EUROPEAN UNIVERSITY INSTITUTE
DEPARTMENT OF ECONOMICS

EUI Working Paper **ECO** No. 2002/27

The Under-Estimated Virtues of the Two-Sector AK Model

Gabriel J. Felbermayr
and
Omar Licandro

BADIA FIESOLANA, SAN DOMENICO (FI)

Published in Italy in December 2002
European University Institute
Badia Fiesolana
I-50016 San Domenico (FI)
Italy

The underestimated virtues of the two-sector AK model

Gabriel J. Felbermayr[(a)] Omar Licandro[(b)*]

December 2002

Abstract

We show that the two-sector AK model proposed by Rebelo (1991) can be read as an endogenous growth extension of Greenwood, Hercowitz and Krusell (1997). By confining constant returns to capital to the investment goods sector, the model generates endogenously the secular downward trend of the relative price of equipment investment and the rising real investment rate observed in US NIPA data. Whereas Jones (1995) criticizes that the one-sector model fails to reconcile the empirical facts of trending real investment rates and stationary output growth, this incompatibility vanishes in the two-sector version.

Keywords: AK model; embodiment; endogenous growth; obsolescence.
JEL - codes: O41, O30.

---

*(a) European University Institute, (b) European University Institute and FEDEA.
We would like to thank Philippe Aghion, Jess Benhabib, Giuseppe Bertola, Larry Jones, Maria Guecheva, Sharon Harrison, Tim Kehoe, Victor Ríos-Rull and Gianluca Violante for helpful comments and suggestions. Moreover, the authors benefitted from discussions at the 7th Macro Workshop at the University of Vigo, the SET Summerschool 2002, the 2002 meeting of LACEA in Madrid and the European Conference on Dynamic Macroeconomics at the EUI.
Authors' address: European University Institute, Economics Department, Badia Fiesolana, Via dei Roccettini 9, 50016 S. Domenico di Fiesole (FI), ITALY, E-Mail: G. Felbermayr: `gfelberm@iue.it`, O. Licandro: `omar.licandro@iue.it`.
Omar Licandro akcnowledges the financial support of the Spanish Ministery of Sciences and Technology (SEC 2000-0260).

1 Introduction

Neoclassical growth theory builds on the famous *Kaldor* [9] stylized facts, which postulate, among other things, the stationarity of aggregate ratios such as the investment to output ratio or the capital to output ratio. However, there is now a rich body of empirical research suggesting that some of these facts are in conflict with U.S. evidence. Based on data from the National Income Product Accounts (NIPA), Whelan [19] documents the following facts for the post world war II period[1]:

(i) The price of equipment investment relative to the price of consumer nondurables and services has been declining permanently.

(ii) Nominal series of consumption and investment share a common stochastic trend with nominal output so that the consumption and investment shares in nominal output are stationary.

(iii) The ratio of real equipment investment to real output is non-stationary. Indeed, the growth rate of real equipment investment has been larger than that of real non-durable consumption, reconciling facts (i), (ii) and (iii).

Recent research addresses the inconsistency of facts (i) and (iii) with standard one-sector neoclassical growth models. Clearly, any growth model that takes the new evidence seriously needs more than just one sector, otherwise relative price trends cannot be addressed properly. Greenwood et al. [5], henceforth abbreviated GHK, based on the seminal contribution of Solow [18], were the first to make an attempt in this direction. Their model consists of a consumption goods sector, which benefits from exogenous disembodied technical progress, and of an investment goods sector, the efficiency of which also grows at an exogenous rate. Advances in the investment sector affect the consumption goods sector to the extent that firms acquire new and more efficient capital goods: technical change is embodied. In this way, GHK are able to generate a permanent decline in the relative price of investment and a rising ratio of real equipment investment to real output, keeping nominal shares constant, as required by the stylized facts cited above.

We study a version of the two-sector AK model originally proposed by Rebelo [16] (section II), where the consumption goods sector features decreasing returns to capital whilst the investment goods sector is described by an AK technology. We argue that this environment yields the simplest

[1] See also Greenwood, Hercowitz and Krusell [5], p. 342.

possible endogenous growth model compatible with the evidence cited above. It complements recent research on endogenous embodied technical change in R&D based models of endogenous growth, see Boucekkine et al. [1], Hsieh [7] or Krusell [10], and the model by Boucekkine et al. [2] where technical change stems from learning-by-doing.

Our approach is guided by the empirical study of Harrison [6] who estimates plant-level and sectoral returns to scale in the consumption and investment sectors. Whereas constant returns to scale cannot be rejected at the *plant*-level in both the consumption and the investment sector, at the *sector*-level there is robust evidence for positive externalities in the investment sector only. This coincides with the widely held view (surveyed, e.g. by OECD [14] and OECD [15]) that spillovers of many sorts are particularly important in investment goods industries such as the information and communication technologies industry, the car and aeronautic industries, or the industrial equipment industry. Thus it is likely that *social* returns to capital are larger in the investment sector than in the consumption sector. Taking these observations to their extreme, the present model assumes constant marginal returns to capital in the investment sector.

