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Abstract
We use a buffer-stock saving model to explain the evolution of the consumer net-worth distribution in the US between 1983 and 2004. We focus on net-worth up to the 90th percentile due to the well-known problem of matching the wealth concentration in the top percentiles. The model .fits the net-worth distribution (up to the 90th percentile) in the Survey of Consumer Finances 1983 well if we account for heterogeneity across consumers, especially in terms of age and education. Most interestingly, the estimated model for 1983 predicts the observed stability of net worth up to the 90th percentile between 1983 and 2004 if we attribute only part of the observed increase in the variance of labor earnings to higher income uncertainty. Quantitatively, the stronger precautionary-saving motive due to the increase in labor income risk is then counterbalanced by the higher cost of buffer-stock savings due to the fall of the real interest rate.

Keywords
Buffer-stock saving, consumer wealth, simulated method of moments.

JEL codes: E21, D91.

Incomplete Markets and the Evolution of the US Consumer Wealth Distribution[1]

Thomas Hintermaier[a] and Winfried Koeniger[b]

1 Introduction

Net worth in the US is quite unequally distributed across consumers where the amount of wealth up to the 90th percentile of the distribution has remained quite stable in the last 20 years (details are provided in the next section). Since many academic and political debates revolve around inequality, it is important to understand what economic models are useful to explain the level of wealth inequality and its evolution or stability over time.

Maybe the most important model in the recent consumption literature that makes predictions about wealth inequality is the classic buffer-stock saving model. In that model incomplete markets imply an endogenous steady-state distribution of wealth across consumers with different histories of uninsurable labor income shocks (Aiyagari, 1994, Carroll 1997, Deaton, 1991). We show that, after accounting for heterogeneity in age or education, the buffer-stock saving model explains the observed cross-sectional distribution of consumer net worth in the US up to the 90th percentile. Most interestingly, the model makes good quantitative predictions for the changes of net worth in the last 20 years if, as suggested by Cunha and Heckman (2007), we attribute only part of the observed increase in the variance of labor earnings to higher earnings uncertainty.

We apply the buffer-stock model to the time period 1983-2004 for which comparable disaggregate data on consumer wealth are available in the US. The focus is on the net-worth distribution up to the 90th percentile due to the well-known problem of matching the wealth concentration in the top percentiles. We first estimate the model by fitting the empirical net-worth distribution (up to the 90th percentile) in 1983 with the simulated method of moments. The model explains the data well: it implies plausible estimates of risk aversion and the discount factor. We then apply the estimated model to predict the evolution of US consumer wealth between 1983 and 2004. We feed

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the changes in labor income risk and the real interest rate into the model to predict the consumer wealth in 2004. The model predicts the stability of net worth up to the 90th percentile of the distribution and the model predictions for 2004 cannot be rejected at the 5% significance level if only part of the observed increase in the variance of labor earnings is unexpected and thus reflects higher labor income risk. Quantitatively, the stronger precautionary-saving motive due to the increase in labor income risk is counterbalanced by the higher cost of buffer-stock savings due to the fall of the real interest rate between 1983 and 2004.

Many other papers have investigated empirical predictions of the buffer-stock saving model (Gourinchas and Parker, 2002, Cagetti, 2003, Castañeda, Díaz-Gímenez and Ríos-Rull, 2003, and the references therein). Gourinchas and Parker (2002) match life-cycle consumption profiles and Cagetti (2003) matches median-wealth profiles to estimate impatience and risk aversion. Instead, in this paper we rely on the shape of the cross-sectional wealth *distribution* as the basis for our analysis and the estimation. Most related in this respect is the quantitative analysis of US wealth inequality by Castañeda et al. (2003) who calibrate an incomplete-markets model to match the earnings and wealth inequality observed in the Survey of Consumer Finances (SCF) 1992. They add additional twists to the basic incomplete-markets model, mixing features of a dynastic and life-cycle economy, to match the concentration of wealth at the *top* percentiles of the distribution. To achieve this quantitatively, one necessary assumption is that there exists a state with very large labor earnings which is attained with small probability: in Castañeda et al. (2003), Table 5, hourly wages in the best state are assumed to be 1,000 times larger than in the worst state and about 100 times larger than in the second-best state. In this paper, we follow an alternative research strategy by abstracting from the very wealth-rich consumers so that we do not need this assumption for the earnings process. Indeed, the very wealth-rich consumers may not be considered *ex ante* identical compared with the rest of the sample and thus may violate the basic assumption in the buffer-stock model which we present below. Our research strategy is sensible if general equilibrium feedbacks on the interest rate from the consumers excluded in the analysis (but exposed to the same changes in the economic environment) are negligible quantitatively. We consider this to be a reasonable working hypothesis given that capital markets have been highly integrated globally in the time period which we consider.

We show that if we abstract from the top percentiles of the wealth distribution, the simple textbook infinite-horizon incomplete-markets model can be quantitatively successful not only in matching the cross-sectional net-worth distribution up to the 90th percentile but also its evolution over time.

Since the first result is achieved to some extent "by construction" (due to the cleaning of the data), the most interesting contribution of this paper is the out-of-sample application where we compare the predictions of the model for the *evolution* of the net-worth distribution between 1983 and 2004 with the data. To the best of our knowledge, we are the first to consider these out-of-sample predictions.

The rest of the paper is structured as follows. In Section 2 we discuss empirical facts for the distribution of US-consumer net worth and its determinants. We present the model and discuss the numerical solution in Section 3. In Section 4 we estimate the model before we analyze in Section 5 how well the model predicts the evolution of US consumer net worth between 1983 and 2004. We conclude in Section 6.

2 Empirical facts

In this section we present the empirical facts which we use for our subsequent analysis. Most of these facts are based on the Survey of Consumer Finances (SCF). The SCF has been widely used as it provides the most accurate information on consumer finances in the US. The data collectors of the Federal Reserve System pay special attention in their sampling procedures to accurately capture the right-skewed wealth distribution (see Kennickell, 2003, and the references therein). The data thus allow us to compute precise statistics for consumer net worth.[2]

We compute the empirical facts for consumers below age 40. This adjusted sample accounts for heterogeneity of consumers with different ages since the model below, in which consumers have an infinite horizon, abstracts from life-cycle considerations. As argued by Gourinchas and Parker (2002), buffer-stock saving can be expected to capture the behavior of consumers below age 40 whereas saving for retirement is important for consumers above that age.[3] Furthermore, we compute per capita labor earnings and net worth in each household to address the concern that some of the variation in the SCF data is due to households of different sizes.[4] We now present the empir-

[2]When computing the statistics in the data we use the sampling weights provided in the SCF.

[3]Increasing the age cutoff reduces the fit of the model, consistent with results in Cagetti (2003) and Gourinchas and Parker (2002). The chosen age limit of 40 also keeps the sample roughly comparable with the sample of 22-36 year olds which we will use to apply results of Cunha and Heckman (2007) for purging the variance of labor earnings from forecastable variation.

[4]We use the equivalence scale proposed by Blundell, Browning and Meghir (1994) where

ical distribution of consumer net worth in the US and then describe changes in its determinants between 1983 and 2004.

2.1 The distribution of consumer net worth

We will estimate our model matching statistics of the distribution of consumer net worth in 1983. We then compare the model's predictions for 2004 with the data. We have chosen these two dates because they span the time period in which detailed comparable data on consumers' net worth is recorded in the triennial SCF. Both years, 1983 and 2004, come after a trough in the US business cycle (1982 and 2001 according to the NBER definition) so that changes reflect long-term trends rather than cyclical variation.

We largely follow Budría Rodríguez, Díaz-Giménez, Quadrini and Ríos-Rull (2002) and Díaz-Giménez, Quadrini and Ríos-Rull (1997) in constructing measures for the distribution of net worth and labor earnings in the US. Net worth is defined as the sum of net financial assets and non-financial assets. To make the empirical data comparable with the data computed within the model, we normalize net worth by average disposable labor earnings in the population. In the next subsection we explain how we compute disposable labor earnings.

Since all assets and liabilities in our model below are riskless, our empirical measure of net financial assets does not include stocks and mutual funds. As we do not allow for default in our model we also exclude unsecured debt. Our main results are not much affected by this cleaning of the data as we mention in one of the robustness checks below. The reason is that the ownership of risky assets is very concentrated at the top of the distribution and thus not very important for our sample of interest. Moreover, secured debt is most important and amounts to more than 80% of total debt in our sample. More detailed information on how we construct net worth is contained in the data appendix.

Figure 1 displays the histograms of consumer net worth in 1983 and 2004. We only plot the distributions up to a value 10 of net worth for expositional purposes (the thin tail of the distribution extends up to levels of net worth of more than 100 average disposable labor income equivalents). The figure suggests that the distribution changes only slightly. The distribution in 2004 has a bit more probability mass at the bottom and top of the support. In particular, there is no household in our sample with negative net worth in

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the size of the household equals the number of adults plus 0.4 the number of children. An alternative approach, which is beyond the scope of the paper, would be a micro-founded adjustment based on modeling the changes in marginal utility due to children or partners in the household.

