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The Problem of Measurement: An Analysis of Money Demand Price Homogeneity, in the Long Run*

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Abstract

The paper focuses on Money Demand for the U.S. The issue of long run money - price proportionality is addressed and bad measurement is identified as the source of its failure. It is possible to show empirically that conventional measures of money and price indexes fail to give rise to a homogeneous money demand function. Instead, using a Divisia index of money and a chained price index, it is empirically demonstrated that money demand becomes homogeneous, hence contrasting the measurement problem. The analysis is performed within the newly developed cointegrated *I(2)* VAR framework, offering some insights about the dynamic links between money and prices.

Keywords: Money Demand, Monetary Services Indexes (Divisia), Chain Price Indexes, Long Run Price Homogeneity, Cointegration, *I(2)* Analysis
JEL Classification: C32, C43, E41

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"Never assume that the available data measure the theory concept the modeler has in mind just because the names are very similar (or even coincide)!"

Spanos, A. (1999) *Probability Theory and Statistical Inference*

1 Introduction

Empirical analyses of money demand have always been at the center of the theoretical controversy of whether the data support one theory or the other. Lately, there has been an increased interest in money demand and its empirical relevance for policy. One route employed in the literature is the use of cointegrated vector autoregressions (VARs), that provide a natural framework for evaluating the effects of temporary and permanent shocks on variables of interest. Furthermore, the use of a cointegrated VAR allows one to formulate a long-run structure that is consistent with theory, while the short-run dynamics of the system are determined empirically. Examples of this approach are included in Hoffman and Rasche (1996) and in Lütkepohl and Wolters (1999). The tradition is to start with a vector of the variables of interest, namely real balances, a variable that measures aggregate activity, inflation and one or two interest rates, and analyze the data using the methods described in Johansen (1995b). The usual assumption, which in most of the cases is also documented by reporting univariate non-stationarity tests, is that real balances are best described as $I(1)$ variables, whereas nominal money and prices are better described as $I(2)$ variables[1].

Lately, using the newly developed statistical analysis for $I(2)$ variables, various papers have documented the failure of long-run price homogeneity in a number of European countries, including Denmark, Italy and Spain[2]. The cointegrated $I(2)$ model allows tests of non-stationarity to be imbedded in a multivariate framework. These multivariate tests have better properties in finite samples than univariate tests because they employ more information. Furthermore, the cointegrated $I(2)$ model allows one to test for the presence of long-run price homogeneity, and if homogeneity is rejected, to identify the sources of the failure. In a recent paper, Kongsted (1999) has proposed the so-called *Nominal to Real* test procedure. Kongsted shows that under certain conditions the transformation of a data vector that includes nominal variables into one where only real variables and growth rates of nominal variables are included entails no loss of information and no invalidation

[1] For definitions of orders of integration see Johansen (1995b). The assumption that underlies these empirical modelling approaches, as well as the one followed here, is that for the purposes of inference one might wish to regard variables as being $I(1)$ or $I(2)$. This is just a "local" (within sample) approximation of the stochastic processes studied rather than a definitive conclusion about the stochastic nature of the variables. To fix ideas, suppose that a variable z_t is described by an autoregressive process with roots of the characteristic polynomial close to unity. Such a process is stationary (in a strict sense), but it converges to its asymptotic distribution at a low rate. Using the approximation that some roots of the charecteristic polynomial are one, can be used as a convenient simplification a in order to conduct inference.

For example, with the short data spans available for empirical analyses, variables such as prices and money can be considered to be $I(2)$, while if one looks over longer data spans, say a century (for U.S. Sweden, Denmark and Italy data are available form various sources), the same variables might be best described as $I(1)$. So the reader should bear in mind that the language and the analysis that previous papers and this one employs, refer to approximations of stochastic processes/economic variables within the available sample.

[2] For the case of Italy see Juselius (2001), Juselius and Gennari (1999), for the case of Spain, see Juselius and Toro (1999), and for the Danish case, see Juselius (1998).

of inference[3].

In this paper, I take a different stance towards the lack of long-run price proportionality. If fundamental properties of economic theory, such as long-run price homogeneity, are not supported by the data, then there might be a serious measurement problem associated with the data. There are two motivations for this observation: mis-measurement of prices and mis-measurement of money. In the U.S. there has been a growing literature on the mis-measurement of prices by the CPI and the potential "correction" of this mis-measurement. According to this literature, the CPI is a biased measure of the aggregate cost of living. Hence, choosing a price index to correct the measurement problem, inherent in the CPI, I propose the use of a superlative, chained price index instead of a fixed base year index usually employed. As shown in Diewert (1976), one should use 'superlative' index numbers to measure the cost of living or to approximate the underlying price-aggregator function.

The second measurement issue concerns the definition of money. It is well known that financial innovation has induced compositional changes in the money aggregates but, until recently, these compositional changes have been widely ignored. This might be partly due to the fact that regulations imposed limitations on the relation between checking and savings accounts, as well as the importance of non-bank competitors. Empirical measures of the money stock have usually tried to identify as components of money those instruments that can be used directly in transactions, whilst in some studies broad monetary aggregates reported by central banks have been employed. To construct a measure of money, using a simple sum aggregate, such as $M2$ or $M3$, the components should be perfect substitutes; but there is evidence that for assets typically included in monetary aggregates, this is not the case.

The seminal work by Barnett (1980, 1987, 1995) has shown that under certain conditions one should use a "monetary services" index number which could, in principle, capture the transaction services yielded by wide range of financial instruments in a "superlative" way. There two main index numbers available for this task: the Divisia index proposed by Barnett (1980) and the Currency Equivalence index proposed by Rotemberg et al. (1995)[4]. The attractive feature of these indexes is that they internalize the substitution effects between components of the potential aggregate, and hence take into account the compositional changes alluded to above. These indexes approximate the optimal aggregator functions for those instruments that should have a property known as "weak separability"[5]. Empirical evidence in Barnett et al. (1984), Belongia and Chalfant (1989), Chrystal and MacDonald (1994) provide support for the superiority of Divisia indexes, while evidence presented

[3] In essense, what a *Nominal to Real* transformation does, is to allow us to use the $I(1)$ analysis for a transformed vector of variables. Kongsted (1999) has shown that under some conditions, this transformation of the vector process entails no loss of information. For example let the data vector be given by $x_t = [m_t, p_t, c_t, i_t]$ where m is the log of money, p is the log of the price index, c is the log of a scale variable (consumption), and i is a nominal interest rate. Let m and p be $I(2)$ variables so that $I(2)$ cointegrated VAR can be employed. A **valid** *Nominal to Real* transformation implies that the analysis can equivalently be performed using either of the trasformed vectors $\bar{x}_t^1 = [m_t - p_t, c_t, i_t, \Delta p_t]'$ or $\bar{x}_t^2 = [m_t - p_t, c_t, i_t, \Delta m_t]'$ which can be analyzed within the convetional $I(1)$ cointegrated VAR framework. But this transformation is a testable hypothesis.

[4] In fact the Divisia indexes proposed by Barnett (1980, 1987) approximate the flow of service of the economic good 'money', while the CE indexes proposed by Rotemberg et al. (1995) approximate the stock of 'money'. It can be shown that under some very restrictive assumtpions the two measures of money would be proportional, but in general this is not the case. See Barnett and Serletis (2000).

[5] For definitions of separability and aggregator functions see e.g. Deaton and Muellbauer (1980).

in Belongia (1996) seems to reverse conclusions reached in earlier studies about various aspects of money. As Crystal and MacDonald put it: "... the problems with tests of money in the economy in recent years may be more due to bad measurement rather than an instability in the link between the true money and the economy. Rather than a problem associated with the Lucas Critique, it could instead be a problem stemming from the 'Barnett Critique' ".

In this paper, I try to bridge the gap between theory and empirical practice, by showing that the failure of zero-degree homogeneity of the demand-for-money can be attributed to bad measurement. I study this property of money demand for the U.S. using the Johansen (1995a,1995b, 1997) procedure of a cointegrated VAR. I show empirically, that the demand-for-money fails basic tests of homogeneity if one measures prices by the CPI and money with a simple sum aggregate. I also show, that when the simple sum definition of money is replaced by a Divisia index and when the price index chosen is a superlative-chained price index, long-run price homogeneity is restored. The approach followed has a firm theoretical justification (aggregation theory and index number theory) for the measurement of money and prices that should be used in the empirical exercise (the variables chosen), while it uses newly developed econometric techniques to draw inferences about the behavior of the economy.

The remainder of the paper is organized as follows: Section 2 describes the theoretical relations that one expects to hold in the long-run and discusses the problem of measurement. Section 3 presents the data used and gives a description of the econometric methodology. The empirical model and the results are laid out in section 4. Finally, section 5 contains some concluding remarks.

2 Measurement Issues and Long-Run Relations

2.1 Some Issues About Measurement

In an innovative paper Diewert (1976) has managed to unite index number theory with aggregation theory by identifying a class of index numbers which not only perform well relative to the desirable properties of statistical index numbers, but also provide high quality approximations to values of the economic quantity and price aggregates of aggregation theory. In general an aggregator function is called exact if it is linearly homogeneous. It can be shown that agents would behave as if exact economic aggregate were elementary goods (see Barnett and Serletis (2000) for some details). The specification of an aggregator function is defined to be 'flexible' if it can provide a second order approximation to any arbitrary aggregator function. According to the definition given in Diewert (1976) an index number is 'superlative' if it has the ability to always give the value of a 'flexible' aggregator function. Following his initial argument, Diewert (1978, 1990) has subsequently argued in favor of superlative index numbers to measure the cost-of-living, or if one wants to approximate the underlying aggregator function. He has shown that the commonly employed Laspeyres and Paasche index numbers, can only provide first order approximations of the underlying aggregator functions, while Divisia or Fisher ideal index numbers can provide second order approximations. Additionally, in his 1990 paper, he has argued that measuring the cost-of-living by a fixed-base index number will result in serious biases as one moves away from the base year. This is usually referred to as the substitution bias. This will be true since, as time goes by and relative prices change, compositional changes in aggregate consumption occur. These of course cannot be internalized because of the use of fixed weights to construct the price index.