In accordance with GHK's findings, in the proposed framework the relative price of investment trends downward and real investment growth outpaces consumption growth. However, the underlying mechanism is different. GHK generate the price trend by exogenous sectoral differences in the rate of technical progress, whereas our model relates the movement in relative prices to the asymmetric sectoral impact of capital accumulation.

As proposed by Whelan [19], we measure the growth rate of real output by the so-called Divisia index, a continuous time approximation to the *chained Fisher index*, officially used in US growth accounting since 1996. This is necessary to account for the substitution effect that trends in relative prices usually bring about. We conclude that real output growth lies above the growth rate of nondurables consumption and below that of equipment investment, which is consistent with fact (iii).

In contrast to its one-sector version, in the two-sector AK model, the user cost of capital is augmented by an obsolescence cost term. Obsolescence costs show up in any model with a trending relative price of investment, as in the models of embodied technical change proposed by Boucekkine et al. [1] and [2], GHK [5], Hsieh [7] or Krusell [10]. In the proposed AK model, the larger the decline rate of the relative price of investment, the larger the obsolescence cost term. This lowers the interest rate perceived by consumers and depresses consumption growth. However, whether the lower interest rate encourages or discourages capital accumulation depends on how the income effect associated with a change in the interest rate relates to the substitution effect, that is, whether the elasticity of intertemporal substitution is smaller or greater than unity. Obsolescence costs reduce (increase) the growth rate of real output if the elasticity of intertemporal substitution is larger (smaller) than the saving rate.

Moreover, the two-sector AK model is isomorphic to the model by Boucekkine et al. [2], which studies embodied technical change in a model of learning-by-doing. In the latter model, reallocating the efficiency of learning from the consumption goods sector to the investment goods sector generates a simultaneous increase in the decline rate of the relative price of investment and a reduction in the growth rate of aggregate output, matching the shift in US series experienced around the first oil shock, as reported by Greenwod and Yorukoglu [4]. In the two-sector AK model, this exercise is equivalent to a reduction of the output elasticity in the consumption sector.

The two-sector AK model has realistic empirical predictions. Jones [8] criticizes that the one-sector AK model fails to reconcile the empirical facts of trending real investment rates and stationary output growth. Interestingly, in the two-sector version this incompatibility vanishes. Furthermore, the model reproduces the negative correlation between GDP and relative price of investment goods, observed in cross-sectional studies (see Restuccia and Urrutia [17] for a recent survey).

The remainder of the paper is organized as follows. Section 2 sets out the analytical framework and derives the main propositions; section 3 provides a discussion of the results and compares them with existing models; finally, section 4 concludes.

2 The Model

In this section we analyze a two-sector version of the AK model introduced by Rebelo [16]. The labor force is constant, and all quantities are in per capita terms. The capital stock per capita k_t is endogenously determined by explicit investment decisions. In the consumption sector, capital is combined with labor in a constant returns to scale production function. As in the models of Boucekkine et al. [1], [2], Hsieh [7] and Krusell [10] the only source of endogenous growth lies in the investment goods sector. We model this in the simplest possible way, assuming that the technology in the investment goods sector features constant returns and capital is the only factor of production. For every point in time, the planner optimally divides the capital stock between investment goods production and consumption goods production.

The planner problem reads

$$\max \int_0^\infty \frac{c_t^{1-\sigma}}{1-\sigma} e^{-\rho t} \mathrm{d}t \tag{1}$$

$$\text{s.t. } c_t = (k_t^c)^\alpha \tag{2}$$

$$\dot{k}_t = (A-\delta)\, k_t - A k_t^c, \tag{3}$$

with $k_0 > 0$ given. c_t denotes per capita consumption and k_t^c is the per capita capital stock used in the production of consumption goods. As usual, σ (with $\sigma > 0$ and $\sigma \neq 1$) is the inverse of the intertemporal elasticity of substitution, $\rho > 0$ denotes the subjective discount rate of the infinitely lived representative individual and $\alpha \in \left]0, 1\right[$ is the output elasticity of capital in the consumption sector. At every point in time, $k_t - k_t^c$ units of capital are used in the investment sector according to the AK technology $i_t = A\,(k_t - k_t^c)$, giving rise to the law of motion of capital (3). $A > 0$ denotes the marginal productivity of capital in the investment sector and $\delta \in \left]0, 1\right[$ its depreciation rate. Note that capital is perfectly intersectorally mobile.

