International Transmission, Firm Entry and Risk Sharing

Marta Arespa Castelló

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

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DEDICATION

A la meva família...
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Part I

Introduction
This dissertation provides a contribution to the literature addressing issues in market dynamics and its implications for the international transmission of shocks. The thesis has been developed with the goal of building a tractable New Open Economics Macroeconomic (NOEM) model with capital accumulation and international portfolio diversification.

In the past decades, capital and goods markets have dramatically increased their international openness. Capital flows deregulation and goods trade globalization have made countries progressively more dependent on neighbours. This tendency has provided them with potential capability to hedge the risk tight to country-specific disturbances, although, at the same time, it has made them more vulnerable to foreign and global instabilities and crises.

Over the last thirty years, the share of imports in US GDP has more than doubled and worldwide trade, measured as the value of trade as a fraction of the value of GDP increased from around 8% in 1950 to more than 15% in 1990. It is crucial to realize that a large part of this enormous increment lays in the enlargement of the number of different goods traded, i.e. the expansion of the so called extensive margin.

On the capital flows side, the 1970s witnessed a remarkable boom to emerging economies. This surge was triggered by the oil shock in 1973-1974, the growth of the Eurodollar market and the remarkable increase in bank lending during 1979-1981. The pace of international lending came to an abrupt end in 1982 with the sharp increase in world real interest rates. Emerging countries were extremely damaged and almost excluded from capital markets. By the late 1980s, there was a revival of international lending. This time, however, the composition of capital flows was mainly on foreign direct investment and portfolio investment. Empirical evidence tells that the allocation of this international portfolio is, in fact, considerably biased towards home equities, contrary to what optimal risk hedging behaviour would suggest in a financial integrated world. This is usually known as the international equity puzzle.

New open-economy macroeconomic models have become a popular way of analyzing international policy issues, starting from Obstfeld and Rogoff (1995, OR from now on.) The main advantage of these kind of models is that they explicit micro-foundations. The baseline model differs from the traditional Mundell-Fleming in two features: first, all agents act under perfect foresight. Households decide their optimal consumption path, asset holdings and labour supply and, although there is no capital, output is not exogenous but depends on the decision of leisure and labour made by individuals. Firms are owned by households and use labour inputs to produce differentiated goods, which provides them with monopoly power. Second, the baseline model assumes that purchasing power parity and the law of one price
Although prices may diverge from marginal costs, there are no restrictions to arbitrage across markets or frictions on international financial markets. Hence, agents can hold foreign and domestic bonds at no costs. Any variation in net wealth of households has an impact on the choice between labour and leisure and, thus, affect output in the long-run. Finally, money markets are fully segmented and only domestic agents hold domestic money.

After OR, the first relevant steps were developed by Corsetti and Pesenti (2001) and Tille (2001), who provided a clear and useful synthesis of both. OR, however, accounted for national markets imperfections and their international implications when the economies open up (e.g. monopoly power.) Corsetti and Pesenti (2001)\(^1\) developed a deep analysis of other distortions directly associated with openness: a country’s power to affect its terms of trade by influencing the level of supply of its products, and explored the interplay between these internal (introduced by OR) and external (monopoly power of a country in trade) sources of economic distortion. Further completing the baseline literature on NOEM, and building on both OR and Corsetti-Pesenti, Tille (2001) set the importance of agents’ preferences on domestic goods. He showed the asymmetric substitutability existent between goods produced in different countries in comparison of that between two domestically produced goods. This enhanced and sustained the claim by Corsetti-Pesenti on the power a country has on its terms of trade.

The First Chapter of this thesis focuses on the importance of the extensive margin and sets the bases of a model with which to tackle the consequences of international shocks in scenarios with an endogenous number of varieties. Households prefer to consume a wider range of varieties rather than a higher quantity of the same ones. Indeed, the chapter studies a simple closed-economy model with monopolistic competition and endogenous entry and exit of firms or varieties and show how these elements have crucial implications for the transmission of real shocks on national variables and welfare. The distinction made between the productivity of creation of new varieties and the productivity of labour becomes crucial for the results: a positive shock on the former causes an extensive effect on the production (i.e. more varieties), whereas a shock on the latter causes an intensive impact. An increment of the market size generates the entry of new firms because a larger demand and, consequently, larger profits are expected. Finally, a more patient population brings the economy to a richer market in terms of the number of varieties, although it cushions the extensive effect in the case of a shock on the productivity of creation.

The Second Chapter uses the main features exploited in Chapter I, i.e. the endogenous

\(^1\) First version, Corsetti and Pesenti (1997) NBER WP 6307.
dynamics of markets and the disentanglement of two kinds of productivity, and provides a
new benchmark for the analysis of the international diversification puzzle in a tractable new
open economics macroeconomy model. It builds on Cole and Obstfeld (1991) and Heathcote
and Perri (2005) and specifies an equilibrium model of perfect risk-sharing with endogenous
portfolio.

On the spirit of New Open Economics Macroeconomy literature, I use a simple model to
isolate, in an analytically and transparent way, the core mechanisms of international trans-
mission. The model is powerful enough to help explain some main international implications
of real shocks that had been only partially understood by previous research.

As in earlier theoretical analyses, some investment is enough to rule out efficient risk-
sharing from terms of trade adjustment. Relative to previous work, here it is shown that
optimal international portfolio diversification is driven by home bias in capital goods, in-
dependently of home bias in consumption, and the share of income accruing to labour.
Most importantly, optimal portfolio shares are independent of market dynamics and nomi-
nal rigidities. Hence the model provides a more general framework to reconsider the main
result from the New Open Economy macroeconomics, in an environment with investment
and firms entry, as well as endogenous portfolio diversification.

Finally, the Third Chapter sheds some light on the implications of financial and trade
globalization on macroeconomic volatilities. It addresses this question by exploring quanti-
tatively the model presented in Chapter II and carries the analysis for a two-country world
under two different financial regimes: a financial autarky which is able to trade across the
borders and the fully integrated economy used in the previous chapter, where risk sharing
is matched regardless of the incompleteness of the capital markets. Moreover, I observe
different levels of international trade in the form of home biases. I find that the terms of
trade and the real exchange rate experience grater volatilities when there is financial integra-
tion, whereas changes on consumption dynamics depend on the source of the shock, either
it comes from labour productivity or from the technology of firms’ creation. However, the
link between trade and volatility, as in previous literature, remains slightly ambiguous but,
in general a positive relation seems to exist.
Part II

Chapters
CHAPTER 1

MACROECONOMICS OF EXTENSIVE MARGINS: A SIMPLE MODEL

1.1 Introduction

This paper focuses on the impact that the gains of productivity and population size have on real variables, such as the size of firms, the variety of goods and the total consumption. Thus, it allows a positive macro-analysis of real shocks. Welfare analysis (see for example Krugman(1991)\(^1\) and Berry and Waldfogel(2002)\(^2\)) typically suggest that those variables influence the aggregate welfare of countries. Furthermore, the present research contributes to bridge international macroeconomics and trade theory. Although the two theories are extremely interrelated, the former usually disregards the effects of changes in the trade sector in taking the pattern of international trade and the structure of markets for goods as given. For example, the exogenous shocks to aggregate productivity induce firms to enter and exit the domestic and foreign markets, thus altering the composition of consumption baskets across countries over time\(^3\).

I take into account the existence of two sources of productivity: the ability of workers in the design and introduction of new goods into the market and the common productivity of the technology of production. This feature is relevant because, as it is shown, each source of productivity affects the variables in different ways.

To the best of my knowledge, these issues have been already analyzed by Corsetti, Martin and Pesenti (2006) in the context of static models. However, there are good reasons to believe that such effects may change considerably when intertemporal decisions and sticky price setting are also included in the model. For instance, economic agents in the real world care about the future when they take decisions, and therefore any productivity shock is likely to affect future values of the aforementioned variables and, consequently, future welfare. Also uncertainty may play a role in affecting the maximizing behaviour of economic agents and thus, the variables of interests. I shall introduce uncertainty through some price rigidities in the economy.

\(^1\)He emphasizes the role of product differentiation on welfare in a spatial-economics context. 
\(^2\)They highlighted the market size as responsible for the creation of more (and of higher quality) varieties. 
\(^3\)See Ghironi and Melitz(2004) for further discussion.
1.1. INTRODUCTION

The main contribution of this paper is the incorporation of dynamics to allow for intertemporal decisions and the analysis of business cycles in the framework of both fully flexible prices and rigid price setting. I follow Corsetti et al.(2006) in constructing the model. To introduce dynamics, I assume "time to build" of a new variety or firm: firms incur in a fixed cost and need one period to set up an operational plant. Although, at first sight, the focus on the closed economy set-up that I give to the present paper may seem to be a step back from Corsetti’s research, this is, instead, a suitable and necessary intermediate step to create solid foundations for my research agenda. Other authors such as Antràs or Melitz also resorted to the autarchic framework before going into further details in some of their open-economy analysis. The development of the closed economy, both with completely flexible prices and, afterwards, with price rigidities is used, here, to observe the variations on the number of varieties due to different shocks on real variables and to the monetary policy target.

The results of the present study agree with the main findings of the two-country version, showing, once more, that the autarky set-up is a useful approach. As in Corsetti et al.(2006), the distinction of two kinds of productivity (on creation and on production) is crucial. The former increases production in an extensive way and enlarges the size of firms, while the latter has a negative impact on the number of varieties. A shock on the size of the population generates an increment on the number of firms, known as the "home market" effect in the open-economy literature (i.e. a country exports the goods for which it offers a relatively large local demand). The presence of the fixed costs generates dependence between the number of firms and the patience of households: when households are more patient, they save more. Thus, the economy reaches a larger number of acting firms. However, this cushions a shock on the ability of creation of new varieties.

When I account for nominal rigidities, the economy is situated in a suboptimal point in terms of welfare because, in comparison with the flexible-price scenario, prices are higher and number of firms lower. The government is able to correct these imperfections by setting a stance of money tied to the level of productivity in production. Finally, the consequences of an insufficient stabilization (i.e. a failure in choosing the suitable monetary policy.) are commented.

Once the behaviour of entry and exit of firms under these frameworks is clear, one is ready to build on an open-economy set-up and investigate, not only the allocation of firms among countries, but also the behaviour of the terms of trade and the exchange rate, and their implications on welfare. The comparison between the closed and the open economy will shed some light on the channels of the transmission of real shocks in autarky and in the
global economy.

This paper is organized as follows: Section 2 presents a brief overview on the existent literature. The next sections delve into building the main analytical tools in a closed economy setting, covering both normative and positive issue. Section 3 establishes the general set-up for the basic model; section 4 develops the flexible-price version; section 5 incorporates rigidities in the prices chosen by the firms and deals with monetary policy and the problems generated by an insufficient stabilization; finally, section 6 concludes. The appendix contains some algebraic details.

1.2 Literature Review

Although there were some clear antecedents, including most notably Svensson and van Wijnbergen (1989), Obstfeld and Rogoff’s (1995) Redux model was the starting point of an outpouring of research on a new class of open-economy macroeconomic models. This surge of literature incorporates a number of distinguishing key features: optimization-based dynamic general-equilibrium modelling; stochastic shocks; imperfect competition; nominal rigidities and evaluation of monetary policies based explicitly on household welfare. The presence of a foundation in microeconomic optimization allows a rigorous welfare analysis of policies and regimes. The approach also invites a rich analysis of alternative product, labour, and asset-market structures, bringing international research closer to the complexity of the real world.

Imperfect competition is a key ingredient in these new models. Monopoly power brings the equilibrium of production below the social optimum, which is a distortion that can potentially be corrected by activist monetary policy intervention. Both nominal rigidities and market imperfections alter the transmission mechanism for shocks. By addressing issues of concern to policymakers, this new strand of research tries to provide an analytical framework that is relevant for policy analysis. In fact, very recent contributions have sought to understand more deeply the positive macroeconomic effects of uncertainty as well as the normative implications for alternative international monetary regimes. As Obstfeld and Rogoff (1998) showed, important effects of uncertainty can compound or offset the more obvious welfare effects of variability, including effects on economic activity levels.

Empirics provide enough motivation for theoretical economists to go further into the analysis of the consequences of the globalization. It is well known that trade has been

---

4 Median elasticity of substitution has been decreasing over time. Thus, trade goods become more and more differentiated. See Broda and Weinstein (2003). Furthermore, focusing on the increase in trade among industrialized countries, Markusen (1986) stressed unequal income elasticity of demands that result from nonhomothetic preferences: demand for differentiated products is superior to that for homogeneous products.
1.2. LITERATURE REVIEW

growing faster than GDP for many decades. Over the last thirty years, the share of imports in US GDP has more than doubled: rising from 4.8 percent in 1972 to 11.5 percent in 2002, while the worldwide trade, measured as the value of trade as a fraction of the value of GDP, increased from 7.9 percent in 1950 to 15.4 percent in 1990, a 94.9 percent increase. Most of the literature claims that the causes for this explosion in trade originate in three interrelated sources: the reduction in trade costs, the relaxation of capital controls, and the relative growth of many East Asian and other economies outside of the United States.

The new trade theory has provided insights for these stylized facts hardly explainable by the traditional trade approach. Helpman and Krugman (1985), for example, point out that conventional trade models such as the Ricardian model and the Heckscher-Ohlin model cannot hope to explain these facts and go on to say: "These (...) empirical weaknesses of conventional trade theory (...) become understandable once economies of scale and imperfect competition are introduced into the analysis."

However, this vast literature called "new-trade theory" often disregards another simultaneous change in the economies: the US trade did not only exploit in terms of total value of imports and exports, it jumped from a range of 74,667 imported varieties in 1972 to 259,215 in 2001. Hence, the introduction of endogenous entry and exit of firms (considering firms as producers of unique and differentiated goods, i.e. one variety for each firm) may represent a crucial feature to capture the real world evolution.

On the theoretical side of the literature, although almost all studies predict that large economies export more in absolute terms than small economies, there is no agreement on how this happens. Models that assume Arlington (1969) national differentiation emphasize the “intensive” margin (i.e. a country which doubles the resources will trade twice as much but will not trade a greater number of goods.) Monopolistic competition models in the vein of Krugman (1980, 1981) stress the “extensive” margin for exports (i.e. economies twice the size will produce and export twice as many goods.) Hummels, Klenow (2002), for instance, analyzed exports in 1995 from 110 countries to 59 importers and decomposed the greater trade of larger economies into contributions from intensive and extensive margins. The main finding was that the extensive margin accounts for two-thirds of the greater exports of larger economies, and one-third of the greater imports of larger economies. Similar results are obtained by Feenstra et al. (1999) and Funke and Ruhwedel (2001). However, Hummels, Klenow (2002) widely extend the results of previous studies regarding the relation between the largeness of an economy, international trade, and product variety i.e. they shed some

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5See Bergoeing and Kehoe 2001
1.2. LITERATURE REVIEW

light on the empirical side of the home market effect.\(^6\)

Mangani and Luini (2004) developed a small empirical analysis with cross-sectional data of the extensive increase in trade. To do so, they use an innovative source of data to estimate the variety of goods and services: the registered trademarks (the Community and the International trademarks protected by the World Intellectual Property Organization.) The use of trademarks probably reduces the scope of the questions that can be examined, but it permitted the authors to carry out a synthetic analysis of product variety. The main result of their research was that, exploiting the mentioned data, they were able to verify the finding obtained by Hummels, Klenow (2002,) that larger economies (in terms of GDP) not only produce and export more in absolute terms, but also produce and trade more goods. This implies that the strong relationship between larger economies and product variety is important in terms of consumer welfare. However, the authors invite us to a cautious interpretation of the findings because of the special characteristics of the type of data they used.

On the theoretical side, nevertheless, the traditional literature always assumed that the number of firms was given or fixed. This fact prevented experts from studying accurately the implications of different shocks on the range of available varieties in different countries and, consequently, on national and international welfare. Broda and Weinstein (2003), through an empirical analysis, offer an extensive discussion of the dramatic consequences this traditional set-up has for research findings. Notice that the suitable import price index for this kind of models (without endogenous variety) is an index that does not take into account the changes in the number of goods. Broda and Weinstein (2003), assuming that Krugman (1980) fits to model the US data, show the relevant mismeasurement caused by the use of an incorrect price index: they conclude that the US welfare increased by 2.83 percent only as a result of the changes in varieties (from 1990-2001). These gains from variety are 3 to 6 times larger than the estimated gains from eliminating protectionism (e.g., Krugman (1990), Feenstra (1992) and Romer (1994)) and around 10 times larger than the estimated gains from eliminating business cycles (Alvarez and Jermann (2000)).

A few “new-open economists” (See for example Ghironi and Melitz (2004) and Corsetti, Martin and Pesenti (2006)) have recently begun to take into account this relevant characteristic. This allows them to consider the relationship between the macroeconomic effects of productivity differential and the substitutability or complementarity of goods, i.e. elasticities matter on determining levels of production and allocations. Ghironi and Melitz (2004) is

\( ^6\)In fact, they found that countries with more workers export higher quantities to each market-category, not at lower prices. This is consistent with a model in which larger countries avoid terms of trade deterioration by enlarging the set and/or increasing the quality of the goods they produce.
the closest paper to my research, on the open economy scenario. They differ from Corsetti et al (2006) mainly in three features: a) they endogenize tradability of goods instead of the creation of new varieties; b) allow for heterogeneity of firms and, thus are able to observe idiosyncratic shocks; and c) they use a more elaborated mechanism to generate dynamics: firms must pay a sunk cost to start their production.

Nevertheless, while our understanding of international economics has somewhat improved, a number of challenges and open questions remain in the international macroeconomics debate and continue to stimulate both theorists and empiricists in the field.

1.3 The Model

I develop a stylized closed-economy macroeconomic model. Hence, there is no external trade in goods or assets. The economy consists of \( L_t \) infinitely-lived households, an endogenously determined number of firms, and a government. The only source of investment lay on the start-up cost of every new monopolistic firm.

1.3.1 Households

The economy is populated by \( L_t \) infinitely-lived households whose utility function is:

\[
E_t \sum_{t=0}^{\infty} \beta^t \left[ C_t^{1-\psi} \left( 1 - \frac{1}{\psi} \right) - k \ell_t \right],
\]

where \( C \) is the index of consumption defined below; \( \ell \) is the labour supply which generates a constant disutility measured by \( k \), thus the labour supply is endogenous; \( \psi \) is the intertemporal elasticity of substitution; and \( \beta \) is the subjective discount factor.

Households can be employed either in the creation of next-period productive firms - with productivity \( v_t \) - or in production tasks of the currently active firms - with productivity \( \alpha_t \). However, they are homogeneous and they receive the same wage regardless of their occupation in the economy.

The index of consumption takes the usual form:

\[
C_t = \left[ \int_{h=0}^{n_t} c_t(h)^{1-\frac{1}{\sigma}} dh \right]^\frac{1}{\frac{1}{\sigma} - 1},
\]

where \( \sigma \) is the elasticity of substitution between varieties. Households’ preferences are assumed to be stated over a very large set of goods, so that their utility is well defined (and increasing) in any new good introduced in the market, given technology and labour endowment.
The budget identity for the consumers is:

\[ s_t I_t + B_t = s_{t-1} \sum_{h=0}^{n_t} \Pi_t(h) dh + (1 + i_t) B_{t-1} + w_t \ell_t - \sum_{h=0}^{n_t} pt(h) c_t(h) dh - T_t, \]  

(1.3)

where \( s \) are savings expressed as a proportion of total investment, \( B \) are riskless bonds and, \( \pi(h) \) are the profits generated by firm \( h \). There are \( n \) monopolistic firms, producing differentiated varieties, \( i \) is the nominal interest rate paid by the bonds, \( w \) represents the wage, \( p(h) \) is the price for variety \( h \) and \( c(h) \) is the demand for variety \( h \), \( T \) is a lump-sum tax paid each period and \( I_t \) denotes total investment, which is equal to:

\[ I_t = \left[ \int_{h=0}^{n_{t+1}} q_t(h) dh \right] = q_t n_{t+1}, \]  

(1.4)

where \( q(h) \) is the cost necessarily paid in order to create a new firm today, which starts to produce a new variety tomorrow. So \( s_t = \frac{1}{L_t} \) due to the homogeneity among households.

1.3.2 Firms

A continuum of \( n_t \) monopolistic firms act in the economy at \( t \). Investment appears under the form of an exogenous start-up cost, \( q_t \), entrepreneurs need to incur in, at time \( t \), to develop their new variety, which enters the market at \( t+1 \). This cost consists in the wages paid to the labour force allocated in creation tasks, that has productivity \( \nu_t \), i.e. \( q_t = \frac{1}{\nu_t} \). Investment is fully depreciated at the end of the productive period, thus all firms die after two periods of existence.

Once created, firms produce a differentiated variety with a homogeneous and linear technology which requires only labour:

\[ y_t(h) = \alpha_t \ell_t(h), \]  

(1.5)

where \( y(h) \) is production of variety \( h \); \( \alpha \) is the productivity parameter, which is completely exogenous in this model; and \( \ell(h) \) is the amount of labour demanded for productive activities of variety \( h \).

1.4 Equilibrium

1.4.1 The Flexible Price Regime

All contracts and prices in the economy are written in nominal terms. Prices are flexible, thus, I only solve for the real variables in the model and do not model the demand for cash
1.4. EQUILIBRIUM

currency, resorting to a cashless economy.

1.4.1.1 The Household’s Problem

The representative household decides on consumption, labour supply and savings. To do so, she maximizes (1.1) subject to (1.3). The first-order conditions are:

\[ c_t(h) : c(h) = \left( \frac{p_t(h)}{P_t} \right)^{-\sigma} C_t, \]  
\[ C_t : \lambda_t = \frac{1}{P_t C_t^\frac{1}{\sigma}}, \]  
\[ \ell_t : w = kP_t C_t^\frac{1}{\sigma}, \]  
\[ B_t : \lambda_t = \beta (1 + i_t) E_t \lambda_{t+1}, \]  
\[ s_t : \lambda_{kt} n_{t+1} = \beta E_t \lambda_{t+1} \Pi_{t+1} n_{t+1}, \]

where \( P \) is the welfare-based price index and \( \lambda \) is the Lagrangian operator.

As regards portfolio choice, focusing on the equilibrium, there will be no borrowing or lending, so that \( B_t = 0 \ \forall t \), and each household will hold an equal share of equities, \( s_t = \frac{1}{L_t} \). Notice that in the case of population growth it is necessary to control for it. This result could be retained by introducing a redistributive scheme with lump sum taxes and transfers such that the government budget is:

\[ T + \int_{a=0}^{\infty} T_a = T = \int G(h), \]

where \( T_a \) is a positive transfer for the generations already working, but negative for the new generation that owns no share of firms. This transfer program is self-financed, so that \( \int_{a=0}^{\infty} T_a = 0 \). The subscript \( a \) refers to the age of the individual.

The welfare-based price index is:

\[ P_t = n_t^{-\frac{1}{\sigma}} p_t, \]

which is decreasing in the number of varieties.

For simplicity, I assume the public demand of the government to be similar to the private demand derived in (1.6) for each specific variety, hence:
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\[ G_t(h) = \left( \frac{p_t(h)}{P_t} \right)^{-\sigma} G_t. \quad (1.13) \]

To solve for \( n_{t+1} \), observe that the last first-order condition implies free entry in the goods market. It tells new firms will be set up until the expected discounted profits of the marginal firm (i.e. the present value of the firm) is equal to the costs of creating it.

\[ q_t = E_t \beta \frac{P_t C_t^{\frac{1}{\psi}}}{P_{t+1} C_{t+1}^{\frac{1}{\psi}}} \Pi_{t+1}, \quad (1.14) \]

which, using the definition

\[ \mu_t = P_t C_t^{\frac{1}{\psi}}, \quad (1.15) \]

can be rewritten as:

\[ q_t = E_t \beta \frac{\mu_t}{\mu_{t+1}} \Pi_{t+1}, \quad (1.16) \]

where \( \mu \) must be interpreted as the monetary policy.

1.4.1.2 The Firms’ Problem

Firms maximise their profits with respect to price and labour and subject to the technology constraint. Profits are proportional to sales so, ex post

\[ \pi_{t+1} = \frac{1}{\sigma} p_{t+1}^{1-\sigma}(h) P_{t+1}^{\sigma-\psi} L_{t+1} \mu_{t+1}^{\psi} + \frac{1}{\sigma} p_{t+1}(h) G_{t+1}(h), \quad (1.17) \]

which is a function of \( n_{t+1} \) via the expression for the CPI. That is,

\[ \Pi_{t+1} = \frac{1}{\sigma} p_{t+1}^{1-\sigma}(h) L_{t+1} \mu_{t+1}^{\psi} + \frac{1}{\sigma} p_{t+1}(h) G_{t+1}(h) = \]

\[ = \frac{1}{\sigma} p_{t+1}^{1-\psi}(h) L_{t+1} \mu_{t+1}^{\psi} + \frac{1}{\sigma} p_{t+1}(h) G_{t+1}(h). \quad (1.18) \]

Literature generally opts to parameterize for the intertemporal elasticity of substitution \( \varphi \) between 1 and 1/2. The suitable choice for \( \sigma \) is less obvious, but in calibration exercises it is typically set around 1. However, values between 5 and 10 are also common. From now on, I will assume that \( \varphi \leq 1 < \sigma \).

Under perfect foresight, \( n_{t+1} \) is the solution to the free entry condition:

\[ q_t = \frac{w_t}{\nu_t} = k \mu_t \frac{\mu_t}{\mu_{t+1}} \left[ \frac{1}{\sigma} p_{t+1}^{1-\psi}(h) L_{t+1} \mu_{t+1}^{\psi} + \frac{1}{\sigma} p_{t+1}(h) G_{t+1}(h) \right], \quad (1.19) \]
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where

\[ p_t = \frac{\sigma}{\sigma - 1} \frac{w_t}{\alpha_t} = \left( \frac{k_P C_t^\frac{1}{\sigma}}{\alpha_t} \right) \frac{1}{\sigma - 1} \frac{k_{\mu t}}{\alpha_t}, \]  

(1.20)

\[ = \frac{\sigma}{\sigma - 1} k_{\mu t} \]

(1.21)

from first-order conditions of the firm’s maximization problem. So that,

\[ \frac{k_{\mu t}}{\nu_t} = \beta \frac{1}{\mu_{t+1}} \frac{\sigma}{\sigma - 1} \frac{1}{\alpha_{t+1}} \left[ \left( \frac{k_{\mu t+1}}{\alpha_{t+1}} \right) \frac{1}{\psi - \sigma} \frac{L_{t+1} \mu_{t+1}^\psi}{\mu_{t+1}^{\psi - \sigma}} + \frac{\sigma}{\sigma - 1} \frac{k_{\mu t+1}}{\alpha_{t+1}} G_{t+1} (h) \right], \]

(1.22)

\[ n_{t+1} = \frac{k_{\mu t}}{\nu_t} \frac{\beta}{\sigma - 1} \frac{1}{\alpha_{t+1}} G_{t+1} (h) \left( \frac{1}{\psi - \sigma} \frac{L_{t+1}^\psi}{L_{t+1}} \right). \]

The relation between the number of active firms and the cost of creation is found using this equality. \( \sigma > \psi \) is needed in order to ensure a decreasing effect of entry cost on the number of available varieties. The reason for requiring this condition is that an increase in the number of firms, which will crucially affect the expected profits, generates two effects on the consumption demand. First, the CPI falls, pushing an intertemporal substitution impact (represented by \( \psi \)), which consists of higher consumption today. Second, since there are more goods to consume, households will split their income among all of them. Thus, this reallocation of private expenditure among goods generates an intratemporal substitution away from existing goods, which is measured by \( \sigma \). Notice that, under this reasoning process, the claimed inequality \( (\sigma > \psi) \) becomes a necessary condition for the steady state equilibrium to be stable.

Due to the existence of the in-advance investment, \( \beta \), the subjective discount factor, affects positively the number of firms. When people are more patient they choose to save more, buying firms’ shares. Thus, the economy will be supplied with a larger range of varieties. Obviously, the cost of creating new firms decreases the quantity of companies acting in the next period.