On the other hand, although the usefulness of aggregation and index number theory in acquiring price indexes is widely known, until recently aggregation theory had not been applied to monetary economics. In a sequel of papers, Barnett (1980, 1987, 1995) has argued that simple sum aggregation of money, which is usually employed by central banks, is implausible because it implies that the elements included in the aggregate are perfect substitutes: this is unlikely to be the case, since different assets included in monetary aggregates have different liquidity characteristics and different returns. Barnett proposed the use of Divisia monetary indexes as an alternative to simple sum aggregates. He has also argued that simple sum aggregates can only be useful as accounting identities, since they cannot approximate the service flow that money provides: they cannot approximate the economic good that we would call money. In fact, the Divisia index proposed by Barnett, possesses some appealing properties. It belongs to the class of 'superlative' index numbers. This implies that the Divisia index for money will be able to track the underlying aggregator function (monetary aggregate) very accurately. In fact, the Divisia index will provide a second order approximation to any given aggregator function (economic monetary aggregate). Hence, the use of such an index will give us a satisfactory approximation of the flow of services that the representative consumer is enjoying out of the economic good, money.

Unfortunately, these measurement issues, especially the measurement of money seem to have received very little attention. In standard money demand studies, it is conventional to use a simple sum aggregate and either the GDP deflator or the CPI, as the price variable. The discussion above indicates that this, due to the bad measurement problem, is bound to lead to flawed results. If neither the money variable nor the price variable approximate the underlying economic variables, is it really meaningful to have any results? And what is the usefulness of these results? It is in part these questions that I try to answer in this paper.

2.2 Methodological Considerations and Long-Run Relations

2.2.1 Methodological Consideration of the Adopted Empirical Approach

In my modelling approach I focus exclusively on domestic relations and I ignore feedback effects from foreign variables. There are two reasons for this. First, I have a *prior belief* that the U.S. is well modelled as a closed economy. Second, I study the long-run properties of money demand using a cointegration approach. The property of cointegration is supposed to be invariant to increases of the information set. So if cointegration is found between domestic variables, the same cointegrating relations should hold in an extended information set; hence the absence of variables measuring foreign effects, will not invalidate my results[6].

There is also a practical reason for focusing only on the domestic economy. I am using a cointegrated VAR approach that has high power when used to analyze relatively small systems. But the extension of this method to larger systems can cause problems of interpretation and may lead to

[6] A specific example of this approach is Juselius (1992).

distorted inferences of the cointegrating rank, when the sample is small[7].

I will now describe how the long-run structure of the model can be related to the cointegration properties of the data. For these hypothetical long-run relations to be supported by the data, a necessary condition is that the corresponding relation is a stationary cointegrating relation with coefficients of the expected magnitude and sign. The dynamics of the model, are empirically determined, since I do not have strong priors to the exact nature of the short-run dynamics.

2.2.2 Long-Run Relations

Here, I construct a small macro model, and examine the relations between money prices, interest rates and aggregate consumption. These relations are similar to those described in Lütkepohl and Wolters (1999).

Money demand, m_t^d, is given by

$$m_t^d - p_t = \lambda_2 c_t + \lambda_3 (i_{mt} - i_{bt}) + \lambda_4 (i_{mt} - \Delta p_t) + v_{mt} \tag{1}$$

where m_t^d is the log of money, p_t is the log of the price level and Δp_t is the inflation rate, c_t is the log of consumption and i_{mt} and i_{bt} denote a short-term and a long-term interest rate (own return of money and return on bonds, in levels). Here, $i_{mt} - i_{bt}$ represents the opportunity cost of holding money relative to a benchmark asset (bond) and $i_{mt} - \Delta p_t$ represents the opportunity cost of holding money relative to stocks of goods[8]. In the above relation, it is real balances that depend on all the other variables. We can rewrite this relationship as

$$m_t^d = \lambda_1 p_t + \lambda_2 c_t + \lambda_3 (i_{mt} - i_{bt}) + \lambda_4 (i_{mt} - \Delta p_t) + v_{mt} \tag{2}$$

In general, we would then expect $\lambda_1 = 1$ for homogeneity of money demand to hold, $\lambda_2 > 0$ so that demand for real balances depends positively on the level of aggregate activity, $\lambda_3 > 0$ and $\lambda_4 > 0$ so that demand for real balances would be higher, the higher the own return on money is. In addition, we would expect that $v_{mt} \sim I(0)$ for the above relationship to qualify as a long-run equilibrium condition.

Real aggregate consumption, c_t, is given by

$$c_t = \lambda_5 t + \lambda_6 i_{bt} + \lambda_7 \Delta p_t + v_{ct} \tag{3}$$

This specification, can be empirically interpreted either as aggregate supply/ Phillips curve relation (if $\lambda_6 = 0$ and $\lambda_5 > 0$, $\lambda_7 > 0$) or as an IS relation (if $\lambda_6 < 0$ and $\lambda_7 = -\lambda_6$) or a combination of the two[9]. The positively sloped Phillips curve interpretation would imply that deviations of c_t from

[7] Gonzalo and Pitarakis (1999) show that increasing the dimension of the vector process analyzed using a VAR with relatively small sample size, in general, leads to highly distorted inferences of the cointegrating rank.

[8] This relationship would be the money-market-equilibrium condition from an optimizing model where there is some sort of storage technology, if the utility function is non-separable in consumption and the good that is used for storage. The return on the goods that are stored is the inflation rate between any two periods.

[9] Implicitely, I am assuming that there is no saving in the model, which implies that we would have $y_t = c_t$. The IS

trend, cointegrate with inflation to stationarity[10]. Similarly, the IS interpretation would imply that deviations of c_t from trend to cointegrate with the real interest rate to stationarity[11]. Empirical support for any such a relation, would require that $v_{ct} \sim I(0)$.

The *short-term interest rate*, i_{mt}, is assumed to be determined by the Fisher parity condition, so that

$$i_{mt} = E_t \Delta p_{t+1} + v_{i_m t} \tag{4}$$

where $E_t \Delta p_{t+1}$ is the expectation at time t of inflation at time $t+1$. Empirical support for the Fisher parity requires $v_{i_m t} \sim I(0)$.

Finally, the *long-term interest rate*, i_{bt}, is assumed to be determined by according to the expectations hypothesis

$$i_{bt} = i_{mt} + v_{i_b t} \tag{5}$$

Hence, for the spread to be stationary we would require that $v_{i_b t} \sim I(0)$. Notice that if the last two relations hold and also $v_{mt} \sim I(0)$, this would also imply that the consumption velocity is also stationary.

These relationships describe the long-run structure that one should aim at uncovering from the data. Notice that all of the above relationships have been formulated under the assumption that only nominal variables m_t and p_t are influenced by the potential $I(2)$ trends in the data, and under the assumption that $\lambda_1 = 1$ in equation (2). Hence all these relations assume that all the other variables in the system might be integrated but at most of order 1. If all four relations hold, then there would be only one common trend (maybe a technology shock). If less than four relationships can be uncovered by the data, then the number of common trends increases. For example one could 'identify' a real and a nominal trend in the data.

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relation may be derived from an optimizing model since it would be given by

$$c_t = E_t c_{t+1} - \sigma E_t \left[i_{bt} - \Delta p_{t+1} \right]$$

Assuming an autoregressive structure for c_t and Δp_t so that $E_t c_{t+1} = \phi_c c_t$ and $E_{t+1} \Delta p_{t+1} = \phi_{\Delta p} \Delta p_t$ we get

$$c_t = -\frac{\sigma}{1 - \phi_c} \left(i_{bt} - \Delta p_t \right) + \left(1 - \phi_{\Delta p} \right) \Delta p_t$$

The Phillips curve specification comes from a similar argument from optimizng behavior. Asusming sticky prices, we have that

$$\Delta p_t = a_1 E_t \Delta p_{t+1} + \alpha_2 y_t$$

Using again the autoregressive structure and the market clearing condition $c_t = y_t$ we have

$$c_t = \frac{1 - a_1 \phi_{\Delta p}}{\alpha_2} \Delta p_t$$

Finally the deterministic trend is motivated in the specification, by the assympption of growth where the real variables are expressed in deviations from their growth rate along a balanced growth path.

[10] With λ_5 and λ_7 positive and with $\lambda_6 = 0$, the aggregate consumption equation can be written as $(c_t - \lambda_5 t) = \lambda_7 \Delta p_t + v_{ct}$.

[11] With $\lambda_6 < 0$, $\lambda_5 > 0$ and $\lambda_7 = -\lambda_6$ then the aggregate consumption equation can be written as $(c_t - \lambda_5 t) = \lambda_6 \left(i_{bt} - \Delta p_t \right) + v_{ct}$.

