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Error Cascades in Observational Learning: An Experiment on the Chinos Game*

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ABSTRACT

The paper reports an experimental study based on a variant of the popular Chinos game, which is used as a simple but paradigmatic instance of observational learning. There are three players, arranged in sequence, each of whom wins a fixed price if she manages to guess the total number of coins lying in everybody's hands. Our evidence shows that, despite the remarkable frequency of equilibrium outcomes, deviations from optimal play are also significant. And when such deviations occur, we find that, for any given player position, the probability of a mistake is increasing in the probability of a mistake of her predecessors. This is what we call an error cascade, which we rationalize by way of a simple model of “noisy equilibrium”.

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1 Motivation

There are many situations of economic interest that involve *public sequential decisions* – that is, choices perfectly observed by others and made in a sequential order in which the position and identity of each player is well anticipated. This is often the case of financial markets daily routine (where the moves of at least some “big players” are known to the market), or the choice of firms on technological adoptions under uncertain market conditions. As both examples suggest, in these situations agents may have some private but incomplete information on which is the profitable decision. Therefore, the action they take (as well as their identity and reputation) may implicitly convey some of this private information to late movers, who can use it as input in their own decisions. Then, it may well happen that the higher the number of agents who have already taken their decision, the lower the level of uncertainty faced by those who still have to do it. This is what is often labelled as *observational* (or *positional*) *learning*, which has been the object of recent attention, both on the theoretical and the experimental side.

This paper reports an experimental study on observational learning based on a traditional parlour game played in many countries, which in Spain is known as *Chinos*.¹ In this game, players start by hiding in their hands a number of coins (or pebbles), from zero to a certain maximum number (often three). Then, in some pre-specified order, each player produces a guess on the *total* number of coins in the hands of every player. When doing so, a player is informed of her own number of coins as well as the guesses produced by all others who preceded her.

Formally, this yields a multi-stage game with incomplete information. In its simplified version played in the lab, the number of coins in the hands of each player is the outcome of an exogenous random mechanism, i.e., a stochastic move by Nature. We further simplified matters by considering just three players and restricting the number of coins in the hands of each player to be either zero or one. Finally, concerning payoffs, we design the game so that players’ incentives do not conflict. Specifically, we allow players to submit the same guess, and the same fixed price is awarded to *all* subjects who guess the total number of coins right.

¹The word “chinos” is a slight modification of the Spanish word “chinas”, which refers to the pebbles that players may hide in their hands when playing the game. This game was first analyzed theoretically by Pastor-Abia *et al.* [28].

As a consequence of this payoff structure, the modified Chinos game that is the object of our experiment turns out to have a unique Perfect Bayesian Equilibrium. In it, after observing any given player’s guess, each subsequent player infers exactly the number of coins lying in the formers’ hands. Therefore, the probability of winning increases with player position, with the last player in the sequence guessing the correct answer with certainty.

In this light, the main objective of our experiment can now be advanced. Succinctly expressed, it is to contrast whether, as theory would unambiguously prescribe, first movers choose clear-cut signalling guesses *and* followers are perfectly able to “decipher” the predecessors’ actions and react accordingly. These are the main regularities we would expect to find in the experimental evidence, possibly entangled by considerations of learning and noise, unavoidable in any real-world context.

In this respect, we find that, qualitatively, our experimental results mimic theoretical predictions, in that we observe that the frequency with which the correct answer is guessed increases with player position. This suggests that late movers use inference to gain information on their predecessors’ signals and thus have a higher chance to guess right. In fact, we find that players’ guesses are always significantly correlated with the guesses of their predecessors. However, we also find that the resulting guessing probabilities are lower than predicted, so players make significant errors along their play. More precisely, it turns out that the higher the player position, the higher the difference between actual and predicted frequency of right guesses. Hence, as it often happens, we find that equilibrium analysis seems to explain the data, but only imperfectly, and that there are systematic deviations from equilibrium. This leads us to propose a simple model – we call it a model of “error cascades” – that happens to deliver interesting insights on players’ behavior.

Our model is motivated by the fact that, in the experimental setup, the same game is repeated 20 rounds among the same group of three subjects, each of which is always made to occupy the same player position. Even though subjects are involved, in effect, in a finitely-repeated game, we are not particularly worried of “repeated-game effects” since, as we already noticed, any given repetition has a unique equilibrium. This is because, since players are rewarded of a fixed prize only if their own guess is correct - independently on the behavior of the other group members- players cannot credibly trade the “quality” of their message in search of higher future rewards. On the other hand, it is reasonable to posit that players can learn

how to interpret the behavior of others on the basis of past experience. Thus, subjects should be able to detect systematic patterns displayed by the behavior of others. In contrast with situations in which groups are formed at random at every round (which is the standard in most experimental settings), here late movers should have the possibility to “tailor” their beliefs to the past observed behavior of their predecessors, whose identity remains the same throughout the experiment. In particular, this should allow learning to occur even when partners’ behavior is suboptimal, as long as it remains “consistently” so (i.e., it displays some regularities). To illustrate the point, take the extreme situation (which, indeed, describes the actual history of some experimental groups) in which, say, player 1 consistently delivers the “wrong” guess. That is, she plays the equilibrium move which corresponds to the signal 1 when she receives signal 0 and vice versa. Since all payoff relevant information (i.e., the actual sequence of private signals) is made public to all group members at the end of every round, we may expect that (rational) players 2 and 3 of the group in question should eventually adjust their beliefs to this situation and optimally react to it. After all, player 1’s strategy is as informative of her private signal as the equilibrium one would have been!

But what if such adjustment is not taking place completely, say on behalf of player 2? Or what if it does not take place at all? In the context of positional learning, “rationality” should allow an agent to entertain and validate hypotheses about others that do not simply make their reasoning process coincide with her own. For, alternatively, *“any other view risks relegating rational players to the role of the ‘unlucky’ bridge expert who usually loses but explains that his play is ‘correct’ and would have led to his winning if only the opponents had played correctly...”* (Binmore [8]).

The Chinos game is much simpler than bridge, and the simplified version played in the lab is even much simpler than the customary one. Nevertheless, even within the version used in our experiment, players who play later in the game should set up (if they are rational) a relatively complex system of beliefs in order to properly specify the “correct way to play” under any contingency. Thus, for example, if

- (a) player 1 makes erratic choices and her guess is little informative of her signal (or, alternatively, consistently delivers the “wrong” guess but its informational content needs to be properly decoded), and/or
- (b) player 2 does not adjust optimally to the former situation (and, probably,

makes some additional errors on her own),

then player 3 will generally find it quite difficult to decipher the guesses of 1 and 2, together with their possibly subtle interplay. Notice that, in principle, player 3 would not need to be concerned with these complications if everybody's play conformed to equilibrium and this were common knowledge. For, in this case, the behavior of every player could be readily decoded by inverting the equilibrium strategy (possibly with some uncertainty if this strategy is not injective). But, if some players fail to play as equilibrium prescribes, then the necessary decoding (if at all possible), must be done on the basis of past observations, a mechanism that should introduce some noise and significant complexity into the players' task. And, naturally, this complexity can only mount for game positions that occur later in the game, where those considerations are compounded by interaction between preceding players. Thus, under such circumstances, we would expect to find increasing number/severity of mistakes as we move later in the game. This is indeed the intuitive basis of what we shall call an *error cascade*, that is, a situation where deviations from optimal learning/behavior by an agent playing later in the game increases with similar deviations incurred by preceding players.

The learning pattern of Binmore's "unlucky bridge expert" is what we may call *notional learning* – that is, overconfidence in equilibrium beliefs independently on how these beliefs match actual experience. Clearly, notional learning in presence of consistent deviations from equilibrium yields error cascades, and we certainly collect consistent evidence of such notional learning in our experimental data (see Section 6). But it may be important to notice that notional learning is simply a possible source of error cascades, not necessarily the only one, or the most important. In any case, the aim of this paper is not to provide a behavioral model to explain *why* error cascades may occur. Rather, our objective is to make a general proposal on *how* to check whether those cascades exist, and then suggest a way of measuring them through a consistent statistical model. In a nutshell, our approach can be conceptually decomposed in two parts. First, we posit that the probability of failing to play optimally by a certain player – which, as explained above, is not necessarily the same as in equilibrium – is a function of the analogous probabilities displayed by her predecessors. Second, we bring in the data coming from our specifically designed experiment on the Chinos game to test the empirical relevance of our stated conclusions – in essence, that the implied coefficients are positive and significant.

The remainder of the paper is organized as follows. Section 2 provides a short survey of the literature on positional learning that is closest to the approach pursued in this paper. Section 3 outlines the theory underlying the experiment. Section 4 describes the experimental design and procedures. Section 5 includes and discusses the summary statistics of our experimental evidence. In Section 6, we present the evidence for error cascades. Finally, Section 7 concludes. The proofs of the theoretical results as well as the experimental instructions are contained in the Appendix.