1.4.2 **Firm’s size:**

It is worth being aware of all the different implications of a change in the number of firms. First, consider that consumers have a love for variety. This means they prefer to consume
1.4. EQUILIBRIUM

a larger range of different goods rather than a large amount of only some goods\(^7\). So, in principle, they may be better off with more firms in the market. However, more firms means more labour used in non-production activities. Each new company needs a previous fixed cost which requires labour. Moreover, this new company will not supply a new good until the second period. During the first period, the workers employed to generate firms are kept away from the production of goods already available to consume. Here there is a traditional trade-off: households must renounce present consumption in order to invest (with the fixed cost) and enjoy more consumption tomorrow\(^8\).

The size of a firm is:

\[
Z_t = \frac{L_t \ell_t - \frac{n_{t+1}}{\alpha_t}}{n_t} = \frac{L_t C_t(\alpha_t) + G_t(\alpha_t)}{\alpha_t n_t}, \tag{1.23}
\]

\(n_t Z_t\) is the quantity of workers not employed in the creation of new firms at \(t\). So, it is the labour force available for the production of final goods in that period. Equation (1.23) represents the size of a firm producing at \(t\). It informs that an increment in the number of currently producing firms, ceteris paribus, reduces the quantity of workers in each one. Obviously, more firms must share the same quantity of workers. If something increases the expected profits for the next period or reduces the cost of creation, \(n_{t+1}\) is higher. Labour force for final goods production is lost today to reach the new level of \(n\); so \(Z_t\) is undoubtedly reduced. Finally, if people experience a larger disutility for their labour effort and decide to increase their leisure time, either the total production per variety must be also reduced or they must renounce future varieties. \(Z\) can be written as a function of the GDP (second part of the equation above), although it does not allow to explain variations on \(Z\). This is because of the two simultaneous effects that an increase on \(\alpha_t\) generates: equation (1.23) tells us that, for a given demand, \(\Delta \alpha_t\) would generate a reduction on the size of firms due to the fact that less labour is necessary to produce the same amount of goods; since \(n\) is given from previous period, the unique instantaneous effect of \(\alpha_t\) improvement is via the reduction of the price index, which produces an increase of the demand that completely offsets the first impact of \(\alpha_t\) movement.

\(^{7}\)There is already some literature which deals with this consideration and explicitly separates the love for variety from the elasticity of substitution. See, for example De Groot and Nahuis (1998) and De Groot (2001).

\(^{8}\)Another consideration may be the existence of economies of scale. In this case, the cost of opportunity of enjoying a new variety would be much more important for scale inefficiencies. But this analysis is out of the scope of our paper.
1.4.3 Change of the Number of Varieties

Let’s differentiate the equilibrium condition (1.22) in the steady state defined above, take the assumption done in (1.13) and set public expenditure to zero \((G = 0)\), then:

\[
\frac{\sigma - \psi}{\sigma - 1} \frac{dn_{t+1}}{n} = \frac{d\nu_t}{\beta} + (\psi - 1) d\alpha_{t+1} + dL_{t+1} + k \left( \frac{\sigma}{\sigma - 1} \right)^\psi n^{1-\psi} dG_{t+1}. \tag{1.24}
\]

Notice, first of all, that the sign of the effects are crucially tied to the relation between \(\sigma\) and \(\psi\). Under the initial assumption, \(\psi \leq 1 < \sigma\), if there is an increase in the efficiency of the creation of firms, i.e. when \(v_t\) goes up, the number of firms also increases. On the contrary, when firms are more productive in their production process, i.e. each firm is able to supply a larger amount of its good \(h\) with the same quantity of inputs \((\Delta \alpha)\), this productivity improvement disincentivizes the creation of new firms. The reason is that a lower marginal cost forces firms to set prices according to it. This is translated into smaller profits when the intertemporal elasticity of substitution is \(\psi < 1\). It is worth noticing the consequences of an overall productivity change, i.e. \(d\alpha = dv\). Although the two shocks acted in opposite directions, in the static model the dominant effect was unambiguously the positive impact of extra efficiency in new creation, except when \(\psi = 0\). In this case, one of the shocks exactly balanced the other. Under the present framework, this is not so clear. It is the relation between the intertemporal elasticity of substitution \((\psi)\) and the patience of the households \((\beta)\) that determines if the total effect leads to entry or to exit of firms. Following the assumption of \(\psi < 1\) and assuming \(0 < \beta \leq 1\), the dominant effect is, as in the static case, the improvement on \(v\). In fact, this positive impact is stronger as \(\beta\) goes down \((\forall \beta < 1)\). Hence, the more patient the consumers, the smaller the positive impact of a shock on the productivity of creation. The intuition behind this result is the following: on the one hand, when people are patient, they always choose to invest more; thus, in general, the economy reaches a range of varieties wider than in the case of impatient consumers (notice that the derivative of (1.22) with respect to \(\beta\) is unambiguously positive). On the other hand, if consumers tend to save more, their demand of each variety is lower and, consequently, the expected profits are also lower, which demotivates new entries. If, instead, \(\psi < 1\) is not imposed the final result is uncertain.

A larger market size, i.e. larger \(L\), generates an increase in the number of firms\(^9\). Public expenditure enlargement means an increase in total demand, so it also motivates the greater range of varieties. Notice that \(\mu\) is not present in the above equation. Monetary policy cannot have any impact on real variables when all prices are completely flexible.

\(^9\)Remember I have controlled for \(L\) growth with redistributive transferences.
1.4. EQUILIBRIUM

To sum up, although most of the conclusions concerning the effects of the endogenous variables on the number of varieties may be quite obvious for the reader, it is important to keep in mind the crucial role of the explicit difference made between the two types of productivity \((v\) and \(\alpha)\).

1.4.4 Changes in Prices

Differentiating equation (1.12), the welfare-based CPI, one can analyze how it moves when the key variables change:

\[
\frac{dP_t}{P} = \frac{1}{1 - \sigma} \frac{dn_t}{n} + d\mu_t - d\alpha_t. \tag{1.25}
\]

Assuming again \(\sigma > 1\), the more firms act in the market, the more the price index \((P)\) decreases. It decreases more rapidly the larger the substitutability among varieties, i.e. the weaker the monopolistic power of firms is. A productivity improvement in the production of goods also reduces CPI. The latter implication has been widely studied in open economy models because it means that countries with better technology (i.e. more developed countries) experience a worsening in terms of trade (defined as the ratio between price of imports over price of exports). Finally, there is the effect of the monetary policy, which is positive, as expected.

1.4.5 Steady State Analysis

In order to define a steady state, let’s set \(\mu = 1, \alpha = \nu = 1, L = 1\). Thus, \(w = k\) and

\[
k = \frac{\beta}{\sigma} \left[ \frac{\sigma}{\sigma - 1} k \right]^{1 - \psi} \frac{1}{n^{\psi - \sigma}} \left[ \frac{\sigma}{\sigma - 1} k \right] G(h) \right]. \tag{1.26}
\]

If, moreover, \(G=0\) then:

\[
n = \left[ \frac{\beta}{\sigma k} \left( \frac{\sigma}{\sigma - 1} k \right)^{1 - \psi} \right]^{\frac{\sigma - 1}{\sigma - \psi}}. \tag{1.27}
\]

In the steady state, when firms are able to charge a high mark-up, new entry is more attracted due to higher expected profits. From the fact that there is (a kind of) investment, represented by the operating cost paid in advance, \(\beta\), the subjective discount factor, affects the number of varieties in equilibrium. The impact will be positive or negative depending, again, on the relation between \(\sigma\) and \(\psi\). If \(\psi > \sigma\) (goods are complements in the Edgeworth-Pareto sense) an increase in households’ patience derives in a reduction of the number of firms in equilibrium. If, instead, \(\psi < \sigma\) (i.e. goods are substitutes), as assumed here, the
more patient people are the more they choose to save by investing in firms’ shares. Thus, a larger number of varieties is produced in equilibrium. On the contrary, as the disutility of labour effort increases ($\Delta k$), consumers give relatively more value to leisure rather than to consumption. Hence, the number of varieties is reduced as labour supply decreases.

From equations (1.9) and (1.10) it is found that,

$$\beta = \frac{\lambda_t}{\lambda_{t+1}} \frac{1}{1 + i_t} = \frac{P_{t+1}C_{t+1}^{1/\psi}}{P_tC_t^{1/\psi}} \frac{1}{1 + i_t} = \frac{\mu_{t+1}}{\mu_t} \frac{1}{1 + i_t}.$$ 

In equilibrium:

$$\beta = \frac{1}{1 + i}.$$ \hfill (1.28)

For the rest of the endogenous variables I find:

$$p = \frac{\sigma}{\sigma - 1} k,$$ \hfill (1.29)

$$P = \left( \frac{1}{\sigma k} \right)^{1/\psi} \left( \frac{\sigma k}{\sigma - 1} \right)^{\psi - 1 \psi - \psi}.$$ \hfill (1.30)

In the steady state, individual prices depend positively on the monopolistic power of firms and marginal cost, measured by $k = w$. The price index value is subject to the size of the mark-up and the level of patience of the households. The effect of variations of these concepts on CPI differs depending on whether goods are substitutes or complements. If varieties are substitutes, a high level of patience reduces the price index: people work to generate more differentiated varieties, which acts to the detriment of $P$.

The consumption in equilibrium is:

$$c = \frac{1}{\beta} (\sigma - 1) = (1 + i) (\sigma - 1),$$ \hfill (1.31)

$$C = P^{-\psi} = \beta^{\psi} (\sigma - 1)^{\psi (\sigma - 1) \psi - \psi} (\sigma k)^{-\psi \psi - \psi}.$$ \hfill (1.32)

Consumption per variety is only affected by patience and the elasticity of substitution. From (1.27) it is known that the elasticity of substitution has a negative relation with the number of firms in equilibrium (when goods are substitutes). As varieties become more substitutable, people are more reluctant to work towards creating new firms (and getting more of the same after paying the fixed cost). Hence, they decide to invest less and consume a larger amount of each already available good. This is the reason that $\sigma$ has a positive effect on $c$. The effects of the parameters on $C$ are of exactly the opposite sign to those for CPI.
Finally,
\[ Y = C, \quad (1.33) \]
\[ \pi = \frac{k}{\beta} \text{ per firm.} \quad (1.34) \]

Notice the extremely simplified form found for profits. As households suffer more from their labour effort, they will renounce creating more firms and so profits per firm will increase. Instead, if consumers have a high level of patience, present consumption has a lower value, savings are larger and the economy is able to support more producers in equilibrium. However, this fact reduces profitability per firm.

\[
\ell = C + n = \left( \frac{\sigma}{\sigma - 1} k \right)^{-\psi} n^{\frac{\psi}{\sigma - 1}} + n; \quad (1.35)
\]
\[
= \beta^{\frac{\psi}{\sigma - \psi}} (\sigma - 1)^{\frac{\psi(\sigma - 1)}{\sigma - \psi}} (\sigma k)^{-\frac{\sigma}{\sigma - \psi}} + \left( \frac{\beta}{\sigma k} \right)^{\frac{\sigma - 1}{\sigma - \psi}} \left( \frac{\sigma}{\sigma - 1} k \right)^{(1-\psi) \frac{\sigma - 1}{\sigma - \psi}}. \quad (1.36)
\]

The last equation of the steady state shows the labour supply level. When the population becomes more patient, it obviously experiences an increment via the increase in \( n \). On the other hand, an increase in the disutility of work reduces the labour supply. For values of \( \sigma \) larger than one, but not extremely high, labour supply decreases with substitutability. This is probably because of the decrease in the number of firms (less creation). However, if \( \sigma \) becomes very large, then labour supply of each individual retrocedes. The reason is that, at this point, the positive effect of \( \sigma \) on \( c \) per variety -which increases total demand and hence, production- dominates the negative effect of the reduction of firms.

### 1.4.6 Analysis of the macro-dynamics:

Let’s log-linearize all the relevant equations: first-order conditions, budget constraints and any equilibrium condition to study the dynamics of the model. There are fourteen unknowns, i.e. endogenous variables: \( P, p, n, C, c, s, B, G(h), \ell, i, w, Y, I, \) and \( \pi \), and fourteen linearized equations.\(^\text{10}\) This system can be reduced to a rearranged expression for the free entry condition:

\[
\left( \frac{1}{\psi} - \frac{1}{\sigma} \right) \frac{n k}{\sigma} \dot{n}_{t+2} = -\dot{v}_{t} - \eta \hat{P}_{t+1} + \chi \dot{L}_{t+1} - \theta \dot{n}_{t+1} - \\
- \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) \frac{k}{\sigma} \left[ -\frac{n}{\beta} \dot{L}_{t} - \dot{\alpha}_{t+1} + n \dot{v}_{t+1} \right], \quad (1.37)
\]

\(^\text{10}\)See appendix.
where $\varpi > 0$, $\theta > 0$, $\chi > 0$ for $0 < \beta$ and $\eta < 0$

Equation (1.37) is a first-order differentiate equation depending on the endogenous variables $n$ and $P$, the exogenous variables $v$, $\alpha$ and $L$ and on constant parameters contained also in $\varpi$, $\theta$, $\eta$ and $\chi$.

Notice that, when the CPI deviates above its steady state value, it has a positive impact on the future number of firms. That is because higher prices are transformed in higher profits. Hence firms will take advantage of this deviation and will enter the market. Moreover, consumption becomes relatively expensive with respect to investment. People expect that $P$ will return to its steady state value in the long term, so they prefer to wait for consumption. If $n_{t+1}$ deviates from $n$ (above), at $t+2$ the economy tends to correct that deviation lowering the number of firms. It is worth observing the effect of $v$ and $L$. Both exogenous variables appear in two lagged periods ($t$ and $t+1$) and affect in opposite directions. An improvement on $v_t$ increases $n_{t+1}$, but in a lower proportion.$^{11}$ Hence, at $t+1$ there is already a high range of varieties available, i.e. in deviations, $n$ is at a level higher than the steady state value. So, again, the negative effect on $\hat{n}_{t+2}$ is the correction of this "excessive" value. $\hat{L}_t$ comes from the substitution of $\hat{C}_{t+1}$ in the budget constraint. Therefore, the impact is generated by the deviations of $C$. If the population was low last period, today, people receive a larger part of total profits generated by the existing firms. Hence, households can consume more. This higher demand gives incentives to businessmen and a higher creation of firms at $t+1$ is observed. So, the deviation in $t+1$ is corrected in $t+2$.

To analyze the dynamic behaviour of the main variables, it’s useful to organize the system to obtain a first-order difference equation. Notice that the unique variable with two-periods lag (a part from population) is the exogenous $\hat{v}_t$.\(^{12}\)

\[
\frac{kn}{\varpi} \hat{n}_{t+2} \simeq \vartheta_n \hat{n}_{t+1} + \gamma \hat{L}_{t+1} - \frac{kn}{\beta \varpi} \hat{L}_t + \frac{kn}{\varpi} \hat{v}_{t+1} + \Omega \frac{\sigma}{\sigma - 1} \hat{v}_t + \vartheta_a \hat{a}_{t+1}. \tag{1.38}
\]

For the standardly accepted values of $\sigma$, $\psi$, $\beta$ and $k$ in the macro literature,\(^{14}\) the intertemporal effects on $n$ are the following: $\vartheta_n > 0$, i.e. positive deviations of $n$ in $t$, increase the number of firms in the next one. The number of varieties in the market helps to reduce the CPI, hence it is cheaper to consume and households agree on saving more to create new

\(^{11}\)Keep in mind that $n_{t+1}$ is decided at $t$, that is the reason why the relevant productivity of creation is $v_t$

\(^{12}\)A further (numerical) analysis should study the roots, considering $L$ and $v$ as parameters from the characteristic polynomial, which, therefore, would depend on time. Here I stick to a simple reasoning about the coefficients of the variables.

\(^{13}\)Check the appendix for detailed coefficients.

\(^{14}\)Standard macroeconomic literature usually sets $\psi$, the risk aversion parameter, around or below 0.5, so that $\frac{1}{\psi} \simeq 2$. The disutility of labour, $k$, in a linear technology is around 0.75. $\beta$ is close to 1 and $\sigma > 1$. The signs of the coefficients in the equation are stable for a large range of values above and below the standard ones.
1.5. NOMINAL RIGIDITIES IN PRICES

Variations in the size of the population has a longer term impact. First, since $\gamma > 0$, a larger population today encourages the creation of firms in the next period more than one to one. This is both due to the fact that there is more labour available and that expected profits are higher because of the extra demand. On the contrary, $\hat{L}$ has a negative effect on $\hat{n}_{t+2}$, lower than one, which helps to correct the excess of creation. $\Omega < 0$ and $\overline{\omega} > 0$, once more, one finds that the change on the variable has opposite intertemporal effects, as it has been explained above. Finally $\vartheta_\alpha > 0$ for $\beta$ really close to one and negative otherwise. A positive deviation on the productivity of production today will become in more varieties tomorrow if population is patient, if it is not, they will consume more. In this case, they expect to benefit, also in the following period, from levels of $\alpha$ over its steady state, so they overconsume and the number of varieties at $t + 1$ decreases.

1.5 Nominal Rigidities in Prices

The recent literature on the new open economy has often been concerned with nominal rigidities. It has studied them as a possible, although partial, explanation of the main economic puzzles which, nowadays, confuses the experts. Therefore, in order to consolidate the bases towards an open-economy analysis, I proceed to develop a version of the basic model: I introduce rigidities in prices of the varieties in this section.

Corsetti and Pesenti (2002) and Obstfeld and Rogo¤ (2000) argue that sticky wages and flexible prices are closer to reality. Despite this point, if prices are set as a constant mark-up over marginal cost, for certain applications it does not matter whether prices or wages are sticky. The introduction of nominal rigidities allows me to analyse monetary policies and the capacity of governments to replicate the flexible-price regime.

Firms cannot flexibly change their prices any more when a shock occurs. Instead, in each period, they must sign contracts setting nominal prices for next year. To do so, they

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Erceg, Henderson, and Levin (2000) show one example of when it does matter—in a closed economy with both staggered price- and staggered wage-setting, the monetary authority can no longer replicate the flexible price equilibrium.
take expectancies.\footnote{Today, a number $n_{t+1}$ of firms (matching condition 1.16) has been created. These firms will start producing tomorrow only if the price they have fixed in the previous period is at least as high as their marginal cost, i.e.

$$p_t - MC_t = \frac{\sigma}{\sigma - 1} \frac{kP_t C_t^{\frac{1}{\sigma}}}{\alpha_t} - \frac{w_t}{\alpha_t} \geq 0$$

$$= \frac{\sigma k}{\sigma - 1} P_t C_t^{\frac{1}{\sigma}} \geq w_t$$

In what follows, one does not need to be concerned about this condition because it is never violated in the present framework.}

Prices crucially depend on expected $\mu$. An expected monetary expansion raises the price level and nominal spending.

In the present model, the government controls the path of short-term rates $i$, providing a nominal anchor for market expectations. A forward-looking measure of monetary stance, $\mu$ is provided by equation (1.9) and the definition in (1.15).

$$\frac{1}{\mu_t} = \beta (1 + i_t) E_t \frac{1}{\mu_{t+1}}.$$  

In a non-stochastic steady state, $\frac{\mu_{t+1}}{\mu_t}$ represents the (gross) inflation target.

1.5.1 The Firms’ Problem

Firms maximize the expected profits with respect to $\ell$ and $p$.

$$\max_{\ell, p} E_t \left[ Q \left( p_{t+1} (L_{t+1} c_{t+1}(h)) + G_{t+1}(h) \right) - w_{t+1} \frac{\alpha_{t+1}}{\alpha_{t+1}} (L_{t+1} c_{t+1}(h) + G_{t+1}(h)) \right],$$

where $Q$ is the discount factor. Thus, the first-order conditions yields:

$$p_{t+1} = \frac{\sigma}{\sigma - 1} E_t \left( \frac{\beta k}{\alpha_{t+1}} \mu_{t+1}^{\psi} \right).$$

(1.39)

Considering the optimal choice of prices and assuming that the public expenditure is zero, the free entry condition (FEC) becomes:

$$q_t = E_t \beta \frac{\mu_t}{\mu_{t+1}} \Pi_{t+1} = \frac{k}{v_t} = \beta E_t \left( \frac{p_{t+1}}{\mu_{t+1}} - \frac{k}{\alpha_{t+1}} \right) L_{t+1} c_{t+1}(h),$$
from where one can deduce the number of varieties in the economy:

\[
\frac{\sigma - \psi}{n_{r,t+1}} = \frac{v_t}{k} \beta L_{t+1} E_t \left[ \left( \frac{p_t+1}{\mu_{t+1}} \right)^{1-\psi} - \frac{k}{\alpha_{t+1}} \left( \frac{p_t+1}{\mu_{t+1}} \right)^{-\psi} \right],
\]

\[
\frac{\sigma - \psi}{n_{r,t+1}} = \frac{v_t}{k} \beta L_{t+1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\psi} k^{1-\psi} \ast E_t \left[ \begin{array}{c}
\mu_{t+1}^\psi \frac{1}{\sigma - 1} \left( \frac{E_t}{E_t(\mu_{t+1}^\psi)} \right)^{1-\psi} - \\
- \mu_{t+1}^\psi \frac{1}{\alpha_{t+1}} \left( \frac{E_t}{E_t(\mu_{t+1}^\psi)} \right)^{-\psi}
\end{array} \right],
\]

where \( n_{r,t+1} \) is the number of varieties available in the market of consumption goods when prices are rigid. The consumption under rigid prices is:

\[
c_{r,t} (h) = \left( \frac{p_t}{P_t} \right)^{-\sigma} C_t = n_t^{\frac{\sigma}{\sigma - \psi}} \left( \frac{\mu_t}{P_t} \right)^\psi = n_t^{\frac{\sigma - \psi}{\sigma}} \left( \frac{\mu_t}{\beta E_{t-1} \left( \frac{\mu_t^\psi}{\nu_t} \right)} \right)^\psi.
\]

In this scenario, the real variables \( n_t \) and \( c_t \) are tied to the expected behaviour of the monetary authorities and the expected shocks on productivity of processes of production. Thus, the credibility of the government becomes relevant in this context.

### 1.5.2 The monetary policy

#### 1.5.2.1 A tool for macroeconomic stabilization

A government may aim to close the output gap and replicate the flexible-price situation. To do so, it should commit a monetary policy \( \mu \) so that \( n_{r,t+1} \) equals \( n_{f,t+1} \), the number of firms in the flexible-price situation. Once more, authorities may have to deal with an important trade-off: it may not be true that by closing the output gap the economy reaches, simultaneously, the consumption of the flexible set-up (i.e. while closing the output gap, the consumption gap may remain.)

If it wants to reach the flexible-price number of firms, the government’s optimal behaviour is to set, taking the CPI as given, a monetary policy \( \mu = \alpha \). This policy rule consists of a commitment to provide a nominal anchor for the economy and deviate from such a stance...
only when productivity shocks shake the economy and destabilize marginal costs. By doing so, the policy eliminates uncertainty in marginal costs and, with it, also in profits.

Now, it is necessary to check the implications of this monetary policy on output. After plugging the optimal policy in (1.40), \( n_{r,t+1} \) becomes, exactly equal to \( n_{f,t+1} \):

\[
n_{r,t+1} = \frac{\psi_t k}{k} \beta L_{t+1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\psi} \left( \frac{1}{\sigma - 1} \right) k^{1-\psi} E_t \left[ \mu_{t+1}^{\psi-1} \right] = n_{f,t+1} \tag{1.42}
\]

Notice that the government is able to catch both the output and consumption levels of the flexible-prices regime by using the defined policy. Consumption, (1.41), becomes \( c_{r,t} (h) = n_t^{\sigma-\psi} (k^\sigma)^{-\psi} \alpha_t^\psi \), which is the same in the flexible version. In the steady state:

\[
n_r = \left( \beta (k^\sigma)^{-\psi} (\sigma - 1)^{\psi-1} \right)^{\frac{\sigma-1}{\sigma-\psi}}.
\]

### 1.5.2.2 The Costs of an Insufficient Stabilization

Although the desires of the monetary authority are to reach the flexible-price situation, it may fail in choosing the correct policy. What would be the consequences of adopting a sub-optimal monetary policy? By addressing this question, one will be able to observe the welfare consequences of macroeconomic uncertainty: insufficient stabilization translates into suboptimal prices and number of varieties.

Let’s assume that the authority incorrectly sets a monetary policy \( \mu = \alpha^\Gamma \) where \( 0 \leq \Gamma \leq 1 \) (\( \Gamma = 1 \) would be the suitable policy for flex-price replication). For any value of \( \Gamma \) different from one, the policy response to shocks will be inefficient for government’s target. Thus, now:

\[
p_{t+1} = \frac{\sigma k}{\sigma - 1} \frac{E_t \left( \alpha_t^{\Gamma - 1} \right)}{E_t \left( \alpha_t^{\Gamma(\psi-1)} \right)}.
\]

By Jensen’s inequality, it is known that \( E_t \left( \alpha_t^{\Gamma - 1} \right) \geq E_t \left( \alpha_t^{\Gamma(\psi-1)} \right) \) and \( E_t \left( \alpha_t^{\Gamma(\psi-1)} \right) \geq E_t \left( \alpha_t^{\Gamma(\psi-1)} \right)^{\Gamma(\psi-1)} \). Without loss of generality, I choose \( \Gamma = 0 \) and check the effect of this insufficient stabilization:

\[
p = \frac{\sigma k}{\sigma - 1} E_t \left( \frac{1}{\alpha} \right) > \frac{\sigma k}{\sigma - 1}.
\]
1.6. CONCLUSIONS

Uncertainty about marginal costs tends to reduce expected discounted profits. That is because people are risk averse: they prefer a certain amount $x$ of income rather than an expected average income of $x$. In order to soften the sensitivity of discounted profits to shocks, firms raise the preset prices. Hence, the economy must stand up to higher prices\textsuperscript{17} and, consequently, a lower level of consumption.

Let’s refer to the number of varieties now. If the government tends to set a monetary stance which does not completely offset the productivity shock, the investors, who are risk averse, feel uncertain about future profits, so they create less firms:

$$n_{r,t+1}^{\alpha_\psi} (\mu = \alpha) - n_{r,t+1}^{\alpha_\psi} (\mu = \alpha)$$ is equal to

$$\delta \left[ \frac{\sigma}{\sigma - 1} E_t \left( \alpha_{t+1}^{\Gamma(-1)} \right) - E_t \left( \alpha_{t+1}^{\Gamma(-1)} \right) \right] - \delta \left[ \frac{\sigma}{\sigma - 1} E_t \left( \alpha_{t+1}^{\psi(-1)} \right) - E_t \left( \alpha_{t+1}^{\psi(-1)} \right) \right]$$

$$= \frac{\sigma}{\sigma - 1} \left[ E_t \left( \alpha_{t+1}^{\Gamma(-1)} \right) - E_t \left( \alpha_{t+1}^{\psi(-1)} \right) \right] + \left[ E_t \left( \alpha_{t+1}^{\psi(-1)} \right) - E_t \left( \alpha_{t+1}^{\Gamma(-1)} \right) \right],$$

where $\delta = \frac{\nu}{\kappa} \beta L_{t+1} \left( \frac{\sigma}{\sigma - 1} \right)^{-\psi} k^{1-\psi}$.

To sum up, when monetary authorities do not manage to use the replication policy,\textsuperscript{18} the economy shows prices which are too high prices and a number of varieties which is too low. The surplus of consumers is seriously damaged, both for consumption level -which is damaged due to lower purchasing power- and for the short range of differentiated goods -people do not satisfy their desire for variety-.

1.6 Conclusions

The paper presents a closed-economy general-equilibrium model with economic dynamics. Both the fully flexible price situation and the case with nominal rigidities in price settings have been analyzed. Within this framework, it has been shown that intertemporal decisions and uncertainty are relevant in determining the economic level of activity: a change in the level of patience of the consumer moves the economy to another steady state, the society enjoys more product diversify the more patient people are. On the other hand, when the society faces nominal rigidities, the firms tend to set higher prices to (partially) offset the losses they would suffer in the case of a negative shock on productivity (of production); managers tend to create less firms because the profits are not certain but risky.

The different types of productivity shocks have effects of opposite signs on the extensive level of production: an improvement of the technology of creation enlarges the number of

\textsuperscript{17}This result coincides with the conclusion in related papers. See, for instance, Corsetti and Pesenti(2004).

\textsuperscript{18}The suitable monetary policy which matches (replicates) the solution in the flexible-price framework.
varieties, while an increase in the productivity of operational firms reduces the number of supplied goods. Finally, an increase in the market size generates a positive effect on varieties.

