EUI Working Papers
ECO 2007/54

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October, 2007

Abstract

A substantial body of empirical works document that exchange rate pass-through to consumer prices is incomplete. This evidence has cast doubts on the ability of flexible exchange rates to generate expenditure switching. In a dynamic stochastic discrete-time duopoly game, non-price competition among firms endogenously originates a degree of exchange rate pass-through close to zero together with an expenditure switching effect stronger than in the standard models.

* I wish to thank Morten O. Ravn and Massimo Motta for helpful comments and suggestions. I am grateful to Giancarlo Corsetti, Renato Faccini, Karel Mertens and seminar participants at the EUI Third Year Forum and 22nd Annual Congress of EEA. e-mail: aurora.ascione@eui.eu.

Keywords: pass-through, non-price competition, expenditure switching
JEL classification: F40, L16
1 Introduction

In the last few years the empirical literature has shown that exchange rate pass-through to consumer prices is close to zero in the short run, and far to be complete in the long run.\(^1\) The degree of exchange rate pass-through is a crucial variable for explaining the role of exchange rates in the process of external adjustment. A limited degree of pass-through can dampen the expenditure switching effect of exchange rate changes on trade volumes, as it forestalls movements in relative trade prices. In this occurrence, a variation of the nominal exchange rate will not orient the demand in favor of the depreciated currency goods, disallowing a trade balance adjustment. A part of the literature indicates that since prices are set in the importer country currency expenditure switching does not occur.\(^2\) A part suggests that even if prices do not adjust at consumer level, they do at producer level, and that exchange rate promotes at producer level expenditure switching.\(^3\) Even if the degree of pass-through is very low, a number of empirical works document that exchange rate changes do indeed redirect global expenditure.\(^4\) In the model proposed in this paper I account for unresponsive prices to exchange rates together with expenditure switching.

This model investigate the relation between exchange rate pass-through and expenditure switching, assuming that firms can compete in price and in non-price dimensions. Building on Hotelling (1929), products are defined as a triple of characteristics, price, and a "types". It is assumed that consumers are heterogenous in the evaluation of types, and homogenous in the evaluation of characteristics. More specifically, for a given price, all consumers prefer higher characteristics content, but they differ in their evaluation of the types.\(^5\) A foreign and a domestic firm compete in a duopoly framework. The domestic firm’s price and costs are expressed in the same currency. The foreign firm, instead sets the price in the local market currency, but sustains costs in a different currency. This feature exposes the market to the fluctuations of the exchange rate. In particular, in a flexible price framework, if products’ characteristic cannot be changed, an exchange rate depreciation of the domestic bilateral exchange rate increases foreign firm costs expressed in domestic currency, increasing its prices and decreasing its market shares.\(^6\) The foreign firm, in order to insulate its profits from exchange rate fluctuations, can find profitable to change its costs, this can be done by changing the characteristics of the product supplied in the domestic market. This intervention can take both the form of substituting the product with a new one that has a smaller content of characteristics, or can take the form of changing some features of the existing product.

The way in which the two firms operate in the market can be described using a two stage game; first firms compete in characteristics, then they compete in prices. The price competition is a static game of complete information, since in each period both firms observe the realization of the exchange rate shock, and because current prices do not have any effect on future demand or profits. The characteristic choice, the non-price competition, is instead a dynamic game,

\(^2\)See for example Devereux and Engel (2003), Engel (2002).
\(^3\)See among others Corsetti and Dedola (2003), Burstein, Neves and Rebelo (2003), Obstfeld (2001).
\(^4\)For an overview see among others Krugman (1991).
\(^5\)Supposing that bicycles are the products of this economy: valuable characteristic is represented by the metal used in production, and the type by their model (for example race bikes, mountain bikes, etc...).
\(^6\)Notice that in a duopoly setting, when the costs of one of the two firms increase, both firms increase their prices, but the firm that has the costs shock increases the price more than its competitor.
since any time the product varies from the previous period, firms must pay an adjustment cost that is assumed to be invariant through time. Given the structure of the problem it is possible to define a unique Markov-Perfect equilibrium that is also a rational expectation equilibrium: the beliefs of a firm about its competitor characteristics in equilibrium are correct and equal to the true amount of characteristics chosen by the competitor.

The main result of the model is that even when the degree of exchange rate pass-through is low, expenditure switching do occur. When a depreciation of the domestic currency hits the economy, the amount of characteristics of the foreign product reduces while its price increases by less than in the standard duopoly models. Expenditure switching occurs even if exchange rate pass-through is moderate. The size of the expenditure switching is even bigger than in the standard duopoly models: this result comes from the fact that demand adjust along price and product characteristic dimension.

The rest of the paper is organized as follows. Sections (2) provides an overview of the empirical evidence relevant to the paper. Section (3) describes the model. Section (4) explains the game. Section (5) describes the results. Section (6) concludes.

