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Competition, Human Capital and Income Inequality
with Limited Commitment

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Abstract

We develop a dynamic general equilibrium model with two-sided limited commitment to study how barriers to competition, such as restrictions to business start-up, affect the incentive to accumulate human capital. We show that a lack of contract enforceability amplifies the effect of barriers to competition on human capital accumulation. High barriers reduce the incentive to accumulate human capital by lowering the outside value of ‘skilled workers’, while low barriers can result in over-accumulation of human capital. This over-accumulation can be socially optimal if there are positive knowledge spillovers. A calibration exercise shows that this mechanism can account for significant cross-country income inequality.

Keywords

Limited commitment; limited enforcement; human capital accumulation; income inequality; innovation; barriers to competition.

Competition, human capital and income inequality with limited commitment*

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1 Introduction

Human capital accumulation plays an important role in the mechanics of economic growth as a complementary factor to physical capital, technological innovations and, with knowledge spillovers, to human capital itself. In turn, economic growth stimulates the accumulation of human capital by raising its return. Such bidirectional effects are at the core of growth theories based on endogenous human capital accumulation (e.g. Nelson & Phelps (1996) and Lucas (2002)). In these theories, higher competitive wages are the usual channel through which human capital is rewarded.

In this paper we focus on human capital accumulation as a key to technological innovation, either in the adoption of existing technologies or in the development of new technologies. We use a dynamic general equilibrium model where contracts are not enforceable, neither for workers nor for firms. The limited commitment of workers means that they can always quit the firm. The limited commitment of firms means that they can renegotiate the payments promised to the workers after the investment in human capital has been made. These contractual frictions affect the accumulation of both human and physical capital.

One contribution of this paper is to show that the way limited enforcement of contracts affects the accumulation of human capital depends on barriers to the mobility of skilled labor. In particular, we show that high barriers discourage the accumulation of human capital while low barriers have a stimulating effect. As a result, differences in ‘barriers to competition’ translate into significant differences in incomes and welfare across economies.

Barriers that limit the value of redeploying human capital outside the firm reduce the outside option of skilled workers. Consequently, if a firm renegotiates the promised payments, the worker does not have a credible mechanism for punishing the firm. Anticipating this, the worker does not provide the effort to acquire the skills. This is a typical time-inconsistency problem.

If the worker could commit to staying with the firm and providing effort (one-sided commitment from the worker) the contractual friction could be resolved by making advance payments to the worker. However, without this commitment, advance payments are not incentive-compatible for the worker. This emphasizes the importance of a double-sided limited commitment to the results of this paper.

In line with existing Contract Theory, one can try to solve the commit-

ment problem with an output-sharing agreement or by transferring total or partial ownership of assets to the workers (e.g. Hart & Moore (1994)). But with two-sided lack of commitment, such arrangements are still open to unverifiable *de facto* renegotiations (or skimming).

There is, however, a natural solution to the time-inconsistency problem: for the firm to invest in complementary factors of production up to the point where there is no discrepancy between *ex-ante* promised payments to the workers and *ex-post* outside values. For example, in our economies the best outside option for a skilled-worker is to enter into a contractual arrangement with a new firm. Therefore, a credible investment policy for an incumbent firm is to mimic the investment decisions of a new firm. However, when the investment cost of new firms is high, their investment is low. Consequently, investment by incumbent firms is also low. In contrast, with full or one-sided commitment, incumbent firms do not mimic the investment decisions of new firms, and only the latter are directly affected by start-up costs.

Our results are first illustrated with a simple two-stage model which is then extended to a dynamic infinite horizon set-up. The parametrization of the infinite horizon model allows us to quantify the ability of one particular barrier—*start-up costs*—to account for different levels of human capital accumulation and innovation, as well as cross-country income differences. The baseline model accounts, roughly, for half of the cross-country income gaps with the US. Even though this number should be taken with caution, given the simplicity of the model, it shows that this mechanism can be quantitatively important, bringing a new perspective on the role of competition as a factor of growth. We deliberately use a semi-endogenous growth model (Jones (1995)) to explain income differences, as opposed to long-run growth differences, since this is what the evidence on the potential role of *start-up costs* suggests. We discuss this evidence in Section 2.

Our results can also be interpreted as saying that *barriers* to competition determine cross-country positions relative to the ‘technology possibility frontier’, without emphasizing a distinction between innovation and technology adoption, which is consistent with the idea that even the implementation of known technologies requires appropriate human capital. In contrast, Acemoglu, Aghion, & Zilibotti (2006) take the ‘technology possibility frontier’ as given and develop a theory where the ‘distance to the frontier’ determines a country’s comparative advantage on innovation vs. adoption. Moreover, while in their theory the cost of barriers depends on the position of a country relative to the frontier, in our framework it is the barriers that determine the

position of a country in relation to the frontier. The causality effect is reversed and the policy implications are very different. They argue the lack of pro-competitive policies becomes more costly as countries approach the world technology frontier, while our theory implies that the lack of pro-competitive policies can determine a country's position away from the frontier, as our computations of the dynamical model show.

We also show how other barriers to mobility such as *covenants* (preventing a skilled-worker from working for a period in the same industry), can be incorporated in our model to account for regional differences. For example, the evolution of the computer industry exemplifies the effects of both types of *barriers* to competition. As Bresnahan & Malerba (2002) emphasize, this industry has gone through different technological stages (from main-frames to PCs and the Internet). Knowledge in this particular industry was geographically spread in many countries including Europe. Yet the United States has persistently been the industry leader. According to them, this dominance can be explained by “...*the existence of a large body of technical expertise in universities and the generally supportive environment for new firm formation in the United States*”, Bresnahan & Malerba (2002, page 69).

While lower barriers to business start-up may have favored the computer leadership of the United States, different enforcement of covenants—and informational linkages across firms—may have determined the shift of regional leadership within the United States. As argued by Saxenian (1996), Gilson (1999) and Hyde (2003), Silicon Valley dominates over Route 128 due to a Californian legal and social tradition of not enforcing post-employment covenants, resulting in high labor mobility and knowledge spillovers.

This paper relates to different strands of literature. In addition to the ones already cited, at least two more should be mentioned. First, the labor literature that studies the accumulation of skills within the firm (e.g. Acemoglu (1997), Acemoglu & Pischke (1999), Acemoglu & Shimer (1999)). In this literature, higher outside values worsen the hold-up problem and lead to lower accumulation of skills. In our framework, instead, higher outside values increase human capital investment. Second, by emphasizing the role of barriers to mobility, our work also relates to the literature that emphasizes the role of barriers to riches in slowing growth and explaining income differences (Mokyr (1990) and Parente & Prescott (2002)).

2 Cross-country evidence on barriers to business start-up

Before describing the theoretical framework, we present here some cross-country data suggesting a relation between the cost of business start-up—which in our theory is one of the barriers to knowledge mobility—and cross-country income. It is important to emphasize that our theory is broader than simply capturing the impact of barriers to business start-up. Here, and in the later application of the theory, we will focus on barriers to business start-up only because this is the data we have available.

A recent publication from the World Bank (2005) provides indicators of the quality of the business environment for a cross-section of countries. It also includes proxies for barriers to business start-up. There are three main variables. The first is the ‘cost of starting a new business’. This is the average pecuniary cost needed to set up a corporation in the country, in percentage of the country per-capita income.¹ The second proxy is the ‘number of bureaucratic procedures’ that need to be filed before starting a new business. The third proxy is the average ‘length of time’ required to start a new business.

Figure 1 plots the level of per-capita GDP in 2004 against these three indicators, where all variables are in log. All panels show a strong negative correlation, indicating that the set-up of a new business is more costly and cumbersome in poor countries.

The cost of business start-up is also negatively correlated with economic growth. To show this, we regress the average growth in per-capita GDP from 2000 to 2004 to the cost of business start-up. We also include the 1999 per-capita GDP to control for the initial level of development. We would like to emphasize that the goal of these regressions is not to establish a causation but only to highlight some key correlations that motivate our study. The estimation results, with t -statistics in parenthesis, are reported in Table 1.

As can be seen from the table, the cost of business start-up is negatively associated with growth even if we control for the level of economic development. Therefore, countries with lower barriers to entry tend to experience faster growth. This finding is robust to the choice of alternative years for

¹The normalization of the cost of business start-up by the level of per-capita income better captures the importance of barriers to business start-up than the absolute dollar cost. What is relevant is the comparison between the cost of business start-up and the value of creating a business. Although the dollar cost is on average higher in advanced economies, the value of a new business is also likely to be higher.

Figure 1: Barriers to business start-up and level of development.

