EUI Working Papers
ECO 2008/28

The Home Bias of the Poor: Terms of Trade Effects and Portfolios across the Wealth Distribution

Tobias Broer
EUROPEAN UNIVERSITY INSTITUTE
DEPARTMENT OF ECONOMICS

The Home Bias of the Poor: Terms of Trade Effects and
Portfolios across the Wealth Distribution

TOBIAS BROER

EUI Working Paper ECO 2008/28
The home bias of the poor: terms of trade effects and portfolios across the wealth distribution

Tobias Broer*

First version: May 2007. This version: September 2008

Abstract

Wealthier people generally hold a larger part of their savings in risky assets. Using the US Survey of Consumer Finances, I show that wealthier households also have a higher portfolio share of foreign assets. This relative home bias of the poor does not seem to be explained by fixed participation costs alone, as the portfolio share of foreign assets increases with financial wealth even among participants in foreign asset markets. This paper shows how both biases of poorer agents’ portfolios, towards safe and home assets, can arise in a simple 2 country economy with income and portfolio heterogeneity. Poor investors are naturally biased against domestic equity when wages and capital returns are positively correlated, making equity a bad hedge against fluctuations in labour income relative to bonds. Moreover poor investors prefer home to foreign bonds if equilibrium terms of trade movements systematically lead to a fall in the purchasing power of domestic assets in periods of high wages. I show that this is likely to be the case if aggregate supply shocks at home are more important than abroad. Finally, the model shows that aggregate home bias in the country portfolio implies relative home bias of the poor and vice versa.

JEL Classification Codes: F36, G11, E21, D11, D31

Keywords: Heterogeneous Agents, Home Bias, Inequality, International Asset Diversification, Portfolio Choice

*European University Institute, Florence, Italy. Contact: tobias.broer@eui.eu.
I would like to thank David Backus, John Leahy, and Gianluca Violante for helpful comments, as well as seminar participants at the Bank of England, the European University Institute, Cornell University, the NYU student Macro Lunch and the 2008 La Pietra Mondragone Workshop. I especially thank Morten Ravn and Giancarlo Corsetti for their support with this work. Finally, I am indebted to Frederico Cepeda at Morningstar for his generous help with US mutual fund data.
1 Introduction

It is well-documented that household portfolios become more diversified as wealth increases. Campbell (2006) and Guiso et al. (2003), for example, show that poor households are less likely to invest in risky assets. Equally, many authors have found that aggregate country-portfolios have surprisingly low shares of foreign assets, the so-called "home bias in portfolios puzzle" (see Lewis, 1997, for a summary of this literature). But little attention has been devoted to the composition of individual household portfolios between domestic and foreign assets, and its relationship with individual wealth. In the empirical part of this paper, I study the US survey of consumer finances (SCF) and show that wealthier investors also seem to invest on average a higher share of their portfolio in foreign assets than investors with lower financial wealth.

A prominent explanation for this bias of poorer investors towards safe and home assets are fixed costs of participating in the markets for risky and foreign assets. Fixed costs, however, cannot explain the relative home bias of the poor among participants in foreign asset markets, for whom the fixed cost is sunk.

In the theoretical part of the paper, I show that without fixed costs, agents with lower financial wealth optimally have a higher portfolio share of assets that hedge against fluctuations in their future income. And I present an environment where home assets are better income hedges than foreign assets. This is because in equilibrium, a positive shock to aggregate home output increases the relative price of foreign goods, and thus reduces the real payoffs of home bonds. Therefore, if individual real incomes comove more strongly with home than foreign output, home bonds provide a hedge against fluctuations in non-diversifiable individual income risk. Wealthy investors, whose future income is less dependent on endowments, care less about this hedging property than poor investors. Therefore, equilibrium portfolios vary across the wealth distribution and poorer investors tend to have a stronger home bias than rich investors. Moreover, this result implies a strong link between the relative home bias of poor agents and home bias of the aggregate country portfolio: the aggregate country portfolio exhibits home bias if and only if there is relative home bias of the poor.

The intuition for these results has similarities to Baxter and Jermann (1997) who show that with income from non-marketable human capital, the optimal portfolio of assets consists of two sub-portfolios, one completely diversified, the other designed to hedge against volatility of human capital returns. I show that
the hedging portfolio can be dominated by safe domestic assets. And its importance relative to the diversified part of the portfolio declines with increases in total wealth. To derive the results, I consider a two country model with incomplete markets and heterogeneous consumers that receive an uncertain amount of a country-specific endowment good every period. I derive analytical portfolio shares by assuming (as Cole and Obstfeld, 1991) that preferences over domestic and foreign goods are unit-elastic and identical across countries and agents. This implies that terms of trade depend only on aggregate country endowments each period.

My paper combines three strands of literature. First, from studies of household finances I take the stylised fact that wealthier individuals have riskier and more diversified portfolios. Using the 2004 wave of the survey of consumer finances, I illustrate how the portfolio share of foreign assets is increasing in investor wealth. Second, I adopt the idea that general equilibrium terms of trade movements can significantly change the optimal portfolios obtained in partial equilibrium (cf. Lewis, 1997, for a survey, or more recently Obstfeld and Rogoff, 2000, van Wincoop and Warnock, 2006, Heathcote and Perri, 2007). Like Baxter and Jermann (1997) I also include non-marketable human capital, but add an idiosyncratic component to its returns. I show that the implications for the composition of individual portfolios across the wealth distribution are consistent with the observed facts. Third, I extend heterogeneous agents models to the open economy.

2 Portfolios across the wealth distribution: evidence from the 2004 SCF

Wealthier and more educated people are more likely to invest in risky assets. This is well-documented for the US (see for example Campbell, 2006, for a review and an illustration using the 2001 SCF data) and a number of European countries (see Guiso et al, 2003, and Carroll, 2002). Equally, it is well-known that average country portfolios have surprisingly low shares of foreign assets - the "home bias in portfolios puzzle". This has been interpreted as a consequence of a more general "local bias" of household portfolios, which outweigh local, regional, and national assets (see e.g. Campbell 2006). But compared to the portfolio shares of risky assets in general, or of domestic equity more in particular, there is very little evidence on the home bias
of individual households and its determinants. Campbell et al (2006) conclude for the case of Sweden that international diversification possibilities exist, but are usually exploited only by wealthier individuals, who have a higher share of investments in mutual funds (with an average portfolio share of 25 percent for foreign assets). However, they provide no evidence on direct holdings of foreign assets.

To document the evolution of foreign asset holdings across the wealth distribution, I examine the 2004 wave of the US survey of consumer finances (SCF). This survey includes information on the US Dollar value of households’ holdings of ”bonds issued by foreign governments or companies” and ”stock in a company headquartered outside of the United States”. In order to control for indirect holdings of foreign assets, I include a measure of foreign assets held via mutual funds. The SCF includes data on households’ investments in several broad categories of mutual funds. I derive a measure of total foreign asset holdings by summing to individuals’ direct investments in foreign equity and bonds the value of their mutual fund shares in US equity, bond and combination funds multiplied by the average portfolio weight of foreign bonds and equity in each type of fund. Figure 1 plots the resulting foreign asset portfolio shares (averaged within every decile of the financial wealth distribution to reduce noise) as

1Question codes x7638 and x7641. An obvious problem of this measure is that it does not refer to non-dollar assets, but to assets issued by foreign issuers, in foreign currency and US dollars.

2In other words, I do not consider pension funds. One reason for this is that individuals’ decisions on pension fund investment are taken under a very different set of constraints than other investment decisions. Also, most shares in pension funds are not actively managed as a part of regular portfolio decisions. However, both these arguments do not apply to individual mutual fund investments.