2 Motivating Evidence

This section is dedicated to a brief review of the macroeconomic literature that document the reaction of imports and exports to exchange rate variations, and to the marketing literature that document non-price competition behaviors.

2.1 Exchange Rate Variation and Expenditure Switching

In the international macroeconomic literature, the scarce responsiveness of prices to the exchange rate, has cast doubts on the capacity of the exchange rate variations to generate expenditure switching, that is, the ability of the exchange rate variations to orient the product demand in favor of the goods whose currency has been depreciated.

In reality, there is evidence that exchange rate changes do indeed redirect global expenditure, though perhaps with lags. Faruqee (2006) finds that in the European countries, even if local currency pricing and pricing to market behavior are in place, expenditure switching effects still operate. In particular, Faruqee finds evidence that exchange rate depreciations, redirecting the demand in favor of the depreciated currency goods, improves the trade balance.

Freund (2000) studies 21 industrialized countries finding evidence on the importance of nominal exchange rate variations for current account adjustment.

Debelle and Galati (2005) analyze 21 industrialized countries for the period that goes from 1974 to 2003, finding that the exchange rate variations contribute significantly to the process of adjustment of the current account imbalances.

Finally, Krugman (1991) presents a wide overview of evidence that support the expenditure switching effect through around 1990.

2.2 Non-price Competition

The evidence of non-price competition used as optimal reaction to shocks by firms, is widely documented in the marketing literature.
Rockoff (1984) studies the price control in the U.S. from the WWII; the author finds that price controls held nominal price of several consumers goods to be constant, with a contemporaneous deterioration of the goods’ quality in response to change in the market prices.

Rotemberg (2005) cites that the October 1994 issue of Consumers Reports discusses a change in the design of the packages for Gütterman thread. The company made its plastic spools fatter so that, when filled with 110 yards of thread, they would have the same outside dimensions than a previous version of 220 yards of thread. The price of the newer version was $1.25 while the previous was $1.45. In the same issue of the Consumers Reports it is declared that Minute Maid significantly reduced the concentration of its 12 oz can of Raspberry Lemonade while keeping the price constant. The firm, questioned on this action, explained that this product variation was motivated by cost increases that would otherwise raise the price.

Koelln and Rush (1993), on a period that goes from 1950 to 1989, collect data of the "text pages" and prices for seven magazine in the U.S. The authors identify the possibility of altering the number of pages of text as a potential means of offsetting declines in real price during the interval between price changes. Koelln and Rush note that the magazine with the most inflexible size over this period also had by far the largest number of nominal price changes. They interpret this result as supporting the hypothesis that variation in valuable characteristic is a potential important alternative in changing the price. The authors find also a statistical significant positive relationship between the number of text pages in a magazine and the real price of the magazine. That is, as inflation erodes the real price of a magazine during the interval between nominal price changes, the number of text pages tend to decline.

In the study of Blinder et al. (1989), firms are asked directly about their pricing behavior; a sample of two hundreds managers and price setters of the Northeastern United States were asked to rank or assign scores to a number of twelve popular economic theories which were explained to them in non-technical terms. The results show that across sector of the economy the theory of "non-price competition" belongs to the top group scores being the third most used way to keep price constant.

With the definition "non-price competition" are named different behavior as using delivery lags, change product quality, change terms of service, or make a different selling effort. When the results are divided by sector, non-price competition is the first method used by firms to avoid changes in prices in the manufacturing sector, the second in the trade sector, the third in the service sector. Quoting Blinder et al.:

"... many firms appear to leave prices fixed in the face of shocks because they prefer to use one or more avenues of nonprice competition to adjust. Rather than cut prices in the face of sagging demand, firms can and do shorten delivery lags, raise product quality, improve term of service, or make greater selling efforts. Non-price competition raises both microeconomic and macroeconomic questions that have barely been addressed by theorists. Why do firms prefer nonprice competition to price competition even though the former appears to entail real resource costs that the latter avoids? What are the welfare implications of markets that clear along non-price dimensions? The answers to these and other questions must be left to future research."
3 The Model

3.1 Products

In the market two firms produce two differentiated products characterized by the triple \((s_{i,t}, p_{i,t}, x_{i,t})\).

\(x_{i,t}\) represents the “type” of the product and is defined as a peculiarity perceived as relevant by consumers, but which does not affect product’s costs (an example of a product’s type is its color). The type \(x_{i,t}\) is defined on a support that ranges between 0 and 1. This index can be represented on a segment of unit length in which firms and consumers are located. It is assumed that through time the positions of the firms along the segment is given and equal to the extreme points of it, and that the consumers are distributed uniformly along the segment.

\(s_{i,t}\) represents the amount of valuable “characteristics” presents in the product and it is defined as a feature on which all consumers agree over the preference ordering: for a given price and a given type, the product with a higher amount of characteristics is preferred by each consumer (an example of characteristics is the product’s raw material). Differently from the type, the characteristic is positively related both to variable and fix firms’ costs. \(s_{i,t}\) and \(x_{i,t}\) are assumed to be independent in order to focus the analysis in the case in which there are not interactions between the two features of the product. Finally \(p_{i,t}\) defines the price of the good.