2 Related literature

In the context of positional learning, herd behavior and information cascades have been first analyzed in the seminal papers of Banerjee [4] and Bikhchandani *et al.* [5]. In the latter, there are two possible states of the world, drawn with the same ex-ante probability. At each round, one of the two states is selected. Then agents, in a fixed order, have to guess the true state after receiving a private signal, with the probability of the true state conditional on the signal being greater than $1/2$. Like in our Chinos game, agents win a fixed price if their guess is right. Given this theoretical setup, they show that the corresponding Bayesian equilibrium yields *information cascades* if late movers disregard their private information when the evidence against it (which they can infer through their predecessors' guesses) is overwhelming. This behavior, although individually rational, may be inefficient, because it overvalues first-movers private information. To the extent to which this information may be misleading, the entire sequence of decisions may be misleading, too. By contrast, in the Chinos game, informational cascades can never occur in equilibrium. This is because, unlike in Bikhchandani *et al.* [5], in the Chinos game every player holds a piece of information over the ruling state of nature which cannot be substituted by anybody else's. In other words, in the Chinos game private signals are *strategic complements* (as opposed to *substitutes*) in revealing the uncertainty which characterizes the environment.²

²In this respect, the closest paper to ours is Çelen and Kariv [11]. They analyse a situation where each agent receives a signal from the continuous space $[-10, 10]$ with uniform probability, and players have to guess sequentially whether the sum over the signals of all players is "positive" or "negative". Their objective is to differentiate *information cascades* from *herd behavior* in the lab.

Turning to the sphere of applications, models of positional learning have been applied to explain a wide variety of social and economic phenomena. To name a few, Kennedy [23] focuses on how firms shape their business strategy; Welch [29] studies consumer behavior; Glaeser *et al.* [20] or Kahan [22] deal with spread of crime; and Lohmann [25] focuses on political action. Finally, a particularly fruitful strand of applications has been concerned with the phenomenon of speculative bubbles in financial markets, on which Avery and Zemsky [3], Lee [24] and Chari and Kehoe [12] have provided useful insights from the perspective of information cascades.³

Positional learning has also been the object of a significant body of experimental work. Anderson and Holt [2] develop an experiment based on Bikhchandani *et al.* [5]. Specifically, they observe a high frequency of equilibrium cascade behavior, although lower than the theoretical prediction. Allsopp and Hey [1] test experimentally Banerjee’s [4] model. Again, they find that herding occurs less frequently than predicted. Cipriani and Guarino [14] test experimentally an environment in which agents can buy or sell an asset whose value depends on the realization of the state of nature. Two experimental treatments are compared: one in which the asset price is fixed (in this sense, this is equivalent to [2]), and one in which the price endogenously fluctuates depending on the information available at the time (this is the setup theoretically analyzed by Avery and Zemsky [3]). While in the former setup equilibrium behavior yields information cascades, in the latter rational agents should always follow their own signal. In this respect, Cipriani and Guarino’s [14] experimental findings conform with these theoretical insights, since more herding is observed in the fixed-price scenario.⁴

3 The Chinos game

In the Chinos game played in the lab, three players, indexed by $i \in N = \{1, 2, 3\}$, privately receive an iid signal $s_i \in \{0, 1\}$, with $s_i = 1$ chosen with probability p uniform across players. Players act in sequence, as indicated

³See the survey Bikhchandani *et al.* [6] for a review of applications of models of information cascades in various economic setups, and the surveys of Camerer [10], Bikhchandani and Sharma [7] and Hirshleifer and Teoh [21] for the financial literature on the subject.

⁴The paper by Drehman *et al.* [15] is quite close to Cipriani and Guarino [14]. They collect a large amount of data (involving more than 6400 subjects) from an internet experiment and find that the presence of a flexible market price prevents herding. They also use an error-based model to explain speculative “contrarian” behavior.

by their indices, and have to guess the sum of signals, $\sigma \equiv s_1 + s_2 + s_3$. By the time player i makes her guess $g_i \in G \equiv \{0, 1, 2, 3\}$, she knows her own signal (s_i) and the guesses of those players $j < i$ who acted before her in the sequence. All players who guess correctly (i.e., those for which $g_i = \sigma$) receive a fixed prize equal to 1, while any incorrect guess yields a payoff of 0. As explained, this means that all players have an unambiguous incentive to maximize their chances of guessing correctly, hence revealing their private signal to later movers.⁵

We focus on behavioral strategies, defined in the conventional fashion as a mapping from information sets to (possibly probabilistic) guesses. Let \mathcal{H}_i denote the collection of player i 's information sets. For player 1, we can simply write $\mathcal{H}_1 \equiv \{h = s_1 : s_1 = 0, 1\}$, since she has only two information sets that can be associated to each of the possible realizations of s_1 . For players 2 and 3, information sets can be defined as $\mathcal{H}_2 \equiv \{h = (g_1, s_2)\}$ and $\mathcal{H}_3 \equiv \{h = (g_1, g_2, s_3)\}$, respectively. Player i 's behavioral strategy is denoted by $\gamma_i : \mathcal{H}_i \rightarrow \Delta(G)$, where $\gamma_i^h(g_i)$ stands for the probability of guessing g_i at information set h .

Next, we define players' beliefs as systems of probabilities of signals conditional on guesses. Given that signals are iid and guesses are publicly observed, we make the simplifying assumption that later movers hold common beliefs of previous signals. First, we have the system $\{\mu^1(g_1)\}_{g_1 \in G}$, where $\mu^1(g_1) \in [0, 1]$ is the probability associated (by players 2 and 3) to $s_1 = 1$ when the guess of player 1 has been g_1 . Analogously, we have $\{\mu^2(g_1, g_2)\}_{g_1, g_2 \in G}$, where $\mu^2(g_1, g_2) \in [0, 1]$ is the probability associated (by player 3) to $s_2 = 1$ when the guesses of players 1 and 2 have been g_1 and g_2 , respectively.

We are now in a position to characterize *players' optimal behavior*. For concreteness, we shall do it under the assumption that $p > \frac{2}{3}$ (in the experiment, we made $p = \frac{3}{4}$). In this case, the distribution over the sum of k signals (binomially distributed as $\text{Bin}(k, p)$ for $k \leq 2$) is unimodal, a feature that greatly simplifies the determination of the unique equilibrium outcome. Specifically, let $M_k(p)$ be the mode of $\text{Bin}(k, p)$, i.e., the most likely realization of the sum of k signals, so that $M_1(p) = 1$ and $M_2(p) = 2$, for all $p > \frac{2}{3}$. Then, given the realized vector of signals $s \equiv (s_1, s_2, s_3)$, we can “solve forward” for the unique equilibrium sequence of guesses $\bar{g}_i(\cdot)$ common to all the

⁵This is in contrast with the traditional version of the Chinos game, where agents' incentives are opposed because guesses have to be distinct, no player allowed to mimic the guess of a predecessor. This fundamental variation of the game is analyzed in a companion work, Feri *et al.* [16].

perfect Bayesian equilibria of the game, which only differ with respect to how out-of-equilibrium beliefs are specified:

$$\begin{aligned}\bar{g}_1(s_1) &= s_1 + M_2(p) = s_1 + 2, \\ \bar{g}_2(g_1, s_2) &= (g_1 - M_2(p)) + s_2 + M_1(p) = g_1 - 1 + s_2 \\ \bar{g}_3(g_2, s_3) &= (g_2 - M_1(p)) + s_3 = g_2 - 1 + s_3,\end{aligned}\tag{1}$$

To see this, remember that, since p is common knowledge, also $M_1(p)$ and $M_2(p)$ are common knowledge. Thus, player 2 and player 3 can infer s_1 from g_1 (i.e., $s_1 = g_1 - M_2(p)$) and, by the same token, player 3 can infer s_2 from g_2 (since $g_2 - M_1(p) = s_1 + s_2$). Therefore, in equilibrium, each player is perfectly informed of the signal received by her predecessors, computes her guess by taking expectations over the signals of her successors and, in doing so, perfectly reveals her own signal. This implies that, the higher the player position, the higher the chances to win the prize. In particular, player 3 guesses right with probability one, while player 2 does so with probability $\Pr(s_3 = M_1(p)) = p$, and player 1 with probability $\Pr(s_2 + s_3 = M_2(p)) = p^2$. Finally, note that, when player 3 computes her optimal guess in (1), she does not need to look at player 1's guess: all relevant information (including that regarding s_1) is subsumed in player 2's guess, g_2 .