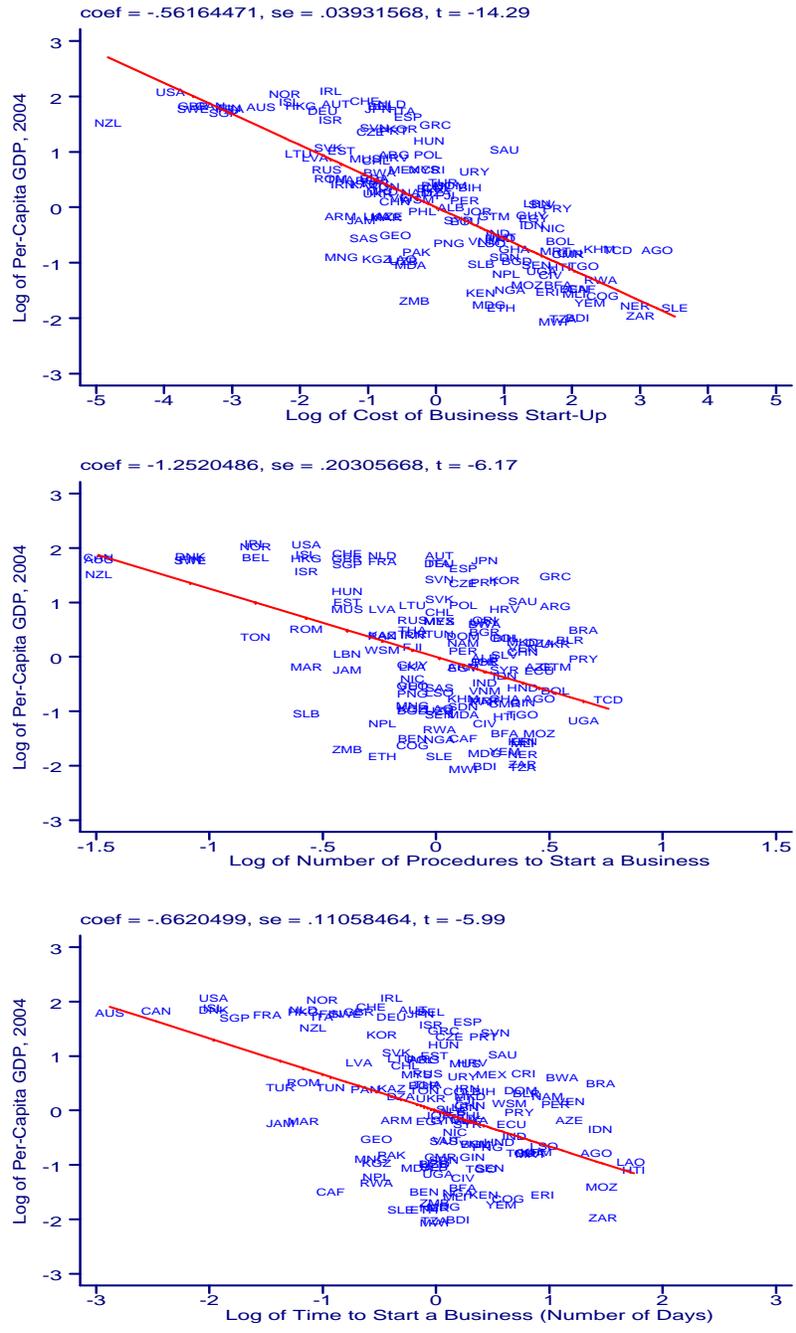


Table 1: Cost of business start-up and growth.

	<i>Constant</i>	<i>Initial Per-Capita GDP</i>	<i>Cost of Business Start-Up</i>
Coefficients	15.55	-1.16	-1.04
<i>t</i> -Statistics	(5.01)	(-3.81)	(-4.92)
<i>R</i> -square	0.150		
<i>N. of countries</i>	140		

NOTES: The dependent variable is the average annual growth rate in per-capita GDP for the five year period 2000-2004. Initial Per-Capita GDP is the log of per-capita GDP in 1999. The cost of business start-up is in percentage of the per-capital Gross National Income as reported in *Doing Business in 2005* (also in log)

compute the average growth rate. The other proxies for barriers to entry—specifically, the number of procedures and the time required to start a new business—are also negatively correlated with growth but they are not statistically significant at conventional levels.

To summarize, the general picture portrayed by the data is that economic development and growth is negatively associated with the cost of starting a business. We have presented simple correlations which, of course, do not imply causation. In the following section we present a model where barriers to entry and, more generally, barriers to the mobility of knowledge or human capital, lead to lower income and growth. We will return to the cross country data presented here in the quantitative analysis of Section 6.

3 The model

There is a continuum of ‘workers’, of total mass 1, each characterized by a level of human or knowledge capital h_t . Their lifetime utility is $\sum_{t=0}^{\infty} \beta^t (c_t - e_t)$, where c_t is consumption and e_t is the ‘effort’ to accumulate knowledge as specified below. In addition to workers there is a continuum of ‘investors’ of total mass $m > 1$. Investors are risk neutral with lifetime utility $\sum_{t=0}^{\infty} \beta^t c_t$. The risk neutrality implies that the equilibrium interest rate is equal to the intertemporal discount rate; that is, $r = 1/\beta - 1$.

Production projects require the input of knowledge capital, h_t , and phys-

ical capital, k_t . They generate output according to:

$$y_t = h_t^{1-\alpha} k_t^\alpha.$$

We assume that workers do not save, and therefore, physical capital must be provided by investors. The goal of this assumption is to differentiate the roles of workers and investors: the first as providers of human capital and the second of financial resources. If workers saved, they would be able to self-finance the purchase of physical capital eliminating the contractual frictions between investors and workers.²

Investors compete to hire workers in a Walrasian market by offering contracts that determine the investment in human and physical capital and the compensation structure. We refer to the contractual arrangement between an investor and a worker as a *firm*. For expositional simplicity we assume that each investor can hire only one worker. However, the investor-worker pair can also be interpreted as a specific project or unit within a large firm with certain common features. First, the relationship with each worker is governed by a specific contract; second, investors behave competitively (for example, they can not collude to prevent workers' mobility); third, workers are in the short side of the market; that is, the ability to hire workers exceeds the number of workers. In the model this is obtained by assuming that the mass of investors is bigger than the number of workers ($m > 1$).

As anticipated above, an investment in knowledge, $h_{t+1} - h_t$, requires effort from the worker. The effort cost function is denoted by:

$$e_t = \varphi(h_t, h_{t+1}; H_t),$$

where H_t is the economy-wide knowledge. The dependence on the aggregate knowledge captures possible leakage or spillover effects.

The function φ is strictly decreasing in H_t and h_t , strictly increasing and convex in h_{t+1} , and satisfies $\varphi(h_t, h_t; H_t) > 0$. It is further assumed to be homogeneous of degree $\rho > 1$. With this homogeneity assumption the model generates only long-term differences in income *levels*, and therefore, this is a semi-endogenous growth model as in Jones (1995). The analysis can be

²Zero savings could also be interpreted as an endogenous outcome if workers discount more heavily than investors. As long as the discount differential is sufficiently high, workers will not save in equilibrium.

easily extended to $\rho = 1$, in which case we would have long-term *growth* differences.³

Physical capital is knowledge-specific. When a worker upgrades the knowledge capital, only part of the existing capital is usable with the new knowledge. Knowledge upgrading is equivalent to the adoption of a new technology that makes part of the existing equipment obsolete. Capital obsolescence increases with the degree of knowledge upgrading. This is formalized by assuming that the depreciation rate of physical capital is:

$$\delta_t = \delta \cdot \left(\frac{h_{t+1}}{h_t} \right).$$

Because of capital obsolescence, there is an asymmetry between *incumbent firms*—whose capital depreciates with the use of more advanced knowledge—and *new firms*, which, without capital in place, have a greater incentive to hire workers with higher knowledge (Arrow’s ‘replacement effect’).

Firms remain productive with probability p . Whether a firm survives is revealed after the investment in knowledge. The assumption guarantees that, after the investment, the mass of workers is larger than incumbent (surviving) firms. As we will see, it also guarantees that the equilibrium is unique when neither the investors nor the workers commit to the contract. Because the uniqueness is achieved for any positive probability of firms’ exit from the market, not matter how small, to facilitate the exposition we first consider p close to 1 and, in the characterization of the individual problems we will ignore it. This would be the limiting equilibrium as p converges to 1. The characterization of the equilibrium with any value of p is provided in Appendix G.

Competitive structure and barriers: In each period there is a mass 1 of investors who are in a contractual relationship with workers, and a mass $m - 1 > 0$ who are idle and could start new firms. Investors can borrow from and lend to each other to finance the capital k at the interest rate r . The labor market is competitive and opens twice, before and after the accumulation

³The model can be interpreted as a detrended version of an economy that grows at the exogenous rate dictated by worldwide knowledge. Let \bar{H}_t be worldwide knowledge growing at rate \bar{g} , with the effort cost function, $e_t = \tilde{\varphi}(h_t, h_{t+1}; H_t, \bar{H}_t)$, homogeneous of degree 1. After normalizing all variables by \bar{H}_t , the effort cost function can be rewritten as $\varphi(h_t, h_{t+1}; H_t)$, which is homogeneous of degree $\rho > 1$.

of knowledge. Active and idle investors can participate, but the effective competition for workers created by potential new firms is limited by several types of barriers. For the moment, we consider only barriers to business start-up. The analysis of other barriers, such as the strict enforcement of covenants, will be conducted in Section 7 with similar results.

Barriers to entry are modeled as a deadweight cost proportional to the initial level of knowledge chosen by the firm. Given the initial knowledge h_{t+1} , the entry cost is $\tau \cdot h_{t+1}$. The key results of the paper are robust to alternative specifications of the entry cost. Our choice is only motivated by its analytical convenience.⁴

4 One-period model

Before studying the general model with infinitely lived agents, we first consider a simplified version with only one period that facilitates the intuition for the key results of the paper. The analysis of the infinite horizon model, however, is still important because it allows us to derive the initial conditions endogenously as steady state values and, more importantly, it allows us to quantitatively explore our model in Section 6.

There are two stages: before and after the investment in knowledge. The states at the beginning of the period are h_0 and k_0 . After making the investment decisions, h_1 and k_1 , the firm generates output $y_1 = h_1^{1-\alpha} k_1^\alpha$ in the second stage. In this simple version of the model we assume that physical capital fully depreciates after production. The worker receives a payment w at the end of the period, i.e. after the choice of h_1 . Payments before the choice of h_1 are not incentive-compatible because of the limited enforcement of contracts for the worker. With only one period, we can ignore discounting, as well as leakage or spillover effects.

The timing of the model is as follows: The firm starts with initial states h_0 and k_0 . At this stage the worker decides whether to stay or quit the firm. If the worker quits, she can be hired either by an incumbent firm or by a new firm (funded by a new investor). If the worker decides to stay, she will exercise effort to upgrade the knowledge capital to h_1 and the investor provides the

⁴For example, we could assume that the cost is proportional to the initial capital k_{t+1} , or to the initial output $h_{t+1}^{1-\alpha} k_{t+1}^\alpha$, or to the discounted flows of outputs. The basic theory and results also apply when the entry cost is a fixed payment. The assumption of proportionality allows for a continuous impact of τ while a fixed cost would have an impact only after it has reached the prohibitive level.

funds to upgrade the physical capital to k_1 . After the investment, the firm pays w . At this stage the worker can still quit, but she cannot change the level of knowledge h_1 . The investor is the residual claimant of the firm's output.

4.1 Equilibrium with one-sided limited commitment

We first characterize the equilibrium when at least one of the parties, either the investor or the worker, commits to the contract. The commitment of one party is sufficient for the implementation of the optimal investment. As we will see, it is the limited commitment of both parties (double-sided limited enforcement) that induces a deviation from the optimal investment. We start with the characterization of the equilibrium when only the investor commits. It will then be trivial to show that this is also the allocation when contracts are enforceable for the worker only, or for both the worker and the investor.