It should be pointed out that these theoretical relations are not going to be further explored in what follows. This is for two reasons: First, the main hypothesis of interest is that of money demand price homogeneity, that is $\lambda_1 = 1$ in equation (2). The rest of the theoretical relations have been provided so as to give the motivation of a real-money demand study, as has been usually done in the literature (see Hoffman and Rasche (1996) and Lütkepohl and Wolters (1999)). Thus for the purposes of the exercise considered, they are redundant. Secondly. limitations in well tested software for the $I(2)$ analysis, would not allow me to further explore hypothesis testing related to the above theoretical specification. With these details in mind, I now move to the description of the data and the econometric methodology.

3 Data and Econometric Methodology

3.1 The Data

The data I am employing are from the Federal Reserve Bank of Saint Louis FRED database. In the empirical application, I am using $M3$ and the monetary services index (Divisia) for $M3$, the consumer price index (CPI) and the deflator of personal consumption expenditures (PPCE) as the "money" and price variables respectively. Then real personal consumption expenditure (PCE) will be the 'scale' variable, the short-term interest rate is the yield on large denomination time deposits and the long-term interest rate is the yield on a five year bond. Some further details about the data are provided in table 1. I am using monthly data and the sample runs from January 1976 to December 1999. All series are seasonally adjusted except for the two interest rates. Plots of the series are reported in figures 1 and 2. The first row of figure 1, reports the log of $M3$, the log of the monetary services index for M3, the log of CPI and the log of personal consumption expenditures deflator. The second row plots the first difference of the variables and the third the second difference. Figure 2 reports the log of personal consumption expenditure, the short term interest rate (LTDY) and the long-term interest rate (BS5Y), both in levels and in first differences. Further details about the construction of the monetary services indexes are provided in Anderson et al. (1997a, 1997b).

[Insert Figures 1 and 2]
[Insert Table 1]

The focus of the paper is on long-run price homogeneity. Juselius (1999b) explains that there seem to be ample empirical evidence of both the measures of prices p_t and of the nominal money stock m_t are best described as variables integrated of order two (i.e. $I(2)$). I provide some initial evidence in favor of such an assumption in table 2 by standard unit root tests. Further evidence will be provided below. The hypothesis of long-run price homogeneity is of interest in empirical modeling for two reasons. Firstly, because imposing long-run price homogeneity without properly testing can create serious problems in the conduct of inference. Recent work by Kongsted (1999) and Jørgensen (2000) indicates that the improper treatment of $I(2)$ variables can lead to serious problems of inference. For example, suppose that there is only one $I(2)$ common trend that is fed into m_t and p_t in the model. If $m_t - p_t$ is CI(2,1) in the sense of Engle and Granger (1987), then inference in the $I(1)$ model using a measure of real balances and the growth rate of one of the nominal variables is valid (see Kongsted (1999)). But if $m_t - p_t$ is still an $I(2)$ variable, then just assuming *a priori* that $m_t - p_t$ is an $I(1)$ variable, will lead to invalid inference since the $I(1)$ model would not be appropriate for the analysis. Any relation between $m_t - p_t$ and the rest of the variables in the system would be spurious. Secondly, because the proportionality between money and prices (homogeneity of degree zero of the money demand) is one of the building blocks of monetary economics, empirical failure of this proposition would imply that we should radically change the way we think about the world.

[Insert Table 2]

3.2 Econometric Methodology

In the econometric analysis I employ a vector autoregression (VAR) with k lags, given by

$$\Delta^2 x_t = \Pi x_{t-1} - \Gamma \Delta x_{t-1} + \sum_{i=1}^{k-2} \Psi_i \Delta^2 x_{t-1} + \mu_0 + \mu_1 t + \varepsilon_t \quad (6)$$

$$\varepsilon_t \sim IID(0, \Omega) \quad (7)$$

where x_t is a $n \times 1$ vector of variables in the system, and t is a deterministic trend. If we allow the parameters $\theta = \{\Pi, \Gamma, \Psi_1, ..., \Psi_{k-2}, \phi, \mu_0, \mu_1, \Omega\}$ to vary unrestricted, then the model (6) corresponds to the *I(0)* model. The *I(1)* and *I(2)* models are obtained as special cases of (6) if certain restrictions are satisfied. Thus, the higher order models are nested within the more general *I(0)* model.

I model the vector of variables x_t as $I(2)$ by testing two reduced rank hypotheses (see Johansen (1992, 1995a))

$$\Pi = \alpha\beta' \quad (8)$$

and

$$\alpha'_\perp \Gamma \beta_\perp = \xi\eta' \quad (9)$$

where α, β are $n \times r$ full rank matrices ($r < n$) and ξ, η are $(n-r) \times s$, also of full rank $(s < n - r)$. Let also $\bar{c} = c(c'c)^{-1}$ for any matrix $n \times r$ of full rank r. Then define the matrices

$$\beta_1 = \bar{\beta}_\perp \eta, \; \beta_2 = \beta_\perp \eta_\perp, \; \alpha_1 = \bar{\alpha}_\perp \xi, \; \alpha_2 = \alpha_\perp \xi_\perp$$

Johansen (1995a, 1997) shows that the space spanned by the vector x_t can be decomposed into r stationary directions, β, and $n - r$ non-stationary directions, $\beta_\perp$, whilst the latter can be further decomposed into the directions (β_1, β_2) where β_1 and β_2 are as defined above[12]. Then, for the

[12] The properties of the process can be described as follows:
$I(2) : \{\beta'_2 x_t\}$
$I(1) : \{\beta' x_t\}, \{\beta'_1 x_t\}$

process to be at most $I(2)$ it should also hold that

$$\alpha_2' \Theta \beta_2 \tag{10}$$

is of full rank $(n - r - s)$, where $\Theta = \left(\Gamma \bar{\beta} \bar{\alpha}' \Gamma + I - \sum_{i=1}^{k-2} \Psi_i\right)$. Following Paruolo (1996), Rahbek et al. (1999), the constant term coefficient μ_0 is restricted to be in $span(\alpha)$ while the linear trend coefficient μ_1 is restricted to be in $span(\alpha, \alpha_1)$, in order to allow for linear trend in all the directions while eliminating any quadratic trends[13]. Then the moving average representation is given by (see Johansen (1992, 1995a, 1995b, 1997), Paruolo (1996) and Rahbek et al. (1999))

$$x_t = C_2 \sum_{j=1}^{t} \sum_{i=1}^{j} \varepsilon_i + C_1 \sum_{j=1}^{t} \varepsilon_j + C_0(L) \varepsilon_t + \vartheta_0 + \vartheta_1 t + A + Bt \tag{11}$$

where

$$C_2 = \beta_2 \left(\alpha_2' \Theta \beta_2\right)^{-1} \alpha_2' \tag{12}$$

The matrix C_1 satisfies the following relationships

$$\beta' C_1 = \left(\bar{\alpha}' \Gamma \bar{\beta}_2\right) \left(\bar{\alpha}_2' \Theta \bar{\beta}_2\right)^{-1} \bar{\alpha}_2' = \bar{\alpha}' \Gamma C_2 \tag{13}$$

$$\beta_1' C_1 = \bar{\alpha}_1 \left(\Theta C_2 - I_n\right) \tag{14}$$

and the parameters A, B are functions of initial conditions.

The hypothesis that x_t is $I(1)$ is formulated as the reduced rank hypothesis (8) and the full rank of (9). Here one assumes that the matrix Π has rank $r < n$. In this case Π can be decomposed as the product of two matrices $\alpha\beta'$ where α, β are each $n \times r$ and have full rank r. In this case, the moving average representation of x_t is given by

$$x_t = C \sum_{i=1}^{t} \varepsilon_i + +\kappa_0 + \kappa_1 t + C^*(L) \varepsilon_t + F \tag{15}$$

where $C = \beta_\perp \left(\alpha_\perp' \Gamma \beta_\perp\right)^{-1} \alpha_\perp'$, $C^*(L)$ is a polynomial in the lag operator, and F is a function of

$I(0)$: $\{\beta_1' \Delta x_t\}, \{\beta_2' \Delta x_t\}, \{\beta' x_t + \delta' \Delta x_t\}$ where δ is a $n \times r$ matrix of weights, designed to pick out the $I(2)$ components of x_t (Johansen, 1995a). We can see that the cointegrating relations $\beta' x_t$ are actually $I(1)$ and require a linear combination of the differenced process Δx_t to achieve stationarity. Of course, Johansen (1992, 1995a) also points out the posibility that there might be, direct cointegrating relations (in the levels of the process), given by $\{\delta_\perp' \beta' x_t\}$.

[13] The constant term can be partitioned into three parts to acount for the intercepts and the linear and quadratic trends:

$$\mu_0 = \alpha \mu_{00} + \alpha_1 \mu_{01} + \alpha_2 \mu_{02}$$

where $\mu_{00} = \bar{\alpha}' \mu_0$, $\mu_{01} = \bar{\alpha}_1' \mu_0$, $\mu_{02} = \bar{\alpha}_2' \mu_0$ and the matrices $\bar{\alpha}, \bar{\alpha}_1$ and $\bar{\alpha}_2$ are as defined in the text. Imposing the restriction of no quadratic trends implies that $\mu_{02} = 0$.

Similarly, the coefficient of the linear term can be decomposed as

$$\mu_1 = \alpha \mu_{10} + \alpha_1 \mu_{11} + \alpha_2 \mu_{12}$$

where $\mu_{10} = \bar{\alpha}' \mu_1$, $\mu_{11} = \bar{\alpha}_1' \mu_1$, $\mu_{12} = \bar{\alpha}_2' \mu_1$, to acount for linear, quadratic and third-order trends. The restriction of having only linear trends in the data would imply that $\mu_{11} = 0$ and $\mu_{12} = 0$.

initial conditions.