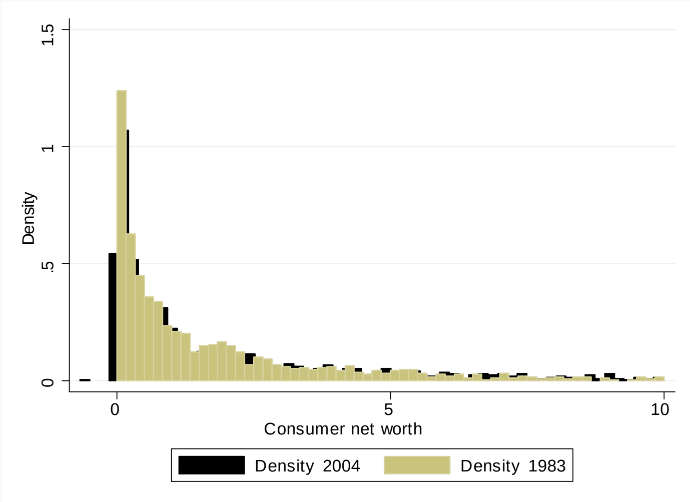

Figure 1: Distribution of consumer net worth in the US in 1983 and 2004. Source: own computations based on the Survey of Consumer Finances. Notes: Data normalized by average disposable labor earnings.

1983 and 0.1% of households have negative net worth in 2004. Moreover, about 4% of households hold zero net worth in 1983 and 2004. Overall, the Kolmogorov-Smirnov test cannot reject at conventional significance levels that both distributions are the same. Indeed Figure 2 shows that net worth in terms of average labor earnings at the nine deciles[5] has remained remarkably stable between 1983 and 2004. Note that average wealth (normalized by average disposable labor earnings) in the sample has increased from 2.8 to 3.3 during the same period, an 18% increase in 20 years. As the data show, this growth has been concentrated mostly in the top percentiles.

2.2 The determinants of consumer net worth

In the buffer-stock saving model, which we present below, the following changes of net-worth determinants are potentially relevant: (i) a higher labor income risk, (ii) a lower real interest rate and (iii) a change in the borrowing limit. In our simple model we do not consider changes in other determinants like changes in demographics, changes in government expenditure, changes in the extent of intergenerational redistribution through the social security system and cohort effects (see, for example, the discussion in Parker, 2000).

[5] Note that when we talk about the nine deciles, we mean net worth at the 10th, 20th, ..., 90th percentile.

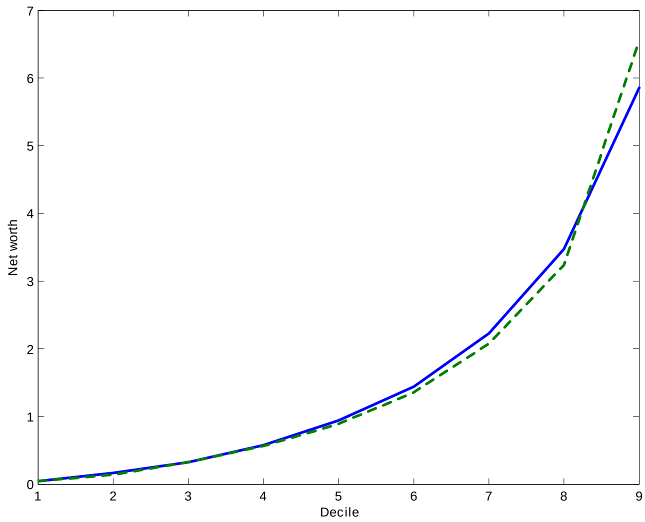

Figure 2: Net worth at each decile. Source: own computations based on the Survey of Consumer Finances. Notes: solid line: SCF data 1983; dashed line: SCF data 2004. Data are normalized by average disposable labor earnings. By deciles we mean net worth *at* the 10th, 20th, ..., 90th percentile.

Some of these explanations would require modeling the life-cycle of consumers which we abstract from here by adjusting the matched data moments accordingly as mentioned above. Moreover, it turns out that the simple model has enough degrees of freedom to capture the stability of net worth up to the 90th percentile in the period 1983-2004. Hence, the benefit of using a more complex model seems limited.

Labor income risk. We use SCF data on gross labor earnings and the NBER tax simulator described in Feenberg and Coutts (1993) to construct a measure for disposable labor earnings after taxes and transfers for each household in 1983 and 2004.[6] Arguably, after-tax rather than pre-tax earnings matter for households' consumption decisions since some of the uninsurable labor earnings risk may be eliminated by redistributive taxes and transfers. Figure 3 displays the histograms of disposable labor income in 1983 and 2004. The histograms are normalized by the mean disposable labor income of the population in the respective year. The figure shows that the

[6] More specifically, we use the programs provided by Kevin Moore on http://www.nber.org/~taxsim/ for constructing the SCF data in 1983 and 2004 which are fed into the tax simulator on the NBER website. For further information see the data appendix.

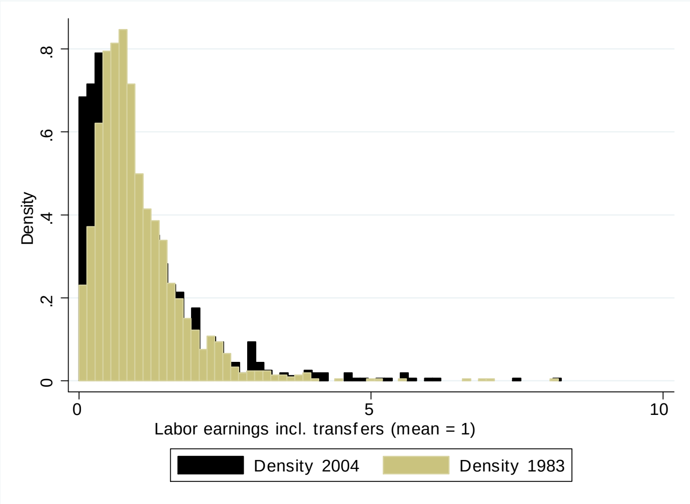

Figure 3: Distribution of disposable labor earnings in the US in 1983 and 2004. Source: own computations based on the Survey of Consumer Finances.

distribution in 2004 has more probability mass at the bottom and top of the support which we capture by an increase in labor income risk in our model. The Kolmogorov-Smirnov test rejects at the 1% significance level that both distributions are the same. We approximate the distributions as log-normal where $\ln(y_{1983}) \sim \mathcal{N}(-0.229, 0.458)$ and $\ln(y_{2004}) \sim \mathcal{N}(-0.327, 0.654)$.[7] Interestingly, the variances of log income are similar to those reported by Krueger and Perri (2006), Figure 1, who use the Consumer Expenditure Survey.

Real interest rate and borrowing limit. We use evidence by Caporale and Grier (2000) and Caballero, Farhi and Gourinchas (2008) which indicates that the real interest rate in the US has fallen by 2 percentage points from 4% to 2% since 1983.

Although the model below implies an endogenous solvency constraint (as shown in Aiyagari, 1994), we cannot exclude *a priori* that consumers' access to borrowing is more restricted. In previous versions of the paper we estimated the borrowing limit for the observed distribution of net worth in 1983 and 2004, respectively, and always estimated that limit to be zero. Given the percentiles of the net worth distribution which we match in our

[7]Although a formal test rejects log-normality due to some skewness, log-normality is a rather good parametric approximation of the data. The assumption of log-normality is attractive because it is convenient when we approximate the AR(1) income process by a Markov chain below.

estimation, reported in Figure 2 above, this may not be surprising since net worth is positive at these percentiles.[8] We thus fix the borrowing limit at zero so that net worth cannot be negative which seems a reasonable assumption in the classic buffer-stock saving model without unsecured debt and default.[9] We now introduce that model briefly.

3 The model

There is a continuum of risk averse consumers who have an infinite time horizon. They derive utility from a non-durable good c. The instantaneous utility is denoted by $u(c)$, where $u(.)$ is a strictly concave and increasing function.

Labor income y is stochastic and evolves according to an m-state Markov chain with transition probability matrix Γ for the income levels $y_1 < y_2 < ... < y_m$. Consumers have access to one risk-free asset a so that markets are incomplete. As consumers cannot fully diversify their risk, they are heterogeneous *ex post* although they are identical *ex ante*: different histories of labor income shocks imply different net worth positions. As has been shown by Aiyagari (1994), the model implies a natural solvency constraint $a_{t+1} \geq -y_1/r$, where r is the risk-free interest rate. The solvency constraint ensures positive consumption in the worst case in which the consumer always receives the lowest possible labor income. We restrict the constraint to $a_{t+1} \geq 0$ since, as explained in the previous section, nothing is gained in our estimation if we allow for a possibly laxer solvency constraint.