3.2 Consumers

Consumers are fully characterized by their location \(x\) in the segment \([0, 1]\). They are thus heterogenous in the evaluation of the type and homogenous in the characteristics evaluation.\(^7\)

I assume that consumers’ utility is decreasing in the distance between the good’s type \(x_i\) and \(x\), decreasing in the price of the product \(p_i\), and increasing in its characteristics \(s_i\). Consumers behavior is static and demand is unitary. At every period \(t\), consumers have full information on the characteristics and price level of the two firms, and decides whether to buy or not a unit of a good, and from which firm to buy it. The utility of consumer \(x\) that buy good \(i\) at time \(t\) is thus:

\[ u_{i,t} = k + s_{i,t} - p_{i,t} - \tau |x - x_i| \]

where \(\tau |x - x_i|\) represents the consumer’s transportation costs, and \(\tau\) is the unit transportation costs. The utility of the outside alternative, i.e. not purchasing the good, is normalized to zero.

A consumer purchases a unit of good \(i\) if \(u_{i,t} \geq 0\) and if \(u_{i,t} \geq u_{-i,t}\). I assume that \(u_{i,t} \geq 0\) always holds, that is the market is always covered. I can rewrite the above condition as \(k + s_{i,t} \geq p_{i,t} + \tau |x - x_i|\) from which clearly it can be seen that a consumer buys the good as long as the effective price, represented by the good’s price plus the transportation costs, is less than the characteristics amount supplied.

I derive now the time invariant consumers’ demands for each product; I omit the time subindex for notation clarity. Let \(\tilde{x} \in [0, 1]\) denote the location of the consumer that is indif-

\(^7\)In the textbook version of the vertical product differentiation, when consumers are homogenous in the evaluation of the characteristics, if two different characteristics are supplied, all consumers always prefer high characteristics if they purchase at all (see Tirole 1988). This means that the low characteristics product is dominated. In my formulation instead, since consumers are concerned also about the type of the good, in equilibrium it is possible to observe different characteristics without involving consumers heterogeneity in characteristics evaluation.
different between purchasing from firm $i$ and firm $-i$. Thus:

$$s_i - p_i - \tau |x_i| = s_{-i} - p_{-i} - \tau |x_{-i}|$$

(1)

since $x_i = 0$, and $x_{-i} = 1$, solving equation (1) for $\hat{x}$ provides:

$$\hat{x} = q_i = \frac{s_i - s_{-i}}{2\tau} + \frac{p_{-i} - p_i + \tau}{2\tau}.$$  

(2)

### 3.3 Firms

In the market a “domestic” and a “foreign” firm compete. I define as domestic the firm whose costs and prices are expressed in the same currency (the so called domestic currency), while I define as foreign the firm who sets prices in domestic currency, but whose costs are expressed in a different currency. The foreign firm costs are converted in the domestic currency through the nominal bilateral exchange rate. The nominal bilateral exchange rate is a stochastic variable that evolves according to a Markov process equal to:

$$e_t = e_{t-1} \exp \varepsilon_t \text{ where } \varepsilon_t \sim N(0, \sigma^2)$$

(3)

Its realization is common knowledge to firms and consumers, and is future realization is the only source of uncertainty of the economy. For a given amount of characteristics, a depreciation (appreciation) of the exchange rate increases (decreases) foreign firm’s costs expressed in domestic currency, but does not affect those of the domestic firm. A variation of the nominal bilateral exchange rate thus leads to fluctuations of the relative costs of the two firms. It is assumed, however, that firms can insulate relative costs fluctuations due to exchange rate variations changing the goods’ content of characteristics. This behavior can be described using the a stationary discrete-time infinite horizon game, that I explain in the following section.

### 4 The Game

Time is discrete. Every period $t$ firms observe the realization of the exchange rate shock. Given this information, in each period $t$ firms choose the amount of characteristics and compete in prices. The Bertrand game is static because current prices do not have any effect on future demand or profits. The equilibrium prices of the Bertrand game resulting determine the equilibrium profits for each firm at time $t$ as a function of the amount of characteristics they choose. Firms decide simultaneously the amount of characteristics that maximize their profits given the optimal response of the competitor. This choice is dynamic because it is assumed that every time firms change the characteristics with respect to the previous period, they sustain an additional quadratic cost of adjustment. The dynamics of the model are thus generated by the exogenous stochastic process of the nominal bilateral exchange rate, and by the choices of the products’ characteristics over time.

