shock (i.e. γ_{min}), the sovereign eventually hits the bottom of the set — owing to immiseration with $\eta < 1$. Hence, in the detrended version of the model, the lower bound is defined by $\underline{x} = \min_{\gamma\in\Gamma}\{x : \widetilde{V}^b(\gamma, x, \tilde{b}_l) = \widetilde{V}^{af}(\gamma)\}$, while the upper bound corresponds to $\overline{x} = \max_{\gamma\in\Gamma}\{x : \widetilde{V}^b(\gamma, x, \tilde{b}_l) = \widetilde{V}^{af}(\gamma)\}$.

To show the existence of a unique stationary equilibrium, one shows that the dynamic of the contract satisfies the conditions given by Stokey et al. (1989, Theorem 12.12). Set $\ddot{x}$ as the midpoint of $[\underline{x}, \overline{x}]$ and define the transition function $\mathcal{Q} : [\underline{x}, \overline{x}] \times \mathcal{X}([\underline{x}, \overline{x}]) \to \mathbb{R}$ as

$$\mathcal{Q}(x, G) = \sum_{\theta'|\theta} \pi(\theta'|\theta)\mathbb{I}\{x' \in G\}$$

We want to show is that $\ddot{x}$ is a mixing point such that for $N \geq 1$ and $\iota > 0$ one has that $\mathcal{Q}(\underline{x}, [x, \overline{x}])^N \geq \iota$ and $\mathcal{Q}(\overline{x}, [\underline{x}, x])^N \geq \iota$. Starting at $\overline{x}$, for a sufficiently long but finite series of γ_{min}, the relative Pareto weight transit to $\underline{x}$. Hence for some $N < \infty$, $\mathcal{Q}(\overline{x}, [\underline{x}, \ddot{x}])^N \geq \pi(\gamma_{min}|\gamma_{min})^N > 0$. Moreover, starting at $\underline{x}$, after drawing $N < \infty$ γ_{max}, the relative Pareto weight transit to $\overline{x}$ meaning that $\mathcal{Q}(\underline{x}, [\ddot{x}, \overline{x}])^N \geq \pi(\gamma_{max}|\gamma_{max})^N > 0$. Setting $\iota = \min\{\pi(\gamma_{min}|\gamma_{min})^N, \pi(\gamma_{max}|\gamma_{max})^N\}$

makes $\ddot{x}$ a mixing point and the above theorem applies. □

Proof of Corollary 2.1

The proof follows the argument of Thomas and Worrall (1994) and Zhang (1997). The participation constraint of the sovereign — i.e. (2.4) — ensures that the value of the sovereign in the contract is at most equal to its outside option. Hence, the sovereign is at most indifferent between defaulting or not and therefore never enters *full* default. □

Proof of Proposition 2.2

We conduct a proof by construction. The combination of the first-order conditions of (2.13) with respect to c and $a'(\theta', b')$ gives the sovereign's Euler equation for the Fund's securities

$$q_f(\theta', \omega'(\theta')|\theta)u_c(c) - \upsilon_b(\theta') = \beta\pi(\theta'|\theta)u_c(c')\left[(1-\delta+\delta\kappa) + \delta\sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\right], \quad \text{(B.19)}$$

where υ_b is the multiplier attached to the sovereign's endogenous borrowing limit in (2.15). Conversely, the first-order conditions with respect to c and b' gives the sovereign's Euler equation for the private bonds

$$q_p(\theta, \bar{\omega}')u_c(c) - \sum_{\theta'|\theta}\upsilon_b(\theta') = \beta\sum_{\theta'|\theta}\pi(\theta'|\theta)u_c(c')\Big[(1-\delta+\delta\kappa) + \delta q_p(\theta', \bar{\omega}'')\Big]. \quad \text{(B.20)}$$

Taking the first-order conditions of (2.17) with respect to b'_l,

$$q_p(\theta, \bar{\omega}') = \frac{\sum_{\theta'|\theta}\pi(\theta'|\theta)(1-\delta+\delta\kappa+\delta q_p(\theta', \bar{\omega}''))}{1+r}, \quad \text{(B.21)}$$

which corresponds to the price without default and without binding constraint of the Fund. Taking the first-order conditions of (2.20) with respect to c and $a'_l(\theta', b'_l)$ gives the Fund's Euler equation

$$q_f(\theta', \omega'(\theta')|\theta) - \varphi_f(\theta') = \tfrac{1}{1+r}\pi(\theta'|\theta)\Big[(1-\delta+\delta\kappa) + \delta\textstyle\sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\Big], \quad \text{(B.22)}$$

where φ_f is the multipliers attached to the Fund's endogenous limit. Given this, we now have to distinguish three cases:[7]

1. The sovereign's and the Fund's participation constraints are not binding. The lenders' Euler equations read respectively

$$q_f(\theta', \omega'(\theta')|\theta) = \frac{\pi(\theta'|\theta)}{1+r}\left[(1-\delta+\delta\kappa) + \delta \sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\right],$$
$$q_p(\theta, \bar{\omega}') = \sum_{\theta'|\theta} \frac{\pi(\theta'|\theta)}{1+r}[(1-\delta+\delta\kappa) + \delta q_p(\theta', \bar{\omega}'')] = \frac{1-\delta+\delta\kappa}{1+r-\delta},$$

and the sovereign's Euler equations are

$$q_f(\theta', \omega'(\theta')|\theta) = \beta\pi(\theta'|\theta)\frac{u_c(c')}{u_c(c)}\left[(1-\delta+\delta\kappa) + \delta \sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\right],$$
$$q_p(\theta, \bar{\omega}') = \beta \sum_{\theta'|\theta} \pi(\theta'|\theta)\frac{u_c(c')}{u_c(c)}[(1-\delta+\delta\kappa) + \delta q_p(\theta', \bar{\omega}'')].$$

If none of the two constraints is ever binding,

$$\begin{aligned}\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta) &= \beta \sum_{\theta'|\theta} \pi(\theta'|\theta)\frac{u_c(c')}{u_c(c)}\left[(1-\delta+\delta\kappa) + \delta \sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\right] \\ &= \sum_{\theta'|\theta} \pi(\theta'|\theta)\frac{1}{1+r}\left[(1-\delta+\delta\kappa) + \delta \sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\right], \\ q_p(\theta, \bar{\omega}') &= \beta \sum_{\theta'|\theta} \pi(\theta'|\theta)\frac{u_c(c')}{u_c(c)}[(1-\delta+\delta\kappa) + \delta q_p(\theta', \bar{\omega}'')] \\ &= \sum_{\theta'|\theta} \pi(\theta'|\theta)\frac{1}{1+r}[(1-\delta+\delta\kappa) + \delta q_p(\theta', \bar{\omega}'')],\end{aligned}$$

It then follows that $Q_p(\theta, \bar{\omega}') = \sum_{\theta'|\theta} Q_f(\theta', \omega'(\theta')|\theta) = \frac{1-\delta+\delta\kappa}{1+r-\delta}$.

2. The sovereign's participation constraint binds and the Fund's participation constraint is not binding.

[7] Recall that, under Assumption 2.1, it is not possible that the two participation constraints bind at the same time.

The lenders' Euler equations are respectively

$$q_f(\theta',\omega'(\theta')|\theta) = \frac{\pi(\theta'|\theta)}{1+r}\left[(1-\delta+\delta\kappa)+\delta\sum_{\theta''|\theta'} q_f(\theta'',\omega''(\theta'')|\theta')\right],$$
$$q_p(\theta,\bar{\omega}') = \sum_{\theta'|\theta}\frac{\pi(\theta'|\theta)}{1+r}[(1-\delta+\delta\kappa)+\delta q_p(\theta',\bar{\omega}'')],$$

and the sovereign's Euler equations are

$$q_f(\theta',\omega'(\theta')|\theta)u_c(c) - \varphi_b(\theta') = \beta\pi(\theta'|\theta)\frac{u_c(c')}{u_c(c)}\left[(1-\delta+\delta\kappa)+\delta\sum_{\theta''|\theta'} q_f(\theta'',\omega''(\theta'')|\theta')\right],$$
$$q_p(\theta,\bar{\omega}')u_c(c) - \sum_{\theta'|\theta}\varphi_b(\theta') = \beta\sum_{\theta'|\theta}\pi(\theta'|\theta)\frac{u_c(c')}{u_c(c)}[(1-\delta+\delta\kappa)+\delta q_p(\theta',\bar{\omega}'')].$$

If the Fund's participation constraint never binds,

$$\sum_{\theta'|\theta} q_f(\theta',\omega'(\theta')|\theta) > \beta\sum_{\theta'|\theta}\pi(\theta'|\theta)\frac{u_c(c')}{u_c(c)}\left[(1-\delta+\delta\kappa)+\delta\sum_{\theta''|\theta'} q_f(\theta'',\omega''(\theta'')|\theta')\right] \quad \text{and}$$
$$\sum_{\theta'|\theta} q_f(\theta',\omega'(\theta')|\theta) = \sum_{\theta''|\theta'}\frac{\pi(\theta'|\theta)}{1+r}\left[(1-\delta+\delta\kappa)+\delta\sum_{\theta''|\theta'} q_f(\theta'',\omega''(\theta'')|\theta')\right],$$

Moreover, $Q_p(\theta,\bar{\omega}') = \sum_{\theta'|\theta} Q_f(\theta',\omega'(\theta')|\theta) = \frac{1-\delta+\delta\kappa}{1+r-\delta}$.

3. The sovereign's participation constraint is not binding and the Fund's participation constraint binds.

   The lenders' Euler equations read respectively

$$q_f(\theta',\omega'(\theta')|\theta) - \varphi_f(\theta') = \frac{\pi(\theta'|\theta)}{1+r}\left[(1-\delta+\delta\kappa)+\delta\sum_{\theta''|\theta'} q_f(\theta'',\omega''(\theta'')|\theta')\right],$$
$$q_p(\theta,\bar{\omega}') = \sum_{\theta'|\theta}\frac{\pi(\theta'|\theta)}{1+r}[(1-\delta+\delta\kappa)+\delta q_p(\theta',\bar{\omega}'')],$$

The sovereign's Euler equations are

$$q_f(\theta', \omega'(\theta')|\theta) = \beta\pi(\theta'|\theta)\frac{u_c(c')}{u_c(c)}\left[(1-\delta+\delta\kappa) + \delta\sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\right],$$

$$q_p(\theta, \bar{\omega}') = \beta\sum_{\theta'|\theta}\pi(\theta'|\theta)\frac{u_c(c(\theta', \omega'))}{u_c(c)}[(1-\delta+\delta\kappa) + \delta q_p(\theta', \bar{\omega}'')].$$

If the sovereign's participation constraint never binds,

$$\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta) = \beta\sum_{\theta'|\theta}\pi(\theta'|\theta)\frac{u_c(c')}{u_c(c)}\left[(1-\delta+\delta\kappa) + \delta\sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\right] \quad \text{and}$$

$$\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta) > \sum_{\theta''|\theta'}\frac{\pi(\theta'|\theta)}{1+r}[(1-\delta+\delta\kappa) + \delta\sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')].$$

However, $Q_p(\theta, \bar{\omega}') < \sum_{\theta'|\theta} Q_f(\theta', \omega'(\theta')|\theta)$ cannot be an equilibrium. At this price the private lender is willing to hold an infinite amount of debt in the Fund and provide an infinite amount of assets to the sovereign. To avoid this arbitrage, it must be that $Q_p(\theta, \bar{\omega}') = \sum_{\theta'|\theta} Q_f(\theta', \omega'(\theta')|\theta) > \frac{1-\delta+\delta\kappa}{1+r-\delta}$.

Hence, in all possible states, $Q_p(\theta, \bar{\omega}') = \sum_{\theta'|\theta} Q_f(\theta', \omega'(\theta')|\theta) \geq \frac{1-\delta+\delta\kappa}{1+r-\delta}$. □

Proof of Proposition 2.3

As shown in Proposition 2.2, when (2.24) binds, $Q_p(\theta, \bar{\omega}') = Q_f(\theta, \bar{\omega}') > \frac{1}{1+r}$. At this price, private lenders do not want to lend to the sovereign as the present net discounted return of one unit of debt is negative. In other words, as the private lenders borrow on the international bond market at r, they are unwilling to save at $r_p(\theta, \bar{\omega}') < r$ because they cannot break even at such rates. There is therefore no trade in the private bond market meaning that $b' \geq \delta b$.

For the second part, we conduct a proof by construction. When (2.24) does not bind, the budget constraint reads

$$c + q_p(\theta, \bar{\omega}')(b' - \delta b) + \sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)(a'(\theta') - \delta a) = \theta f(n) + (1-\delta+\delta\kappa)(b+a).$$

Given that $\sum_{\theta'|\theta} q_f(\theta', \omega'(\theta')|\theta)\hat{a}(\theta') = 0$ and Proposition 2.2, it can be rewritten as

$$c + q_p(\theta, \bar{\omega}')(b' - \delta b) + q_f(\theta, \bar{\omega}')(\bar{a}' - \delta \bar{a}) = \theta f(n) + (1 - \delta + \delta\kappa)(b + a),$$

$$c + q(\theta, \bar{\omega}')(\bar{\omega}' - \delta(b + a)) = \theta f(n) + (1 - \delta + \delta\kappa)(b + a),$$

where $q(\theta, \bar{\omega}') \equiv q_p(\theta, \bar{\omega}') = q_f(\theta, \bar{\omega}')$ by Proposition 2.2. Having the same price and being equally accessible, private and Fund-provided bonds are prefect substitute, so that the decomposition of $\bar{\omega}'$ between b' and $\bar{a}'$ is indeterminate. $\square$

Proof of Proposition 2.4

The proof has the following steps: first, following Alvarez and Jermann (2000), we complete the characterization of the Fund contract asset structure; second we map the state in the Fund problem with the state in that the decentralized economy; third we map the initial conditions and participation constraints, and fourth, we complete the mapping between policies and value functions.

First, Fund's assets have prices given by (2.25); i.e.

$$q_f(\theta', x', b'|\theta) = \frac{\pi(\theta'|\theta)}{1+r}\left[(1 - \delta + \delta\kappa) + \delta \sum_{\theta''|\theta'} q_f(\theta'', x'', b''|\theta')\right] \max\left\{\frac{u_c(c(\theta', x', b'))}{u_c(c(\theta, x, b))}\eta, 1\right\}.$$

As shown in Lemma B.2, iterating over the budget constraint of the sovereign gives

$$a(\theta^t) + b(\theta^t) =$$
$$\mathbb{E}_t \sum_{j=0}^{\infty} Q(\theta^{t+j}, x(\theta^{t+j}), b(\theta^{t+j})|\theta^t)[c(\theta^{t+j}, x(\theta^{t+j}), b(\theta^{t+j})) - Y(\theta^{t+j}, x(\theta^{t+j}), b(\theta^{t+j}))], \quad \text{(B.23)}$$

where, $Y(\theta^t, x(\theta^t), b(\theta^t)) = \theta(\theta_t) f(n(\theta^t, x(\theta^t), b(\theta^t)))$ for all t and θ^t. Similarly, for $\bar{a}_p(\theta^t) = 0$, iterating over the consolidated budget constraint of the two lenders and denoting $c_l \equiv c_f + c_p$ leads to

$$
\begin{aligned}
a_l(\theta^t) + b_l(\theta^t) &= \mathbb{E}_t \sum_{j=0}^{\infty} Q(\theta^{t+j}, x(\theta^{t+j}), b(\theta^{t+j})|\theta^t) c_l(\theta^{t+j}, x(\theta^{t+j}), b(\theta^{t+j})) \qquad \text{(B.24)} \\
&= \mathbb{E}_t \sum_{j=0}^{\infty} Q(\theta^{t+j}, x(\theta^{t+j}), b(\theta^{t+j})|\theta^t)[Y(\theta^{t+j}, x(\theta^{t+j}), b(\theta^{t+j})) \\
&\quad - c(\theta^{t+j}, x(\theta^{t+j}), b(\theta^{t+j}))] \\
&= \; - a(\theta^t) - b(\theta^t).
\end{aligned}
$$

The market clearing conditions in the Fund and the private bond market implies that $a_l(\theta^t) + a(\theta^t) = 0$ and $b(\theta^t) + b_l(\theta^t) = 0$, respectively, for all t and θ^t.

Second, up to this point, θ is a short-hand for (θ, x, b) and we need to map (s, x) into (θ, ω, b) or, equivalently, into (θ, a, b) or (θ, a_l, b_l), depending on whether we are referring to the sovereign or the Fund perspective. This map is given by the identification of the consumption policies and the Fund's consumption first-order condition:

$$
u_c(c(\theta, a, b)) = u_c(c(\theta, x, b_l)) = \frac{1 + \nu_l(\theta, x, b_l)}{1 + \nu_b(\theta, x, b_l)} \frac{1}{x}.
$$

This equality has three implications: *i*) since $u_c(c(\theta, a, b)) = \varkappa(\theta, a, b)$ the above equality also defines the Lagrange multiplier of the sovereign's budget constraint (2.14); *ii*) defines $c(\theta, a, b) = c(\theta, x, b_l)$, *i*), and *iii*) since the right hand side of the above equality is, by (2.8), equal to η / x' the law of motion of the co-state variable x maps into the borrower's Euler equation.

Third, we establish the correspondence between the initial conditions and participation constraints, between the Fund problem and the RCE. For the former, given (B.23) and (B.24) evaluated at $t = 0$, one can determine $\bar{a}'$ and b' using Definition 2.2, the budget constraint

$$
\begin{aligned}
c(\theta_0, a_0, b_0) + q_f(\theta_0, \omega_1)(\bar{a}' - \delta a_0) + \sum_{\theta_1|\theta_0} q_f(\theta_1, \omega_1(\theta_1)|\theta_0) \hat{a}'(\theta_1) + q_p(\theta_0, \bar{\omega}_1)(b' - \delta b_0) \\
\leq \theta_0 f(n) + (1 - \delta + \delta\kappa)(a_0 + b_0).
\end{aligned}
$$

and the fact that $\sum_{\theta_1|\theta_0} q_f(\theta_1, \omega_1(\theta_1)|\theta_0)\hat{a}'(\theta_1) = 0$. Once, $\bar{a}'$ and b' are determined, one can find the holdings of Arrow securities $\hat{a}'(\theta', \theta_0, a_0, b_0)$ for all $\theta' \in \Theta$. We can then retrieve the entire portfolio recursively for $t > 0$.