3To my knowledge, these average portfolio shares of mutual funds are not readily available from published sources. But Morningstar kindly provided data on portfolio shares of non-US assets for more than 4700 US mutual funds, not including funds of funds. From this I calculated weighted averages for portfolio shares of foreign bonds and equity for the three categories of funds for the year 2003. Since equity (bond) funds seem to often not report zero foreign bond (equity) holdings, I made an adjustment by setting missing observations to zero for all funds that reported portfolio shares summing to at least 99.5 percent. The resulting sample included around 2800 observations for shares of international equity and slightly less for bonds. Using this sample, the average US equity mutual fund invested 17.1 percent in foreign shares, while the average bond fund (disregarding funds of government / municipal bonds) invested 3.6 percent abroad. Combination funds invested on average 10.7 percent in non-US assets.
a function of individual financial wealth. The figure shows that the portfolio shares of foreign assets are monotonically increasing across deciles of the financial wealth distribution. So richer households seem to have lower home bias on average.

The evidence presented in Figure 1, however, raises several questions. First, under- or misreporting of foreign assets by households, and the use of average mutual fund portfolios might introduce measurement error in foreign assets. An appendix argues that this is in fact likely to bias any positive relationship between portfolio shares and financial wealth towards zero. This is because off-shore investments for tax evasion are likely to make underreporting more severe for foreign assets, and average mutual fund portfolio shares are likely to under-represent the foreign asset holdings by wealthy households’ if these systematically choose mutual funds with higher foreign exposure. A second question is whether the rise in average portfolio shares across the wealth distribution could merely be due to a higher participation rate of wealthy individuals in the foreign asset market, rather than a rise in individual portfolio shares of participants as they become richer. One factor that could cause such a pattern are fixed costs of entering sophisticated financial markets. An appendix presents a simple model that shows that this implies a non-linear relationship between financial wealth and participation, in the form of a threshold value of assets below which individuals do not hold any foreign assets. Optimal portfolios above the threshold value, however, would not be affected by sunk fixed costs. Thus, any variation in portfolio shares above the threshold value has to be attributed to other factors.

Both the deciles and the averages take account of the fact that the SCF oversamples parts of the population, by applying the weights suggested by Kennickell (1999), and the multiple imputation procedure used for the SCF. This is because, to eliminate inconsistencies and missing values, the SCF imputes some values from the other information provided by a household. However, rather than simply reporting one best guess for the imputed values, the SCF provides 5 draws per observation from the distribution of the missing values conditional on observables.

The portfolio shares of foreign assets are low relative to those calculated from aggregate US data. But keep in mind that the SCF measure of financial wealth, the denominator of the ratio, includes a large range of assets such as insurance contracts, liquid retirement funds, etc., while the numerator only considers bonds and stocks held directly and via mutual funds. Also, the aggregate shares of foreign assets in the country portfolio cannot directly be read from the graph. The ratios of foreign to total assets of the implied weighted aggregate portfolio are 2.75, 4.08, 3.99 percent for bonds, equities and their total respectively.

[Figure 1 somewhere here.]
Finally, one might suspect that financial wealth simply captures the effects of other important variables, such as education, age, or income, on portfolios. In this case we would expect an analysis that controls for these variables to yield significantly different results.

To answer the last two questions, I perform a more formal econometric analysis. I estimate jointly the probability of participation and the optimal portfolio share of participants with the Heckman (1979) method, conditioning on other variables that were found to be important for portfolio decisions of individuals in previous research. To be precise, I estimate the parameters of the following 2 equation system

\[
\text{SHARE} = \begin{cases} 
\alpha + \beta_1 \ln(\text{FIN}) + \beta_2 \ln(\text{INCOME}) + \epsilon_1 & \text{if } H > 0 \\
0 & \text{otherwise} 
\end{cases} 
\tag{1}
\]

\[
H = a + b_1 \text{AGE} + b_2 \text{COL} + b_3 \text{FIN}_2 + b_4 \text{FIN}_3 + b_5 \text{FIN}_4 + \epsilon_2 
\tag{2}
\]

Here, \(\text{SHARE}\) is the portfolio share of foreign assets, \(\text{FIN}\) is the SCF definition of gross financial wealth and \(\text{INCOME}\) is the sum of salaries, wages and income or losses from a professional practice, business, limited partnership, or farming. \(H\) is an indicator variable that captures the probability of participation in foreign asset markets. This probability is a function of age, a dummy variable "COL" that equals 1 when the household head holds a college degree, and a set of dummies \(\text{FIN}_x\) that capture financial wealth, taking the value 1 when total financial assets of the household fall in the (weight-adjusted) xth quartile. Only when \(H\) is above a threshold, normalised to 0, agents participate in foreign asset markets and we observe the variable \(\text{SHARE}\), their portfolio share of foreign assets. Conditional on participation the portfolio share is a function of income and financial wealth. The errors \(\epsilon_1\) and \(\epsilon_2\) are assumed to follow a joint normal distribution. The equations are estimated jointly with full maximum-likelihood adjusted for sampling weights. Identification is achieved by restricting the effects of financial wealth to be linear in logs in (1), and constant within quartiles in (2), which I take to be a proxy for different possible participation thresholds.\(^6\)

Results are reported in table 1, where numbers in italics are standard errors.\(^7\)

\(^6\)I also estimated an alternative specification that included income quartiles in the participation equation. While, in the presence of fixed costs of entering foreign asset markets, we would expect financial wealth to determine the participation threshold and not income, current income could act as a proxy for future financial wealth. However, the income quartile dummies turned out to be insignificant, so I excluded them from the final specification.

\(^7\)Again, an additional complication is the use in the SCF of multiple imputations for missing values. To account for this, I estimate the same model for each of the 5 im-
The effect of financial wealth is significant (at the 1 percent level) in both equations. Ceteris paribus, individuals in the bottom quartile of the financial wealth distribution are least likely to invest in foreign assets. But after a jump in the likelihood of participation between the first and second quartile, moving further up the wealth distribution has much smaller, and non-monotonous effects. This is in line with a threshold value of assets beyond which a rise in wealth does not systematically raise the probability of participation. However, higher financial wealth increases significantly the portfolio share of participants in equation (1), which cannot be attributed to fixed costs. The effect of age on the probability of participation is insignificant, but college graduates have on average a higher probability of investing in foreign assets. Finally, for participants the effect of rising income on the portfolio share of foreign assets is insignificant.

This section has shown that individual portfolio shares of foreign assets increase with financial wealth. There is a significant jump in the probability of participation in foreign asset markets between the first and second financial wealth quartiles, consistent with fixed participation costs. But fixed costs cannot explain the significant positive relationship between portfolio shares and financial wealth for participants. The next section presents a simple model of the international economy, where general equilibrium movements in the relative price of home and foreign goods can make home assets better hedges against income fluctuations, and thus lead to the observed pattern of portfolios: poor individuals have a stronger taste for home bonds as in general equilibrium their real payoffs hedge against volatile endowments, which are their dominant source of income.

3 A 2 country heterogeneous agents endowment economy

I consider an economy with 2 countries, home (H) and foreign (F). In each country there is a large number of agents with unit mass. Individual agents are indexed by h, f at home and abroad respectively. They live for two periods, complicates separately and then aggregate the estimation results. For the coefficients and standard errors reported in Table 1, I use the formulae suggested in the SCF codebook (http://www.federalreserve.gov/PUBS/oss/oss2/2004/codebk2004.txt). For the chi² value I report a simple average of the following individual values: 75.66, 70.18, 81.42, 69.49, 43.22.
and receive endowments of a country-specific perishable good H or F. Agents’ preferences are described by a von Neumann-Morgenstern utility function that is homogeneous across agents and countries, with constant relative risk aversion over sequences of a Cobb-Douglas aggregate of country-specific goods as for example in Cole and Obstfeld (1991)

\[ U_k = U(c_k) + \beta E[U(c'_{k})] \]  
\[ U(c_k) = \frac{c_k^{1-\gamma} - 1}{1 - \gamma} \]  
\[ c_k = c_k^H c_k^F \]

where \( c_{k,I} \) denotes consumption by agent k of good I and \( k \in \{h,f\} \). More generally, notation is as follows: Capital letters H,F denote country-specific variables or goods, small letters h,f denote individual variables that can vary across agents of country H,F. First subscripts denote agents or countries, second subscripts goods. Second period values of a variable x are denoted as x', its distribution as \( \Psi^x \).

