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Delegation and Coordination
in Fiscal-Monetary Policy Games
Implementation of the Best Feasible Equilibrium

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Delegation and Coordination in Fiscal-Monetary Policy Games: Implementation of the best feasible equilibrium.*

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Abstract

We show how linear contracts and/or targets on inflation and output (or deficit) implement the optimal outcomes in fiscal-monetary policy games (with rational expectations of the private sector) with no need for any binding agreements. The *centralised equilibrium* is optimal (if policymaker shares the social welfare function) and dynamically consistent. We argue for *decentralisation of policy* and study a cooperative and commitment equilibrium defined as a second best. *Commitment* alone, albeit by both authorities, can only achieve a third best and discretionary Nash equilibrium is fourth best. Both the second best and the first best (if feasible) can be implemented by simple *state-independent linear penalties* and/or *targets* defined over the macroeconomic outcomes for both authorities, imposed by a *same principal* to avoid coordination problems at the delegation stage. This is shown both in a Nash game and in a Stackelberg game (with sequential moves). The implications for policy design in the European Monetary Union are straightforward: e.g., an appropriately designed inflation target for the ECB and an SGP-like system of linear excessive deficit penalties for each fiscal authority can implement the first or second best.

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1 Introduction

In this paper we address the question of institutional design in monetary-fiscal policy games as a mean to implement the desired (socially optimal or cooperative second-best - to be defined below) economic outcomes. The analysis has straightforward implications for the design of policy institutions in the European and Monetary Union and for interpreting existing arrangements.

The main institutional and operational context is by now common knowledge. The monetary policy in the EMU is delegated to the European Central Bank (ECB), having as a precise goal the maintenance of price stability (we abstract from the ambiguities concerning the definition price stability and institutional difficulties faced by the ECB - a survey is provided by, i.a. Svensson 2000). The fiscal policies, however, remain to be conducted at each country's government level, immediately raising issues related to the way in which interactions between either monetary and fiscal policy, or individual fiscal policies take place. The Stability and Growth Pact (SGP) can be discussed in this framework as intending to strengthen the Excessive Deficit Procedure for countries inside the EMU. Its main characteristic is that it imposes sanctions to EMU countries that run a deficit larger than 3% of GDP (a full description of these issues may be found in Artis and Winkler (1998) or Buti et al. (2000).

In a purely descriptive game-theoretical language, it is easy to see that given the current institutional set-up, the monetary policy of the ECB and the fiscal policy at governmental level are decided in a non-cooperative manner. The nature (and efficiency - in the Pareto sense) of the achieved equilibrium will depend on two factors: (1) the degree of cooperation and (2) the degree of commitment to a monetary and/or fiscal rule. Absent any of these, different mechanisms acting as substitutes for them would be needed in the form of delegation to, e.g. an inflation-targeting central bank.

A terminological note is in order right at the outset. By *centralisation*, we mean that policy decisions are taken by the same authority, albeit using more than one instrument (two, in our initial case). *Cooperation* is a situation in which there is decentralisation, but the two separate authorities decide, before actually playing the game, to minimise a weighted average of their loss functions (that is we take the game-theoretical view of the term). This implies *binding commitments* and when these are not possible the game

is *non-cooperative*. *Coordination* will mean (using Canzoneri and Henderson's (1991) definition) the mechanism by which one particular equilibrium is chosen in the non-cooperative game and it can be enforced by an external player. *Delegation* would mean that an agent is chosen by the principal (with a particular compensation scheme) to play the game on its behalf. In the game we will consider delegation and coordination will be similar in a sense: delegation being done by the same principal, it is a just a mean to achieve coordination.

Using the reduced form of a model with nominal rigidities and monopolistic competition (developed in Dixit and Lambertini 2000a) we first show that centralised policy can implement what we will define as a *first best equilibrium (social optimum)* without any need to commit. Providing arguments for decentralisation, we then move to analyse a more realistic situation where there are two policymakers with different preferences and playing is non-cooperative. The main goal is to define a *second best cooperative and commitment equilibrium* as a benchmark solution and then to study non-cooperative means to achieve the optimal (or best feasible) outcome.

Commitment in this model cannot achieve much. We show that when authorities play Nash between themselves but commit with respect to the private sector, the equilibrium is far from being even second best (in contrast with Dixit and Lambertini, 2000a), though it dominates the fourth best Nash-discretionary equilibrium.

We find that delegation can implement the first or second best (where the second best is considered for feasibility reasons) when it is done by contracts or targets and prove this result both in a Nash game, where fiscal and monetary authorities move simultaneously, and in a Stackelberg game, where either of the authorities is the leader. This resembles the argument in Alesina et al. (2001) that as long as authorities 'keep their houses in order' there is no need for coordination. What we argue is that this is indeed the case as long as each authority has been assigned the appropriate house to take care of. That is because initially there is also a 'common space', and absent coordination the policymakers do not do the best job at 'keeping it in order'.

A marginal result in the leadership game is that no matter what the sequence, the Stackelberg equilibrium is dynamically consistent, i.e. the rational expectations constraint is not binding so commitment cannot improve on the discretionary equilibrium. This is in clear contrast with the 'fiscal discretion destroys monetary commitment' result of Dixit and Lambertini (2000a), as:

(i) one could say 'monetary discretion destroys fiscal commitment'; (ii) discretion does not destroy anything in this model as there is nothing commitment can additionally achieve in this game.

The main desirable feature of our delegation result is that contract or targets are state-independent (i.e. independent of the realisation of the shocks). As long as the contracts/targets are imposed by the same principal, the critique in Bilbiie (2000) targeted at delegation in multiple-principal policy games is not binding. It is worth emphasising that this preserves the independence of the two authorities and makes the day-to-day feared coordination of fiscal and monetary authorities unnecessary. These results are easily extendable to a monetary union and we do so in Bilbiie (2001b).

We proceed as follows: Section 2 presents a literature review, section 3 the reduced-form model and the centralised policy case, section 4 analyses the cooperative, Nash discretionary and Nash with commitment equilibria; section 5 presents the delegation results in the Nash simultaneous move game; section 6 deals with equilibria in the leadership game and implementing the optimum there; section 7 presents our results in an institutional context and discusses implications for policy institutions design in the EMU; section 8 summarises and concludes.

2 Literature review

The approach we use in this paper brings together a few branches of macroeconomics and due to space constraints we cannot review them in detail. First, it touches on issues of dynamic inconsistency. Secondly, it deals with externalities and cooperation in macroeconomic policy. Thirdly, it deals with delegation and institutional design to achieve a desirable outcome that eliminates both spillovers and the time inconsistency problem. Fourthly, its roots in terms of microfoundations and social welfare come from models like the one studied by Blanchard and Kiyotaki (1987) and, respectively, Woodford (1999). Fifthly, its real-world policy implications relate to the literature on monetary and fiscal policy design in the European Monetary Union.

To start with, when one considers policy interactions in the presence of the private sector that forms rational expectations two biases usually appear, making individualistic policymaking sub-optimal. First, in a world with *only*

monetary policy, as in Barro and Gordon (1983), a combination of a short-run Philips curve and a natural rate of output that is suboptimally low due, e.g., to imperfect competition, make the monetary authority try to increase the output above the natural rate by creating surprise inflation. There is a distortion of the discretionary equilibrium with respect to the commitment equilibrium (where authority commits to follow a policy rule). This is known as the *inflation bias* and there are by now some widely accepted solutions to this problem. A conservative central banker (i.e. inflation averse in the sense of Rogoff, 1985a) cannot implement the commitment outcome as it leads to suboptimal response to shocks and so do simple rules with escape clauses such as the ones considered in Lohmann (1991). Instead, an inflation contract (a linear penalty in inflation as proposed by Walsh, 1995 and Persson and Tabellini, 1993) or an inflation target (as proposed by Svensson, 1997) implement the commitment outcome by delegation in a static one-period model.¹

A second bias usually studied in the literature is *strategic interaction between or among policymakers*. This literature can be traced to Cooper (1969) and Hamada (1976), who were the first to analyse policy interactions with game theoretical tools. Generally, in a monetary policy game between countries, non-cooperative behaviour in the presence of shocks leads to Pareto sub-optimal outcomes when externalities are present. The nature of the bias will depend on the sign of the externalities. Binding agreements between policymakers would be here the ideal solution that would achieve the Pareto optimal outcome. Given that commitments are infeasible, this branch of the literature too moved to analysing solutions leading to implementation of the optimal equilibrium in a non-cooperative fashion. One branch we will not focus on here regards trigger mechanisms and reputation in repeated games (so it is supposed policymakers interact repeatedly over time). The argument relying on the Folk Theorem in repeated games is well known but so are its shortcomings (amongst them, the policymaker's horizon would have to be infinite-though there are extensions to finite horizon models, there are multiple equilibria arising and the trigger strategy should be credible - more details on applications to this field are given in Canzoneri and Henderson, 1991).

Putting together the two approaches (i.e. a time inconsistency prob-

¹In a dynamic setting, where there is persistence in output or unemployment, the contract has to be state-contingent as another type of bias appears, as shown by Lockwood, 1997. Moreover, in the same dynamic setting, a target has to be augmented with a conservative central banker in order to eliminate all the biases (for more detail see Svensson, 1997).

lem in each country and policy spillovers between countries) leads to different results regarding optimality of cooperation. Rogoff (1985b) shows that cooperation between policymakers in such a game can even be harmful, when there is a domestic commitment problem with respect to the private sector. Oudiz and Sachs (1985) obtain a somehow converse result in a different model, where commitment with respect to the private sector is counterproductive when cooperation between policymakers does not take place. Relatedly, Canzoneri and Henderson (1991) show that in a three-country world with no inflation bias, cooperation between two policymakers (and them playing Nash against the third) is even worse than pure Nash playing. All these are just different illustrations of the same general result in terms of coalitions: coalitions among a subset of players may be counterproductive if commitment with respect to the other players is infeasible. This is potentially relevant also for our problem and we shall return to this issue when we study in more detail fiscal-monetary interactions.

When both types of distortions are present, and given the inability to commit, one can again think about *non-cooperative means of achieving the optimum*. The optimum in this case would mean both cooperation between policymakers and commitment with respect to the private sector. A second mechanism to enforce the cooperative outcome here is based on institutional design and was pioneered by Persson and Tabellini (1995, 1996). The institutions could be either domestic or international. The main idea starts from the Folk Theorem for Delegation Games of Fershtman, Judd and Kalai (1991), stating that the cooperative outcome can be implemented in a decentralised manner in the game with delegation, subject to the condition that each principal is fully committed to its agent and once signed, contracts become public information. Implementation can be done by target compensation functions .

Persson and Tabellini (1995) apply this result to show how domestic institutions can be designed by delegating monetary policy to achieve the cooperative (or cooperative and commitment) optimum and how various existing policy arrangements can be reinterpreted in this framework. Persson and Tabellini (1996) apply these general results to a more specific problem, i.e. optimal monetary policy arrangements for the countries inside and outside EMU (where all the countries are however members of the EU) but based on implementation considerations they conclude that a system of symmetric zero-inflation-targets achieves a second-best solution.

In Bilbiie (2000), we show that a non-zero inflation target can implement exactly the optimal policy with cooperation and ex-ante commitment, being equivalent to the linear contracts. We then move to analysing the incentives that governments have to delegate in the first place, i.e. we analyse the sub-game perfect equilibrium contracts and targets of the game with delegation. We obtain that these are usually different from the optimal contracts and targets an international social planner would like to implement, questioning the implementation of cooperative policies by purely noncooperative mechanisms. We conclude that in the Persson and Tabellini framework, there is an implicit assumption about the ability of governments to commit and cooperate at the first stage, when delegating to the monetary authority. We feel this is not consistent with the search for noncooperative means to implement cooperative outcomes.