Now, given Lemma B.1, we can define the relative Pareto weight for which the sovereign's and the Fund's participation constraints bind in (θ, b) as $\underline{\underline{x}}(\theta)$ and $\overline{\overline{x}}(\theta, b)$, respectively. Then set the endogenous borrowing limits such that

$$\begin{aligned}
\mathcal{A}_b(\theta) &= a(\theta, \underline{\underline{x}}(\theta), b) + b(\theta, \underline{\underline{x}}(\theta), b), \\
\mathcal{A}_f(\theta, b) &= a_l(\theta, \overline{\overline{x}}(\theta, b), b) + b_l(\theta, \overline{\overline{x}}(\theta, b), b).
\end{aligned}$$

This definition implies that $a'(\theta', \theta, a, b) + b' \geq \mathcal{A}_b(\theta')$ and $a_l'(\theta', \theta, a, b) + b_l' \geq \mathcal{A}_f(\theta', b')$. Hence, the constructed asset holdings satisfy the competitive equilibrium constraints for both the lenders and the sovereign.

Therefore, if we identify $W^b(\theta, a, b) = V^b(\theta, s, b_l)$ we have shown that the Fund allocation of consumption, leisure and asset holdings is a solution to the sovereign's problem (2.13).

Fourth, we complete the mapping of policies and value functions. For the lenders, consumption is optimal if the asset portfolio is optimally determined. For this observe that

$$\begin{aligned}
q_f(\theta', \omega'(\theta')|\theta) &= \frac{1}{1+r}\pi(\theta'|\theta)\frac{u_c(c')}{u_c(c)}\eta\left[(1-\delta+\delta\kappa) + \delta\sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\right] \\
&\geq \frac{1}{1+r}\pi(\theta'|\theta)\left[(1-\delta+\delta\kappa) + \delta\sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\right] \\
&\quad \text{if } a'(\theta', \theta, a, b) + b' > \mathcal{A}_b(\theta'), \\
q_f(\theta', \omega'(\theta')|\theta) &= \frac{1}{1+r}\pi(\theta'|\theta)\left[(1-\delta+\delta\kappa) + \delta\sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\right] \\
&\geq \frac{1}{1+r}\pi(\theta'|\theta)\frac{u_c(c')}{u_c(c)}\eta\left[(1-\delta+\delta\kappa) + \delta\sum_{\theta''|\theta'} q_f(\theta'', \omega''(\theta'')|\theta')\right] \\
&\quad \text{if } a_l'(\theta', \theta, a, b) + b_l' > \mathcal{A}_f(\theta', b')
\end{aligned}$$

It we then identify $W^b(\theta,a,b) = V^b(\theta,x,b)$ and $W^p(\theta,a,b)+W^f(s,a,b) = V^l(s,x,b)$, the corresponding portfolios solve the private lenders' problem (2.17) and the decentralized Fund problem (2.20).

In sum, we obtain a map between (x,b) and $\omega = a+b$ for a given θ. More precisely, $B(\theta,x,b) = B(\theta,a,b)$, $c(\theta,a,b) = c(\theta,x,b)$, $c_p(\theta,a,b) = \tau_p(\theta,x,b)$, $c_f(\theta,a,b) = \tau_f(\theta,x,b)$, $c_p(\theta,a,b) + c_f(\theta,a,b) = \tau(\theta,x,b)$ and $n(\theta,a,b) = n(\theta,x,b)$. Moreover the endogenous limits of the sovereign and the Fund bind uniquely and exclusively when the participation constraints of the sovereign and the Fund bind, respectively. $\square$

Proof of Proposition 2.5

As we said, the starting point is the first-order condition of the sovereign's problem (2.13): $u_c(c(\theta,a,b)) = \varkappa(\theta,a,b)$, where $\varkappa(\theta,a,b)$ is the Lagrange multiplier of the budget constraint (2.14). From the sovereign's and Fund's Euler equations we obtain the following intertemporal relation between these multipliers:

$$\varkappa(\theta,a,b) = \eta\frac{1+\mathring{\nu}_b(\theta',a',b')}{1+\mathring{\nu}_l(\theta',a',b')}\varkappa'(\theta',a',b'), \tag{B.25}$$

where $\mathring{\nu}_b(\theta',a',b')$ and $\mathring{\nu}_l(\theta',a',b')$ are normalized Lagrange multipliers of the endogenous limit constraints (2.15) and (2.22).

The proof has two steps: first, we derive (B.25) and map it into the co-state x and, second, we map policies and value functions from RCE to the Fund's problem (B.1).

First, note that (B.19) and (B.22) can be read as the Euler equations of (2.13) and (2.20), where $\upsilon_b(\theta',a',b')$ and $\varphi_f(\theta',a'_l,b'_l)$ are the Lagrange multipliers of the endogenous constraints (2.15) and (2.22). Let $A(\delta) \equiv \left[(1-\delta+\delta\kappa) + \delta\sum_{\theta''|\theta'} q_f(\theta'',\omega''(\theta'')|\theta')\right]$, and define $\mathring{\nu}_b(\theta',a',b')$ and $\mathring{\nu}_f(\theta',a',b')$ by

$$\upsilon_b(\theta',a'_l,b'_l) = \frac{\pi(\theta'|\theta)\beta A(\delta)}{\varkappa'(\theta',a',b')}\mathring{\nu}_b(\theta',a',b') \text{ and } \varphi_f(\theta',a'_l,b'_l) = \frac{\pi(\theta'|\theta)A(\delta)}{(1+r)\varkappa'(\theta',a',b')}\mathring{\nu}_f(\theta',a',b').$$

Then, substituting the first-order condition $\varkappa(\theta, a, b) = u_c(c(\theta, a, b))$ into (B.19) and (B.22), these Euler equations read

$$q_f(\theta', \omega'(\theta')|\theta)\varkappa(\theta, a, b) = \pi(\theta'|\theta)\beta A(\delta)(1 + \mathring{\nu}_b(\theta', a', b'))\varkappa'(\theta', a', b') \tag{B.26}$$

$$q_f(\theta', \omega'(\theta')|\theta) = \pi(\theta'|\theta)\frac{A(\delta)}{1+r}(1 + \mathring{\nu}_l(\theta', a', b')), \tag{B.27}$$

where we have used the fact that by Proposition 2.2, $\varphi_l(\theta', a_l', b_l') = \varphi_f(\theta', a_l', b_l')$. Dividing (B.26) by (B.27) we obtain (B.25). Therefore, we can define

$$x = \frac{1 + \mathring{\nu}_l(\theta, a, b))}{1 + \mathring{\nu}_b(\theta, a, b)}\frac{1}{\varkappa(\theta, a, b)},$$

which also defines the map from state (θ, a, b) to state (s, x, b), as well as the policy identities: $c(s, x, b) = c(\theta, a, b)$, $n(s, x, b) = (\theta, a, b)$ and transfers by $\tau = \theta f(n) - c$.

Second, the split of transfers $\tau_p(s, x, b)$ and $\tau_f(s, x, b)$ is given by (2.12) and (2.11) and since the limited enforcement values, $V^{af}(\theta)$ and $\theta^- Z + b_l$ are already specified in the decentralized economy, it only remains to identify the value functions: $V^b(\theta, x, b_l) = W^b(\theta, a, b)$, $V^l(\theta, x, b_l) = W^p(\theta, a_l, b_l) + W^f(\theta, a_l, b_l)$, to finally define $FV(s, x, b_l) = xV^b(\theta, x, b_l) + V^l(\theta, x, b_l)$. It follows the allocation and assets of the RCE uniquely maps into the solution of the Fund problem (B.1).

□

Proof of Proposition 2.6

Assume that there exists an $-\underline{\underline{a}}(\theta) = \ddot{\bar{a}}' < 0$ where, according to Definition 2.3, in a given state, say $\ddot{\theta}'$, for an announcement $\bar{\omega}' + \hat{a}'(\theta', d_p')$,

$$V^{ap}(\ddot{\theta}, \ddot{\bar{a}}' + \hat{a}'(\ddot{\theta}, 1)) = W^b(\ddot{\theta}, \ddot{\bar{a}}' + \hat{a}'(\ddot{\theta}, 0), \bar{\omega}' - \ddot{\bar{a}}', 0),$$

while for the remaining $\theta' \in \Theta \setminus \ddot{\theta}'$ for which $\pi(\theta'|\theta) > 0$,

$$V^{ap}(\theta', \ddot{a}' + \hat{a}'(\theta', 1)) \leq W^b(\theta', \ddot{a}' + \hat{a}'(\theta', 0), \bar{\omega}' - \ddot{a}', 0).$$

As a result, if the Fund's MIP satisfies Definition 2.3, there is no *partial* default. Also, by monotonicity of the value function, if $\bar{a}' > \ddot{a}'$, then it is optimal for the sovereign to enter *partial* default in at least $\ddot{\theta}'$. □

Proof of Proposition 2.7

The first part of the proposition is a direct corollary of Proposition 2.3.

For the second part of the proposition, we want to show that, for all θ' for which $\pi(\theta'|\theta) > 0$ and for which the Fund's participation constraint does not bind,

$$V^{ap}(\theta', \ddot{a}' + \hat{a}'(\theta', 1)) = W^b(\theta', \ddot{a}' + \hat{a}'(\theta', 0), \bar{\omega}' - \ddot{a}', 0). \tag{B.28}$$

If this is true, then by a simple monotonicity argument, when $\bar{a}' > \ddot{a}'$,

$$V^{ap}(\theta', \bar{a}' + \hat{a}'(\theta', 1)) > W^b(\theta', \bar{a}' + \hat{a}'(\theta', 0), \bar{\omega}' - \bar{a}', 0).$$

Conversely, when $\bar{a}' \leq \ddot{a}'$,

$$V^{ap}(\theta', \bar{a}' + \hat{a}'(\theta', 1)) < W^b(\theta', \bar{a}' + \hat{a}'(\theta', 0), \bar{\omega}' - \bar{a}', 0).$$

Thus, the proof of the second part of the proposition relies on whether (B.28) holds with equality for all θ' for which $\pi(\theta'|\theta) > 0$ and for which the Fund's participation constraint does not bind.

In the Appendix 2.1, we show that the optimal consumption and leisure policies satisfy the first-order conditions of problem (B.1), $u_c(c) = \frac{1+\nu_l}{1+\nu_b}\frac{1}{x} = \frac{\eta}{x'}$, and $\theta f_n(n) = \frac{h_n(1-n)}{u_c(c)}$. As one can see, optimal consumption solely depends on x', while optimal labor depends on both x' and θ — and is therefore subject to change in *partial* default owing to the output penalty $\theta^d \leq \theta$ for the same x'.

Given this, we consider two cases: when labor is exogenous (i.e. $U(c, n) = U(c)$) and when it is endogenous.

We start with the case of exogenous labor. One way to make the borrower indifferent between repayment and *partial* default is to construct a path of consumption, say $\{c_t(\theta^t)\}_{t=0}^{\infty}$, which remains unchanged irrespective of the repayment decision in any state θ. Given the aforementioned first-order condition and the fact that labor is exogenous, the sequence $\{c_t(\theta^t)\}_{t=0}^{\infty}$ directly relates to a unique sequence of relative Pareto weight, say $\{x_t(\theta^{t-1})\}_{t=0}^{\infty}$. From Proposition 2.4, we know that there is a direct correspondence between $x_{t+1}(\theta^t)$ and $\omega_t(\theta^t)$ for all t and θ^t. Moreover, as long as the the Fund's participation constraint does not bind, $x_{t+1}(\theta^t)$ for any t and θ^t is independent of the level of private debt b_t as shown in Lemma B.1.

Assume that the Fund's participation constraint never binds. From $\{x_t(\theta^{t-1})\}_{t=0}^{\infty}$, we can find the underlying assets under *partial* default, say $\{a_t^{ap}(\theta^t)\}_{t=0}^{\infty}$. That is, for a specific $x_{t+1}(\theta^t)$, we have an underlying $a_t^{ap}(\theta^t) = \bar{a}_t^{ap} + \hat{a}(\theta^t, 1)$. Similarly, we can find the underlying assets under repayment, say $\{\omega_t(\theta^t)\}_{t=0}^{\infty}$, where for a specific $x_{t+1}(\theta^t)$ we have a corresponding $\omega_t(\theta^t) = \bar{a}_t + b_t + \hat{a}(\theta^t, 0)$. As the Fund's constraint does not bind, $x_{t+1}(\theta^t)$ is independent on the split of $\bar{\omega}_t$ between b_t and $\bar{a}_t$. Now observe that in a *partial* default at t, the sovereign reneges b_t and repays $\bar{a}_t$. That is, by entering *partial* default, for $b_t < 0$, the sovereign would end up with an indebtedness of $\bar{a}_t$ instead of $\bar{a}_t + b_t$ and an insurance of $\hat{a}(\theta^t, 1)$ instead of $\hat{a}(\theta^t, 0)$. Thus, if one sets $\bar{a}_t > \bar{a}_t^{ap}$ for all t, then entering *partial* default, the sovereign has a lower liability towards the Fund than $-\bar{a}_t^{ap}$ which corresponds to a larger relative Pareto weight than $x_{t+1}(\theta^t)$ and therefore a larger consumption than under repayment. In opposition, if one sets $\bar{a}_t = \bar{a}_t^{ap}$ and $b_t = \ddot{b}_t = \bar{\omega}_t - \bar{a}_t^{ap}$ for all t, $x_{t+1}(\theta^t)$ — and therefore $c_t(\theta^t)$ — remains the same irrespective of the repayment decision.[8] Most importantly, this holds true for any state θ as $\bar{a}_t$ and b_t are not state contingent.[9] There is therefore a perfect indifference in entering into *partial* default for any state, as repayment and *partial* default are related to the same sequence of relative Pareto weights for any t and θ^t.

[8]Note that $\bar{a}_t^{ap} > \bar{\omega}_t$ as we equalize the consumption in repayment and *partial* default and $b_t < 0$.

[9]Particularly, $\bar{\omega}' = \frac{\sum_{\theta'|\theta} q_f(\theta', a'(\theta', b', 0), b'|\theta)(a'(\theta', b', 0) + b')}{q_f(\theta, \bar{a}', b')}$ and $\bar{a}^{ap\prime} = \frac{\sum_{\theta'|\theta} q_f(\theta', a'(\theta', b', 1), 0|\theta) a'(\theta', 0, 1)}{q_f(\theta, \bar{a}', 0)}$.

We turn to the case of endogenous labor analyzed in the main text. Given the default penalty upon *partial* default, the same sequence of relative Pareto weight, $\{x_t(\theta^{t-1})\}_{t=0}^{\infty}$, would lead to the same consumption sequence but a different labor sequence between repayment and *partial* default. In other words, the sovereign would not anymore be indifferent between repayment and *partial* default. To correct this, we have to generate two sequences of relative Pareto weight — one for repayment, say $\{x_t^r(\theta^{t-1})\}_{t=0}^{\infty}$, and one for *partial* default, say $\{x_t^d(\theta^{t-1})\}_{t=0}^{\infty}$ — such that the sequence of instantaneous utility in repayment, say $\{U(c_t^r(\theta^t), n_t^r(\theta^t))\}_{t=0}^{\infty}$, exactly equates the sequence of instantaneous utility in *partial* default, say $\{U(c_t^d(\theta^t), n_t^d(\theta^t))\}_{t=0}^{\infty}$. As in the case of exogenous labor, we then use the correspondence between $x_{t+1}(\theta^t)$ and $\omega_t(\theta^t)$ given in Proposition 2.4 and apply the same reasoning as before with the only difference that we need to consider the two sequences of relative Pareto weight instead of one.

As a result, (B.28) holds with equality for all θ' for which $\pi(\theta'|\theta) > 0$ and for which the Fund's participation constraint does not bind meaning that the *partial* default decision — being optimal whenever $0 > \bar{a}' > \ddot{\bar{a}}'$ — is not state contingent when the DSA does not bind.

Given this, it holds that for all θ', and $\bar{a}'$ and b' such that $\bar{a}' \leq -\underline{\underline{a}}(\theta)$, $D_p(\theta', a', b') = 0$, and under Corollary 2.1, $D_f(\theta', a', b') = 0$. Moreover, when the Fund's participation constraint does not bind, for all θ' and for all $\bar{a}'$ and $b' < 0$ such that $\bar{a}' > -\underline{\underline{a}}(\theta)$, $D_p(\theta', a', b') = 1$ and $D_f(\theta', a', b') = 0$, which implies from (2.28) that for all θ and for all $\tilde{b}' < \bar{\omega}' + \underline{\underline{a}}(\theta)$, $q_p(\theta, \bar{a}', \tilde{b}') = 0$. □

Proof of Proposition B.1

The first order condition on consumption reads $u_c(c) = \frac{1+\nu_l}{1+\nu_b}\frac{1}{x}$. The law of motion of the relative Pareto weight is given by $x' = \frac{1+\nu_b}{1+\nu_l}\eta x$. Combining those two equations one obtains

$$x' = \frac{1+\nu_b(\theta, x, b)}{1+\nu_l(\theta, x, b)}\eta x = \frac{1}{u_c(c(\theta', x', b'))}\frac{1+\nu_l(\theta', x', b')}{1+\nu_b(\theta', x', b')}. \tag{B.29}$$

Moreover, observe that using the above first-order condition

$$\frac{1+\nu_b(\theta,x,b)}{1+\nu_l(\theta,x,b)}\eta x = \eta\left[\frac{1}{u_c(c(\theta,x,b))}\frac{1+\nu_b(\theta,x,b)}{1+\nu_l(\theta,x,b)}\frac{1+\nu_l(\theta,x,b)}{1+\nu_b(\theta,x,b)}\right] = \eta\frac{1}{u_c(c(\theta,x,b))}.$$

Hence, one can rewrite (B.29) as

$$\eta\frac{1}{u_c(c(\theta,x,b))} = \frac{1}{u_c(c(\theta',x',b'))}\frac{1+\nu_l(\theta',x',b')}{1+\nu_b(\theta',x',b')}.$$

Taking expectations on both sides with respect to θ' leads to

$$\eta\frac{1}{u_c(c(\theta,x,b))} = \mathbb{E}\left[\frac{1}{u_c(c(\theta',x',b'))}\frac{1+\nu_l(\theta',x',b')}{1+\nu_b(\theta',x',b')}\middle|\theta\right].$$

This equation is the inverse Euler equation. It gives the dynamic of consumption over time and therefore the extent of insurance. If none of the constraint ever binds and $\eta = 1$, then the contract achieves full insurance. However, whenever one of those two point is no true, consumption is not constant across states. Insurance is thus only partial in our environment. □

Proof of Lemma B.1

Recall that, in the detrended version of the model, the lower bound is defined by $\underline{x} = \min_{\gamma\in\Gamma}\{x : \widetilde{V}^b(\gamma,x,\tilde{b}) = \widetilde{V}^{af}(\gamma)\}$, while the upper bound corresponds to $\overline{x} = \max_{\gamma\in\Gamma}\{x : \widetilde{V}^b(\gamma,x,\tilde{b}) = \widetilde{V}^{af}(\gamma)\}$.