**Heterogeneity and uncertainty**

Heterogeneity in our economy comes from differences in endowments. More precisely, agents in country K receive individual endowments \( \epsilon_k, \epsilon'_k \) of their specific good in period 1 and 2 respectively. Initial endowments \( \epsilon_k \) are known at the beginning of period 1 before agents choose consumption and portfolios. Income inequality in country K is summarised by the distribution of period 1 endowments across agents \( \Psi_{\epsilon_k} \), which is common knowledge. \( \epsilon'_k \), the endowment of individual k in period 2, is the product of two terms: an ”individual endowment share” \( e'_k \), and a country-specific ”aggregate endowment” \( Y'_K \)

\[ \epsilon'_k = e'_k \cdot Y'_K \]

”Idiosyncratic risk” is given by the probability distribution of \( e'_k \), the period 2 endowment shares of individual k, which I denote \( \Psi_{e'_k} \). For simplicity I assume that second period endowments are i.i.d. across agents and independent of all aggregate variables. Also I normalise \( \int e'_k \Psi_{e'_k} = 1 \) - expected period 2 individual endowment is 1. By the iid assumption and the law of large numbers this means the sum of realised endowment shares is always 1 and aggregate period 2 output in country K simply equals \( Y'_K \).  

---

*For the derivation of a law of large numbers for continuum economies, see Uhlig (1996).*
"Aggregate risk" is summarized by the probability distribution of $Y'_H$ and $Y'_F$, the aggregate endowments in period 2, denoted $\Psi Y'_H$, $\Psi Y'_F$. I assume that these are independent of individual random variables, but possibly correlated among each other.

I assume that all period 2 random variables are log-normally distributed:

$$(\hat{e}'_h, \hat{e}'_f, \hat{Y}'_H, \hat{Y}'_F)' \sim N((\hat{e}'_h, \hat{e}'_f, \hat{Y}'_H, \hat{Y}'_F)', \Sigma),$$

where a hat denotes natural logarithms $\hat{z} = \ln(z)$ and $\Sigma$ is a matrix with diagonal entries $V_{eH}, V_{eF}, V_{H}, V_{F}$ whose only off diagonal entry is $\text{Cov}_{HF}$, the covariance of aggregate log-endowments.

### Incomplete asset markets and borrowing constraints

I impose the simplest structure of asset markets that allows me to analyse two kinds of trade-offs in optimal portfolios: the choice between safe and risky assets on the one hand, and between home and foreign assets on the other.

Like Hugget (1993), agents trade "IOUs" that are in zero net supply and denominated in domestic goods. These are "safe" assets in the sense that for 1 unit of $H$ goods invested today, IOUs in $H$ always pay $R_bH$ units of good $H$ next period (where "b" stands for "bonds"). Equivalently, foreign IOUs pay $R_bF$ units of $F$ goods.

In contrast to Hugget’s (1993) economy, however, agents can also trade shares in national mutual funds, and are allowed to buy shares and IOUs from foreigners.

Shares are also in zero net supply, and risky in the sense that their payoffs are proportional to the stochastic aggregate endowment. Thus the return on home shares is $R_sH Y'_H$ per unit of $H$ goods invested, equivalently for $F$. One obvious implication of the exogenous incompleteness of asset markets is that individual claims to future endowments are non-tradable, and that the resulting risk thus is non-diversifiable.

I denote h’s holdings of home and foreign IOUs by $a^b_{h,H}$ and $a^b_{h,F}$ respectively, and her holdings of shares by $a^s_{h,H}$ and $a^s_{h,F}$. Asset quantities are denoted in endowment goods of the owner. So if h holds a portfolio $a^b_{h,H}, a^b_{h,F}$, she owns $a^b_{h,H}$ units of $H$ IOUs and $a^b_{h,F}$ units of $F$ IOUs. I denote the vector of returns as $\overline{R}$, the vectors of assets held by individuals in $H$, $F$ as $\overline{a}_H, \overline{a}_F$, and the total value, in terms of their domestic good, of their assets at the end of period 1 as $a_h, a_f$.

I assume both IOUs and shares have zero default probability. Consistent with this, agents can credibly promise to repay only in units of their income - so borrowing contracts are always written in the endowment good of the issuer.

This means agents can issue only domestic assets, but invest both at home.
and abroad. One consequence of the no-default assumption are individual borrowing constraints: agents in country K can only issue IOUs and mutual fund shares up to maximum amounts $B^b_K, B^s_K$. The borrowing limits play no further role in the discussion as I concentrate on interior portfolios.\footnote{The "natural" limit to total borrowing in riskless assets would equal the present discounted value of minimum future income $B_K = b_K \frac{1}{1-\theta} \min \{K \} R$, which is the highest amount agents can repay for sure. But with log-normal endowments there is a positive probability of having endowment realisations arbitrarily close to 0, such that this formulation does not lead to a non-zero borrowing limit. The problem can be avoided by introducing a positive non-stochastic minimum endowment level for all agents in a country. This can be chosen such that the resulting natural borrowing limit equals the sum of $B^b_K$ and $B^s_K$ above. As I concentrate on interior portfolios, I prefer the simpler formulation in terms of $B^b_K$ and $B^s_K$ directly.}

**The Household’s problem**

A typical home Household h maximises expected lifetime utility by choosing in period 1 consumption and a vector of assets $a^i_h$ subject to her budget constraint, borrowing constraints for domestic assets and the non-negativity of foreign asset holdings, taking as given the relative price of home goods (in units of the foreign good) $p$ this period and the vector of returns $\bar{R}$. h’s problem is thus given as:

$$\max_{c_h, c'_h, a^i_h} \left\{ \frac{c^\gamma_h - 1}{1 - \gamma} + \beta E \left\{ \frac{c^{1-\gamma}_h - 1}{1 - \gamma} \right\} \right\}$$ (7)

Subject to the constraints

$$c_h = \sum_{i \in \{b, s\}} a^i_h \frac{\epsilon_h - \sum_{j \in \{b, s\}} a^j_h}{p^i_H}$$
$$c'_h = \sum_{i \in \{b, s\}} a^i_h + R^b_h a^b_h + R^s_h Y^s_H a^s_h + \frac{(R^b_F a^b_h + R^s_F Y^s_H a^s_h) p'^i}{p^i_H}$$
$$a^i_h \geq B^i_H, \text{ for } i \in \{b, s\}$$
$$a^j_h \geq 0, \text{ for } j \in \{b, s\}$$
$$c'_h = c'Y^i_H$$

where $p_H = \theta - \theta (1 - \theta) (1 - \theta) p^{1-\theta}$ is the home consumption price index. The problem of a typical foreign household is symmetric.

Note that the expectation in (7) is taken across the joint distribution of the random variables $c'_h, c'_f, Y^i_H, Y^i_F, p'$. Thus, to solve her problem, h needs to know the distribution of the equilibrium relative price next period, as this determines the real value of her assets and endowments. But in equilibrium, $p'$ potentially depends on the demand functions of all individual agents, and thus on the joint
distribution of endowments and the distribution of equilibrium asset holdings at the end of period 1.

Note also that in (7), households can be constrained by any combination of the borrowing limits on assets, and in most equilibria there will be some constrained households. In the analysis of portfolios, I concentrate on unconstrained households with interior portfolios. For simplicity, I specify the vector of assets in two different ways, allowing either cross-border trade in equity or in IOUs.

4 Competitive equilibrium and the terms of trade

This section defines a competitive equilibrium and discusses the properties of the equilibrium terms of trade, before the following section looks at portfolios. An appendix discusses existence and uniqueness.

4.1 Definition of Competitive equilibrium

A competitive equilibrium is

1. A Consumption Allocation:
   For every agent \( k \), a consumption sequence of both goods for both periods: \( c_{k,H}, c_{k,F}, c'_{k,H}, c'_{k,F} \), where \( c'_{k,J} \) is a random variable depending on the realisation of period 2 uncertainty.

2. A set of Portfolios:
   For every agent \( k \), a vector \( \overline{a}_k \) specifying holdings of all assets in the economy at the end of period 1.