Since we aim at constructing a statistical model of error cascades, it is helpful at this stage to compute players' optimal responses, in and out-of-equilibrium. For a given behavioral strategy profile $\gamma = \{\gamma_i^h\}$ and a given system of beliefs $\mu^j(\cdot)$, $j < i$, let θ_i^h be the probability that player i plays a best response at $h \in \mathcal{H}_i$, that is,

$$\theta_i^h = \gamma_i^h(g_i^*),\tag{2}$$

where $g_i^* = \arg \max_{g_i} \pi_i^h(g_i \mid (\mu^j)_{j < i})$ and $\pi_i^h(g_i \mid (\mu^j)_{j < i})$ is player i 's expected payoff (evaluated at h) associated to guess g_i , given player i 's system of beliefs $(\mu^j)_{j < i}$.⁶

We start with player 1, for whom matters are straightforward since beliefs play no role. For each $s_1 \in \{0, 1\}$ and $g_1 \in G$, expected payoffs $\pi_1^{(s_1)}(g_1)$ are

⁶Notice that, there are (non generic) combinations of guesses and beliefs for which best-replies may not be unique. Since this *never* happens with our experimental evidence, we shall abstract to the problem of multiple best-replies here.

as follows:

	$g_1 = 0$	$g_1 = 1$	$g_1 = 2$	$g_1 = 3$
$s_1 = 0$	$(1-p)^2$	$2p(1-p)$	p^2	0
$s_1 = 1$	0	$(1-p)^2$	$2p(1-p)$	p^2

(3)

This, in turn, implies that, to maximize her chances to win the prize, player 1 should simply add $M_k(p) = 2$ to her own signal:

$$\theta_1^{(s_1)} = \gamma_1^{(s_1)}(s_1 + 2). \quad (4)$$

For future reference, also notice that the payoff loss incurred by player 1 off the equilibrium path (i.e., when $g_1 < 2$) is lower when the signal $s_1 = 0$ than when the signal is $s_1 = 1$.

We move to player 1's followers. From the perspective of observational learning, player 2 and 3's optimal behavior depend on their respective systems of beliefs, $\mu^j(\cdot)$, as these are derived from the behavioral strategies perceived to be played by their predecessors $j' \leq j$.

First, consider the computation of the beliefs $\{\mu^1(g_1)\}_{g_1 \in G}$, supposing that the strategy on the part of 1 (commonly) perceived by 2 and 3 is some given $\hat{\gamma}_1$. Then, for any $g_1 \in G$ such that $\hat{\gamma}_1^{(s_1)}(g_1) > 0$ for some s_1 , we can readily apply Bayes Rule and compute:⁷

$$\mu^1(g_1) = \frac{p\hat{\gamma}_1^1(g_1)}{(1-p)\hat{\gamma}_1^0(g_1) + p\hat{\gamma}_1^1(g_1)}.$$

Now, as a magnitude derived from $\mu^1(\cdot)$, we want to compute a related variable that will play a key role in our analysis, namely, the estimated probability $\beta_1^{(g_1)}$ that player 1 is "behaving optimally" conditional on delivering any given guess, $g_1 \in G$. When $g_1 \in \{2, 3\}$, what this means is clear: we should simply make $\beta_1^{(3)} = \mu^1(3)$ and $\beta_1^{(2)} = 1 - \mu^1(2)$, given that player 1 should guess 2 (3) when $s_1 = 0$ ($s_1 = 1$). Instead, for any $g_1 < 2$, there is no way to rationalize the guess as optimal (i.e., payoff maximizing), for any possible signal, s_1 . In that case, we extend naturally our approach and

⁷If the prior probability $(1-p)\hat{\gamma}_1^0(g_1) + p\hat{\gamma}_1^1(g_1)$ equals zero, the corresponding beliefs are not defined. In Section 6, in which behavioral strategies are empirically estimated as relative frequencies of use, this only happens the first time a guess is submitted by a particular player. This is why, in our regressions, these observations have been dropped by our database.

label any guess $g_1 < 2$ as “optimal” if it is submitted under the signal s_1 for which the expected payoff loss (relative to behaving in a strictly optimal way) is minimized. Specifically, this means that, when $g_1 < 2$, we must make $\beta_1^{(g_1)} = 1 - \mu^1(g_1)$ since, by (3), the expected loss is minimal when $s_1 = 0$.⁸ To sum up, a compact specification of the complete array of the probabilities $\beta_1^{(g_1)}$ is given by:

$$\beta_1^{(g_1)} = \begin{cases} \frac{(1-p)\hat{\gamma}_1^0(g_1)}{(1-p)\hat{\gamma}_1^0(g_1)+p\hat{\gamma}_1^1(g_1)} & \text{if } g_1 < 3, \\ \frac{p\hat{\gamma}_1^1(g_1)}{(1-p)\hat{\gamma}_1^0(g_1)+p\hat{\gamma}_1^1(g_1)} & \text{if } g_1 = 3. \end{cases} \quad (5)$$

By analogy with (3), we are now in the position to specify player 2’s expected payoff, $\pi_2^{(g_1, s_2)}(g_2)$. When $s_2 = 1$, we have

$s_2 = 1$	$g_1 < 3$	$g_1 = 3$
$g_2 = 0$	0	0
$g_2 = 1$	$\beta_1^{(g_1)}(1-p)$	$(1-\beta_1^{(g_1)})(1-p)$
$g_2 = 2$	$\beta_1^{(g_1)}p + (1-\beta_1^{(g_1)})(1-p)$	$\beta_1^{(g_1)}(1-p) + (1-\beta_1^{(g_1)})p$
$g_2 = 3$	$(1-\beta_1^{(g_1)})p$	$\beta_1^{(g_1)}p$

(6)

while, when $s_2 = 0$, for all $g_1 \in G$, $\pi_2^{(g_1, 0)}(3) = 0$ and $\pi_2^{(g_1, 0)}(g_2) = \pi_2^{(g_1, 1)}(g_2 + 1)$ for all $g_2 < 3$.

As we did for player 1 in (4), our empirical analysis in Sections 5 and 6 relies on assessing the probability that, if player 2 *actually follows* a particular behavioral strategy γ_2 , her induced guess is optimal. This probability, which we denote by θ_2^h for each $h = (g_1, s_2) \in \mathcal{H}_2$, is characterized by the following

Proposition 1 *Let γ_2 be the behavioral strategy played by player 2 and define the function $\phi : [0, 1] \rightarrow \mathbb{R}$ by $\phi(x) = \frac{1-2x}{1-3x}$. Then, the determination of each $\theta_2^{(g_1, s_2)}$ can be divided into two cases:⁹*

⁸In this respect, notice that for the cases where $g_1 \in \{2, 3\}$, payoff-loss minimality and payoff-gain maximality coincide, so we can view the former criterion as subsuming the latter.

⁹To understand the form of $\theta_2^{(g_1, s_2)}$ in either case, note that if either $g_1 < 3$ and $\beta_1^{(g_1)} = \phi(p)$ or $g_1 = 3$ and $\beta_1^{(g_1)} = 1 - \phi(p)$, then the expected payoff of choosing $g_2 = s_2 + 1$ equals the expected payoff of $g_2 = s_2 + 2$ and is strictly higher than the expected payoff of choosing any other guess. Hence, at thresholds given by the corresponding values $\phi(p)$ and $1 - \phi(p)$ we have $\theta_2^{(g_1, s_2)} = \gamma_2^{(g_1, s_2)}(s_2 + 1) + \gamma_2^{(g_1, s_2)}(s_2 + 2)$. Also note that, for our experimental sessions where $p = 3/4$, we have $\phi(3/4) = 2/5$.

$$\begin{aligned}
(a) \text{ Suppose } g_1 < 3. \text{ Then } \theta_2^{(g_1, s_2)} &= \begin{cases} \gamma_2^{(g_1, s_2)}(s_2 + 1) & \text{if } \beta_1^{(g_1)} > \phi(p) \\ \gamma_2^{(g_1, s_2)}(s_2 + 2) & \text{if } \beta_1^{(g_1)} < \phi(p) \end{cases} \\
(b) \text{ Suppose } g_1 = 3. \text{ Then } \theta_2^{(g_1, s_2)} &= \begin{cases} \gamma_2^{(g_1, s_2)}(s_2 + 1) & \text{if } \beta_1^{(g_1)} < 1 - \phi(p) \\ \gamma_2^{(g_1, s_2)}(s_2 + 2) & \text{if } \beta_1^{(g_1)} > 1 - \phi(p) \end{cases}
\end{aligned}$$

Proof. See the Appendix. ■

Finally, we turn to the situation faced by player 3. We first obtain, by analogy with (5), the probabilities $\beta_2^{(g_1, g_2)}$ with which player 2 is estimated to behave optimally when the guesses observed for player 3's predecessors are g_1 and g_2 . These probabilities are now a function of the strategies $\hat{\gamma}_1$ and $\hat{\gamma}_2$ followed by players 1 and 2 that are perceived by player 3. Or, in an equivalent way, they are a function of the perceived strategy $\hat{\gamma}_2$ and the probabilities $\beta_1^{(g_1)}$ that player 1 is perceived to play optimally after every possible g_1 . The exact form of $\beta_2^{(g_1, g_2)}$ is provided by

Proposition 2 *Let $\hat{\gamma}_1$ and $\hat{\gamma}_2$ be the strategies of players 1 and 2 perceived by player 3 and let $\beta_1^{(g_1)}$ be computed as in (5). Given $(g_1, g_2) \in G^2$, $\beta_2^{(g_1, g_2)}$ is computed as follows:*

$$\beta_2^{(g_1, g_2)} = \begin{cases} \frac{p\hat{\gamma}_2^{(g_1, 1)}(g_2)}{p\hat{\gamma}_2^{(g_1, 1)}(g_2) + (1-p)\hat{\gamma}_2^{(g_1, 0)}(g_2)} & \text{if } \begin{cases} g_2 = 3 \text{ or} \\ g_2 = 2, g_1 < 3 \text{ and } \beta_1^{(g_1)} > \phi(p) \text{ or} \\ g_2 = 2, g_1 = 3 \text{ and } \beta_1^{(g_1)} < 1 - \phi(p) \end{cases} \\ \frac{(1-p)\hat{\gamma}_2^{(g_1, 0)}(g_2)}{p\hat{\gamma}_2^{(g_1, 1)}(g_2) + (1-p)\hat{\gamma}_2^{(g_1, 0)}(g_2)} & \text{otherwise.} \end{cases} \quad (7)$$

Proof. See the Appendix. ■

We can then determine, by analogy with Proposition 1, the probabilities $\theta_3^{(g_1, g_2, s_3)}$ with which player 3 chooses a best response if she plays according to some given strategy γ_3 , as a function of the values of $\beta_1^{(g_1)}$ and $\beta_2^{(g_1, g_2)}$ specified in (5-7). A formal statement of the result, as well as a comprehensive account of the technical details, are relegated to the Appendix (see Proposition 3 and its proof). However, Figure 1 carries out graphically a complete description of the situation.