The recursive formulation of the household problem. We specify our model in discrete time. Rearranging the budget constraint,

$$c_t = (1+r)a_t - a_{t+1} + y_t \ ,$$

and defining cash-on-hand as

$$x_t = (1+r)a_t + y_t$$

[8] Matching lower percentiles would be problematic, as their measurement turns out to be very sensitive to a small number of observations. Moreover, a non-trivial problem of discontinuity would arise if we included percentiles at the mass point at zero where 4% of consumers are located. As we show in the appendix, the asymptotic properties of the estimator rely on a continuous and differentiable distribution function.

[9] Obviously, this is not inconsistent with the observed large levels of household debt in US data because most of that debt is secured by collateral.

we can write the Bellman equation as

$$V(x_t, y_t) = \max_{a_{t+1} \geq 0} \left[u(\underbrace{x_t - a_{t+1}}_{c_t}) + \beta E_t V(x_{t+1}, y_{t+1}) \right] .$$

It is well known that in an environment with incomplete markets, existence of finite steady-state wealth requires consumers to be impatient, $\beta < 1/(1+r)$, where β is the discount factor. Since the problem satisfies Blackwell's sufficient conditions (monotonicity and discounting) for a contraction mapping, standard dynamic programming techniques can be applied to solve for the steady state. Because of stationarity, we drop time indexes unless necessary and use primes " $'$ " to denote a one-period lead.

The steady state definition. A steady state is given by the policy functions for consumption $c(x, y)$, the accumulation equation $a'(x, y)$ and the evolution of the state variable $x'(x, y)$ so that for a given price r

(i) the value function $V(x, y)$ attains its maximal value,

(ii) the distribution measure χ is stationary[10] on the state space of x and y where, for a transition matrix $\Gamma(y'|y)$,

$$\chi(x', y') = \Sigma_y \Sigma_{\{x:\ x'=x'(x,y)\}} \chi(x, y) \Gamma(y'|y) .$$

Note that we assume that domestic changes in the supply of assets do not affect the interest rate. As in a small-open economy, this price is determined exogenously (on world markets).[11]

3.1 Numerical solution

It is well known that problems like ours do not have a closed-form solution for the optimal policies. Therefore, we solve the model numerically. We apply the endogenous grid-point method (EGM) proposed by Carroll (2006) which speeds up the computations by avoiding root-finding procedures. This helps to *estimate* the model since we then have to solve the model thousands of times on the grid of the parameter space.

[10] See Ríos-Rull (1999) for further discussion on the restrictions of admissible income processes which satisfy monotone mixing or the American-dream / American-nightmare condition.

[11] This assumption is not restrictive since we *observe* the price change in our *ex post* analysis for the period 1983-2004. Hence, the supply of assets and the observed price suffice to determine the equilibrium change of asset quantities. This allows us to be agnostic about the price elasticity of asset demand.

The EGM requires that we specify an exogenous grid for the asset holdings a_{t+1} in the *next* period. For a given income state y_{t+1} this determines future cash-on-hand $x_{t+1} = (1 + r^j)a_{t+1} + y_{t+1}$. More importantly, c_t is then retrieved simply by inverting the first-order condition $u'(c_t) = \beta E_t \partial V(x_{t+1}, y_{t+1})/\partial a_{t+1}$. For isoelastic utility functions the optimal c_t's are determined in closed form which speeds up computations. The endogenous grid for x_t is then found by using the identity $x_t = a_{t+1} + c_t$.

The smallest exogenous grid point for a_{t+1} is zero in which case the constraint $a_{t+1} \geq 0$ is binding. Then, $c_t = x_t$. The values for the 1,000 exogenous grid points above the constraint are chosen so that the grid is much finer where the strict concavity of the policy function is more pronounced. We interpolate the consumption function linearly between the points. We choose the upper bound of the exogenous grid so that, for any realization of income, the equilibrium policy implies that the consumer decumulates assets.

After an initial guess for the policy function, we iterate until convergence of that function is achieved at precision of 10^{-8}. As has become standard in the literature (see, e.g., Judd, 1992, and Aruoba et al., 2006), we evaluate the accuracy of our solutions by the normalized Euler equation error implied by the policy function. For the results below, these errors are always smaller than 10^{-5} over the entire range where the Euler equation applies with equality.

4 Model estimation

We first fit the model to the empirical distribution of consumer net worth in 1983 by using the simulated method of moments. We then apply the model with the estimated deep parameters to predict the evolution of the US consumer wealth distribution between 1983 and 2004. Since the "moments" we use are percentiles and thus non-standard, we show in the computational appendix that we can still apply standard methods to derive consistency and asymptotic normality of the estimates.

Parameter values for 1983. Based on the empirical facts discussed in Section 2, the interest rate $r = 0.04$. We assume that the natural logarithm of labor income follows an AR(1) process with a persistence of 0.95. This is close to the estimates in Storesletten, Telmer and Yaron (2004) and Heathcote, Storesletten and Violante (2004) for persistent earnings shocks in the US.[12]

[12] Storesletten et al. (2004) and Heathcote et al. (2004) estimate a richer income process than the AR(1) process we assume so that the use of their estimate is not fully satisfactory in our application. Given the different nature of our data set (cross-sectional data rather

We approximate the distribution of labor income in the SCF data as log-normal where $\ln(y_{1983}) \sim \mathcal{N}(-0.229, 0.458)$ and the AR(1) income process is discretized by a 31-state Markov chain using Tauchen's (1986) method.

We set the minimum labor income $y_1 = 0.05$ which is the average transfer income received in the SCF data in 1983 through unemployment insurance, food stamps or child support. We specify the 31 values of *log*-labor earnings symmetrically around the mean, where mean income (not in logs) is normalized to 1. For the given log-normal distribution of income, we then compute the transition matrix $\Gamma(y'|y)_{1983}$ implied by the Tauchen method. We check that the resulting Markov chain with 31 states approximates the log-normally distributed AR(1) process well.[13]

We choose a functional form of instantaneous utility with constant relative risk aversion (CRRA)

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

where the parameter for risk aversion, σ, will be estimated together with the discount factor β.

Estimation. We apply the simulated method of moments to estimate two parameters: the discount factor β and risk aversion σ. We solve the model for values of $\beta \in [0.75, 0.95]$ and $\sigma \in [0.1, 11]$ where we use 41 and 110 equi-spaced grid points for each interval and the adjacent grid points have a distance of 0.005 and 0.1, respectively. This results in solving and simulating 4,510 model cases. The upper bound for β is implied by the economic structure of the model, i.e., by restricting our attention to solutions with a finite stock of assets in the steady state. The restrictions on the parameter space for the lower bound of β and the interval for σ are based on economic plausibility.

than panel data), it is not obvious how to adjust their estimate without further strong assumptions. Interestingly, however, Figure 2 in Heathcote et al. (2004) shows that a substantial part, if not most, of the increase in the cross-sectional variance of log-wages since 1983 is due to the persistent component of the stochastic process. The variance of the permanent component increased already before 1983 and not so much thereafter and there is some indication that the transitory component increased at the beginning of the 1990s. Overall, our focus on the persistent component does seem to be a reasonable approximation and we view the persistence-parameter estimate as a best guess in our application. We provide robustness checks for a smaller persistence below.

[13] Although the Markov chain with 31 states approximates the log-normally distributed AR(1) process very well, we implement a bias correction which ensures that the discrete Markov chain implies *exactly* the same mean and variance. The idea is to choose the standard deviation which we use to compute the transition matrix so that the implied standard deviation of the Markov chain is exactly equal to the one in the data.

We estimate the parameters by matching the nine deciles[14] of the net worth distribution of the model with the data. These deciles make good moments since the parameters we estimate – impatience and risk aversion – are important determinants of the net worth distribution.

The estimation proceeds in subsequent steps. In the first step, we search for the combination of the two parameters which minimizes the distance between the model and the data for the nine deciles of the net worth distribution in 1983.[15] The search on the full parameter grid avoids complications in the estimation which otherwise might arise due to local minima. In the second step, we exploit the consistency of the estimates obtained in the first step to compute the variance-covariance matrix of the moments by bootstrapping 10,000 samples with the size equal to the respective selected sample in the SCF 1983. We then use the inverse of this matrix as weights in the second step of the estimation. Hence, moments which are less precisely estimated receive less weight in the estimation. We continue to update the weighting matrix and reestimate the model until the model estimates have converged. Convergence of the estimates typically occurs after about 5 steps.

Since we want to explain changes in the distribution in the next section, it is essential that the model matches the distribution well for the baseline year 1983. We test the model's performance using the seven overidentifying restrictions as we estimate two parameters matching nine moments. For further details on the implementation of the simulated method of moments and the computation of standard errors, we refer to the appendix.

Table 1 displays the estimates for 1983, with standard errors in brackets, and the statistic for the test of the overidentifying restrictions. The estimate of risk aversion is within the range of economically plausible values. The discount factor 0.85 is below commonly calibrated values but sensibly so given our different data targets. We elaborate on this in the next paragraph. Although the test of overidentifying restrictions rejects the model at the 1% level, Figure 4 shows that the net worth distribution is matched rather well: the model (the dotted line) matches the SCF data (the solid line) quite closely. Furthermore, Figure 5 shows that the distributions of the deciles (normalized by the respective mean) are well behaved and nicely bell shaped, in accordance with the asymptotically normal distribution derived in the appendix.