In the present paper, as in most of the literature, the elasticity of substitution between goods and the love for varieties are perfectly tied (in the set-up I present, love for variety equals the mark-up of firms). It may be interesting to relax this assumption and consider the explicit separation of them. De Groot and Nahuis (1998) suggested that the disentanglement of these two elasticities may have important implications on the economic growth and welfare.

Finally, it is worth stressing the lack of empirical research. The separation between both kinds of productivity in empirics is a difficult task. Debaere and Lee (2003) made a first attempt to identify them separately, but there is still much work to do.
1.7 Appendix

1.7.1 Flexible Price Regime

Profits expression is:

\[ \Pi_t = p_t(h) [L_t c_t(h) + G_t(h)] - w_t \ell_t \]  

(1.44)

or

\[ \Pi_t = \left(p_t(h) - \frac{w_t}{\alpha_t}\right) [L_t c_t(h) + G_t(h)], \]  

(1.45)

where the first bracket on the RHS determines the marginal profit and the second bracket is total demand for this variety.

The expression of profits becomes a fraction of total revenue:

\[ \Pi_{t+1} = \frac{1}{\sigma} p_{t+1}(h) \left( \int c_{t+1}(h) dj + G_{t+1}(h) \right) = \]  

(1.46)

\[ = \frac{1}{\sigma} p_{t+1}(h) \left( L_{t+1} \left( \frac{p_{t+1}(h)}{P_{t+1}} \right)^{-\sigma} \left( \frac{\mu_{t+1}}{P_{t+1}} \right)^{\psi} + G_{t+1}(h) \right). \]

The complete model:

The model may be summarized into fourteen nonlinear equations depending on fourteen endogenous variables (P, p, n, C, c, s, B, G(h), l, i, w, Y, I and \( \pi \)) which, together, determines the equilibrium. After using all of them, I can reduce the system to a second-order nonlinear difference equation. This depends on n, the exogenous variables and parameters.

The BC depends only on the endogenous variables n, P and G:

\[ BC : \ n_{t+1} = n_t^{\frac{\psi-2}{\psi}} \frac{\nu_t L_t}{k} \left( \frac{\alpha_t}{k \sigma} \right)^{\psi-1} \left( \frac{L_t}{\sigma} \left( \frac{n_t}{L_{t-1}} + \frac{\sigma}{\alpha_t} \right) - \frac{n_t^{\frac{1}{\psi}}}{} \right) + \]

\[ + G_t \frac{\nu_t L_t}{k} \frac{n_t^{\frac{1}{\psi}}}{\alpha_t^{\frac{1}{k \sigma}}} \]  

\[ \left( \frac{1}{\sigma L_{t-1}} + \frac{\sigma - 1}{\alpha_t} \frac{n_t^{\frac{1}{\psi}}}{\sigma} - \frac{1}{P_t L_t} \right). \]

The free entry condition becomes an expression which depends on n and G. Notice that the previous period affects only through the fixed cost \( q_t = \frac{k}{\nu_t} \). With flexible prices, CPI does not affect the amount of varieties in the market:

\[ \frac{k}{\nu_t} = E_t \beta \left( \frac{L_{t+1} n_{t+1}^{\frac{\psi-1}{\psi}}}{\alpha_{t+1}^{\frac{1}{k \sigma}}} \left( \frac{\alpha_{t+1}}{k \sigma} \right)^{\psi} + \frac{n_{t+1}^{\frac{1}{\psi}} G_{t+1}}{\alpha_{t+1}^{\frac{1}{k \sigma}}} \right). \]
If G=0,

\[ F_{\text{EC:}}: n_{t+1}^{2-\psi} = E_t\beta k^{\alpha_{t+1} \frac{\psi}{\alpha_{t+1}} + 1} \left( \frac{\alpha_{t+1} - 1}{k\sigma} \right)^{\psi}; \]

\[ B_{\text{C:}}: n_{t+1} = \frac{L^3_t n_t}{k} \left( \frac{\alpha_1 - 1}{k\sigma} \right)^{-1} \left( \frac{n_t}{L_{t-1}} + \frac{\sigma - 1}{\alpha_1} - \frac{n_t^{\frac{1}{\alpha_1}}}{L_t} \right). \]

1.7.2 Dynamic analysis

The fourteen log-linearized equations are:

1. Euler equation: \( 0 \approx \hat{r}_t - \frac{1}{\psi} (\hat{C}_{t+1} - \hat{C}_t) - (\hat{P}_{t+1} - \hat{P}_t) \),

2. \( \hat{w}_t \approx \frac{1}{\psi} \hat{C}_t + \hat{P}_t = \hat{C}_t \),

3. CPI: \( \hat{P}_t \approx \frac{1}{1 - \sigma} \hat{n}_t + \hat{\mu}_t \),

4. \( \hat{c}_t \approx -\sigma \hat{p}_t + \sigma \hat{P}_t + \hat{C}_t \),

5. \( \hat{G}_t(h) \approx -\sigma \hat{p}_t + \sigma \hat{P}_t + \hat{G}_t \),

6. \( \hat{p}_t \approx \hat{w}_t - \hat{\alpha}_t \),

7. B.C.: \( sqn(\hat{s}_t + \hat{q}_t + \hat{n}_{t+1}) \approx s\pi n(\hat{s}_{t-1} + \hat{\pi}_t + \hat{n}_t) + \frac{k\beta C^1 \hat{w}_t}{\hat{\ell}_t} - n_{pc}(\hat{n}_t + \hat{\mu}_t + \hat{c}_t) \),

8. FEC: \( \hat{q}_t = \hat{P}_t - \hat{P}_{t+1} + 1 \frac{1}{\psi} (\hat{C}_t - \hat{C}_{t+1}) + \hat{\pi}_{t+1} \approx \hat{\mu}_t - \hat{\mu}_{t+1} + \hat{\pi}_{t+1} \),

9. \( \hat{\mu}_t \approx \hat{P}_t + \hat{C}_t \),

10. \( \hat{\pi}_{t+1} \approx \frac{1}{\sigma} (\hat{p}_{t+1} + \hat{L}_{t+1} + \frac{\sigma}{1 - \sigma} \hat{n}_{t+1} + \hat{C}_{t+1}) \),

11. \( \hat{Y}_t \approx \hat{\alpha}_t + \hat{\ell}_t = \hat{L}_t + \hat{C}_t \),

12. \( \hat{C}_t \approx \frac{\sigma}{\sigma - 1} \hat{n}_t + \hat{c}_t \rightarrow \hat{P}_t = \hat{p}_t - \frac{1}{\sigma - 1} \hat{n}_t \) (2),

13. \( \hat{s}_t = -\hat{L}_t \),

14. \( \hat{I}_t = \hat{q}_t + \hat{n}_{t+1} \),

where \( r_t = 1 + i_t \). Notice that, from government’s budget constraint, which is not allowed to incur in deficits, \( B_t = 0 \). That is the reason B is not present in the linearized equations.

Proceeding to substitute one into the other in the above system of fourteen equations in order to rearrange expressions for the free entry condition and the budget constraint for
1.7. APPENDIX

households, one obtains:

\[ \frac{s\pi n}{\sigma} \left( -\hat{L}_{t-1} + \frac{1}{\sigma} \left( \dot{\hat{P}} + \hat{L}_t \right) + \left( 1 - \frac{1}{\sigma} \right) \hat{n}_t \right) + \]
\[ + \frac{kPC}{\sigma} \left( +\dot{\hat{P}} + \hat{L}_t - \hat{\alpha}_t \right) - \frac{npc}{\sigma} \dot{\hat{P}}_t - \frac{s\pi n}{\sigma} \left( -\hat{L}_t + \dot{\hat{P}}_t - \hat{v}_t + \hat{n}_{t+1} \right); \]

(8) FEC : \[ -\hat{v}_t = \left( \frac{1}{\sigma} - 1 \right) \hat{\dot{P}}_{t+1} + \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) \dot{\hat{C}}_{t+1} + \]
\[ + \frac{1}{\sigma} \hat{L}_{t+1} + \left( \frac{1}{1 - \sigma} - \frac{1}{\sigma (1 - \sigma)} \right) \hat{n}_{t+1}, \]

where \( \sigma = sqn \frac{1}{\psi} - s\pi n \frac{1}{\sigma} - kPC \frac{1}{\psi} \frac{1+\psi}{\psi} + npc = k \left[ \left( \frac{1}{\psi} + \frac{\sigma}{\beta} - \frac{1}{\sigma \beta} \right) n - \frac{1+\psi}{\psi} \right] > 0 \)

Plugging the budget constraint in the free entry condition:

(8) FEC : \[ \theta \hat{n}_{t+1} = -\hat{v}_t - \eta \ddot{\hat{P}}_{t+1} + \chi \hat{L}_{t+1} \]
\[ - \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) \frac{k}{\sigma} \left[ -n\hat{n}_{t+2} - \frac{n}{\beta} \hat{L}_t - \hat{\alpha}_{t+1} + n\hat{v}_{t+1} \right], \]

where \( \theta = \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) \frac{k\pi n}{\sigma} \frac{\sigma - 1}{\sigma} > 0 \) and \( \chi = \frac{k}{\omega} \left( \frac{1}{\psi} - \frac{1}{\sigma} \right) + \frac{n}{\beta} \left( n + 1 \right) - \frac{1}{\sigma} > 0, \)

because \( 0 > \frac{1+\sigma (1+n)}{n} + \frac{1-\beta}{\beta} - \left( 1 + \frac{1}{\sigma} \right) \psi \) for \( 0 < \beta \) and finally,

\[ \eta = \left( \frac{1}{\sigma} - \frac{1}{\psi} + \frac{n}{\sigma} \right) \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) \left( \frac{\pi}{\sigma} + \frac{kPC}{\sigma} \frac{1}{n} - \frac{pc - k}{n} \right) = \]
\[ = \frac{1}{\sigma} - \frac{1}{\sigma} + \frac{k}{\sigma} \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) + \frac{nk}{\omega \beta} \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) \left( \frac{1}{\sigma} - \sigma + \beta \right) < 0, \]

because \( \sigma^2 > \frac{n-1}{n} \beta \sigma. \)

Now there is one equation and two (endogenous) unknowns \( n \) and \( P. \) So, a system is needed to work it out. Remember that \( \dot{\hat{C}}_t \simeq \left[ \hat{\alpha}_t + \frac{1}{\sigma - 1} \hat{n}_t \right] \psi. \) Then,

B.C. : \[ \left[ \hat{\alpha}_{t+1} + \frac{1}{\sigma - 1} \hat{n}_{t+1} \right] \psi \simeq \frac{kn}{\beta \sigma} \left( -\hat{L}_t + \frac{1}{\sigma} \left( \dot{\hat{P}}_{t+1} + \hat{L}_{t+1} \right) + \left( 1 - \frac{1}{\sigma} \right) \hat{n}_{t+1} \right) + \]
\[ + \frac{k}{\sigma} \left( +\dot{\hat{P}}_{t+1} + \hat{L}_{t+1} - \hat{\alpha}_{t+1} \right) - \frac{n \sigma k}{\omega \beta} \hat{P}_{t+1} - \frac{kn}{\sigma} \left( -\hat{L}_{t+1} + \dot{\hat{P}}_{t+1} - \hat{v}_{t+1} + \hat{n}_{t+2} \right); \]

FEC : \[ -\hat{v}_t = \left( \frac{1}{\sigma} - 1 \right) \hat{P}_{t+1} + \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) \left[ \hat{\alpha}_{t+1} + \frac{1}{\sigma - 1} \hat{n}_{t+1} \right] \psi + \]
\[ + \frac{1}{\sigma} \hat{L}_{t+1} + \left( \frac{1}{1 - \sigma} - \frac{1}{\sigma (1 - \sigma)} \right) \hat{n}_{t+1}, \]
Rearranging and plugging the free entry condition into the budget constraint (substituting $P$):

\[
\text{B.C. : } \frac{kn}{\omega} \hat{n}_{t+2} \simeq -\frac{kn}{\beta \omega} \hat{L}_t + \frac{k}{\omega} \left[ \frac{n}{\sigma \beta} + 1 \right] \hat{L}_{t+1} + \Omega \hat{P}_{t+1} + \\
+ \frac{kn}{\omega} \left( \hat{L}_{t+1} + \hat{v}_{t+1} \right) - \left[ \psi + \frac{k}{\omega} \right] \hat{\alpha}_{t+1} - \left[ \frac{\psi}{\sigma - 1} - \frac{kn}{\beta \omega} \frac{\sigma - 1}{\sigma} \right] \hat{n}_{t+1}; \\
\text{FEC : } \hat{P}_{t+1} = \frac{\sigma}{\sigma - 1} \hat{v}_t + \psi \frac{\sigma}{\sigma - 1} \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) \hat{\alpha}_{t+1} + \\
+ \frac{\sigma}{(\sigma - 1)^2} \left[ \psi \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) + \frac{1 - \sigma}{\sigma} \right] \hat{n}_{t+1} + \frac{1}{\sigma - 1} \hat{L}_{t+1},
\]

where $\Omega = \frac{nk}{\omega} \left[ \frac{1}{n} + \frac{1}{\sigma \beta} - \frac{\sigma}{\beta} - 1 \right]$. Then:

\[
(7) \text{ B.C. : } \frac{kn}{\omega} \hat{n}_{t+2} \simeq -\frac{kn}{\beta \omega} \hat{L}_t + \left( \frac{k}{\omega} \left[ \frac{n}{\sigma \beta} + 1 \right] + \Omega \hat{P}_{t+1} \right) \hat{L}_{t+1} + \\
+ \Omega \frac{\sigma}{\sigma - 1} \hat{v}_t + \frac{kn}{\omega} \hat{v}_{t+1} + \left( \Omega \psi \frac{\sigma}{\sigma - 1} \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) - \left[ \psi + \frac{k}{\omega} \right] \right) \hat{\alpha}_{t+1} + \\
+ \left( \Omega \frac{\sigma}{(\sigma - 1)^2} \left[ \psi \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) + \frac{1 - \sigma}{\sigma} \right] - \left[ \frac{\psi}{\sigma - 1} - \frac{kn}{\beta \omega} \frac{\sigma - 1}{\sigma} \right] \right) \hat{n}_{t+1}.
\]

Changing names:

\[
\text{B.C. : } \frac{kn}{\omega} \hat{n}_{t+2} \simeq -\frac{kn}{\beta \omega} \hat{L}_t + \gamma \hat{L}_{t+1} + \\
+ \Omega \frac{\sigma}{\sigma - 1} \hat{v}_t + \frac{kn}{\omega} \hat{v}_{t+1} + \vartheta_{\alpha} \hat{\alpha}_{t+1} + \vartheta_n \hat{n}_{t+1},
\]

\[
\vartheta_{\alpha} = \left( \Omega \psi \frac{\sigma}{\sigma - 1} \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) - \left[ \psi + \frac{k}{\omega} \right] \right); \\
\vartheta_n = \left( \Omega \frac{\sigma}{(\sigma - 1)^2} \left[ \psi \left( \frac{1}{\sigma} - \frac{1}{\psi} \right) + \frac{1 - \sigma}{\sigma} \right] - \left[ \frac{\psi}{\sigma - 1} - \frac{kn}{\beta \omega} \frac{\sigma - 1}{\sigma} \right] \right); \\
\gamma = \left( \frac{k}{\omega} \left[ \frac{n}{\sigma \beta} + 1 \right] + \frac{\Omega}{\sigma - 1} + \frac{kn}{\omega} \right).
\]
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CHAPTER 2

A NEW OPEN ECONOMY MACROECONOMIC (NOEM) MODEL WITH ENDOGENOUS PORTFOLIO DIVERSIFICATION AND FIRMS ENTRY

2.1 Introduction

Home bias in international investment is one of the main puzzles in international finance. Investors tend to invest mostly in domestic assets, apparently without taking advantage of the possibilities of international risk diversification. Placed in the border between international macroeconomics and finance, the home bias in portfolio has important implications for economic analysis and policy-making.

Traditional theory, starting from Lucas’ (1982) seminal paper, tends to claim that in a frictionless world with perfectly-mobile factors, portfolio should be allocated following perfect pooling. However, when people talk about home bias in portfolio formation, the crucial issue is: "What is the benchmark for perfect diversification?" It may be that a little proportion of foreign equity is, indeed, efficient.

This paper, drawn on Heathcote and Perri (2005, henceforth H&P,) explores the demand for diversification due to investment fluctuations in a Cole and Obstfeld economy. Indeed, terms of trade (TOT) mechanism to hedge risk works only when there is no possibility of intertemporal transmission of consumption. i.e. the mere existence of some investment kills out the power of TOT in offsetting productivity shocks. It provides insurance only in a "static sense". If households account for expectations of the future they need something else to ensure perfect risk sharing: some portfolio diversification.

I build a new benchmark for the analysis of the international equity puzzle in a tractable New Open Economics Macroeconomy Model. Unlike in H&P, I disentangle the technology of the consumption goods from that of the capital goods. Second, I introduce nominal rigidities in prices and provide robustness for the main result. Third, I introduce dynamics to the markets.

The main features and findings of my analysis are the following:

1. First, I differentiate the cobb-douglas aggregator for the goods used in the creation of
the firms from that of the consumption goods. It is useful to ensure that one is not overlooking the role of the preferences in this model. Indeed, I show that it is the parameter tied to investment demand which deals the optimal bias in portfolio.

2. Second, I explore the interrelation between investment allocation and firms allocation, by adding the extensive margin in the markets. To do so, I assume that the introduction of a new variety requires an initial sunk cost and some time to build-up the plant before starting the production. I find that the allocation of the firms -and so, the number of varieties supplied in the market- is independent of the ownership of their shares and, consequently, that the constant allocation of investment is optimal even with market dynamics.

3. Third, I show that, when assuming a monetary policy which replicates the flexible price equilibrium allocation, the previous result holds independently of the price regime of the economy. I introduce nominal rigidities in prices, first assuming producer currency pricing (PCP) and then local currency pricing (LCP). In both cases, the optimal level of diversification coincides with that of the flexible price regime.\(^1\) The analysis of the general case, which allows for any monetary policy, is a work in progress with Giancarlo Corsetti. When the authorities apply a general monetary policy, the endogenous portfolio arising from the benchmark setup is no longer constant. A generic forward contract with pay-offs tied to the monetary stance and realization of shocks is needed to ensure perfect risk diversification. By adding it, the same constant portfolio is still optimal and, together with the asset, provides perfect risk sharing.

4. Finally, the role of the undiversifiable labour income must not be dismissed. I agree with H&P on its relevance and, like in their paper, the technology parameter (i.e. the labour income share) affects crucially the degree of diversification, with a negative relation. I stick to the complete-market framework\(^2\) and all my goods are tradable.

The optimal proportion of diversification resultant from this theoretical model is compatible with European actual data, for the parameterization usually used in the literature. A bias on capital goods between 70 and 75%, an elasticity of substitution of 5 and a labour income share of 2/3, implies a diversification of the portfolio around 33%-37%. EU-15 portfolio in 2003 was 65% biased towards home equities.\(^3\) Concerning the bias on capital goods,

\(^1\)Like in H&P, I find that a constant, homely-biased portfolio exists for an equilibrium with perfect risk sharing, although my result differs from that of H&P.

\(^2\)I write "complete" in the sense it provides perfect risk sharing, although it is not a complete market offering a full set of arrow-debreu securities.

\(^3\)Source: Bruegel estimates based on OECD and IMF CPIS.
2.2. LITERATURE REVIEW

A large part of the literature agrees on the fact that physical capital is mostly bought or built domestically. It is not difficult to defend this claim: first of all, construction (of the plants and some equipment installation) is almost entirely local and it represents a large proportion of total set-up costs; moreover, equipment trade is tight to costs arising from marketing overseas, the negotiations for foreign purchases, transportation, tariffs and non-tariff barriers, the distribution in foreign markets, adaptations to foreign conditions and standards, installation in foreign production facilities, the need to train foreign workers to use the equipment and the provision of parts, maintenance and customer service from abroad. All these features make capital home bias even greater than that of consumption goods.4

The roadmap for the remainder of the paper is the following: section two provides a small literature review. Section three presents the setup of the model under fully flexible prices. Section four gives us the equilibrium results. Section five introduces nominal rigidities. First, I consider a monetary authority who uses an optimal policy to replicate the flexible prices equilibrium allocation. I analyse it under a producer currency price (PCP) regime and, later, under locally currency price (LCP). Second, I assume a general monetary policy, following the work in progress with Giancarlo Corsetti. The conclusions and plans for future research are in section seven. The appendix contains some algebraic details which are not necessary for the understanding of the text and conclusions of the paper.

2.2 Literature Review

The first question which is worth addressing is whether diversification raises the level of consumer’s welfare. If this is not the case, the lack of diversification of the international portfolio would not be such a puzzle but simply the result of agents’ optimal decisions. Van Wincoop (1999) performed an accurate empirical estimation of the magnitude of these gains and explained the main reason for which past literature disagrees dramatically on the ranges where these benefits fall. He found that welfare gains increase with the level of risk aversion and that they are between 1.1% and 3.5% in a fifty-year horizon and between 2.5-7.9% for a horizon of a hundred years. These are very large values. Thus, as Van Wincoop argued, if potential gains are so significant, the natural question that must be analysed by economic researchers is why financial markets have not achieved more risk sharing. One needs to better understand both why investors do not take diversified positions in existing stock and

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4See Eaton and Kortum (2001) for an empirical study on equipment trade. Notice, however, that the analysis refers only to equipment and disregards construction. In the model I present, one must consider "construction goods" to be aggregated in the composites for consumption and capital. Thus, the correct proportion of capital produced domestically must be, necessarily higher than the levels indicated by Eaton and Kortum.
bond markets, and why markets that allow for trade in broad claims on national income (macro markets) have not yet developed.

In Lucas’ (1982) seminal paper, households optimally split the portfolio half and half to each country. They live in a one-good endowment economy. Baxter and Jermann (1997) went a step forward and introduced production with non-diversifiable labour. They conclude that the international equity puzzle was even worse that what was claimed by Lucas: households should go short in home assets in order to hedge the extra risk generated by the undiversifiable factor.

Economic research has moved in several directions to explain the home biased equities puzzle which still remains an unexplained behaviour. Gehrig (1993), Brennan and Cao (1997) and Martínez-García (2005), for instance, focus on the existence of informational asymmetries as the principal source of the bias, whereas Pesenti and van Wincoop (1996) found empirical evidence against this theory. Another strand of literature quite in line with the latter is that focusing on the costs of diversification as relevant investment allocation barriers.\footnote{See Obstfeld and Rogoff 2000 for a view in favour of this explanation. They claim that trading costs may be relevant if modelled right. See French and Poterba 1991 or Tesar and Werner 1995 for arguments against it.} It is also argued that investment may principally be an issue of control, instead of having the scope of risk sharing. The concentration of the ownership of savings in a relatively small number of individuals may be evidence in favour of this explanation. It seems that it is the familiarity that investors have with a (local) firm rather than the preferences on the aggregated domestic portfolio that makes them bias their savings towards home assets.\footnote{See, for example, Kang and Stulz 1997; and Mankiw and Zeldes 1991 for a discussion on equity holdings concentration on a small number of better-off individuals.} The role of non-tradable goods was a well-known direction of research by the nineties. Tesar (1993) showed that the high correlation between savings and investment, the low cross-country correlation between consumption growth rates and the home bias in investment portfolios are consistent with complete financial markets when agents face stochastic fluctuations in the output of non-traded goods. Consumer preferences over traded and non-traded goods and over the intertemporal allocation of consumption may skew portfolios toward claims on domestic output. Recently, some authors have expressed the conviction that home bias in data is due to a mere error of misspecification (Coeurdacier and Guibaud 2005).

Cole and Obstfeld (1991) depart from the widespread view taken by most of the authors that home bias is the result of market frictions or agents’ unoptimal behaviour. They presented an extreme case where the lack of diversification was efficient. The hedge of risk went via terms of trade movements: any variation in the relative value of home output was compensated by a change in relative prices, keeping nominal intercountry difference of con-
sumption equal to zero. Hence, the effect of country-specific productivity shocks could be perfectly offset through the international transmission. However, they limited the analysis to a labour-economy set-up, missing the potentially relevant role of investment.

H&P and a small bunch of quite recent papers argue that the home bias corresponds to optimal rational agents’ strategies for portfolio diversification. They go one step further and include capital in the model. As Cole and Obstfeld did, they rely on relative international prices adjustment after shocks as the main mechanism to ensure the diversification of risk. Their main finding is that a time-invariant share of investment on home and foreign firms yields perfect risk sharing. Their model is built on Backus, Kehoe and Kydland (1992, 1995), assuming that households only trade shares in domestic and foreign firms. They allow for capital investment dynamics and imperfect substitutability between traded goods.

Finally, the recent paper by Coeurdacier et al. (2007) addresses three main stylised facts on international portfolios and exchange rate in an incomplete markets scenario. The first one of these empirical facts is, precisely, the home equity puzzle. They argue that previous literature fails in accounting only for supply shocks. Indeed, they are the first to introduce two extra types of shocks: redistributive and relative demand shocks, which produce a home biased portfolio in equilibrium. They do so in a two-country two-good world.

2.3 The Model

The world consists of two symmetric countries, denoted by $H$ (home) and $F$ (foreign) and an endogenously determined number of varieties, all of them perfectly tradable. Home (foreign) country is inhabited by a continuum of homogeneous households who sum up to 1 and supply their labour to domestic firms. There is no capital accumulation but only a cost to entry into the market. Firms and agents are homogeneous within countries. However, preferences are symmetrically biased towards domestically-produced goods. The monopolistic firms set prices flexibly, by maximizing profits.

2.3.1 Households

Each country is populated by a continuum of households, whose preferences are defined over the consumption of $n_t + n_t^*$ goods: a composite of home + foreign final produced varieties. The preferences of home households are represented by

$$U_t = E_t \Sigma_{t=0}^{\infty} \beta^t [ \ln C_t - \kappa \ell_t (j) ],$$

(2.1)

where $0 < \beta < 1$ is the discount factor and $U(\cdot)$ is a utility function defined over the consumption of a basket $C_t$ and a linear disutility of labour effort, $\kappa \ell_t (j)$. The consumption
2.3. THE MODEL

basket is given by a Cobb-Douglas aggregator over the bundles of tradables produced in the home \( C_H \) and foreign \( C_F \) country (i.e. a CES basket with unit elasticity),

\[
C_t = C_H^\gamma C_F^{1-\gamma},
\]

where \( \gamma < 1 \). \( C_H \) and \( C_F \) are CES aggregators over the \( n \) \( (n^*) \) varieties produced in the home(foreign) country. For simplicity, I assume identical elasticity of substitution, \( \sigma \).

\[
C_H = \left( \int_{h=0}^{n_H} c_t(h)^{1-\frac{1}{\sigma}} dh \right)^{\frac{\sigma}{\sigma-1}};
\]

\[
C_F = \left( \int_{f=0}^{n_F} c_t(f)^{1-\frac{1}{\sigma}} df \right)^{\frac{\sigma}{\sigma-1}}.
\]

Here, \( h \) and \( f \) denote a specific variety of the corresponding country. Households all over the world finance the creation of firms in both countries. In order to construct her portfolio of investment, the home household purchases a fraction \( \lambda_{H,t+1} \), of the shares issued by foreign-country firms and \( \lambda_{H,t+1} \) of the domestic firms, which will start producing next period. She affords her consumption expenditure and investment with the dividends received from currently active firms at home and abroad, proportionally to her current portfolio allocation: \( \lambda_{H,t} \), \( \lambda_{F,t} \) and her labour income. The budget constraint is

\[
B_{t+1} + \lambda_{H,t+1} \int q_t(h) dh + e_t \lambda_{F,t+1} \int q_t^*(f) df + \int p_t(h) c_t(h) dh + \int p_t(f) c_t(f) df = \lambda_{H,t} \int \pi_t(h) dh + e_t \lambda_{F,t} \int \pi_t^*(f) df + w_t \ell_t(j) + (1 + i_t) B_t
\]

where labour supply \( (\ell_t(j)) \) is elastic, \( \kappa \) being the linear disutility for the effort of working; \( \pi_t(h) \) are the profits of firm \( h \). An initial investment is needed for a new firm to start producing. \( q_t(h) \) \( (q_t^*(f)) \) is the cost necessary for the creation of a firm at home (foreign). \( \pi_t(h) \) \( (\pi_t^*(f)) \) are the profits of a single home (foreign) firm in home (foreign) currency; \( e_t \) is the nominal exchange rate \( (p_t(h) = e_t P_t^*(h)) \), \( c_t(h) \) the domestic demand for good \( h \), \( n_t \) is the number of firms allocated at home and \( w_t \) is the wage. \( B_t \) is the international riskless bond. Finally, \( \gamma \) indicates the home-bias on consumption preferences. The super script *, \( x^* \), stands for the foreign country.
2.3. THE MODEL

2.3.2 Firms

A continuum of \( n(n^*) \) tradable goods firms in the home (foreign) country act in a monopolistically competitive economy. All of them sell their products in both home and foreign markets. A sunk cost is paid at time \( t \) to develop a new variety, which will enter the market at \( t+1 \) and disappear at the end of that period (full amortization.) This cost is financed by issuing equities in the international stock market, i.e. both home and foreign agents have access to shares of any firm created all over the world.