With the investor's commitment, the optimal contract can be characterized by choosing all variables at the beginning of the period to maximize the total surplus, subject to the enforcement constraints for the worker. Let $D(h_0)$ be the repudiation value for the worker before choosing h_1 , and $\widehat{D}(h_1)$ the repudiation value after choosing h_1 . These functions are endogenous and will be derived below as the values that the worker would get by quitting the firm. From now on we will use the *hat* sign to denote the functions that are defined *after* the investment in knowledge (second stage). The participation of the worker requires that the value of staying is greater than the repudiation value before and after the knowledge investment; that is,

$$\begin{aligned} w - \varphi(h_0, h_1) &\geq D(h_0) \\ w &\geq \widehat{D}(h_1). \end{aligned}$$

As we will show, the second constraint is always satisfied if the first constraint is satisfied. Therefore, in the derivation of the optimal policy, we can neglect the second constraint and write the optimization problem as:

$$\max_{h_1, k_1, w} \left\{ -\varphi(h_0, h_1) - k_1 + \left[1 - \delta \cdot \left(\frac{h_1}{h_0} \right) \right] k_0 + h_1^{1-\alpha} k_1^\alpha \right\} \quad (1)$$

s.t.

$$w - \varphi(h_0, h_1) \geq D(h_0)$$

$$-w - k_1 + \left[1 - \delta \cdot \left(\frac{h_1}{h_0}\right)\right] k_0 + h_1^{1-\alpha} k_1^\alpha \geq 0,$$

where the second constraint is the participation condition for the investor.

A quick glance at the optimization problem reveals that the investment choices are independent of the payment w . The value of w is determined by the division of the surplus which, at this stage, we do not have to specify. As long as w does not violate the enforcement and participation constraints, it does not affect the investment in human and physical capital.

To determine the repudiation value before the choice of h_1 , we have to solve for the value that the worker would get by switching to a new firm. Because of competition among potential entrants, the value received by the worker is the surplus created by a new firm. This is given by:

$$S(h_0) = \max_{h_1, k_1, w} \left\{ -\varphi(h_0, h_1) - \tau h_1 - k_1 + h_1^{1-\alpha} k_1^\alpha \right\} \quad (2)$$

s.t.

$$w - \varphi(h_0, h_1) \geq D(h_0)$$

$$-w - \tau h_1 - k_1 + h_1^{1-\alpha} k_1^\alpha \geq 0.$$

Therefore, $D(h_0) = S(h_0)$ and, if the worker stays with the incumbent firm, the payment w must be at least $\varphi(h_0, h_1) + S(h_0)$. Formally, the participation constraint in problem (1) becomes $w - \varphi(h_0, h_1) \geq S(h_0)$.

Problems (1) and (2) show the different incentive to invest for an incumbent versus a new firm. On the one hand, new firms do not have any physical capital and knowledge upgrading does not generate capital obsolescence. On the other, they must pay the entry cost τh_1 , which discourages knowledge and capital accumulation. This is clearly shown by the first order conditions in problems (1) and (2), with respect to h_1 . These can be written as:

$$(1 - \alpha) \left(\frac{k_1}{h_1}\right)^\alpha = \varphi_{h_1}(h_0, h_1) + \delta \cdot \left(\frac{k_0}{h_0}\right) \quad (3)$$

$$(1 - \alpha) \left(\frac{k_1}{h_1}\right)^\alpha = \varphi_{h_1}(h_0, h_1) + \tau \quad (4)$$

where the subscripts denote derivatives. The left-hand terms are the marginal productivity of knowledge. The right-hand terms are the marginal costs. For an incumbent firm, the marginal cost derives from the effort incurred by the worker plus the obsolescence of physical capital. For a new firm the obsolescence cost is replaced by the entry cost.

Let h_1^{Old} be the optimal knowledge investment of an incumbent (old) firm and h_1^{New} the optimal investment of a new firm. The following proposition formalizes the relation between barriers to entry and knowledge investment.

Proposition 1 *The knowledge investment of a new firm, h_1^{New} , is strictly decreasing in the entry cost τ and there exists $\bar{\tau} > 0$ such that $h_1^{New} = h_1^{Old}$.*

Proof 1 *The first order condition for the choice of k_1 is $\alpha(k_1/h_1)^{\alpha-1} = 1$ for both incumbent and new firms. Using this condition, (3) and (4) become:*

$$\begin{aligned} (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}} &= \varphi_{h_1}(h_0, h_1^{Old}) + \delta \cdot \left(\frac{k_0}{h_0}\right) \\ (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}} &= \varphi_{h_1}(h_0, h_1^{New}) + \tau. \end{aligned}$$

The proposition follows directly from these two conditions.

Q.E.D.

In equilibrium there is no entrance of new firms at the beginning of the period and the investment in knowledge is $h_1 = h_1^{Old}$, where h_1^{Old} is determined by (3). The potential entrance of new firms only affects the payment received by the worker. In the second stage there will be the entrance of new firms (although the number is negligible because $p \simeq 1$). However, the level of knowledge cannot be changed at this stage.

Before continuing we show that the equilibrium investment does not change if both parties (or the investor only) commit. Because h_1^{Old} maximizes the total surplus, this must also be the equilibrium investment if both parties commit to the contract. The same result applies if it is the worker who commits. In this case the investor can renege on the promised payments after the investment in knowledge. However, this problem can be solved by making the payment w before the investment in knowledge. As long as the contract is enforceable for the worker, there is no risk that she runs away or does not exercise the effort to acquire knowledge.

4.2 Equilibrium with double-sided limited commitment

We want to show first that, when the investor can not commit to fulfill his promises, he will renegotiate the contract after the choice of h_1 . To see this, we must derive the value that the worker would get by quitting the firm when her knowledge has already been chosen to be h_1 . This is the surplus generated by a new firm that hires a worker with knowledge h_1 ; that is,

$$\widehat{S}(h_1) = \max_{k_1, w} \left\{ -\tau h_1 - k_1 + h_1^{1-\alpha} k_1^\alpha \right\} \quad (5)$$

s.t.

$$w \geq \widehat{D}(h_1)$$

$$-w - \tau h_1 - k_1 + h_1^{1-\alpha} k_1^\alpha \geq 0.$$

Because of competition among potential entrants, the worker gets the whole surplus, and therefore, $\widehat{D}(h_1) = \widehat{S}(h_1)$. An incumbent firm will renegotiate the promised payment w if this is higher than $\widehat{S}(h_1)$. The renegotiation threat after the accumulation of knowledge is credible because the firm can always replace the current worker with other workers. This could be either a worker still employed by an incumbent (surviving) firm, or a worker who separated from an exiting firm. Because in the second stage there are only $p < 1$ firms that are still alive but the mass of workers is 1, workers are in the long side of the market relatively to incumbent firms. This implies that they only get the reservation value.⁵

Given that the ex-post payment received by the worker is $w = \widehat{S}(h_1)$, the total utility from staying with the firm is:

$$-\varphi(h_0, h_1) + w = -\varphi(h_0, h_1) + \widehat{S}(h_1). \quad (6)$$

If instead the worker quits at the beginning of the period, she will get the surplus $S(h_0)$ generated by a new firm which started before making the investment in knowledge. This is given by:

$$S(h_0) = \max_h \left\{ -\varphi(h_0, h) + \widehat{S}(h) \right\} = -\varphi(h_0, h_1^{New}) + \widehat{S}(h_1^{New}) \quad (7)$$

⁵We have ignored this probability in the contractual problem because we are looking at the limiting case of $p \simeq 1$. The explicit analysis of $p < 1$ will be done in Appendix G.

Equations (6) and (7) show that the value of quitting at the beginning of the period, $S(h_0)$, is greater than the value of staying, as long as $h_1 \neq h_1^{New}$. Therefore, the worker will quit unless the firm agrees to the same knowledge investment chosen by a new entrant firm; that is, $h_1 = h_1^{New}$. In this way the worker keeps the repudiation value high and prevents the firm from renegotiating.⁶

Proposition 2 *Suppose that all firms have the same initial states (k_0, h_0) . Then there is a unique equilibrium with aggregate knowledge $H_1 = h_1^{New}$.*

Proof 2 *See Appendix A.*

Because h_1^{New} is decreasing in τ (see Proposition 1), the accumulation of knowledge decreases with the cost of entry. Therefore, with double-sided limited enforcement, there is a negative correlation between barriers to entry and the accumulation of knowledge.

To summarize, greater competition (lower barriers to entry) leads to higher investment in knowledge. Because the investment is determined by the optimality condition of new firms, this level is not necessarily efficient for incumbent firms. In particular, if τ is small, incumbent firms accumulate too much knowledge. The presence of spillovers, however, may make the higher investment socially desirable. We will re-introduce spillovers in the analysis of the infinite horizon model.

Remarks: There are two points to be emphasized. First, the importance of $p < 1$. If p was equal to 1, we would have the same number of workers as incumbent firms in the second stage. This may lead to multiple equilibria. Figure 2 illustrates the issue. The left-hand panel plots the demand and supply of workers when $p = 1$. The supply is equal to 1 because there is a mass one of workers. The demand comes from incumbent firms, which is in this case is equal to 1, and from potential entrants. Incumbent firms are willing to hire for any wage between $h_1^{1-\alpha}k_1^\alpha$ and $\widehat{S}(h_1)$. Potential entrants, instead, are willing to hire only if the wage is not greater than $\widehat{S}(h_1)$. This gives rise to the dotted line which overlaps with the supply of workers for a range of wages. This implies that the wage is not uniquely determined

⁶This proves that, if the enforcement constraint for the worker is satisfied at the beginning of the period, it is also satisfied after the investment in knowledge.

in the second stage of the model. The right-hand panel, instead, shows the market equilibrium when $p < 1$. In this case there are only $p < 1$ firms that are willing to hire for a wage greater than $\widehat{S}(h_1)$. As a result, the aggregate demand of workers intersects the supply only once and the equilibrium wage is uniquely determined at $w = \widehat{S}(h_1)$.

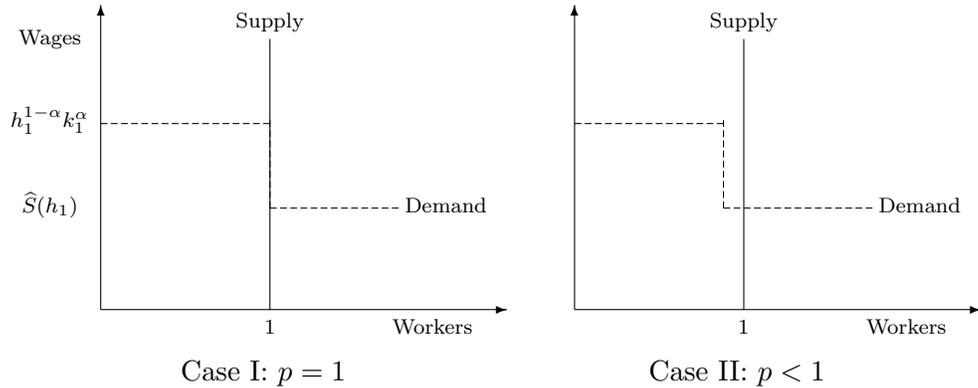


Figure 2: Labor market equilibrium after the accumulation of knowledge.