3. A Price System, consisting of
   - \( p, p' \), the relative prices of \( F \) goods in terms of \( H \) goods in period 1 and 2, where \( p' \) is a random variable with distribution \( \Psi p' \).
   - \( \overline{R} \), the vector of asset returns

such that

1. Agents allocate their funds optimally across goods in period 2 given a particular realisation \( p' \).

10Summed across all agents individual quantities imply an Aggregate consumption allocation for consumption of good \( K \) in country \( J \) \( C_{J,K} = \int c_{J,K} d\Psi_j \), \( C'_{J,K} = \int c'_{J,K} d\Psi'_j \), as well as a Country portfolio of gross and net asset holdings, and a Net asset position once net holdings of all assets in a country are summed at period 1 prices.
2. the allocation solves every household’s problem (7) in period 1 given a relative price \( p \), a distribution \( \Psi^{p'} \), and rates of return \( \overline{R} \).

3. markets clear:
   - for goods: \( \int c_{h,H}dh + c_{f,H}df = Y_H \), \( \int c_{h,F}dh + c_{f,F}df = Y_F \) in both periods
   - and assets: \( \int a_{h,J}dh + \int pa_{f,J}df = 0 \), \( \forall i \in \{b, s\}, J \in \{H, F\} \) (each asset is in zero net supply)

4. The distribution of the future relative price \( \Psi^{p'} \) is consistent with the joint distribution of random variables \( e'_h, e'_f, Y'_H, Y'_F \), and individual asset holdings at the end of period 1.

I call a competitive equilibrium "symmetric" if all home and foreign distributions are identical, and \( \theta = \frac{1}{2} \).

Note that, in order to determine the distribution of relative prices \( \Psi^{p'} \) that affects portfolio decisions, agents need to map today’s distribution of endowments into tomorrow’s excess demand functions via individual savings decisions and the joint law of motion of aggregate and individual endowments.\(^{11}\) As the next section shows, the assumption of identical homothetic preferences for all agents in a particular country implies that a country’s demand for goods only depends on the relative price and aggregate savings and endowment, not individual savings or endowments. But individual uncertainty and heterogeneity still matter for aggregate savings and net asset positions. Identical preferences across countries ensure that even the aggregate net asset positions do not matter for excess demands, so aggregate endowments tomorrow completely determine aggregate demand for goods and thus market-clearing prices.

4.2 Equilibrium terms of trade movements

An important consequence of the identical homothetic preferences across goods is that the optimal expenditure shares are identical for all agents independent of endowment income. Since assets are in zero net supply, this implies that excess demands are independent of the heterogeneity in the economy. It follows

\(^{11}\)This is similar to the recursive framework with capital accumulation presented by Krusell and Smith (1998), where agents need to know the law of motion for the joint distribution of individual asset holdings and (aggregate and idiosyncratic) shocks, as this determines aggregate savings and thus the returns to capital tomorrow.
from this that aggregate endowments $Y_h, Y_f$ map directly into a market clearing relative price $p$ independent of the heterogeneity in the economy.\footnote{To see this, denote by $s_h (s_f)$ agents’ total expenditure in terms of $H$ ($F$) goods in any period. Cobb-Douglas preferences imply constant budget shares, so optimality requires}

$$p = \frac{1 - \theta}{\theta} \frac{Y_h}{Y_F} \forall \Psi^e_F, \Psi^e_H, \Psi^r_F, \Psi^r_H$$

(13)

As a consequence of the unit-elasticity of the consumption basket, for consumers with constant relative risk aversion, these equilibrium price movements yield complete insurance against country-endowment shocks: prices move against endowments to optimally spread risk across countries. So with representative agents, there is no incentive for asset trade (see Cole and Obstfeld (1991)). However, in an environment with heterogeneity in incomes, agents do have incentives to trade assets to smooth consumption and engage in precautionary savings. Moreover, their asset portfolios will generally depend on their period 1 income, which, given the assumption of i.i.d. period 2 endowments, maps monotonically into lifetime wealth.

A feature of unit-elastic demand for goods closely related to the perfect insurance result is that claims to country-endowments, or national mutual fund shares, must have equal stochastic consumption payoffs in equilibrium

$$\frac{R^s_H Y_H' p'_H}{p'_H} = \frac{R^s_F Y_F' p'_F}{p'_F} = R^s_F Y_F^\theta p^\theta$$

(14)

where I set the period 1 relative price of goods to 1 for simplicity. So with international trade in shares agents are always indifferent between home and

\footnote{To see this, denote by $s_h (s_f)$ agents’ total expenditure in terms of $H$ ($F$) goods in any period. Cobb-Douglas preferences imply constant budget shares, so optimality requires}

$$c_{h,H} = \theta s_h, \ c_{h,F} = (1 - \theta) \frac{1}{p} s_h \quad (8)$$

$$c_{f,H} = \theta p s_f, \ c_{f,F} = (1 - \theta) s_f \quad (9)$$

where $s_k$ denotes expenditure of agent $k$ (endowments minus net asset investment) in domestic goods. When agents spend $a_h, a_f$ units of their endowment good on assets, the \textbf{excess demand for $H$ goods} in the first period is:

$$\int \theta (\epsilon_h - a_h) dh + \int \theta p (\epsilon_f - a_f) df = Y_H$$

(10)

Zero net assets implies that individual asset holdings sum to 0 across countries once expressed in the same currency:

$$\int a_h dh + \int pa_f df = 0$$

(11)

So the excess demand function becomes

$$\int \theta \epsilon_h dh + \int \theta p \epsilon_f df = \theta Y_H + \theta p Y_F = Y_H$$

(12)
foreign mutual fund shares. In this sense, the equilibrium portfolio is never unique with international trade in shares. Also, there may be two equilibria with high and low rates of interest in some cases. An appendix discusses conditions for uniqueness and existence of equilibrium.

5 Optimal portfolios

I now derive the optimal asset allocation when domestic agents can invest in assets issued abroad. Asset holdings differ across individuals for 2 reasons: first, although the distributions of their future endowment income are the same (due to the i.i.d. assumption), domestic agents differ in current income. To smooth consumption, richer agents, with higher current income, save more than poorer agents. Second, poor agents, with low or negative savings, have tomorrow’s consumption determined largely by tomorrow’s endowment income. Thus, they prefer assets that are good hedges against fluctuations of endowment income, to limit consumption volatility. One main result of this paper is that, due to terms of trade movements, home IOUs can be better hedges against income risk than foreign assets or mutual fund shares. Richer agents, whose consumption is mainly determined by asset returns, care relatively less about this hedging, and thus have a lower portfolio share of home IOUs.

For the results in this section, the distinction between wealth portfolios and financial portfolios is important (see also Campbell 2007, section 2.4). Wealth portfolio shares are simply percentages of total wealth, the sum of financial assets and the present value of future endowment income. Since total wealth is strictly positive for all individuals, wealth portfolio shares are always bounded and sum to 1. Also, this section shows that wealth portfolios are approximately the sum of 2 sub-portfolios: first, a ”hedge portfolio”, which is the same for all individuals, designed to optimally sell off individual income risk. And second, a ”balanced portfolio”, determined only by the joint distribution of asset returns, that invests the proceeds of this sale and additional savings, making it proportional to total wealth. Thus, for ”wealth poor” individuals with low total wealth, the hedge portfolio always dominates, leading to ”home bias” in their asset allocation, when home IOUs are better income hedges.