All the work reviewed above deals only with monetary policy, fiscal policy being ignored or assumed exogenous. However, interactions between monetary and fiscal policy have been analysed with this kind of tools. As an early example, Alesina and Tabellini (1987) analyse a one-country world where the monetary authority chooses the inflation rate, the fiscal authority chooses the tax rate and government expenditure is financed by tax revenues. Authorities have the same targets but different preference parameters over policy goals. Their main conclusion is that coordination (what we call cooperation) can be harmful in absence of commitment with respect to the private sector. We see this just as another example (similar to the 'Rogoff critique') of coalitions among subsets of players being Pareto inefficient. Debelle and Fisher (1994) consider a similar model, where the monetary authority has no preferences over government expenditures and study different types of equilibria that might arise, but all the rules they consider are state-independent.

Dixit and Lambertini (2000a), whose approach we will follow herein, study the same type of problem. In their framework, fiscal policy is introduced in a Barro-Gordon type of model. That is, there is a type of short-run Philips curve but fiscal policy can increase output through, e.g., a production subsidy but it also influences inflation. This structure hints at a time consistency problem, as the output objectives are indeed larger than the natural rate (due to monopolistic competition, in their microfounded model). The two authorities differ in both the values of the economic goals and the parameters in the loss function (with the monetary authority being assumed as

more conservative). They show that the Nash Equilibrium is Pareto inefficient (which is not surprising given existence of externalities) and that a form of leadership (first-mover advantage) by any of the authorities improves on the Nash (which, again, is not very surprising but is also problematic due to the inability of the authorities to commit and the ambiguity of the authorities' incentives to act as leader or followers). Their central argument is that the value of monetary commitment is completely negated in the presence of fiscal discretion as the kind of equilibrium that is achieved is exactly equivalent to monetary leadership (i.e. the rational expectations constraint is not binding). Another emphasised result is that what they call optimum is achieved when authorities agree on targets' levels.

We will start from the same microfounded model presented in Dixit and Lambertini (2000a). However, we will try to look more carefully at what optimal monetary and fiscal policy mean in this framework and then argue with their results as we described in the introduction. We study incentives to delegate and decentralise policy and given that, incentives of each authority to deviate from what we will call the optimal policy. Defining the optimal cooperative equilibrium and supposing the game is non-cooperative we will further study non-cooperative means of achieving the optimum. These would take the form of delegation and imposing penalties on the authorities, both in the Nash and in the Stackelberg game. The main result here would be that delegation can eliminate the distortions that give rise to deviations from optimality and no formal cooperation would be needed should these be implemented. The main advantage of these schemes would be their simplicity, as they are state-independent.

The type of arrangements we propose here can be easily related to the Stability and Growth Pact. The existence of the SGP was addressed by some researchers in the context of monetary-fiscal policy coordination. The type of restrictions imposed by the SGP (concluded at the Amsterdam Summit in 1997) were criticised by, i.a., Buiter et al. 1993, as imposing too severe restrictions on countries whose economies are in recession (and usually run a high deficit) and would just aggravate their economic situation. Beetsma and Jensen (2000) show how a stability pact can eliminate the exacerbation of debt accumulation occurring in a monetary union and the Buiter et al. critique is solved by making the sanctions contingent on the observed state of the economy. However, moral hazard appears due to unobservability of the polit-

ical cost of the fiscal effort which directly influences the state of the economy. They view the strictness of the SGP in allowing sanctions waivers as being directly related to this moral hazard problem. An earlier modeling of the SGP was pursued by Beetsma and Uhlig (1999), who show the overburden on future monetary policy induced by excessive debt accumulation (in a two-period model) which occurs due to the inability of the governments to internalise the effect of public debt accumulation on the common inflation rate. Their conclusion is that a pact that limits the accumulation of debt is desirable. This, however, is argued to depend on commitment ability of the monetary authority by Chari and Kehoe (1998); more specifically they argue that the SGP would just act as a substitute for commitment of the monetary authority. Buti et al. (2000) develop a stylised theoretical model encompassing a game between the monetary and fiscal authority (where the central bank is Rogoff conservative) in which they also include the rule imposed by the SGP, arguing that it leaves room for fiscal stabilisation, although it imposes budgetary restrictions. The strategic complementarity degree of the two policies is found to be state-contingent. They find (for both one fiscal authority and many fiscal authorities) that a deficit bias arises under non-cooperation, while under cooperation there is a deficit bias and an inflation bias (which disappear in the unlikely case that the government pursues only 'cyclical stabilisation'). We will try to argue with this point in a more formal setup in a subsequent section.

Finally, we would like to contrast our view and results with the opinions expressed in Alesina et al. (2001) in section 2, 'Monetary and fiscal policy coordination in EMU'. Their answer to the question 'Is there a need for (macroeconomic policy) coordination in EMU?' is 'No' (p. 8). Bluntly said, our answer, subject to the limitations of the model we use, is: 'cooperation IS optimal when defined appropriately and can be implemented by simple institutional arrangements that make the much feared pressures on the ECB by fiscal authorities unnecessary and, moreover, impossible'. We argued in more detail with this approach in Bilbiie (2001a) and will not reformulate the arguments here. We perform this exercise in section 7 in more detail.

3 The Model

Our modelling approach follows initially Dixit and Lambertini (2000a) in that it uses the reduced forms derived there from a fully microfounded model with monopolistic competition of the Blanchard-Kiyotaki (1987) type and staggered price setting as in, e.g. Woodford (1999). Monopolistic competition gives rise to a natural rate of output that is suboptimally low, whereas nominal rigidities imply there is a role for policy. The reduced form of this type of model consists of a short-term Philips curve and an equation for inflation as a function of both monetary and fiscal policy. These can be thought of as log-linearisations around the steady state in the microfounded model, thus holding close to the steady state.

$$y = af + b(\pi - \pi^e) - \varepsilon \quad (1)$$

$$\pi = m + cf + v \quad (2)$$

y represents deviation of real output from the natural rate (normalised for convenience to zero, hence $\bar{y} = 0$) and π is the inflation rate. There are two policy instruments: m is the monetary policy instrument and f is the fiscal policy instrument. m can be thought of as the monetary base as an increase in m indicates a more expansionary policy. An increase in f means a more expansionary fiscal policy, that is a subsidy or a reduction in taxes, or an increase in deficit. a, b, c are parameters and are not stochastic, as in Dixit and Lambertini. We decided to add the two stochastic shocks ε and v for tractability of the model for the problem at hand. Non-linearity in the stochastic shocks does not bring any insights for the problem we address but would just complicate the algebra. We will examine implications of this hypothesis when analysing the results. a represents the direct effect of fiscal policy and in the microfounded model of Dixit and Lambertini is a production subsidy that counterbalances the distortion created by imperfect competition that makes the natural rate too low. This would mean an expansionary effect (abstracting from deadweight losses for the moment). However, at least algebraically the possibility of crowding out is allowed, since we can also have $a < 0$. $b > 0$ is of course the usual coefficient of surprise inflation in aggregate supply. c is the impact of fiscal policy on inflation and can have either sign. A

subsidy as the one described earlier would stimulate supply and reduce prices. However, a demand-side fiscal expansion could lead to an increase in prices. We allow for both signs of c but do, however, maintain the assumption that fiscal policy has an overall expansionary effect on output, i.e. $a + bc > 0$.

The shocks ε and v are supposed to be independently and identically distributed, of zero mean and constant variance. The first is an adverse supply shock and the second a velocity shock (an aggregate demand shock). We denote the vector of shocks $\omega = (\varepsilon, v)$ and $\tilde{\omega} = (\tilde{\varepsilon}, \tilde{v})$ a realisation of the shocks. The private sector forms expectations rationally before shocks ω are realised and policies implemented (we will spell out the timing more explicitly later). The rational expectations rule is:

$$\pi^e = E_\omega [\pi(\omega)] \equiv \int \pi(\omega) d\Phi(\omega) \quad (3)$$

Expected inflation is the expected value of inflation taken over the joint distribution (which is equal to the product as shocks are independent) of the stochastic shocks $\Phi(\omega)$. Note that inflation is a function of the shocks as policies are implemented and thus outcomes realised after observing the shocks.

Society's preferences can be derived from maximisation of the representative agent's welfare in line with Woodford (1999) and this is done in Dixit and Lambertini. Here we suppose these preferences are defined over inflation and output deviations and the loss function of society is given by:

$$L^* = \frac{1}{2} \left[\lambda^* (y - y^*)^2 + (\pi - \pi^*)^2 \right] \quad (4)$$

The interpretation is standard: λ^* indicates society's preferences over output deviations versus inflation, π^* is the socially desirable level of inflation (equal to the average level of pre-set prices) and y^* is the socially optimal output level, and we suppose it is greater than zero as more output than the natural rate is desirable due to the monopolistic distortion. Suppose for the moment that there are no deadweight losses, there is no decentralisation of policy and no delegation.

3.1 Optimal and dynamically consistent centralised policy

An authority minimising the loss function in eq.4 can achieve the socially ideal levels of output and inflation (y^*, π^*) . In doing so, if its commitment is fully credible, that is there is no dynamic inconsistency (alternatively, the Nash Equilibrium is also subgame perfect as it is the unique one!). The derivation can be found in Appendix A. Intuitively, there are two policy instruments (control variables) and two targets, so the system can be controlled perfectly to achieve the objective for any realisation of shocks. We will call this an ideal equilibrium, though rigorously it would be dominated by an equilibrium of the form (\bar{y}, π^*) , implying elimination of monopolistic distortions.

The result means that in this model there is no need for commitment or inflation-conservativeness as there is no inflation bias in the discretionary equilibrium! The authority (or two authorities sharing the same loss function (4)) does not need to create surprise inflation to increase output above the natural rate. This is substituted in this model by having a second policy instrument, namely the fiscal policy, with which it can counter the monopolistic distortion. The perfect credibility of commitment can also be trivially seen by solving for the commitment equilibrium (whereby authority not only minimises (4) but also takes as a constraint (3), i.e. commits with respect to the private sector to follow a state-contingent policy rule $m^*(\omega, \tau), f^*(\omega, \tau)$ -not reported in appendix). This of course delivers the same equilibrium, and the Lagrange multiplier on the rational expectations constraint (3) is zero. Intuitively, the discretionary Nash equilibrium is also subgame perfect as it is unique, the constraint is not binding and the authority has no incentive to cheat (no time inconsistency problem).

3.2 Arguments for decentralisation

This section is aimed at providing some arguments for decentralisation in a world in which it seems that centralised policy can achieve the first best with no commitment. The main arguments are: (i) institutional constraints for modelling (i.e. if real policy is what we want to model then we are bound to take decentralisation as a constraint) and (ii) political economy reasons that

make the targets of the centralised authority different from the social optimum and thus centralisation undesirable.

Dixit and Lambertini (2000a, section 3.1), instead argue for delegation in a model in which fiscal policy has no effect on inflation (i.e. $c = 0$). But that is a totally different model as it ignores spillovers between the two authorities and it is natural that in a Barro-Gordon type model one obtains a need for delegation. Arguing for decentralisation in a model where fiscal policy does not influence inflation is by now trivial. Analysing 'strategic interactions' then in a model in which fiscal policy does influence inflation (which, we showed, makes delegation apparently unnecessary) but monetary policy has been delegated seems unrigorous.