The second point to be emphasized is that output sharing is equivalent to promised payments. Thus, the assumption of limited enforcement for the investor also applies to the promise of a share of output.

5 The infinite horizon model

In this section we generalize the model to an infinite horizon set-up. There are two important gains. First, it allows us to derive the initial conditions k_0 and h_0 endogenously as steady state values. Second, the infinite horizon structure is better suited to the quantitative analysis of Section 6.

We first characterize the equilibrium with commitment and then we turn to the case of double-sided limited commitment. The comparison between these two environments clarifies the importance of double-sided limited enforcement for barriers to entry affecting the accumulation of knowledge. To present the results more compactly, we relegate most of the technicalities and proofs to the appendix.

Before continuing, it will be convenient to define the gross output func-

tion, inclusive of undepreciated capital, as follows:

$$\pi(h_t, k_t, h_{t+1}) = \left[1 - \delta \cdot \left(\frac{h_{t+1}}{h_t} \right) \right] k_t + h_t^{1-\alpha} k_t^\alpha. \quad (8)$$

5.1 Equilibrium with one-sided commitment

We start by characterizing the environment where only the investor commits. As in the one-period model, the equilibrium allocation with investor's commitment is equivalent to the allocation achieved when the worker commits (with or without commitment from the investor). What changes the equilibrium outcome is the limited commitment of both.

The analysis of the infinite horizon model will concentrate on steady state equilibria. Therefore, in the analysis that follows we ignore the aggregate states as an explicit argument of the value functions.

Although in equilibrium there is no entrance of firms (more precisely the number of firms entering is negligible), we still need to solve for the dynamics of a new firm in order to determine the outside or repudiation value for the worker. Even though the analysis is limited to steady states, newly created firms do experience a transition to the long-term level of physical and knowledge capital.

Let $V(h_t)$ be the repudiation value for the worker at the beginning of the period, before investing in knowledge. This is the value that a worker with knowledge h_t would receive by switching to a new firm. Similarly, let $\widehat{V}(h_{t+1})$ be the value of quitting after the investment in knowledge, and therefore, after exercising the effort to accumulate knowledge. The optimization problem solved by a new firm that hires a worker with knowledge capital h_0 at the beginning of period 0 is:

$$V(h_0) = \max_{\{w_t, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[w_t - \varphi(h_t, h_{t+1}; H) \right] \quad (9)$$

subject to

$$\begin{aligned} \sum_{j=t}^{\infty} \beta^{j-t} \left[w_j - \varphi(h_j, h_{j+1}; H) \right] &\geq V(h_t), & \text{for } t > 0 \\ w_t + \sum_{j=t+1}^{\infty} \beta^{j-t} \left[w_j - \varphi(h_j, h_{j+1}; H) \right] &\geq \widehat{V}(h_{t+1}), & \text{for } t \geq 0, \text{ and} \end{aligned}$$

$$-\tau h_1 - w_0 - k_1 + \sum_{t=1}^{\infty} \beta^t \left[\pi(h_t, k_t, h_{t+1}) - w_t - k_{t+1} \right] \geq 0.$$

The optimal contract maximizes the value for the worker, subject to the enforcement constraints for the worker (the first two conditions) and the participation constraint for the investor (non-negative profits). The problem is also subject to a non-negative constraint for w_t .

For a worker hired by a new firm at time 0, *after* the investment in knowledge, the value of the contract is:

$$\widehat{V}(h_1) = \max_{\{w_t, k_{t+1}, h_{t+2}\}_{t=0}^{\infty}} \left\{ w_0 + \sum_{t=1}^{\infty} \beta^t \left[w_t - \varphi(h_t, h_{t+1}; H) \right] \right\}, \quad (10)$$

subject to the same constraints as problem (9).

The key difference, with respect to the problem solved by a new firm entering at the beginning of the period, is that the effort to accumulate knowledge has already been exercised and h_1 is given at this point. Hence, the current flow of utility for the worker is only w_0 . This also explains why the choice of knowledge starts in the next period.

Appendix B derives the first order conditions for problem (9). Because of the entry cost and the obsolescence of physical capital, the optimality conditions in the entry period, that is when $t = 0$, are different from the optimality conditions in subsequent periods. The first order conditions at $t = 0$ are:

$$V(h_t) \leq w_t - \varphi(h_t, h_{t+1}; H) + \beta V(h_{t+1}) \quad (11)$$

$$\beta \pi_2(h_{t+1}, k_{t+1}, h_{t+1}) = 1 \quad (12)$$

$$\tau + \varphi_2(h_t, h_{t+1}; H) = \beta \left[\pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H) \right], \quad (13)$$

where subscripts denote derivatives. A transversality condition must also be satisfied.

The first condition says that the value of quitting the current employer cannot be greater than the current flow of utility plus the discounted value of quitting in the next period. The second condition equalizes the gross marginal return of capital to its marginal cost, which is 1. The last condition

equalizes the marginal cost of accumulating knowledge to the discounted value of its return (greater production and lower cost of future knowledge investment).

The first order conditions after entering, that is, for $t > 0$, are similar to the ones derived above with the exception of condition (13), which becomes:

$$-\pi_3(h_t, k_t, h_{t+1}) + \varphi_2(h_t, h_{t+1}; H) = \beta \left[\pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H) \right]. \quad (14)$$

Conditions (13) and (14) show the asymmetry between new and incumbent firms. While the marginal benefit from investing in knowledge (the right-hand side) is the same, the marginal cost (the left-hand side) differs. For new firms this includes the entry cost, τ . For incumbent firms the entry cost is replaced by the depreciation of physical capital, $-\pi_3(h_t, k_t, h_{t+1})$.

We can now characterize the steady state equilibrium. Because in equilibrium there is no entrance, all firms have the economy-wide knowledge H . The convergence to the economy-wide average is the result of the spillovers in the accumulation of knowledge. Because of this, firms with lower than average knowledge tend to invest more. Thanks to the complementarity of knowledge and physical capital, all firms accumulate the economy-wide level of physical capital. The values of H and K are determined by conditions (12) and (14) after imposing the steady state conditions, that is:

$$\beta \pi_2(H, K, H) = 1 \quad (15)$$

$$-\pi_3(H, K, H) + \varphi_2(H, H; H) = \beta \left[\pi_1(H, K, H) - \varphi_1(H, H; H) \right] \quad (16)$$

Appendix C shows that the steady state values of H and K are unique. After solving for H and K , we can then solve for the steady state payment w . This requires us to solve for the whole transition experienced by a 'new firm', as characterized by the first order conditions (11)-(14). Even if in equilibrium workers do not quit and new firms are not created, the payment w depends on the value of a new firm $V(H)$.

Conditions (15) and (16) also reveal that the entry cost τ does not affect the steady state values of K and H . We will see in the next section that this does not hold in the case of double-sided limited commitment.⁷

⁷As we will show in Appendix G, when p is not arbitrarily close to 1, the steady state with investor's commitment does depend on τ . In this case the limited enforcement from the investor amplifies the negative effects of barriers to entry.

5.2 Equilibrium with double-sided limited commitment

Let us start with the enforcement constraints imposed on the previous problem with investor's commitment. These constraints, before and after the investment in knowledge, can be written as:

$$\sum_{j=t}^{\infty} \beta^{j-t} [w_j - \varphi(h_j, h_{j+1}; H)] \geq V(h_t) \quad (17)$$

$$\sum_{j=t}^{\infty} \beta^{j-t} [w_j - \varphi(h_j, h_{j+1}; H)] \geq -\varphi(h_t, h_{t+1}; H) + \widehat{V}(h_{t+1}). \quad (18)$$

Appendix B shows that $V(h_t) > -\varphi(h_t, h_{t+1}; H) + \widehat{V}(h_{t+1})$. This implies that the investor has an ex-post incentive to renegotiate the promised payments. That is, the lack of credibility in the one-period economy is recurrent in the infinite horizon economy.

Let $h_{t+1}^{New} = f(h_t)$ be the investment in knowledge chosen by a new firm in the entry period, when the initial knowledge of the worker is h_t and the investor does not commit to the contract. The next proposition establishes that, with double-sided limited commitment, incumbent firms choose the same knowledge investment as new firms.

Proposition 3 *With double-sided limited commitment, the knowledge investment chosen by an incumbent firm is equal to the knowledge investment chosen by a new firm; that is, $h_{t+1}^{Old} = h_{t+1}^{New} = f(h_t)$.*

Proof 3 *See Appendix D.*

Since the firm can renegotiate the promised payments after the investment in knowledge, the worker would not stay unless the firm agreed to the same knowledge investment chosen by a new firm. In this way, the worker keeps the outside value high and prevents the firm from renegotiating.

Let $J(h_t)$ be the repudiation value for the worker when neither the investor nor the worker commit to the contract. Furthermore, let $\widehat{J}(h_{t+1})$ be the corresponding value after the investment in knowledge. Given the above proposition, the optimization problem for a new firm, which started at $t = 0$,

can be written as:

$$J(h_0) = \max_{h_1, \{w_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[w_t - \varphi(h_t, h_{t+1}; H) \right] \quad (19)$$

subject to

$$\sum_{j=t}^{\infty} \beta^{j-t} \left[w_j - \varphi(h_j, h_{j+1}; H) \right] \geq J(h_t), \quad \text{for } t \geq 0$$

$$-\tau h_1 - w_0 - k_1 + \sum_{t=1}^{\infty} \beta^t \left[\pi(h_t, k_t, h_{t+1}) - w_t - k_{t+1} \right] \geq 0,$$

$$\text{and } h_{t+1} = f(h_t), \quad \text{for } t > 0.$$

Notice that only the initial knowledge h_1 is chosen in this problem. Future values are determined by the investment policy of future new firms; that is, $h_{t+1} = f(h_t)$. We have not included the enforcement constraint after the investment in knowledge since it is already imbedded in $f(h_t)$.