#### 4.1 Price Competition: the Spot Market

In this subsection I omit the time subindex for notational simplicity. Every period firms compete in prices. This price competition game is a static game of complete information. Firms maximize profits taking as given the value of the nominal bilateral exchange rate and both their own and
their competitor’s previous period characteristics level. In order to restrict the analysis on the duopoly outcome the following assumption must hold:

\( A.1 \forall s \in S, \quad |C_i(s_i, e) - C_{-i}(s_{-i}, e) + s_{-i} - s_i| < 3\tau; \)

where \( C_i(s_i, e) (C_{-i}(s_{-i}, e)) \) represents firm \( i \) (-i) marginal costs that are function of the own characteristics amount and the nominal bilateral exchange rate \( e \). If assumption (A.1) holds the marginal costs of the two firms are not too different relative to the consumers’ unit transportation costs \( \tau \). Indeed if \( C_i(s_i, e) > C_{-i}(s_{-i}, e) - (s_{-i} - s_i) + 3\tau \) firm \( i \) would have too high marginal cost with respect to firms \(-i\) for competing in that market. It also states that \( \forall s \) we can observe the duopoly outcome, that is, there does not exist a characteristics level for which one of the two firms has an absolute advantage such that the firm can be the monopolist for that given characteristics amount. Since in the dynamic game firms choose the level of characteristics and therefore their marginal costs, at that stage it must checked that the firms endogenously choose levels of characteristics that satisfy assumption (A.1). In appendix (B) I show that this indeed will be the case for each given level of characteristics \( s \).

Given assumption (A.1) I can define the Bertrand-Nash equilibrium of the static game:

**Proposition 1** Suppose assumption (A.1) holds, then firm \( i \) and \(-i\) play a duopoly price competition game characterized by the following prices, quantities and profits:

\[
\begin{align*}
p_i(e, s_{-i}; s_i) &= \frac{s_i - s_{-i}}{3} + \frac{3\tau + C_{-i}(s_{-i}, e) + 2C_i(s_i, e)}{3} \\
q_i(e, s_{-i}; s_i) &= \frac{s_i - s_{-i}}{6\tau} + \frac{3\tau + C_{-i}(s_{-i}, e) - C_i(s_i, e)}{6\tau} \\
\Pi_i(e, s_{-i}; s_i) &= \frac{(s_i - s_{-i} + 3\tau + C_{-i}(s_{-i}, e) - C_i(s_i, e))^2}{18\tau}
\end{align*}
\]

**Proof.** Each firm maximizes profits taking the price of the other firm as given. It follows from equation (2) that the static maximization problem of the two firms is:

\[
\max_{p_i} \left( p_i - C_i(s_i, e) \right) \left( \frac{s_i - s_{-i}}{2\tau} + \frac{p_{-i} - p_i + \tau}{2\tau} \right)
\]

Each firm maximizes its profit by choosing its best-response price. The best-response of firm \( i \) is characterized by the first order condition:

\[
\frac{s_i - s_{-i} + p_{-i} - 2p_i + \tau}{2\tau} - \frac{C_i(s_i, e)}{2\tau} = 0
\]

that combined with the reaction function of the competitor delivers the equilibrium prices, and hence the quantities and the profits stated in Proposition 1.

For each level of characteristics in the characteristics state space I find a unique Bertrand-Nash equilibrium given by that described in Proposition 1.

4.2 Non-Price Competition: the Dynamic Analysis

4.2.1 The Primitives

The interaction among the two firms in the dynamic setting can be described using a set of primitives which are common knowledge to the two firms:

\[ \{(S, S), (E, E), \Omega, \Gamma, \Pi_i(e, s_{-i}; s_i), R_i(s_{i-1}, s_{-i}, e; s_i), e, \beta\} \]
(S, S) and (E, E) are measurable spaces of possible values for the endogenous and exogenous state variables, s and e, respectively; in particular:

A.4 \( s \in S \subset \mathbb{R}_+ ; \quad e \in E \subset \mathbb{R}_+ ; \)

A.5 \( S \) is a convex Borel set in \( \mathbb{R}_+ ^2 \), with its Borel subsets \( S ; \)

The state space is \( \Omega = S \times S \times E \subset \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ ; \) \( \Gamma : \Omega \to S \) is a correspondence describing the feasibility constraint, it is assumed that:

A.6 \( \Gamma \) is nonempty, compact valued, and continuous;

\( \Pi _i (e, s_{-i}; s_i) \) gives the payoff or profit of a firm from its current production and sales activities. It reflects the equilibrium in the spot market, and is equal to the equilibrium profit of the firm’s static problem stated in Proposition 1. It is assumed that characteristics can be changed in each period, but any time characteristics vary from the previous period, firms must pay an adjustment cost \( B \) that is assumed to be invariant through time. In each period the objective function of each firm is therefore given by the optimal profit derived in the price competition in Proposition 1, minus the fix costs associated with the characteristics level, minus the quadratic adjustment cost associated with the characteristics variation; that is:

\[
R_i (s_{i,-1}, s_{-i}, e; s_i) = \Pi_i (e, s_{-i}; s_i) - A_i (s_i) - B (s_i - s_{i,-1})^2 .
\]

for each \( e \in E \) I assume that \( R_i (s_{i,-1}, s_{-i}, e; s_i) \) is strictly concave. This specification of the one period return allows to obtain a unique steady state equilibrium. In particular the term \( A_i (s_i) \) guarantees the intersection of the reaction functions of the two firms with respect to characteristics; finally \( \beta \) is the common discount factor of the two firms in the model that:

A.7 \( \beta \in (0, 1) ; \)

### 4.2.2 Dynamic Game

At any time \( t \) the dynamic problem of the two firms is to choose simultaneously the optimal characteristics levels that maximize their expected present value of net cash flows given the state of the economy \((s_{i,-1}, s_{-i,-1}, e)\). Firms choose their amount of characteristics simultaneously after the exchange rate shock is realized. Both firms have complete information on the realization of the shock and on their own and their competitor’s costs. The intertemporal problem of the two firms can be written as:

\[
\max _{s_i} \mathbf{L}_{i,t} (e, s_{-i}; s_i) = E_0 \left( \sum _{t=0} ^{\infty } \beta ^t R_i (s_{i,t-1}, s_{-i,t}, e_t; s_{i,t}) \right)
\]

s.t. \(
\max _{s_i} \mathbf{L}_{-i,t} (e, s_{-i}; s_i) = E_0 \left( \sum _{t=0} ^{\infty } \beta ^t R_{-i} (s_{-i,t-1}, s_{i,t}, e_t; s_{i,t}) \right)
\)

For any given \( s_{i,t} \) the distribution used to form expectation in (5) can be derived from the firm’s perception of the Markov transition kernel for its competitor, and the controlled Markov stated in (3). This formulation has a stationary Markovian structure. That is the current states \((s_{i,-1}, s_{-i,-1}, e)\) and the current decision, \( s_i \) are sufficient to determine the evolution to
the next state \((s_i, s_{-i}, e')\). This implies that for each of the two firms the optimal characteristics strategies, if it exists, can be chosen from the class of stationary Markov strategies. Therefore the optimal characteristics choice can be defined as \(G_i(s_{i-1}, s_{-i-1}, e)\), that means that characteristics are stationary function of only the current state \((s_{i-1}, s_{-i-1}, e)\). This means that the problem can be formulated as a recursive problem and if a solution exists to the firm’s problem it must satisfy the Bellman equation:

\[
V_i(s_{i-1}, s_{-i-1}, e) = \max_{s_i \in S} \left\{ R_i(s_i, s_{-i}, e) + \beta E_t V_i(s_{i-1}, s_{-i-1}, e') \right\} \tag{6}
\]

subject to:

\[
s_{-i} = G_{-i}(s_{i-1}, s_{-i-1}, e) \tag{7}
\]

Therefore, in any state, for each firm, the optimal policy involves choosing a level of characteristics that maximizes the above Bellman equation given the symmetric policy function of the competitor.

### 4.2.3 The Equilibrium

I study the dynamic equilibrium of the market arising from the competitive interaction between the domestic and foreign firms. The equilibrium is one of “rational expectation”, in the sense that, in equilibrium, the beliefs of firm \(i\) on the optimal decisions of its competitor, must be equal to firm \(-i\) true policy function. The two firms solve a dynamic programming problem that is interdependent only through the states of the economy \((s_{i-1}, s_{-i-1}, e)\), such that their quality strategies remain optimal at every state, regardless of how that state was reached, against the optimal decision of the competitor. Thus we can say that the equilibrium is also a Markov-Perfect Nash Equilibrium in the sense of Maskin and Tirole (1988) and (2001). Given Assumptions (A.4)-(A.7) and equation (3) we can therefore define:

**DEFINITION** \(\forall(s_{i-1}, s_{-i-1}, e) \in \Omega\) The Markov Perfect Equilibrium for this market is given by a pair of value functions \(V_i(s_{i-1}, s_{-i-1}, e)\) and a pair of policy function \(G_i(s_{i-1}, s_{-i-1}, e)\) such that:

1. Given \(G_{-i}(s_{i-1}, s_{-i-1}, e)\), \(V_i(s_{i-1}, s_{-i-1}, e)\) satisfies (6);
2. The policy function \(G_i(s_{i-1}, s_{-i-1}, e)\) solves (6) and satisfies:

\[
\frac{\partial}{\partial s_i} R_i(s_{i-1}, s_{-i}, e; s_i) + \beta E_t \frac{\partial}{\partial s_i} V_i(s_i, s_{-i}, e') = 0.
\]

Given our definition of the equilibrium we want now to show that the equilibrium exists and is unique.