Notice that for a vector of variables to be characterized as an $I(2)$ process it is not necessary that all the variables in the vector process are $I(2)$. The above reduced rank conditions are applicable for the vector process; this vector process could include only $I(2)$, or any mixture of $I(2)$, $I(1)$ and even $I(0)$ variables. In essence, including a stationary variable in the system would result in a trivial unit cointegrating vector. In contrast with the definition of cointegration in Engle and Granger (1987), where it would be a necessary for all variables to be $I(2)$ for the analysis to 'make sense', in the approach of Johansen (1995a, 1997) this is not a prerequisite for the analysis. I can thus proceed with testing the two reduced rank hypotheses (8) and (9), establishing the cointegrating ranks (or 'integration indexes' according to Paruolo (1996)). Then from (12) it can easily be seen that whether any variable is $I(2)$ or not depends on β_2, the loading matrix of the $I(2)$ common trends. The hypothesis that any components x_{it} of the vector process x_t is at most $I(1)$ instead of $I(2)$ can be formulated as zero row of β_2. Additionally, the coefficients α_2 show which (reduced form) shocks contribute to the common $I(2)$ trends, since we can define these common trends as

$$\alpha_2' \sum_{j=1}^{t} \sum_{i=1}^{j} \varepsilon_i.$$

Hence, the hypothesis that the shocks from any given equation do not contribute to the common $I(2)$ trends, can be formulated as zero row of the α_2 matrix.

4 The Empirical Model: Is Long-Run Price Homogeneity Supported by The Data?

I chose a VAR model with a linear trend restricted in the cointegrating space. The linear trend is included in the cointegrating space following the argument of Rahbek et al. (1999)[14]. I analyze the following vectors of variables:

$$\begin{aligned} \tilde{x}_t &= \begin{bmatrix} \tilde{m}_t & \tilde{p}_t & c_t & i_{mt} & i_{bt} \end{bmatrix}', \; t = 1976:01 - 1999:12 \\ \text{and } x_t &= \begin{bmatrix} m_t & p_t & c_t & i_{mt} & i_{bt} \end{bmatrix}', \; t = 1976:01 - 1999:12 \end{aligned}$$

where $\tilde{m}_t$ is the log of $M3$, $\tilde{p}_t$ is the log of CPI, c_t is real personal consumption expenditure, i_{mt} is a short-term interest rate (yield on large denomination time deposits at commercial banks and thrift institutions), i_{bt} is the yield on a long-term bond (5 year treasury bond with constant maturity), while m_t is the log of the monetary services index for $M3$, and p_t is the deflator of personal consumption expenditure (a chained price index). In the following subsection I analyze the first vector of variables, which I will call the 'conventional money demand' analysis and demonstrate that long-run price homogeneity fails; next I analyze the second vector of variables with the monetary services (Divisia) index as the measure of money and a superlative-chained price index instead of

[14] Rahbek et al. (1999), extending previous results of Paruolo (1996), argued that for similarity of inference one should restrict the trend to lie in the cointegrating space. The interested reader should consult their paper for details.

the CPI, which I will call the 'divisia approach to money demand'[15].

4.1 Long-Run Price Homogeneity: Lost ...

4.1.1 Lag-Length Selection and Specification Tests

To proceed with the statistical analysis I chose the lag-length using a general-to-specific reduction of the VAR model. The AIC and the reduction process indicated that a lag order of 7 was necessary. As a general check of the statistical adequacy of the model both univariate and multivariate mis-specification tests are reported in table 3. Significant test statistics are given in bold face. As can be seen from the table, the model seems to be mis-specified. Strong ARCH effects are present, and non-normal error terms. There is also some autocorrelation present, although the equation-by-equation statistics give a different indication. Using fewer lags, the used empirical model becomes even more mis-specified, since most the tests statistics are significant even at a 1% level. I finally chose a lag-length of seven since, in my judgement, further reducing the lag length would have resulted in higher order autocorrelation[16]. A more serious issue is that of parameter constancy. To ensure that the reduced form parameters remain constant within the sample, I recursively estimated the models and no strong evidence of parameter non-constancy was found[17].

[Insert Table 3]

4.1.2 Cointegration Rank Inference

I can now turn to the investigation of relationship between $M3$ and the CPI in the $I(2)$ model by means of (6). I first present the two rank/integration indices and then, I investigate long-run price proportionality.

The joint determination of the two "integration indices" can be based either on the two-step method (henceforth 2SI2) derived by Johansen (1995a) or using the maximum likelihood method (henceforth ML, Johansen (1997)). To facilitate comparisons with other studies and since the available software is still being developed, I used both methods to obtain the integration indices. The results should be treated with caution since, my findings do not clearly support only one pair of integration indices.

The model I estimate included a constant and a trend restricted, so that a linear term is allowed in all directions. The results are summarized in tables 4 and 5.

[15] Here I am running the danger of abusing slightely the language since the 'divisia' approach would require the use of the user cost of money instead of the two interest rates that I have included in the model. This is done solely for the sake of comparability of the results. The interested reader should consult Barnett and Serletis (2000) for details on the 'divisia approach' to money demand.

[16] Gonzalo (1994) argues that for the $I(1)$ model, deviations from normality and ARCH effects are not so harmful for the purpose of statistical inference (cointegration), and documents the superiority of the Johansen/ ML estimator. A similar argument probably carries over to the $I(2)$ ML or Two-Step Procedure, which are used in this paper. The reason for the equivalence of these two methods is that, as Paruolo (2000) shows, the Two-Step estimator is asymptotically efficient, as is the ML estimator.

[17] The likelihood ratio tests of model reduction, the information criteria and the recursive tests are not presented in the paper for brevity, but are available upon request.

[Insert Tables 4 & 5]

- For the uninitiated reader some explanations would be helpful. The testing sequence starts from the most restricted model $\{r = 0, s = 0\}$ which implies that there is no cointegration and there are five $I(2)$ trends in the system. The sequence goes on reading each table from left to right and from top to bottom. The last column in each table, labeled Q_r corresponds to the so-called trace statistic for the $I(1)$ model (see Johansen (1995b)). For example, from both tables we can see that the hypothesis of no cointegration ($r = 0$) with four $I(1)$ trends ($s = 4$) and one $I(2)$ trend (i.e. $\{r = 0, s = 4\}$ and $n - r - s = 1$) can be rejected (first row, fifth column). The testing procedure stops, when one hypothesis cannot be rejected. The 2SI2 procedure works as follows: First one chooses the cointegrating rank r i.e. the rank of β and α. This is done by reduced rank regression (see Johansen (1995b)). Having fixed r, one can then determine the rank of the matrices ξ and η, i.e. s, again by reduced rank regression. The ML procedure of on the other hand, works by simultaneously determining the two ranks r and s.

- As we can see from the tables 4 and 5, one cannot reject the hypothesis $\{r = 2, s = 1\}$ using both the 2SI2 and the ML procedure. Similarly the hypothesis $\{r = 2, s = 2\}$ can only marginally be rejected. Both the 2SI2 and the ML procedure indicate that there may be two independent $I(2)$ trends in the system ($\{r = 2, s = 1\}$). Such a finding is also consistent with number of roots close to unity after the choice of the cointegration rank r in the first step of the 2SI2 procedure[18] (see table 3). As a further indication, I used the recursively estimated the eigenvalues of the first step from the 2SI2 procedure, which indicated that a choice of $r = 2$ is appropriate[19]. The more economically meaningful choices are $\{r = 1, s = 3\}$, $\{r = 2, s = 2\}$ and $\{r = 3, s = 1\}$, since such choices imply one common $I(2)$ trend in the data, and economic intuition suggests that this is the common trend of the nominal variables (money and prices). When I test for long-run price homogeneity I will use all three economically meaningful choices, with $\{r = 2, s = 2\}$ being the preferred one. The choice of $\{r = 2, s = 2\}$ is economically appealing: Such a choice implies two cointegrating vectors and hence three common trends, one of which is the $I(2)$ trend and the remaining are $I(1)$. We then have three "autonomous" permanent shocks to the system, say $\varepsilon_{1t}, \varepsilon_{2t}$ and ε_{3t} that can be interpreted as a domestic nominal shock (that gives rise to the $I(2)$ trend), a real shock, and another 'nominal' shock that influences the system via the long-term rate rate.

- Further notice that the integration indexes (cointegration ranks) r and s that can be chosen as the outcome of the above testing procedure indicate that there are some $I(2)$ variables in the system. Deciding in favor of two $I(2)$ common trends would imply that there are at least two $I(2)$ variables in the system and the natural candidates are $\tilde{m}_t$ and $\tilde{p}_t$, which would be

[18] In the case of the $I(2)$ model, the total number of unit roots of the process is $2(p - r) - s$ (See Johansen (1995b) pp. 61, *Corollary 4.7)*. Thus knowing the number of unit roots consistent with choices of r and s, provides an indirect, although *ad hoc* way of evaluating whether the ranks / integration indices obtained via the test statistics are "correct".

[19] These figures are not reported in the paper for brevity, but are available upon request.

driven by these two independent $I(2)$ trends. Deciding that there is one $I(2)$ common trend in the system $(n-r-s=1)$, it still provides evidence that there are some $I(2)$ variables in the system, and the more likely are again $\tilde{m}_t$ and $\tilde{p}_t$. But having found one $I(2)$ trend does not necessarily imply that a linear combination $m_t - p_t$ cancels this common trend!