The solution to this problem involves a non-trivial fixed point problem. First, as in the previous problem, the enforcement constraints include the outside value $J(h_t)$, which is derived from the optimization problem solved by a new firm. Second, the policy function $f(h_t)$, which is taken as given by an incumbent firm, is also the policy function obtained as the solution to the same optimization problem. Solving for endogenous participation constraints is relatively new in the literature since they are often imposed exogenously by assuming autarky values.

A detailed characterization of the solution is given in Appendix E. It should be noticed that conditions (11) and (12), derived in the environment with investor's commitment, are also valid in the case with double-sided limited commitment. The optimality condition for the accumulation of knowledge, however, is different. For new firms this is given by:

$$\begin{aligned} \tau + \varphi_2(h_t, h_{t+1}; H) &= \beta \left\{ \pi_1(h_{t+1}, k_{t+1}, f(h_{t+1})) - \varphi_1(h_{t+1}, f(h_{t+1}); H) \right. \\ &\quad \left. + \left[\pi_3(h_{t+1}, k_{t+1}, f(h_{t+1})) + \tau \right] f_1(h_{t+1}) \right\} \quad (20) \end{aligned}$$

For incumbent firms there is no optimality condition for the investment in knowledge since they take as given the investment policy $f(h_t)$.

Imposing the steady state conditions $h_t = h_{t+1} = H$ and $k_t = k_{t+1} = K$, conditions (12) and (20) become:

$$\beta\pi_2(H, K, H) = 1 \tag{21}$$

$$\tau + \varphi_2(H, H; H) = \beta \left\{ \pi_1(H, K, f(H)) - \varphi_1(H, f(H); H) + f_1(H) \left[\pi_3(H, K, f(H)) + \tau \right] \right\}. \tag{22}$$

Unlike the case in which the investor commits to the contract, these two conditions are no longer sufficient to determine the steady state values of H and K . The unknown function $f(H)$ also needs to be determined. This requires us to solve for a fixed point problem. Denote by $h' = \psi(h; f)$ the policy function that solves problem (19), for given f . The policy function satisfies the first order condition (20) and in equilibrium $f(H) = \psi(H; f)$.

Because incumbent firms innovate at the same rate as new firms, condition (20) also determines the investment in knowledge of incumbent firms. Therefore, in order to determine whether lack of commitment leads to higher or lower investment in knowledge, we have to compare condition (22) to the optimality condition when the investor commits to the long-term contract, that is, condition (14).

Let H^C be the steady state knowledge in the economy in which the investor commits, and H^{NC} the steady state knowledge without commitment. We then have the following proposition:

Proposition 4 *Suppose that $f_1(H) \leq 1$. Then the steady state value of H^{NC} is strictly decreasing in τ and there exists $\bar{\tau} > 0$ such that $H^{NC} > H^C$ for $\tau < \bar{\tau}$ and $H^{NC} < H^C$ for $\tau > \bar{\tau}$.*

Proof 4 *See Appendix F.*

Notice that the proof is based on the assumption that $f_1(H) \leq 1$; that is, the derivative of the policy function at the steady state equilibrium is not greater than one. We have checked this condition numerically. Therefore, when contracts are not enforceable for either party, neither for the worker nor

for the investor, the start-up cost is harmful for the accumulation of knowledge. With low barriers, the economy experiences a higher level of income than in the economy with commitment. This could be welfare-improving if there are spillovers in the accumulation of knowledge.

6 Quantitative application

In this section we use the model to quantify the contribution of the cost of business start-up in generating cross-country income inequality. In the quantitative application we focus on the ‘cost of business start-up’ because of data availability. It should be clear, however, that our theory applies more broadly to other barriers affecting the mobility of knowledge.

We calibrate the economy to the United States and then we ask how much of the cross-country income gap from the US can be accounted for by the observed cost of business start-up. The discount factor, β , the production parameter, α , and the depreciation parameter, δ , are calibrated to replicate the following moments: an interest rate of 5 percent, a capital income share of 33 percent, and a capital-output ratio of 3. This implies $\beta = 0.9524$, $\alpha = 0.33$, and $\delta = 0.06$. Notice that the three moments are invariant to the entry barrier τ , and therefore, they are constant across countries.⁸

The effort cost function is derived from the accumulation equation for the stock of knowledge, which is assumed to take the form:

$$h_{t+1} = (1 - \phi)h_t + (H_t^\theta e_t^{1-\theta})^\nu,$$

where H_t is the average level of knowledge, e_t is the effort cost of accumulating knowledge and ϕ is the depreciation rate. The parameter $\nu < 1$ captures the return to scale in the accumulation of knowledge and $\theta < 1$ the leakage or spillover effects. Inverting, we get the cost function:

$$e_t = \varphi(h_t, h_{t+1}; H_t) = \frac{[h_{t+1} - (1 - \phi)h_t]^{\frac{1}{(1-\theta)\nu}}}{H_t^{\frac{\theta}{1-\theta}}},$$

⁸While it is easy to see the mapping between the first two moments and the first two parameters ($\beta = 1/(1+r)$ and $\alpha = rK/Y$), less obvious is the mapping between δ and the capital-income ratio. From condition(12), evaluated at the steady state, we have $\beta\pi_2(H, K, H) = \beta[1 - \delta + \alpha(K/H)^{\alpha-1}] = 1$. Given the output function $Y = H^{1-\alpha}K^\alpha$, the capital-output ratio can be written as $K/Y = (K/H)^{1-\alpha}$. Using this expression to eliminate K/H in the previous condition, we get $\beta[1 - \delta + \alpha/(K/Y)] = 1$. Therefore, after choosing β and α , the parameter δ is uniquely determined by the capital-output ratio.

which is homogeneous of degree $\rho = (1 - \theta\nu)/(1 - \theta)\nu$.

The depreciation of knowledge results from working directly with the stationary version of the model, detrended by the rate of worldwide knowledge. The parameter ϕ is then approximately equal to the exogenous rate of growth.⁹ Assuming that the economy grows at 1.8 percent per year, we set $\phi = 0.018$. This is about the average growth rate in per-capita GDP experienced by the US during the last century.

The values of the other two parameters, θ and ν , are more controversial. Manuelli & Seshadri (2005) uses a similar specification of the investment function, within an overlapping generation model, but without externalities. In order to generate some key properties of the life-time profile of earnings, they choose a return to scale of 0.93. This is also the value estimated by Heckman, Lochner, & Taber (1998). We use this value to calibrate ν on the assumption that there is sufficient intergenerational transmission of human capital.¹⁰ For the baseline parametrization we also follow Manuelli & Seshadri (2005) and assume no externalities, that is, $\theta = 0$. The sensitivity analysis will clarify how the results depend on the choice of θ and ν .

6.1 Results

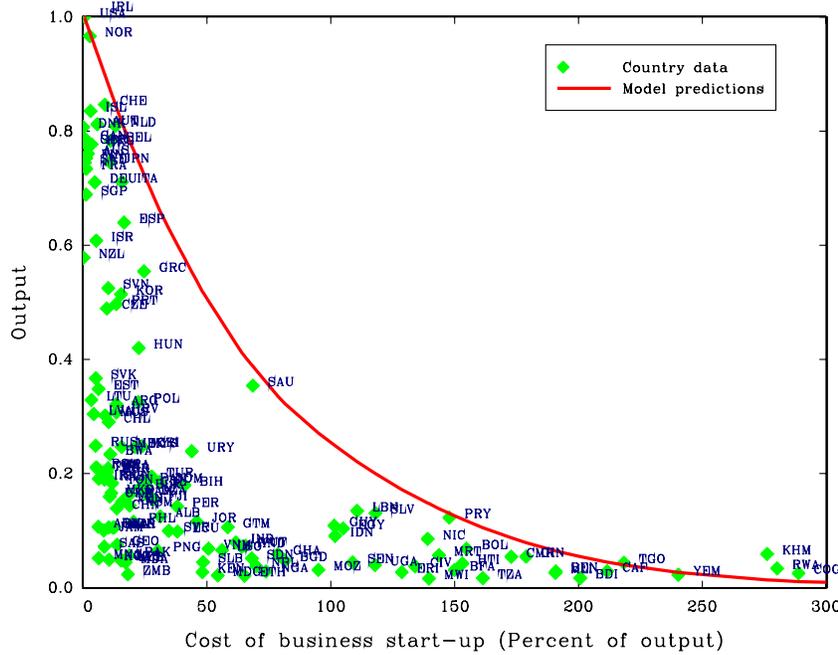
Figure 3 plots the values of per-capita GDP and start-up costs for different countries. The figure also plots the values predicted by the model. As can be seen, the cost of business start-up captures a substantial amount of cross-country income variability.

To compute the average income gap from the US captured by the model,

⁹The original (undetrended) function for the accumulation of knowledge is $h_{t+1} = h_t + \bar{H}_t^{1-\nu} (H_t^\theta e_t^{1-\theta})^\nu$, where \bar{H}_t is the worldwide knowledge, external to an individual country, which grows at the constant rate \bar{g} . Normalizing all terms by \bar{H}_t , the investment function becomes $h_{t+1} = (1 - \phi)h_t + A(H_t^\theta e_t^{1-\theta})^\nu$, where $\phi = \bar{g}/(1 + \bar{g}) \simeq \bar{g}$ and $A = 1/(1 + \bar{g})$. Because A acts as a rescaling factor, we can set $A = 1$.

¹⁰In Manuelli & Seshadri (2005) the cost of human capital investment has two components: time and expenditures. Our specification does not distinguish between these components and the investment cost is captured by the single variable e . However, this does not alter in important ways the main properties of the model. As Manuelli and Seshadri show, the key parameter to replicate the life-time earning profile is not the relative importance of the two inputs but the return-to-scale parameter. Notice also that the depreciation rate $\phi = 0.018$ is also equal to the value chosen by Manuelli and Seshadri.

Figure 3: Entry cost and cross-country output per capita - Data and model.



we compute the following index:

$$\text{Index} = 1 - \frac{\sum_i |\hat{y}_i - y_i|}{\sum_i |y_{US} - y_i|},$$

where y_i is the actual income of country i , \hat{y}_i is the income predicted by the model, given the observed cost of business start-up, and y_{US} is the US income. The model has been normalized so that it replicates US income; that is, $\hat{y}_{US} = y_{US}$. The index is 1 if the model replicates perfectly the actual cross-country incomes, that is, $\hat{y}_i = y_i$. It is zero if the cost of business start-up has no impact on the equilibrium income; that is, $\hat{y}_i = y_{US}$. For the baseline calibration the index is 0.51. Therefore, the model accounts for roughly half of the cross-country income gaps from the US.