After substituting (2) in the objective (1), the control variable to the planner problem is k_t^c and the state variable is k_t. Denote by $\lambda_t e^{-\rho t}$ the co-state. Then, the first order condition for the control can be written as

$$\alpha\,(k_t^c)^{-(1-\alpha(1-\sigma))} = A\lambda_t, \tag{4}$$

the Euler equation is

$$\frac{\dot{\lambda}_t}{\lambda_t} = \delta + \rho - A, \tag{5}$$

and the transversality condition associated to the problem reads

$$\lim_{t\to\infty} \lambda_t k_t e^{-\rho t} = 0. \tag{6}$$

From (5), the growth rate of λ_t is constant from $t = 0$ on. Denoting the growth rate of variable x by g_x, from equations (2) and (4) we derive the growth rates for consumption and the capital stock employed in the consumption sector:

$$g_c = \frac{\alpha}{\omega}\,(A - \rho - \delta)\,, \tag{7}$$

$$g_k = \frac{1}{\omega}\,(A - \rho - \delta)\,, \tag{8}$$

where $\omega \equiv 1 - \alpha\,(1-\sigma) > 0$. Both g_k and g_c are constant from $t = 0$ onwards.

Notice that from equation (4), $1/\omega$ is the intertemporal elasticity of substitution associated to k^c. Given our assumptions on technology, we refer to $1/\omega$ as to the *elasticity of intertemporal substitution in foregone investment.* It measures the easiness of substitution between using capital to produce consumption goods today and producing investment goods today to produce consumption goods in the future.

Assumption 1. *Let the following parameter restriction hold:*

$$A - \delta > \rho > \alpha \left(1 - \sigma\right) \left(A - \delta\right).$$

The first inequality in Assumption 1 is required for the growth rate of consumption, as given by equation (7), to be strictly positive. The second inequality ensures that the utility representation in equation (1) remains bounded at equilibrium.

Proposition 1. *Under Assumption 1, for all $t \geq 0$, the time path of capital is given by*

$$k_t = k_0 \, e^{g_k t}$$

and the initial value of the control is $k_0^c = \left(1 - \frac{\delta + g_k}{A}\right) k_0 > 0$.

Proof. See the appendix.■

As in the standard AK model, the economy is on its balanced growth path from $t = 0$; i.e. there are no transitional dynamics. The equilibrium path of the model economy features $g_k = g_i$ and $g_c = \alpha g_k$. Since $\alpha \in \left]0, 1\right[$, consumption grows at a slower pace than investment and capital.

As stated in the introduction, in the U.S. three important secular trends are in sharp contradiction with Kaldor's stylized facts. First, the relative price of investment exhibits a secular downward trend. Second, the share of nominal investment in nominal output is constant, and, third, the ratio of equipment investment to real output is steadily increasing. The standard one-sector AK growth model cannot account for these facts. The main proposition of this paper shows that, in contrast, a two-sector model with endogenous AK-type growth in the investment goods sector has predictions consistent with these empirical regularities.

Proposition 2. *In the proposed two-sector growth model, (i) the relative price of investment p_t is decreasing at rate $\left(1 - \alpha\right) g_k$, (ii) the nominal saving rate is constant and (iii) the ratio of investment to output is increasing. The growth rate of output, g, defined by a Divisia quantity index, is constant and* *lies in the interval* $]g_c, g_k[$, *its exact position being determined by the saving rate.*

Proof. (i) In a competitive equilibrium, the marginal rate of transformation between the investment good and the consumption good has to be equal to the relative price of the investment good at every point in time. The feasibility constraint of our economy is described by the sectoral production functions and the aggregate endowment of capital. Thus, in per capita terms, at time t, the transformation curve of the economy is defined by the expression

$$c_t = (k_t - i_t/A)^{\alpha}.$$

Denoting the relative price of investment by p_t and equalizing it with the marginal rate of transformation (MRT) (for a given level of k_t^c) gives

$$p_t = \left| \frac{\mathrm{d}c_t}{\mathrm{d}i_t} \right| = \frac{\alpha}{A} \left(k_t^c\right)^{\alpha - 1}.$$

Clearly, as k_t^c grows over time, p_t has to decrease. More precisely,

$$\frac{\dot{p}_t}{p_t} = (\alpha - 1)\, g_k < 0. \tag{9}$$

(ii) The share of nominal investment in nominal output is given by the saving rate $s_t \equiv p_t i_t / (c_t + p_t i_t)$. Using the results derived above, the saving rate is constant and reads

$$s = \frac{\alpha\,(\delta + g_k)}{A - (1 - \alpha)\,(\delta + g_k)}. \tag{10}$$

(iii) In our context, using the Divisia index amounts to writing the growth rate of real output, g, as the weighted sum of the rates of growth of consumption and investment:

$$g = (1 - s)\, g_c + s g_i. \tag{11}$$

By Assumption 1, $s \in]0, 1[$ and hence $g \in]g_c, g_k[$, which implies a permanently increasing investment to output ratio.■

In Figure 1 we provide a graphical illustration of the equilibrium in the space (i, c). The expansion path Φ shows all pairs of c_t and i_t compatible with the equilibrium in Proposition 1. Φ is found by writing the policy functions $c_t = \phi_c(k_t)$ and $i_t = \phi_i(k_t)$, which are combined by substituting k_t out. In $(i, c)-$ space, Φ can easily be shown to be strictly concave and increasing in

i_t. As the economy accumulates capital, it moves north-east along Φ. Equilibrium is identified at the intersection of the transformation curve and the expansion path and denoted by E. The relative price of investment goods is found as the slope of the transformation curve at point E. Due to the assumed differences in the sectoral production functions, capital accumulation affects sectors asymmetrically and the transformation curve shifts out in an uneven way. Consequently, on the way from E to E', the relative price of investment falls from p to p' at a rate proportional to the rate at which the economy accumulates capital.