More usually, however, the term portfolio refers to financial portfolios, a difference that becomes important when talking about portfolio shares. Financial portfolio shares are generally defined as percentages of gross financial assets. In order to have financial portfolio shares that are positive and bounded, the
finance literature often imposes a no-short-selling constraint for assets. In my
general equilibrium framework, on the other hand, agents can short-sell domes-
tic assets. But in the discussion of financial portfolios, I concentrate on net
savers, who have well-defined financial portfolio shares. Portfolios of poor net
savers, with a low but positive net financial asset position and thus a signifi-
cant level of total wealth, are a combination of the hedge and balanced portfolio.
Importantly, they typically feature positive investment in only 1 asset that has
superior combined hedging and return properties, and whose financial portfolio
share is 1 (or higher if the alternative asset is short-sold). As financial (and thus
total) wealth rises, financial portfolio shares converge as before to the balanced
portfolio. Whether financial portfolio shares have home bias relative to individ-
uals with higher savings thus depends not only on hedging properties, but also
on the joint distribution of returns, which determines the balanced portfolio.
For simplicity this section considers two binary portfolio problems: first, to
show how the importance of hedging against income risk declines with wealth,
I analyse portfolio decisions between domestic IOUs and mutual fund shares.
There, agents can easily hedge future income fluctuations by selling shares and
investing the proceeds, plus any other financial wealth they hold, in a balanced
portfolio. As agents get richer portfolios become more diversified as the bal-
anced investment portfolio dominates the hedging sub-portfolio. Second, I show
how this effect can lead to home bias of the poor when agents trade domestic
and foreign IOUs. Propositions 1 and 2 characterise wealth portfolios including
endowment wealth. Proposition 3 considers financial portfolios across home
and foreign IOUs.

Since real payoffs to home and foreign mutual fund shares are always equalised
by equilibrium terms of trade movements, in this section I call both of them
shares in an "international mutual fund". This allows me to drop superscripts
on returns and asset quantities in this section: returns on home and foreign
IOUs are from now on simply denoted as $R_H, R_F$, those on shares as $R_S$, and
h’s corresponding asset holdings as $a_{h,H}, a_{h,F}, a_{h,S}$.

5.1 The bias of the poor against risky assets

To show how hedging against non-diversifiable income risk affects the portfolios
of the poor differently, this section briefly looks at portfolio decisions between
domestic bonds and (international) mutual fund shares. The "safe asset bias"
of investors with lower financial wealth ("poorer investors") arises because in-
dividual real endowments $(e'_h Y_H, e'_F Y_F)$ and real share returns $(r_s Y_H, r_s Y_F)$ co-
move in exactly the same way with aggregate endowments. To hedge against endowment risk, investors thus simply take a short position in mutual funds equal to their endowment wealth. The proceeds, and any net financial wealth, are invested in a balanced portfolio of bonds and shares. For poor investors, the hedge effect dominates portfolio composition. For rich investors with high financial wealth, the balanced portfolio of positive investments in bonds and shares dominates the short position in the hedge portfolio.

**Proposition 1: Portfolio weight of risky assets**

*Both wealth and financial portfolio shares of mutual funds increase along the wealth distribution.*

**Proof of proposition 1**

h agents solve their problem (7) by choosing holdings of IOUs \((a_{h,H})\) and international mutual fund shares \((a_{h,S})\) subject to \(a_{h,H} \geq B^h_H, a_{h,S} \geq B^s_H\).

Given log-normality of the returns, a log-linear approximation to the arbitrage condition for interior portfolios

\[
E\left[c^\sigma h \frac{RB - RSY_H}{p'_H}\right] = 0
\] (15)
yields the wealth portfolio shares as a function of expected returns and the variance-covariance-structure of aggregate endowments

\[
\tilde{\alpha}_{h,H} = \frac{r_B - y_H - r_S}{\sigma \sqrt{V_H}} + \frac{1}{2} + \theta(\sigma - 1) + \frac{(1 - \theta)(\sigma - 1)}{\sigma} \frac{\text{Cov}_{HF}}{V_H}
\] (16)

\[
\tilde{\alpha}_{h,S} = \frac{r_S + y_H - r_B}{\sigma \sqrt{V_H}} + \frac{1}{2} + \frac{(1 - \theta)(\sigma - 1)}{\sigma} - \frac{(1 - \theta)(\sigma - 1)}{\sigma} \frac{\text{Cov}_{HF}}{V_H} - \tilde{c}^\sigma h
\] (17)

where a tilde denotes ratios with respect to total wealth \(\tilde{c}^\sigma h Y_H + a_h\), \(r_s\) is the inverse of the share price and \(y_I\) denote growth rates.

According to equation (16), the wealth portfolio share of domestic IOUs \(\tilde{\alpha}_{h,H}\) is independent of endowment wealth. The share of mutual funds in the wealth portfolio \(\tilde{\alpha}_{h,S}\), on the other hand, is the sum of two terms: the first term in brackets in equation (17) reflects the balanced sub-portfolio independent of endowment income, the counterpart of the constant bond portfolio share. The positions of this balanced portfolio are more responsive to return differentials (adjusted for average growth in endowments) when return differentials are less volatile or investors less risk averse. The second term in the expression for the portfolio share of mutual funds (17) reflects a sub-portfolio that hedges against endowment risk by taking a short position in shares of equal magnitude as endowment wealth. As a fraction of total wealth, this hedging term becomes less
important as agents become richer and (constant) endowment wealth declines relative to total wealth, i.e. as $\tilde{c}_h$ decreases. Multiplying equation (17) through by the ratio of asset to total wealth $\frac{a_h}{a_h + \pi}$ gives the same result for financial portfolio shares.

This result is similar to that obtained by Baxter and Jermann (1997), who show that the optimal portfolio of a representative agent can be split in the sum of a diversified portfolio and one that hedges against endowment risk. In my framework with heterogeneous endowments, this effect is more important for poorer agents whose balanced investment portfolio is smaller.

5.2 The Home Bias of the poor

I now look at optimal portfolios in the second case, when agents can, in addition to home IOUs, also invest in foreign IOUs, but not in mutual fund shares. As before, poor agents have stronger incentives to invest in good hedges against the volatility of endowments. But unlike mutual fund shares, home and foreign IOUs do not have the same hedging properties. Real payoffs of home IOUs $(R_{HH}Y_{HH}^{-\theta}Y_{HF}^{1-\theta})$ are low when home endowment is high relative to foreign endowment (yielding a high home consumption price). And the inverse holds for real returns to foreign IOUs $(R_{HF}Y_{HH}^{-\theta}Y_{HF}^{1-\theta})$. Endowment income $(\epsilon_h Y_{HH}^{-\theta}Y_{HF}^{1-\theta})$ on the other hand comoves positively with both home endowment (due to the co-movement of individual incomes and aggregate output) and foreign endowment (due to lower home consumption prices when aggregate foreign endowment is high). If the volatility of real labour income is dominated by the volatility of endowments agents with a higher share of labour income have a stronger preference for home IOUs that hedge against this volatility. "Wealthy" individuals on the other hand, whose future income is dominated by asset returns, have a diversified portfolio. This leads to a relative bias of portfolios held by poorer agents defined below.

**Definition 1: Relative home bias**

Agents with lower financial wealth ("poor agents") are said to have relative home bias, if, moving down the wealth distribution, the portfolio share of foreign assets declines faster than that of home assets.

**Proposition 2**

With international trade in IOUs, home bias of wealth portfolios changes across...
the wealth distribution. There is relative home bias of the poor, if the volatility of aggregate home endowment is higher than that abroad.

Proof of proposition 2

Agents now choose their holdings of home and foreign IOUs, i.e. \( \overline{a_h} = a_{h,H}, a_{h,F} \), to solve their problem (7) subject to \( a_{h,H} \geq B_H^0, a_{h,F} \geq 0 \). Again, solutions to h’s problem may be interior for none, one or both assets. For interior portfolios, the same approximation as in the previous section yields the following expressions for the portfolio share of home and foreign assets

\[
\tilde{a}_{h,F} = \alpha \frac{r_F - y_f - r_H + y_H}{\sigma(V_H + V_F - 2Cov_{HF})} + \frac{1}{2} (1 - \theta)(\sigma - 1) - \tilde{e}_h \frac{V_H - Cov_{HF}}{V_H + V_F - 2Cov_{HF}} \tag{18}
\]

\[
\tilde{a}_{h,H} = \beta \frac{r_H - y_h - r_F + y_f}{\sigma(V_H + V_F - 2Cov_{HF})} + \frac{1}{2} \theta(\sigma - 1) - \tilde{e}_h \frac{V_F - Cov_{HF}}{V_H + V_F - 2Cov_{HF}} \tag{19}
\]

Here, both portfolio shares are the sum of a two terms: the first reflects the balanced sub-portfolio and depends on relative mean returns, whose effect is greater the lower risk aversion or the volatility of the return differential; the second term reflects the hedge sub-portfolio, and is proportional to the share of the future endowment in total wealth. Thus, portfolios differ across the wealth distribution, as \( \tilde{e}_h \) falls with agents’ total assets. More particularly, there is relative home bias of the poor whenever the variance of aggregate home endowments is larger. And interestingly, this effect becomes larger as the covariance of aggregate endowments increases: as increasing covariance makes payoffs to home and foreign IOUs more similar, the positions necessary to achieve the optimal hedge become more extreme.