**Proposition 2** Consider the firm decision problem (6). \(\forall s_i \in [\bar{s}_i, \underline{s}_i] \subset S\) and \(\forall e \in [\underline{e}, \bar{e}] \subset E\) under the Assumptions (A.4)-(A.7):

a. there exist a unique \(V_i(s_{i-1}, s_{-i-1}, e)\), \(V : \mathbb{R}^3_+ \rightarrow \mathbb{R}_+\), monotonic increasing in \(s_i\), uniformly bounded, and satisfying (6);

b. there exist a unique optimal policy function, \(G_i(s_{i-1}, s_{-i-1}, e)\), \(G : \mathbb{R}^3_+ \rightarrow \mathbb{R}_+\).
Proof. a. Monotonicity:
Let $\max_{s \in T} \{ R_i(s_{i-1}, s_{-i}, e; s_i) + \beta E_t V_i(s_i, s_{-i}, e') \} \geq \max_{s \in T} \{ R_i(s_{i-1}, s_{-i}, e; s_i) + \beta E_t W_i(s_i, s_{-i}, e') \}$
and:
$$TV_i(s_{i-1}, s_{-i}, e) \geq TW_i(s_{i-1}, s_{-i}, e)$$
Discounting:
Take an $a \geq 0$. We have:
$$T(V_i + a)(s_{i-1}, s_{-i}, e) = \max_{s \in T} \{ R_i(s_{i-1}, s_{-i}, e; s_i) + \beta E_t [V_i(s_i, s_{-i}, e') + a] \}$$
$$= \max_{s \in T} \{ R_i(s_{i-1}, s_{-i}, e; s_i) + \beta E_t V_i(s_i, s_{-i}, e') + \beta a \}$$
$$= \max_{s \in T} \{ R_i(s_{i-1}, s_{-i}, e; s_i) + \beta E_t V_i(s_i, s_{-i}, e') \} + \beta a$$
$$= TV_i(s_{i-1}, s_{-i}, e) + \beta a$$
that satisfies the discounting property.

b. The proof of uniqueness of the policy function $G_i(s_{i-1}, s_{-i}, e)$ is straightforward since the concavity of the one period return with respect to the endogenous state variable ensures that the sequence of approximated policy function $\{G_n\}$ converges to the optimal policy function $G_i(s_{i-1}, s_{-i}, e)$.

5 Results

In this section I describe the main results of the model. The algorithm I have used to solve the model is described in appendix (A). The results discussed in this section have an absolute error of $10^{-9}$ and a relative error of $10^{-8}$. Solving for the optimal policy function of the two firms shows that the characteristics choice of each firm depends positively on their previous period characteristics, and negatively by the previous period amount of competitor’s characteristics. The first result can be explained by the presence of adjustment costs, the second by the fact that firms tend to slightly differentiate their products also on the characteristics dimension. The exchange rate value affect positively the amount of characteristics of the domestic firm and negatively that of foreign firm. Indeed the higher the exchange rate the higher the costs of the foreign firm, the more the foreign firm will find optimal to reduce its characteristics content in order to reduce its costs.

I have performed an impulse response analysis of a one percent depreciation of the domestic exchange rate. I have analyzed a symmetric equilibrium in which, before the shock, the two firms have the same costs, prices and characteristics. In order to analyze the role of the persistence of the shock, I have analyzed a case in which $\rho = 0.7$, defined as high persistency, and a case in which $\rho = 0.2$, defined as low persistency. The pictures of the impulse responses are in Appendix (C.1) and Appendix (C.2) respectively.

The results obtained using two different presidencies are qualitatively the same. Comparing the results for $\rho = 0.2$ with those for $\rho = 0.7$ one can see that in the first case the economy comes back to the steady state much quicker and characteristics, prices and market shares react less than in the case of $\rho = 0.7$. This result highlights the importance of the persistence of the
shock, showing that the bigger is the persistence of the shock, the bigger are the effects on the economy. The results reported in the two Appendix are robust to a sensitivity analysis on fix and adjustment costs.

For a given level of characteristics, a depreciation of the exchange rate increases the marginal costs of foreign firm expressed in domestic currency. The impulse response shows that the foreign firm reduces the amount of characteristics while increases the prices. The domestic firm slightly increases the characteristics level and increases more the price. Overall the two firms increase prices less than in the case in which characteristics stay constant, leading to a smaller exchange rate pass-through than the standard models with strategic interaction. The domestic firm’s market shares increase more than the standard duopoly models. This effect is due to the fact that consumers observe the goods’ characteristics and reward the domestic firm that does not vary them. In this case, it is observed an higher degree of expenditure switching even if prices vary less than in the duopoly models without non-price competition.

The foreign firm decreases the characteristics of the product supplied even if this means a lower market share with respect to the one it would get not changing its product. Once the depreciation occurs, marginal revenues, expressed in the foreign currency, reduces. The foreign producer optimally curbs her profits reduction, reducing costs. This can be done, by diminishing the characteristics content of the good. The decrease of the characteristics content, as well as the increase in prices, have a negative effect on the foreign firm market shares. This is a mechanism that shows how it is possible to observe a low degree of exchange rate pass-through together with an expenditure switching effect. This result come from the producer costs structure and from the non-price competition among firms.