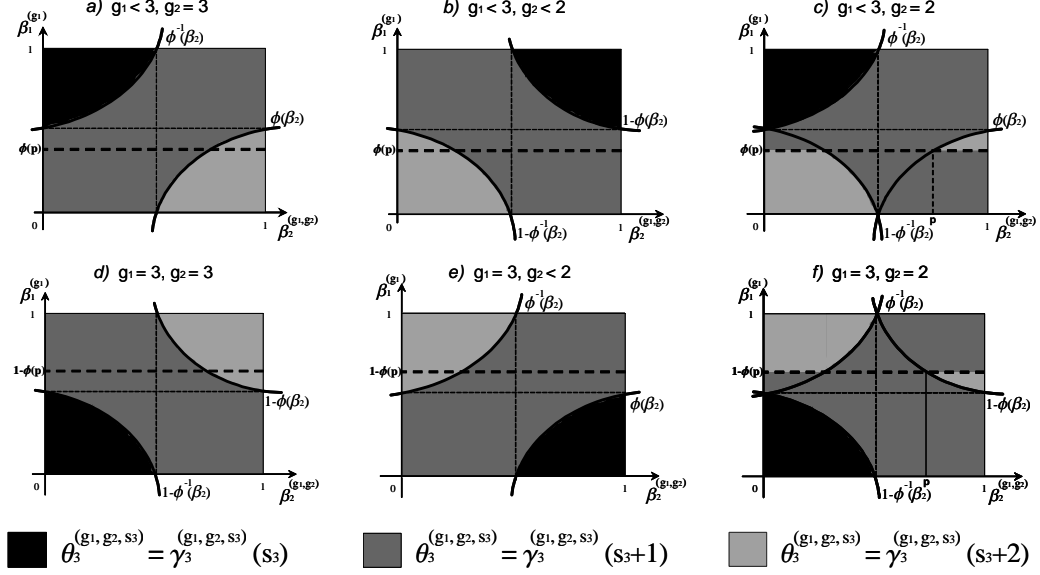


Figure 1. Player 3's best response

To understand Figure 1, bear in mind that player 3 does not directly observe s_1 and s_2 . However, given $\beta_1^{(g_1)}$ and $\beta_2^{(g_1, g_2)}$, player 3 assigns a subjective probability to $s_1 + s_2$ being 0, 1 or 2. Clearly, for any realization of $\beta_1^{(g_1)}$ and $\beta_2^{(g_1, g_2)}$ for which the mode of $s_1 + s_2$ is $z \in \{0, 1, 2\}$, we must have $\theta_3^{(g_1, g_2, s_3)} = \gamma_3^{(g_1, g_2, s_3)}(s_3 + z)$. This is represented in Figure 1 by identifying, for all possible pair of predecessors' guesses (g_1, g_2) , the regions of the $(\beta_1^{(g_1)}, \beta_2^{(g_1, g_2)})$ -space where the mode of $s_1 + s_2$ is 0, 1, or 2. The function ϕ and its transformations ϕ^{-1} , $1 - \phi$, and $1 - \phi^{-1}$, determine the boundaries of the corresponding regions.

4 Experimental design

In what follows, we describe the features of the experiment in detail.

1. *Sessions.* The 4 experimental sessions were run in a computer lab.¹⁰ A total of 48 students (12 per session) were recruited among the student population of the Universidad de Alicante – mainly, undergraduate students from the Economics Department with no (or very little)

¹⁰The experiment was programmed and conducted using z-Tree (Fischbacher [17]).

prior exposure to game theory. Instructions were provided by a self-paced, interactive computer program that introduced and described the experiment. Subjects were also provided with a written copy of the experimental instructions, identical to what they were reading on the screen.¹¹

2. *Matching.* In each session, subjects played 20 rounds of the Chinos game described in Section 3. In all 20 rounds, subjects played anonymously in groups of 3 players. Each group consisted of the same subjects throughout (that is, group composition was kept constant) and each of them occupied the same position. Both of these important features of the experimental design were publicly announced at the beginning of each session. In every round, each player’s signal was the outcome of an iid random draw with $p = \frac{3}{4}$. Given these experimental conditions, we were able to collect 16 independent observations of our experimental environment.¹²
3. *Payoffs.* All monetary payoffs in the experiment were expressed in Spanish pesetas (1 euro is approx. 166 pesetas).¹³ All subjects received 1000 pesetas just to show up. The fixed prize for each round was set equal to 50 pesetas. On average, subjects received about 15 euros for a 75’ experiment.
4. *Ex-post information.* After each round, all subjects were informed of all payoff-relevant information, that is, the correct guess (and, therefore, their individual payoff), as well as the individual guesses and signals of all subjects in their group. In addition, they were provided with a “history table,” to better track the sequence of signals and guesses of the other members of their group in all previous rounds.

¹¹A copy of the instructions, translated into English, can be found in the Appendix.

¹²Since subjects interact with each other within groups but not across groups, each group can be considered as a statistically independent observation.

¹³It is standard practice, for all experiments run in Alicante, to use (obsolete) Spanish pesetas as experimental currency. The reason for this design choice is twofold. First, it mitigates integer problems, compared with other currencies (USD or euros, for example). On the other hand, although Spanish pesetas are no longer in use (substituted by the euro in the year 2002), Spanish people still use pesetas to express monetary values in their everyday life. Thus, by using a “real” (as opposed to artificial) currency, we avoid the problem of framing the incentive structure of the experiment using a scale (*e.g.* “experimental currency”) with no cognitive content.

5 Results I: descriptive statistics

This section is divided in two parts. First, we look at winning distributions, that is, the frequency with which players win the prize. Then, we turn to analyze behavior – we check, specifically, the extent to which players’ behavior in the experiment adjusts to the theoretical (i.e., equilibrium) prediction.

5.1 Winning distributions

Table 1 shows players’ winning frequencies (i.e., the fraction of times when their guess coincided with the sum of signals), disaggregated by player position. We report within brackets the corresponding theoretical prediction, that is, the probability of guessing right (or, equivalently, winning the prize) if all players conformed to the equilibrium strategy (1).

Player	Frequency of guessing right
1	40.51 (56)
2	50.32 (75)
3	61.08 (100)

Table 1. Winning distribution

First, we can observe these probabilities, although lower than the corresponding equilibrium levels, are qualitatively consistent with the theoretical predictions, since the probability of winning is increasing with player position. We also observe that the difference between actual and theoretical frequency is increasing with player position (15.49 for player 1, 24.68 for player 2 and 38.92 for player 3).

5.2 Aggregate behavior

Now we turn our attention to subjects’ behavior. Since we run 4 sessions of 20 rounds each, with 4 groups of 3 players in each session, our panel database contains $4 \times 4 \times 20 = 320$ guessing sequences for $4 \times 4 = 16$ independent observations. The focus here will be on behavioral strategies along the equilibrium path, while we postpone to the next section our analysis of out-of-equilibrium behavior.

Tables 2.1*a*), 2.2*a*), and 2.3 report behavioral strategies of players 1 to 3, respectively. In all tables, each row (column) corresponds to an information

set (a guess conditional on the information set being reached). In each cell of the three matrices, we report absolute (top) and relative (bottom) frequency of use of a particular guess – where the latter can be seen as the “aggregate behavioral strategy” empirically observed. In all tables, we also highlight in light (dark) grey the equilibrium path corresponding to $g_1 = 2$ ($g_1 = 3$). On the other hand, Tables 2.1*b*) and 2.2*b*) look at aggregate choice frequencies of players 1 and 2 from the perspective of information decoding; that is, they calculate the *relative frequencies of signals conditional on a particular guess*.¹⁴ In a strict sense, the latter are the only relevant regularities that players need to extract from their predecessors’ strategies, namely, the extent to which players’ guesses reveal their private signals.