The key insight is to see that the sovereign's outside option is independent of the level of indebtedness, while the sovereign's value increases with the relative Pareto weight by definition. Assume now by contradiction that the lower bound $\underline{x}(\gamma,\tilde{b})$ is a function of γ and the level of debt $\tilde{b}$. That is for some $\ddot{b} \neq \tilde{b}$, $\underline{x}(\gamma,\tilde{b}) \neq \underline{x}(\gamma,\ddot{b})$. This implies that either $\widetilde{V}^b(\gamma,\underline{x}(\gamma,\tilde{b}),\tilde{b}) > \widetilde{V}^b(\gamma,\underline{x}(\gamma,\ddot{b}),\ddot{b})$ or $\widetilde{V}^b(\gamma,\underline{x}(\gamma,\tilde{b}),\tilde{b}) < \widetilde{V}^b(\gamma,\underline{x}(\gamma,\ddot{b}),\ddot{b})$ depending on which of the two relative Pareto weight is the largest. The former case leads to $\widetilde{V}^b(\gamma,\underline{x}(\gamma,\tilde{b}),\tilde{b}) > \widetilde{V}^{af}(\gamma)$, while the latter case leads to $\widetilde{V}^b(\gamma,\underline{x}(\gamma,\tilde{b}),\tilde{b}) < \widetilde{V}^{af}(\gamma)$. Both cases contradict the fact that $\underline{x}(\gamma,\tilde{b})$ is the relative Pareto weight for which the sovereign's constraint binds. It must therefore be that for all $\ddot{b} \neq \tilde{b}$, $\underline{x}(\gamma) = \underline{x}(\gamma, \tilde{b}) = \underline{x}(\gamma, \ddot{b})$. The same reasoning applies to the upper bound. $\square$

Proof of Lemma B.2

Under Proposition 2.2, define

$$q(\theta^t, \bar{\omega}(\theta^t)) \equiv \sum_{\theta^{t+1}|\theta^t} q_f(\theta^{t+1}, \omega(\theta^{t+1})|\theta^t) = q_p(\theta^t, \bar{\omega}(\theta^t)),$$
$$Q(\theta^t, \bar{\omega}(\theta^t)) \equiv \sum_{\theta^{t+1}|\theta t} Q_f(\theta^{t+1}, \omega(\theta^{t+1})|\theta^t) = Q_p(\theta^t, \bar{\omega}(\theta^t)),$$

for all t and θ^t. Furthermore, the transversality condition of the borrower is:[10]

$$\lim_{j\to\infty} \mathbb{E}_t Q(\theta^{t+j}, \omega(\theta^{t+j})|\theta^t)[a(\theta^{t+j}) + b(\theta^{t+j})] = 0,$$

where

$$Q(\theta^{t+j}, \omega(\theta^{t+j})|\theta^t) = Q(\theta^{t+j}, \omega(\theta^{t+j})|\theta^{t+j-1}) \cdots Q(\theta^{t+1}, \omega(\theta^{t+1})|\theta^t).$$

Using the borrower's budget constraint and the price relationship, one gets

$$\begin{aligned}&(a(\theta^t) + b(\theta^t))(1 - \delta + \delta\kappa + \delta q(\theta^t, \bar{\omega}(\theta^{t+1}))) = \\ &\qquad c(\theta^t, a(\theta^t), b(\theta^t)) + q(\theta^t, \bar{\omega}(\theta^{t+1}))a(\theta^{t+1}) + q(\theta^t, \bar{\omega}(\theta^{t+1}))b(\theta^{t+1}) - Y(\theta^t, a(\theta^t), b(\theta^t)),\end{aligned}$$

where, $Y(\theta^t, a(\theta^t), b(\theta^t)) = \theta(\theta_t) f(n(\theta^t, a(\theta^t), b(\theta^t)))$ for all t and θ^t. Iterating forward the budget constraint and using the transversality condition as well as the equilibrium price relationship, one obtains

$$a(\theta^t) + b(\theta^t) =$$

[10]The differentiability and strict concavity and convexity assumptions of the functional forms guarantee the local uniqueness of the policy and value functions. This in turn implies that the transversality conditions are satisfied.

$$\mathbb{E}_t \sum_{j=0}^{\infty} Q(\theta^{t+j}, \omega(\theta^{t+j})|\theta^t)[c(\theta^{t+j}, a(\theta^{t+j}), b(\theta^{t+j})) - Y(\theta^{t+j}, a(\theta^{t+j}), b(\theta^{t+j}))].$$

Similarly, the transversality condition of the lenders is:

$$\lim_{t\to\infty} \mathbb{E}_t Q(\theta^{t+1}, \omega(\theta^{t+1})|\theta^t)[a_l(\theta^{t+1}) + b_l(\theta^{t+1})] = 0.$$

Using the consolidated budget constraint of both lenders, one gets

$$\begin{aligned}
&(a_l(\theta^t) + b_l(\theta^t))(1 - \delta + \delta\kappa + \delta q(\theta^t, \bar{\omega}(\theta^{t+1}))) = \\
&\quad c_f(\theta^t, a(\theta^t), b(\theta^t)) + c_p(\theta^t, a(\theta^t), b(\theta^t)) + q(\theta^t, \bar{\omega}(\theta^{t+1}))a_l(\theta^{t+1}) + q(\theta^t, \bar{\omega}(\theta^{t+1}))b_l(\theta^{t+1}).
\end{aligned}$$

Note that we only consider the case in which $\bar{a}_p(\theta^t) = 0$. Iterating forward the budget constraint and using the transversality condition as well as the equilibrium price relationship, one obtains

$$\begin{aligned}
a_l(\theta^t) + b_l(\theta^t) &= \mathbb{E}_t \sum_{j=0}^{\infty} Q(\theta^{t+j}, \omega(\theta^{t+j})|\theta^t) c_l(\theta^{t+j}, a(\theta^{t+j}), b(\theta^{t+j})) \\
&= \mathbb{E}_t \sum_{j=0}^{\infty} Q(\theta^{t+j}, \omega(\theta^{t+j})|\theta^t)[Y(\theta^{t+j}, a(\theta^{t+j}), b(\theta^{t+j})) - c(\theta^{t+j}, a(\theta^{t+j}), b(\theta^{t+j}))] \\
&= a(\theta^t) + b(\theta^t).
\end{aligned}$$

The market clearing conditions in the Fund and the private bond market implies that $a_l(\theta^t) + a(\theta^t) = 0$ and $b(\theta^t) + b_l(\theta^t) = 0$, respectively, for all t and θ^t. □

Proof of Corollary B.1

Observe that if $\delta = 0$, the entire part of b matures today. Hence, following Proposition 2.3, if there is a sudden stop of funding from private lenders, $b' \geq 0$ meaning that the Fund's participation constraint becomes independent of the value of the debt held in the private bond market. We are therefore back to the standard case of Ábrahám et al. (2022).

In opposition, when $\delta > 0$, only a faction $1 - \delta$ of b matures today. Hence, if there is a sudden stop of funding from private lenders, $b' \geq \delta b$ for $b < 0$ following Proposition 2.3. As a result, the Fund's participation constraint depends on the value of the debt held in the private bond market. Moreover, the larger is $-b$, the tighter is the Fund's constraint. In other words, the more debt is held in the private bond market, the lower is the risk sharing provided by the Fund contract. In this case, $\delta = 0$ is the average maturity that maximizes the risk-sharing in the Fund contract.

Focusing on the steady state, Lemma B.1 states that the bounds of the ergodic set are independent of b. Hence, if (2.24) does not bind in steady state for $\delta > 0$, δ is irrelevant. □

2.5 Additional Details of the Calibration

2.5.1 Data Sources and Measurement

We calibrate the model for Italy. The main data sources and definitions of data variables are listed in Table B.1. The data frequency is quarterly, and the time periods are from 1992Q1 to 2019Q4, avoiding the interruption caused by COVID-19. Whenever the data souces contain the seasonally adjusted series for the relevant data variables, we use the them directly; otherwise, we seasonally adjust the data series using X11 algorithm with R package `seasonal`. For debt service and average maturity, we use annual series since quarterly ones are unavailable meanwhile we only need the sample avearge for our calibration.

To map the data to the model, we construct model consistent data measures as below.

Labor input For the aggregate labor input n_t, we use two series, the aggregate working hours H_t and the total employment E_t. We calculate the normalized labor input as $n_t = H_t/(E_t \times 5200)$, assuming 100 hours of allocatable time per worker per week. However, for second order data moment computations, we use H_t directly, since the per worker annual working hours do not show a significant cyclical pattern and both the level and the trend do not affect the computation of the moments.

Fiscal position and private consumption We hold the premise of fitting the *observed* fiscal behavior of Italy, so that we use directly the *data measures* of primary surplus to calibrate the

Table B.1: Data Sources and Definitions

Series	Sources	Unit
Output	ECB[a]	1 million 2010 constant Euro
Total working hours	ECB[b]	1 thousand hours
Employment	Eurostat[c]	1000 persons
Government debt	Eurostat[d]	end-of-quarter percentage
Debt service	AMECO[e]	end-of-year percentage of GDP, annual
Fiscal surplus	Eurostat, Bank of Italy[f]	million Euro
Long-term bond yields	Eurostat[g]	percentage, nominal
Debt maturity	OECD, EuroStat, ESM[h]	years, annual
Labor share	AMECO[i]	percentage, annual

[a] Real GDP, chain linked volume; data in 1991Q1–2014Q2 under ESA95, and data in 2014Q3–2019Q4 under ESA10, with the latter series adjusted to match the former in the overlapping periods 1995Q1–2014Q2.
[b] Hours for total employment; same adjustment to data under ESA95 and ESA10 as for output.
[c] Total employment (Eurostat label `lfsi_emp_q_h`).
[d] General government consolidated gross debt (Eurostat label `gov_10q_ggdebt`); quarterly series available for 2000Q1 onwards, and for 1992Q1-1999Q4, interpolate annual series instead; measured as end-of-quarter debt stock to total GDP of previous 4 quarters.
[e] AMECO (label `UYIGE`) for 1995–2015; European Commission *General Government Data* (GDD 2002) for 1992–1995.
[f] Eurostat (net lending, label `gov_10q_ggnfa`) 1999Q1–2019Q4; Bank of Italy (financing of the gross borrowing requirement, including privatization receipts) 1992Q1–1998Q4.
[g] EMU convergence criterion bond yields (label `irt_lt_mcby_q`).
[h] See text below; ESM data are obtained from private correspondance.
[i] Compensation of employees (UWCD) plus gross operating surplus (UOGD) minus gross operating surplus adjusted for imputed compensation of self-employed (UQGD), then divided by nominal GDP (UVGD).

model, and correspondingly, define the model consistent measure of consumption as the difference between output and primary surplus, since in the model, primary surplus ps is equal to output y minus consumption c. We have raw data on quarterly fiscal *surplus* instead of *primary surplus*. To arrive the latter from the former, we add back interest payment of the government to fiscal surplus. To be more precise, we first calculate fiscal suplus to GDP ratio (nominal quarterly GDP obtained from CEIC for Italy). Second, we obtain quarterly interest payment to GDP ratio from Eurostat (label `gov_10q_ggnfa`) for 1999Q1 onwards, and use the end-of-year annual value (obtained from AMECO and European Commission *General Government Data*) for each quarter in the year as a proxy for 1992Q1–1998Q4. Third, we add fiscal surplus to GDP and interest payment to GDP to arrive at primary surplus to GDP, and conduct seasonal adjustment to the series. And finally, we obtain the level of quarterly (*real*) primary surplus by multiplying the seasonally adjusted primary surplus to GDP ratio to (*real*) output in the same quarter.

Government debt, spread, and maturity Following Bocola et al. (2019) and Ábrahám et al. (2022), we calibrate the model to match the total public debt of Italy.

For the nominal risk free rate, we use the annualized short-term (3M) interest rates in the Euro money market (obtaied from EuroStat with label `irt_st_q`) for 1999Q1–2019Q4, and the annulized short-term (3M) bond return of Germany (obtained from EuroStat with label `irt_h_mr3_q`) for 1992Q1–1998Q4, before the start of Euro. To convert the nominal risk-free rate into real rate, we subtract GDP deflator of Germany from the former series. To arrive at a meaningful measure of the *real* spread, i.e., a spread unaffected by expected inflation hence rightly reflecting credit risk, we split the sample into to two parts. After the introduction of Euro, we can directly use the spread between the long-term nominal bond yields and the nominal risk-free rate, since all rates are denominated in Euro and thus subject to the same inflation expectation. For the period before Euro, we follow Ábrahám et al. (2022) and use spot and forward exchange rates (retrieved from Datastream) to convert the German nominal risk free rate into Italy's local currency, hence deriving a synthetic local currency risk free rate, and finally take the difference between the local nominal long-term bond yield with the synthetic risk free rate.

The information on the maturity structure of the government debt for Italy is not comprehensive. We manage to obtain government debt maturity data over 1990–2015 for Italy from all sources listed in Table B.1.

2.5.2 Estimation Results

Panel (a) of Figure B.1 plots the sample productivity series for Italy used for our calibration of the productivity shock process. It is clear that the during the 2008 Global Financial Crisis, there was prominent negative growth in productivities. This distinctive feature in the productivity dyanmics is also the main motivation for the use of Markov regime switching model (2.29) to calibrate the productivity shock. Correspondingly, Panel (b) shows that a 2-regime specification capture the crisis dynamics very well, with the smoothed regime probabilities reach almost 1 during the sudden drop periods observed in Panel (a).

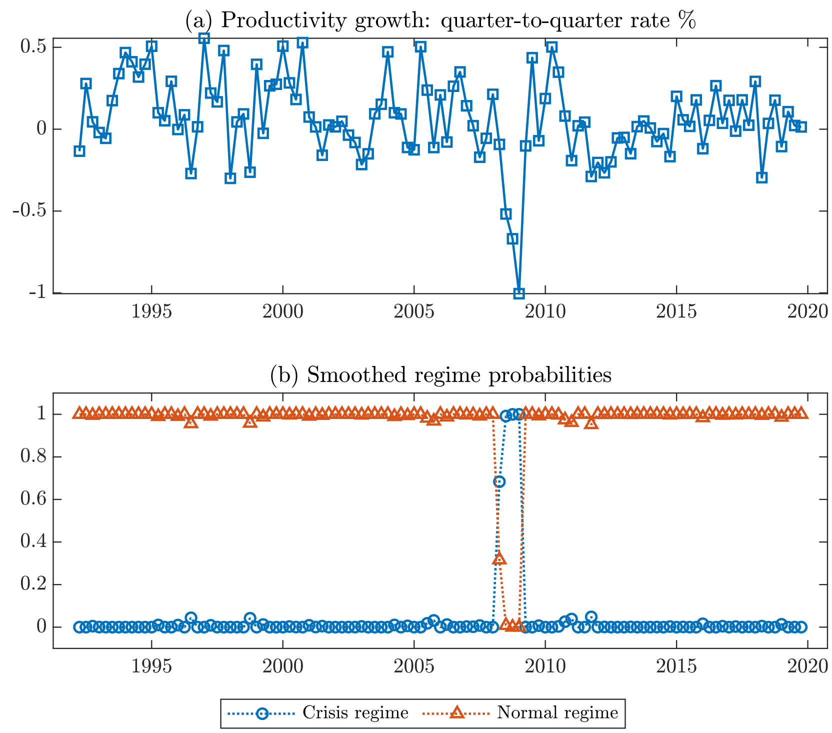


Figure B.1: Data sample and the estimated smoothed regime probabilities

The final estimation results are summarized in Table B.2. Note that we identify regime 1 as the crisis regime, and regime 2 as the normal regime. To overcome the local maximum problem of the highly nonlinear likelihood function, we randomize initializations of the EM algorithm of 1,000 times.

Table B.2: Parameters of the regime switching productivity process

	$\mu(\varsigma)$	$\rho(\varsigma)$	$\sigma(\varsigma)$	P	$\varsigma' = 1$	$\varsigma' = 2$	invariant dist.
$\varsigma = 1$	-0.0336	0.9018	0.0009	$\varsigma = 1$	0.6633	0.3367	0.0372
$\varsigma = 2$	0.0009	0.2167	0.0020	$\varsigma = 2$	0.0130	0.9870	0.9628

Notes: ς denotes the current regime of growth shock, and ς' denotes that of the next period. We consider two regimes, $\varsigma \in \{1, 2\}$, with transition matrix Π. $\varsigma = 1$ captures the period of the Great Financial Crisis. The regime-specific autocorrelation, mean and variance of the process are denoted by $\rho(\varsigma_t)$, $\mu(\varsigma_t)$, and $\sigma(\varsigma_t)$, respectively.

2.6 Welfare Calculations

This section describes how the welfare gains depicted in Table 2.3 are computed. Similar to Ábrahám et al. (2022), define value of the sovereign for a sequence $\{c(\theta^t), n(\theta^t)\}$ starting from an initial state at $t = 0$ as

$$V^b(\{c(\theta^t), n(\theta^t)\}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c(\theta^t), n(\theta^t)) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c(\theta^t)) + \gamma \frac{(1 - n(\theta^t))^{\sigma_n} - 1}{1 - \sigma_n} \right],$$

where the last equality is obtained from the functional form considered in Section 2.6. We denote the sovereign's allocations with the Fund by $\{c^f(\theta^t), n^f(\theta^t)\}$ and the allocations without the Fund by $\{c^i(\theta^t), n^i(\theta^t)\}$. The value for the borrower with and without the Fund is given by $W^{bf}(\theta, \omega) = W^{bf}(\{c^f(\theta^t), n^f(\theta^t)\})$ and $V^{bi}(\theta, b) = V^{bi}(\{c(^i\theta^t), n^i(\theta^t)\})$, respectively.[11] To properly compare the two economies, we consider the point where $\omega = b \equiv o$. Thus (θ, o) represents the initial state for both economies. Now define $V^{bi}(\theta, o; \chi) \equiv V^{bi}(\{(1 + \chi)c^i(\theta^t), n^i(\theta^t)\})$, where $\chi(\theta, o)$ represents the consumption-equivalent welfare gain of the Fund's intervention. It then directly follows that the welfare gain is computed in the following way $V^{bi}(\theta, o; \chi) = W^{bf}(\theta, o)$. Given the above functional form, we have that $\frac{\log(1+\chi)}{1-\beta} + V^{bi}(\theta, o) = V^{bf}(\theta, o)$. The welfare gain therefore boils down to $\chi(\theta, o) = \exp\left[(V^{bf}(\theta, o) - V^{bi}(\theta, o))(1 - \beta)\right] - 1$. We concentrate our analysis to the case in which $o = 0$.