However the centralised equilibrium is an ideal and far from realistic situation. A straightforward relaxation is that the fiscal policy has deadweight losses associated with it. This would imply an additional linear, say, penalty in (4) for the authority as changing the fiscal instrument would be costly (this is the way it is modelled in Dixit and Lambertini). We choose to model deadweight losses as thinking about the authority minimising a different loss function, where the output target is $y^F \leq y^*$. There are more relaxations we wish to allow as the implications of the simple set-up above are not very comforting and do not conform with what we see in real life. Though here there is no argument for delegating monetary policy (unless fiscal policy is absent, but that would be a different model) we do see independent central banks in real life. There is quite a consensus on central bank independence (instrument independence, following DeBelle and Fisher, 1994) being associated with low inflation empirically and we will not insist on that. Thus, there might be the case that the uselessness of delegation is a feature of the model. The key assumption here is that fiscal policy increases output above the natural rate without a need for surprise inflation, which gives rise to the result above. What the model does not capture, for example, are political economy reasons that make the targets different from the social optima. Moreover, present deadweight losses, the authority will want to expand output more than the achievable level that would in this case be y^F (just replace y^* with y^F in Appendix A).

The main argument for decentralisation would be that, if the authority can implement the socially desirable equilibrium (y^*, π^*) , it is equally true that it can implement any desired equilibrium, as policymakers are short-

sighted and have short-term goals which might or, more likely, might not be the same as society's. For example, consider a policymaker comes into office and has preferences given by $L^G = \frac{1}{2} \left[\lambda^G (y - y^G)^2 + (\pi - \pi^G)^2 \right] \neq L^*$. By the same argument as before, he can implement the equilibrium (y^G, π^G) with no credibility problem. Decentralisation might be a solution to exactly this type of problems but that of course implies further modelling, which we leave for future research.

Henceforth, the institutional set-up we consider consists of two authorities: a monetary authority choosing m and a fiscal authority choosing f . The loss functions of the two authorities differ from the social loss function and between themselves, and we like to impose as much heterogeneity as possible. Let the two losses be given by:

$$L^F = \frac{1}{2} \left[\lambda^F (y - y^F)^2 + (\pi - \pi^F)^2 \right] \quad (5)$$

$$L^M = \frac{1}{2} \left[\lambda^M (y - y^M)^2 + (\pi - \pi^M)^2 \right] \quad (6)$$

Interpretation is as before, now each parameter or variable with a superscript being specific for the respective authority (M for monetary, F for fiscal). There is heterogeneity in both target levels and preference parameters and we do not wish to impose any restrictions on the goals as all the biases in this model come from this heterogeneity of goals (between authorities and with respect to society).² The only restriction is that the output target of the fiscal authority is the one described above (i.e. less than society's due to dead-weight losses). This model encompasses all the possible combinations whereby the authorities agree on targets or any of them shares the same target(s) as society. This decentralisation will be a potential source for inefficiencies. This is no argument for centralising policymaking, as this would bring further inefficiencies not captured by the model and is impossible due to the shortly mentioned reasons above. Instead, the challenge is to find the institutional

²It could be assumed that monetary authority cares more about inflation than the fiscal (i.e. $\lambda^M \leq \lambda^F$) and/or it has been delegated with an inflation target smaller than the society's, i.e. $\pi^M \leq \pi^*$ but that would just be a particular case of our analysis.

mechanisms through which the optimal solution can be implemented without giving up Central Bank independence, decentralisation and even national sovereignty in a multi-country model studied elsewhere (Bilbiie, 2001b).

4 Optimal policy, Nash equilibrium and incentives to deviate

The game we study throughout the paper is the following: there are two authorities (monetary and fiscal) with the loss functions given by (4) and (5). There is also a private sector forming expectations about the inflation rate (e.g. incorporating them into binding contracts-wage contracts, for example) and the structure of the economy is given by the reduced forms in (1) and (2). The stochastic part of the world is chosen by nature, which leads to the realisation of shocks ω . The timing of the game is: (i) targets $\tau \equiv (y^F, \pi^F, y^M, \pi^M) \equiv (\tau^F; \tau^M)$ are revealed; (ii) expectations π^e are formed; (iii) shocks ω hit the economy; (iv) authorities choose their respective instruments m, f and they do so simultaneously (if not specified otherwise); (v) equilibrium outcomes $(y(\tau, \omega), \pi(\tau, \omega))$ are realised.

4.1 Cooperative and commitment equilibrium

As a benchmark and purely hypothetical solution we consider the case where authorities decide, before stage (i) above, to cooperate and commit ex-ante with respect to the private sector (we shall relax this further). Thus, they commit to implement state-contingent policy rules $m^c(\tau, \omega), f^c(\tau, \omega)$ that minimise the expected value of a joint loss function

$$L^A = \alpha L^M + (1 - \alpha) L^F \quad (7)$$

taking as a constraint (3).

α and $1 - \alpha$ can be regarded as the Pareto weights in a linear social welfare function or as coefficients indicating the bargaining powers of the two

authorities³. We emphasise the result here by stating it in Lemma 1:

Lemma 1 *There exists a unique **dynamically consistent** cooperative and commitment equilibrium of the monetary-fiscal policy game whereby authorities implement the state-contingent rules $m^c(\tau, \omega), f^c(\tau, \omega)$ that deliver the outcomes $(\pi^c, y^c), \forall \tilde{\omega}$:*

$$\begin{aligned}\pi^c &= \alpha\pi^M + (1 - \alpha)\pi^F \\ y^c &= \frac{\alpha\lambda^M y^M + (1 - \alpha)\lambda^F y^F}{\alpha\lambda^M + (1 - \alpha)\lambda^F}\end{aligned}$$

Equivalent ways to state dynamic consistency are that this is the same as the cooperative discretionary equilibrium, or the rational expectations constraint is not binding, or the Nash Equilibrium is Subgame Perfect as it is the unique one, or there is no incentive to cheat.

Proof. Please find appendix B. ■

Note that the first order conditions of the authorities after eliminating expectations are (where derivation is in Appendix B), $\forall \tilde{\omega}$ a realisation of ω :

$$\alpha b\lambda^M (y - y^M) + (1 - \alpha)b\lambda^F (y - y^F) + \alpha(\pi - \pi^M) + (1 - \alpha)(\pi - \pi^F) = 0 \quad (8)$$

$$\alpha(a + bc)\lambda^M (y - y^M) + (1 - \alpha)(a + bc)\lambda^F (y - y^F) +$$

$$+ \alpha c(\pi - \pi^M) + (1 - \alpha)c(\pi - \pi^F) = 0 \quad (9)$$

Hence, just by cooperating and without any need to commit, the two authorities can achieve a linear combination of their two bliss points, depending on the bargaining coefficients. We will call this the *cooperative optimum* or

³One could think of L^A as the logarithm of a Nash bargaining product but we do not model bargaining explicitly here

second-best equilibrium. It might coincide with the first best⁴ described above for specific parameters of the loss functions. For example, given that $\pi^M \leq \pi^*$ and $\pi^F \geq \pi^*$ the social optimum can be achieved for certain values of α .

Intuitively, there is no need to commit with respect to the private sector as again there is no time inconsistency in the cooperative discretionary equilibrium. This happens as aggregating the loss functions means in fact aggregating also the target levels and hence what the authorities do is to use two policy instruments to achieve two targets⁵.

There is also a direct correspondence in terms of policy rules in the two cases (centralisation and cooperation), which can be easily observed by comparing expressions A3 and B10. The two rules imply the same response to shocks and same elasticity coefficients in equilibrium outcomes.

The case we considered here is hypothetical and is in essence the same as the centralised policymaking case. Our goal, however, is to find the mechanisms that insure achieving the second best cooperative equilibrium when the two authorities play non-cooperatively, that is given decentralisation and heterogeneity in loss functions. To do that, we need to look at the Nash equilibrium and at the incentives of authorities to deviate. This is more realistic as in practice binding agreements are improbable (again Central Bank independence being one of many arguments as to why this happens) and even undesirable. After all, if the two policymakers meet and cooperate, the equilibrium has similar features to that in the centralised case. This gives them full power in selecting the equilibrium as no commitment is needed, nothing prevents them from choosing a 'bad' equilibrium resulting from short-term political goals.

4.2 Nash equilibrium and discretionary policy

Suppose, more realistically, that the two authorities do not cooperate in the policy-design stage (i.e. before stage (i) in our game) and play non-cooperatively

⁴At most in one policy dimension if we think fiscal policy has deadweight losses ($y^F \leq y^*$) and monetary authority has an even smaller output target than fiscal.

⁵A marginal thing to note is that what Dixit and Lambertini (2000) call fiscal-monetary policy 'symbiosis' is just a very special case of this result. In fact, when two authorities share the same target levels they can be thought of as one single authority (as the inflation aversion parameters do not matter in this type of equilibrium).

both with respect to each other and the private sector. This is bound to lead to inefficiencies and we expect them to arise from two sources: one is non-internalisation of externalities between the two authorities; the other is non-internalisation of externalities on private sector's expectations. The second one appears as in decentralised non-cooperative policymaking there is also a credibility problem that was not apparent before. This will become clearer later.

The timing of the game is the same as before and at stage (iv) the authorities move simultaneously. Each authority solves $\min_m E[L^M]$ and $\min_f E[L^F]$, both taking π^e as given. This equilibrium is studied in Dixit and Lambertini (2000a) and solving the formulated problem straightforwardly leads to the two equilibrium outcomes (where derivation is in Appendix C for completion), $\forall \tilde{\omega}$:

$$\begin{aligned}\pi^{NE} &= \frac{[(a+bc)\lambda^F\pi^M - bc\lambda^M\pi^F] + b(a+bc)\lambda^F\lambda^M(y^F - y^M)}{(a+bc)\lambda^F - bc\lambda^M} \quad (10) \\ y^{NE} &= \frac{c(\pi^F - \pi^M) + (a+bc)\lambda^F y^F - bc\lambda^M y^M}{(a+bc)\lambda^F - bc\lambda^M}\end{aligned}$$

We will call this equilibrium *fourth best* (as would arguably be dominated by any commitment or leadership - see below). It is also useful to note the first order conditions of this problem, holding $\forall \tilde{\omega}$:

$$b\lambda^M (y - y^M) + (\pi - \pi^M) = 0 \quad (11)$$

$$(a+bc)\lambda^F (y - y^F) + c(\pi - \pi^F) = 0 \quad (12)$$

Comparing this with the second-best cooperative outcomes in Lemma 1 we see they are different. The nature of the inefficiency of the Nash Equilibrium depends on the relationship between the targets of the authorities, τ^F and τ^M . The source of the inefficiency can be seen more clearly when comparing the two sets of first order condition in the cooperative and Nash case, respectively given by the systems (8,9) and (11, 12). There are two types of

distortions present here. One is obviously due to the fact that policymakers ignore the effects their own instruments have on the other's loss functions. Thus, the second and the fourth term of eqs 8 and 9 are absent in the Nash conditions. Moreover, each policymaker reacts too much to deviation of variables from his own targets (11 and 12 as compared to first and third term in 8 and 9). The second distortionary source, however, comes from non-commitment. Commitment would make a difference in the non-cooperative simultaneous-move game and we study this equilibrium below. The two externalities (on each other and on the private sector) combine in creating the inefficiency of the Nash Discretionary equilibrium. To see the difference from the second best cooperative equilibrium more clearly, we rewrite the first-order conditions for cooperation, 8 and 9, like:

$$\begin{aligned}
b\lambda^M (y - y^M) + (\pi - \pi^M) &= (1 - \alpha)b (\lambda^M - \lambda^F) y + (1 - \alpha)(\pi^F - \pi^M) + \\
&\quad + (1 - \alpha)b(\lambda^F y^F - \lambda^M y^M) \\
(a + bc)\lambda^F (y - y^F) + c(\pi - \pi^F) &= \alpha(a + bc) (\lambda^F - \lambda^M) y + \alpha c(\pi^M - \pi^F) + \\
&\quad + \alpha(a + bc)(\lambda^M y^M - \lambda^F y^F)
\end{aligned}$$

Here we see that in both equations the first term is related to the incentive to expand output above the natural rate which has a coefficient proportional to each authority's ability to do it (b for monetary and $a + bc$ for fiscal) and to the difference in output preference coefficients. Second incentive comes from difference in the output desired goals themselves and is related both to the inflation bias and individualistic policymaking. Third term comes from difference in inflation goals and generates a distortion due to non-cooperation purely. It does so for the fiscal authority depending on its impact on inflation. For each authority the incentives are proportional to the other's weight in the aggregate loss function (which again shows non-internalisation of externalities in the Nash Equilibrium).