As Whelan [19] points out, to be consistent with NIPA data, the appropriate way to compute the growth rate of real output is to use the "*ideal chain index*" proposed by Irving Fisher. This index is the geometric average of a Paasche and a Laspeyres index and can be accurately approximated by the so-called Divisia index, which weights the growth rate of each component of output by its current share in the corresponding nominal aggregate (see Deaton and Muellbauer ([3]), pp. 174-5 for more details).

Since expression (11) plays a major role in our analysis, some additional remarks on the appropriate definition of real output growth may be useful. Typically, a base year quantity index is computed as the average growth rate of real output in the different sectors, where base year prices are used as fixed weights. As Whelan [19] explains, in an economy with different sectoral growth rates, such a fixed-weight methodology results in *unsteady* aggregate growth. The growth rate will tend to increase, converging towards the growth rate of the faster growing sector. The reason for this pattern is the so-called substitution bias introduced by holding relative prices fixed over time. Those categories of output which exhibit faster growth in quantities typically also experience declining relative prices. Measured in prices of a base year, current output will become more and more expensive, as the fast-growing components are still weighted with high historical prices. Moreover, not only does this fixed-weight definition lead to unsteady growth, but the size of the substitution bias, and thus the computed growth rate, depend on the choice of the base-year: the farther in the past, the larger the error. In our model, since the weights used in the computation of the Divisia index are constant, the problem of unsteady growth is avoided.

This is why NIPA data is computed using a chained-type index equivalent to the Divisia index in continuous time. This method amounts to continually updating the prices used to calculate real output and to chain the index forward from an arbitrary base year on, in which nominal magnitudes have been set equal to real magnitudes. Moreover, Licandro et al. [12] provide theoretical support for the use of such a chained index in the framework of a two-sector exogenous growth model with embodied technical change (the

GHK model). They compute a true quantity index, use official NIPA data to calibrate it for different parameter values, and show that their results come very close to what is obtained by applying NIPA's methodology.

3 Discussion

Obsolescence costs. At this point of our argument, it is useful to make the role of the relative price of investment goods explicit in the Euler equation. Let firms own the capital stock and the representative consumer own the firms. The consumer's wealth is given by the value of her asset holdings, a_t. If we define r_t as the rate of return to this asset, the consumer's wealth evolves according to $\dot{a}_t = y_t + r_t a_t - c_t$, where y_t denotes labor income. y_t is given for the consumer since labor supply is perfectly inelastic and the labor market is competitive. The consumer maximizes lifetime utility (1) subject to the law of motion of wealth. The Euler equation associated with the optimal consumption path writes $\dot{c}_t/c_t = \sigma^{-1}\left[r_t - \rho\right]$. Let firms sell their output at the ongoing market price and use some of their revenue to purchase investment goods. They maximize the present value of their profits where the relevant discount rate is r_t. In this context, the user cost of capital is

$$u_t \equiv r_t + \delta - \frac{\dot{p}_t}{p_t}. \tag{12}$$

Then, the optimality conditions of firms in both sectors require that the marginal product of capital be equal to u_t. The sectoral allocation of capital is governed by the efficiency condition $(p_t)^{-1}\alpha\left(k_t^c\right)^{\alpha-1} = A = u_t$. Consequently, the user cost of capital is constant along the balanced growth path and identical to A. Moreover, the growth rates of k_t^c and p_t must have opposite signs and any increase in $\dot{p}_t/p$ must be entirely offset by a corresponding reduction in r_t. We can express the asset return rate by

$$r_t = A - \delta + \frac{\dot{p}_t}{p_t} \tag{13}$$

and the consumer's Euler equation by

$$g_c = \frac{1}{\sigma}\left(A - \delta - \rho + \frac{\dot{p}_t}{p_t}\right). \tag{14}$$

Substituting expression (9) for $\dot{p}_t/p_t$ in the Euler equation (14) yields exactly the centralized counterpart (7). Except for the term $\dot{p}_t/p_t$ equation (14) is identical to the Euler equation of the standard one-sector AK model.