Welfare decomposition

Following Ábrahám et al. (2022), we can decompose the welfare gains into four main components. As the Fund avoids default, it avoids the output penalty and the market exclusions. Those are the first two sources of welfare gains. In addition, as one can see from the two last columns of Table 2.3, the Fund enlarges the debt capacity of the sovereign. Finally, the Fund provides state-contingent transfer, whereas the economy without the Fund only has access to non-contingent bonds. Table B.3, presents the decomposition of the welfare gains for each of the depicted growth

[11] Note that in equilibrium $W^{bf}(\theta, a, b) \equiv W^{bf}(\theta, \omega)$.

states and zero initial debt. As one can see, the main source of welfare gains is the larger debt capacity followed by the state contingency and the circumvention of output penalty. Note that debt capacity and state contingency are closely linked one another. Without state-contingent transfers, the sovereign could not sustain a larger indebtedness.

Table B.3: Welfare Decomposition at Zero Initial Debt

State	No penalty (%)	Immediate return to market (%)	Greater debt capacity (%)	State-contingent insurance (%)
$\gamma = \gamma_{min}$	4.57	2.00	85.43	8.00
$\gamma = \gamma_{med}$	4.02	1.51	88.18	6.28
$\gamma = \gamma_{max}$	3.91	1.26	86.82	8.01

2.7 Additional Tables and Figures

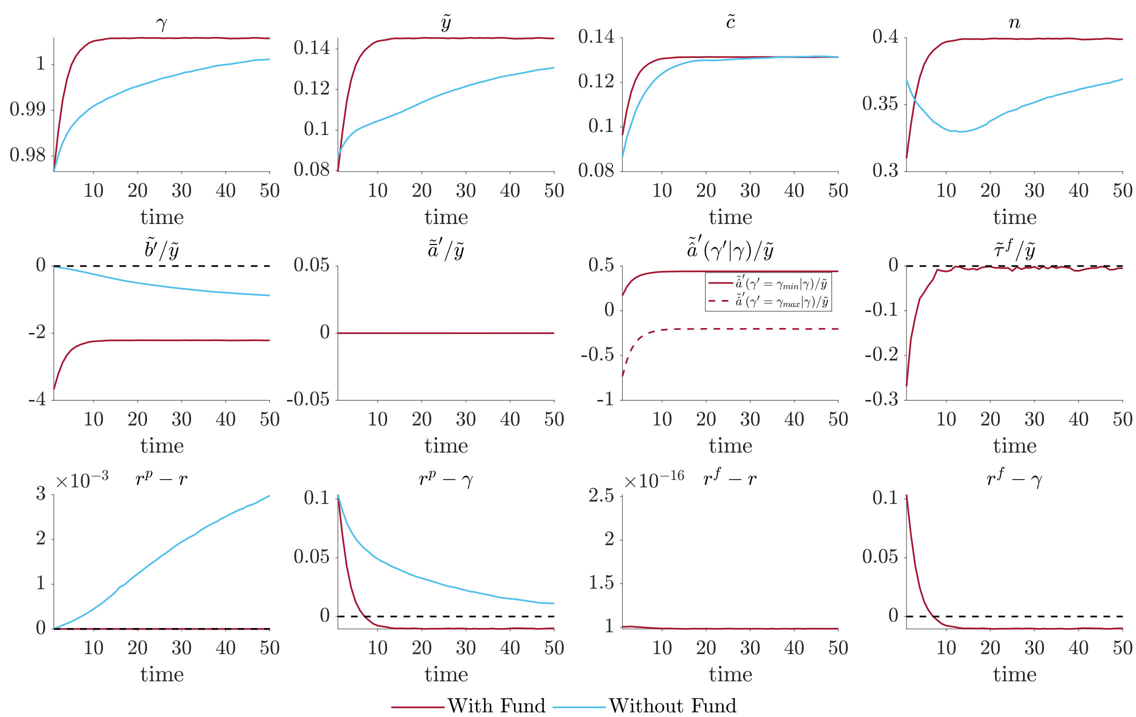


Figure B.2: Impulse Response Functions — Negative γ Shock

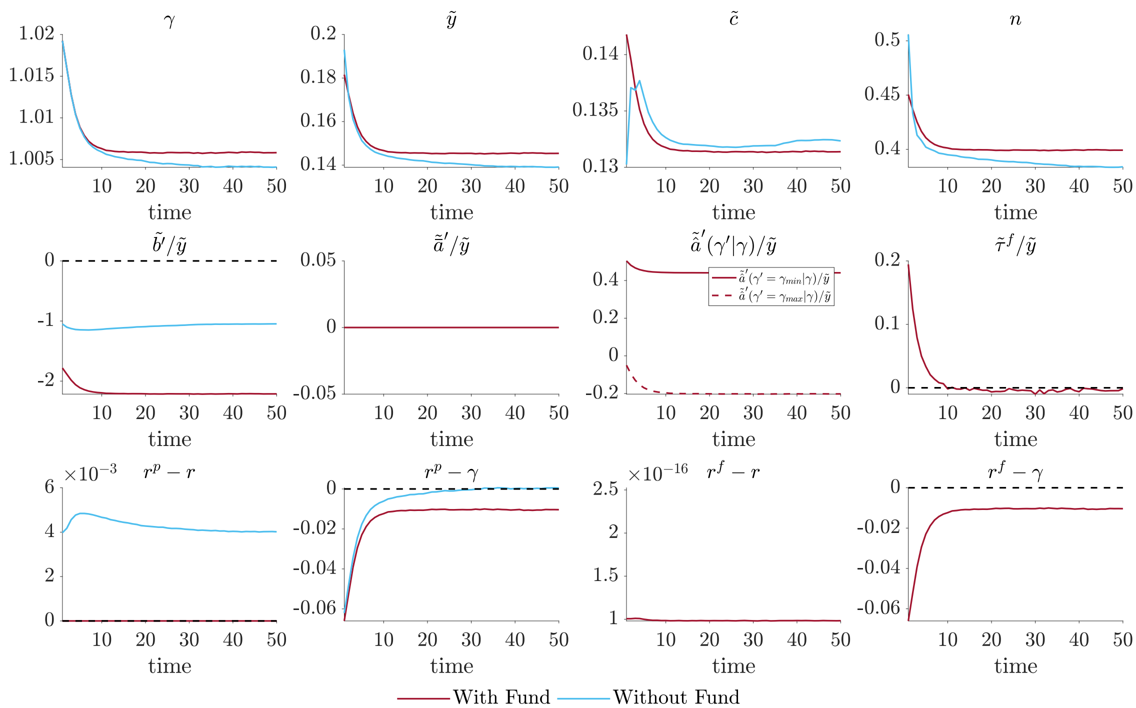


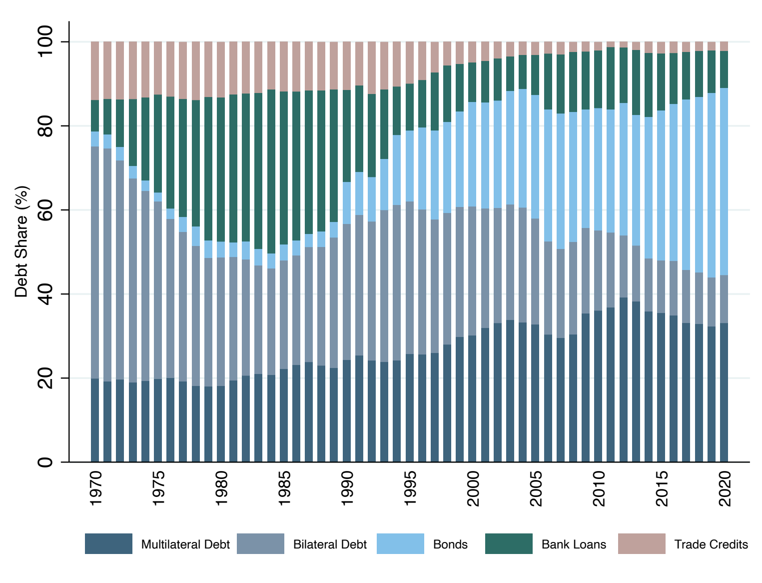
Figure B.3: Impulse Response Functions — Positive γ Shock

Appendix C

Appendix to Chapter 3

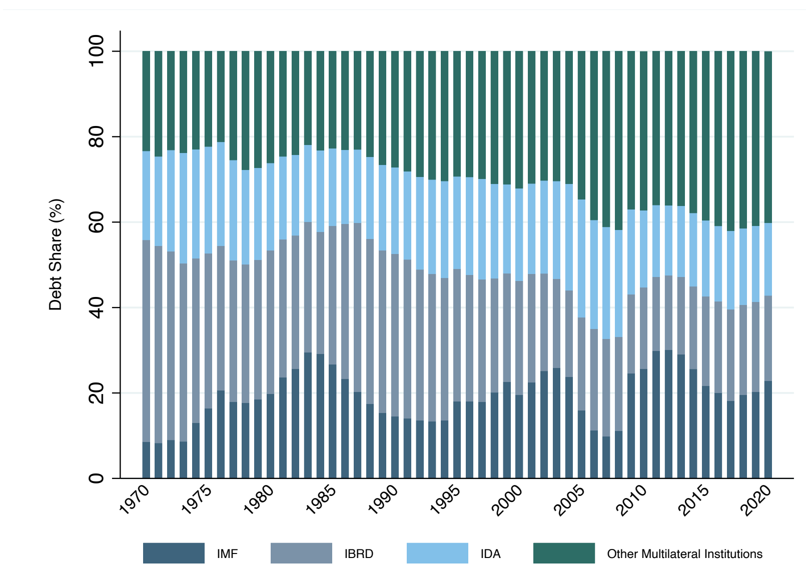
3.1 Additional Tables and Figures

This section presents additional tables and figures. Figure C.1 presents the composition of the sovereign debt excluding advanced economies. One observes significant changes over the years. While in the 1970s, bilateral loans represented the biggest share of the pie, it is now the smallest with bank loans and trade credits. In opposition, bonds which were rare in the 1970s are now the largest part of the sovereign debt. The switch appeared in the 1990s after the numerous defaults on bank loans especially in Latin American and the emergence of Brady bonds. The multilateral debt has always been important representing 20% of the total in the 1970s. It has followed a growing trend over the past decades and amounts now roughly 35% of the total sovereign debt.



Note: Multilateral debt refers to loans from official institutions such as the IMF, the IBRD, the IDA, regional development bank and other intergovernmental agencies. Bilateral debt refers to loans from other sovereign governments.
Source: Schlegl et al. (2019), WB, author's calculation.

Figure C.1: Structure of Sovereign Debt

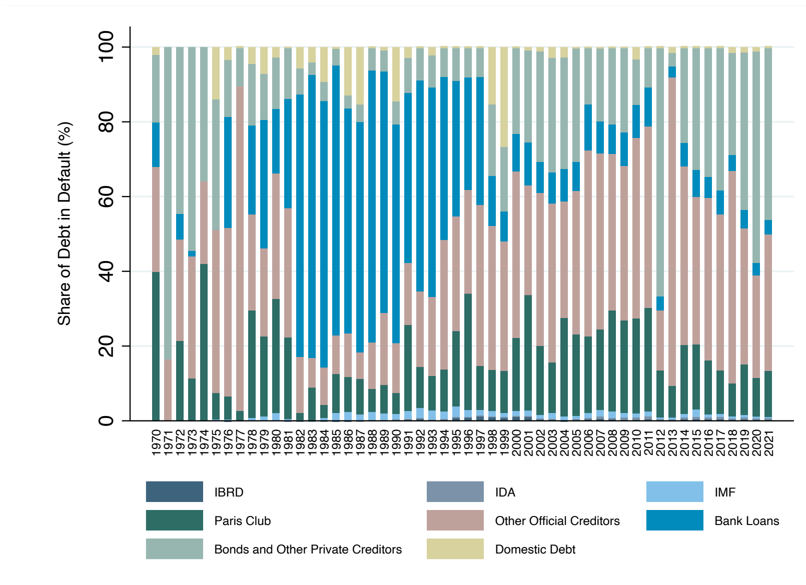


Note: Other Multilateral Institutions refer to loans from regional development bank and other intergovernmental agencies different from the IMF, the IBRD and the IDA.

Source: Schlegl et al. (2019), WB, author's calculation.

Figure C.2: Structure of Multilateral Sovereign Debt

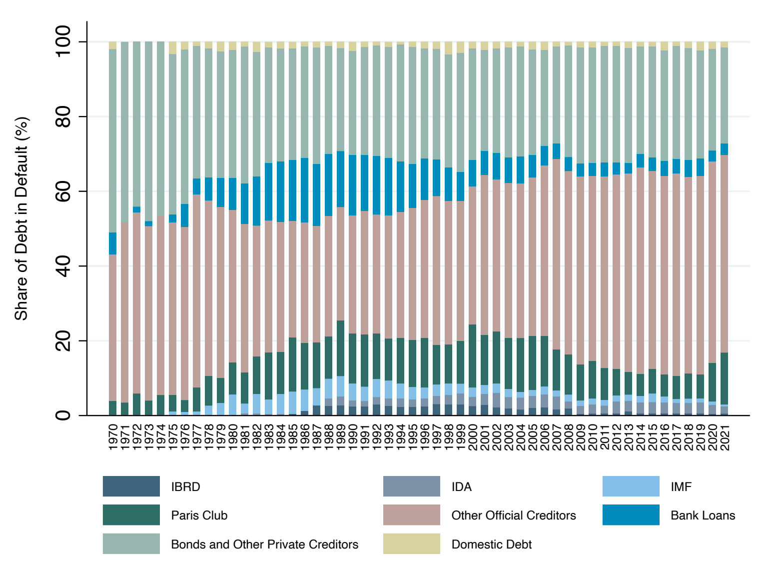
Figure C.2 presents the composition of the multilateral sovereign debt excluding advanced economies. Two main elements deserve to be noted. First, the share of debt held by the IMF and WB (i.e. the IBRD and the IDA) represents the majority of the total. Notably, one observes that the share of the IMF was the largest in the 1980s, while the WB has dominated the scene of multilateral lending until the beginning of the 21st century. For the IMF specifically, one sees a large drop of its share in the second half of the Great Moderation before rebounding with the Great Financial crisis of 2007-2008.



Source: Beers et al. (2022), author's calculation.

Figure C.3: Debt in Default by Creditors

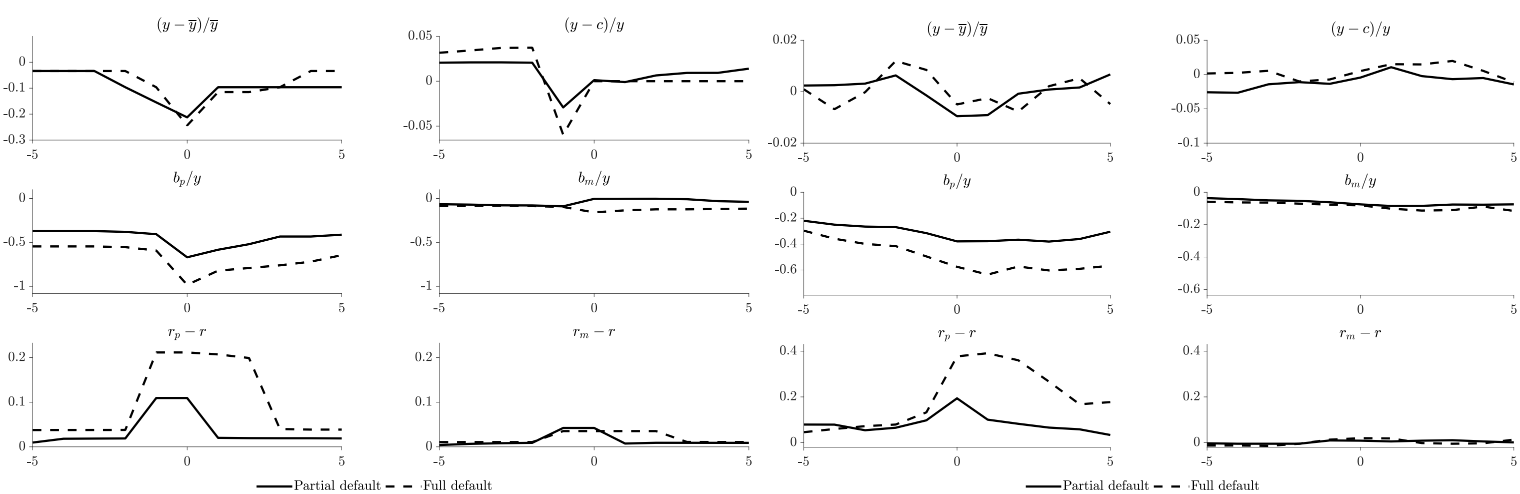
Figure C.3 presents the breakdown of debt in default by creditors. One directly sees that the IMF and the WB represent a negligible share throughout the entire sample. The two entities combined never represented more than 4% of the total amount of debt in default. This is however not the case for the Paris Club and the other official creditors which account for a large share of defaulted debt in the 1970s and in the last two decades depicted. Another large share of the pie goes to the private creditors especially in the 1980s through bank loans and in the 1990s-2000s through bonds.



Source: Beers et al. (2022), author's calculation.

Figure C.4: Countries in Default by Creditors

Figure C.4 presents the breakdown of countries in default by creditors. As in the previous figure, I note very few countries in default on the IMF and the WB. The two institutions combined never accounted for more than 11% of the countries in default. In opposition, the Paris Club and the other official creditors are involved once more in a big part of the defaults. The same holds true for private creditors.



(a) Model (b) Data

Figure C.5: Event Analysis – Median

Figure C.5 presents the event analysis described in Section 3.10 where I take the median instead of the average. The dynamic is similar to the one depicted in Figure 3.6 for both the model and the data.