4.3 Nash Equilibrium with commitment

When policymakers cooperate, they have no time inconsistency problem, so commitment and discretionary equilibria are the same. However, when they play Nash against each other, commitment does make a difference as part of the inefficiency of the Nash discretionary equilibrium above comes from non-commitment. Thus, authorities have an incentive to cheat the private sector

to improve on the Nash outcomes. This can be seen by solving for the Nash equilibrium with commitment. Each authority minimises the expected value of its own loss function but also takes as a constraint the rational expectations of the private sector given by (3), i.e. it internalises effects of its actions on expectations but not externalities on the other authority. The result is stated in Lemma 2

Lemma 2 *There exists a unique equilibrium of the monetary-fiscal policy game with commitment with respect to the private sector, whereby authorities implement state-contingent rules delivering the outcomes (y^{nc}, π^{nc}) given $\forall \tilde{\omega}$ by:*

$$\begin{aligned}\pi^{nc} &= \pi^M \\ y^{nc} &= y^F + \frac{c}{a\lambda^F} (\pi^F - \pi^M)\end{aligned}$$

Proof. Please find Appendix D ■

It is also useful to note the first order conditions (again, $\forall \tilde{\omega}$) after accounting for expectations and rearranging:

$$b\lambda^M (y - y^M) + (\pi - \pi^M) = b\lambda^M (y^F - y^M) + \frac{cb\lambda^M}{a\lambda^F} (\pi^F - \pi^M) \quad (13)$$

$$(a + bc)\lambda^F (y - y^F) + c(\pi - \pi^F) = \frac{bc^2}{a} (\pi^F - \pi^M) \quad (14)$$

We can easily see that commitment of both authorities is quite far from achieving the second best outcome (as argued in Dixit and Lambertini, 2000a), let alone the first best. The intuition is that in choosing its own policy authorities ignore spillovers on each other, though not on the private sector. When monetary authority does not recognise that by cooperating it could increase output above the natural rate by use of the fiscal instrument, it has to resort to surprise inflation to try and achieve y^M . But fiscal authority also ignores the influence of m on its loss function.

The difference is easily seen when looking at the first order conditions above and comparing them with the Nash discretionary first order conditions. The same expressions on the left hand side of eqs 11, 12 are now equal, in

eqs 13, 14 with a quantity that is different from zero when we allow for heterogeneity in targets (otherwise we are back in the centralised policy case). The additional terms, however, are not enough to eliminate all the incentive constraints present in the Nash discretionary equilibrium and implement the cooperative second-best (this is seen by comparing 13, 14 with 8 and 9). We note that the incentives to deviate coming from discretionarity, which we found in the Nash discretionary case, disappeared now (e.g. there is no term depending on y on the right hand side).

Here the monetary authority can achieve its bliss point inflation rate and the fiscal authority can expand output above the suboptimal level with dead-weight losses, y^f (that is if it has an inflationary effect, there is no crowding out and $\pi^F > \pi^M$), the difference being dependent on the level of disagreement upon inflation targets.

Having the same inflation targets would insure implementation of the monetary bliss inflation and fiscal bliss output, but this would be both sub-optimal and time inconsistent. Authorities could do better by cooperating as described in Lemma 1 and would have no need to commit (or if they do commit, they would face no credibility problem). In that situation they would achieve the second best by internalising the externalities they impose on each other when pursuing independent policy.

5 Implementing the cooperative second-best and the centralised first best non-cooperatively and with discretion

The results in this section are, to the best of our knowledge a first attempt to study the problem of delegation in monetary-fiscal policy games⁶. The optimality (or second-best nature, that is optimality given systematic distortions) of the cooperative equilibrium, combined with either the inability or even the undesirability of policymakers to form binding agreements raises the issue of implementing the desired equilibrium non-cooperatively. Alternatively, one can think of implementing the socially optimal equilibrium (y^*, π^*) . Technically, this is the same but we present both in parallel. We choose to study

⁶We formulated the problem in Bilbiie (2001a) but did not solve for optimal delegation parameters.

implementation of the second best as well since implementation of the first best would again be contingent upon the principal sharing the social welfare function, L^* and there might be dissatisfactions with this.

As we abstract here from the repeated nature of the game, we focus on institutional design, or delegation. Institutions do matter in this framework and we show how they can be designed to achieve the desired outcome in terms of macroeconomic outcomes. This section draws on the microeconomic literature on contracts and its applications to other policy questions. For the latter, the most representative references are Persson and Tabellini (1995, 2001) for two principal-two agents international policy games, and Walsh (1995) or Svensson (1997) for delegation in monetary policy games at one-country level.

In our specific context, we think about the monetary and fiscal policymakers as being two independent policy agencies. That is, they do act as agents, but in contrast with the work in Persson and Tabellini (1995) or Bilbiie (2000), delegation is done by the same principal. This preserves decentralisation and independence of the two policymakers. We think of the principal as being the authority to which both policymakers are accountable (e.g., the Parliament) but further political economy models can deal with this issue.

Of course one can argue that if the same principal makes the delegation, then he could do the same job just by centralising policy. We view the main advantages of our contracts and targets as being simplicity to implement and enhancement of credibility. Moreover, this form of implementing the optimum preserves independence of the two authorities and prevents, e.g. ex-post accommodation of monetary policy to fiscal shocks. This becomes more important in a model with many fiscal authorities of the type we study in Bilbiie (2001b), where national sovereignty in conducting fiscal policy is needed.

We anticipate in saying that this will make our early critique of the applications of the Folk Theorem in Delegation Games (see Fershtman et al., 1991) non-binding. When there are two principals delegating to two agents by contracts, implementation of the Pareto Optimum in a purely non-cooperative manner is not insured. The contracts implementing the optimum are not subgame perfect, i.e. are not chosen if we consider the principals play Nash in the delegation game, when choosing contracts. The subgame perfect contracts will be different from the optimal ones, hence the cooperation problem is relocated at the principals' level. We proved this in Bilbiie (2000) for international monetary policy games with domestic credibility problems.

The game described earlier is thus modified only in one respect: at stage 0, there is a principal imposing a form of delegation to both authorities (i.e. distorting their loss function) by contracts or targets (we will describe these in more detail later). He can select as an equilibrium in the non-cooperative monetary-fiscal policy game the second-best or first-best outcome. The result is stated in Proposition 3 (where D superscript denotes 'delegated').

Proposition 3 *Both second-best cooperative equilibrium and the first best social optimum can be implemented in the non-cooperative monetary-fiscal policy game by imposing **linear contracts** on the two authorities, distorting their loss functions in the form:*

$$\begin{aligned} L^{FD}(y, \pi, \tau, \omega) &= L^F(y, \pi, \tau, \omega) + t^F(\tau, \omega) y(\tau, \omega) \\ L^{MD}(y, \pi, \tau, \omega) &= L^M(y, \pi, \tau, \omega) + t^M(\tau, \omega) \pi(\tau, \omega) \end{aligned}$$

Both the contracts implementing the cooperative equilibrium (y^c, π^c) and the social optimum (y^, π^*) are state-independent and have the marginal penalty (or reward) given, respectively for each authority and $\forall \tilde{\omega}$ by:*

$$\begin{aligned} \widehat{t^M}(\tau, \omega) &= \widehat{t^M}(\tau) = \frac{b(1-\alpha)\lambda^M\lambda^F}{\alpha\lambda^M + (1-\alpha)\lambda^F} [y^M - y^F] + (1-\alpha) [\pi^M - \pi^F] \\ \widehat{t^F}(\tau, \omega) &= \widehat{t^F}(\tau) = \frac{\alpha\lambda^M\lambda^F}{\alpha\lambda^M + (1-\alpha)\lambda^F} [y^F - y^M] + \frac{\alpha c}{a+bc} [\pi^F - \pi^M] \end{aligned}$$

$$\begin{aligned} t^{M*}(\tau, \omega) &= t^{M*}(\tau) = b\lambda^M [y^M - y^*] + \pi^M - \pi^* \\ t^{F*}(\tau, \omega) &= t^{F*}(\tau) = \lambda^F [y^F - y^*] + \frac{c}{a+bc} [\pi^F - \pi^*] \end{aligned}$$

Proof. Please find Appendix E. ■

The marginal penalties in Proposition 3 have some desirable properties. First, they are state-independent, which increases the ease of implementation (state-dependent contracts or targets are usually criticised for implementation reasons). Their state-independence shows once more that in the original game there is only one distortion, namely the non-cooperation between the two authorities. Usually, as a rule of thumb, state-dependent contracts appear when there are more than one distortion to counteract: that happens in Lockwood (1997) for inflation contracts with an inflation bias and persistence in output or in Persson and Tabellini (1995, 1996) for inflation bias and

spillovers between policymakers. Here, there is a need for penalties only if there is no agreement on target levels (otherwise contracts would be naturally zero as bliss points are achieved anyway).

Interpretation of the contracts depends, of course, on the parameters of the model and on the relationship between τ^F and τ^M . For example, the marginal 'penalty' on inflation deviations for the central bank is in fact a reward if, but not only if, $y^M < y^F$ and $\pi^M < \pi^F$. That is, the principal has to bribe the central bank to induce more inflation in order to get the cooperative second best, as in this case there is a deflationary bias of the Nash equilibrium. It has to do so since otherwise the central bank can obtain its bliss point inflation $\pi^{nc} = \pi^M$ (by committing to the private sector and playing Nash with the fiscal authority) but this is suboptimally low when compared to π^c .

Under the same assumptions, the marginal penalty on the fiscal authority is positive if fiscal policy has inflationary effects: too much fiscal expansion has to be tempered. However, in the particular case where $c < 0$ and the second term is large enough in absolute value to dominate the first, this can be a 'reward' too. The intuition is that in the Nash Equilibrium the fiscal authority could do better by expanding more but it does not recognise this as it does not take into account its deflationary effects (strategic complementarity), i.e. it disregards externalities on monetary authority.

When the *first best equilibrium* is implemented, contracts are easier to interpret. For the monetary authority there is a penalty for additional inflation if its targets are bigger than the social optima. The penalty on excessive inflation is one-to-one and the one on excessive output is proportional to the coefficient of output stabilisation and to the coefficient on surprise inflation in aggregate supply. That happens as the spillovers from monetary to fiscal policy come from the former influencing inflation directly (one-to-one - see eq 2) and output through surprise inflation (depending on b and λ^M). This second term also leads to elimination of the inflation bias which is present in Nash discretionary policymaking. For the fiscal authority interpretation is similar. The marginal penalty depends on additional inflation and the coefficient depends proportionally on c , fiscal policy's direct spillover on monetary policy through inflation and inversely on $a + bc$, as a larger capacity to expand output increases strategic substitutability. It also depends on more than optimal output and this is of course related to the weight on output stabilisation.