**Creation of new firms**

To produce a new home variety at time \( t+1 \), entrepreneurs must incur a startup cost of \( q_t(h) = P_{k,t} K_t \) today. Firms are fully depreciated after one year of production. \( K_t \) is a composite good containing both home and foreign varieties following a Cobb-Douglas aggregator, the size of which is randomly determined every period,

\[
K_t = K_{H,t}^{\delta} K_{F,t}^{(1-\delta)},
\]

where \( K_{H,t} \) and \( K_{F,t} \) are the baskets of home and foreign final goods used in capital. The lower the \( K_t \) \((K_t^*)\) the more efficient home (foreign) country in the creation of new firms or varieties. \( P_{k,t} \) is the CPI for the basket \( K_t \).\(^7\) Finally, \( \delta \) indicates the bias in the preferences of capital goods. And,

\[
K_{H,t} = \left( \int_{h=0}^{n_t} k_t(h)^{1-\frac{1}{\sigma}} dh \right)^{\frac{1}{1-\sigma}}; \quad K_{F,t} = \left( \int_{f=0}^{n^*_t} k_t(f)^{1-\frac{1}{\sigma}} df \right)^{\frac{1}{\sigma-1}}. \tag{2.6}
\]

with * on all the K and k for the foreign country.\(^8\) Hence, total investment at home is

\[
I_{H,t} = n_{t+1} q_t(h) = n_{t+1} P_{K,t} K_t
\]

**Production**

Once created, firms produce a differentiated variety with an homogeneous technology which requires only labour:

\[
Y_t(h) = A_{H,t} \ell_t(h)^\theta.
\]

The state of the economy is

\[
\{A_{H,t}, A_{F,t}\}. \tag{2.7}
\]

\(^7\)One may easily have different CES aggregators and/or an extra parameter of productivity for \( K \) in the model (e.g. of the type \( K_{i,t} = A_{K_{i,t}} \left( \int_{h=0}^{n_t} k_t(h)^{1-\frac{i}{\sigma}} dh \right)^{\frac{1}{\sigma-1}} \), where \( i = H, F \) and \( \theta \) stands for the elasticity of substitution between capital goods, which may differ from \( \sigma \), the elasticity between consumption goods). However, the set-up presented in the paper disregards this alternative to concentrate only on the scope explained in the introduction. In this case, the closed-economy version would have \( P = P_K \), since the unique differentiation between \( C \) and \( K \) would be, by assumption, the Cobb-Douglas parameter \((\delta \neq \gamma)\).

\(^8\)See the appendix for details.
2.4. EQUILIBRIUM

\( \theta \) is, indeed, the share of output going to labour. The \((1 - \theta)\), which belongs to capital, is distributed among investors via dividends. \( Y_t(h) \) is the production of one firm, and \( k_t(h) \) is the demand of the final good \( h \) by new entrants to build up their plants. \( p_t(h) \) is the price of variety \( h \) which is flexibly set by the monopolistic firm and \( \ell_t(h) \) is labour demand for good \( h \).

2.4 Equilibrium

2.4.1 The Household’s Problem

Households maximize utility subject to the budget constraint. The first-order conditions are:

\[
\frac{\kappa}{w_t} = \xi_t = \frac{1}{P_t C_t} \rightarrow w_t = \kappa P_t C_t; \tag{2.8}
\]

\[
C_{H,t} = \gamma \frac{P_t C_t}{P_{H,t}}; \quad C_{F,t} = (1 - \gamma) \frac{P_t C_t}{P_{F,t}}; \tag{2.9}
\]

\[
c_t(h) = C_{H,t} \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\sigma}; \quad c_t(f) = C_{F,t} \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\sigma}; \tag{2.10}
\]

\[
B_{t+1}: \quad \frac{1}{P_t C_t} = \beta (1 + \i_t) E_t \frac{1}{P_{t+1} C_{t+1}}; \tag{2.11}
\]

\[
\lambda_{H,t+1}: \quad q_{H,t} = E_t Q_{t,t+1} \Pi_{H,t+1}; \tag{2.12}
\]

\[
\lambda_{F,t+1}: \quad e_t q_{F,t}^* = E_t Q_{t,t+1}^* e_{t+1} \Pi_{F,t+1}; \tag{2.13}
\]

where \( Q_{t,t+1} \) is the discount factor of future dividends and \( q_{H,t} (q_{F,t}^* \) is the country aggregate of \( q_t(h) (q_t^* (f)) \). Equation (2.8) is the endogenous supply of hours of labour; (2.9) shows the allocation of the consumption expenditure among home and foreign-produced goods which is constant due to the Cobb-Douglas assumption; (2.12) and (2.13) provide us with the free entry conditions for new firms. Firms will enter the market whilst the initial fixed cost is lower or equal to the expected profits. \( \Pi_{H,t} \) are the aggregate profits of all domestic firms. Finally, (2.11) is the usual Euler equation, the intertemporal rate of substitution between the consumption in period \( t \) and \( t + 1 \). The welfare-based price index is

\[
P_t = \frac{P_{H,t}^{\gamma} P_{F,t}^{1-\gamma}}{\Gamma}, \tag{2.14}
\]

where \( \Gamma = \gamma (1 - \gamma)^{1-\gamma} \). And,

\[
Q_{t,t+1} = \frac{1}{1 + \i_t} = \beta E_t \left( \frac{P_t C_t}{P_{t+1} C_{t+1}} \right) = \beta E_t \left( \frac{\mu_t}{\mu_{t+1}} \right),
\]
is the intertemporal rate of substitution between the consumption in period \( t \) and \( t + 1 \). Foreign households solve an analogous problem with symmetric preferences, i.e. they prefer the foreign-produced goods, \( f \), as much as home households prefer home-produced ones, \( h \).^9

### 2.4.2 The Firm’s Problem

**Creation of new varieties:**

During the creation of the variety, home firms choose the demand of each capital good, \( k_t(h) \) and \( k_t(f) \), by solving the following minimization problems:

\[
\min_{k_t(h)} \int_0^{n_t} p_t(h)k_t(h)dh - \zeta_t \left( \left( \int_0^{n_t} k_t(h)^{1-\frac{1}{\delta}} dh \right)^{\frac{\sigma}{\sigma-1}} - K_{H,t} \right)
\]

The first-order condition is:

\[
k_t(h) = \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} K_{H,t}, \tag{2.15}
\]

and

\[
\min_{k_t(f)} \int_0^{n_t} p_t(f)k_t(f)df - \zeta_t^* \left( \left( \int_0^{n_t} k_t(f)^{1-\frac{1}{\delta}} df \right)^{\frac{\sigma}{\sigma-1}} - K_{F,t} \right)
\]

thus,

\[
k_t(f) = \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} K_{F,t},
\]

where the shadow price, \( \zeta_t = P_{H,t} = n_t^{\frac{1}{\delta}} p_t(h) \) and \( \zeta_t^* = P_{F,t} \). The optimal baskets of home and foreign capital are

\[
K_{H,t} = \delta \frac{P_{K,t}K_t}{P_{H,t}} \; ; \; K_{F,t} = (1 - \delta) \frac{P_{K,t}K_t}{P_{F,t}}.
\]

Firm \( h \) today has a demand of variety \( h \), to be used in building firms, of \( n_{t+1}k_t(h) \). \( \sigma > 1 \), is the intratemporal elasticity of substitution between goods and the price indexes for capital are,

\[
P_{K,t} = \frac{(P_{H,t})^{\delta} (P_{F,t})^{1-\delta}}{\Gamma_\delta} ; \; P_{K,t}^* = \frac{(P_{H,t}^*)^{1-\delta} (P_{F,t}^*)^{\delta}}{\Gamma_\delta},
\]

where \( \Gamma_\delta = \delta^\delta (1 - \delta)^{1-\delta} \).^10

**Cost Minimization and Optimal Prices:**

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^9See the appendix for details.

^10The condition for stability requires that \( 1 > \theta^{\frac{\sigma-1}{\sigma}} \). See appendix for details.
2.4. EQUILIBRIUM

Firms choose the amount of labour which minimizes costs,

$$\min w_t \ell_t (h),$$

subject to the technology constraint. Thus, the first order condition is,

$$\phi_t = \frac{w_t}{\theta A_{H,t}} \ell_t (h)^{1-\theta} = \text{mg cost},$$

where $\phi_t$ is the lagrange multiplier. Once operative, firms maximize profits:\(^{11}\)

$$\max_{p_t(h)} p_t(h) Y_t(h) - w_t \ell_t (h),$$

subject to the technology restriction and demand. Thus, the optimal price is

$$p_t(h) = \frac{\sigma}{\sigma - 1} \frac{1}{A_{H,t}^\theta} Y_t(h)^{\frac{1}{\theta} - 1}.$$  

Prices consist of a constant mark-up over the expression of marginal costs which depends crucially on the level of production, due to the non-linear technology.

2.4.3 Markets Clearing

The clearing conditions for the domestic and foreign goods markets are:

$$c_t (h) + c^*_t (h) + n_{t+1} k_t (h) + n_{t+1}^* k^*_t (h) = Y_t (h);$$  

$$c_t (f) + c^*_t (f) + n_{t+1} k_t (f) + n_{t+1}^* k^*_t (f) = Y_t (f).$$  

A firm satisfies four sources of demand: those of the home and the foreign households and those of the firms which will produce next year in the home and foreign country.

The labour market is emptied when:

$$n_t \ell_t (h) = \ell_t (j);$$  

$$n_t^* \ell_t^* (f) = \ell_t^* (j^*).$$

Finally, the financial markets in equilibrium must fulfill:

$$B_t = -B_t^*;$$

$$\lambda_{H,t} = 1 - \lambda_{H,t}^*;$$

$$\lambda_{F,t} = 1 - \lambda_{F,t}^*.$$  

\(^{11}\) See appendix for details.
2.4. EQUILIBRIUM

Under this non-linear technology, one can write home aggregate profits as a constant fraction of total revenue, although this fraction is different from that found under constant returns to scale (with linear technology $\Pi^\text{CRS}_H = \frac{1}{\sigma} P_H Y_H < \Pi^\text{DRS}_H$). This depends both on the elasticity of substitution and the technological parameter. Hence,

$$\Pi_{H,t} = P_{H,t} Y_{H,t} \left(1 - \frac{\sigma - 1}{\sigma} \theta\right) = \left(\frac{\sigma (1 - \theta) + \theta}{\sigma}\right) P_{H,t} Y_{H,t}. \tag{2.24}$$

Notice that $\frac{\sigma (1 - \theta) + \theta}{\sigma} - \frac{1}{\sigma} > 0$. The amount of profits over total income is higher due to the diminishing returns to scale in the technology. One can also write the labour cost as a fraction of the output of the firms:

$$w_t \ell_t (h) = \frac{\sigma - 1}{\sigma} \theta p_t (h) Y_t (h). \tag{2.25}$$

2.4.4 Solving the optimal diversification level, $\lambda$

Let’s conjecture that an equilibrium allocation exists with $B = 0$ and constant portfolio demand $\lambda^*_t = \lambda_{F,t} = \lambda$ for symmetric countries ($\ell_t = \ell^*_t = 1$) such that:

$$P_tC_t = e_t P^*_t C^*_t, \tag{2.26}$$

i.e. where households get perfect risk sharing. So that $Q_t = e_t Q^*_t$, stochastic discount rates are the same across countries. Hereafter it is shown that this is, indeed, an equilibrium allocation, by characterizing the associated vector of equilibrium prices, and verifying that prices and quantities satisfy households’ first-order conditions, market clearing conditions and the resource constraints.

Let’s define the following relative variables in nominal terms:

$$\begin{align*}
\triangle C &= P_tC_t - e_t P^*_t C^*_t, \tag{2.27} \\
\triangle k &= n_{t+1} P_{k,t} K_t - e_t n^*_t P^*_t K^*_t = I_{H,t} - e_t I^*_t \\
\triangle Y &= P_{H,t} Y_{H,t} - e_t P^*_{F,t} Y^*_{F,t}
\end{align*}$$

These are the intercountry differences in consumption, investment and output in nominal

\footnote{See appendix for profits aggregation.}
2.4. EQUILIBRIUM

terms. Moreover, from goods market clearing conditions,

Home Output
\[ P_{H,t} Y_{H,t} = P_{H,t} \gamma P_t C_t + e_t P_{H,t}^* (1 - \gamma) \frac{P_t^* C_t^*}{P_{H,t}} + n_{t+1} P_{H,t} \frac{\delta P_{k,t} K_t}{P_{H,t}} + n_{t+1} e_t P_{H,t}^* (1 - \delta) \frac{P_{k,t}^* K_t^*}{P_{H,t}^*}. \]

Foreign Output
\[ P_{F,t} Y_{F,t} = \gamma P_t^* C_t^* + \frac{P_t^* C_t^*}{e_t} (1 - \gamma) + n_{t+1} P_{H,t} \frac{\delta P_{k,t} K_t^*}{P_{H,t}} + n_{t+1} P_{H,t}^* \frac{\delta P_{k,t}^* K_t^*}{P_{H,t}^*}. \]

By taking differences, I have an expression for the output absorption in the economy: \[ \Delta \Upsilon = (2\gamma - 1) \Delta C + (2\delta - 1) \Delta k. \tag{2.28} \]

The difference in nominal output is due to the differences in consumption and in investment. The size of each of them in \( \Delta \Upsilon \) depends on the corresponding parameter of the Cobb-Douglas aggregator in C or K, \( \delta \) or \( \gamma \). Hence, in the conjectured equilibrium,

\[ \Delta \Upsilon |_{\Delta C = 0} = (2\delta - 1) \Delta k. \tag{2.29} \]

Let’s take the home and foreign households’ aggregate budget constraints

\[ P_t C_t = w_t \ell_t L_t + (1 - \lambda) n_t \pi_t (h) + \lambda e_t m_t^* \pi_t^* (f) - (1 - \lambda) I_{H,t} - e_t \lambda I_{F,t}^* \]

\[ P_t^* C_t^* = w_t^* \ell_t^* L_t^* + \frac{1}{e_t} \lambda \Pi_{H,t} + (1 - \lambda) \Pi_{F,t}^* - \frac{1}{e_t} \lambda I_{H,t} - (1 - \lambda) I_{F,t}^* \]

and substitute the expressions for profits and labour income as a function of GDP (eq. 2.24 and 2.25).

\[ P_t C_t = \frac{\sigma - 1}{\sigma} \theta P_{H,t} Y_{H,t} + (1 - \lambda) \left(1 - \frac{\sigma - 1}{\sigma} \theta\right) P_{H,t} Y_{H,t} + \lambda e_t \left(1 - \frac{\sigma - 1}{\sigma} \theta\right) P_{F,t}^* Y_{F,t} - (1 - \lambda) I_{H,t} - \lambda e_t I_{F,t}^* + (1 + i_t) B_t - B_{t+1} \]

\[ \text{This equation is the equivalent of number 27 in H&P.} \]
and

\[ P^*_t C^*_t = \frac{\sigma - 1}{\sigma} \theta P^*_{F,t} Y_{F,t} + \lambda \frac{1}{\epsilon_t} \left( 1 - \frac{\sigma - 1}{\sigma} \theta \right) P_{H,t} Y_{H,t} + \]

\[ + (1 - \lambda) \left( 1 - \frac{\sigma - 1}{\sigma} \theta \right) P^*_{F,t} Y_{F,t} - \lambda \frac{1}{\epsilon_t} I_{H,t} - (1 - \lambda) I^*_t \]

\[ + (1 + i^*_t) B^*_t - B^*_{t+1}, \]

where \( P_{H,t} Y_{H,t} \) is the nominal domestic output (\( Y \)) and \( P^*_{F,t} Y_{F,t} \) the foreign output (\( Y^* \)) and \( I \) is the investment of the current period. By imposing \( P_t C_t - \epsilon_t P^*_t C^*_t = 0, \)

\[ \Delta C = \Delta Y \left[ \frac{\sigma - 1}{\sigma} \theta + (1 - 2\lambda) \left( 1 - \frac{\sigma - 1}{\sigma} \theta \right) \right] - \Delta k (1 - 2\lambda) \]

\[ + ((1 + i_t) B_t - B_{t+1}) - \epsilon_t \left( 1 + i^*_t \right) B^*_t - B^*_{t+1} \right). \]

Plugging equation (2.29) and setting the gross and net holding of bonds identically equal to zero, \( B = 0, \)

\[ \Delta C = \left( 1 + 2\lambda \left( \frac{\sigma - 1}{\sigma} \theta - 1 \right) \right) (2\delta - 1) \Delta k - \Delta k (1 - 2\lambda). \quad (2.30) \]

### 2.4.5 Derivation of the Terms of Trade

Terms of trade are defined as the price of a country’s exports in terms of their imports, i.e. \( TOT = \frac{P_{H,t}}{P_{F,t}}. \) One can derive TOT from the resource constraints. In the case of symmetric countries I have assumed, \( L_t = L^*_t = 1, \)

\[ C_{H,t}^* + C_{H,t} + n_{t+1} K_{H,t} + n_{t+1} K^*_{H,t} = Y_{H,t}; \]

\[ C_{F,t}^* + C_{F,t} + n_{t+1} K_{F,t} + n_{t+1} K^*_{F,t} = Y_{F,t}. \]

Taking the ratio of the two equations on \( \frac{P C_t}{P_{F,t}} \) and \( \frac{P^*_t C^*_t}{P^*_{H,t}}, \)

\[ TOT_{H,t} = \frac{Y_{F,t} - n_{t+1} K^*_{F,t} - n_{t+1} K_{F,t}}{e_t Y_{H,t} - n_{t+1} K^*_{H,t} - n_{t+1} K_{H,t}}. \]

---

14 This equation is the equivalent of 32 in H&P.

15 See appendix for computations in detail.

16 If I allow for \( L_t \neq L^*_t, \)

\[ L_t C_{H,t} + L_t^* C_{H,t}^* + n_{t+1} K_{H,t} + n_{t+1} K^*_{H,t} = Y_{H,t}; \]

\[ L_t C_{F,t} + L_t^* C_{F,t}^* + n_{t+1} K_{F,t} + n_{t+1} K^*_{F,t} = Y_{F,t}. \]

Then,

\[ TOT_{H,t} = \frac{1}{e_t} \left[ \gamma L_t + (1 - \gamma) L_t^* \right] \left[ Y_{F,t} - n_{t+1} K^*_{F,t} - n_{t+1} K_{F,t} \right] \]

\[ - e_t \left[ (1 - \gamma) L_t + \gamma L_t^* \right] \left[ Y_{H,t} - n_{t+1} K_{H,t} - n_{t+1} K^*_{H,t} \right]. \]
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The terms of trade depend on the relative supply of output net of investment. Given investment, the international transmission is positive: an increase in net home output benefits foreign households by lowering the home output prices. At the same time, a positive productivity shock at Home raises investment.

Let \( \Gamma = \gamma (1 - \gamma)^{1-\gamma} \). Hence, the price indexes can be rewritten as

\[
P_t = \frac{P_{H,t}}{\Gamma} \left( \frac{Y_{H,t} - n_{t+1}K_{H,t} - n_{t+1}K_{H,t}^*}{Y_{F,t} - n_{t+1}K_{F,t} - n_{t+1}K_{F,t}^*} \right)^{1-\gamma};
\]

\[
P_t^* = \frac{P_{H,t}}{e_t \Gamma} \left( \frac{Y_{H,t} - n_{t+1}K_{H,t} - n_{t+1}K_{H,t}^*}{Y_{F,t} - n_{t+1}K_{F,t} - n_{t+1}K_{F,t}^*} \right)^{\gamma}.
\]

For \( \gamma = 1/2 \), households’ preferences are identical, the real exchange rate \( P/P^* \) is identically equal to 1. In other words, purchasing power parity holds.\(^{17}\) For \( \gamma \neq 1/2 \), instead, home bias in consumption implies that the real exchange rate is not constant, but moves with the terms of trade:

\[
RER_t = \frac{P_t}{e_t P_t^*} = \left( \frac{P_{F,t}}{P_{H,t}} \right)^{1-2\gamma}.
\]

With perfect risk sharing, it follows that the ratio between consumption levels is also equal to the RER:

\[
\frac{C_t^*}{C_t} = \left( \frac{P_{F,t}}{P_{H,t}} \right)^{1-2\gamma}.
\]

2.4.6 Transmission Mechanism

Equation (2.30), which I recall below, is the key equation yielding the explanations for investors’ behaviour.

\[
\Delta C = \underbrace{(2\delta - 1)}_{\text{indirect effect}} \underbrace{\left(1 - 2\lambda \left(1 - \frac{\sigma - 1}{\sigma} \bar{\theta} \right)\right)}_{\text{direct effect}} \Delta k - (1 - 2\lambda) \Delta k.
\]

To make the mechanism clear, let’s assume that a fully anticipated shock consistent in a rise in relative investment occurs\(^{18}\) (i.e. \( \Delta I_{H,t+1} \) whereas \( I^*_{F,t+1} \) keeps constant.) First, consider an environment where the basket of capital goods is biased towards domestic varieties (\( \delta > \frac{1}{2} \)). In this case, \( \lambda < \frac{1}{2} \).

\(^{17}\)Cole and Obstfeld case.

\(^{18}\)A typical example of \( \Delta I_H \) is the expectancy of a future increase in home productivity, so that agents want to create more firms to take advantage of such improvement.
In brief, one can say that the $\Delta I_{H,t+1}$ causes a quantity and a valuation effect and these disturb perfect risk sharing. The shock generates an increment in home households’ wealth whereas foreign wealth decreases. This is reflected on the valuation of home output via the increase in prices.

I may split the overall impact into two simultaneous effects which move in opposite directions. On the one hand, $\Delta k$ has a negative direct effect on $\Delta C$. The relative demand of home goods increases because they are used to satisfy the extra investment. Although part of this cost is financed by foreigners through ownership ($\frac{1}{2} > \lambda > 0$), the home household is forced to reduce its relative consumption. This impact on $\Delta C$ helps to regulate the financial flows and thus avoids disturbing perfect risk sharing.

Notice that when $\lambda = 0$ (no diversification), the term $- (1 - 2\lambda)$ equals $-1$. So, an increment of one euro in domestic investment directly implies a one euro reduction in domestic consumption because it generates a one unit decline in dividends received by home households. By contrast, if $\lambda = 1$ (home households only own foreign assets) the direct term equals $1$. Thus, an extra euro of home investment generates a reduction of one euro in the foreign consumption because it is the foreign households who finance the whole cost of it.

On the contrary, the indirect effect, the impact of $\Delta \Psi$ on $\Delta C$, is positive. This can also be separated into two parts. The first, $(2\delta - 1)$, captures the extent to which an increase in domestic absorption (in this case, investment) increases the relative value of home output. The second part, $(1 - 2\lambda (1 - \frac{\sigma-1}{\sigma}\theta))$, reflects the impact of a change in relative output on relative consumption. It shows the fact that an increment in relative demand for home goods has a positive effect on the terms of trade for the domestic economy. This effect is negatively related to $\lambda$ and positively to $\theta$. This is the case because, the larger the non-diversifiable labour’s share, the larger the impact of an improvement in the domestic economy’s terms of trade on relative consumption, given $\lambda$. Similarly, the smaller the diversification level $\lambda$, the larger the impact of a variation in relative prices on $\Delta C$.

To sum up, when the shock is anticipated, home output has a higher relative value due to the increment of the demand. In consequence, the distributed dividend, which belongs partly to foreign households, is larger. The increase in the output demand pushes the quantity of labour up and so the total labour income increases, making households become richer.

Indeed, the magnitude of this general equilibrium effect is greater than the magnitude of the direct effect when $\lambda$ (the proportion of foreign assets) is inefficiently high and vice versa.

In order to compensate a situation like this and re-establish perfect risk sharing, $\lambda$

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19 This is due to the fact that most of the revenue goes directly to labour, via wages. Real wages are affected by the changes on relative prices. On the contrary, when a large part of the household’s income comes from dividends, TOT loses its capacity of offsetting the impact of the shocks.
must increase. Therefore, a larger proportion of dividends is redistributed to the foreign households in order to get a smooth consumption. In this way home households pass part of their wealth to the other country and, simultaneously, reduce the demand effect. The latter occurs because, although they are importing more, these imports are partly financed by giving extra ownership to the foreigners.

In the case in which capital goods are mostly composed by foreign varieties ($\delta < \frac{1}{2} \Rightarrow \delta > \frac{1}{2}$), all the effects act in the opposite direction. The direct effect is positive, whereas the indirect effect becomes negative. This change is reasonable since the demand generated by the extra investment, now, must be mostly covered by foreign goods and so foreign output increases its value with respect to home production.

This result yields a basic conclusion: diversification is not a tool to redistribute the purchasing power, but to control the excess of demand.20 And it is the existence of investment which makes diversification necessary, because terms of trade are not able to neutralize the consequences of the shocks.

By setting $\Delta C = 0$ I solve for $\lambda$,

$$\lambda = \frac{1 - \delta}{1 + (2\delta - 1) \left( \frac{\sigma}{\sigma + \theta} - 1 \right)}.$$  

(2.35)

Equation (2.35) is the equilibrium value for $\lambda$, i.e. the diversification level for which the direct and indirect effects of a shock disturbing relative consumption (for instance, a shock in investment) are exactly offset.

Households allocate a positive part of their portfolios on foreign assets, $0 \leq \lambda \leq 1$. Notice that it is not the parameter from the preferences on consumption ($\gamma$) which plays a role in the diversification, but the parameter of the preferences on capital goods ($\delta$). Hence, it is important to disentangle these two, allowing them to be different. This is not done in H&P. The larger the home bias in the preferences for capital goods, the less they diversify. $\lambda$ decreases with $\delta$ and is kept above $\frac{1}{2}$ for $\delta < \frac{1}{2}$ and below $\frac{1}{2}$ for $\delta > \frac{1}{2}$. Thus, as H&P did, I find a portfolio biased towards home assets as the optimal allocation for households to reach perfect risk sharing. A larger trade share (smaller $\delta$) in capital goods implies a weaker terms of trade response to changes in relative final demand. So, for any given diversification level, the indirect effect of demand changes on relative consumption that works through prices is going to be smaller. Moreover, whilst $\delta \geq \frac{1}{2}$, $\lambda$ decreases with labour income share. This is

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20 Notice that $\delta$ and $\lambda$ appear multiplied by 2. When investment at home goes up, home country increases both the demand for domestic goods (by $\delta$) and for foreign goods (by $1 - \delta$) and $e_tP_{K,t}K_{t}^{*} = (1 - \delta) P_{K,t}K_{t}^{*} = (1 - \delta) P_{H,t}K_{H,t}^{*} = \delta P_{K,t}K_{t}$. Thus, $\Delta$ includes the term $(2\delta - 1)$. By the same token, $\Delta C = \sum (1 - \lambda) P_{K,t}K_{t} - (-\lambda) P_{K,t}K_{t} = (2\lambda - 1) P_{K,t}K_{t}$. Arespa Castelló, Marta (2008), International Transmission, Firm Entry and Risk Sharing European University Institute

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because when \( \theta \) is high terms of trade does most of the job in equalizing consumptions. A smaller diversification is needed to match perfect risk sharing.