3.2 Data

This section presents the different sources of data used in the empirical analysis and for the calibration of the model. Generally, there are three main sources of data.

First, for the duration of the default, I rely on the restructurings' dates contained in Asonuma and Trebesch (2016). A restructuring starts whenever a sovereign misses some payments beyond any contract-specified grace period, or if the sovereign undergoes renegotiations of the original debt contract.[1] Conversely, a restructuring ends with the official settlement announcement or the implementation of the debt exchange.[2]

Second, given the above default duration, I retrieve the creditors involved in each default by means of the database of Beers et al. (2022). The dataset specifies 9 types of foreign creditors: the IMF, the IBRD, the IDA, the Paris Club, China, other official creditors, banks, bondholders and other private creditors. I merge the IMF, the IBRD and the IDA together under the label of multilateral creditors. I also group China together with other official creditors. Finally, I add bondholders and other private creditors together.[3] I therefore end up with 5 dummies: multilateral creditors, Paris Club, other official creditors, bank loans and bonds and other private creditors. Table C.1 indicates the default episodes with multilateral lenders.

Finally, haircut statistics on private creditors are retrieved from Cruces and Trebesch (2013).[4] The database contains information about defaulted amounts and haircuts of defaults on external private debt from 1970 to 2014. I use two specifications of the haircut. The first one is the market

[1]This definition follows the one of Standard & Poor's (Beers and Chambers, 2006).

[2]This definition may differ from the one of Standard & Poor's which defines the end of a restructuring when a settlement occurs with no prospects of further resolutions (Beers and Chambers, 2006).

[3]Results do not significantly change if I consider those two categories separately.

[4]I use the database updated in 2014. In addition to revised computations, the update contains new default cases. Note that the haircut of Greece follows the estimation of Zettelmeyer et al. (2014).

haircut and is the one used by many financial institutions such as credit rating agencies as well as official lenders. The second one is computed according to Sturzenegger and Zettelmeyer (2008) and is becoming the standard in the empirical literature on sovereign defaults. The haircuts account for private creditors (i.e. bondholders and banks) and disregard official creditors (e.g. the IMF, the WB, the Paris Club).

Table C.1: Sample

Country	Default Start	Default End	Duration	SZ Haircut	Multilateral Creditor Default	Country	Default Start	Default End	Duration	SZ Haircut	Multilateral Creditor Default	Country	Default Start	Default End	Duration	SZ Haircut	Multilateral Creditor Default
Albania	01.11.1991	31.08.1995	3.8	80.4	No	Honduras	01.06.1981	01.10.1989	8.4	73.2	Yes	Philippines	01.07.1988	01.02.1990	1.7	42.8	No
Algeria	01.10.1990	01.03.1992	1.5	8.7	No	Honduras	01.06.1990	01.08.2001	11.3	82.0	Yes	Philippines	01.07.1990	01.12.1992	2.5	25.4	No
Algeria	01.12.1993	17.07.1996	2.7	23.5	No	Iraq	01.09.1986	01.01.2006	19.4	89.4	Yes	Poland	01.03.1981	06.04.1982	1.2	40.6	No
Argentina	01.07.1982	27.08.1985	3.2	30.3	No	Jamaica	01.06.1977	01.09.1978	1.3	2.2	No	Poland	01.01.1982	04.11.1982	0.9	62.9	No
Argentina	01.08.1985	21.08.1987	2.1	21.7	No	Jamaica	01.05.1978	01.04.1979	1.0	3.5	No	Poland	01.12.1982	04.11.1983	1.0	52.5	No
Argentina	01.01.1988	07.04.1993	5.3	32.5	No	Jamaica	01.03.1980	20.06.1981	1.3	15.2	Yes	Poland	01.12.1983	13.07.1984	0.7	26.9	No
Argentina	01.11.2001	10.06.2005	3.7	76.8	Yes	Jamaica	01.06.1983	01.06.1984	1.1	18.1	Yes	Poland	01.01.1986	01.09.1986	0.8	37.5	No
Belize	02.08.2006	20.02.2007	0.6	23.7	No	Jamaica	01.07.1984	01.09.1985	1.3	31.7	No	Poland	01.10.1986	20.07.1988	1.8	24.4	No
Belize	31.08.2012	01.03.2013	0.7	31.5	No	Jamaica	01.09.1986	07.05.1987	0.8	32.8	Yes	Poland	01.08.1988	01.07.1989	1.0	12.0	No
Bolivia	01.09.1980	17.03.1988	7.6	92.7	Yes	Jamaica	01.01.1990	26.06.1990	0.5	44.0	No	Poland	01.10.1989	27.10.1994	5.1	49.0	No
Bolivia	01.04.1988	01.04.1993	5.1	76.5	Yes	Jordan	01.02.1989	23.12.1993	4.9	54.6	No	Rep. Of Congo (Brazzaville)	01.06.1983	27.02.1988	4.8	42.3	No
Bosnia & Herzegovina	01.06.1992	09.12.1997	5.6	89.6	Yes	Kenya	01.01.1992	02.06.1998	6.5	45.7	No	Rep. Of Congo (Brazzaville)	01.03.1988	14.12.2007	19.8	90.8	Yes
Brazil	01.12.1982	25.02.1983	0.3	-9.8	No	Liberia	01.11.1980	01.12.1982	2.2	35.7	No	Romania	01.09.1981	07.12.1982	1.3	32.9	Yes
Brazil	01.01.1983	27.01.1984	1.1	1.7	No	Liberia	01.12.1981	01.04.2009	27.4	97.0	Yes	Romania	01.01.1983	20.06.1983	0.5	31.7	No
Brazil	01.06.1984	05.09.1986	2.3	19.2	No	Macedonia	01.05.1992	26.03.1997	4.9	34.6	Yes	Romania	01.06.1986	01.09.1986	0.3	12.3	Yes
Brazil	01.09.1986	11.11.1988	2.3	18.4	No	Madagascar	01.05.1981	01.11.1981	0.6	19.0	No	Russia	01.08.1991	01.12.1997	6.4	26.2	No
Brazil	01.06.1989	20.11.1992	3.5	27.0	No	Madagascar	01.06.1982	25.10.1984	2.4	41.3	No	Russia	17.08.1998	07.05.1999	0.8	46.0	No
Brazil	01.06.1989	15.04.1994	4.9	29.3	No	Madagascar	01.06.1985	15.06.1987	2.1	13.7	No	Russia	20.11.1998	25.08.2000	1.8	50.8	No
Bulgaria	01.03.1990	29.06.1994	4.3	56.3	No	Madagascar	01.06.1987	10.04.1990	2.9	52.7	No	Russia	20.04.1999	03.02.2000	0.9	51.5	No
Cameroon	01.06.1985	01.08.2003	18.3	85.5	No	Malawi	12.07.1982	06.03.1983	0.8	28.5	No	Senegal	01.05.1981	01.02.1984	2.8	28.8	No
Chile	01.01.1983	01.11.1983	0.9	0.7	No	Malawi	01.08.1987	04.10.1988	1.3	39.2	No	Senegal	01.06.1985	07.05.1985	0.1	31.3	No
Chile	01.01.1983	25.01.1984	1.1	8.4	No	Mauritania	01.06.1992	01.08.1996	4.3	90.0	No	Senegal	01.06.1990	28.09.1990	0.3	35.7	No
Chile	01.08.1984	14.04.1986	1.8	31.7	No	Mexico	01.08.1982	27.08.1983	1.1	-0.2	No	Senegal	01.06.1992	18.12.1996	4.6	92.0	No
Chile	01.10.1986	17.06.1987	0.8	14.3	No	Mexico	01.05.1984	29.03.1985	0.9	2.2	No	Serbia	01.06.1992	22.07.2004	12.2	70.9	Yes
Chile	01.04.1990	12.12.1990	0.8	17.0	No	Mexico	01.05.1984	29.08.1985	1.3	5.4	No	Seychelles	01.07.2008	11.02.2010	1.7	56.2	No
Costa Rica	15.07.1981	10.09.1983	2.3	39.4	No	Mexico	02.09.1986	01.03.1987	0.6	18.1	No	Sierra Leone	01.06.1980	01.08.1995	15.3	88.6	Yes
Costa Rica	01.10.1984	27.05.1985	0.7	35.6	No	Mexico	01.08.1987	01.03.1988	0.7	56.3	No	Slovenia	01.06.1992	12.03.1996	3.8	3.3	No
Costa Rica	01.05.1986	21.05.1990	4.1	71.9	No	Mexico	01.12.1988	04.02.1990	1.3	30.5	No	South Africa	01.09.1985	24.03.1987	1.6	8.5	No
Croatia	01.12.1991	31.07.1996	4.7	11.0	No	Moldova	01.06.2001	17.06.2004	3.1	56.3	No	South Africa	01.06.1989	18.10.1989	0.4	12.7	No
Cuba	01.09.1983	30.12.1983	0.3	42.9	No	Moldova	12.06.2002	29.10.2002	0.4	36.9	No	South Africa	01.01.1992	27.09.1993	1.8	22.0	No
Cuba	01.01.1984	24.12.1984	1.0	44.2	No	Morocco	25.08.1983	01.02.1986	2.6	23.5	No	St. Kitts & Nevis	01.06.2011	01.04.2012	0.9	62.9	No
Cuba	01.01.1985	19.09.1985	0.8	49.5	No	Morocco	22.10.1985	23.09.1987	2.0	21.3	No	Sudan	01.06.1975	01.10.1985	10.4	54.6	Yes
Côte d'Ivoire	01.06.1983	01.03.1998	14.8	62.8	No	Morocco	01.02.1989	01.09.1990	1.7	40.3	No	São Tomé and Príncipe	01.06.1984	01.08.1994	10.3	90.0	No
Côte d'Ivoire	01.03.2000	16.04.2010	10.2	55.2	Yes	Mozambique	01.06.1983	27.12.1991	8.6	90.0	No	Tanzania	01.06.1981	01.01.2004	22.7	88.0	Yes
Côte d'Ivoire	31.01.2011	12.11.2012	1.9	6.1	No	Mozambique	01.03.1993	01.09.2007	14.6	91.0	No	Togo	01.06.1987	01.05.1988	1.0	46.0	No
Dem. Rep. of Congo (Kinshasa)	01.06.1975	12.04.1980	4.9	29.6	Yes	Nicaragua	01.09.1978	01.12.1980	2.3	26.1	No	Togo	01.06.1991	01.12.1997	6.6	92.3	No
Dem. Rep. of Congo (Kinshasa)	01.04.1982	29.01.1983	0.8	38.2	Yes	Nicaragua	01.06.1981	01.12.1981	0.6	48.5	No	Trinidad & Tobago	01.09.1988	20.12.1989	1.3	15.5	No
Dem. Rep. of Congo (Kinshasa)	01.02.1983	01.06.1984	1.4	30.1	Yes	Nicaragua	01.06.1982	01.03.1982	-0.2	56.3	No	Turkey	01.12.1976	01.06.1979	2.6	22.2	No
Dem. Rep. of Congo (Kinshasa)	01.09.1984	01.05.1985	0.8	37.0	No	Nicaragua	01.03.1983	01.02.1984	1.0	41.7	Yes	Turkey	02.12.1976	22.08.1979	2.8	19.5	No
Dem. Rep. of Congo (Kinshasa)	01.06.1985	01.05.1986	1.0	35.4	No	Nicaragua	01.04.1985	01.11.1995	10.7	92.0	Yes	Turkey	01.01.1981	01.08.1981	0.7	8.6	No
Dem. Rep. of Congo (Kinshasa)	01.06.1986	20.05.1987	1.0	26.8	No	Nicaragua	01.01.1995	01.12.2007	13.0	95.5	No	Turkey	01.01.1981	13.03.1982	1.3	17.0	No
Dem. Rep. of Congo (Kinshasa)	01.06.1987	01.06.1989	2.1	50.6	Yes	Niger	01.06.1983	09.03.1984	0.8	37.4	No	Uganda	01.06.1979	26.02.1993	13.8	88.0	No
Dominica	01.07.2003	15.06.2004	1.0	54.0	No	Niger	01.06.1984	01.04.1986	1.9	45.8	No	Ukraine	12.08.1998	21.09.1998	0.2	11.8	No
Dominican Republic	01.06.1982	24.02.1986	3.8	49.9	No	Niger	01.06.1986	08.03.1991	4.8	82.0	No	Ukraine	12.08.1998	20.10.1998	0.3	14.7	No
Dominican Republic	01.06.1987	30.08.1994	7.3	50.5	No	Nigeria	01.08.1982	01.07.1983	1.0	2.1	No	Ukraine	18.05.1999	20.08.1999	0.3	-8.3	No
Dominican Republic	01.04.2004	11.05.2005	1.2	4.7	No	Nigeria	01.08.1982	01.09.1983	1.2	1.2	No	Ukraine	10.01.2000	07.04.2000	0.3	18.0	No
Dominican Republic	01.08.2004	18.10.2005	1.3	11.3	No	Nigeria	01.10.1983	01.04.1984	0.6	-2.8	No	Uruguay	01.01.1983	29.07.1983	0.6	0.7	No
Ecuador	08.10.1982	14.10.1983	1.1	6.3	No	Nigeria	01.01.1986	23.11.1987	1.9	19.3	No	Uruguay	01.04.1985	10.07.1986	1.3	24.3	No
Ecuador	01.12.1983	09.08.1984	0.8	5.7	No	Nigeria	01.10.1987	01.01.1988	0.3	41.5	No	Uruguay	01.05.1987	04.03.1988	0.9	20.3	No
Ecuador	01.08.1984	11.12.1985	1.4	15.4	No	Nigeria	01.03.1988	01.06.1989	1.3	30.1	No	Uruguay	01.07.1989	31.01.1991	1.6	26.3	No
Ecuador	01.08.1986	28.02.1995	8.6	42.2	No	Nigeria	01.06.1989	20.12.1991	2.6	40.1	No	Uruguay	11.03.2003	29.05.2003	0.3	9.8	No
Ecuador	28.01.1999	23.08.2000	1.7	38.3	No	Pakistan	01.07.1998	12.12.1999	1.5	11.6	No	Venezuela	01.03.1983	27.02.1986	3.0	9.9	No
Ecuador	14.11.2008	03.06.2009	0.7	67.7	No	Pakistan	30.01.1999	13.12.1999	1.0	15.0	No	Venezuela	24.04.1986	18.09.1987	1.5	4.3	No
Ethiopia	01.06.1990	16.01.1996	5.7	92.0	No	Panama	01.11.1984	01.10.1985	1.0	12.0	No	Venezuela	12.01.1989	05.12.1990	2.0	36.7	No
Gabon	15.09.1986	01.12.1987	1.3	7.9	No	Panama	01.03.1987	01.08.1994	7.5	15.1	Yes	Vietnam	01.01.1982	05.12.1997	16.0	52.0	Yes
Gabon	01.06.1989	16.05.1994	5.0	16.2	No	Panama	01.03.1987	17.04.1996	9.2	34.9	Yes	Yemen	01.06.1983	01.02.2001	17.8	97.0	No
Gambia	01.06.1984	15.02.1988	3.8	49.3	Yes	Paraguay	01.01.1986	01.07.1993	7.6	29.2	No	Yugoslavia	01.01.1983	09.09.1983	0.8	6.5	No
Greece	01.07.2011	13.03.2012	0.8	64.6	No	Peru	01.03.1976	01.12.1978	2.8	-7.2	No	Yugoslavia	01.09.1983	16.05.1984	0.8	-7.5	No
Grenada	01.10.2004	16.11.2005	1.2	33.9	No	Peru	01.09.1979	01.01.1980	0.4	-4.6	No	Yugoslavia	01.06.1984	18.12.1985	1.6	14.5	No
Guinea	01.06.1985	20.04.1988	2.9	26.1	No	Peru	01.03.1983	01.07.1983	0.4	6.3	No	Yugoslavia	01.07.1987	21.09.1988	1.3	19.7	No
Guinea	01.06.1991	01.12.1998	7.6	87.0	No	Peru	01.06.1984	07.03.1997	12.8	63.9	Yes	Zambia	07.01.1983	14.09.1994	11.8	89.0	Yes
Guyana	01.06.1982	24.11.1992	10.5	89.2	Yes	Philippines	01.10.1983	01.04.1986	2.6	42.6	No						
Guyana	01.01.1993	01.12.1999	7.0	91.0	No	Philippines	01.09.1986	01.12.1987	1.3	15.4	No						

With the above data, I obtain a dataset containing the start and the end date of each default in months with the underlying haircut on private creditors and that for a total of 187 default episodes between 1970 and 2014. Furthermore, for each default episode, I identify which types of creditor is involved. I find that overall 33 default episodes involve multilateral creditors. That is the country defaulted on such creditors.[5] Table C.1 depicts the sample used in the analysis.

I complement my datasets with other data presented in Table C.2. First, I use UN data for national accounting statistics. For many of the countries covered in my analysis the default's start coincides with a major political revolution (e.g. Yemen), a civil war (e.g. Liberia and Ethiopia), an independence or a dismantlement (e.g. former Yugoslavia). The UN keeps track record of the different political entities and their evolution. Hence, compared to the WB's WDI database it is possible to obtain data on former political entities.

Second, statistics on the countries' external debt comes mainly from the WB's WDI and IDS. The WB provides a breakdown of debt by creditor types: multilateral, bilateral and private. However, the time and geographic coverage is imperfect. Regarding private debt, complementing the dataset with the IMF's historical public debt database of Abbas et al. (2010) does not fill all the missing values. Hence, I do not integrate such variable in the regression analysis. Regarding multilateral debt, I retrieve the level of IMF debt by means of the "use of IMF credit". The WB debt is simply formed by the sum of IBRD loans and IDA credits. Missing values are filled by the joint BIS-IMF-OECD-WB Statistics and newspaper articles from the New York Times archives.[6]

As the focus of the analysis is the IMF and the WB, it is important to account for their respective programs and projects financing in the sample countries. For this purpose, I extend the dataset of Dreher and Gassebner (2012) by means on the IMF MONA database and the WB Projects & Operations listing.[7] The two aforementioned authors propose three variables. The first one is a dummy taking value one if the sovereign is under a IMF's Structural Adjustment Facility (SAF)

[5]As noted by Cordella and Powell (2021), multilateral lenders do not identify these episodes as defaults but simply as arrears because they eventually expect full repayment. I nevertheless use the term default as it corresponds to a missed payment consistent with the definition of Cruces and Trebesch (2013).