6 Conclusion

Empirical evidence show that the exchange rate pass-through to consumer prices is far from be complete in the short run. This model proposes an explanation of this evidence alternative to the existing theories. A producer hit by an exchange rate shock, prefers to supply a less valuable product, instead of increasing its price. When an impulse response is performed the results show that the foreign firm characteristics react much more than the foreign firm price. Domestic demand rewards the domestic firm which even if increases its price, keeps constant or slightly increases its amount of characteristics. Even if prices do not respond much to the variation of the exchange rate, the dynamics of the model show expenditure switching in favor of the domestic product, whose currency depreciate.

This result supports the literature that attribute to exchange rate variations the ability of working as adjustment mechanisms. This adjustment mechanism occurs through the real channel of product adjustment instead of through the nominal channel of price changes. These results suggest that non-price competition is a key element in the analysis of exchange rate pass-through, and, more in general, costs pass-through.

The model can be extended in several directions. Currently I am solving the model in a general equilibrium framework, and I am providing some empirical evidence to this mechanism. I leave instead for future research a proper dynamic empirical analysis of the model.
Appendix

A Numerical Solution

In this section I report the solution of the model and the impulse response functions of a one percent depreciation of the nominal bilateral exchange rate. In order to analyze the dynamics of the model I must find the policy functions $G_i$ and $G_{-i}$ of the two firms. The usual way of solving dynamic game is implementing the Linear Quadratic algorithm (L-Q). In order to solve the L-Q algorithm, the one period return function must be quadratic, or, if it is not, it should be possible to determine the deterministic steady state before the algorithm is computed in order to compute the second order Taylor approximation around it to reshape the problem in a quadratic form. The one period return function of the problem described above neither can be expressed as quadratic, nor is possible to know a priori the steady-state equilibrium around which make the Taylor expansion, I therefore use the collocation method to solve the model. I describe in the next subsection the algorithm used to find the policy functions.

A.1 The Algorithm

The collocation method gives a straightforward strategy to solve functional equations. Using the collocation method the policy functions of the two firms can be considered as an unknown functions that can be approximated using a linear combination of $n$ basis functions.

$$
\hat{G}_{-i}(s_{i,-1}, s_{-i,-1}, e) = \sum_{j_1=0}^{1} \sum_{j_2=0}^{1} \sum_{j_3=0}^{1} d_{j_1j_2j_3} \phi_{j_1j_2j_3}(s_{i,-1}, s_{-i,-1}, e).
$$

where the policy function is expressed as a linear combination of a set of $n$ linearly independent basis functions $\phi_{j_1j_2j_3}$ and $d_{j_1j_2j_3}$ basis coefficients that are the unknowns of the function. In tensor notation, the approximant can be written:

$$
\hat{G}_{-i}(s_{i,-1}, s_{-i,-1}, e) = \left[ \phi_i(s_i) \otimes \phi_{-i}(s_{-i}) \otimes \phi_e(e) \right] d
$$

where $d$ is an $1 \times n$ column vector and each $\phi_i$ is the $1 \times n_t$ row vector of basis functions over the dimension $t$. $n = \prod_{i=1}^{l} n_i$ basis coefficients are fixed by requiring the approximant to satisfy the functional equation, not at all possible points of the domain, but rather at $n$ prescribed point in the approximation space called the collocation nodes. Given the multivariate nature of the problem and the values of the relative residual function that I obtain, I report the results that I get for $n_t = 2$ for each of the states of the economy. $n_t = 2$ for each of the states implies eight collocation nodes, and eight unknown basis coefficients for each policy function.

Step 1: Definition of functional forms and parameter values. In order to solve the firms’ maximization problem, I need to specify functional forms for the model. Marginal and fixed costs functional equations are defined as:

$$
C(s_{i,t}, e_t) = \begin{cases} 
  c_i s_{i,t} & \text{for } i = d \\
  c_i^* s_{i,t} e_t & \text{for } i = f 
\end{cases}
$$

$$
A_{i,t}(s_{i,t}) = a_i s_{i,t}^2 & \text{for } i = d, f.
$$
Foreign firm’s marginal costs are expressed in foreign currency, while foreign firm’s fix costs are expressed in domestic currency. This specification allows to analyze just the effects that an exchange rate shock has on the spot market. If both the foreign firm’s costs of adjustment and the fix costs were expressed in foreign currency, the foreign firm’s decision of changing characteristics would depend exclusively on the importance of these costs with respect to revenues. Also in the case of a small depreciation, the costs of selling high characteristics goods would not compensate by the associated revenues, and foreign firm will always decrease the amount of characteristics. When only marginal costs are function of the exchange rate instead, I am able to capture how selling an extra unit of the good in the domestic market increase the costs of the foreign firm, and thus to consider only the direct effect of selling an extra unit of characteristics.

In each period, given the costs specification, the concavity of $R_i$ is guaranteed if:

$$(1 - c_i)^2 < 18\tau (a_i + B) e$$

with $e = 1$ if $i = d$.