In the extreme case of \( \delta \to 1 \), i.e. the country uses only domestically produced goods as capital in the creation of new firms, households do not diversify at all, \( \lambda \to 0 \). This shows, again, that the home bias on consumption preferences is not relevant for diversification, but also that the size of the labour income share alone, without the presence of some bias in demand (here in capital goods), is not important either. This is easy to understand: when home agents use only their own goods to create firms, the first term of the indirect effect, the one explaining the impact of relative output on relative consumption, is zero. There is no valuation of home output because flexible prices react one to one to the excess of demand, compensating the shock and ensuring perfect risk sharing. Thus, it agrees with Cole and Obstfeld’s result. Finally, when the bundle of capital goods is equally divided between home and foreign varieties, households need perfect pooling (i.e. they perfectly divide their portfolios between home and foreign equities) in order to get perfect risk sharing, as in Heathcote and Perri’s paper.

In H&P, the share of diversification of the portfolio is

\[
[1 - \lambda^{HP}] = \left[\frac{1 - \omega}{1 + \theta - 2\omega \theta}\right]_{\omega = \frac{1}{2}} = \frac{1}{2},
\]

where \( \theta \) is the capital income share, \( \omega \) is the parameter of the Cobb-Douglas aggregator in consumption (i.e. the indicator of the bias in consumption) and \( 1 - \lambda^{HP} \) is the level of diversification in the portfolio (i.e. the equivalent of \( \lambda \) in this paper.)

This happens because, when the demand on capital goods is equally allocated on home and foreign goods, any increase of either home or foreign investment pushes the demand for domestic and foreign varieties in the same proportion, keeping terms of trade invariable (the indirect effect is zero). So, if agents rely on perfect pooling, they share the weight of the financing whichever country is affected by the shock.

The model provides an example of complementarity between terms of trade movements and income transfers via asset holdings in insuring against consumption risk from productivity fluctuations.

Relative price movements already provide some consumption risk insurance, but this is not perfect. The reason lays on the fact that trade flows among countries move terms of trade in response, not only to consumption but also, to investment needs. These needs are possibly driven by expectations of future returns to capital. Portfolio diversification provides a way to insulate terms of trade from the components of demand due to investment. Hence,
income flows from assets cover the demand for local inputs by foreign firms: the higher the proportion of investment which is local, the lower the need to diversify.

2.4.7 Allocation of firms

The free entry conditions (FECs) provide us with a system of two difference equations to solve for \( n \) and \( n^* \). At Home, the FEC is,

\[
P_{K,t}K_t = P_{H,t}K_{H,t} + P_{F,t}K_{F,t} = E_tQ_{t,t+1}\pi_{t+1}(h).
\]

After some algebra,\(^{21}\) one finds a system of two non-linear differentiated equations on \( n \) and \( n^* \),

\[
n_{t+1}^* \left[ K_{H,t} \frac{Y_t(h) \frac{1}{h-1}}{\theta A_{H,t}} \left( 1 - \beta \left( 1 - \frac{1 - \sigma - 1}{\sigma} \right) \right) + K_{F,t} \frac{Y_t(f) \frac{1}{f-1}}{\theta A_{F,t}} \right] = \beta \left( 1 - \frac{1 - \sigma - 1}{\sigma} \right) \frac{\theta \sigma - 1}{\sigma K}
\]

\[
E_t \left[ L_{t+1} + L_{t+1}^* (1 - \gamma) \frac{A_{H,t+1}}{A_{H,t+1}} \right] \frac{Y_{t+1}(h)}{Y_{t+1}(f)} \frac{1}{\theta A_{H,t+1}^*} n_{t+1}^* + \frac{1}{\theta \sigma - 1} \frac{Y_{t+1}(f) \frac{1}{f-1}}{\theta A_{F,t+1}^*} n_{t+1}^* K_{H,t+1}
\]

and, symmetrically,

\[
n_{t+1} \left[ K_{F,t} \frac{Y_t(f) \frac{1}{f-1}}{\theta A_{F,t}} \left( 1 - \beta \left( 1 - \frac{1 - \sigma - 1}{\sigma} \right) \right) + K_{H,t} \frac{Y_t(h) \frac{1}{h-1}}{\theta A_{H,t}} \right] = \beta \left( 1 - \frac{1 - \sigma - 1}{\sigma} \right) \frac{\theta \sigma - 1}{\sigma K}
\]

\[
E_t \left[ L_{t+1}^* + L_{t+1} (1 - \gamma) \frac{A_{F,t+1}}{A_{F,t+1}} \right] \frac{Y_{t+1}(f)}{Y_{t+1}(h)} \frac{1}{\theta A_{F,t+1}^*} n_{t+1} + \frac{1}{\theta \sigma - 1} \frac{Y_{t+1}(h) \frac{1}{h-1}}{\theta A_{H,t+1}^*} n_{t+1} K_{F,t+1}
\]

Although an analytical solution for \( n \) and \( n^* \) cannot be provided, it is worth noticing that the expressions above do not depend on \( \lambda \) at all. Hence, the decision on the allocation of plants of production is completely disconnected from the decision on the ownership of firms made by agents in the home and foreign country. So, it follows that the dynamics of markets do not invalidate H&P result, found in a perfectly competitive world.

2.4.8 The labour demand in equilibrium.

The f.o.c. for home firms was

\[
\ell_t(h) = \left[ \frac{w_t \frac{1}{\theta \phi_t}}{A_{H,t} \frac{1}{\theta}} \right]^{\frac{1}{\theta}}.
\]

\(^{21}\) See appendix for the computations.
I use the technology restriction to get the lagrangian multiplier, \( \phi_t = p_t(h) \frac{\sigma - 1}{\sigma} \). So that,

\[
\ell_t(h) = \left[ \frac{\sigma - 1}{\sigma} \frac{A_{H,t}}{w_t} p_t(h) \right]^{\frac{1}{1-\theta}}.
\]

Households supply an elastic amount of labour. It increases with the increment of the returns to scale of their effort, i.e. the higher \( \theta \) is, the more productive labour is and the more they are willing to work. Labour supply goes up for higher levels of \( A_{H,t} \) -the productivity of technology- and for higher prices, since, in this case, they need more income to be able to consume the same amount of goods. Finally, they supply less labour when wages are high, given prices.

### 2.5 Nominal rigidities

This subsection checks whether the main result of the model holds under the assumption that prices are sticky. Suppose the simplest case of nominal rigidities: firms must set their prices one period in advance, at the moment they are created, whilst the optimal level of labour is chosen when the production starts.

#### 2.5.1 The Optimal Monetary Policy to Replicate Flexible Prices Allocation

To start with the analysis of an economy with price stickiness, let’s assume that the monetary authority decides to commit to a simple monetary policy. It determines a monetary stance tied to productivity shocks in such a way it is able to optimally obtain the equilibrium allocation we found when prices were perfectly flexible. Under this restrictive assumption, the optimal portfolio derived in the first part of the chapter holds.

#### 2.5.1.1 Production with Producer Currency Pricing -PCP-

The Firm’s Problem  At time \( t \), the firm maximizes the discounted value of future profits subject to the technology restriction. Let’s assume that they set prices in the currency of the country of production. They are paid \( p_t(h) \) for any unit sold at home or abroad, although foreign households account for the exchange rate, \( e_t \), in deciding the level of consumption of variety \( h \) (\( p^*_t(h) = \frac{1}{e_t} p_t(h) \), so

\[
c^*_t(h) = C^*_{H,t} \left[ \frac{p^*_t(h)}{P^*_t} \right]^{-\sigma} = C^*_{H,t} \left[ \frac{p_t(h)}{P^*_t} \right]^{-\sigma}.
\]

However, since the law of one price holds also for the price indexes (\( P^*_t = \frac{1}{e_t} P_{H,t} \)),

\[
\left[ \frac{p^*_t(h)}{P^*_t} \right]^{-\sigma} = \left[ \frac{p_t(h)}{P_{H,t}} \right]^{-\sigma}.
\]

Hence, one can simply maximize,

\[
\max_{p_{t+1}(h)} E_t Q_{t,t+1} [p_{t+1}(h) Y_{t+1}(h) - w_{t+1} \ell_{t+1}(h)],
\]

\[ (2.36) \]
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which yields the optimal price-setting,\(^\text{22}\)

\[ p_{t+1}(h) = \frac{1}{\theta \sigma - 1} \frac{\kappa}{\theta} \left( E_t \left( \frac{Y_{t+1}(h)}{\mu_{t+1}} \right)^{\frac{\sigma}{2}} \frac{Y_{H,t+1}}{Y_{t+1}(h)^{N-1}} \right) E_t \left( \frac{Y_{H,t+1}}{Y_t} \right), \]

(2.37)

where \( \mu_{t+1} = P_{t+1} C_{t+1} \). Prices crucially depend on expected \( \mu \). An expected monetary expansion raises the price level and nominal spending.

Since, in equilibrium \( P_{H,t} = n_{t}^{\frac{1}{\sigma}} p_{t} (h) \) and recalling that \( Y_{H,t+1} = Y_{t+1}(h) \left( \frac{p_{t+1}(h)}{P_{H,t+1}} \right)^{-\sigma} \),

\[ p_{t+1}(h) = \frac{1}{\theta \sigma - 1} \frac{\kappa}{\theta} \frac{E_t \left( \frac{\mu_{t+1}^{1/2}}{Y_{H,t+1}} Y_{t+1}(h)^{\frac{1}{2}} \right)}{E_t \left( \frac{1}{\mu_{t+1}} Y_{t+1}(h) \right)}. \]

Let’s assume a monetary stance \( \mu_{t} = A_{H,t}^{1/2} \). Thus, the government reacts one to one to any productivity shock affecting the domestic economy. In this case, the preset price is a constant proportion of the expected output \( (E_t Y_{t+1}(h)) \) weighted by the technology parameter,

\[ p_{t+1}(h) = \frac{\sigma \kappa}{\sigma - 1} E_t Y_{t+1}(h)^{\frac{1}{\sigma} - 1}. \]

The government controls the path of short-term rates \( i \), providing a nominal anchor for market expectations. A forward-looking measure of monetary stance, \( \mu \) is provided by equation (2.11) and the definition \( \mu_{t+1} = P_{t+1} C_{t+1} \).

At time \( t+1 \), the firm created at \( t \) chooses \( \ell_{t+1}(h) \) by minimizing costs. The f.o.c. does not change from the flexible-price case,

\[ \phi_{t+1} = \frac{w_{t+1}}{\theta A_{H,t+1}} \ell_{t+1}(h)^{1-\theta} = mgC. \]

**Determinator of \( \lambda \)**

In order to compute \( \lambda \), I use the expressions of profits and labour income as a proportion of domestic output, as well as the definitions in (2.27). All of them are in aggregate terms. Since the individual price is not used, but only the price indexes and I have assumed that firms are atomistic, the optimal \( \lambda \) is not affected by the nominal rigidities in the economy, when prices are set in the currency of the producer. For example, from

\[ w_{t} \ell_{t}(h) = \theta \frac{\sigma - 1}{\sigma} p_{t}(h) Y_{t}(h), \]

\(^{22}\)Go to the appendix for details.
The aggregate is

\[ w_t \int_{\ell_t}^{n_t} \ell_t (h) \, dh = \theta \frac{\sigma - 1}{\sigma} \int_{\ell_t}^{n_t} p_t (h) \, Y_t (h) \, dh \]

\[ n_t w_t \ell_t (h) = \theta \frac{\sigma - 1}{\sigma} Y_{H,t} P_{H,t}. \]

The same expression from the fully-flexible price version applies, as happens with that of profits.\(^{23}\)

### 2.5.1.2 Production with Local Currency Pricing -LCP-

In this subsection, I show how fundamental equations change when prices are set on the local currency (i.e. on the currency of the market where the variety is consumed). However, the conclusions for the optimal portfolio allocation do not vary. In this case, firms maximize,

\[
\max_{p_{t+1}(h), p_{t+1}^*(h)} E_t Q_{t+1} \left[ \frac{p_{t+1}(h) (L_{t+1} c_{t+1} (h) + n_{t+2} k_{t+1} (h)) + e_{t+1} p_{t+1}^* (h) (L_{t+1}^* c_{t+1}^* (h) + n_{t+2}^* k_{t+1}^* (h))}{\frac{1}{A_{H,t+1}} Y_{H,t+1}^1 (h)} - \frac{w_{t+1}}{A_{H,t+1}} Y_{H,t+1} (h) \right].
\]

(2.38)

When prices are set per market, firms must suffer from the uncertainty generated by the exchange rate volatility. The new f.o.c. yield the following optimal price setting,

\[
p_{t+1} (h) = \frac{\sigma}{\sigma - 1} \hat{\phi} \frac{E_t Y_{H,t+1}^{-1} (h)}{A_{H,t+1}} (C_{H,t+1} + n_{t+2} K_{H,t+1}) \]

(2.39)

and

\[
p_{t+1}^* (h) = \frac{\sigma}{\sigma - 1} \hat{\phi} \frac{E_t Y_{H,t+1}^{-1} (h)}{A_{H,t+1}} (C_{H,t+1}^* + n_{t+2}^* K_{H,t+1}) \]

(2.40)

Notice that the price of variety \( h \) at home depends also on the part of the production effectively used in the home country (\( L_{t+1} C_{H,t+1} + n_{t+2} K_{H,t+1} \)); and the price of the same variety \( h \) in the foreign country, on the part used in that foreign country (\( L_{t+1}^* C_{H,t+1}^* + n_{t+2}^* K_{H,t+1}^* \)),\(^{24}\) both for consumption and investment.

---

\(^{23}\) However, this is true under the most simple example of price stickiness. If I introduce costs of adjustment, the parameter of the quadratic form may appear in a new expression of \( \lambda \).

\(^{24}\) Under flexible prices would be: \( p_t (h) = \frac{\sigma}{\sigma - 1} \hat{\phi} \frac{E_t Y_{H,t}^{-1}}{A_{H,t}} \) and \( p_{t+1}^* (h) = \frac{1}{\hat{\phi}} \frac{\sigma}{\sigma - 1} \hat{\phi} \frac{E_t Y_{H,t}^{-1}}{A_{H,t}} \) as it was in our model.
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Under this setup, equations (2.24) and (2.25) are no longer true. Here instead,

\[ \pi_t(h) = \left( 1 - \theta \frac{\sigma - 1}{\sigma} \right) p_t(h) V_t + \left( 1 - \theta \frac{\sigma - 1}{\sigma} \right) e_t p_t^* (h) D_t, \]

where \( V_t = c_t(h) \) and \( n_{t+1} k_t(h) \), \( D_t = c_t(h) + n_{t+1} k_t(h) \) and, symmetrically, \( V_t^* = c_t(f) + n_{t+1} k_t^*(f) \), \( D_t^* = c_t^*(f) + n_{t+1} k_t^*(f) \). Moreover,

\[ w_t \ell_t(h) = \theta \frac{\sigma - 1}{\sigma} p_t(h) V_t + \theta \frac{\sigma - 1}{\sigma} e_t p_t^* (h) D_t \]

and using the new expressions for (2.24) and (2.25) in the difference of the budget constraints,

\[ \Delta \mathcal{C} = \left[ \theta \frac{\sigma - 1}{\sigma} + (1 - 2 \lambda) \left( 1 - \theta \frac{\sigma - 1}{\sigma} \right) \right] \Delta \Psi - (1 - 2 \lambda) \Delta k \]

where \( \Psi_t = p_t(h) V_t + e_t p_t^* D_t \) and \( \Psi_t^* = \frac{\nu(l)}{e_t} V_t^* + p_t^* (f) D_t^* \). Let’s define \( \Delta \Psi = \Psi_t - e_t \Psi_t^* \). Thus, the new equation for output absorption in the economy is,

\[ \Delta \Psi = (2 \gamma - 1) \Delta \mathcal{C} + (2 \delta - 1) \Delta k. \]

Notice that \( \Delta \Psi = \Delta \mathcal{Y} \) and, so, \( \Delta \mathcal{C} \) is equal to that in the previous cases. Hence, the expression of \( \lambda \) is still true.

2.5.2 The General Monetary Policy (with Giancarlo Corsetti)

In this section, we allow authorities to introduce a general monetary policy. If we keep the previous setup mainly unchanged and we only introduce one-period preset prices, the optimal portfolio is not constant anymore. Instead, it is linked to the present discounted values of nominal labour income and nominal output.

The presence of nominal rigidities in prices, causes a negative correlation between labour income and dividends. After a positive shock on labour productivity, prices do not adjust. Hence, demand remains untouched and firms release some labour force because they are able to produce the same within less hours of work.

However, nominal rigidities do not prevent households from getting perfect risk sharing in equilibrium. We just need to create a new asset per country, a part from firm shares.

Let’s introduce two new assets \( X \), one denominated in Home currency, the other in Foreign currency. The number of assets exchanged across the border are \( X_H \) and \( X_F \) and their prices, \( q_H \) and \( q_F \).
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Let’s define these assets as generic one-period forward contracts (i.e. there is no exchange of cash at the time when the contract is entered.) The Euler equation must satisfy:

$$q_{X,t} = 0 = \beta E \frac{P_tC_t}{P_{t+1}C_{t+1}} \Theta_{X,t+1}, \tag{2.41}$$

where $\Theta_X$ is the payoff of the contract.

Using the budget constraint under our educated conjecture of efficient risk sharing with constant shares $\lambda$ and $X$ we get:

$$PC = (1 - \lambda) \left[ 1 - \frac{\mu}{A_H^*P_H} \right] P_H Y_H + \frac{\mu}{A_H^*P_H} P_H Y_H + \lambda \left[ 1 - \frac{\mu^*}{A_F^*P_F^*} \right] E P_F^* Y_F^* + \tag{2.42}$$

$$+X_{H,t} \Theta_{X,t} - \varepsilon X_{F,t} \Theta_X^* - q_{X,t} X_H + \varepsilon q_{F,t} X_F - (1 - \lambda) P_K I - \lambda E P_F^* I^*$$

and

$$E P^* C^* = \lambda \left[ 1 - \frac{\mu}{A_H^*P_H} \right] P_H Y_H + \frac{\mu^*}{A_F^*P_F^*} E P_F^* Y_F^* + \frac{\mu}{A_H^*P_H} (1 - \lambda) \left[ 1 - \frac{\mu^*}{A_F^*P_F^*} \right] E P_F^* Y_F^* + \tag{2.43}$$

$$-X_H \Theta_X + \varepsilon X_F \Theta_X^* + q_{X,t} X_H - \varepsilon q_{F,t} X_F - \lambda P_K I - (1 - \lambda) E P_F^* I^*.$$

If we take differences under the maintained hypothesis of perfect risk sharing and use the equilibrium condition for forward contracts:

$$0 = (1 - \lambda) \left[ 1 - \frac{\mu}{A_H^*P_H} \right] P_H Y_H + \frac{\mu}{A_H^*P_H} P_H Y_H + \lambda \left[ 1 - \frac{\mu^*}{A_F^*P_F^*} \right] E P_F^* Y_F^*$$

$$-\lambda \left[ 1 - \frac{\mu}{A_H^*P_H} \right] P_H Y_H - \frac{\mu^*}{A_F^*P_F^*} E P_F^* Y_F^* - (1 - \lambda) \left[ 1 - \frac{\mu^*}{A_F^*P_F^*} \right] E P_F^* Y_F^* +$$

$$+2 X_H \Theta_t - 2 \varepsilon X_F \Theta_t^* - (1 - 2\lambda \mu) \Delta P_K I,$$

which yields:

$$0 = (1 - 2\lambda) \left[ 1 - \frac{\mu}{A_H^*P_H} \right] P_H Y_H + \frac{\mu}{A_H^*P_H} P_H Y_H -$$

$$(1 - 2\lambda) \left[ 1 - \frac{\mu^*}{A_F^*P_F^*} \right] E P_F^* Y_F^* - \frac{\mu^*}{A_F^*P_F^*} E P_F^* Y_F^* +$$

$$+2 [X_H \Theta_t - \varepsilon X_F \Theta_t^*] - (1 - 2\lambda) \Delta P I.$$

Collecting terms we derive:

$$0 = P_H Y_H \left\{ (1 - 2\lambda) \left[ 1 - \frac{\mu}{A_H^*P_H} \right] + \frac{\mu}{A_H^*P_H} + \frac{\mu}{A_H^*P_H} \right\} -$$

$$E P_F^* Y_F \left\{ (1 - 2\lambda) \left[ 1 - \frac{\mu^*}{A_F^*P_F^*} \right] + \frac{\mu^*}{A_F^*P_F^*} + \frac{\mu^*}{A_F^*P_F^*} \right\} +$$

$$- (1 - 2\lambda) \Delta P I. \tag{2.44}$$
2.5. NOMINAL RIGIDITIES

From the resource constraint, we know that under perfect risk sharing:

\[ 0 = P_H Y_H - \mathcal{E} P_F^* Y_F - (2\delta - 1) \Delta PI. \]  

(2.45)

Equating these two expressions:

\[
(1 - 2\lambda) \left[ 1 - \frac{\mu}{A_H P_H} \right] + \frac{\mu}{A_H P_H} + 2 \frac{X_H \Theta_t}{P_H Y_H} = \frac{(1 - 2\lambda)}{(2\delta - 1)} = (2.46)
\]

\[
= (1 - 2\lambda) \left[ 1 - \frac{\mu^*}{A_F P_F^*} \right] + \frac{\mu^*}{A_F P_F^*} + 2 \frac{X_F \Theta_t^*}{\mathcal{E} P_F^* Y_F}.
\]

Given that the cash flow of the forward contract is endogenous, we can set arbitrarily the \( X_s \) equal to 1. We then have 3 unknowns \( \lambda, \Theta_t \) and \( \Theta_t^* \) and four equations:

\[
(1 - 2\lambda) \left[ 1 - \frac{\mu}{A_H P_H} \right] + \frac{\mu}{A_H P_H} + 2 \frac{X_H \Theta_t}{P_H Y_H} = \frac{(1 - 2\lambda)}{(2\delta - 1)}; \quad (2.47)
\]

\[
(1 - 2\lambda) \left[ 1 - \frac{\mu^*}{A_F P_F^*} \right] + \frac{\mu^*}{A_F P_F^*} + 2 \frac{X_F \Theta_t^*}{\mathcal{E} P_F^* Y_F} = \frac{(1 - 2\lambda)}{(2\delta - 1)}; \quad (2.48)
\]

\[
 q_{X_H,t} = 0 = E \beta \frac{\mu_t}{\mu_{t+1}} \Theta_{t+1}; \quad (2.49)
\]

\[
 q_{X_F} = 0 = E \beta \frac{\mu^*}{\mu_{t+1}} \Theta_{t+1}^* . \quad (2.50)
\]

But, by symmetry, one of the two last equations is redundant.\(^{25}\)

2.5.2.1 General solution

In the general case, for \( \Theta_t \) we get:

\[
\Theta_t = \frac{P_H Y_H}{2X_H} \left[ (1 - 2\lambda) \left( \frac{1 - 2\lambda}{2\delta - 1} \right) - (1 - 2\lambda) \left[ 1 - \frac{\mu}{A_H P_H} \right] - \frac{\mu}{A_H P_H} \right] = \frac{P_H Y_H}{X_H} \left[ (1 - 2\lambda) \left( \frac{1 - 2\lambda}{2\delta - 1} \right) - 1 \right] - \frac{\mu}{A_H P_H}. \quad (2.51)
\]

Note above that the size of \( X \) only enters multiplicatively to the cash flow, which confirms that it is irrelevant.

\(^{25}\)To find the flexible-price solution presented in the previous section from the first two equations above, see the appendix.
2.5. NOMINAL RIGIDITIES

To derive \( \lambda \), we use:

\[
0 = E \beta^{\mu t-1} \Theta_t = E \beta^{\mu t-1} P_H Y_H \frac{\lambda}{X_H} \left[ (1 - 2\lambda) \frac{1}{2\lambda} - 1 - \frac{\mu}{A_H P_H} \right].
\]

Simplifying:

\[
0 = (1 - 2\lambda) \frac{1}{2\lambda} P D V (P Y) - P D V (W \ell).
\]

Hence,

\[
0 = (1 - \lambda) \left( \frac{2(1 - \delta)}{2\delta - 1} + \frac{P D V (W \ell)}{P D V (P Y)} \right) - \left( \frac{(1 - \delta)}{2 - 1} + \frac{P D V (W \ell)}{P D V (P Y)} \right).
\]

The equilibrium share holding of home assets is:\(^{26}\)

\[
1 - \lambda = \frac{(1 - \delta) + (2\delta - 1) \frac{P D V (W \ell)}{P D V (P Y)}}{2 (1 - \delta) + (2\delta - 1) \frac{P D V (W \ell)}{P D V (P Y)}}.
\]

This share is not constant. It depends on the present discounted value of labour income relative to the present discounted value of output. The case of flexible prices, instead, implies that the share of labour income in total income is constant. Thus,

\[
1 - \lambda = \frac{(1 - \delta) + (2\delta - 1) \frac{E \beta^{\mu t-1} \frac{\sigma}{\sigma - 1} P Y}{E \beta^{\mu t-1} \frac{\sigma}{\sigma - 1} P Y}}{2 (1 - \delta) + (2\delta - 1) \frac{E \beta^{\mu t-1} \frac{\sigma}{\sigma - 1} P Y}{E \beta^{\mu t-1} \frac{\sigma}{\sigma - 1} P Y}} = \frac{\sigma \delta - 2\delta + 1}{\sigma - 2\delta + 1}.
\]

This value makes the pay-off of the forward contract equal to Aero:

\[
P_H Y_H \frac{(1 - S_H)}{X_H} \left[ (2S_H - 1) \frac{1}{2(1 - S_H)} - 1 - \frac{\mu}{A_H P_H} \right] = P_H Y_H \frac{(1 - S_H)}{X_H} \left[ (2\sigma \delta - 2\delta + 1 - 1) \frac{(2\delta - 1)}{\sigma - 2\delta + 1} - 1 \right] = 0.
\]

\(^{26}\)Observe that, without home bias \( \delta = 1/2 \), the solution is again perfect pooling, i.e. \( \lambda = 1/2 \) always.
2.5. NOMINAL RIGIDITIES

2.5.2.2 Interpreting the solution

Equity shares With nominal rigidities, \( \lambda \) depends crucially on the properties of the shock and on monetary policy:

\[
1 - \lambda = \frac{(1 - \delta) + (2\delta - 1) \frac{EY_H}{E_H^t \chi_H}}{2 (1 - \delta) + (2\delta - 1) \frac{EY_H}{E_H^t \chi_H}}. \tag{2.55}
\]

If monetary policy targets the flex-price equilibrium \( \mu = A_H \), we have:

\[
1 - \lambda = \frac{(1 - \delta) + (2\delta - 1) \frac{EY_H}{E_H^t \chi_H}}{2 (1 - \delta) + (2\delta - 1) \frac{EY_H}{E_H^t \chi_H}} = \frac{(1 - \delta) + (2\delta - 1)}{2 (1 - \delta) + (2\delta - 1)}. \tag{2.56}
\]

For a generic distribution of \( \mu \):

\[
E \frac{1}{\mu_t} \mu_t \ell = E \frac{1}{\mu_t} E \mu_t \ell + Cov \left( \frac{1}{\mu_t}, \mu_t \ell \right). \tag{2.57}
\]

Hence,

\[
1 - \lambda = \frac{(1 - \delta) + (2\delta - 1) \frac{E \frac{1}{\mu_t} E \mu_t \ell + Cov \left( \frac{1}{\mu_t}, \mu_t \ell \right)}{E \frac{1}{\mu_t} Y_H}}{2 (1 - \delta) + (2\delta - 1) \frac{E \frac{1}{\mu_t} E \mu_t \ell + Cov \left( \frac{1}{\mu_t}, \mu_t \ell \right)}{E \frac{1}{\mu_t} Y_H}}.
\]

Other things equal, with home bias in investment \( \delta > 1/2 \) the share \( (1 - \lambda) \) is increasing in the covariance between the labour income and the inverse of the monetary stance:

\[
\frac{\partial S_H}{\partial Cov} = \frac{\partial}{\partial Cov} \frac{(1 - \delta) + (2\delta - 1) \frac{A + Cov \left( \frac{1}{\mu_t}, \mu_t \ell \right)}{B}}{2 (1 - \delta) + (2\delta - 1) \frac{A + Cov \left( \frac{1}{\mu_t}, \mu_t \ell \right)}{B}} = \frac{(2\delta - 1) \frac{1}{B} [1 - \delta]}{(\cdot)^2} > 0 \text{ if } \delta > 1/2. \tag{2.58}
\]

and decreasing otherwise.