Next we show how the values of θ and ν affect the results. Table 2 reports the income gaps accounted for by the model for alternative values of these parameters. The general finding is that the model is more successful the higher the return to scale, ν , and the lower the externalities, θ . The

sensitivity is especially high for the return to scale. However, even for small returns to scale, the model accounts for a non-negligible fraction of cross-country income gaps. Even if we take the extreme parametrization chosen by Parente & Prescott (2002), $\nu = 0.6$, the model still accounts for about 11 percent of the income gaps.

Table 2: Income gaps accounted for by the model.

Value of θ	Value of ν					
	<i>0.97</i>	<i>0.93</i>	<i>0.90</i>	<i>0.80</i>	<i>0.70</i>	<i>0.60</i>
<i>0.0</i>	0.68	0.51	0.42	0.26	0.18	0.13
<i>0.1</i>	0.66	0.48	0.40	0.25	0.17	0.12
<i>0.2</i>	0.64	0.46	0.37	0.23	0.15	0.11
<i>0.3</i>	0.62	0.43	0.35	0.21	0.14	0.10
<i>0.4</i>	0.59	0.40	0.32	0.19	0.12	0.09

We have also calculated the ‘domestic socially optimal’ steady-state level of output, that is, the output resulting from solving the problem of a country’s planner. This differs from the competitive output because of the externality at the domestic level, represented by θ . It also differs from the ‘global socially optimal’ steady-state level of output, which is the solution to the problem of a ‘global planner’, which internalizes the worldwide leakage, or spillover, represented by ν . The steady-state values of H and K in the domestic planner allocation are found by solving the first order conditions:

$$\beta\pi_2(H, K, H) = 1$$

$$\varphi_2(H, H; H) - \pi_3(H, K, H) = \beta \left[\pi_1(H, K, H) - \varphi_1(H, H; H) - \varphi_3(H, H; H) \right].$$

These are similar to conditions (15) and (16) except for the additional term $\varphi_3(H, H; H)$ in the second equation. This term captures the externality taken into account by the planner but ignored by the atomistic agents.

Table 3 reports the ‘competitive’ output as a fraction of the ‘domestic socially optimal’ output when there are no barriers to entry. A value greater than 1 means that there is over-accumulation of knowledge compared to the socially optimal level. As expected, this arises when the spillovers are small or zero; that is, when θ is small. In this case moderate barriers to business start-up would be welfare improving. On the other hand, values smaller than 1 mean that there is under-accumulation of knowledge compared to the socially optimal level. In this case barriers to entry are always suboptimal, while moderate subsidies could improve welfare. As can be seen from the table, the under-accumulation of knowledge arises for moderate spillovers.

Table 3: Steady-state output when contracts are not enforceable and there are no barriers to entry. Numbers are relative to the socially optimal output.

Value of θ	Value of ν					
	<i>0.97</i>	<i>0.93</i>	<i>0.90</i>	<i>0.80</i>	<i>0.70</i>	<i>0.60</i>
<i>0.0</i>	1.81	1.28	1.18	1.08	1.04	1.03
<i>0.1</i>	0.80	0.92	0.95	0.99	1.00	1.00
<i>0.2</i>	0.41	0.71	0.80	0.92	0.96	0.98
<i>0.3</i>	0.25	0.58	0.70	0.88	0.94	0.97
<i>0.4</i>	0.17	0.51	0.64	0.84	0.92	0.96

7 Covenants and other barriers to mobility

Other barriers to the mobility of knowledge capital may have an effect in our model which is similar to the cost of business start-up. As we have discussed in the Introduction, even within a similar legal and economic environment—resulting in similar costs for business start-up—there may be differences in other barriers. Covenants is one of them. A covenant which is ex-post enforced prevents the worker from using her acquired knowledge if she moves to another firm.

A natural way to model non-competitive covenants is by assuming that

a quitting worker can only use a fraction ξ of her accumulated knowledge in a new firm. This formulation also captures the case in which part of the knowledge can not be used by the worker due to the enforcement of IPR if she does not have full control of the patent. In our formulation, a more stringent enforcement of covenants (or IPRs) is captured by a lower fraction ξ .

To keep the presentation brief, we limit the analysis to the one-period model. The extension to an infinite horizon will follow the same logic as in the analysis with entry costs. The problem solved by a new firm which started at the beginning of the period can be written as:

$$S(h_0) = \max_{h_1, k_1, w} \left\{ -\varphi(h_0, h_1) - k_1 + (\xi h_1)^{1-\alpha} k_1^\alpha \right\} \quad (23)$$

s.t.

$$w - \varphi(h_0, h_1) \geq D(h_0)$$

$$-w - k_1 + (\xi h_1)^{1-\alpha} k_1^\alpha \geq 0.$$

The problem solved by an incumbent firm is as in problem (1). The first order conditions with respect to h_1 , for incumbent and new firms respectively, are:

$$(1 - \alpha) \left(\frac{k_1}{h_1} \right)^\alpha = \varphi_{h_1}(h_0, h_1) + \delta \cdot \left(\frac{k_0}{h_0} \right) \quad (24)$$

$$(1 - \alpha) \left(\frac{k_1}{h_1} \right)^\alpha = \varphi_{h_1}(h_0, h_1) \cdot \xi^{\alpha-1}. \quad (25)$$

Because $\xi < 1$ and $\alpha < 1$, the term $\xi^{\alpha-1} > 1$. Therefore, covenants have the effect of increasing the cost of accumulating knowledge and act similarly to the entry cost τ . Proposition 1 becomes:

Proposition 5 *The knowledge investment of a new firm h^{New} is strictly increasing in ξ and there exists $\bar{\xi} > 0$ such that $h^{New} = h^{Old}$.*

Proof 5 *Using the first order condition for the choice of physical capital, which is $\alpha(k_1/h_1)^{\alpha-1} = 1$ for both incumbent and new firms, the above first*

order conditions can be rewritten as:

$$\begin{aligned}(1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}} &= \varphi_{h_1}(h_0, h^{Old}) + \delta \cdot \left(\frac{k_0}{h_0}\right) \\ (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}} &= \varphi_{h_1}(h_0, h^{New})\xi^{-1}\end{aligned}$$

The proposition follows directly from these two conditions.

Q.E.D.

All the results obtained in Section 4 trivially extend to the case of covenants and other similar barriers to the mobility of knowledge.

8 Conclusion

We have developed a theory in which *barriers to knowledge mobility* affect the accumulation of knowledge, and therefore the level of income. The theory does not simply say that “competition enhances income”. It shows how different forms of contract enforcement affect the relation between competition, accumulation of human capital and economic development. In particular, when both investors and workers can not commit to long-term contracts, the accumulation of human capital is determined by those firms that value human capital the most, that is, start-up firms. As a result, high levels of human capital accumulation are associated with low barriers to the mobility of knowledge. In the absence of barriers the accumulation of knowledge can be suboptimal at the firm level but could be welfare improving if there are spillovers.

In a semi-endogenous growth model we have shown that *barriers to business start-up* have the potential to explain significant cross-country income differences. This is the first step to bringing our theory to the data. We have also shown that other *barriers to knowledge mobility*, such as strict enforcement of Covenants or Intellectual Property Rights, can have similar effects, suggesting a wide scope for the empirical application of the theory.

A Proof of Proposition 2

We show that there is a unique equilibrium in which all firms implement $h_1 = h_1^{New}$, with the exception of one firm implementing the policy $h_1 = h_1^{Old}$. Because each firm is of mass zero, the deviating firm is negligible to the aggregate outcome.

Let h_1 be the policy choice given h_0 . The promised payment associated with this policy is $w(h_1) = S(h_0) + \varphi(h_0, h_1)$, where we make explicit the dependence of the payment on the investment policy h_1 . Define $\tilde{h}(h_1)$ the knowledge that makes the worker ex-post indifferent between keeping the promised payment $w(h_1)$, or moving to a new firm with initial knowledge $\tilde{h}(h_1)$. Formally, $\widehat{S}(\tilde{h}(h_1)) = w(h_1)$. From the definition of \widehat{S} provided in equation (5), we have that $\tilde{h}(h_1^{New}) = h_1^{New}$. Moreover, from (4), (6) and (7) we have that, if $h_1 \neq h_1^{New}$, then $w(h_1) > \widehat{S}(h_1)$. Because $\widehat{S}(h_1)$ is strictly increasing in h_1 , this implies that $\tilde{h}(h_1) > h_1$.

Now consider a firm implementing the policy $h_1 = h_1^{Old}$. If $h_1^{Old} \neq h_1^{New}$, as long as there is one worker outside the firm with $h_1 \in [h_1^{Old}, \tilde{h}(h_1^{Old})]$, the investor will not pay the promised $w(h_1^{Old})$ since he can always hire such worker with a lower payment. Anticipating this, the worker will not participate because the ex-ante value of quitting the firm, $S(h_0)$, is higher (recall (7)). On the other hand, if no other worker outside the firm has knowledge $h_1 \in [h_1^{Old}, \tilde{h}(h_1^{Old})]$, then the investor's promise $w(h_1^{Old})$ is credible.

The above argument shows that the unique optimal policy (up to removal of weakly dominated strategies) is:

$$h_1 = \begin{cases} h_1^{Old} & \text{if } \nexists \text{ another firm with } h_1 \in [h_1^{Old}, \tilde{h}(h_1^{Old})] \\ h_1^{New} & \text{if } \exists \text{ another firm with } h_1 \in [h_1^{Old}, \tilde{h}(h_1^{Old})] \end{cases}$$

This policy induces a unique equilibrium where only one firm implements $h_1 = h_1^{Old}$, while all other firms implement $h_1 = h_1^{New}$.