4.1.3 Testing Long-Run Price Homogeneity

Under the assumption that only nominal variables are affected by the $I(2)$ common trend in the system, the hypothesis of long-run price homogeneity can be formulated as[20]:

$$\beta_i = [a_i, -a_i, *, *, *]', \; i = 1, ..., r \tag{16a}$$
$$\beta_{1j} = [b_j, -b_j, *, *, *]', \; j = 1, ..., s \tag{16b}$$

and respectively

$$\beta_{2l} = [c_l, c_l, 0, 0, 0]', \; l = 1 \tag{17}$$

where I have assumed that the data vector is $x_t = [\tilde{m}_t, \tilde{p}_t, c_t, i_{mt}, i_{bt}]$, where β and β_1 define the CI(2,1) relations, β may also define CI(2,2) relations, and β_2 is the loading matrix of the $I(2)$ common trends in the system, while $n-r-s=1$, so that β_2 is of dimension $n \times (n-r-s) = n \times 1$.

To get a feeling of the parameter estimates in table 6, I have reported the parameter estimates for the matrices β, β_1 and β_2 for the case where $\{r=2, s=2\}$. What one can observe from β_2 is that the loading coefficients of the two nominal variables are similar and of similar magnitude, while c_t does not seem to be influenced by the $I(2)$ trends. Lastly, the two interest rates, seem to be equally influenced the common $I(2)$ trend, but the magnitude of the influence is rather limited.

[Insert Table 6]

The results for the tests of long-run price homogeneity are reported in table 7. I test the joint hypothesis on the columns of β and β_1. From the results it can inferred that price homogeneity of the 'traditional' money demand fails dramatically! In all three economically meaningful cases, the $p-value$ of the hypothesis of long-run price homogeneity is virtually zero. It follows directly that, using $\tilde{m}_t - \tilde{p}_t$ as a variable in the system and preforming an $I(1)$ analysis on the transformed system

$$\bar{x}_t = \begin{bmatrix} \tilde{m}_t - \tilde{p}_t & \Delta\tilde{p}_t & c_t & i_{mt} & i_{bt} \end{bmatrix}'$$

would lead to invalid inference. One could also argue that, using standard monetary aggregates and a price index which, in general, do not provide a good approximation to the economic good 'money' and the true cost-of-living, fails to give rise to a homogeneous money demand function. This argument will become crystal-clear below.

[Insert Table 7]

[20] Some technical details are provided in Appendix I.

4.2 Long-Run Price Homogeneity: ... and Found

4.2.1 Lag-Length Selection and Specification Tests

Using similar steps as above I have chosen a lag length of 7, that was selected by likelihood ratio reduction tests and the AIC. Some univariate and multivariate mis-specification tests are reported in table 8, with significant test statistics given in bold face. As can be seen again, the model seems to be mis-specified, with some autocorrelation present, while again the equation-by-equation statistics give a different indication. Recursive test statistic also indicated no serious parameter non-constancy, so I decided to keep the model with seven lags[21].

[Insert Table 8]

4.2.2 Cointegration Rank Inference

I can now look at the relationship between divisia index $M3$ and the chained price index in the $I(2)$ model. The model specification is identical to the one used above: it includes seven lags, a constant and a trend restricted, so that a linear term is allowed in all directions. The results are summarized in tables 9 and 10.

[Insert Tables 9 & 10]

- As it can be read from tables 9 and 10, again one cannot reject the hypothesis $\{r = 2, s = 1\}$ using the 2SI2 while the hypothesis $\{r = 2, s = 2\}$ can marginally be rejected at the 10% significance level. In contrast,the ML procedure indicates that the first non-rejection occurs for the pair $\{r = 2, s = 0\}$. This pair of cointegration ranks is economically unappealing since there is no straightforward economic interpretation that can be attached to it. If one chooses $\{r = 2, s = 2\}$ then the modulus of the next largest eigenvalue of the companion matrix is roughly 0.95, implying a half-life of roughly a year. That is although persistent, the deviation from the steady-state dies out pretty quickly. Again the recursively estimated the eigenvalues of the first step from the 2SI2 procedure, indicated that a choice of $r = 2$ is appropriate[22]. In what follows, I will focus again on the more economically meaningful choices, which are $\{r = 1, s = 3\}$, $\{r = 2, s = 2\}$ and $\{r = 3, s = 1\}$, since such choices imply one common $I(2)$ trend in the data; $\{r = 2, s = 2\}$ will be the preferred one, for the same reasons that were described before in the 'traditional approach to money demand' framework.

4.2.3 Testing Long-Run Price Homogeneity

Under the assumption that only nominal variables are affected by the $I(2)$ common trend in the system, the hypothesis of long-run price homogeneity can be formulated by equations (16a) and (16b) or (17). In table 11, I have reported the parameter estimates for the matrices β, β_1 and β_2 for the case where $\{r = 2, s = 2\}$. What one can again observe from β_2 is that the loading coefficients

[21] These results are available upon request.

[22] These figures are not reported in the paper for brevity, but are available upon request.

of the two nominal variables are similar and of similar magnitude, while c_t does not seem to be influenced by the $I(2)$ trends. Finally, the two interest rates, seem to be equally influenced the common $I(2)$ trend, but the magnitude of the influence is rather limited.

[Insert Table 11]

The results for the tests of long-run price homogeneity are reported in table 12. I test the joint hypothesis on the columns of β and β_1. From the results it can easily seen that when a Divisia index measure of money and a superlative, chained price index is used, long-run price homogeneity can be attained. In two out of three cases we have a large $p-value$ supporting the hypothesis. The fact that the hypothesis is rejected for the case of $\{r = 3, s = 1\}$ is not a serious problem, taking into account that according to the 2SI2 and the ML procedure, the choice of the cointegration ranks should be $\{r = 2, s = 2\}$. In contrast with the 'traditional' use of simple sum monetary aggregate (the 'traditional money demand'), the use of monetary aggregation theory and of index number theory to choose the appropriate measure of money and prices, gives rise to a homogeneous money demand function. This should come as no surprise. The measurements for money and prices used in this case, are very good approximations (up to a third order remainder term) of the underlying aggregator functions. An additional implication of this finding is that under the economically meaningful cases, the system can be transformed to one using a measure of real balances and inflation or the money growth rate as the system variables[23]. That is the analysis can be based on a transformed vector of variables being given by

$$\hat{x}_t = \left[\begin{array}{ccccc} m_t - p_t & \Delta p_t & c_t & i_{mt} & i_{bt} \end{array} \right]'$$

without any loss of information regarding the long-run relationships between the variables.

[Insert Table 12]

In contrast, resorting to the traditional money measures like $M3$ and CPI as the price variable, such a transformation would not be valid! The argument here can easily be generalized. It is not just CPI, but it would be any price index that does not belong to the class of 'superlative' index numbers defined in Diewert (1976). The specification using $M3$ and CPI, is supposed to serve merely as an example. Additionally, it has been repeatedly argued that simple sum monetary aggregates do not provide reasonable approximations to the underlying economic aggregates (see Barnett and Serletis (2000) and the references therein). The results obtained here, point in the same direction. Unless a divisia money index and a chained-superlative price index are used, money and prices would not be cointegrated CI(2,1), in the sense of Engle and Granger (1997).

5 Concluding Remarks

The main purpose of this paper was to show that using good approximations to variables that are included in economic models, is most likely to bring about desired economic relationships that are

[23] This the the Nominal-to-Real transformation proposed in Kongsted (1999).

both theoretically and empirically appealing. More specifically I have empirically demonstrated that employing Divisia monetary aggregates and superlative-chained price indexes for the U.S., has restored long-run price homogeneity in the money demand. In contrast, employing simple sum aggregates and fixed base price indexes long-run price homogeneity fails dramatically! This is not surprising taking into account the fact that the divisia and the chained price index used here belong to the class of superlative index numbers defined in Diewert. This means that these index numbers can provide second order approximation to the underlying - unknown aggregator functions. Additionally, the work of Barnett has shown that, in continuous time, the divisia monetary aggregate will track the underlying aggregator function *without any error.* In discrete time with uncertainty, the divisia index number used, although somewhat to a lesser extent, it will track the underlying aggregator function fairly well.

Using the newly developed statistical analysis of $I(2)$ variables, I have shown that there is at least one $I(2)$ trend in the systems of variables employed and I have pointed out that the natural candidates are money and prices. After establishing that the there were two cointegrating relations, I have examined the hypothesis of long-run price homogeneity for the more economically interesting cases, and I have established that the system formed using simple sum $M3$ and the CPI does not satisfy the assumption of long-run price homogeneity. Conversely, a similar system that uses a Divisia measure of $M3$ and a chained price index, satisfies the hypothesis of long-run price homogeneity and the system can be further analyzed by a conventional $I(1)$ analysis.

I have thus demonstrated that using superlative measures of money and prices restores long-run price homogeneity. What comes out as the main result of the paper is that the measurement problem turns out to be more serious than people in the profession imagine. The use of superlative, chained measures moderates the mis-measurement problem to a certain extent. The reason is that even if an ideal superlative index number is used, this index is bound not to take explicitly into account product quality improvements, or different technologies adopted in different sectors, having an impact both on the quality of the supplied services and on the relative price change for each good produced within that sector. Still, such superlative indexes do provide more accurate measures of the underlying economic goods and in any case they provide a better approximation than the usually employed fixed-base indexes.