[6]Multilateral debt in newspaper articles are for the following countries: Cuba (1983-1985), Iraq (1986-2006), Poland (1981-1994) and all the former republics of Yugoslavia (1983-1988 and 1992-1997).

[7]Link to the WB Projects & Operations listing is available here.

or Poverty Reduction and Growth Facility (PRGF) program for at least five months. The second variable is also a dummy taking value one if the sovereign is under a IMF's Stand-by Agreement (SBA) program for at least five months. I merge those two dummies together under the label of IMF program. Finally, the two authors propose a variables counting the number of WB's loans given for structural adjustment in effect for at least five months. I label this variable as WB adjustment loan.

Table C.2: Data Source

Item	Details	Source	Comments
National accounts	Nominal GDP (USD)	UN	Consistent with PWT and World Bank data
	Real GDP (2015 USD)		
	Real GDP per capita (2015 USD)		
	Real consumption (2015 USD)		
	Real exports of goods and services (2015 USD)		
	Real imports of goods and services (2015 USD)		
Debt	External debt stock (current USD)	World Bank	
	Debt service on external debt (current USD)		
	PPG, commercial banks (current USD)		
	PPG, bonds (current USD)		
	PPG, other private creditors (current USD)		
	PPG, IBRD (current USD)		
	PPG, IDA (current USD)		
	PPG, bilateral (current USD)		
	PPG, multilateral (current USD)		
	IBRD loans and IDA credits		
	Use of IMF credit (current USD)		
	External debt stock (% of GDP)	Abbas et al. (2010)	Covers some missing World Bank data
	IMF Loans (current USD)	Joint BIS, IMF, OECD and World Bank	Covers IMF debt missing data
	Multilateral Loans (current USD)	Joint BIS, IMF, OECD and World Bank	Covers multilateral debt missing data
	Multilateral debt	New York Times Archives	Covers remaining missing data for
			Cuba (1983-1985), Iraq (1986-2006),
			Poland (1981-1994) and all the former
			republics of Yugoslavia (1983-1988, 1992-1997)
EMBI+ spread		World Bank, Global Financial Data	Only available from 1997
IBRD lending rate		World Bank	Average across sample, when no rate
			available take 5-year Libor plus 1.5 spread
IDA service charge		World Bank	
IMF adjusted rate of charge		IMF	
Yield on 1-year US government bond	Real Yield curve rate	US Treasury	Only available from 1990
10-year US Treasury rate	10-year US Treasury constant maturity rate	Federal Reserve Bank of St. Louis	
US PCE inflation rate	Personal Consumption Expenditures inflation rate	Federal Reserve Bank of St. Louis	
Federal Funds rate		Federal Reserve Bank of St. Louis	
Haircut, private creditors	Preferred haircut (Sturzenegger and Zettelmeyer, 2008)	Cruces and Trebesch (2013)	With 2014 update
	Market haircut		
Creditors involved	Default by creditor groups	Beers et al. (2022)	
IMF program	IMF's SAF, PRGF or SBA program in effect for at least 5 months	Dreher and Gassebner (2012)	Extended to 2014 using IMF MONA database
World Bank adjustment loan	World Bank's adjustement loans in effect for at least 5 months		and World Bank Projects & Operations
Default duration	Default's start and end date (month and year)	Asonuma and Trebesch (2016)	
		Beers and Chambers (2006)	Definition of Standard & Poor's
War	Inter-state wars	Sarkees and Wayman (2010)	Dummy variables
	Intra-state wars		
Political Regime	Regime type (communist, dictatorship)	Bjørnskov and Rode (2020)	Dummy variables
	Regime change (election, postponed election, coup)		

Regarding the IMF's and WB's charged interest rate, I retrieve the IMF adjusted rate of charge and the IDA service charge directly from the IMF's and the WB's websites. For the IBRD lending rate, I gather the historical data on IBRD Statement Of Loans. I take the average rate over the entire set of loans. For loans which do not report interest rates, I take the 5-year Libor rate to which I add the standard front-end fee of 0.25%, the commitment fee of 0.25%, the contractual spread of 0.50% and the excess borrowing charge of 0.50%. Spreads are calculated as the rate charged minus 1-year US government bonds yield.

To control for the political situation of each sovereign I add two main sources of data. First, I use the database of Bjørnskov and Rode (2020) who propose a set of dummies to account for the type of and the change in political regimes. I would have liked to have a single variable controlling for the political risk. Unfortunately, the variables developed by Political Risk Services Group – which is the standard in the empirical literature and has the most comprehensive coverage – only starts in 1984 and does not cover all the countries in my sample. Finally I obtain dummies for the irruption of inter and intra-state wars using the database of Sarkees and Wayman (2010).

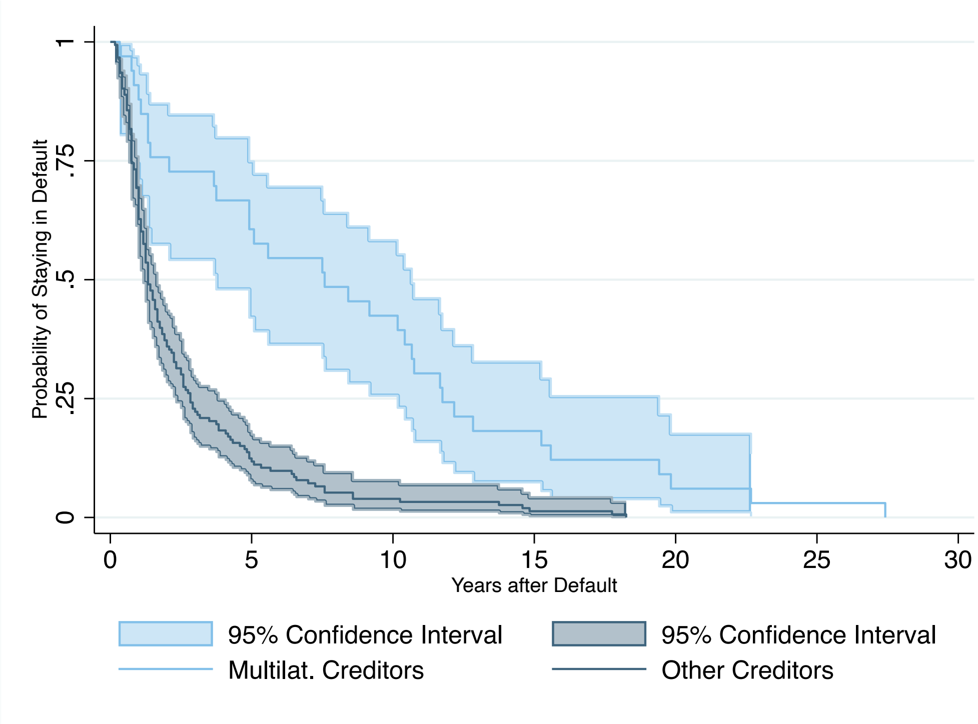
3.3 Regression Analysis

This section assesses the robustness of the empirical facts presented in Section 3.4. While Facts I and IV can be directly imputed to the multilateral creditors, Facts II and III might be associated to different factors.[8]

I start with Fact II and analyze the probability of remaining in default when defaulting on multilateral creditors. For this purpose, I conduct three analytical exercises. First, I estimate the survival function using a non-parametric estimator. Second, I conduct a cross-sectional analysis controlling for the default's and the country's specificities using an OLS estimator. Finally, I run a longitudinal analysis with similar control variables using a semi-parametric Cox proportional hazard model.

The non-parametric estimate of the survival function is presented in Figure C.6. It indicates a lower probability of leaving the default's state in the case of default on multilateral creditors. Most

[8]The following regression analyses are not necessarily causal.



Note: Kaplan-Meyer estimates of the unconditional survival function.

Figure C.6: Non-Parametric Survival Function

notably, default episodes without multilateral creditors have a 75% probability of successfully exiting the default state within 3 to 4 years, while for defaults with multilateral creditors this same probability amounts roughly 25%.

It is entirely plausible that some other factors that are at the source of lengthy defaults also explain the default on multilateral debt. That is why I estimate both an OLS and a semi-parametric proportional hazard model. Both models treat the default duration as functions of the types of creditor involved in the default alongside a number of control variables. For the OLS regression, I estimate the following equation

$$y_i^k = \alpha + \mathbf{D}_i\beta + \mathbf{X}_i\delta + v,$$

where i refers to a specific default episode, y is the default duration in years with $k \in \{\text{A\&P}, \text{S\&P}\}$ defined momentarily, $\mathbf{D}$ is a vector of 5 dummy variables accounting for the type creditors involved in the default (multilateral creditors, Paris Club, other official creditors, banks and bonds and other private creditors), $\mathbf{X}$ is a vector of controls, α is a constant and the remaining variable is the error term, v.

I consider two specifications for the default duration to ensure the robustness of my analysis. On the one hand, I take the definition Asonuma and Trebesch (2016) (i.e. A&T) which accounts for the duration of individual restructurings. On the other hand, I follow the definition of Standard & Poor's (i.e. S&P) which often aggregates restructurings together (Beers and Chambers, 2006).

For the choice of control variables I follow the literature on the determinants of default.[9] More precisely, I account for three sets of control variables. The first one relates to the specificity of the default episode and includes the total amount of debt defaulted, a dummy variable taking value one in case of a Brady deal and the private creditors' SZ haircut.

The second set of controls accounts for the economic condition of the country in default. I first add the standard control variables such as the debt held at the IMF as a share of GDP, the debt held at the WB as a share of GDP, the real GDP growth, the real GDP per capita growth, the net export per GDP, the inflation rate and the US Federal Funds Rate. Furthermore, I account for the trade openness of the economy by the sum of exports and imports as a share of GDP. Drawing on Reinhart and Rogoff (2004), I generate a dummy for serial defaulters taking value one if the country defaulted more than twice in the period under study. Finally, I introduce a dummy to account for whether the country is eligible for the HIPC or IDA programs following Allen (2008). Once a country enters such program, it becomes qualified for some automatic debt relief and other concessional actions. In a similar logic, Reinhart and Trebesch (2016) show that defaults often overlap with an IMF program. I therefore include a dummy taking values one if an IMF program (SAF, PRGF or SBA) is in effect for at least five months. Besides this, I introduce a variable counting the number of WB adjustment loans in effect for at least five months.

[9] See for instance Dell'Ariccia et al. (2006), Trebesch (2008), Cruces and Trebesch (2013), Asonuma and Trebesch (2016) and Asonuma and Joo (2020).

Table C.3: OLS Duration Regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	A&T	A&T	A&T	A&T	A&T	S&P	S&P	S&P	S&P	S&P
Multilateral Creditors	5.32***	3.31***	2.90***	2.98***	3.18***	6.66***	4.53***	3.77**	4.01**	5.27***
	[1.32]	[1.05]	[1.01]	[1.06]	[1.05]	[1.66]	[1.49]	[1.57]	[1.62]	[1.73]
Paris Club	0.85	-0.23	-0.29	0.13	0.09	-0.52	-0.82	-1.12	-0.82	-1.00
	[0.58]	[0.54]	[0.54]	[0.53]	[0.60]	[1.73]	[1.58]	[1.47]	[1.41]	[1.35]
Other Official Creditors	1.83***	0.25	-0.12	-0.25	-0.41	4.32**	1.65	1.34	1.27	1.75
	[0.59]	[0.54]	[0.64]	[0.66]	[0.72]	[1.79]	[1.57]	[1.78]	[1.72]	[2.11]
Bank Loans	-1.22	-3.09***	-2.78**	-4.03***	-4.16***	-2.00	-4.03**	-3.97*	-6.35***	-6.77***
	[0.85]	[1.12]	[1.09]	[1.33]	[1.46]	[1.86]	[1.86]	[2.05]	[2.13]	[2.06]
Bonds and Other Private Creditors	0.27	0.88	1.12*	1.27*	1.33*	1.19	0.41	-0.14	0.91	0.86
	[0.68]	[0.56]	[0.64]	[0.66]	[0.74]	[2.00]	[1.49]	[1.60]	[1.54]	[1.73]
Debt Restructured		0.00***	0.00**	0.00	0.00*		0.00*	0.00	0.00	-0.00
		[0.00]	[0.00]	[0.00]	[0.00]		[0.00]	[0.00]	[0.00]	[0.00]
SZ Haircut		0.08***	0.09***	0.10***	0.10***		0.09***	0.11***	0.11***	0.11***
		[0.01]	[0.02]	[0.02]	[0.02]		[0.02]	[0.03]	[0.03]	[0.03]
Brady Deal		2.79***	2.66***	2.45**	2.19**		3.21**	2.50*	2.65*	3.24*
		[1.04]	[0.95]	[0.97]	[1.03]		[1.46]	[1.36]	[1.57]	[1.68]
HIPC or IDA Eligibility			-1.62	-0.95	-0.57			-0.86	0.96	0.76
			[1.04]	[1.10]	[1.15]			[1.64]	[1.79]	[1.92]
Real GDP per Capita Growth, Start			-0.54	-0.70*	-0.82**			-1.05*	-1.33**	-1.35**
			[0.36]	[0.37]	[0.36]			[0.57]	[0.65]	[0.59]
Real GDP Growth, Start			0.50	0.69*	0.83**			0.92*	1.22*	1.31**
			[0.34]	[0.36]	[0.36]			[0.54]	[0.62]	[0.58]
Federal Fund Rate, Start			1.69***	1.85***	1.77***			2.08**	2.56***	1.93**
			[0.56]	[0.56]	[0.62]			[0.84]	[0.86]	[0.79]
Serial Defaulter			0.73	1.04	0.98			-1.64	-0.93	-1.46
			[1.03]	[1.03]	[1.03]			[1.25]	[1.27]	[1.35]
Inflation, Start			0.01	0.00	0.00			0.08*	0.06	0.07
			[0.02]	[0.02]	[0.02]			[0.04]	[0.04]	[0.04]
Trade Openness, Start			0.02*	0.02**	0.02**			0.02	0.02	0.02
			[0.01]	[0.01]	[0.01]			[0.02]	[0.02]	[0.02]
Net Exports (% GDP), Start			0.01	0.01	0.01			0.03	0.05	0.03
			[0.02]	[0.02]	[0.02]			[0.03]	[0.04]	[0.04]
IMF Program, Start				-1.49	-2.01				-1.00	-0.59
				[1.46]	[1.49]				[1.12]	[1.24]
WB Adjustment loans, Start				0.07	0.08				-0.11	-0.25
				[0.21]	[0.24]				[0.34]	[0.35]
IMF Debt (% GDP), Start				-0.03	-0.03				0.02	-0.07
				[0.06]	[0.06]				[0.10]	[0.10]
WB Debt (% GDP), Start				-0.14**	-0.17**				-0.29**	-0.34**
				[0.06]	[0.07]				[0.14]	[0.16]
Regime Change to Dictatorship, Start					-2.94					-5.70
					[2.30]					[3.99]
Dictatorial Regime, Start					-0.10					2.19
					[0.68]					[1.39]
War, Start					1.16					1.45
					[1.20]					[1.76]
Civil War, Start					-2.42*					-0.02
					[1.37]					[3.58]
Legislative Election, Start					-0.39					0.32
					[0.61]					[1.48]
Postponed Legislative Election, Start					2.77					5.32
					[1.80]					[3.40]
Coup, Start					0.65					2.16
					[1.23]					[2.18]
Communist Regime, Start					-0.36					-3.32
					[1.03]					[2.60]
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Region FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	187	187	187	187	187	104	104	104	104	104
R^2 adjusted	0.34	0.56	0.58	0.59	0.59	0.42	0.60	0.63	0.65	0.65

Note: *** $p < .01$, ** $p < .05$, * $p < .10$. Robust standard errors in brackets.

The last set of control variables accounts for the political situation of the country under default.

A default often coincides with a major political disruption or the outbreak of a war. Hence, I add a batterie of dummy variables accounting for such events. I control for the outbreak of inter and intra-state wars in the year of the default and the year preceding it using two dummies. For the political system, I add a set of dummy variables accounting for whether the defaulting country is a communist regime, whether it is a dictatorial regime, whether it changed to a dictatorial regime the year of the default or the year preceding it, whether there has been legislative elections or those elections have been postponed in the year of the default or the year preceding it and whether there has been a coup in the year of the default or the year preceding it.[10]

Finally, following Cruces and Trebesch (2013), I introduce year and region fixed effects. The latter accounts for the fact that defaults of Latin American countries have very different characteristics (including unobservables) compared to defaults in Europe or Asia. Conversely, the year fixed effects control for potential issues in the timing of restructuring as defaults often happen in waves (Reinhart and Rogoff, 2009).

The outcome of the OLS duration regressions is depicted in Table C.3. There is a strong and positive association between defaults with multilateral creditors and the length of the default duration. A default on multilateral debt is associated with a default's duration between 3 and 7 additional years depending on the model's specification. In opposition, the association between defaults on the Paris Club and the default's length is ambiguous as it reverses across the different specifications. The same holds true for other official creditors. Regarding private creditors, it seems that defaults on bank loans are settled more quickly.

Regarding the control variables, there are some significant results. First and foremost, the SZ haircut is positively associated with the default's duration. This stark relationship is consistent with the findings of Benjamin and Wright (2013). Besides this, countries involved in a Brady deal record a longer default's duration. However, there is mixed evidence regarding WB adjustment loans and HIPC or IDA programs, while IMF programs are associated with shorter default duration. Interestingly, the WB debt is related to a reduced default length, while there is almost no effect

[10] There is no transition to a communist regime in the sample at hand. I therefore do not include a dummy for that.

associated with the IMF debt.

I now turn to the Cox proportional hazard model. The major advantage of this model compared to an OLS regression is that it can integrate both constant and time-varying covariates. While the OLS specification relied on a cross-sectional structure of the data, the Cox model builds on longitudinal datasets. In other words, the latter can control for the evolution of the economic and political variables throughout the default's duration. More precisely, I estimate the following equation

$$g_i^k(t) = g_0^k(t)\exp(\mathbf{D}_i\beta + \mathbf{X}_i\delta),$$

where i refers to a specific default episode and t indicates the survival time (i.e. the time in default), $g^k(t)$ is the hazard function and g_0^k is the baseline hazard for $k \in \{\text{A\&P}, \text{S\&P}\}$. Using the duration jargon, a failure corresponds to the moment in which the country exits the default state. That is, the dependent variable is a dummy taking value 1 if the country exits the default and zero otherwise. The period of observation spans from the moment the country enters the default to the moments it exits. As I solely consider settled default episodes, there is no censoring.