This result confirms our intuition that agents with lower financial wealth have stronger incentives to hedge against the main source of real endowment volatility, which is simply the more volatile aggregate endowment.

[Figures 2 to 4 somewhere below here.]

Does relative home bias in wealth portfolios imply relative home bias in financial portfolios? Not necessarily, as illustrated in figures 2 to 4. Figure 2 shows the levels of wealth components (financial assets and endowment wealth), as a function of total wealth \( W = a_h + e \). The thin dashed lines show the case without endowment wealth: financial assets are approximately linear in wealth, with a slope equal to their share in the balanced portfolio, determined by return differentials, risk aversion and the variance-covariance of aggregate endowments.
For a given level of total wealth, adding endowment wealth, the thick dotted line, obviously reduces financial assets. But importantly, the downward move in the level of financial assets is not the same across assets, but determined by their relative hedging properties against fluctuations in endowment income. The graph shows the situation where \( V_H > V_F \), so hedging reduces foreign IOU holdings by more than home IOUs. According to proposition 2, this implies relative home bias in wealth portfolios, shown in figure 2: wealth portfolio shares of financial assets are approximately equal to those without endowment wealth for very high levels of total wealth, but the share of foreign IOUs decreases faster as wealth declines and its inferior hedging properties become more important. However, the behaviour of financial portfolio shares shown in figure 3 depends on the behaviour of gross assets. I show a situation where agents with low but positive net financial wealth invest only in home IOUs, giving the former a portfolio share of 1. But this depends on the relative mean returns as well as the hedging properties. Particularly, for high excess returns on foreign IOUs, poorer agents will typically first invest in these despite their inferior hedging characteristics, leading to an increasing financial portfolio share of foreign assets as we move down the wealth distribution. Thus, proposition 3 conditions on mean returns, determined by many factors that my model is not designed to capture.

**Proposition 3**

*With international trade in IOUs, financial portfolio shares change across the wealth distribution. Poorer agents have relative home bias if the volatility of aggregate home endowments is high enough for the hedging effect to dominate portfolio decisions. Particularly, in a symmetric equilibrium there is relative home bias of the poor if credit constraints are more binding abroad.*

**Proof of proposition 3**

Multiplying equation (17) through by the ratio of total wealth to financial wealth \( \frac{\tilde{e}^f_h Y_H + a_h}{a_h} \), we get an expression for the share of IOUs in the financial portfolio:

\[
\tilde{a}_{h, F}^{fin} = \frac{r_F - y_f - r_H + y_h}{\sigma (V_H + V_F - 2\text{Cov}_{HF})} + \frac{1}{2} + \frac{(1 - \theta)(\sigma - 1)}{\sigma} (1 + \tilde{e}_h^{fin}) - \tilde{e}_h^{fin} \frac{V_H - \text{Cov}_{HF}}{V_H + V_F - 2\text{Cov}_{HF}} (20)
\]

\[
\tilde{a}_{h, H}^{fin} = \frac{r_H - y_h - r_F + y_f}{\sigma (V_H + V_F - 2\text{Cov}_{HF})} + \frac{1}{2} + \frac{\theta(\sigma - 1)}{\sigma} (1 + \tilde{e}_h^{fin}) - \tilde{e}_h^{fin} \frac{V_F - \text{Cov}_{HF}}{V_H + V_F - 2\text{Cov}_{HF}} (21)
\]

Contrary to the wealth portfolio shares considered in proposition 2, the effect
of changing relative endowment wealth \( \tilde{e}_h \) is determined by the sum of two terms: the second term is simply the hedge portfolio share we saw before: agents sell short foreign and home IOUs depending on their relative volatility. But the first term describes the increase from investing these short sale proceeds in a balance investment portfolio. If the balanced portfolio has a large share of foreign IOUs because their mean return is higher, this may more than offset their inferior hedging properties. So portfolios of "poor savers" with low \( a_h \) may not have home bias, despite the home bias in the hedge portfolio. Inversely, there is relative home bias of poorer agents (with a larger \( \tilde{e}_h \)) whenever either home volatility is relatively important, giving home agents incentives to hold home assets for hedging purposes, and / or when the weight of home assets in the balanced portfolio is large. In either case portfolios change across the wealth distribution, unless the hedge and balanced portfolio exactly offset each other. Particularly, we know that in a completely symmetric equilibrium with equal borrowing limits, returns on IOUs issued by different countries must be equal. If we decrease the relative supply of foreign bonds, e.g. by a tighter borrowing limit abroad, the equilibrium return on foreign bonds unambiguously falls. According to equation (21), this leads to relative home bias of the poor.

This section has shown the main results of this paper. In my model, the portfolio share of risky assets declines with wealth. And as long the volatility of home endowments is high enough to dominate the effect of return differentials, financial portfolio shares of home IOUs also fall with wealth, in line with the home bias of poor investors found in the data.

6 Aggregate and individual Home Bias

So far, this paper has looked at relative home bias, or the portfolio share of domestic assets across the wealth distribution. But it turns out that the model I consider creates a one-to-one link between relative home bias of individuals’ portfolios on the one hand and aggregate home bias of country portfolios on the other. For this, we first need a definition of absolute home "bias", relative to some "unbiased" aggregate portfolio. The literature offers 2 such benchmarks: in Finance, actual portfolios are often compared to an optimal hedge portfolio given the observed payoff structure of assets (see e.g. Lewis 1999). The macroeconomic literature on the other hand is often concerned with showing that home bias is an optimal outcome in general equilibrium. Thus, this literature tends
to define home bias relative to a more ad hoc benchmark, for example the portfolio consistent with completely diversified portfolios in all countries, implying portfolio shares equal to the share of a country’s assets in world supply (see e.g. Van Wincoop and Warnock (2006)). As I am concerned with the impact of non-diversifiable income risk, I take as a benchmark the optimal portfolio of a financial investor without future endowment income.

Definition 2: Aggregate home bias
A country portfolio has aggregate home bias if the sum of individuals’ gross holdings of home assets as a share of total gross assets exceeds the home asset share in the optimal portfolio of a purely financial investor.

Note that the definition is in terms of gross asset holdings. This is intuitive, as a country’s net holdings of domestic assets is bounded above by 0 (as no non-domestic agents can issue domestic assets, but domestic agents can sell their assets abroad). Proposition 4 states that aggregate home bias implies, and requires, relative home bias of individuals. The intuition comes from figure 4: financial portfolio shares are monotone in wealth and asymptote to the balanced portfolio, which equals the benchmark portfolio of a purely financial investor without endowment wealth. So when there is relative home bias, the home asset share of all diversified portfolios is bounded by the unbiased portfolio of a purely financial investor from below, which implies aggregate home bias.13 Equally, relative foreign bias implies aggregate foreign bias. To conclude the proof formally, however, we need to show that the portfolio decisions of constrained agents do not change this intuition, as (21) only holds for unconstrained agents with an interior portfolio.

Proposition 4: Aggregate and individual home bias
With international trade in IOUs there is relative home bias of poorer households if and only if the country portfolio has aggregate home bias.

Proof of proposition 4
Only if
According to equation (21), if \((\hat{a}_{h,F} \mid e_h - \frac{V_{h}}{V_{tot}}) < 0\), portfolio shares of home bonds increase as we move down the wealth distribution. In other words, all

13While the aggregate portfolio share of home bonds is not equal to the average of individual shares, all portfolios having higher home bias than the balanced portfolio is sufficient for aggregate home bias.
agents with interior portfolios have more home biased portfolios than the limiting portfolio.