We estimate a lower discount factor than the often calibrated value of $\beta = 0.96$ (for example, Aiyagari, 1994) which is typically set to match aggregate

[14] Recall that by "nine deciles" we mean net worth at the 10th, 20th, ..., 90th percentile.

[15] To compute the model moments for the steady-state distributions, we simulate a history of 101,000 observations and discard the first 1,000.

Risk aversion σ
3.50
(0.1503)
Discount factor β
0.845
(0.0087)
Test of overidentifying restrictions
19.389
(5% crit.value of $\chi^2(7)$: 14.067
1% crit.value of $\chi^2(7)$: 18.475)

Table 1: Estimation results for 1983. Standard errors are reported in brackets.

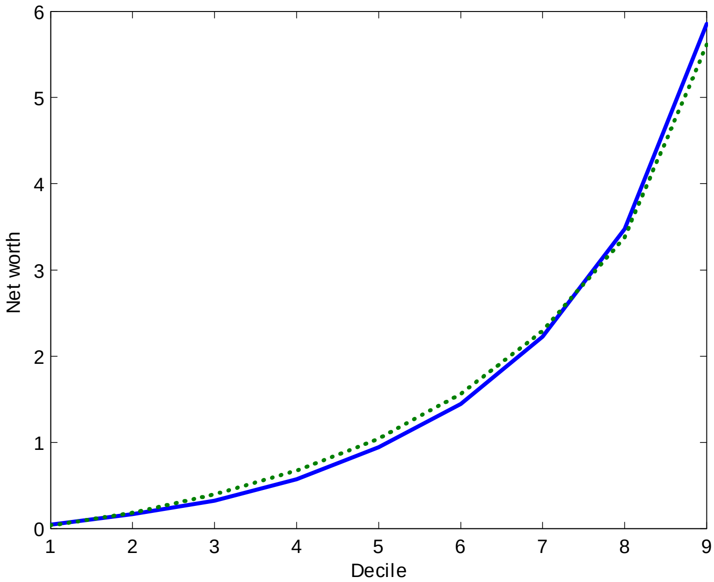

Figure 4: Net worth at each decile for consumers below age 40 in 1983. Notes: solid line: SCF data; dotted line: model data.

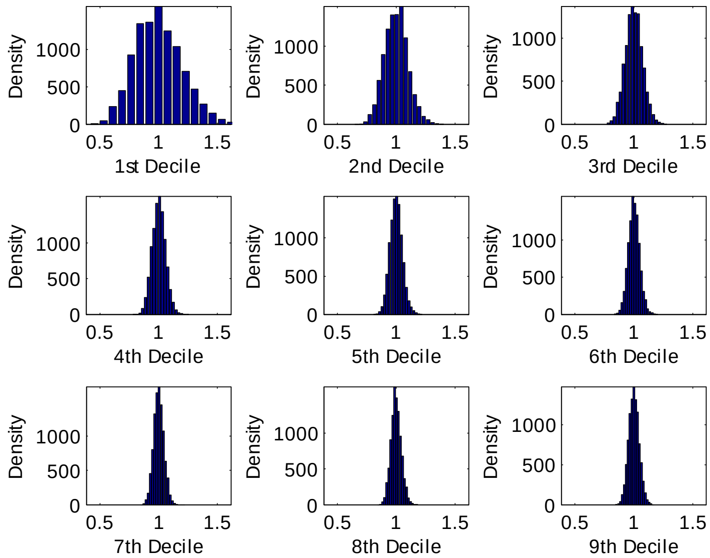

Figure 5: Distributions of the nine deciles (normalized by the respective mean).

wealth holdings. This is so for two reasons: (i) our population of interest is less than 40 years old and thus has a lower average wealth-income ratio, (ii) we match the wealth distribution up to the 90th percentile and not the top percentiles where a significant proportion of the wealth is concentrated.

Let us further discuss the selection of the sample and the data moments in the context of the estimation results. The focus on consumers below age 40 shall "control" for life-cycle effects in a non-parametric way so that a simple infinite-horizon buffer-stock saving model has a chance to fit the data. As mentioned before, Gourinchas and Parker (2002) have found that saving for retirement is an important saving motive for consumers above age 40. The chosen age limit of 40 also keeps the sample roughly comparable with the sample of 22-36 year olds which we use to apply results in Cunha and Heckman (2007) to purge the variance of labor earnings from forecastable variation. Furthermore, the focus on the distribution up to the 90th percentile allows us to circumvent the well-known failure of the simple buffer-stock saving model to account for the concentration of wealth at the top percentiles of the wealth distribution. As we have discussed in the Introduction, strong assumptions need to be imposed on the income process to capture this empirical wealth concentration. Hence, we try to answer the

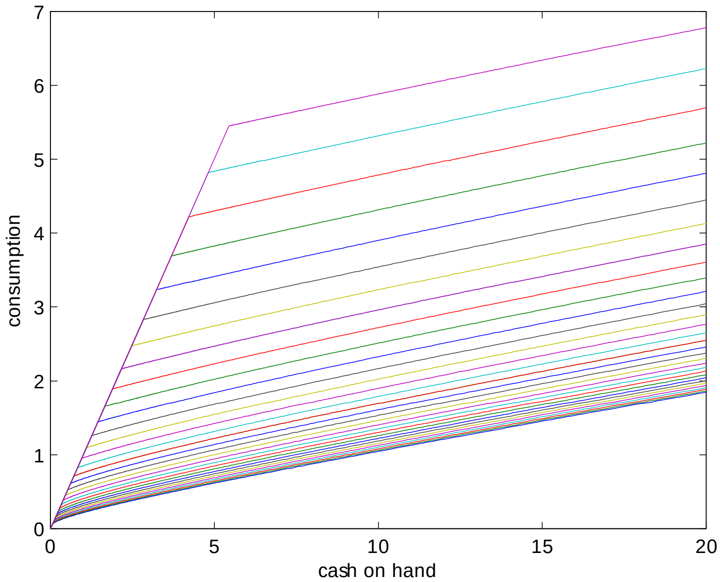

Figure 6: Consumption as a function of cash-on-hand for the 31 income states.

more modest but interesting question whether a simple buffer-stock saving model is useful to predict changes in the wealth distribution relevant for most consumers, that is for 90% of the population. We now characterize the model results in more detail using the parameter estimates in Table 1, before we check the robustness of the model performance.

Consumption policy. Figure 6 plots consumption as a function of cash-on-hand for the 31 different income states. Not surprisingly, higher income shifts the consumption function upward and the consumption function is more concave at lower income levels. For levels of cash-on-hand which imply that consumers are at the borrowing constraint, consumption is a linear function of cash-on-hand.

Simulation. We simulate our economy and plot the simulation results for an arbitrary sample of 300 periods. As can be seen in Figure 7, the exogenous income shocks are quite persistent. In the good income states, the consumer accumulates substantial wealth. If the consumer is in the bad income states long enough, he decumulates wealth until he is borrowing constrained. Consumers are borrowing constrained 3.8% of the time which is close to the 4%

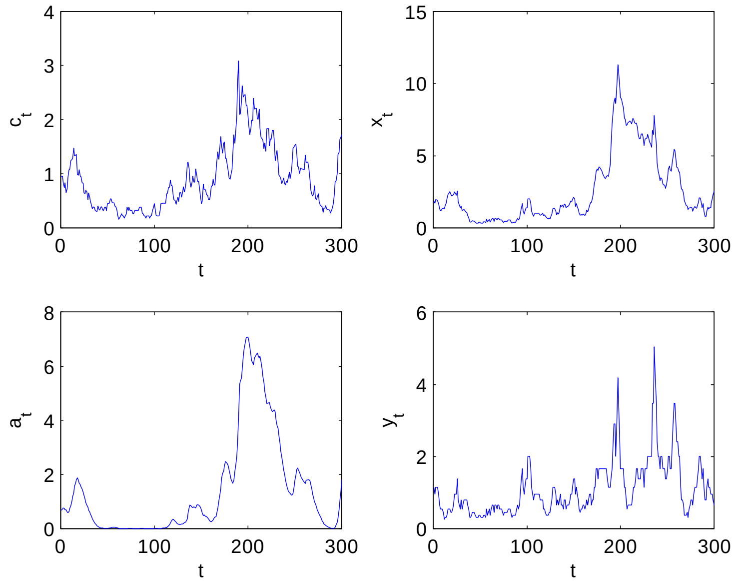

Figure 7: Simulation of the model plotted for an arbitrary subset of 300 periods.

of households with zero net wealth in our SCF sample, reported in Section 2, but below Jappelli's (1990) estimate of 20% for the US based on consumers' survey answers. The possibility of being constrained, however, affects behavior of consumers in our model more often than at the occasion when they are at the constraint. The value function incorporates the expectation that the borrowing constraint can bind in the future with some probability, especially for low values of cash-on-hand.

Note that the presence of uninsurable income risk prevents impatient consumers to be permanently at the borrowing constraint. Income risk induces a buffer-stock saving motive so that, if income is persistently good, consumers accumulate a positive amount of financial assets $a > 0$. This self-insurance allows consumers to smooth part of the income fluctuations.