To complete the proof we only need to show that, if $h_1^{New} > h_1^{Old}$, then $h_1^{New} > \tilde{h}(h_1^{Old})$. This ensures that workers with $h_1 = h_1^{New}$ are not used to replace workers with $h_1 = h_1^{Old}$. To show this, consider condition (7). This implies that $w(h_1^{Old}) = \varphi(h_0, h_1^{Old}) - \varphi(h_0, h_1^{New}) + \widehat{S}(h_1^{New})$. From this we see that, if $h_1^{New} > h_1^{Old}$, then $w(h_1^{Old}) < \widehat{S}(h_1^{New})$. Because $w(h_1^{Old}) = \widehat{S}(\tilde{h}(h_1^{Old}))$, we must have $h_1^{New} > \tilde{h}(h_1^{Old})$. *Q.E.D.*

B First order conditions with investor's commitment

We first prove the following lemma:

Lemma 1 *The enforcement constraint ‘after’ the investment in knowledge is satisfied if the enforcement constraint is satisfied ‘before’ the investment in knowledge.*

Proof 1 *The enforcement constraints can be rewritten as:*

$$\begin{aligned} \sum_{j=t}^{\infty} \beta^{j-t} [w_j - \varphi(h_j, h_{j+1}; H)] &\geq V(h_t) \\ \sum_{j=t}^{\infty} \beta^{j-t} [w_j - \varphi(h_j, h_{j+1}; H)] &\geq -\varphi(h_t, h_{t+1}; H) + \widehat{V}(h_{t+1}). \end{aligned}$$

Therefore, to show that the second constraint is satisfied when the first constraint is satisfied, it is enough to show that $V(h_t) \geq -\varphi(h_t, h_{t+1}; H) + \widehat{V}(h_{t+1})$ for any value of h_{t+1} . Because $V(h_t) = \max_h \{-\varphi(h_t, h; H) + \widehat{V}(h)\}$, we have that:

$$V(h_t) = \max_h \left\{ -\varphi(h_t, h; H) + \widehat{V}(h) \right\} \geq -\varphi(h_t, h_{t+1}; H) + \widehat{V}(h_{t+1})$$

for any h_{t+1} .

Q.E.D.

Let us now consider problem (9). Thanks to the above lemma we can ignore the enforcement constraint after the investment in knowledge. Let γ_t be the Lagrange multiplier associated with the enforcement constraint before the investment in knowledge and λ_0 the Lagrange multiplier associated with the participation constraint for the investor. The Lagrangian can be written as:

$$\begin{aligned} \mathcal{L} &= \sum_{t=0}^{\infty} \beta^t [w_t - \varphi(h_t, h_{t+1}; H)] \\ &+ \sum_{t=0}^{\infty} \beta^t \gamma_t \left\{ \sum_{j=t}^{\infty} \beta^{j-t} [w_j - \varphi(h_j, h_{j+1}; H)] - V_t(h_t) \right\} \\ &+ \lambda_0 \left\{ -w_0 - \tau h_1 - k_1 + \sum_{t=1}^{\infty} \beta^t [\pi(h_t, k_t, h_{t+1}) - w_t - k_{t+1}] \right\}. \end{aligned}$$

Define μ_t recursively as follows: $\mu_{t+1} = \mu_t + \gamma_t$, with $\mu_0 = 0$. Using this variable and rearranging terms, the Lagrangian can be written as:

$$\begin{aligned} \mathcal{L} &= \sum_{t=0}^{\infty} \beta^t \left\{ (1 + \mu_{t+1}) [w_t - \varphi(h_t, h_{t+1}; H)] - (\mu_{t+1} - \mu_t) V(h_t) \right\} \\ &+ \lambda_0 \left\{ -w_0 - \tau h_1 - k_1 + \sum_{t=1}^{\infty} \beta^t [\pi(h_t, k_t, h_{t+1}) - w_t - k_{t+1}] \right\}. \end{aligned}$$

This problem becomes recursive at any $t > 0$. Therefore, we can rewrite it as follows:

$$\mathcal{L} = \min_{\mu_1 \geq 0} \max_{\substack{w_0 \geq 0, \\ k_1, h_1}} \left\{ \lambda_0 \left[-w_0 - \tau h_1 - k_1 \right] + (1 + \mu_1) \left[w_0 - \varphi(h_0, h_1; H) \right] \right. \quad (26) \\ \left. - \mu_1 V(h_0) + \beta W(\mu_1, h_1, k_1) \right\},$$

with the function W is defined recursively as follows:

$$W(\mu_t, h_t, k_t) = \min_{\mu_{t+1} \geq \mu_t} \max_{\substack{w_t \geq 0, \\ k_{t+1}, h_{t+1}}} \left\{ \lambda_0 \left[\pi(h_t, k_t, h_{t+1}) - w_t - k_{t+1} \right] \quad (27) \right. \\ \left. + (1 + \mu_{t+1}) \left[w_t - \varphi(h_t, h_{t+1}; H) \right] \right. \\ \left. - (\mu_{t+1} - \mu_t) V(h_t) + \beta W(\mu_{t+1}, h_{t+1}, k_{t+1}) \right\},$$

for all $t > 0$.

The first optimization problem (equation (26)) is the problem solved by a new firm with initial state h_0 and for a given λ_0 . The Lagrange multiplier λ_0 is determined such that the participation constraint for the investor is satisfied. The tighter this constraint is, the higher the value of λ_0 . The second optimization problem (equation (27)) is the one solved after entering. Therefore, this is the problem solved by an incumbent firm that starts with states μ_t , h_t and k_t .

Taking derivatives in problem (26) gives:

$$V(h_t) \leq -\varphi(h_t, h_{t+1}; H) + w_t + \beta V(h_{t+1}) \quad (28)$$

$$1 + \mu_{t+1} \leq \lambda_0 \quad (29)$$

$$\beta \pi_2(h_{t+1}, h_{t+1}, k_{t+1}) = 1 \quad (30)$$

$$\lambda_0 \tau + (1 + \mu_{t+1}) \varphi_2(h_t, h_{t+1}; H) = \beta W_2(\mu_{t+1}, h_{t+1}, k_{t+1}), \quad (31)$$

for $t = 0$ and with the envelope term given by:

$$W_2(\mu_t, h_t, k_t) = \lambda_0 \pi_1(h_t, k_t, h_{t+1}) - (1 + \mu_{t+1}) \varphi_1(h_t, h_{t+1}; H) - (\mu_{t+1} - \mu_t) V_1(h_t).$$

The first-order conditions in problem (27) are (28)-(30) and

$$-\lambda_0 \pi_3(h_t, k_t, h_{t+1}) + (1 + \mu_{t+1}) \varphi_2(h_t, h_{t+1}; H) = \beta W_2(\mu_{t+1}, h_{t+1}, k_{t+1}). \quad (32)$$

As emphasized above, the value of λ_0 depends on the tightness of the participation constraint for the investor. Assume that a new firm can choose $h_1 < h_0$

without any cost. This is equivalent to assuming that the worker chooses to destroy part of her knowledge. We can then prove that the investor is able to break even if the contract chooses the unconstrained sequence of h . This implies that $\lambda_0 = 1$ and, from condition (29), $\mu_t = 0$ for all t . Using this and substituting the envelope term, conditions (28), (30), (31) and (32) become (11)-(14). *Q.E.D.*

C Steady state equilibrium when the investor commits

Proposition 6 *There is a unique steady state equilibrium in which all firms have the same knowledge H and physical capital K .*

Proof 6 *Consider condition (16), which we rewrite here as follows:*

$$\varphi_2(H, H; H) + \beta\varphi_1(H, H; H) = \pi_3(H, K, H) + \beta\pi_1(H, K, H).$$

The right-hand term remains constant for any value of H . In fact, taking into account the functional form of π (see equation (8)), we have $\pi_3(H, K, H) = -\delta(K/H)$ and $\pi_1(H, K, H) = \delta(K/H) + (1 - \alpha)(K/H)^\alpha$. These two terms only depend on the ratio K/H . From condition (15) we have $\beta\pi_2(H, K, H) = \beta[1 + \alpha(K/H)^{\alpha-1}] = 1$, which uniquely determines the ratio K/H .

Let us now look at the left-hand term. Because φ is homogenous of degree $\rho > 1$, the derivatives φ_1 and φ_2 are homogeneous of degree $\rho - 1$. Therefore, the left-hand-side term can be written as

$$\varphi_2(H, H; H) + \beta\varphi_1(H, H; H) = \left[\varphi_2(1, 1; 1) + \beta\varphi_1(1, 1; 1) \right] H^{\rho-1}.$$

*Because $\rho > 1$, this term is strictly increasing in H , converges to zero as $H \rightarrow 0$ and to infinity as $H \rightarrow \infty$. Therefore, there exists a unique value of H that solves this condition. The uniqueness of H then implies the uniqueness of K . *Q.E.D.**

D Proof of Proposition 3

Suppose that the knowledge investment chosen by a new firm is different from that chosen by an incumbent firm. Denote by h_{t+1}^{New} and h_{t+1}^{Old} the investment of new and incumbent firms, respectively. Because h_{t+1}^{New} solves the problem $V_t(h_t) = \max_{h_{t+1}} \{-\varphi(h_t, h_{t+1}; H) + \widehat{V}_t(h_{t+1})\}$, it follows that:

$$V_t(h_t) > -\varphi(h_t, h_{t+1}^{Old}; H) + \widehat{V}_t(h_{t+1}^{Old}),$$

if $h_{t+1}^{Old} \neq h_{t+1}^{New}$. But then constraints (17) and (18) cannot both be satisfied. Therefore, the only feasible solution is $h_{t+1} = h_{t+1}^{New}$. *Q.E.D.*

E Derivation of the first order condition (20)

Following the same steps as Appendix B, we can show that in a steady state equilibrium, problem (19) can be reformulated as:

$$\mathcal{L} = \min_{\mu_1 \geq 0} \max_{\substack{w_0 \geq 0, \\ k_1, h_1}} \left\{ \lambda_0 \left[-w_0 - \tau h_1 - k_1 \right] + (1 + \mu_1) \left[w_0 - \varphi(h_0, h_1; H) \right] \right. \\ \left. - \mu_1 J(h_0) + \beta W(\mu_1, h_1, k_1) \right\}, \quad (33)$$

with the function W is defined recursively as follows:

$$W(\mu_t, h_t, k_t) = \min_{\mu_{t+1} \geq \mu_t} \max_{\substack{w_t \geq 0, \\ k_{t+1}}} \left\{ \lambda_0 \left[\pi(h_t, k_t, f(h_t)) - w_t - k_{t+1} \right] \right. \\ \left. + (1 + \mu_{t+1}) \left[w_t - \varphi(h_t, f(h_t); H) \right] \right. \\ \left. - (\mu_{t+1} - \mu_t) J(h_t) + \beta W(\mu_{t+1}, f(h_t), k_{t+1}) \right\} \quad (34)$$

for all $t > 0$.