As we move down the wealth distribution, however, at some level of total assets $a_h^*$, foreign assets will be optimally zero. All agents with $a_h < a_h^*$ are constrained by the non-negativity of foreign assets. However, this means their home asset share is simply 1, which strictly dominates that of a diversified limiting portfolio. As I focus on gross asset holdings, I can neglect agents with negative holdings of home bonds.

If I prove this by showing the contrapositive. Assume that there is relative foreign bias at home. The same reasoning as in the previous section goes through for unconstrained agents. However, as the home asset share now reaches zero faster as assets drop, some agents can have positive holdings of foreign bonds and issue home bonds at the same time. But again, negative holdings of home bonds do not affect gross asset holdings. So we can simply set the foreign asset share to 1 for all issuers of home bonds. Again, all individual gross asset shares are bound by the limiting portfolio share from below. So relative foreign bias implies no aggregate home bias.

7 Robustness of the results

7.1 International trade in both shares and IOUs

An interesting extension of the above analysis is to look at international trade of both IOUs and shares. In this case, h’s problem is to solve (7) with $\bar{a}_h = a_{h,H}, a_{h,F}, a_{h,S}$, subject to $a_{h,H} \geq B^H_0, a_{h,F} \geq 0, a_{h,S} \geq B^H_0$. Using the same approximation as above, and setting the covariance of aggregate endowments to zero for simplicity, the wealth portfolio shares are now, for interior portfolios,

$$\bar{a}_{h,F} = \frac{r_F + y_f - r_S}{\sigma V_F} + \frac{1}{2} \frac{(1 - \theta)(\sigma - 1)}{\sigma}$$

$$\bar{a}_{h,S} = \left( \frac{r_S + y_h - r_H}{\sigma V_H} + \frac{r_S + y_f - r_F}{\sigma V_F} \right) - \bar{a}_h$$

$$\bar{a}_{h,H} = \frac{r_H - y_h - r_S}{\sigma V_H} + \frac{1}{2} \frac{\theta(\sigma - 1)}{\sigma}$$

Again, given the perfect comovement of endowment and shares in terms of aggregate fluctuations, the hedge portfolio is just a short position in mutual fund shares. So, as before, there is risky asset bias of the poor. To determine how the share of total foreign assets evolves with individual wealth, we have to
bear in mind that $a_{h,S}$ comprises both home and foreign equity. In my model their individual portfolio weights $a_{h,H,S}$, $a_{h,F,S}$ are not unique, as their payoff distribution is exactly equal. Assuming that adjusted mean returns to IOUs are equal and that the fraction of home mutual fund shares is independent of wealth, it is easy to show that there is relative home bias of the poor in wealth portfolios whenever there is home bias in mutual fund portfolios. Thus again, aggregate home bias implies, and requires, individual relative home bias.

7.2 Persistent endowment shares

Above I assumed that idiosyncratic risk is i.i.d. across agents, implying that having high income today does not have any implications for my expected future income. It is interesting to examine the consequences of relaxing this assumption. Note first that the variance of future endowments is not a direct argument of the arbitrage equations. Nevertheless, it determines precautionary savings and thus the size of the portfolio. Second, with i.i.d. endowments, future expected income is the same for all agents. So higher current income translates to higher assets and a lower ratio of expected future income to the sum of total claims $e_{h}$. This yielded the link between current wealth and portfolio shares of propositions 1, 2 and 3. But of course, if we distinguish agents directly by their $e_{h}$, all results of section 5 still hold true. Thus, independently of the structure of uncertainty, agents with a higher share of future endowments in total wealth will have a higher portfolio share of domestic and safe assets.

8 Conclusion

In this paper I have shown that, according to the Survey of Consumer Finances, wealthier US Households invest a higher share of their portfolio in international assets. This result continues to hold when I take account of the fact that poorer households are less likely to participate in more sophisticated financial markets. Fixed costs of participating in foreign asset markets do not explain the rising portfolio shares for participants. So I constructed a simple 2 country model with incomplete markets and income heterogeneity that can account for this finding. Agents in the model receive stochastic endowments of a country-specific tradable good which are affected by idiosyncratic and country-specific shocks. Agents are prevented from access to a complete set of asset markets but can trade in riskless assets and/or in equity. Assuming log-normal returns, I derived asset portfolios under alternative assumptions regarding the structure of asset
markets but maintaining the assumption of no insurance against idiosyncratic risk.

In this model, terms of trade movements imply that poorer households can partly insure against income volatility by holding domestic or foreign bonds. Wealthier investors, whose income share of endowments is less important, care less about this hedging property than poor investors and therefore hold a more diversified portfolio. Equally they have lower aversion against equity, which due to terms of trade movements has real payoffs that comove perfectly with individual endowments. Thus, portfolio shares of equity rise with financial wealth. The same holds true for bonds as long as their superior hedging properties are important enough, relative to e.g. return differentials.

With regards to policy this study implies that the welfare loss from poorer households’ non-participation in sophisticated financial markets may be less important than thought. In future research it would be interesting if this result also holds in different environments. Particularly, one could try to relax the assumptions of unit-elastic preferences, and explore how the model deals with shocks to demand, rather than the supply shocks to endowments this study has looked at.
9 References


Appendix 1: Measurement error in foreign assets

The measure of total foreign asset holdings used in the empirical part of this study potentially suffers at least from two kinds of measurement error. First, the responses of households to questions on their asset holdings are accurate only insofar individuals both know the accurate Dollar value of their assets, and truthfully report it. Since I only look at portfolio investments (in other words I disregard directly owned foreign companies), market values of investments are in principle available, and individuals should report their Dollar values at current exchange rates. This may be a strong assumption not only as individuals might not be aware of up-to-date market values for long-term investments or exchange rates, but also, for example, if some of them underreport systematically off-shore investments used to evade tax payments. In the latter case, however, the resulting measurement error would tend to dilute the correlation between wealth and the foreign asset share of the portfolio. So a rejection of the Null hypothesis of no relation would be less likely in the presence of this kind of measurement error. To see this, suppose all individuals were to invest \( x \) percent of their foreign asset holdings in unreported offshore vehicles. In this case, the difference between true portfolio shares \( \tilde{a}_{true} \) and those calculated from reported asset holdings can easily be shown to be

\[
\frac{x}{1-x} (\tilde{a}_{true} (1 - \tilde{a}_{true})
\]

Thus portfolio shares of foreign assets calculated from individual reports are always smaller than the true shares, and the difference is greatest for intermediate portfolios. As we see foreign asset shares rising from zero to single-digit percentages in Figure 1, the bias will increase along the wealth distribution.

A second source of measurement error results from the use of average portfolio shares in the imputation of households’ indirect foreign asset holdings via mutual funds. If rich individuals systematically invest in funds with different exposures to foreign assets, this might distort the observed wealth effect on total foreign assets. But again, this error is likely to dampen the observed relationship between wealth and the portfolio share of foreign assets. To see this, suppose all individuals have the same portfolio share of mutual funds, but richer individuals choose funds with a higher (lower) share of foreign assets. Using average mutual fund portfolios introduces measurement error that is negative for rich (poor) individuals, positive for poor (rich) individuals. This biases the wealth effect estimated from observed data towards zero. The bias will be even stronger when richer individuals also have a higher portfolio share of mutual funds. So again, we would be less likely to reject the null of no wealth effect on portfolios in the presence of measurement error, than we would be without it.
11 Appendix 2: Fixed costs and home bias

This section shows that higher costs of investing in foreign assets alone cannot explain relative home bias of poorer market participants found in the data. Consider the 2 period problem of a home investor that receives a stochastic share \( e \) of aggregate home endowment \( Y_H \), and can invest in home bonds at a return \( R_H \), or foreign assets, yielding \( R_F \) units of foreign currency for bonds and \( R_S Y_F \) for shares. Assume \( e, Y_H, Y_F \) are independent log-normal random variables. To abstract from the general equilibrium terms of trade movements at the basis of the results in the main text, assume that the exchange rate \( S \), defined as the price of foreign currency in units of the home currency, is simply also a mean zero independent log-normal variable. In addition, assume that to buy \( a^* \) units of foreign assets, the investor has to pay a cost of \( K = k_0 + k_1 a^* \), i.e. there are fixed and proportional costs of investing abroad. The investor’s problem can be expressed as

\[
\max_{c_h, c'_h, a_h} \frac{c_h^{1-\sigma} - 1}{1 - \sigma} + \beta E_Y \left( \frac{c'_h^{1-\sigma} - 1}{1 - \sigma} \right)
\]

Subject to the constraints

\[
c_h = eY_H - a^h_{h,H} - \sum_{j \in \{b,s\}} a^j_{h,F} \\
c'_h = e'Y'_H + R'_H a^b_{h,H} + \frac{S'}{S} (R'_F a^b_{h,F} + R'_F Y'_F a^s_{h,F}) - K
\]

where \( K = 0 \) if the investor has 0 foreign asset holdings.