4.1 Ex-ante heterogeneity

We investigate the robustness of the model's performance by further controlling for some possibly relevant *ex ante* heterogeneity of consumers. So far, we have controlled for differences in age by focussing on consumers below age 40 but otherwise we have taken the extreme stand that all of the remaining heterogeneity in the data reflects *ex post* differences of *ex ante* identical agents. This assumption is not as strong as it may seem since we consider an infinite-horizon model and attributes like ability may not be fully persistent across generations. These philosophical considerations notwithstanding, we check the robustness of our results if we condition on further important sources of heterogeneity across consumers such as education, gender or race (see Cagetti, 2003, and Gourinchas and Parker, 2002, for similar robustness checks).[16]

[16] Note that we can consider these subpopulations separately since the assumption of a small open economy precludes general-equilibrium interactions between them through changes in the interest rate.

	College (1)	No-College (2)	College White males (3)	No-College White males (4)
$\ln y_{1983}$	$\mathcal{N}(-0.205, 0.410)$	$\mathcal{N}(-0.216, 0.432)$	$\mathcal{N}(-0.078, 0.156)$	$\mathcal{N}(-0.043, 0.086)$
Net worth percentile	Net worth	Net worth	Net worth	Net worth
10th	0.088	0.019	0.145	0.097
20th	0.224	0.124	0.343	0.250
30th	0.456	0.276	0.601	0.434
40th	0.756	0.477	0.897	0.658
50th	1.130	0.739	1.397	0.893
60th	1.763	1.264	1.918	1.439
70th	2.642	1.926	2.750	2.021
80th	3.973	2.881	4.014	2.870
90th	6.706	4.904	5.983	4.530
Sample size	796	759	435	381

Table 2: Labor earnings and net worth data for subsamples in 1983.

We do this sequentially and first distinguish between consumers who have at most a high-school degree and consumers who have at least some college. This is a commonly used approximation for differences in skills in the literature on wage inequality (see Autor, Katz and Kearney, 2005, and references therein) and, in the SCF 1983, 51% of the consumers below age 40 have at least some college education.

In a second step we only consider white males within these groups. Restricting the sample to consumers with age 22-36, our sample is then comparable to Cunha and Heckman (2007) so that we can use their estimates for the proportion of the variance of labor income which reflects unexpected changes and thus true labor income risk.

We approximate the income process and match the net-worth decile targets separately for all considered samples. Table 2 shows the log-normal distribution of disposable labor income and the net worth percentiles for the different samples in 1983. Comparing households at the respective percentiles of their distribution, college-educated households not surprisingly hold more net worth than households without college degrees. The dispersion of disposable labor earnings is slightly larger for the population without college education (the mean of disposable labor earnings, not in logs, is normalized to 1 for each subpopulation).[17] In columns (3) and (4) we try to purge the dispersion of labor earnings from forecastable variation by using results of Cunha and Heckman (2007). They combine choice data with earnings data to compute the variance of the forecastable and unforecastable fraction of future earnings for two samples: white males born between 1941 and 1952 from the NLS/1966 survey and white males born between 1957 and 1964 from the NLSY/1979 survey. Since most of the labor market career of the latter individuals is after 1983, we use results based on the NLSY/1979 survey to adjust the earnings variance in the SCF 2004 and the results for the earlier sample to correct the earnings variance in the SCF 1983.[18] Cunha and Heckman's results in Table 3 show that in their early NLS sample the variance of unforecastable components accounts for 39% of the total variance for college-educated workers and for 23% of the total variance for high-school

[17] This is consistent with Budría Rodríguez et al. (2002) who find that the earnings dispersion is higher for consumers with no high-school degree. Note that we only distinguish two education groups in order to have a large enough sample size when computing decile statistics.

[18] Cunha and Heckman (2007) assume risk neutrality. Navarro (2007) allows for risk aversion in a related paper but only considers the NLSY/1979 sample. Since we are interested in the evolution of income risk over time, we use the results of Cunha and Heckman (for the NLS/1966 and NLSY/1979), as they are more suitable for our application.

College	No-College	College White males	No-College White males
(1)	(2)	(3)	(4)
	Risk aversion σ		
4.1	3.6	5.8	8.4
(0.232)	(0.188)	(0.441)	(0.649)
	Discount factor β		
0.830	0.835	0.865	0.845
(0.013)	(0.012)	(0.013)	(0.016)
	Test of overidentifying restrictions		
23.19	7.75	13.63	20.99
	(5% crit.value of $\chi^2(7)$: 14.067		
	1% crit.value of $\chi^2(7)$: 18.475)		

Table 3: Estimation results for subsamples in 1983.

educated workers.[19] After using these factors to correct the variance of labor earnings in the SCF 1983, the remaining, arguably unforecastable, dispersion of labor earnings for our samples displayed in columns (3) and (4) is much smaller than in the other columns.

Table 3 displays the estimation results for the different subsamples. Remarkably, the estimates for the college and no-college subpopulation in columns (1) and (2) are rather similar compared with the benchmark specification in which we pooled all consumers below age 40. Moreover, the test statistics in the last row of the table show that the model performs much better for the no-college subpopulation whereas the model continues to be rejected for the college subpopulation.

When we further restrict the sample to white males and try to purge the variance of labor earnings of forecastable components in columns (3) and (4), the model passes the test of overidentifying restrictions at the 5% level for the college subpopulation. The estimates imply that agents are more patient ($\beta = 0.87$) and more risk averse ($\sigma = 5.8$) in this case. The reason is that the smaller dispersion of labor earnings reduces the precautionary saving motive so that patience and risk aversion need to increase to match the moments of net worth. For the no-college population income risk is much smaller (see Table 2) which makes it more difficult for the model to match the net worth data. Figure 8 shows the 95% confidence region of the estimates for both subsamples. The confidence ellipses are tilted in an intuitive way: if

[19] In the later NLSY sample these fractions are 29% for both subpopulations.

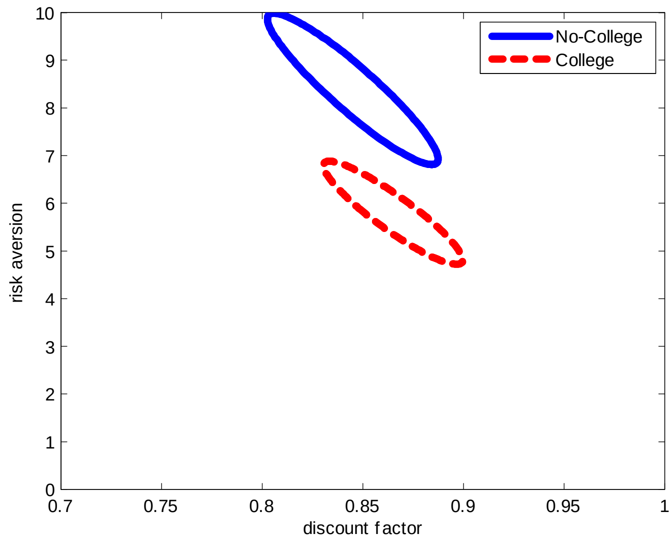

Figure 8: 95% confidence region of the estimates for the white male population with and without college education.

agents are estimated to be more patient then their risk aversion needs to be lower to match the data. Interestingly, we find that the college population is less risk averse and slightly more patient than the population without college education.[20] These differences are rather similar to Cagetti (2003)'s estimates but differ from results in Gourinchas and Parker (2002) who find that consumers with college education are more risk averse.

Overall, the results in Table 3 show that the performance of the simple model is rather good and robust if we control for some heterogeneity and downward adjust the estimates of income risk, although the model is sometimes rejected by a small margin. In the next section we explore the model's predictions for 2004.

[20] If we restrict the model parameters to be the same across both subpopulations in the estimations of columns (3) and (4), the model is rejected by a wide margin. The model performance also worsens substantially (with a test statistic above 80 compared with the 1% critical value of 31.99 for the $\chi^2(16)$ distribution) if we restrict parameters to be the same in the estimations of columns (1) and (2). We need some heterogeneity in preference parameters to match the data for both subpopulations.

5 The evolution of US consumer net worth

We now apply the models which we estimated in the previous section to predict changes in the distribution of consumers' net worth during the last 20 years. As in the previous section we start with predictions for the sample of all consumers below age 40 and then compare the performance with the models based on the other subsamples.

Parameter values for 2004. As discussed in Section 2, the interest rate for 2004 is lower at $r = 0.02$. The higher dispersion in labor earnings in 2004, for the sample of all consumers below age 40, implies that the natural logarithm of labor income in the SCF 2004 is approximated by the log-normal distribution $\ln(y_{2004}) \sim \mathcal{N}(-0.326, 0.652)$. Keeping the income grid constant at the 1983 values, we use Tauchen's (1986) method as before to compute the transition matrix.