An equivalent way to implement the cooperative or centralised equilibria is through delegating with targets, where there is no linear penalty imposed, instead the loss functions of the authorities are distorted in their bliss points⁷. This might be more interesting as it does not involve pecuniary rewards or penalties, hence it should not suffer the political problems⁸ usually associated with implementing contracts of the type studied before. The result is stated in Proposition 4 (where T superscript denotes 'target delegation').

Proposition 4 *Both the second-best cooperative equilibrium and the first-best social optimum can be implemented in the non-cooperative monetary-fiscal policy game by imposing **targets** on the two authorities, distorting their loss functions in the form:*

$$\begin{aligned} L^{FT}(y, \pi, \tau, \omega) &= \frac{1}{2} \left[\lambda^F (y - y^T)^2 + (\pi - \pi^F)^2 \right] \\ L^{MT}(y, \pi, \tau, \omega) &= \frac{1}{2} \left[\lambda^M (y - y^M)^2 + (\pi - \pi^T)^2 \right] \end{aligned}$$

where the two targets implementing the cooperative equilibrium are state-independent (i.e. $\forall \tilde{\omega}$) of the form:

$$\begin{aligned} \pi^T &= \hat{\pi}(\tau, \omega) = \hat{\pi}(\tau) = \alpha \pi^M + (1 - \alpha) \pi^F + \frac{b(1 - \alpha) \lambda^M \lambda^F}{\alpha \lambda^M + (1 - \alpha) \lambda^F} [y^F - y^M] \\ y^T &= \hat{y}(\tau, \omega) = \hat{y}(\tau) = \frac{\alpha \lambda^M y^M + (1 - \alpha) \lambda^F y^F}{\alpha \lambda^M + (1 - \alpha) \lambda^F} - \frac{\alpha c}{\lambda^F (a + bc)} [\pi^F - \pi^M] \end{aligned}$$

The optimal targets implementing the first best centralised equilibrium are:

$$\begin{aligned} \pi^T &= \pi^{M*}(\tau, \tau^*, \omega) = \pi^{M*}(\tau, \tau^*) = \pi^* - b \lambda^M [y^M - y^*] \\ y^T &= y^{F*}(\tau, \tau^*, \omega) = y^{F*}(\tau, \tau^*) = y^* - \frac{c}{\lambda^F (a + bc)} [\pi^F - \pi^*] \end{aligned}$$

Proof. Please find Appendix F ■

First look at the targets implementing the second best. State-independentness appears again as a desirable features for the motives described above. Interpretation of the targets is straightforward if we also note that using Lemma 1

⁷Svensson (1997) was the first to show this in Barro-Gordon one-country model. In Bilbiie (2000) we apply the same idea to international monetary policy games.

⁸A synthesis of implementation problems related to inflation contracts is provided by Svensson (1997).

we can write them as:

$$\hat{\pi} = \pi^c + \frac{b(1-\alpha)\lambda^M\lambda^F}{\alpha\lambda^M + (1-\alpha)\lambda^F}[y^F - y^M] \quad (15)$$

$$\hat{y} = y^c - \frac{\alpha c}{\lambda^F(a+bc)}[\pi^F - \pi^M] \quad (16)$$

The inflation target with which monetary policy is delegated to the central bank is higher than the cooperative optimum (constrained optimum, that is) to eliminate the deflationary bias of the Nash Equilibrium present when $y^F \geq y^M$. It depends proportionally on the degree of disagreement about the output ideal levels of the authorities, since this is what generates the deflationary bias. It also depends proportionally on the effect of surprise inflation on output, b , and the bargaining power (or weight in the aggregate loss function) of the fiscal authority. Quite naturally, if any of these is zero, the inflation target is just the cooperative equilibrium level of inflation.

Under the same assumption of $\pi^F \geq \pi^M$, say the output target of the fiscal authority is lower than the cooperative level if fiscal policy is a strategic complement of monetary policy in inflation ($c < 0$) and higher if they are strategic substitutes ($c > 0$). For strategic complementarity, fiscal expansion reduces inflation but also increases output. Thus, it would increase the deflationary bias of the Nash equilibrium in the non-cooperative game when trying to increase output. The magnitude of this is dependent on its preferred inflation level, π^F . If policies are strategic substitutes in inflation, then expansionary fiscal policy has inflationary effects apart from output-expansionary effects. The output target of the fiscal authority has then to be higher than the optimum, since it would eliminate the deflationary bias of the Nash Equilibrium. The amount by which the target differs from optimal output is of course proportional to the difference in the inflation goals. More disagreement about the preferred inflation level is, bigger is the distortion and hence larger the amount by which the output optimal target differs from the cooperative optimum. Also, a zero weight of the monetary authority in the aggregate loss function makes the output target be the cooperative level. Again, given agreement on preferred output and inflation levels, the two targets would just be the cooperative optima directly, equal in this case with the common preferred levels, whichever they are.

When the first best is implemented, targets have a very intuitive interpretation. The optimal inflation target of the monetary authority is equal

to the socially desirable level reduced by a term depending on disagreement on output goals. Thus, if the Central Bank has a higher preferred output than society, it has to aim to a lower inflation rate (this is consistent with Svensson, 1997 in a different setting). Fiscal authority is delegated with the optimal output level minus a term depending on excessive aimed inflation. This again depends on strategic complementarity/substitutability of the two policies given by c and on the ability of fiscal policy to expand output. It can be higher or lower than the target with deadweight losses, y^F .

The advantage of decentralisation might be the easiest to observe here. There are two authorities with totally different loss functions and an external authority imposing some simple distortion of these. This leads to implementation of the first best when the two play non-cooperatively by means of internalising the externalities, i.e. eliminating the deviation incentives.

A cautionary note is in order: both the uniqueness and the state-independency of the equilibrium contracts and targets implementing the optimum cannot be regarded independently from the assumptions we made. Notably, these desirable properties might be due to the linearity assumptions and to the existence of only one principal that we imposed. In the Folk Theorem of Fershtman et al. (1991), where contracts are more general and there are two principals, multiple equilibria can occur. This is potentially an interesting theoretical question but in real world non-linear contracts might be very difficult both to understand and implement.

One could argue that this just relocates centralisation one step ahead as there is again the issue of benevolence of the principal who is to implement delegation. This is one of the reasons why we also consider implementation of the second best. We view, however, as less restrictive a world in which simple penalties as the ones above are imposed on independent policymakers than one where an omnipotent policymaker may choose the equilibrium it desires. This can be modelled further drawing on the political economy literature, e.g. with voting models and accountability of the principal itself with respect to voters, but we leave that important question for future research.

6 Equilibria in the leadership game

Up to now we have assumed that authorities either play a pre-game and decide to cooperate and/or both commit before stage (i)-at stage 0, we said-or

play the first time simultaneously in choosing their policies at stage (iv). In both situations, policy instruments are chosen (so policies are implemented) simultaneously at stage (iv). Real world policy making, however, often sees non-simultaneity, due to various reasons. Different policy lags, various institutional setups making implementation of the two policies radically different are among the most important motives. There is, however, no consensus as to who has a leadership role, i.e. who moves first. Let us start by noting that if only one of the two policies is committed, and the other acts in a discretionary way, it is clear this implies leadership of the former policymaker. Commitment will take place at stage 0, and at stage (iv) the fully predictable policy would be implemented, whereas the discretionary policy is chosen just at stage (iv). If there is no commitment, the timing of the policies is decided at stage (iv).

Dixit and Lambertini (2000a) study these types of equilibria. Their main result is that when only monetary policy is committed, this equilibrium is the same as the one in which there is leadership of the monetary authority and discretion on both sides. They conclude that '*fiscal discretion destroys monetary commitment*' as the rational expectations constraint was not binding.

First, we should argue with this point noting that also '*monetary discretion destroys fiscal commitment*'! This is only a feature of the model: given leadership by any authority, commitment cannot bring additional improvements towards achieving the second best equilibrium. It also means that once an authority committed, it is credible, i.e. it faces no time inconsistency. The equilibrium survives as subgame perfect - the respective authority has no incentive to cheat (by cheating it cannot do better). Furthermore, this feature of the model does not mean that discretion of one authority destroys commitment of the other. It just means that once one of them committed it is fully credible as has no incentives to cheat given this particular timing of the game. 'Destroying commitment' would allegedly mean bringing the equilibrium from a high ranked one to a low ranked one. But what can commitment achieve in this model? Here discretion does not *destroy* anything, as there is nothing commitment achieves (this will become clearer later). This argument is showed in Remark 1. Even in the Nash game, commitment by both is far from the optimal solution, though it does improve on the Nash (Lemma 2). But when the authorities cooperate it does not make a difference either.

Secondly, as a corollary of Propositions 3 and 4 we will show that there exist optimal contracts and targets that can implement the cooperative second-

best in the leadership game, no matter which of the authorities is the leader.

6.1 Perfect credibility or dynamic consistency of commitment given leadership: or how '*monetary discretion "destroys" fiscal commitment*' also.

Imposing leadership in our non-cooperative game would mean that at stage (iv) there are two sub-stages: the two authorities move sequentially. Imposing commitment by only one authority would mean that at stage 0, before (i), the respective authority commits to a state-contingent rule with respect to the private sector. This makes timing at stage (iv) irrelevant. Dixit and Lambertini study leadership by both authorities and commitment by only the monetary authority, obtaining the abovementioned 'destruction of commitment' result. We study commitment by the fiscal authority and show it is equivalent to fiscal leadership with no commitment. In the leadership game the appropriate equilibrium concept is again subgame perfectness. The Stackelberg leader takes the follower's reaction function (derived in the Nash game) as a constraint and minimises its expected loss subject to the constraint. When there is also commitment, it also takes as a constraint the rational expectations rule in eq. 3. We emphasise our equivalence result in Remark 1.

Remark 1 *The leadership equilibrium is dynamically consistent: commitment by one (and any) of the two authorities is equivalent to leadership by the same authority, i.e. the rational expectations constraint is not binding given leadership. Hence, commitment is perfectly credible.*

Proof. Please find Appendix G. ■

The leadership game is obviously inefficient (the first order conditions are different from the cooperative ones). Commitment cannot make a difference here for achieving the desired outcomes. By committing, the leader authority cannot do better than the discretionary leadership and the unique Stackelberg equilibrium is trivially also subgame perfect. Thus, we look again at institutional design as a solution of implementing the cooperative optimum in the non-cooperative leadership game.

6.2 Implementing the cooperative second-best and the centralised first best in the leadership game

This is a straightforward extension of the results in Propositions 3 and 4⁹. While it might not present great theoretical interest, we view it as policy-relevant given the timing of policy decisions in reality. The timing in the stage game is as described in Appendix D: at stage (iv) there are two substages and the two authorities move sequentially. There is however a pre-stage, again 0, say, where a principal (again the Parliament, say) imposes a delegation scheme to the two authorities, that is it distorts their loss functions in the same manner as described in section 5. The idea applies no matter who is the leader and who the follower. Since the motivation for this exercise is mostly policy relevance, we just study the game where fiscal authority is the leader (the reverse-order exercise is of course trivial). This is more likely to be the case since tax rates are harder to change and monetary policy can adjust more quickly. The results, being a corollary of Proposition 3 and 4, are emphasised in Corollaries 5 and 6.