Another thing that should be pointed out is that the empirical specification used in what I have called the 'divisia approach to money demand' is somewhat in contrast with the results form the theory of monetary aggregation. For a proper analysis, one should use an index of user cost of the monetary aggregate, instead of two interest rates. Knowing the limitations of the approach followed here, this choice was made just in order to have a direct comparison of the results. Of course similar results can be obtained using the user cost of the monetary aggregate instead of a short-term and a long-term interest rate (Konstantinou, 2002). Nevertheless, I think that the results are encouraging and the possibilities of using indexes for monetary aggregates in a similar framework needs to be further investigated.

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Appendix I: Some Details

In this appendix I provide some details regarding the statistical analysis of the cointegrated $I(2)$ model. The model, disregarding deterministic and lagged terms and for simplicity, is given by

$$\Delta^2 x_t = \Pi x_{t-1} - \Gamma \Delta x_{t-1} + \varepsilon_t \tag{A.1}$$

where the two reduced rank conditions $\Pi = \alpha\beta'$ and $\alpha'_\perp \Gamma \beta_\perp = \xi\eta'$ hold. Define as in the text $\bar{c} = c(c'c)^{-1}$ for any matrix $n \times r$ of full rank r such that $c'\bar{c} = \bar{c}'c = I_r$; define also projection into the space spanned by the columns of c as $P_c = \bar{c}c' = c\bar{c}' = c(c'c)^{-1} c'$, and the matrices

$$\beta_1 = \bar{\beta}_\perp \eta,\ \beta_2 = \beta_\perp \eta_\perp,\ \alpha_1 = \bar{\alpha}_\perp \xi,\ \alpha_2 = \alpha_\perp \xi_\perp$$

Then the full rank condition $\alpha'_2 \left(\Gamma\bar{\beta}\bar{\alpha}'\Gamma + I - \sum_{i=1}^{k-2} \Psi_i \right) \beta_2$ should also hold. Note that $(\beta, \beta_1, \beta_2)$ are mutually orthogonal and the same holds for $(\alpha, \alpha_1, \alpha_2)$. Following Johansen (1997) the MA representation of x_t is given by

$$x_t = C_2 \sum_{j=1}^{t} \sum_{i=1}^{j} (\varepsilon_i + \phi D_i) + C_1 \sum_{j=1}^{t} (\varepsilon_j + \phi D_j) + C_0(L)(\varepsilon_t + \phi D_t) + \vartheta_0 + \vartheta_1 t + A + Bt \tag{A.2}$$

where

$$C_2 = \beta_2 \left(\alpha'_2 \Theta \beta_2 \right)^{-1} \alpha'_2 \tag{A.3}$$

and the matrix C_1 satisfies the following relationships

$$\beta' C_1 = \left(\bar{\alpha}'\Gamma\bar{\beta}_2 \right) \left(\bar{\alpha}'_2 \Theta \bar{\beta}_2 \right)^{-1} \bar{\alpha}'_2 = \bar{\alpha}'\Gamma C_2 \tag{A.4a}$$

$$\beta'_1 C_1 = \bar{\alpha}_1 (\Theta C_2 - I_n) \tag{A.4b}$$

It follows form (A.2) that x_t needs to be difference twice to be made stationary. It is also clear form (A.2) and (A.3) that $\beta'_2 x_t$ is $I(2)$ and no linear combination of this process has a lower order of integration. Let $\tau = (\beta, \beta_1)$ and hence $\tau_\perp = \beta_2$. Since $(\beta, \beta_1)' \beta_2 = 0$ we have that $(\beta, \beta_2)' C_2 = 0$ and hence $\tau' x_t = (\beta, \beta_1)' x_t$ is $I(1)$ in general. In fact Johansen (1995b) shows that $\beta' x_t - \bar{\alpha}\Gamma\bar{\beta}_2\beta'_2\Delta x_t = \beta' x_t - \bar{\alpha}\Gamma P_{\beta_2}$ and hence also the process $\beta' x_t - \bar{\alpha}\Gamma\Delta X_t = \beta' x_t - \bar{\alpha}\Gamma \left(P_\beta + P_{\beta_1} + P_{\beta_2} \right) \Delta x_t = \beta' x_t - \bar{\alpha}\Gamma \left(P_\tau + P_{\beta_2} \right) \Delta x_t$ are stationary, since $\tau'\Delta x_t = (\beta, \beta_1)' \Delta x_t$ is stationary. Hence the representation (A.2) implies that τ reduces the order of integration from 1 to 2. The process $\beta' x_t$ cointegrates with the $I(1)$ process Δx_t to stationarity, a phenomenon referred to as polynomial cointegration.

Following Johansen (1997) the following re-parametrization of the model can be used

$$\Delta^2 x_t = a \left(\rho'\tau' x_{t-1} + \psi \Delta x_t \right) + \Omega\alpha_\perp \left(\alpha'_\perp \Omega \alpha_\perp \right)^{-1} \kappa'\tau'\Delta x_{t-1} + \varepsilon_t \tag{A.5}$$

where $\beta = \tau\rho, \psi' = -\left(\alpha'\Omega^{-1}\alpha \right)^{-1} \alpha'\Omega^{-1}\Gamma$ and $\kappa = -\alpha'_\perp \Gamma \left(\bar{\beta}, \bar{\beta}_1 \right)$. Another re-parameterization of the model (A1) give in Johansen (1995a) is

$$\Delta^2 x_t = \alpha \left(\beta' x_{t-1} + \delta\beta'_2 \Delta x_{t-1} \right) + (\phi_1, \phi_2) \tau' \Delta x_{t-1} + \varepsilon_t \tag{A.6}$$

where $\phi_1 = -\Gamma\bar{\beta}, \phi_2 = -\Gamma\bar{\beta}_1$ and $\delta = -\bar{\alpha}'\Gamma\bar{\beta}_2$. It can directly be seen that (A.5) and (A.6) can be combined to give

$$\Delta^2 x_t = \alpha \begin{pmatrix} \rho \\ \delta \end{pmatrix}' \begin{pmatrix} \tau' x_{t-1} \\ \tau'_\perp \Delta x_{t-1} \end{pmatrix} + \zeta\tau'\Delta x_{t-1}$$

where $\zeta = \left[\alpha\psi\bar{\tau} + \Omega\alpha_\perp (\alpha'_\perp \Omega \alpha_\perp)^{-1} \kappa' \right] = (\phi_1, \phi_2)$ and where I have used the definition $\tau_\perp = \beta_2$ and the fact that $\psi' = \bar{\alpha}\Gamma - \bar{\alpha}\Omega\alpha_\perp \left(\alpha'_\perp \Omega \alpha_\perp\right)^{-1} \kappa'\tau'$ so that $\psi'\bar{\beta}_2 = -\bar{\alpha}\Gamma\bar{\beta}_2 = \delta$ hence $\left(\psi'\bar{\beta}_2\right)\beta_2 = \delta\beta_2$.

The hypothesis of long-run price homogeneity (assuming the presence of only one $I(2)$ common trend, i.e. $(n - r - s) = 1$) amounts to testing that

$$\tau_\perp = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \varphi_{1,\tau_\perp}$$

or

$$\tau = H\varphi = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \varphi_{11} & \varphi_{21} & \varphi_{31} & \varphi_{41} \\ \varphi_{12} & \varphi_{22} & \varphi_{32} & \varphi_{42} \\ \varphi_{13} & \varphi_{23} & \varphi_{33} & \varphi_{43} \\ \varphi_{14} & \varphi_{24} & \varphi_{34} & \varphi_{44} \\ \varphi_{15} & \varphi_{25} & \varphi_{35} & \varphi_{45} \end{pmatrix}$$

The asymptotic distribution of the test is a χ^2 with four degrees of freedom. Notice that the last coefficient in each vector corresponds to the trend that has been restricted to lie in the cointegrating space. (Following Johansen (1995b), instead of the vector x above, we would use the vector $x_t^* = \begin{pmatrix} x_t \\ t \end{pmatrix}$).