Let us now consider in turn each of the three player positions in some detail. First, observe in Table 2.1*a*) that player 1 guesses consistently with equilibrium 58.43% of the time $((78 + 109)/320)$. Also notice that the equilibrium guess corresponds to the modal choice in both information sets, although this frequency is higher when player 1 gets signal 0 (72% *vs.* 52%). The evidence that player 1 seems to play better when $s_1 = 0$ is also confirmed when we calculate the expected probabilities of winning given observed behavioral strategies (46% *vs.* 37% for $s_1 = 0$ and $s_1 = 1$, respectively). As for out-of-equilibrium guesses notice that, in both information sets, relative frequencies of use of suboptimal actions are aligned with expected payoffs (3). In other words, the higher the expected payoff of each guess, the higher the corresponding frequency associated to it.¹⁵ Finally, concerning the information content of player 1’s guesses, we notice that, even though she “plays better” when $s_1 = 0$, her “message” is much clearer when $g_1 = 3$. For, as the right-bottom cell of Table 2.1*b*) shows, it is *always* the case that $s_1 = 1$ whenever $g_1 = 3$ – and we can reasonably assume that, sooner or later, players 2 and 3 have come to realize this “lucky coincidence”.

¹⁴Since player 3 is the last in line, what would be Table 2.3*b*) is omitted here. Also notice that player 2 never guessed 0. The symbol “N/A” in Table 2.2*b*) simply indicates that, in this case, conditional probabilities cannot be calculated.

¹⁵This empirical evidence supports our, somehow arbitrary, treatment of out-of-equilibrium “optimal behavior” implicit in (5).

$\begin{smallmatrix} g_1 \\ s_1 \end{smallmatrix}$	0	1	2	3
0	1	29	78	0
%	0.93	26.85	72.22	0.00
1	0	20	79	109
%	0.00	9.62	37.98	52.40

a)

$\begin{smallmatrix} g_1 \\ s_1 \end{smallmatrix}$	0	1	2	3
0	1	29	78	0
%	100	59.18	49.68	0.00
1	0	20	79	109
%	0.00	40.82	50.32	100.00

b)

Table 2.1. Player 1’s behavioral strategy

We now move to player 2, whose aggregate behavior is reported in Table 2.2. First, Table 2.2*a*) shows that player 2 conforms to the equilibrium strategy 65.78% of the time, $((20 + 61 + 22 + 72)/266)$, where 266 is the number of times player 1 guessed as in equilibrium after a history consistent with it, i.e., whenever $g_1 \geq 2$). The distortion detected for player 1 (namely, that the frequency of equilibrium behavior depends on her own signal, s_1) also occurs for player 2, but in the opposite direction: conformity to equilibrium is now higher for the higher signal $s_2 = 1$. On the other hand, we also find that conformity to equilibrium behavior is much stronger after $g_1 = 3$ than after any other guess. This may be due to the fact that, in that case, player 1’s message is “crystal clean”, as $g_1 = 3$ is always associated with $s_1 = 1$. We may then conjecture that experience should lead player 2 and 3 to reach this same conclusion. As a consequence, adherence to equilibrium behavior on behalf of player 2 is higher when $g_1 = 3$ (86.23%) than with $g_1 = 2$ (51.59%).

Obviously, for player 1, guessing according to equilibrium behavior (1) always coincides with optimal behavior, independently her successors’ behavior. Indeed, the same happens for player 2 when she reacts as in (1) after observing $g_1 = 3$, since this guess always happens to be an accurately revealing message, just as in equilibrium. Concerning the optimality of player 2’s response to $g_1 = 2$, however, matters are much more intricate. For, as we observe in Table 2.1*b*), this guess was almost equally likely to be delivered for $s_1 = 0$ and $s_1 = 1$. This entails a message decoding different from that at equilibrium, which therefore needs to be learned by player 2 through experience. But such a learning introduces additional complexity into player 2’s decision problem, possibly leading to his suboptimal behavior and, in turn, increasing complexity and entailed suboptimality as well on player 3’s behavior. This is what we shall call an error cascade in Section 6. To study the phenomenon, we need to consider separately each of the 16 experimental matching groups (i.e., disaggregate the observations) and, most importantly,

account properly for how player 2's and 3's beliefs *evolve* (or should evolve), *redefining over time what is to be considered optimal behavior*. We postpone a detailed discussion of these issues to the following section, where error cascades are the focus of the analysis.

g_1	$s_1 \backslash g_2$	0	1	2	3
		0	1	2	3
2	0	0	20	31	0
	%	0.00	39.22	60.78	0.00
	1	0	8	61	37
	%	0.00	7.55	57.55	34.91
3	0	0	6	22	1
	%	0.00	20.69	75.86	3.45
	1	0	0	8	72
	%	0.00	0.00	10.00	90.00

a)

g_1	$s_1 \backslash g_2$	0	1	2	3
		0	1	2	3
2	0	0	20	31	0
	%	N/A	71.43	33.70	0.00
	1	0	8	61	37
	%	N/A	28.57	66.30	100.00
3	0	0	6	22	1
	%	N/A	100.00	73.33	1.37
	1	0	0	8	72
	%	N/A	0.00	26.67	98.63

b)

Table 2.2. Player 2's behavioral strategy (along the equilibrium path)

Finally, consider Table 2.3, which summarizes player 3's aggregate behavior. There we find that player 3 follows the equilibrium strategy (1) with a frequency of 63.23%. And, similarly to player 2, player 3 is more likely to play the equilibrium strategy when her predecessors' guesses are higher. Overall, player 3's likelihood to mimic the equilibrium grows as *i*) her own signal s_3 is higher and *ii*) players 1 and 2's guesses are also higher. The effect in *i*) corresponds to the systematic distortion tailored to the player's own signal that was already encountered for players 1 and 2. The bias, in this case, is in the same direction as for player 2: player 3 is more likely to play the equilibrium strategy when her signal is higher. On the other hand, the effect in *ii*) seems to reflect that, on average, "higher guesses are associated to clearer messages." As a case in point, consider a situation in which both players 1 and 2 produce a common guess $g_1 = g_2 = 3$. Then, it is intuitive that player 3 must expect (as also confirmed by the experience she will keep gathering on ongoing play) that both player 1 and 2 received a signal equal to 1. But, in general, to carry out a proper discussion of player 3's rationality we must address the same problem encountered for player 2, i.e., the average statistics contained in Table 2.3 are not enough. We need a richer understanding of the dynamics of *learning and adjustment* that take place over the repeated play unfolding each round. As advanced, this is the issue undertaken in the next section.

g ₁	g ₂	g ₃		0	1	2	3
		s ₁					
2	1	0		1	8	2	0
		%		9.09	72.73	18.18	0.00
		1		0	5	10	2
		%		0.00	29.41	58.82	11.76
	2	0		0	5	24	0
		%		0.00	17.24	82.76	0.00
		1		0	0	45	18
		%		0.00	0.00	71.43	28.57
3	2	0		0	6	9	1
		%		0.00	37.50	56.25	6.25
		1		0	0	9	5
		%		0.00	0.00	64.29	35.71
	3	0		0	1	21	2
		%		0.00	4.17	87.50	8.33
		1		0	0	0	49
		%		0.00	0.00	0.00	100.00

Table 2.3. Player 3’s behavioral strategy (along the equilibrium path)

6 Results II: Error cascades

The aim of this section is to go beyond the aggregate information contained in Tables 2.1-2.3 to obtain a richer (dynamic) understanding of how information decoding is unraveling along the individual (and completely independent) histories of our 16 experimental matching groups. Specifically, we would like to explore the conjecture that is suggested by the heuristic discussion closing the previous section. Namely, we want to formalize and check precisely the idea of whether the correct interpretation of a message (i.e., a submitted guess) is negatively affected by its lack of consistency with optimality. Or, to be more precise, our conjecture can be succinctly described as follows: as the behavior of a player deviates from optimality, the messages she sends (even if systematic and thus readily decodable) are interpreted more poorly; this, in turn, affects the effectiveness of subsequent messages, thus triggering an error cascade that exacerbates the phenomenon.

To carry out the test, we first estimate, at each round t and for each experimental group, behavioral strategies $\hat{\gamma}_{it}^h = \{\hat{\gamma}_{it}^h(g_i)\}$, $i = 1, 2, 3$, as the relative frequency of use of each possible guess at each information set. These play the role of the *perceived* strategies introduced in the theoretical framework of Section 3. For information sets never reached at t , we posit

uniform play, i.e., we assign equal probability to each guess in G . All this leads to the full-fledged behavioral strategies estimated at the beginning of round t , which are constructed as follows:

$$\hat{\gamma}_{jt}^h(g_j) = \begin{cases} \frac{\sum_{\tau=1}^{t-1} \chi_{\tau}(h \wedge g_j)}{\sum_{\tau=1}^{t-1} \chi_{\tau}(h)} & \text{if } \sum_{\tau=1}^{t-1} \chi_{\tau}(h) > 0 \\ \frac{1}{4} & \text{otherwise,} \end{cases} \quad (8)$$

where $\chi_{\tau}(\Xi) = 1$ if the event Ξ occurs in round τ , and 0 otherwise. In words, to estimate player j 's behavioral strategy at h , player i simply counts the number of times player j has guessed g_j at h , conditional on h being visited sometime in the past. Otherwise, we assume that i assigns a uniform probability distribution over j 's behavioral strategies at h .