The first order condition with respect to h_1 in problem (33) gives:

$$\lambda_0 \tau + (1 + \mu_1) \varphi_2(h_0, h_1; H) = \beta W_2(\mu_1, h_1, k_1), \quad (35)$$

with the envelope condition given by:

$$W_2(\mu_t, h_t, k_t) = \lambda_0 \pi_1(h_t, k_t, f(h_t)) + \lambda_0 \pi_3(h_t, k_t, f(h_t)) f_1(h_t) \\ - (1 + \mu_{t+1}) \varphi_1(h_t, f(h_t); H) - \mu_{t+1} \varphi_2(h_t, f(h_t); H) f_1(h_t) \\ - (\mu_{t+1} - \mu_t) J_1(h_t) + \beta W_2(\mu_{t+1}, h_{t+1}, k_{t+1}) f_1(h_t). \quad (36)$$

With limited enforcement, condition (35) must be satisfied at any point in time. Substituting this condition in (36), we get:

$$W_2(\mu_t, h_t, k_t) = \lambda_0 \pi_1(h_t, k_t, f(h_t)) - (1 + \mu_{t+1}) \varphi_1(h_t, f_t(h_t); H) \\ - (\mu_{t+1} - \mu_t) J_1(h_t) + \lambda_0 \left[\pi_3(h_t, k_t, f(h_t)) + \tau \right] f_1(h_t).$$

Also, in this case we can prove that the unconstrained investment in knowledge capital allows the investor to break even. Therefore, $\lambda_0 = 1$ and $\mu_t = 0$. Using this result and substituting the envelope in (35) we get condition (20). *Q.E.D.*

F Proof of Proposition 4

In the steady state without commitment, potential new firms start with the same knowledge H as incumbents. Because $H = f(H)$, (22) can be written as:

$$\tau + \varphi_2(H, H; H) = \beta \left[\pi_1(H, K, H) - \varphi_1(H, H; H) \right] + \beta f_1(H) \left[\pi_3(H, K, H) + \tau \right],$$

which determines the steady state knowledge for incumbent and new firms when the investor does not commit (double-sided limited enforcement).

This condition must be compared to the optimality condition that determines the steady state knowledge when the investor commits to the contract (one-side limited enforcement). This is given by equation (16), which we rewrite as:

$$\varphi_2(H, H; H) = \beta \left[\pi_1(H, K, H) - \varphi_1(H, H; H) \right] + \pi_3(H, K, H).$$

The homogeneity of degree ρ of the cost function φ implies that the derivatives are homogeneous of degree $\rho - 1$. Therefore, the above two conditions can be rewritten as:

$$\begin{aligned} \left[\varphi_2(1, 1; 1) + \beta \varphi_1(1, 1; 1) \right] H^{\rho-1} &= \beta \pi_1(H, K, H) + \beta f_1(H) \pi_3(H, K, H) \quad (37) \\ &- \tau \left[1 - \beta f_1(H) \right] \end{aligned}$$

$$\left[\varphi_2(1, 1; 1) + \beta \varphi_1(1, 1; 1) \right] H^{\rho-1} = \beta \pi_1(H, K, H) + \pi_3(H, K, H). \quad (38)$$

Because $\rho - 1 > 0$, the left-hand terms are strictly increasing in H , converge to zero as $H \rightarrow 0$ and to infinity as $H \rightarrow \infty$. We further observe that, as shown in the proof of Proposition 6, the terms π_1 and π_3 only depend on the ratio K/H . This term is uniquely pinned down by condition (12), which is the same for both economies. Therefore, $\pi_1(H, K, H)$ and $\pi_3(H, K, H)$ do not change as H changes.

Consider first the case with zero start-up cost, that is, $\tau = 0$. If $f_1(H) \leq 1$, as postulated in the proposition, the term $\beta f_1(H) < 1$. Because $\pi_3(H, K, H) < 0$ and $\beta f_1(H) < 1$, the right-hand side of (37) is bigger than the right-hand side of (38) for a given H . This implies that the value of H in the first equation must be bigger than in the second, that is, $H^{NC} > H^C$. Without capital obsolescence, $\pi_3(H, K, H) = 0$, and therefore, (37) and (38) are indistinguishable if $\tau = 0$.

Let us now consider the case $\tau > 0$. This variable only affects condition (37). Because $\beta f_1(H) < 1$, an increase in τ reduces the right-hand side of (37), which requires a lower value of H . For a sufficiently large τ , the steady-state level of knowledge declines to the point in which $H^{NC} < H^C$. *Q.E.D.*

G The role of barriers when the probability of survival is $p < 1$

We keep the assumption that the survival of the firm is observed after the investment in knowledge. Therefore, the level of h_{t+1} is predetermined for new firms. The physical capital, however, is only chosen after it is known that the firm will survive. This is not essential for the properties of the equilibrium but it simplifies its characterization.

If an incumbent firm survives, the worker receives w_t and stays with her current employer. If the firm exits, the worker is hired by a new firm and receives the lifetime utility $\widehat{V}(h_{t+1})$. We can then define the pseudo utility flow for the worker as follows:

$$U(h_t, h_{t+1}, w_t; H) \equiv -\varphi(h_t, h_{t+1}; H) + pw_t + (1-p)\widehat{V}(h_{t+1}).$$

The problem with investor's commitment is similar to (9) after making the following changes: the term $w_t - \varphi(h_t, h_{t+1}; H)$ is replaced with $U(h_t, h_{t+1}, w_t; H)$; future flows are discounted by $p\beta$; the firm pays $w_t + k_{t+1}$ only in case of survival.

After making these changes, we repeat the steps used in Appendix B for the case with $p = 1$. The first order conditions for a new firm at $t = 0$ are:

$$V(h_t) \leq -\varphi(h_t, h_{t+1}; H) + pw_t + (1-p)\widehat{V}(h_{t+1}) + p\beta V(h_{t+1}) \quad (39)$$

$$1 + \mu_{t+1} \leq \lambda_0 \quad (40)$$

$$\beta\pi_2(h_{t+1}, k_{t+1}, h_{t+1}) = 1 \quad (41)$$

$$\begin{aligned} \lambda_0\tau + (1 + \mu_{t+1})\varphi_2(h_t, h_{t+1}; H) = \\ (1 + \mu_{t+1})(1-p)\widehat{D}_1(h_{t+1}) + p\beta W_2(\mu_{t+1}, h_{t+1}, k_{t+1}), \end{aligned} \quad (42)$$

with the envelope term given by:

$$W_2(\mu_t, h_t, k_t) = \lambda_0\pi_1(h_t, k_t, h_{t+1}) - (1 + \mu_{t+1})\varphi_1(h_t, h_{t+1}; H) - (\mu_{t+1} - \mu_t)V_1(h_t).$$

The first order conditions for $t > 0$ (incumbent firm) are (39)-(41) and

$$\begin{aligned} -\lambda_0\pi_3(h_t, k_t, h_{t+1}) + (1 + \mu_{t+1})\varphi_2(h_t, h_{t+1}; H) = \\ (1 + \mu_{t+1})(1-p)\widehat{V}_1(h_{t+1}) + p\beta W_2(\mu_{t+1}, h_{t+1}, k_{t+1}). \end{aligned} \quad (43)$$

Also, in this case the investor breaks even with unconstrained knowledge. Therefore, $\lambda_0 = 1$ and $\mu_t = 0$. With all μ set to 0, the function W is the surplus generated by an incumbent firm. Using this, the surplus generated by a new firm, after the investment in knowledge and after the realization of survival, can be written as:

$$\widehat{V}(h_t) = -\tau h_{t+1} - k_{t+1} - w_t + \beta W(1, h_{t+1}, k_{t+1}),$$

from which we have $\widehat{V}_1(h_{t+1}) = -\tau + \beta W_2(1, h_{t+1}, k_{t+1})$. Therefore, conditions (42) and (43) can be rewritten as:

$$(1-p)\tau + \tau + \varphi_2(h_t, h_{t+1}; H) = \beta \left[\pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H) \right] \quad (44)$$

$$(1-p)\tau - \pi_3(h_t, k_t, h_{t+1}) + \varphi_2(h_t, h_{t+1}; H) = \beta \left[\pi_1(h_{t+1}, k_{t+1}, h_{t+2}) - \varphi_1(h_{t+1}, h_{t+2}; H) \right]. \quad (45)$$

The conditions for the accumulation of knowledge when the investor commits to the contract are similar to the corresponding conditions derived earlier (see (13) and (14)), with the exception of the constant term $(1-p)\tau$. The most important difference with respect to the case with $p = 1$ is that now the entry cost negatively affects the steady state value of H even if the investor commits to the contract.

Higher values of p (higher survival) increase the steady state value of knowledge because they reduce the term $(1-p)\tau$. This corresponds to a reduction in the marginal cost of accumulating knowledge for both new and incumbent firms.

When both parties are unable to commit, the optimization problem can be written as in (19), once we replace $w_t - \varphi(h_t, h_{t+1}; H)$ with $U(h_t, h_{t+1}, w_t; H)$, discount future flows by $p\beta$, and take into account that the firm pays $w_t + k_{t+1}$ only in case of survival. The first order condition for the accumulation of knowledge of a new firm ($t = 0$) can be written as:

$$(1-p)\tau + \tau + \varphi_2(h_t, h_{t+1}; H) = \beta \left\{ \pi_1(h_{t+1}, k_{t+1}, f(h_{t+1})) - \varphi_1(h_{t+1}, f(h_{t+1}); H) + f_1(h_{t+1}) \left[\pi_3(h_{t+1}, k_{t+1}, f(h_{t+1})) + \tau \right] \right\}, \quad (46)$$

which differs from (20) only in the constant term $(1-p)\tau$.

Let H^C be the steady state knowledge when the investor commits and H^{NC} when the investor does not commit. Proposition 4 can be reformulated as follows:

Proposition 7 *Assume $p \in (0, 1)$. The steady state values of H^C and H^{NC} are both strictly decreasing in τ . Moreover, there exists $\bar{\tau} > 0$ such that $H^{NC} > H^C$ for $\tau < \bar{\tau}$ and $H^{NC} < H^C$ for $\tau > \bar{\tau}$.*

Barriers to entry affect the accumulation of knowledge even when the investor commits to the contract. However, their negative impact is stronger with double-sided limited commitment. The proof of the proposition, which is omitted for economy of space, uses the same logic as the proof of Proposition 4.

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