Appendix II

A Figures

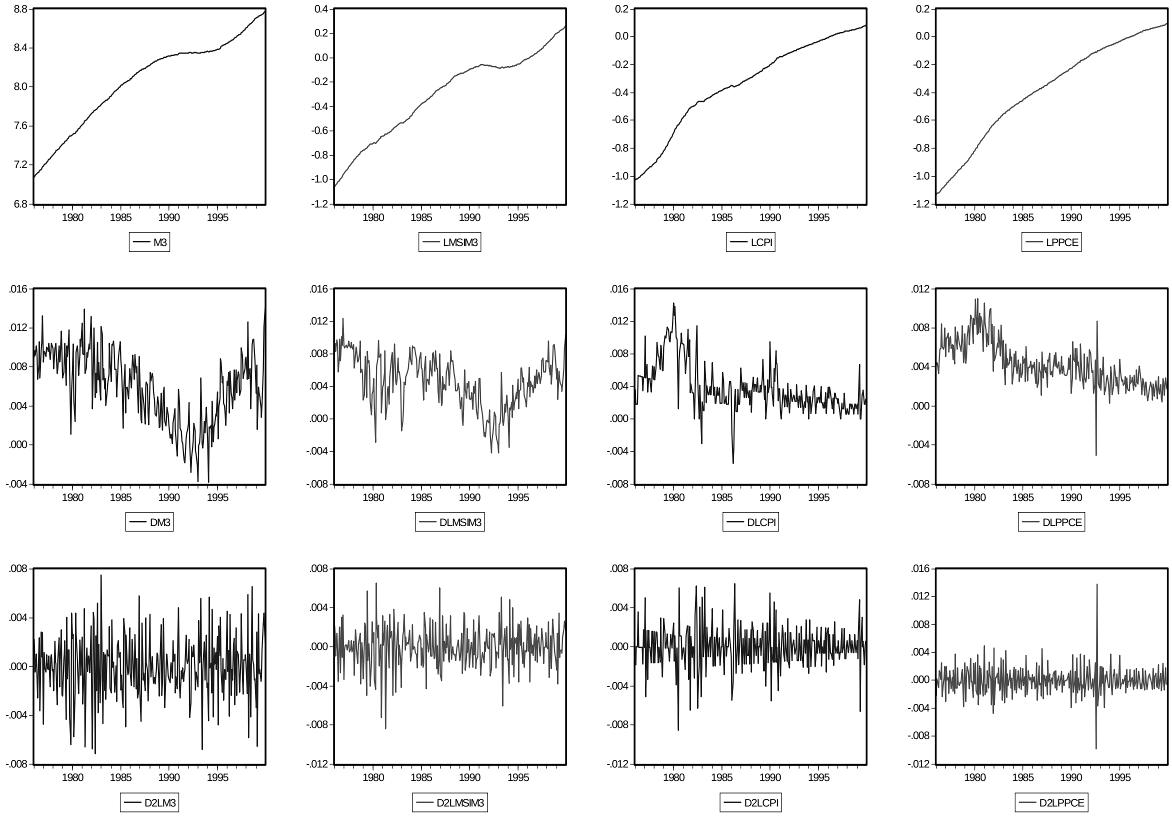

Figure1: Money and Price Variables: Levels, Fisrt and Second Differences

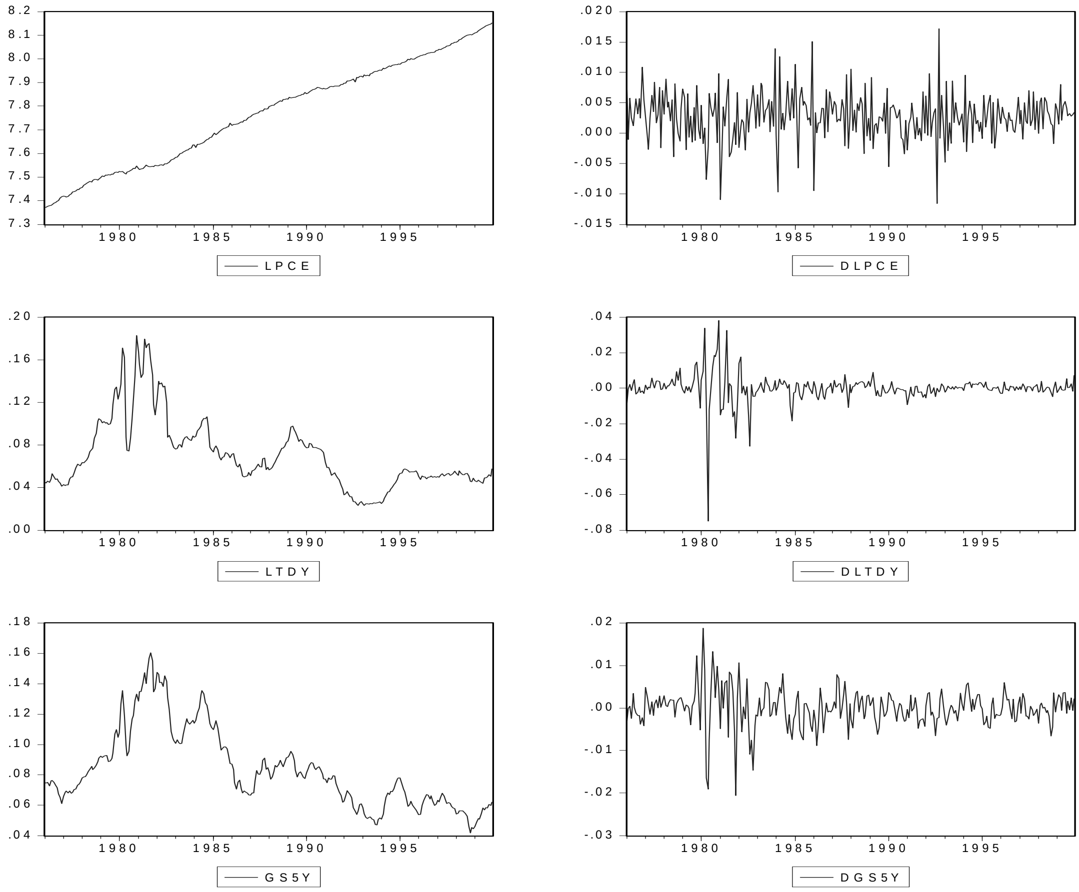

Figure2: Real Personal Consumption Expenditure and Interest Rates: Levels and First Differences

B Description of the Data

Table 1: Data Definitions

Variable.	*Definition*	*Source*
$\tilde{m}_t$	(log of) M3	
$\tilde{p}_t$	(log of) Consumer Price Index (CPI), base year 1996	
m_t	(log of) Nominal (Törnqvist-Theil) Monetary Services Index For M3[a]	FRED
p_t	(log of) Personal Consumption Expenditure Deflator, base year 1996	FRED
c_t	(log of) Real Personal Consumption Expenditure[b]	FRED
i_{mt}	Yield on Total Large Denomination Time Deposits	Constructed[c]
i_{bt}	Five-year Treasury Constant Maturity Rate	FRED

a. Base year 1996

b. Billions of Dollars, Chained 1996 prices

c. From the nominal user cost formula $\pi_{it} = \frac{R_t - r_{it}}{1+R_t}$ obtained as $r_{it} = R_t - \pi_{it}(1 + R_t)$ where the user cost for Large Denomination Time Deposits π_{it} and the benchmark asset return R_t were obtained from the data set from the data set of Anderson et. al. (1997)/ St. Louis FED available at `http://www.stls.frb.org/research`

Table 2: Univariate Unit Root Tests

Var	Constant & Trend	Constant	Var.	Constant	No Det.	Var.	Constant	No Det.
$\tilde{m}_t$	-2.172 (8)	-0.6036 (8)	$\Delta\tilde{m}_t$	-1.877 (7)	-0.9216 (7)	$\Delta^2\tilde{m}_t$	-9.772 (6)**	-9.824 (6)**
$\tilde{p}_t$	-3.194 (9)	-2.713 (9)	$\Delta\tilde{p}_t$	-1.656 (8)	-1.118 (8)	$\Delta^2\tilde{p}_t$	-10.88 (7)**	-10.93 (7)**
m_t	-1.984 (8)	-0.4190 (8)	Δm_t	-2.531 (7)	-1.400 (7)	$\Delta^2 m_t$	-9.330 (6)**	-9.409 (6)**
p_t	-1.935 (6)	-3.463 (6)**	Δp_t	-1.624 (5)	-1.270 (5)	$\Delta^2 p_t$	-12.37 (4)**	-12.42 (4)**
c_t	-1.857 (9)	0.1152 (9)	Δc_t	-4.484 (8)**	-1.504 (9)			
i_{mt}	-2.920 (8)	-1.926 (8)	Δi_{mt}	-5.953 (7)**	-5.968 (7)**			
i_{bt}	-2.692 (6)	-1.442 (6)	Δi_{bt}	-7.215 (5)**	-7.224 (5)**			

Number of lags in parentheses. The methodology of Campbell and Perron (1991) has been followed in selecting the number of lags for the ADF tests. Significance at 5% and 1% level indicated by * and **
Critical values used in ADF test with Constant and Trend are: 5%=-3.428, 1%=-3.995
Critical values used in ADF test with Constant are: 5%=-2.872, 1%=-3.456
Critical values used in ADF test with no deterministic terms are: 5%=-1.941, 1%=-2.573

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C Tables

Table 3: Misspecification Statistics and Characteristic Roots for the VAR(7): M3 and CPI

Multivariate Statistics							
LM_1	$\chi^2(25) =$	56.688	[0.000]	LM_7	$\chi^2(175) =$	313.02	[0.000]
LM_4	$\chi^2(100) =$	176.00	[0.000]	LM_{12}	$\chi^2(300) =$	458.4	[0.000]
Normality	$\chi^2(10) =$	349.898	[0.000]				

Univariate Tests							
	$\Delta^2\tilde{m}_t$	$\Delta^2\tilde{p}_t$	$\Delta^2 c_t$	$\Delta^2 i_{mt}$	$\Delta^2 i_{bt}$		
ARCH(7)	0.4982	1.9460	1.6316	**3.7170**	**6.7365**		
Norm. $\chi^2(2)$	1.1520	**12.711**	**15.906**	**289.21**	**17.389**		
$\hat{\sigma} \times 100$	0.21797	0.16523	0.34923	0.61087	0.33818		
F_{AR}-LM_4	2.6349	0.2250	1.5261	2.1550	1.8745		
F_{AR}-LM_7	2.5109	1.7330	1.2657	1.3177	1.1802		
F_{AR}-LM_{12}	1.9834	1.3979	0.84671	1.5206	0.88873		
R^2	0.613	0.692	0.857	0.794	0.755		
Modulus of the 7 largest roots							
Unrestricted Model	0.9918	0.9918	0.9807	0.9807	0.8813	0.8813	0.8625
$r = 3$	1.0000	1.0000	0.9867	0.9581	0.9090	0.9090	0.8615
$r = 2$	1.0000	1.0000	1.0000	0.9870	0.9632	0.8632	0.8632
$r = 1$	1.0000	1.0000	1.0000	1.0000	0.9864	0.9462	0.8637

The F_{AR}-LM tests are distributed as: F_{AR}-$LM_4 \sim F(4, 241)$; F_{AR}-$LM_7 \sim F(7, 238)$ and F_{AR}-$LM_{12} \sim F(12, 233)$ respectively, while LM vector tests refer to tests for autocorrelation.