\( ^{27} \)Which is the same policy I assumed in the previous subsection, for the case of \( \theta = 1 \).
Let’s suppose now that $\mu$ is held constant. For instance, arbitrarily set $\mu = 1$:

$$1 - \lambda = \frac{(1 - \delta) + (2\delta - 1) \frac{E_H Y_H}{E_Y^H}}{2 (1 - \delta) + (2\delta - 1) \frac{E_H}{E_Y^H}}$$

$$= \frac{(1 - \delta) + (2\delta - 1) \frac{E_Y^H - \text{Cov}(A_H, \ell)}{E_Y^H}}{2 (1 - \delta) + (2\delta - 1) \frac{E_A H}{E_Y^H}}$$

$$= \frac{(1 - \delta) + (2\delta - 1) \left( \frac{1}{E_A H} - \frac{\text{Cov}(A_H, \ell)}{E_Y^H E_A H} \right)}{2 (1 - \delta) + (2\delta - 1) \left( \frac{1}{E_A H} - \frac{\text{Cov}(A_H, \ell)}{E_Y^H E_A H} \right)}.$$

With a constant monetary stance, the covariance between productivity shocks and employment is negative. Hence, provided $\delta > 1/2$, the share of domestic portfolio invested in domestic equities is increasing in the absolute value of this covariance.\(^{28}\)

Cash flow of the forward contract  We have seen that:

$$\Theta_t = P_H Y_H \left[ (1 - 2\lambda) \frac{1}{2 (2\delta - 1)} - 1 - \lambda \frac{\mu}{A_H P_H} \right] =$$

$$= P_H Y_H \left[ \frac{(1 - 2\lambda)}{2} \left( \frac{1}{2 (2\delta - 1)} - 1 \right) - \lambda \frac{W \ell}{Y_H P_H} \right].$$

With nominal rigidities, unless monetary policy stabilizes the economy at the flex-price equilibrium, a positive productivity shock raises markups above the flex-price level and lowers labour income. The last term of the cash flow above implies that, by entering a forward, foreign investors ‘pay back’ the excess profits in proportion of the share they own of the domestic capital (and domestic investors receive it.) Labour income share goes down, hence the cash flow is high:

\(^{28}\)Another way of looking at it:

In the expression,

$$1 - \lambda = \frac{(1 - \delta) + (2\delta - 1) \frac{E_H Y_H}{E_Y^H} \frac{\mu}{\bar{z}^H P_H}}{2 (1 - \delta) + (2\delta - 1) \frac{E_H}{E_Y^H} \frac{\mu}{\bar{z}^H P_H}},$$

the term $\frac{\mu}{\bar{z}^H P_H}$ is just marginal costs over prices. Hence it is the inverse of the markup. The term $\frac{P_H Y_H}{P_C}$ is total value of output over total value of consumption, hence it is equal to 1 plus the ratio of the value of investment goods produced in the economy (also for export) to total value of consumption.
2.6. Conclusions

I developed a stylized two-period two-country model with perfect risk sharing. The dynamic number of firms and the international portfolio diversification is endogenously determined. The model builds on Heathcote and Perri’s idea of the compatibility of the home bias in portfolio found in actual data and perfect risk sharing.

The model presented here confirms Heathcote and Perri’s result in the sense that an equilibrium exists where a home-biased and constant portfolio allocation is able to provide households with perfect risk sharing. It shows that terms of trade play an important role in neutralizing the effects of country-specific shocks on relative consumption, as Cole and Obstfeld (1991) claimed. However, they are not able to offset the disturbances on investment. One needs to diversify assets to control for these.

The main contributions of this analysis are the following: first, it highlights the need to distinguish between the preferences of demand on capital and those on consumption goods. Contrary to H&P’s result, it is the home-bias parameter in the Cobb-Douglas aggregator for capital demand that determines the level of diversification. Second, I checked the role of the endogenous number of firms or varieties in the determination of the portfolio allocation. I find that these two endogenous variables are completely independent when the economy has flexible determination of prices, i.e. in the long run horizon. Finally, it is shown that, when the monetary authority imposes a monetary policy to replicate the flexible price allocation in equilibrium, the result for the optimal portfolio is still true, whatever the price regime applied to the economy. When a more general monetary policy is allowed, we are still able to provide perfect risk sharing under our incomplete-markets scenario.\footnote{We do not need a complete arrow-debreu security set to get perfect risk sharing.}

However, in this case, we need to add a generic forward contract which pay-off must allow foreign investors to
2.6. CONCLUSIONS

"pay back" the excess profits generated by the negative correlation between dividends and labour income. This section is a work-in-progress with Giancarlo Corsetti.

This chapter offers a powerful framework to explore different dimensions of the world economy. With it, one is able to study the disturbances generated by shocks both on the productivity of production ($A_H, A_F$) and on the productivity of creation ($K, K^*$) and analyse the role of the monetary policy in their stabilization. One can consider the differences between PCP and LCP and observe the transmission of shocks in an environment with market dynamics. Finally, one can take into account and compare both the set-up with perfect risk sharing and the financial autarky version. The simulations of these alternative economies may provide extremely interesting results to go one step further in international macroeconomics research.
2.7 Appendix

2.7.1 The households' problem in the foreign country under fully flexible prices

Utility maximization

\[
\max E_t \sum \beta^t C^*_t = E_t \sum \beta^t \left[ \ln \left( C^*_{H,t} \right)^{1-\gamma} \left( C^*_{F,t} \right)^\gamma \right] - \kappa \ell^*_t (j), \tag{2.62}
\]

subject to the budget constraint,

\[
B^*_{t+1} + \lambda^*_{H,t+1} \frac{1}{e_t} \int q_t (h) \, dh + \lambda^*_{F,t+1} \int q^*_t (f) \, df + P^*_{H,t} C^*_t + P^*_{F,t} C^*_F =
\]

\[
= \frac{1}{e_t} \lambda^*_t \int \pi_t (h) \, dh + \lambda^*_f \int \pi^*_f (f) \, df + w^*_t \ell^*_t (j) + (1 + i^*_t) B^*_{t+1}.
\tag{2.63}
\]

Symmetrically to those in the Home country, the first-order conditions are:

\[
\kappa \frac{w^*_t}{e^*_t} = \xi^*_t = \frac{1}{P^*_t C^*_t} \to w^*_t = \kappa P^*_t C^*_t; \tag{2.65}
\]

\[
C^*_{H,t} = (1 - \gamma) \frac{P^*_t C^*_t}{P^*_H,t}; C^*_{F,t} = \gamma \frac{P^*_t C^*_t}{P^*_F}; \tag{2.66}
\]

\[
c^*_t (h) = C^*_{H,t} \left( \frac{p_t (h)}{P^*_H,t} \right)^{-\sigma}; c^*_t (f) = C^*_{F,t} \left( \frac{p_t (f)}{P^*_F} \right)^{-\sigma}; \tag{2.67}
\]

\[
\frac{1}{P^*_t C^*_t} = \beta (1 + i^*_t) E_t \frac{1}{P^*_{t+1} C^*_t}; \tag{2.68}
\]

\[
q^*_{H,t} = n_{t+1} q_t (h) \frac{1}{e_t} = Q^*_{t+1} \frac{1}{e_{t+1}} \Pi_{H,t+1}; \tag{2.69}
\]

\[
q^*_{F,t} = Q^*_{t+1} \Pi_{F,t+1};
\]

where the welfare-based price index is symmetric to the domestic one:

\[
P^*_t = \left( \frac{P^*_H,t}{P^*_F} \right)^{1-\gamma} \left( \frac{P^*_F}{P^*_F} \right)^\gamma \tag{2.70}
\]

and

\[
Q^*_{t+1} = \beta E_t \left( \frac{P^*_t C^*_t}{P^*_{t+1} C^*_{t+1}} \right) = \beta E_t \left( \frac{\mu^*_t}{\mu^*_{t+1}} \right). \tag{2.71}
\]
This set-up implies that, for given home-currency prices of the varieties, \( p_t(h) \) and \( p_t(f) \), the utility-based CPI, \( P_t \) is as defined by (2.14) in the text, whereas:

\[
P_{H,t} = \left( \int p_t(h)^{1-\sigma} \, dh \right)^{\frac{1}{1-\sigma}}; \quad P_{F,t} = \left( \int p_t(f)^{1-\sigma} \, df \right)^{\frac{1}{1-\sigma}}.
\]

2.7.2 The firms’ problem in the foreign country under fully flexible prices:

**Creation**

The basket of capital goods necessary for the creation of a firm in the Foreign country is

\[
K^*_t = (K^*_{H,t})^{1-\delta} (K^*_{F,t})^\delta.
\]

Following the same process explained in the text for Home country, the optimal demands for capital are

\[
\text{Foreign: } K^*_{H,t} = (1-\delta) \frac{P^*_{k,t}K^*_t}{P^*_{H,t}}; \quad K^*_{F,t} = \delta \frac{P^*_{k,t}K^*_t}{P^*_{F,t}}.
\]

Foreign firms choose the demand of each capital good, \( k^*_t(h) \) and \( k^*_t(f) \), by solving the minimization problems presented in the text for home. Thus,

\[
k^*_t(h) = \left( \frac{p^*_t(h)}{P^*_{H,t}} \right)^{-\sigma} K^*_{H,t}; \quad k^*_t(f) = \left( \frac{p^*_t(f)}{P^*_{F,t}} \right)^{-\sigma} K^*_{F,t}
\]

and the price index for capital is,

\[
P^*_{K,t} = \frac{(P^*_{H,t})^{1-\delta} (P^*_{F,t})^\delta}{\Gamma_\delta}.
\]

**Production**

**Home**

Once operative, firms maximize profits:

\[
\max_{p_t(h)} p_t(h) Y_t(h) - w_t \ell_t(h),
\]

subject to the technology restriction and demand. So, one can write,

\[
\max_{p_t(h)} p_t(h) \left[ L_t c_t(h) + n_{t+1} k_t(h) + e_t L^*_t c^*_t(h) + n^*_t k^*_t(h) \right] -
\]

\[
- w_t \left( \frac{L_t c_t(h) + n_{t+1} k_t(h) + e_t L^*_t c^*_t(h) + n^*_t k^*_t(h)}{A_{H,t}} \right)^{\frac{1}{\delta}}.
\]
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Home firms choose the optimal price:

\[ p_t(h) : Y_t(h) + p_t(h) \frac{\partial Y_t(h)}{\partial p_t(h)} - \frac{w_t}{A_{H,t}^\theta} \frac{\partial p_t(h)}{\partial p_t(h)} = 0. \]

By substituting the optimal demands and deriving:

\[
\left[ \frac{p_t(h)}{P_{H,t}} \right]^{-\sigma} F + p_t(h) (-\sigma) \frac{p_t(h)^{-\sigma-1}}{P_{H,t}} F - (-\sigma) \frac{1}{A_{H,t}^\sigma} w_t Y_t(h)^{\frac{1}{\theta}} - 1 \frac{p_t(h)^{-\sigma-1}}{P_{H,t}} F = 0,
\]

where \( F = [L_tC_{H,t} + n_{t+1}K_{H,t} + e_tL_t^*C_{H,t}^* + n_{t+1}^*K_{H,t}^*] \). Thus,

\[ p_t(h) = \frac{\sigma}{\sigma - 1} \frac{1}{\frac{1}{A_{H,t}^\sigma}} Y_t(h)^{\frac{1}{\theta} - 1}. \]

One may also write

\[ p_t(h) = \frac{\sigma}{\sigma - 1} \frac{1}{\frac{1}{A_{H,t}^\sigma}} \left( \ell_t(h)^{\theta} \right)^{\frac{1}{\theta} - 1}. \]

\[ p_t(h) = \frac{\sigma}{\sigma - 1} \phi_t = \frac{\sigma}{\sigma - 1} mgC. \]

Symmetrically, in the foreign country:

As at home, firms produce a differentiated variety with an homogeneous technology which requires only labour:

\[ Y_t(f) = A_{F,t} \ell_t(f)^{\theta}. \]

The state of the economy is

\[ \{ A_{H,t}, A_{F,t} \} . \]

Firms choose the amount of labour which minimizes costs

\[ \min w_t^* \ell_t(f) . \]

The optimal price is:

\[ p_t^*(f) = \frac{\sigma}{\sigma - 1} \frac{1}{\frac{1}{A_{F,t}^\sigma}} Y_t(f)^{\frac{1}{\theta} - 1} = \frac{\sigma}{\sigma - 1} \phi_t^* . \]
2.7.3 Stability Condition

In aggregate, 
\[ Y = \left[ \int_0^n n^{-\theta \rho} dn \right]^\frac{1}{\rho}, \]
where \( \rho = \frac{\sigma}{\sigma - 1} \). So, 
\[ Y = n^{\frac{1-\theta\rho}{\rho}}. \]

In order to ensure stability, the first derivative of output with respect to the number of firms should be positive and, the second, negative. Here, 
\[ \frac{\partial Y}{\partial n} = \frac{1-\theta\rho}{\rho} n^{\frac{1-\theta\rho}{\rho}}. \]

Which is positive for 
\[ 1 > \theta \frac{\sigma - 1}{\sigma}. \]

This is always true, since the labour income share, \( \theta \in (0, 1) \) and \( \frac{\sigma - 1}{\sigma} < 1 \). Moreover, 
\[ \frac{\partial^2 Y}{\partial n^2} = \frac{1-\theta\rho}{\rho} \frac{1-\theta\rho - \rho}{\rho} n^{\frac{1-\theta\rho -2\rho}{\rho}}. \]

Hence, 
\[ 1 - \theta\rho - \rho < 0 \]
is needed. This is also always true, since 
\[ 1 - \theta \frac{\sigma}{\sigma - 1} - \frac{\sigma}{\sigma - 1} < 0 \]
\[ 1 < \frac{\sigma}{\sigma - 1} (\theta + 1) \]
\[ \frac{\sigma - 1}{\sigma} - 1 < \theta \]
\[ -\frac{1}{\sigma} < \theta \]

and \( \sigma \) and \( \theta \) are both positive. The stability condition ensures the inexistence of increasing returns to scale in investment, which would make the model explosive.

2.7.4 Profits Aggregation

Firm’s profits are 
\[ \pi_t (h) = p_t (h) y_t (h) - w_t \ell_t (h). \]
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One can express them in terms of firm’s revenue,

\[ \pi_t(h) = \left( 1 - \frac{\sigma - 1}{\sigma} \right) p_t(h) y_t(h). \]

The price index and the aggregate output equations are needed in order to aggregate profits over the \( n \) homogeneous firms producing at home. These are,

\[ P_{H,t} = n_t^{\frac{1}{\sigma}} p_t(h) \]

and

\[ Y_t = n_t^{\frac{\sigma}{\sigma - 1}} y_t(h). \]

The latter is due to the fact that the bundles of capital and consumption goods are structured equally and have the same elasticity of substitution and, consequently, firm charges the same price to both consumers and new firms buying its variety \( h \). Using these two equations I find that,

\[ \int_0^{n_t} \pi_t(h) \, dh = \left( 1 - \frac{\sigma - 1}{\sigma} \right) \int_0^{n_t} p_t(h) y_t(h) \, dh \]

becomes

\[ \Pi_{H,t} = \left( 1 - \frac{\sigma - 1}{\sigma} \right) n_t^{\frac{1}{\sigma}} \frac{P_{H,t} Y_{H,t}}{n_t^{\frac{1}{\sigma}} n_t^{\frac{\sigma}{\sigma - 1}}} \]

\[ \Pi_{H,t} = \left( 1 - \frac{\sigma - 1}{\sigma} \right) P_{H,t} Y_{H,t}, \]

where \( n_t \) has cancelled out.

2.7.5 Allocation of firms

The free entry conditions (FECs) provide us with a system of two difference equations to solve for \( n \) and \( n^* \). At Home, the FEC is,

\[ P_{K,t} K_t = E_t Q_{t,t+1} \pi_{t+1}(h) \]

or

\[ P_{H,t} K_{H,t} + P_{F,t} K_{F,t} = E_t Q_{t,t+1} \pi_{t+1}(h) \]

\[ P_{H,t} K_{H,t} + P_{F,t} K_{F,t} = E_t \beta \frac{P_t C_t}{P_{t+1} C_{t+1}} \left( 1 - \frac{\sigma - 1}{\sigma} \right) p_{t+1}(h) Y_{t+1}(h) \]

\[ = E_t \beta \frac{P_t C_t}{P_{t+1} C_{t+1}} \left( 1 - \frac{\sigma - 1}{\sigma} \right) p_{t+1}(h) \left[ L_{t+1} c_{t+1}(h) + n_{t+2} k_{t+1} + L_{t+1} c^*_t + n_{t+2} k^*_t \right]. \]
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Use the optimal demands for the variety $h$. Substitute the expressions for prices in $\frac{p_{t+1}(h)}{P_{H,t+1}}$ and $\frac{p_{t+1}(h)}{P_{H,t+1}}$. Cancel PC in $\frac{P_{t+1}(C_{t+1})}{P_{H,t+1}}$. Get a common factor $\theta^{\frac{\sigma-1}{\sigma\kappa}}$. Multiply by $n_{t+1}$. And, finally, let’s suppose that all shocks are iid. So one can write,

$$E_{t-1} \left( n_{t+1} K_{H,t} \frac{Y_t(h)^{\frac{1}{\sigma}}}{A_{H,t}^\frac{1}{\sigma}} \right) = E_t \left( n_{t+2} K_{H,t+1} \frac{Y_{t+1}(h)^{\frac{1}{\sigma}-1}}{A_{H,t+1}^\frac{1}{\sigma}} \right).$$

Hence,

$$n_{t+1} \left[ K_{H,t} \frac{Y_t(h)^{\frac{1}{\sigma}-1}}{A_{H,t}^\frac{1}{\sigma}} \left( 1 - \beta \left( 1 - \frac{\sigma-1}{\sigma} \theta \right) \right) + K_{F,t} \frac{Y_t(f)^{\frac{1}{\sigma}-1}}{A_{F,t}^\frac{1}{\sigma}} \right] = \beta \left( 1 - \frac{\sigma-1}{\sigma} \theta \right) \frac{\theta \sigma - 1}{\sigma \kappa}$$

$$E_t \left[ L_{t+1} \gamma + L_{t+1}^* (1 - \gamma) \left[ \frac{A_{F,t+1}^*}{A_{H,t+1}^*} \right]^{\frac{1}{\sigma}} \left[ \frac{Y_{t+1}(h)}{Y_{t+1}(f)} \right]^{\frac{1}{\sigma}-1} n_{t+1}^{\frac{1}{\sigma}} + \frac{1}{\theta - 1} \frac{Y_{t+1}(h)^{\frac{1}{\sigma}-1}}{A_{H,t+1}^\frac{1}{\sigma}} n_{t+2} K_{H,t+1}^* \right]$$

and, symmetrically,

$$n_{t+1}^* \left[ K_{F,t}^* \frac{Y_t(f)^{\frac{1}{\sigma}-1}}{A_{F,t}^\frac{1}{\sigma}} \left( 1 - \beta \left( 1 - \frac{\sigma-1}{\sigma} \theta \right) \right) + K_{H,t}^* \frac{Y_t(h)^{\frac{1}{\sigma}-1}}{A_{H,t}^\frac{1}{\sigma}} \right] = \beta \left( 1 - \frac{\sigma-1}{\sigma} \theta \right) \frac{\theta \sigma - 1}{\sigma \kappa}$$

$$E_t \left[ L_{t+1} \gamma + L_{t+1}^* (1 - \gamma) \left[ \frac{A_{F,t+1}}{A_{H,t+1}^*} \right]^{\frac{1}{\sigma}} \left[ \frac{Y_{t+1}(f)}{Y_{t+1}(h)} \right]^{\frac{1}{\sigma}-1} n_{t+1}^{\frac{1}{\sigma}} + \frac{1}{\theta - 1} \frac{Y_{t+1}(f)^{\frac{1}{\sigma}-1}}{A_{F,t+1}^\frac{1}{\sigma}} n_{t+2} K_{F,t+1} \right]$$

2.7.6 Derivation of the Terms of Trade

Terms of trade may also be derived in the following way:

$$\frac{P_{H,t}}{P_{F,t}} = \frac{n_t^{\frac{1}{\sigma}} p_t^* (h)}{n_t^{\frac{1}{\sigma}} p_t (f)} = \frac{n_t^{\frac{1}{\sigma}}}{n_t^{\frac{1}{\sigma}}} \frac{1}{e_t} \frac{1}{\sigma-1} \frac{1}{\theta} \frac{w_t}{A_{H,t}^\frac{1}{\sigma}} \frac{Y_t(h)^{\frac{1}{\sigma}-1}}{A_{H,t}^\frac{1}{\sigma}}$$

Arranging it,

$$TOT_t = \frac{1}{e_t} \frac{w_t}{w_t^*} \left( A_{F,t} \right)^{\frac{1}{\sigma}} \left( Y_t(h) \right)^{\frac{1}{\sigma}-1} \left( \frac{n_t^{\frac{1}{\sigma}}}{n_t^{\frac{1}{\sigma}}} \right) \left( \frac{1}{\sigma} \right)$$

and,

$$\text{if } \theta = 1 \rightarrow TOT_{H,t} = \frac{w_t}{w_t^*} \frac{A_{F,t}}{A_{H,t}} \left( \frac{n_t^{\frac{1}{\sigma}}}{n_t^{\frac{1}{\sigma}}} \right)^{\frac{1}{\sigma}}.$$
One can also derive **TOT from the resource constraints.** For symmetric countries and population equal to 1,

\[
C_{H,t} + C^*_{H,t} + n_{t+1}K_{H,t} + n^*_{t+1}K^*_{H,t} = Y_{H,t};
\]

\[
C_{F,t} + C^*_{F,t} + n_{t+1}K_{F,t} + n^*_{t+1}K^*_{F,t} = Y_{F,t}.
\]

Plugging the optimal demands, one is able to write supply equals demand with the following expressions:

\[
\frac{PC_t}{P_{H,t}} + (1 - \gamma) \frac{P^*_t C^*_t}{P^*_{H,t}} + n_{t+1}K_{H,t} + n^*_{t+1}K^*_{H,t} = Y_{H,t};
\]

\[
(1 - \gamma) \frac{PC_t}{P_{F,t}} + \gamma \frac{P^*_t C^*_t}{P^*_{F,t}} + n^*_{t+1}K_{F,t} + n^*_{t+1}K^*_{F,t} = Y_{F,t}.
\]

Rearranging by using (2.26),

\[
\frac{P_t C_t}{P^*_{H,t}} + (1 - \gamma) \frac{P^*_t C^*_t}{P^*_{H,t}} + n_{t+1}K_{H,t} + n^*_{t+1}K^*_{H,t} = Y_{H,t}.
\]

\[
\gamma \frac{P_t C_t}{P^*_{H,t}} + (1 - \gamma) \frac{P^*_t C^*_t}{P^*_{H,t}} + n_{t+1}K_{H,t} + n^*_{t+1}K^*_{H,t} = Y_{H,t}.
\]

By the same token,

\[
PC = Y_{F,t} - n^*_{t+1}K^*_{F,t} - n^*_{t+1}K^*_{F,t}.
\]

Taking the ratio of the two equations above yields an expression for the terms of trade:

\[
TOT_{H,t} = \frac{1}{e_t} \frac{Y_{F,t} - n^*_{t+1}K^*_{F,t} - n^*_{t+1}K^*_{F,t}}{Y_{H,t} - n_{t+1}K_{H,t} - n^*_{t+1}K^*_{H,t}}.
\]

### 2.7.7 Nominal Rigidities in Prices
#### 2.7.7.1 PCP

Firms maximize,

\[
\max_{p_{t+1}(h)} E_t Q_{t,t+1} \left[p_{t+1}(h)Y_{t+1}(h) - w_{t+1} \ell_{t+1}(h)\right],
\]

which yields the f.o.c.

\[
E_t Q_{t,t+1} \left[\left(\frac{p_{t+1}(h)}{P_{H,t+1}}\right)^{-\sigma} Y_{H,t+1} \left[1 - \sigma + \frac{\sigma}{A^*_{H,t+1}} w_{t+1} \frac{Y_{t+1}(h) - \ell_{t+1}(h)}{p_{t+1}(h)}\right]\right] = 0;
\]
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\[
E_t \left( Q_{t,t+1} (\sigma - 1) \left[ \frac{p_{t+1} (h)}{P_{H,t+1}} \right]^{-\sigma} Y_{H,t+1} \right) =
\]

\[
E_t \left( Q_{t,t+1} \left[ \frac{p_{t+1} (h)}{P_{H,t+1}} \right]^{-\sigma} Y_{H,t+1} \frac{\sigma}{\theta} \frac{w_{t+1}}{A_{H,t+1}^{\frac{1}{\theta}}} \frac{Y_{t+1} (h)^{\frac{1}{\theta} - 1}}{p_{t+1} (h)} \right).
\]

Substituting the definition of the discount factor, the expression for wages and keeping in mind that both \( p_{t+1} (h) \) and \( n_{t+1} \) are set at \( t \) and, hence, these go out of the expectations brackets:

\[
\frac{(\sigma - 1)}{\sigma} \theta \beta \mu_t E_t \left( \frac{Y_{H,t+1}}{\mu_{t+1}} \right) = \beta \kappa \mu_t E_t \left( \frac{Y_{H,t+1} Y_{t+1} (h)^{\frac{1}{\theta} - 1}}{A_{H,t+1}^{\frac{1}{\theta}} p_{t+1} (h)} \right).
\]

One can isolate \( p_{t+1} (h) \),

\[
p_{t+1} (h) = \frac{k}{\theta \sigma - 1} \frac{E_t \left( \frac{Y_{H,t+1}}{A_{H,t+1}^{\frac{1}{\theta}} Y_{t+1} (h)} \right)}{E_t \left( \frac{Y_{H,t+1}}{p_{t+1}} \right)},
\]

where \( \mu_{t+1} = P_{t+1} C_{t+1} \). Prices crucially depend on expected \( \mu \). An expected monetary expansion raises the price level and nominal spending.

Since, in equilibrium \( P_{H,t} = p_t (h) \) and recalling that \( Y_{H,t+1} = Y_{t+1} \left( \frac{p_{t+1} (h)}{P_{H,t+1}} \right)^{-\sigma} \),

\[
p_{t+1} (h) = \frac{1}{\theta \sigma - 1} \frac{E_t \left( \frac{k}{A_{H,t+1}^{\frac{1}{\theta}}} Y_{t+1} (h)^{\frac{1}{\theta}} \right)}{E_t \left( \frac{1}{p_{t+1}} Y_{t+1} (h) \right)}.
\]

**Determination of \( \lambda \)**

From

\[
w_t \ell_t (h) = \theta \frac{\sigma - 1}{\sigma} p_t (h) Y_t (h),
\]

the aggregate is

\[
w_t \int_{\ell_t}^{n_t} \ell_t (h) dh = \theta \frac{\sigma - 1}{\sigma} \int_{\ell_t}^{n_t} p_t (h) Y_t (h) dh
\]

\[
= \theta \frac{\sigma - 1}{\sigma} \int_{\ell_t}^{n_t} p_t (h) dh \int_{\ell_t}^{n_t} \left( \frac{p_t (h)}{P_{H,t}} \right)^{-\sigma} Y_{H,t} dh
\]

\[
= \theta \frac{\sigma - 1}{\sigma} \frac{Y_{H,t}}{P_{H,t}^{-\sigma}} \int_{\ell_t}^{n_t} p_t (h)^{1-\sigma} dh
\]

\[
= \theta \frac{\sigma - 1}{\sigma} \frac{Y_{H,t}}{P_{H,t}^{-\sigma}} P_{H,t}^{1-\sigma}
\]

\[
n_t w_t \ell_t (h) = \theta \frac{\sigma - 1}{\sigma} Y_{H,t} P_{H,t}
\]
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The same expression from the fully-flexible price version applies, as happens with that of profits.