Corollary 5 *The cooperative second best equilibrium can be implemented in the leadership game (where fiscal authority is leader) by delegating with contracts as in Proposition 3, where the marginal penalties are now given by (for monetary, respectively fiscal authority and $\forall \tilde{\omega}$):*

$$t^{MS}(\tau) = \widehat{t^M}(\tau)$$

$$t^{FS}(\tau) = \frac{\alpha \lambda^M \lambda^F}{\alpha \lambda^M + (1 - \alpha) \lambda^F} [y^F - y^M] - \alpha b \lambda^M [\pi^F - \pi^M]$$

When delegation is done by targets as in Proposition 4, the target values are:

$$\widehat{\pi^S}(\tau) = \widehat{\pi}(\tau)$$

$$\widehat{y^S}(\tau) = \frac{\alpha \lambda^M y^M + (1 - \alpha) \lambda^F y^F}{\alpha \lambda^M + (1 - \alpha) \lambda^F} + \frac{\alpha b \lambda^M}{\lambda^F} [\pi^F - \pi^M]$$

⁹I thank to Karl Schlag for suggesting me this extension.

Corollary 6 *Implementation of the **centralised first best** is done in the same manner, with marginal contracts and targets given, $\forall \tilde{\omega}$, by:*

$$t^{MS*}(\tau) = t^{M*}(\tau)$$

$$t^{FS*}(\tau) = \lambda^F [y^F - y^*] - b\lambda^M [\pi^F - \pi^*]$$

$$\pi^{MS*}(\tau) = \pi^{M*}(\tau)$$

$$y^{FS*}(\tau) = y^* + \frac{b\lambda^M}{\lambda^F} [\pi^F - \pi^*]$$

Proof. Please find Appendix H ■

The interpretation of the monetary authority's inflation contract/target is the same as before. However, delegation to the fiscal policy has now to take into account for the first mover advantage. In Corollary 5 we observe that, compared to the expressions in Propositions 3 and 4, the first term (the one depending on the heterogeneity in output goals) is the same. The only difference comes from the second term, depending on the inflation goals differential. Comparing the two values we see that the relation between the two contracts (and targets, for that matter) depends directly on whether

$$\frac{c}{a + bc} \begin{matrix} \geq \\ < \end{matrix} -b\lambda^M$$

Note that the right hand side is always negative. If the two are equal, first mover advantage of fiscal policy would not matter and the penalty would be the same. A necessary (and not sufficient) condition for the LHS to be lower is that fiscal policy has deflationary effects ($c < 0$), otherwise the left hand side term would be positive. If the left hand side is larger-e.g. but not only if fiscal policy has inflationary effects-the penalty in the Nash game is larger. That is because some of the negative externality that fiscal policy imposes on monetary policy in this case is already internalised by the former in the Stackelberg game through leadership. The penalty imposed in the Nash game should be larger to account for that.

We should, however, keep in mind that here, apart from the sensitive points mentioned in the Nash case (where state-independtness of optimal contracts was due to linearity of the model) there is an additional potential

problem one should model. Any uncertainty of the fiscal authority regarding the best response function of the monetary authority propagates, and will influence the delegation parameters. This does not matter in the simultaneous move case but does matter here as one of the two authorities 'backward inducts' the best response of the other. A model with less rationality and with uncertainty regarding parameters and targets is an important direction for development as it could, for example, assess the role of transparency of the Central Bank in such models.

7 Policy design implications

Due to the fact that in Propositions 3 and 4 and Corollaries 5 and 6, contracts for the two authorities (and targets) are independent between them one can think about a mixed strategy which is more resembling of real life institutions. For example the delegating (or coordinating) principal imposes an inflation target for the Central Bank and a linear penalty for the fiscal authority, say as a function of excessive deficit. The fiscal penalty can be thought of as a way to model the Stability and Growth Pact in our framework. In fact, in Bilbiie (2001b) we show that a similar scheme with linear deficit penalties can achieve the first best in a monetary union with n fiscal authorities. The analogy with the EMU and the SGP as a method to discipline fiscal policymakers goes without saying.

For the one-country case, the arrangement we see as mirroring the current situation (of course given the simplifying assumptions of the model) would be: at stage 0, before (i) a principal (again the Parliament, the European Parliament maybe for the EMU) delegates policy to two policymakers. It does so by imposing an inflation target to the Central Bank (as in Proposition 4) and a linear penalty as in proposition 3, but this time as a function of deficit. The parameters of delegation are chosen such that either the first or the second best is implemented, in either the Nash or the Stackelberg game.

So the loss functions of the authorities, after delegation, are, for the monetary and fiscal authority respectively:

$$L_{ECB}^M(y, \pi, \tau, \omega, \pi^{ECB}) = \frac{1}{2} \left[\lambda^M (y - y^M)^2 + (\pi - \pi^{ECB})^2 \right] \quad (17)$$

$$L_{SGP}^F(y, \pi, \tau, \omega, t_{SGP}) = \frac{1}{2} \left[\lambda^F (y - y^F)^2 + (\pi - \pi^F)^2 \right] + t_{SGP} (f - f^*) \quad (18)$$

Note that the linear penalty is a function of deviations of the fiscal instrument (which can be thought of as deficit) from a benchmark level chosen by the principal too. The form of the loss function is the same no matter which equilibrium is to be implemented and no matter the timing of the game. What is different are the inflation target levels and the marginal penalty t_{SGP} .

The inflation targets π^{ECB} corresponding to different equilibria are as shown before and we just restate them here for completion (first in the Nash simultaneous move game -for the second best and the first best-, then for the Stackelberg game with fiscal leadership - for the second and first best, respectively) with the same notation as before and $\forall \tilde{\omega}$:

$$\begin{aligned} \widehat{\pi}(\tau) &= \alpha \pi^M + (1 - \alpha) \pi^F + \frac{b(1 - \alpha) \lambda^M \lambda^F}{\alpha \lambda^M + (1 - \alpha) \lambda^F} [y^F - y^M] \quad (19) \\ \pi^{M*}(\tau, \tau^*) &= \pi^* - b \lambda^M [y^M - y^*] \\ \widehat{\pi^S}(\tau) &= \widehat{\pi}(\tau) \\ \pi^{MS*}(\tau, \tau^*) &= \pi^{M*}(\tau, \tau^*) \end{aligned}$$

The marginal excessive deficit penalties are, $\forall \tilde{\omega}$, with the same ordering of equilibria:

$$\begin{aligned} \widehat{t_{SGP}}(\tau) &= \frac{\alpha(a + bc) \lambda^M \lambda^F}{\alpha \lambda^M + (1 - \alpha) \lambda^F} [y^F - y^M] + \alpha c [\pi^F - \pi^M] \\ t_{SGP}^*(\tau) &= (a + bc) \lambda^F [y^F - y^*] + c [\pi^F - \pi^*] \\ t_{SGP}^S(\tau) &= \frac{\alpha(a + bc) \lambda^M \lambda^F}{\alpha \lambda^M + (1 - \alpha) \lambda^F} [y^F - y^M] - \alpha(a + bc) b \lambda^M [\pi^F - \pi^M] \\ t_{SGP}^{S*}(\tau) &= (a + bc) \lambda^F [y^F - y^*] - b(a + bc) \lambda^M [\pi^F - \pi^*] \end{aligned}$$

Derivations are ignored since they are a trivial extension of the derivation of contracts on y , given the linear dependence of f and y . Hence, these are essentially the same as the 'output contracts' (and interpretation is equivalent), just that regarded in a more realistic way. It is more realistic to assume that the fiscal authority is penalised for an excessive deficit as this is more easily controllable in practice and also more easily observable/measurable. Note

that the threshold level of the 'deficit', f^* , is left to the discretion of the principal. Hence, in equilibrium it does not matter which is the benchmark level¹⁰. Aside the other simplifications of the model, the question of who should be the principal appears and this can be subject to further debate.

We would like to relate our policy implications and results with the arguments in Alesina et al. (2001, section 2). First of all, we view their answer to 'is there a need for coordination?' (which is 'no') as being totally contingent upon the way one defines coordination. As they do not provide a definition and one has to infer it from the content, and as what we can infer as a definition is what we would call a form of discretionary sub-ordination of the ECB, we agree with this point as long as the way we understand the definition is correct.

Their first argument is that coordination is not necessary when 'houses are in order' (i.e. the ECB targets inflation and the fiscal authority maintains a cyclically adjusted balanced budget). This is what we indeed showed formally here in Propositions 3 and 4 and Corollaries 5 and 6. But it is an argument FOR and not against coordination. As 'keeping houses in order' means here independent policymaking by each authority, but after it has been delegated by principal with appropriate delegation parameters, the need for what we call coordination immediately arises. Coordination for us means that there is one principal who imposes both delegation schemes (e.g. inflation target and SGP-like excessive deficit penalty). Then there is no need for *cooperation* in the sense of Lemma 1 (minimisation of a joint loss function), which would be akin to centralisation and jeopardise independence. Should there be two principals who delegate and no coordination, then the critique we formulated in Bilbiie (2000) and we briefly described here will immediately apply. Thus, coordination of policies is needed but it does not mean interference with the independence of the two authorities. "Keeping houses in order' is enough as long as each authority has been given the appropriate house to take care of.

Their fears (expressed throughout section 2.3 and summarised in the end) that 'formal meetings between the ECB and fiscal authorities would become an opportunity for the latter to put pressure on the former' are also taken care of in our framework. Once implementation of the scheme proposed

¹⁰In Bilbiie (2001b), in the context of a monetary union, we show that such 'contracts' can be designed depending only on the respective country's parameters. As they will have different values for every country this is a way of getting around the Buiters et al (1993) critique mentioned earlier, alternative to the solution of Beetsma and Jensen (2000) who propose state-contingent sanctions.

above took place, total independence is needed and no political intrusions in the ECB's decisionmaking are possible. The case of multiple fiscal authorities which Alesina et al. analyse in section 2.5 is studied in Bilbiie (2001b) in the same framework as here. There, we propose the same type of arrangement for a monetary union. We argue that collusion of fiscal authorities and no cooperation with the ECB is sub-optimal but an inflation target and excessive deficit penalties for each fiscal authority, all imposed by the same principal, can achieve the first-best outcome without affecting the ECB independence and sovereignty of fiscal policymaking. As to the meetings between finance ministers and the ECB, we indeed find they are not necessary unless for informal exchange of information. They would bring no benefit *as long as delegation has taken place in the appropriate form*.

The scheme we propose here has among its main advantages simplicity and transparency, which might enhance credibility and strengthen independence of both the monetary and fiscal authorities.

8 Conclusions

As any model, this is of course just an approximation of reality and misses many important aspects. Keeping that in mind, we would like to summarise here our results.

1. When there are two policy instruments available for one single authority, the *centralised equilibrium* achieves *first best* and is *dynamically consistent* (no need to commit).

2. Assuming centralisation is unrealistic and might be harmful. We see two main arguments for decentralising policymaking: (i) decentralisation is a *constraint* for modelling as we do have different and independent institutions in real life; (ii) *political economy* reasons not modelled here might make the unique policymaker willing to implement a 'bad' equilibrium, affected by some political distortion. The model predicts it could do that, and this is not a very desirable feature idea. We propose these arguments as an alternative to Dixit and Lambertini's, where arguments for monetary policy delegation are constructed in a model where fiscal policy does not influence inflation, which is the Barro Gordon (1983) case, quite different from what is modelled here.