Table 4: Two Step Procedure Test Statistics for Integration Indices

$n-r$	r	2SI2 Algorithm					
		$S_{r,s}$					Q_r
5	0	**336.96**	**256.03**	**202.04**	**161.95**	**154.13**	**150.67**
		(191.9/198.2)	(161.9/167.9)	(137.0/142.1)	(114.9/119.8)	(96.5/101.5)	(83.2/87.3)
4	1		**195.97**	**142.92**	**102.64**	**95.01**	**92.85**
			(132.0/137.0)	(107.9/113.0)	(87.9/92.2)	(71.3/75.3)	(59.1/63.0)
3	2			**101.91**	58.91	**54.08**	**51.93**
				(82.3/86.7)	(64.2/68.2)	(49.7/53.2)	(39.1/42.4)
2	3				30.99	25.69	23.15
					(44.5/47.6)	(31.6/34.4)	(22.8/25.3)
1	4					8.78	6.20
						(17.6/19.9)	(10.5/12.3)
$n-r-s$		5	4	3	2	1	0

Table 5: Maximum Likelihood Procedure Test Statistics for Integration Indices

$n-r$	r	ML Algorithm					
		$S_{r,s}$					Q_r
5	0	**336.95**	**256.03**	**202.04**	**161.95**	**154.13**	**150.67**
		(191.9/198.2)	(161.9/167.9)	(137.0/142.1)	(114.9/119.8)	(96.5/101.5)	(83.2/87.3)
4	1		**183.44**	**142.23**	**102.48**	**94.94**	**92.85**
			(132.0/137.0)	(107.9/113.0)	(87.9/92.2)	(71.3/75.3)	(59.1/63.0)
3	2			**86.41**	58.20	**54.01**	**51.93**
				(82.3/86.7)	(64.2/68.2)	(49.7/53.2)	(39.1/42.4)
2	3				30.15	25.57	23.15
					(44.5/47.6)	(31.6/34.4)	(22.8/25.3)
1	4					8.55	6.20
						(17.6/19.9)	(10.5/12.3)
$n-r-s$		5	4	3	2	1	0

NOTE FOR TABLES

The tables are read using the test procedure proposed by Paruolo (1996): From left to right and from top to bottom. This way we have two indices corresponding to each cell: (1) the index r : which shows the number of cointegrating relations (either direct or polynomially cointegrating relations), (2) the index s : which shows the number of relations that can be made stationary only by differencing once. Finally, $(n-r-s)$ shows the number of linear combinations that need to be differenced twice to become stationary. So, $(n-r-s)$ = number of $I(2)$ trends, s = number of $I(1)$ trends and r = number of cointegrating relations

Table 6: The Loading Matrix and the Cointegrating Vectors

	$\boldsymbol{\tau}$				$\boldsymbol{\tau}_{\perp}$
	$\boldsymbol{\beta}$		$\boldsymbol{\beta}_1$		$\boldsymbol{\beta}_2$
	β^1	β^2	β_1^1	β_1^2	
$\tilde{m}_t$	2.42	-31.27	-0.115	-0.584	0.799
$\tilde{p}_t$	-19.65	39.42	-0.107	0.729	0.569
c_t	-32.24	138.06	0.176	-0.347	0.007
i_{mt}	111.34	61.61	0.447	0.065	0.144
i_{bt}	-53.36	-59.38	0.862	0.050	0.123
t	0.12	-0.39	0.0007	0.002	

Table 7: Tests of Long-Run Price Homogeneity: M3 and CPI

Pairs of Integration Indices for the model		
$\{r=1, s=3\}$	$\{r=2, s=2\}$	$\{r=3, s=1\}$
$\chi^2(4) = 12.880$ [0.0119]	$\chi^2(4) = 13.642$ [0.008]	$\chi^2(4) = 27.519$ [0.000]

Table 8: Misspecification Statistics and Characteristic Roots for VAR(7)

Multivariate Statistics							
LM_1	$\chi^2(25) =$	40.548	[0.00]	LM_7	$\chi^2(175) =$	311.43	[0.000]
LM_4	$\chi^2(100) =$	149.19	[0.001]	LM_{12}	$\chi^2(300) =$	457.55	[0.000]
Normality	$\chi^2(10) =$	435.54	[0.000]				

Univariate Tests					
	$\Delta^2 m_t$	$\Delta^2 p_t$	$\Delta^2 c_t$	$\Delta^2 i_{mt}$	$\Delta^2 i_{bt}$
ARCH(7)	1.7730	**4.0139**	0.8805	**3.8383**	**5.3449**
Norm. $\chi^2(2)$	5.0126	**55.442**	**18.381**	**289.20**	**17.084**
$\hat{\sigma} \times 100$	0.1832	0.1354	0.3464	0.5845	0.3349
F_{AR}-LM_4	1.000	0.1366	0.5522	1.1739	1.7168
$F_{AR} - LM_7$	0.7599	0.6146	0.6116	1.4398	1.4156
$F_{AR} - LM_{12}$	0.8667	0.6041	0.6545	1.6168	0.9284
R^2	0.584	0.775	0.859	0.814	0.762

Modulus of the 7 largest roots$^{\neq)}$							
Unrestricted Model	0.9821	0.9761	0.9761	0.9684	0.9684	0.8695	0.8695
$r = 3$	1.0000	1.0000	0.9865	0.9436	0.9304	0.9304	0.8716
$r = 2$	1.0000	1.0000	1.0000	0.9840	0.9575	0.9575	0.8728
$r = 1$	1.0000	1.0000	1.0000	1.0000	0.9923	0.9595	0.8587

The F_{AR}-LM tests are distributed as: F_{AR}-$LM_4 \sim F(4, 241)$; F_{AR}-$LM_7 \sim F(7, 238)$ and F_{AR}-$LM_{12} \sim F(12, 233)$ respectively, while LM vector tests refer to tests for autocorrelation.

Table 9: Two Step Procedure Test Statistics for Integration Indices

$n-r$	r	2SI2 Algorithm					
				$S_{r,s}$			Q_r
5	0	**331.36**	**248.23**	**181.89**	**142.75**	**135.68**	**133.04**
		(191.9/198.2)	(161.9/167.9)	(137.0/142.1)	(114.9/119.8)	(96.5/101.5)	(83.2/87.3)
4	1		**211.55**	**145.18**	**93.79**	**83.48**	**78.87**
			(132.0/137.0)	(107.9/113.0)	(87.9/92.2)	(71.3/75.3)	(59.1/63.0)
3	2			**108.24**	57.07	**49.80**	**45.42**
				(82.3/86.7)	(64.2/68.2)	(49.7/53.2)	(39.1/42.4)
2	3				30.37	22.59	16.96
					(44.5/47.6)	(31.6/34.4)	(22.8/25.3)
1	4					10.54	4.54
						(17.6/19.9)	(10.5/12.3)
$n-r-s$		5	4	3	2	1	0

Table 10: Maximum Likelihood Procedure Test Statistics for Integration Indices

$n-r$	r	ML Algorithm					
		$S_{r,s}$					Q_r
5	0	**331.36**	**248.22**	**181.89**	**142.75**	**135.68**	**133.04**
		(191.9/198.2)	(161.9/167.9)	(137.0/142.1)	(114.9/119.8)	(96.5/101.5)	(83.2/87.3)
4	1		**169.94**	**116.29**	**91.46**	**83.15**	**78.87**
			(132.0/137.0)	(107.9/113.0)	(87.9/92.2)	(71.3/75.3)	(59.1/63.0)
3	2			74.68	55.38	48.90	**45.42**
				(82.3/86.7)	(64.2/68.2)	(49.7/53.2)	(39.1/42.4)
2	3				28.56	21.99	16.96
					(44.5/47.6)	(31.6/34.4)	(22.8/25.3)
1	4					9.83	4.54
						(17.6/19.9)	(10.5/12.3)
$n-r-s$		5	4	3	2	1	0

Table 11: The Loading Matrix and the Cointegrating Vectors:Divisia Money Demand

	$\boldsymbol{\tau}$				$\boldsymbol{\tau}_\perp$
	$\boldsymbol{\beta}$		$\boldsymbol{\beta}_1$		$\boldsymbol{\beta}_2$
	β^1	β^2	β_1^1	β_1^2	
m_t	11.40	27.69	0.043	-0.855	0.477
p_t	-18.15	-8.17	-0.341	0.355	0.803
c_t	-41.35	-106.73	0.096	-0.300	-0.040
i_{mt}	36.57	-56.29	0.257	0.163	0.283
i_{bt}	-13.51	23.05	0.898	0.161	0.214
t	0.11	0.21	0.0006	0.0036	

Table 12: Tests of Long-Run Price Homogeneity

Pairs of Integration Indices for the model		
$\{r=1,s=3\}$	$\{r=2,s=2\}$	$\{r=3,s=1\}$
$\chi^2(4) = 3.199$ [0.525]	$\chi^2(4) = 5.892$ [0.207]	$\chi^2(4) = 17.139$ [0.002]