Predictions for net worth in 2004. We hold the deep parameters for risk aversion σ and impatience β constant at their estimated 1983 values. Figure 9 illustrates the predicted changes of the net-worth deciles in the period 1983 to 2004 due the changes in interest rates and income risk. We decompose the overall prediction (the dotted line) into the effect of higher income risk (the dashed-dotted line) and lower interest rates (the dashed line). The lower interest rate implies that buffer-stock savings are more costly so that consumers accumulate less wealth. Hence net worth is lower across all deciles. The figure also shows that more volatile labor income increases net worth due to the precautionary saving motive. Quantitatively the effect of labor income risk is much stronger so that the model predicts net worth at the deciles in 2004 (the dotted line) which is higher than in 1983 (the solid line).

How do these predictions compare with actual net worth in the 2004 data? This is illustrated in Figure 10. The model (the dotted line) predicts much too high net-worth across all deciles compared with the data (the solid line). The test of overidentifying restrictions shows that these predictions for 2004 are rejected by the data. The test statistic is 81.32 which is well above the critical value of 18.475 at which the model would be rejected at the 1% significance level.[21] Hence, the buffer-stock saving model for all consumers below age 40 does a poor job of predicting the stability of the net worth distribution up to the 90th percentile if we attribute all changes in the cross-

[21] We use the variance-covariance matrix for the model with the interest rates and income risk in 2004 to compute the critical value. When simulating we match the selected sample size 1,159 in the SCF 2004, as for the decomposition in Figure 9.

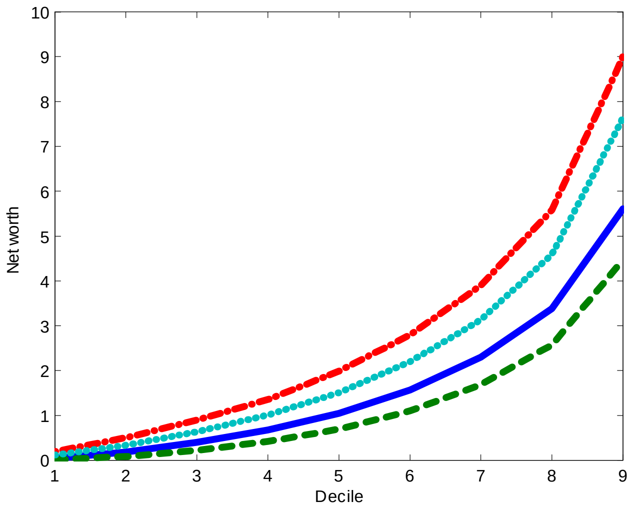

Figure 9: Net worth at each decile. Notes: solid line: model 1983; dashed line: lower interest rate; dashed-dotted line: higher income risk; dotted line: model prediction for 2004.

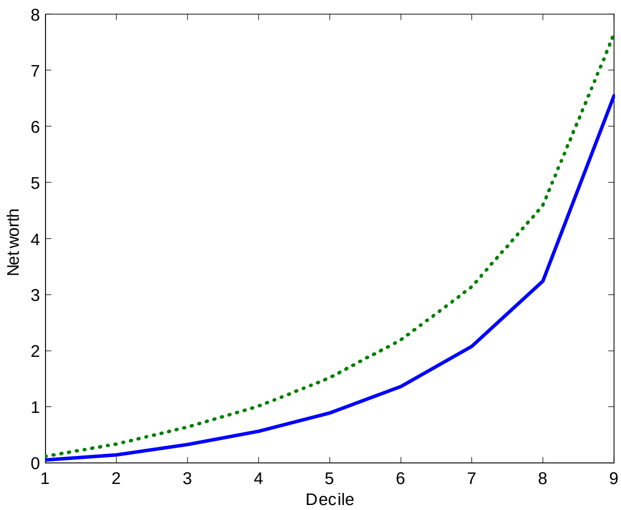

Figure 10: Net worth at each decile for 2004. Notes: solid line: SCF data; dotted line: model predictions.

sectional variance of labor earnings to changes in income risk. We thus now investigate whether (i) accounting for some further *ex ante* heterogeneity among consumers and (ii) attributing only some of the increase in the labor-earnings dispersion to unexpected increases improves these results.

In fact, one may wonder how many of the changes in the interest rate and the cross-sectional variance of labor earnings, which we feed into our model, had been anticipated by consumers. So far, we have assumed implicitly that all of these changes have been unexpected in 1983. *Ex post* we know that the real interest rate increased substantially before 1983 and then slowly decreased in the next two decades. Hence, it may not be implausible to assume that the fall in the real interest rate has been unexpected. The cross-sectional variance of labor earnings started to increase already before 1983, however. The results of Cunha and Heckman (2007) allow us to correct the variance of labor earnings downwards for 1983 and 2004 (deducting the variance of forecastable components). We will now gauge the quantitative importance of this correction and compare it to the changes in income risk which are implied by our data and model structure.

5.1 Ex-ante heterogeneity

Table 4 summarizes the income distributions and net worth moments which we use for 2004 and the last row displays the test of overidentifying restrictions of the predictions for 2004. We now discuss these result in some detail.

Comparison of Table 2 and Table 4 shows that the dispersion of labor earnings has increased between 1983 and 2004, and more so for those without college education when we correct for the variance of forecastable components of labor earnings in columns (3) and (4). Instead, net worth up to the 90th percentile remained remarkably stable in the same time period.

The last row of the table displays the test of overidentifying restrictions for the model predictions. The test statistics improve if we distinguish between households with and without college education but the model is still rejected (see columns (1) and (2)). If we attribute only part of the variance of labor earnings to labor income risk, then the model predictions can no longer be rejected for the college subpopulation but the predictions are worse for the sample without college education.

	College (1)	No-College (2)	College White males (3)	No-College White males (4)
$\ln y_{2004}$	$\mathcal{N}(-0.304, 0.608)$	$\mathcal{N}(-0.302, 0.604)$	$\mathcal{N}(-0.092, 0.184)$	$\mathcal{N}(-0.089, 0.178)$
Net worth percentile	Net worth	Net worth	Net worth	Net worth
10th	0.086	0.013	0.144	0.072
20th	0.268	0.095	0.347	0.248
30th	0.466	0.207	0.601	0.395
40th	0.735	0.393	0.906	0.766
50th	1.103	0.677	1.184	1.160
60th	1.598	0.994	1.568	1.581
70th	2.436	1.703	2.448	2.199
80th	4.145	2.744	3.707	3.234
90th	7.082	4.697	5.573	4.420
Sample size	665	494	266	164
	Test of overidentifying restrictions for 2004 predictions (5% crit.value of $\chi^2(7)$: 14.067, 1% crit.value of $\chi^2(7)$: 18.475)			
	56.81	41.66	10.86	106.05

Table 4: Labor earnings and net worth data for the subsamples in 2004, and the test results of the model predictions for 2004.

Cunha and Heckman (2007)'s estimates imply that the variance of labor earnings increases by 50% for the sample without college education and by only 10% for those with college education. After accounting for the effect of the lower interest rate, this implies that the model predicts an increase of net worth across the distribution for the no-college population (which is rejected by the data) and stable net worth for the college sample (which is consistent with the data). Thus, the performance out-of-sample is much better if we "control" for some relevant dimensions of heterogeneity but worse for the no-college sample if we use Cunha and Heckman (2007)'s results to compute the increase in labor income risk.

The interesting question remains to determine what increase in income risk would generate predictions of the buffer-stock saving model which are consistent with the data. We illustrate this in Figure 11 for the subsamples of white males with and without college education. We plot the respective test statistic for the 2004-predictions (the solid graph for the sample without college and the dotted graph for the sample with college education) against the percentage increase of the standard deviation of log labor earnings. The test statistics are asymptotically $\chi^2(7)$ distributed and we display the 5% and 1% critical value as the dashed and dash-dotted line, respectively. We find that the predictions of the model cannot be rejected at the 1% level for an increase of labor income risk between 4% and 20% for the college sample and 5% and 16% for the no-college sample.

Put differently, the net worth data for 1983 and 2004 together with the assumption of a buffer-stock saving model imply bounds for the increase in labor income risk. The increases in labor income risk that are accepted, applying our model to wealth data, include Cunha and Heckman (2007)'s estimated value for the college sample but not for the sample without college education. Whereas Cunha and Heckman (2007) find that the income risk for those without college education has increased much more, our analysis suggests that the increase in income risk has been rather similar across these education groups.[22] We now perform some further robustness checks before we conclude by discussing extensions which may help to improve further the match between the model and the data.

[22]If we do the same exercise for the whole population and the college and no-college subsamples of Table 4, columns (1) and (2), we find that the respective models perform best if unexpected income risk increased by 7.5%, 9.5% and 5%, respectively. Thus, all of our estimated models suggest rather similar increases in income risk which are substantially smaller than the changes in the variance of log-wages reported in Heathcote et al. (2004), Figure 2.