3. As a benchmark in the decentralised case, we solve for the *cooperative*

and commitment equilibrium - which is similar to the centralised case. We formulate the result in Lemma 1 and show the equilibrium is dynamically consistent (commitment is perfectly credible). We view Dixit and Lambertini's result on 'symbiosis' of policies when they share the same target levels as a very particular case of this.

4. When solving for the *Nash-discretionary equilibrium* we find inefficiency and show that it comes from two sources: one is spillovers between the two policymakers and the other credibility (which is binding here). Commitment alone cannot achieve second best (as argued in Dixit and Lambertini, 2000a) and we show this by solving for the *Nash equilibrium with commitment* of both policymakers. In this case externalities between policymakers are still not internalised and give incentives to deviate.

5. Our main result is that both the *cooperative second best* and *centralised first best* can be implemented by delegation by one common principal. We show this can be done by delegating with contracts (linear penalties or rewards) or targets in Propositions 3 and 4. Here delegation is akin to coordination by an external authority who imposes the appropriate delegation parameters. Once this is done, however, (instrument-)independence of the two authorities is unaffected.

6. Considering more realistic sequences of the game, we find the Stackelberg with commitment equilibrium is dynamically consistent, no matter who moves first. Dixit and Lamertini's (2000a) result on '*fiscal discretion destroys monetary commitment*' is a misinterpretation of the rational expectations constraint not being binding. We show how one could also say '*monetary discretion destroys fiscal commitment*' and be equally wrong. In this game, *commitment cannot achieve anything* additional given leadership as given the structure of game, any leadership is equivalent to commitment (as the latter implies also leadership by the authority that commits).

7. In Corollaries 5 and 6 we find the *optimal contracts respectively targets*, that can implement the cooperative second best or the centralised first best in the leadership game. This is similar to the previous delegation result, just that penalties are less strict (rewards less generous) as part of the externality is already internalised by leadership.

8. This leads us to formulate policy recommendations for the EMU institutional framework. We show how *an inflation target for the ECB* and a *linear excessive deficit penalty* (or penalties, in a monetary union) *resembling*

the SGP for the fiscal authority can implement the desired equilibrium. This would preserve independence and sovereignty of the policymakers as it does not imply any formal cooperation. We link our results to the arguments in Alesina et al. (2001) supporting the non-necessity of coordination. We find arguments for coordination, not consisting in day-to-day adjustment but delegation by an external authority. We view this as an argument for 'keeping houses in order' as something that substitutes cooperation, but only if each authority has been given the appropriate 'house' and the appropriate share of the 'common space'.

Our proposed scheme is an alternative (though close in spirit, as draws on microeconomic solutions to inefficiencies coming from the presence of externalities) to Casella's (1999, 2001), based on tradable deficit permits.

Our model misses important aspects that cannot be ignored in the design of fiscal rules. First, it is static, thus there is no debt present. Introducing debt would introduce an additional channel of interaction of the two policies as debt is linked to deficit and is influenced by monetary policy through the latter's manipulation of interest rates. Thus, in our model a debt-based SGP cannot be modelled, though there is an increasing amount of arguments in favour of it (see, e.g. Beetsma 2000).

However, the simplicity of the policy arrangements we propose is an advantage as it enhances credibility and helps building independence of both authorities. Moreover, the argument we make here for an SGP-like arrangement is not dependent on fiscal spillovers. One of the main counterarguments for the SGP is the absence of fiscal spillovers. Here, as we have only one authority the desirability of the excessive deficit penalty comes from fiscal-monetary interactions and not from cross-country externalities¹¹.

A Efficiency of discretionary equilibrium with centralised policymaking

The timing of the game is simple here: (i) targets $(y^*, \pi^*) \equiv \tau^*$ are revealed, (ii) expectations π^e are formed; (iii) shocks ω hit the economy; (iv) authority chooses the instruments m, f ; (v) outcomes (y, π) are realised.

The authority solves the problem:

¹¹This argument extends to a monetary union case in Bilbiie (2001b).

$$\min_{m,f} E_\omega[L^*(y, \pi, \lambda, \tau)] \text{ s.t. } \pi^e \text{ given}$$

It does not internalise the effect of its decisions on expectations, i.e. it plays Nash with respect to the private sector. The first order conditions are, $\forall \tilde{\omega}$:

$$\frac{\partial E_\omega[L^*]}{\partial f} = (a + bc)\lambda^*(y - y^*) + c(\pi - \pi^*) = 0 \quad (\text{A1})$$

$$\frac{\partial E_\omega[L^*]}{\partial m} = b\lambda^*(y - y^*) + (\pi - \pi^*) = 0 \quad (\text{A2})$$

Solving the two together delivers the policy rules:

$$(m^*(\tau, \omega), f^*(\tau, \omega)) = (\pi^* - \frac{c}{a}y^* - \frac{c}{a}\varepsilon - v, \frac{1}{a}y^* + \frac{1}{a}\varepsilon) \quad (\text{A3})$$

and so the equilibrium outcomes as the bliss point of authority (y^*, π^*) , $\forall \tilde{\omega}$. This is the unique equilibrium given the structure of the problem (loss functions are quasi-concave functions and level curves are ellipses) and second order conditions are trivially satisfied.

B Proof of Lemma 1

Proof. Consider first the case where authorities commit before stage (i) both to cooperate between them and with respect to the private sector. The appropriate equilibrium concept is subgame perfectness, given commitment and the sequential nature of the game. So the solution method is backward induction, i.e. the best response of the private sector given by equation (3) is taken as a constraint. That is equivalent to a social planner solving the constrained aggregate social loss minimisation problem:

$$\begin{aligned} & \min_{m,f,m^e,f^e} E_\omega[\alpha L^M + (1 - \alpha) L^F] \\ \text{s.t. } & \pi^e = E_\omega[\pi(\omega)] \end{aligned} \quad (\text{B1})$$

The Lagrangean of this problem is:

$$\begin{aligned}\Lambda &= E_\omega[\alpha L^M + (1 - \alpha) L^F] - \rho[\pi^e - E_\omega(\pi)] \\ &= \int[\alpha L^M + (1 - \alpha) L^F + \rho\pi]d\Phi(\omega) - \rho\pi^e\end{aligned}\quad (\text{B2})$$

The first order conditions for the two authorities, for each of the two controls respectively, are, $\forall \tilde{\omega}$:

$$\begin{aligned}\frac{\partial \Lambda}{\partial m} &= \alpha b \lambda^M (y - y^M) + (1 - \alpha) b \lambda^F (y - y^F) + \\ &+ \alpha(\pi - \pi^M) + (1 - \alpha)(\pi - \pi^F) + \rho = 0\end{aligned}\quad (\text{B3})$$

$$\frac{\partial \Lambda}{\partial m^e} = E_\omega[-\alpha b \lambda^M (y - y^M) - (1 - \alpha) b \lambda^F (y - y^F)] - \rho = 0 \quad (\text{B4})$$

$$\begin{aligned}\frac{\partial \Lambda}{\partial f} &= \alpha(a + bc)\lambda^M (y - y^M) + (1 - \alpha)(a + bc)\lambda^F (y - y^F) + \\ &+ \alpha c(\pi - \pi^M) + (1 - \alpha)c(\pi - \pi^F) + c\rho = 0\end{aligned}\quad (\text{B5})$$

$$\frac{\partial \Lambda}{\partial f^e} = E_\omega[-\alpha bc \lambda^M (y - y^M) - (1 - \alpha) bc \lambda^F (y - y^F)] - c\rho = 0 \quad (\text{B6})$$

Eliminating the multiplier from B3 and B4, respectively from B5 and B6, noting that $y^e = E_\omega[y] = aE_\omega[f] = af^e$ and slightly rearranging using also eq1 and eq2 we get:

$$b[\alpha \lambda^M + (1 - \alpha) \lambda^F][a(f - f^e) + b(\pi - \pi^e) - \varepsilon] +$$

$$+\alpha(\pi - \pi^M) + (1 - \alpha)(\pi - \pi^F) = 0 \quad (\text{B7})$$

$$\begin{aligned} \alpha a \lambda^M (y - y^M) + (1 - \alpha) a \lambda^F (y - y^F) + c \alpha (\pi - \pi^M) + c (1 - \alpha) (\pi - \pi^F) + \\ + bc [\alpha \lambda^M + (1 - \alpha) \lambda^F] [a (f - f^e) + b (\pi - \pi^e) - \varepsilon] = 0 \end{aligned} \quad (\text{B8})$$

Taking expectations of the (B7, B8) system we solve for π^e and f^e :

$$\begin{aligned} \pi^e &= \alpha \pi^M + (1 - \alpha) \pi^F \\ f^e &= \frac{\alpha \lambda^M y^M + (1 - \alpha) \lambda^F y^F}{a [\alpha \lambda^M + (1 - \alpha) \lambda^F]} \end{aligned} \quad (\text{B9})$$

Substituting these back in B7 and B8 we get the policy outcomes in the cooperative and commitment equilibrium as in Lemma 1. The policy rules are also obtained as:

$$\begin{aligned} m^c(\tau, \omega) &= \alpha \pi^M + (1 - \alpha) \pi^F - \frac{c}{a} \frac{\alpha \lambda^M y^M + (1 - \alpha) \lambda^F y^F}{[\alpha \lambda^M + (1 - \alpha) \lambda^F]} - \frac{c}{a} \varepsilon \\ f^c(\tau, \omega) &= \frac{1}{a} \frac{\alpha \lambda^M y^M + (1 - \alpha) \lambda^F y^F}{[\alpha \lambda^M + (1 - \alpha) \lambda^F]} + \frac{1}{a} \varepsilon \end{aligned} \quad (\text{B10})$$

Again given the structure of the problem this is a minimum and it is the unique one (we do not present the formal argument due to space constraints).

It is also useful for further reference to note the first order conditions of the authorities after substituting for expected variables:

$$\alpha b \lambda^M (y - y^M) + (1 - \alpha) b \lambda^F (y - y^F) + \alpha (\pi - \pi^M) + (1 - \alpha) (\pi - \pi^F) = 0 \quad (\text{B11})$$

$$\begin{aligned}
& \alpha(a + bc)\lambda^M (y - y^M) + (1 - \alpha)(a + bc)\lambda^F (y - y^F) + \\
& + \alpha c(\pi - \pi^M) + (1 - \alpha)c(\pi - \pi^F) = 0
\end{aligned} \tag{B12}$$

This already shows the lagrange multiplier is zero. To prove that the rational expectations constraint does not bind we can either solve for the Lagrange multiplier or solve the unconstrained problem (i.e. just with cooperation). Solving for the Lagrange multiplier from, e.g. B4 and substituting for f^e we get, $\forall \tilde{\omega}$:

$$\begin{aligned}
\rho &= -\alpha b \lambda^M \left(a \frac{\alpha \lambda^M y^M + (1 - \alpha) \lambda^F y^F}{a[\alpha \lambda^M + (1 - \alpha) \lambda^F]} - y^M \right) - \\
& - (1 - \alpha) b \lambda^F \left(a \frac{\alpha \lambda^M y^M + (1 - \alpha) \lambda^F y^F}{a[\alpha \lambda^M + (1 - \alpha) \lambda^F]} - y^F \right) \\
& \Leftrightarrow \rho = 0
\end{aligned}$$

This proves the cooperative and commitment equilibrium and the cooperative-only equilibrium are the same. ■

C Nash equilibrium with decentralised policymaking

Timing and problems of the two authorities are described in the text. The first order conditions are, $\forall \tilde{\omega}$:

$$\frac{\partial E_\omega[L^M]}{\partial m} = b \lambda^M (y - y^M) + (\pi - \pi^M) = 0 \tag{C1}$$

$$\frac{\partial E_\omega[L^F]}{\partial f} = (a + bc)\lambda^F (y - y^F) + c(\pi - \pi^F) = 0 \tag{C2}$$

Solving these two together for (y, π) delivers the outcomes stated in the text, where again second order conditions are trivially satisfied. The policy rules could also be obtained then straightforwardly.