The problem can be seen as a two-stage decision. First, the investor determines the optimal portfolios with and without foreign assets; then she compares expected utility for both and decides wether or not to hold foreign assets. For simplicity consider binary portfolios where the investor either invests in shares or bonds. Given log-normality and independence, and approximating marginal utility from investing in foreign assets at zero real returns \( R^b_F = R^s_F = \frac{S'}{S} = 1 \), the approximate share of foreign bonds in a diversified portfolio is

\[
\tilde{a}_f = \frac{1}{2} + \frac{r^b_F - k_1 - r^b_H}{\sigma \text{Var}_s} 
\]

where lower case letters denote logs and \( \text{Var}_s \) is the variance of the log exchange rate. Equivalently, the portfolio share of foreign shares is

\[
\tilde{a}_s = \frac{1}{2} + \frac{r^s_F - k_1 - r^s_H}{\sigma (\text{Var}_F + \text{Var}_s)}
\]
So optimal portfolios are a function of risk aversion, excess returns and the variances of payoffs. But importantly, they do not include individual wealth. Also, proportional costs simply show up as a proportionate reduction of returns that affects all portfolios equally. So portfolio do not change with wealth among participants. Fixed costs on the other hand mean that only investors with a large enough portfolio diversify into foreign assets, where for investment in foreign bonds say, the threshold value of total assets is defined as that for which losses from fixed costs exactly offset those from sub-optimal portfolios

$$E[u(e' - aR^b_H)] = E[u(e' - a^*_h R^b_H - a^*_f (\frac{S'}{S} R^b_F - k_1) - k_0)]$$ (29)

12 Appendix 3: Existence and uniqueness of equilibrium

In section 4, I showed that the equilibrium relative price of goods is independent of heterogeneity and the allocation of assets. As long as agents have some preference for both goods ($0 < \theta < 1$), (13) thus describes a non-empty, single-valued mapping from the two-dimensional space of aggregate endowments into a market-clearing price. In other words a market-clearing price of goods always exists and is unique for any combination of $Y_H, Y_F$.

The excess demands for assets are the sum of the quantities solving (7), integrated across the distribution of unconstrained agents in both countries, plus maximum borrowing multiplied by the measure of constrained agents. For example, for Home IOUs, remembering that these can only be issued by Home agents and that asset quantities are denoted in terms of domestic goods for home and foreign agents, we get

$$a_H = \int a_{h,H} d\Psi^e_H + p \int a_{f,H} d\Psi^r_F$$ (30)

$$= \int_{-\infty}^{e_{h}^*} a_{h,H} B^b_H \Psi^f_H + \int_{e_{h}^*}^{\infty} a_{h,H} d\Psi^r_H + p \int_0^{\infty} a_{f,H} d\Psi^r_F$$ (31)

where $e^*$ denotes the level of current home endowment share that solves the first order condition for borrowing at the maximum level $B^b_H$.

Under financial autarky, existence of an equilibrium price vector $R = (R^b_H, R^s_H)$ is easy to prove by a fixed point argument. Local uniqueness of both consumption allocation and portfolios can also be shown.

However, global uniqueness is more difficult to prove as individual asset demands are not necessarily monotone in relative returns. Two special cases where the equilibrium can be shown to be globally unique are when $b_i = 0, i \in \{b, s\}$
(only domestic trade in either bonds, or shares), and either \( \sigma \leq 1 \) (substitution effects dominate income effects) or \( b_j = \infty \) (unconstrained issuance of assets). This is because with one asset only, total excess demand shows no inter-asset substitution effects. Then, for \( \sigma < 1 \), all individual asset demands, and therefore total excess demand for assets, are monotone in returns as the substitution effect dominates. For \( \sigma > 1 \) savers may have decreasing asset demand (as the income effect dominates). But borrowers’ asset demand is always increasing in returns, with an elasticity higher than that of savers at optimal borrowing levels as long as everybody faces the same period 2 uncertainty. So if all borrowers are unconstrained the total excess demand is again upward sloping in returns, and the equilibrium globally unique. However, even with only one asset, when a lot of borrowers are constrained, there may be multiple equilibria, as the non-monotonous asset demands of savers can dominate total excess demand.

With more than 1 asset, possibly traded across countries, the equilibrium is not generally globally unique. But conditions for global uniqueness can be derived for example by imposing the gross substitution property on the system of individuals’ arbitrage equations. For the analysis here this is not a problem, however, as I only look at interior portfolios, given an equilibrium vector of returns \( \overline{R} \). I do not solve for the equilibrium explicitly, which will be a function of the particular specification of distributions and borrowing constraints in both countries.
Table 1: Heckman model for participation and portfolio share of foreign assets

<table>
<thead>
<tr>
<th>Equation (1)</th>
<th>const</th>
<th>ln(FIN)</th>
<th>ln(INCOME)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-15.16</td>
<td>0.92</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>2.28</td>
<td>0.14</td>
<td>0.13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation (2)</th>
<th>const</th>
<th>AGE</th>
<th>COL</th>
<th>FIN₂</th>
<th>FIN₃</th>
<th>FIN₄</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2.65</td>
<td>0.000</td>
<td>0.11</td>
<td>1.73</td>
<td>1.20</td>
<td>2.15</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.0016</td>
<td>0.045</td>
<td>0.27</td>
<td>0.29</td>
<td>0.26</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>No of obs</th>
<th>Censored:</th>
<th>(\chi^2)(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4519</td>
<td>3378</td>
<td>68.00</td>
</tr>
</tbody>
</table>

FIN is the SCF measure of total gross financial wealth; INCOME the sum of salaries, wages and income or losses from a professional practice, business, limited partnership, or farming; AGE the age of the household head in years; FINₓ a dummy variable that takes the value 1 when financial wealth falls in the (weight-adjusted) xth quartile of the cumulative distribution; and COL a dummy variable that equals 1 if the head of the household has a college degree. Numbers in italics are standard errors.
Figure 1: Portfolio share of total foreign assets (decile average) across the financial wealth distribution
Without endowment wealth and thus no hedging (thin dashed lines), financial assets increase linearly in total wealth from 0. With endowment wealth (thick lines), financial asset holdings are necessarily lower at any given level of total wealth. But foreign IOUs, with inferior hedging properties against endowment risk, are lowered by more than home IOUs. Below total wealth level $w^*$ the short-selling constraint on foreign assets binds, below $e$ net financial assets are negative.

Without endowment wealth (thin dashed lines), shares of financial assets in total wealth are constant and equal to those of the balanced portfolio. With endowment wealth (thick lines), portfolio shares converge to the balanced portfolio, with an initial gap that is larger for foreign IOUs, which have inferior hedging properties against endowment risk.

Without endowment wealth (thin dashed lines), shares of financial assets in gross financial wealth are constant, equal to those of the balanced portfolio. With endowment wealth (thick lines), financial portfolio shares also converge to the balanced portfolio, but are initially 1 for home IOUs (with superior combined hedging properties and mean return) and 0 for foreign IOUs (where the short-selling constraint binds).