Once (assumed common) perceptions on behavioral strategies $\hat{\gamma}_{it}^h$ are derived, we can evaluate the induced probabilities, $\beta_1^{(g_1)}$ and $\beta_2^{(g_1, g_2)}$ computed as in (5-7), with all observations for which Bayes rule could not be applied being omitted (cf. footnote 7). As explained in Section 3, those probabilities are identified with the beliefs (as held by other players) that players 1 and 2 hold the signal that minimizes payoff loss, conditional on their guesses. For our purposes, these beliefs measure the extent of optimality embodied by the estimated strategies of players 1 and 2.

Given the beliefs $\beta_1^{(g_1)}$ and $\beta_2^{(g_1, g_2)}$ induced by the empirical behavioral strategies computed in (8), we are in a position to assess whether the behavior of players 2 and 3 qualifies as optimal, i.e., maximizes expected payoffs given those beliefs. For each subject (in player position) i , we construct an index variable $b_{it}^h \in \{0, 1\}$, which is equal to 1 if and only if player i selects the optimal guess at the information set h visited at t . For players 1 to 3, optimal guesses are derived by (4) and Propositions 1 and 3, respectively.

Thus, as is standard in the learning literature (cf. for example the so-called fictitious play), our approach implicitly assumes that players are (a) *myopic*, as they only care about their current expected payoff in the game, and (b) *adaptive*, as they adjust their beliefs on the opponents' strategies by matching the observed frequencies.¹⁶

Figure 2 tracks the relative frequency $b_{it} = \frac{\sum_{\tau=1}^t b_{i\tau}^h}{t}$ with which, for each experimental group and up to any round $t = 1, \dots, 20$, each player $i = 1, 2, 3$ submitted her optimal guess (i.e., had $b_{it}^h = 1$). It shows that different groups

¹⁶The interested reader can find, for example, in the monograph by Fudenberg and Levine [18] an extensive discussion of this literature.

experienced rather heterogenous paths in this respect. The striking feature that transpires from the evidence is that there is a significant tendency for the different b_{it} prevailing in each group to converge towards each other as the session advances. That is, as t approaches the end value of 20, the differences $|b_{it} - b_{jt}|$ ($i, j = 1, 2, 3$) approach relatively small values even if they witnessed much larger differences early on. This suggests a relationship between them, probably mediated through the effect that optimal behavior has on delivering clear, and thus useful, messages to successors. It points, in other words, to a dependence of each b_{it} on an overall assessment of the past optimality (and thus “cleanness”) of the behavior by predecessors. And in order to quantify these latter considerations at any given t , the prevailing values of $\beta_{1t}^{(g_1)}$ and $\beta_{2t}^{(g_1, g_2)}$ – depending on whether the player i in question is a second or third mover – are the natural candidates highlighted by our theoretical framework.

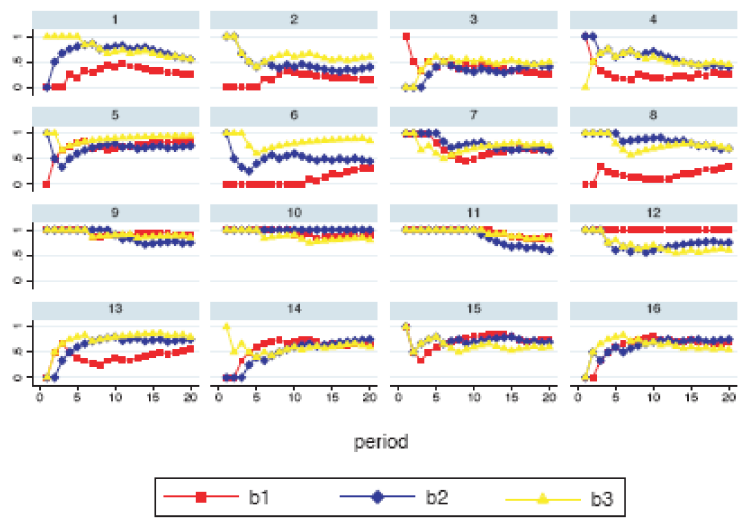


Figure 2. Average frequency of subjects’ optimal responses

The results of regressions motivated by our former discussion are reported in Table 3. In these regressions, we estimate the $\text{Prob}(b_{it}^h = 1)$ -basically, what we define as θ_i^h in (2)- as a logit function of s_i and the estimated β_{jt} of player i ’s predecessors. In case of player 3, we also include an interaction term, $\beta_1\beta_2$, to the regression. All the regressions of Table 3 also include round dummies (whose estimated coefficients are not reported here), with the reported standard errors taking also into account group clustering.

	Var.	Coef.	Std. Err.	z	P > z	[95% Conf.	Interval]
1)	s_1	-1.337	0.371	-3.610	0.000	-2.064	-0.611
	Cons.	1.054	0.740	1.420	0.154	-0.396	2.504
2)	β_1	2.402	0.475	5.050	0.000	1.470	3.334
	s_2	0.049	0.335	0.150	0.883	-0.609	0.707
	Cons.	0.240	0.922	0.260	0.795	-1.568	2.084
3)	β_1	-1.112	1.202	-0.930	0.355	-3.469	1.244
	β_2	0.306	0.761	0.400	0.688	-1.186	1.798
	$\beta_1 \cdot \beta_2$	2.286	1.436	1.590	0.112	-0.529	5.101
	s_3	1.582	0.390	4.050	0.000	0.817	2.347
	Cons	-0.057	0.833	-0.070	0.945	-1.689	1.575
	$d\text{Prob} / d\beta_1$	0.214	0.544	0.390	0.694	-0.853	1.281
	$d\text{Prob} / d\beta_2$	1.982	0.649	3.060	0.002	0.711	3.254

Table 3. Error cascades

First, a simple consequence following from regression 1) confirms the evidence already drawn from Table 2.1: the estimated coefficient for s_1 is negative and significant, indicating that player 1's behavior is closer to equilibrium after a low signal ($s_1 = 0$), than it is after a high one ($s_1 = 1$). By the same token, as we already observed in the previous section, the estimated coefficients associated with s_2 and s_3 in regressions 2) and 3) are negative, although they are not statistically significant. However, the novel and more interesting conclusions are those derived from looking at the coefficients for $\beta_{1t}^{g_1}$ and $\beta_{2t}^{(g_1, g_2)}$ in regressions 2) and 3). In 2) we find strong (positive) dependency of b_{2t}^h on $\beta_{1t}^{g_1}$. As for regression 3), the analysis on the dependence of b_{3t}^h on $\beta_{1t}^{(g_1)}$ and $\beta_{2t}^{(g_1, g_2)}$ is less transparent to read, given the interaction term included in the regression. This is why, in 3), we also estimate the corresponding marginal effects (evaluated at average values of $\beta_{1t}^{(g_1)}$ and $\beta_{2t}^{(g_1, g_2)}$). As Table 3 shows, all marginal effects have the expected (positive) signs, with that associated to $\beta_{2t}^{(g_1, g_2)}$ highly significant. This result seems to suggest that the impact of predecessors' mistakes vanishes as you move further down in the sequence.

Overall, the dynamic analysis of the evidence provides support for the idea that error cascades are a salient feature of subjects' behavior in the Chinos game played in the lab. As behavior becomes less aligned with optimality

(either because it deviates from equilibrium or because it fails to adjust optimally to such deviations by others) the messages being sent are harder to “understand.” Indeed, this even happens when the deviations are systematic and therefore, in principle, could lend themselves to straightforward decodification. The natural interpretation of the situation is that mistakes somehow build up, intensifying the implications of what might otherwise be limited to isolated deviations from rationality.

7 Summary and conclusion

The paper reports on an experiment designed to study whether agents, in a sequential and repeated setup, are able to extract valuable information from others’ decisions and react optimally. The first observation has been that, qualitatively, players conform to the theoretical prediction of our game – i.e., in the aggregate, modal play coincides with equilibrium play. It has been found, however, that there are also significant deviations from equilibrium. Motivated by this finding, a statistical model has been proposed to analyze the extent to which deviations from optimality by a certain player impairs the ability of her successors to play optimally. This has led to identifying the phenomenon that we call error cascades, namely, suboptimal behavior by a player increases the likelihood of others to play suboptimally. The presence of such cascades suggests that early errors makes the decoding task of successors more difficult, thus intensifying the tendency of subsequent players to commit analogous mistakes.

The paper provides experimental evidence that could prove useful for the design of dynamic models aiming to understand human behavior in processes of information transmission. It shows that such models should not only accommodate the fact that players make errors, but also the possibility that such errors may accumulate, or grow, along the decision sequence. The experimental results, moreover, can be viewed as a warning for real-world contexts where these considerations might be important. The fact that, in our simple setup – with binary signals and an exogenous guessing sequence – players mistakes are amplified quite sharply lead us to conjecture that similar phenomena could also be quite prevalent in more complex real-world situations. In financial markets, for example, this issue should be probably taken into account by both participants and regulators.