2.7.7.2 LCP

New expressions for equations (2.24) and (2.25) in the text:

\[
\pi_t (h) = p_t (h) (L_t c_t (h) + n_{t+1} k_t (h)) + \\
+ e_t p_t^* (h) (L_t^* c_t^* (h) + n_{t+1} k_t^* (h)) - w_t \left( \frac{Y_t (h)}{A_{H,t}} \right)^{1/2}
\]

or

\[
\pi_t (h) = \left( 1 - \theta \frac{\sigma - 1}{\sigma} \right) p_t (h) V_t + \\
+ \left( 1 - \theta \frac{\sigma - 1}{\sigma} \right) e_t p_t^* (h) D_t
\]

where \( V_t = L_t c_t (h) + n_{t+1} k_t (h) \), \( D_t = L_t^* c_t^* (h) + n_{t+1} k_t^* (h) \) and, symmetrically, \( V_t^* = L_t c_t (f) + n_{t+1} k_t (f) \), \( D_t^* = L_t^* c_t^* (f) + n_{t+1} k_t^* (f) \). Moreover,

\[
w_t \ell_t (h) = \theta \frac{\sigma - 1}{\sigma} p_t (h) V_t + \theta \frac{\sigma - 1}{\sigma} e_t p_t^* (h) D_t
\]

Using the previous equations in the budget constraints:

\[
P_t C_t = \theta \frac{\sigma - 1}{\sigma} \Psi_t + (1 - \lambda) \left( 1 - \theta \frac{\sigma - 1}{\sigma} \right) \Psi_t + \lambda e_t \left( 1 - \theta \frac{\sigma - 1}{\sigma} \right) \Psi_t^* \\
- (1 - \lambda) I_{H,t} - \lambda e_t I_{F,t};
\]

\[
P_t^* C_t^* = \theta \frac{\sigma - 1}{\sigma} \Psi_t^* + \frac{\lambda}{e_t} \left( 1 - \theta \frac{\sigma - 1}{\sigma} \right) \Psi_t + (1 - \lambda) \left( 1 - \theta \frac{\sigma - 1}{\sigma} \right) \Psi_t^* \\
- \frac{\lambda}{e_t} I_{H,t} - (1 - \lambda) I_{F,t},
\]

where \( \Psi_t = p_t (h) V_t + e_t p_t^* D_t \) and \( \Psi_t^* = \frac{v(k_t^*)}{v_t^*} V_t^* + p_t^* (f) D_t^* \). Let’s define \( \Delta \Psi = \Psi_t - e_t \Psi_t^* \).

Hence,

\[
\Delta \tilde{C} = \left[ \theta \frac{\sigma - 1}{\sigma} + (1 - 2\lambda) \left( 1 - \theta \frac{\sigma - 1}{\sigma} \right) \right] \Delta \Psi - (1 - 2\lambda) \Delta k
\]

and

\[
\Delta \Psi = \left[ p_t (h) (L_t c_t (h) + n_{t+1} k_t (h)) + e_t p_t^* (h) (L_t^* c_t^* (h) + n_{t+1} k_t^* (h)) \right] - \\
e_t \left[ \frac{p_t (f)}{e_t} (L_t c_t (f) + n_{t+1} k_t (f)) + p_t^* (f) (L_t^* c_t^* (f) + n_{t+1} k_t^* (f)) \right];
\]
\[ \Delta \Psi = (2\gamma - 1) \Delta C + (2\delta - 1) \Delta k. \]

Notice that \( \Delta \Psi = \Delta Y \) and, so, \( \Delta C \) is equal to the previous cases. Hence, the expression of \( \lambda \) is still true.

2.7.8 Nominal Rigidities with a General Monetary Policy

2.7.8.1 Checking the flexible-price solution

If we set \( \Theta_t = 0 \) and \( \frac{\mu}{Z_t \mu_t} = \frac{\sigma - 1}{\sigma} \), it is easy to see that the above equations become identical to the case of the flexible-price economy.31

\[
(1 - 2\lambda) \left[ 1 - \frac{\sigma - 1}{\sigma} \right] + \frac{\sigma - 1}{\sigma} = \frac{(1 - 2\lambda)}{(2\delta - 1)};
\]

\[
(1 - 2\lambda) \left[ 1 - \frac{\sigma - 1}{\sigma} \right] + \frac{\sigma - 1}{\sigma} = \frac{(1 - 2\lambda)}{(2\delta - 1)}.
\]

With solution:

\[
1 - \lambda = \frac{1 - 2\delta + \sigma \delta}{1 - 2\delta + \sigma}; \tag{2.77}
\]

\[
\lambda = \frac{1 - \delta}{1 + \frac{1 - 2\delta}{\sigma}}. \tag{2.78}
\]

Some interesting cases:

- With no home bias, for \( \delta = 1/2 \), \( \lambda = 1/2 \). This is the perfect pooling solution.
- When goods become more and more substitute, \( \lim_{\sigma \to \infty} (1 - \lambda) = \delta \). The share coincides with Home bias.
- In general, the share of portfolio allocated on home assets, \( (1 - \lambda) \), is increasing in \( \delta \) and decreasing in \( \sigma \):

\[
\frac{\partial (1 - \lambda)}{\partial \delta} = \frac{(\sigma - 2) (1 - 2\delta + \sigma) + 2 (1 - 2\delta + \sigma \delta)}{(1 - 2\delta + \sigma)^2} = \frac{\sigma (\sigma - 1)}{(1 - 2\delta + \sigma)^2} > 0.
\]

\[
\frac{\partial (1 - \lambda)}{\partial \sigma} = \frac{\delta (1 - 2\delta + \sigma \delta) - (1 - 2\delta + \sigma)}{(1 - 2\delta + \sigma)^2} = \frac{(\delta - 1) (\sigma + \sigma \delta + 1 - 2\delta)}{(1 - 2\delta + \sigma)^2} < 0.
\]

31 Imposing \( \theta = 1 \) for simplicity in this section.
BIBLIOGRAPHY


CHAPTER 3

GAINS FROM FINANCIAL INTEGRATION AND TRADE OPENNESS: A QUANTITATIVE ANALYSIS

3.1 Introduction

International financial integration is believed to have two major potential benefits: it enhances an efficient allocation of capital and helps countries sharing risk by reducing consumption and income volatilities. So far, however, neither empirical nor theoretical studies are able to clearly demonstrate to what extent these claims are true. This paper uses the New Open Economics Macroeconomy (NOEM) model developed in chapter II to shed some light on the theoretical side of this literature. It addresses the open debate on the interrelation between different degrees of financial integration and trade openness with international volatilities.

The second chapter of this dissertation offered a stylized set-up to analyse international transmission. In Chapter three I exploit this set-up to develop a quantitative analysis with the tools provided by this framework. I experiment with the economy assuming two different financial structures. The first one presents a set-up of incomplete financial markets. Individuals do not have access to a complete set of arrow-debreu contingent-claims, but only to shares of home and foreign firms, as well as to an international riskless bond. However, as it has been proved in the previous chapter, these are sufficient to provide with perfect-risk-sharing. This scenario is called RS (risk sharing) or FI (fully integrated) in the text. The second one is a financial isolated economy. Home and Foreign countries can freely trade on goods, but individuals are not able to invest abroad.

These two environments show the effects of international integration of capital markets and trade openness on macroeconomic volatilities. On the one side, as a large part of empirical analysis shows, the model reports an increase of the volatilities of the real exchange rate and the terms of trade as the country opens its capital market. On the other side, consumption volatility is affected differently by the financial integration process depending on the source of the real shock.

Concerning the links between trade openness and volatilities, neither the causality direction nor its strength is clear. Empirical studies, often claim that this relationship is extremely
3.1. INTRODUCTION

weak. At a theoretical level, real exchange rate (RER) uncertainty may discourage international trade but, simultaneously, inter-country trade favours international transmission of shocks via RER and terms of trade (TOT), affecting their volatilities. Herwartz and Weber (2007) argue that the link is quite heterogeneous among countries and that it might be non-linear.

Traditional literature claims the existence of a direct and indirect effect of trade openness over output growth: on the one hand, openness generates a direct positive effect on output growth; on the other, however, it enhances aggregate production volatility, which causes a reduction of output growth. For this triangular relation to be possible, either the direct effect must outweigh the indirect one or the relation between openness to trade and output volatility has been incorrectly observed. This paper sustains the first explanation, since it finds that, in general, international trade increases output and consumption volatilities under any level of financial integration.

The analysis is carried out in a set-up with market dynamics. The introduction of the extensive margin allows for a distinction to be made between different types of productivity disturbances. One can shock either the productivity of manufacturing or that of the technology of creation of new varieties. Indeed, the first chapter of this dissertation studies the opposite effects generated by these two shocks on the allocation of firms. I do it in a simple closed-economy model with firms’ entry. The presence of endogenous entry can alter the dynamic response to shocks, leading to greater persistence in the effects of monetary and real shocks. Entry may have notable welfare effects, to the degree that households derive utility from greater variety (the love of variety in consumption and investment) and because the entry of new firms raises competition in a market. Thus, it is relevant to take into account the behaviour of the economy after these different impacts, as well as, to observe the welfare effects in terms of the variations in the range of varieties available in a specific market.

The rest of the paper is organized as follows: section two provides a brief literature review to situate the paper and inform of the blueprint on the topic. Section three presents the simplest analytical version of the model, which replicates the closed economy of chapter II. Real shocks, on the productivity of creation and of production, are simulated with Schmitt-Grohé algorithm both under flexible and one-period-in-advance price setting. The scope of this section is, first of all, to get in touch with the more general model, as well as to provide a tool to observe how trade openness alone affects volatilities. To do the latter, one can compare the closed economy with a financially isolated country. Section four presents

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1 See Corsetti et al. (2004.)
2 See Bergin and Corsetti (2006.)
3.2. BRIEF LITERATURE REVIEW

the quantitative results for the open economy: first under financial integrated markets and, afterwards under the financial autarky case. Section five summarizes the results for different degrees of trade openness. Finally, section six concludes. An appendix with some extra explanatory plots and the parametrization used in simulations can be found at the end.

3.2 Brief Literature Review

3.2.1 Financial Integration and Volatility

Understanding the links between globalization and the dynamics of macroeconomic volatility has recently come to the forefront. This is, mainly, due to a burgeoning literature that describes the first-order effects volatility has on welfare and, more recently, to the apparition of a number of papers documenting the declining volatility of output in the US and most industrial economies since the mid-eighties. However, existent literature disagrees considerably on the strength and directions of such relationships. Available empirical evidence on the effects of financial integration on volatility is very far limited. Hence, at theoretical level, the effects of increased integration on business cycle volatility are not clear either: on the one hand, increased financial integration allows households to cushion against adverse domestic shocks by lending and borrowing abroad. This would cause a decline in the volatility of consumption. On the other hand, financial integration increases the potential for the magnification of domestic financial markets when foreign capital enters them. If this happens, output and investment volatilities will increase.

Kose et al. (2003) address these questions at the empirical level and find that financial openness, as measured by gross capital flows as a ratio to GDP, is associated with an increase in the ratio of consumption volatility to income volatility, opposite to the theoretical risk-sharing benefits of capital globalization. However, this relationship is found to be non-linear.3 Above a certain threshold the ratio starts to decrease again. Moreover, capital account openness is associated with higher output volatility, although the coefficient is only marginally significant.

The model I present compares a non-integrated economy with a fully integrated one, i.e. obviously, this full integration would be above any measured threshold. As Kose et al. found in the data, this highly integrated economy provides households with more capacity of consumption smoothness. The ratio of consumption to output volatility falls with the jump from the autarkic regime to the open one. In addition, I also observe higher output volatility in the risk sharing economy. These quantitative matches give some robustness to the model

3Evans and Hnatkovska (2006) explored an economy under three different levels of financial integration and also found a non-linearity on this relation.
3.2. BRIEF LITERATURE REVIEW

to be considered a suitable tool to explore industrialized economies’ responses to shocks and their pass-through.

An older paper by Mendoza (1994) develops a stochastic dynamic business cycle model and concludes that quantitative variations in the volatility of output and consumption are quite small in response to the changes in the degree of financial integration. Moreover, it seems that larger and more persistent shocks enhance output volatility for higher levels of financial integration. On the contrary, Baxter and Crucini (1995) show that consumption volatility decreases, although output volatility increases as the level of financial integration goes up. They argue that these differences lie on the wealth effects and their interaction with different capital market regimes. Differently, Bekaert et al. (2004) find that financial liberalization tends to be associated with lower consumption volatility. Finally, Buch and Yener (2005) provide another proof of empirical ambiguity. They show that, in spite of the fact that G7 countries have become more open for financial capital in legal terms over the past decades and that capital flows have increased rapidly, there has been no consistent pattern for consumption volatility to increase or decrease. Indeed, they found that the change over time of the ratio of volatilities often depends on the country and the period of time. This result is completely compatible with those reported in this paper: since the link between consumption and output volatility with financial globalization and its sign depends on the source of the shock, the presence of different shocks to the economy may perfectly be the explanation of the unclear correlation.

3.2.2 Trade Integration and Volatility

The debate is equally open for the link between openness of the goods market and volatility. Although there seems to be some consensus on the fact that more open economies are more volatile, not all the specialists agree. Part of the literature retains that the effects are tight to the kind of shock -sector specific or common- and to the patterns of trade specialization. If this association is true and trade openness, nowadays, consists in the increase in interindustry specialization across countries, industry-specific shocks would result in a rise in output volatility, as Krugman (1993) set. Moreover, in the case of highly persistent shocks, consumption volatility would increase as well. If trade increment is done via intraindustry because of the higher country-specialization in specific parts of the production process chain, the volatility of output could decline. This evidence can be interpreted as proof of the relation between volatilities and both financial and trade integration, i.e. globalization. Some theoretical research suggests that output volatility has a positive interaction

4See Razin and Rose (1994.)
with trade openness in the developing countries while it maintains a negative link in developed economies. Prasad et al. (2003) document that recent decades data on consumption volatility shows a decline in developed economies. Furthermore, the level of consumption volatility in developing countries is above that of the developed ones.

One can refer to Easterly et al. (2001) and Kose et al. (2003) to find some empirical evidence of what the present model reports on the side of trade openness. Easterly et al. (2001) carry an exploration of the sources of macroeconomic volatility using data for a sample of 74 countries over the period 1960-97 and conclude that an increase in the degree of trade openness leads to an increase in the volatility of output and consumption. However, the significance of these two measures of international globalization on macroeconomic volatility is not very relevant. Kose et al. (2003) give some attention to trade openness too and suggest the existence of a positive effect of international trade on volatilities due to the higher vulnerability of more open economies to external shocks.

Indeed, recent research tends to sustain the complementarity of trade integration and financial integration on macroeconomics volatility.\(^5\)

### 3.3 Reduced Version: the Closed Economy

This simple closed economy version becomes the key for the present research to decouple the effects of trade openness from those of financial integration on macroeconomic volatilities, for the home country. To do such a comparison I confront the quantitative results of this section with the case of financial autarky developed in section four.

#### 3.3.1 Set-up

I concentrate on a completely autarkic economy with monopolistic competition and an endogenous number of firms or varieties. Firms set prices flexibly.

##### 3.3.1.1 Households

Households maximize the following logarithmic utility on consumption and leisure with respect to \(\ell_t(j)\) and \(n_{t+1}\):

\[
U = \sum_{t=0}^{\infty} \beta^t \left( \ln C_t + \kappa \ln (N - \ell_t(j)) \right),
\]

subject to the budget constraint

\[
C_t = w_t \ell_t(j) + r_t K_{t-1} n_t - K_t n_{t+1},
\]

where $\beta$ is the discount factor, $K_t$ is the exogenous requirement for the creation of a new firm for next period. It is a basket of investment on all the final goods produced that period. $C_t$ is the basket of consumption of the produced varieties, $N$ is the total endowment of time per period, $\ell_t(j)$ is household’s elastic labour supply, $\kappa$ is the disutility of labour, $r_tK_{t-1}n_t$ is the total amount of dividends the household receives today as payment of her last-year investment. $r$ measures the returns to capital, which depend on the profits generated by firms and $n$ is the number of varieties. The composition of the baskets are:

$$K_t = \left[ \int_0^{n_t} k_t(h)^{\sigma-1} dh \right]^{\frac{\sigma}{\sigma-1}};$$
$$C_t = \left[ \int_0^{n_t} c_t(h)^{\sigma-1} dh \right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ is the intratemporal elasticity of substitution between goods. Notice that, for simplicity, CES aggregators are identical, so that I can write:

$$Y_t = C_t + n_{t+1}K_t$$

$$= \left[ \int_0^{n_t} y_t(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}.$$

Moreover, $r_t$ is the payment made by firms to investors for the capital lent in the creation. It is equivalent to the dividends. The price index has been normalized to 1. Thus, all the prices are in terms of final goods of consumption.

A justification regarding the functional form of the utility is in order. Per-capita consumption enters the instantaneous utility function log-linearly because, in the presence of separability between consumption and labour and of a Cobb-Douglas production function (the aggregator is a CES), it is the only formulation consistent with stationary labour supply in a growing economy. Moreover, the disutility of labour is modelled as nonlinear to allow us to derive an analytical expression for the labour supply function.

### 3.3.1.2 Firms

**Final-Goods Firms** do not use labour or capital. They only aggregate goods to construct the basket of the so called final good, $Y_t = \left[ \int_0^{n_t} y_t(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}}$, which is used both for consumption and capital for creation of next-period firms. They maximize profits in a competitive market by choosing $y_t(h)$:

$$\Pi_{F,t} = \left[ \int_0^{n_t} y_t(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} - \int_0^{n_t} p_t(h) y_t(h) dh = 0,$$
where $p_t(h)$ is the monopolistic price that the intermediate-good firms $h$ set for its variety. The f.o.c. is

$$y_t(h) = p_t(h)^{-\sigma} Y_t.$$ 

Since firms are homogeneous, it is known that

$$Y_t = n_t^{\frac{\sigma}{\sigma-1}} y_t(h).$$ \hspace{1cm} \text{(3.2)}$$

Hence,

$$p_t(h) = n_t^{\frac{1}{\sigma-1}}.$$ \hspace{1cm} \text{(3.3)}$$

This tells us that the price per variety expressed in units of consumption increases when the number of firms does. For a given labour supply, if $n$ increases, the amount of labour per firm must decrease. By the decreasing returns to scale, this labour is more productive, so that the real wage, which enters the marginal cost, is higher, pushing the prices up. Schematically, $\uparrow p_t(h) = \frac{\sigma}{\sigma-1} \frac{1}{A_t^\theta} y_t(h)^{\frac{1}{1-\theta}} = \frac{\sigma}{\sigma-1} \frac{1}{A_t^\theta}$. However, the price index will decrease:

$$\downarrow P_t = \downarrow n_t^{\frac{1}{\sigma-1}} \uparrow p_t(h).$$

**Intermediate-good firms** produce differentiated varieties and maximize profits choosing the amount of labour, $\ell_t(h)$:

$$\pi_{I,t} = p_t(h) y_t(h) - w_t \ell_t(h) - r_t = 0,$$ \hspace{1cm} \text{(3.4)}$$

subject to the technology constraint:

$$y_t(h) = A_t \ell_t(h)^\theta.$$ \hspace{1cm} \text{(3.5)}$$

The first order condition is:

$$p_t(h) \frac{\partial y_t(h)}{\partial \ell_t(h)} + y_t(h) \frac{\partial p_t(h)}{\partial \ell_t(h)} = w_t.$$ 

Being aware of $p_t(h) = \left( \frac{Y_t}{y_t(h)} \right)^{\frac{1}{\sigma}}$ while solving $\frac{\partial p_t(h)}{\partial \ell_t(h)}$, the derivation yields:

$$w_t = A_t \partial h \ell_t(h)^{\theta-1} p_t(h) \left(1 - \frac{1}{\sigma} \right).$$ \hspace{1cm} \text{(3.6)}$$

Finally, labour market clearing condition, assuming that the total population is 1, is

$$\ell_t(j) = \ell_t(h) n_t,$$ \hspace{1cm} \text{(3.7)}$$
and both productivity shock follow AR1 processes:

\[ A_t = \phi_t A_{t-1} + (1 - \phi_t) A_0 + \eta_t; \tag{3.8} \]

\[ K_t = \phi_{k,t} K_{t-1} + (1 - \phi_{k,t}) K_0 + \eta_{k,t}, \tag{3.9} \]

where \( \eta_t \) is iid.

### 3.3.2 Impulse responses

#### 3.3.2.1 Shock on Technology of Production, \( A_H \)

By modelling the above set-up and shocking the economy with a 1% increase of \( A_t \), one obtains the following impulse responses:

**Notation 3.1**: in the plots, \( Y \) is aggregate output; \( Y_H \), output per firm; \( PH \) is the price per variety and the CPI, \( P \), is normalized to one. \( LH \) is labour demand and \( LJ \) labour supply, \( R \) is the interest rate (\( R = 1 + i \)) and it is the inverse of the \( Q \) of the open economy. A variation of 1 means a deviation of 1% from the steady state value.

In the first period, when the shock occurs, the production per firm increases. The extensive margin cannot react because firms need one period to build-up. Since total output is higher in \( t=1 \) and productivity is expected to be still above its steady state value, households
save more in order to create more varieties. Because of the higher output, the household’s effort for renouncing to part of the consumption in favour of \( n \) is relatively lower. Hence, they want to create even more firms in the third and fourth period. Moreover, while \( n \) increases, the CPI decreases,\(^7\) making the nominal cost of creation \( P \times K \) relatively low.

3.3.2.2 Shock on technology of creation, \( K \)

These are the impulse responses of a negative shock on \( K \), i.e. a decrease (improvement) in the amount of capital needed in the creation of a new variety or firm. Most of the effects are reflected in the economy from time \( t + 1 \) onwards. That is due to the one-period time-to-build needed by firms. The number of firms goes up because of the relatively high efficiency of creation. Consequently, every firm produces less \( (y(h) \text{ goes down}) \) and the total product increases. The latter occurs because \( \theta < 1 \), so that, technology experiences decreasing returns to scale on the labour input. For the same reason, firms demand less labour \( (\ell(h) \text{ decreases}) \) to be able to reduce their production. Since the marginal product of labour has increased, wages go up, as prices do. Households must sacrifice less in order to create an extra firm and, simultaneously, their purchasing power improves \( (w \text{ goes up and } P \text{ would decrease thanks to the upward in } n.) \) Hence, consumption is higher.

\(^7\)The CPI, \( P \) can not be plotted since it has been normalized to 1. The reasoning here is developed following the theoretical set-up explained above.
3.4 The Open Economy: Two Financial Regimes

This section explores the open economy I presented in Chapter two from the point of view of the international transmission. For this reason, I do not offer a detailed presentation of the set-up and the equilibrium solution. These can be consulted in Chapter II. Here, I modify slightly the utility function to introduce logarithmic disutility of labour.

I start by repeating the experiments carried out for the closed economy, i.e. a study of the impact of shocks in the country-specific productivity of production and of technology is carried on. I concentrate on the flexible-price regime with full integration first; afterwards, the case of financial autarky is developed.

3.4.1 General Set-up

Households maximize the following logarithmic utility on consumption and leisure with respect to \( \ell_t(j) \) and \( \lambda_{t+1} \):

\[
U = \sum_{t=0}^{\infty} \beta^t \left( \ln C_t + \kappa \ln (N_t - \ell_t(j)) \right),
\]

where \( 0 < \beta < 1 \) is the discount factor and \( U(.) \) is a utility function defined over the consumption of a basket \( C_t \) and a logarithmic disutility of the labour effort, \( \kappa \). \( N \) is the total endowment of time a household can allocate between leisure and work per period and \( \ell_t(j) \) is the household’s elastic labour supply. The consumption basket is given by a Cobb-Douglas aggregator over the bundles of tradables produced in the home (\( C_H \)) and foreign (\( C_F \)) country (i.e. a CES basket with unit elasticity),

\[
C_t = C_{H,t}^{\gamma}C_{F,t}^{1-\gamma},
\]

where \( \gamma < 1 \). \( C_H \) and \( C_F \) are CES aggregators over the \( n(n^*) \) varieties produced in the home(foreign) country. For simplicity, I assume identical elasticity of substitution, \( \sigma \):

\[
C_{H,t} = \left( \int_{h=0}^{n^*} c_t(h) \left( 1 - \frac{1}{\sigma} \right) dh \right)^{\frac{\sigma}{\sigma-1}};
\]

\[
C_{F,t} = \left( \int_{f=0}^{n^*} c_t(f) \left( 1 - \frac{1}{\sigma} \right) df \right)^{\frac{\sigma}{\sigma-1}}.
\]

Here, \( h \) and \( f \) denote a specific variety of the corresponding country. Households all over the world finance the creation of firms in both countries. In order to construct her portfolio of investment, home household purchases a fraction \( \lambda_{F,t+1} \), of the shares issued by foreign-country firms and \( \lambda_{H,t+1} \) of the domestic firms, which will start producing next period.
3.4. THE OPEN ECONOMY: TWO FINANCIAL REGIMES

She affords her consumption expenditure and investment with the dividends received from currently active firms at home and abroad, proportionally to her current portfolio allocation: $\lambda_{H,t}$, $\lambda_{F,t}$ and her labour income. The budget constraint is

$$B_{t+1} + \lambda_{H,t+1} \int q_t^h(h) \, dh + e_{t+1} \lambda_{F,t+1} \int q_t^f(f) \, df +$$

$$+ \int p_t(h) c_t(h) \, dh + \int p_t(f) c_t(f) \, df =$$

$$= \lambda_{H,t} \int \pi_t(h) \, dh + e_{t+1} \lambda_{F,t} \int \pi_t^f(f) \, df + w_t \ell_t(j) + (1 + i_t) B_t. \tag{3.13}$$

$\pi_t(h)$ are the profits of firm $h$. An initial investment is needed for a new firm to start producing. $q_t(h)$ ($q_t^f(f)$) is the cost necessary for the creation of a firm in the home (foreign) country. $\pi_t(h)$ ($\pi_t^f(f)$) are the profits of a single home (foreign) firm in home (foreign) currency, i.e. the total amount of dividends the household receives today as payment of her last-year investment; $e_t$ is the nominal exchange rate ($p_t(h) = e_t p_t^*(h)$); $c_t(h)$, the domestic demand for good $h$; $n_t$ is the number of firms allocated at home and $w_t$ is the wage. $B_t$ is the international riskless bond. Finally, $\gamma$ indicates the home-bias on consumption preferences. The super script $^*$, $x^*$, stands for the foreign country.

The households’ problem yields the first order conditions I have already presented and detailed in Chapter two, except for that of labour. Due to the variation in the utility function, it becomes:

$$\frac{w_t}{P_t C_t} = \frac{\kappa}{N_t - \ell_t(j)}, \tag{3.14}$$

I do not specify the firms problem either, since it is exactly that explained in Chapter II.

3.4.2 Full Financial Integration

Let’s analyse the international transmission in an economy with market dynamics. To do so, the model is treated with Schmitt-Grohè algorithm. The price indexes, $P$ and $P^*$, have been taken as numeraires. Hence, all the variables are expressed in terms of final goods of consumption. I report, below, the plots produced for the scenario with financial integration, assuming a home bias in consumption of 0.65, whereas the demand for capital lays a 75% on home goods. The latter parameterization, as argued in Chapter II, has been chosen to match real data for home bias in portfolio. Indeed, 0.75-0.70 implies an equity bias towards home goods of 0.67-0.63, a range in which the average EU-15 level of home bias in portfolio laid in...
3.4. THE OPEN ECONOMY: TWO FINANCIAL REGIMES

The actual share of imports of equipment goods over total acquisitions of equipment varies considerably for developed countries, but almost all of them show home bias\(^9\).

The quantitative experiment has been repeated for shocks on the productivity of production, \(A_H\) and \(A_F\), and of creation, \(K\) and \(K^*\), as well as for several levels of home bias in both capital and consumption. When relevant, the differences in the impacts caused by these different levels of bias are explained, although the graphs are not included in the text for space reasons.

**Notation 3.2** Notation in the plots works as follows: \(LH\) is \(\ell(h)\) in the text, \(LF\) is \(\ell(f)\), \(LJ\) is \(\ell(j)\) and \(N\) is \(n\). A subscript \(s\) is sometimes added to indicate foreign variables, \(RER\) is the real exchange rate and \(TOT\), the terms of trade. In the figures reporting savings and investment dynamics, \(h\) stands for home and \(f\) for foreign, finally, \(I\) is investment and \(S\), savings. The rest of the variables keep the names used in the analytical analysis.