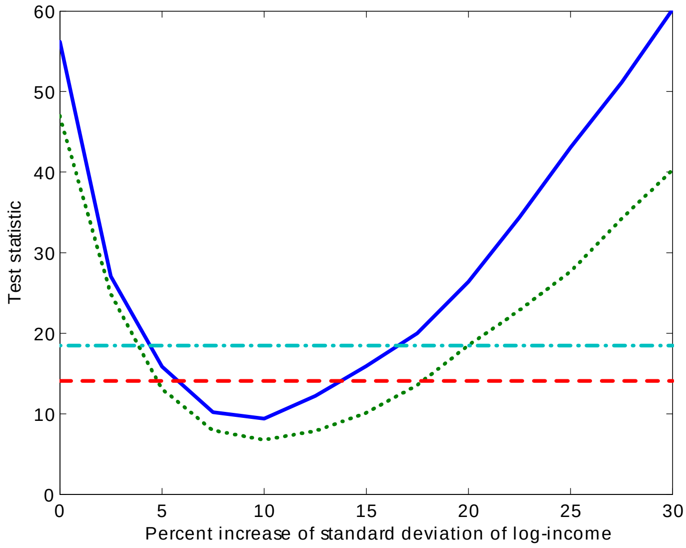

Figure 11: Test statistic of the prediction as a function of the increase in income risk, for the white male sample *with* and *without* college education. Notes: solid line: test statistic no-college; dotted line: test statistic college; dashed line: 5% critical value; dashed-dotted line: 1% critical value.

College White males (1)	No-College White males (2)
Risk aversion σ	
1.5	1.9
(0.457)	(0.374)
Discount factor β	
0.945	0.945
(0.006)	(0.004)
Test of overidentifying restrictions (5% crit.value of $\chi^2(7)$: 14.067 1% crit.value of $\chi^2(7)$: 18.475)	
1983: 21.926	1983: 27.449
2004: 906.97	2004: 9.230

Table 5: Estimation results for 1983 with a lower autocorrelation of income shocks. Standard errors are reported in brackets.

5.2 Robustness

We first check the robustness of the model if we reduce the persistence of the labor income shocks. We then discuss the steady-state assumption in our analysis before we briefly return to measurement issues for net worth.

5.2.1 Persistence of the income shocks

We probe the robustness of our results using a lower value, 0.8, for the autocorrelation of labor income shocks. This is interesting for at least two reasons. Firstly, the persistence of labor income shocks is important for buffer-stock savings. If shocks are less persistent, less precautionary wealth needs to be accumulated to self-insure against these shocks. It is thus interesting to assess the importance of persistence for the fit between the model and data. Secondly, a lower autocorrelation of 0.8 is still within in the range of commonly used values in the literature.

Table 5 displays the estimation results if we assume an autocorrelation of 0.8. Since the motive of accumulating precautionary wealth decreases with less persistence, the discount factor is estimated to be higher than in the benchmark case, for the model to fit the data. Note that risk aversion is estimated to be smaller which mitigates the effect of the higher discount factor. Overall the model with lower persistence does not fit the data in 1983 as well as the benchmark does. The test statistics increase by 30-50% compared with Table 3, columns (3) and (4). This is because lower persistence makes it more difficult to match the concentration of wealth observed in the data. The model overestimates wealth holdings at the median but underestimates wealth at the 90th percentile for both subpopulations.

Concerning the predictions for 2004, the higher estimated discount factor makes the model much more sensitive to changes in the real interest rate whereas the lower risk aversion makes it less sensitive to changes in labor income risk. Thus, the increase of income risk for white males with college education estimated by Cunha and Heckman (2007) is quantitatively too small to outweigh the effect of the lower real interest rate. Hence, the model for the college subpopulation underestimates net worth in 2004. For white males *without* college education, lower persistence improves the model predictions for 2004. Whereas the model with a persistence of 0.95 overestimated net worth for this subpopulation due to the large increase in income risk, the new parameter estimates for a lower persistence reduce the positive effect of income risk and increase the negative effect due to the fall of the real interest rates so that net worth in 2004 is predicted rather well. In other words, the increase in income risk (for which the model matches the data in 2004) needs to be 5 times higher if the first-order autocorrelation of the income process is 0.8 instead of 0.95 (but the increase in risk still needs to be rather similar across education groups). Given the better fit of the model for our base year 1983 with a persistence of 0.95, we prefer the smaller increase in income risk around 10% implied by that specification.[23]

5.2.2 Transition

In our analysis we have focussed on steady states. It turns out that the steady-state comparison is not restrictive since we find very similar results if we start from the steady state in 1983, change the model parameters to their values in 2004 and compute the transition for 21 periods. Since there are no costs which hamper adjustment to the new steady state and the net-worth distribution changes very little, the model is close to its new steady state already after 21 years. The model moments after 21 years of transition are not significantly different (at the 5% level) from the steady state moments.

[23] An alternative interpretation of these results is that the model is consistent with the stronger increase of labor-income risk for consumers without college education reported by Cunha and Heckman (2007) if the persistence of labor income shocks is lower for that subpopulation.

5.2.3 Measurement

If we include unsecured debt and risky assets in the net-worth distribution for the whole sample in 1983, the estimate of β increases to 0.89 and σ falls to 2.5. The performance of the model in terms of matching the net-worth distribution in 1983 are slightly better but its predictions for 2004 are worse. Overall, including unsecured debt and risky assets in our measure of net worth does not affect the main conclusions about the performance of the model.

Not surprisingly, the model performance deteriorates substantially if we add the means of net worth within each decile as moments. This is a challenge for the model since the mean in the top decile is hard to match as some consumers hold large amounts of net worth.

6 Conclusion

We have studied whether a buffer-stock saving model can explain the evolution of consumer net worth in the US between 1983 and 2004. We have argued that the model requires to focus on the net worth distribution up to the 90th percentile. That part of the distribution is matched reasonably well with economically plausible estimates for risk aversion and impatience if we account for heterogeneity across consumers in terms of age and education.

Most interestingly, the estimated model for 1983 predicts the observed stability of net worth up to the 90th percentile between 1983 and 2004 if we attribute only part of the observed increase in the variance of labor earnings to higher income uncertainty. Quantitatively, the stronger precautionary-saving motive due to the increase in labor income risk is then counterbalanced by the higher cost of buffer-stock savings due to the fall of the real interest rate.

It would be interesting in further research to extend the model by distinguishing different components of net worth like financial and non-financial wealth (as Fernández-Villaverde and Krueger, 2005, or Nakajima, 2005). This would introduce changes in the relative price of durables as an additional determinant of the wealth portfolio.

Appendices

Data appendix.

The variables used in the paper are constructed in the following way:

Gross *labor income* is the sum of wage and salary income. As in Budría Rodríguez et al. (2002) we add a fraction of the business income where the fraction is the average share of labor income in total income in the SCF. *Disposable labor income* is computed using the NBER tax simulator. We use the programs provided by Kevin Moore provided on http://www.nber.org/~taxsim/ to construct disposable labor earnings for each household in the SCF 1983 and 2004. Following the standardized instructions on the NBER website, we feed the following required data of the SCF into the NBER tax simulator (we then later apply the age restrictions to obtain our sample of interest): the US state (where available, otherwise we use the average of the state tax payments across states), marital status, number of dependents, taxpayers above age 65 and dependent children in the household, wage income, dividend income, interest and other property income, pensions and gross social security benefits, non-taxable transfer income, rents paid, property tax, other itemized deductions, unemployment benefits, mortgage interest paid, short and long-term capital gains or losses. We then divide the resulting federal and state income tax payments as well as federal insurance contributions of each household by the household's gross total income in the SCF. This yields the implicit average tax rate for each household in 1983 and 2004. The mean of that average tax rate for consumers in the SCF below age 40 is 24% in 1983 and 20% in 2004. Finally, we compute household net labor earnings as (1 - household average tax rate) * household gross labor earnings (including taxable transfers) and then add non-taxable transfers.

Financial assets are defined as the sum of money in checking accounts, savings accounts, money-market accounts, money-market mutual funds, call accounts in brokerages, certificates of deposit, bonds, account-type pension plans, thrift accounts, the current value of life insurance, savings bonds, other managed funds, other financial assets. We exclude risky assets like stocks and mutual funds in the benchmark and check robustness of the results when we include these assets.

Gross secured financial debt is defined as the sum of mortgage and housing debt, other lines of credit and debt on residential and nonresidential property, debt on non-financial business assets.

Net-Financial assets are defined as *financial assets - gross secured financial debt.*

Non-financial assets are defined as the sum of residential property, vehicles, other durables like jewelry or antiques, owned non-financial business assets.

Net worth is defined as the sum of net-financial and non-financial assets.

Computational appendix.

In this appendix we describe the implementation of the simulated method of moments and the computation of standard errors since the moments in our application are percentiles and thus not standard. Denote as $\theta = (\beta, \sigma)$ the parameters to be estimated, $m(\theta)$ the model moments and μ the corresponding moments in the data. The estimation method proceeds in steps.