Our experiment on the Chinos game is a first step in the analysis of

positional learning when agents' private signals are strategic complements. The results that we have described illustrate the potential of our approach, and also suggest the interest of exploring different variations of it. In this vein, a companion paper (Feri *et al.* [16]) studies experiments carried out on two alternative versions of the Chinos game where only one player gets the prize and therefore individual incentives *do* conflict.

In one of them, the *first-win game*, the first player who guesses correctly wins the prize (or the last player, in case no one guesses correctly). In contrast, the alternative *last-win game* has the prize going to the last player who guesses correctly (or to the first player, in case no one guesses correctly). The experimental data reveal that subjects react very differently to such fully opposed kind of incentives. Whereas in the last-win game players tend to hide their private signal by using pooling strategies, in the first-win game players use separating strategies that reveal their information to their successors. Another variation of the Chinos game has been studied by Carbone and Ponti [9]. They introduce a smart device that allows one to control the intensity of the payoff received by each player, and show that a Quantal Response Equilibrium model (McKelvey and Palfrey, [26] and [27]) closely explains the behavior observed in the lab.

Finally, as a further avenue for future research, an important development would be to allow for a richer interaction (network) structure that could better reflect the complex pattern of information transmission that prevails in real economic environments. Setups of this sort have been theoretically studied by the important contribution of Gale and Kariv [19]. In the experimental realm, interesting results for that model are reported by Choi, Gale, and Kariv [13]. They consider contexts with just three agents and three directed-network architectures: the star, the circle, and the complete network. Interestingly, they find that both the network architecture as well as the information structure have an important bearing on subjects' behavior.

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Appendix

Experimental Instructions

Welcome to the experiment! This is an experiment to study how people solve decision problems. Our unique goal is to see how people act on average; not what you in particular are doing. That is, we do not expect any particular behavior of you. However, you should take into account that your behavior will affect the amount of money you will earn throughout the experiment. These instructions explain the way the experiment works and the way you should use your computer. Please do not disturb the other participants during the course experiment. If you need any help, please, raise your hand and wait quietly. You will be attended as soon as possible.

How to get money! This experimental session consists of 20 rounds in which you and two additional persons in this room will be assigned to a group, that is to say, including you there will be a *total of three people* in the group. You, and each of the other two people, will be asked to make a choice. Your choice (and the choices of the other two people in your group) will determine the amount of money that you will earn after each round. Your group will remain the same during the whole experiment. Therefore, you will be always playing with the same people. During the experiment, your earnings will be accounted in pesetas. At the end of the experiment you will be paid the corresponding amount of Euros that you have accumulated during the experiment.

The game. Notice that you have been assigned a *player number*. Your player number is displayed at the right of your screen. This number represents your player position in a sequence of 3 (player 1 moves first, player 2 moves after player 1 and player 3 moves after players 1 and 2). Your position in the sequence will remain the same during the entire experiment. At the beginning of each round, the computer will assign to each person in your group (including yourself) either 0 tokens or 1 token. Within each group, each player is assigned 0 tokens with a probability of 25% and is assigned 1 token with a probability of 75%. The fact that a player is assigned 0 tokens or 1 token is independent of what other players are assigned; that is to say, the above probabilities are applied separately for each player.

At each round, the computer executes again the process of assignment of tokens to each player as specified above. The number of tokens that

each player is assigned at any particular round does not depend at all on the assignments that he/she had in other rounds. You will only know the number of tokens that you have been assigned (0 or 1), and you will not know the number of tokens assigned to the other persons in your group. The same rule applies for the other group members (each of them will only know his/her number of tokens).

At each round you will be asked to make a guess over the *total* number of tokens distributed among the three persons in your group (including yourself) at the current round. The other members of your group will also be asked to make the same guess. The order of the guesses corresponds to the sequence of the players in the group. That is to say: player 1 makes his/her guess first, then player 2 makes his/her guess and, finally, player 3 makes his/her guess. Moreover, you will make your guess knowing the guesses of the players in your group that moved *before* yourself. Therefore, player 2 will know player 1's guess and player 3 will know both player 1 and player 2's guesses.

At each round, if you make the correct guess you will win a prize of 50 pesetas and if your guess is not the correct one you will earn nothing.

Proofs

Proof of Proposition 1

Let $g_1 < 3$. From (5), s_1 and s_3 take value 1 with probabilities $(1 - \beta_1^{(g_1)})$ and p , respectively. Hence, player 2's subjective probability distribution on $s_1 + s_3$ is:

$$\begin{aligned} \Pr(s_1 + s_3 = 0) &= \beta_1^{(g_1)}(1 - p) \\ \Pr(s_1 + s_3 = 1) &= \beta_1^{(g_1)}p + (1 - \beta_1^{(g_1)})(1 - p) \\ \Pr(s_1 + s_3 = 2) &= (1 - \beta_1^{(g_1)})p \end{aligned} \tag{9}$$

Player 2's optimal strategy simply consists in adding s_2 to the mode of (9). First, we claim that the mode of (9) is not 0. Assume not. Then $\beta_1^{(g_1)}(1 - p) \geq \beta_1^{(g_1)}p + (1 - \beta_1^{(g_1)})(1 - p)$, i.e., $\beta_1^{(g_1)}(2 - 3p) \geq 1 - p$, a contradiction with $p \in (2/3, 1)$.

We claim that the unique mode of (9) is 2 if and only if $\beta_1^{(g_1)} < \phi(p) = \frac{1-2p}{1-3p}$. Since $p > 2/3$, $1 - 2p < 0$ and $1 - 3p < 0$. Hence, $\beta_1^{(g_1)} < \frac{1-2p}{1-3p}$ becomes $\beta_1^{(g_1)}(1 - 3p) > 1 - 2p$, which is equivalent to $\Pr(s_1 + s_3 = 2) >$

$\Pr(s_1 + s_3 = 1)$. Since 0 is not a mode of (9), the claim follows. By analogous arguments, it can be shown that the unique mode of (9) is 1 if and only if $\beta_1^{(g_1)} > \phi(p)$.¹⁷

Let $g_1 = 3$. From (5), s_1 and s_3 take value 1 with probabilities $\beta_1^{(3)}$ and p , respectively. Hence, player 2's subjective probability distribution on $s_1 + s_3$ is: $\Pr(s_1 + s_3 = 0) = (1 - \beta_1^{(3)})(1 - p)$, $\Pr(s_1 + s_3 = 1) = (1 - \beta_1^{(3)})p + \beta_1^{(3)}(1 - p)$ and $\Pr(s_1 + s_3 = 2) = \beta_1^{(3)}p$. Properly accounting for the fact that, if $(1 - \beta_1^{(3)})$ is relabelled by $\hat{\beta}_1$, we get (9), the proof follows. ■

Proof of Proposition 2

We just need to calculate player 2's loss minimizing signal conditional on her guess, given the common belief $\beta_1^{(g_1)}$.

Let $g_1 = 3$. By (6): (i) $\pi_2^{(3,1)}(3) = \beta_1^{(3)}p > \pi_2^{(3,0)}(3) = 0$. Hence, $\beta_2^{(3,3)} = \mu^2(3, 3)$. (ii) $\pi_2^{(3,0)}(2) = \beta_1^{(3)}p > \pi_2^{(3,1)}(2) = \beta_1^{(3)}(1-p) + (1-\beta_1^{(3)})p$ if and only if $\beta_1^{(3)} > \frac{p}{3p-1}$. Hence, if $\beta_1^{(3)} > \frac{p}{3p-1}$ ($\beta_1^{(3)} < \frac{p}{3p-1}$), $\beta_2^{(3,2)} = 1 - \mu^2(3, 2)$ ($\beta_2^{(3,2)} = \mu^2(3, 2)$). (iii) $\pi_2^{(3,0)}(1) = \beta_1^{(3)}(1-p) + (1-\beta_1^{(3)})p > \pi_2^{(3,1)}(1) = (1-\beta_1^{(3)})(1-p)$. Hence, $\beta_2^{(3,1)} = 1 - \mu^2(3, 1)$. (iv) $\pi_2^{(3,0)}(0) = (1 - \beta_1^{(3)})(1 - p) > \pi_2^{(3,1)}(0) = 0$. Hence, $\beta_2^{(3,0)} = 1 - \mu^2(3, 0)$.