The parameter values have been chosen in order that Assumption (A.1)-(A.7) hold given the functional forms of the model and the above condition on the concavity of the one period return holds as well.

**Step 2: Definition of the approximation space.** I have to define a compact set of $(s_i, s_{-i}, e)$ pairs over which solve the algorithm. The choice consists in four parameters $\bar{s}_i, \underline{s}_i, \bar{e}, \underline{e}$. The choice of the upper and lower bound of $s$ must be done in order to obtain a steady state value that is contained in the interval, similarly for the value of $e$.

**Step 3: Computation of the Euler equations.** Once the control variable of the competitor is defined as the unknown function, I compute the Euler equations of the two firms. For each firm the Euler equation is equal to:

$$0 = (s_i(1 - c_i s_{i,t}) + G_{-i}(s_{i_{-1}}, s_{-i_{-1}}, e)(c_{-i} e_t - 1) + 3\tau) - a_i s_{i,t} - B(s_{i} - s_{i_{-1}}) + \frac{18\tau}{\beta E_t} \left\{ \frac{(s_{i,t+1}(1 - c_i s_{i,t}) + G_{-i}(s_{i}, s_{-i}, e_{t+1})(c_{-i} e_{t+1} - 1) + 3\tau)}{18\tau} \frac{\partial G_{-i}(s_{i}, s_{-i}, e_{t+1})}{\partial s_i} (c_{-i} e_{t+1} - 1) + B(s_{i,t+1} - s_i) \right\}$$

I thus obtain a system of two equations (the two Euler), in two unknowns (the two policy functions). In each Euler equation I approximate the expectation operator using a Gauss-Hermite quadrature choosing the number of nodes equal two.\(^8\)

**Step 4: Solution of the collocation equations** I solve the collocation equations in order to find the given Chebicev nodes.

**Step 5: Computation of the basis coefficients.** Given the Euler equations evaluated at the Chebicev nodes, I find, by Newton’s method, the sixteen basis coefficients that solve for the zeros of the equations.

**B Time Consistency**

In this subsection I analyze the conditions under which assumption (A.1) holds in the static and the dynamic game; this is equivalent to say that when firms have the possibility of reoptimize their characteristics in the dynamic game, they choose an amount of characteristics that satisfies

\(^8\)A higher number of nodes does not change the results and the relative error.
assumption (A.1). In the dynamic game firms could find profitable to deviate from assumption (A.1), reducing their characteristics and hence their marginal costs such that assumption (A.1) does not hold anymore; in this occurrence the firm that deviates becomes the monopolist. This scenario however does not realize if the following assumptions hold:

(A.2) follows from the static maximization problem of the firms and guarantees that \( \Pi_i(e; g(s_1); s_i) \) is strictly increasing in \( s_i \). This assumption leads to the possibility of observing in equilibrium a characteristics level different from the lowest possible level. If the intratemporal profits were decreasing in \( s_i \) indeed, firms would offer always the lowest possible characteristics and it would never be observed characteristics changes.

(A.3) follows from (A.1) and (A.2), and relates the expected stream of profit of a firm that reduces its characteristics in order to be the monopolist in the market (L.H.S.), and the expected stream of profits of a firm always playing a duopoly in the highest possible characteristics level (R.H.S.). It is assumed that given the realization of the exchange rate, once a firm chooses a characteristics lower than the highest possible in order to get the monopoly profits \( R^m_i(s_{i,t-1}, s_{i,t-1}, e_t; s_{i,t}) \), the firm gets them for just one period but from the next period on, it receives the lowest possible characteristics duopoly profits. This is due to the reaction of the firm that has not deviate by the duopoly strategy who punish the competitor supplying the lowest possible characteristics for ever \( R^d_i(s_{i,t-1}, s_{i,t-1}, e_t; s_{i,t}) \).

Even when an exchange rate shock occurs such that assumption (A.1) does not hold anymore, the firms’ best strategy is to choose a new level of characteristics that restores assumption (A.1). Suppose that after a large depreciation of the exchange rate, foreign firm’s marginal costs are too high with respect to those of the domestic firms, to compete in the market. In order to sell a positive quantity, the foreign firm will reduce its amount of characteristics, but by (A.2) and (A.3), it will choose the highest possible characteristics level compatible with the new value of the nominal bilateral exchange rate. On the other hand, due to assumption (A.3) the domestic firm, whose marginal costs are not increased, cannot do better than choose a new characteristics level compatible with (A.1). This can be equal or lower than that of the period before.
C  Pictures

C.1  High Persistency

The plots presented in this and in the following section represent, the percentage deviation from the steady state of the characteristic, the domestic market share and the aggregate prices, with respect to a one percent exchange rate depreciation. The light blue lines represent the results obtained by allowing firms to compete in the non-price dimension. The yellow lines represent the results obtained by running the standard duopoly model in which firms compete just in prices.
C.2 Low Persistence

- Characteristics (LP)
- Expenditure Switching
- Price Index
References