In terms of controls, I use the same sort of variables as before. The major difference with the OLS regression is that most control variables are time-varying. The only exceptions are the IMF-debt-to-GDP ratio and the WB-debt-to-GDP ratio as the time series are incomplete for many countries. I therefore integrate those two variable as constant over time and add their value both at the beginning and at the end of the default episode. The other variables that are not time-varying are: the creditor's dummies, the HIPC or IDA eligibility and the SZ haircut. Note that the political dummies referring to legislative elections, postponed elections and coups take value one in the year of occurrence of such event and the year preceding it and zero otherwise. Finally, similar to the previous set of regressions, I introduce year and region fixed effects.

Note that the Cox model cannot account for defaults starting and ending in the same year as the failure coincides with the observation's start. I therefore lose 6 episodes for the S&P definition and 27 episodes for the A&T definition.

Table C.4: Cox Duration Regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	A&T	A&T	A&T	A&T	A&T	S&P	S&P	S&P	S&P	S&P
Multilateral Creditors	0.37***	0.52***	0.63**	0.63**	0.64*	0.31***	0.39***	0.42***	0.43***	0.40***
	[0.20]	[0.21]	[0.21]	[0.22]	[0.23]	[0.24]	[0.26]	[0.27]	[0.30]	[0.33]
Paris Club	0.82	0.94	0.95	0.81	0.82	0.65*	0.69	0.68	0.68	0.72
	[0.17]	[0.17]	[0.19]	[0.20]	[0.20]	[0.26]	[0.28]	[0.29]	[0.31]	[0.32]
Other Official Creditors	0.69**	0.93	1.05	1.08	1.08	0.67	0.85	0.78	0.88	0.90
	[0.19]	[0.23]	[0.25]	[0.23]	[0.24]	[0.32]	[0.38]	[0.43]	[0.41]	[0.43]
Bank Loans	1.23	2.21**	2.56**	2.56**	2.67***	1.03	1.54	1.47	1.85	1.93
	[0.30]	[0.37]	[0.40]	[0.39]	[0.36]	[0.35]	[0.43]	[0.43]	[0.46]	[0.44]
Bonds and Other Private Creditors	0.89	0.89	0.85	0.81	0.80	0.49***	0.71	0.89	0.66	0.57
	[0.16]	[0.18]	[0.17]	[0.16]	[0.17]	[0.27]	[0.31]	[0.34]	[0.33]	[0.36]
SZ Haircut		0.98***	0.97***	0.97***	0.97***		0.98***	0.98***	0.97***	0.97***
		[0.00]	[0.00]	[0.00]	[0.00]		[0.00]	[0.01]	[0.01]	[0.01]
Debt Restructured		1.00	1.00**	1.00***	1.00**		1.00	1.00	1.00	1.00
		[0.00]	[0.00]	[0.00]	[0.00]		[0.00]	[0.00]	[0.00]	[0.00]
Brady Deal		0.72	0.73	0.70	0.69		0.52**	0.48***	0.44***	0.45***
		[0.26]	[0.24]	[0.25]	[0.26]		[0.30]	[0.27]	[0.30]	[0.31]
HIPC or IDA Eligibility			0.91	0.89	0.81			0.71	0.48*	0.54
			[0.25]	[0.29]	[0.30]			[0.30]	[0.38]	[0.42]
Serial Defaulter			1.08	1.19	1.16			1.11	1.08	1.11
			[0.23]	[0.23]	[0.24]			[0.27]	[0.27]	[0.28]
Federal Funds Rate			0.01***	0.01***	0.01***			0.01***	0.01***	0.01***
			[0.06]	[0.06]	[0.07]			[0.07]	[0.08]	[0.09]
Real GDP per Capita Growth			1.03	1.02	1.03			1.01	0.97	0.97
			[0.04]	[0.04]	[0.04]			[0.06]	[0.06]	[0.07]
Real GDP Growth			0.98	0.98	0.98			0.99	1.02	1.02
			[0.04]	[0.04]	[0.04]			[0.06]	[0.06]	[0.07]
Net Exports (% GDP)			0.99	0.99	0.99			0.99*	0.99*	0.99
			[0.00]	[0.00]	[0.01]			[0.01]	[0.01]	[0.01]
Inflation			1.00	1.00	1.00			1.00	1.00	1.00
			[0.00]	[0.00]	[0.00]			[0.01]	[0.01]	[0.01]
Trade Openness			0.38***	0.31***	0.31***			0.54**	0.54**	0.54**
			[0.27]	[0.32]	[0.33]			[0.31]	[0.32]	[0.31]
IMF Program				1.54***	1.58***				2.29***	2.33***
				[0.14]	[0.15]				[0.25]	[0.27]
WB Adjustment loans				1.04	1.05				1.05	1.05
				[0.05]	[0.05]				[0.05]	[0.05]
IMF Debt (% GDP), Start				1.03**	1.03**				1.03*	1.03*
				[0.01]	[0.01]				[0.02]	[0.02]
WB Debt (% GDP), Start				0.99	0.99				0.99	0.98
				[0.02]	[0.02]				[0.02]	[0.03]
IMF Debt (% GDP), End				1.00	1.00				1.01	1.01
				[0.01]	[0.01]				[0.01]	[0.01]
WB Debt (% GDP), End				1.00	1.00				1.02	1.03
				[0.01]	[0.01]				[0.02]	[0.02]
Coup					1.30					1.67
					[0.32]					[0.61]
Communist Regime					1.21					1.21
					[0.24]					[0.46]
Dictatorial Regime					1.08					0.97
					[0.19]					[0.33]
Postponed Legislative Election					0.83					1.21
					[0.57]					[0.83]
Legislative Election					0.83					1.10
					[0.14]					[0.21]
War					0.77					2.25
					[0.74]					[0.70]
Civil War					1.04					0.77
					[0.22]					[0.35]
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Region FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	654	654	654	654	654	670	670	670	670	670
Episodes	160	160	160	160	160	98	98	98	98	98
Pseudo R^2	0.06	0.09	0.09	0.10	0.10	0.15	0.16	0.17	0.19	0.19

Note: *** $p < .01$, ** $p < .05$, * $p < .10$. Robust standard errors in brackets. Hazard ratios are reported.

The outcome of the Cox duration regressions is depicted in Table C.4. I find similar results as in the OLS estimation. Nevertheless, the interpretation of the coefficient is here different as I report the hazard ratios. An hazard ratio above one means that the variable is associated with a greater probability of exiting default, while a ratio below one indicates the opposite. As before, a default implicating multilateral creditors is related to a longer default. More precisely, such event is associated with a reduced probability of exiting default between 36% and 69% depending on the model's specification. Moreover, defaults involving the Paris Club and other official creditors seem to reduce the probability of exiting default, but the coefficients lack robustness. Regarding private creditors, it seems that defaults on bank loans are settled more quickly. However, the magnitude and the statistical significance of the coefficients vary a great deal across the different specifications.

Regarding the control variables, greater haircuts are associated with a reduced probability of exiting the default state. The same holds true for the HIPC or IDA eligibility and Brady deals. The IMF debt is associated to an increasing probability of ending default – at least at the default's start – while there is little effects related to the WB debt. Finally, the participation to both an IMF program or a WB adjustment loan are associated to an increasing probability of ending default. Only the former is economically and statistically significant, though.

In view of the results presented above, it seems that Fact II is relatively robust. Controlling for the specificity of each default episodes and the country's characteristics does not reduce the strong association between the default's length and multilateral creditors.

I now assess the robustness of Fact III. The aim is to gauge whether greater private creditors' losses are due to the presence of multilateral lenders or are simply a by-product of other factors. For this purpose, I conduct OLS regressions with similar controls and fixed effects as before. The equation, I estimate is the following

$$H_i^k = \mathbf{D}_i\beta + \mathbf{X}_i\delta + u_i,$$

where i refers to a specific restructuring episode, H_i^k is the haircut's specification of $k \in \{M, SZ\}$ defined below and the remaining variable is the error term, u_i.

Table C.5: Haircut Regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	H^M	H^M	H^M	H^M	H^M	H^{SZ}	H^{SZ}	H^{SZ}	H^{SZ}	H^{SZ}
Multilateral Creditors	15.19***	9.13*	9.05**	9.07*	9.60*	15.25***	7.97	7.68*	8.18*	8.93*
	[5.27]	[5.48]	[4.36]	[4.64]	[4.89]	[5.38]	[5.29]	[4.27]	[4.41]	[4.62]
Paris Club	8.37**	7.49*	7.97***	5.98**	5.71**	7.74*	6.89*	7.56**	5.66*	5.29*
	[4.04]	[3.97]	[3.04]	[2.84]	[2.84]	[4.09]	[3.90]	[3.09]	[2.97]	[2.91]
Other Official Creditors	9.26**	9.06*	10.15**	9.86**	11.83***	7.93*	7.82*	8.71*	8.20*	10.59**
	[4.64]	[4.79]	[4.55]	[4.15]	[4.32]	[4.62]	[4.70]	[4.59]	[4.22]	[4.49]
Bank Loans	21.53**	13.48	4.27	14.14*	15.56**	25.01***	14.94*	6.07	16.18**	18.02**
	[8.50]	[8.25]	[8.30]	[7.17]	[7.21]	[8.54]	[8.61]	[8.32]	[7.46]	[7.61]
Bonds and Other Private Creditors	-10.02**	-10.53**	-11.41***	-10.94***	-11.09***	-8.54*	-9.15**	-9.83**	-9.36**	-9.46**
	[4.46]	[4.62]	[4.29]	[4.12]	[4.22]	[4.63]	[4.61]	[4.42]	[4.43]	[4.54]
Private Debt Restructured		0.00	0.00	0.00*	0.00**		0.00	0.00	0.00	0.00*
		[0.00]	[0.00]	[0.00]	[0.00]		[0.00]	[0.00]	[0.00]	[0.00]
Default Duration		1.74***	1.35***	1.17**	1.04**		2.06***	1.70***	1.54***	1.38***
		[0.64]	[0.49]	[0.47]	[0.51]		[0.66]	[0.52]	[0.48]	[0.52]
Brady Deal		-9.85	-4.81	4.93	3.83		-15.96**	-10.94*	-1.27	-2.78
		[6.92]	[5.40]	[4.96]	[5.17]		[7.19]	[6.09]	[5.77]	[6.06]
HIPC or IDA Eligibility			26.54***	16.53***	13.99**			24.74***	14.41***	11.45**
			[5.10]	[5.05]	[5.50]			[5.30]	[5.35]	[5.69]
Serial Defaulter			8.45	5.38	3.96			9.30*	6.80	5.21
			[5.23]	[5.04]	[5.28]			[5.55]	[5.41]	[5.67]
Federal Funds Rate, End			-6.32***	-8.41***	-8.01***			-7.06***	-9.00***	-8.44***
			[1.80]	[1.71]	[1.82]			[1.81]	[1.83]	[1.89]
Real GDP Growth, End			-4.71**	-4.88**	-4.70**			-4.77**	-5.12**	-4.86**
			[1.90]	[1.92]	[1.95]			[1.96]	[1.97]	[2.00]
Real GDP per Capita Growth, End			5.36***	5.43***	5.27***			5.42***	5.67***	5.44***
			[1.88]	[1.92]	[1.92]			[1.94]	[1.96]	[1.96]
Net Exports (% GDP), End			-0.11	-0.04	-0.06			-0.13	-0.06	-0.08
			[0.12]	[0.10]	[0.10]			[0.12]	[0.10]	[0.10]
Inflation, End			0.19**	0.15*	0.17**			0.18**	0.14	0.17*
			[0.08]	[0.08]	[0.09]			[0.08]	[0.09]	[0.09]
Trade Openness, End			-0.16***	-0.16***	-0.15***			-0.14**	-0.12***	-0.12**
			[0.05]	[0.04]	[0.05]			[0.05]	[0.05]	[0.05]
IMF Program, End				0.35	1.09				0.49	1.32
				[2.83]	[2.85]				[3.05]	[3.08]
WB Adjustment loans, End				-1.50*	-1.53*				-1.05	-1.08
				[0.90]	[0.88]				[0.97]	[0.94]
IMF Debt (% GDP), End				-0.32*	-0.27				-0.44***	-0.39**
				[0.16]	[0.19]				[0.16]	[0.19]
WB Debt (% GDP), End				1.40***	1.40***				1.44***	1.42***
				[0.23]	[0.24]				[0.25]	[0.25]
Communist Regime, End					2.73					3.53
					[4.99]					[5.71]
Dictatorial Regime, End					2.25					2.40
					[3.69]					[3.75]
Legislative Election, End					-3.51					-4.55
					[3.10]					[3.19]
Postponed Legislative Election, End					-4.58					-2.26
					[7.20]					[8.40]
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Region FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	187	187	187	187	187	187	187	187	187	187
R^2 adjusted	0.46	0.50	0.64	0.70	0.70	0.44	0.49	0.61	0.68	0.68

Note: *** $p < .01$, ** $p < .05$, * $p < .10$. Robust standard errors in brackets.

I consider two specifications of the haircut. The first one is the market haircut, H^M, and is the one computed by rating agencies and official lenders. It however tends to overestimate the level of creditor's losses. That is why I consider a second haircut specification based on the estimation method of Sturzenegger and Zettelmeyer (2008), H^{SZ}.

In terms of controls, I add similar variables as for the previously exposed regressions. First, I control for the default's specificity by including the amount of private debt the country defaulted on, a dummy for the presence of a Brady deal and the default's duration in year. Second, I control for the economic situation of the country at the default's end using the same control variables as for the OLS duration regressions. Furthermore, I account for the political system of the economy at the moment of the restructuring. More precisely, I add a dummy controlling whether the country is a communist or a dictatorial regime as well as two dummies to control for legislative elections and postponed legislative elections in the year of the restructuring or the year preceding it. Finally, in accordance to what has been done previously, I introduce year and region fixed effects.

Table C.5 presents the results of the haircut regressions. The coefficient related to multilateral defaults is economically important. Defaulting on multilateral creditors is associated with an increase of the private creditors' haircut between 8 and 15 percentage points depending on the model's specification. However, the statistical significance is on average lower than in the previous regression analyses. Defaults involving the Paris Club creditors are also associated with larger haircuts. The same holds true for the other official creditors with a greater economic and statistical significance than for multilateral creditors. Regarding private creditors, defaults on bonds and other private creditors are associated with lower haircuts, while the opposite holds true for bank loans.

Regarding the control variables, one observes many significant results. Especially, the duration has a a strong and positive association with the private haircut. Once added to the regression, the effect attributed to the multilateral lenders decreases. Similarly, the HIPC or IDA eligibility have a strong and positive association with the private haircut. This was to be expected as such programs automatically provide substantial debt reliefs. Besides this, the coefficients attached to the real

GDP growth, the Federal Funds Rate and the trade openness are strongly and negatively associated with the private haircut. This indicates that better recovery of the economy tend to be associated with lower haircuts. Finally, the level of WB debt to GDP is positively associated with the haircut, while the opposite is true for the level of IMF debt to GDP. Note however that neither an IMF program nor a WB adjustment loan seem to significantly affect the haircut.

Hence, in view of those results, it seems that there is a link between private creditor's losses and the presence of multilateral lenders. Even though the statistical significance is less pronounced than for Fact II, the economic significance of this link is important and remains relatively stable across the different specifications.

3.4 Competitive Equilibrium

In this section, I define the competitive equilibrium. On the sovereign's side, the equilibrium is composed of two components. First, given the prices and the outcome of the renegotiation problem, the sovereign determines its repayment decision. Second, given the prices and the outcome of the repayment problem, the sovereign sets its restructuring decision. On the lenders' side, the equilibrium is governed by the break even assumption.

Definition C.1 (Recursive Equilibrium)**.** *A recursive equilibrium in this environment consists of*

- *Policy functions for the sovereign's consumption,* $c(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$*, private bond holdings,* $b_p(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$*, multilateral bond holdings,* $b_m(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$*, default,* $D^{DP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$ *and* $D^{DF}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$*, proposed settlement,* $W_b^{RP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$ *and* $W_b^{RF}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$*, and stopping functions,* $A^{DP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W)$ *and* $A^{DF}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W)$*.*
- *Policy functions for the lenders' proposed settlement,* $W_l^{RP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$ *and* $W_l^{RF}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$*.*
- *Price schedules for the multilateral debt,* $q_m(z, b_m^i, b_p^i)$*, and the private debt,* $q_p(z, b_m^i, b_p^i)$*.*

such that

1. *Taking the above prices as given,*
   (a) *and taking the solution to the renegotiation problem as given, the policy functions* $c(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$*,* $b_p(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$*,* $b_m(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$*,* $D^{DP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$ *and* $D^{DF}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$

solve the sovereign's repayment problem in (3.1)-(3.4).

(b) and taking the solution to the repayment problem as given, the policy functions $W_l^{RP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$, $W_b^{RP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$, $W_l^{RF}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$, $W_b^{RF}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)$, $A^{DP}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W)$ *and* $A^{DF}(z, \boldsymbol{\epsilon}, b_m^i, b_p^i, W$ *solve the sovereign's renegotiation problem in (3.5)-(3.9).*

2. *The price charged by the private and multilateral lenders correctly reflects the default probability and the expected recovery rate and is consistent with zero expected profit.*

3.5 Numerical Solution

In this section, I present the different value functions, policies and prices after taking the expectations over the utility shock $\boldsymbol{\epsilon}$. I then describe how the model is solved.