Thus, it is really the relative price change that makes the crucial difference. In this setting, the expected evolution of the relative price of investment is irrelevant for the firms' investment decision since the user cost of capital always remains fixed at A. However, the net interest rate r_t is affected by the growth rate of p_t. This results in a flatter consumption path (a lower growth rate g_c), giving rise to the typical intertemporal substitution effect. Thus, the term $\dot{p}_t/p_t$ acts as a cost and is referred to as capturing the so-called *obsolescence costs* typically associated with embodied technical change. Obsolescence costs lower the value of installed capital, therefore reducing the consumer's wealth. Higher obsolescence costs, do not, however, necessarily reduce aggregate output growth, as a simple inspection of equation (14) might suggest.

Differences to the one-sector version. Setting $\alpha = 1$, the two-sector AK model collapses to the standard one-sector AK model. Then $\dot{p}_t/p_t$ is clearly zero and there are no obsolescence costs. Does this imply that the one-sector AK-type economy grows faster than its two-sector version?

Denote the growth rate of the standard one-sector AK model by g_{AK}. Then the following proposition can be made:

Proposition 3.

$$\begin{aligned} (i) \quad & g_{AK} && > && g_c, \\ (ii) \quad & g_{AK} && \gtreqless && g_k \Longleftrightarrow \frac{1}{\sigma} \gtreqless 1, \\ (iii) \quad & g_{AK} && \gtreqless && g \Longleftrightarrow \frac{1}{\sigma} \gtreqless s. \end{aligned}$$

Proof. In the one-sector AK model the growth rates of consumption, output and capital are all equal to

$$g_{AK} \equiv \frac{1}{\sigma} (A - \rho - \delta). \tag{15}$$

Parts (i) and (ii) of the proof involve comparing g_{AK} to the growth rates given by equations (7) and (8), which is obvious. To show part (iii), note that g can be expressed as

$$g = [(1 - s)\alpha + s] \frac{1}{\omega} (A - \delta - \rho). \tag{16}$$

Then $g_{AK} \gtreqless g \Longleftrightarrow \sigma^{-1} \gtreqless \omega^{-1} [(1 - s)\alpha + s] \Longleftrightarrow \sigma^{-1} \gtreqless s$. From (8), (11) and the definition of ω, s is a function of σ so that it is not *a priori* clear whether there are values for σ for which the above inequalities hold. The appendix shows that the equation $\sigma^{-1} = s(\sigma)$ has a unique interior solution $\sigma^* > 1$, so that the above inequalities can go either way.■

Note the role of the intertemporal elasticity of substitution in the comparison between g_{AK} and g_k. In the two-sector model, the interest rate being weighed down by obsolescence costs, the agent chooses g_k larger than g_{AK} if the income effect outweighs the substitution effect, that is, if the intertemporal elasticity of substitution is smaller than unity.

In the comparison of (15) and (16) two differences stand out. The first relates to the terms σ^{-1} and ω^{-1}. These terms measure how ready agents are to forego consumption today in order to increase consumption tomorrow. Clearly, with identical production functions for consumption and investment goods, ω^{-1} and σ^{-1} coincide, which is the case in the one-sector model. The second difference relates to the term $(1-s)\alpha + s$, which shows how the marginal effect of a change in g_k effects g, as implied by our definition of output growth (11). A fraction $1-s$ of capital is allocated to the consumption goods sector where it encounters decreasing returns given by α; the complementary fraction goes to the investment sector where returns are constant. In the one-sector model, this term is equal to unity.

Hence, for $g > g_{AK}$ two conditions must be satisfied: first, the elasticity of intertemporal substitution must lie below unity so that the income effect outweighs the substitution effect, generating a larger growth rate of capital; second, the saving rate has to be large enough so as to give sufficient weight to the investment sector when it comes to determining aggregate output growth.

A model of learning-by-doing. Next, we compare our model to the one developed by Boucekkine et al. [2] (hereafter BdL), where endogenous growth is due to learning-by-doing in both the consumption and the investment goods sectors. The technological description of their model is given by

$$c_t + x_t = z_t k_t^{\eta}, \tag{17}$$

$$i_t = q_t x_t, \tag{18}$$

where the efficiency of production increases with cumulated net investment so that $z_t = k_t^{\gamma}$ and $q_t = Ak_t^{\lambda}$. The parameters $\lambda > 0$ and $\gamma > 0$ describe the efficiency of learning in the consumption and in the investment sector, respectively. In order to generate sustained growth, BdL assume $\lambda+\gamma+\eta = 1$. Thanks to constant returns to scale in the production of consumption goods, equations (17) and (18) can be written as

$$\begin{aligned} c_t &= \left(k_t^c\right)^{\gamma+\eta}, \\ i_t &= A\left(k_t - k_t^c\right)^{\lambda+\gamma+\eta} = A\left(k_t - k_t^c\right). \end{aligned}$$

After the change $\alpha = \gamma + \eta$, the two-sector AK model perfectly coincides with the optimal growth version of the learning-by-doing model. Thus, the two-sector AK model can be seen as the reduced form of a learning-by-doing model where firms internalize the learning externality.