Let $g_1 < 3$. By (6): (i) $\pi_2^{(g_1,1)}(3) = (1 - \beta_1^{(g_1)})p > \pi_2^{(g_1,0)}(3) = 0$. Hence, $\beta_2^{(g_1,3)} = \mu^2(g_1, 3)$. (ii) $\pi_2^{(g_1,0)}(2) = (1 - \beta_1^{(g_1)})p > \pi_2^{(g_1,1)}(2) = \beta_1^{(g_1)}p + (1 - \beta_1^{(g_1)})(1 - p)$ if and only if $\beta_1^{(g_1)} < \frac{1-2p}{1-3p}$. Hence, if $\beta_1^{(g_1)} < \frac{1-2p}{1-3p}$ ($\beta_1^{(g_1)} > \frac{1-2p}{1-3p}$), $\beta_2^{(g_1,2)} = 1 - \mu^2(g_1, 2)$ ($\beta_2^{(g_1,2)} = \mu^2(g_1, 2)$). (iii) $\pi_2^{(g_1,0)}(1) = \beta_1^{(g_1)}p + (1 - \beta_1^{(g_1)})(1 - p) > \pi_2^{(g_1,1)}(1) = \beta_1^{(g_1)}(1 - p)$. Hence, $\beta_2^{(g_1,1)} = 1 - \mu^2(g_1, 1)$. (iv) $\pi_2^{(g_1,0)}(0) = \beta_1^{(g_1)}(1 - p) > \pi_2^{(g_1,1)}(0) = 0$. Hence, $\beta_2^{(g_1,0)} = 1 - \mu^2(g_1, 0)$. ■

Proposition 3 *Let γ_3 be the behavioral strategy followed by player 3.*

1. If $g_1 < 3$ and $g_2 = 3$, then

$$\theta_3^{(g_1, g_2, s_3)} = \begin{cases} \gamma_3^{(g_1, g_2, s_3)}(s_3) & \text{if } \beta_2^{(g_1, g_2)} < \frac{1}{2} \text{ and } \beta_1^{(g_1)} > \phi^{-1}(\beta_2^{(g_1, g_2)}) \\ \gamma_3^{(g_1, g_2, s_3)}(s_3 + 2) & \text{if } \beta_2^{(g_1, g_2)} > \frac{1}{2} \text{ and } \beta_1^{(g_1)} < \phi(\beta_2^{(g_1, g_2)}) \\ \gamma_3^{(g_1, g_2, s_3)}(s_3 + 1) & \text{if } \begin{cases} \beta_2^{(g_1, g_2)} \leq \frac{1}{2} \text{ and } \beta_1^{(g_1)} < \phi^{-1}(\beta_2^{(g_1, g_2)}) \\ \text{or } \beta_2^{(g_1, g_2)} \geq \frac{1}{2} \text{ and } \beta_1^{(g_1)} > \phi(\beta_2^{(g_1, g_2)}) \end{cases} \end{cases}$$

¹⁷Hence, if $\beta_1^{(g_1)} = \phi(p)$, (9) has two modes: 1 and 2.

2. If $g_1 < 3$ and $g_2 < 2$, then

$$\theta_3^{(g_1, g_2, s_3)} = \begin{cases} \gamma_3^{(g_1, g_2, s_3)}(s_3) & \text{if } \beta_2^{(g_1, g_2)} > \frac{1}{2} \text{ and } \beta_1^{(g_1)} > 1 - \phi(\beta_2^{(g_1, g_2)}) \\ \gamma_3^{(g_1, g_2, s_3)}(s_3 + 2) & \text{if } \beta_2^{(g_1, g_2)} < \frac{1}{2} \text{ and } \beta_1^{(g_1)} < 1 - \phi^{-1}(\beta_2^{(g_1, g_2)}) \\ \gamma_3^{(g_1, g_2, s_3)}(s_3 + 1) & \text{if } \begin{cases} \beta_2^{(g_1, g_2)} \geq \frac{1}{2} \text{ and } \beta_1^{(g_1)} < 1 - \phi(\beta_2^{(g_1, g_2)}) \\ \text{or } \beta_2^{(g_1, g_2)} \leq \frac{1}{2} \text{ and } \beta_1^{(g_1)} > 1 - \phi^{-1}(\beta_2^{(g_1, g_2)}) \end{cases} \end{cases}$$

3. If $g_1 < 3$ and $g_2 = 2$, then $\theta_3^{(g_1, g_2, s_3)}$ coincides with case 1. if $\beta_1^{(g_1)} > \phi(p)$ and coincides with case 2. if $\beta_1^{(g_1)} < \phi(p)$.

4. If $g_1 = 3$ and $g_2 = 3$, then

$$\theta_3^{(g_1, g_2, s_3)} = \begin{cases} \gamma_3^{(g_1, g_2, s_3)}(s_3) & \text{if } \beta_2^{(g_1, g_2)} < \frac{1}{2} \text{ and } \beta_1^{(g_1)} < 1 - \phi^{-1}(\beta_2^{(g_1, g_2)}) \\ \gamma_3^{(g_1, g_2, s_3)}(s_3 + 2) & \text{if } \beta_2^{(g_1, g_2)} > \frac{1}{2} \text{ and } \beta_1^{(g_1)} > 1 - \phi(\beta_2^{(g_1, g_2)}) \\ \gamma_3^{(g_1, g_2, s_3)}(s_3 + 1) & \text{if } \begin{cases} \beta_2^{(g_1, g_2)} \leq \frac{1}{2} \text{ and } \beta_1^{(g_1)} > 1 - \phi^{-1}(\beta_2^{(g_1, g_2)}) \\ \text{or } \beta_2^{(g_1, g_2)} \geq \frac{1}{2} \text{ and } \beta_1^{(g_1)} < 1 - \phi(\beta_2^{(g_1, g_2)}) \end{cases} \end{cases}$$

5. If $g_1 = 3$ and $g_2 < 2$, then

$$\theta_3^{(g_1, g_2, s_3)} = \begin{cases} \gamma_3^{(g_1, g_2, s_3)}(s_3) & \text{if } \beta_2^{(g_1, g_2)} > \frac{1}{2} \text{ and } \beta_1^{(g_1)} < \phi(\beta_2^{(g_1, g_2)}) \\ \gamma_3^{(g_1, g_2, s_3)}(s_3 + 2) & \text{if } \beta_2^{(g_1, g_2)} < \frac{1}{2} \text{ and } \beta_1^{(g_1)} > \phi^{-1}(\beta_2^{(g_1, g_2)}) \\ \gamma_3^{(g_1, g_2, s_3)}(s_3 + 1) & \text{if } \begin{cases} \beta_2^{(g_1, g_2)} \geq \frac{1}{2} \text{ and } \beta_1^{(g_1)} > \phi(\beta_2^{(g_1, g_2)}) \\ \text{or } \beta_2^{(g_1, g_2)} \leq \frac{1}{2} \text{ and } \beta_1^{(g_1)} < \phi^{-1}(\beta_2^{(g_1, g_2)}) \end{cases} \end{cases}$$

6. If $g_1 = 3$ and $g_2 = 2$, then $\theta_3^{(g_1, g_2, s_3)}$ coincides with case 4. if $\beta_1^{(g_1)} < 1 - \phi(p)$ and coincides with case 5. if $\beta_1^{(g_1)} > 1 - \phi(p)$.

Proof. Let $z \in [0, 1]$ and $y \in [0, 1]$ be the subjective probabilities that player 3 assigns to the events $s_1 = 0$ and $s_2 = 1$, respectively. Then, player 3's subjective probability distribution on $s_1 + s_2$ is as follows:

$$\begin{aligned} \Pr(s_1 + s_3 = 0) &= z(1 - y) \\ \Pr(s_1 + s_3 = 1) &= zy + (1 - z)(1 - y) \\ \Pr(s_1 + s_3 = 2) &= (1 - z)y \end{aligned} \tag{10}$$

After some algebra, we get that:

i) If $y < \frac{1}{2}$ and $z > \frac{1-y}{2-3y} \equiv \phi^{-1}(y)$, the unique mode of (10) is 0.

ii) If $y > \frac{1}{2}$ and $z < \frac{1-2y}{1-3y} \equiv \phi(y)$, the unique mode of (10) is 2.

iii) If either $y \leq \frac{1}{2}$ and $z < \phi^{-1}(y)$ or $y \geq \frac{1}{2}$ and $z > \phi(y)$, the unique mode of (10) is 1.

Let $g_1 < 3$. By (5), $z = \beta_1^{(g_1)}$. If $g_2 = 3$, by Proposition 2, $y = \beta_2^{(g_1, g_2)}$. Substituting z and y in *i)-iii)* we prove case 1. If $g_2 < 2$, by Proposition 2, $y = 1 - \beta_2^{(g_1, g_2)}$. Substituting z and y in *i)-iii)*, we prove case 2. To prove case 3, we just need to note that, if $g_2 = 2$, then, by Proposition 2, if $\beta_1^{(g_1)} > \phi(p)$, then $y = \beta_2^{(g_1, g_2)}$ whereas, if $\beta_1^{(g_1)} < \phi(p)$, then $y = 1 - \beta_2^{(g_1, g_2)}$.

Let $g_1 = 3$. By (5), $z = 1 - \beta_1^{(g_1)}$. If $g_2 = 3$, by Proposition 2, $y = \beta_2^{(g_1, g_2)}$. Substituting z and y in *i)-iii)* we prove case 4. If $g_2 < 2$, by Proposition 2, $y = 1 - \beta_2^{(g_1, g_2)}$. Substituting z and y in *i)-iii)* we prove case 5. Finally, to prove case 6, we just need to note that, if $g_2 = 2$, then, by Proposition 2, if $\beta_1^{(g_1)} < 1 - \phi(p)$, then $y = \beta_2^{(g_1, g_2)}$ whereas, if $\beta_1^{(g_1)} > 1 - \phi(p)$, then $y = 1 - \beta_2^{(g_1, g_2)}$. ■