The use of of extreme value shocks simplifies the computation of the model. Following Rust (1988) and Dvorkin et al. (2021), the continuation value upon repayment is given by

$$\begin{aligned} V(z, b_m^i, b_p^i) =& \omega \ln \Bigg\{ \left(\sum_{j=1}^{\mathcal{J}} \exp(u(c_{i,j}(z)) + \beta \mathbb{E}_{z'|z} V(z', b_m^j, b_p^j))^{\frac{1}{\omega\nu}} \right)^{\nu} \\ &+ \left(\exp(u(y^{DP}(z) + (1-\delta+\delta\kappa) b_m^i) + \beta \mathbb{E}_{z'|z} V^{RP}(z', \delta b_m^i, b_p^i)) \right)^{\frac{1}{\omega}} \\ &+ \left(\exp(u(y^{DF}(z)) + \beta \mathbb{E}_{z'|z} V^{RF}(z', b_m^i, b_p^i)) \right)^{\frac{1}{\omega}} \Bigg\} \\ \text{s.t. } & c_{i,j}(z) = y(z) + [1-\delta+\delta\kappa] (b_m^i + b_p^i) - q_m(z, b_m^j, b_p^j)(b_m^j - \delta b_m^i) - q_p(z, b_m^j, b_p^j)(b_p^j - \delta b_p^i). \end{aligned} \tag{C.1}$$

The probability of choosing the portfolio $\{b_m^j, b_p^j\}$ is then given by

$$B(b_m^j, b_p^j; z, b_m^i, b_p^i) = \frac{\exp\left(u(c_{i,j}(z)) + \beta \mathbb{E}_{z'|z} V(z', b_m^j, b_p^j) \right)^{\frac{1}{\omega\nu}}}{\sum_{k=1}^{\mathcal{J}} \exp\left(u(c_{i,k}(z)) + \beta \mathbb{E}_{z'|z} V(z', b_m^k, b_p^k) \right)^{\frac{1}{\omega\nu}}}. \tag{C.2}$$

The probability of a *partial* and *full* default are respectively

$$
\begin{aligned}
D^{DP}(z, b_m^i, b_p^i) &= \frac{\mathcal{X}(z, b_m^i, b_p^i)}{\mathcal{X}(z, b_m^i, b_p^i) + \mathcal{Y}(z, b_m^i, b_p^i) + \mathcal{Z}(z, b_m^i, b_p^i)}, \\
D^{DF}(z, b_m^i, b_p^i) &= \frac{\mathcal{Y}(z, b_m^i, b_p^i)}{\mathcal{X}(z, b_m^i, b_p^i) + \mathcal{Y}(z, b_m^i, b_p^i) + \mathcal{Z}(z, b_m^i, b_p^i)},
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{X}(z, b_m^i, b_p^i) &= \exp\left(u(y^{DP}(z) + (1-\delta+\delta\kappa)b_m^i) + \beta\mathbb{E}_{z'|z}V^{RP}(z', \delta b_m^i, b_p^i)\right)^{\frac{1}{\omega}}, \\
\mathcal{Y}(z, b_m^i, b_p^i) &= \exp\left(u(y^{DF}(z)) + \beta\mathbb{E}_{z'|z}V^{RF}(z', b_m^i, b_p^i)\right)^{\frac{1}{\omega}}, \\
\mathcal{Z}(z, b_m^i, b_p^i) &= \left(\sum_{k=1}^{\mathcal{J}} \exp\left(u(c_{i,k}(z)) + \beta\mathbb{E}_{z'|z}V(z', b_m^k, b_p^k)\right)^{\frac{1}{\omega\nu}}\right)^{\nu}.
\end{aligned}
$$

The value of renegotiation after a *partial* default is given by

$$
\begin{aligned}
V^{RP}(z, b_m^i, b_p^i) =& \omega\phi \ln\Bigg\{\Bigg(\sum_{j,\tau_j\geq 0, b_m^j=\delta b_m^i} \exp\left(u(c_{i,j}(z, W_l^{RP}) + \beta\mathbb{E}_{z'|z}V(z', b_m^j, b_p^j)\right)^{\frac{1}{\omega\nu}}\Bigg)^{\nu} \\
& + \exp\left(u(y^{DP}(z) + (1-\delta+\delta\kappa)b_m^i) + \beta\mathbb{E}_{z'|z}V^{RP}(z', \delta b_m^i, b_p^i)\right)^{\frac{1}{\omega}}\Bigg\} \\
& + \omega(1-\phi) \ln\Bigg\{\Bigg(\sum_{j,\tau_j\geq 0, b_m^j=\delta b_m^i} \exp\left(u(c_{i,j}(z, W_b^{RP}) + \beta\mathbb{E}_{z'|z}V(z', b_m^j, b_p^j)\right)^{\frac{1}{\omega\nu}}\Bigg)^{\nu} \\
& + \exp\left(u(y^{DP}(z) + (1-\delta+\delta\kappa)b_m^i) + \beta\mathbb{E}_{z'|z}V^{RP}(z', \delta b_m^i, b_p^i)\right)^{\frac{1}{\omega}}\Bigg\} \\
& \text{s.t. } c_{i,j}(z, W) = y(z) + [1-\delta+\delta\kappa]\, b_m^i - W - q_p(z, b_m^j, b_p^j) b_p^j.
\end{aligned}
\tag{C.3}
$$

The related probability of accepting a restructuring offer, for $k \in \{l, b\}$, is

$$A^{RP}(z, b_m^i, b_p^i, W_k^{RP}) = \frac{\left(\sum_{j,\tau_j \geq 0, b_m^j = \delta b_m^i} \exp\left(u(c_{i,j}(z, W_k^{RP}) + \beta \mathbb{E}_{z'|z} V(z', b_m^j, b_p^j)\right)^{\frac{1}{\omega\nu}}\right)^{\nu}}{\left(\sum_{j,\tau_j \geq 0, b_m^j = \delta b_m^i} \exp\left(u(c_{i,j}(z, W_k^{RP}) + \beta \mathbb{E}_{z'|z} V(z', b_m^j, b_p^j)\right)^{\frac{1}{\omega\nu}}\right)^{\nu} + \mathcal{X}(z, b_m^i, b_p^i)}.$$

The value of renegotiation after a *full* default is given by

$$\begin{aligned} V^{RF}(z, b_m^i, b_p^i) =& \omega\phi \ln\left\{\left(\sum_{j,\tau_j \geq 0, b_m^j = 0} \exp\left(u(c_{i,j}(z, W_l^{RF}) + \beta \mathbb{E}_{z'|z} V(z', b_m^j, b_p^j)\right)^{\frac{1}{\omega\nu}}\right)^{\nu}\right. \\ &\left. + \exp\left(u(y^{DF}(z)) + \beta \mathbb{E}_{z'|z} V^{RF}(z', b_m^i, b_p^i)\right)^{\frac{1}{\omega}}\right\} \\ &+ \omega(1-\phi) \ln\left\{\left(\sum_{j,\tau_j \geq 0, b_m^j 0} \exp\left(u(c_{i,j}(z, W_b^{RF}) + \beta \mathbb{E}_{z'|z} V(z', b_m^j, b_p^j)\right)^{\frac{1}{\omega\nu}}\right)^{\nu}\right. \\ &\left. + \exp\left(u(y^{DF}(z)) + \beta \mathbb{E}_{z'|z} V^{RF}(z', b_m^i, b_p^i)\right)^{\frac{1}{\omega}}\right\} \\ &\text{s.t. } c_{i,j}(z, W) = y(z) + b_m^i \bar{q} - W - q_p(z, b_m^j, b_p^j) b_p^j. \end{aligned} \tag{C.4}$$

The related probability of accepting a restructuring offer, for $k \in \{l, b\}$, is

$$A^{RF}(z, b_m^i, b_p^i, W_k^{RF}) = \frac{\left(\sum_{j,\tau_j \geq 0, b_m^j = 0} \exp\left(u(c_{i,j}(z, W_k^{RF}) + \beta \mathbb{E}_{z'|z} V(z', b_m^j, b_p^j)\right)^{\frac{1}{\omega\nu}}\right)^{\nu}}{\left(\sum_{j,\tau_j \geq 0, b_m^j = 0} \exp\left(u(c_{i,j}(z, W_k^{RF}) + \beta \mathbb{E}_{z'|z} V(z', b_m^j, b_p^j)\right)^{\frac{1}{\omega\nu}}\right)^{\nu} + \mathcal{Y}(z, b_m^i, b_p^i)}.$$

The private bond price therefore reduces to

$$\begin{aligned} q_p(z, b_m^j, b_p^j) = \frac{1}{1+r} \mathbb{E}_{z'|z} \Bigg[&\left(1 - D^{DP}(z', b_m^j, b_p^j) - D^{DF}(z', b_m^j, b_p^j)\right) \times \\ &\left(1 - \delta + \delta\kappa + \delta \sum_{k=1}^{\mathcal{J}} q_p(z', b_m^k, b_p^k) B(b_m^k, b_p^k; z', b_m^j, b_p^j)\right) + \end{aligned} \tag{C.5}$$

$$D^{DP}(z', b_m^j, b_p^j) q_p^{DP}(z', b_m^j, b_p^j) + D^{DF}(z', b_m^j, b_p^j) q_p^{DF}(z', b_m^j, b_p^j) \Bigg].$$

with recovery value

$$q_p^{DP}(z, b_m^i, b_p^i) = \frac{1}{1+r} \mathbb{E}_{z'|z} \Big[(1 - \phi A^{RP}(z', \delta b_m^i, b_p^i, W_l^{RP})) q_p^{DP}(z', \delta b_m^i, b_p^i) + \phi A^{RP}(z', \delta b_m^i, b_p^i, W_l^{RP}) \frac{W_l^{RP}(z', \delta b_m^i, b_p^i)}{-b_p^i \bar{q}} \Big],$$

and

$$q_p^{DF}(z, b_m^i, b_p^i) = \frac{1}{1+r} \mathbb{E}_{z'|z} \Big[(1 - \phi A^{RF}(z', b_m^i, b_p^i, W_l^{RF})) q_p^{DF}(z', b_m^i, b_p^i) + \phi A^{RF}(z', b_m^i, b_p^i, W_l^{RF}) \frac{W_l^{RF}(z', b_m^i, b_p^i)}{-b_p^i \bar{q}} \Big].$$

Conversely, the multilateral debt price reduces to

$$q_m(z, b_m^j, b_p^j) = \frac{1}{1+r} \mathbb{E}_{z'|z} \Bigg[\Big(1 - D^{DF}(z', b_m^j, b_p^j) \Big) \times \Big(1 - \delta + \delta\kappa + \delta \sum_{k=1}^{\mathcal{J}} q_m(z', b_m^k, b_p^k) B(b_m^k, b_p^k; z', b_m^j, b_p^j) \Big) + D^{DF}(z', b_m^j, b_p^j) q_m^{DF}(z', b_m^j, b_p^j) \Bigg]. \tag{C.6}$$

with recovery value

$$q_m^{DF}(z, b_m^i, b_p^i) = \frac{1}{1+r} \mathbb{E}_{z'|z} \Big[(1 - A^{RF}(z', b_m^i, b_p^i, W_l^{RF})) q_m^{DF}(z', b_m^i, b_p^i) + A^{RF}(z', \delta b_m^i, b_p^i, W_l^{RF}) \bar{q} \Big].$$

I solve the model using value function iterations on a discretized grid for output, private and multilateral debts. Following Hatchondo et al. (2010), both the value functions and the prices are iterated in the same loop.

The process starts with a guess of the value function V as well as of the prices q_p and q_m corresponding to the limit of finite horizon. Given those guesses, I first determine the repayment value given by (3.2). I compute the value for each combination of multilateral and private debts. I also compute the bond choice probability through (C.2).

For the autarky values (3.3)-(3.4), I first solve the optimal lenders' offer over a W-grid. For each point on the W-grid, I determine the value of reentering the market given in (3.7) and (3.9) by means of a grid search.[11] I subsequently generate the values of renegotiation using (C.3)-(C.4) and compute the different sovereign's acceptance probabilities.

Having calculated the value under repayment and the value under default, I retrieve the new value of V from equation (C.1) and generate the different default probabilities.

With the acceptance probabilities and the lender's offer, I can calculate the recovery price for each debt instrument and for each default case as specified above. Once this is done, I compute the new bond prices q_p and q_m by means of equations (C.5) and (C.6), respectively.

Subsequently, I compare the initial guesses with the new outcome. I compute the maximal absolute distance between the newly-computed and previously-computed prices of private and multilateral debts. The same is done for the value V. If convergence is not attained, guesses are updated using a relaxation parameter and the whole process starts again.

Once the model is solved, I run simulations for 2000 countries and 600 years. The first 200 years are discarded to ensure that the initial conditions do not matter. The model-generated moments are computed as averages across countries. Business cycle moments are HP filtered with a smoothing parameter of 6.25.

3.6 Sensitivity to Utility Shocks

The utility shocks ease the numerical computation of the model. In fact without such shocks, I cannot always solve the model using standard value function iteration. It is possible to obtain some convergence under the refinement suggested by Chatterjee and Eyigungor (2012) but not for

[11] For computational efficiency, this step takes place at the same stage as the grid search for the repayment value.

all specifications of the model.

Table C.6: Sensitivity to ω and ν

	Baseline	$\omega \times 0.70$	$\omega \times 1.12$	$\nu \times 0.85$	$\nu \times 1.1$
Default length (year) (with multilateral lenders)	5.0	5.3	4.8	5.0	4.9
Default length (year) (without multilateral lenders)	3.1	2.9	2.7	2.7	2.7
Private creditors' haircut (%) (with multilateral lenders)	49.3	55.8	47.6	52.4	48.1
Private creditors' haircut (%) (without multilateral lenders)	32.1	36.0	31.8	34.1	31.8
Debt-to-GDP ratio (%)	40.8	39.1	41.2	40.1	41.4
Multilateral-debt-to-GDP ratio (%)	6.3	6.7	6.2	6.5	6.2
Share *full* default (%)	18.5	27.0	21.2	23.2	18.4
Default rate (%)	2.5	2.2	2.6	2.4	2.6
Debt increase prior to default (percentage point)	26.8	31.0	27.6	29.6	27.2
Standard deviation debt-to-GDP ratio	9.8	9.7	10.1	9.9	10.1
Standard deviation duration	3.4	3.8	3.5	3.6	3.4

Nevertheless, utility shocks are likely to affect the solution of the model. Especially, as shown by Dvorkin et al. (2021), they mainly affect the choices regarding debt and default. That is why I calibrated the variance parameter ω and the correlation parameter ν to the standard deviation debt-to-GDP ratio and the default duration, respectively.

As one can see in Table C.6, changes in ω and ν affect the main moments of the model in a negligible manner. Most notably, the variance parameter seems to affect the default duration and the debt dynamic. Conversely, the correlation parameter affects the duration and the default dynamic. The share of *full* default seems also to be sensitive to changes in both ω and ν.

3.7 Maturity Differential

The benchmark model assumes that the same maturity for the private and the multilateral debt. I now relax this assumption and consider that the multilateral debt has a maturity δ_m, while the private debt has a maturity δ_p. In particular, I consider two settings: $\delta_m = 0.89 < 0.9 = \delta_p$ and

$\delta_m = 0.9 < 0.91 = \delta_p$. I find that the model is very sensitive to such changes.

Table C.7: Maturity Differential

	Benchmark	$\delta_s = 0.89$	$\delta_p = 0.91$
Default length (year) (with multilateral lenders)	4.95	5.12	6.12
Default length (year) (without multilateral lenders)	2.68	2.70	2.44
Private creditors' haircut (%) (with multilateral lenders)	49.34	46.58	57.50
Private creditors' haircut (%) (without multilateral lenders)	32.14	30.54	31.80
Share *full* default (%)	18.52	16.17	98.17
Default rate (%)	2.46	2.49	2.32
Total debt increase (percentage point) (prior to default)	26.84	25.58	53.30
Total debt to GDP (%)	40.77	41.15	43.91
Multilateral debt to GDP (%)	6.27	6.24	6.79
Private debt spread (%)	1.17	1.10	2.00
Multilateral debt spread (%)	0.44	0.43	0.76

Allowing for a shorter maturity of the multilateral debt has two opposite consequence. On the one hans, as argued in Section 3.6, when $\delta_m \to 0$, a *partial* default becomes less attractive because the multilateral debt service is greater in the first few periods spent in autarky. Especially, when $\delta_m = 0$, the multilateral debt has to be repaid in one instalment at the beginning of the *partial* default. On the other hand, as argued by Arellano and Ramanarayanan (2012) and Niepelt (2014), shorter maturity enhances the willingness to repay and therefore can reduce the share of *full* defaults. As depicted in Table C.7, this second channel dominates as the share of *full* default decreases but the default rate increases.

Allowing for a longer maturity of the private debt reinforces the seniority cost to the repayment incentive. On the other hand, longer maturities are more sensitive to the default risk. On the other hand, more multilateral debt continues to depress the recovery value under a *full* default because of subordination. As one can see, when I set $\delta_p = 0.91$, the share of *full* default gets close to 100%. This is the opposite of when I reduce δ_m.

3.8 Welfare Analysis

In this section, I present how welfare gains are calculated. To compute the sovereign's welfare, first define the value of the sovereign for a sequence of consumption $\{c(z^t, \boldsymbol{\epsilon}^t)\}$ starting from an initial state at $t = 0$ as

$$V(\{c(z^t, \boldsymbol{\epsilon}^t)\}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c(z^t, \boldsymbol{\epsilon}^t)) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c(z^t, \boldsymbol{\epsilon}^t)^{1-\varrho}}{1-\varrho},$$

where the last equality is obtained from the functional form considered in Section 3.9. I denote the sovereign's consumption allocation in the benchmark model by $\{c^b(z^t, \boldsymbol{\epsilon}^t)\}$ and the consumption allocation in the alternative model by $\{c^a(z^t, \boldsymbol{\epsilon}^t)\}$. The sovereign's value in the benchmark model in state $(z, \boldsymbol{\epsilon}, b^i_m, b^i_p)$ is given by

$$V_b(z, \boldsymbol{\epsilon}, b^i_m, b^i_p) \equiv V(\{c^b(z^t, \boldsymbol{\epsilon}^t)\}).$$

Conversely, the sovereign's value under the alternative model in the exact same state $(z, \boldsymbol{\epsilon}, b^i_m, b^i_p)$ reads

$$V_a(z, \boldsymbol{\epsilon}, b^i_m, b^i_p) \equiv V(\{c^a(z^t, \boldsymbol{\epsilon}^t)\}).$$

Now define the consumption-equivalent welfare gain of the alternative model with respect to the benchmark model by χ such that

$$V(\{(1+\chi)c^b(z^t, \boldsymbol{\epsilon}^t)\}) = V(\{c^a(z^t, \boldsymbol{\epsilon}^t)\}).$$

Given the functional form of the instantaneous utility one obtains

$$(1+\chi)^{1-\varrho} V_b(z, \boldsymbol{\epsilon}, b_m^i, b_p^i) = V_a(z, \boldsymbol{\epsilon}, b_m^i, b_p^i).$$

The welfare gain therefore boils down to

$$\chi(z, \boldsymbol{\epsilon}, b_m^i, b_p^i) = \left[\frac{V_a(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)}{V_b(z, \boldsymbol{\epsilon}, b_m^i, b_p^i)}\right]^{\frac{1}{1-\varrho}} - 1.$$