Note that the efficiency of learning in the investment goods sector in BdL is inversely related to the elasticity of capital in the consumption sector in the two-sector AK model since $\lambda = 1 - \alpha$. From equations (8) and (9) and after substituting $\alpha = 1 - \lambda$, the decline rate of the relative price of investment can be written as

$$\frac{\dot{p}}{p} = -\frac{A - \delta - \rho}{1 + \sigma \left(\frac{1}{\lambda} - 1\right)},$$

which is an increasing function of the efficiency of learning in the investment goods sector relative to the consumption goods sector. Therefore, an adverse shock on the parameter α (as in the numerical example above) in the two-sector AK model is equivalent to a positive shock on λ in BdL's framework, with returns to capital in the investment sector kept constant. If returns to capital in the investment sector are kept constant, reducing the learning efficiency in the consumption sector comes with increasing it in the investment sector. BdL note that such a *reassignment* of learning efficiencies can account for the change in the US growth pattern observed around the year 1974 and discussed by Greenwood and Yorukoglu [4]: a deceleration in the growth rates of output and consumption, an increase in the growth rate of investment and an acceleration of the decline of the relative price of investment goods.

Consider an unexpected negative shock on the output elasticity of capital in the production function of the consumption goods sector. From equation (9) it can be seen that the decline rate of the relative price of investment increases if α is reduced, since $\partial \left|\dot{p}_t/p_t\right| / \partial\alpha = -g_k\sigma/\omega < 0$. The growth rate of consumption, g_c, is also reduced since $\partial g_c/\partial\alpha = g_k\omega^{-1} > 0$. The effect on g_k is ambiguous and depends on σ being smaller or larger than unity: $\partial g_k/\partial\alpha = g_k(1-\sigma)/\omega$. If the income effect of a reduced interest rate dominates the substitution effect ($\sigma > 1$), the drop in α increases g_k. However, from equation (11), in order for the shock to generate a reduction in g, σ must not be lower than some threshold $\underline{\sigma}$. Thus, if $\sigma \in \left]\underline{\sigma}\ , 1\right[$, a reduction in α reproduces the facts put forward by Greenwood and Yorukoglu and can be interpreted, following BdL, as a reassignment of learning efficiency.

A model of technical change. GHK [5], p. 349, take the observed downward trend of the price of investment goods relative to the price of consumption goods as evidence for investment specific technical change. In their theoretical model, this price trend is generated by sectoral differences in the exogenous rates of technical progress. Boucekkine et al. [1], Hsieh [7] or Krusell [10] take up this idea and write up models where R&D driven endogenous technical progress is confined to the investment goods sector.

In our model, instead, capital accumulation plays a key role. As the economy becomes ever more capital abundant, the capital intensive investment goods sector must expand overproportionally, leading to a decrease in the relative price of investment goods: along the balanced growth path, the marginal rate of transformation falls at a steady rate, as Figure 1 makes clear. This mechanism can be interpreted in two ways. Taking the AK technology in the investment sector literally, the transformation curve changes over time due to capital accumulation and not to technical progress. However, recognizing that the AK description is a reduced form production function of some more complicated model, as in the model of BdL discussed above, technical change occurs through changes in the efficiency parameter q_t.

Note that along the balanced growth path the marginal productivity of capital in the consumption goods sector falls, whereas that of the investment goods sector remains constant at A. Therefore, in stark contrast to the abovementioned R&D models, in the two-sector AK model the relative price trend is driven not by decreasing marginal costs in the investment goods sector, but by increasing marginal costs in the consumption goods sector. This is why we prefer to talk about technical *change* rather than technical *progress*.

Remarks on the empirical evidence. Using data from 1950 to 1987 for 15 OECD countries, Jones [8] criticizes that the standard AK model cannot account for the observed coincidence of stationary growth rates and upward-trending real investment rates. The two-sector AK model reconciles stationary output growth with trending investment rates and overcomes Jones' criticism. At the same time, it is a defense of the AK model.[2]

In a recent contribution, Restuccia and Urrutia [17] review the well-known negative and surprisingly robust correlation between the relative price of equipment investment and output per capita for a wide range of countries. In our model the relative price of investment goods is a decreasing function of the aggregate capital stock which is itself an increasing function of time. Thus, our model correctly predicts that countries which have entered the modern process of economic growth later than others, in autarky should exhibit a higher relative investment price. Moreover, again in line with evidence reviewed in Restuccia and Urrutia, our model shows a positive relationship between the relative price of equipment goods and the real investment ratio but an *a priori* ambiguous relation between aggregate growth and the decline

[2]See McGrattan [13] and Li [11] for recent attempts to challenge Jones' critique in econometric models of the standard AK model.

rate of the relative investment price.