D Proof of Lemma 2

Proof. Authorities solve: ■

$$\begin{aligned} & \min_{m, m^e} E_\omega[L^M] & (D1) \\ \text{s.t. } \pi^e & = E_\omega[\pi(\omega)] \end{aligned}$$

$$\begin{aligned} & \min_{f, f^e} E_\omega[L^F] & (D2) \\ \text{s.t. } \pi^e & = E_\omega[\pi(\omega)] \end{aligned}$$

The first order conditions are, for each authority (where ρ^M, ρ^F are Lagrange multipliers of the constraint for each of the policymakers) and $\forall \tilde{\omega}$:

$$\frac{\partial E_\omega[L^M]}{\partial m} = b\lambda^M (y - y^M) + (\pi - \pi^M) + \rho^M = 0 \quad (D3)$$

$$\frac{\partial E_\omega[L^M]}{\partial m^e} = E_\omega[-b\lambda^M (y - y^M)] - \rho^M = 0 \quad (D4)$$

$$\frac{\partial E_\omega[L^F]}{\partial f} = (a + bc)\lambda^F (y - y^F) + c(\pi - \pi^F) + c\rho^F = 0 \quad (D5)$$

$$\frac{\partial E_\omega[L^F]}{\partial f^e} = E_\omega[-bc\lambda^F (y - y^F)] - c\rho^F = 0 \quad (D6)$$

Eliminating the Lagrange multipliers using the same method as in the proof for Lemma 1, we get:

$$b\lambda^M [a(f - f^e) + b(\pi - \pi^e) - \varepsilon] + (\pi - \pi^M) = 0 \quad (D7)$$

$$a\lambda^F (y - y^F) + bc\lambda^F [a(f - f^e) + b(\pi - \pi^e) - \varepsilon] + c(\pi - \pi^F) = 0 \quad (\text{D8})$$

Taking expectations of this system we get solutions for π^e and y^e as:

$$\begin{aligned} \pi^e &= \pi^M \\ y^e &= y^F + \frac{c}{a\lambda^F} (\pi^F - \pi^M) \end{aligned}$$

Substituting these back in D7 and D8 we get the first order conditions after taking into account expectations, which we slightly rearrange to compare with the discretionary first order conditions:

$$b\lambda^M (y - y^M) + (\pi - \pi^M) + b\lambda^M (y^M - y^F) - \frac{cb\lambda^M}{a\lambda^F} (\pi^F - \pi^M) = 0 \quad (\text{D9})$$

$$(a + bc)\lambda^F (y - y^F) + c(\pi - \pi^F) - \frac{bc^2}{a} (\pi^F - \pi^M) = 0 \quad (\text{D10})$$

Solving these two we can get both (as we accounted for expected variables) the policies m^{nc}, f^{nc} or the outcomes π^{nc}, y^{nc} as stated in Lemma 2. Given the structure of the problem the solution is indeed a minimum and is the unique one.

E Proof of Proposition 3

Proof. Suppose delegation took place at stage 0, before stage (i) in our game and now the two policymakers face the loss functions described in Proposition 1 and play non-cooperatively and discretionarily. That is, each of them solves the problem

$$\begin{aligned} \min_m E_\omega [L^{MD}] \\ \text{s.t. } \pi^e \text{ given} \end{aligned} \quad (\text{E1})$$

$$\begin{aligned} \min_f E_\omega[L^{FD}] & \quad (E2) \\ \text{s.t. } \pi^e & \text{ given} \end{aligned}$$

We thought about the contract on the monetary authority being imposed on inflation and for the fiscal on output deviations. Alternative forms will be considered later.

The first order conditions of this problem are, $\forall \tilde{\omega}$:

$$\frac{\partial E_\omega[L^{MD}]}{\partial m} = b\lambda^M (y - y^M) + (\pi - \pi^M) + t^M = 0 \quad (E3)$$

$$\frac{\partial E_\omega[L^{FD}]}{\partial f} = (a + bc)\lambda^F (y - y^F) + c(\pi - \pi^F) + (a + bc)t^F = 0 \quad (E4)$$

The solution of this problem is exactly the cooperative one (y^c, π^c) if the new terms in the first order conditions E3 and E4 lead to elimination of the incentive constraints present in the Nash equilibrium (i.e. making the first order conditions in the Nash discretionary delegated playing to be the same as in the cooperative (and commitment, for that matter) equilibrium. Comparing E3 and E4 with the first order cooperative conditions B11 and B12 we see that the marginal penalties/rewards that satisfy this condition are, $\forall \tilde{\omega}$:

$$\begin{aligned} t^{F*} &= \frac{1}{a + bc} [(a + bc) \alpha \lambda^M (y^c - y^M) - \\ & (a + bc) \alpha \lambda^F (y^c - y^F) - c\alpha (\pi^c - \pi^M) - c\alpha (\pi^c - \pi^F)] \end{aligned} \quad (E5)$$

$$\begin{aligned} t^{M*} &= b(\alpha - 1) \lambda^M (y^c - y^M) + b(1 - \alpha) \lambda^F (y^c - y^F) \\ & + (\alpha - 1) (\pi^c - \pi^M) + (1 - \alpha) (\pi^c - \pi^F) \end{aligned} \quad (E6)$$

Substituting for (y^c, π^c) from Lemma 1 we obtain the expressions in Proposition 1. The obtained expressions do not depend on the vector of stochastic shocks $\omega = (\varepsilon, v)$ and so marginal penalty are the same for any state of the world. The result for implementation of the first best is obtained similarly, just compare E3 and E4 with the centralised first order conditions A1 and A2. We omit details as this is the same solution method. ■

F Proof of Proposition 4

Proof. This is very similar to the proof of proposition 3. The initial assumptions are the same, authorities minimising now the respective loss functions described in Proposition 4 in the text. There is an inflation target for the monetary authority and an output target for the fiscal authority. The first order conditions for each of the authority's problem are, $\forall \tilde{\omega}$:

$$\frac{\partial E_{\omega}[L^{MT}]}{\partial m} = b\lambda^M (y - y^M) + \pi - \hat{\pi} = 0 \quad (\text{F1})$$

$$\frac{\partial E_{\omega}[L^{FT}]}{\partial f} = (a + bc)\lambda^F (y - \hat{y}) + c(\pi - \pi^F) = 0 \quad (\text{F2})$$

Using the same solution method as in the proof of Proposition 3, i.e. using Lemma 1, we get the targets both for implementing the cooperative and the centralised equilibrium as in Proposition 4. We omit details here. ■

G Proof of Remark 1

Proof. The equivalence of monetary leadership and monetary commitment is proven in Dixit and Lambertini (2000a) and we shall use their result. We deal with the fiscal leadership and commitment case.

Stackelberg leadership of fiscal authority

This is also studied in Dixit and Lambertini and we restate it here in our context. Here the fiscal authority at stage (iv)1 takes the monetary Nash reaction function C1 (derived at stage (iv)2) as a constraint since we solve for the subgame perfect equilibrium by backward induction. Thus, it solves:

$$\begin{aligned} & \min_f E_{\omega}[L^F] \\ \text{s.t.} \quad & b\lambda^M (y - y^M) + (\pi - \pi^M) = 0 \text{ and } \pi^e \text{ given} \end{aligned}$$

The Lagrangean is:

$$\Lambda^F = E_{\omega}[L^F] - \theta^F [b\lambda^M (y - y^M) + (\pi - \pi^M)]$$

θ^F is the Lagrangean multiplier of the monetary constraint. Since at (iv)1 monetary policy fulfills the constraint, we regard the problem as the fiscal authority choosing directly y and π . The first order conditions are, $\forall \tilde{\omega}$:

$$\frac{\partial \Lambda^F}{\partial y} = \lambda^F (y - y^F) - \theta^F b \lambda^M = 0 \quad (\text{G1})$$

$$\frac{\partial \Lambda^F}{\partial \pi} = \pi - \pi^F - \theta^F = 0 \quad (\text{G2})$$

Eliminating the multiplier θ^F we get the fiscal first order condition for the Stackelberg game, $\forall \tilde{\omega}$:

$$\lambda^F (y - y^F) - (\pi - \pi^F) b \lambda^M = 0 \quad (\text{G3})$$

We do not carry the analysis of the equilibrium values further as this is all we need to prove the point.

Fiscal commitment

In this case, the backward induction is not only with respect to the monetary authority but also the private sector. Hence, additionally to the constraint above, there is also the rational expectations constraint and the authority solves:

$$\min_f E_\omega[L^F]$$

$$\text{s.t.} \quad b \lambda^M (y - y^M) + (\pi - \pi^M) = 0$$

$$\text{and} \quad \pi^e = E_\omega[\pi]$$

The Lagrangean in this case is:

$$\Lambda^{F*} = E_\omega[L^F] - \theta^{F*} [b \lambda^M (y - y^M) + (\pi - \pi^M)] - \rho^F (\pi^e - E_\omega[\pi])$$

Applying the same solution method as above, the first order conditions are, $\forall \tilde{\omega}$:

$$\frac{\partial \Lambda^{F*}}{\partial y} = \lambda^F (y - y^F) - \theta^{F*} b \lambda^M = 0 \quad (\text{G4})$$

$$\frac{\partial \Lambda^{F*}}{\partial \pi} = \pi - \pi^F - \theta^{F*} + \rho^F = 0 \quad (\text{G5})$$

$$\frac{\partial \Lambda^{F*}}{\partial \pi^e} = E_\omega[-b\lambda^F (y - y^F) + \theta^{F*} b^2 \lambda^M] - \rho^F = 0 \quad (\text{G6})$$

Eliminating the Lagrange multiplier θ^{F*} from first two equations we get, $\forall \tilde{\omega}$:

$$\lambda^F (y - y^F) - (\pi - \pi^F + \rho^F) b \lambda^M = 0 \quad (\text{G7})$$

Substituting G7 in G6 we get:

$$\rho^F = 0, \quad \forall \tilde{\omega}$$

So the constraint is not binding and the two equilibria are equivalent.

Given also that the equilibria with monetary leadership/commitment are equivalent Remark 1 is proven. ■

H Proof of Corollary 5

Proof. We will be succinct as the solution method is the same as in Appendix E. Consider delegation has taken place at stage 0 by contracts, and the new loss functions are those in Proposition 3. Given the timing of the game, the first order conditions are obtained as in the proof of Remark 1 (Appendix G) taking into account delegation.

As monetary authority plays still Nash it has the same reaction function given by E3, hence delegation has the same parameters as there.

The first order condition for the fiscal authority after eliminating the multiplier of the monetary constraint is, $\forall \tilde{\omega}$:

$$\lambda^F (y - y^F) - b \lambda^M (\pi - \pi^F) + t^{FS} = 0 \quad (\text{H1})$$

Although fiscal authority takes monetary reaction as a constraint, the monetary contract does not appear in this expression due to the linearity hypothesis.

As in Proof of Proposition 3, comparing this with the first order cooperative fiscal condition B12, we find that the contract given in Corollary 5 eliminates the deviation incentives in the Stackelberg Equilibrium and implements the cooperative optimum.

The proof for targets (Corollary 6) is omitted since it is trivially derived from the above and Appendix F just replacing contract delegation with target delegation. ■

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