Step 1: In the first step we search for the parameter values which minimize the distance between the model and data moments. That is

$$\widehat{\theta}^1 = \arg\min[(m(\theta) - \mu)' I(m(\theta) - \mu)],$$

where the weighting matrix in the first step is the identity matrix I. These estimates are consistent and asymptotically normally distributed. Since the moments in our application are not standard, we now derive this explicitly. An important requirement is a continuous and differentiable distribution function of wealth.

i) Asymptotic distribution of the quantile "moments"

Define the empirical wealth quantile that is to be matched as $\mathbf{q} \equiv \mu$ and the simulated quantile in the model $\widehat{q} \equiv m(\widehat{\theta})$. The simulated $\widehat{q}$ is implicitly defined by the sample probability

$$F_n(\widehat{q}) = \frac{1}{N} \Sigma_{n=1}^{N} I_{\{x_n \leq \widehat{q}\}} = \alpha \ ,$$

where $\alpha \in (0,1)$ is a percentile and $I_{\{x_n \leq \widehat{q}\}}$ is the indicator function which equals unity if wealth is below the simulated wealth quantile $\widehat{q}$. For a continuous and well-behaved distribution function, we can invert that function and express the quantile as

$$\widehat{q} = F_n^{-1}(\alpha) = \inf\{x \ ; \ F_n(x) \geq \alpha\} \ .$$

That is, we can express quantiles as the inverse of standard sample moments (the sample probabilities in our application). In fact,

$$\{\widehat{q} \leq x\} = \{\alpha \leq F_n(x)\} \ . \quad \text{(A1)}$$

We now use this insight to derive the asymptotic distribution of $\widehat{q}$. Suppose there exists an $\varepsilon > 0$ so that

$$\sqrt{N}\left(\widehat{q} - \mathbf{q}\right) \leq \varepsilon \ .$$

Rearranging this to $\widehat{q} \leq \mathbf{q} + \varepsilon/\sqrt{N}$, we use (A1) to express this as

$$\alpha \leq F_n(\mathbf{q} + \varepsilon/\sqrt{N}) \ .$$

Subtracting $F(\mathbf{q} + \varepsilon/\sqrt{N})$ from both sides and multiplying by $\sqrt{N}$ yields

$$\sqrt{N}\left(\alpha - F(\mathbf{q} + \varepsilon/\sqrt{N})\right) \leq \sqrt{N}\left(F_n(\mathbf{q} + \varepsilon/\sqrt{N}) - F(\mathbf{q} + \varepsilon/\sqrt{N})\right) \ .$$

If F is differentiable,

$$\begin{aligned} F(\mathbf{q} + \varepsilon/\sqrt{N}) &= F(\mathbf{q}) + D_q F(\mathbf{q}) \frac{1}{\sqrt{N}}(\varepsilon - 0) + O(\varepsilon^2) \\ &\simeq F(\mathbf{q}) + D_q F(\mathbf{q}) \frac{1}{\sqrt{N}}\varepsilon \end{aligned}$$

for small ε. Using this approximation on the left-hand side of the inequality and $\alpha = F(\mathbf{q})$, we get

$$-D_q F(\mathbf{q})\varepsilon \leq \sqrt{N}\left(F_n(\mathbf{q} + \varepsilon/\sqrt{N}) - F(\mathbf{q} + \varepsilon/\sqrt{N})\right) \ .$$

Since the state space is bounded in our application and the Markov chain in our model has the properties of uniform ergodicity and uniform mixing, we can apply a version of the central limit theorem to the right-hand side of this inequality (for example, Jones, 2004, corollary 5). Hence, for $N \to \infty$,

$$\sqrt{N}\left(F_n(\mathbf{q} + \varepsilon/\sqrt{N}) - F(\mathbf{q} + \varepsilon/\sqrt{N})\right) \to \mathcal{N}\left(0, \Sigma\right) \ ,$$

where Σ is the variance-covariance matrix of the probabilities $F(\cdot)$. Hence, asymptotically

$$-D_q F(\mathbf{q})\varepsilon \leq \mathcal{N}\left(0, \Sigma\right)$$

or

$$\mathcal{N}\left(0, \Omega\right) \leq \varepsilon, \text{ with } \Omega \equiv \left[D_q F(\mathbf{q})\right]^{-1} \Sigma \left[D_q F(\mathbf{q})'\right]^{-1}$$

and it follows that

$$\sqrt{N}\left(\widehat{q} - \mathbf{q}\right) \xrightarrow{d} \mathcal{N}\left(0, \Omega\right) \ .$$

ii) Asymptotic distribution of the parameter estimates

The parameter estimates $\widehat{\theta}^1$ are obtained by minimizing the objective function $(q(\theta) - \mathbf{q})' I (q(\theta) - \mathbf{q})$. The first-order condition is

$$D_\theta q(\theta)' I (q(\theta) - \mathbf{q}) = 0 \; . \tag{A2}$$

For this approach to be valid we need that

$$q(\theta) = F^{-1}(\alpha, \theta)$$

is differentiable in θ. Since

$$D_\theta q(\theta) = -\left[D_q F\right]^{-1} D_\theta F$$

this requires that the distribution function of wealth is differentiable in q and θ. For example, there must be no mass points in the region around the quantiles. If this holds true, we can subtract $D_\theta q(\theta)' I (q(\widehat{\theta}) - \mathbf{q})$ on both sides of (A2) so that

$$D_\theta q(\theta)' I (q(\theta) - \mathbf{q}) - D_\theta q(\theta)' I (q(\widehat{\theta}) - \mathbf{q}) = -D_\theta q(\theta)' I (q(\widehat{\theta}) - \mathbf{q})$$

which can be simplified to

$$D_\theta q(\theta)' I (q(\widehat{\theta}) - q(\theta)) = D_\theta q(\theta)' I (q(\widehat{\theta}) - \mathbf{q}) \; .$$

Since the estimates are asymptotically consistent, $q(\widehat{\theta}) \to q(\theta)$ and $\widehat{\theta} \to \theta$, we can approximate

$$q(\widehat{\theta}) \simeq q(\theta) + D_\theta q(\theta)(\widehat{\theta} - \theta)$$

so that

$$D_\theta q(\theta)' I D_\theta q(\theta)(\widehat{\theta} - \theta) = D_\theta q(\theta)' I (q(\widehat{\theta}) - q)$$

and

$$\sqrt{N}(\widehat{\theta} - \theta) = \left[D_\theta q(\theta)' I D_\theta q(\theta)\right]^{-1} D_\theta q(\theta)' I \underbrace{\sqrt{N}(q(\widehat{\theta}) - q)}_{\to \mathcal{N}(0,\Omega)} \; .$$

It follows that

$$\sqrt{N}(\widehat{\theta} - \theta) \xrightarrow{d} \mathcal{N}\left(0, \Theta\right),$$

where

$$\Theta \equiv \left[D_\theta q(\theta)' I D_\theta q(\theta)\right]^{-1} D_\theta q(\theta)' \; \Omega \; D_\theta q(\theta) \left[D_\theta q(\theta)' I D_\theta q(\theta)\right]^{-1} \; .$$

Using the estimates from the first step $\widehat{\theta}^1$, we simulate the model for $100,000$ periods, with the same seed to draw random numbers for the income shocks. We use the resulting cross-sectional distribution to draw (with replacement) $S = 10,000$ independent samples of size N, the size of the respective selected sample in the SCF 1983. We then compute the model moments $m(\widehat{\theta}^1)$ for each sample $s = 1, ..., S$ and use the S independent observations of the model moments to compute an estimate for their variance-covariance matrix W. We compute the matrix as

$$W(\widehat{\theta}^1) = \frac{1}{S}\left(q_s(\widehat{\theta}^1) - \frac{1}{S}\Sigma_{s=1}^S q_s(\widehat{\theta}^1)\right)\left(q_s(\widehat{\theta}^1) - \frac{1}{S}\Sigma_{s=1}^S q_s(\widehat{\theta}^1)\right)' ,$$

where $q_s(\widehat{\theta}^1)$ depends on N as it is computed for the empirical sample size N in the SCF data.

Step 2: In the second step we reestimate the parameters using the inverse of W as a weighting matrix so that

$$\widehat{\theta}^2 = \arg\min[(m(\theta) - \mu)' W(\widehat{\theta}^1)^{-1}(m(\theta) - \mu)].$$

Moments which have more variance receive less weight in the estimation.

Steps 3 to k: We continue these steps of updating the weighting matrix and reestimating model until the model estimates converge. This is typically the case after 5 steps in our application. After the final k-th step, we compute the asymptotic standard errors for the parameter estimates, using results by Gourieroux and Monfort (1996), Proposition 2.4,

$$Q = \left[\frac{\partial m(\widehat{\theta}^k)}{\partial \theta}' W(\widehat{\theta}^k)^{-1} \frac{\partial m(\widehat{\theta}^k)}{\partial \theta}\right]^{-1} .$$

The more important are changes in the parameters for the moments, the more precisely are they estimated, *ceteris paribus.*

The test statistic for the test of overidentifying restrictions is then easily computed as

$$(m(\widehat{\theta}^k) - \mu)' W(\widehat{\theta}^k)^{-1}(m(\widehat{\theta}^k) - \mu)$$

which has 7 degrees of freedom when we estimate 2 parameters by matching 9 moments and is asymptotically $\chi^2(7)$-distributed.

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