4 Conclusion

We analyze a version of the two-sector AK model proposed by Rebelo [16] (section II), where constant aggregate returns to capital are confined to the investment goods sector. We show that this setup, an endogenous growth extension to the model of Greenwood et al. [5], fits the following empirical observations. It provides an endogenous growth rationale for the secular downward trend of the price of investment relative to consumption and the increasing ratio of real investment to real output observed in US NIPA data. Using a Divisia quantity index, we show that real output grows faster than consumption but more slowly than investment. Moreover, since the model is compatible at the same time with stationary output growth and upward-trending real investment, it overcomes Jones' [8] well-known critique of the AK model. Finally, the model predicts that less developed countries should feature higher relative prices of equipment investment relative to more advanced countries, a fact documented in several recent empirical papers.

Therefore, despite the extreme simplicity of the two-sector AK model, it does surprisingly well in capturing relevant empirical facts and lends to interesting interpretations. In that sense, we view our paper as a contribution towards a defense of AK-type models of endogenous growth.

References

[1] Boucekkine, Raouf, Fernando del Río and Omar Licandro. Obsolescence and modernization in the growth process. *EUI-ECO Working Papers*, 2001/18, 2001.

[2] Boucekkine, Raouf, Fernando del Río and Omar Licandro. Embodied technological change, learning-by-doing and the productivity slowdown. *Scandinavian Journal of Economics*, forthcoming.

[3] Deaton, Angus and John Muellbauer. *Economics and Consumer Behavior.* Cambridge University Press, Cambridge, UK, 1980.

[4] Greenwood, Jeremy and Mehmet Yorukoglu. 1974. *Carnegie-Rochester Conference Series on Public Policy*, 46:49–95, 1997.

[5] Greenwood, Jeremy, Zvi Hercowitz and Per Krusell. Long run implications of investment-specific technological growth. *American Economic Review*, 87(3):342–362, 1997.

[6] Harrison, Sharon G. Returns to Scale and Externalities in the Consumption and Investment Sectors. *Review of Economic Dynamics*, forthcoming.

[7] Hsieh, Chang-Tai. Endogenous Growth and Obsolescence. *Journal of Development Economics*, 66:153–171, 2001.

[8] Jones, Charles I. Time Series Tests of Endogenous Growth Models. *Quarterly Journal of Economics*, 110(5):495–525, 1995.

[9] Kaldor, Nicholas. A Model of Economic Growth. *Economic Journal*, 67:591–624, 1957.

[10] Krusell, Per. Investment-specific R and D and the decline of the relative price of capital. *Journal of Economic Growth*, 3(2):131–142, 1998.

[11] Li, Dajin. Is the AK Model Still Alive? The Long-run Relation between Growth and Investment Re-examined. *Canadian Journal of Economics*, 35:92–114, 2002.

[12] Licandro, Omar, Javier Ruiz-Castillo and Jorge Durán. The measurement of growth under embodied technical change. *Recherches Economiques de Louvain*, 68(1-2):7–19, 2002.

[13] McGrattan Ellen R. A Defense of AK Growth Models. *Federal Reserve Bank of Minneapolis Quarterly Review*, 22(4):13–27, 1998.

[14] Organization for Economic Cooperation and Development. *A New Technology? The Changing Role of Innovation and Information Technology in Growth.* OECD, Paris, 2000a.

[15] Organization for Economic Cooperation and Development. *OECD Economic Outlook No. 67.* OECD, Paris, 2000b.

[16] Rebelo, Sergio. Long-Run Policy Analysis and Long-Run Growth. *Journal of Political Economy*, 99(3):500–521, 1991.

[17] Restuccia, Diego and Carlos Urrutia. Relative Prices and Investment Rates. *Journal of Monetary Economics*, 47:93–121, 2001.

[18] Solow, Robert. Investment and Technological Progress. In Kenneth J. Arrow, Samuel Karlin, and Patrick Suppes, editor, *Mathematical Methods in the Social Sciences*, pages 89–104, Stanford, CA, 1960. Stanford University Press.

[19] Whelan, Karl. A Two-Sector Approach to Modeling U.S. NIPA Data. *Journal of Money, Credit, and Banking*, forthcoming.

A Proof of proposition 1

We start from equation (8):

$$\dot{k}_t = (A - \delta) k_t - A k_t^c.$$

Using $e^{-(A-\delta)t}$ as the integrating factor, substituting $k_t^c = k_0^c e^{g_k t}$ and rearranging terms yields

$$e^{-(A-\delta)t} \left[\dot{k}_t - (A - \delta) k_t \right] = -e^{-(A-\delta-g_k)t} A k_0^c.$$