3.4.2.1 Shocks on the productivity of production, \(A_H\)

When the Home country is impacted by an improvement on its technology of production, every produced unit of variety \(h\) becomes cheaper. Firms are able to produce as much with lower levels of labour. Hence, \(\ell(h)\) decreases and every single household takes some holidays (i.e. \(\ell(j)\) decreases simultaneously, in the first period) due to the wealth effect. Notice, however, that \(\ell(h)\) experiences an immediate negative jump, followed by further decrease in the subsequent periods. Although firms need less labour to produce at the same level, households desire to take advantage of the shock to enlarge the number of available varieties in the market. Thus, the number of firms goes up altogether with \(\ell(j)\), from the second period. As intuition tells us, the economy adjustment works mostly via the intensive margin (more production per firm), but the set-up of new firms is also cheaper, since the same goods are used for both consumption and investment, and that is the reason for having some adjustment via extensive margin too. Indeed, consumption increases (even more in the second period, when part of the extensive margin adjustment has already been done) but it does not absorb the total variation of aggregate output \((Y_H)\). Finally, the home price index goes strictly down due to two effects: first, the decrease of the price per variety \(p(h)\) and, second, the increase in \(n\).

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\(^8\) I decided to be consistent with the previous chapter in choosing the values of the parameters here. Equity home bias for EU-15 in 2003 was around 65%. However, I do not mention the transition of \(\lambda\), the level of portfolio diversification since it has been proved that a constant value is optimal regardless of the shocks impacting the economy and the passage of time. There are no transition movements in it.

\(^9\) See Eaton and Kortum (2001) and the explanation in the introduction of Chapter II. Denmark is an exception to this empirical fact.
3.4. THE OPEN ECONOMY: TWO FINANCIAL REGIMES

Home Country

Foreign Country

Figure 3
3.4. **THE OPEN ECONOMY: TWO FINANCIAL REGIMES**

International Variables and others

Let’s focus, now, on the foreign country. Terms of trade worsen for home, making imports cheaper for the neighbours. However, due to the home bias in consumption and capital, the foreign country experiences a lower upwards in consumption during the transition back to
3.4. THE OPEN ECONOMY: TWO FINANCIAL REGIMES

the steady state. Demand for goods $f$ is higher and expected to be over the steady state value for some time. To create more varieties in this country would not be a good strategy, since most of the goods used in the building-up are produced there (due to the presence of home bias), at the higher price the extra demand has generated and with a relatively low productivity. In fact, $n^*$ goes downward. For this reason, the foreign aggregate output decreases in spite of the fact that every firm contracts more labour ($\ell(f)$ is higher) and that, consequently, per firm product is higher. To put it in different words, initially, the expenditure switching effect is high and turns demand towards home cheaper goods. This forces foreign firms to reduce their outputs and, consequently, the amount of contracted labour. Foreign households enjoy more leisure in this period. Later, the recovering starts. The number of firms at home can be adjusted. Foreign profits are above the equilibrium from the moment the extensive margin adapts ($n^*$ reduces) to the new situation and the increase in wages encourages foreign households to choose this higher leisure ($\ell^*(j)$ decreases). Notice that the shock on the productivity generates more volatility on home extensive margin than in the foreign one. This is the cause of the differences on wage transitions. In the moment of the impact and during the first period, the increase in $P_F$ is compensated by the drop in $P_H$. Terms of trade worsen for home. In the second period, the increase of $n$ and decrease of $n^*$ generate a new decrease in $P_H$ and the increase in $P_F$, which explains the extra deterioration of the terms of trade. All this permits the non-shocked (foreign) economy to benefit from the technology improvement too. It is able to consume more and to perceive higher wages, which enhance the demand for foreign goods.

The intertemporal adjustment identified in most of the variables, i.e. the fact that $\ell(j)$, $p(h)$... change the sign of the distance from the steady state between first periods and the subsequent ones, and that some of them like $\ell(h)$, $Y_H$,... experience an initial jump, followed by a further downward/upward movement in the same direction, can be interpreted as a consequence of the adjustment cost of investment. Indeed, in the first period, in which the number of firms is already set, any shock bumps in an economy where the costs of adapting the level of investment are infinite. Although here, the rigidity comes from $n$ instead of capital itself. Afterwards, it can start walking again towards the equilibrium. In a typical economy with costs of capital adjustment, it would not suffer a bang after one period, as $n$ does, but it would smoothly move up. Moreover, it would not reach such a high maximum(minimum) level due to the fact that the remission of the effects of the positive(negative) shock would make such a large reaction worthless.

Finally, the real exchange rate depreciates. This is explained by the kick the consumption price indexes receive from variations of $n$. The depreciation of the real exchange rate (RER)
generates the distance one sees between the two interest rates \( Q_t = \frac{1}{1+i_t} \) and \( Q^* = \frac{1}{1+i^*_t} \), generally called uncovered interest parity.

When \( \delta \) is high enough, the home price index for capital responds downwards to a positive shock of \( A_H \). Mathematically, a heavier weight is given to \( P_H \), which decreases with the shock:

\[
\downarrow P_{K,t} | \delta | = \left( \frac{P_{H,t}}{P_{F,t}} \right)^{\delta} \left( \frac{P_{F,t}}{P_{F,t}} \right)^{1-\delta}.
\]

Intuitively, entrepreneurs are able to buy most of the inputs necessary for the setup of a new variety at a lower price (because the composite \( K \) contains basically home goods). This explains that \( n \) reacts even more positively for high levels of \( \delta \).

In the case both countries have identical preferences on consumption and capital goods, i.e. \( \gamma = \delta = 0.5 \), the foreign country would enjoy an increase in real terms of consumption equal to that experienced at home. All the insurance goes via terms of trade, since \( P_H \) and \( P_F \) vary exactly equal but in opposite directions. Any improvement in a country is transmitted to the other via relative prices movements, keeping exchange rates (real and nominal) and the aggregate price indexes (for consumption, \( P \) and \( P^* \), and for capital, \( P_K \) and \( P^*_K \)) untouched. Once more, the number of firms in the foreign country decreases, whereas production per firm is higher. In contrast to the case with home biases, foreign profits go down. Compared to the economy with home biases, in the \( \gamma = \delta = 0.5 \), every foreign firm contract in more workers and wages are more expensive due to the further increase in consumption.

The last group of plots in Figure 4 report the dynamics of savings \((S = P_H Y_H - PC)\) and \(S^* = P_F Y_F - P^* C^*\) and investment \((I = n_{t+1} P_K K^* \) and \(I^* = n^*_{t+1} P^*_K K^*\)) in both countries, as well as the part of investment financed by the other country \((S - I \) and \(S^* - I^*.)\)

Foreign savings describe an upward hooked form caused by foreign output response to the shock, whereas investment decreases immediately due to the higher price index of capital goods and the consequent reduction of the number of firms. These two movements favour an improvement of \( S^* - I^* \) abroad. At home, total investment goes up despite of the reduction in the price index. Do not forget to keep in mind that the share of investment financed by the foreign country is always constant, regardless of the shock, i.e. the international portfolio is at its optimum, maintaining a time-invariant proportion of diversification, as it has been analytically proved in Chapter II.

\[\text{A textbook definition of the current account balance says it is the change in the value of its net claims on the rest of the world -the change in its net foreign assets- over a period. The plots do not show this change but, after setting the international bond to zero, shares on } n + n^* \text{ are the set of available claims in the economy. The home net possession of foreign claims is measured by the difference between } (S - I) - (S^* - I^*), \text{ i.e. the part of investment not covered by home savings, which is financed by the foreign country minus the part of investment made abroad and not financed by foreign savings during the period.}\]
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3.4.2.2 Shocks on the productivity of creations, $K$

Opposite to a positive shock on the productivity of production, an unanticipated improvement of the technology of creation in the home country, i.e. a decrease of $K$, has a positive effect on the number of firms at home. This is consistent to the findings in the Chapter I of this dissertation and the small experiment carried out in section two. Since the creation of varieties is relatively cheap in the home country, entrepreneurs can take advantage of their efficiency and enlarge the range of products in the market. For the same reason, only few foreign entrepreneurs decide to enter the market, indeed, $n^*$ decreases.

An interesting feature of the effects of a shock in this economy, either on $A$ or on $K$, is the intertemporal reaction of labour supply. $\ell(j)$ is below its steady state value in the first period but goes up afterwards. This is surprising at first sight, since one expects consumption and labour to be positively correlated. Indeed, they are from the second period onwards, but at the moment of the shock the wealth effect wins. With the positive shock,\footnote{‘Positive’ in the sense it is beneficial for the Home country. It is an improvement in technology, although, mathematically, it is a negative shock on $K$, since $K$ decreases by 1%.} the number of firms dramatically increases after the first period in which it is fixed. Hence, working today or tomorrow is really different for households: today, the labour effort is spread among a smaller range of varieties. The total labour is more productive when it is allocated in more firms, due to the decreasing returns to scale. Moreover, there is a wealth effect via wages: they increase humbly when the shock occurs but steep upwards in the following periods. Labour supply accompanies this increase and households offer more labour during the transition. However, while the rigidity in $n$ prevents the economy to adjust perfectly, they enjoy more leisure and wait.

The extensive margin suffers in favour of the intensive one: investment is cheap, so households do not need to save so much ($I$ decreases.) Initially, the wealth effect pushes labour supply down, increasing the cost of work for firms, which have less incentives to contract ($\ell(h)$ goes down) and decrease their outputs. In aggregate, the impact in the intensive margin is stronger than that of the extensive, thus total output is lower after the shock. Prices are above the steady state value due to the expensiveness of labour, but the price index is below because, on it, the impact from the increment of $n$ is higher than the one coming from the upwards prices.
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Home Country

Foreign Country

Figure 5

Arespa Castelló, Marta (2008), International Transmission, Firm Entry and Risk Sharing
European University Institute

DOI: 10.2870/14337
3.4. THE OPEN ECONOMY: TWO FINANCIAL REGIMES

International Variables and others

Investment and Savings

Figure 6

Arespa Castelló, Marta (2008), International Transmission, Firm Entry and Risk Sharing
European University Institute

DOI: 10.2870/14337
Since creation is less costly, more firms enter the market. That pushes profits down at home: firms are built-up until expected profits do not cover the set-up cost. In the foreign country, profits vary much less during the transition, compared to home profits. The small downward movement is generated by the higher wages and the initially lower production motivated by the expectation that less output will be allocated in capital goods -and so, demand decreases-, because the creation of firms is less efficient there. The foreign country absorbs part of the positive impact via terms of trade. Imports are cheaper now and this, altogether with the higher revenue from dividends -although home profits per firm are down, the aggregate is higher because of the greater number of active firms-, causes the improvement of foreign consumption.

For $\gamma > 0.5$, i.e. home bias in consumption, a decrease of the price of the basket of home goods, $P_H$, makes the real exchange rate lower. Hence, home experiences a depreciation in front of the foreign country which enhances the benefits to the other country. However, compared to the case without home biases, the main effects are qualitatively the same.

### 3.4.3 Financial Autarky

A large part of the experts in macroeconomics agrees on the incompleteness of the capital markets. I do not want to disregard this reality, here. The scope of this section is to study how the international transmission works under the simplest example of extreme incomplete markets: the case of financial autarky. The two countries are completely isolated in terms of capital flows, although they are able to trade in consumption and capital goods, as before.

Home (foreign) households must finance the total investment necessary to build up next period producing firms in the home (foreign) country. So far, the budget constraint was

$$ B_{t+1} + \lambda_{H,t+1} \int_{n^t_{t+1}} q_t(h) \, dh + e_t \lambda_{F,t+1} \int_{n^f_{t+1}} q^f_t(f) \, df + \int_{n^t_t} p_t(h) c_t(h) \, dh + \int_{n^f_t} p_t(f) c_t(f) \, df = \lambda_{H,t} \int_{n^t_t} \pi_t(h) dh + e_t \lambda_{F,t} \int_{n^f_t} \pi^f_t(f) df + w_t \ell_t(j) + (1 + i_t) B_t, $$

where, for the financial autarky scenario, $\lambda_H = 1$ (all firms set-up at home are financed by home agents), $\lambda_F = 0$ (home households do not invest abroad) and $B$, the international
riskless bond, does not exist anymore. Hence,

\[ \int_{n_{t+1}}^{n_t} q_t(h) \, dh + \int_{n_t}^{n_t} p_t(h) c_t(h) \, dh + \int_{n_t}^{n_t} p_t(f) c_t(f) \, df = \int_{n_t}^{n_t} \pi_t(h) \, dh + w_t \ell_t(j). \]  

(3.16)

Moreover, the economy must keep a balanced trade, since it can no longer cover any temporary excess of imports with international assets:

\[ c_t(h) p_t^*(h) n_t = c_t(f) p_t^*(f) n_t^*. \]  

(3.17)

The nominal value of home exports have to be equal to the nominal value of home imports, period by period.

Obviously, households have to withdraw the perfect risk sharing target. They are affected by shocks that impact the home economy, as well as, by those of the foreign, via terms of trade movements and the financial isolation prevent them from any kind of international insurance. They can only vary their level of investment to smooth some consumption.

### 3.4.3.1 Relative Volatilities: Financial Autarky vs Risk Sharing

The main point to be emphasized in the following analysis of shocks’ effects is the magnitude of the volatilities relative to those found in the fully integrated case. Regardless of the source or the direction of the shock, the economy with a high degree of financial integration shows, unambiguously, more volatility during the transition back to the steady state. Thus, when a country moves from a financial isolation to an integrated economy, shocks generate larger volatilities on terms of trade and the exchange rate. i.e. it experiences a deeper depreciation or appreciation. The effects of a positive and a negative impact are exactly symmetric, both for disturbances on \( A_H \) and on \( K \). This is true despite the current set-up, which is concerned with households’ love of variety (i.e. the price indexes include the number of available products in the market, \( P_H = n \frac{k}{1-k} p(h) \) and \( P_F = n^* \frac{k}{1-k} p(f) \)).

One may expect to observe asymmetric responses of the economy in front of shocks going in different directions, due to the fact that an increase or a decrease of \( n \) and \( n^* \) has a different degree of impact on CPI and, consequently, on \( P_K \) and \( P_K^* \). However, the asymmetric disturbances generated in any price index are corrected by the movements of the other.

Due to their structure, everything else equal, the optimal price indexes have asymmetric responses in front of decreases or increases of the number of firms:
The slope of \( P_H \) reduces with \( n \). Hence, when a shock on \( A_i \) or on \( K_i \) - where \( i \) stands for country \( H \) or \( F \) - causes an increase in \( n \), \( P_H \) experiences a stronger downwards movement than the upwards reaction after a decrease of \( n \). Moreover, for an equal variation of \( n \), the degree of volatility varies with the initial level of the number of firms.

This may make one think that national and international macroeconomic volatilities should be asymmetric after a positive or a negative shock on productivities. However, exactly the same asymmetry is present in the price index of the foreign country, \( P_F \). Following a shock in the home country, \( n^* \) has a reaction in the opposite direction and, in general, smaller than the response of \( n \). Thanks to the symmetry between \( P \) and \( P^* \) and between \( P_K \) and \( P_K^* \), this counter-movement perfectly offsets the asymmetric effect that, otherwise, one would see in plots, and permits us to affirm that volatilities do not depend on the direction of the shock.

Going back to the volatility comparison between regimes, the differences are intuitively clear. With financial autarky, a large depreciation makes imported goods expensive. Relative to the integrated economy case, the lower depreciation helps to contain the cost of investment that would be, otherwise, quite unsustainable in a financially closed economy. With the financial integration, however, investment is high and partially financed by foreign capital, which covers part of the negative consequences of the depreciation. Under financial autarky, instead, a large depreciation would force a huge decrease in investment; which would trigger a worsening in both economies due to the reduction in the number of traded varieties and the consequent increase of prices at home.\(^{12}\)

\(^{12}\)See the appendix for plots on compared volatilities.
3.4. THE OPEN ECONOMY: TWO FINANCIAL REGIMES

The relative variability of consumption depends on the kind of shock. If labour productivity, $A_H$, receives a shock, either positive or negative, consumption reacts more in the economy with full integration, whereas it is the financial isolated economy which describes a larger movement of consumption after a shock on capital.

The reason lies, once more, on the characteristics of investment funding. When the shock is (positive) on $A_H$, imports are more expensive in the RS because of the further depreciation on the TOT. This is true both for consumption and capital goods. However, in this case, foreign investors afford part of the extra cost of home capital and reallocate some investment from the foreign less efficient country to Home. This allows for a larger adjustment in the extensive margin ($n$ increases and $n^*$ decrease by more) under RS, which moderates aggregate profits. Thus, the amount of dividends is smaller for this scenario and households decide to take less and shorter vacations due to this wealth effect.

On the contrary, when the shock is negative on $K$ (i.e. decreases), investment is already cheap, so the size of the depreciation matters less for capital goods. In fact, $n$ and $n^*$ volatilities are almost the same under both regimes ($n$ is a bit more volatile for RS and $n^*$ is slightly more variable for FA). Consequently, home profits vary similarly. Hence, the differences in consumption volatility come from the level of depreciation: households lose less purchasing power when they are in an isolated economy and this allows them to consume more compared to the RS case.

3.4.3.2 Shocks on the productivity of production, $A_H$

At first sight, a positive impact on labour productivity, $A_H$, either in an economy with full integration or in a financially isolated country, is almost the same for the country where the shock occurs. However, the magnitude of the benefits in terms of production per firm and in aggregate is lower in the latter. Per variety price volatility is extremely small after the first period and needs few periods, compared to the rest of the variables, to get back to the steady state value and it does not decrease as much. This is due to the faster return of labour supply to its equilibrium level, which affects prices via individual production.

On the one hand, the volatility of the scale of production, $\ell (h)$ is larger, although the extensive margin gets a slightly higher peak (i.e. $n^{FA} > n^{PRS}$). One would expect just the opposite from the differences in the transitions for the cost of capital. $P_K$ does not decrease as much as in the fully integrated markets. So, to build-up a new firm is more expensive now. It is clear that under financial autarky, the wealth effect is enhanced. Home profits increase by more and the totality of them go to home households. This explains the extensive margin movement and the fact that $\ell (j)$, from the second period onwards, moves upwards

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less than in the case of full integration: households take more vacation or, put differently, relative to the efficient allocation with risk sharing, people work too little. When going out from the Cole and Obstfeld case, both terms of trade and real exchange rate depreciates by less. That makes the cost of the foreign goods relatively cheaper than with risk sharing. Investment increases more than previously, which forces consumption to increase by less due to the unavoidable trade-off. Hence, households invest more and save more thanks to the lower depreciation of RER, that makes imports cheaper than before. To sum up, the integration of financial markets exacerbates the volatility in real terms.

For the foreign country, TOT and RER deteriorate. The wealth effect produces an upward movement when the shock occurs. Firms work relatively more and there is a milder destruction of varieties. This permits the foreign economy to maintain total output above the steady state value during the transition, contrary to the adjustment in the integrated economy. The non-linear transitions for consumption and wages can be explained by the lower impact of the home shock on foreign wealth during the first period. In general, under the financial autarky, both the substitution and wealth effects are smaller. This is the reason for having all the variables a bit over the values of the previous case.
3.4. THE OPEN ECONOMY: TWO FINANCIAL REGIMES

Home Country

Foreign Country

Figure 8
3.4. **THE OPEN ECONOMY: TWO FINANCIAL REGIMES**

International Variables and others

![Graphs of various variables including Home Profits, Foreign Profits, PK, PKs, TOT, RER, XR, Q, Qs, h savings, h investment, home S-I, f savings, f investment, f S-I.]

![Figure 9](https://example.com/figure9.png)

Arespa Castelló, Marta (2008), International Transmission, Firm Entry and Risk Sharing
European University Institute

DOI: 10.2870/14337
3.4. THE OPEN ECONOMY: TWO FINANCIAL REGIMES

3.4.3.3 Shocks on the productivity of creation, $K$

Home Country

Foreign Country

Figure 10

Arespa Castelló, Marta (2008), International Transmission, Firm Entry and Risk Sharing
European University Institute

DOI: 10.2870/14337
3.4. **THE OPEN ECONOMY: TWO FINANCIAL REGIMES**

A negative shock on $K$, i.e. a decrease in the quantity of capital required for the creation of a new firm, under financial isolation, generates a milder expansion of the extensive margin in comparison with the RS case. $p(f)$ goes up by more. Hence, the decrease on $P_K$, caused by the upwards movement in $n$, is slightly cushioned by the high $p(f)$. In general, the qualitative results are those of the fully integrated economy, with some variations on volatilities that have already been mentioned.

Home investment stays below the steady state during all the transition period. This differs from the case with RS, where the smaller effect on the extensive margin and the faculty of relying on some foreign investment permits home investment ($I = n_{t+1}P_KK$) to stay above the equilibrium level. Moreover, with FA, where the depreciation of terms of trade is lower, home investment decreases further more due to the cheapness of creation. Notice that the $S - I$ jumps down when the shock occurs and continues to decrease for several periods. It is easy to realise that the reason lies on the steeper upwards transition of investment in front of savings.
3.4. THE OPEN ECONOMY: TWO FINANCIAL REGIMES

International Variables and others

Investment and Savings

Figure 11
3.4.4 Openness and Volatility

Although some comments on trade openness have been made in the text of the above sections, a more detailed observation is offered here.

I start by comparing the closed economy with the financial autarky. On the one side, the closed economy is, indeed, the extreme case in which trade openness is zero, as it is 'financial integration', due to the inexistence of a foreign world. On the other side, the financial autarky scenario studied above is an economy which keeps the financial isolation and introduces some international trade: 25% of the capital goods used in the building-up of new firms and 35% of the consumption goods are produced in a foreign country.

It is easy to see that volatility increases with trade in front of a shock on labour productivity: consumption, output, the number of active firms... all aggregate variables experience exacerbated reactions when the autarky is only on the financial sector. This is reasonable: when another country exists, international transmission permits the foreign country to receive part of the shock. Hence, for instance, when the shock is positive, they are also benefited and able to increase their demand, investment etc., providing an extra push to both economies. On the contrary, when the shock occurs on the technology of creation, i.e. on investment, the economy opened to trade is less volatile. Despite of the financial closure, any shock at home motivates a depreciation (or appreciation) in terms of trade and the real exchange rate. When the shock is on \( K \), the depreciation is smaller because only investment is cheaper. Instead, when \( A_H \) improves, investment cost decreases as well as consumption. Hence, consumption goods, which are produced by those cheaper firms, are doubly efficiently generated. This engine motivates a further depreciation in RER that enhances volatility and makes the difference. So, the transition from a completely isolated economy to another one with some trade favours the increase (decrease) of volatility after a shock on labour productivity (investment.)

For the second experiment, I first keep the average trade openness in consumption found in EU-15 and check volatility behaviour for different levels of trade in the capital goods market under both financial structures. Afterwards, I set the same degree of openness for both consumption and capital goods markets in order to value the effects of total openness on volatility.

The results are quite ambiguous. Terms of trade experiences an exacerbated level of volatility as total trade integration increases. This is true regardless of the financial regime and the type of shock. However, when one keeps \( \gamma = 0.65 \) and observes several degrees of trade in capital goods, neither the volatility of TOT nor the real exchange rate is clear. Avoiding values too close to 50%, which would set \( RER = 1 \) and volatility to zero, changes
on RER volatility seem to be slightly positive related to increases in capital goods trade, whilst one keeps $\gamma = 0.65$, whereas it changes when accounting for total trade openness. Finally, consumption shows a clear increment in volatility as international trade goes up.

Some authors consider the relevance of shock persistency as a determinant of the potential impact of openness on macroeconomic volatilities. This model produces higher volatilities the more persistent the shock is, regardless of the source.

### 3.5 Conclusion

The paper has addressed a crucial open debate on international macroeconomics: the effects of financial and trade openness on macroeconomic volatilities after two different sources of shocks: on labour productivity and on investment productivity. It has been completed using a quantitative study and a NOEM model with firms entry.

The main conclusions are the following:

- Both financial integration and trade openness seem to have a role on macroeconomic volatilities.

- When the economy gains integration of capital markets, the real exchange rate and the terms of trade, as well as output and investment, are affected by higher volatilities after any kind of shock, whereas consumption variability increases only after a shock on labour productivity. When the impact comes from the productivity of investment (i.e. the technology of creation of new firms or varieties), more financial integration reduces consumption volatility.

- Trade openness has an unambiguous positive effect on consumption volatility, although its implications for terms of trade and the real exchange rate are unclear when the economy starts with some degree of trade, either under financial autarky or with full integration. However, when one compares a closed economy with the financial autarky case, the effect of the appearance of international trade generates an increase on all variables’ volatilities after a shock on labour productivity, but a decrease after a shock on capital.

The results of the present research show the key importance of the introduction of market dynamics in the model, as well as the observation of the two types of shocks: on labour productivity and on creation of firms, when addressing the following question: what are the interrelations between financial integration and trade openness with international volatilities
and the transmission mechanism? Moreover, the findings are in line with recent empirical literature: Kose et al (2003) on the side of financial integration and Easterly et al. (2001) suggest that data reports results that are compatible with the present paper on the trade openness side.
3.6 Appendix

3.6.1 Volatility Comparison

The four plots reported below show the transitions for the real exchange rate under financial autarky (FA) and full integration or risk sharing (RS), for the four different shocks which can impact the home country. Notice how the RS line takes always more extreme values in its path back to the equilibrium, regardless of the direction of the shock (positive or negative) or its source (shock on capital requirements or in technology productivity). Moreover, the distances between FA and RS are identical within each of the two kinds of shocks.
Figure 12

RER transition - decrease in AH
The same occurs for terms of trade, consumption... volatilities.
3.6.2 Parametrization

The baseline case uses the following parametrization:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Bias in capital</td>
<td>$\delta$</td>
<td>.75</td>
</tr>
<tr>
<td>Home bias in consumption</td>
<td>$\gamma$</td>
<td>.65</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>.98</td>
</tr>
<tr>
<td>Technol, labour income share</td>
<td>$\theta$</td>
<td>.66</td>
</tr>
<tr>
<td>Intratemp elast of substit</td>
<td>$\sigma$</td>
<td>5</td>
</tr>
<tr>
<td>Home labour productivity at t=0</td>
<td>$A_{H,0}$</td>
<td>1</td>
</tr>
<tr>
<td>Foreign labour productivity at t=0</td>
<td>$A_{F,0}$</td>
<td>1</td>
</tr>
<tr>
<td>Disutility of labour</td>
<td>$\kappa$</td>
<td>1.75</td>
</tr>
<tr>
<td>Home productivity creation t=0</td>
<td>$K$</td>
<td>.7</td>
</tr>
<tr>
<td>Foreign productivity creation t=0</td>
<td>$K^*$</td>
<td>.7</td>
</tr>
<tr>
<td>Total population at home</td>
<td>$L$</td>
<td>1</td>
</tr>
<tr>
<td>Total population abroad</td>
<td>$L^*$</td>
<td>1</td>
</tr>
<tr>
<td>Shock persistency for $A_H$</td>
<td>$\phi$</td>
<td>.95</td>
</tr>
<tr>
<td>Shock persistency for $A_F$</td>
<td>$\phi^*$</td>
<td>.95</td>
</tr>
<tr>
<td>Shock persistency for $K$</td>
<td>$\phi_K$</td>
<td>.95</td>
</tr>
<tr>
<td>Shock persistency for $K^*$</td>
<td>$\phi_{K^*}$</td>
<td>.95</td>
</tr>
<tr>
<td>Portfolio diversification</td>
<td>$\lambda$</td>
<td>((1-\delta)/(1+(2*\delta-1)<em>(((\sigma-1)/\sigma)^</em>\theta-1)))</td>
</tr>
</tbody>
</table>

To perform the experiments, I checked different combinations of values for $\delta$ and $\gamma$ from .55 to .85; $\sigma$ from 2 to 8; lower persistency of shocks and several levels of disutility of labour and initial values for $K$ and $K^*$.

The parameters for $\delta$ and $\gamma$ are argued in the text and perfectly reasonable for actual data. $\beta$ and $\theta$ take values standardly used in macro literature, as well as $\kappa$ does. $L$ and $L^*$ are set to 1 to assume symmetric countries. This is a simplification that have important implications for the results, as explained in the text. Finally, $\lambda$ takes the optimal value obtained in chapter II.
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[19] International Monetary Fund (2002.) World Economic Outlook, September (Washington.)