The LHS can easily be recognized as $\frac{d}{dt}\left[e^{-(A-\delta)t} k_t\right]$ and the RHS as $\frac{d}{dt}\left[\frac{1}{(A-\delta-g_k)} e^{-(A-\delta-g_k)t} A k_0^c\right]$. Integrating and dividing by $e^{-(A-\delta)t}$ gives

$$k_t = \frac{1}{(A - \delta - g_k)} e^{g_k t} A k_0^c + C e^{(A-\delta)t} \tag{A1}$$

where C and k_0^c are constants which can be determined using the initial condition $k_0 > 0$ and the transversality condition 6. Using expression (A1) and the law of motion for the state variable 5 in the transversality condition we get

$$\lim_{t \to \infty} \left\{ \frac{A}{(A - \delta - g_k)} e^{-(A-\delta-g_k)t} + C \right\} = 0. \tag{A2}$$

The first term in the braced brackets converges towards zero since $A - \delta - g_k > 0$. Therefore, the TVC requires the constant C to be zero. From (A1) we get $k_0 = A k_0^c / (A - \delta - g_k)$.■

B Proof of proposition 3(iii)

It still needs to be shown that there exist intervals of values for which $\sigma^{-1} \lesseqgtr s$. In particular, it is not clear a priori whether there are values for σ such that

$g > g_{AK}$. We need to prove that the equation $\sigma^{-1} = s$ has a solution σ^*. Clearly, s is a continuous function. Moreover it is decreasing in σ since

$$\frac{\partial s}{\partial \sigma} = -\left[\frac{\alpha}{A - (1-\alpha)(\delta + g_k)}\right]^2 \frac{A}{\omega} g_k < 0. \tag{A3}$$

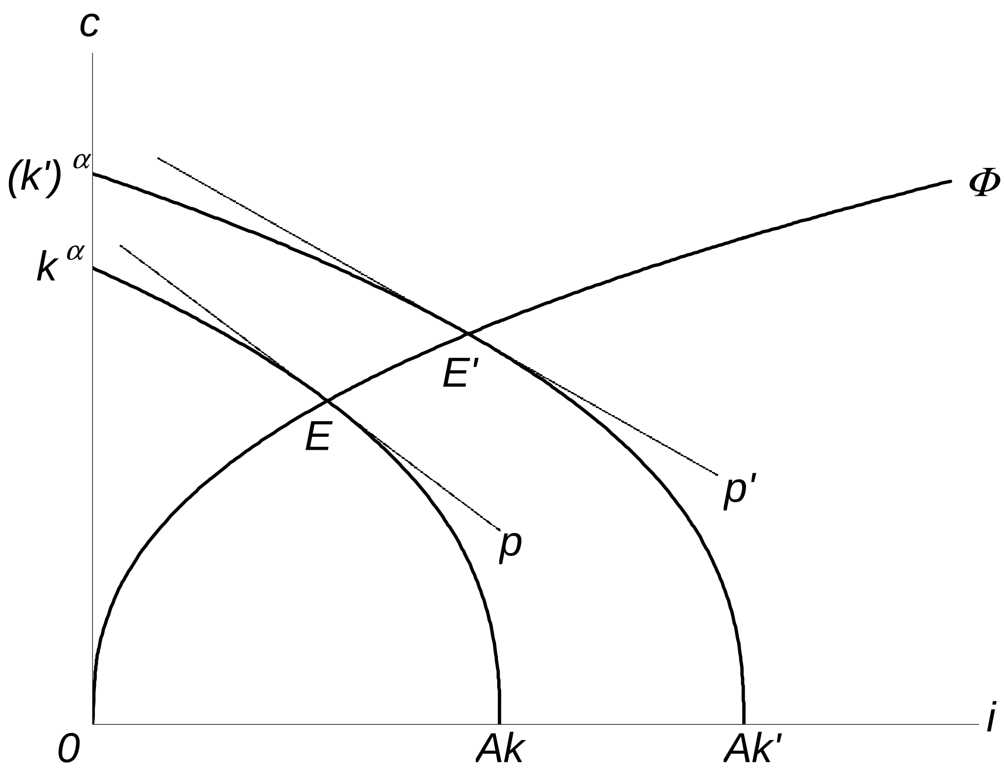
Moreover, as σ tends towards infinity, s converges towards the constant

$$s^- = \frac{\alpha\delta}{A - \delta + \alpha\delta} \tag{A4}$$

strictly larger than zero (by Assumption 1) and as σ tends towards zero, s converges towards the constant

$$s^+ = \frac{\alpha}{1-\alpha} \frac{A - (\rho + \alpha\delta)}{\rho + \alpha\delta} > s^-. \tag{A5}$$

Therefore it is clear that the there is a solution σ^* to the equation $\sigma^{-1} = s$ and that this solution is bounded below by $(s^+)^{-1}$. Since $\partial^2 s / \partial \sigma^2 > 0$, we can exclude oscillating behavior of $s(\sigma)$ around σ^{-1}, which is necessary and sufficient for unicity of σ^*. We conclude that if $\sigma < \sigma^* \iff \sigma^{-1} > s \iff g_{AK} > g$ and if $\sigma > \sigma^* \iff \sigma^{-1} < s \iff g_{AK} < g$.■



The economy on its expansion path Φ from E to E' $(k' > k)$.